NBER WORKING PAPER SERIES

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Working Paper 27041
http://www.nber.org/papers/w27041

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2020

We are grateful to several anonymous referees, to the audiences at Sciences Po, and to Gerry Jaynes for helpful comments on an earlier draft, to Michael Rothschild and Richard Arnott, long time collaborators, to Byoung Heon Jun, Debarati Ghosh, Andrea Gurwitt, and Lim Nayeon for research and editorial assistance, and to the Institute for New Economic Thinking and the Ford Foundation and Fulbright Foundation for financial support. The companion paper "Characterization, Existence, and Pareto Optimality in Insurance Markets with Asymmetric Information with Endogenous and Asymmetric Disclosures; Revisiting Rothschild-Stiglitz" (Stiglitz et al 2018) contains more results. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. D43,D82,D86

ABSTRACT

We study insurance markets with nonexclusive contracts, introducing bilateral endogenous information disclosure about insurance sales and purchases by firms and consumers. We show that a competitive equilibrium exists under remarkably mild conditions, and characterize the unique equilibrium outcome. With two types of consumers the outcome consists of a pooling contract which maximizes the well-being of the low risk type (along the zero profit pooling line) plus a supplemental (undisclosed and nonexclusive) contract that brings the high risk type to full insurance (at his own odds). We show that this outcome is extremely robust and constrained Pareto efficient. Consumer disclosure and asymmetric equilibrium information flows are critical in supporting the equilibrium.

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1 Introduction

In 1976 Rothschild and Stiglitz characterized equilibrium in a competitive market with exogenous information asymmetries in which market participants had full knowledge of insurance purchases. Self-selection constraints affected individual choices; but unlike the monopoly equilibrium (Stiglitz (1977)), no single firm framed the set of contracts among which individuals chose. There never existed a pooling equilibrium (in which the two types bought the same policy); if there existed an equilibrium, it entailed the high risk getting full insurance, and the low risk individual only getting partial insurance; and under plausible conditions—e.g. if the two types were not too different—a competitive equilibrium did not exist; finally, the single-crossing condition on preferences was necessary but not sufficient for an equilibrium. The results were disquieting, as in reality equilibrium seemed to exist, and often entailed pooling.

A vast literature has applied the Rothschild and Stiglitz (1976) model (henceforth simply RS), to labor, capital, and product markets in a variety of contexts, with many empirical applications. A smaller literature focused on remedying the deficiencies in the underlying framework by formalizing the insurance "game", by changing the information assumptions, and by changing the equilibrium concept.

This paper introduces bilateral endogenous information disclosure about insurance purchases: each firm and each consumer make a decision about what information to disclose to whom. Thus, information about contract purchases is both endogenous, and potentially asymmetric - a firm may disclose information about a consumer to some firms, but not others, depending on what the consumer discloses. We are motivated by the following observations: the outcomes with full information disclosure (exclusivity is enforceable, so the RS model applies, and a pooling equilibrium is impossible), and with no information disclosure (in which case pooling again cannot be an equilibrium), are known. Without consumer disclosure, any disclosure that firms make has to be symmetric, since they have no basis for differentiation; as we show later such disclosure cannot underpin an equilibrium. The question of what happens if disclosure is bilateral, endogenous, and thus potentially asymmetric, is thus natural.

The results are surprising: (i) asymmetries in information about insurance purchases turn out to be nontrivial and important; (ii) equilibrium exists under mild assumptions (notably, the single crossing property need not hold); and (iii) equilibrium always entails a pooling component. The unique insurance allocation (an allocation describes the sum of benefits and premia over all insurance companies for each individual) consists of the pooling contract which maximizes the well-being of the low risk individual subject to the zero-profit constraint plus, for the high-risk individual, a supplemental contract that brings him to full insurance at his own odds. While the equilibrium allocation is unique, it can be supported by alternative information disclosure strategies.

We begin by characterizing the set of constrained Pareto efficient (CPE) allocations in the presence of a secret contract. We then show that the CPE allocation which maximizes the well-being of the low risk individual is the unique equilibrium allocation and can be supported by simple yet illuminating information disclosure strategies. As in RS, firms offer insurance contracts, but now they have an option to reveal (possibly partial) information about insurance purchases to other firms. In RS, it was assumed that contracts were exclusive, e.g. implicitly, that if a firm discovered a purchaser had violated the exclusivity restriction, the coverage would be cancelled. (It was also assumed that the insurance firm had the information necessary to enforce exclusivity.) Here we consider a broader range of possible restrictions and under a broader set of assumptions concerning the information available to insurance firms. Obviously, the enforceability of any conditions imposed is dependent on information available to the insurance firm. Consumers, too, have a slightly more complicated life: they have to decide which policies to buy, aware of the restrictions in place and the information that the firm may have to enforce those restrictions. And they also have to decide on what information to reveal to which firms. A competitive equilibrium in this model is a set of insurance contracts, such that no one can offer an alternative contract or set of contracts and make positive profits. A contract is defined by the benefit, the premium, any restrictions associated with the contract, and the firm's disclosure policy. And in assessing the consequences of offering an alternative contract, each firm takes into account the consumers' response to the set of contracts on offer, both with respect to insurance purchases and disclosures.

The intuition behind our result is this: in RS, a pooling equilibrium can always be broken by a deviant policy which will be purchased only by low risk individuals, and as a result, is profitable. But that deviant contract will be purchased only by low risk individuals because the deviant firm can enforce exclusivity. If high risk individuals can supplement the deviant contract (one breaking the putative pooling equilibrium) with secret insurance at their own odds, that policy will be purchased by high risk individuals, and thus make a loss. Hence, the deviant policy will not be offered and the pooling contract can be sustained. The trick is to find an information disclosure strategy which ensures that a deviant firm can't enforce exclusivity, but which also ensures that the firms selling insurance at the pooling odds (which we refer to as "established" firms) don't "oversell": high risk individuals would like to buy more insurance at the pooling odds than low risk individuals. If they did so, the pooling contract would lose money. Accordingly, there has to be information disclosure among the established firms to prevent the high risk individuals from doing so. Thus, supporting the equilibrium allocation requires an *intermediate* amount of disclosure: one needs some information sharing (enough to prevent overselling), but not too much (not enough to enforce exclusivity). Furthermore, disclosure has to be asymmetric in that established firms must have sufficient information, but deviant firms (which, of course, deviate secretly) must not. But firms by themselves have no basis for such asymmetric disclosures: without further information, they only know whether they themselves have sold insurance to an individual. [1]

This is where consumer disclosure becomes critical: firms base the asymmetries in disclosure on consumer-revealed information. The presence of consumer disclosure is an essential feature distinguishing our paper from other work in this area. The equilibrium firm information disclosure strategy that we analyze induces truth-telling by consumers to established firms, and this in turn enables asymmetries in firm disclosures of information about insurance purchases. Thus, endogenizing consumer disclosure is not just a natural modeling postulate - it is necessary for the outcome we characterize.

One can obtain this result by formalizing this setting as a game with appropriately defined strategy spaces, and focusing on the outcome in a perfect Bayesian equilibrium; however, doing so would introduce unnecessary complexity[2], thus detracting attention from the basic insight of our analysis. For this reason, we pursue the route of the original Rothschild-Stiglitz paper, positing only the elements that are absolutely necessary to make the point in the simplest possible setting that nevertheless has all of the features we are interested in, using an equilibrium concept that is in the spirit of competitive equilibrium.

The paper is organized in six sections. Section two lays out the model; we characterize the set of CPE contracts in the presence of secret insurance in section three. Section four provides details on contracts, information disclosure strategies, the equilibrium concept, and shows that there is a unique allocation that an equilibrium if it exists, has to implement. Such an equilibrium is explicitly constructed in section five, while section six relates our results to previous literature.

We note that while this paper demonstrates the existence of a robust asymmetric information equilibrium with endogenous information disclosure in a simple context with two types of individuals and where deviant firms attempt to break the equilibrium by offering a single alternative contract, the results can, with considerable increase in complexity, be extended to multiple (or even a continuum) of types and to contexts where deviant firms offer a menu of contracts. The equilibrium insurance allocation described in this paper can be shown to be supported by alternative endogenous disclosure rules, including rules that entail sequential disclosure, i.e. where firms may disclose information to other firms that has been revealed to them by still other firms.

[1] This is essentially the point that Hellwig (1988) makes in criticism of Jaynes (1978) argument that with endogenous information, there always exists an equilibrium. In contrast, he shows that "there does not exist a sequential equilibrium…" He shows that Jaynes' equilibrium requires that each firm's communication strategy be conditioned on the set of contracts that are offered by other firms, making the equilibrium a reactive equilibrium, like that of Wilson, not a competitive equilibrium as in RS.

[2] We work through a model along these lines in Stiglitz et al. 2016.

