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IS THEORY REALLY AHEAD OF MEASUREMENT? CURRENT REAL BUSINESS CYCLE THEORIES AND AGGREGATE LABOR MARKET FLUCTUATIONS

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ABSTRACT

In the 1930s, Dunlop and Tarshis observed that the correlation between hours and wages is close to zero. This classic observation has become a litmus test by which macroeconomic models are judged. Existing real business cycle models fail this test dramatically. Based on this result, we argue that technology shocks cannot be the sole impulse driving post-war U.S. business cycles. We modify prototypical real business cycle models by allowing government spending shocks to influence labor market dynamics in a way suggested by Aschauer (1985), Barro (1981, 1987) and Kormendi (1983). This modification can, in principle, bring the models into closer conformity with the data. While the empirical performance of the models is significantly improved, they still fail to account for the Dunlop-Tarshis observation. Accounting for that observation will require futher advances in model devlopment. Consequently, we conclude that theory is behind, not ahead of, business cycle measurement.

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1. Introduction

This paper assesses the quantitative implications of existing RBC models for the time series properties of real wages and hours worked using postwar aggregate US data. We find that the single most salient shortcoming of RBC models lies in their predictions for the correlation between real wages and hours worked. Existing RBC models predict a correlation between real wages and hours that is well in excess of .9. The actual correlation which obtains in the aggregate data is roughly zero.

The ability to account for the observed correlation between real wages and hours worked is a traditional litmus test by which aggregate models are judged. For example Dunlop (1938) and Tarshis' (1939) critique of the classical and Keynesian models was based on the implications of those models for the correlation between real wages and employment. Both models share the common assumption that real wages and hours lie on a stable downward sloped marginal productivity of labor curve.[1] Consequently, they predict, counterfactually, a strong negative correlation between real wages and hours worked.[2] This conflict between theory and evidence stimulated a great deal of research activity. For example, Lucas (1970) suggested that the puzzle could be resolved by modeling variations in the rate of capital utilization. Modigliani (1977) and Phelps and Winter (1970) explored the potential of noncompetitive behavior to account for the Dunlop–Tarshis observation, while Barro and Grossman (1976) actually abandoned equilibrium theories altogether.

In contrast to the classical and Keynesian models which *understate* the correlation

[1]In Keynes' own words: "Thus I am not disputing this vital fact which the classical economists have (rightly) asserted as indefeasible. In a given state of organisation, equipment and technique, the real wage earned by a unit of labour has a unique (inverse) correlation with the volume of employment." (Keynes [1964,p.17].)

[2]Subsequent investigations, which tended to corroborate the Dunlop—Tarshis findings, include Bodkin (1969), Lucas (1980), Geary and Kennan (1982), Schor (1985), and Bils (1985). Summarizing this evidence, Fischer (1988,p.310) concludes "...the weight of the evidence by now is that the real wage is slightly procyclical."

between hours worked and real wages, existing RBC models are inconsistent with the Dunlop–Tarshis observation because they grossly *overstate* that correlation. The reason for this failing can be best understood by recalling that, according to existing RBC models, the *only* impulses generating fluctuations in aggregate employment are stochastic shifts in the marginal product of labor. Loosely speaking, the time series on hours worked and real wages are modeled as the intersection of a stochastic labor demand curve with a fixed labor supply curve. It is therefore not surprising that these theories predict a strong positive correlation between real wages and hours of work.

In view of the traditional interest in the Dunlop–Tarshis observations it is surprising that they have played so little role in the recent debate about RBC models.[3] Instead, attention has centered on the observation that hours are very volatile relative to real wages.[4] For example, Fischer (1988) claims that the degree of intertemporal substitution required to render RBC models consistent with this fact exceeds what is plausible based on micro studies. In contrast, Hansen (1985) and Rogerson (1988) argue that, given sufficiently large nonconvexities in labor supply, it is possible to reconcile *infinite* intertemporal substitution at the level of the representative agent with *any* degree of intertemporal substitution on the part of individual agents at the microeconomic level. Nevertheless it is still the case that these models assume that the only impulse to business cycles are shifts to labor demand. Consequently, RBC models which incorporate nonconvexities in labor supply are also grossly inconsistent with Dunlop–Tarshis type

[3]Two important exceptions are Kennan (1988) and Barro and King (1984).

