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MONETARY POLICY WITH OPINIONATED MARKETS

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ABSTRACT

We build a model in which the Fed and the market disagree about future aggregate demand. The market anticipates monetary policy "mistakes," which affect current demand and induce the Fed to partially accommodate the market's view. The Fed expects to implement its view gradually. Announcements that reveal an unexpected change in the Fed's belief provide a microfoundation for monetary policy shocks. Tantrum shocks arise when the market misinterprets the Fed's belief and overreacts to its announcement. Uncertainty about tantrums motivates further gradualism and communication. Finally, disagreements affect the market's expected inflation and induce a policy trade-off similar to "cost-push" shocks.

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1. Introduction

The Fed and the financial markets often disagree about future interest rates. Figure 1 documents this observation by plotting the evolution of the Fed funds rate over time (thin black line), along with predicted paths. The dotted lines plot the Fed's predictions—either the Fed staff's assumption for the Greenbook (the left panel) or the FOMC members' median dot forecast (the right panel). The solid lines plot the forward interest rates that reflect the financial market's predictions. Each color-matched pair of lines plots data from the same FOMC meeting.[1] Similar disagreements are observed in other countries where central banks publish their expected interest rate paths, e.g., Sweden, Norway, and New Zealand (see Ubide (2015); Couture (2021)). Empirical evidence suggests these disagreements are at least partly driven by different opinions about future economic activity (see Section 2). There is also plenty of anecdotal evidence that market participants often have their own opinions and disagree with the Fed about appropriate interest rate policy.[2] These opinionated disagreements are a source of concern for the Fed, as they suggest that the market might perceive the Fed's policy decisions as "mistakes."

In this paper we build a model in which the Fed and the market have opinionated disagreements about future aggregate demand. We obtain several positive and normative results: First, we show that these types of disagreements can explain the differences in interest rate predictions between the Fed and the market depicted in Figure 1. Second, we find that the Fed's *optimal* interest rate policy partially reflects the market's view. The Fed expects to implement its view *gradually*: it waits for the data to change the market's belief toward the Fed's belief before fully implementing its view. Third, we provide a microfoundation for monetary policy shocks: Policy announcements that reveal a surprise change in the Fed's belief affect financial markets like textbook policy shocks, even though they are optimal under the Fed's belief. Fourth, we show that more damaging *tantrum shocks* arise when the market misinterprets the Fed's belief and overreacts to its announcement. Uncertainty about tantrums justifies *(prudential) gradualism* and *communication* policies. Finally, we show that disagreements affect the market's expected

[1] The forward interest rates in Figure 1 embed a risk premium. However, the estimates of this risk premium are small relative to the observed disagreements (see, e.g., Diercks, Carl et al. (2019)). Moreover, as we show in Section 2, the disagreement patterns remain when we measure the market's predictions using survey data. There is also a more subtle issue on whether the dot forecasts (on the right panel) represent the FOMC's *predictions* or its *wishes*. On this, we note that to the extent that these "wishes" are linked to the FOMC's view of the state of the economy, their comparison with the market's predictions is still useful for gauging disagreements between the Fed and the market.

[2] To illustrate how opinionated the market can be, consider the FOMC meeting in December 2007—the run-up to the financial crisis—in which the Fed cut interest rates by 25 basis points. The market was expecting a larger interest rate cut, so this was a "hawkish" policy surprise that led to a decline in stock prices. According to media coverage, some market participants were quite pessimistic that deteriorating financial conditions would adversely affect the economy, and they thought the Fed did not realize the scope of the problem. The day after the FOMC meeting the Wall Street Journal wrote: "Some on Wall Street yesterday criticized the Fed's actions so far as inadequate. 'From talking to clients and traders, there is in their view no question the Fed has fallen way behind the curve,' said David Greenlaw, economist at Morgan Stanley. *'There's a growing sense the Fed doesn't get it.'* Markets believe a weakening economy will force the Fed to cut rates even more than they expected before yesterday, Mr. Greenlaw said."

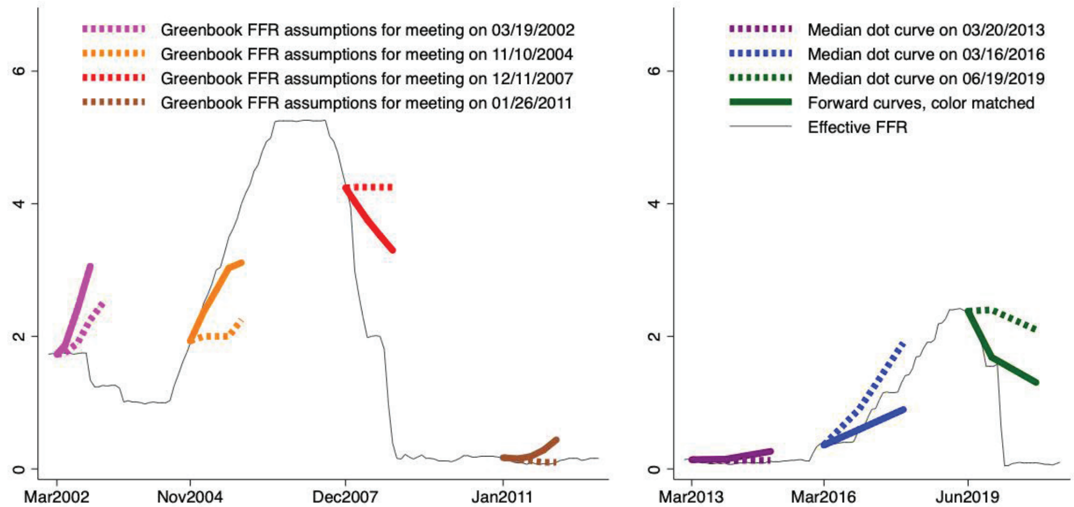

Figure 1: Dotted lines: the Fed prediction for Fed funds rates for select FOMC meetings—from either the Greenbook assumptions (the left panel) or the FOMC dots (the right panel). Solid lines: the forward Fed funds rates for the same meetings. Thin black line: the Fed funds rate. See Online Appendix D.1 for the data details and sources.

inflation and induce a policy trade-off similar to "cost-push" shocks.

Our model is a variant of the canonical New Keynesian model (e.g., Clarida, Gali and Gertler (1999); Galí (2015)). Nominal prices are fully sticky in our baseline setup (and partially sticky in an extension). There is a representative household (the market) that makes consumption-saving and labor supply decisions. The Fed sets the risk-free interest rate in an attempt to insulate the economy from *aggregate demand shocks* (shocks that affect spending without changing current potential output). Due to policy lags, the Fed cannot fully stabilize the output gap (output relative to potential). Instead, the Fed targets a zero output gap "on average" *according to the Fed's belief.*

The central insight of our paper is that the market considers the Fed's interest rate decisions *that do not match the market's belief* to be *"mistakes"* (we use quotes to remind the reader that these are mistakes under the market's belief, not under the Fed's belief or the objective belief). To capture this insight, we assume the market and the Fed can have *opinionated belief disagreements* about aggregate demand shocks. In our baseline setup, agents know each others' beliefs and *agree to disagree.* Agents also learn over time, as they observe new public signals and data, but learning is gradual and disagreements can persist. Persistent disagreements imply that the market anticipates monetary policy "mistakes" that affect forward interest rates, economic activity, and the Fed's optimal interest rate policy.

Concretely, suppose the Fed becomes more optimistic than the market about a permanent component of demand. Since the Fed is optimistic about demand, it raises the interest rate to stabilize the output gap. However, since the market doesn't share the Fed's optimism, it considers the interest rate hike to be a "mistake" and expects the output gap to be negative. Moreover, since the disagreements disappear gradually, the market expects "mistakenly high" interest rates in future periods as well. The forward interest rates immediately increase and put downward pressure on *current* economic activity. Therefore, even though the Fed is optimistic about aggregate demand, it does not need to raise the current interest rate by much to stabilize the output gap under its belief. In equilibrium, the Fed raises the interest rate by a relatively small amount, which—together with the increase in the forward rates—reduces aggregate demand just enough to counteract the increase in the Fed's optimism. The Fed also expects to continue raising rates over time, since it expects the data to support its view and persuade the market to move closer to its view—a form of *expected gradualism* in monetary policy.

Conversely, when the market becomes more optimistic than the Fed about demand, the market thinks the Fed sets "mistakenly low" interest rates in the current and future periods. This puts upward pressure on current economic activity. Therefore, even though the Fed's belief did not change, it hikes the interest rate to some extent—i.e., there is *initial overshooting*. The Fed also *expects to gradually* undo this overshooting as the data supports its view and reduce the market's optimism.

Our first set of results formalizes the logic in these examples. We show that the Fed's optimal interest rate reflects a *weighted average of the Fed's belief and the market's belief*. That is, the Fed cannot set interest rates by focusing only on its own view of aggregate demand—it also puts some weight on the market's view, even though it disagrees with the market. This weighted-average policy rule, together with agents' learning, also explains the observed differences between the market's and the Fed's expectations for future interest rates (see Figure 1). Agents agree on how the policy rule will respond to beliefs, but they disagree over what future beliefs will be. For sufficiently distant horizons, the market's expected rates reflect the market's *current* belief, because the market thinks *the Fed* will learn from data and come to the *market's* belief. Conversely, the Fed thinks the market will learn from data and it will be able to set future interest rates reflecting its current belief.

Our second set of results provides a microfoundation for textbook monetary policy shocks (which are typically modelled as ad-hoc random variations around a policy rule). In our model, beliefs and disagreements change over time because agents heterogeneously interpret new public signals. The idiosyncratic part of the Fed's belief is then naturally revealed to the market at discrete times, e.g., via a policy announcement or a communication. This revelation leads to microfounded monetary policy shocks that we call *Fed belief surprises*. These surprises affect financial markets like textbook policy shocks: for instance, after an interest rate hike that reveals a more optimistic Fed than the market expected, the forward interest rates increase and the market's expected output gaps decrease. The market revises its view of monetary policy "mistakes" in the direction of higher interest rates. However, unlike textbook shocks, Fed belief surprises are *optimal* under the Fed's belief. Therefore, their implications for subsequent economic outcomes are subtle and depend on the data generating process (DGP). For instance, if the market's belief is correct (the same as the DGP), then a positive interest rate shock driven by a Fed belief surprise is on average followed by negative output gaps. However, if the Fed's belief is correct, then the interest rate shock is on average followed by zero output gaps (the Fed's target), despite the market's negative reaction. Overall, the Fed belief surprises are relatively benign policy shocks, at least under the Fed's belief.

Policy shocks are potentially more damaging *if the Fed is uncertain about how the market will react to its announcements*. Suppose the market can interpret an interest rate hike as an increase in either long-term or short-term Fed optimism, and the Fed does not know how the market will interpret the rate hike. This allows for *tantrum shocks* in which the forward rates *overreact* to the Fed's rate hike relative to what is optimal under the Fed's belief. After a tantrum shock, the Fed misses its output gap target *even under its own belief.* In practice, the Fed is likely to be aware of contexts where tantrum shock are more likely. We show that the fear of tantrum shocks induces the Fed to act *more gradually* than in our baseline setting. We also show that communication between the Fed and the market is useful, *not* to persuade the market—the market is opinionated—but to reduce the likelihood of a tantrum shock.

In the final part of the paper we extend the model to allow for partial price flexibility, which gives rise to a standard New Keynesian Phillips curve. This extension strengthens our mechanism, in the sense that the Fed accommodates the market's belief even more than with fully sticky prices. For optimal policy purposes, disagreements closely resemble the cost-push shocks in the textbook New-Keynesian model. Consider the earlier example with an optimistic Fed in which the market expects the Fed to set high interest rates and induce negative output gaps. With partially flexible prices, the market also expects disinflation which, via the Phillips curve, reduces *current* inflation. The Fed is then pushed to set a lower interest rate than before—closer to the market's pessimistic belief—to induce a positive output gap (under its belief) and fight the disinflationary pressure. In fact, the "divine coincidence" breaks down and the Fed faces a trade-off between stabilizing the current inflation and the current output gap.

The rest of the paper is organized as follows. After discussing the related literature, we start in Section 2 by documenting facts about interest rate disagreements between professional forecasters and the Fed that motivate our modeling ingredients. Section 3 introduces our general environment, describes the belief structure, and derives the equilibrium conditions. Section 4 shows how disagreements affect optimal interest rate policy and (together with learning) explain the gap between the Fed's and the market's expected interest rates. Section 5 introduces the market's uncertainty about the Fed's belief and derives our results about microfounded monetary policy shocks. Section 6 introduces the Fed's uncertainty about the market's reaction to its announcements and derives our results about tantrum shocks. Section 7 analyzes the extension with partial price flexibility. Section 8 provides final remarks. The (online) appendices contain the omitted derivations and proofs as well as the details of our empirical analysis.

Related literature. Our paper has normative and positive components, each related to multiple literatures about monetary policy. The distinctive feature of our model is *belief disagreements* between the Fed and the market. In particular, the market has its own belief and *does not* consider the Fed to have superior information about economic activity.

Our policy analysis contributes to a large literature that investigates *gradualism* in monetary policy: the idea that the Fed tends to adjust interest rates in small steps in the same direction (see, e.g., Woodford (2003); Bernanke (2004); Stein and Sunderam (2018)). Our model features a novel form of *expected* gradualism. When the Fed becomes more optimistic than the market, it hikes the interest rate by a small amount—partially accommodating the market's view—but it also *expects* to continue to hike rates. *The market does not expect the rate hikes to continue*, which might help explain why gradualism has been difficult to detect from the term structure of interest rates (e.g., Rudebusch (2002)). With tantrum shocks, our model features a second, more standard rationale for gradualism (similar to Brainard (1967); Sack (1998)): the Fed adjusts the policy rate conservatively because it is afraid of a large market reaction. In contrast, our model generates rapid policy responses when there is no disagreement between the Fed and the market.

Our policy analysis is also related to the growing literature on central bank communication (see Blinder et al. (2008) for a review). The literature documents that central bank transparency has increased in recent years, and that the common forms of communication have made monetary policy more predictable. Our model is consistent with these findings and provides a rationale for Fed communication. As anticipated by Blinder (1998), the Fed in our setup communicates *to let the market know its own belief.* This transparency improves the Fed's ability to predict how the market will react to its actions and devise appropriate policy. In particular, the Fed can avoid tantrum shocks in which the market overreacts to its announcements.[3]

Similarly, *forward guidance* is a common form of communication by which the central bank reveals the path of interest rates it is likely to set. The recent literature has mostly focused on the role of forward guidance as a commitment device that might help circumvent the effective lower bound (ELB), e.g., Eggertsson and Woodford (2003). We show that forward guidance can be useful even if the economy is away from the ELB, since it can reveal the central bank's belief and mitigate tantrum shocks (see Bassetto (2019) for a related mechanism and Campbell et al. (2012); Woodford (2013*a*); Svensson (2014) for perspectives on the role of forward guidance in facilitating communication vs. commitment).

More broadly, our normative analysis is part of a large literature that investigates optimal macroeconomic policy without rational expectations (see Woodford (2013*b*) for a review). This

[3] See Woodford (2005) for other arguments for Fed communication and Amato, Morris and Shin (2002) for a model in which Fed communication might be excessive. A parallel debate concerns the best practices for central bank communication; for instance, whether the central bank should speak with a single voice or with many voices—reflecting the differences of opinion among policymakers (see, e.g., Ehrmann and Fratzscher (2007)). In recent work, Vissing-Jorgensen (2019) analyzes "the quiet cacophony of voices": *informal* communication by multiple FOMC members. She argues that market beliefs *do* influence actual monetary policy decisions (as in our model), and the FOMC members know this and selectively reveal information to influence the market's belief. In her model, informal communication resembles a Prisoner's dilemma and is welfare reducing.

literature typically assumes the planner is rational, but agents are boundedly rational due to frictions such as learning (e.g., Evans and Honkapohja (2001); Eusepi and Preston (2011)), level-k thinking (e.g., García-Schmidt and Woodford (2019); Farhi and Werning (2019); Angeletos and Sastry (2018)), or cognitive discounting (Gabaix (2020)).[4] The focus is on designing policies that address or are robust to agents' bounded rationality. Our approach has two key differences. First, we do not take a stand on who has rational beliefs: in fact, the market thinks it has correct beliefs and the Fed has incorrect beliefs—the opposite of the typical assumption. Second, our agents are not boundedly rational in the usual sense: both the market and the Fed have *dogmatic* beliefs about exogenous states and understand how those states map into endogenous outcomes. These assumptions lead to a different policy analysis and results. In our setting, the Fed's main non-standard concern is to mitigate the macroeconomic impact of the monetary policy "mistakes" perceived by the market.

Our positive analysis contributes to the large empirical literature that investigates the effects of monetary policy shocks on economic activity (see Ramey (2016) for a recent review). We introduce *Fed belief surprises* as microfounded monetary policy shocks. These surprises generate some of the asset price responses observed by the literature that uses high-frequency event study methods to identify monetary policy shocks (e.g., Bernanke and Kuttner (2005); Gürkaynak, Sack and Swanson (2005*a*,*b*); Hanson and Stein (2015); Goodhead and Kolb (2018)).

The Fed belief surprises are also related to *the Fed information effect* emphasized in the recent literature (see, e.g., Romer and Romer (2000); Campbell et al. (2012); Melosi (2017); Nakamura and Steinsson (2018*a*); Andrade et al. (2019); Gürkaynak et al. (2021); Hillenbrand (2021); Rungcharoenkitkul and Winkler (2022))—the idea that the Fed's policy announcements might signal information about fundamentals. We highlight a different effect. In our model, *the market does not think policy announcements have information about fundamentals.* Instead, the market updates its belief about the Fed's belief. Similar to this literature, we emphasize that monetary policy shocks can be driven by the Fed's belief about fundamentals, which can confound the standard empirical approaches estimating the effects of monetary policy shocks. However, we do not *necessarily* assume the Fed's belief is correct, and we characterize when (and how) the standard regressions will be confounded (see Section 5).

A strand of the literature documents that the high-frequency "policy surprises" are predictable from information *publicly available* before the announcement (see, e.g., Miranda-Agrippino (2016); Miranda-Agrippino and Ricco (2018); Cieslak (2018)). In recent work, Sastry (2019); Bauer and Swanson (2020) investigate this puzzle and find that the Fed has reacted to public data about the state of the economy more than the market had anticipated. The evidence further suggests that, at the time of the announcement, the market learns the Fed's belief (or reaction) and disagrees with it. Instead of adopting the Fed's belief, the market independently

[4] A related literature assumes agents are rational but lack common knowledge of each other's beliefs, and shows coordination problems can lead to aggregate behavior that resembles bounded rationality (e.g., Woodford (2001); Angeletos and La'O (2010); Morris and Shin (2014); Angeletos and Lian (2018); Angeletos and Huo (2018)).

updates its own belief from the same public data—possibly at a different time. These findings are consistent with our key ingredients, disagreements and learning from data.

Our analysis with partial price flexibility is related to the New Keynesian literature on the limits of inflation stabilization policy. In the textbook model, stabilizing inflation also replicates the flexible-price outcomes. This *divine coincidence* applies for supply shocks as well as demand shocks and implies that the central bank does not face a policy trade-off (e.g., Goodfriend and King (1997); Blanchard and Galí (2007); Galí (2015)). This feature seems counterfactual, which has led the literature to introduce "cost-push" shocks—often motivated by markup fluctuations or wage rigidities—that create a policy trade-off. We show that disagreements between the Fed and the market (the price setters) create a policy trade-off *even without cost-push shocks*. Intuitively, perceived policy "mistakes" shift agents' inflation expectations and affect their price setting (the Phillips curve) *as-if* there is a cost-push shock.

Our empirical analysis of interest rate disagreements is related to a literature that uses survey data to document belief distortions about macroeconomic outcomes. Much of the recent literature focuses on whether agents over- or underreact to data (e.g., Coibion and Gorodnichenko (2015); Bordalo et al. (2020); Broer and Kohlhas (2018); Angeletos, Huo and Sastry (2020); Ma et al. (2020)). In contrast, we focus on the relationship between disagreements on different macroeconomic variables (see also Andrade et al. (2016); Giacoletti, Laursen and Singleton (2021); Bauer and Chernov (2021)). We show that, consistent with our model, disagreements between the Fed and the market about future interest rates correlate with disagreements about future inflation.

Finally, this paper is related to a large literature that studies the implications of belief disagreements for financial markets and the macroeconomy (see Simsek (2021) for a recent survey). We analyze the disagreements between a policymaker (the Fed) and investors, whereas the literature mostly focuses on the disagreements among investors (see, e.g., our previous work, Caballero and Simsek (2020, 2021*b*)).

2. Motivating facts on Fed-market disagreements

In this section, we present evidence for our two main modeling ingredients: i) disagreements between the Fed and the market about expected interest rates are driven by *disagreements about expected aggregate demand*; and ii) these disagreements are somewhat *persistent*.

For our baseline analysis, we measure the market's beliefs from Blue Chip Financial Forecasts (Blue Chip). Blue Chip is a monthly survey of several major financial institutions. Forecasters report predictions about interest rates and other outcomes for up to five quarters ahead. We use the consensus (average) prediction. We also average the predictions made in each month of the quarter and construct a quarterly time series. We are interested in predictions for the future policy interest rate and for future aggregate demand. We measure the beliefs for the policy rate from the predictions for the Fed funds rate (FFR), reported as the quarterly average. We

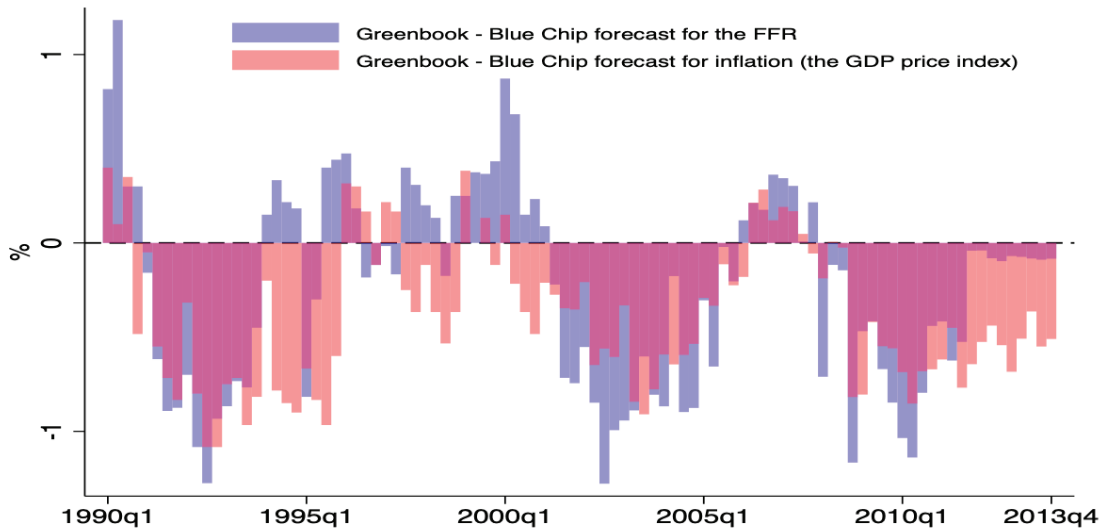

Figure 2: The bars denote the difference between the Fed's Greenbook/Tealbook forecast and the consensus Blue Chip forecast for 4 quarters ahead. The blue (resp. red) bars correspond to forecasts for the FFR (resp. the GDP price index growth).

proxy the beliefs for aggregate demand from the predictions for inflation (the GDP price index), reported as the annualized quarterly growth rate. We analyze predictions for the fourth quarter (beyond the current quarter), but the results are similar for other forecast horizons.

We measure the Fed's beliefs from the Greenbook (which subsequently became the Tealbook) and its supplements. Greenbooks are documents produced by the Fed research staff before each FOMC meeting to assist policymakers (they are released to the public after a five-year delay). They contain the Fed staff's predictions for several macroeconomic variables. We focus on predictions for the FFR and inflation (the GDP price index) at the same horizon and in the same units as the Blue Chip. We construct a quarterly time series by averaging the predictions made in each FOMC meeting within the quarter. We mainly rely on the digitized data from the Philadelphia Fed, which provides the predictions for inflation made until the end of 2013 and for the FFR made until late 2008. We also hand-collect data on the FFR predictions made between 2008 and 2013. Our baseline sample combines the Greenbook and Blue Chip data from 1990-Q1 until 2013-Q4.[5] Online Appendix D.1 contains details about data sources and construction.

Figure 2 plots the *difference* between the Fed's and the market's predictions for the FFR (blue bars) and inflation (red bars). The frequent coincidence in the direction of blue and red bars shows that the Fed tends to predict a higher policy rate when it predicts higher aggregate demand (proxied by inflation). In the early-1990s and during the recovery from the 2001 recession, the Fed had a more pessimistic view of demand than the market and predicted lower interest rates.

[5] Our data is available from 1986-Q1, but we start the baseline sample in 1990-Q1 since, by this time, the Fed was able to stabilize inflation around a target level on average, as in our model. In contrast, the Fed in the 1980s was focused on reducing inherited inflation. Our results extend to the brief period 1986-1989 but they are noisier.

Before the Great Recession, the Fed briefly turned more optimistic and predicted higher rates. During the recovery from the Great Recession, the Fed was once again more pessimistic and predicted lower rates.[6] These patterns are consistent with the left panel of Figure 1.

Figure 2 also highlights that disagreements are persistent. Once the Fed (or the market) forms a substantially more pessimistic view of demand, it remains pessimistic for many quarters. The persistence of beliefs and disagreements plays an important role in our analysis.

Online Appendix D.2 shows that the patterns highlighted in Figure 2 hold in a regression analysis (see Table D.1) and for other prediction horizons (see Figure D.1). One concern with this analysis is that the Blue Chip forecasts might not be representative of the dominant belief in financial markets (the belief that determines asset prices). Online Appendix D.2 also addresses this concern and shows that the results are robust to measuring the market's beliefs from asset price data. We measure the market's interest rate predictions from forward interest rates (as in Figure 1) and the inflation predictions from the inflation breakevens in the TIPS market. This approach leads to qualitatively similar results as in Figure 2, except for the financial crisis period during which the inflation breakevens were confounded by a large liquidity premium (see Figure D.2).

We next turn to our theory, where we equip the Fed and the market with persistent disagreements about aggregate demand and investigate the implications for monetary policy.

3. Environment, equilibrium, and beliefs

In this section we introduce our model, characterize the equilibrium conditions, and describe agents' beliefs. We also solve for the equilibrium in a benchmark case with common beliefs.

3.1. The model

The model is similar to a textbook New Keynesian model, with the novelty that the Fed and the market may disagree about future aggregate demand. We also assume that policy actions affect the economy with a lag. With transmission lags, belief disagreements about future aggregate demand matter because the market expects the Fed to make "mistakes." We start by describing the general environment and the equilibrium conditions. We then describe the aggregate demand process and belief disagreements.