2 Model

We employ the standard insurance model with adverse selection. An individual, indexed by $i \in [0,1]$, is faced with the risk of an accident. The two types of individuals - high risk ($t = H$) and low-risk ($t = L$) - differ only in the probability of accident, P_t, with $P_H > P_L$. The type is privately known to the individual, while the proportion θ of high-risk types is common knowledge. The average probability of accident for an individual is $\overline{P}$, where

$$\overline{P} = \theta P_H + (1-\theta)P_L \tag{1}$$

An accident involves damages. The cost of repairing the damage in full is d. An insurance firm pays a part of the repair cost, $\alpha \leq d$. The benefit is paid in the event of accident, whereas the insurer is paid an insurance premium β when no accident occurs. The price of insurance is $q \equiv \frac{\beta}{\alpha}$. The expected utility for an individual with a policy (α, β) is

$$V_t(\alpha,\beta) = P_t U(w-d-\alpha) + (1-P_t)U(w-\beta) \tag{2}$$

We assume that U is continuously differentiable and increasing. Sometimes we refer to a policy $A = \{\alpha, \beta\}$, and to the expected utility generated by that policy as $V_t\{A\}$. A policy A, with insurance level α and price q can also be described by the vector $\{\alpha, \alpha q\}$. We do not require the preferences to be convex for our results on the existence of equilibrium, nor that the single-crossing property of preferences be satisfied. The key property of $V_t(\alpha, \beta)$, which we assume is satisfied throughout the paper, is that the income consumption curve at the insurance price $\frac{P_t}{1-P_t}$ is the full insurance line[3], implying that at full insurance, the slope of the indifference curve equals the relative probabilities,

$$-\frac{\frac{\partial V_t(\alpha,\beta)}{\partial \alpha}}{\frac{\partial V_t(\alpha,\beta)}{\partial \beta}} = \frac{P_t}{1-P_t} \equiv q_t \tag{3}$$

so that will full information, equilibrium would entail full insurance for each type at their own odds.

The profit π_t of a contract (α, β) that is chosen by type t is $\pi_t(\alpha,\beta) = (1-P_t)\beta - P_t\alpha$. $\pi_t(\alpha,\beta) = 0$ is defined as the t-type's zero profit locus (the line along which firms selling to type-t make zero profit). We sometimes write the profits associated with policy A purchased by type t as $\pi_t\{A\}$. Figure 1 illustrates the zero-profit locus for a firm selling insurance to a t-type (OB and OC, respectively) or both types of individuals (OD) by a line from the origin with the slope being $q_t (\equiv \frac{P_t}{1-P_t})$ or $\overline{q} \left(\equiv \frac{\overline{P}}{1-\overline{P}}\right)$, respectively. The latter is referred to as the "zero profit pooling line." There are N firms and the identity

[3] That is, even if the indifference curve is not quasiconcave, after being tangent to a given isocline with slope $\frac{P_t}{1-P_t}$, at full insurance, it never touches the isocline again.

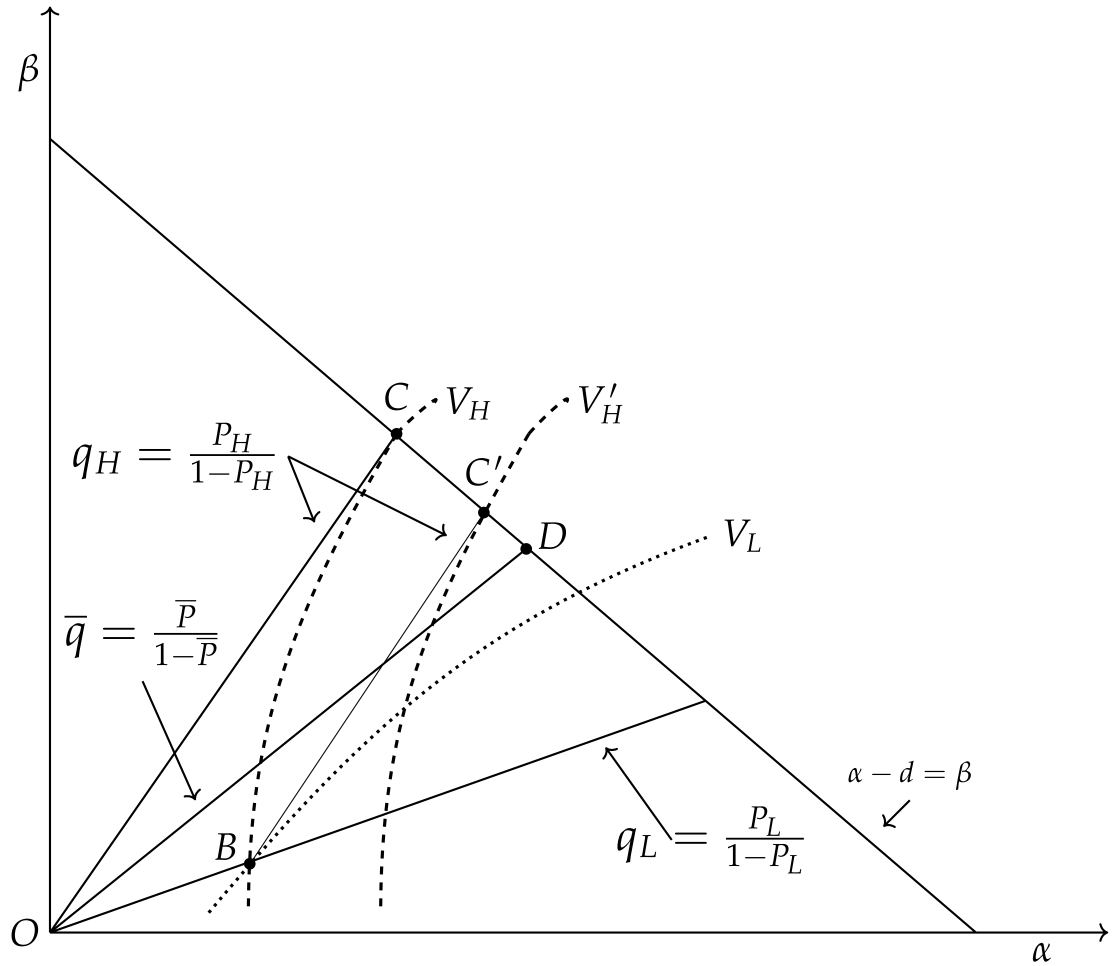

Figure 1: Breaking the RS separating equilibrium (B, C) in the presence of undisclosed contracts at high-risk odds. V_H is an indifference curve of type H, V_L is an indifference curve of type L, the line $\alpha - d = \beta$ is the full insurance line.

of a firm is represented by j, with $j \in M(\equiv 2, \ldots, N)$[4]. An individual may purchase multiple policies. A set of benefits and premiums of the insurance policies purchased in the aggregate by each type of individual, denoted by $E = \{(\alpha_t, \beta_t)_{t=L,H}\}$ is called an *allocation*, with $\alpha_t = \sum_j \alpha_{t,j}$.

3 Pareto Efficiency with Secret Contracts, and RS without Exclusivity

Central to RS was the assumption that there was sufficient information to enforce exclusivity; an individual could not buy insurance from more than one firm. As Rothschild and Stiglitz realized, once we introduce into the analysis unobservable contracts in ad-

[4] If there is only one firm, it can trivially impose exclusivity, being a monopolist.

dition to observable ones, the whole RS framework collapses, because exclusivity cannot be enforced. We now discuss briefly the consequences of introducing secret contracts; the interested reader is referred to the companion paper Stiglitz et al. (2018) for details.

Exclusivity in RS implied, among other things, the existence of contracts that break a putative pooling equilibrium. Without exclusivity some of these contracts no longer do so, because they will be taken up by not only low risk individuals, but also high risk individuals who will supplement the given contract with undisclosed insurance at price q_H. Such secret insurance breaks even and so will always be on offer. Yet it turns out that there always exists some contract that even with secret insurance breaks a pooling equilibrium. But without exclusivity, the separating contracts from RS are also not equilibrium contracts, as illustrated in figure 1. The RS separating contracts are $\{A, C\}$, where C provides full insurance to the high risk individual at his own odds; and A is the contract at the low risk individual's odds which just satisfies the self-selection constraints, i.e. will not be purchased by the high risk individual. But obviously, if the high risk individual can supplement A with secret insurance at the high risk odds, he will purchase A. But if high and low risk individuals both purchase A it makes a loss. It can easily be shown that there exists no separating equilibrium. Since there can neither exist a pooling or a separating equilibrium it follows that with a fixed information structure (where firms that disclose their sales always do so and those that don't never do so) there never exists a RS competitive equilibrium.

