CONFIDENCE MANAGEMENT IN TOURNAMENTS

Shanglyu Deng
Hanming Fang
Qiang Fu
Zenan Wu

## ABSTRACT

An incumbent employee competes against a new hire for bonus or promotion. The incumbent's ability is commonly known, while that of the new hire is private information. The incumbent is subject to a perceptional bias: His prior about the new hire's type differs from the true underlying distribution. He can be either ex ante overconfident or underconfident. We first explore whether a firm that aims to maximize aggregate effort would benefit or suffer from the bias. It is shown that debiasing may not be productive in incentivizing efforts. We then study the optimal information disclosure policy. The firm is allowed to ex ante commit to whether an informative signal—which allows the incumbent to infer the new hire's type—will be disclosed publicly. We fully characterize the conditions under which transparency or opacity will prevail. We further take a Bayesian persuasion approach to optimally design the firm's evaluation and feedback structure. We also consider an alternative context in which the manager is concerned about the expected winner's effort. We demonstrate that the insights obtained from the baseline setting remain intact. Our results shed light on the extensive discussion of confidence management in firms and the debate about organizational transparency.

Shanglyu Deng
Department of Economics
University of Maryland
3114 Tydings Hall
7343 Preinkert Dr.
College Park, MD 20742
sdeng1@umd.edu

Hanming Fang
Department of Economics
Ronald O. Perelman Center
for Political Science and Economics
133 South 36th Street
Suite 150
Philadelphia, PA 19104
and NBER
hanming.fang@econ.upenn.edu

Qiang Fu
Department of Strategy and Policy
NUS Business School
15 Kent Ridge Drive
Singapore
bizfq@nus.edu.sg

Zenan Wu
School of Economics
Peking University
Beijing, China
zenan@pku.edu.cn

*"Attempt easy tasks as if they were difficult, and difficult as if they were easy; in the one case that confidence may not fall asleep, in the other that it may not be dismayed."*

—Baltasar Gracián

*"Perhaps a successful life, like a successful company, needs both optimism and at least occasional pessimism, and for the same reason a corporation does."*

—Martin Seligman

# 1  Introduction

The internal labor markets inside firms are widely viewed to resemble a tournament (Lazear and Rosen, 1981; Rosen, 1986): Workers strive for bonus or to climb hierarchical ladder toward higher rungs (Brown and Minor, 2014); they are rewarded or punished based on their performance relative to competitors or benchmarks instead of absolute output metrics (Chen and Lim, 2013). A plethora of anecdotal and empirical observations have documented the prevalence of tournament incentives and relative performance evaluation (RPE) schemes (see, e.g., Eriksson, 1999; Henderson and Fredrickson, 2001; Belzil and Bognanno, 2008; Connelly, Tihanyi, Crook, and Gangloff, 2014; and Lazear, 2018). Consider, for instance, the popular practice of *vitality curve*—or stack ranking—that are pioneered by Jack Welch and have proliferated in the modern corporate landscape.[1] As argued by DeVaro (2006), promotion tournaments are an integral component of firms' HR practice to advance their strategic interests.

The conventional wisdom tells that the incentive of the agents involved in tournament situations crucially depends on their relative competitiveness and their perception of each other's competency (Brown, 2011). However, one's knowledge about his opponent is often limited, and their perception can be systematically biased. Consider the usual scenario in which a new hire joins an organization and competes—under an RPE scheme—against incumbent employees for bonus or promotion. The competency of the incumbents can be inferred from their established track record, while that of the new hire often remains to be ascertained, which gives rise to the typical problem of information asymmetry (see, e.g., Hurley and Shogren, 1998; Wärneryd, 2003; Zhang and Zhou, 2016; and Denter, Morgan, and Sisak, 2020). Furthermore, incumbent employees may have misperceptions about the new

---

[1]A vitality curve is a performance management practice that ranks or rates individuals against their coworkers. It is also called stack ranking, forced ranking, and rank and yank. The concept of a "vitality curve" has been used to justify the "rank-and-yank" system of management at GE, whereby 10% of workers are fired at each evaluation.

hire. A large economics and psychology literature has identified the prevalence of perceptional biases, by which people "misplace" themselves in comparison with others or population mean, being either overconfident or underconfident (see, e.g., Larwood and Whittaker, 1977; Cooper, Woo, and Dunkelberg, 1988; Malmendier and Tate, 2005, 2015; Moore and Cain, 2007; Moore and Healy, 2008; and Muthukrishna, Henrich, Toyokawa, Hamamura, Kameda, and Heine, 2018). Such phenomena are pervasive in workplaces. Consider the following examples.

  i. A startup recruits a high-profile executive poached from an industry leader; incumbent employees may presumably overestimate the external hire.

 ii. Optimism typically arises in a rapidly growing firm; incumbent workers would arguably underestimate newbies, as they attribute the firm's success to their own superior competency.

iii. A corporate culture that champions workplace Darwinism—e.g., that at Enron—typically boosts workers' ego and breeds overconfidence, which also lead them to look down on newcomers.[2]

In this paper, we aim to explore two main questions. Suppose that the firm cares about aggregate effort supply in the workplace. First, does a firm benefit or suffer from its employee's perceptional bias? Second, suppose that the firm is able to conduct an evaluation to acquire an informative signal about the new hire's true ability, is the firm willing to disclose it to employees, which manipulates their beliefs and, in turn, influences the performance of the competition?

To answer these questions, we adopt a standard lottery contest setting—as in Denter, Morgan, and Sisak (2020) and Zhang and Zhou (2016)—to model a promotion tournament in a firm. Two employees—an incumbent worker and a new hire—are involved in the competition. They differ in their valuations of the "prize"—i.e., promotion to a higher rung along the corporate ladder—which can conveniently be interpreted as a measure of one's ability or strength: A larger valuation incentivizes more efforts. It is noteworthy that the model can alternatively but equivalently be set up in a way that employees have common valuation of the prize—i.e., bonus package with monetary value—but bear different effort costs. The ability of the incumbent is common knowledge, while that of the new hire is privately known to himself. The new hire's ability can take either a high or a low value. We allow the incumbent employee to possess a different prior about the new hire than the true underlying distribution. The uncommon priors thus depict the incumbent employee's misperception of his relative competitiveness in the tournament. A manager—e.g., HR director—can secure

---

[2]See Netessine and Yakubovich (2012).

an informative signal about the new hire's true ability through an evaluation exercise. The manager decides on the firm's information disclosure policy: She ex ante commits to either disclosing the signal or concealing it, with the latter to be equivalent to foregoing the evaluation exercise.

The questions posed in this paper are not only theoretically interesting, but also practically relevant. First, successful confidence management is broadly viewed in practice as a key to boosting productivity. The economics literature has espoused the motivation effect of (over)confidence, as a positive self-image could incentivize efforts and catalyze success (see, e.g., Bénabou and Tirole, 2002; Compte and Postlewaite, 2004; Gervais and Goldstein (2007); Chen and Schildberg-Hörisch, 2019). However, the usual motivation effect arises in settings of a stand-alone decision making or a principal-agent relationship. We nevertheless demonstrate subtler impact of overconfidence on effort supply in a tournament setting. We show that both overconfidence and underconfidence can benefit or harm effort provision depending on the parameters. Imagine that the incumbent is ex ante a favorite. Overconfidence would stifle the competition, as the complacency entices him to further slack off; in contrast, underconfidence by the incumbent can prevent shirking. Conversely, imagine that the incumbent is ex ante an underdog, overconfidence would help avoid discouragement, and thus debiasing would weaken the competition. The ramifications result from (i) the relative-performance based reward structure in tournaments, and (ii) players' nonmonotone best response correspondence in the strategic interactions in such competitive events (Lazear and Rosen, 1981; and Dixit, 1987). To the best of our knowledge, such effects have yet to be formally delineated in the literature.

Second, firms' internal information management—i.e., the information accessible to their employees—has spawned extensive discussion in both academic studies and practice. A large portion of leading firms in Europe and the United States have established internal knowledge system or built competency models that contain and reveal to workers the performance of their peers (Nafziger and Schumacher, 2013). Eli Lilly & Co. allows its employees to access their rankings in the succession planning system. In National University of Singapore (NUS) Business School, faculty members are allowed to access colleagues' student feedback reports.[3] The informative signal, if disclosed, allows the uninformed incumbent to make inference about the type of his opponent: It not only ameliorates information asymmetry, but also varies his perception of relative competitiveness, which may either mitigate or strengthen his perceptional bias. This update, by the same logic laid out above, would indeterminately affect his incentive in the competition and trigger ambiguous strategic response from the new hire.

---

[3]NUS conducts annual performance review for faculty members. Each department sets aside a bonus pool to reward teaching excellence, and only top ranked performers receive the monetary reward.

The results of our analysis can be summarized as follows. We first fully characterize the necessary and sufficient conditions under which the persistence of the incumbent's misperception benefits/harms the firm in terms of aggregate effort. We then proceed to explore the optimal information disclosure policy. When the quality—i.e., the precision—of the signal obtained through the evaluation is fixed, two effects loom large when the incumbent observes the signal with misperception in place. The informative signal serves two roles. First, it catalyzes an *information effect* due to information asymmetry. The updating alleviates information asymmetry ex post, but causes dispersed tournament outcomes across different states ex ante. Second, it gives rise to a *morale effect* because of the perceptional bias. The additional information leads the biased incumbent to revise his perception of the relative competitiveness. The *direction* and *magnitude* of his response to the signal depends on the nature of his initial perceptional bias and the realization of the signal. The morale effect reconciles with the information effect in the presence of overconfidence, but a tension emerges with underconfidence in place. Either disclosing the signal or concealing it can be optimal, and we identify the conditions and interpret the underlying logic. The comparison between biased and unbiased beliefs and that between transparency and opacity sensitively depend on employees' ex ante relative competitiveness, the underlying distribution of the new hire's ability, as well as the incumbent employee's perceptional bias. Our theoretical results yield novel and useful managerial implications for firms' confidence and internal information management, which we elaborate on in Sections 2.4 and 3.2.

We further explore two variations of the model. First, we take a Bayesian persuasion approach to endogenize the information structure. In the baseline setting, we assume that the quality of the signal is fixed and that the firm can either disclose or conceal it. In the extension, we endow the firm to design flexibly its evaluation. We show that the optimum requires either fully revealing or completely non-informative evaluations. This result corroborates our findings in the baseline setting. Second, we allow the firm to maximize the expected winner's effort instead of the aggregate effort. Again, we demonstrate that the main findings in the baseline setting remain qualitatively intact.

**Related Literature**   Our paper contributes to the literature on information transmission in contests/tournaments. One stream of this literature assumes that a designer possesses superior information about contenders and explores her optimal disclosure policy, e.g., Fu, Jiao, and Lu (2014), Zhang and Zhou (2016), Serena (2018), Lu, Ma, and Wang (2018), and Boosey, Brookins, and Ryvkin (2020). The other stream of work studies contenders' strategic action to reveal private information. Denter, Morgan, and Sisak (2020) and Fu, Gürtler, and Münster (2013) let the informed party take a costly action to signal his private type prior to the competition. Kovenock, Morath, and Münster (2015) and Wu and Zheng

(2017) study contenders' voluntary information disclosure. All these studies assume common priors and rational beliefs. Our paper belongs to the former class of studies, as it allows the firm to conduct evaluation and decide whether to disclose an informative signal. However, the extant literature does not allow for perceptionally biased players; as a result, the morale effect—which plays a subtle and important role in determining the optimum and looms large because of the perceptional bias in our setting—is absent in the existing literature. Our study thus complements this literature.