[4]Two models of aggregate fluctuations, those of Lucas (1977) and Taylor (1980), specify a constant real wage, in recognition of this fact.

observations.[5]

One strategy for reconciling RBC models with Dunlop–Tarshis type observations is to find measurable economic impulses that shift the labor supply function.[6] With impulses impacting on both the labor supply and demand functions there is no a priori reason for real wages and hours worked to display any sort of marked correlation. To us, an obvious candidate for a labor supply shifter are shocks to government spending. By ruling out any role for government spending shocks in labor market dynamics, existing RBC models implicitly assume that public and private consumption have the same impact on the marginal utility of private spending. Aschauer (1985), Barro (1981, 1987) and Hall (1980) argue that when $1 dollar of additional public consumption drives the marginal utility of private consumption down by less than does $1 of additional private consumption, then shocks to government consumption in effect shift the labor supply curve. Coupled with diminishing labor productivity, these type of impulses will, absent technology shocks, generate a negative correlation between hours and the real wage in RBC models.

Our empirical results indicate that shocks to government purchases do have an important quantitative impact on the performance of RBC models. Accounting for these shocks helps generate additional volatility in hours worked relative to the volatility of output and the real wage. Moreover, we find that letting government consumption play a role in labor market dynamics has at least as large an impact on the empirical performance

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[5]Although Prescott (1986) and Kydland and Prescott (1982) never explicitly examine the hours/real wage correlation implication of the RBC, Prescott (1986) nevertheless implicitly acknowledges that failure to account for the Dunlop—Tarshis observation is *the* key remaining deviation between "economic theory" and observations. He states (p.21): "The key deviation is that the empirical labor elasticity of output is less than predicted by theory." Denote the empirical labor elasticity by η. By definition, $\eta \equiv \rho(y,n)\sigma_y/\sigma_n$, where $\rho(i,j)$ is the correlation between i and j, σ_i is the standard deviation of i, y is log detrended output and n is log hours. Simple arithmetic yields $\rho(y-n,n) = [\eta-1](\sigma_n/\sigma_{y-n})$. If—as Prescott claims—the magnitude of σ_n/σ_{y-n} in the RBC is empirically accurate, then saying that the RBC overstates η is equivalent to stating that it overstates $\rho(y-n,n)$. We argue below that this correlation is exactly the same as the hours worked/real wage correlation implied by existing RBC models.

[6]An alternative strategy is pursued by Bencivenga (1987), who allows for shocks to labor suppliers' preferences. Shapiro and Watson (1988) also allow for unobservable shocks to the labor supply function.

of RBC models as does allowing for nonconvexities in labor supply of the type stressed by Hansen (1985) and Rogerson (1988). At the same time, our results suggest that actual government consumption has not been sufficiently volatile in the post war U.S. to significantly offset the sources of positive correlations between hours worked and the real wage embedded in existing RBC models. We reached this conclusion by incorporating government shocks into prototypical RBCs in a manner consistent with Aschauer, Barro and Hall. We find that under these circumstances the correlation between the real wage and hours worked still exceeds .6.

Our results leave us puzzled as to why households choose such large variations in hours given the time series properties of real wages. We suspect that Lucas may have been correct when he wrote:

> Observed real wages are not constant over the cycle, but neither do they exhibit consistently pro— or countercyclical tendencies. This suggests that any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure. (Lucas [1981], p.226.)

Our analysis indicates that existing RBC models fall prey to this (less well known) Lucas critique. Since we believe the Dunlop–Tarshis puzzle will ultimately be resolved by further developments in theory and not by more refinements in data measurement, we, unlike Prescott (1986), conclude that theory is behind, not ahead, of measurement.

The remainder of this paper is organized as follows. In section 2 we describe a general equilibrium model which nests as special cases a variety of existing RBC models. In section 3 we discuss our method of assigning parameter values to the model. Section 4 presents our central result, namely, the difficulty existing RBC models have in accounting for the Dunlop–Tarshis observations. Throughout, we measure the real wage by labor's average productivity rather than, for example, average compensation rates. We do this for several reasons. First, existing RBC models imply that the shadow wage is proportional to average productivity, so the two should be interchangeable for the calculations we perform.