For the rest of this section we consider the set of efficient insurance allocations under the premise that there exists a secret (undisclosed) policy being offered at the price q_H. We characterize the set of "constrained Pareto Efficient" allocations - where the constraint is that the government cannot proscribe the secret provision of insurance, unlike the Pareto Efficient allocations associated with the RS model, where government could restrain such provision. We use the following ex-interim variant[5] of constrained Pareto efficiency:

Definition 1. *An allocation $E = \{(\alpha_t, \beta_t)_{t=L,H}\}$ is constrained Pareto-efficient (CPE for short) if the government cannot force disclosure and there does not exist another allocation that at least breaks even, and leaves each type of consumer as well off and at least one type strictly better off.*

In the presence of a contract that secretly offers any amount of insurance at a price q_H, therefore, a high-risk individual with a less-than-full insurance policy (say, A or B in figure 2) would always supplement it by purchasing additional insurance at q_H to reach a full-insurance policy C or C' in figure 2. Thus, the only CPE pooling allocation is full insurance along the pooling line (D in figure 1).

[5] See also Prescott and Townsend (1984), Hammond (1987), Bisin and Gottardi (2006), and Attar, Mariotti, and Salanie (2019) for important discussions of Pareto efficiency in related contexts; earlier versions of the present paper were the first to explicitly consider the "constrained Pareto efficiency" concept introduced here, allowing for secret contracts as well as disclosed contracts.

The set of CPE can now be easily described: it consists of a pooling contract, i.e. a contract along the pooling line OA^*D in figure 3) plus a supplemental contract, for the high risk individual only, bringing him to full insurance at the high risk odds. $\{A', C'\}$ is a typical CPE. A^* is the pooling contract that maximizes the utility of the low risk individual. The set of CPE entails equal or more insurance than A^*, i.e. the pooling policy lies between A^* and D. Later, we will show that $\{A^*, C\}$ is the unique competitive equilibrium allocation.

In characterizing CPE allocations, we will first provide an analytic representation of the possible set of allocations for a high risk individual, given that he can purchase secret insurance, and prove a general result concerning his level of expected utility. Next, we narrow down the set of allocations that satisfy the zero-profit constraint to those satisfying the self-selection constraints in the presence of the undisclosed contracts, which enables us to fully characterize the set of CPE.

If a high risk individual can purchase a policy $(\alpha, \alpha q)$, he will supplement it by purchasing additional insurance at q_H to reach a full-insurance policy $\{\gamma(\alpha, \alpha q), \delta(\alpha, \alpha q)\}$, where

$$\gamma(\alpha, \alpha q) = \frac{1}{1+q_H}[d + \alpha(q_H - q)] \tag{4}$$

and

$$\delta(\alpha, \alpha q) = d - \gamma(\alpha, \alpha q) = \frac{1}{1+q_H}[q_H d - \alpha(q_H - q)] \tag{5}$$

The derivations of (4) and (5) can be found in the appendix. Denoting by $H(\alpha, q)$ the expected utility that a high-risk individual with a less-than-full insurance policy $(\alpha, \alpha q)$ (where $\alpha(1+q) < d$) can obtain by supplementing it with the desired amount of insurance at a price q_H, we show the following lemma:

Lemma 3.1. *$H(\alpha, q)$ is decreasing in q while it is increasing (respectively, decreasing) in α if $q <$(respectively, $>$) q_H.*

Proof. Using (4) and (5), we have

$$H(\alpha, q) \equiv V_H(\gamma(\alpha, \alpha q), \delta(\alpha, \alpha q)) = \tag{6}$$

$$= U(W - d + \gamma(\alpha, \alpha q)) = U(W + \frac{1}{1+q_H}[(q_H - q)\alpha - q_H d]) \tag{7}$$

from which lemma 3.1 follows by inspection. □

Lemma 3.1, which plays a critical role for the results in this paper, implies that a high-risk individual would always like to purchase more insurance (up to full insurance) at a price lower than q_H in the presence of the undisclosed supplemental purchase of insurance at q_H.

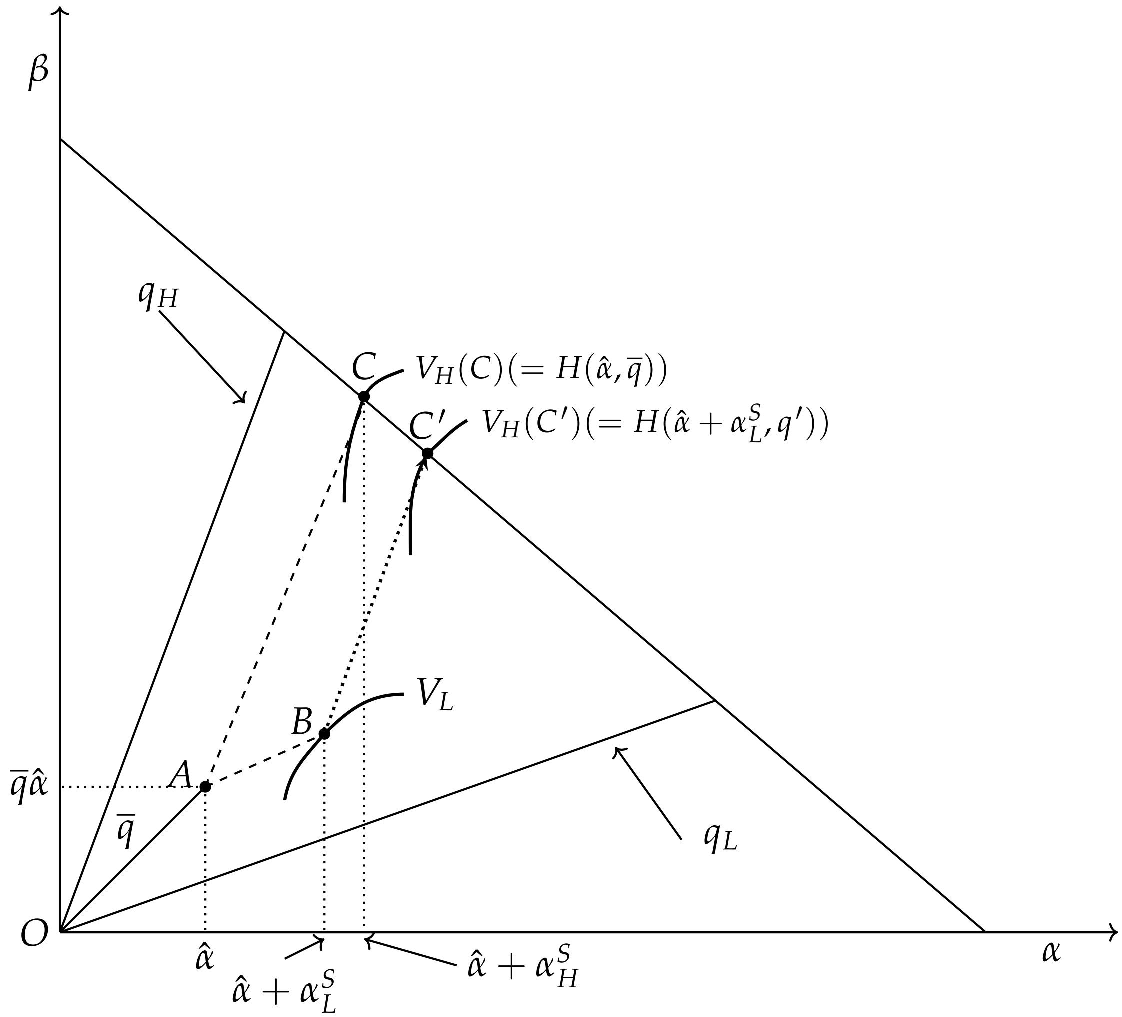

Figure 2: An allocation (B, C) can be decomposed as A (a pooling contract) and (AC, AB) (supplemental insurance at the individual's odds). Allocation (B, C) is not feasible as it does not satisfy the self-selection constraints in the presence of undisclosed policies at high-risk odds q_H, while (A, C) is feasible; $q' =$ slope of $OB(< \overline{q})$.

Lemma 3.2 provides a characterization of all zero profit allocations.

Lemma 3.2. *Any allocation $\{(\alpha_t, \beta_t)_{t=L,H}\}$ yielding zero profit can without loss of generality be represented as a sum of a pooling allocation $(\hat{\alpha}, \hat{\beta})$ and a set (α_t^S, β_t^S) of type-specific supplemental allocations:*

$$\alpha_t = \hat{\alpha} + \alpha_t^S, \beta_t = \hat{\beta} + \beta_t^S \text{ where } \hat{\beta} = \overline{q}\hat{\alpha}, \beta_t^S = q_t \alpha_t^S, \text{ and } \alpha_t^S = \alpha_t - \hat{\alpha} \tag{8}$$

The proof can be found in the appendix. Lemma 3.2 is illustrated in figure 2, which shows how an allocation (B, C) (that yields zero profit) can be decomposed into a pooling allocation A and the two supplemental allocations (AC, AB). When (α_t, β_t) is a pooling full insurance allocation, $\hat{\alpha} = \frac{1}{1+\overline{q}}d$, $\hat{\beta} = \frac{\overline{q}}{1+\overline{q}}d$, $\alpha_t^S = \beta_t^S = 0$. Lemma 3.2 implies that an allocation yielding zero profit can be characterized by the three parameters $\hat{\alpha}, \alpha_H^S$ and α_L^S.

