NBER WORKING PAPER SERIES

ON SOCIALIZING AND SOCIAL DISTANCING IN MARKETS: IMPLICATIONS FOR RETAIL PRICES, STORE-LEVEL CONSUMER DENSITY, AND DISEASE TRANSMISSION

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Working Paper 27724
http://www.nber.org/papers/w27724

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2020

I gratefully acknowledge research support from the C. V. Starr Center of Applied Economics at NYU, and thank Matías Moretti for outstanding research assistance. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. D43,I12

ABSTRACT

I generalize the "noisy search" model of Burdett and Judd (1983) to settings where individual buyers have preferences over the number of other buyers who visit the same seller as them. I consider a version in which buyers have a preference for social distancing derived from the risk of contracting a disease from other buyers, and use it to study the two-way equilibrium interaction between supply-side considerations (such as the distribution of prices posted by sellers) and individual buyers' behavioral responses to the risk of contagion. I find that the price response to the buyers' shift toward social distancing can be an important determinant of the degree to which buyers' individual behavioral responses to the risk of contagion can mitigate the spread of the disease.

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1. Introduction

In the context of in-person retail transactions, the utility a buyer gets from purchasing a good or service is oftentimes influenced by the presence of other buyers at the seller's store or venue. Most people would prefer to attend a sporting event in a crowded stadium rather than sitting alone in the stands; to drink in the company of others rather than in an empty bar; to dance in a crowded night club rather than in a half-empty one; to share a restaurant with other patrons rather than being surrounded by empty tables. There are other shopping activities where buyers prefer to have fewer buyers around. Your experience in a supermarket, for example, may be negatively affected by having other buyers nearby inspecting and handling the apples and tomatoes you intend to buy. Another, perhaps higher-stake example, is the preference for shopping at stores with lower buyer density to reduce the risk of becoming infected with a pathogen transmitted through close contact with other people.[1] In all these examples, buyers' preferences over the proximity to other buyers are an inherent part of the satisfaction they derive from the purchase. It is therefore natural to think that the prices for these goods and services, and the resulting densities of clients at the establishments that sell them, reflect individual attitudes toward socializing, and in turn feed back into the choices that buyers make regarding these market-based social interactions.

This paper has two objectives. The first is to generalize the classic equilibrium price-posting model of Burdett and Judd (1983) to accommodate individual buyers' preferences over the density of other buyers who visit the seller where they choose to buy.[2] As argued above, this seems like a relevant driver of consumer behavior for a wide range of goods and services—yet it does not appear to have been studied in the literature. The second objective is to deploy a version of the theory (in which preferences for social distancing are derived from the risk of contracting a disease) to analyze the two-way interaction between disease propagation and retail-market outcomes (the distribution of prices posted by sellers, and the distribution of density of buyers across sellers).

The paper is organized as follows. Section 2 lays out the model. Section 3 defines equilibrium

[1] In the midst of the year 2020, SARS-CoV-2 and the risk of COVID-19 is an obvious example.

[2] Burdett and Judd (1983) is a natural starting point for the question at hand since it is a standard workhorse model of sellers' explicit competition for clients: some firms choose to charge high prices and attract few buyers, while others choose to charge low prices and attract many buyers. The model has found applications in many fields, including Macroeconomics, Microeconomics, Industrial Organization, International Economics, Labor Economics, Monetary Economics, and the Economics of Crime.

and shows how to characterize it. Section 4 discusses how the equilibrium of the Burdett-Judd benchmark responds to changes in individual buyers' attitudes toward socializing and social distancing. The two main insights are intuitive. First, introducing a preference for socializing or social distancing changes the utility level buyers associate with purchasing the good, and this typically has a first-order effect on sellers' profit and the distribution of prices. Second, buyers' preferences for socializing heighten competition among sellers, while preferences for social distancing weaken it. The reason is that facing sociable buyers boosts sellers' incentives to undercut each other by lowering prices, while a seller's incentive to undercut other sellers in order to attract more buyers is weakend if buyers prefer distancing themselves from other buyers. As a result, keeping the (average) level of utility the same as in the Burdett-Judd benchmark, prices in the economy with sociable buyers tend to be lower than in the Burdett-Judd economy, while prices in the economy where buyers have a preference for social distancing tend to be higher than in the Burdett-Judd benchmark.

Section 5 turns the focus toward the epidemiological application. In this section I do two things. First, I show that a situation with positive prevalence of a disease transmitted from person to person that reduces the utility of those infected, leads to an expected utility function that fits the utility specification with a preference for social distancing studied in Section 3 and Section 4. Second, in this section I study how the prevalence of the disease affects market-based contagion and consumer welfare. The most immediate effect of the probability of infection is to reduce buyers' expected utility of visiting sellers, and keeping all else constant, this force induces buyers to cut back on shopping activities that may lead to contagion. In the full equilibrium, however, all else is not constant: sellers respond to this change in customer behavior by changing prices, and as a result, the contagion function reflects the changes in the best-responses of both sides of the market. This section reports several numerical examples that show that taking into account the sellers' pricing responses to the buyers' shift toward social distancing is critical to assess the degree to which consumers' individual behavioral responses to the risk of contagion can mitigate the spread of the disease. I propose a decomposition of the full equilibrium response in contagion and welfare into two components: One that captures the change in buyers' strategies keeping sellers' behavior fixed at the no-disease equilibrium, and another that arises as a result of the (full equillibrium responses triggered by the) pricing responses of sellers. The typical finding in the numerical examples is that by abstracting from the supply-side response one would overestimate the effectiveness of buyers' individual incentives to distance themselves from other buyers, and therefore predict that private incentives are more effective in reducing contagion than they are in the full equilibrium. The idea is simple: sellers respond by accommodating prices in a way that draws consumers back into their stores. This logic applies as long as the additonal expected cost of shopping introduced by the risk of infection is not so large that it dissipates all gains from trade between buyers and sellers. Otherwise, the market simply shuts down.

In Section 6 I characterize the efficient allocation of buyers to sellers by solving the problem of a fictitious benevolent social planner. The difference between the planner's decision of where to allocate a buyer, and that buyer's private decision of which seller to visit in the decentralized equilibrium is that the buyer only considers the effect that her trading decision has on her own contagion probability, while the planner, in addition, internalizes the effect that this decision has on the contagion probability of every other buyer.[3] The planner and the decentralized solutions generically differ along two margins: an *extensive margin* (the number of buyers who participate in the market), and an *assignment margin* (how buyers are allocated to sellers conditional on participating in the market). The numerical work I report suggests that the former is quantitatively more relevant than the latter. If the disease is very costly to an individual, or the contagion probability is very high, the planner's solution excludes more buyers from market activity than the equilibrium. This result can be inerpreted as society's desire to implement social distancing practices that are more aggressive than the ones that would be implemented by decentralized self-interested individual decision making. In this section I also explore the role of the density of population, e.g., as measured by the size of the population of buyers relative to the number of stores. The general insight that emerges is that the planner's solution and the equilibrium outcome tend to coincide for relatively low population densities, and become different once population density exceeds a threshold that depends on the cost of contracting the disease and the probability of contagion.

The second half of this paper is related to work in Behavioral Epidemiology, see, e.g., Manfredi and D'Onofrio (2013), and in Economics, e.g., Posner et al. (1993), Geoffard and Philipson (1996), Kremer (1996), Gersovitz and Hammer (2004), Reluga (2010), Toxvaerd (2019, 2020), Bethune and Korinek (2020), Brotherhood et al. (2020), Eichenbaum et al. (2020), Garibaldi et al. (2020), Jarosch et al. (2020), Keppo et al. (2020), Rachel (2020), that emphasizes

[3]This kind of health externality is standard in the literature, see, e.g., Kremer (1996), Toxvaerd (2019, 2020), Brotherhood et al. (2020), Garibaldi et al. (2020), Jarosch et al. (2020), Keppo et al. (2020).

the importance of modeling individual agents' behavioral responses to the risk of contagion. In these contributions, the model of contacts between agents is highly stylized, i.e., typically uniform random matching between everyone in the economy, and the contacts that lead to contagion are either modeled as non-market interactions, or when a connection is made to market activity, it is by assuming a probability of contagion that is a reduced-form function of the consumption or labor supply chosen by the agent. Relative to these contributions, the novelty here is twofold. First, I offer an explicit micro-level model in which contagion takes place between buyers who seek to purchase goods from a common seller. Second, I highlight the relevance of the two-way interaction between buyers' initial inclination toward social distancing and the supply-side price setting responses that in turn feed back into buyers' incentives to expose themselves to the disease.

2. Model

There is a set $\mathcal{B} = [0, B]$ of buyers and a set $\mathcal{S} = [0, S]$ for sellers, with $B, S \in \mathbb{R}_{++}$, and $\theta \equiv B/S$. There is a single good and each seller can sell any quantity of it at a constant marginal cost $c \in \mathbb{R}_{+}$. Each buyer wishes to purchase one unit of the good, and their utility depends on the number of other buyers who buy from the same seller. Specifically, a buyer who buys from a seller who is visited by $b \in \mathbb{R}_{+}$ buyers gets utility $U(b)$, where the function $U : \mathbb{R}_{+} \to \mathbb{R}$ is continuously differentiable, with $c < \underline{u} \equiv \min_{b \in \mathbb{R}_{+}} U(b) \leq \max_{b \in \mathbb{R}_{+}} U(b) \equiv \overline{u}$. The special case $U' = 0$ corresponds to the "noisy search" model in Burdett and Judd (1983), but the point here is to consider more general specifications. For example, $U' < 0$ captures an individual preference for *social distancing*. The case $U' > 0$ captures an individual preference for *socialization* associated with purchasing the good. Each seller posts and commits to the price it will charge any buyer who wishes to buy the good. Let $\Pi(p)$ denote the expected profit of a seller who posts price p. Different sellers may offer different prices. The cumulative distribution of posted prices is denoted by F. Let $\underline{p} = \inf_{\{p \in \mathbb{R}: F(p)=0\}} p$ and $\overline{p} = \sup_{\{p \in \mathbb{R}: F(p)=1\}} p$, and let $\mathbb{P}$ denote the convex hull of the support of F, i.e., $\mathbb{P} = [\underline{p}, \overline{p}]$. For the analysis I assume $c \leq \underline{p}$ and $\overline{p} \leq \overline{u}$.[4] A buyer's search process is denoted $\{q_k\}_{k \in \mathbb{N}}$, where $\mathbb{N} \equiv \{0, 1, ...\}$, $q_k \in [0, 1]$, and $\sum_{k \in \mathbb{N}} q_k = 1$. For each $k \in \mathbb{N}$, q_k denotes the probability a buyer receives k independent random draws from the distribution of prices posted by sellers. A buyer has the option to buy

[4]This is with no loss of generality, since no profit maximizing seller of has an incentive to post $p < c$ (since this implies selling at a loss) or $p > \overline{u}$ (since this implies making no sales).

the good from any one of the sampled sellers. The payoff of a buyer is

$$V(p, b) = U(b) - p \tag{1}$$

if she purchases the good from a seller with client density b at price p, and 0 if she does not purchase the good.

3. Equilibrium

Consider a buyer $i \in \mathcal{B}$ who draws a sample of prices $\boldsymbol{p}_i = \{p_i^k\}_{k=1}^{\eta(i)}$, where $p_i^k \in \mathbb{R}$ denotes the k^{th} price drawn by i (an independent draw from the distribution of prices posted by sellers), and $\eta(i) \in \mathbb{N}$ is the random number of prices sampled by i (with probability distribution given by the search process $\{q_k\}_{k\in\mathbb{N}}$). Let $\mathcal{P}$ denote the set of all possible price samples a buyer can draw, and let H denote the cumulative distribution function for the random variable $\boldsymbol{p}_i$ on $\mathcal{P}$ that is implied by the search process. A *purchase strategy* for buyer i is a function $\xi_i : \mathcal{P} \to \mathbb{P}$ that satisfies $\xi_i(\boldsymbol{p}_i) \in \boldsymbol{p}_i$ for any $\boldsymbol{p}_i \in \mathcal{P}$. Intuitively, for any price sample $\boldsymbol{p}_i \in \mathcal{P}$ that buyer i may draw, $\xi_i(\boldsymbol{p}_i)$ represents the single price (among the $\eta(i)$ sampled prices) at which buyer i chooses to purchase. Let $\boldsymbol{\xi} = (\xi_i)_{i\in\mathcal{B}}$ and $\boldsymbol{\xi}_{-i} = (\xi_i)_{i\in\mathcal{B}\setminus\{i\}}$ denote a purchase strategy profile for all buyers, and for all buyers other than buyer i, respectively. Given a strategy profile $\boldsymbol{\xi}$, the number of buyers who visit a seller who posts price p can be written as

$$b(p) = \sum_{k=0}^{\infty} \frac{1}{S} k q_k \int\int \mathbb{I}_{\left\{\xi_i\left(\{p_i^n\}_{n=1}^k\right)=p\right\}} dH\left(\{p_i^n\}_{n=1}^k |k\right) di,$$

where $H\left(\{p_i^n\}_{n=1}^k |k\right)$ denotes the cumulative distribution function for the sample of prices drawn by buyer i, conditional on i drawing k prices. From (1), a buyer's payoff depends not only on the price she pays, but also on the number of buyers at the store where she buys. Hence, when buyer i samples a posted price p, in order to decide whether to visit that price, buyer i has to form a belief, $\hat{b}_i(p)$, for the number of buyers she expects to find at that seller. I will restrict attention to equilibria in which these beliefs are rational, i.e., $\hat{b}_i(p) = b(p)$. A *posting strategy* for a seller is a price, p, at which he commits to sell any quantity of the good.

Definition 1. *Given a search process, $\{q_k\}_{k\in\mathbb{N}}$, a price posting equilibrium is a list, $\left(F, \bar{\Pi}, \boldsymbol{\xi}\right)$, such that: (1) given $\left(F, \bar{\Pi}, \boldsymbol{\xi}_{-i}\right)$, ξ_i is a best response for every buyer $i \in \mathcal{B}$, i.e., $\xi_i(\boldsymbol{p}_i) = \arg\max_{p\in\boldsymbol{p}_i} [U(b(p)) - p]$ for every $\boldsymbol{p}_i \in \mathcal{P}$; and (2) every seller maximizes profit given $\boldsymbol{\xi}$ and* *all other sellers' price posting decisions, i.e., (a)* $\Pi(p) = \bar{\Pi}$ *for all* p *in the support of* F, *and (b)* $\Pi(p) \leq \bar{\Pi}$ *for all* p.