Second, this paper is concerned with the implications of RBC models for shadow wage rates and these need not coincide with average compensation rates. In particular, RBC theories do not imply that wages actually paid and labor services coincide in time. In any event, our empirical results are not very sensitive to whether wage or productivity data are used. Using average productivity data, we obtain essentially the same results as Dunlop and Tarshis, who used wage data. Section 5 contains concluding remarks and suggestions for future research. In Appendix A we examine the correlation between real wages and hours worked as measured in existing RBC studied, eg., Kydland and Prescott (1982) and Hansen (1985). Using this data set we find a substantial negative correlation between the variables. We show that these results are consistent with the view that they reflect the impact of measurement error and that the true correlation is close to zero. First, when we adjust the raw correlation under the assumption that the hours data are mismeasured in the way suggested by Prescott (1986), the results are considerably closer to zero. The second source of measurement error we consider is the misalignment in the coverage of the hours and output series used in existing empirical RBC studies. When the coverage is aligned by considering only the private business sector, then the correlation between wages and hours worked is also close to zero.

2. Two Prototypical Real Business Cycle Models

In this section we present two prototypical real business cycle models, both of which assume steady state growth is generated by exogenous technical change. The first model corresponds to a stochastic version of the standard one sector growth model (see, eg., Kydland and Prescott [1980,p.174].) The second model corresponds to a version of the model considered by Hansen (1985) in which labor supply is indivisible. In addition to allowing for random shocks to the aggregate production possibility set, we relax the implicit assumption in existing RBC models that public and private spending have identical effects on the marginal utility of private consumption. Under these circumstances, government shocks have a nontrivial impact on the labor market.

2.1 The Models

Consistent with existing real business cycle models we assume that the time series on the beginning-of-period t per capita stock of capital, k_t, private time t consumption c_t^p, and hours worked at time t, n_t, correspond to the solution of a social planning problem which can be decentralized as a Pareto optimal competitive equilibrium. For pedagogical purposes we consider the following planning problem which nests both RBC models as special cases. Let T be a positive scalar which denotes the time t endowment of the representative consumer and let γ be a positive scalar. At time t the social planner ranks streams of consumption services, c_t, leisure, $T-n_t$ and publically provided goods and services, g_t according to the criterion function:

$$(2.1) \quad E_0 \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \gamma V(T-n_t) + \phi(g_t)\}.$$

where $\phi(\cdot)$ is some quasi concave function. We follow Kormendi (1983), Aschauer (1985) and Barro (1981,1987) in supposing that consumption services are related to private and public consumption as follows:

(2.2) $c_t = c_t^p + \alpha g_t$,

where α is a parameter which governs the sign and magnitude of the derivative of the marginal utility of c_t^p with respect to g_t. When $\alpha = 1$, then c_t^p and g_t have identical effects on the marginal utility of c_t^p. For values of α less than 1, a unit increase in g_t drives the marginal utility of c_t^p down by less than does a unit increase in c_t^p. When $\alpha = 0$, g_t has no effect, and when $\alpha < 0$ an increase in g_t increases the marginal utility of c_t^p.[7] Throughout this paper we assume that agents view g_t as an uncontrollable stochastic process. Consequently we are free to set $\phi(\cdot) \equiv 0$ without affecting the competitive equilibrium.

We consider two specifications for the function $V(\cdot)$. In what we refer to as the *divisible labor model*, $V(\cdot)$ is given by,

(2.3) $V(T-n_t) = \ln(T-n_t)$ for all t.

In what we refer to as the *indivisible labor model*, $V(\cdot)$ is given by

(2.3)′ $V(T-n_t) = (T-n_t)$ for all t.

There are at least two interpretations of specification (2.3)′. First, it may just reflect the assumption that individual utility functions are linear in leisure. The second interpretation builds on the assumption that there are indivisibilities in labor supply. Here individuals can

[7]When α is negative, then for suitable choice of $\phi(\cdot)$ the marginal utility of g_t is positive, as long as $c_t > 0$.

either work some positive number of hours or not at all. Rogerson (1988) shows that a market structure in which individuals choose lotteries rather than hours worked will support a Pareto optimal allocation of consumption and leisure. The lottery determines whether individuals work or not. Under this interpretation (2.3)′ represents a reduced form preference ordering over hours worked which can be used to derive the Pareto optimal allocation using a fictitious social planning problem. This is the specification used by Hansen (1985) who notes that it is consistent with any degree of intertemporal substitutability of leisure at the individual level.