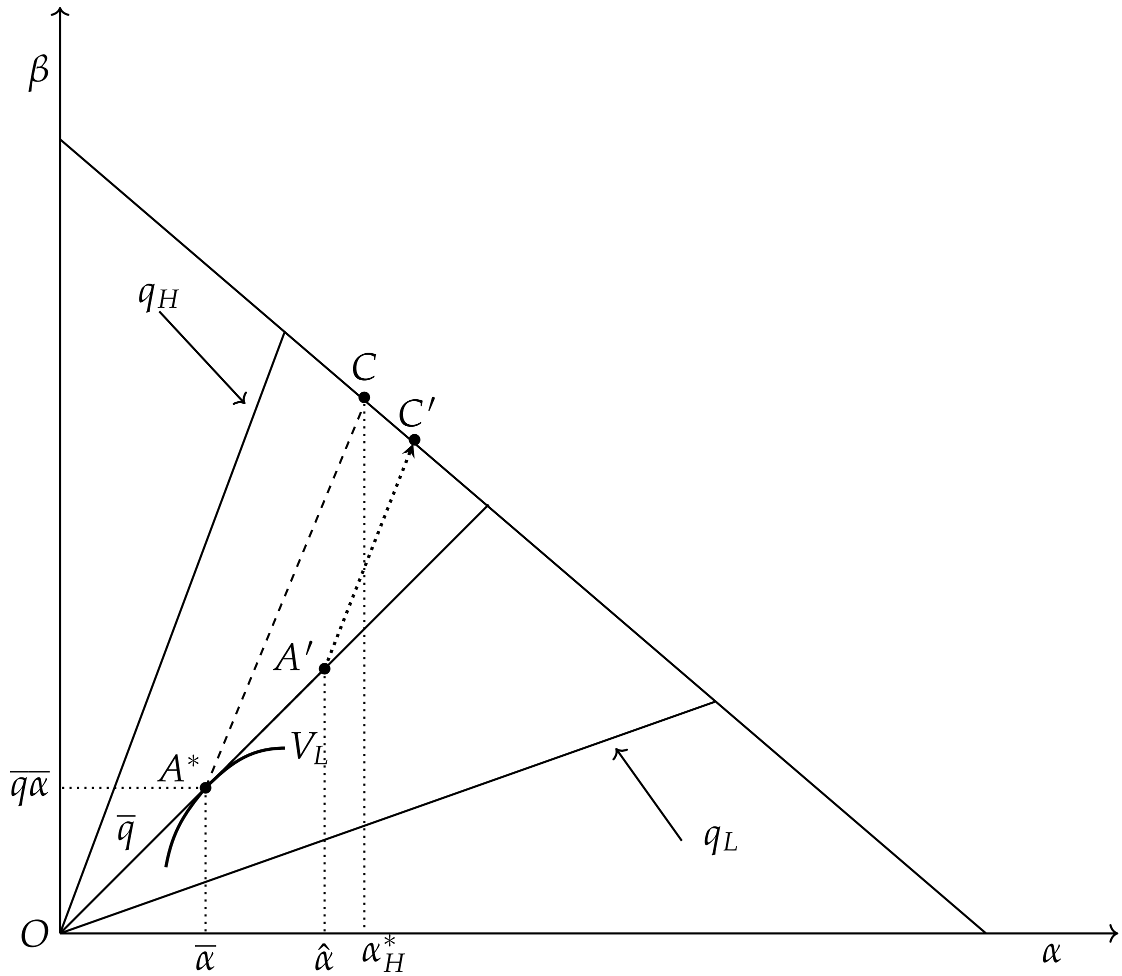

Figure 3: Pareto Efficient Allocations $\{Z_t(\hat{\alpha})\}_t$, denoted by (A', C'), and the equilibrium allocation $\{Z_t(\overline{\alpha})\}$ denoted by (A^*, C)

An allocation is said to be *feasible* if it satisfies self-selection constraints - i.e. if given that allocation, neither type will choose to deviate to another allocation, given the presence of undisclosed insurance at price q_H. Lemma 3.3 shows that the self-selection constraints in the presence of the undisclosed policies result in the following restrictions on allocations:

Lemma 3.3. *Any feasible allocation must satisfy*

i) $\alpha_H^S = \frac{1}{1+q_H}(d - \hat{\alpha}(1+\overline{q}))$ *for* $\hat{\alpha} \leq \frac{1}{1+\overline{q}}d$.

ii) $\alpha_L^S = 0$.

Lemma 3.3 i) follows directly from eq. (4). To see lemma 3.3 ii), note that if $\alpha_L^S > 0$ the allocation would not satisfy the self-selection constraint, since then high-risk individuals, after choosing $(\hat{\alpha}, \hat{\alpha}\overline{q})$ (A in figure 2), would supplement it by choosing supplemental policy AB $(= \alpha_L^S, \alpha_L^S q_L)$, and supplement that with secret insurance BC', bringing them to full insurance which yields $H(\hat{\alpha} + \alpha_L^S, q')(= V_H(C')$ in figure 2), where $q' = \frac{1}{\hat{\alpha}+\alpha_L^S}(\overline{q}\hat{\alpha} + q_L\alpha_L^S) < \overline{q}$. By lemma 3.1, $H(\hat{\alpha} + \alpha_L^S, q') > H(\hat{\alpha}, \overline{q})$, because $q' < \overline{q}$ and $\hat{\alpha} + \alpha_L^S > \hat{\alpha}$. Of course, lemma 3.ii implies that $(\alpha_L, \beta_L) = (\hat{\alpha}, \hat{\beta})$.

A feasible allocation for type t is denoted by $Z_t(\hat{\alpha})$, which is completely characterized by the parameter $\hat{\alpha}$ by lemma 3.3:

$$Z_H(\hat{\alpha}) = (\gamma(\hat{\alpha}, \hat{\alpha}\overline{q}), \delta(\hat{\alpha}, \hat{\alpha}\overline{q})) \tag{9}$$

$$Z_L(\hat{\alpha}) = (\hat{\alpha}, \hat{\alpha}\overline{q}) \tag{10}$$

where $\hat{\alpha} \in \left[0, \frac{1}{1+\overline{q}}d\right]$. The feasible allocation $\{Z_t(\hat{\alpha})\}_t$ with $\hat{\alpha} = \frac{1}{1+\overline{q}}d$ is the full-insurance allocation, whereas that with $\hat{\alpha} = 0$ implies that the low risk individual gets no insurance.

Only a subset of the feasible allocations are CPE. Define $\overline{\alpha}$ as

$$\overline{\alpha} = \arg\max_{\hat{\alpha}} V_L(\hat{\alpha}, \hat{\alpha}\overline{q}) \tag{11}$$

$\overline{\alpha}$ is the amount of insurance that is the most preferred by low-risk individuals given a price $\overline{q}$, as illustrated by A^* in figure 3. Lemma 3.1 implies that $V_H(Z_H(\hat{\alpha}))$ or $H(\hat{\alpha}, \overline{q})$ is increasing in $\hat{\alpha}$ because $\overline{q} < q_H$. It also implies that a feasible allocation $\{Z_t(\hat{\alpha})\}_t$ is not CPE if $\hat{\alpha} < \overline{\alpha}$, because both types of individuals could be made better off as $\hat{\alpha}$ increases to $\overline{\alpha}$. Also, a feasible allocation $\{Z_t(\hat{\alpha})\}$ with $\hat{\alpha} > \overline{\alpha}$ does not Pareto-dominate the allocation $\{Z_t(\overline{\alpha})\}$, implying that $\{Z_t(\hat{\alpha})\}$ is also CPE. Under the assumption of convex preferences, a feasible allocation $\{Z_t(\hat{\alpha})\}_t$ is also CPE if $\hat{\alpha} > \overline{\alpha}$ because $V_H(Z_H(\hat{\alpha})) > V_H(Z_H(\overline{\alpha}))$ while $V_L(Z_L(\hat{\alpha}))$ is decreasing in $\hat{\alpha}$ for $\hat{\alpha} > \overline{\alpha}$ (see figure 3). This establishes the following:

Proposition 1. *With convex preferences, the set of CPE allocations is $\{Z_t(\hat{\alpha})\}_t$, with $\hat{\alpha} \geq \overline{\alpha}$.*

In particular, the allocation $\{Z_t(\overline{\alpha})\}_t$ is CPE. It entails a pooling policy $(\overline{\alpha}, \overline{\alpha q})$ for the high-risk type, and the pooling policy supplemented by additional insurance at price q_H up to full insurance for the high-risk type.

4 Contracts and Equilibrium

Individuals are allowed to purchase one or more policies from one or more firms. An individual or his insurer may disclose to other firms all or some information about the set of policies purchased or sold, respectively. Information revealed must be truthful, but individuals or firms may choose not to reveal some or all information. What is critical about the information disclosure in the model is that they cannot reveal the fact that they have not purchased a particular policy.[6]

4.1 Setting

We employ a two-stage framework consistent with the conventional setting of a screening model.