We incorporate transmission lags by making the Fed set the policy rate in each period *before* the realization of an aggregate demand shock (a shock that determines spending and output within the period). Figure 3 illustrates the timeline of events. In the first phase, agents observe a public signal and draw their interpretations (as described subsequently), after which the Fed

[6] Starting with 2008, the Fed implemented large-scale asset purchases (LSAPs) that provide a substitute to conventional monetary policy. The LSAPs help explain why the correlation illustrated by Figure 2 does not apply in 2012 and 2013. During this period, there was broad agreement that the policy rate would remain near zero (the effective lower bound) but there was disagreement about the LSAPs, e.g., regarding the timing and the size of the Fed's "tapering" of asset purchases.

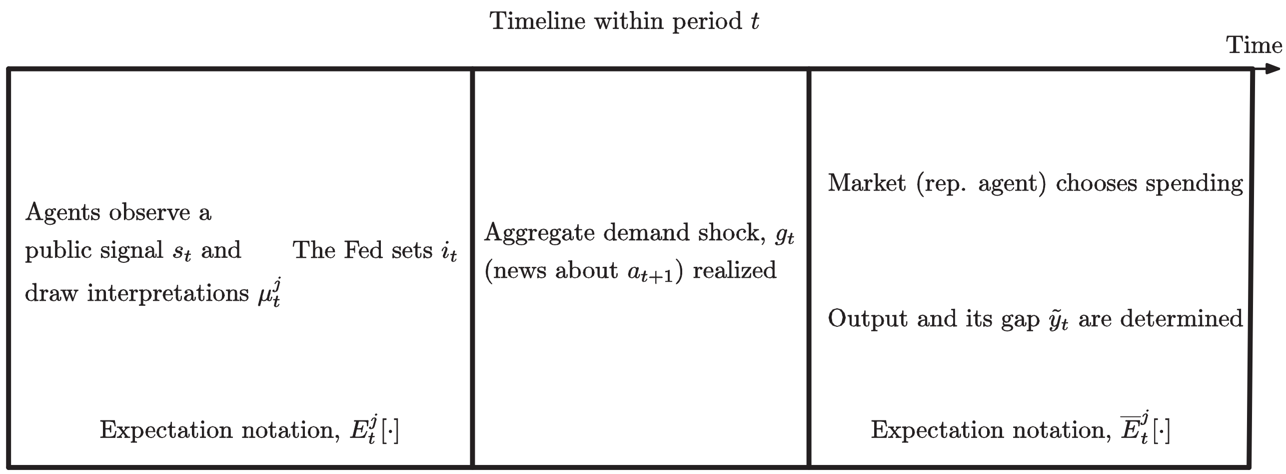

Figure 3: The timeline of events and the summary of the model.

sets the risk-free interest rate. Then, the aggregate demand shock is realized. Finally, in the last phase, the market chooses optimal allocations, markets clear, and the equilibrium level of output is determined. Throughout, we denote the Fed and the market with the superscript $j \in \{F, M\}$. We use $E_t^j[\cdot]$ to denote agent j's expectation in period t *before* the realization of the aggregate demand shock (in the first phase), and we use $\overline{E}_t^j[\cdot]$ to denote the corresponding updated beliefs *after* the realization of the shock (in the last phase).[7]

Preferences and technology. The economy is set in discrete time $t \in \{0, 1, ..\}$. The demand side features a representative household (*the market*) that maximizes utility in the last phase of each period,

$$\overline{E}_t^M \left[\sum_{h=0}^{\infty} \beta^h \left(\log C_{t+h} - \frac{N_{t+h}^{1+\eta}}{1+\eta} \right) \right].$$

The market observes the current aggregate demand shock (which we describe subsequently) and solves a standard problem that we relegate to Online Appendix A.

The supply side features a competitive final goods sector and monopolistically competitive

[7] Our policy-timing assumption might create the impression that the market has an informational advantage relative to the Fed. This "advantage" is a by-product of the fact that the market (the representative agent) *is* the economy, whereas the policymaker tries to influence the economy with lags. As noted by Greenspan (1995):

> Monetary policy acts with a lag... The oft-cited analogy to a barge travelling on a river is apt. To successfully navigate a bend in the river, the barge must begin the turn well before the bend is reached. Even so, currents are always changing, and even an experienced crew cannot foresee all the events that might occur as the river is being navigated.

We capture policy lags with a within-period sequence of actions, rather than the more explicit version where the Fed sets the interest rate for period t in period $t-1$. We do this to isolate our insights from commitment issues that arise with dynamic linkages. For instance, the Fed might want to set policy for period t partly to improve objectives in period $t-1$. Aside from this commitment issue, the two timing structures lead to a similar analysis.

intermediate goods firms that produce according to

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } Y_t(\nu) = A_t N_t(\nu)^{1-\alpha}.$$

If nominal prices were fully flexible, the equilibrium labor and output would be equal to their potential levels denoted by N^* and $Y_t^* = A_t (N^*)^{1-\alpha}$ (see Eq. ($A.12$) in the online appendix).

Nominal rigidities. We assume that a fraction of the intermediate goods firms have sticky nominal prices. We use the standard Calvo setup. In each period a randomly selected fraction of firms reset their nominal prices, whereas the remaining fraction leave their prices unchanged. For small aggregate demand shocks, this setup implies aggregate output is determined by aggregate demand, $Y_t = C_t$.

In Online Appendix A, we log-linearize the equilibrium around allocations that feature potential (flexible-price) real outcomes and zero nominal inflation. We show that our price setting assumption implies the New-Keynesian Phillips Curve (NKPC),

$$\pi_t = \kappa \tilde{y}_t + \beta \overline{E}_t^M [\pi_{t+1}], \tag{1}$$

where $\tilde{y}_t = \log(Y_t/Y_t^*)$ denotes the output gap relative to potential and $\pi_t = \log(P_t/P_{t-1})$ denotes inflation. The coefficient, κ, is a price flexibility parameter (see Eq. ($A.21$) in the online appendix).

Aggregate demand shocks. We capture aggregate demand shocks with *news about potential growth.* Formally, log productivity, $a_t = \log A_t$, follows the process

$$a_{t+1} = a_t + g_t,$$

where g_t denotes the growth rate of productivity between periods t and $t+1$, which is *realized in period* t. In particular, by the time the economy reaches period t, there is no uncertainty about the potential output of the economy in the current period: $a_t = a_{t-1} + g_{t-1}$, and g_{t-1} is already determined. However, there is uncertainty about potential growth between this period and the next period, g_t.

In the online appendix, we log-linearize the Euler equation for the market to obtain the IS equation,

$$\tilde{y}_t = -\left(i_t - \overline{E}_t^M [\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M [\tilde{y}_{t+1}], \tag{2}$$

where $i_t - \overline{E}_t^M [\pi_{t+1}]$ corresponds to the (market-expected) real interest rate and $\rho = -\log \beta$ is the discount rate. Eq. (2) illustrates that, for a given real interest rate, the equilibrium output gap increases one-to-one with the potential growth rate, g_t, as well as with the expected future output gap. Hence, we refer to g_t as the *aggregate demand* shock in period t.

Monetary policy. The interest rate is set by the monetary authority (the Fed). To capture policy transmission lags, the Fed sets the interest rate at the beginning of the period, *before* observing the aggregate demand shock for the current period. Otherwise, the Fed minimizes a standard objective function $E_t^F\left[\sum_{h=0}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2+\pi_{t+h}^2\right)\right]$, where γ denotes the weight on the output gap relative to inflation. We assume the Fed sets policy *without commitment*. We can then write the Fed's problem as

$$\min_{i_t} E_t^F\left[\gamma\tilde{y}_t^2+\pi_t^2\right]+E_t^F\left[V_{t+1}^F\right] \text{ where } V_{t+1}^F=\sum_{h=1}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2+\pi_{t+h}^2\right) \tag{3}$$

subject to (1) and (2). In the equilibria we will analyze, the Fed's expected continuation value, $E_t^F\left[V_{t+1}^F\right]$, will be exogenous to its interest rate decision in period t (because the model has no endogenous state variables). Therefore, the Fed effectively solves a sequence of static problems. In each period and state, it takes the future values of output gaps and inflation as given and sets the policy rate i_t to minimize the expected quadratic gaps in the current period.

3.2. Equilibrium conditions

Except for Section 7, we focus on the special case with fully sticky prices, $\kappa=0$. In this case, inflation is zero, $\pi_t=0$, and the Fed focuses on stabilizing current output. In particular, using (2) and (3) and assuming $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t}=0$ (which we will verify), the Fed's optimality condition is

$$E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right]=0, \text{ where } \frac{d\tilde{y}_t}{di_t}=-1+\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t}. \tag{4}$$

For most of our analysis, belief specifications are such that the policy rate has a constant impact on the market's expected future output gap: $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t}$ is constant. This is either because $\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$ does not depend on the current interest rate (Section 4), or because it has a constant slope with respect to the current interest rate (Section 5 and the benchmark case in Section 6). In these cases, the Fed's optimality condition simplifies to

$$E_t^F\left[\tilde{y}_t\right]=0. \tag{5}$$

The Fed targets a zero output gap *in expectation* and *according to its own belief.*

We can then combine Eqs. (2) and (5) to solve for the optimal interest rate as

$$i_t=\rho+E_t^F\left[g_t\right]+E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right]. \tag{6}$$

The Fed sets a higher interest rate when it expects greater aggregate demand, $E_t^F\left[g_t\right]$. More subtly, the Fed also sets a higher interest rate if it expects *the market* to be more optimistic about the *subsequent* output gap (higher $E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right]$). As illustrated by Eq. (2), the market's optimism about future output increases *current* output, and the Fed increases the interest rate to offset this effect. This mechanism plays an important role for our results.

Substituting Eq. (6) into Eq. (2), we solve for the equilibrium output gap as

$$\tilde{y}_t = g_t - E_t^F [g_t] + \overline{E}_t^M [\tilde{y}_{t+1}] - E_t^F \left[\overline{E}_t^M [\tilde{y}_{t+1}] \right]. \tag{7}$$

In equilibrium, the output gap depends on surprises relative to *the Fed's* expectations. The first two terms capture surprises to the aggregate demand shock, g_t. When aggregate demand is higher than the Fed expected when it set the interest rate, the output gap is higher. The last two terms capture the Fed's surprise about the market's expectation about the output gap in the next period. This second surprise will play no role until Section 6 (on tantrum shocks).

Finally, we also characterize risky asset prices along the equilibrium path. We focus on "the market portfolio," which we define as a financial asset (in zero net supply) whose payoff is equal to output in subsequent periods, $\{Y_{t+h}\}_{h \geq 1}$. In the online appendix, we show that the log (real) price of this asset satisfies

$$q_t = q^* + a_t + \tilde{y}_t, \tag{8}$$

where q^* is a constant. Under log-utility, the price of the market portfolio is proportional to output (see Eq. $(A.23)$). Therefore, this price moves either when productivity changes or when the output gap changes. In subsequent analysis, we focus on characterizing the output gap, $\tilde{y}_t$, and refer to Eq. (8) to describe the impact on asset prices.

Eqs. $(6-8)$ provide a generally applicable characterization of equilibrium (when prices are fully sticky and $\frac{d\overline{E}_t^M [\tilde{y}_{t+1}]}{di_t}$ is deterministic). We next specify the agents' beliefs and disagreements.

3.3. Aggregate demand process and belief disagreements

We focus on a setup in which disagreements emerge from *heterogeneous interpretations of public signals,* although our results generalize to other sources of disagreements (see Remark 1 at the end of the section). We start with our baseline model in which agents know each others' interpretations and beliefs (they agree to disagree).

Formally, aggregate demand follows

$$\begin{aligned} g_t &= \mathbf{g}_t + v_t, \text{ where } v_t \sim N\left(0, \sigma_v^2\right) \\ \mathbf{g}_t &= \mathbf{g}_{t-1} + \varepsilon_t \text{ and } \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right) \end{aligned} \tag{9}$$

The term v_t captures transitory demand shocks that are i.i.d. across periods. The term $\mathbf{g}_t$ captures a persistent component of demand, which can be interpreted as the underlying state of the economy. Agents do not observe $\mathbf{g}_t$ and therefore need to estimate it given the available information. We assume $\mathbf{g}_t$ follows a random walk , although we could allow for richer dynamics (see Remark 4). Henceforth, we refer to $\mathbf{g}_t$ as the *permanent* component of demand.

At the beginning of each period t, agents receive a *public* signal, s_t, that might be informative about $\mathbf{g}_t$. Similar to Sethi and Yildiz (2016), agents interpret this signal heterogeneously. Specifically, after observing the public signal, each agent $j \in \{F, M\}$ forms an idiosyncratic interpretation, μ_t^j. Given this interpretation, the agent believes the public signal is drawn from

$$s_t =^j \mathbf{g}_t - \mu_t^j + e_t \text{ where } e_t \sim N\left(0, \sigma_e^2\right).$$

The noise term, e_t, is i.i.d. across periods. The notation $=^j$ captures that the equality holds under agent j's belief. For now, we also assume agents observe each others' interpretations.

The upshot of these assumptions is that each agent j effectively receives an *interpreted* signal of the permanent component,

$$s_t + \mu_t^j =^j \mathbf{g}_t + e_t. \tag{10}$$

Moreover, agent j thinks the other agent (denoted by j') has a *garbled* version of her own interpreted signal,

$$s_t + \mu_t^{j'} =^j \mathbf{g}_t + e_t - \mu_t^j + \mu_t^{j'}. \tag{11}$$

Agents do *not* consider each others' idiosyncratic interpretations to be informative, conditional on their own interpretation. Consequently, when agents' interpretations differ, *they will form heterogeneous beliefs even if they had no prior disagreements.*

Finally, we assume that the agents' idiosyncratic interpretations are drawn from a joint Normal distribution that is i.i.d. across periods (and both agents know this distribution):

$$\mu_t^F, \mu_t^M \sim N\left(0, \sigma_\mu^2\right) \text{ and } corr\left(\mu_t^F, \mu_t^M\right) = \rho_\mu. \tag{12}$$

Here, ρ_μ captures the correlation between agents' interpretations. When $\rho_\mu = 1$, the interpretations are the same and there are no disagreements. When $\rho_\mu < 1$, there can be disagreements.

Recall that by the end of period t agents also observe the current period's demand realization. This provides them with additional information about the permanent component since $g_t = \mathbf{g}_t + v_t$. Agents are Bayesian given their own interpretations of signals. Therefore, they update their beliefs about the permanent component using a Kalman filter.

Kalman filtering of beliefs. Formally, let $\mathbf{g}_t^j = E_t^j[\mathbf{g}_t]$ denote the agent's conditional mean belief for the permanent component *before* the realization of g_t but after the realization of interpreted signals (see Figure 3). Let $\overline{\mathbf{g}}_t^j = \overline{E}_t^j[\mathbf{g}_t]$ denote the conditional mean belief *after* the realization of g_t. The following lemma describes the evolution of these conditional beliefs. We are mainly interested in the pre-shock belief, $\mathbf{g}_t^j$, but we also characterize the post-shock belief, $\overline{\mathbf{g}}_t^j$, because the Kalman filter for the post-shock belief is simpler (and easier to interpret).

Lemma 1. *Suppose sufficient time has passed that agents are in a learning steady state.* Before *and* after *observing g_t, agent j believes $\mathbf{g}_t \sim N\left(\mathbf{g}_t^j, \sigma_{\mathbf{g}}^2\right)$ and $\mathbf{g}_t \sim N\left(\overline{\mathbf{g}}_t^j, \sigma_{\overline{\mathbf{g}}}^2\right)$, respectively. The* *variance of the pre-shock belief solves* $\frac{1}{\sigma_{\mathbf{g}}^2} = \frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2}$*, where the variance of the post-shock belief is the unique positive solution to* $\frac{1}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}$*. The conditional mean of the pre- and post-shock beliefs evolve according to*

$$\mathbf{g}_t^j = \frac{\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2}\overline{\mathbf{g}}_{t-1}^j}{\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2}} + \frac{\frac{1}{\sigma_e^2}\left(s_t + \mu_t^j\right)}{\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2}}, \tag{13}$$

$$\overline{\mathbf{g}}_t^j = \frac{\left(\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2}\right)\mathbf{g}_t^j}{\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}} + \frac{\frac{1}{\sigma_v^2}g_t}{\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}}.$$

The conditional mean of the pre-shock belief satisfies the recursive equation

$$\mathbf{g}_t^j = \varphi \mathbf{g}_{t-1}^j + \omega^s \left(s_t + \mu_t^j\right) + \omega^g g_{t-1} \tag{14}$$

for appropriate coefficients $\varphi, \omega^s, \omega^g > 0$ *that sum to one* $\varphi + \omega^s + \omega^g = 1$*. The coefficient on the past belief,* $\varphi = \frac{\sigma_{\overline{\mathbf{g}}}^2}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} \in (0,1)$*, is increasing in the variance of the public signal,* σ_e^2*, and the variance of the transitory component,* σ_v^2*; and decreasing in the variance of the permanent component,* σ_ε^2*.*

The result follows from standard Kalman filtering techniques (see Online Appendix B.1). For a sketch proof, fix a period $t-1$ and suppose at the end of this period the agent has the prior belief $\mathbf{g}_{t-1} \sim N\left(\overline{\mathbf{g}}_{t-1}^j, \sigma_{\overline{\mathbf{g}}}^2\right)$. Then, the agent believes the permanent component of demand in the next period, $\mathbf{g}_t = \mathbf{g}_{t-1} + \varepsilon_t$, has the distribution $N\left(\overline{\mathbf{g}}_{t-1}^j, \sigma_{\overline{\mathbf{g}}}^2 + \sigma_\varepsilon^2\right)$. Starting with this prior, Eq. (13) describes the Bayesian posterior in two steps. The first equation incorporates the interpreted signal, $s_t + \mu_t^j =^j \mathbf{g}_t + e_t$, and describes the pre-shock belief. The second equation incorporates the demand shock, $g_t = \mathbf{g}_t + v_t$, and describes the post-shock belief. In a learning steady state, the precision of the post-shock belief is the same as in the last period, $\frac{1}{\sigma_{\overline{\mathbf{g}}}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2} = \frac{1}{\sigma_{\overline{\mathbf{g}}}^2}$.[8]

Eq. (14) combines the two equations in (13) to provide a recursive formulation for agents' pre-shock belief (our focus). The pre-shock belief is a weighted average of the most recent belief, the most recent interpreted signal, and the most recent demand shock. We refer to the weight on the most recent belief, $\varphi \in (0,1)$, as the *persistence* of beliefs. The last part of the lemma describes the comparative statics of φ. As expected, the agents hold more persistent beliefs for the permanent component when this component does not change much from period-to-period (low σ_ε^2) and when the signal or the demand shock are not very informative (high $\sigma_\varepsilon^2, \sigma_v^2$).

Recall that the equilibrium depends on the agents' conditional belief about aggregate demand, $E_t^j[g_t]$ (see Section 3.2). This belief is the same as the conditional pre-shock belief for

[8] In general, if the agents start with a prior belief with precision $\frac{1}{\sigma_{\overline{\mathbf{g}},t-1}^2}$, then they would have a posterior belief with precision $\frac{1}{\sigma_{\overline{\mathbf{g}},t}^2} = \frac{1}{\sigma_{\overline{\mathbf{g}},t-1}^2+\sigma_\varepsilon^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}$. The proof in the online appendix shows $\lim_{t\to\infty} \sigma_{\overline{\mathbf{g}},t}^2 = \sigma_{\overline{\mathbf{g}}}^2 > 0$. We assume sufficient time has passed so that the variance of the post-shock belief has converged to its steady-state level.

the permanent component, $E_t^j[g_t] = \mathbf{g}_t^j$ (because $g_t = \mathbf{g}_t + v_t$ and the transitory component has mean zero). We establish two additional properties of these pre-shock beliefs that facilitate the subsequent analysis. The first result describes the evolution of disagreements. The second result describes the higher order beliefs that matter for the equilibrium: in particular, the agents' expectations in period t about the conditional beliefs they will have in a future period.

Lemma 2. *Disagreements evolve according to*

$$\mathbf{g}_t^M - \mathbf{g}_t^F = \varphi\left(\mathbf{g}_{t-1}^M - \mathbf{g}_{t-1}^F\right) + \omega^s\left(\mu_t^M - \mu_t^F\right), \tag{15}$$

where $\mu_t^M - \mu_t^F \sim N\left(0, 2\left(1-\rho_\mu\right)\sigma_\mu^2\right)$ *(see (12)).*

Disagreements are somewhat persistent and follow an AR(1) process *according to either agent*. On average, disagreements decline over time either because agents update from the same demand shock, g_t, or from ex-ante unbiased interpretations of the same public signal, $s_t + \mu_t^j$. However, shocks to interpretation differences, $\mu_t^M - \mu_t^F$, regenerate disagreements.

Lemma 3. *Consider the (pre-shock) beliefs in period t about the conditional mean of the (pre-shock) beliefs in a subsequent period $t+h \geq t$. For each agent $j \in \{F, M\}$ and $j' \neq j$,*

$$E_t^j\left[\mathbf{g}_{t+h}^j\right] = \mathbf{g}_t^j, \tag{16}$$

$$E_t^j\left[\mathbf{g}_{t+h}^{j'}\right] = \varphi^h \mathbf{g}_t^{j'} + \left(1-\varphi^h\right)\mathbf{g}_t^j. \tag{17}$$

Each agent expects their own conditional belief about aggregate demand in a future period to be the same as their current belief. In contrast, each agent expects *the other agent's* conditional belief in a future period to be a *weighted average* of the other agent's current belief and their own current belief. The weights depend on the persistence of beliefs, φ. Intuitively, each agent expects the future data (the demand shocks and the interpreted signals) to be centered around her conditional belief. Therefore, the agent expects the other agent to learn from data and to come toward her own view. The expected speed of learning is decreasing in (the other agent's) belief persistence. This implication of learning will be important for our results.

Remark 1 (Other sources of disagreements)**.** *While we focus on disagreements driven by heterogeneous interpretations of public signals, our results are robust to the source of disagreement. The key ingredients we need are: (i) disagreements are somewhat persistent over time, as agents learn slowly, and (ii) each agent expects the other agent to learn from data and come toward her own view. These ingredients apply quite generally. In an earlier version of the paper, we assumed agents* start *with heterogeneous prior beliefs, perhaps because they received the news of a rare event (e.g., a financial crisis) and history does not provide enough guidance about how these events affect the economy. Our main results also hold in this alternative setup, but disagreements disappear over time, while they are regenerated in our current setup.*

3.4. Benchmark with common beliefs

We end this section by solving for the equilibrium in a benchmark scenario with no disagreement between the Fed and the market. Specifically, suppose $\rho_\mu = 1$ so that agents always have the same interpretation and hence the same conditional belief, $\mathbf{g}_t^F = \mathbf{g}_t^M \equiv \mathbf{g}_t^{com}$.

Since agents share the same belief and the Fed sets output gaps to zero in expectation [see (5)], Eqs. (6) and (7) imply

$$i_t = \rho + \mathbf{g}_t^{com}, \tag{18}$$

$$\tilde{y}_t = g_t - E_t^{com}[g_t] = g_t - \mathbf{g}_t^{com}. \tag{19}$$

With common beliefs, the market knows the Fed will, on average, stabilize *future* output gaps, $E_{t+1}^{com}[\tilde{y}_{t+1}] = 0$. Therefore, there are no perceived "mistakes" and the Fed sets an interest rate that reflects its expected aggregate demand. Naturally, surprises relative to the Fed's belief still shift the *current* output gap.

Next consider the expected future interest rates according to the market's and the Fed's beliefs, respectively. Using Eq. (18) and Lemma 3, we obtain

$$E_t^M[i_{t+h}] = E_t^F[i_{t+h}] = \rho + \mathbf{g}_t^{com} \quad \text{for } h \geq 0. \tag{20}$$

With common beliefs, the market's and the Fed's expected rates are the same and they reflect agents' *current* beliefs about aggregate demand.

Remark 2 (Forward interest rates)**.** *With a slight abuse of terminology, we will also refer to the market's expected future interest rates as "the forward rates." Consider the h-period-ahead (one-period) forward interest rate, $f_{t,t+h}$—the rate that an investor can obtain in period t for a (one-period) risk-free investment to be made in a future period, $t+h$. In our model, up to a log-linear approximation, the forward rate is equal to the market's expected interest rate in the future period, $f_{t,t+h} = E_t^M[i_{t+h}]$:* the expectations hypothesis holds under the market's belief. *In general, or without the log-linear approximation, the forward rate also contains a risk premium that could make it smaller or larger than the market's expectation for the future rates. We study the implications of disagreements for the forward-rate risk premium in a companion paper.*

4. Disagreements and optimal monetary policy

We next turn to disagreements. Our first result describes how disagreements affect the optimal interest rate and expected interest rates.

Proposition 1. *Consider the setup with interpretation differences, $\rho_\mu < 1$, so that conditional beliefs $\mathbf{g}_t^M$ and $\mathbf{g}_t^F$ are not necessarily the same (see Lemma 2).*

(i) The optimal interest rate and the corresponding equilibrium output gap are given by

$$i_t = \rho + (1-\varphi)\,\mathbf{g}_t^F + \varphi \mathbf{g}_t^M, \tag{21}$$
$$\tilde{y}_t = g_t - \mathbf{g}_t^F. \tag{22}$$

The optimal rate set by the Fed depends on a weighted average of the Fed's and the market's beliefs, with the weight on the market's belief given by the persistence of beliefs, φ.

(ii) The market's expected future rates (the forward rates) and the Fed's expected future rates in period t are given by

$$E_t^M[i_{t+h}] = \rho + \mathbf{g}_t^M + \varphi^h (1-\varphi)\left(\mathbf{g}_t^F - \mathbf{g}_t^M\right), \tag{23}$$
$$E_t^F[i_{t+h}] = \rho + \mathbf{g}_t^F + \varphi^{h+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \tag{24}$$

*Expected rates reflect the **corresponding agent's current belief** about aggregate demand, with an adjustment toward the other agent's belief that declines with the horizon. For sufficiently long horizons, expected rates reflect **only** the corresponding agent's belief, $\lim_{h\to\infty} E_t^j[i_{t+h}] = \rho + \mathbf{g}_t^j$, and the difference reflects the level of current disagreement, $\lim_{h\to\infty}\left(E_t^M[i_{t+h}] - E_t^F[i_{t+h}]\right) = \mathbf{g}_t^M - \mathbf{g}_t^F$.*

The first part of Proposition 1 characterizes the equilibrium outcomes. The output gap is similar to the benchmark with common beliefs [cf. (19)]. The interest rate is different and partly reflects the market's belief about aggregate demand. The Fed cannot set interest rates by focusing only on its own view of aggregate demand—it also needs to take into account the market's view and the extent of disagreement. Moreover, the more persistent is the disagreement (the higher is φ), the more the Fed ignores *its own* view.

The second part of Proposition 1 shows that, unlike in the benchmark case, *the market's and the Fed's expected future rates trace out different paths* [cf. (20)]. The market's expected rates reflect the market's belief for aggregate demand with an adjustment toward the Fed's belief—and vice versa for the Fed's expected rates. Therefore, disagreements about aggregate demand translate into disagreements about expected rates.