Focus on a buyer $i \in \mathcal{B}$ who is considering whether to visit a seller $s \in \mathcal{S}$ who has posted price p. Suppose i expects s to be visited by $b(p)$ buyers. Then, from (1), the payoff (or *value*) i expects to get from visiting s is

$$v = U(b(p)) - p. \tag{2}$$

For any $\boldsymbol{p}_i \in \mathcal{P}$, use (2) to define the *implied sample of values*, $\boldsymbol{v}_i = \{v_i^k\}_{k=1}^{\eta(i)}$, with $v_i^k = U(b(p_i^k)) - p_i^k$, let $\mathcal{V}$ be the set of all buyer payoffs associated with all possible price samples from $\mathcal{P}$, let $\tilde{H}$ denote the cumulative distribution function for the random variable $\boldsymbol{v}_i$ on $\mathcal{V}$, and let $\mathbb{V}$ be the set of buyer payoffs associated with all prices in $\mathbb{P}$. Since buyers care only about their payoff, v, it is natural to cast buyer i's purchase strategy in *value space*, i.e., as a function $\gamma_i : \mathcal{V} \to \mathbb{V}$ that satisfies $\gamma_i(\boldsymbol{v}_i) \in \boldsymbol{v}_i$. Intuitively, for any implied value sample $\boldsymbol{v}_i \in \mathcal{V}$ that buyer i may draw, $\gamma_i(\boldsymbol{v}_i)$ represents the single value (among the $\eta(i)$ sampled values) that is offered by the seller whom buyer i chooses to visit. Let $\boldsymbol{\gamma} = (\gamma_i)_{i\in\mathcal{B}}$ and $\boldsymbol{\gamma}_{-i} = (\gamma_i)_{i\in\mathcal{B}\setminus\{i\}}$ denote a purchase strategy profile in value space for all buyers, and for all buyers other than buyer i, respectively. Given a strategy profile in value space, $\boldsymbol{\gamma}$, the number of buyers who visit a seller who offers value v is

$$\textsc{b}(v) = \sum_{k=0}^{\infty} \frac{1}{S} k q_k \int \int \mathbb{I}_{\left\{\gamma_i\left(\{v_i^n\}_{n=1}^k\right)=v\right\}} d\tilde{H}\left(\{v_i^n\}_{n=1}^k |k\right) di,$$

where $\tilde{H}\left(\{v_i^n\}_{n=1}^k |k\right)$ denotes the cumulative distribution function for the sample of values drawn by buyer i, conditional on i drawing k prices. Clearly, a buyer should visit the seller who delivers her the highest payoff among the sellers she sampled, so buyer i's *best-response condition in value space* is

$$\gamma_i(\boldsymbol{v}_i) = \max \boldsymbol{v}_i \tag{3}$$

for any random sample $\boldsymbol{v}_i \in \mathcal{V}$. By thinking of buyers' purchase strategies in value space, we know that if a seller who posts price p is delivering value v, then v must satisfy

$$v = U(\textsc{b}(v)) - p. \tag{4}$$

Condition (4) defines the p that a seller needs to post in order to deliver value v to buyers, i.e., $p = U(\textsc{b}(v)) - v \equiv \textsc{p}(v)$.

To find a price posting equilibrium, it is convenient to begin by characterizing the *distribution of posted values* implied by the price posting equilibrium. To this end, let G denote the equilibrium cumulative distribution of posted values induced by (2) and the equilibrium cumulative distribution of prices posted by sellers, F. Let $\underline{v} = \inf_{\{v \in \mathbb{R}: G(v)=0\}} v$ and $\overline{v} = \sup_{\{v \in \mathbb{R}: G(v)=1\}} v$, and let $\mathbb{G}$ denote the convex hull of the support of G, i.e., $\mathbb{G} = [\underline{v}, \overline{v}]$. Assume that $0 \leq \underline{v}$ and $\overline{v} \leq \bar{u} - c$.[5] Hereafter, I will assume the search process satisfies $q_1 \in (0,1)$ and will focus the analysis on a class of equilibria in which G is strictly increasing and continuous. In this case, the buyer best-response condition (3) implies

$$\mathrm{B}(v) = \sum_{k=0}^{\infty} \theta k q_k G(v)^{k-1}. \tag{5}$$

From (5), it is clear that the focus on equilibrium with $G'(v) > 0$ entails $\mathrm{B}'(v) > 0$. To find the equilibrium, I will also conjecture that $\mathrm{P}'(v) < 0$ for all $v \in \mathbb{G}$.[6] Condition (5) implies the numbers of buyers who visit the sellers who deliver the lowest, and the highest utilities, respectively, are

$$\mathrm{B}(\underline{v}) = \theta q_1 \tag{6}$$

$$\mathrm{B}(\overline{v}) = \sum_{k=0}^{\infty} \theta k q_k. \tag{7}$$

If a seller offers value v, this value must deliver the seller the equilibrium level of profit, i.e.,

$$\bar{\Pi} = [\mathrm{P}(v) - c]\,\mathrm{B}(v) \equiv \Psi(v). \tag{8}$$

The first observation is that in any equilibrium, $\underline{v} = 0$.[7] Then (8) implies the equilibrium level of profit is

$$\bar{\Pi} = [U(\mathrm{B}(0)) - c]\,\mathrm{B}(0) \tag{9}$$

[5] This is with no loss of generality, since $\overline{v} > \bar{u} - c$ requires a seller to post $p < c$ (and no profit maximizing seller is willing to sell at a loss), and no seller would post a p that delivers negative value to buyers (since this would imply making no sales).

[6] Notice that $\mathrm{P}'(v) = U'(\mathrm{B}(v))\mathrm{B}'(v) - 1$, so since $\mathrm{B}(v)$ is an equilibrium object yet to be found, the conjecture $\mathrm{P}'(v) < 0$, as well as the conjecture that $G(v)$ (or $\mathrm{B}(v)$) is strictly increasing and continuous, need to be verified after the equilibrium has been constructed. Proposition 1 provides sufficient conditions on primitives that ensure these conjectures are indeed verified in the equilibrium.

[7] To see this, notice that $\Psi(\underline{v}) = [U(\theta q_1) - \underline{v} - c]\,\theta q_1$ is decreasing in $\underline{v}$, so any seller who posts a price that delivers $\underline{v} > 0$ can increase profit by reducing the value he offers buyers to zero (i.e., by sufficiently increasing his posted price), which delivers profit $\Psi(0) = [U(\theta q_1) - c]\,\theta q_1 > 0$. The equilibrium cannot have $\underline{v} < 0$ because any seller who offers value $v < 0$ would attract no buyers and earn profit equal to $0 < \Psi(0)$.

with $\text{B}(0) = \text{B}(\underline{v})$ as given in (6). Since $\bar{\Pi}$ is now known, the equal-profit condition (8) can be evaluated at $\overline{v}$ to obtain $\bar{\Pi} = [U(\text{B}(\overline{v})) - \overline{v} - c]\text{B}(\overline{v})$, which implies

$$\overline{v} = \left\{1 - \frac{[U(\text{B}(0)) - c]\,\text{B}(0)}{[U(\text{B}(\overline{v})) - c]\,\text{B}(\overline{v})}\right\}[U(\text{B}(\overline{v})) - c] \tag{10}$$

with $\text{B}(\overline{v})$ as given in (7). Also, given $\bar{\Pi}$, the equal-profit condition (8) can now be regarded as a single equation in the unknown $\text{B}(v)$, i.e.,

$$\bar{\Pi} = [U(\text{B}(v)) - v - c]\,\text{B}(v) \tag{11}$$

can be used to solve for $\text{B}(v)$ for all $v \in [0, \overline{v}]$ (see proof of Proposition 1 for details). Once (11) has been solved for $\text{B}(v)$, condition (5) can be solved for $G(v)$ for all $v \in [0, \overline{v}]$ (see proof of Proposition 1 for details). Finally, $G(v)$ and the equilibrium mapping from values to prices, $p = \text{P}(v)$ from (4), can be combined to recover the underlying distribution of posted prices

$$F(p) = 1 - G(\text{P}^{-1}(p)) \tag{12}$$

with

$$\underline{p} = \text{P}(\overline{v}) = \frac{\text{B}(0)}{\text{B}(\overline{v})}U(\text{B}(0)) + \left(1 - \frac{\text{B}(0)}{\text{B}(\overline{v})}\right)c \tag{13}$$

$$\overline{p} = \text{P}(0) = U(\text{B}(0)). \tag{14}$$

The number of buyers who visit a seller who posts price p is $b(p) = \text{B}(\text{P}^{-1}(p))$. Proposition 1 (in Appendix A) gives an existence result.

To conclude this section, notice that if sellers' profits are rebated to buyers lump sum, then market welfare can be defined as the equally weighted average of buyers' utility (net of seller's profit), and equals

$$\mathcal{W} = \frac{1}{\theta}\int_0^{\overline{v}} \text{B}(v)\,[U(\text{B}(v)) - c]\,dG(v). \tag{15}$$

4. Market responses to attitudes toward socializing and social distancing

To fix ideas, suppose

$$U(b) = \frac{u + \tilde{u}}{2} - \frac{\tilde{u} - u}{2}\left(1 - e^{-\sigma b}\right),$$

where $u, \tilde{u}, \sigma \in \mathbb{R}_+$, with $c < \min(u, \tilde{u})$. If either $\tilde{u} = u$ or $\sigma = 0$, buyers' payoffs are independent of their social interactions while shopping, and the model reduces to Burdett and Judd (1983).

If $u < \tilde{u}$, buyers are *rivalrous*, i.e., they have a *preference for social distancing* in the sense that their utility is decreasing in the number of other buyers who shop at the same seller. The parameter σ indexes the intensity of the buyer's preference for social distancing: a larger value of σ reduces an individual buyer's utility from purchasing the good at a store with a given number of other buyers. If $\tilde{u} < u$, buyers are *sociable*, i.e., they have a *preference for socializing* in the sense that their utility is increasing in the number of other buyers who shop at the same seller. In this case, the parameter σ indexes the intensity of the buyer's preference for socializing: a larger value of σ increases an individual buyer's utility from purchasing the good at a store with a given number of buyers.

To delineate how the model works, consider two economies: one where buyers are rivalrous and have utility function

$$U^R(b) = \frac{\overline{u} + \underline{u}}{2} - \frac{\overline{u} - \underline{u}}{2}\left(1 - e^{-\sigma b}\right), \tag{16}$$

and another where buyers are sociable and have utility function

$$U^S(b) = \frac{\overline{u} + \underline{u}}{2} + \frac{\overline{u} - \underline{u}}{2}\left(1 - e^{-\sigma b}\right), \tag{17}$$

with $\underline{u}, \overline{u}, \sigma \in \mathbb{R}_+$, and $c < \underline{u} \leq \overline{u}$. To illustrate, set $B = 100$, $S = 10$, $c = 0$, $\underline{u} = 1$, $\overline{u} = 3$, $\sigma = 0.01$, and $q_1 = 1 - q_2 \in (0, 1)$. Figure 1 assumes $q_1 = 0.7$ and displays the key equilibrium variables for these two economies. If $\sigma = 0$, both economies reduce to the Burdett-Judd economy, which is also depicted in Figure 1. In this case the utility a buyer gets from purchasing the good is independent of the density of buyers across stores, and equal to $\frac{\overline{u}+\underline{u}}{2}$. When $\sigma \neq 0$, a seller is not only offering a buyer the good, but also proximity to other buyers, e.g., in the case of (16) and (17), this implies $U^R(b) < \frac{\overline{u}+\underline{u}}{2} < U^S(b)$ for all $b \in \mathbb{R}_{++}$. That is, the gain from trade available to any buyer-seller pair in the Burdett-Judd economy is higher than in the economy where buyers value social distancing, and lower than in the economy where buyers value socializing. As a result, the equilibrium of the market with sociable buyers delivers higher utility at higher prices than the Burdett-Judd equilibrium, which in turn delivers higher utility at higher prices than the equilibrium of the market with rivalrous buyers.[8] Figure 2 displays the same economies as Figure 1, but with $q_1 = 0.9$. As is to be expected, for each

[8]Let G^R, G^{BJ}, and G^S denote the equilibrium cumulative distribution functions for posted values in the economies with rivalrous, Burdett-Judd, and sociable buyers, respectively (and similarly for the distributions of posted prices, F^R, F^{BJ}, and F^S). Then, the figures show that $G^R \prec G^{BJ} \prec G^S$ and $F^R \prec F^{BJ} \prec F^S$, where for any pair of cumulative distribution functions, F_1 and F_2, "$F_1 \prec F_2$" means that F_2 first-order stochastically dominates F_1.

of the three economies the cumulative distribution function of offered values, G, in the more competitive parametrization of Figure 1 first-order stochastically dominates the corresponding distribution of values in the less competitive parametrization of Figure 2, while the opposite is true for F, i.e., the distribution of posted prices that implements the posted values.

To illustrate how the preferences toward socializing and social distancing interact with the forces that drive competition, it is instructive to control for the effects that these preferences have on the size of the gain from trade for buyer-seller pairs. To this end, Figure 3 considers the same economy with rivalrous buyers depicted in Figure 1 and compares it with a Burdett-Judd economy that is the same as the one depicted in Figure 1, except for the fact that $\overline{u}$ in the Burdett-Judd economy has been calibrated to match the average buyer-seller gain from trade implied by the equilibrium of the economy with rivalrous buyers.[9] Figure 4 makes the analogous comparison between the economy with sociable buyers depicted in Figure 1, and the Burdett-Judd economy parametrized as in Figure 1, but with $\overline{u}$ calibrated to match the average buyer-seller gain from trade implied by the equilibrium of the economy with sociable buyers.