Per capita output, y_t, is produced using the Cobb–Douglas production function

$$(2.4) \quad y_t = (z_t n_t)^{(1-\theta)} k_t^{\theta},$$

where $0 < \theta < 1$ and z_t is an aggregate shock to technology. We suppose that z_t has the time series representation

$$(2.5) \quad z_t = z_{t-1}\exp(\lambda_t),$$

where λ_t is a serially uncorrelated iid process with mean λ and standard error σ_λ. The national income identity is given by

$$(2.6) \quad c_t^p + g_t + k_{t+1} - (1-\delta)k_t \leq y_t,$$

according to which per capita consumption and investment cannot exceed per capita output.

At date 0 the social planner chooses contingency plans for $\{c_t^p, k_{t+1}, n_t\colon t \geq 0\}$ to maximize (2.1) subject to (2.4) – (2.6) and (2.3) or (2.3)′, k_0 and a law of motion for g_t

which remains to be specified. Before continuing it is useful to substitute out several of the constraints. First, because of the nonsatiation assumption implicit in (2.1) we can, without loss of generality, impose strict equality in (2.6). Using (2.2), (2.4) and this version of (2.6) we obtain the following *planning problem for the divisible labor economy*:

Maximize

$$(2.7) \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[(z_t n_t)^{(1-\theta)} k_t^{\theta} + (1-\delta)k_t - k_{t+1} + (\alpha-1)g_t\right] + \gamma \ln(T-n_t) \right\},$$

subject to k_0 given and a law of motion for g_t to be specified, by choice of contingency plans for $\{k_{t+1}, n_t: t \geq 0\}$.

The corresponding *planning problem for the indivisible labor economy* is:

Maximize

$$(2.7)' \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[(z_t n_t)^{(1-\theta)} k_t^{\theta} + (1-\delta)k_t - k_{t+1} + (\alpha-1)g_t\right] + \gamma(T-n_t) \right\},$$

subject to k_0 given and a law of motion for g_t to be specified, by choice of contingency plans for $\{k_{t+1}, n_t: t \geq 0\}$.

2.2 Stationary Representations of the Model

Before discussing the solutions to the two models, it is convenient to represent the social planning problems (2.7) and (2.7)′ in a different manner. These alternative, equivalent, representations have the property that all of the planner's decision variables converge in nonstochastic steady state. We refer to these alternative representations as the "stationary representations" of the two models.

It is convenient to define the following detrended variables:

$$(2.8)\quad \bar{k}_{t+1} = k_{t+1}/z_t,\quad \bar{y}_t = y_t/z_t,\quad \bar{c}_t = c_t/z_t,\quad \bar{g}_t = g_t/z_t.$$

The variables with a bar over them are well defined because $z_t > 0$ for all t. To complete our specification of agents' environment we assume that $\bar{g}_t$ evolves according to

$$(2.9)\quad \ln(\bar{g}_t) = (1-\rho)\ln(\bar{g}) + \rho\ln(\bar{g}_{t-1}) + \mu_t,$$

where $\ln(\bar{g})$ is the mean of $\ln(\bar{g}_t)$, $|\rho| < 1$ and μ_t is the innovation in $\ln(\bar{g}_t)$ with standard deviation σ_g. Notice that g_t has two components, z_t and $\bar{g}_t$. Movements in the former give rise to permanent changes in the level of government consumption, whereas perturbations in the latter produce effects which die out at a geometric rate, and so are temporary. With this specification, the factors that give rise to permanent shifts in government spending are the same as those which permanently enhance the economy's productive ability.

Substituting (2.8) and (2.9) into (2.7) and (2.7)′ we obtain the criterion function:

$$(2.10)'\quad \kappa + E_0\sum_{t=0}^{\infty}\beta^t r(n_t,\bar{k}_t,\bar{k}_{t+1},\bar{g}_t,\lambda_t),$$

where

$$(2.11)'\quad \kappa = E_0\sum_{t=0}^{\infty}\beta^t \ln(z_t).$$

For the divisible labor model,

(2.12) $r(n_t,\bar{k}_t,\bar{k}_{t+1},\bar{g}_t,\lambda_t) =$

$$\left\{\ln\left[n_t^{(1-\theta)}\bar{k}_t^{\theta}\exp(-\theta\lambda_t) + \exp(-\lambda_t)(1-\delta)\bar{k}_t - \bar{k}_{t+1} + (\alpha-1)\bar{g}_t\right] + \gamma\ln(T-n_t)\right\}.$$