Our paper is naturally linked to the literature on the motivational effect of over(under)-confidence, such as Bénabou and Tirole (2002), Compte and Postlewaite (2004), Fang and Moscarini (2005), and Chen and Schildberg-Hörisch (2019). However, these studies focus on the stand-alone decision making of a single agent or in a principal-agent setting. Fang (2001) instead explores the role of perceptional bias in a team-production setting. In contrast, we explore the role played by the perceptional bias in a tournament in which reward is based on relative performance. Gervais and Goldstein (2007) show that overconfidence reduces free-riding and benefits teamwork, as an overconfident agent works hard. Kyle and Wang (1997) demonstrate in a Cournot duopoly setting the commitment value of overconfidence. They interpret overconfidence as one's excessively optimistic perception about his signal's precision; in contrast, we focus on players' over(under)-placement (Moore and Healy, 2008), by which one over(under)-estimates his relative competitiveness.

Crutzen, Swank, and Visser (2013) demonstrate that manager may refrain from differentiation among employees, as differentiation may lead them to downgrade their self-ratings and dampen incentives. Nafziger and Schumacher (2013) show that revealing peer's performance can be counterproductive as a worker can infer the impact of his effort on the probability of success. However, these settings do not involve competition or perceptional biases.

In one of our extensions, we take a Bayesian persuasion approach pioneered by Kamenica and Gentzkow (2011) to endogenize the information structure of the internal evaluation. Zhang and Zhou (2016) study the optimal information design in a similar setting but with common prior. Alonso and Camara (2016) explore Bayesian persuasion while allowing the sender and (single) receiver to possess heterogeneous beliefs. We borrow their approach and apply it to a tournament setting.

The rest of our paper is organized as follows. In Section 2, we set up an asymmetric-information tournament model with uncommon priors, characterize the equilibrium, and elaborate on the impact of perceptional bias. In Section 3, we explore the optimal information disclosure policy in the tournament and interpret the results. In Section 4, we explore two variations to the baseline setting: (i) optimally designed evaluation and (ii) a setting in which the firm cares about the expected winner's effort. In Section 5, we conclude.

# 2 Asymmetric-Information Tournament with Uncommon Priors

We model the competition between two employees inside an organization as a tournament. In this part, we first spell out the fundamentals of the tournament model and solve for the equilibrium, which lays a foundation for the analysis of optimal information policy.

## 2.1 Model

We consider a firm with a manger and two risk-neutral employees, index by $i \in \{A, B\}$. The two employees compete for a prize—i.e., promotion—by exerting irreversible efforts simultaneously. We assume a lottery *contest success function* (CSF) to model the competition in the inside-firm labor market: For an effort profile $(x_A, x_B) \geq (0, 0)$, an employee $i$ wins with a probability[4]

$$p_i(x_A, x_B) = \begin{cases} x_i/(x_A + x_B) & \text{if } x_A + x_B > 0, \\ 1/2 & \text{if } x_A + x_B = 0. \end{cases}$$

An employee $i$ values the win for $v_i > 0$, with $v_A$ to be commonly known and $v_B$ to be a piece of private information. Specifically, $v_B$ is a random variable on the set $\Omega = \{v_B^L, v_B^H\}$ with $0 < v_B^L < v_B^H$ and $\Pr(v_B = v_B^H) = \mu \in (0, 1)$. For ease of exposition, we interpret one's value for the win as his ability: A more motivated worker is better incentivized to engage in effort. We impose the following assumption throughout the paper:

**Assumption 1** $v_B^L \geq v_A/4$.

Assumption 1 is intuitive. It ensures that the competition would not be excessively lopsided even if employee $B$ is of the low type, which rules out the case of corner solution in which a low-ability employee $B$ is completely discouraged to exert effort in equilibrium.

One's effort $x_i$ entails a unity marginal effort cost. An employee $i$ chooses his effort to maximize his expected payoff

$$\pi_i(x_i, x_j) = p_i(x_A, x_B)v_i - x_i, \ i, j \in \{A, B\}, i \neq j.$$

We assume that the manager possesses the correct prior $\mu$ while employee $A$ believes

---

[4]A closed-form equilibrium solution to the model is not available if we assume a CSF in the form of $(x_i)^\gamma/[(x_A)^\gamma + (x_B)^\gamma]$, with $\gamma \in (0, 1]$. Simulation shows that our results remain qualitatively unchanged if $0 < \gamma < 1$. The analysis is available from the authors upon request.

$\Pr(v_B = v_B^H) = \tilde{\mu} \in [0, 1]$.[5] It is common knowledge that the manager and employee $A$ may hold different priors about $v_B$; that is, they "agree to disagree." When $\tilde{\mu} < \mu$, employee $A$ underestimates his opponent, and we say that employee $A$ exhibits overconfidence; when $\tilde{\mu} > \mu$, he overestimates his opponent, which alludes to underconfidence.

**Remark 1** *It is useful to note that the model is isomorphic to an alternative setting in which employees commonly value the prize from winning the tournament—i.e., $v_i = v$ for $i \in \{A, B\}$—but bear different (linear) effort costs (e.g., Moldovanu, Sela, and Shi, 2007; Taylor and Yildirim, 2011; Brown and Minor, 2014). One's payoff function is given by*

$$\tilde{\pi}_i(x_i, x_j) = p_i(x_A, x_B)v - c_i x_i, \ i, j \in \{A, B\}, i \neq j,$$

*and maximizing $\tilde{\pi}_i(x_i, x_j)$ is equivalent to maximizing*

$$\frac{\tilde{\pi}_i(x_i, x_j)}{c_i} = p_i(x_A, x_B)\frac{v}{c_i} - x_i,$$

*which restores the original game considered in our paper.*

## 2.2 Equilibrium in Tournament

Zhang and Zhou (2016) fully characterize the equilibrium of a lottery contest game with one-sided incomplete information, and the analysis extends to our setting. In the equilibrium, employee $A$ exerts effort

$$x_A = \left( \frac{\frac{1-\tilde{\mu}}{\sqrt{v_B^L}} + \frac{\tilde{\mu}}{\sqrt{v_B^H}}}{\frac{1}{v_A} + \frac{1-\tilde{\mu}}{v_B^L} + \frac{\tilde{\mu}}{v_B^H}} \right)^2,$$

and employee $B$ has a type-dependent effort strategy indeterminately, which is given as follows:

$$x_B(v_B) = \sqrt{v_B x_A} - x_A, \text{ for } v_B \in \{v_B^H, v_B^L\}.$$

For notational convenience, we define $K(\tilde{\mu}) := \sqrt{x_A}$. It is straightforward to write down the ex ante expected total effort of the tournament, which we denote by $TE(\mu, \tilde{\mu})$, as

$$TE(\mu, \tilde{\mu}) = \mathbb{E}_\mu \left[ x_B(v_B) + x_A \right] = \mathbb{E}_\mu \left[ \sqrt{v_B x_A} \right] = \left[ (1-\mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\tilde{\mu}). \quad (1)$$

We use the notation $\mathbb{E}_\mu[\cdot]$ to denote the expectation under belief $\mu$. It is noteworthy that employees' equilibrium efforts, $x_A$ and $x_B(v_B)$, involve only employee $A$'s perceived belief $\tilde{\mu}$.

---

[5]Note that employee $B$'s belief about $v_B$ does not matter in our model because (i) he has private information about $v_B$; and (ii) he only cares about employee $A$'s effort.

However, both $\mu$ and $\tilde{\mu}$ enter the expression of the an ex ante expected total effort $TE(\mu, \tilde{\mu})$, as it is aggregated over the true distribution described by $\mu$.

We now explore the property of $K(\tilde{\mu})$. Taking derivative of $K(\tilde{\mu})$ with respect to $\tilde{\mu}$ yields

$$K'(\tilde{\mu}) = \frac{\left(\sqrt{v_B^H} - \sqrt{v_B^L}\right)\left(v_A - \sqrt{v_B^L v_B^H}\right)}{v_A v_B^L v_B^H \left[\frac{1}{v_A} + \frac{v_B^H(1-\tilde{\mu}) + v_B^L \tilde{\mu}}{v_B^L v_B^H}\right]^2}.$$

Note that the sign of $K'(\tilde{\mu})$ depends on that of $v_A - \sqrt{v_B^L v_B^H}$. Further, we can obtain that

$$K''(\tilde{\mu}) = \frac{2\left(\sqrt{v_B^H} - \sqrt{v_B^L}\right)^2 \left(v_A - \sqrt{v_B^L v_B^H}\right)}{v_A \left(v_B^L v_B^H\right)^2 \left[\frac{1}{v_A} + \frac{v_B^H(1-\tilde{\mu}) + v_B^L \tilde{\mu}}{v_B^L v_B^H}\right]^3}.$$

Again, the sign of $K''(\tilde{\mu})$ depends on that of $v_A - \sqrt{v_B^L v_B^H}$. It is straightforward to obtain the following.

**Lemma 1** *The function $K(\cdot)$ is strictly increasing with its argument and convex if $v_A > \sqrt{v_B^L v_B^H}$, and is strictly decreasing and concave if $v_A < \sqrt{v_B^L v_B^H}$.*

## 2.3 Desirability of Persistent Misperception

Employees' efforts accrue to the benefit of the manager in our context. The equilibrium result allows us to explore one natural question: Does the firm benefit from employee $A$'s misperception, i.e., $\mu \neq \tilde{\mu}$? Specifically, does the persistence of the uncommon priors boost the productivity of organization in terms of its expected total effort $TE(\mu, \tilde{\mu})$? Recall by (1) that the tournament generates an expected total effort

$$TE(\mu, \tilde{\mu}) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right] K(\tilde{\mu}).$$

With common prior, the expected total effort boils down to

$$TE(\mu, \mu) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right] K(\mu),$$

as in Zhang and Zhou (2016). Therefore, the comparison hinges on the monotonicity of $K(\cdot)$. We obtain the following.

**Proposition 1 (Value of Persistent Misperception)** *Suppose that the firm aims to maximize the expected total effort in the tournament. Then the following statements hold:*

i. When $v_A < \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $TE(\mu, \tilde{\mu}) > TE(\mu, \mu)$—if and only if employee A exhibits overconfidence—i.e., $\tilde{\mu} < \mu$;

ii. When $v_A > \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $TE(\mu, \tilde{\mu}) > TE(\mu, \mu)$—if and only if employee A exhibits underconfidence—i.e., $\tilde{\mu} > \mu$;

iii. When $v_A = \sqrt{v_B^H v_B^L}$, employee A's belief does not affect the expected total effort, i.e., $TE(\mu, \tilde{\mu}) = TE(\mu, \mu)$.