The main takeaway from these figures is that buyers' preferences for social distancing weaken competition among sellers, while preferences for socializing heighten it. The reason is that facing sociable buyers boosts sellers' incentives to undercut each other by lowering prices, while a seller's incentive to undercut other sellers in order to attract more buyers is weakend if buyers prefer distancing themselves from other buyers. Therefore, keeping the average equilibrium level of utility the same as in the Burdett-Judd benchmark, prices (values) in the economy with sociable buyers tend to be lower (higher) than in the Burdett-Judd economy, while prices (values) in the economy where buyers have a preference for social distancing tend to be higher (lower) than in the Burdett-Judd benchmark.

5. Market contagion

In the previous sections I specified agents' preferences over socializing, $U(b)$, as a primitive. In this section I focus on the case with $U'(b) < 0$ to show that these preferences can be regarded as a reduced-form representation of an environment in which buyers are concerned about the possibility of becoming sick from interacting with other buyers while visiting a seller's location.

Assume a buyer can be in one of two health states: S or I, that stand for *susceptible*, and

[9] This can be done by setting $\overline{u} = 2\int U^R(b(p))\,dF(p) - \underline{u}$ in the Burdett-Judd economy, where $b(\cdot)$ and $F(\cdot)$ are the buyer density and distribution of posted prices for the equilibrium of the economy with rivalrous buyers.

infected, respectively. Let N_h denote the number of buyers whose pre-trade health state is $h \in \{S, I\}$, with $N_S + N_I = B$ and $\mu \equiv N_I/B$. Buyers in state S may transit to state I if they come into contact with infected buyers.[10] Let $h, h' \in \{S, I\}$ denote a buyer's health states before and after visiting a seller, respectively. The transition of the health state of a susceptible buyer depends on her market interactions. Specifically, a buyer who is in state S when she visits a seller who is visited by b other buyers has $\Pr(h' = I|h = S) = 1 - \Pr(h' = S|h = S) = \iota(b)$, where $\iota : \mathbb{R}_+ \to [0, 1]$ is given by

$$\iota(b) = 1 - e^{-\sigma\mu b}. \tag{18}$$

The parameter $\sigma \in \mathbb{R}_+$ represents the likelihood that a buyer contacts another while at a seller's location, as well as the probability that the disease is transmitted conditional on a contact with an infected buyer.[11]

In an epidemiological model in which agents make decisions that influence their exposure to other individuals it is necessary to specify the information available to them regarding their own health state. I work with the following own-health information structure. Agents do not know if their healh state is S or I, but they have a pre-trade belief $\phi \in [0, 1]$ that they are infected (i.e., ϕ is the probability the agent assigns to being in state I before engaging in market activity). If the agent visits a seller visited by b other buyers, then her post-trade belief of being infected is

$$\varphi(b, \phi) = \phi + (1 - \phi)\,\iota(b).$$

I assume all buyers share the same prior belief of being infected.[12] This belief could be treated as a free paramter, or it could be set to equal the prevalence of the disease in the population of buyers, i.e., $\phi = \mu$.

Let $\overline{u} \in \mathbb{R}_{++}$ denote the utility any agent gets from consuming the good. The payoff of a buyer with prior belief ϕ from purchasing the good at price p from a seller with buyer density $b \in \mathbb{R}_+$ is given by $\overline{u} - p - \varphi(b, \phi)\lambda$, where $\lambda \equiv (\overline{u} - \underline{u}) \in \mathbb{R}_{++}$ is the disutility associated with being in state I. The payoff of a buyer with prior belief ϕ who does not participate in the

[10] The assumption that agents are either in state S or I would be accurate for a disease that does not confer immunity to those who recover (i.e., a "SIS" model in epidemiological jargon). Otherwise, it would be a reasonable approximation to the early stages of an epidemic when the number of recovered is relatively small.

[11] In Appendix B (Section B.1) I describe two micro-level contact processes among the buyers who visit a particular seller that give rise to the contagion probability (18).

[12] The assumption that all agents share the same prior belief of infection seems like a reasonable approximation for the early stages of the epidemic. As time passes, however, priors would branch out as a result of different histories of individual actions.

market is equal to $\mathcal{V}_0(\phi) \equiv -\varphi(0,\phi)\lambda$. Hence, the buyer's gain from purchasing the good at price p is

$$\mathcal{V}(p,b,\phi) = \overline{u} - p - \varphi(b,\phi)\lambda - \mathcal{V}_0(\phi). \tag{19}$$

Notice (19) is a special case of (1) with

$$U(b) = \overline{u} - (\overline{u} - \underline{u})(1-\phi)\left(1 - e^{-\sigma\mu b}\right). \tag{20}$$

The number of new infections at a seller with $\mu b \in [0, b]$ infected buyers is $(1-\mu)\, b\iota(b)$.[13] Hence, the total number of new infections is

$$\mathcal{C} = S\int_0^{\overline{v}} (1-\mu)\,\mathrm{B}(v)\,\iota(\mathrm{B}(v))\,dG(v). \tag{21}$$

5.1. Equilibrium decomposition: consumer behavior and the role of prices

I now turn to the question of how the prevalence of the disease, μ, affects market-based contagion and consumer welfare. The immediate effect of the probability of infection is to reduce buyers' expected utility from visiting sellers. If we abstract from the supply-side response by sellers, the result would be that buyers cut back on shopping activities that may result in contagion. In the equilibrium, however, sellers respond to the change in buyers' behavior by changing prices, and as a result, the contagion function reflects the changes in the best-responses of both sides of the market. As mentioned in the introduction, existing work in this area emphasizes the importance of accounting for the endogenous choices of agents in the determination of the contagion rate. My goal here is to go a step further and bring into the analysis the role of the endogenous response of market prices in shaping buyers' incentives to expose themselves to the disease. To this end, it is useful to decompose the overall equilibrium response in the contagion function (21) into two components: One that is due to the change in buyers' strategies keeping sellers' behavior fixed at the no-disease equilibrium, and another that arises as a result of the (full equillibrium responses triggered by the) pricing responses of sellers.

To carry out this decomposition, I calculate contagion and welfare in two counterfactual economies. The first is an economy in which all buyers and sellers ignore the disease, i.e., they all act as if the environment was one with $\mu = 0$ even though it has $\mu > 0$. The second is

[13] In Appendix B (Section B.2) I show that if the number of infected buyers is small, then the number of new infections at a seller's location visited by b buyers, i.e., $(1-\mu)\, b\iota(b)$, is approximately equal to the number of new infections that would be obtained by assuming quadratic contacts between susceptible and infected buyers at the seller's location. (Quadratic contacts is a common specification in the simplest epidemiological SIR models.)

an economy in which prices remain fixed as if $\mu = 0$ even though $\mu > 0$, but buyers optimize taking as given the posted prices and the true prevalence of the disease, μ. The former is a conventional Burdett-Judd economy in which buyers are oblivious to the disease, and the latter is a fixed-price economy in which buyers are aware of the disease and will therefore express a preference for social distancing. These two counterfactual economies will allow me to decompose the full equilibrium response of contagion and consumer welfare to the level of μ into a component attributed to the change in buyers' behavior in response to the disease keeping prices fixed, and a component attributed to the change in firms' pricing strategies in response to the disease. I will use $\bar{\mathcal{C}}$ and $\bar{\mathcal{W}}$ to denote the number of new infections and welfare in the Burdett-Judd economy with oblivious buyers, and $\hat{\mathcal{C}}$ and $\hat{\mathcal{W}}$ to denote the number of new infections and welfare in the economy with fixed prices and buyers wary of contagion. I will use $\tilde{\mathcal{C}} \equiv \mathcal{C} - \hat{\mathcal{C}}$ and $\tilde{\mathcal{W}} \equiv \mathcal{W} - \hat{\mathcal{W}}$ to denote the components of equilibrium contagion, and welfare, respectively, that are due to price responses.

In the Burdett-Judd economy in which buyers are oblivious to the disease (obtained by solving the model with $\mu = 0$ in (20)), contagion is

$$\bar{\mathcal{C}} = S \int_{\underline{p}}^{\overline{p}} (1-\mu)\, \bar{b}(p)\, \iota\left(\bar{b}(p)\right) d\bar{F}(p),$$

and average welfare is

$$\bar{\mathcal{W}} = \frac{1}{\theta} \int_{\underline{p}}^{\overline{p}} \bar{b}(p) \left[U\left(\bar{b}(p)\right) - c\right] d\bar{F}(p),$$

where for each $p \in [\underline{p}, \overline{p}]$, $\bar{F}(p)$ is the number that solves

$$0 = (p-c) \sum_{k=0}^{\infty} \theta k q_k \left[1 - \bar{F}(p)\right]^{k-1} - (\overline{u} - c)\, \theta q_1, \tag{22}$$

with $\underline{p} = c + \frac{\theta q_1}{\sum_{k=0}^{\infty} \theta k q_k} (\overline{u} - c)$, $\overline{p} = \overline{u}$, and

$$\bar{b}(p) = \sum_{k=0}^{\infty} \theta k q_k \left[1 - \bar{F}(p)\right]^{k-1} \text{ for } p \in [\underline{p}, \overline{p}]. \tag{23}$$

In the economy in which prices are fixed but buyers are aware of the disease and optimize accordingly, contagion is

$$\hat{\mathcal{C}} = S \int_{\underline{p}}^{\overline{p}} (1-\mu)\, \hat{b}(p)\, \iota(\hat{b}(p)) d\bar{F}(p)$$

and average welfare is

$$\hat{\mathcal{W}} = \frac{1}{\theta}\int_{\underline{p}}^{\overline{p}} \hat{b}\left(p\right)\left[U(\hat{b}\left(p\right)) - c\right]d\bar{F}\left(p\right),$$

where $\bar{F}\left(p\right)$ is given by (22), and

$$\hat{b}\left(p\right) = \begin{cases} \bar{b}\left(p\right) & \text{if } \underline{p} \leq p \leq \hat{p} \\ \bar{b}_0\left(p\right) & \text{if } \hat{p} < p \leq \overline{p}, \end{cases} \tag{24}$$

where $\bar{b}\left(p\right)$ is given by (23), the cutoff $\hat{p}$ satisfies

$$\hat{p} = U(\hat{b}\left(\hat{p}\right)), \tag{25}$$

and for each $p \in (\hat{p}, \overline{p}]$, $\bar{b}_0\left(p\right)$ is the number that satisfies

$$p = U\left(\bar{b}_0\left(p\right)\right). \tag{26}$$

The total number of buyers who visit sellers to purchase the good equals

$$\hat{\mathrm{B}} = \int_{\underline{p}}^{\overline{p}} \hat{b}\left(p\right)d\bar{F}\left(p\right).$$

Conditions (24)-(26) summarize buyers' optimal purchase behavior in an economy where they fear contagion but sellers post the same prices they would post in the disease-free equilibrium. To see why, consider the following. In the economy with positive prevalence, a buyer's payoff internalizes the risk of infection, so given any purchase behavior followed by other buyers (as summarized by a resulting density of buyers at every price posted by sellers), the value delivered to any buyer by a price p on the support of $\bar{F}$ is lower than the value that same p would deliver to a buyer in the disease-free environment for which $\bar{F}$ constitutes an equilibrium. For example, in the disease-free equilibrium, given optimal buyer purchase behavior summarized by the function $\bar{b}\left(p\right)$, every posted price $p \in [\underline{p}, \overline{p})$ delivers strictly positive value to buyers, while $\overline{p}$ delivers value equal to 0. In the equilibrium with risk of contagion, however, there is a cutoff price, $\hat{p} < \overline{p}$ such that—given other buyer's optimal search strategies imply density function $\bar{b}\left(p\right)$ for $p \in [\underline{p}, \hat{p}]$—an individual buyer only gets positive value from prices in the interval $[\underline{p}, \hat{p})$.[14] Buyers whose lowest sampled price is from a seller who posts $p \in (\hat{p}, \overline{p}]$ only

[14]The reason buyers' optimal purchase strategy given $\bar{F}$ in the economy with positive prevalence is still characterized by the density function $\bar{b}\left(p\right)$ for $p \in [\underline{p}, \hat{p}]$ (i.e., the same density function that characterizes optimal purchase strategy in the disease-free equilibrium) can be understood as follows. Having sampled prices from $k \in \{1, 2, ...\}$ sellers, a buyer's optimal strategy is always to purchase from the seller whose combination of

visit the seller if the equilibrium density of buyers who visit a seller who posts p implies their individual value from buying the good at this price p is nonnegative. Thus, for buyers whose best sampled price is in $(\hat{p}, \overline{p}]$, the optimal purchase strategy with positive disease prevalence but prices given by $\bar{F}$ is given by the consumer density $\bar{b}_0(p)$, which is the maximum number of buyers visiting price $p \in (\hat{p}, \overline{p}]$ consistent with a nonnegative value from purchasing the good.

To illustrate the effect of different levels of prevalence of the disease (μ) with utility function (20), assume $\phi = \mu$ and set $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -2.3$, $\overline{u} = 1$, $\sigma = 0.05$, $q_1 = 1 - q_2 = 0.7$. Figure 5 displays, for $\mu = 0$ (or equivalently, $\mu = 1$), and for $\mu = 0.5$, the equilibrium distributions of posted values and prices, and the density of buyers per seller. Figure 6 shows the contagion and welfare for all possible levels of prevalence, $\mu \in [0, 1]$. The left panel shows the behavior of $\bar{\mathcal{C}}$ along with $\mathcal{C}$ and its decomposition into $\hat{\mathcal{C}}$ and $\tilde{\mathcal{C}}$. The right panel shows the behavior of $\bar{\mathcal{W}}$ along with $\mathcal{W}$ and its decomposition into $\hat{\mathcal{W}}$ and $\tilde{\mathcal{W}}$.[15]

Interestingly, contagion and welfare do not differ much between the economy with oblivious buyers (in which neither buyers nor sellers change their behavior in response to the risk of contagion), and the economy in which buyers and sellers play best responses that incorporate the risk of contagion. The decomposition of the equilibrium contagion rate $\mathcal{C}$, however, reveals that the sellers' pricing response is critical to understand the full equilibrium effect: if prices remained fixed, contagion would be lower (up to 1/3 lower, in this particular example). The reason is that in the economy with fixed prices, sellers who post prices in the upper part of the support of $\bar{F}$, i.e., $(\hat{p}, \overline{p}]$, attract fewer buyers when $\mu > 0$ than when $\mu = 0$. Formally, $\hat{\text{B}}S < B$, i.e., the total number of buyers who visit sellers in the setting with $\mu > 0$ and fixed prices is lower than the total number of buyers who visit sellers in the economy in which both buyers and sellers optimize in response to the disease. This market-induced response is reminiscent of a kind of endogenous voluntary social distancing by buyers. The right panel of Figure 6 shows that while it generates lower contagion, the economy with fixed prices also delivers lower

posted price and implied consumer density deliver the highest value. Any price $p \in [\underline{p}, \hat{p}]$ delivers strictly positive value to an individual buyer given other buyer's strategies are characterized by $\bar{b}(p)$ on $[\underline{p}, \hat{p}]$ (this is how $\hat{p}$ is defined). The value is decreasing in the price, so buyers' optimal purchase behavior implies any seller who posts a price in the interval $[\underline{p}, \hat{p}]$ attracts a density of buyers equal to $\bar{b}(p)$, i.e., the same density of buyers they would attract in a disease-free equilibrium with price distribution $\bar{F}$. This is because, conditional on offering a positive value to buyers, the number of buyers that a seller manages to attract only depends on the position of the seller's posted price in the distribution of prices that the buyers are sampling from, which in the economy with positive prevalence and fixed prices is still $\bar{F}$.