- First stage: each firm offers a contract specifying a set of policy offers and a rule of information disclosure. The policy offers come with restrictions on what insurance individuals can buy from other insurers, but the implementation of those restrictions depends on the information available to the insurer.

- Second stage: consumers purchase policies and disclose information about them (possibly selectively) to their potential insurers, following which each firm executes its contract for its consumer as announced in the first stage by disclosing information as specified by its disclosure rule. Consumers whose insurance purchases are found to be inconsistent with the policy restrictions have their policies cancelled.

Firms disclose their information simultaneously in the second stage, implying the disclosure rule of a firm may be made conditional only upon consumer-revealed information, in particular, on information about which firm(s) individuals have purchased insurance from[7]. As a policy offer is subject to cancellation once a firm receives information from other firms, on the other hand, the enforcement of the restrictions imposed within the policy offers can rely on information disclosed by consumers and firms.

Consumer disclosure is absolutely essential to our analysis. Another critical aspect of this setting is that a contract offered by a firm does not depend upon those offered by other firms - i.e., it is non-reactive.

4.2 A Simple Illustration of the Equilibrium Contract

Before conducting a formal analysis of an equilibrium we will describe how an equilibrium contract we propose works in a simple context to highlight the core logic of the main argument on the existence of an equilibrium. The equilibrium we propose involves two kinds of firms: a given number of ”established” firms selling insurance at the pooling odds, and many other ”secret” firms, offering unlimited amount of insurance at price q_H without disclosure. An established firm sells a consumer insurance at the price $\overline{q}$ with the following restriction on additional insurance purchases, and with the following disclosure rule:

[6] In terms of the literature on strategic communication, this is a setting of verifiable disclosure, or hard information (Milgrom (1981), Grossman (1981)).

[7] In the more general analyses of Stiglitz and Yun (2016) and Stiglitz et al. (2019) sequential disclosure rules are also discussed.

- Restriction: the total amount of revealed purchases is not greater than $\overline{\alpha}$ (the amount most preferred by the low-risk consumers at price $\overline{q}$).
- Disclosure Rule: disclose its sale to all the other firms but those who are revealed by the consumer to be his insurers.

For these contracts to sustain the equilibrium allocation E^*, they should be able to do the two things: 1) prevent over-purchases by high-risk individuals and 2) deter a cream-skimming deviant contract from breaking an equilibrium. The central result of this paper is to show that the allocation $\{Z_t(\overline{\alpha})\}_t$ can be sustained by the above set of contracts. Assume, for ease of exposition, that individuals honestly reveal to all the firms from whom they have purchased insurance all of their purchases from other firms.[8] Such honesty directly prevents anyone from overpurchasing the pooling contract. (It can easily be seen why a high risk individual would not lie. Assume he tried to over-purchase, say by purchasing $\frac{1}{2}\overline{\alpha}$ from 3 different firms, firms A, B, and C; but disclosed only one of his other purchases to each, say only his purchase from B to A, but not that from C to A, and symmetrically for the other firms. Then firm A discloses his sales to C. But then C knows that the individual's total purchases are $\frac{3}{2}\overline{\alpha}$ and his insurance is cancelled.)

More subtle is how the asymmetric disclosure rule prevents a deviant contract from breaking the pooling contract. Whenever a deviant firm, say A, charges a price lower than $\overline{q}$, the policy offered by A is always purchased, regardless of the restriction imposed by A, by both types of consumers, yielding losses for the firm A. This is because a consumer could always purchase the same amount $\overline{\alpha}$ in total from the deviant firm A and another established firm B, hence at an average price lower than $\overline{q}$. And if the high risk individual can always buy $\overline{\alpha}$ at this lower price, he will always then want to supplement it with secret insurance. Honest consumer revelation implies that the consumer discloses his purchase from A to B, and that means that B does does not disclose to A its sale (to that consumer) so that any restriction imposed by firm A can't be implemented. Thus, the asymmetric disclosure rule of the established firms can deter any cream-skimming deviant contract from upsetting an equilibrium while preventing over-purchases of the pooling contract by high-risk individuals.

We note the importance of asymmetric disclosure by firms based on the consumer disclosure - which is only possible, as we have noted, because of consumer disclosure. Without consumer disclosure, there would be no basis for the asymmetry of the firm disclosure in a non-reactive framework where a disclosure rule of a firm does not depend upon that of another firm. On the other hand, if the firm disclosure is symmetric and complete, we would obtain RS results, where a pooling allocation cannot be sustained in equilibrium.

[8] In the later analysis, we both show that this is the case and that our disclosure rule supports the equilibrium, even if there is not full disclosure.

4.3 Contracts

In this and the following sections we formalize these intuitions. An insurance contract consists of two components (i) a policy, defined by a price, q, a benefit, α, and a set of restrictions that have to be satisfied (to the knowledge of the insurer) if the policy is to go into effect; and (ii) an information disclosure rule. The set of contracts (policies with their restrictions, and disclosure rules) that are conceivable is quite rich; all that is required is that firms can only disclose a subset of what they know, and can impose restrictions that can only be implemented based on the knowledge of the insurer. But we show that there exists an equilibrium with a simple set of contracts and disclosure rules (one which naturally implements the allocation described in section 3). In this equilibrium, the firms are divided into two groups. What we call the established firms offer insurance at price $\overline{q}$ (the pooling price), with a restriction that no one (to their knowledge) purchases in aggregate more than $\overline{\alpha}$, the amount that maximizes the utility of low risk individuals at that price. The firms' contracts are such as to induce everyone to comply and to reveal that information. The remaining firms (the secret firms) sell undisclosed insurance at price q_H, which is purchased only by high risk individuals, and brings these individuals to full insurance.

To simplify the notation and exposition, we begin by assuming all firms have to offer insurance with a single price[9], while possibly imposing a constraint on aggregate purchases, and then show that, given the equilibrium contracts described, no firm would want to offer any other contract(s) (e.g. with any other set of restrictions or disclosure rules).[10]

1. Policies: (α, β) is given by $(\alpha, \alpha q) \in \mathbb{R}^2_+$ (with $q = \frac{\beta}{\alpha}$ indicating the price of insurance offered). With the index i being suppressed for simplicity, a policy purchased by individual i from firm j is represented by $x(j) \in \mathbb{R}^2_+$:

$$x(j) \equiv (\alpha_j, \alpha_j q_j) \tag{12}$$

while the set of policies purchased from all of the established insurers is denoted by $\hat{X} \equiv \{x(j)\}_{j \in K}$ where $K \subset M$ is the set of the established insurers from which individual i purchases insurance. The amount α_j of insurance offered by a firm j may be required to satisfy a restriction, which can be in general represented by a set of insurance amounts allowed, denoted by $\psi_j(X_j^T)$, where $X_j^T \subseteq \hat{X}$, as defined by

[9]See Stiglitz et al. (2017) for a generalization to the case with multiple prices and possible cross-subsidization.

[10]Similarly, while we allow disclosure of any information available to firms, the equilibrium entails only disclosure of information revealed to it by its consumers plus what it knows from its own sales. In more general models with sequential revelation of information (that is, firms can reveal information that they have from other firms to still other firms), disclosure rules can be more complex.

(17) below, is the total information (about the individual's purchases) available to the firm j. That is,

$$\alpha_j \in \psi_j(X_j^T) \tag{13}$$

A policy offer by a firm j may thus be represented by $(q_j, \psi_j(X_j^T))$.

2. Disclosure Rules: an information disclosure rule by a firm j, denoted DIS_j, specifies a set $RE_j(\subset M)$ of firms receiving information from j about a particular individual, and information $INF_{jk}(\subseteq X_j)$ to be disclosed to a firm $k(\in RE_j)$, where X_j (defined by (14) below) combines the information the firm has directly about j with the information disclosed by a consumer to firm j about his purchases (including the purchase from j). The information disclosed is obviously a subset of X_j.