Sketch of proof. We sketch the proof of the proposition, since it helps develop intuition. We conjecture (and verify in Online Appendix B.2) an equilibrium in which the Fed's continuation value, $E_t^F\left[V_{t+1}^F\right]$, or the market's conditional belief about the subsequent output gap, $\overline{E}_t^M[\tilde{y}_{t+1}]$, do not depend on the current policy rate i_t. Thus, the baseline characterization in Section 3.2 applies. In addition, we conjecture that agents know $\overline{E}_t^M[\tilde{y}_{t+1}]$ before the realization of the demand shock for the current period, which implies $E_t^F\left[\overline{E}_t^M[\tilde{y}_{t+1}]\right] = \overline{E}_t^M[\tilde{y}_{t+1}]$. Eq. (7) then immediately implies (22). The Fed can still adjust the interest rate appropriately to hit its output target on average, according to its own belief. Disagreements manifest themselves in the *interest rate* that the Fed must set to achieve this outcome.

Next consider the IS curve (2) for the case without inflation,

$$\tilde{y}_t = -(i_t - \rho) + g_t + \overline{E}_t^M [\tilde{y}_{t+1}].$$

All terms except for g_t are determined before the realization of the demand shock. Taking the expectations according to each agent and using $E_t^F[\tilde{y}_t] = 0$, we obtain

$$E_t^M [\tilde{y}_t] - E_t^F [\tilde{y}_t] = E_t^M [\tilde{y}_t] = \mathbf{g}_t^M - \mathbf{g}_t^F. \tag{25}$$

The market does not expect the output gap to be zero since it thinks the Fed makes "mistakes." The extent of these "mistakes" depends on *disagreements*. For instance, when $\mathbf{g}_t^F > \mathbf{g}_t^M$, the market thinks the Fed is too optimistic about demand and therefore sets an interest rate that is too high, which will on average induce negative output gaps, $E_t^M[\tilde{y}_t] < 0$.

Using Eq. (25), we also obtain

$$\overline{E}_t^M [\tilde{y}_{t+1}] = \overline{E}_t^M \left[E_{t+1}^M [\tilde{y}_{t+1}]\right] = \overline{E}_t^M \left[\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right] = \varphi \left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \tag{26}$$

The second equality uses the law of iterated expectations and the last equality uses Lemma 2. *Expected* "mistakes" in the future, $\overline{E}_t^M[\tilde{y}_{t+1}]$, depend on current disagreements, $\mathbf{g}_t^M - \mathbf{g}_t^F$, as well as on their persistence, φ.

We next use (6) to solve for the optimal interest rate,

$$\begin{aligned} i_t &= \rho + \mathbf{g}_t^F + \overline{E}_t^M [\tilde{y}_{t+1}] \\ &= \rho + \mathbf{g}_t^F + \varphi \left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \end{aligned}$$

This proves Eq. (21). The anticipation of future "mistakes" affects *current* activity and induces the Fed to adjust the interest rate in the direction of the market's belief. For instance, when $\mathbf{g}_t^F > \mathbf{g}_t^M$, the market thinks the Fed will remain optimistic in the next period and induce a negative output gap, $\overline{E}_t^M[\tilde{y}_{t+1}] < 0$. This exerts downward pressure on the current output gap. Consequently, the Fed sets a lower interest rate than implied by its own (more optimistic) belief. The extent to which the Fed accommodates the market's belief depends on the persistence of disagreements, φ, because this determines the size of future "mistakes."

Finally, we establish the second part of Proposition 1. Consider the market's expected future interest rates. Taking the expectation of Eq. (21) under the market's belief, we obtain

$$E_t^M [i_{t+h}] = \rho + (1-\varphi) E_t^M \left[\mathbf{g}_{t+h}^F\right] + \varphi E_t^M \left[\mathbf{g}_{t+h}^M\right].$$

Substituting the higher order belief from Lemma 3 proves Eq. (23). For intuition, recall that the higher order belief, $E_t^M\left[\mathbf{g}_{t+h}^F\right]$, monotonically converges to $\mathbf{g}_t^M$ as the horizon h increases. The market expects the Fed to learn over time and to converge to the market's belief. Therefore, the market expects future interest rates to be determined by its current belief, $\mathbf{g}_t^M$. A symmetric argument proves Eq. (24).

Illustration. Figure 4 illustrates the results from Proposition 1 and provides further intuition. In each panel, the thin dashed line corresponds to the (overlapping) expected interest rates with a common baseline belief. The thin solid line shows the expected rates when the common belief becomes more optimistic. The thicker purple and blue lines show the Fed's and the market's expected rates, respectively, when one agent becomes more optimistic and the other agent remains with the more pessimistic baseline belief.

First consider the case in which the Fed becomes more optimistic. The top panels of Figure 4 illustrate that this shifts upward both the Fed's and the market's expected rates, but with a larger effect on the Fed's expected rates [see $(23-24)$]. This gap arises because the Fed expects the market to learn. Hence, over longer horizons, the Fed expects to set interest rates that reflect its optimism (whereas the market expects that the *Fed* will learn instead).

These panels also illustrate that the Fed raises the interest rate by *less* than the increase in its optimism would imply in isolation [see (21)]. For a complementary intuition, note that the market's expected future interest rates also increase—illustrated by the shaded area in the figure. Moreover, the market considers these increases a "mistake." These "mistakenly high" forward rates exert downward pressure on *current* output. Hence, even though the Fed has become more optimistic, it only needs to increase the current interest rate slightly to achieve its target output gap. In fact, the Fed can be thought of as targeting an overall increase in current and forward interest rates—the current rate hike plus the shaded area—that is just enough to counteract the increase in its current optimism. Consistent with this intuition, the Fed increases the interest rate by more when beliefs are less persistent. In this case, disagreements disappear faster and the market expects the interest rate hike to decline more quickly (see the top right panel of Figure 4).

Next consider the case in which the market becomes more optimistic. The bottom panels of Figure 4 show that this also shifts upward both the Fed's and the market's expected rates, but with a larger effect on the market's expected rates. In particular, the Fed raises the initial interest rate even though its own belief did not change. In this case, the market thinks the Fed is too pessimistic and will set interest rates too low in future periods—illustrated by the shaded area in the figure. These "mistakenly low" forward rates (together with the market's optimism) exert upward pressure on *current* output. Therefore, the Fed is forced to increase the interest rate to achieve its target output gap. In fact, the Fed can be thought of as hiking the current rate just enough to counteract the expected "shortfall" in the forward rates—the shaded area. Consistent with this intuition, the Fed increases the interest rate by less when beliefs are less persistent. In that case, disagreements disappear faster and the market expects the interest rate to catch up with its optimism more quickly (see the bottom right panel of Figure 4).

Remark 3 (Expected gradualism and initial overshooting)**.** *The top row of Figure 4 illustrates that our model features a novel form of expected gradualism in monetary policy. When the Fed*

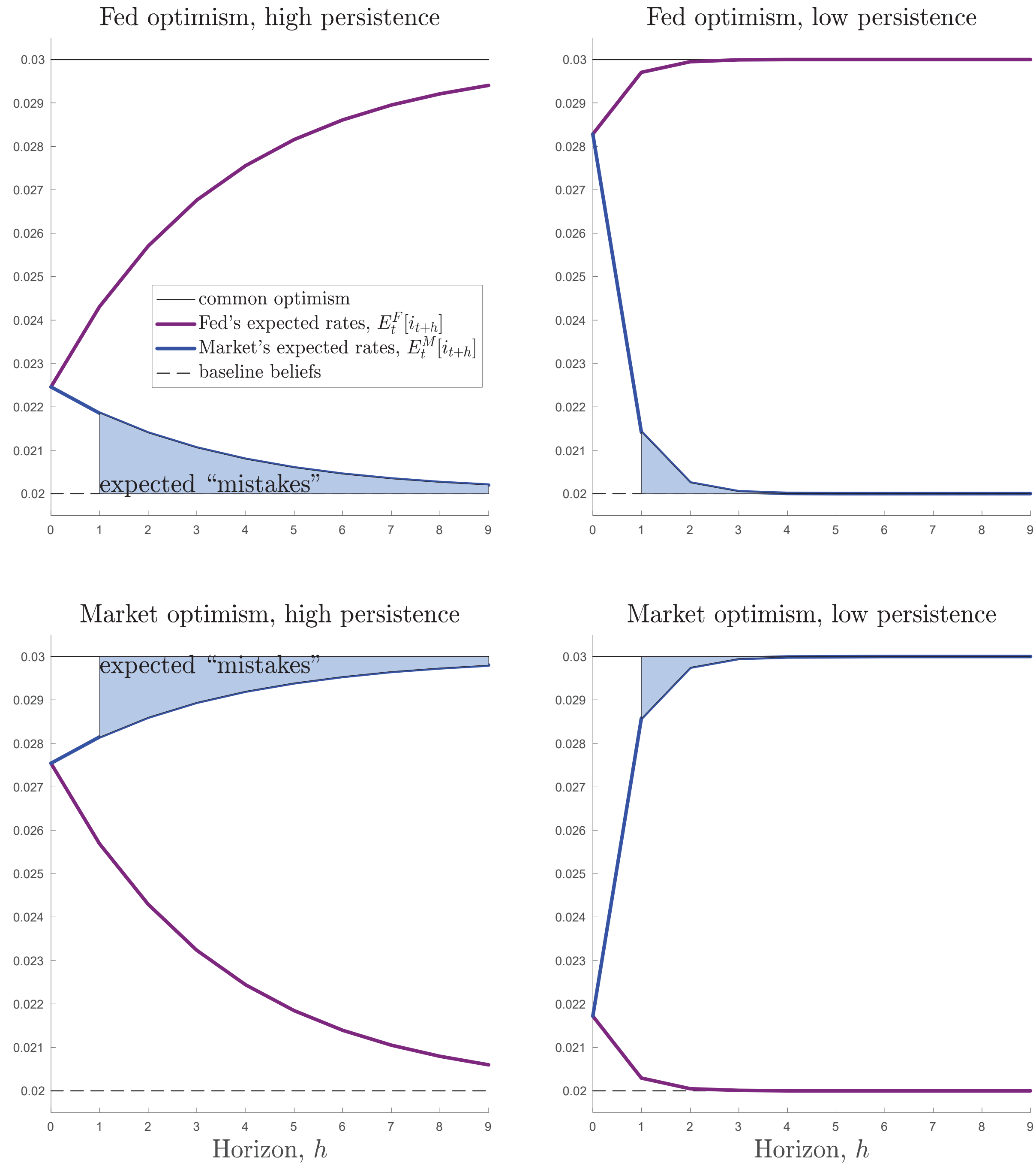

Figure 4: Top (resp. bottom) panels illustrate expected interest rates when the Fed (resp. the market) becomes more optimistic while the other agent remains with the baseline belief. Left (resp. right) panels feature a higher (resp. lower) persistence of beliefs, φ.

becomes more optimistic than the market, it chooses not to increase the policy rate by the full amount of its optimism (it partially accommodates the market's view), but it also expects to continue raising rates since it expects the data to sway the market toward the Fed's view over time (as reflected in the Fed's expected rates). On the other hand, the bottom row of Figure 4 highlights that our model can also feature initial overshooting *in monetary policy, followed by expected gradualism. Specifically, when the market becomes more optimistic than the Fed, the Fed initially raises the policy rate to partially accommodate the market's view, but it also expects to gradually undo the initial overshooting as the market learns from data.*

Remark 4 (Richer dynamics)**.** *Figure 4 shows that our model can explain the disagreements in interest rate predictions between the Fed and the market depicted in Figure 1. However, our model is not designed to capture the* slopes *of these predictions by forecast horizon. In our model, one agent's prediction is upward-sloping whereas the other agent's prediction is downward-sloping, whereas in practice the Fed's and the market's interest rate predictions tend to be either both upward or downward-sloping (see Figure 1). This discrepancy arises because we focus on fully persistent aggregate demand shocks (see* (9)*). We could generate other patterns for interest rate predictions by allowing for richer shock dynamics. For instance, consider a recovery scenario in which a negative shock is expected to mean revert. Under common beliefs, this scenario would lead to upward-sloping interest rate predictions. With disagreements, the scenario may still induce upward-sloping interest rate predictions according to each agent, but with a gap between the predictions similar to Figure 4.*

5. Fed belief surprises as "optimal" monetary policy shocks

In this section, we show that our setup generates microfounded monetary policy shocks. In our model, agents' disagreements change over time as agents heterogeneously interpret new public data. So far, we have assumed that the agents observe each others' interpretations and know each others' beliefs. In practice, while the Fed might observe the market's belief through asset prices (albeit with noise), it is harder for the market to observe the Fed's belief. We next consider the baseline model with the only difference that the market is uncertain about the Fed's belief. In this setup, the idiosyncratic part of the Fed's belief is naturally revealed to the market via a policy announcement or a communication. This revelation leads to microfounded monetary policy shocks that we refer to as *the Fed belief surprises.* While these surprises affect financial markets like textbook monetary policy shocks, they are *optimal* under the Fed's belief and have more subtle implications for subsequent economic outcomes. In particular, an interest rate hike driven by a Fed belief surprise is not *necessarily* followed by negative output gaps on average.

For concreteness, we focus on shocks driven by the Fed's policy announcements (we discuss the role of broader Fed communication at the end of this section). Recall that in the baseline model agents observe a public signal about the permanent component of demand, s_t, form interpretations of the signal, μ_t^F, μ_t^M, and observe each others' interpretations. Consider the same model with the only difference that *the market does not observe the Fed's interpretation,* μ_t^F. The Fed still observes the market's interpretation, μ_t^M. The rest of the model is the same.

Notice that the agents' conditional beliefs, $\mathbf{g}_t^F, \mathbf{g}_t^M$, still evolve according to Lemma 1. The difference is that the market does not know the Fed's conditional belief, $\mathbf{g}_t^F$—as it depends on the Fed's interpretations in the current and past periods that the market does not observe. Nonetheless, we conjecture an equilibrium that is *the same* as in Section 4. That is, the Fed sets the interest rate

$$i_t = \rho + (1-\varphi)\,\mathbf{g}_t^F + \varphi \mathbf{g}_t^M.$$

Note that this rate is a one-to-one function of the Fed's belief, $\mathbf{g}_t^F$. Therefore, after observing the interest rate, the market infers the Fed's belief as $\mathbf{G}_t^F(i_t) \equiv \frac{i_t-\rho-\varphi \mathbf{g}_t^M}{1-\varphi}$. Along the equilibrium path, the market's inference is correct, $\mathbf{G}_t^F(i_t) = \mathbf{g}_t^F$. Once the market learns $\mathbf{g}_t^F$ (and therefore μ_t^F), the analysis is the same as in Section 4. In Online Appendix B.3, we verify that it is optimal for the Fed to set the same policy rate as before and reveal its belief.

In this equilibrium, the *Fed belief surprise*—the revelation of the Fed's belief via the interest rate—affects *the market's* expected equilibrium outcomes. To characterize the impact, first consider the expected outcomes *before* the interest rate decision. Proposition 1 implies the market's expected interest rates are

$$E_t^M[i_{t+h}] = \rho + \left(1-\varphi^h(1-\varphi)\right)\mathbf{g}_t^M + \varphi^h(1-\varphi)\,E_t^M\left[\mathbf{g}_t^F\right]. \tag{27}$$

Likewise, we calculate the market's expected output gaps as

$$E_t^M[\tilde{y}_{t+h}] = E_t^M\left[g_{t+h} - \mathbf{g}_{t+h}^F\right] = E_t^M\left[\mathbf{g}_{t+h}^M - \mathbf{g}_{t+h}^F\right] = \varphi^h\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F\right]\right). \tag{28}$$

The first equality substitutes Eq. (22), the second equality uses the law of iterated expectations, and the last equality follows from Lemma 2. Before the policy decision, the market's expectations depend on *the market's ex-ante belief for the Fed's belief*, $E_t^M\left[\mathbf{g}_t^F\right]$.

Next consider the expected outcomes *after* the interest rate decision, denoted by $E_t^M[i_{t+h}|i_t]$ and $E_t^M[\tilde{y}_{t+h}|i_t]$. These are given by Eqs. $(27-28)$ with $E_t^M\left[\mathbf{g}_t^F\right]$ replaced by $\mathbf{g}_t^F$. After the policy decision, the market's expectations depend on *the Fed's actual belief*, $\mathbf{g}_t^F$.

Finally, using Eqs. (12) and (14) we calculate

$$\mathbf{g}_t^F - E_t^M\left[\mathbf{g}_t^F\right] = \omega^s \tilde{\mu}_t^F \text{ where } \tilde{\mu}_t^F = \mu_t^F - \rho_\mu \mu_t^M \sim N\left(0, \left(1-\rho_\mu^2\right)\sigma_\mu^2\right). \tag{29}$$

Here, $\tilde{\mu}_t^F$ is *the Fed's residual interpretation after controlling for the market's interpretation* (it is the residual term in a regression of μ_t^F on μ_t^M). Thus, the Fed belief surprises are driven by shocks to the Fed's residual interpretation. This leads to the following result.

Proposition 2. *Suppose the market does not observe the Fed's interpretation of the public signal, μ_t^F. Let Δx_t denote the surprise change of a variable in period t relative to its* ex-ante *expectation by the market. For instance,* $\Delta \mathbf{g}_t^F \equiv \mathbf{g}_t^F - E_t^M \left[\mathbf{g}_t^F\right]$ *denotes the Fed belief surprise and* $\Delta E_t^M \left[i_{t+h}\right] \equiv E_t^M \left[i_{t+h}|i_t\right] - E_t^M \left[i_{t+h}\right]$ *denotes the expected interest rate surprise under the market's belief.*

The equilibrium is the same as in Proposition 1. The Fed's interest rate announcement in each period t fully reveals its interpretation, μ_t^F*, and its conditional belief,* $\mathbf{g}_t^F$*. The Fed belief surprises depend on shocks to the Fed's residual interpretation according to* (29)*. A positive Fed belief surprise increases the current and forward interest rates,*

$$\frac{\Delta i_t}{\Delta \mathbf{g}_t^F} = 1 - \varphi \quad \text{and} \quad \frac{\Delta E_t^M \left[i_{t+h}\right]}{\Delta \mathbf{g}_t^F} = \varphi^h \left(1 - \varphi\right). \tag{30}$$

The surprise reduces the market's expectation for the output gap and the price of the market portfolio,

$$\frac{\Delta E_t^M \left[\tilde{y}_{t+h}\right]}{\Delta \mathbf{g}_t^F} = \frac{\Delta E_t^M \left[q_{t+h}\right]}{\Delta \mathbf{g}_t^F} = -\varphi^h. \tag{31}$$

Eq. (30) says that a Fed belief surprise affects the expected rates as we described previously (see Figure 4). Eq. (31) shows that the surprise reduces the market's expected output gap. After an interest hike, the market revises its view of monetary policy "mistakes" in the direction of higher interest rates. This also reduces the expected price of the market portfolio, which is a one-to-one function of the output gap [see (8)].

These results highlight that Fed belief surprises affect interest rates and (the market's) expected economic activity like the textbook monetary policy shocks—typically modeled as random fluctuations around an interest rate rule (see, e.g., Galí (2015)). Finding an empirical counterpart to these shocks is challenging and requires a structural interpretation.[9] Proposition 2 describes *microfounded* monetary policy shocks driven by the idiosyncratic component of the Fed's belief. In fact, these shocks are *"optimal"* under the Fed's belief, whereas they behave as monetary policy shocks under the market's belief.

Implications for subsequent outcomes and monetary policy shock regressions. While a Fed belief surprise generates conventional effects on *financial market outcomes* that depend on *the market's belief*, its implications for *subsequent economic outcomes* are more subtle. To fix ideas, consider an empirical regression of output gaps on interest rate shocks driven by Fed belief surprises. There is a large empirical literature that analyzes regressions along these lines. Our next result characterizes when these regressions recover the conventional (negative) coefficient.

To interpret regressions, we need to consider the DGP—the belief that will be reflected in

[9] For instance, Ramey (2016) notes: "Because monetary policy is typically guided by a rule, most movements in monetary policy instruments are due to the systematic component of monetary policy rather than to deviations from that rule. We do not have many good economic theories for what a structural monetary policy shock should be."

the data on average. Suppose under the DGP the public signal is drawn from

$$s_t + \mu_t^{DGP} = \mathbf{g}_t + e_t \text{ where } e_t \sim N\left(0, \sigma_e^2\right). \tag{32}$$

Here, μ_t^{DGP} is the actual interpretation of the signal [cf. (10)]. Suppose under the DGP this interpretation is related to the Fed's and the market's interpretations according to the multivariate regression,

$$\mu_t^{DGP} = \beta^F \mu_t^F + \beta^M \mu_t^M + \varepsilon_t^{DGP}. \tag{33}$$

Here, ε_t^{DGP} is a zero-mean random variable uncorrelated with μ_t^F, μ_t^M. Recall from (12) that μ_t^F, μ_t^M are i.i.d. random draws from a joint Normal distribution with zero mean and correlation ρ_μ.

With these assumptions, while agents' *expected* interpretations are unbiased under the DGP (μ_t^j, μ_t^{DGP} both have a zero mean), their *realized* interpretations can be biased to capture several possibilities. For instance, when $\beta^F = 1, \beta^M = 0$ and $\varepsilon_t^{DGP} = 0$, the Fed's belief is *always* correct. In this case, a shock to the Fed's residual interpretation (after controlling for the market's interpretation) is *always* unbiased under the DGP: it implies the actual interpretation received the same shock. Conversely, when $\beta^F = 0, \beta^M = 1$ and $\varepsilon_t^{DGP} = 0$, the market's belief is *always* correct. In this case, a shock to the Fed's residual interpretation is *always* too large under the DGP: it implies the actual interpretation received no shock. More generally, $\beta^F = 1$ implies that a shock to the Fed's residual interpretation is *on average* unbiased under the DGP, whereas $\beta^F < 1$ implies it is *on average* too large under the DGP.

Proposition 3. *Consider the setup in Proposition 2. Suppose under the DGP the correct interpretation μ_t^{DGP} is drawn from (33) and the signal is drawn from (32). Let $\beta^{DGP}(y,x) = \frac{cov^{DGP}(y,x)}{var^{DGP}(x)}$ denote the beta coefficient between two variables under the DGP. Then, we have*

$$\beta^{DGP}\left(\tilde{y}_{t+h}, \Delta i_t\right) = \frac{\varphi^h}{1-\varphi}\left(\beta^F - 1\right).$$

A regression of the output gap $\tilde{y}_{t+h}$ on the interest rate shock Δi_t produces the conventional (negative) coefficient if and only if a shock to the Fed's residual interpretation (after controlling for the market's interpretation) is on average too large under the DGP, $\beta^F < 1$.

For intuition, recall from Eq. (29) that the interest rate shock is driven by the Fed's residual interpretation. Recall also that the Fed sets the interest rate to achieve a zero expected output gap under its belief, $E_t^F[\tilde{y}_t] = 0$. If the Fed's residual interpretation shocks are on average unbiased under the DGP ($\beta^F = 1$), then the Fed's residual belief revisions on average imply equivalent revisions under the DGP. Therefore, a positive interest rate shock is on average followed by zero output gaps also under the DGP. The interest rate shock is followed by negative output gaps only if the Fed's residual interpretation shocks (and the resulting residual belief revisions) are on average too large under the DGP, $\beta^F < 1$. In this case, the magnitude of the

Fed's surprise interest rate change is also a "mistake" under the DGP.

The following corollary characterizes the regression coefficient for the special cases in which either the market or the Fed always has the correct belief.

Corollary 1. *If the market always has the correct belief,* $\beta^M = 1, \beta^F = 0, \varepsilon_t^{DGP} = 0$*, then* $\beta^{DGP}(\tilde{y}_{t+h}, \Delta i_t) = -\frac{\varphi^h}{1-\varphi} < 0$*. If instead the Fed always has the correct belief,* $\beta^F = 1, \beta^M = 0, \varepsilon_t^{DGP} = 0$*, then* $\beta^{DGP}(\tilde{y}_{t+h}, \Delta i_t) = 0$*.*

This result provides further intuition for why a Fed belief surprise shock *always* generates the conventional effects *under the market's belief* (see Proposition 2). Naturally, the market thinks its belief *is* the DGP. Therefore, the market thinks a shock to the Fed's residual interpretation is always too large, leading to excessive belief revisions and "mistaken" interest rate decisions. Conversely, the Fed thinks its belief *is* the DGP. So the Fed thinks its interpretation shocks are unbiased and its interest rate decisions are appropriate to stabilize the output gap.[10]

Remark 5 (Fed belief surprises driven by communication)**.** *For concreteness, we focused on the role of the policy rate announcements in conveying the news about the Fed's interpretation,* μ_t^F*. In practice, the Fed has many other mechanisms to communicate its views, such as the official statements or the policymakers' speeches. If credible, these communications will have effects and policy implications similar to those we have analyzed in this section.*

6. Tantrum shocks, gradualism, and communication

In this section, we investigate monetary policy shocks that are more damaging than in the previous section and derive their policy implications. A key feature of the analysis so far is that *the Fed knows how the market will react to its policy announcements.* This allows the Fed to shock the interest rates "optimally" and achieve its objective under its own belief, despite its disagreements with the market. We next consider a setup in which *the Fed is uncertain about how the market will react to its announcements.* This setup allows for *tantrum shocks*: policy announcements can trigger an overreaction of the forward rates (relative to what is ideal under the Fed's belief). Tantrum shocks are damaging in the sense that the Fed misses its output gap target *even under its own belief.* These shocks have two important policy implications. First, the fear of tantrum shocks induces the Fed to act *even more gradually* than in our baseline model. Second, communication policies between the Fed and the market can be useful to mitigate these tantrum shocks. To simplify the exposition, we relegate the formal results in this section to Online Appendix C, and focus on the key equations and intuitions.

To capture tantrum shocks, we extend the baseline model to make the market uncertain about the Fed's belief change *even after observing the current policy rate.* Fix a period t and suppose in (only) this period the Fed and the market can also disagree about the transitory

[10] In recent decades, the Fed and the market seem to have similar forecasting performance (see, e.g., Cieslak (2018); Bauer and Swanson (2020); Couture (2021)), which suggests neither of them always has the correct belief.

demand shock, v_t. Specifically, the Fed believes $v_t \sim N\left(\Delta \mathbf{v}_t^F, \sigma_v^2\right)$, whereas the market still believes $v_t \sim N\left(0, \sigma_v^2\right)$ [see (9)]. Here, $\Delta \mathbf{v}_t^F$ captures a change in the Fed's belief about the short-term component of demand. The Fed's belief about the long-term component of demand might also change due to the Fed's interpretation of the public signal in period t. As in the previous section, the market does not observe the Fed's interpretation.