[15]The welfare component attributable to the endogenous response of prices, $\tilde{\mathcal{W}}$, can be read as the vertical difference between the line labeled $\mathcal{W}$ and the line labeled $\hat{\mathcal{W}}$ in the right panel of Figure 6.

consumer welfare. This is because in this example, there are allocations of buyers to sellers such that the private gain from trade $U(b) - c$ can be positive for all buyer-seller pairs, and the full equilibrium implements one of these allocations. Traditional epidemiological models are sometimes criticized on the basis that they ignore the behavioral responses of the agents who bear the risk of infection. The economy displayed in Figure 6 shows it may be just as important to also incorporate the endogenous responses of the markets where suceptible and infected agents interact. In fact, notice that in the example in Figure 6, ignoring all endogenous responses as in the traditional epidemiological models would be closer to the "correct" fully optimizing model than only endogenizing the behavioral responses of buyers.

6. Contagion externalities and market efficiency

When a buyer decides to visit a seller in the price posting equilibrium, she understands her decision will affect her own probability of infection and therefore her own utility, but fails to internalize the fact that her decision affects the probability of contagion of other buyers and consequently their utilities as well. In other words, the possibility of contagion introduces an externality in the price posting equilibrium. In this section I characterize the efficient allocation of buyers to sellers by solving the problem of a fictitious social planner who wishes to maximize the equally weighted sum of buyer's utilities (under the assumption that seller's profits are redistributed to buyers lump sum).

The planner's problem consists of finding an *optimal assignment of buyers to sellers*, i.e., a nonnegative real-valued (piecewise continuous) function $\boldsymbol{a} = [a(s)]_{s \in [0,S]}$ that solves

$$\max \int_0^S a(s) [U(a(s)) - c] \, ds, \text{ s.t. } \int_0^S a(s) \, ds \leq B. \tag{27}$$

Intuitively, $a(s)$ represents the density of buyers that the planner assigns to an individual seller $s \in \mathcal{S}$. Proposition 2 (in Appendix A), shows that the solution to the planner's problem (27) is the function $\boldsymbol{\theta}^* = [\theta^*(s)]_{s \in [0,S]}$, with

$$\theta^*(s) = \theta^* \equiv \arg \max_{a \in [0,\theta]} a [U(a) - c] \text{ for all } s \in [0, S]. \tag{28}$$

According to (28), depending on the parametrization, the planner's solution could take one of two forms. First, it could have per-seller buyer density $\theta^* < \theta$ (in this case the costate associated with the planner's Hamiltonian is zero, i.e., the shadow price of placing an additional buyer in the market is zero at the optimum). This is the solution that would obtain when the health externality that buyers impose on other buyers is relatively strong, and it could be interpreted as the planner's decision to prevent some buyers from engaging in shopping activities. In other words, $\theta^* < \theta$ can be construed as the planner wanting to restrict the number of buyers who go shopping in order to achieve a desired level of *social distancing*. Conversely, the planner's solution could have $\theta^* = \theta$ (in this case the shadow price of a buyer in the planner's Hamiltonian is positive). This is the solution that would obtain when the expected losses from the buyer's health externality are relatively mild, so the planner still wants all buyers to participate in the market. The average welfare achieved by the planner's solution is

$$\mathcal{W}^* = \frac{1}{\theta}\theta^* \left[U\left(\theta^*\right) - c\right], \tag{29}$$

and the efficient contagion function is

$$\mathcal{C}^* = S\left(1-\mu\right)\theta^* \iota\left(\theta^*\right). \tag{30}$$

There are two margins along which the planner and the decentralized solutions may differ: an *extensive margin* having to do with the number of buyers who participate in the market, and an *assignment margin* having to do with how buyers are allocated to sellers conditional on participating in the market.

6.1. Assignment inefficiency

Figure 7 compares the contagion and welfare implied by the planner's solution to the market-based contagion and welfare, for different levels of disease prevalence (for the same economy displayed in Figures 5 and 6). In this example there is no difference between the market outcome and the planner's solution along the extensive margin (i.e., all buyers participate in both). The differences along the assignment margin imply contagion is a somewhat lower under the planner's solution, but the difference in welfare appears to be negligible.

6.2. Participation inefficiency

To illustrate the difference between the market outcome and the planner's allocation along the extensive margin, consider the model with preferences given by (20), but set $q_1 = 1$, and generalize the formulation to allow for exit of buyers. Specifically, suppose the number of buyers who choose to participate in the market is $B^e \leq B$, and let $\theta^e \equiv B^e/S$. (So far I have been assuming all buyers choose to participate in the market, i.e., that $B^e = B$.) The fact that all buyers get one price offer implies all firms post the price that delivers value $v = 0$ to every buyer they might contact. Hence, the equilibrium is characterized by a single price, namely

$$p = U(\theta^e), \tag{31}$$

and an individual seller's profit equals

$$\bar{\Pi} = \theta^e \left[U(\theta^e) - c \right]. \tag{32}$$

For parametrizations with $0 < U(\theta) - c$, the gain from trade between any buyer-seller pair is positive even if all buyers choose to participate in the market, so the equilibrium is that all buyers choose to participate, i.e., $B^e = B$, and sellers earn a positive profit given by (32). This will also be the equilibrium if $U' = 0 < U(0) - c$, i.e., if there is no risk of contagion and there are gains from trade in the contagion-free economy. For economies with $U(\theta) < c < U(0)$ some buyers will prefer not to participate in the market. In this case, the equilibrium is that the number of buyers who choose to participate in the market, B^e, satisfies $U(\theta^e) = c$, and every seller's profit and every buyer's utility is equal to zero (whether they choose to participate or not). The equilibrium therefore consists of the posted price, (31), the seller's profit, (32), and the per-seller density of buyers, θ^e, resulting from buyers' participation decision, i.e.,

$$\theta^e = \begin{cases} 0 & \text{if } \overline{u} \leq c \\ U^{-1}(c) & \text{if } U(\theta) < c < \overline{u} \\ \theta & \text{if } c \leq U(\theta). \end{cases}$$

Equilibrium welfare and contagion are given by

$$\mathcal{W} = \frac{1}{\theta} \theta^e \left[U(\theta^e) - c \right]$$

and

$$\mathcal{C} = S(1-\mu)\,\theta^e \iota(\theta^e),$$

respectively. The efficient levels of welfare and contagion are still given by (29) and (30).

To compare the key prices and allocations implemented by the price-posting equilibrium with those chosen by the social planner for different levels of prevalence of the disease (μ), assume the utility function is given by (20) with $\phi = \mu$, and set $B = 100$, $S = 10$, $c = 0$, $\overline{u} = 1$, and $q_1 = 1$. Given this baseline parametrization, Figure 8 assumes $\underline{u} = -0.2$ and $\sigma = 0.5$. In this case the cost of contracting the disease and the contagion rate are relatively mild, and the equilibrium allocation and prices coincide with those chosen by the planner. The top-left panel, for example, shows that $S\theta^e = S\theta^* = B$ for all prevalence levels μ, i.e., all buyers participate in the market, and this is efficient. Similarly, for all prevalence levels μ, the bottom-left panel shows that $\mathcal{C} = \mathcal{C}^*$, the top-right panel that $U(\theta^e) = U(\theta^*)$, and the bottom-right panel that $\mathcal{W} = \mathcal{W}^*$. Figure 9 keeps the same baseline parametrization, but sets $\underline{u} = -0.8$ and $\sigma = 0.5$; i.e., the cost of contracting the disease is higher than in the economy of Figure 8, but all else is equal. In this economy the equilibrium outcome diverges from the planner's solution. The top-left panel shows that the planner prescribes that a large percentage of buyers (more than 50% for intermediate values of μ) stay away from the stores where they shop, i.e., $S\theta^* \leq B$, with "$<$" for intermediate values of μ. The planner's recommendation could be interpreted as a kind of social distancing. In the equilibrium, however, even though buyers are aware of the risk of infection, they all find it privately optimal to continue to shop, i.e., $S\theta^e = B$ for all μ. This different behavior along the extensive margin manifests itself in a much higher market-based infection rate than is socially optimal, i.e., $\mathcal{C}^* \leq \mathcal{C}$, with "$<$" for intermediate values of μ, as shown in the bottom-left panel. By restricting participation, the planner keeps buyer's valuations higher than in the equilibrium, i.e., $U(\theta^e) \leq U(\theta^*)$, with "$<$" for intermediate values of μ, as shown in the top-right panel. Finally, the bottom-right panel shows that the planner's social distancing prescriptions mitigate the welfare loss from the disease. Figure 10 has the same baseline parametrization as Figure 8 and Figure 9, but assumes $\underline{u} = -1$ and $\sigma = 0.9$, so the disease is even more costly and more contagious than in the economy depicted in Figure 9. The top-left and bottom-left panels show that in this case the market can curb contagion by inducing a significant amount of privately-chosen social distancing, while the bottom-right panel shows that the more aggressive use of the participation margin by the planner can somewhat mitigate the welfare losses of the market economy, but arguably not buy much.

6.3. The role of population density

The probability a buyer becomes infected when visiting a seller is increasing in the number of buyers who visit that same seller. Hence, an increase in the aggregate density of buyers per seller, θ, will affect the contagion rate, the participation of buyers, buyer's equilibrium valuations, and welfare. Figure 11 illustrates this result by varying the size of the population of buyers, B, in an economy parametrized by (20) with $\phi = \mu$, $S = 10$, $c = 0$, $\underline{u} = -0.8$, $\overline{u} = 1$, $\mu = 0.25$, $\sigma = 0.5$, and $q_1 = 1$. The planner's solution and the equilibrium outcome tend to coincide for relatively low population densities, but diverge once the aggregate number of buyers per seller exceeds a threshold that depends on the cost of contracting the disease and the probability of contagion. Beyond this threshold, the planner's solution calls for more severe social distancing than the decentralized equilibrium.

7. Conclusion

Section 3 focused on an equilibrium characterized by a continuous and strictly increasing cumulative distribution function of posted values. This class of equilibria is the most natural generalization of the Burdett-Judd equilibrium. It would be interesting to learn more about uniqueness and existence of equilibria in a more general class.

In Sections 5 and 6 I brought into the analysis the endogenous marketwide responses that shape buyers' incentives to expose themselves to the disease. In this regard I have gone a step further than the exising work that focuses on individual behavioral responses but abstracts from the explicit role that markets play in disease transmission. I have, however, also taken a step back relative to this existing work in my treatment of time. To introduce these new mechanisms it was sufficient to work in a static environment. But disease transmission is an eminently dynamic phenomenom, so embedding the market-mediated model of interpersonal contacts proposed here into a dynamic econ/epi framework seems like a natural next step. The background environment could borrow elements from some of the recent contributions of the Economics literature spurred by the COVID-19 pandemic, e.g., Acemoglu et al. (2020), Alvarez et al. (2020), Atkeson (2020), Berger et al. (2020), Brotherhood et al. (2020), Eichenbaum et al. (2020), Garibaldi et al. (2020), Jarosch et al. (2020), Keppo et al. (2020), Rampini (2020), Toxvaerd (2020).