Proposition 1 states that the firm may either benefit or suffer from the incumbent employee's perceptional bias; neither overconfidence nor underconfidence necessarily harms the firm. To interpret its logic, recall that the new hire's type-dependent equilibrium effort is given by

$$x_B(v_B) = \sqrt{v_B x_A} - x_A, \text{ for } v_B \in \left\{v_B^H, v_B^L\right\},$$

which leads to the expected total effort

$$TE(\mu, \tilde{\mu}) = \mathbb{E}_\mu\left[x_B(v_B) + x_A\right] = \sqrt{x_A} \times \mathbb{E}_\mu\left(\sqrt{v_B}\right) = K(\tilde{\mu})\left[(1-\mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right].$$

To explore the impact of the incumbent employee's belief on the total effort $TE(\mu, \tilde{\mu})$, it suffices to focus on how $x_A$ varies with $\tilde{\mu}$, i.e., the property of $K(\tilde{\mu})$. The conventional wisdom in the contest/tournament literature is that a more level playing field fuels competition. In the case of $v_A < \sqrt{v_B^H v_B^L}$, employee A can be viewed as an ex ante underdog. Proposition 1(i) shows that in this case, his overconfidence turns out to boost his morale, which narrows the gap in terms of ability and fuels the competition. Conversely, Proposition 1(ii) states that, if employee A is the favorite in the sense that $v_A > \sqrt{v_B^H v_B^L}$, the the firm suffers from his overconfidence: Employee A underestimates his opponent, which softens the competition and entices himself to slack off. In the knife-edge case of $v_A = \sqrt{v_B^H v_B^L}$, these balancing forces cancel out in the ex ante even race.

The contest/tournament literature has conventionally espoused the productive role played by various design instruments that manipulate the balance of competition—e.g., favoritisms (Epstein, Mealem, and Nitzan, 2011; Franke, Kanzow, Leininger, and Schwartz, 2013, 2014; Fu and Wu, 2020, among others), headstarts (Kirkegaard, 2012; Konrad, 2002; Siegel, 2009; Drugov and Ryvkin, 2017, among others), and bidding caps (Che and Gale, 1998; Gavious, Moldovanu, and Sela, 2002; Olszewski and Siegel, 2019, among others).[6] Our analysis implies that the same can alternatively be achieved by a perceptional bias, and debiasing may turn out to weaken the competition and mute employees' incentives.

---

[6] See Mealem and Nitzan (2016), Chowdhury, Esteve-González, and Mukherjee (2019), and Fu and Wu (2019a) for comprehensive surveys on discrimination in contests.

## 2.4    Managerial Implications of Proposition 1

Our analysis demonstrates the subtle roles played by employees' perceptional biases. It is broadly championed that confidence catalyzes success, and managers should build confidence in his staff. The economics and psychology literature has also identified the motivational effect that advocates the positive incentive effect of overconfidence. We, however, show that employees' incentives and productivities depend indeterminately on their (mis)perception about relative competitiveness when they engaged in internal competitions, which are pervasive in modern workplace (Netessine and Yakubovich, 2012).

Proposition 1 demonstrates that employees' (mis)perception can be either productive or counterproductive, depending on the actual relative competitiveness between the incumbent and the new worker. The firm may sometimes benefit from persistent underconfidence. Consider, for instance, a startup that rose from successful grassroots innovations. Its early employees could underestimate their own abilities relative to better educated junior recruits, despite the extensive experience and knowhow they possess. Proposition 1 suggests that the firm may not have to "debias" even if it is able to: For instance, if the firm is confident in the value of its early employees' human capital—i.e., $v_A > \sqrt{v_B^H v_B^L}$—which might have been critical in helping the firm navigate the startup stages, then underconfidence would turn out to incentivize employees and fuel more competition. In contrast, consider an ambitious academic institution in the process of aggressive expansion by recruiting from more prestigious peers. Its faculty members may be on average disadvantaged in their research capacity ex ante, i.e., $v_A < \sqrt{v_B^H v_B^L}$, but also underconfident about their skills relative to the new hires. Proposition 1 then suggests that it is helpful to restore the confidence of the incumbent faculty.

# 3    Internal Evaluation and Information Disclosure

In this section, we expand the model to explore the optimal information disclosure policy that modifies the information environment. The firm sets an information disclosure policy prior to the competition. For the moment, we assume that the firm equally values employees' contribution and the policy is chosen to maximize the expected total effort.[7]

The firm conducts an internal evaluation on employee $B$ and obtains a noisy signal $s \in \{H, L\}$ regarding his ability. Specifically, we assume that the signal is drawn as follows:

$$\Pr\left(s = H \mid v_B = v_B^H\right) = \Pr\left(s = L \mid v_B = v_B^L\right) = q, \tag{2}$$

---

[7]We consider an extension in which the manager cares about the expected winner's effort in Section 4.2.

where $q \in \left(\frac{1}{2}, 1\right]$ indicates the quality of the signal.[8] When $q = 1$, the signal perfectly reveals employee $B$'s ability. In the extreme case that $q = 1/2$, the firm's signal is completely uninformative. The manager commits prior to the competition her disclosure policy, i.e., whether the result of her private evaluation about employee $A$'s ability—i.e., the realized signal $s$—is to be disclosed publicly or concealed from employee $A$ before the competition takes place.

The signal would allow the manager and employee $A$ to update their beliefs based on their own prior. For the manager, she would infer that employee $B$ is of high type with a posterior probability $\mu_s$, as given by

$$\mu_s = \frac{\mu \Pr\left(s | v_B = v_B^H\right)}{\mu \Pr\left(s | v_B = v_B^H\right) + (1 - \mu) \Pr\left(s | v_B = v_B^L\right)}, \text{ for } s = H, L. \tag{3}$$

Similarly, employee $A$'s posterior belief, denoted by $\tilde{\mu}_s$, is given by

$$\tilde{\mu}_s = \frac{\tilde{\mu} \Pr\left(s | v_B = v_B^H\right)}{\tilde{\mu} \Pr\left(s | v_B = v_B^H\right) + (1 - \tilde{\mu}) \Pr\left(s | v_B = v_B^L\right)}, \text{ for } s = H, L. \tag{4}$$

It is straightforward to verify that both $\mu_s$ and $\tilde{\mu}_s$ strictly increase with the priors, $\mu$ and $\tilde{\mu}$, respectively, for $q < 1$. When the signal is perfectly informative—i.e., $q = 1$—both parties' posterior belief would jump to one upon receiving $s = H$ and drop to zero upon receiving $s = L$, independent of their priors.

## 3.1 Optimal Information Disclosure Policy

The manager sets the information disclosure policy to maximize employees' expected aggregate effort. We denote by $TE^C(\mu, \tilde{\mu})$ the expected total effort when the signal $s$ is withheld, where the superscript $C$ indicates "concealment." The expected total effort $TE^C(\mu, \tilde{\mu})$ is the same as (1) and given by

$$TE^C(\mu, \tilde{\mu}) = \left[(1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right] K(\tilde{\mu}). \tag{5}$$

When the signal $s \in \{H, L\}$ is disclosed, the expected total effort is given by

$$TE\left(\mu_s, \tilde{\mu}_s\right) = \left[(1 - \mu_s)\sqrt{v_B^L} + \mu_s\sqrt{v_B^H}\right] K(\tilde{\mu}_s).$$

---

[8]Note that $q$ is exogenous in this section. We will generalize the model and endogenize the information structure using a Bayesian persuasion approach (e.g., Kamenica and Gentzkow, 2011, Alonso and Camara, 2016) in Section 4.1.

Further, the actual probabilities that $s = H$ and $s = L$ occur amount to $\mu q + (1 - \mu)(1 - q)$ and $\mu(1 - q) + (1 - \mu)q$, respectively. This allows us to calculate the expected equilibrium total effort when the manager commits to disclosing the signal, $TE^D(\mu, \tilde{\mu})$, where we use superscript $D$ to indicate "disclosure":

$$
\begin{aligned}
TE^D(\mu, \tilde{\mu}) \;=\; & \left[\mu q + (1 - \mu)(1 - q)\right] \times \left[(1 - \mu_H)\sqrt{v_B^L} + \mu_H\sqrt{v_B^H}\right] K(\tilde{\mu}_H) \\
& + \left[\mu(1 - q) + (1 - \mu)q\right] \times \left[(1 - \mu_L)\sqrt{v_B^L} + \mu_L\sqrt{v_B^H}\right] K(\tilde{\mu}_L), \quad (6)
\end{aligned}
$$

where $\mu_s$ and $\tilde{\mu}_s$, with $s \in \{H, L\}$, are given by (3) and (4), respectively.

We then investigate the manager's incentive to disclose the result of her noisy evaluation to employee $A$, holding fixed the quality of the signal, $q$. For expositional convenience, we define $\Theta$ as

$$
\Theta := \left[\sqrt{v_B^H v_B^L} - v_A\right] \times \left[\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)}\right]. \quad (7)
$$

**Proposition 2 (Concealment vs. Disclosure)** *Suppose $q \in (\frac{1}{2}, 1]$ and that the manager aims to maximize the expected total effort in the tournament. Then the following statements hold:*

*i. When $\Theta > 0$, it is optimal to commit to disclosing her private signal, i.e., $TE^D(\mu, \tilde{\mu}) > TE^C(\mu, \tilde{\mu})$;*

*ii. When $\Theta < 0$, concealing the signal is optimal to the manager, i.e., $TE^D(\mu, \tilde{\mu}) < TE^C(\mu, \tilde{\mu})$;*

*iii. When $\Theta = 0$, the firm is indifferent between disclosing the signal and concealing it, i.e., $TE^D(\mu, \tilde{\mu}) = TE^C(\mu, \tilde{\mu})$.*

Proposition 2 states that the optimal information disclosure policy hinges on the sign of $\Theta$. To interpret this proposition, it is key to identify the condition that determines the sign of $\Theta$. Note that that the second term in (7) is always negative when employ $A$ exhibits (weak) overconfidence, i.e., $\tilde{\mu} \le \mu$. To see that, note that $[\mu(1 - \tilde{\mu})]/[\tilde{\mu}(1 - \mu)] \ge 1$ in this case, which in turn implies that

$$
\frac{(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)} - \frac{\mu(1 - \tilde{\mu})}{\tilde{\mu}(1 - \mu)} \;\le\; \frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - 1 = \frac{v_A(v_B^L - v_B^H)}{v_B^H(v_A + v_B^L)} < 0.
$$

This observation allows us to infer that with overconfidence ($\tilde{\mu} < \mu$) or rational belief ($\tilde{\mu} = \mu$), disclosure is optimal if $\sqrt{v_B^H v_B^L} - v_A < 0$, or equivalently, employee $A$ is an ex ante favorite;

12

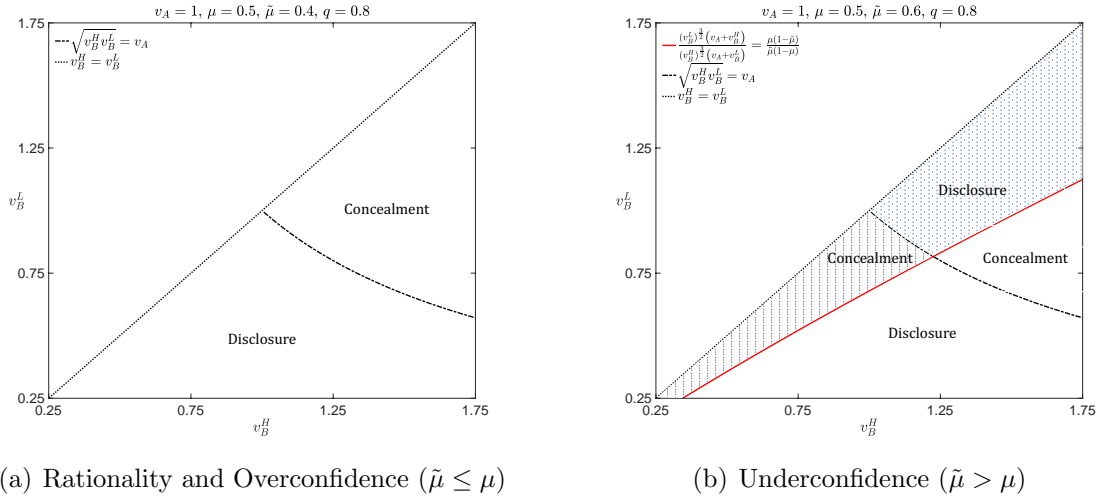(a) Rationality and Overconfidence ($\tilde{\mu} \leq \mu$)  (b) Underconfidence ($\tilde{\mu} > \mu$)

Figure 1: Optimal Effort-Maximizing Information Disclosure Policy

conversely, concealment is optimal if $\sqrt{v_B^H v_B^L} - v_A > 0$, or equivalently, employee $A$ is an ex ante underdog.