These assumptions lead to a *signal extraction problem*: when the market sees a one-dimensional policy signal in period t, such as the policy rate, it does not know if the surprise in the signal reflects a short-term or a long-term belief change. The market reacts to the signal according to its prior belief. Suppose the market's prior is that the Fed's short-term belief change has mean zero and is drawn independently of all other variables, $\Delta \mathbf{v}_t^F \sim N\left(0, \sigma_{\mathbf{v}^F}^2\right)$. Recall also that the market thinks the Fed's long-term belief change, $\Delta \mathbf{g}_t^F = \mathbf{g}_t^F - E_t^M\left[\mathbf{g}_t^F\right]$, has mean zero and variance $(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2$ [see (29)]. Then, standard Bayesian techniques imply that the market interprets a one-dimensional belief signal according to the composite parameter, $\tau = \frac{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2}{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2+\sigma_{\mathbf{v}^F}^2}$ (see Online Appendix C). We refer to $\tau \in [0,1]$ as *the market's reaction type*. When $\tau = 1$, the market interprets the signal as a long-term belief change, as in the baseline model. When $\tau = 0$, the market interprets the signal as a short-term belief change. The key assumption of this section is that *the Fed does not know the market's reaction type* τ.

Benchmark when the Fed knows the market's reaction. As a benchmark, consider what happens if the Fed knows τ. Proposition 5 in Online Appendix C.1 shows that the optimal interest rate in period t is given by

$$\begin{aligned} i_t &= E_t^M\left[i_t\right] + (1-\tau\varphi)\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right), \\ \text{where } E_t^M\left[i_t\right] &= \rho + (1-\varphi) E_t^M\left[\mathbf{g}_t^F\right] + \varphi \mathbf{g}_t^M. \end{aligned} \tag{34}$$

$E_t^M\left[i_t\right]$ captures the predictable part of the policy rate (determined as in Proposition 1). When the Fed is more optimistic than the market predicts, $\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F > 0$, it adjusts the interest rate according to the market's reaction type, τ. If the market is reactive ($\tau = 1$), the Fed hikes the policy rate by a fraction of its optimism, as in the baseline model, because it expects the forward rates to also increase. When the market is unreactive ($\tau = 0$), the Fed hikes the policy rate by the full amount of its optimism because it does not expect the forward rates to increase. In either case, the Fed shocks the current and forward rates "optimally" and achieves a zero expected output gap under its belief as before.

Mechanics of tantrum shocks. We next turn to an extreme scenario in which the Fed underestimates τ, which is useful to illustrate the mechanics of tantrum shocks. Formally, suppose the Fed thinks the market is unreactive ($\tau = 0$), whereas the market is actually reactive ($\tau = 1$). Suppose also that the market thinks the Fed knows its type.

In this case, the Fed sets the interest rate according to (34) with $\tau = 0$, which implies

$$i_t = E_t^M [i_t] + \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F. \tag{35}$$

An optimistic Fed hikes the policy rate by the full amount of its optimism, because it anticipates that the forward rates will not react to the rate hike. However, since the market is reactive (and thinks the Fed knows this), it expects the Fed to set the policy rate according to (34) with $\tau = 1$. Therefore, after observing the interest rate in (35), the market extracts a *large* optimism signal that it attributes to a *long-term* belief change, $E_t^M \left[\mathbf{g}_t^F | i_t\right] - E_t^M \left[\mathbf{g}_t^F\right] = (\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F)/(1-\varphi)$. Consequently, the market's expected interest rates and output gaps change by a large amount (see Online Appendix C.2 for derivations),

$$\Delta E_t^M [i_{t+h}] = \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h (1-\varphi), \tag{36}$$

$$\Delta E_t^M [\tilde{y}_{t+h}] = -\frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h. \tag{37}$$

In fact, unlike in the baseline model, the Fed misses its output gap target in period t *even under its own belief,*[11]

$$E_t^F [\tilde{y}_t | \tau = 1] = -\frac{\varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)}{1-\varphi}. \tag{38}$$

Figure 5 illustrates these results. The interest rate hike raises the forward rates substantially more than the Fed anticipated when it set the policy rate (the top panel). This implies that the expected output gap becomes negative *even under the Fed's belief* (the bottom panel).

In this extreme case, the Fed operates under the assumption that the market is unreactive and will interpret its interest rate change as temporary. Thus, the Fed is (ex-post) surprised when the market is revealed to be reactive. While driven by extreme assumptions (in particular, the Fed is confused about the market's type), this case might be relevant for episodes in which the market reacts to a policy decision very differently than the Fed anticipated. One example is the 2013 "Taper Tantrum" episode, where the market seems to have interpreted a one-time policy tightening as the first installment of a tightening cycle.[12]

[11] Here, $E_t^F [\tilde{y}_t | \tau = 1]$ denotes the Fed's expected output gap for period t once it observes the market's actual type. For subsequent horizons the Fed expects to hit its target, $E_t^F [\tilde{y}_{t+h} | \tau = 1] = 0$ for $h \geq 1$, because the Fed does not have a short-term belief change in those periods (by assumption) and the equilibrium is the same as in the previous sections.

[12] On May 23, 2013, the day after Fed Chairman Bernanke's testimony to Congress that touched off the "Taper Tantrum" episode, the WSJ wrote: "...The next step by the Fed could be especially tricky. One worry at the central bank is that a single small step to shrink the size of the program could be interpreted by investors as the first in a larger move to end it altogether. Mr. Bernanke sought to dispel that view, part of a broader effort by Fed officials to manage market expectations. If the Fed takes one step to reduce the bond buying, it won't mean the Fed is 'automatically aiming towards a complete wind-down,' Mr. Bernanke said. 'Rather we would be looking beyond that to seeing how the economy evolves and we could either raise or lower our pace of purchases going forward. Again that is dependent on the data,' he said."

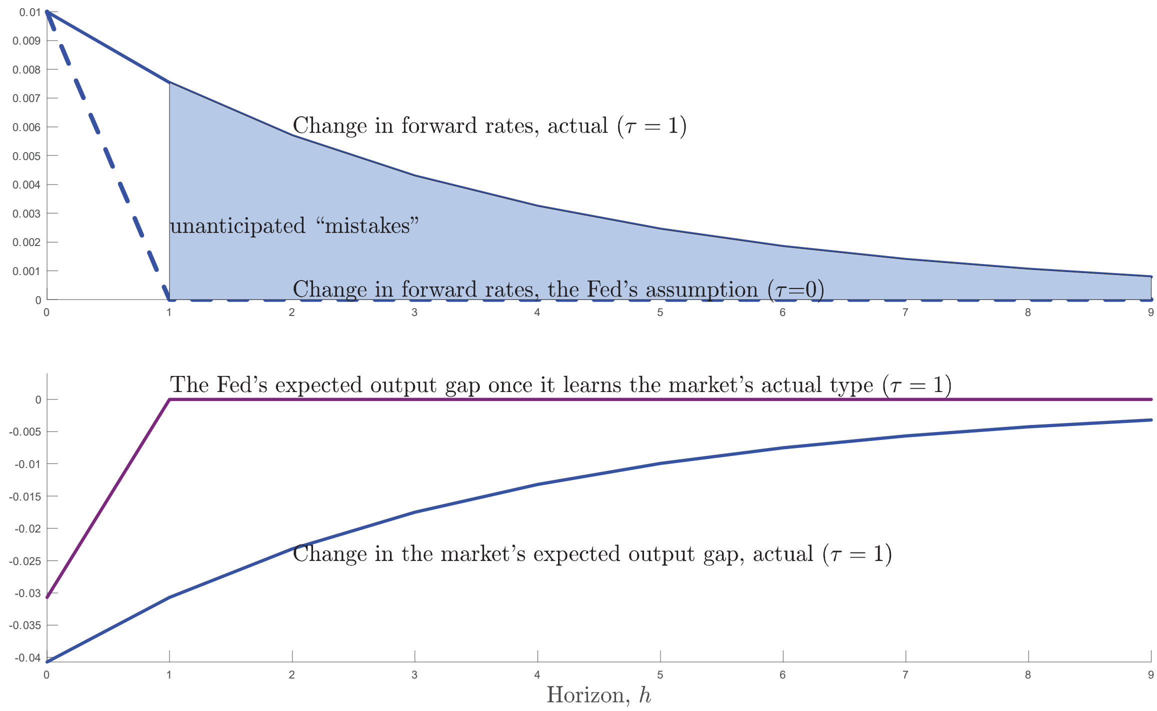

Figure 5: **Tantrum shock.** The Fed becomes more optimistic (by 1pp) and thinks the market is not reactive, $\tau = 0$, when the market is actually reactive, $\tau = 1$.

Policy implication of tantrums: Gradualism. We next investigate the policy implications of tantrum shocks. To analyze policy, we consider a less extreme scenario: the Fed does not make a mistake about the market's type but instead sets the policy rate under *uncertainty* about the market's type. The market knows that the Fed is uncertain (so neither agent has incorrect views). In this case, the equilibrium features milder tantrum shocks. Importantly, the anticipation of tantrum shocks induces the Fed to change interest rates more gradually than in our baseline model. As emphasized by Brainard (1967), since the Fed faces uncertainty about how a change in the policy rate will affect the economy, it prefers to behave more conservatively.

Formally, suppose the Fed believes (in period t) that the market has the reactive type, $\tau = 1$, with probability $\delta \in (0, 1)$, and the unreactive type, $\tau = 0$, with probability $1 - \delta$. The market knows δ. The rest of the model is as before. Proposition 6 in Online Appendix C.3 shows that the optimal interest rate in period t is given by the following analogue of Eq. (34),

$$i_t = E_t^M [i_t] + \left(1 - \tilde{\delta}\varphi\right) \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right) \text{ where } \tilde{\delta} > \delta = E_t^F [\tau] . \tag{39}$$

The parameter $\tilde{\delta} \in (0, 1)$ is the solution to a quadratic equation that we relegate to the online appendix. When the Fed is more optimistic than the market expected, $\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F > 0$, it hikes the interest rate according to a *weighted-average* over the cases in which the market is reactive and unreactive. However, the Fed *overweights* the case in which the market is reactive relative to its perceived prior probability of this case, $\tilde{\delta} > \delta$, which implies $1 - \tilde{\delta}\varphi < 1 - \delta\varphi$ where $\delta = E_t^F [\tau]$.

That is, *the Fed hikes the interest rate more gradually than in a "certainty-equivalent" benchmark in which the market's reaction type is certain and equal to the Fed's ex-ante expectation of the market's type* [cf. (34)].

Related, and unlike the cases we have analyzed so far, Eq. (7) does not apply: the Fed *does not hit its output target on average*. Instead, the Fed's *ex-ante* expected output gap satisfies

$$E_t^F\left[\tilde{y}_t\right] = \left(\tilde{\delta} - \delta\right)\varphi\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \tag{40}$$

An optimistic Fed (with $\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F > 0$) induces a positive output gap on average *under its own belief*.

Why does an optimistic Fed hike the interest rate more cautiously and induce a positive output gap on average? Intuitively, the Fed sacrifices the mean to reduce the variance of the output gap (recall that the Fed minimizes $E_t^F\left[\tilde{y}_t^2\right]$). A cautious rate hike reduces the variance because the Fed is uncertain about how its rate hike will affect the output gap. If the market is reactive, the rate hike increases the forward rates and has a large impact on the current output gap, $\frac{d\tilde{y}_t[\tau=1]}{di_t} < -1$. This is a milder version of the tantrum shock illustrated in Figure 5. If instead the market is unreactive, then the rate hike does not affect the forward rates and has a smaller impact on the current output gap, $\frac{d\tilde{y}_t[\tau=0]}{di_t} = -1$. Since the economy is more sensitive to a rate hike when the market is reactive, an optimistic Fed overweights that case in deciding how much to hike, $\tilde{\delta} > \delta$ [see Eq. (4)]. By acting conservatively, the Fed misses its output gap on average but it mitigates the tantrum shock that exacerbates its miss when the market is revealed to be reactive.

Policy implication of tantrums: Communication between the Fed and the market. The possibility of tantrum shocks increases the Fed's ex-ante expected gaps in (3). When the market is uncertain about the Fed's belief, its reaction type τ becomes a key parameter for policy. This creates a natural role for communication between the Fed and the market. First, the Fed can try to figure out the market's reaction type τ. Second, and perhaps more simply, the Fed can try to reveal its own belief to the market—making the market's reaction predictable as in the baseline model and therefore eliminating the tantrum shocks. In an early and insightful analysis, Blinder (1998) emphasized this mechanism as the key benefit of central bank communication:

> Greater openness might actually improve the efficiency of monetary policy... [because] expectations about future central bank behavior provide the essential link between short rates and long rates. A more open central bank... naturally conditions expectations by providing the markets with more information about its own view of the fundamental factors guiding monetary policy..., thereby creating a virtuous circle. By making itself more predictable to the markets, the central bank makes market reactions to monetary policy more predictable to itself. And that makes it possible to do a better job of managing the economy.

Proposition 7 in Online Appendix C.4 formalizes Blinder's insight. In our model with two belief types, the Fed can reveal its belief by announcing the average interest rate it expects to set in the next period in addition to the current rate. This result provides a rationale for the enhanced Fed communication that we have seen in recent years, e.g., "forward guidance" or "the dot curve". In our model, the role of these policies is *not* to persuade the market—the market is opinionated. Rather, communication is useful because it helps reveal the Fed's belief to the market, reducing the chance of tantrum shocks in which the market misinterprets the Fed's belief and overreacts to policy announcements.

7. Disagreements and inflation

So far, we have assumed nominal prices are fully sticky, $\kappa = 0$. In this section, we consider the case with partial price flexibility, $\kappa > 0$. We show that disagreements affect the market's expected inflation and create a policy trade-off for the Fed that reinforces our earlier findings. In particular, the Fed accommodates the market's belief more than in the earlier sections with fully sticky prices. We further show that, for optimal policy purposes, disagreements closely resemble the cost push shocks in a textbook New Keynesian model. For simplicity, we focus on the baseline setup from Section 3 in which agents know each others' beliefs and "agree-to-disagree."[13]

In this section, inflation is determined by the NKPC in (1), $\pi_t = \kappa \tilde{y}_t + \beta \overline{E}_t^M [\pi_{t+1}]$. Similar to Section 4, we conjecture an equilibrium in which the Fed and the market both know $\overline{E}_t^M [\pi_{t+1}]$ and $\overline{E}_t^M [\tilde{y}_{t+1}]$ *before* the realization of the demand shock in period t. This implies Eq. (25) still applies, $E_t^M [\tilde{y}_t] = E_t^F [\tilde{y}_t] + \mathbf{g}_t^M - \mathbf{g}_t^F$. Combining this expression with the NKPC (for period $t+1$), we obtain

$$
\begin{aligned}
E_{t+1}^M [\pi_{t+1}] &= E_{t+1}^F [\pi_{t+1}] + \kappa \left(E_{t+1}^M [\tilde{y}_{t+1}] - E_{t+1}^F [\tilde{y}_{t+1}] \right) \\
&= E_{t+1}^F [\pi_{t+1}] + \kappa \left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F \right). \qquad (41)
\end{aligned}
$$

That is, the market can expect inflation or disinflation, $E_{t+1}^M [\pi_{t+1}] \neq 0$, even if the Fed sets expected inflation to zero according to its own belief. For instance, when the market is more optimistic than the Fed, it expects positive output gaps ("too low interest rates") as in our earlier analysis. Expectations of positive output gaps translate into expected inflation.

[13] There is an important caveat for this section. Until now we have referred to the private sector as *the market*, because we think in practice financial market participants' expectations, rather than households' or firms' expectations, determine interest rates and asset prices (and households' consumption decisions are affected indirectly through asset prices). Instead, the new results in this section depend on the expectations of price setters, which in practice correspond to a different set of agents, e.g., firms, workers, or labor unions. We ignore this distinction and leave this dimension of heterogeneity for future work.

We next solve for the NKPC (for period t) and obtain

$$\begin{aligned}\pi_t &= \kappa\tilde{y}_t + \beta\overline{E}_t^M[\pi_{t+1}] \\ &= \kappa\tilde{y}_t + \beta\overline{E}_t^M\left[E_{t+1}^M[\pi_{t+1}]\right] \\ &= \kappa\tilde{y}_t + \beta\left(\overline{E}_t^M\left[E_{t+1}^F[\pi_{t+1}]\right] + \kappa\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)\right) \end{aligned} \tag{42}$$

The second line substitutes Eq. (41) along with Lemma 2. Since price setters are forward looking, expected inflation in the next period creates inflationary pressure in the current period as well, captured by $\beta\overline{E}_t^M[\pi_{t+1}]$. The extent of the inflationary pressure depends on *current* disagreements, $\mathbf{g}_t^M - \mathbf{g}_t^F$, and on the persistence of beliefs, φ. Consequently, when there are disagreements, the divine coincidence breaks down: the Fed cannot simultaneously set average inflation and output to zero (under its belief).

Next consider how the Fed trades off inflation and output. Similar to Section 4, we conjecture an equilibrium in which the Fed's continuation value, $E_t^F\left[V_{t+1}^F\right]$, or the market's conditional belief about the subsequent inflation or the output gap, $\overline{E}_t^M[\pi_{t+1}], \overline{E}_t^M[\tilde{y}_{t+1}]$, do not depend on the current policy rate i_t. Then, the Fed's problem (3) is effectively static and the optimality conditions imply

$$E_t^F[\pi_t] = \frac{\gamma}{\gamma+\kappa^2}\beta\overline{E}_t^M[\pi_{t+1}], \tag{43}$$

$$E_t^F[\tilde{y}_t] = -\frac{\kappa}{\gamma+\kappa^2}\beta\overline{E}_t^M[\pi_{t+1}]. \tag{44}$$

The Fed responds to the inflationary pressure by choosing a particular split between expected inflation and expected output. The Fed focuses on stabilizing inflation relatively more when it puts less weight on the output gap (smaller γ) and when nominal prices are more flexible (greater κ).

Eqs. (42) and (43) provide a recursive characterization for the Fed's expected inflation,

$$E_t^F[\pi_t] = \frac{\gamma}{\gamma+\kappa^2}\beta\left(\overline{E}_t^M\left[E_{t+1}^F[\pi_{t+1}]\right] + \kappa\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)\right).$$

We guess-and-verify that the solution is proportional to current disagreements, $E_t^F[\pi_t] = \Pi^F\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$ for an appropriate coefficient Π^F. Using this guess for period $t+1$ along with Lemma 2, we verify the guess for period t and obtain the solution,

$$E_t^F[\pi_t] = \Pi^F\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right), \text{ where } \Pi^F = \frac{\gamma\beta\varphi}{\gamma+\kappa^2-\gamma\beta\varphi}\kappa. \tag{45}$$

Using Eq. (41), we obtain a similar expression for the market's expected inflation,

$$E_t^M[\pi_t] = \Pi^M\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right), \text{ where } \Pi^M = \Pi^F + \kappa = \frac{\gamma+\kappa^2}{\gamma+\kappa^2-\gamma\beta\varphi}\kappa. \tag{46}$$

Using Lemma 2 once more, the market's expected inflation for the next period is given by,

$$\overline{E}_t^M[\pi_{t+1}] = \overline{E}_t^M\left[E_{t+1}^M[\pi_{t+1}]\right] = \Pi^M \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \tag{47}$$

Each agent believes expected inflation is proportional to current disagreements. When the market is more optimistic than the Fed, *both the Fed and the market expect positive inflation.*

Finally, we solve for the market's and the Fed's expected output gaps along the equilibrium path. Using Eqs. (44) and (47) along with $E_t^M[\tilde{y}_t] = E_t^F[\tilde{y}_t] + \mathbf{g}_t^M - \mathbf{g}_t^F$, we obtain

$$E_t^F[\tilde{y}_t] = \Gamma^F\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right), \text{ where } \Gamma^F = -\frac{\kappa^2\beta\varphi}{\gamma+\kappa^2-\gamma\beta\varphi} < 0, \tag{48}$$

$$E_t^M[\tilde{y}_t] = \Gamma^M\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right), \text{ where } \Gamma^M = \Gamma^F + 1 = \frac{\left(\gamma+\kappa^2\right)(1-\beta\varphi)}{\gamma+\kappa^2-\gamma\beta\varphi} > 0. \tag{49}$$

When the market is more optimistic than the Fed, *the Fed expects a negative output gap but the market expects a positive output gap* (consistent with the market expecting positive inflation). The following result summarizes this discussion (see Online Appendix B.4 for the proof).

Proposition 4. *Suppose prices are partially flexible,* $\kappa > 0$*. In equilibrium, the Fed's and the market's expected inflation are given by Eqs.* $(45-47)$ *and their expected output gaps are given by Eqs.* $(48-49)$*. When the market is more optimistic than the Fed,* $\mathbf{g}_t^M > \mathbf{g}_t^F$*, the market expects positive output gaps and future inflation,* $E_t^M[\tilde{y}_t] > 0, \overline{E}_t^M[\pi_{t+1}] > 0$*, and the Fed responds by inducing positive inflation and negative output gaps under its belief,* $E_t^F[\pi_t] > 0, E_t^F[\tilde{y}_t] < 0$*. Conversely, when the market is more pessimistic,* $\mathbf{g}_t^M < \mathbf{g}_t^F$*, the market expects negative output gaps and future disinflation,* $E_t^M[\tilde{y}_t] < 0, \overline{E}_t^M[\pi_{t+1}] < 0$*, and the Fed responds by inducing disinflation and positive output gaps under its belief,* $E_t^F[\pi_t] < 0, E_t^F[\tilde{y}_t] > 0$*.*

We next characterize the equilibrium interest rate and generalize our main result from Section 4. Taking the expectation of Eq. (2) under the Fed's belief, we obtain

$$r_t \equiv i_t - \overline{E}_t^M[\pi_{t+1}] = \rho + \mathbf{g}_t^F + \overline{E}_t^M[\tilde{y}_{t+1}] - E_t^F[\tilde{y}_t]. \tag{50}$$

Here, r_t denotes the *real* interest rate the Fed targets to achieve its desired output gap. The first three terms on the right side are similar to their counterparts with fully sticky prices [cf. (6)]. The last term is no longer necessarily zero and captures the Fed's concerns with stabilizing inflation. Combining this expression with Eqs. $(48-49)$ that characterize the output gaps, we obtain the following corollary.

Corollary 2. *The* real *interest rate corresponding to the equilibrium in Proposition 4 is*

$$r_t = \rho + (1-\tilde{\varphi})\,\mathbf{g}_t^F + \tilde{\varphi}\mathbf{g}_t^M, \text{ where } \tilde{\varphi} = \left(1 + \frac{\kappa^2\beta(1-\varphi)}{\gamma+\kappa^2-\gamma\beta\varphi}\right)\varphi.$$

The interest rate reflects the market's belief more than in the case with fully sticky prices, i.e.,

$\tilde{\varphi} \in (\varphi, 1)$.

For intuition, consider the case in which the market is more optimistic than the Fed. In this case, the market expects inflation, $\overline{E}_{t+1}^{M}[\pi_{t+1}] > 0$, and the Fed targets a negative output gap to respond to the inflationary pressure, $E_t^F[\tilde{y}_t] < 0$. This induces the Fed to set a higher real interest rate than in the baseline model—closer to the level implied by the market's optimistic belief. Conversely, when the market is pessimistic and expects disinflation, the Fed targets a positive output gap to fight the disinflationary pressure. This leads to a lower interest rate than before—closer to the level implied by the market's pessimistic belief.

This result reinforces our earlier analysis and provides a complementary reason for why the Fed needs to accommodate the market's belief. With fully sticky prices, the market's perceived monetary policy "mistakes" translate into expected future output gaps. This exerts pressure on the current output gap (via the IS curve) and forces the Fed's hand. With partially flexible prices, perceived "mistakes" translate into expected future inflation. This exerts pressure on the current inflation (via the NKPC) and forces the Fed's hand through a second channel.

Relationship to cost-push shocks. In the textbook New Keynesian model, the NKPC is usually augmented with cost-push shocks: a catchall term for factors other than output gaps (relative to an "effficient" benchmark) and inflation expectations that might affect firms' price setting. In our model, disagreements closely resemble cost-push shocks *from the Fed's perspective.* Therefore, our model inherits the optimal policy implications of cost-push shocks.

To illustrate this connection, note that along the equilibrium path we have $\overline{E}_t^M\left[E_{t+1}^F[\pi_{t+1}]\right] = \overline{E}_t^F\left[E_{t+1}^F[\pi_{t+1}]\right]$ (they are both equal to $\Pi^F\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$). Substituting this into Eq. (42), we obtain

$$\pi_t = \kappa\tilde{y}_t + \beta\overline{E}_t^F[\pi_{t+1}] + u_t \text{ where } u_t \equiv \beta\kappa\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

Therefore, the NKPC *under the Fed's belief* features an "as-if" cost-push shock—even though the actual NKPC (under the market's belief) features no such shock. The as-if cost-push shock is positive, $u_t > 0$ (resp. negative $u_t < 0$) when the market is more optimistic, $\mathbf{g}_t^M > \mathbf{g}_t^F$ (resp. more pessimistic, $\mathbf{g}_t^M < \mathbf{g}_t^F$). This provides a complementary intuition for Proposition 4.

Observe also that the as-if cost push shock follows an AR(1) process with persistence φ (see Lemma 2). This implies a tighter relationship between our model and the textbook model with cost-push shocks. In particular, the equilibrium is identical to a corresponding equilibrium analyzed by Clarida, Gali and Gertler (1999) with an appropriate as-if cost-push shock.

Corollary 3. *Consider the equilibrium characterized in Proposition 4. The Fed's expected output* *gap and inflation are given by*

$$E_t^F[\tilde{y}_t] = \frac{-\kappa}{\gamma + \kappa^2 - \gamma\varphi\beta} u_t,$$
$$E_t^F[\pi_t] = \frac{\gamma}{\gamma + \kappa^2 - \gamma\varphi\beta} u_t \text{ where } u_t = \beta\kappa\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

These expressions are the same as Eqs. (3.4) and (3.5) in Clarida, Gali and Gertler (1999) (after appropriately adjusting the notation).

This result implies that the optimal policy in our model shares some of the properties in Clarida, Gali and Gertler (1999). In particular, the Fed can benefit from committing to put a higher relative weight on inflation than implied by its own preferences in (3). Intuitively, since inflation is forward looking, committing to aggressively stabilize future inflation helps stabilize current inflation. We leave a more complete analysis of the benefits of commitment in our setting for future work.

8. Final remarks

Summary. We analyzed a macroeconomic model in which the Fed and the market have opinionated disagreements about future aggregate demand. The key feature of our environment is that the market expects the Fed to make "mistakes" (under the market's belief). The anticipation of future "mistakes" affects forward interest rates and current output, and these effects induce the Fed to partially accommodate the market's belief to stabilize the output gap. In particular, the optimal policy rate ("rstar") reflects the extent of disagreement and the persistence of beliefs. Partial price flexibility strengthens this result since perceived "mistakes" create inflationary or disinflationary pressures that induce the Fed to accommodate the market's belief by even more. The Fed plans to implement its own belief gradually, as it expects the market to learn over time and move closer to the Fed's belief. At times, when the market becomes more optimistic or more pessimistic than the Fed, the Fed initially overshoots interest rates in the direction of the market's view, but it also expects to reverse this overshooting gradually. The Fed and the market disagree about future interest rates, as in the data, because both agents expect the other agent to come to their own belief.