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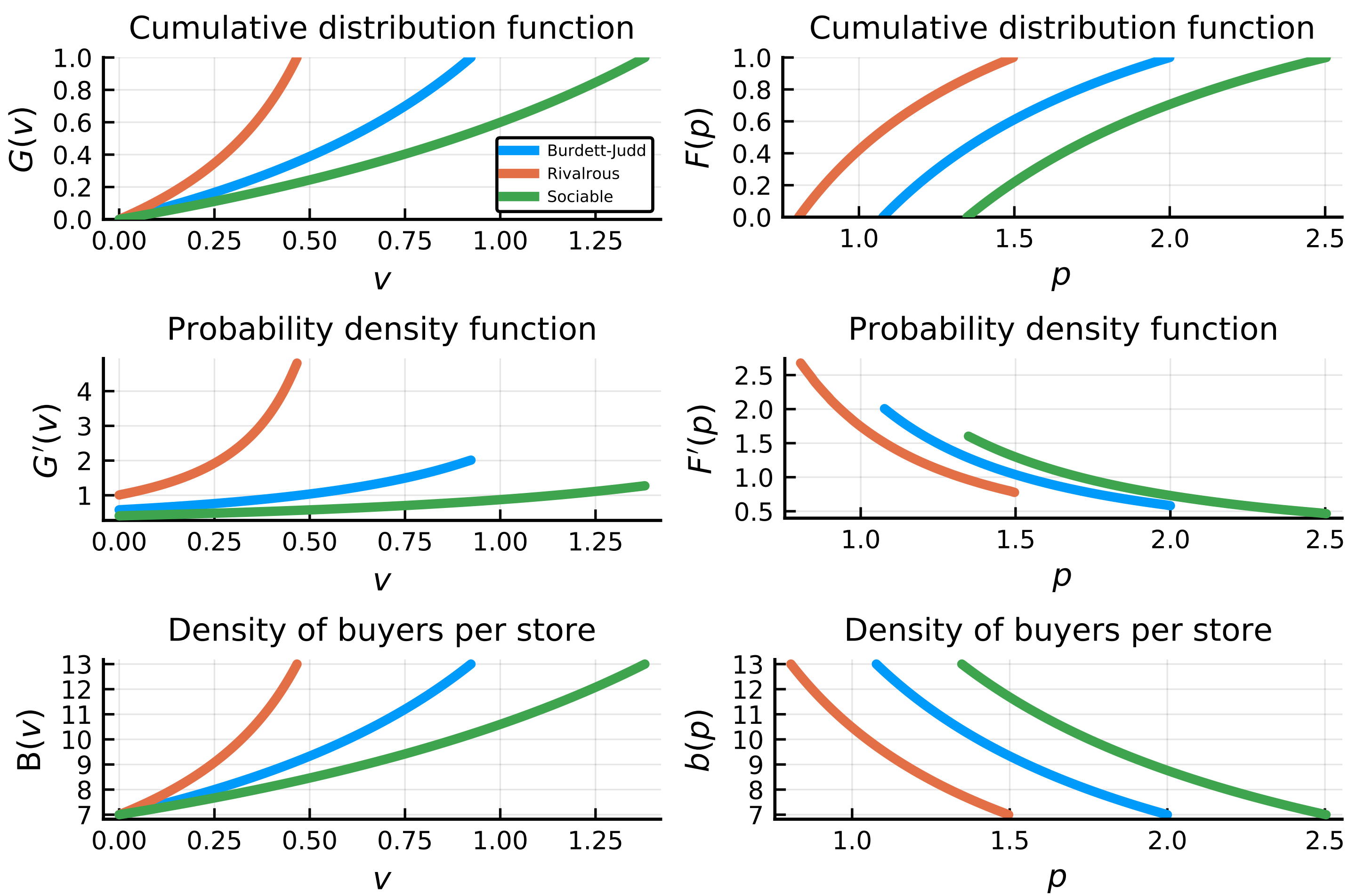

Figure 1: Equilibrium implications of preferences toward socializing and social distancing. Parameter values: $B = 100$, $S = 10$, $c = 0$, $\underline{u} = 1$, $\overline{u} = 3$, $q_1 = 1 - q_2 = 0.7$. In every panel, the line on the left corresponds to an economy with rivalrous buyers ($\sigma = 0.01$ and preferences given by (16)), and the line on the right corresponds to an economy with sociable buyers ($\sigma = 0.01$ and preferences given by (17)). The middle line corresponds to a Burdett-Judd economy (i.e., preferences given by (16) or (17), and $\sigma = 0$).

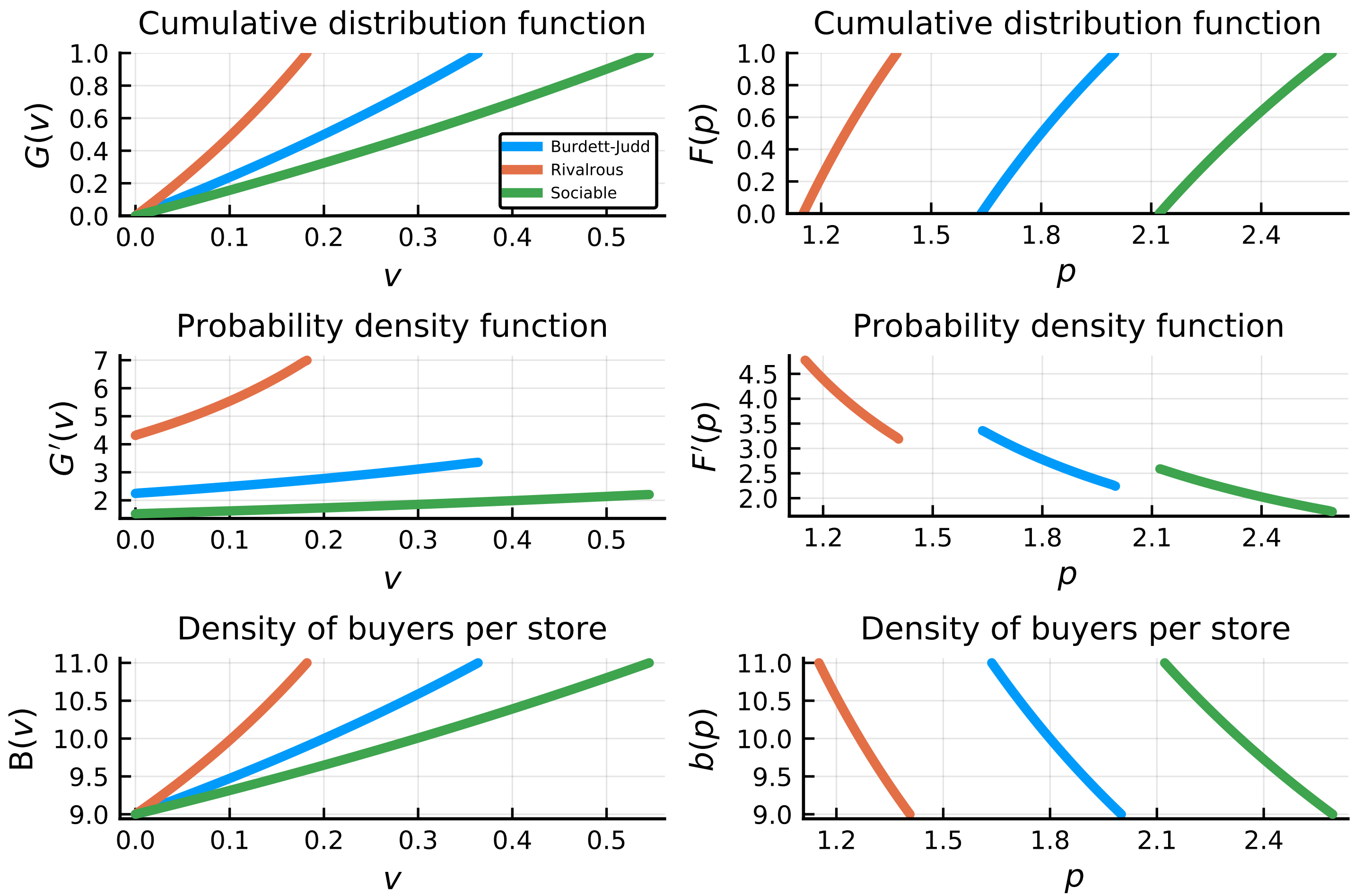

Figure 2: Equilibrium implications of preferences toward socializing and social distancing in an economy with low degree of competition. Parameter values: $B = 100$, $S = 10$, $c = 0$, $\underline{u} = 1$, $\overline{u} = 3$, $q_1 = 1 - q_2 = 0.9$. In every panel, the line on the left corresponds to an economy with rivalrous buyers ($\sigma = 0.01$ and preferences given by (16)), and the line on the right corresponds to an economy with sociable buyers ($\sigma = 0.01$ and preferences given by (17)). The middle line corresponds to a Burdett-Judd economy (i.e., preferences given by (16) or (17), and $\sigma = 0$).

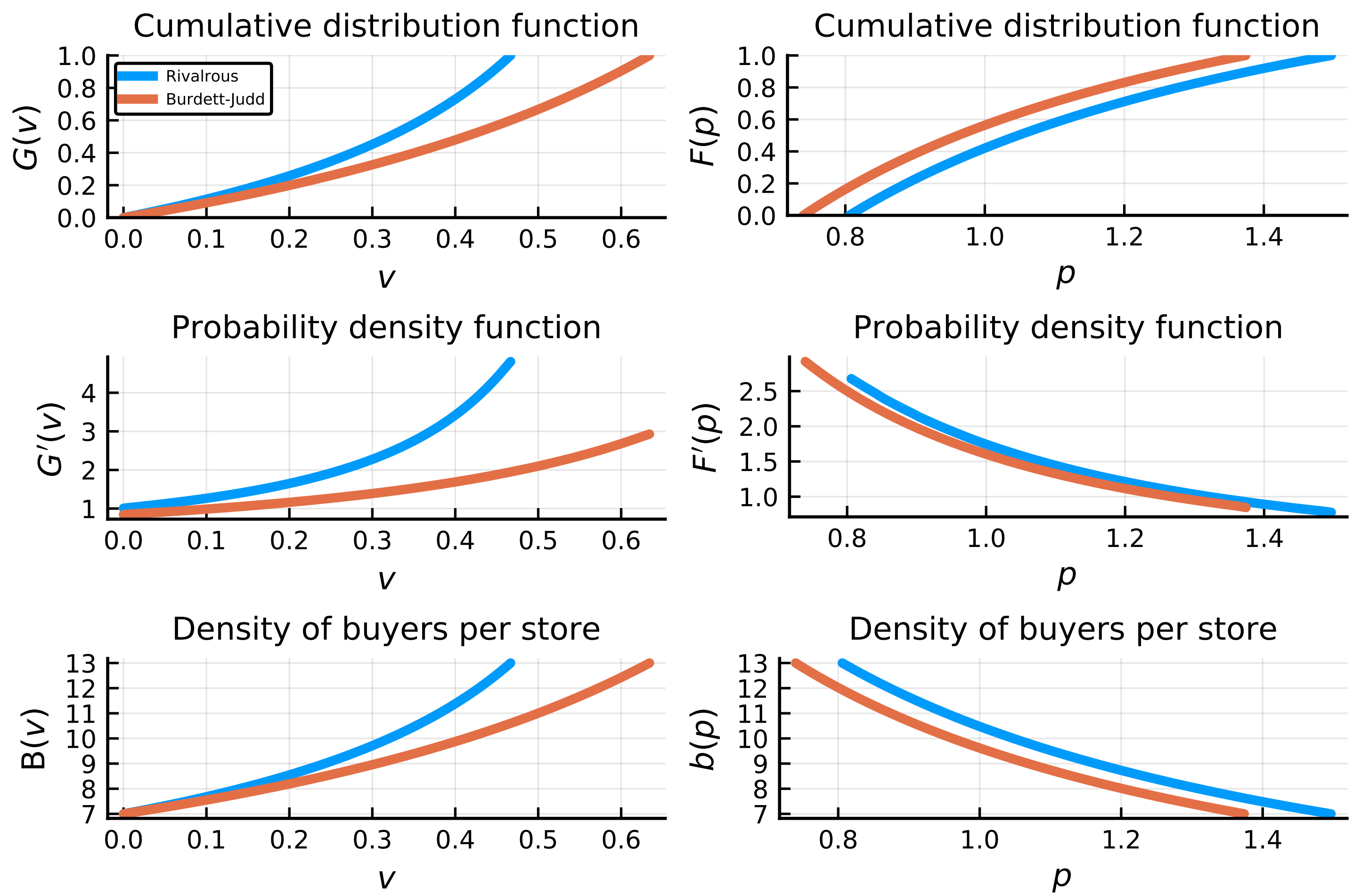

Figure 3: Comparison between economy with rivalrous buyers, and Burdett-Judd economy (keeping average market utility constant). Parametrization with rivalrous buyers: $B = 100$, $S = 10$, $c = 0$, $\underline{u} = 1$, $\overline{u} = 3$, $q_1 = 1 - q_2 = 0.7$, $\sigma = 0.01$, and preferences given by (16). The Burdett-Judd economy has utility of consumption equal to $\frac{\overline{u}+\underline{u}}{2}$ with $\underline{u} = 1$ and $\overline{u} = 2\int U^R(b(p))dF(p) - \underline{u}$.

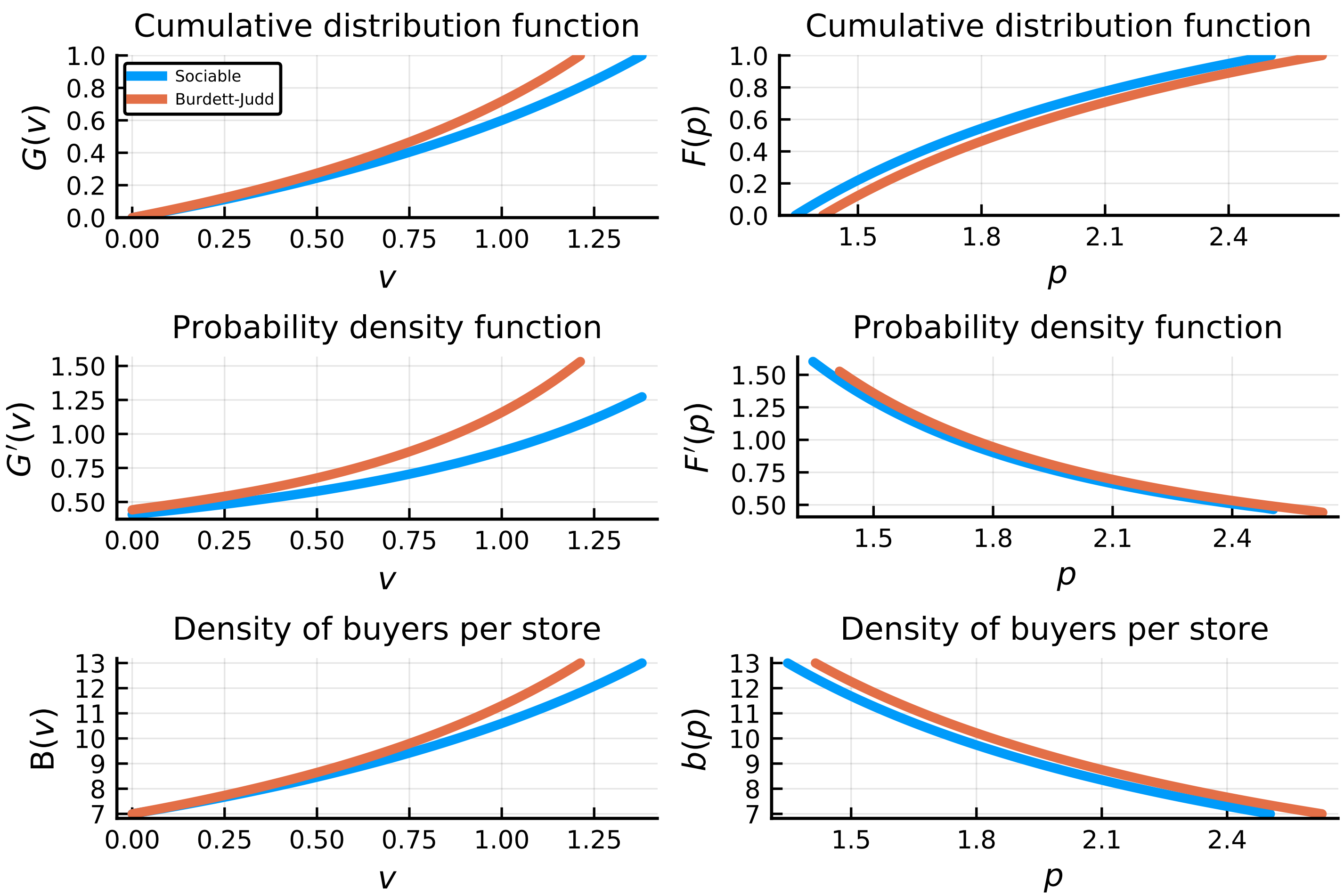

Figure 4: Comparison between economy with sociable buyers, and Burdett-Judd economy (keeping average market utility constant). Parametrization with sociable buyers: $B = 100$, $S = 10$, $c = 0$, $\underline{u} = 1$, $\overline{u} = 3$, $q_1 = 1 - q_2 = 0.7$, $\sigma = 0.01$, and preferences given by (17). The Burdett-Judd economy has utility of consumption equal to $\frac{\overline{u}+\underline{u}}{2}$ with $\underline{u} = 1$ and $\overline{u} = 2\int U^S(b(p))dF(p) - \underline{u}$.