   The information disclosed by an individual i to his insurer j about purchases from others is denoted by $X_j^o(\subseteq \hat{X})$, indicating that an individual cannot disclose a policy that he does not purchase[11] although he may withhold from his insurer information about some policies purchased. Thus, the information set of firm j about individual i, X_j, before receiving information from other firms is

$$X_j = x(j) \cup X_j^o \tag{14}$$

   We suppose that whenever a policy $x(j)$ is disclosed, the identity j of the insurer is also disclosed. Thus, the set $I(\subset M)$ of firms (including firm j) disclosed as providing insurance by the consumer is given by:

$$I \equiv \{k \in M | x(k) \in X_j\} \tag{15}$$

   Now a disclosure rule DIS_j of firm j may be represented as follows:

$$DIS_j(X_j) = (RE_j(X_j), INF_{jk}(X_j)) \tag{16}$$

   specifying what firms will be disclosed to, and, given that there is some disclosure to firm k, what information is disclosed. Given the disclosure rules $\{DIS_j\}_j$ of all the firms, the aggregate of them will determine the information disclosed to firm j by all the other firms, denoted by X_j^{-j}. Thus, all the information X_j^T available to a firm is that disclosed to firm j by the consumer, by other firms, and what it knows directly from its own sales:

[11] That is, the individual cannot lie about purchases he has not made; in our model, so long as there is no negative insurance, individuals would have no incentive to make such lies.

$$X_j^T \equiv X_j \cup X_j^{-j} \tag{17}$$

A contract C_j offered by a firm j is thus represented by a policy, characterized in turn by a price and a possible constraint on quantities purchased, and a disclosure rule:

$$C_j = \{q_j; \psi_j(X_j^T); DIS_j(X_j)\} \tag{18}$$

4.4 Consumer Response

We now analyze in greater detail how consumers respond to the set of offers. An individual consumer i responds optimally to any given set of contracts offered by firms in the first stage. Formally, given a set $\{C_j\}_{j\in M}$ of contracts offered by firms, a consumer i optimally chooses a set K of his established insurers from which to purchase insurance, the set $\hat{X}(= \{x(j)\}_{j\in K})$ of policies to be purchased from them, $\{X_j^o\}_{j\in M}$ specifying which information about his purchases to disclose to whom, and amounts (if any) of insurance to purchase from other (the non-established) firms. If indifferent across multiple contracts, the consumers randomly chooses one. Further, we assume that consumers tell the truth (disclose information) unless it is in their interests not to do so, which we refer to as the assumption of predilection for truth. It is important to emphasize that we do not assume that consumers are always truthful – we only assume that if they are indifferent between truth telling any anything else, they tell the truth. In other words, this is a tie-breaking rule, not an assumption requiring truth-telling.

We can formalize the optimization the consumers: they choose a set K (or K') of established (or secret) firms, a set of policies $x(k)(= (\alpha_k, \beta_k))$ to purchase from them, and their disclosure rules $\{X_j^o\}_{j\in(K\cup K')})$ to maximize

$$\max_{\{\alpha_k,\beta_k\}_{k\in K\cup K'}} \sum_k V_t\left(\alpha_k, \beta_k\right) \tag{19}$$

$$s.t.\ \alpha_k \in \psi_k(X_k^T), \forall k \in K \cup K' \tag{20}$$

We say that a consumer's choice$\{\{x(j)\}_{j\in M}, \{X_j^o\}_{j\in M}\}$ and disclosure rule is optimal if given $\{x(j)\}_{j\in M}, X_j^o$, and$\{DIS_j(X_j)\}_{j\in M}$ no policy is ever cancelled, and $\{x(j)\}_{j\in M}$ solves the above problem

4.5 Equilibrium Allocations

An equilibrium is defined as follows:

Definition 2. *An equilibrium is a set* $\{C_j^*\}_j$ *of contracts offered by firms such that, given the contracts offered by other firms* $\{C_j^*\}_j$*, there does not exist any other contract that a firm can offer to make positive profits once consumers optimally respond to firms' announced contracts*

We allow a deviant firm to offer any policy $x(j)$ with any restriction and to choose any disclosure rule based on any or all the information available to the firm. In this section, we show that the only possible equilibrium allocation is $\{Z_t(\overline{\alpha})_t\}$, the CPE allocation in the presence of undisclosed insurance which maximizes the well-being of the low risk individual, denoted by E^*.

$$E^* \equiv \{\{\alpha_t^*, \beta_t^*\}_{t=L,H}\} = \{\{Z_t(\overline{\alpha})\}_{t=L,H}\} \tag{21}$$

For any other posited equilibrium allocation, it is possible for an entrant to attract all of the low risk consumers and make a profit; hence that allocation could not be an equilibrium allocation.

While a formal proof is in the appendix, the result is almost trivial: assume that there were some other equilibrium allocation, generated by any set of contracts purchased from any array of insurance firms, that was not E^*. It cannot be preferred to the contract E^* by the low risk individual, for if it were it would have been purchased by high risk individuals as well; unless the contracts purchased by the high risk individuals make them even better off. But there cannot exist such a set of contracts that make both the high and low risk individuals better off than E^*, because we know that E^* is CPE. And it should be obvious that it cannot generate a lower level of utility for the low risk individuals, because an insurance firm that offered E^* would then attract all the low risk individuals, and at least break even. The low risk individuals would purchase that contract regardless of its information disclosure and cancellation provisions, since they will not purchase supplemental insurance and will not be affected by these provisions. The putative equilibrium can thus be broken. This establishes:

Theorem 4.1. *There exists a unique allocation* E^* *that an equilibrium, if it exists, has to implement.*

5 Equilibrium with Endogenous Information Revelation

In this section we show that the contracts described in section 4 support the allocation E^* as an equilibrium. (There may, of course, exist other equilibrium contracts. Our objective is simply to demonstrate the existence of an equilibrium with endogenous information that is implemented using simple, interesting, and illuminating contracts.)

In showing that the equilibrium set of contracts C_j^* implements E^*, we first prove the following lemma[12]:

Lemma 5.1. *Given the set of contracts C_j^*, no individual purchases more than $\overline{\alpha}$ from the established firms.*

This in turn implies that all purchase just $\overline{\alpha}$. While a formal proof is given in the appendix, the intuition is clear. Assume an individual purchased more than $\overline{\alpha}$ in the aggregate from the established firms. Given C_j^* he cannot disclose that he has purchased more than $\overline{\alpha}$ (to any of his insurers) because were he to do so, the policy would be cancelled. So there must not be full disclosure. If the consumer does not disclose one of his insurers, say purchases from firm j, then all the other insurers disclose to the firm j what they know about the consumer's purchases (i.e., their sales to the consumer, and what the consumer reveals to them), and then the firm knows that the individuals has purchased more than $\overline{\alpha}$, so j cancels its policy. But the individual would have known that, and so would not have purchased a policy from j.

There is one important corollary of lemma 5.1: all individuals reveal their purchases from established firms to all established firms, since they have no reason not to (using the assumption of predilection for truth). We now prove the main theorem of the paper:

Theorem 5.2. *Suppose that the income consumption curve for insurance for type t individual at price $\frac{P_t}{1-P_t}$ is the full insurance line, and let $C_j^* = \{q_j^*; \psi_j^*(X_j^*); DIS_j^*(X_j)\}$ be defined as follows:*

1. *For established firms $(j = 1, 2, 3, \ldots, n)$, $q_j^* = \overline{q}$, with $\psi_j^*(X_j^T)$ given by*

$$\psi_j^*(X_j^T) = \{[0, \overline{\alpha}] | T(X_j^T) \leq \overline{\alpha}\} \tag{22}$$

where $T(X_j^T) = \sum_{x(k) \in X_j^T} \alpha_k$. The information disclosure rule of established firms is given by

$$DIS_j^*(X_j) = (RE_j^*(X_j), INF_{jk}^*(X_j)) \tag{23}$$

   a) *Disclose to all of the firms that have not been disclosed by the consumers as insurers, i.e. to*

$$RE_j^*(X_j) = M \setminus I(X_j) \tag{24}$$

   b) *All the information that a firm j has about a consumer that he has obtained from the consumer plus his own sales:*

$$INF_{jk}^*(X_j) \equiv X_j, \forall k \in RE_j \tag{25}$$

[12] Note that, in equilibrium, no established firm sells more than one contract to an individual. It would be only the high-risk individuals that may be interested in purchasing multiple contracts from a firm, because by so doing they might underreport their purchases from the insurer to another established firm (disclosing one policy but not another) to be able to purchase more than $\overline{\alpha}$ at $\overline{q}$. Knowing that the only individual who would wish to buy multiple policies is a high risk individual, an established firm would not sell multiple contracts to an individual without charging a price equal to or higher than q_H, which, however, would not be taken by any individual.

A policy is cancelled if

$$T(X_j^T) > \overline{\alpha} \tag{26}$$

2. *For secret firms* $(j = n+1, n+2, \ldots, N), q_j^* = q_H$ *with no restrictions i.e., offering unlimited insurance at* $q_H : \psi_j^*(X_j^T) = \mathbb{R}_+$*, and make no disclosure:*

$$RE_j^*(X_j) = INF_{jk}^*(X_j) = \emptyset \tag{27}$$

Then C_j^* *implements allocation* E^*.

The equilibrium contract imposes restrictions on the amounts of insurance which an individual can purchase from other firms. The intent of these restrictions is the same as the exclusivity provision in RS: if possible, to exclude high risk individuals from buying insurance, and barring that, at least to limit the amount of insurance purchased by high risk individuals, who will be those who want to buy high levels of insurance.

In equilibrium, the established firm succeeds in the latter, but not the former, i.e. succeeds in limiting the total amount of insurance that the high-risk individual can buy to $\overline{\alpha}$. More precisely, the firm offers a consumer any amount of insurance so long as aggregate purchases of insurance (that it knows about) including its own sales is equal to or less than $\overline{\alpha}$. Because it knows that high risk individuals will never disclose their purchases from the secret firms (and the secret firms won't disclose their sales), it can focus its restrictions on (what it knows about) purchases from the established firms. If the restriction is violated, then the policy is cancelled.