For the indivisible labor model,

(2.12)′ $r(n_t,\bar{k}_t,\bar{k}_{t+1},\bar{g}_t,\lambda_t) =$

$$\left\{\ln\left[n_t^{(1-\theta)}\bar{k}_t^{\theta}\exp(-\theta\lambda_t) + \exp(-\lambda_t)(1-\delta)\bar{k}_t - \bar{k}_{t+1} + (\alpha-1)\bar{g}_t\right] + \gamma(T-n_t)\right\}.$$

Since κ is beyond the planner's control, it can be dropped from (2.11)′ to obtain the criterion function

(2.10) $E_0\sum_{t=0}^{\infty}\beta^t r(n_t,\bar{k}_t,\bar{k}_{t+1},\bar{g}_t,\lambda_t).$

Consequently the original planning problems for the divisible and indivisible labor economies are equivalent to the social planning problems of maximizing (2.10), subject to $\bar{k}_0$, (2.9) and (2.12) and (2.12)′ respectively.

Since the date t state variables in (2.10) are $\bar{k}_t$, $\bar{g}_t$ and λ_t, the solution to both problems is a set of functions:

(2.13) $\bar{k}_{t+1} = f[\bar{k}_t,\bar{g}_t,\lambda_t]$

(2.14) $n_t = q[\bar{k}_t,\bar{g}_t,\lambda_t].$

The solution to the original problems of interest are then given by $k_{t+1} = z_t f[\bar{k}_t, \bar{g}_t, \lambda_t]$ and $n_t = z_t q[\bar{k}_t, \bar{g}_t, \lambda_t]$.

2.3 Approximate Solutions.

The only case in which it is possible to obtain an analytical solution for the models just discussed is when $\alpha = \delta = 1$ and the function $V(\cdot)$ is given by (2.3). This case is analyzed in, among other places, Long and Plosser (1982). For general values of α and δ analytical solutions are not available. Here we use Christiano's (1988b) log linear modification of the procedure used by Kydland and Prescott (1982) to obtain an approximate solution to our social planning problems. In particular we approximate the functions f and q by decision rules that solve the linear quadratic problem obtained by replacing the function r in (2.12) and (2.12)′ by a function R which is quadratic in $\ln(n_t)$, $\ln(\bar{k}_t)$, $\ln(\bar{k}_{t+1})$, $\ln(\bar{g}_t)$ and λ_t. The function R is the second order Taylor expansion of $r[\exp(A_1), \exp(A_2), \exp(A_3), \exp(A_4), A_5]$ about the point $[A_1, A_2, A_3, A_4, A_5] = [\ln(n), \ln(\bar{k}), \ln(\bar{k}), \ln(\bar{g}), \lambda]$. Here n and $\bar{k}$ denote the steady state values of n_t and $\bar{k}_t$ in the nonstochastic version of (2.10) is obtained by setting $\sigma_\lambda = \sigma_g = 0$.

It follows from results in Christiano (1988b) that the decision rules which solve this problem are of the form:

$$(2.15) \quad \bar{k}_{t+1} = \bar{k}(\bar{k}_t/\bar{k})^{r_k}(\bar{g}_t/\bar{g})^{d_k}\exp[e_k(\lambda_t - \lambda)],$$

and

$$(2.16) \quad n_t = n(\bar{k}_t/\bar{k})^{r_n}(\bar{g}_t/\bar{g})^{d_n}\exp[e_n(\lambda_t - \lambda)].$$

In (2.15) and (2.16) r_k, d_k, e_k, r_n, d_n and e_n are scalar functions of the models' underlying structural parameters.[8]

The approximate decision rules (2.15) and (2.16) are appealing for a number of reasons. First, for sufficiently small values of the vector $(\sigma_\lambda, \sigma_g)$ and for $(\bar{k}_t, \bar{g}_t)$ sufficiently close to $(\bar{k}, \bar{g})$ relations (2.15) and (2.16) approximate (2.13) and (2.14) arbitrarily well. Second, for the case in which $\alpha = \delta = 1$ and V is given by (2.3)' the log linear approximation is exact. Third, Christiano (1987b,1988a) studies versions of our models in which $\alpha = 1$ and δ is close to zero and shows that the log linear approximation is quite accurate.