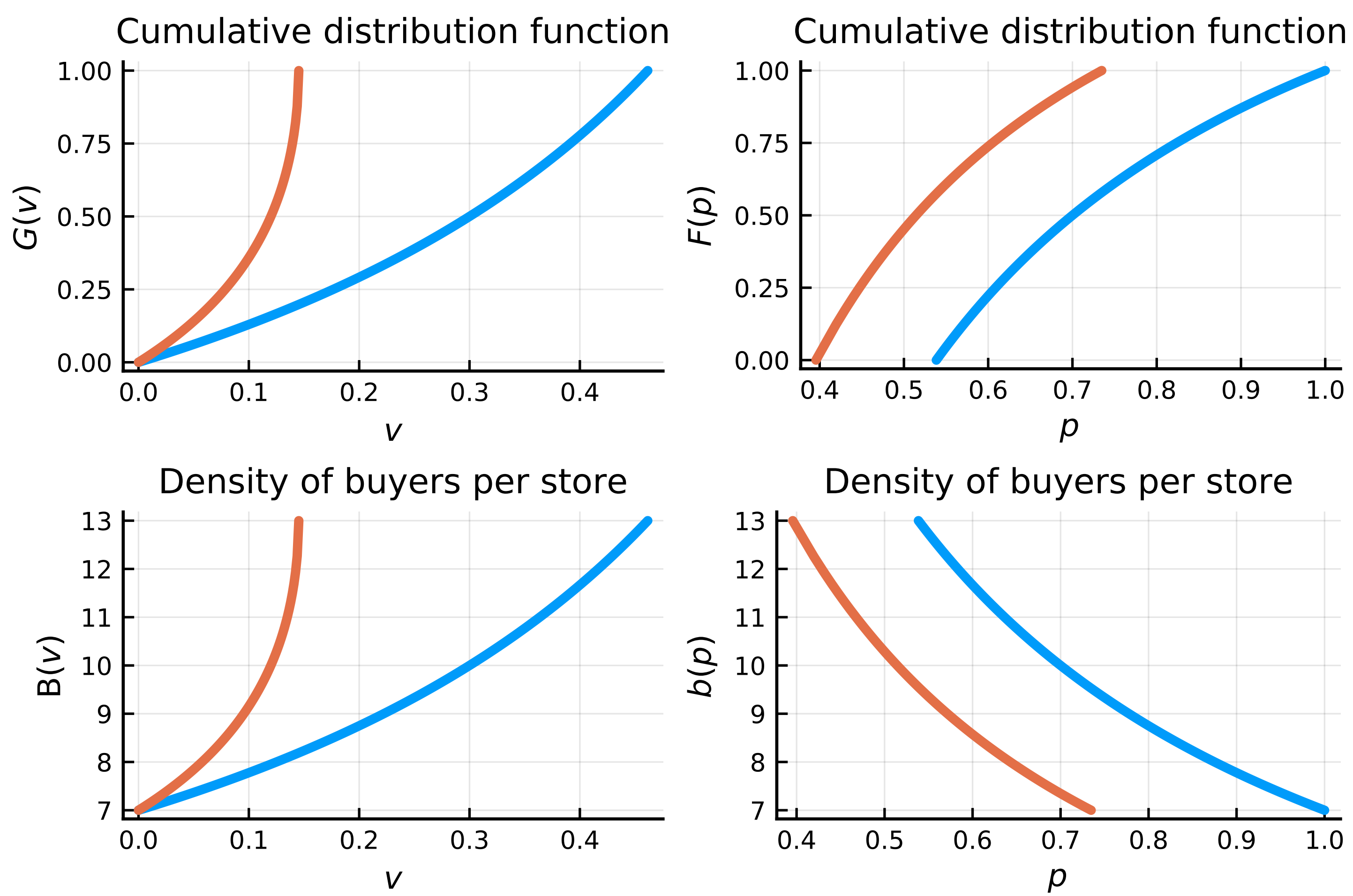

Figure 5: Equilibrium implications of a disease-induced preference for social distancing. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -2.3$, $\overline{u} = 1$, $\sigma = 0.05$, and $q_1 = 1 - q_2 = 0.7$. In every panel, the line on the left and the line on the right correspond to an economy with $\mu = 0.5$ and $\mu = 0$, respectively.

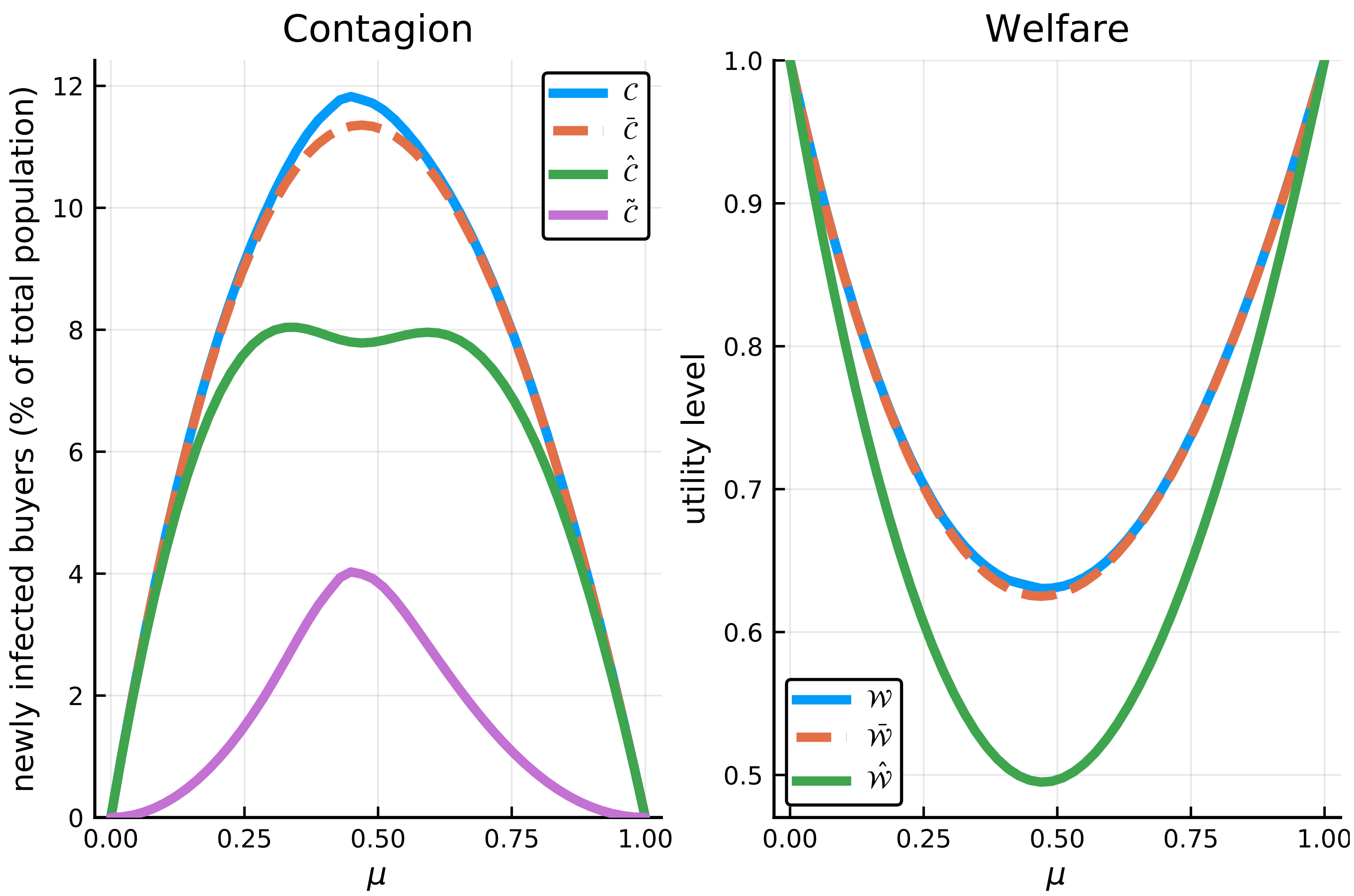

Figure 6: Market-based contagion and welfare for different levels of disease prevalence. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -2.3$, $\overline{u} = 1$, $\sigma = 0.05$, and $q_1 = 1 - q_2 = 0.7$.

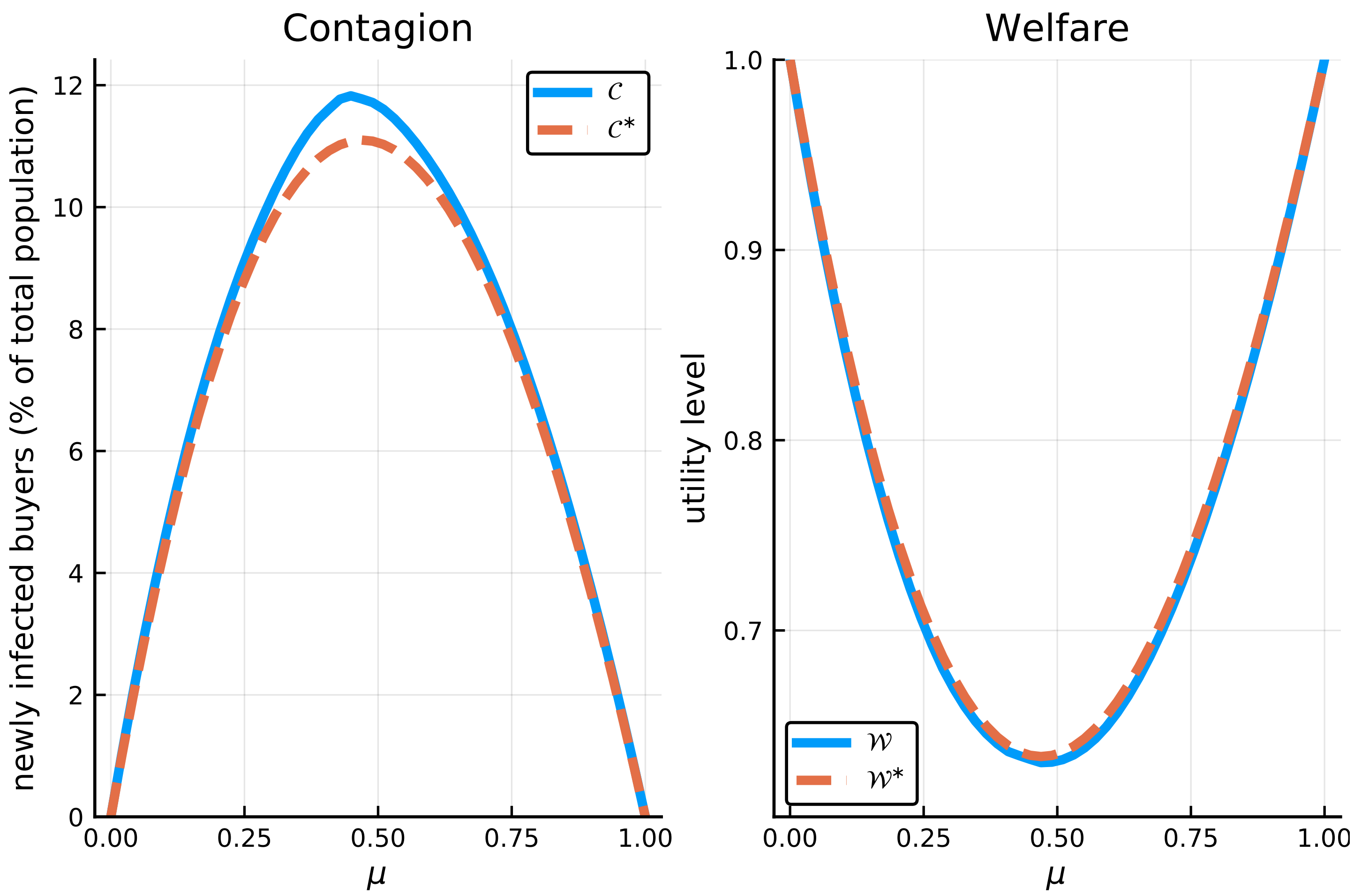

Figure 7: Efficient versus market-based contagion and welfare for different levels of disease prevalence. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -2.3$, $\overline{u} = 1$, $\sigma = 0.05$, and $q_1 = 1 - q_2 = 0.7$.