The model generates microfounded monetary policy shocks because disagreements change over time, as agents heterogeneously interpret new public data, and the market learns the idiosyncratic part of the Fed's belief from policy announcements. Fed belief surprises arise when the market learns the Fed has a different belief than it expected and revises its view of future "mistaken" interest rate changes. These surprises affect financial markets like textbook policy shocks, but their implications for subsequent economic activity depend on the DGP. If the Fed's residual interpretation of data (after controlling for the market's interpretation) is on average unbiased under the DGP, then a positive interest rate shock driven by a Fed belief surprise is followed by zero output gaps on average, despite the adverse reaction in financial markets. The model also allows for more damaging shocks when the Fed is uncertain about how the market will react to its policy announcements. Tantrum shocks arise when the market overreacts to the Fed's announcement, and they motivate additional gradualism and communication policies that reveal the Fed's belief. The goal of communication is not to persuade the market, but to reduce the likelihood of tantrum shocks in which the market misinterprets the Fed's belief.

Broader interpretations. While we focus on disagreements about aggregate demand, a broader interpretation of our model is that the Fed and the market *disagree about the interest rate required to close future output gaps*, i.e., about future "rstar." In practice, there are many reasons for disagreements about future "rstar" beyond the particular aggregate demand channel we emphasize. For example, disagreements about future risk premia or discount rates, or even disagreements about certain supply-side factors, could generate similar mechanisms.

Likewise, while we focus on disagreements about the state of the economy, rather than on the policy framework or objectives, our model can capture certain types of disagreements about "the policy reaction function." In our model, the optimal interest rate depends on expected *future* aggregate demand, and agents interpret differently how *current* macroeconomic data will translate into future demand. Thus, disagreements can stem from heterogeneous reactions to current data, such as the current output gap or current inflation, not because there is disagreement on the Fed's objectives or tactics, but because agents disagree on the implications of the current data for future activity. In particular, our model can naturally accommodate "the Fed's response to news" channel emphasized by the recent empirical work on the origins of monetary policy shocks (e.g., Bauer and Swanson (2020); Sastry (2019)).

Clarifications. The optimal policy we have characterized does *not* mean that the Fed should "surrender" to the market and avoid surprises at all costs. Instead, the optimal policy says that disagreements and surprises are normal, as long as the policy itself considers the effect of disagreement and surprises on output and inflation stabilization. Concretely, suppose that the Fed (but not the market) receives divine information that the long run "rstar" has risen by 100 basis points. If the market had received the same information or fully trusted the infinite wisdom (or divine connections) of the Fed, the optimal policy would be to immediately hike the target rate by 100bps. Instead, in our environment the market is opinionated, so the Fed knows that if it raises the rate by 100bps in one shot, it will trigger a much larger contraction in aggregate demand than it seeks. Thus, the Fed optimally raises the target rate by only 25bps today and it anticipates that it will continue raising rates by 25bps for three more meetings. This expected gradualism arises because the Fed expects the future data to confirm its belief. The Fed thinks that, by the next meeting, the market will update toward the Fed's belief and will expect smaller "mistakes" than it did after the previous hike, which will create more room for the Fed to raise rates in subsequent meetings. Rather than "surrendering" to the market, the Fed plans to implement its own belief more gradually.

While we illustrated the optimality of expected gradualism in a highly stylized model that abstracts from many realistic ingredients, the essence of the argument would extend to richer settings. For example, if there are dynamic linkages across periods, e.g., due to investment, a Fed that is optimistic about the permanent component of activity might need to engineer an even larger decrease in demand than in our (quasi-static) model, but the Fed would still need to hike the policy rate by a smaller amount than in the absence of disagreements. Similarly, if the transmission from monetary policy to real activity has substantial delays, then the Fed may need to frontload interest rate hikes (see Caballero and Simsek (2021a))—our insight in this context is that the Fed, again, would need to frontload the interest adjustment by less than in the absence of disagreements.

Empirical implications. Our analysis speaks to the large empirical literature that regresses output gaps or inflation on monetary policy shocks. For shocks driven by Fed belief surprises, these regressions will uncover the conventional (negative) coefficient if the Fed's residual belief revisions about future aggregate demand (after controlling for the market's belief revisions) are on average too large under the DGP. In particular, the coefficient is negative if the market has the correct belief, but it is zero if the Fed has the correct belief. For tantrum shocks, the coefficient can be negative under both agents' beliefs. While we do not test these empirical predictions, our results are in line with the empirical findings that monetary policy shocks seem to have a smaller effect on economic activity—and sometimes with flipped signs—after the mid-1980s (see, e.g., Boivin and Giannoni (2006); Barakchian and Crowe (2013); Ramey (2016)). One interpretation is that greater central bank transparency in recent years has made tantrum shocks rarer. It is also possible that the Fed's belief has become more accurate over time.

More broadly, our analysis is mostly qualitative, but it hints at several empirical questions worthy of future exploration. To what extent does monetary policy accommodate the market's beliefs? Does the degree of accommodation rise when these beliefs are more persistent? Are policies more gradual when the Fed and the market disagree (relative to situations in which they share the same view), or when the market response to policy becomes more unpredictable? Are contractionary monetary policy shocks more likely to occur when the Fed becomes more optimistic about demand (and the market does not share this change of view)? Do disagreements affect the market's (price setters') expected inflation and create inflationary pressures that are similar to cost-push shocks?

Finally, we focus on monetary policy but our analysis also speaks to the broader empirical literature on "economic policy uncertainty" (e.g., Baker, Bloom and Davis (2016)). While policy uncertainty seems large in practice, its origins are not fully understood. As a benchmark, imagine a benevolent policymaker that chooses a policy that maximizes a representative agent's expected utility. If the agent's preferences were common knowledge, and the policymaker and the agent held common beliefs, then the policymaker's choice would be predictable and there would be no policy uncertainty. The theoretical literature on policy uncertainty typically departs from this benchmark by introducing uncertainty about the policymaker's objectives (e.g., Pástor and Veronesi (2013)). We show that *disagreements* and *uncertainty about the policymaker's* *belief* provide another rationale for policy uncertainty. In our model, the policymaker (the Fed) maximizes a commonly-known objective function under its own belief, but the agent (the market) does not necessarily hold the same belief and does not necessarily know the policymaker's belief. The revelation of the policymaker's belief induces policy shocks that affect financial markets and economic activity. From an ex-ante perspective, the agent's uncertainty about the policymaker's belief induces economic policy uncertainty.[14] We leave the broader implications of this type of *belief-driven economic policy uncertainty* for future work.

[14] In the current paper, the market's ex-ante uncertainty about the Fed's belief does not play a central role, because the model is log-linearized. In a companion paper, we analyze a nonlinear version of the model and study how this type of *monetary policy uncertainty* affects the risk premia in financial markets.

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Online Appendix for: Monetary Policy with Opinionated Markets

Ricardo J. Caballero Alp Simsek

A. Model and the log-linearized equilibrium

In this appendix, we describe the details of the model and derive the log-linearized equilibrium conditions that we use in our analysis. The model and the analysis closely follows the textbook treatment in Galí (2015), except that the central bank sets the interest rate *before* observing the aggregate demand shock within the period (to capture monetary policy transmission lags in a simple way).

Representative household (the market). The economy is set in discrete time with periods $t \in \{0, 1, ...\}$. In each period, a representative household ("the market") makes consumption-savings and labor supply decisions. Formally, the market solves,

$$\max_{\{C_{t+h}, N_{t+h}\}_{h=0}^{\infty}} \overline{E}_t^M \left[\sum_{h=0}^{\infty} \beta^h \left(\log C_{t+h} - \frac{N_{t+h}^{1+\eta}}{1+\eta} \right) \right] \tag{A.1}$$

$$\text{s.t. } P_t C_t + \frac{B_{t+1}}{R_t^f} = B_t + W_t N_t + \int_0^1 \Pi_t(\nu)\, d\nu.$$

C_t denotes consumption, N_t denotes the labor supply, and η is the inverse labor supply elasticity. The market has log utility—we describe the role of this assumption subsequently. The expectations operator $E_t^M[\cdot]$ corresponds to the market's belief after the realization of the demand shock in period t (see Figure 3).

In the budget constraint, R_t^f denotes the gross risk-free nominal interest rate between periods t and $t+1$. The term B_t denotes the one-period risk-free bond holdings. In equilibrium, the risk-free asset is in zero net supply, $B_t = 0$. The term $\Pi_t(\nu)$ denotes the profits from intermediate good firms (that we describe subsequently). For simplicity, we do not allow households to trade the firms (in equilibrium, there would be no trade since this is a representative household).

The optimality conditions for problem $(A.1)$ are standard and given by,

$$\frac{W_t}{P_t} = \frac{N_t^{\eta}}{C_t^{-1}} \tag{A.2}$$

$$C_t^{-1} = \beta R_t^f \overline{E}_t^M \left[\frac{P_t}{P_{t+1}} C_{t+1}^{-1} \right]. \tag{A.3}$$

Final good firms. There is a competitive final good sector that combines intermediate inputs from a continuum of monopolistically competitive firms indexed by $\nu \in [0,1]$. The final good sector produces according to the technology,

$$Y_t = \left(\int_0^1 Y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{A.4}$$

This firm's optimality conditions imply a demand function for the intermediate good firms,

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t} \right)^{-\varepsilon} Y_t \tag{A.5}$$

$$\text{where } P_t = \left(\int_0^1 P_t(\nu)^{1-\varepsilon} d\nu \right)^{1/(1-\varepsilon)}. \tag{A.6}$$

P_t denotes the ideal price index.

Intermediate good firms. Each intermediate good firm produces according to the technology

$$Y_t(\nu) = A_t N_t(\nu)^{1-\alpha}. \tag{A.7}$$

Firms take the demand for their goods as given and set price to $P_t(\nu)$ to maximize the current market value of their profits, as we describe subsequently.

Market clearing conditions. The aggregate goods and labor market clearing conditions are given by,

$$Y_t = C_t \tag{A.8}$$

$$N_t = \int_0^1 N_t(\nu) d\nu. \tag{A.9}$$

Potential (flexible-price) outcomes. We start by characterizing a potential (flexible-price) benchmark around which we log-linearize the equilibrium conditions. In this benchmark, firms are symmetric and set the same price P_t^*. The optimal price solves [cf. $(A.5)$ and $(A.7)$]:

$$\max_{P_t^*, Y_t^*, N_t^*} P_t^* Y_t^* - W_t N_t^* \tag{A.10}$$
$$\text{s.t. } Y_t^* = A_t \left(N_t^*\right)^{1-\alpha} = \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} Y_t.$$

Y_t denotes the aggregate output that firms take as given. The solution is

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{(1-\alpha) A_t \left(N_t^*\right)^{-\alpha}}. \tag{A.11}$$

Hence, firms operate with a constant markup over their marginal cost. By symmetry, aggregate output is given by $Y_t = Y_t^* = A_t \left(N_t^*\right)^{1-\alpha}$. Combining these observations with Eqs. $(A.2)$ and $(A.8)$, we solve for the potential labor supply

$$N^* = \left(\frac{\varepsilon - 1}{\varepsilon} (1-\alpha)\right)^{1/(1+\eta)}. \tag{A.12}$$

Likewise, potential output is

$$Y_t^* = A_t \left(N^*\right)^{1-\alpha}. \tag{A.13}$$

Note that potential output is determined by current productivity and is independent of expectations about the future.

Nominal rigidities. We next describe the nominal rigidities. In each period, a randomly selected fraction, $1-\theta$, of firms reset their nominal prices. The firms that do not adjust their price in period t, set their labor input to meet the demand for their goods (since firms operate with a markup and we focus on small shocks). Consider the firms that adjust their price in period t. These firms' optimal price, P_t^{adj}, solves

$$\max_{P_t^{adj}} \sum_{h=0}^{\infty} \theta^h \overline{E}_t^M \left\{ M_{t,t+h} \left(Y_{t+h|t} P_t^{adj} - W_{t+h} N_{t+h|t} \right) \right\} \tag{A.14}$$
$$\text{where } Y_{t+h|t} = A_{t+h} N_{t+h|t}^{1-\alpha} = \left(\frac{P_t^{adj}}{P_{t+h}}\right)^{-\varepsilon} Y_{t+h}$$
$$\text{and } M_{t,t+h} = \beta^h \frac{1/C_{t+h}}{1/C_t} \frac{P_t}{P_{t+h}}.$$

The terms, $N_{t+h|t}, Y_{t+h|t}$, denote the input and the output of the firm (that resets its price in period t) in a future period $t+h$. The term, $M_{t,t+h}$, is the stochastic discount factor between periods t and $t+h$ (determined by the firm owners' preferences). Note that firms share the same belief as the representative household. The optimality condition is

$$\sum_{h=0}^{\infty} \theta^h \overline{E}_t^M \left\{ M_{t,t+h} P_{t+h}^{\varepsilon} Y_{t+h} \left(P_t^{adj} - \frac{\varepsilon}{\varepsilon - 1} \frac{W_{t+h}}{(1-\alpha) A_{t+h} N_{t+h|t}^{-\alpha}} \right) \right\} = 0 \tag{A.15}$$

$$\text{where } N_{t+h|t} = \left(\frac{P_t^{adj}}{P_{t+h}} \right)^{\frac{-\varepsilon}{1-\alpha}} \left(\frac{Y_{t+h}}{A_{t+h}} \right)^{\frac{1}{1-\alpha}}.$$

The New-Keynesian Phillips curve. We next combine Eq. $(A.15)$ with the remaining equilibrium conditions to derive the New-Keynesian Phillips curve. Specifically, we log-linearize the equilibrium around the allocation that features real potential outcomes and zero inflation, that is, $N_t = N^*, Y_t = Y_t^*$ and $P_t = P^*$ for each t. Throughout, we use the notation $\tilde{x}_t = \log(X_t / X_t^*)$ to denote the log-linearized version of the corresponding variable X_t. We also let $Z_t = \frac{W_t}{A_t P_t}$ denote the normalized (productivity-adjusted) real wage.

We first log-linearize Eq. $(A.2)$ (and use $Y_t = C_t$) to obtain

$$\tilde{z}_t = \eta \tilde{n}_t + \tilde{y}_t. \tag{A.16}$$

Log-linearizing Eqs. $(A.4 - A.7)$ and $(A.9)$, we also obtain

$$\tilde{y}_t = (1-\alpha) \tilde{n}_t. \tag{A.17}$$

Finally, we log-linearize Eq. $(A.15)$ to obtain

$$\sum_{h=0}^{\infty} (\theta\beta)^h \overline{E}_t^M \left\{ \tilde{p}_t^{adj} - \left(\tilde{z}_{t+h} + \alpha \tilde{n}_{t+h|t} + \tilde{p}_{t+h} \right) \right\} = 0, \tag{A.18}$$

$$\text{where } \tilde{n}_{t|t+h} = \frac{-\varepsilon \left(\tilde{p}_t^{adj} - \tilde{p}_{t+h} \right)}{1-\alpha} + \tilde{n}_{t+h}.$$

The second line uses $\tilde{y}_t = (1-\alpha) \tilde{n}_t$.

We next combine Eqs. $(A.16 - A.18)$ and rearrange terms to obtain a closed-form solution for the price set by adjusting firms

$$\begin{aligned} \tilde{p}_t^{adj} &= (1-\theta\beta) \sum_{h=0}^{\infty} (\theta\beta)^h \overline{E}_t^M \left[\Theta \tilde{y}_{t+h} + \tilde{p}_{t+h} \right], \\ \text{where } \Theta &= \frac{1+\eta}{1-\alpha+\alpha\varepsilon} \end{aligned}$$

Since the expression is recursive, we can also write it as a difference equation

$$\tilde{p}_t^{adj} = (1-\theta\beta)(\Theta \tilde{y}_t + \tilde{p}_t) + \theta\beta \overline{E}_t^M \left[\tilde{p}_{t+1}^{adj} \right]. \tag{A.19}$$

Here, we have used the law of iterated expectations, $\overline{E}_t^M[\cdot] = \overline{E}_t^M\left[\overline{E}_{t+1}^M[\cdot]\right]$.

Next, we consider the aggregate price index $(A.6)$

$$\begin{aligned} P_t &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon} + \int_{S_t} (P_{t-1}(\nu))^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)} \\ &= \left((1-\theta)\left(P_t^{adj}\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}, \end{aligned}$$

where we have used the observation that a fraction θ of prices are the same as in the last period. The term, S_t, denotes the set of sticky firms in period t, and the second line follows from the assumption that adjusting terms are randomly selected. Log-linearizing the equation, we further obtain $\tilde{p}_t = (1-\theta)\tilde{p}_t^{adj} + \theta\tilde{p}_{t-1}$. After substituting inflation, $\pi_t = \tilde{p}_t - \tilde{p}_{t-1}$, this implies

$$\pi_t = (1-\theta)\left(\tilde{p}_t^{adj} - \tilde{p}_{t-1}\right). \tag{A.20}$$

Hence, inflation is proportional to the price change by adjusting firms.

Finally, note that Eq. $(A.19)$ can be written in terms of the price change of adjusting firms as

$$\tilde{p}_t^{adj} - \tilde{p}_{t-1} = (1-\theta\beta)\Theta\tilde{y}_t + \tilde{p}_t - \tilde{p}_{t-1} + \theta\beta\overline{E}_t^M\left[\tilde{p}_{t+1}^{adj} - \tilde{p}_t\right].$$

Substituting $\pi_t = \tilde{p}_t - \tilde{p}_{t-1}$ and combining with Eq. $(A.20)$, we obtain the New-Keynesian Phillips curve (1) that we use in the main text

$$\begin{aligned} \pi_t &= \kappa\tilde{y}_t + \beta\overline{E}_t^M[\pi_{t+1}] \\ \text{where } \kappa &= \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1+\eta}{1-\alpha+\alpha\varepsilon}. \end{aligned} \tag{A.21}$$

Aggregate demand shocks. We focus on aggregate demand shocks, which we capture by assuming log productivity, a_{t+1}, follows the process

$$a_{t+1} = a_t + g_t. \tag{A.22}$$

g_t denotes the growth rate of productivity between periods t and $t+1$, which is realized in period t.

The IS curve. Finally, we log-linearize the Euler equation $(A.3)$ to obtain Eq. (2) in the main text,

$$\tilde{y}_t = -\left(i_t - \overline{E}_t^M[\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M[\tilde{y}_{t+1}].$$

$i_t = \log R_t^f$ denotes the nominal risk-free interest rate and $\rho = -\log\beta$ is the discount rate. We have used the market clearing condition, $Y_t = C_t$ [cf. Eq. $(A.8)$], the definition of the potential output, $Y_t^* = A_t(N^*)^{1-\alpha}$ [cf. $(A.13)$], and the evolution of productivity, $A_{t+1} = A_t e^{g_t}$ [cf.

$(A.22)$]. The equation illustrates g_t has a one-to-one effect on aggregate spending and output in period t. Hence, we refer to g_t as the *aggregate demand shock* in period t.

Monetary policy and equilibrium. To capture transmission lags, the Fed sets the interest rate at the beginning of the period, *before* observing the aggregate demand shock for the current period. Following much of the literature, we assume the Fed's objective function is given by $E_t^F\left[\sum_{h=0}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2+\pi_{t+h}^2\right)\right]$. Here, γ denotes the relative weight on the output gap. Recall also that we also assume the Fed sets policy *without commitment*. Thus, the Fed solves the problem in (3)

$$\min_{i_t} E_t^F\left[\gamma\tilde{y}_t^2+\pi_t^2\right]+E_t^F\left[V_{t+1}^F\right] \text{ where } V_{t+1}^F=\sum_{h=1}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2+\pi_{t+h}^2\right),$$
$$\text{s.t. (1) and (2)}.$$

As long as $E_t^F\left[V_{t+1}^F\right]$ is exogenous to the Fed's current policy decision, which will be the case for the equilibria we will analyze, the Fed effectively solves a sequence of static problems. This completes the equilibrium conditions.

Price of the market portfolio. For future reference, we also derive the equilibrium price of "the market portfolio." Specifically, in every period t, agents can also invest in a security in zero net supply whose payoff is proportional to output in subsequent periods, $\{Y_{t+h}\}_{h\geq 1}$. We let ω_t denote the market's holding of this security and modify the budget constraint as follows

$$P_tC_t+\frac{B_{t+1}}{R_t^f}+\omega_tP_tQ_t=B_t+\omega_{t-1}P_t\left(Y_t+Q_t\right)+W_tN_t+\int_0^1\Pi_t\left(\nu\right)d\nu.$$

Q_t denotes the *ex-dividend* and *real* price of this security (excluding the current dividends and adjusted for the nominal price level). Using the optimality condition for ω_t, we obtain

$$Q_t=\overline{E}_t^M\left[\beta\frac{C_{t+1}^{-1}}{C_t^{-1}}\left(Q_{t+1}+Y_{t+1}\right)\right].$$

Solving the equation forward, and using the transversality condition, we further obtain

$$Q_t=\overline{E}_t^M\left[\sum_{h\geq 1}\beta^h\frac{\left(C_{t+h}\right)^{-1}}{\left(C_t\right)^{-1}}Y_{t+h}\right].$$

After substituting $Y_t=C_t$ and simplifying, we find

$$Q_t=\frac{\beta}{1-\beta}Y_t. \tag{A.23}$$

Hence, in view of log utility, the equilibrium price of the market portfolio is proportional to output. Substituting $Y_t = \exp(\tilde{y}_t)\, Y_t^*$ and $Y_t^* = A_t\, (N^*)^{1-\alpha}$ and taking logs, we obtain Eq. (8) in the main text

$$q_t = q^* + a_t + \tilde{y}_t, \text{ where } q^* = \log\left(\frac{\beta}{1-\beta}\,(N^*)^{1-\alpha}\right).$$

In equilibrium, asset prices are proportional to output. Therefore, asset prices change either when productivity (a_t) changes or when the output gap ($\tilde{y}_t$) changes.

B. Omitted derivations

This appendix presents the derivations omitted from the main text.

B.1. Omitted derivations in Section 3.3

Proof of Lemma 1. We characterize agents' beliefs more generally, even when they might not have yet reached a learning steady state. We obtain the beliefs along the learning steady state by taking the limit of the variance of agents' beliefs as $t \to \infty$.

Fix a period $t-1$ and suppose that at the end of this period the agent has the prior belief $\mathbf{g}_{t-1} \sim N\left(\overline{\mathbf{g}}_{t-1}^{j}, \sigma_{\overline{\mathbf{g}},t-1}^{2}\right)$. Using $\mathbf{g}_t = \mathbf{g}_{t-1} + \varepsilon_t$, the agent's prior about $\mathbf{g}_{t-1}$ implies a prior about $\mathbf{g}_t$ given by $N\left(\overline{\mathbf{g}}_{t-1}^{j}, \sigma_{\overline{\mathbf{g}},t-1}^{2} + \sigma_{\varepsilon}^{2}\right)$. Note also that the agent has the following signals about the permanent component in period t:

$$s_t + \mu_t^j =^j \mathbf{g}_t + e_t$$

$$g_t = \mathbf{g}_t + v_t,$$

where recall that $=^j$ implies equality under agent j's belief. Combining these observations, the agent's pre-shock and post-shock beliefs in the next period are given by $\mathbf{g}_t \sim N\left(\mathbf{g}_t^{j}, \sigma_{\mathbf{g},t}^{2}\right)$ and $\mathbf{g}_t \sim N\left(\overline{\mathbf{g}}_t^{j}, \sigma_{\overline{\mathbf{g}},t}^{2}\right)$, where the variances satisfy

$$\begin{aligned} \frac{1}{\sigma_{\mathbf{g},t}^{2}} &= \frac{1}{\sigma_{\overline{\mathbf{g}},t-1}^{2} + \sigma_{\varepsilon}^{2}} + \frac{1}{\sigma_e^2} \\ \frac{1}{\sigma_{\overline{\mathbf{g}},t}^{2}} &= \frac{1}{\sigma_{\overline{\mathbf{g}},t-1}^{2} + \sigma_{\varepsilon}^{2}} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2}, \end{aligned} \tag{B.1}$$

and the conditional means satisfy

$$\mathbf{g}_t^j = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}\overline{\mathbf{g}}^j_{t-1}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}} + \frac{\frac{1}{\sigma^2_e}\left(s_t+\mu^j_t\right)}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}, \tag{B.2}$$

$$\overline{\mathbf{g}}_t^j = \frac{\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}\right)\mathbf{g}^j_t}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}} + \frac{\frac{1}{\sigma^2_v}g_t}{\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}}.$$

Next note that the second equation in $(B.1)$ implies

$$\frac{1}{\sigma^2_{\overline{\mathbf{g}},t}} = f\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}},t-1}}\right) \text{ where } f(x) = \frac{1}{\frac{1}{x}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}.$$

We have $\frac{d(f(x)-x)}{dx} = \frac{1}{(1+x\sigma^2_\varepsilon)^2}-1<0$ for each $x>0$. We also have $\lim_{x\to 0} f(x)-x>0$ and $\lim_{x\to\infty} f(x)-x<0$. These observations imply that $f(x)$ has a unique fixed point over the range $x>0$ denoted by $x^*>0$. Moreover, starting with any $x_0>0$, the sequence, $x_t = f(x_{t-1})$, converges to the unique fixed point, $\lim_{t\to\infty} x_t = x^*$. It follows that $\lim_{t\to\infty}\sigma^2_{\overline{\mathbf{g}},t} = \sigma^2_{\overline{\mathbf{g}}}>0$ where $\sigma^2_{\overline{\mathbf{g}}}$ is the unique positive solution to

$$\frac{1}{\sigma^2_{\overline{\mathbf{g}}}} = f\left(\frac{1}{\sigma^2_{\overline{\mathbf{g}}}}\right) = \frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}. \tag{B.3}$$

Taking the limit of the first equation in $(B.1)$, we also obtain $\lim_{t\to\infty}\sigma^2_{\mathbf{g},t} = \sigma^2_{\mathbf{g}}$ where $\frac{1}{\sigma^2_{\mathbf{g}}} = \frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}$.

Next suppose sufficient time has passed and agents have already reached a learning steady state in which the variances of their beliefs are constant, $\sigma_{\overline{\mathbf{g}},t} = \sigma_{\overline{\mathbf{g}}}$ and $\sigma_{\mathbf{g},t} = \sigma_{\mathbf{g}}$. Substituting these expressions into $(B.2)$, we establish the two equations in (13). Combining these equations, we also obtain Eq. (14) with coefficients

$$\varphi = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}} = \frac{\sigma^2_{\overline{\mathbf{g}}}}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon},$$

$$\omega^s = \frac{\frac{1}{\sigma^2_e}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}$$

$$\text{and } \omega^g = \frac{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}}\frac{\frac{1}{\sigma^2_v}}{\frac{1}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}+\frac{1}{\sigma^2_e}+\frac{1}{\sigma^2_v}}.$$

It remains to establish the comparative statics of the persistence of beliefs, $\varphi = \frac{\sigma^2_{\overline{\mathbf{g}}}}{\sigma^2_{\overline{\mathbf{g}}}+\sigma^2_\varepsilon}$.