2.4 The Dynamic Effects of Government Spending Shocks.

Notice that, when $\alpha = 1$, the only way in which c_t^p and g_t enter into the social planner's preferences and constraints is via their sum, $c_t^p + g_t$. Thus, exogenous shocks to g_t induce one–for–one offsetting shocks in c_t^p, leaving other variables like y_t, k_{t+1} and n_t unaffected. This implies that the coefficients d_n and d_k in the planner's decision rules for k_{t+1} and n_t both equal to zero. Consequently, the absence of a role for g_t in existing RBC

[8]At this point, we can give some indication as to why the Kydland–Prescott linear approximation is inappropriate in our context. Their method delivers an approximation to the f function in (2.13) that is linear in its arguments, ie., it is a function $A_t + \alpha_3(\lambda_t - \lambda)$, say, where $A_t = \alpha_0 + \alpha_1 \bar{k}_t + \alpha_2 \bar{g}_t$. Thus, the implied approximate decision rule for k_{t+1} is f, where $f = z_{t-1}\exp(\lambda_t)[A_t + \alpha_3(\lambda_t - \lambda)]$. Since the linear approximation is arbitrarily accurate for $\lambda_t - \lambda$ sufficiently close to zero, it follows that for such values of $\lambda_t - \lambda$, f is positive and increasing in $\lambda_t - \lambda$. However, since α_3 is negative (in the stationary version of the model, a *positive* perturbation in λ_t is a *negative* technology shock and a positive innovation to capital depreciation), it follows that f must become negative for $\lambda_t - \lambda$ sufficiently large. This non–monotonicity in f has the implication that a large technology shock induces a fall in capital investment and, via the resource constraint, a surge in private consumption. Christiano (1987a;1988b,ftn 9,18) documents that these perverse dynamics are sufficiently large, even for plausible shock variances, to significantly distort second moment properties. It is easily confirmed that the log–linear approximate decision rule for k_{t+1} implied by (2.15) is monotone in $\lambda_t - \lambda$. The key feature of our context that accounts for the difference between the log–linear and linear approximations is our model for z_t, (2.5). When z_t is modelled as covariance stationary about a deterministic trend (as is done implicitly in Kydland and Prescott [1982] and Hansen [1985], see footnote 15), then results in Christiano (1988a) suggest that the difference between the log linear and the linear approximations is small.

models can be rationalized by the assumption that $\alpha = 1$.

In simulation experiments with our models, we found that reducing α below 1 increases d_n, so that hours worked become more responsive to movements in $\bar{g}_t$. This in turn reduces the correlation between hours worked and average productivity. In addition, we found that this correlation is systematically affected by the parameter ρ, which governs the serial correlation of shocks to $\bar{g}_t$. The longer lasting these shocks are (ie., the larger ρ is) the larger is d_n and the smaller is the predicted correlation between hours worked and average productivity. We now discuss the intuition behind these results.

To understand the role played by α, consider the following suboptimal, benchmark policy in which the planner responds to shocks in $\bar{g}_t$ by leaving all labor market variables unchanged. Formally, under the benchmark policy: $\nabla n_s = \nabla y_s = \nabla k_{s+1} = 0$ for $s \geq t$, where ∇ signifies the response of the associated variable to a shock in $\bar{g}_t$. Feasibility of this policy requires $\nabla c_s^p = -\nabla g_s$ $s \geq t$, so that,

$$\text{(2.17)} \qquad \nabla c_s = (\alpha - 1)\nabla g_t.$$

To see why this benchmark policy response is suboptimal when $\alpha < 1$, it is useful to focus on the first order condition requiring that the marginal rate of substitution between leisure and consumption equal the real wage:

$$\text{(2.18)} \qquad MPL_t u'(c_t) = \gamma V'(T - n_t).$$

Here, $u(\cdot)$ is the period utility function of consumption services ($u(\cdot) = \log(\cdot)$) and MPL_t is the marginal product of labor with $MPL_t = z_t[(1-\theta)\exp(-\theta\lambda_t)(\bar{k}_t/n_t)^{\theta}]$. When $\alpha < 1$ the benchmark policy implies that an exogenous jump in government spending produces a rise in $u'(c_t)$ by reducing consumption services. From (2.18), we see that the benchmark policy cannot be optimal because it implies that the marginal utility of leisure is less than the marginal return to working as measured by $MPL_t u'(c_t)$. The rise in hours that is actually optimal assures the equality in (2.18). Relative to the benchmark policy, this involves a lower value of MPL_t and, when V is given by (2.3), a larger value of $V'(T-n_t)$.[9] Notice that the smaller is α, the larger is the initial increase in $u'(c_t)$ associated with a given increase in $\bar{g}_t$ under the benchmark policy. Consequently, the sensitivity of n_t to $\bar{g}_t$ rises as α falls. Finally, because $\bar{g}_t$ has no direct effect on the production function, a smaller α also implies a larger negative response of productivity. We conclude that by magnifying the opposing movements in hours and productivity associated with a shock in $\bar{g}_t$, smaller values of α lead to a smaller correlation between these two endogenous variables.

To understand the role played by the degree of permanence in exogenous government shocks it is useful to consider the two extremes: $\rho = 0, 1$. In the first case, the effect of a shock to $\bar{g}_t$ lasts only one period. Concavity of the utility function suggests that households will accommodate the one period increase in $\bar{g}_t$ with a small increase in n_t and a small decrease in c_t^p sustained over a number of periods. The increased y_t and reduced c_t^p in the period of the shock, along with a small reduction in capital investment, make room for the government spending shock. Future periods' reduced c_t^p and increased y_t permit the increased investment required to gradually return the capital stock to its unchanged steady state growth path. This reasoning suggests that when $\rho = 0$, $d_k < 0$ and $d_n > 0$, but small in absolute value.

The negative income effect associated with a permanent increase in $\bar{g}_t$ causes steady state n_t and $\bar{k}_t$ to increase. By itself, the smoothing motive associated with the concavity

[9] We find that d_n is larger when $V(\cdot)$ is linear in $T-n_t$ than when it is defined by (3.3). This is not surprising since in this case $V'(T-n_t)$ has no role to play in restoring equality in (3.18) relative to the benchmark policy. Put differently, the indivisible labor model increases the income effect on leisure by reducing the income effect on consumption to zero.

of the planner's preferences now induces a *strong* positive response of hours worked.[10] The immediate response is even larger. This follows from the well known property of the one sector growth model that adjustment of capital to steady state is not instantaneous. During the transition period, when capital is below its steady state growth path, hours worked is above its steady state value. This is the reason why the intital response of hours worked to a permanent increase in $\bar{g}_t$ is even larger than the steady state response. This reasoning suggests that the larger ρ is the larger d_n and d_k are, with the latter eventually becoming positive. Finally, because of the small short term response in the stock of capital, the large increase in n_t generates a large fall in average productivity. With larger values of ρ generating larger values of d_n, we expect a smaller correlation between hours worked and productivity.

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[10]In our models the steady state rate of interest is independent of $\bar{g}$, as is $\bar{k}/n$. It follows that $\bar{y} = \psi n$, where ψ is independent of $\bar{g}$, so that a useful, unit free measure of the income effect on n is given by the output multiplier, $d\bar{y}/d\bar{g}$. After some algebra, it can be shown that, for the divisible labor economy,

$$\frac{d\bar{y}}{d\bar{g}} = \frac{1-\alpha}{a_0 + a_1(\alpha)},\quad a_0 = \frac{\bar{c}^P + \bar{g}}{\bar{y}},\quad a_1(\alpha) = \frac{\bar{c}^P + \alpha\bar{g}}{\bar{y}}\,\frac{n}{T-n}.$$

Here, a_0 and $a_1(\alpha)$ are independent of $\bar{g}$. In the case of a_0, this is because it is 1 minus the ratio of gross investment to output, which is determined by $\bar{k}/n$. In the case of a_1, this is because $\gamma a_1(\alpha)$ is the product of the steady state marginal rate of substitution between leisure and consumption and $n/\bar{y}$, neither of which is related to $\bar{g}$. To evaluate the magnitude of the output multiplier, we replaced $\bar{c}^P/\bar{y}$, $\bar{g}/\bar{y}$ and n by their post war sample averages, .55, .177, and 320.2, respectively. In addition, we set T = 2190, the number of hours in a quarter. The output multiplier is approximately linear in α, with slope −1.2. For the following values of α: 1.0, 0.5, 0.0, −0.5, −1.0, −1.5, the output multiplier is, respectively, 0.00, 0.60, 1.22, 1.86, 2.53, 3.22.