The optimum is illustrated in Figure 1(a). In the figure, the horizontal axis traces $v_B^H$, while the vertical axis measures $v_B^L$. Therefore, the area under the diagonal collects all the relevant parameterizations with $v_B^L < v_B^H$. Assuming $(v_A, \mu, \tilde{\mu}) = (1, 0.5, 0.4)$, the dashed curve splits the area into two regions: The upper portion depicts the case of $\sqrt{v_B^H v_B^L} - v_A > 0$ such that $\Theta < 0$, in which concealment policy is preferred; while the lower portion represents $\sqrt{v_B^H v_B^L} - v_A < 0$ such that $\Theta > 0$, in which case full disclosure prevails.

Complexity arises in the scenario of underconfidence. The sign of $\sqrt{v_B^H v_B^L} - v_A$ alone cannot predict the sign of $\Theta$, as the second term in the expression of (7) is indeterminate when $\tilde{\mu} > \mu$. The optimal disclosure policy is depicted by Figure 1(b). The division between $\sqrt{v_B^H v_B^L} - v_A < 0$ and $\sqrt{v_B^H v_B^L} - v_A > 0$ is insufficient to predict the optimal disclosure policy. The terms of $\sqrt{v_B^H v_B^L} - v_A$ and $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ jointly determine the sign of $\Theta$, with a total of four scenarios. Fixing $(v_A, \mu, \tilde{\mu}) = (1, 0.5, 0.6)$, the solid solid curve in the figure traces all parameterizations that satisfy $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] = [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$. The area above the solid curve depicts the case of $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] > 0$, in which case the optimum with overconfidence or rational belief is overturned. In the area below the solid curve, the term continues to be negative, which retains the prediction under overconfidence or rational belief.

Proposition 2 and Equation (7) enable comparative statics with respect to the degree of
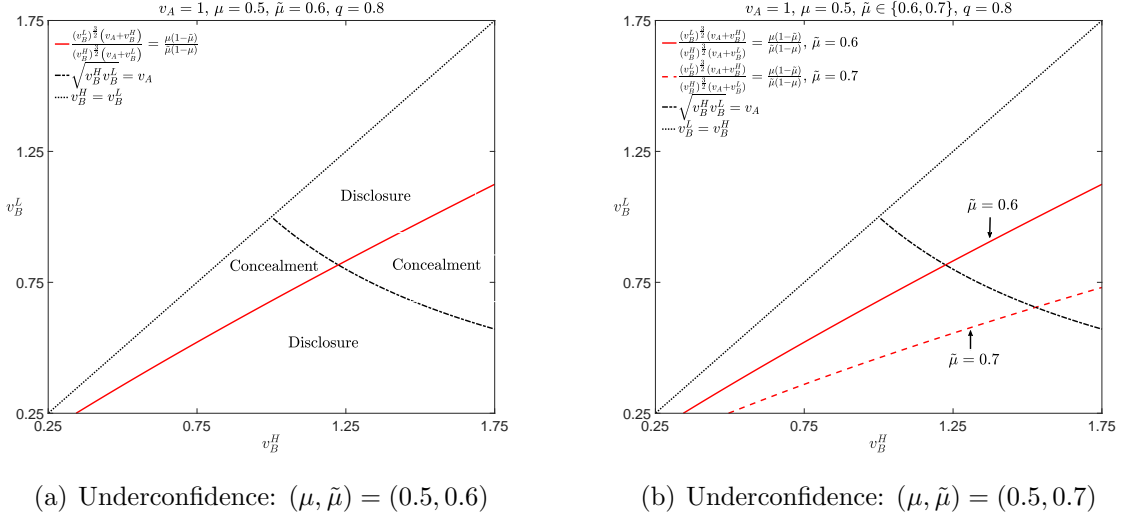
(a) Underconfidence: $(\mu, \tilde{\mu}) = (0.5, 0.6)$

(b) Underconfidence: $(\mu, \tilde{\mu}) = (0.5, 0.7)$

Figure 2: Impact of Underconfidence on Optimal Information Disclosure Policy

employee underconfidence. Fix $v_A$ and $\tilde{\mu} > \mu$. Let us define

$$
\Upsilon(\tilde{\mu}) := \left\{ (v_B^H, v_B^L) \,\middle|\, \frac{(v_B^L)^{\frac{3}{2}} \left(v_A + v_B^H\right)}{(v_B^H)^{\frac{3}{2}} \left(v_A + v_B^L\right)} - \frac{\mu \left(1 - \tilde{\mu}\right)}{\tilde{\mu} \left(1 - \mu\right)} > 0, v_B^H > v_B^L \geq \frac{v_A}{4} \right\},
$$

as the set of parameters $(v_B^H, v_B^L)$ under which the optimal information disclosure policy with underconfidence differ from that with overconfidence or rational belief. The following proposition can be obtained:

**Proposition 3 (Impact of Increasing Underconfidence)** *Suppose that $\tilde{\mu}^\dagger > \tilde{\mu} > \mu$. Then $\Upsilon(\tilde{\mu}) \subset \Upsilon(\tilde{\mu}^\dagger)$ and the inclusion is strict.*

By Equation (7), for given $(v_A, v_B^L, v_B^H)$, the sign of $\Theta$ would be determined by the size of $\tilde{\mu}$ relative to $\mu$ in the case of underconfidence. For a $\tilde{\mu}$ mildly above $\mu$, i.e., moderate underconfidence, the optimum is more likely to coincide with that under overconfidence or rational belief, as the sign of $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ remains negative. With severe underconfidence in place, i.e., a large $\tilde{\mu}$ relative to $\mu$, the sign would turn positive, and the optimum under overconfidence or rational belief would be overturned.

Figure 2 illustrates how a change in the degree of underconfidence affects the optimal information disclosure policy, which confirms the observation from Proposition 3. Figure 2(a) depicts the same scenario as in Figure 1(b), which shows the optimum under underconfidence with $(\mu, \tilde{\mu}) = (0.5, 0.6)$. Recall that the area above the solid curve depicts the case of $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] > 0$, which causes the optimum to divert from that with overconfidence and rational belief. In Figure 2(b), we demonstrate

14

the comparative statics when $\tilde{\mu}$ increases from from 0.6 to 0.7. The curve that defines $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)] = 0$ is shifted downward, with the lower dashed curve representing the case of $\tilde{\mu} = 0.7$. Because $[\mu(1 - \tilde{\mu})]/[\tilde{\mu}(1 - \mu)]$ strictly decreases with $\tilde{\mu}$, the term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ is more likely to be positive following such an increase in $\tilde{\mu}$, which enlarges the set of parameterizations under which the optimal information disclosure policy differs from that when the incumbent is overconfident or has rational belief.

## 3.2 Managerial Implications of Propositions 2 and 3

Our results in Section 3.1 provide a playbook for firms' internal information management. The optimal information disclosure policy sensitively depends on the specific environment, which can be summarized as follows:

| | Overconfidence | Moderate Underconfidence | Significant Underconfidence |
|---|---|---|---|
| Weak Incumbent $(v_A < \sqrt{v_B^H v_B^L})$ | Concealment | Concealment | Disclosure |
| Strong Incumbent $(v_A > \sqrt{v_B^H v_B^L})$ | Disclosure | Disclosure | Concealment |

The table demonstrates that the optimal disclosure policy depends solely on employees' ex ante relative competitiveness—i.e., the comparison between $v_A$ and $\sqrt{v_B^H v_B^L}$—when the incumbent employee is overconfident or has rational beliefs. However, additional cautions are required when the incumbent is underconfident: Mild underconfidence preserves the optimum under the previous case, while significant underconfidence overturns that.

Let us first consider the scenario of overconfidence. Imagine a rapidly-growing firm whose employees excessively attribute the firm's success to their own talent and contribution, and thus exhibit overconfidence. If the firm is confident in the quality of its search effort, i.e., $v_A < \sqrt{v_B^H v_B^L}$, then Proposition 2 would recommend that the firm refrain from granting to its employees the access to information about their peers, as the table shows. Conversely, imagine a seasoned teaching star in a business school: The wealth of classroom experience and industry knowledge accumulated over the years not only ensures reliable delivery in teaching, but also breeds complacency. Proposition 2, as well as the table, clearly indicates that allowing the faculty members to access peers' teaching feedback report may increase the school's aggregate teaching quality.[9]

Next, let us consider a case of underconfidence. Consider the example of a startup that poaches a veteran executive from an industry leader to upgrade its managerial talent. The early employees may grossly overestimate the external hire who possesses a stellar career

---

[9]The practice of NUS business school exemplifies a transparent internal feedback and competitive performance evaluation system. See Introduction and Footnote 3 for details.

record, which alludes to severe underconfidence; by Propositions 2 and 3, the firm should embrace a transparent internal information management. The recommendation appears to be counterintuitive at the first glance. In this scenario, an early employee suffers from both deficiency in competency and a severe lack of confidence. When additional observation from the evaluation allows him to infer more about relative competitiveness, his morale can either be elevated or degraded, depending on the realization of the signal. The possible boost in his confidence, however, ex ante outweighs the possible "bust." The logic will be further unveiled when we delve in depth the underlying logic for our results in the next subsection.

## 3.3 Intuition for Propositions 2 and 3

We now interpret the logic that underlies Propositions 2 and 3. We mainly focus on the economic forces that drive Proposition 2; the intuition for Proposition 3 naturally ensues. Recall by Proposition 2, the optimal information disclosure policy under rational belief coincides with that under overconfidence, but may not for the case of underconfidence. We begin with the benchmark case of common prior and consider the role played by information disclosure without the complications caused by employee's misperception. We then elaborate on the role played by misperception. We label the effect from the former source an *information effect*, while that from the latter a *morale effect*. The combination of the two forces determines the optimum depicted in Proposition 2. Equation (7)—which explains how the optimum with underconfidence may depart from that with overconfidence or rational belief—is underpinned by the morale effect. A rationale about the morale effect would then shed light on Equation (7), which we elaborate on at the end of this section.

**Common Prior: Information Effect**   Upon observing the signal $s \in \{H, L\}$, employee $A$ updates his belief about employee $B$'s type; the additional information leads his belief to be revised either upward or downward, depending on the realization of the signal. The Bayesian updating causes the equilibrium in the tournament to diverge across states.