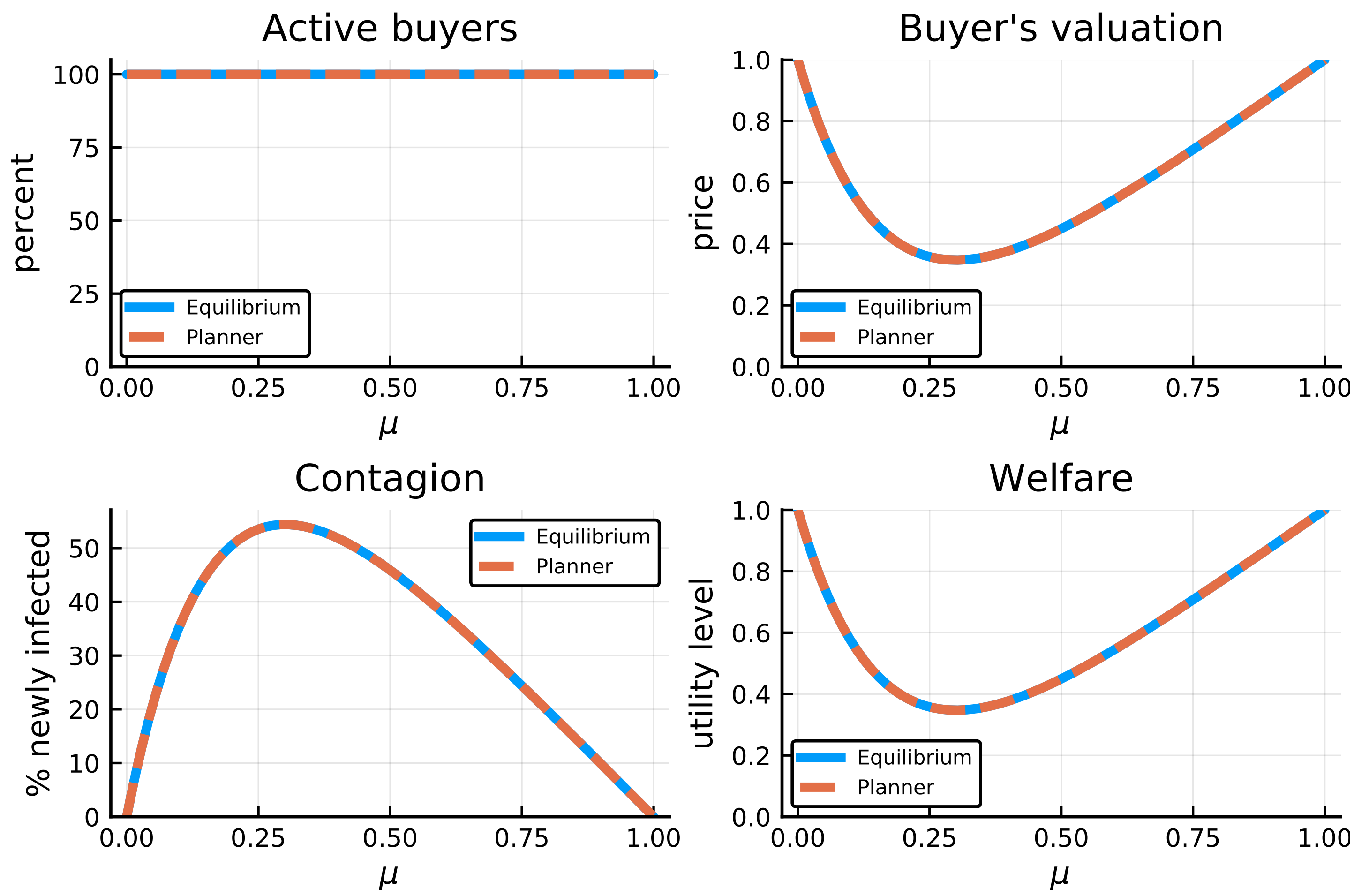

Figure 8: Efficient versus market-based allocations, prices, contagion, and welfare for different levels of disease prevalence. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -0.2$, $\overline{u} = 1$, $\sigma = 0.5$, and $q_1 = 1$.

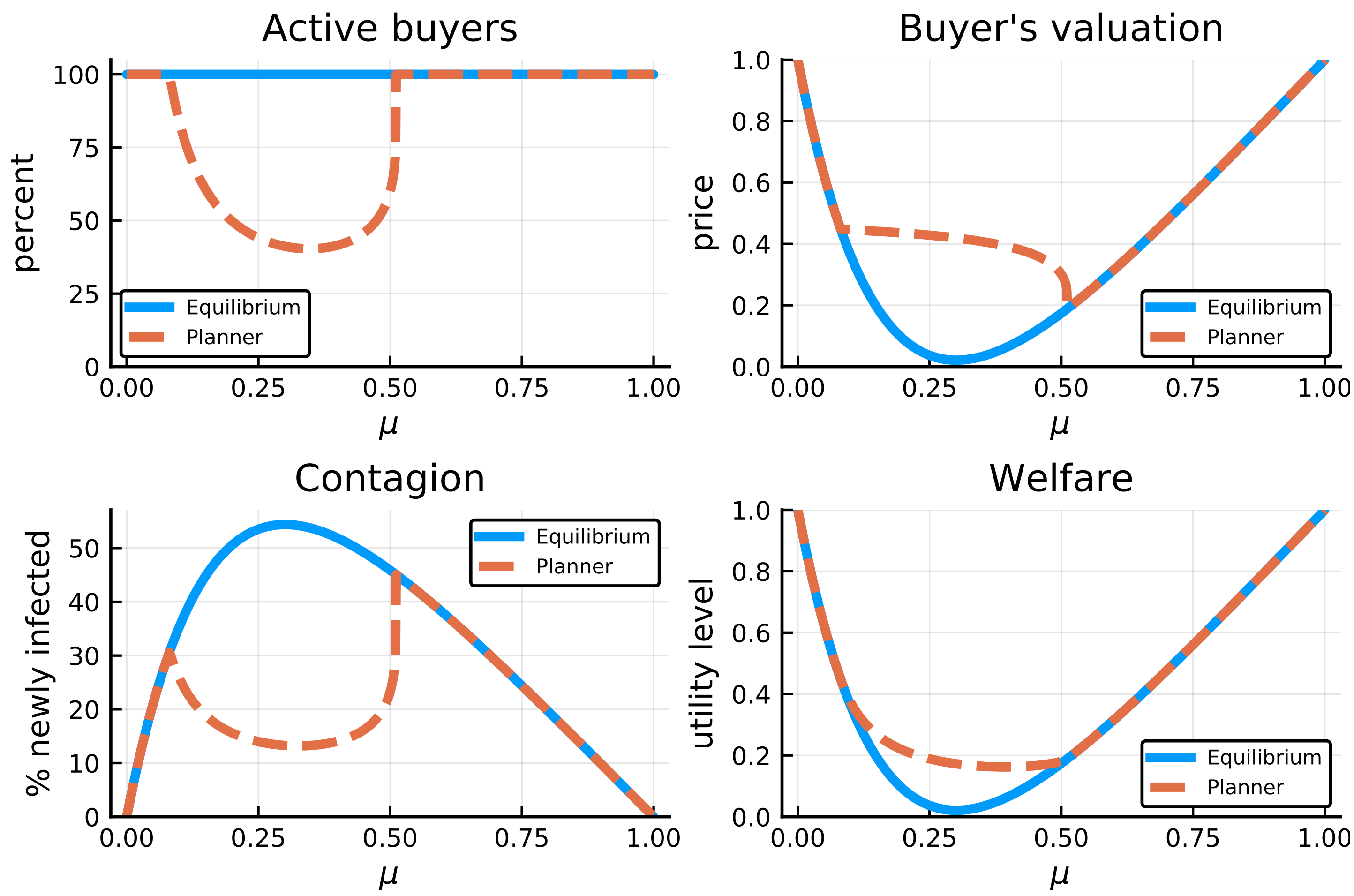

Figure 9: Efficient versus market-based allocations, prices, contagion, and welfare for different levels of disease prevalence. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -0.8$, $\overline{u} = 1$, $\sigma = 0.5$, and $q_1 = 1$.

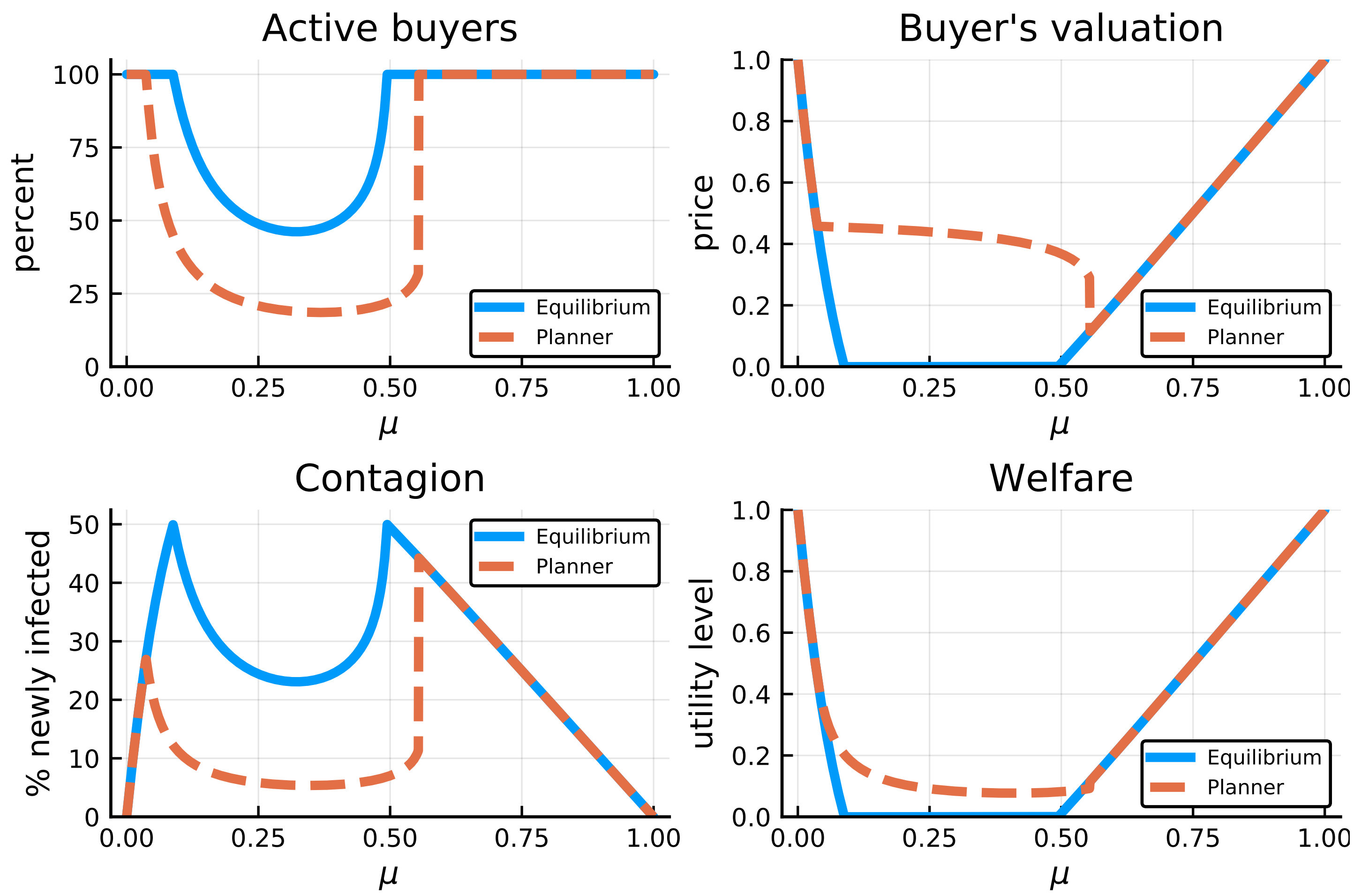

Figure 10: Efficient versus market-based allocations, prices, contagion, and welfare for different levels of disease prevalence. Preferences given by (20) with $\phi = \mu$, $B = 100$, $S = 10$, $c = 0$, $\underline{u} = -1$, $\overline{u} = 1$, $\sigma = 0.9$, and $q_1 = 1$.