Rewriting this expression, we obtain $\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1-\varphi}{\varphi}$. Note also that Eq. $(B.3)$ implies,

$$\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1}{\frac{\sigma_{\overline{\mathbf{g}}}^2}{\sigma_\varepsilon^2} + 1} + \sigma_\varepsilon^2 \left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2} \right).$$

After substituting $\frac{\sigma_\varepsilon^2}{\sigma_{\overline{\mathbf{g}}}^2} = \frac{1-\varphi}{\varphi}$ and rearranging terms, we characterize φ as the unique solution (in the range $(0,1)$) to:

$$\frac{(1-\varphi)^2}{\varphi} = \sigma_\varepsilon^2 \left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2} \right).$$

Since the left side is a decreasing function of φ, the solution is decreasing in σ_ε^2 and increasing in σ_v^2 and σ_e^2. This completes the proof. □

Proof of Lemma 2. Follows from Eq. (14). □

Proof of Lemma 3. Note that the identities trivially hold when $h = 0$. Suppose $h > 0$. First consider an agent's belief about their own belief in period $t+h$. For each agent j, we have

$$E_t^j \left[\mathbf{g}_{t+h}^j \right] = E_t^j \left[E_{t+h}^j \left[\mathbf{g}_{t+h} \right] \right] = E_t^j \left[\mathbf{g}_{t+h} \right] = E_t^j \left[\mathbf{g}_t + \sum_{\tilde{h}=1}^{h} \varepsilon_{t+\tilde{h}} \right] = E_t^j \left[\mathbf{g}_t \right] = \mathbf{g}_t^j.$$

The first and the last equalities substitute the definition of $\mathbf{g}_{t+h}^j$ and $\mathbf{g}_t^j$. The second equality applies the law of iterated expectations. The third equality substitutes the dynamics for the permanent component of demand from (9). The fourth equality uses the fact that $\varepsilon_{t+\tilde{h}}$ has zero mean for each period $t + \tilde{h}$. This proves Eq. (16).

Next consider an agent's belief about the other agent's belief in period $t+h$. For each agent j and $j' \neq j$, we have

$$E_t^j \left[\mathbf{g}_{t+h}^{j'} \right] = E_t^j \left[\mathbf{g}_{t+h}^j + \mathbf{g}_{t+h}^{j'} - \mathbf{g}_{t+h}^j \right] = \mathbf{g}_t^j + E_t^j \left[\mathbf{g}_{t+h}^{j'} - \mathbf{g}_{t+h}^j \right] = \mathbf{g}_t^j + \varphi^h \left(\mathbf{g}_t^{j'} - \mathbf{g}_t^j \right).$$

The first equality rewrites $\mathbf{g}_{t+h}^{j'}$, the second equality applies Eq. (16), and the third equality uses Lemma 2 (which implies that disagreements follow an AR(1) process according to each agent j). This implies Eq. (17) and completes the proof. □

B.2. Omitted derivations in Section 4

Proof of Proposition 1, part (i). Most of the proof is provided in the main text. It remains to verify the conjectures we have made for the Fed's expected continuation value, $E_t^F \left[V_{t+1}^F \right]$, and the market's expected output gap for the next period, $\overline{E}_t^M \left[\tilde{y}_{t+1} \right]$.

The Fed's expected continuation value is given by $E_t^F \left[V_{t+1}^F \right] = E_t^F \left[\sum_{h=1}^{\infty} \beta^h \left(\gamma \tilde{y}_{t+h}^2 \right) \right]$. In view of Eq. (22), future output gaps, $y_{t+h} = g_{t+h} - \mathbf{g}_{t+h}^F$, are exogenous to the current policy rate. This verifies our conjecture that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$. In particular, the Fed's problem (3) is effectively static and the optimality condition is given by (4).

Using (26), the market's expected output gap for the next period is given by $\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$. This verifies our conjectures that $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t} = 0$ and that agents know $\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$ before the realization of the demand shock for the period, completing the proof. □

Proof of Proposition 1, part (ii). The derivation of the market's expected interest rates is presented in the main text. Here, we derive the Fed's expected interest rates. Taking the expectation of Eq. (21) according to the Fed's belief, we obtain,

$$\begin{aligned} E_t^F\left[i_{t+h}\right] &= \rho + (1-\varphi) E_t^F\left[\mathbf{g}_{t+h}^F\right] + \varphi E_t^F\left[\mathbf{g}_{t+h}^M\right] \\ &= \rho + (1-\varphi)\mathbf{g}_t^F + \varphi\left(\varphi^h \mathbf{g}_t^M + \left(1-\varphi^h\right)\mathbf{g}_t^F\right) \\ &= \rho + \mathbf{g}_t^F + \varphi^{h+1}\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \end{aligned}$$

The second line uses Lemma 3. This proves Eq. (24). Taking the limit as $h \to \infty$, we also obtain $\lim_{h\to\infty} E_t^M\left[i_{t+h}\right] = \rho + \mathbf{g}_t^M$, completing the proof. □

B.3. Omitted derivations in Section 5

Proof of Proposition 2. We verify that it is optimal for the Fed to follow the interest rate rule in (21),

$$i_t = \rho + (1-\varphi)\mathbf{g}_t^F + \varphi \mathbf{g}_t^M.$$

Recall that, after seeing this interest rate, the market thinks the Fed's belief is

$$\mathbf{G}^F\left(i_t\right) \equiv \frac{i_t - \rho - \varphi \mathbf{g}_t^M}{1-\varphi}.$$

Along the equilibrium path, the market's inference is correct, $\mathbf{G}^F\left(i_t\right) = \mathbf{g}_t^F$.

Consider a continuation path in which the Fed sets an arbitrary policy rate i_t in period t and follows the interest rate rule in (21) starting period $t+1$ onward. Since the Fed reveals its belief in period $t+1$, the equilibrium in future periods is the same as in Section 4. In particular, as in the proof of Proposition 1, future output gaps are exogenous to the current policy rate. This verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$ and ensures the Fed's optimality condition is still given by (4).

After seeing the policy rate i_t, the market *thinks* the equilibrium in future periods will be the same as in Section 4 given the Fed's period-t belief, $\mathbf{G}^F\left(i_t\right)$. Therefore, the market's expected output gap in period $t+1$ depends on the current policy rate. In particular, using Eq. (26), we have

$$\overline{E}_t^M\left[\tilde{y}_{t+1}|i_t\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{G}^F\left(i_t\right)\right) \text{ where } \mathbf{G}^F\left(i_t\right) = \frac{i_t - \rho - \varphi \mathbf{g}_t^M}{1-\varphi}.$$

This implies $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}|i_t\right]}{di_t} = -\frac{\varphi}{1-\varphi}$. Substituting this into the Fed's optimality condition (4), we obtain $E_t^F\left[\left(1+\frac{\varphi}{1-\varphi}\right)\tilde{y}_t\right]=0$. The optimality condition simplifies to Eq. (5) as before, $E_t^F[\tilde{y}_t]=0$. Thus, the Fed's optimal interest rate is still given by Eq. (6)

$$\begin{aligned} i_t &= \rho + E_t^F[g_t] + E_t^F\left[\overline{E}_t^M[\tilde{y}_{t+1}|i_t]\right] \\ &= \rho + \mathbf{g}_t^F + \varphi\left(\mathbf{g}_t^M - \mathbf{G}^F(i_t)\right). \end{aligned}$$

The second line substitutes $\overline{E}_t^M[\tilde{y}_{t+1}|i_t]$ as well as the Fed's belief, $E_t^F[g_t]=\mathbf{g}_t^F$. Substituting the equilibrium condition, $\mathbf{G}^F(i_t)=\mathbf{g}_t^F$, we obtain the interest rate rule in (21). This verifies that it is optimal for the Fed to follow the interest rate rule.

Finally, note that Eq. (30) follows from considering Eq. (27) before and after the Fed's interest rate decision. To establish Eq. (31), first note that Eq. (8) implies

$$\Delta E_t^M[\tilde{q}_{t+h}] = \Delta E_t^M[q^* + a_{t+h} + \tilde{y}_{t+h}] = \Delta E_t^M[\tilde{y}_{t+h}].$$

Here, the last line follows because $\Delta E_t^M[a_{t+h}]=0$ (the Fed belief surprise does not change the market's expectation for future productivity). Eq. (31) then follows from considering Eq. (28) before and after the interest rate decision. This completes the proof. □

Proof of Proposition 3. Note that Lemma 2 also applies after replacing the market's belief with the DGP. Using this observation, we characterize the expected future output gap under the DGP

$$\begin{aligned} E_t^{DGP}[\tilde{y}_{t+h}] = E_t^{DGP}\left[g_{t+h} - \mathbf{g}_{t+h}^F\right] &= E_t^{DGP}\left[\mathbf{g}_{t+h}^{DGP} - \mathbf{g}_{t+h}^F\right] \\ &= \varphi^h\left(\mathbf{g}_t^{DGP} - \mathbf{g}_t^F\right) \\ &= \varphi^h\left[\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right) + \omega^s\left(\mu_t^{DGP} - \mu_t^F\right)\right]. \end{aligned}$$

The first equality uses Eq. (22), the second equality uses the law of iterated expectations, and the last two lines use Lemma 2. This implies the future output gap follows

$$\tilde{y}_{t+h} = \varphi^h\left[\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right) + \omega^s\left(\mu_t^{DGP} - \mu_t^F\right)\right] + \tilde{\varepsilon}_{t+h}.$$

Here, $\tilde{\varepsilon}_{t+h}$ is a random variable that has zero mean and is uncorrelated with all information available before the demand shock in period t, including $\mu_t^{DGP}, \mu_t^F, \mu_t^M$. On average, future output gaps depend on the *past* belief differences between the DGP and the Fed, $\varphi\left(\mathbf{g}_{t-1}^{DGP} - \mathbf{g}_{t-1}^F\right)$, and on the *current* interpretation differences between the DGP and the Fed, $\omega^s\left(\mu_t^{DGP} - \mu_t^F\right)$.

We next use Eqs. (29−30) to characterize the interest rate shock

$$\Delta i_t = (1-\varphi)\,\Delta\mathbf{g}_t^F = (1-\varphi)\,\omega^s\tilde{\mu}_t^F \text{ where } \tilde{\mu}_t^F = \mu_t^F - \rho_\mu\mu_t^M.$$

The interest rate shock depends on the Fed's residual interpretation after controlling for the market's interpretation.

Next, we combine the expressions for the future output gap and the interest rate shock to obtain:

$$\begin{aligned}\beta^{DGP}\left(\tilde{y}_{t+h},\Delta i_t\right) &= \frac{cov^{DGP}\left(\varphi^h\omega^s\left(\mu_t^{DGP}-\mu_t^F\right),\left(1-\varphi\right)\omega^s\tilde{\mu}_t^F\right)}{var^{DGP}\left(\left(1-\varphi\right)\omega^s\tilde{\mu}_t^F\right)} \\ &= \frac{\varphi^h}{1-\varphi}\frac{cov\left(\mu_t^{DGP}-\mu_t^F,\tilde{\mu}_t^F\right)}{var\left(\tilde{\mu}_t^F\right)}. \end{aligned} \tag{B.4}$$

The regression coefficient depends on the covariance between the interpretation differences between the DGP and the Fed, $\mu_t^{DGP}-\mu_t^F$, and the Fed's residual interpretation after controlling for the market's interpretation, $\tilde{\mu}_t^F$. To calculate this covariance, we rewrite Eq. (33) to obtain

$$\begin{aligned}\mu_t^{DGP}-\mu_t^F &= \left(\beta^F-1\right)\mu_t^F+\beta^M\mu_t^M+\varepsilon_t^{DGP} \\ &= \left(\beta^F-1\right)\left(\tilde{\mu}_t^F+\rho^M\mu_t^M\right)+\beta^M\mu_t^M+\varepsilon_t^{DGP} \\ &= \left(\beta^F-1\right)\tilde{\mu}_t^F+\left(\beta^M+\left(\beta^F-1\right)\rho^M\right)\mu_t^M+\varepsilon_t^{DGP} \end{aligned} \tag{B.5}$$

Here, the second line substitutes the Fed's residual interpretation from Eq. (29) and the last line rearranges terms. By construction, μ_t^M and ε_t^{DGP} are both uncorrelated with $\tilde{\mu}_t^F=\mu_t^F-\rho_\mu\mu_t^M$. Therefore, substituting $(B.5)$ into $(B.4)$, we obtain

$$\beta^{DGP}\left(\tilde{y}_{t+h},\Delta i_t\right)=\frac{\varphi^h}{1-\varphi}\frac{cov\left(\left(\beta^F-1\right)\tilde{\mu}_t^F,\tilde{\mu}_t^F\right)}{var\left(\tilde{\mu}_t^F\right)}=\frac{\varphi^h}{1-\varphi}\left(\beta^F-1\right).$$

This completes the proof. □

B.4. Omitted derivations in Section 7

Proof of Proposition 4. Most of the proof is provided in the main text. Here we complete the remaining steps. We first describe the processes for the output gap and inflation associated with the equilibrium characterized in the proposition.

Consider the IS equation (2)

$$\tilde{y}_t=-\left(i_t-\overline{E}_t^M\left[\pi_{t+1}\right]-\rho\right)+g_t+\overline{E}_t^M\left[\tilde{y}_{t+1}\right].$$

Recall our conjecture that the agents know $\overline{E}_t^M\left[\pi_{t+1}\right],\overline{E}_t^M\left[\tilde{y}_{t+1}\right]$ before the realization of the demand shock in period t. Then, the IS equation implies

$$\begin{aligned}\tilde{y}_t &= E_t^M\left[\tilde{y}_t\right]+g_t-\mathbf{g}_t^M \\ &= g_t-\mathbf{g}_t^M+\Gamma^M\left(\mathbf{g}_t^M-\mathbf{g}_t^F\right)\text{ where }\Gamma^M=\frac{\left(\gamma+\kappa^2\right)\left(1-\beta\varphi\right)}{\gamma+\kappa^2-\gamma\beta\varphi}. \end{aligned} \tag{B.6}$$

Here, the second line substitutes (49). This characterizes the process for the output gap. The first line also implies $E_t^F[\tilde{y}_t] = E_t^M[\tilde{y}_t] + \mathbf{g}_t^F - \mathbf{g}_t^M$, which verifies our conjecture that Eq. (25) still applies.

Next consider the NKPC (1)

$$\begin{aligned}\pi_t &= \kappa\tilde{y}_t + \kappa\overline{E}_t^M[\pi_{t+1}] \\ &= \kappa\tilde{y}_t + \kappa\beta\Pi^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \\ &= \kappa\left(g_t - \mathbf{g}_t^M\right) + \left(\kappa\Gamma^M + \beta\varphi\Pi^M\right)\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \text{ where } \Pi^M = \frac{\gamma+\kappa^2}{\gamma+\kappa^2-\gamma\beta\varphi}\kappa. \quad \text{(B.7)}\end{aligned}$$

Here, the second line substitutes (47) and the last line substitutes (B.6). This characterizes the process for inflation.

We next verify that the equilibrium satisfies the conjectures we have made for the Fed's expected continuation value, $E_t^F\left[V_{t+1}^F\right]$, and the market's expected inflation and output gap for the next period, $\overline{E}_t^M[\tilde{y}_{t+1}], \overline{E}_t^M[\pi_{t+1}]$.

The Fed's expected continuation value is given by $E_t^F\left[V_{t+1}^F\right] = E_t^F\left[\sum_{h=1}^{\infty}\beta^h\left(\gamma\tilde{y}_{t+h}^2 + \pi_{t+h}^2\right)\right]$. In view of Eqs. ($B.6 - B.7$), future output gaps and inflation, y_{t+h}, π_{t+h}, do not depend on the Fed's current policy rate. This verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$.

Using Eq. (47), the market's expected inflation in the next period is given by

$$\overline{E}_t^M[\pi_{t+1}] = \Pi^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right).$$

Likewise, using Eq. (49), the market's expected output in the next period is given by,

$$\overline{E}_t^M[\tilde{y}_{t+1}] = \overline{E}_t^M\left[E_{t+1}^M[\tilde{y}_{t+1}]\right] = \overline{E}_t^M\left[\Gamma^M\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right)\right] = \Gamma^M\varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right). \quad \text{(B.8)}$$

The last equality uses Lemma 2. These expressions verify our conjectures that $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = \frac{d\overline{E}_t^M[\pi_{t+1}]}{di_t} = 0$ and that agents know $\overline{E}_t^M[\pi_{t+1}], \overline{E}_t^M[\tilde{y}_{t+1}]$ before the realization of the demand shock in period t.

Finally, we derive the Fed's optimality conditions (43) and (44). Using (3), the Fed's problem is

$$\begin{aligned} &\min_{i_t} \gamma E_t^F\left[\tilde{y}_t^2\right] + E_t^F\left[\pi_t^2\right] + E_t^F\left[V_{t+1}^F\right] \\ \text{s.t.} \quad &\tilde{y}_t = -\left(i_t - \overline{E}_t^M[\pi_{t+1}] - \rho\right) + g_t + \overline{E}_t^M[\tilde{y}_{t+1}] \\ &\pi_t = \kappa\tilde{y}_t + \beta\overline{E}_t^M[\pi_{t+1}].\end{aligned}$$

Using $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = \frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = \frac{d\overline{E}_t^M[\pi_{t+1}]}{di_t} = 0$, the Fed's problem is effectively static and the optimality condition is given by

$$\gamma \frac{d\tilde{y}_t}{di_t} E_t^F [\tilde{y}_t] + \frac{d\pi_t}{di_t} E_t^F [\pi_t] = -\gamma E_t^F [\tilde{y}_t] - \kappa E_t^F [\pi_t] = 0.$$

Rearranging this expression, we obtain $E_t^F [\tilde{y}_t] = -\frac{\kappa}{\gamma} E_t^F [\pi_t]$. Substituting this into the NKPC under the Fed's belief (1), $E_t^F [\pi_t] = \kappa E_t^F [\tilde{y}_t] + \beta \overline{E}_t^M [\pi_{t+1}]$, we obtain Eqs. (43) and (44). This completes the characterization of equilibrium. □

Proof of Corollary 2. Substituting Eq. $(B.8)$ and (48) into Eq. (50), we obtain $r_t = \rho + (1 - \tilde{\varphi}) \mathbf{g}_t^F + \tilde{\varphi} \mathbf{g}_t^M$ where

$$\tilde{\varphi} = \Gamma^M \varphi - \Gamma^F = \left(\frac{\left(\gamma + \kappa^2\right)(1 - \beta\varphi) + \kappa^2 \beta}{\gamma + \kappa^2 - \gamma\beta\varphi} \right) \varphi = \left(1 + \frac{\kappa^2 \beta (1 - \varphi)}{\gamma + \kappa^2 - \gamma\beta\varphi} \right) \varphi.$$

Note that $\tilde{\varphi} > \varphi$. We also have $\tilde{\varphi} < 1$ since $\gamma + \kappa^2 - \gamma\beta\varphi > \kappa^2 \beta\varphi$. This establishes $\tilde{\varphi} \in (\varphi, 1)$ and completes the proof. □

Proof of Corollary 3. The expressions for $E_t^F [\tilde{y}_t]$ and $E_t^F [\pi_t]$ follow from Eqs. (48) and (45) after substituting $u_t = \beta\kappa\varphi \left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$. These expressions are the same as Eqs. (3.4) and (3.5) in Clarida, Gali and Gertler (1999); Galí (2015) after appropriately adjusting the notation: specifically, by setting $E_t^F [\tilde{y}_t] = x_t, E_t^F [\pi_t] = \pi_t, \kappa = \lambda, \gamma = \alpha$, and $\varphi = \rho$. □

C. Tantrum shocks, gradualism, communication

Section 6 extends the baseline model to analyze tantrum shocks and gradualism. In the main text we present only the key equations and the intuitions. In this appendix, we present the formal results omitted form the main text along with the proofs.

Recall that, to capture tantrums, we allow the market and the Fed to disagree about the short-term component of demand. Fix a period t and suppose in (only) this period the Fed and the market can disagree about the transitory demand shock, v_t. The Fed believes $v_t \sim N\left(\Delta \mathbf{v}_t^F, \sigma_v^2\right)$, whereas the market still believes $v_t \sim N\left(0, \sigma_v^2\right)$ [see (9)]. Here, $\Delta \mathbf{v}_t^F$ captures the Fed's belief change for the short-term component in period t. The market does not observe the Fed's belief change and thinks it has mean zero and is drawn independently of all other variables, $\Delta \mathbf{v}_t^F \sim N\left(0, \sigma_{\mathbf{v}^F}^2\right)$.

As before, the Fed and the market can also disagree about the long-term component due to different interpretations of the public signal. As in Section 5, the market does not observe the Fed's interpretation. Recall that at the beginning of period t the market thinks the Fed's long-term belief change, $\Delta \mathbf{g}_t^F = \mathbf{g}_t^F - E_t^M \left[\mathbf{g}_t^F\right]$, has mean zero and variance $(\omega^s)^2 \left(1 - \rho_\mu^2\right) \sigma_\mu^2$ [see (29)].

These assumptions create a signal extraction problem for the market. Unlike in Section 5, the interest rate does not fully reveal the Fed's belief, because the market is uncertain about both dimensions of the Fed's belief. Instead, we will establish that the market interprets a surprise interest rate change according to the parameter

$$\tau = \frac{(\omega^s)^2 \left(1-\rho_\mu^2\right) \sigma_\mu^2}{(\omega^s)^2 \left(1-\rho_\mu^2\right) \sigma_\mu^2 + \sigma_{\mathbf{v}^F}^2}. \tag{C.1}$$

We refer to $\tau \in [0,1]$ as *the market's reaction type*.

The key assumption of this appendix is that *the Fed does not know the market's reaction type* τ. We start with a benchmark case in which the Fed knows τ. We then analyze an extreme case in which the Fed underestimates τ, which is useful to illustrate the mechanics of tantrum shocks. We then consider a more common case in which the Fed is uncertain about τ, which is useful to analyze the policy implications of tantrum shocks. We show that the fear of tantrums induces the Fed to act more gradually than in the baseline model. Finally, we discuss how communication policies between the Fed and the market can help mitigate tantrums.

C.1. Benchmark when the Fed knows τ

Our next result characterizes the equilibrium for the benchmark case in which the Fed knows the market's reaction type.

Proposition 5. *Consider the setup in which in (only) period t the Fed believes the short-term component is distributed according to, $v_t \sim N\left(\Delta \mathbf{v}_t^F, \sigma_v^2\right)$. Suppose the market believes the Fed's short-term belief change is drawn from the distribution, $\Delta \mathbf{v}_t^F \sim N\left(0, \sigma_{\mathbf{v}^F}^2\right)$, and is independent of other random variables. Let $\tau \in (0,1)$ denote the market's reaction type given by (C.1). Suppose the Fed knows τ.*

In period t, the equilibrium interest rate is given by (34)

$$\begin{aligned} i_t &= E_t^M[i_t] + (1-\varphi\tau)\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right), \\ \text{where } E_t^M[i_t] &= \rho + (1-\varphi) E_t^M\left[\mathbf{g}_t^F\right] + \varphi \mathbf{g}_t^M. \end{aligned} \tag{C.2}$$

The market's posterior belief after observing the interest rate is given by

$$E_t^M\left[\mathbf{g}_t^F | i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau \frac{i_t - E_t^M[i_t]}{1-\varphi\tau}, \tag{C.3}$$

which is equal to $\tau\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$ along the equilibrium path. A surprise increase in Fed optimism in period t increases the current and the forward rates according to the market's reaction type:

$$\frac{\Delta i_t}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = 1 - \varphi\tau \text{ and } \frac{\Delta E_t^M[i_{t+h}]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = \tau\varphi^h(1-\varphi) \text{ for } h \geq 1. \tag{C.4}$$

The surprise reduces the market's expectation for the current and future gaps as follows:

$$\frac{\Delta E_t^M [\tilde{y}_t]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = -1 \text{ and } \frac{\Delta E_t^M [\tilde{y}_{t+h}]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = -\tau \varphi^h \text{ for } h \geq 1. \tag{C.5}$$

From period $t+1$ onward, the equilibrium is the same as in Proposition 2.

Eq. $(C.2)$ describes the Fed's *optimal* interest rate policy. The market's expected interest rate is determined as in Proposition 1. If the Fed is more optimistic than what the market expected, then it adjusts the interest rate according to the market's reaction type, τ. Notice that an optimistic Fed hikes the interest rate by the same amount, $(1-\varphi\tau)\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$, regardless of whether that optimism concerns the long-term or the short-term belief.

Eq. $(C.3)$ describes the market's posterior belief given the interest rate it observes. Along the equilibrium path, the interest rate is given by Eq. $(C.2)$. Thus, the market's posterior belief is *Bayesian* and given by $E_t^M\left[\mathbf{g}_t^F | i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$. Higher-than-expected interest rates reveal a bundled signal of Fed optimism, but not whether the optimism is short term or long term. The market interprets the signal according to its reaction type, $\tau = \frac{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2}{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2 + \sigma_{\mathbf{v}^F}^2}$. When τ is high, the market is more uncertain about the long-term belief and attributes high interest rates to long-term optimism.

Eq. $(C.4)$ describes how the current and the forward interest rates respond to a surprise increase in Fed optimism. The interest rate responses are determined by the market's reaction type—rather than by the Fed's actual belief type. When the market is reactive, $\tau = 1$, the responses are the same as in the baseline model with long-term optimism (see Proposition 2). When the market is unreactive, $\tau = 0$, the current rate increases substantially (by the amount of the Fed's optimism) but the forward rates remain unchanged.

Why does the market's reaction type drive the current and forward interest rates? Forward rates are naturally determined by the market's reaction, as these rates reflect the market's belief. The current rate is also determined by the market's reaction, because the Fed *optimally* responds to the market's reaction. As before, the Fed targets an overall increase in the current and forward interest rates that counteracts its initial optimism. When the market is more reactive, the forward rates increase by more and the Fed hikes the current interest rate by less—closer to the baseline case in which it has long-term optimism [cf. Figure 4].

Finally, Eq. $(C.5)$ describes how the market's expected output gaps respond to a surprise increase in Fed optimism. For the current period ($h = 0$), the market's expected output gap decreases one-to-one with the Fed's optimism, as in the baseline model (see Proposition 2). The reason is that the Fed targets an overall change in the current and forward rates that exactly counteracts its optimism. Therefore, a surprise increase in Fed optimism reduces the market's expectation of the current output gap one-to-one (and it leaves the Fed's expectation of the current output gap unchanged). For future periods ($h > 1$), the market's expected output gap responds according to its reaction type. When the market is reactive, $\tau = 1$, it expects a decline also in future output gaps. When the market is unreactive, $\tau = 0$, it does not expect a decline in future output gaps.