The dispersion across states triggered by the signal occurs regardless of the perceptional bias. We thus focus on the case of common prior—i.e., $\mu = \tilde{\mu}$—to illustrate its nuance. Our rationale is largely aligned with that in Zhang and Zhou (2016). Define

$$TE_R^C(\mu) := TE^C(\mu, \mu) = \left[ (1 - \mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\mu),$$

the expected total effort for the case of concealment, where the subscript $R$ indicates the rational benchmark. When the signal is revealed, employee $A$'s belief will be revised to either $\mu_H$ or $\mu_L$, and the expected total effort of the tournament ends up as either $TE_R^C(\mu_H)$

or $TE_R^C(\mu_L)$; the corresponding ex ante expected total effort with common prior—which is similarly defined as $TE_R^D(\mu, \tilde{\mu}) := TE^D(\mu, \mu)$—must aggregate over the states.

Simple algebra would verify that

$$\frac{d\,TE_R^C}{d\mu} = \left(\sqrt{v_B^H} - \sqrt{v_B^L}\right) K(\mu) + \left[(1-\mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right] K'(\mu),$$

and

$$\frac{d^2\,TE_R^C}{d\mu^2} = 2\left(\sqrt{v_B^H} - \sqrt{v_B^L}\right) K'(\mu) + \left[(1-\mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H}\right] K''(\mu).$$

Recall from Lemma 1 that (i) $K(\mu)$ is increasing if $v_A > \sqrt{v_B^L v_B^H}$, and decreases if $v_A < \sqrt{v_B^L v_B^H}$; and (ii) $K(\mu)$ is convex if $v_A > \sqrt{v_B^L v_B^H}$ and concave if $v_A < \sqrt{v_B^L v_B^H}$. Hence, $TE_R^C(\mu)$ perfectly reserves the concavity/convexity of $K(\mu)$.

Carrying out the algebra, we can obtain the ex ante expected total effort

$$TE_R^D(\mu) = \left[\mu q + (1-\mu)(1-q)\right] TE_R^C(\mu_H) + \left[\mu(1-q) + (1-\mu)q\right] TE_R^C(\mu_L).$$

The informative signal causes the posterior to disperse and deviate from the prior $\mu$, with $\mu_H > \mu > \mu_L$. Because $\left[\mu q + (1-\mu)(1-q)\right]\mu_H + \left[\mu(1-q) + (1-\mu)q\right]\mu_L \equiv \mu$ by the martingale property of beliefs, the function $TE_R^D(\mu)$ is simply a weighted average of $TE_R^C(\mu)$ over two different states. As a result, the comparison depends on the concavity/convexity of the function $TE_R^C(\mu)$. We can immediately infer the following by Jensen's inequality.

**Remark 2** $TE_R^D(\mu) > (<)TE_R^C(\mu)$ if and only if $TE_R^C(\mu)$ is strictly convex (concave).

That is, full disclosure (concealment) outperforms concealment (full disclosure) if and only if employee $A$ is an ex ante favorite (underdog), which explains Proposition 2 for the case of $\mu = \tilde{\mu}$.

**Uncommon Priors: Morale Effect** We now explore the case of uncommon priors, i.e., $\mu \neq \tilde{\mu}$. We need to compare $TE^C(\mu, \tilde{\mu})$ as in (5) to $TE^D(\mu, \tilde{\mu})$ as in (6). For the sake of expositional convenience, we focus on the case of $\mu = 1/2$, which implies that the ex ante probabilities of receiving $s = H$ and $s = L$ do not depend on $q$, and are both equal to $1/2$. As a result, $TE^C(\mu, \tilde{\mu})$ and $TE^D(\mu, \tilde{\mu})$ can be, respectively, simplified as

$$TE^C\left(\frac{1}{2}, \tilde{\mu}\right) = \frac{1}{2}\left[\sqrt{v_B^L} + \sqrt{v_B^H}\right] K(\tilde{\mu}), \text{ and}$$

$$TE^D\left(\frac{1}{2}, \tilde{\mu}\right) = \frac{1}{2}\left[(1-q)\sqrt{v_B^L} + q\sqrt{v_B^H}\right] K(\tilde{\mu}_H) + \frac{1}{2}\left[q\sqrt{v_B^L} + (1-q)\sqrt{v_B^H}\right] K(\tilde{\mu}_L).$$

The comparison boils down to

$$
TE^D\left(\frac{1}{2},\tilde{\mu}\right) - TE^C\left(\frac{1}{2},\tilde{\mu}\right)
$$

$$
= \frac{1}{2}\left\{
\begin{array}{l}
\left[(1-q)\sqrt{v_B^L} + q\sqrt{v_B^H}\right] \times \left[K(\tilde{\mu}_H) - K(\tilde{\mu})\right] \\[2mm]
- \left[q\sqrt{v_B^L} + (1-q)\sqrt{v_B^H}\right] \times \left[K(\tilde{\mu}) - K(\tilde{\mu}_L)\right]
\end{array}
\right\}.
$$

Upon observing the signal $s$, employee $A$ updates his belief $\tilde{\mu}$, which affects his effort incentive in the tournament. His perception can be shifted either upward or downward. In other words, employee $A$'s morale can be either boosted—i.e., $\tilde{\mu}$ dropping to $\tilde{\mu}_L$—or be busted—i.e., $\tilde{\mu}$ rising to $\tilde{\mu}_H$. The comparison highlighted above hinges on the change of $\left[K(\tilde{\mu}_H) - K(\tilde{\mu})\right]$ vis-à-vis $\left[K(\tilde{\mu}) - K(\tilde{\mu}_L)\right]$. The magnitude of his belief adjustment in response to a given signal depends on the nature of his initial misperception, i.e., whether employee $A$ exhibits overconfidence or underconfidence.

Suppose that employee $A$ is overconfident, so he underestimates his opponent, i.e., $\tilde{\mu} < \mu$. His posterior tends to respond to a high signal more sensitively—i.e., with a significant jump above from the initially underestimated $\tilde{\mu}$ to $\tilde{\mu}_H$—compared to the response to a low signal, i.e., a relatively mild decrease from $\tilde{\mu}$ to $\tilde{\mu}_L$. This follows from the properties of Bayesian updating: new signal impacts the posterior more if it is more unexpected under the prior.[10] Thus in the case of overconfidence, the incumbent's perception about the competitor would be substantially revised upward when a high signal refutes his initial underestimate of the competitor, while the revision would be more incremental when a low signal simply reinforces the existing bias. The opposite holds for the case of underconfidence with $\tilde{\mu} > \mu$, but the intuition is analogous. The upward revision of the posterior in response to a high signal tends to be muted compared to that in the presence of a low signal. A low signal would sharply overturn the initial overestimates, causing a significant drop from $\tilde{\mu}$ to $\tilde{\mu}_L$; in contrast, a high signal only confirms the initial overestimate, so the rise from $\tilde{\mu}$ to $\tilde{\mu}_H$ tends to be moderate.

For expositional efficiency, let us focus on the case of $v_A > \sqrt{v_B^L v_B^H}$, as the case of $v_A < \sqrt{v_B^L v_B^H}$ is simply its mirror image. Recall that in this case $K(\cdot)$ is strictly increasing in its argument by Lemma 1, and $K(\tilde{\mu}_H) - K(\tilde{\mu})$ and $K(\tilde{\mu}) - K(\tilde{\mu}_L)$ are both positive. Further, $TE^D\left(\frac{1}{2},\tilde{\mu}\right) - TE^C\left(\frac{1}{2},\tilde{\mu}\right)$ is positive when $\tilde{\mu} = \mu = 1/2$ by the information effect.

With overconfidence, the argument laid out above implies that $K(\tilde{\mu}_H) - K(\tilde{\mu})$ tends to outweigh $K(\tilde{\mu}) - K(\tilde{\mu}_L)$: A high signal overturns his initial misperception, while a low signal marginally confirms his bias. This implies $TE^D(\frac{1}{2},\tilde{\mu}) - TE^C(\frac{1}{2},\tilde{\mu})$ tends to be positive, and thus information disclosure outperforms concealment. Clearly, when employee $A$ is an ex

---

[10]This property of Bayesian updating is also exploited in Fang and Moscarini (2005) in a principal-agent setting, where they refer to this effect the *morale hazard*.

ante favorite, the effect caused by asymmetric response in his morale triggered by a high or a low signal coincides with the information effect laid out above. Therefore, the comparison between disclosure and concealment under overconfidence remains the same as that under rationality, as Figure 1(a) shows.

Consider, alternatively, the case of underconfidence. Although both $K(\tilde{\mu}_H) - K(\tilde{\mu})$ and $K(\tilde{\mu}) - K(\tilde{\mu}_L)$ are positive, $K(\tilde{\mu}_H) - K(\tilde{\mu})$ tends to be outsized by $K(\tilde{\mu}) - K(\tilde{\mu}_L)$: In this case, a low signal tends to overturn the initial underconfidence, whereas a high signal only mildly endorses the misperception. As a result, $TE^D(\frac{1}{2}, \tilde{\mu}) - TE^C(\frac{1}{2}, \tilde{\mu})$ is less likely to be positive, and thus concealment is more likely to prevail. The morale effect runs into conflicts with the aforementioned information effect and could outweigh the latter and overturn the optimum, as Figure 1(b) depicts.

Intuitively, the more biased the belief, the stronger this morale effect. This rationale thus sheds light on the observation of Proposition 3: The result implies that a larger $\tilde{\mu}$ relative to $\mu$—which alludes to more significant underconfidence—may overturn the optimum under overconfidence or rational belief. Recall, again, that the optimum depends on the sign of $\Theta$ as defined in (7). As mentioned previously, the term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ is always negative with overconfidence, which is in line with the case of rational belief. The optimum under rational belief depends entirely on the sign of $\sqrt{v_B^H v_B^L} - v_A$, which captures the information effect. The term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] - [\mu(1 - \tilde{\mu})/\tilde{\mu}(1 - \mu)]$ encapsulates the morale effect, and it plays a nontrivial role when a larger $\tilde{\mu}$ relative to $\mu$ is present, in which case the significant underconfidence amplifies the morale effect and more than offsets the information effect, as shown in Proposition 3.

# 4    Extensions and Discussion

In this section, we consider two variations to the baseline model. We first apply a Bayesian persuasion approach to endogenize the information structure of the internal evaluation. We then explore a setting in which the manager is concerned about the expected winner's effort instead of total effort. Finally, we elaborate on the managerial implications our results may contribute.

## 4.1    Optimal Design of Internal Evaluation

Thus far, we have assumed that the quality of the internal evaluation—i.e., $q$—is exogenous. In practice, a firm has the discretion to set the scope and format of the evaluation in workplace or choose the evaluator, which presumably affects the quality of the exercise. For instance, a more experienced supervisor can assess his employee's ability more accurately.

19

We now allow the firm to flexibly design and precommit to the information structure of the evaluation exercise before the tournament begins, which is referred to as the Bayesian persuasion approach in the literature, pioneered by Kamenica and Gentzkow (2011). An information structure consists of a signal space $\mathcal{S}$ and a pair of likelihood distributions $\left\{\pi(\cdot|v_B^H), \pi(\cdot|v_B^L)\right\}$ over $\mathcal{S}$. We allow the manager to freely set the information structure of the evaluation; she is thus endowed with full control over the amount of information to be revealed through the evaluation and the form of signal to be disclosed to employees. Obviously, the evaluation exercise depicted in Section 3 involves a simple information structure with a binary signal space $\mathcal{S} = \{H, L\}$ and a conditional likelihood distribution for each underlying state—i.e., $v_B^H$ or $v_B^L$—parametrized by a variable $q$ [see Equation (2)].