Proof. It is obvious that by lemma 5.1, the set of contracts C_j^* generates the equilibrium allocation E^*. Because of lemma 5.1 and its corollary, every established firm has the information required to effectively implement the allocation. There is no over-insurance by high risk individuals. They just purchase $\overline{\alpha}$ from the established firms and supplement it with undisclosed insurance at price q_H, bringing them to full insurance. We now show the set of contracts C_j^* sustains E^* against any deviant contract. Note first that a deviant firm, indexed by d, cannot make profits by attracting only high-risk individuals in the presence of firms offering secretly any amount of insurance at q_H. This is because then no individual would pay a price higher than q_H since a deviant firm cannot induce the established firms (with C_j^*) to sell more than $\overline{\alpha}$ at $\overline{q}$ under any circumstance. If the deviant firm d attracts both high and low risk individuals, its policy would have to charge a price q_d equal to or lower than $\overline{q}$, yielding zero profit at best. A deviant firm d can thus make positive profits only by attracting low-risk types only, i.e., only by a cream-skimming contract C_d. We will now show that in the presence of undisclosed insurance at price q_H, the contract C_d always attracts high-risk individuals. To attract low risk individuals, we must have $q_d < q_H$. It is obvious from lemma 1 that, the high risk individual, if he could, would purchase the contract OD in figure 4 plus additional pooling insurance DB in figure 4 up to $\overline{\alpha}$ plus supplemental undisclosed insurance (BF in figure 4) at q_H, rendering the deviant contract unprofitable. The deviant firm knows this, and hence must put a constraint on the amount of supplemental pooling insurance that the individual can purchase. The problem is that no such constraint can be enforced. The high risk individual obviously will not disclose directly that he has made the supplemental pooling purchases. If the high risk individual discloses his purchase of the deviant contract to the established firms and limits his total purchases (combining what he has purchased from the deviant firm and amounts purchased from other established firms) to $\overline{\alpha}$, no established firm will cancel insurance that it has sold, and, by its disclosure rule of C_j^*, no established firm will reveal to the deviant firm its sales to the individual. Thus, firm d cannot enforce any restriction entailing total purchases from itself, plus from the established firms, being less than or equal to $\overline{\alpha}$. Accordingly, high risk individuals will purchase the deviant contract, and it loses money. (Because any supplemental pooling insurance purchased from another established firm would not change the information set of the deviant firm d, the deviant firm could not enforce any restriction on such purchases. Also, similarly, the deviant firm cannot make its disclosure rule effectively depend on such supplemental insurance, and so can't use the disclosure rule to deter purchases.) □

6 Previous Literature

In the more than four decades since RS appeared, its disquieting results have given rise to a large literature, which we can divide into two major strands. The first looked for alternative equilibrium concepts or game forms, under which equilibrium might always exist, or under which a pooling equilibrium might exist. Rothschild and Stiglitz (1997) and Mimra and Wambach (2014) reviewed the literature as it existed to those points, with Rothschild and Stiglitz (1997) suggesting that proposed seeming resolution of their nonexistence result contravened plausible specifications of what a competitive market equilibrium should look like in the presence of information asymmetries. For instance, in the ”reactive” equilibrium of Riley (1979) contracts are added, while in ”anticipatory” equilibrium (Wilson (1977)), the entry of even a very small firm induces all firms to withdraw their pooling contracts, making the deviant contract unprofitable, and enabling the pooling equilibrium to be sustained. Miyazaki (1977) (in the case of two types) and Spence (1978) (in the case of n types) extend this equilibrium concept to allow for menus of contracts; the Miyazaki-Wilson-Spence (MWS) outcome is separating, jointly zero-profit contracts with cross-subsidization. Dasgupta and Maskin (1986) provided a game theoretic formulation for the RS model, showing that there was a mixed strategy equilibrium; Farinha Luz (2017) provides a full characterization of equilibria in this setting, and allows for

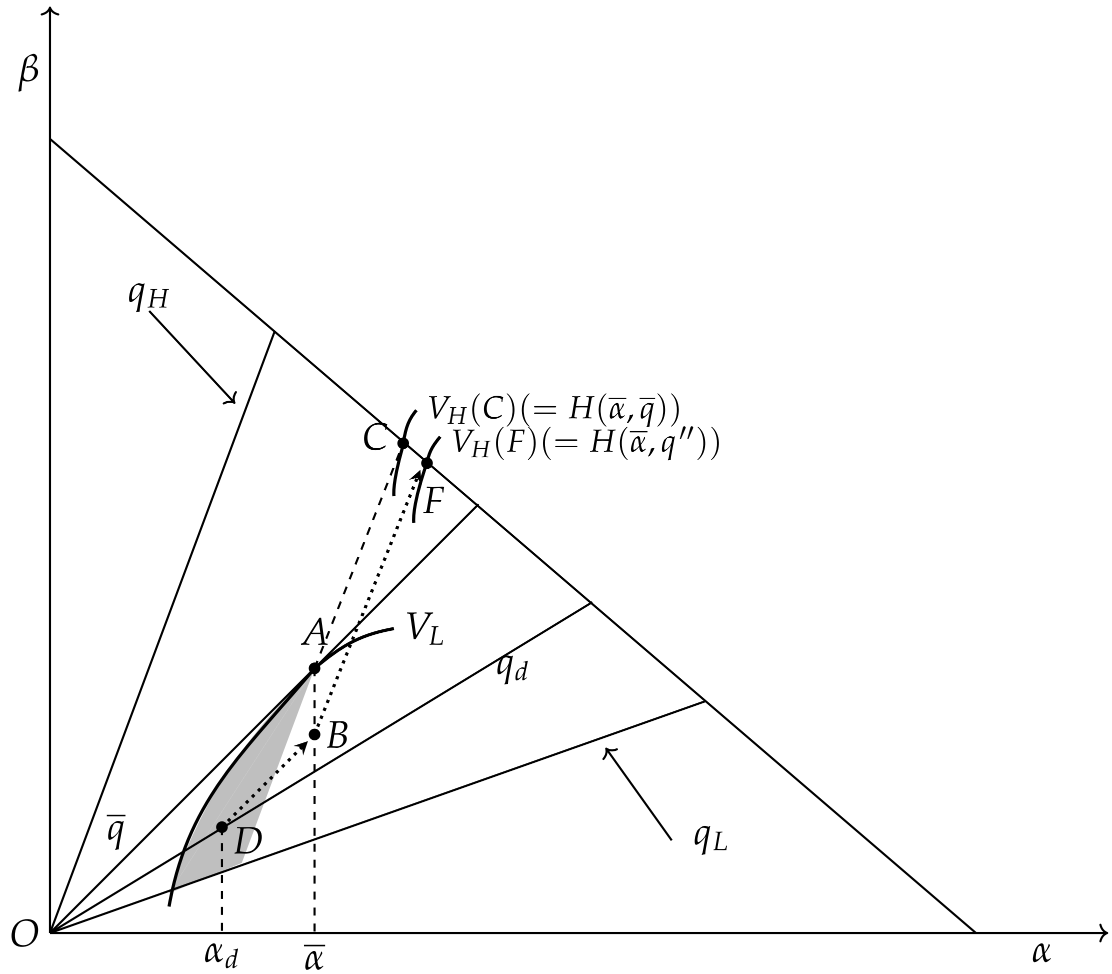

Figure 4: Sustaining an equilibrium against C_d (offering D); $q'' =$ slope of $OB(< \overline{q})$. High risk individuals would always supplement D with pooling insurance (DB) (disclosed only to non-deviant established firms) and secret insurance (BF). As a result $V_H(F) > V_H(C)$, and accordingly, the deviant contract would make losses.

stochastic contracts, while retaining exclusivity. Ales and Maziero (2014) study nonexclusive competition in this framework and obtain a separating equilibrium outcome, relying on latent (i.e. off the equilibrium path) contracts; they do not allow for information sharing. Mimra and Wambach (2017) endogenize insurer capital by having firms choose capital levels before playing the RS game. Mimra and Wambach (2019) add a contract withdrawal stage to the RS model, relying on latent contracts. Netzer and Scheuer (2014) also tackle the same model, but add another twist - firms may "opt out" of the market after observing the contract offers of other firms; the previous three papers obtain the MWS outcome. The equilibrium we identify shares some features with the equilibrium of the limit-order book studied by Glosten (1994), although the context is somewhat different.

A second, more related, strand has explored the consequences of different information structures, allowing for nondisclosed contracts. Most notable are the series of papers by Attar, Mariotti, Salanie (2011, 2014, 2016). The 2014 model, employing strictly convex preferences, provides necessary conditions for the existence of an equilibrium. Applied to the insurance market, the equilibrium (when it exists) turns out to be the allocation where no one but the highest-risk individuals purchase insurance. The difference between their results and ours, where we have focused on endogeneity of information disclosure, are marked. In their 2016 model they showed that an equilibrium entailing the allocation E^* which we identified as the PE allocation which maximizes the welfare of the lowest risk individuals may exist only under a very restrictive set of preferences. When equilibrium exists, it entails the use of latent policies. Again, the difference between our results, where an equilibrium always exists and does not entail latent contracts, and theirs, which argues that undisclosed contracts makes existence less likely than in RS, is marked, and is attributable to our assumption of information endogeneity.