Proof of Proposition 5. First consider the equilibrium from period $t+1$ onward. Since the Fed does not have a short-term belief change in these periods, the equilibrium is the same as in Proposition 2. In particular, in subsequent periods the Fed's interest rate decision fully reveals its belief. This also verifies that $\frac{dE_t^F\left[V_{t+1}^F\right]}{di_t} = 0$. As before, the Fed's problem (3) in period t is effectively static and the optimality condition is given by (4).

Next consider the equilibrium in period t. We conjecture that the interest rate rule in Eq. $(C.2)$ is optimal for the Fed and the belief updating rule in Eq. $(C.3)$ is Bayesian for the market along the equilibrium path.

Consider the Fed's optimal interest rate decision in period t. As before, this depends on the market's expected output gap for the next period $t+1$. The output gap in period $t+1$ is still given by Eq. (22), $\tilde{y}_{t+1} = g_{t+1} - \mathbf{g}_{t+1}^F$. Combining this with Lemma 2, the market's expected output gap in period t *after* the interest rate decision is given by

$$\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \overline{E}_t^M\left[\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right] = \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t\right]\right), \tag{C.6}$$
$$\text{where } E_t^M\left[\mathbf{g}_t^F|i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\frac{i_t - E_t^M\left[i_t\right]}{1-\varphi\tau}.$$

The second line substitutes the market's belief updating rule from $(C.3)$.

Eq. $(C.6)$ implies $\frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t} = -\frac{\varphi\tau}{1-\varphi\tau}$. Substituting this into the Fed's optimality condition (4), we obtain $E_t^F\left[\left(1+\frac{\varphi\tau}{1-\varphi\tau}\right)\tilde{y}_t\right] = 0$. Since $\frac{\varphi\tau}{1-\varphi\tau}$ is constant, this simplifies to Eq. (5) as before, $E_t^F\left[\tilde{y}_t\right] = 0$. Thus, the Fed's optimal interest rate is still characterized by Eq. (6)

$$\begin{aligned} i_t &= \rho + E_t^F\left[g_t\right] + E_t^F\left[\overline{E}_t^M\left[\tilde{y}_{t+1}\right]\right] \\ &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t\right]\right) \\ &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\left(\mathbf{g}_t^M - \left(E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)\right)\right) \\ &= E_t^M\left[i_t\right] + (1-\varphi\tau)\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

The second line substitutes Eq. $(C.6)$ along with the Fed's belief for period t, $E_t^F\left[g_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F$. The third line substitutes $E_t^M\left[\mathbf{g}_t^F|i_t\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$, which holds along the equilibrium path. The last line simplifies the expression. This proves that the interest rate rule in Eq. $(C.2)$ is optimal for the Fed.

Next consider the market's belief updating rule in period t. Along the equilibrium path, the interest rate policy in $(C.2)$ provides the market with an imperfect signal of the Fed's long-term belief change,

$$\frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau} = \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F \sim N\left(\Delta\mathbf{g}_t^F, \sigma_{\mathbf{v}^F}^2\right).$$

The market combines the signal with its prior belief, $\Delta \mathbf{g}_t^F \sim N\left(0, (\omega^s)^2 \left(1-\rho_\mu^2\right) \sigma_\mu^2\right)$ [see (29)]. The Bayesian posterior is then given by,

$$E_t^M\left[\Delta \mathbf{g}_t^F | i_t\right] = \frac{\frac{1}{\sigma_{\mathbf{v}^F}^2}}{\frac{1}{(\omega^s)^2\left(1-\rho_\mu^2\right)\sigma_\mu^2} + \frac{1}{\sigma_{\mathbf{v}^F}^2}} \frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau} = \tau \frac{i_t - E^M\left[i_t\right]}{1-\varphi\tau},$$

where τ is given by Eq. $(C.1)$. In particular, the belief updating rule in Eq. $(C.3)$ is Bayesian for the market along the equilibrium path. This verifies the conjectured equilibrium.

Next consider Eq. $(C.4)$ that describes the impact of a Fed belief surprise on the current and forward interest rates. The impact on the current rate follows from Eq. $(C.2)$. Consider the impact on the forward rates for horizons $h \geq 1$. From period $t+1$ onward, the equilibrium is the same as before. Taking expectations of Eq. (21) in period $t+1$ conditional on $\mathbf{g}_{t+1}^F$ and using Lemma 3, we obtain

$$\begin{aligned} E_{t+1}^M\left[i_{t+h}|\mathbf{g}_{t+1}^F\right] &= \rho + (1-\varphi) E_{t+1}^M\left[\mathbf{g}_{t+h}^F|\mathbf{g}_{t+1}^F\right] + \varphi\left(E_{t+1}^M\left[\mathbf{g}_{t+h}^M\right]\right) \\ &= \rho + (1-\varphi)\left(\varphi^{h-1}\mathbf{g}_{t+1}^F + \left(1-\varphi^{h-1}\right)\mathbf{g}_{t+1}^M\right) + \varphi \mathbf{g}_{t+1}^M. \end{aligned}$$

Taking the expectation of this expression in period t *after the interest rate decision*, and using Lemma 3 once more, we obtain

$$\begin{aligned} E_t^M\left[i_{t+h}|i_t\right] &= \rho + (1-\varphi)\varphi^{h-1} E_t^M\left[\mathbf{g}_{t+1}^F|i_t\right] + \left(1-(1-\varphi)\varphi^{h-1}\right) E_t^M\left[\mathbf{g}_{t+1}^M\right] \\ &= \rho + (1-\varphi)\varphi^{h-1}\left(\varphi E_t^M\left[\mathbf{g}_t^F|i_t\right] + (1-\varphi)\mathbf{g}_t^M\right) + \left(1-(1-\varphi)\varphi^{h-1}\right)\mathbf{g}_t^M \\ &= \rho + (1-\varphi)\varphi^h E_t^M\left[\mathbf{g}_t^F|i_t\right] + \left(1-(1-\varphi)\varphi^h\right)\mathbf{g}_t^M. \end{aligned}$$

Taking the expectation in period t *before the interest rate decision*, we have the same expression with $E_t^M\left[\mathbf{g}_t^F|i_t\right]$ replaced by $E_t^M\left[\mathbf{g}_t^F\right]$. Combining these observations, and using $E_t^M\left[\mathbf{g}_t^F|i_t\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$, we obtain

$$\frac{\Delta E_t^M\left[i_{t+h}\right]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = \tau(1-\varphi)\varphi^h \text{ for } h \geq 1.$$

This establishes Eq. $(C.4)$.

We finally establish Eqs. $(C.5)$ that describe the impact of a Fed belief surprise on the market's expectation for the current and future output gaps.

First consider the market's expected output gaps in future periods $(h \geq 1)$. From period $t+1$ onward, the equilibrium is the same as before. Therefore, taking the expectation of Eq. (22) conditional on $\mathbf{g}_{t+1}^F$ and using Lemma 2, we obtain

$$E_{t+1}^M\left[\tilde{y}_{t+h}|\mathbf{g}_{t+1}^F\right] = E_{t+1}^M\left[\mathbf{g}_{t+h}^M - \mathbf{g}_{t+h}^F\right] = \varphi^{h-1}\left(\mathbf{g}_{t+1}^M - \mathbf{g}_{t+1}^F\right).$$

Taking the expectation of this expression in period t *after* the interest rate decision, and using Lemma 2 once more, we obtain

$$E_t^M \left[\tilde{y}_{t+h} | i_t\right] = \varphi^h \left(\mathbf{g}_t^M - E_t^M \left[\mathbf{g}_t^F | i_t\right]\right) \text{ for } h \geq 1.$$

Taking the same expectation *before* the interest rate decision, we obtain the same expression with $E_t^M \left[\mathbf{g}_t^F | i_t\right]$ replaced by $E_t^M \left[\mathbf{g}_t^F\right]$. Taking the difference between these expressions and substituting $E_t^M \left[\mathbf{g}_t^F | i_t\right] = E_t^M \left[\mathbf{g}_t^F\right] + \tau \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$, we prove Eq. $(C.5)$ for future output gaps, $\Delta E_t^M \left[\tilde{y}_{t+h}\right] = -\tau \varphi^h$ for $h \geq 1$.

Next consider the market's expected output gap in the current period ($h = 0$). Recall that Eq. (7) still applies,

$$\tilde{y}_t = g_t - E_t^F \left[g_t\right] + \overline{E}_t^M \left[\tilde{y}_{t+1}\right] - E_t^F \left[\overline{E}_t^M \left[\tilde{y}_{t+1}\right]\right].$$

From the previous analysis, we have $\overline{E}_t^M \left[\tilde{y}_{t+1}\right] = \varphi \left(\mathbf{g}_t^M - E_t^M \left[\mathbf{g}_t^F | i_t\right]\right)$. This in turn implies $\overline{E}_t^M \left[\tilde{y}_{t+1}\right] = E_t^F \left[\overline{E}_t^M \left[\tilde{y}_{t+1}\right]\right]$, because the Fed knows $E_t^M \left[\mathbf{g}_t^F | i_t\right] = E_t^M \left[\mathbf{g}_t^F\right] + \tau \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$. Combining these observations and substituting $E_t^F \left[g_t\right] = \mathbf{g}_t^F + \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F$, the current output gap satisfies

$$\tilde{y}_t = g_t - \left(\mathbf{g}_t^F + \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right).$$

Taking the expectation of the output gap under the market's belief *before* and *after* the interest rate decision, we obtain

$$E_t^M \left[\tilde{y}_t | i_t\right] = \mathbf{g}_t^M - \left(\mathbf{g}_t^F + \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right) \text{ and } E_t^M \left[\tilde{y}_t\right] = \mathbf{g}_t^M - \mathbf{g}_t^F.$$

Taking the difference between these expressions, we establish Eq. $(C.5)$ for the current period, $\frac{\Delta E_t^M [\tilde{y}_t]}{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F} = -1$. This completes the proof of the proposition. □

C.2. Mechanics of tantrum shocks

We next analyze an extreme case in which the Fed thinks the market is unreactive ($\tau = 0$), whereas the market is actually reactive ($\tau = 1$). This case helps illustrate the mechanics of tantrum shocks. The analysis of this case is mostly presented in the main text. Here, we derive Eqs. $(36 - 38)$.

As we explain in the main text, the Fed sets the policy rate in (35) because it believes the market is unreactive, $\tau = 0$. However, the market is actually reactive, $\tau = 1$, and thinks the Fed knows this. Therefore, after seeing the policy rate in (35), the market's posterior belief for the Fed's long-term belief becomes

$$E_t^M \left[\mathbf{g}_t^F | i_t\right] - E_t^M \left[\mathbf{g}_t^F\right] = \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1 - \varphi}. \tag{C.7}$$

Proposition 5 then applies for the market reaction type, $\tau = 1$, and the "as-if" Fed belief change in $(C.7)$. In particular, Eq. $(C.4)$ implies

$$\Delta E_t^M [i_{t+h}] = \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h (1-\varphi) \text{ for } h \geq 1.$$

This proves Eq. (36). Likewise, Eqs. $(C.5)$ imply

$$\Delta E_t^M [\tilde{y}_t] = -\frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \text{ and } \Delta E_t^M [\tilde{y}_{t+h}] = -\frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi} \varphi^h \text{ for } h \geq 1.$$

This proves Eq. (37).

Finally, we establish Eq. (38) that describes the Fed's expected output gap in the current period. The output gap is still given by (7),

$$\tilde{y}_t = g_t - E_t^F [g_t] + \overline{E}_t^M [\tilde{y}_{t+1} | \tau = 1] - E_t^F \left[\overline{E}_t^M [\tilde{y}_{t+1} | \tau = 0] \right]. \tag{C.8}$$

Here, we have written conditional expectations to incorporate the fact that the Fed sets the output gap thinking the market is unreactive, $\tau = 0$, but the market actually is reactive, $\tau = 1$. Note also that that Eq. $(C.6)$ applies conditional on the market's reaction type, τ, and its posterior belief, $E_t^M \left[\mathbf{g}_t^F | i_t \right]$. Applying the equation for $\tau = 1$, we obtain

$$\overline{E}_t^M [\tilde{y}_{t+1} | \tau = 1] = \varphi \left(\mathbf{g}_t^M - E_t^M \left[\mathbf{g}_t^F | i_t \right] \right) = \varphi \left(\mathbf{g}_t^M - E_t^M \left[\mathbf{g}_t^F \right] \right) - \varphi \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi}.$$

Applying the equation for $\tau = 0$, we obtain

$$\overline{E}_t^M [\tilde{y}_{t+1} | \tau = 0] = \varphi \left(\mathbf{g}_t^M - E_t^M \left[\mathbf{g}_t^F \right] \right).$$

Substituting these expressions into Eq. $(C.8)$, we obtain

$$\tilde{y}_t = g_t - E_t^F [g_t] - \varphi \frac{\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F}{1-\varphi}.$$

Taking the expectation under the Fed's belief, we prove Eq. (38), $E_t^F [\tilde{y}_t | \tau = 1] = -\frac{\varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F \right)}{1-\varphi}$. This completes the derivations omitted from the main text.

In this extreme case, the Fed operates under the assumption that the market is unreactive and will interpret its interest rate change as temporary. Thus, the Fed is ex-post surprised when the market is revealed to be reactive and misses its output gap target even under its own belief.

C.3. Policy implication of tantrums: Gradualism

We next turn to the policy implications of tantrums. To analyze policy, we analyze a less extreme case in which the Fed is uncertain about the market's reaction type. Our next result characterizes the equilibrium for this case and shows that the Fed acts *even more gradually* than in our baseline model.

Proposition 6. *Consider the setup in Proposition 5 with the difference that in period t the market can have one of two types, $\tau \in \{0,1\}$. The Fed believes $\tau = 1$ with probability $\delta \in (0,1)$. The market knows δ.*

In period t, the equilibrium interest rate is given by Eq. (39)

$$i_t = E_t^M [i_t] + \left(1 - \varphi\tilde{\delta}\right) \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right), \tag{C.9}$$

where $\tilde{\delta}$ is the unique root of the following quadratic over the range $x \in (\delta, 1)$:

$$P(x) = x^2\varphi - x(1 + 2\delta\varphi) + \delta(\varphi + 1). \tag{C.10}$$

The market's posterior belief after observing the interest rate is given by

$$E_t^M \left[\mathbf{g}_t^F | i_t\right] - E_t^M \left[\mathbf{g}_t^F\right] = \tau \frac{i_t - E_t^M [i_t]}{1 - \varphi\tilde{\delta}}, \tag{C.11}$$

which is equal to $\tau\left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$ along the equilibrium path. The Fed's ex-ante expected output gap in period t is given by Eq. (40)

$$E_t^F [\tilde{y}_t] = \left(\tilde{\delta} - \delta\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \tag{C.12}$$

The Fed's output gap conditional on the market's type is given by

$$E_t^F [\tilde{y}_t | \tau = 1] = -\left(1 - \tilde{\delta}\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right) \text{ and } E_t^F [\tilde{y}_t | \tau = 0] = \tilde{\delta}\varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \tag{C.13}$$

From period $t+1$ onward, the equilibrium is the same as in Proposition 2.

Eq. (C.11) is the same as in Section C.1 in which the Fed knows the market's reaction type (see (C.3)). Along the equilibrium path, the market extracts a bundled optimism signal and forms a posterior belief that depends on its reaction type. Eq. (39) is different and says that the Fed acts as if the market is more reactive than implied by its ex-ante mean, $\tilde{\delta} > \delta = E_t^F [\tau]$ (cf. (C.2)).

Eq. (C.12) characterizes the Fed's expected output gap. The Fed misses its *ex-ante* output gap target *even under its own belief.* For instance, when the Fed is more optimistic than what the market expected, $\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F > 0$, it leaves a positive output gap on average. Eqs. (C.13) characterize the Fed's expected output gap *conditional on the market's type.* An optimistic Fed expects a negative output gap when the market is revealed to be reactive—a milder version of the tantrum shocks from the earlier Section C.2. Conversely, the Fed expects a positive output gap when the market is unreactive.

Why does the Fed hike the interest rate more cautiously and leave a positive output gap on average? Unlike in the previous versions of the model, the Fed is uncertain about how a *change* in its policy interest rate i_t will affect the output gap. If the market is reactive, an interest rate hike increases the market's perception for the Fed's long-term optimism. Consequently, the interest rate hike also reduces the market's expected future output gap, $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}|i_t,\tau=1]}{di_t} < 0$. In view of the IS curve (2), this creates a large impact on the current output gap, $\frac{d\tilde{y}_t[\tau=1]}{di_t} < -1$. In contrast, if the market is unreactive, an interest rate hike does not change the market's expected future output gap, $\frac{d\overline{E}_t^M[\tilde{y}_t|i_t,\tau=0]}{di_t} = 0$, and it has a smaller impact on the current output gap, $\frac{d\tilde{y}_t[\tau=0]}{di_t} = -1$. Since the economy is more sensitive to the Fed's interest rate decision when the market is reactive, the Fed overweights that case in its decision, $\tilde{\delta} > \delta$ [cf. Eq. (4)]. Therefore, the Fed acts as if the market is more reactive than implied by its prior mean belief, and adjusts the interest rate by a small amount. By acting conservatively, the Fed misses its output gap on average but it mitigates the tantrum shock that exacerbates its miss when the market is revealed to be reactive.

Proof of Proposition 6. We first check that the quadratic in $(C.10)$ has a unique root over the range, $x \in (\delta, 1)$. Note that $P(\delta) = \delta(1-\delta)\varphi > 0$ and $P(1) = (1-\delta)(\varphi - 1) < 0$. Since $P(\cdot)$ is an upward sloping parabola, these conditions imply that $P(x)$ has exactly one root that falls in the interval $(\delta, 1)$.

Next consider the equilibrium from period $t+1$ onward. Once the Fed sets the interest rate and observes the forward interest rate's reaction to it, the Fed learns the market's reaction type τ (see Proposition 5). Therefore, the equilibrium in subsequent periods is the same as in Proposition 2. This also verifies that $\frac{dE_t^F[V_{t+1}^F]}{di_t} = 0$. As before, the Fed's problem (3) in period t is effectively static and the optimality condition is given by (4).

Consider the equilibrium in period t. We conjecture that the interest rate rule in Eq. $(C.9)$ is optimal for the Fed and the belief updating rule in Eq. $(C.11)$ is Bayesian for the market along the equilibrium path.

Consider the Fed's optimal interest rate decision in period t. Once the Fed learns τ, the analysis is the same as in Section C.1. Following the steps in the proof of Proposition 5, we obtain the following analogue of $(C.6)$:

$$\overline{E}_t^M[\tilde{y}_{t+1}|\tau] = \varphi\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right]\right) \tag{C.14}$$
$$\text{where } E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right] - E_t^M\left[\mathbf{g}_t^F\right] = \tau\frac{i_t - E_t^M[i_t]}{1-\varphi\tilde{\delta}}.$$

This expression implies $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}|\tau]}{di_t} = -\frac{\varphi\tau}{1-\varphi\tilde{\delta}}$. Substituting this into the Fed's optimality condition (4), we obtain,

$$E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right] = 0 \text{ where } \frac{d\tilde{y}_t}{di_t} = -\left(1 + \frac{\varphi\tau}{1-\varphi\tilde{\delta}}\right). \tag{C.15}$$

The marginal policy impact, $\frac{d\tilde{y}_t}{di_t}$, depends on the market's reaction type, τ. Since the Fed is uncertain about the market's type, this term does *not* drop out of the expectation. Therefore, unlike the equilibria we analyzed so far, the Fed's expected output gap, $E_t^F[\tilde{y}_t]$, is not necessarily zero.

To characterize the optimal policy further, we rewrite Eq. $(C.15)$ in terms of conditional expectations

$$0 = -E_t^F\left[\frac{d\tilde{y}_t}{di_t}\tilde{y}_t\right] = \delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)E_t^F[\tilde{y}_t|\tau=1] + (1-\delta)E_t^F[\tilde{y}_t|\tau=0].$$

Note that the root of the quadratic in Eq. $(C.10)$ satisfies

$$\tilde{\delta} = \frac{\delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)}{\delta\left(1+\frac{\varphi}{1-\varphi\tilde{\delta}}\right)+1-\delta}.$$

Therefore, the Fed's optimality condition can be equivalently written as

$$0 = \tilde{\delta}E_t^F[\tilde{y}_t|\tau=1] + \left(1-\tilde{\delta}\right)E_t^F[\tilde{y}_t|\tau=0]. \tag{C.16}$$

Hence, the Fed targets a *weighted average* of the output gap over the cases in which the market is reactive and unreactive. The weight for the reactive case is given by the endogenous parameter, $\tilde{\delta}$, which exceeds the prior probability of this case, $\tilde{\delta} > \delta$.

We next solve for the optimal interest rate. Substituting the IS curve (2) into $(C.16)$, we obtain

$$i_t = \rho + E_t^F[g_t] + \tilde{\delta}\overline{E}_t^M[\tilde{y}_{t+1}|\tau=1] + \left(1-\tilde{\delta}\right)\overline{E}_t^M[\tilde{y}_{t+1}|\tau=0]. \tag{C.17}$$

This in turn implies

$$\begin{aligned} i_t &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\begin{pmatrix} \tilde{\delta}\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,\tau=1\right]\right) \\ +\left(1-\tilde{\delta}\right)\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F|i_t,,\tau=0\right]\right)\end{pmatrix} \\ &= \rho + E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F + \varphi\begin{pmatrix} \tilde{\delta}\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F\right] - \left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)\right) \\ +\left(1-\tilde{\delta}\right)\left(\mathbf{g}_t^M - E_t^M\left[\mathbf{g}_t^F\right]\right)\end{pmatrix} \\ &= E_t^M[i_t] + \left(1-\varphi\tilde{\delta}\right)\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right). \end{aligned}$$

Here, the first line substitutes Eq. $(C.14)$ along with the Fed's belief for period t, $E_t^F[g_t] = E_t^M\left[\mathbf{g}_t^F\right] + \Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F$. The second line substitutes $E_t^M\left[\mathbf{g}_t^F|i_t,\tau\right] = E_t^M\left[\mathbf{g}_t^F\right] + \tau\left(\Delta\mathbf{g}_t^F + \Delta\mathbf{v}_t^F\right)$ for each $\tau \in \{0,1\}$, which holds along the equilibrium path. The last line simplifies the expression. This proves that the interest rate rule in $(C.9)$ is optimal for the Fed.

Next consider the market's belief updating rule in period t. Along the equilibrium path, the interest rate policy in $(C.9)$ provides the market with an imperfect signal of the Fed's long-term belief change, $\frac{i_t - E^M[i_t]}{1-\varphi\tilde{\delta}} = \Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F$. Following the same steps as in the proof of Proposition 5, we establish that the belief updating rule in $(C.11)$ is Bayesian for the market along the equilibrium path. This verifies the conjectured equilibrium path.

We next establish Eqs. $(C.12 - C.13)$. To this end, we substitute the interest rate from $(C.17)$ into the IS curve (2) to obtain

$$\tilde{y}_t = g_t - E_t^F [g_t] + \overline{E}_t^M [\tilde{y}_{t+1}|\tau] - \left\{ \tilde{\delta} \overline{E}_t^M [\tilde{y}_{t+1}|\tau = 1] + \left(1 - \tilde{\delta}\right) \overline{E}_t^M [\tilde{y}_{t+1}|\tau = 0] \right\}.$$

Taking the Fed's expectation conditional on $\tau = 1$, we obtain

$$\begin{aligned} E_t^F [\tilde{y}_t|\tau = 1] &= \left(1 - \tilde{\delta}\right) \left(\overline{E}_t^M [\tilde{y}_{t+1}|\tau = 1] - \overline{E}_t^M [\tilde{y}_{t+1}|\tau = 0]\right) \\ &= -\left(1 - \tilde{\delta}\right) \varphi \begin{pmatrix} \left(\overline{E}_t^M \left[\mathbf{g}_t^F|i_t, \tau = 1\right] - E_t^M \left[\mathbf{g}_t^F\right]\right) \\ -\left(\overline{E}_t^M \left[\mathbf{g}_t^F|i_t, \tau = 0\right] - E_t^M \left[\mathbf{g}_t^F\right]\right) \end{pmatrix} \\ &= -\left(1 - \tilde{\delta}\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \end{aligned}$$

The second line substitutes Eqs. $(C.14)$ and the third line substitutes $E_t^M \left[\mathbf{g}_t^F|i_t, \tau\right] = E_t^M \left[\mathbf{g}_t^F\right] + \tau \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right)$ for each $\tau \in \{0, 1\}$. Likewise, taking the Fed's expectation conditional on $\tau = 0$, we obtain

$$\begin{aligned} E_t^F [\tilde{y}_t|\tau = 0] &= \tilde{\delta} \left(\overline{E}_t^M [\tilde{y}_{t+1}|\tau = 0] - \overline{E}_t^M [\tilde{y}_{t+1}|\tau = 1]\right) \\ &= -\tilde{\delta}\varphi \begin{pmatrix} \left(\overline{E}_t^M \left[\mathbf{g}_t^F|i_t, \tau = 0\right] - E_t^M \left[\mathbf{g}_t^F\right]\right) \\ -\left(\overline{E}_t^M \left[\mathbf{g}_t^F|i_t, \tau = 1\right] - E_t^M \left[\mathbf{g}_t^F\right]\right) \end{pmatrix} \\ &= \tilde{\delta}\varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \end{aligned}$$

This proves Eq. $(C.13)$. Finally, note that the unconditional expectation is given by

$$\begin{aligned} E_t^F [\tilde{y}_t] &= \delta E_t^F [\tilde{y}_t|\tau = 1] + (1 - \delta) E_t^F [\tilde{y}_t|\tau = 0] \\ &= \left(-\delta \left(1 - \tilde{\delta}\right) + (1 - \delta) \tilde{\delta}\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right) \\ &= \left(\tilde{\delta} - \delta\right) \varphi \left(\Delta \mathbf{g}_t^F + \Delta \mathbf{v}_t^F\right). \end{aligned}$$

This establishes Eq. $(C.12)$ and completes the proof of the proposition. □

C.4. Policy implication of tantrums: Communication

Proposition 6 shows that, despite acting conservatively, the Fed misses its output gap conditional on the market's type. Therefore, the possibility of tantrum shocks increases the Fed's ex-ante expected gaps in (3). When the market is uncertain about the Fed's belief, its reaction type τ becomes a key parameter for policy. If the Fed is confused about τ, there can be extreme tantrum shocks as in Section C.2. If the Fed is uncertain about τ, there are still (milder) tantrum shocks that make the Fed miss its output target more often than without these shocks.

The welfare losses induced by tantrum shocks create a natural role for communication between the Fed and the market. First, the Fed can try to figure out the market's reaction type τ. Second, the Fed can try to reveal its own belief to the market—making the market's reaction type irrelevant to the equilibrium and therefore mitigating the tantrum shocks. The following result formalizes the second point. In our model with two belief types, the Fed can reveal its belief by announcing the average interest rate it plans to set in the next period in addition to the current rate.