In a seminal paper, Kamenica and Gentzkow (2011) show that searching for the optimal disclosure policy is equivalent to solving the concave closure of a value function defined on the set of all posteriors—i.e., $\mu_s$ with our notation—assuming that all agents share a common prior (i.e., $\tilde{\mu} = \mu$) over the underlying states. Recently, Alonso and Camara (2016) generalize the tools in Kamenica and Gentzkow (2011) and allow for heterogeneous priors. According to Alonso and Camara (2016), it is without loss of generality to consider a binary signal space in our setting, i.e., $\mathcal{S} = \{H, L\}$; the search for the optimal effort-maximizing signal structure $\left\{\pi(\cdot|v_B^H), \pi(\cdot|v_B^L)\right\}$ can be reduced to the following optimization problem:

$$\max_{\{\lambda, \mu_L, \mu_H\}} \lambda TE(\mu_H, \tilde{\mu}_H) + (1 - \lambda)TE(\mu_L, \tilde{\mu}_L) \tag{8}$$

subject to

$$\lambda \mu_H + (1 - \lambda)\mu_L = \mu, \tag{9}$$

$$\tilde{\mu}_s = \frac{t\mu_s}{t\mu_s + r(1 - \mu_s)}, \text{ for } s \in \{H, L\}, \tag{10}$$

$$0 \leq \lambda, \mu_H, \mu_L \leq 1, \tag{11}$$

where $r$ and $t$ are defined as $r := (1 - \tilde{\mu})/(1 - \mu)$ and $t := \tilde{\mu}/\mu$ respectively and capture the likelihood ratios of prior beliefs. As defined above, the variable $\mu_s$ in the objective function (8) is the manager's posterior about employee $B$'s ability inferred upon observing signal $s \in \{H, L\}$; $\tilde{\mu}_s$ in expression (10), accordingly, refers to employee $A$'s posterior.

Given the disagreed priors $(\mu, \tilde{\mu})$ and manager's belief $(\mu_L, \mu_H)$, employee $A$'s posterior belief can be derived from (10). When the manager and the employees share a common prior (i.e., $\tilde{\mu} = \mu$), we have $r = t = 1$ and they share the same Bayesian update (i.e., $\mu_s = \tilde{\mu}_s$ for $s \in \{H, L\}$). Condition (9) requires $\mathbb{E}_\mu(\mu_s) = \mu$, which is identical to the one in Kamenica and Gentzkow (2011) and is commonly referred to as the Bayes-plausibility (BP) constraint. Condition (11) simply requires that the posterior belief $\mu_H$ and $\mu_L$ and the probability $\lambda$ be bounded between zero and one. It is useful to point out a perfectly informative evaluation

corresponds to $(\mu_H, \mu_L) = (1, 0)$ with $\lambda = \mu$, and a completely uninformative evaluation (i.e., no information disclosure) corresponds to $(\mu_H, \mu_L) = (\mu, \mu)$ with $\lambda \in [0, 1]$.

Kamenica and Gentzkow (2011) and Alonso and Camara (2016) show that the indirect value function from the above maximization problem boils down to the value of the concave closure of $TE\big(\mu_s, \tilde{\mu}_s(\mu_s)\big)$ at the firm's prior $\mu$. Simple algebra yields the following:

$$TE\left(\mu_s, \tilde{\mu}_s(\mu_s)\right) = \left[(1 - \mu_s)\sqrt{v_B^L} + \mu_s\sqrt{v_B^H}\right] \times K\left(\frac{t\mu_s}{t\mu_s + r(1 - \mu_s)}\right).$$

**Proposition 4 (Optimal Design of Evaluation with Heterogeneous Priors)** *Suppose that the manager aims to maximize the expected total effort in the tournament and can flexibly design the internal evaluation. Then the following statements hold:*

*i. When $\Theta > 0$, full disclosure with a perfectly revealing evaluation—i.e., $(\mu_H, \mu_L) = (1, 0)$—is optimal;*

*ii. When $\Theta < 0$, a completely uninformative evaluation—i.e., $(\mu_H, \mu_L) = (\mu, \mu)$—is optimal;*

*iii. When $\Theta = 0$, the expected total effort is the same across all evaluation designs.*

Proposition 4 states that the optimal evaluation is either perfectly revealing or completely uninformative. The firm has a polarized preference regarding its evaluation, either maximizing the transparency in the tournament or simply minimize it: The firm can forgo the evaluation when preferring no disclosure. The condition for perfect revelation or no evaluation coincides with that for fully disclosing or concealing a noisy signal of quality $q \in (\frac{1}{2}, 1]$ in Proposition 2.

## 4.2 Maximizing the Expected Winner's Effort

Next, we consider an alternative context in which the manager is concerned about the expected winner's effort instead of the total effort (e.g., Moldovanu and Sela, 2006; Serena, 2017; and Barbieri and Serena, 2019). This objective is sensible in many scenarios. For instance, when a firm solicits a technical solution internally, only the quality of the chosen entry accrues to its benefit. A CEO succession race motivates candidates to develop their managerial skills when carrying out assigned tasks: Large public firms—e.g., GE and HP— often have difficulty retaining losing candidates, which would lead them to focus only on the acquisition of human capital from the winner (Fu and Wu, 2019b).

Denote the expected winner's effort fixing $(\mu, \tilde{\mu})$ by $WE(\mu, \tilde{\mu})$. Similar to Equation (1), $WE(\mu, \tilde{\mu})$ can be derived as

$$WE(\mu, \tilde{\mu}) = \mathbb{E}_\mu\left[\frac{(x_A)^2 + [x_B(v_B)]^2}{x_A + x_B(v_B)}\right] = \mathbb{E}_\mu\left[x_A + x_B(v_B) - 2\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)}\right].$$

Because total effort $TE(\mu, \tilde{\mu})$ is simply given by $\mathbb{E}_\mu[x_A + x_B(v_B)]$, the expression can alternatively be written as

$$WE(\mu, \tilde{\mu}) = TE(\mu, \tilde{\mu}) - 2\mathbb{E}_\mu\left[\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)}\right].$$

Thus, maximizing $WE(\mu, \tilde{\mu})$ is equivalent to maximizing the total effort minus the term $2\mathbb{E}_\mu\left[\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)}\right]$. The additional non-linear term adds complications. However, we show below that the prediction under total effort maximization remains qualitatively robust to a large extent.

We first evaluate the desirability of persistent misperception, as in Section 2.3. The following can be obtained.

**Proposition 5 (Value of Persistent Misperception)** *Suppose that the firm is concerned about the expected winner's effort in the tournament. Then the following statements hold:*

*i. When $v_A < \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $WE(\mu, \tilde{\mu}) > WE(\mu, \mu)$—if and only if employee A exhibits overconfidence—i.e., $\tilde{\mu} < \mu$;*

*ii. When $v_A > \sqrt{v_B^H v_B^L}$, the firm strictly benefits from employee A's misperception—i.e., $WE(\mu, \tilde{\mu}) > WE(\mu, \mu)$—if and only if employee A exhibits underconfidence—i.e., $\tilde{\mu} > \mu$;*

*iii. When $v_A = \sqrt{v_B^H v_B^L}$, employee A's prior does not affect the expected total effort, i.e., $WE(\mu, \tilde{\mu}) = WE(\mu, \mu)$.*

Proposition 5 states that the prediction of Proposition 1 is perfectly preserved in this alternative setting. Further, we explore the question that leads to Proposition 2: Suppose that an informative signal of quality $q \in (\frac{1}{2}, 1]$ is available; would the manager disclose it to the employees? We resort to numerical exercises and hereby report the observations. Specifically, we compare the expected winner's effort between disclosure and concealment. To proceed, we set $(v_A, \mu, q) = (1, 0.5, 0.8)$.

Figure 3 illustrates our numerical results for different cases. Three observations deserve to be highlighted. First, a comparison between Figure 3(a) and Figure 1(a) show that the
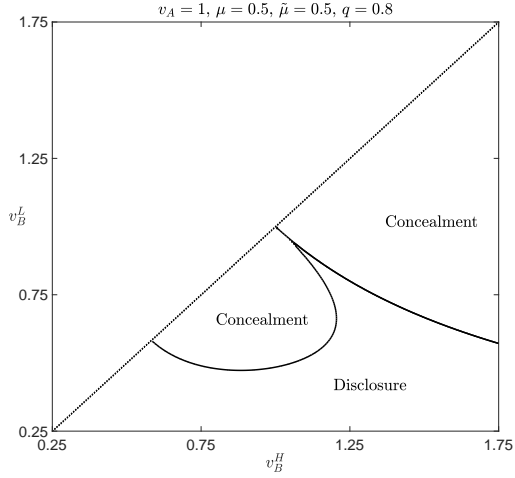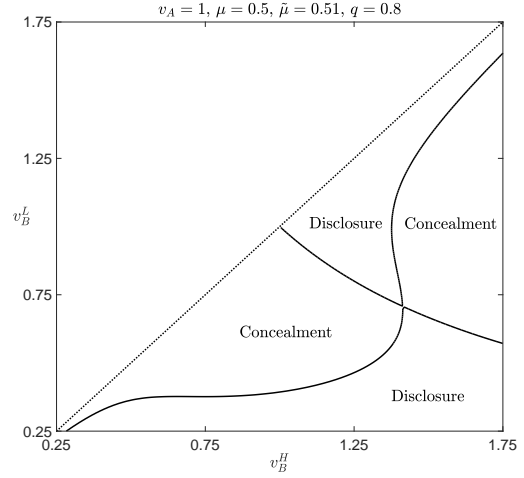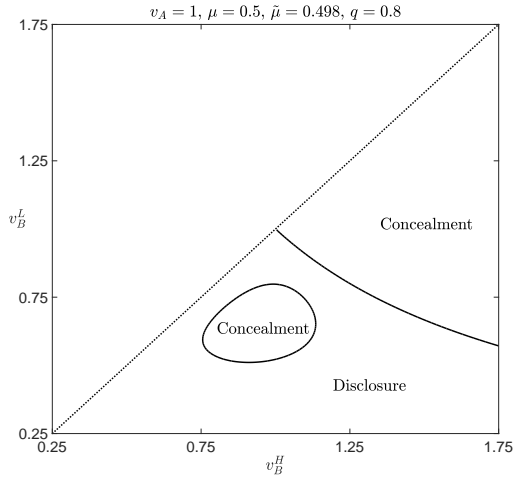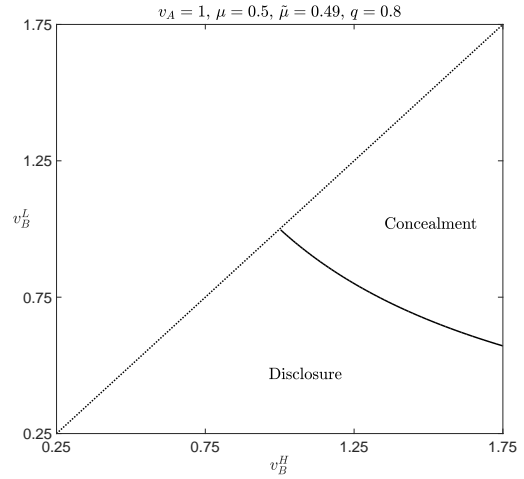
Figure 3: Optimal Information Disclosure Policy: Maximizing Winner's Effort

manager is more likely to hide information under the rational benchmark when maximizing the expected winner's effort vis-à-vis total effort. Second, as employee $A$ becomes more overconfident, the manager tends to disclose information more often, which can be seen by comparing Figure 3(c) to Figure 3(d), i.e., $\tilde{\mu}$ dropping from 0.498 to 0.49: In the latter case, the resultant pattern for the optimum coincides with that in the case of maximizing total effort as is depicted in Figure 1(a). Third, when employee $A$ exhibits underconfidence, the pattern for the optimum is similar to that in the case of total effort, which can be seen by comparing Figure 3(b) to Figure 1(b). In summary, the result of Section 3 qualitatively remain in place, despite the fact that the objective function of expected winner's effort causes nonlinearity.