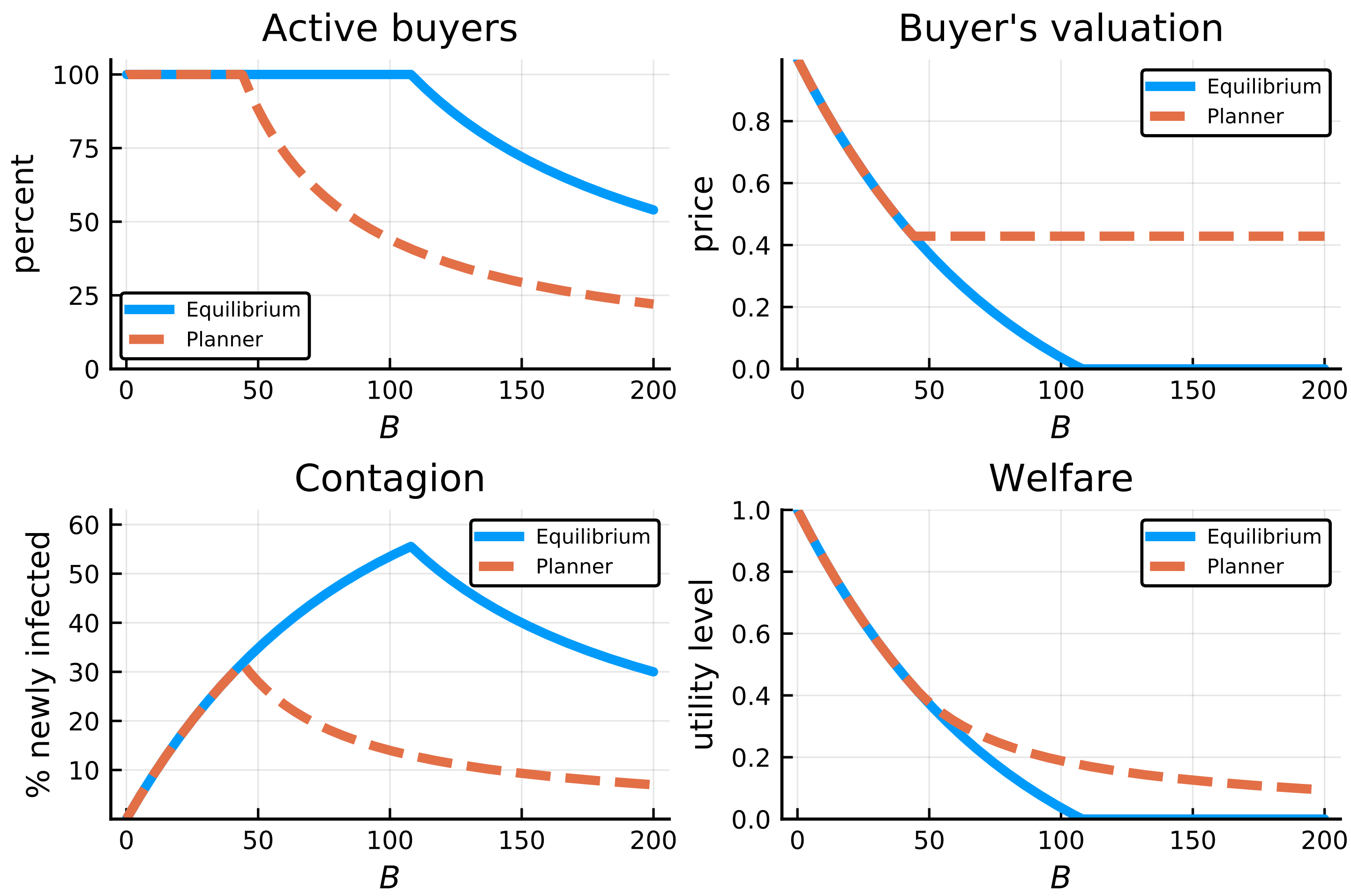

Figure 11: Efficient versus market-based allocations, prices, contagion, and welfare for different levels of aggregate density of buyers per seller. Preferences given by (20) with $\phi = \mu$, $S = 10$, $c = 0$, $\underline{u} = -0.8$, $\overline{u} = 1$, $\mu = 0.25$, $\sigma = 0.5$, and $q_1 = 1$.

A. Proofs

Proposition 1. *Suppose $U' < 0$ with*

$$\sup_{b \in [\mathrm{B}(0), \mathrm{B}(\overline{v})]} \left| U'(b)\, b^2 \right| < \left[U(\mathrm{B}(0)) - c \right] \mathrm{B}(0) < \left[U(\mathrm{B}(\overline{v})) - c \right] \mathrm{B}(\overline{v}), \tag{33}$$

or alternatively, $U' \geq 0$ with

$$\sup_{b \in [\mathrm{B}(0), \mathrm{B}(\overline{v})]} \left| \frac{U'(b)\, b}{U'(b)\, b^2 + \left[U(\mathrm{B}(0)) - c \right] \mathrm{B}(0)} \right| < 1. \tag{34}$$

Then, there exists a unique equilibrium with a continuous and strictly increasing distribution of posted prices. Equilibrium profit is given by (9) and the price distribution is

$$F^*(p) = F(p)\, \mathbb{I}_{\{\underline{p} \leq p \leq \overline{p}\}} + \mathbb{I}_{\{\overline{p} < p\}},$$

with F given by (12), and $\underline{p}$ and $\overline{p}$ given by (13) and (14), respectively.

Proof of Proposition 1. The proof consists of checking that the conjectures made in the construction of the equilibrium are indeed true in the equilibrium.

(1) [Prove that $\underline{v} < \overline{v}$.] For G to be continuous and strictly increasing, it is necessary that $\overline{v} > 0 = \underline{v}$. The equilibrium value of $\overline{v}$ is given in (10). From this it is clear that $U' \geq 0$ immediately implies $\overline{v} > 0$. If $U' < 0$, $\overline{v} > 0$ follows from the second inequality in (33).

(2) [Prove that $\mathrm{B}(v)$ and $G(v)$ are strictly increasing continuous functions on $[0, \overline{v}]$.] The second step is to prove that (11) implicitly defines a continuous and strictly increasing function $\mathrm{B}(v)$ on $[0, \overline{v}]$. Condition (11) can be written as $T(\mathrm{B}, v) = 0$, where $T(\mathrm{B}, v) \equiv [U(\mathrm{B}) - v - c]\mathrm{B} - \bar{\Pi}$. The condition $T(\mathrm{B}, v) = 0$ implies

$$\begin{aligned} \mathrm{B}'(v)\big|_{T(\mathrm{B},v)=0} &= -\frac{T_v}{T_{\mathrm{B}}} \\ &= \frac{1}{U'(\mathrm{B}(v))\, \mathrm{B}(v) + U(\mathrm{B}(v)) - v - c} \\ &= \frac{1}{U'(\mathrm{B}(v))\, \mathrm{B}(v) + \frac{\bar{\Pi}}{\mathrm{B}(v)}} \\ &= \frac{1}{U'(\mathrm{B}(v))\, \mathrm{B}(v) + \frac{[U(\mathrm{B}(0)) - c]\mathrm{B}(0)}{\mathrm{B}(v)}}. \end{aligned} \tag{35}$$

Hence, $\mathrm{B}'(v) > 0$ if and only if

$$-U'(\mathrm{B}(v))\, \mathrm{B}(v)^2 < \left[U(\mathrm{B}(0)) - c \right] \mathrm{B}(0) \tag{36}$$

for all $\text{B}(v) \in [\text{B}(\underline{v}), \text{B}(\overline{v})]$. The right side is strictly positive because of the maintained assumption $0 < \min_{b \in \mathbb{R}_+} U(b) - c$. Hence, (36) is immediately satisfied if $U' > 0$. If $U' < 0$, the first inequality in (33) is a sufficient condition on primitives for (36) to hold. Hence, in either case the condition $T(\text{B}, v) = 0$ implicitly defines a strictly increasing continuous function $\text{B}(v)$ on $[0, \overline{v}]$. Notice that the values of this implicit function at the endpoints 0 and $\overline{v}$, are the values $\text{B}(0)$ and $\text{B}(\overline{v})$, respectively, defined in (6) and (7). This can be checked by verifying that

$$T(\text{B}(0), 0) = 0 = T(\text{B}(\overline{v}), \overline{v}),$$

where the first equality follows from (9), and the second from (10). Since $\text{B}(v)$ is a strictly increasing continuous function on $[0, \overline{v}]$, so is $G(v)$ (by (5)).

(3) [Prove that $\rho'(v) < 0$.] The construction of equilibrium presumes that

$$\rho'(v) \equiv U'(\text{B}(v))\,\text{B}'(v) - 1 < 0. \tag{37}$$

First, consider the model with $U' < 0$. In this case, as shown above, $\text{B}'(v) > 0$ follows from the first inequality in assumption (33). Given $\text{B}'(v) > 0$, the inequality (37) is immediately implied by $U' < 0$. Next, consider the model with $U' > 0$. In this case, as shown above, no additional assumptions are needed to ensure that $\text{B}'(v) > 0$. However, in order to ensure the inequality (37) holds, we need to ensure that $U'(\text{B}(v))\text{B}'(v) - 1 < 0$. With (35), this condition can be written as

$$\frac{U'(\text{B}(v))}{U'(\text{B}(v))\,\text{B}(v) + \frac{[U(\text{B}(0)) - c]\text{B}(0)}{\text{B}(v)}} - 1 < 0. \tag{38}$$

Condition (34) is sufficient for (38) to hold for all $v \in [0, \overline{v}]$.

(4) [Prove that given $\text{B}(v)$ (5) can be solved for $G(v)$.] Condition (5) can be written as $\text{T}(G) = 0$, where $\text{T}(G) \equiv \sum_{k=0}^{\infty} \theta k q_k G^{k-1} - \text{B}$. Notice that T is continuous with $\text{T}'(G) > 0$, and

$$\text{T}(0) = \text{B}(0) - \text{B} \leq 0 \leq \text{B}(\bar{v}) - \text{B} = \text{T}(1),$$

for all $\text{B} \in [\text{B}(0), \text{B}(\bar{v})]$, so there exists a unique G that solves $\text{T}(G) = 0$ for each $\text{B} \in [\text{B}(0), \text{B}(\bar{v})]$. Thus, $\text{T}(G) = 0$ implicitly defines $G(v)$ as a continuous strictly increasing function of $\text{B}(v)$. ■

Proposition 2. *The solution to the planner's problem (27) is* $\boldsymbol{a}^* = [a^*(s)]_{s \in [0,S]}$ *with* $a^*(s)$ *given by (28).*

Proof of Proposition 2. The planner's problem can be cast as an optimal control problem with $a(s)$ as the control, $x(s) = \int_0^s a(z)\,dz$ as the state, and $\lambda(s)$ as the costate. The corresponding Hamiltonian is

$$H = a(s)\left[U(a(s)) - c\right] + \lambda(s)\,a(s),$$

and the necessary conditions for optimization are

$$U(a(s)) - c + U'(a(s))\,a(s) + \lambda(s) = 0 \tag{39}$$

$$\dot{\lambda}(s) = 0. \tag{40}$$

Condition (40) implies, $\lambda(s) = \lambda$ for all s, and then condition (39) implies

$$U(a(s)) - c + U'(a(s))\,a(s) = -\lambda \text{ for all } s,$$

so the solution must satisfy $a(s) = a$ for all s. Hence, at the optimum, $x(S) = aS$, and the constraints $0 \leq a(s)$ and $\int_0^S a(s)\,ds \leq B$ can be written as $0 \leq a \leq \theta$, and the solution to the planner's problem is given by (28). ■

B. Supplementary material

This appendix consists of two sections. In Section B.1 I describe two micro-level contact processes among the buyers who visit a particular seller that give rise to the contagion probability (18). In Section B.2 I show the relationship between the new infections at a given seller with buyer density b implied by the contagion probability (18), and the quadratic matching specification often used in epidemiological SIR-style models.

B.1. Micro-level meeting processes

B.1.1. Micro model I

Consider a seller who is visited by b buyers, a fraction μ of whom are infected. Assume the rate at which an individual buyer contacts another buyer at the seller's location is equal to αb. Suppose that a contact with an infected buyer causes a susceptible buyer to become infected with probability $\kappa \in [0, 1]$. Hence the rate at which an individual susceptible buyer contacts an *infected* buyer at a location with b buyers and becomes infected is $\alpha\mu\kappa b$. Assume buyers coincide at the seller's location during a time period of length τ. If we divide this time interval into τ/Δ discrete subintervals of small length Δ, then $\alpha\mu\kappa b\Delta$ is the probability that an individual susceptible buyer contacts and becomes infected by an infected buyer while both visit a seller's location during the subinterval of length Δ. Then the probability that an individual buyer *does not become infected* in the period of length τ is $(1 - \alpha\mu\kappa b\Delta)^{\tau/\Delta}$. Thus, as $\Delta \to 0$, the probability a buyer comes in contact with an infected agent is

$$\lim_{\Delta \to 0} \left[1 - (1 - \alpha\mu\kappa b\Delta)^{\tau/\Delta}\right] = 1 - e^{-\alpha\kappa\tau\mu b}.$$

This is the same as (18) if we set $\sigma = \alpha\kappa\tau$.

B.1.2. Micro model II

Consider a setup in which b large (non-atomistic) buyers visit a seller, and assume $\mu b \in [0, b]$ of these buyers are infected. These sellers are "large" in the sense that they occupy some nonnegligible amount of space in the seller's store. In this context, consider (a deterministic adaptation of) the Reed-Frost formulation of incidence (e.g., as discussed in Sattenspiel (1990)). That is, suppose an individual buyer comes in contact with each one of the other buyers in the store with independent probability $\varpi \in [0, 1]$ (i.e., think of the contacts of buyer i with all the other buyers at the store as being a sequence of independent Bernoulli trials). Assume each between a susceptible agent and an infected agent results in contagion with probability $\kappa \in [0,1]$. Hence, the probability that a susceptible buyer becomes infected at a store with μb infected buyers is given by $1-(1-\varpi\kappa)^{\mu b}$. Next, consider the limit as each individual buyer becomes small, while keeping the collective size of all buyers equal to b. A way to do this is to "partition" every agent into N replicas, each of size $1/N < 1$. The probability of meeting each of the infected buyers is scaled accordingly, to $\varpi\frac{1}{N}$. Then, the probability that a susceptible is infected is given by $1-\left(1-\varpi\kappa\frac{1}{N}\right)^{\mu b N}$. Thus, as agents become infinitesimally small, we get the probability that a susceptible is infected is

$$\lim_{N\to\infty}\left[1-\left(1-\varpi\kappa\frac{1}{N}\right)^{\mu b N}\right]=1-e^{\varpi\kappa\mu b}.$$

This is the same as (18) if we set $\sigma=\varpi\kappa$.

B.2. Quadratic matching

The standard SIR model often assumes quadratic matching between the infected and susceptible agents. Consider a given location (e.g., a seller's store) in which there are b buyers, and the number of infected and susceptible buyers are $n_I=\mu b$ and $n_S=(1-\mu)\,b$, respectively, for some $\mu\in[0,1]$. Then the typical SIR quadratic matching assumption, the number of new infections at this seller's location would equal

$$\mathcal{C}_Q \equiv \sigma n_S n_I = \sigma\,(1-\mu)\,\mu b^2. \tag{41}$$

To make the connection between (41) and the number of new infections at a given seller with buyer density b implied by the contagion probability (18), first notice that we can write $\iota\,(b)$ as

$$\iota\,(b)=1-e^{-\sigma\mu b}=1-e^{-\sigma n_I}\equiv\hat{\iota}\,(N_I)\,.$$

Notice that for $N_I\approx 0$ (i.e., if the infection has just started), $\hat{\iota}\,(N_I)\approx\sigma N_I=\sigma\mu b$, so $(1-\mu)\,b\iota\,(b)=(1-\mu)\,b\hat{\iota}\,(N_I)\approx\mathcal{C}_Q$.