The closest works to our paper within the adverse selection literature are Pauly (1974), and especially Jaynes (1978, 2011) and Hellwig (1988), who analyze a model with a kind of strategic communication among firms about customers' contract information. Jaynes (1978) analyzes the same allocation E^* that we do. However, as Hellwig clarified, in Jaynes (1978) two-stage framework, the strategy of firms, including the associated strategic communication, is not a Nash but a reactive equilibrium, with firms responding to the presence of particular deviant contracts, and thus Jaynes' formulation was subject to the same objections to reactive equilibria raised earlier. Hellwig formulated a four-stage game, in which E^* emerges as the sequential equilibrium, but as he emphasizes, it has the unattractive property that firm behavior (in the final two stages) is conditioned on knowing the offers of all firms, including the deviant firm. Thus, in contrast to our model, a firm cannot offer a contract in secret. Moreover, as Hellwig observes, "it is not the endogenous treatment of interfirm communication that solves the existence problem of Rothschild, Stiglitz, and Wilson. Instead the existence problem is solved by the sequential specification of firm behavior which allows each firm to react to the other firms' contract offers."

While our work differs from that of Jaynes (1978, 2011) and Hellwig (1988) in several ways, perhaps most important is that we consider information revelation by consumers as well as firms. This allows the creation of asymmetries of information about insurance purchases between established firms and deviant firms, which, in turn, enables the pooling contract to be sustained. As we have noted, there is a delicate balance: on one hand, one has to prevent overinsurance by high risk individuals purchasing pooling contracts (which requires established firms to know certain information), and on the other, one has to prevent a deviant firm from having enough information to enforce an exclusive contract that would break the pooling equilibrium. The consumer and firm information strategies which we describe achieve this. In contrast, at least in a simple setting, models relying on just firm information strategies cannot do this, because they do not have any basis on which to engage in this necessary kind of selective disclosure.

Appendix

Derivations of equations (4) and (5):

Assume an individual with a policy $(\alpha, q\alpha)$ supplements it by purchasing $\alpha^S(>0)$ at q_H to reach full insurance, $(\alpha+\alpha^S, q\alpha+q_H\alpha^S)$, where $q\alpha+q_H\alpha^S = d-(\alpha+\alpha^S)$. Note that $\alpha^S>0$ if $\alpha+q\alpha<d$. Then

$$\gamma(\alpha, q\alpha) = \alpha+\alpha^S = \frac{1}{1+q_H}[d+(q_H-q)\alpha] \tag{28}$$

which determines $\delta(\alpha, \alpha q)$ as $B(\alpha, \alpha q) = d-\gamma(\alpha, \alpha q)$.

Proof. **(Lemma 3.2)**: Consider any set $\{(\alpha_t, \beta_t)_{t=L,H}\}$ of policies that yield zero profit, i.e.,

$$\theta\pi_H(\alpha_H, \beta_H)+(1-\theta)\pi_L(\alpha_L, \beta_L) = \theta\{(1-P_H)\beta_H - P_H\alpha_H\}+(1-\theta)\{(1-P_L)\beta_L - P_L\alpha_L\} = 0 \tag{29}$$

Let each policy (α_t, β_t) be represented as the sum of $(\hat{\alpha}_t, \hat{\beta}_t)$ and (α_t^S, β_t^S), where

$$\alpha_t = \hat{\alpha}_t+\alpha_t^S, \beta_t = \hat{\beta}_t+\beta_t^S \text{ while } \hat{\beta}_t = \overline{q}\hat{\alpha}_t, \beta_t^S = q_t\alpha_t^S. \tag{30}$$

It will then suffice to show that $\hat{\alpha}_H = \hat{\alpha}_L$. Since $\pi_t(\alpha_t^S, \beta_t^S) = 0$ for $t = H, L$ we have

$$\theta\pi_H(\hat{\alpha}_H, \hat{\beta}_H)+(1-\theta)\pi_L(\hat{\alpha}_L,)\hat{\beta}_L = 0. \tag{31}$$

Using $\overline{\beta}_t = \overline{q\alpha}_t$ and rearranging the terms, we have

$$\theta[\overline{P}-P_H]\hat{\alpha}_H+(1-\theta)[\overline{P}-P_L]\hat{\alpha}_L = 0. \tag{32}$$

Using the result that $\theta[\overline{P}-P_H]+(1-\theta)[\overline{P}-P_L] = 0$, this implies that

$$\theta[\overline{P}-P_H](\hat{\alpha}_H-\hat{\alpha}_L) = 0 \tag{33}$$

i.e., $\hat{\alpha}_H = \hat{\alpha}_L$. □

Proof. **(Theorem 4.1)**: Suppose, to the contrary, there were another equilibrium allocation $\tilde{E} \neq E^*$, implemented by an arbitrary set of contracts, and let $\tilde{E} = \{(\tilde{\alpha}_L, \tilde{q}_L), (\tilde{\alpha}_H, \tilde{q}_H)\}$. $\tilde{E}$ may be pooling, fully separating, or hybrid; let $\tilde{q}_t$ be the average price for type t.

Decompose $\tilde{E}$ using lemma 3.2:

$$\tilde{\alpha}_L = \hat{\alpha}+\tilde{\alpha}_L^S \tag{34}$$

$$\tilde{\alpha}_H = \hat{\alpha} + \tilde{\alpha}_H^S \tag{35}$$

If $\tilde{\alpha}_L \neq \overline{\alpha}$, by lemma 3.3, the allocation is infeasible. So it must be that $\tilde{\alpha}_L = \overline{\alpha}$ (or equivalently, $\tilde{\alpha}_L^S = 0$). Suppose that $\tilde{q}_L > \overline{q}$. Then a firm could enter, offer a contract $(\overline{\alpha}, \overline{q})$, selling at most one policy for each individual. $V_L(\overline{\alpha}, \overline{q}) > V_L(\overline{\alpha}, \tilde{q}_L)$, and so the low-risk types prefer this to their putative equilibrium allocation, purchase it, so $\pi(\overline{\alpha}, \overline{q}) \geq 0$. Thus, $\tilde{E}$ cannot be an equilibrium.

Suppose now that $\tilde{q}_L < \overline{q}$. By lemma 3.2, $\tilde{\alpha}_H = \overline{\alpha} + \tilde{\alpha}_H^S$, by lemma 3.1, $\tilde{\alpha}_H^S > 0$, and thus both types purchase $\overline{\alpha}_L$. Then $\pi(\tilde{\alpha}_L, \tilde{q}_L) < 0$, and $\tilde{E}$ cannot be an equilibrium, since firms selling this policy are making a loss.

So we must have $\tilde{q}_L = \overline{q}$. But then by lemma 3.1, $\tilde{E}$ coincides with E^*, which concludes the proof. □

Proof. **(Lemma 5.1)**: Given the equilibrium contract, a consumer purchasing more than $\overline{\alpha}$ must not reveal his full purchases to any firm. We first prove the following result: given the equilibrium disclosure rules, in spite of this non-disclosure by consumers, there is at least one firm that knows all the firms from whom the individual has purchased insurance. Assume a consumer purchases more than $\overline{\alpha}$ from K firms, and suppose the consumer makes any set of disclosures. Pick up first the firm that is the most informed (by the consumer) , say firm $j_1 (< K)$, who knows about the consumer's purchases from firms $1, \ldots, j_1$ (including his own sales) and does not know about his purchases from firms $j_1 + 1, \ldots, K$, a group of firms undisclosed to j_1. (When there is a tie in which firm is the most informed, choose any of those;$j_1 = 1$ if a consumer does not disclose anything to any firm). Focus then upon the firms $(j_1 + 1, \ldots, K)$ undisclosed to j_1, and consider a firm who is the most informed of the purchases from those firms, say j_2, who knows about the purchases from $j_1 + 1, \ldots, j_2$. Similarly, we consider the most informed of the firms undisclosed to j_2 and j_1, say j_3. We can continue until we get j_k, where $k = K$. Then, clearly, the purchase from firm j_k is undisclosed to firms $j_1, j_2, \ldots, j_{k-1}$. Now consider the disclosures by firms. As a firm discloses to any other firm that is undisclosed by the consumer as his insurer, all the firms $j_1, j_2, \ldots, j_{k-1}$ (at least) will disclose to the firm j_k their own sales and information received from the consumer, implying that the firm j_k knows all of the K purchases.

The result of lemma 5.1 is now immediate: since that firm knows all of the individual's purchases, it knows that the individual has purchased more than $\overline{\alpha}$, and so cancels the policy. But the individual would not make those purchases, knowing that they would be cancelled. □

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