# 5  Concluding Remarks

In this paper, we investigate the impact of perceptional bias—i.e., overconfidence or underconfidence—in a promotion tournament and the optimal information disclosure in a firm. Rich implications can be inferred from our results.

First, we demonstrate that a persistent misperception may either benefit or harm the firm's performance. As a result, debiasing its employees can be potentially counterproductive to a firm. Second, we fully characterize the conditions under which disclosing an informative signal of an employee's ability, or concealing it, can prevail.

The intricate role played by the perceptional bias sheds light on the extensive discussion of confidence or morale management and workplace culture building, which casts doubt into any universal recipe given the complexity. The analysis also speaks to the debate about organizational transparency. The information fed to employees varies their belief and perception, which in turn affect their incentives subtly and indeterminately.

In this paper, we primarily focus on the maximization of aggregate effort. A firm can be subject to other concerns and could calibrate its information management practice to achieve other goals. It would be interesting to explore the impact of perceptional bias and the optimal information management policy that addresses other objective functions—e.g., selection efficiency (Ryvkin and Ortmann, 2008; Brown and Minor, 2014)—which should be attempted in future research.

# References

Alonso, Ricardo and Odilon Camara, "Bayesian persuasion with heterogeneous priors," *Journal of Economic Theory*, 2016, *165*, 672–706.

Barbieri, Stefano and Marco Serena, "Winners' efforts in team contests," *Working Paper*, 2019.

Belzil, Christian and Michael Bognanno, "Promotions, demotions, halo effects, and the earnings dynamics of American executives," *Journal of Labor Economics*, 2008, *26* (2), 287–310.

Bénabou, Roland and Jean Tirole, "Self-confidence and personal motivation," *Quarterly Journal of Economics*, 2002, *117* (3), 871–915.

Boosey, Luke, Philip Brookins, and Dmitry Ryvkin, "Information disclosure in contests with endogenous entry: An experiment," *Management Science*, 2020, *forthcoming*.

Brown, Jennifer, "Quitters never win: The (adverse) incentive effects of competing with superstars," *Journal of Political Economy*, 2011, *119* (5), 982–1013.

‗ and Dylan B. Minor, "Selecting the best? Spillover and shadows in elimination tournaments," *Management Science*, 2014, *60* (12), 3087–3102.

Che, Yeon-Koo and Ian L. Gale, "Caps on political lobbying," *American Economic Review*, 1998, *88* (3), 643–651.

Chen, Hua and Noah Lim, "Should managers use team-based contests?," *Management Science*, 2013, *59* (12), 2823–2836.

Chen, Si and Hannah Schildberg-Hörisch, "Looking at the bright side: The motivational value of confidence," *European Economic Review*, 2019, *120*, 103302.

Chowdhury, Subhasish M., Patricia Esteve-González, and Anwesha Mukherjee, "Heterogeneity, leveling the playing field, and affirmative action in contests," *Working Paper*, 2019.

Compte, Olivier and Andrew Postlewaite, "Confidence-enhanced performance," *American Economic Review*, 2004, *94* (5), 1536–1557.

Connelly, Brian L., Laszlo Tihanyi, T. Russell Crook, and K. Ashley Gangloff, "Tournament theory: Thirty years of contests and competitions," *Journal of Management*, 2014, *40* (1), 16–47.

Cooper, Arnold C., Carolyn Y. Woo, and William C. Dunkelberg, "Entrepreneurs' perceived chances for success," *Journal of Business Venturing*, 1988, *3* (2), 97–108.

Crutzen, Benoît S.Y., Otto H. Swank, and Bauke Visser, "Confidence management: On interpersonal comparisons in teams," *Journal of Economics & Management Strategy*, 2013, *22* (4), 744–767.

Denter, Philipp, John Morgan, and Dana Sisak, "Showing off or laying low? The economics of psych-outs," *American Economic Journal: Microeconomics*, 2020, *forthcoming*.

DeVaro, Jed, "Strategic promotion tournaments and worker performance," *Strategic Management Journal*, 2006, *27* (8), 721–740.

Dixit, Avinash, "Strategic behavior in contests," *American Economic Review*, 1987, *77* (5), 891–898.

Drugov, Mikhail and Dmitry Ryvkin, "Biased contests for symmetric players," *Games and Economic Behavior*, 2017, *103*, 116–144.

Epstein, Gil S., Yosef Mealem, and Shmuel Nitzan, "Political culture and discrimination in contests," *Journal of Public Economics*, 2011, *95* (1), 88–93.

Eriksson, Tor, "Executive compensation and tournament theory: Empirical tests on Danish data," *Journal of Labor Economics*, 1999, *17* (2), 262–280.

Fang, Hanming, "Confidence management and interpersonal strategic interactions," *Mimeo*, 2001.

__ and Giuseppe Moscarini, "Morale hazard," *Journal of Monetary Economics*, 2005, *52* (4), 749–777.

Franke, Jörg, Christian Kanzow, Wolfgang Leininger, and Alexandra Schwartz, "Effort maximization in asymmetric contest games with heterogeneous contestants," *Economic Theory*, 2013, *52* (2), 589–630.

__ , __ , __ , and __ , "Lottery versus all-pay auction contests: A revenue dominance theorem," *Games and Economic Behavior*, 2014, *83*, 116–126.

Fu, Qiang and Zenan Wu, "Contests: Theory and topics," *Oxford Research Encyclopedia of Economics and Finance*, 2019a.

__ and __ , "Disclosure and favoritism in hierarchical competitions," *Working Paper*, 2019b.

__ and __ , "On the optimal design of biased contests," *Theoretical Economics*, 2020, *forthcoming*.

__ , Oliver Gürtler, and Johannes Münster, "Communication and commitment in contests," *Journal of Economic Behavior & Organization*, 2013, *95*, 1–19.

__ , Qian Jiao, and Jingfeng Lu, "Disclosure policy in a multi-prize all-pay auction with stochastic abilities," *Economics Letters*, 2014, *125* (3), 376–380.

Gavious, Arieh, Benny Moldovanu, and Aner Sela, "Bid costs and endogenous bid caps," *RAND Journal of Economics*, 2002, *33* (4), 709–722.

Gervais, Simon and Itay Goldstein, "The positive effects of biased self-perceptions in firms," *Review of Finance*, 2007, *11* (3), 453–496.

Henderson, Andrew D. and James W. Fredrickson, "Top management team coordination needs and the CEO pay gap: A competitive test of economic and behavioral views," *Academy of Management Journal*, 2001, *44* (1), 96–117.

Hurley, Terrance M. and Jason F. Shogren, "Effort levels in a Cournot Nash contest with asymmetric information," *Journal of Public Economics*, 1998, *69* (2), 195–210.

Kamenica, Emir and Matthew Gentzkow, "Bayesian persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Kirkegaard, René, "Favoritism in asymmetric contests: Head starts and handicaps," *Games and Economic Behavior*, 2012, *76* (1), 226–248.

Konrad, Kai A., "Investment in the absence of property rights; the role of incumbency advantages," *European Economic Review*, 2002, *46* (8), 1521–1537.

Kovenock, Dan, Florian Morath, and Johannes Münster, "Information sharing in contests," *Journal of Economics & Management Strategy*, 2015, *24* (3), 570–596.

Kyle, Albert S. and F. Albert Wang, "Speculation duopoly with agreement to disagree: Can overconfidence survive the market test?," *Journal of Finance*, 1997, *52* (5), 2073–2090.

Larwood, Laurie and William Whittaker, "Managerial myopia: Self-serving biases in organizational planning," *Journal of Applied Psychology*, 1977, *62* (2), 194–198.

Lazear, Edward P., "Compensation and incentives in the workplace," *Journal of Economic Perspectives*, 2018, *32* (3), 195–214.

_ and Sherwin Rosen, "Rank-order tournaments as optimum labor contracts," *Journal of Political Economy*, 1981, *89* (5), 841–864.

Lu, Jingfeng, Hongkun Ma, and Zhe Wang, "Ranking disclosure policies in all-pay auctions," *Economic Inquiry*, 2018, *56* (3), 1464–1485.

Malmendier, Ulrike and Geoffrey Tate, "CEO overconfidence and corporate investment," *Journal of Finance*, 2005, *60* (6), 2661–2700.

_ and _ , "Behavioral CEOs: The role of managerial overconfidence," *Journal of Economic Perspectives*, 2015, *29* (4), 37–60.

Mealem, Yosef and Shmuel Nitzan, "Discrimination in contests: A survey," *Review of Economic Design*, 2016, *20* (2), 145–172.

Moldovanu, Benny and Aner Sela, "Contest architecture," *Journal of Economic Theory*, 2006, *126* (1), 70–96.

_ , _ , and Xianwen Shi, "Contests for status," *Journal of Political Economy*, 2007, *115* (2), 338–363.

Moore, Don A. and Daylian M. Cain, "Overconfidence and underconfidence: When and why people underestimate (and overestimate) the competition," *Organizational Behavior and Human Decision Processes*, 2007, *103* (2), 197–213.

_ and Paul J. Healy, "The trouble with overconfidence," *Psychological Review*, 2008, *115* (2), 502–517.

Muthukrishna, Michael, Joseph Henrich, Wataru Toyokawa, Takeshi Hamamura, Tatsuya Kameda, and Steven J. Heine, "Overconfidence is universal? Elicitation of Genuine Overconfidence (EGO) procedure reveals systematic differences across domain, task knowledge, and incentives in four populations," *PLOS ONE*, 2018, *13* (8), 1–30.

Nafziger, Julia and Heiner Schumacher, "Information management and incentives," *Journal of Economics & Management Strategy*, 2013, *22* (1), 140–163.

Netessine, Serguei and Valery Yakubovich, "The Darwinian workplace," *Harvard Business Review*, 2012, *90* (5), 25.

Olszewski, Wojciech and Ron Siegel, "Bid caps in large contests," *Games and Economic Behavior*, 2019, *115*, 101–112.

Rosen, Sherwin, "Prizes and incentives in elimination tournaments," *American Economic Review*, 1986, *76* (4), 701–715.

Ryvkin, Dmitry and Andreas Ortmann, "The predictive power of three prominent tournament formats," *Management Science*, 2008, *54* (3), 492–504.

Serena, Marco, "Quality contests," *European Journal of Political Economy*, 2017, *46*, 15–25.

_ , "Harnessing beliefs to optimally disclose contestants' types," *Working Paper*, 2018.

Siegel, Ron, "All-pay contests," *Econometrica*, 2009, *77* (1), 71–92.

Taylor, Curtis R. and Huseyin Yildirim, "Subjective performance and the value of blind evaluation," *Review of Economic Studies*, 2011, *78* (2), 762–794.

Wärneryd, Karl, "Information in conflicts," *Journal of Economic Theory*, 2003, *110* (1), 121–136.

Wu, Zenan and Jie Zheng, "Information sharing in private value lottery contest," *Economics Letters*, 2017, *157*, 36–40.