Proposition 7. *Consider the setup in Proposition 6 in which the Fed can have a short-term belief change in period t as well as a long-term belief change, and the Fed is uncertain about the market's reaction type τ. Suppose in period t the Fed announces both the current interest rate, i_t, and the interest rate it expects to set in the next period, $i^F_{t+1} \equiv E^F_t\left[i_{t+1}\right]$. In equilibrium, the Fed's announcements are truthful and given by*

$$\begin{aligned} i_t &= \rho + (1-\varphi)\,\mathbf{g}^F_t + \varphi \mathbf{g}^M_t + \Delta \mathbf{v}^F_t \qquad (C.18) \\ i^F_{t+1} = E^F_t\left[i_{t+1}\right] &= \rho + \left(1-\varphi^2\right)\mathbf{g}^F_t + \varphi^2 \mathbf{g}^M_t . \end{aligned}$$

These announcements fully reveal both dimensions of the Fed's belief change, $\Delta \mathbf{g}^F_t = \mathbf{g}^F_t - E^M_t\left[\mathbf{g}^F_t\right]$ and $\Delta \mathbf{v}^F_t$. The Fed achieves a zero expected output gap under its belief regardless of the market's reaction type, $E^F_t\left[\tilde{y}_t\right] = E^F_t\left[\tilde{y}_t|\tau\right] = 0$.

Intuitively, by announcing two interest rates, the Fed can fully reveal both dimensions of its belief change. The expected rate in the next period reveals the Fed's long-term belief change, $\Delta \mathbf{g}^F_t$. The current rate then reveals the Fed's short-term belief change, $\Delta \mathbf{v}^F_t$. Once the market learns the Fed's belief, the equilibrium is similar to the baseline setting in which the market's reaction type does not play a role (cf. Proposition 1). We only need to adapt the analysis to incorporate the fact that the Fed's short-term belief, $\Delta \mathbf{v}^F_t$, is not necessarily zero. The current interest rate increases one-to-one with $\Delta \mathbf{v}^F_t$, since this belief change does not persist into future periods and the market knows this.

Proposition 7 provides a rationale for the enhanced Fed communication that we have seen in recent years, e.g., "the forward guidance" or "the dot curve". In our model, the role of these policies is *not* to persuade the market—the market is opinionated. Rather, communication is useful because it helps reveal the Fed's belief to the market, reducing the chance of tantrum shocks in which the market misinterprets the Fed's belief.

Proof of Proposition 7. We verify that it is optimal for the Fed to announce the interest rates in (C.18) in period t. After seeing the announcements, $\left(i_t, i^F_{t+1}\right)$, the market infers the

Fed's long-term and short-term beliefs as

$$\begin{aligned} \mathbf{G}^F\left(i_t, i_{t+1}^F\right) &\equiv \frac{i_{t+1}^F - \rho - \varphi^2 \mathbf{g}_t^M}{1-\varphi^2}, \\ \mathbf{\Delta V}^F\left(i_t, i_{t+1}^F\right) &\equiv i_t - \rho - (1-\varphi)\,\mathbf{G}^F\left(i_t, i_{t+1}^F\right) - \varphi \mathbf{g}_t^M. \end{aligned} \tag{C.19}$$

Along the equilibrium path, the market's inferences are correct, $\mathbf{G}^F\left(i_t, i_{t+1}^F\right) = \mathbf{g}_t^F$ and $\mathbf{\Delta V}^F\left(i_t, i_{t+1}^F\right) = \Delta \mathbf{v}_t^F$.

As before, the equilibrium in subsequent periods is the same as in Proposition 2. This implies the Fed's expected continuation value $E_t^F\left[V_{t+1}^F\right]$ does not depend on its policy choice in period t. Thus, the Fed's problem (3) is effectively static and its optimality conditions are given by the following analogues of Eq. (4),

$$E_t^F\left[\left(-1 + \frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_t}\right)\tilde{y}_t\right] = E_t^F\left[\left(-1 + \frac{d\overline{E}_t^M\left[\tilde{y}_{t+1}\right]}{di_{t+1}^F}\right)\tilde{y}_t\right] = 0. \tag{C.20}$$

After seeing the Fed's policy announcements, the market *thinks* the continuation equilibrium will be the same as in Proposition 2 given the Fed's beliefs in (C.19). In particular, Eq. (26) implies

$$\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{G}^F\left(i_t, i_{t+1}^F\right)\right).$$

Using Eq. (C.19), we obtain $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_t} = 0$ and $\frac{d\overline{E}_t^M[\tilde{y}_{t+1}]}{di_{t+1}^F} = -\frac{\varphi}{1-\varphi^2}$. Substituting these expressions into (C.20), the Fed's optimality conditions simplify to $E_t^F\left[\tilde{y}_t\right] = 0$ as in the baseline model (see (5)).

We next show that the policy announcements in (C.18) achieve a zero output gap under the Fed's belief, $E_t^F\left[\tilde{y}_t\right] = 0$, and therefore are optimal. Along the equilibrium path, the market infers the Fed's belief correctly and thinks, $\overline{E}_t^M\left[\tilde{y}_{t+1}\right] = \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right)$. Substituting this into the IS curve (2) along with the expression for the current interest rate we obtain

$$\begin{aligned} \tilde{y}_t &= -\left((1-\varphi)\,\mathbf{g}_t^F + \varphi \mathbf{g}_t^M + \Delta \mathbf{v}_t^F\right) + g_t + \varphi\left(\mathbf{g}_t^M - \mathbf{g}_t^F\right) \\ &= g_t - \mathbf{g}_t^F - \Delta \mathbf{v}_t^F. \end{aligned}$$

Taking the expectation under the Fed's belief establishes $E_t^F\left[\tilde{y}_t\right] = 0$. Thus, it is optimal for the Fed to follow the announcements in (C.18). This analysis also implies $E_t^F\left[\tilde{y}_t\right] = E_t^F\left[\tilde{y}_t|\tau\right] = 0$, completing the proof. □

D. Data details and omitted empirical results

This appendix contains the details of our data sources and variable construction, and the empirical results omitted from the main text.

D.1. Data sources

Federal funds rate (FFR). In Figure 1, we plot the Federal funds rate (FFR). These data are public and obtained from the Fed Board, retrieved through FRED. The corresponding FRED ticker is "FEDFUNDS". We use the monthly version of the series.

Dates of FOMC meetings: These data are public and obtained from Nakamura and Steinsson (2018*a*,*b*).

The Fed's Greenbook/Tealbook forecasts. In the left panel of Figure 1 as well as in Figure 2, we use Greenbook/Tealbook forecasts for the FFR and for the GDP price index. These forecasts are produced by the Fed research staff before each FOMC meeting. The data come from two sources:

- **Digital Greenbook/Tealbook data from the Philadelphia Fed.** These data are public and obtained from the Philadelphia Fed.
  - The predictions for the GDP price index inflation come from the main Greenbook data set (available at `https://www.philadelphiafed.org/-/media/frbp/assets/surveys-and-data/greenbook-data/documentation/gbweb_row_format.xlsx`). This data set is at the FOMC-meeting frequency and quarterly forecasting horizon. For most of our sample period, the data report the Greenbook projections for the annualized quarterly growth rate in the price index for GDP.[1] The data are available until the end of 2013.
  - The predictions for the FFR come from the supplement data set (available at `https://www.philadelphiafed.org/-/media/frbp/assets/surveys-and-data/greenbook-data/greenbook_financial_assumptions_interestrates_web.xls`). This data set contains the Fed staff's assumptions for the future values of financial variables, including the FFR, other interest rates, and equity prices. We focus on the assumptions for the FFR. These assumptions are available at a quarterly forecasting horizon. Each assumption corresponds to the quarterly average effective federal funds rate. The data are available until the sixth meeting of 2008.[2]
- **Hand-collected Greenbook/Tealbook data:** We collected these data from public sources (Caballero and Simsek (2022)). Specifically, since the digital data for the FFR

[1] Before December 11, 1991, the data report the projections for the price index growth for the GNP implicit deflator. Between December 11, 1991 and March 21, 1996, the data report the projections for the price index growth for the GDP implicit deflator.

[2] In some cases, the assumption is appended with a "+" or "-", suggesting that the value was likely to be a bit higher (in the case of a "+") or a bit lower (in the case of a "–") than the value to which they were appended. We have ignored these additional suffixes. For further information on this data set, see the notes on the Philadelphia Fed's website: `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/gap-and-financial-data-set`

predictions are available for a shorter time period than the digital data for inflation predictions, we hand-collected the FFR predictions for the missing meetings to obtain a longer time series. We obtained the predictions from either the corresponding Greenbook PDF file (from the sixth meeting of 2008 until the third meeting of 2010) or the corresponding Tealbook PDF file (from the fourth meeting of 2010 until the last meeting of 2015).[3] The hand-collected data are at the FOMC meeting-frequency and reflect *yearly* forecasts. To match the forecasting horizon of the digital data set, we linearly interpolate these yearly forecasts to obtain quarterly forecasts. For the nearby quarters, we use the FFR in the last quarter as the interpolation anchor.

It might be useful to give one example. Consider the FOMC meeting on August 10, 2010. The data for this meeting come from the Tealbook PDF file dated August 4, 2010. On page 32, there is a table that contains the forecasts for several macroeconomic variables:

The Long-Term Outlook

(Percent change, Q4 to Q4, except as noted)

Item	2010	2011	2012	2013	2014
Real GDP	2.7	3.6	4.8	5.0	4.6
Civilian unemployment rate[1]	9.7	8.9	7.6	6.2	5.3
PCE prices, total	1.3	1.1	1.0	1.2	1.4
Core PCE prices	1.1	.9	1.0	1.1	1.4
Federal funds rate[1]	.1	.1	.4	2.1	3.3

1. Percent, average for the final quarter of the period.

We hand-collect all of these data but focus on the predictions for the FFR. For the last two quarters of 2010, we interpolate the average FFR in Q2 (which was 0.19) with the prediction for the last quarter of 2010. For the subsequent quarters, we interpolate the predictions for the last quarters of the neighboring years. This results in the following forecasts at quarterly frequency (q0 corresponds to the current quarter):

fomc_date	ffrFq0	ffrFq1	ffrFq2	ffrFq3	ffrFq4	ffrFq5	ffrFq6	ffrFq7	ffrFq8	ffrFq9
10aug2010	0.15	0.10	0.10	0.10	0.10	0.10	0.18	0.25	0.33	0.40

We combine the hand-collected data with the digital data to obtain a time series for the FFR and inflation predictions that runs until the end of 2013. The combined data set is at the FOMC-meeting frequency and quarterly forecasting horizon. In the left panel of Figure 1, we plot the FFR predictions for select FOMC meetings up to four-quarters ahead (for q0-q4). Since the date of the Greenbook/Tealbook is slightly earlier than that of the FOMC meeting,

[3]We collected the predictions for several additional macroeconomic variables even though we use the hand-collected data only for the Fed funds rate. We include all of our hand-collected data in our replication package.

the predictions in Figure 1 are matched to the date of the corresponding FOMC meeting. For Figure 2 as well as for the robustness analyses in Online Appendix D.2, we convert the data into a quarterly time series by averaging the predictions made in each FOMC meeting within the quarter.

The Fed's SEP and the dot curve. Beginning with the October 2007 FOMC meeting, FOMC meeting participants submit individual forecasts of various economic variables in conjunction with four FOMC meetings a year. The Summary of Economic Projections (SEP) provides a summary of these forecasts such as the range, the mean, and the median. These summary forecasts are released to the public shortly after the corresponding FOMC meeting (beginning in April 2011, it is released with the Chairman's post-meeting press conference). Individual forecasts are made available to the public after five years. Beginning in 2012, the SEP began to include the forecasts for the FFR—also known as "the dot curve." Each dot corresponds to an FOMC member's forecast for the FFR.

In the right panel of Figure 1, we plot the median SEP prediction for the FFR for select FOMC meetings. We collected this data from public sources (Caballero and Simsek (2022)). Specifically, we collect the individual participants' forecasts for the available years (between 2012-2015) from the Fed's SEPs published on `https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm`. We collect the median participant's forecast for the more recent years (between 2016-2021Q2) from the advance releases of the SEPs published on `https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`.[4] These data have a *yearly* forecasting horizon: specifically, we have the prediction for the FFR for the end of the current year, for the end of the next year, and so on. In the right panel of Figure 1, we plot the median prediction for the end of the current and the next year.

The forward rates extracted from the FFR futures. In Figure 1, we also plot the forward interest rates corresponding to select FOMC meetings. These data are proprietary and obtained from Bloomberg. We use the Bloomberg Terminal to download the daily FFR futures prices for up to 35 months ahead. The corresponding Bloomberg tickers are "FF1 Comdty-FF36 Comdty." We obtain these data from March 1, 2002 until February 28, 2020.[5]

We then convert the futures prices to implied forward rates. Each futures contract settles at the end of the month at 100 minus the average FFR observed in the corresponding month. We extract the implied forward interest rate for the corresponding month using the conversion,

$$(\text{Forward interest rate})_{t,h} = 100 - (\text{FFR futures price})_{t,h}.$$

Here, t is a trading day and $h \in \{0, .., 35\}$ is the monthly horizon ($h = 0$ corresponds to the

[4] As with the Greenbook/Tealbook data, we hand-collected more data than we use. We include all of our hand-collected data in our replication package.

[5] Upon inspection, two prices (93.425, 93.245) seem to be filler for misising data. We change all instances of these prices to missing.

current month). We use end-of-day futures prices: that is, the forward interest rates reflect the market's predictions *after* the FOMC meeting.

We adjust the forward interest rates to match the format of the Fed's predictions in each panel of Figure 1. For the right panel, where we plot the SEP predictions for the end of the current and the next year, we plot the forward interest rate for the last month of the current year and the last month of the next year. For the left panel, where we plot the Greenbook/Tealbook predictions at a quarterly forecasting horizon (up to four quarters ahead), we plot the forward interest rates for the corresponding quarter. Specifically, for the current quarter (q0), we average over the forward rates for the remaining months in the current quarter. For subsequent quarters (q1-q4), we average over the forward rates for all of the months in the corresponding quarter.

Blue Chip Financial Forecasts. In Figure 2, we also use data from Blue Chip Financial Forecasts. This is a proprietary database that contains the forecasts made by member financial institutions for future interest rates and economic activity. The source data set is at a monthly frequency and quarterly forecasting horizon. It contains individual as well as the consensus (average) forecasts up to five quarters ahead.

We have access to the digital Blue Chip data from January 2001 until February 2020. We have access to PDF files for years before 2001. Blue Chip starts in 1983 but it has the forecasts for the GDP price index starting in 1986. Therefore, we hand-collect the consensus predictions for the FFR and the GDP price index from 1986 to 2000. We combine the hand-collected and the digital data to obtain a time series for consensus predictions that runs from 1986 to 2020. For Figure 1 as well as for the robustness analyses in Online Appendix D.2, we convert the data into a quarterly time series by averaging the predictions made in each month within the quarter, and we use the subset of the data from 1990 to 2013.

Inflation breakevens from the TIPS market. In Figure D.2 of Online Appendix D.2, we show that the correlations illustrated in Figure 2 are robust to measuring the market's beliefs from asset price data as opposed to survey data. To this end, we need asset-price-based measures of interest rate and inflation predictions.

For the interest rate predictions, we use the forward rates data that we described earlier. We use the version of the data at a trading-day frequency and a quarterly-forecasting horizon. We convert this data to quarterly frequency by averaging over all trading days within the quarter.

For the inflation predictions, we use the inflation breakevens implied by the TIPS market. To this end, we obtain data for both the nominal yield curve and the TIPS yield curve. These data are public and come from the Fed Board, estimated based on the approach by Gürkaynak, Sack and Wright (2007, 2010). We use these data to calculate the nominal and real (inflation-adjusted) rates at the appropriate forecasting horizon. We then obtain the inflation breakevens by taking the difference between the nominal rate and the real rate.

More specifically, Gürkaynak, Sack and Wright (2007, 2010) estimate yield curve by properly tuning the parameters of the Nelson-Siegel-Svensson yield curve. This approach assumes the instantaneous forward rates n years ahead are characterized by the formula:

$$f_t(n) = \beta_{0,t} + \beta_{1,t} \exp(-\frac{n}{\tau_{1,t}}) + \beta_{2,t} \frac{n}{\tau_{1,t}} \exp\left(-\frac{n}{\tau_{2,t}}\right). \tag{D.1}$$

The approach then estimates the parameters $\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_{1,t}, \tau_{2,t}$ for each trading day t to fit the actual market data on that day. We obtain these daily parameters for both the nominal yield curve (available from `https://www.federalreserve.gov/data/nominal-yield-curve.htm`) and for the TIPS yield curve (available from `https://www.federalreserve.gov/data/tips-yield-curve-and-inflation-compensation.htm`). This enables us to construct the instantaneous nominal and TIPS forwards rates at arbitrary horizons.

Figure D.2 requires data at a quarterly forecasting horizon. We therefore use the formula in (D.1) to construct implied predictions at a quarterly forecasting horizon. We associate the predictions for a particular quarter with the instantaneous forward rate for the last day of the quarter. In particular, for each trading day t and forecasting horizon $h \in \{0, 1, ..\}$, we first calculate the (yearly) distance between the current day and the last day of the forecasted quarter: that is, $n_{t,h} = (t(q_h) - t + 1)/365$, where $t(q_h)$ denotes the last day of h quarters ahead. We then apply Eq. (D.1) with the parameters for the trading day t along with $n = n_{t,h}$. We use this approach to calculate both the nominal forward rate and the TIPS forward rate. We calculate the breakeven inflation by taking the difference between the two forward rates, $\pi_{t,h}^{bkeven} = f_t^{nom}(n_{t,h}) - f_t^{tips}(n_{t,h})$. The resulting data has a trading-day frequency and a quarterly forecasting horizon. We convert this data to quarterly frequency by averaging over all trading days within the quarter. Our data set for inflation breakevens runs from 2004-Q1 until 2021-Q2 (because the TIPS yield curve parameters are available starting in 2004).

D.2. Robustness of Fed-market disagreement patterns

In Section 2, we show that the disagreements between the Fed and the market about future interest rates are correlated with the disagreements about future aggregate demand (proxied by inflation). We also show that these disagreements about demand are persistent over time. In this appendix, we show that these patterns are robust to using a regression analysis, focusing on different prediction horizons, and measuring the market's belief from asset price data as opposed to survey data.

Regression analysis. In the main text, we focus on a graphical analysis. Table D.1 shows that the results illustrated by Figure 2 also hold in a regression analysis. Column 1 shows that the disagreement between the Fed and market on the future interest rate is correlated with the disagreement on future inflation. Column 2 shows that the disagreement on future inflation is correlated with its lagged value. The coefficient on the lag term is large, indicating that disagreements are quite persistent.

Table D.1: Fed-market disagreements on interest rates vs. inflation

	(1) FFR disagreement b/se	(2) Inflation disagreement b/se
Inflation disagreement	0.87**	
	(0.16)	
Inflation disagreement last quarter		0.70**
		(0.06)
R^2 (adjusted)	0.37	0.48
Observations	96	96

Note: The sample is a quarterly time series of Greenbook and Blue Chip forecasts between 1990-2013. Disagreement is the difference between the Greenbook and the Blue Chip forecast for 4 quarters ahead. FFR is the quarterly average (percent) and inflation is the annualized quarterly growth rate of the GDP price index (percent). Estimation is via OLS. Newey-West standard errors with a bandwidth of 4 quarters are in parentheses. +, *, and ** indicate significance at 0.1, 0.05, and 0.01 levels, respectively.

Alternative forecast horizons. In the main text, we focus on forecasts for the fourth quarter (beyond the current quarter). Figure D.1 shows that the patterns also hold when we use alternative forecast horizons. However, the correlation between disagreements on interest rates and inflation becomes weaker for shorter horizons. This might be because disagreements about aggregate demand are smaller over shorter time horizons, since macroeconomic uncertainty tends to grow with time.[6]

Measuring the market's belief using asset price data. In the main text, we measure the market's belief using the Blue Chip survey data. One concern is that the survey data might not be fully representative of financial markets. For instance, the dominant belief that determines asset prices might be different than the consensus (average) belief that we use (in models with disagreements, the dominant belief is typically a *wealth-weighted* average belief). To address this concern, we next redo the analysis by measuring the market's belief from asset price data. We measure the market's interest rate predictions from forward interest rates (as in Figure 1) and the inflation predictions from inflation breakevens in the TIPS market. As before, we measure the Fed's beliefs from Greenbook/Tealbook. The combined data set is at a quarterly frequency and quarterly forecasting horizon, and it runs from 2004-Q1 to 2013-Q4. Online Appendix D.1 contains details about data sources and construction.

Figure D.2 shows that the correlations illustrated in Figure 2 apply also when we measure the market's belief from asset price data. The direction of the disagreements implied by asset prices is generally similar to the direction of the disagreements implied by the Blue Chip survey data. The exception is the Global Financial Crisis (GFC) period 2008-2009, during which the inflation disagreement implied by asset prices is very large and has the opposite sign of the inflation disagreement implied by survey data. However, during the GFC inflation breakevens experienced

[6]Figure D.1 shows that there are sizeable disagreements about inflation also for the first horizon. This is arguably driven by unmodeled factors, e.g., disagreement about relative prices in the GDP price index.

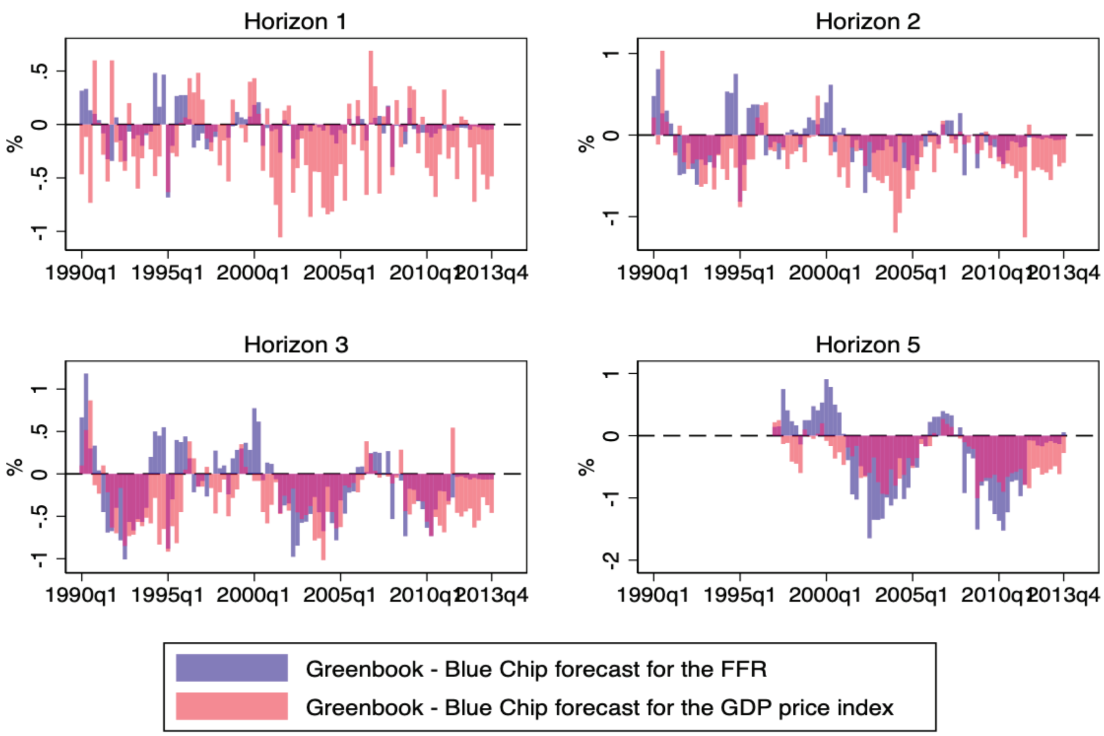

Figure D.1: The bars denote the disagreement between the Fed's Greenbook forecast and the consensus BlueChip forecasts. Each panel corresponds to forecasts for a different horizon. The blue (resp. red) bars correspond to disagreements on the FFR (resp. GDP price index).

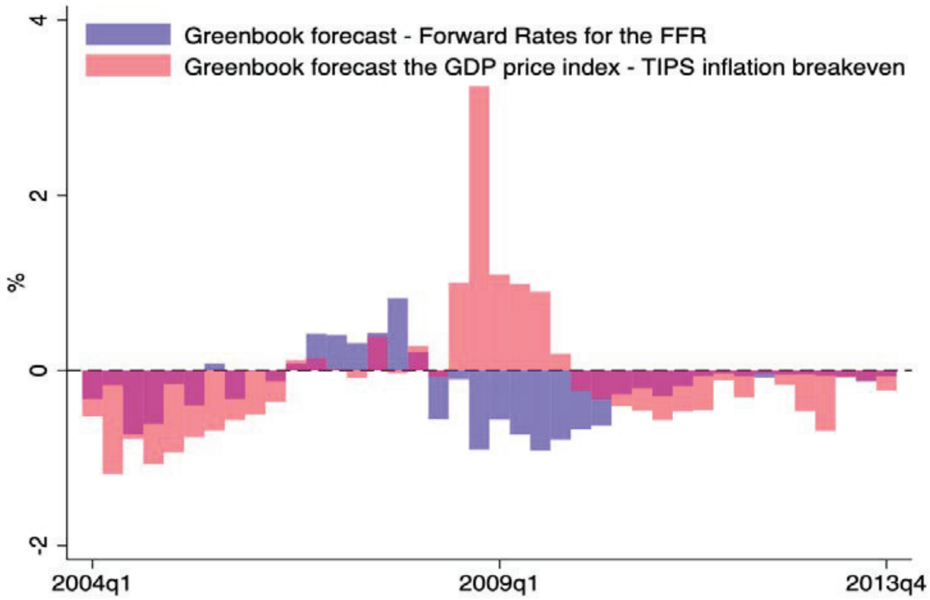

Figure D.2: The bars denote the disagreement between the Fed's Greenbook forecast and the market's forecast inferred from asset prices. The forecasts are for 4 quarters ahead. The blue (resp. red) bars correspond to disagreements on the FFR (resp. inflation). The market's FFR forecast is inferred from the FFR futures prices. The market's inflation forecast is obtained as the breakeven inflation rate in the TIPS market (see Online Appendix D.1 for details).

a large liquidity premium spike. The TIPS rates increased relative to the corresponding nominal treasury rates, not because of a change in inflation expectations, but because the market became very illiquid. This imported a downward bias to the market's expected inflation and an upward bias to the Fed-market inflation disagreements.

The breakdown of the correlation patterns during the GFC highlights the shortcomings of using asset prices to measure the market's predictions. While the asset price data might more accurately reflect the dominant belief, it can be confounded by liquidity premia or risk premia. These confounding premia tend to be especially large when financial markets are in distress. Therefore, we adopt the analysis with the survey data as our baseline approach and present the analysis with asset price data as a robustness check.