Zhang, Jun and Junjie Zhou, "Information disclosure in contests: A Bayesian persuasion approach," *Economic Journal*, 2016, *126* (597), 2197–2217.

# Appendix: Proofs

**Proof of Proposition 1**

**Proof.** Recall that

$$TE(\mu, \tilde{\mu}) = \left[ (1-\mu)\sqrt{v_B^L} + \mu\sqrt{v_B^H} \right] K(\tilde{\mu}).$$

The result immediately follows from the monotonicity of $K(\cdot)$, which is characterized by Lemma 1. ∎

**Proof of Proposition 2**

**Proof.** For notational ease, we include $q$ as an argument of $TE^D(\mu, \tilde{\mu})$,

$$TE^D(\mu, \tilde{\mu}; q) = \left[ \mu q + (1-\mu)(1-q) \right] \times \left[ (1-\mu_H)\sqrt{v_B^L} + \mu_H\sqrt{v_B^H} \right] K(\tilde{\mu}_H)$$
$$+ \left[ \mu(1-q) + (1-\mu)q \right] \times \left[ (1-\mu_L)\sqrt{v_B^L} + \mu_L\sqrt{v_B^H} \right] K(\tilde{\mu}_L).$$

Note that concealment is equivalent to disclosure with $q = \frac{1}{2}$: $TE^C(\mu, \tilde{\mu}) = TE^D\left(\mu, \tilde{\mu}; \frac{1}{2}\right)$.

Define $G(q)$ as

$$G(q) := \left[ \mu q + (1-\mu)(1-q) \right] \times \left[ (1-\mu_H(q))\sqrt{v_B^L} + \mu_H(q)\sqrt{v_B^H} \right] K(\tilde{\mu}_H(q)).$$

Recall that $\mu_H = \frac{\mu q}{\mu q + (1-\mu)(1-q)}$ and $\tilde{\mu}_H = \frac{\tilde{\mu} q}{\tilde{\mu} q + (1-\tilde{\mu})(1-q)}$. In defining $G(\cdot)$, we treat $\mu_H$ and $\tilde{\mu}_H$ as functions of $q$.

It is easy to verify that $TE^D(\mu, \tilde{\mu}; q) = G(q) + G(1-q)$. Then,

$$\frac{\partial TE^D(\mu, \tilde{\mu}; q)}{\partial q} = G'(q) - G'(1-q), \text{ and } \frac{\partial^2 TE^D(\mu, \tilde{\mu}; q)}{\partial q^2} = G''(q) + G''(1-q).$$

Simple algebra yields that

$$G''(q) = \left[ K'(\tilde{\mu}_H)\tilde{\mu}_H''(q) + K''(\tilde{\mu}_H)[\tilde{\mu}_H'^2] \times \left[ \mu q \sqrt{v_B^H} + (1-\mu)(1-q)\sqrt{v_B^L} \right] \right.$$
$$+ 2K'(\tilde{\mu}_H)\tilde{\mu}_H'(q) \left[ \mu\sqrt{v_B^H} - (1-\mu)\sqrt{v_B^L} \right]$$
$$= -\underbrace{\frac{2\sqrt{v_B^H}\left(\frac{1}{v_B^L} + \frac{1}{v_A}\right)\tilde{\mu}(1-\mu)}{\frac{(1-\tilde{\mu})(1-q)+\tilde{\mu}q}{v_A} + \frac{(1-\tilde{\mu})(1-q)}{v_B^L} + \frac{\tilde{\mu}q}{v_B^H}}}_{>0} \times \underbrace{\tilde{\mu}_H'(q)}_{>0} \times K'(\tilde{\mu}_H) \times \left[ \frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)} \right].$$

29

It can be verified that $\tilde{\mu}'_H(q) = \frac{(1-\tilde{\mu})\tilde{\mu}}{[(1-\tilde{\mu})(1-q)+\tilde{\mu}q]^2} > 0$. Moreover, it follows from Lemma 1 that $K'(\tilde{\mu}_H) \gtreqqless 0$ is equivalent to $v_A - \sqrt{v_B^H v_B^L} \gtreqqless 0$. Therefore, $G''(q) \gtreqqless 0$ is equivalent to

$$\Theta := \left[ \sqrt{v_B^H v_B^L} - v_A \right] \times \left[ \frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)} \right] \gtreqqless 0.$$

Similarly, we can show that $G''(1-q) \lesseqqgtr 0$ is equivalent to $\Theta \gtreqqless 0$. Therefore, we can obtain that

$$\frac{\partial^2 TE^D (\mu, \tilde{\mu}; q)}{\partial q^2} \gtreqqless 0 \Leftrightarrow \Theta \gtreqqless 0.$$

Next, note that $\frac{\partial TE^D(\mu, \tilde{\mu}; \frac{1}{2})}{\partial q} = G'(\frac{1}{2}) - G'(\frac{1}{2}) = 0$. Consequently, when $\Theta > 0$, $TE^D (\mu, \tilde{\mu}; q)$ is strictly increasing in $q$ and hence $TE^D (\mu, \tilde{\mu}; q) > TE^D (\mu, \tilde{\mu}; \frac{1}{2}) = TE^C (\mu, \tilde{\mu})$ for all $\frac{1}{2} < q \leq 1$. When $\Theta < 0$, $TE^D (\mu, \tilde{\mu}; q)$ is strictly decreasing in $q$ and hence $TE^D (\mu, \tilde{\mu}; q) < TE^D (\mu, \tilde{\mu}; \frac{1}{2}) = TE^C (\mu, \tilde{\mu})$ for all $\frac{1}{2} < q \leq 1$. When $\Theta = 0$, $TE^D (\mu, \tilde{\mu}; q)$ is constant in $q$ and thus the firm is indifferent between disclosure and concealment. ∎

## Proof of Proposition 3

**Proof.** In the case of underconfidence, for every given $\mu \in (0, 1)$, the term $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]$ strictly decreases with $\tilde{\mu}$, with $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]\big|_{\tilde{\mu}=\mu} = 1$ and $[\mu(1-\tilde{\mu})]/[\tilde{\mu}(1-\mu)]\big|_{\tilde{\mu}=1} = 0$. Note that the term $[(v_B^L)^{\frac{3}{2}}(v_A + v_B^H)]/[(v_B^H)^{\frac{3}{2}}(v_A + v_B^L)] < 1$. Therefore, fixing $(v_A, v_B^L, v_B^H)$, there exists a unique cutoff $\tilde{\mu}^* \in (\mu, 1)$ such that

$$\frac{(v_B^L)^{\frac{3}{2}} (v_A + v_B^H)}{(v_B^H)^{\frac{3}{2}} (v_A + v_B^L)} - \frac{\mu (1 - \tilde{\mu})}{\tilde{\mu} (1 - \mu)} \lesseqqgtr 0, \text{ if and only if } \tilde{\mu} \gtreqqless \tilde{\mu}^*. \tag{12}$$

Proposition 3 follows instantly from (12) and Proposition 2. ∎

## Proof of Proposition 4

**Proof.** Recall that

$$\tilde{\mu}_s(\mu_s) = \frac{t\mu_s}{t\mu_s + r(1 - \mu_s)}.$$

It follows immediately that

$$\tilde{\mu}'_s(\mu_s) = \frac{rt}{[t\mu_s + r(1 - \mu_s)]^2} > 0, \text{ and } \tilde{\mu}''_s(\mu_s) = \frac{-2(t - r)rt}{[t\mu_s + r(1 - \mu_s)]^3}.$$

Denote $TE(\mu_s, \tilde{\mu}_s(\mu_s))$ by $\widehat{TE}_s(\mu_s)$. The second order derivative of $\widehat{TE}_s(\mu_s)$ with respect to $\mu_s$ is

$$
\begin{aligned}
\widehat{TE}_s''(\mu_s) &= \left\{ K''\left(\tilde{\mu}_s(\mu_s)\right)\left[\tilde{\mu}_s'(\mu_s)\right]^2 + K'\left(\tilde{\mu}_s(\mu_s)\right)\tilde{\mu}_s''(\mu_s)\right\} \times \left[(1-\mu_s)\sqrt{v_B^L} + \mu_s\sqrt{v_B^H}\right] \\
&\quad + 2K'\left(\tilde{\mu}_s(\mu_s)\right)\tilde{\mu}_s'(\mu_s)\left(\sqrt{v_B^H} - \sqrt{v_B^L}\right) \\
&= -\underbrace{\frac{2\tilde{\mu}_s'(\mu_s)\sqrt{v_B^H}\left(\frac{1}{v_B^L} + \frac{1}{v_A}\right)t}{\frac{t\mu_s + r(1-\mu_s)}{v_A} + \frac{r(1-\mu_s)}{v_B^L} + \frac{t\mu_s}{v_B^H}}}_{>0} \times K'\left(\tilde{\mu}_s(\mu_s)\right) \times \left[\frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)}\right].
\end{aligned}
$$

$$(13)$$

It follows from Lemma 1 that $K'\left(\tilde{\mu}_s(\mu_s)\right) \gtreqqless 0$ is equivalent to $v_A \gtreqqless \sqrt{v_B^H v_B^L}$. Therefore, $\widehat{TE}_s''(\mu_s) \gtreqqless 0$ is equivalent to

$$
\Theta = \left[\sqrt{v_B^H v_B^L} - v_A\right] \times \left[\frac{v_B^L(v_A + v_B^H)}{v_B^H(v_A + v_B^L)} - \frac{\mu(1-\tilde{\mu})}{\tilde{\mu}(1-\mu)}\right] \gtreqqless 0.
$$

When $\Theta > 0$, $TE_s(\mu_s)$ is strictly convex in $\mu_s$, indicating the optimality of perfectly revealing signals. When $\Theta < 0$, $TE_s(\mu_s)$ is strictly concave in $\mu_s$, indicating the optimality of completely uninformative signals. When $\Theta = 0$, $TE_s(\mu_s)$ is linear in $\mu_s$, and thus all information disclosure policies lead to the same expected total effort. ∎

**Proof of Proposition 5**

**Proof.** First we simplify $WE(\mu, \tilde{\mu})$.

$$
\begin{aligned}
WE(\mu, \tilde{\mu}) &= \mathbb{E}_\mu\left[x_A + x_B(v_B) - 2\frac{x_A \cdot x_B(v_B)}{x_A + x_B(v_B)}\right] \\
&= \mathbb{E}_\mu\left[\sqrt{v_B x_A} - 2\frac{x_A\left(\sqrt{v_B x_A} - x_A\right)}{\sqrt{v_B x_A}}\right] \\
&= \mathbb{E}_\mu\left[F\left(v_B, K(\tilde{\mu})\right)\right],
\end{aligned}
$$

where $F(v_B, K) := \frac{2K^3}{\sqrt{v_B}} + \sqrt{v_B}K - 2K^2$. Note that

$$
\frac{\partial F(v_B, K)}{\partial K} = \frac{6K^2}{\sqrt{v_B}} + \sqrt{v_B} - 4K \geq \left(2\sqrt{6} - 4\right)K > 0.
$$

Therefore, $WE(\mu, \tilde{\mu})$ is increasing in $K$. From Lemma 1, $K(\cdot)$ is strictly decreasing in $\tilde{\mu}$ if $\sqrt{v_B^H v_B^L} > v_A$ and $K(\tilde{\mu})$ is strictly increasing in $\tilde{\mu}$ otherwise. This completes the proof. $\blacksquare$