3. Assigning Values to the Models' Parameters

In this section we describe our strategy for assigning values to the models' parameters. These are estimated using Hansen's (1982) Generalized Method of Moments (GMM) procedures. Apart from the balanced growth implications we do not impose any of the models' overidentifying restrictions. We proceed this way because a variety of authors, including Mankiw, Rotemberg and Summers (1985), Altug (1986), Christiano (1988b) and Eichenbaum, Hansen and Singleton (1988) have already rejected, using formal statistical methods, versions of the RBC models discussed in section 3. Here we are more interested in documenting the models' performance along specific dimensions. Our GMM strategy amounts, in practice, to requiring that the model fit selected first moments of the data. In this sense the procedure is consistent with the way in which Prescott (1986) uses growth observations for pinning down values of a subset of his model's parameters. As it turns out, our GMM estimates are essentially identical to those obtained using the procedure described in Christiano (1988b) which chooses values for the structural parameters that equate an approximation of the models' first moment implications with appropriate sample moments of the data. An important advantage of the GMM procedure is that we can obtain standard errors for our point estimates.

3.1 Methodology

This subsection discusses how we assigned values to the structural parameters of the model:

(3.1) $\quad T,\beta,\alpha,\delta,\theta,\gamma,\lambda,\sigma_{\lambda},\bar{g},\rho,\sigma_{g}.$

Throughout the paper we assume that T equals 2190, the total number of hours in a quarter and set the parameter β a priori so as to imply a 3% annual subjective discount rate, i.e. $\beta = (1.03)^{-.25}$.

Consider the parameter δ. Let dk_t denote gross investment at time t. According to our model, $dk_t = [k_{t+1} - (1-\delta)k_t]$, so that,

$$(3.2) \quad \delta = 1 - dk_t/k_t - k_{t+1}/k_t.$$

Let δ^* denote the unconditional mean of the time series $[1 - dk_t/k_t - k_{t+1}/k_t]$, so that,

$$(3.3) \quad E\{\delta^* - (1 + dk_t/k_t - k_{t+1}/k_t)\} = 0.$$

We identify δ with a consistent estimate of the parameter δ^*.

The time t first order necessary condition for capital accumulation in our models states that the time t expected value of the marginal rate of substitution of goods in consumption equals the time t expected value of the marginal return to physical investment in capital,

$$(3.4)' \quad E_t\{\beta^{-1} c_{t+1}/c_t - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]\} = 0.$$

It follows from (3.4)′ that

$$(3.4) \quad E\{\beta^{-1} c_{t+1}/c_t - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]\} = 0.$$

This is the moment restriction that underlies our estimate of θ.

The time t first order necessary condition for hours worked require that the time t expected value of the marginal productivity of labor times the marginal utility of consumption equals the time t expected value of the fictitious representative consumer's marginal disutility of working. Given our assumptions regarding the aggregate production technology and the social planner's criterion function this condition can be written as

$$(3.5) \quad (1-\theta)[y_t/n_t]/c_t = \gamma V'(T-n_t)$$

where

$$(3.6) \quad V'(T-n_t) = (T-n_t)^{-1},$$

or

$$(3.6)' \quad V'(T-n_t) = 1.$$

Let γ^* denote the unconditional expected value of the time series $(1-\theta)[y_t/n_t]/[c_t V'(T-n_t)]$, so that

$$(3.7) \quad E\{\gamma^* - (1-\theta)y_t c_t^{-1} n_t^{-1}/V'(T-n_t)\} = 0.$$

We identify γ with a consistent estimate of the parameter γ^*.

Given a value of θ we can compute a time series on the Solow residuals z_t using the (2.4) and observations on (y_t, n_t, k_t). Let λ and σ_λ denote the unconditional expected value and standard error of the time series process $\lambda_t = \ln(z_t) - \ln(z_{t-1})$. By assumption,

$$(3.8) \quad E[\lambda_t - \lambda] = 0,$$

and

(3.9) $E[\lambda_t^2 - 2\lambda_t\lambda - \lambda^2 + \sigma_\lambda^2] = 0,$

where $\lambda_t = \Delta[\ln(y_t)/(1-\theta) - \ln(n_t) - \theta\ln(k_t)/(1-\theta)]$ and Δ is the first difference operator.[11]

Let μ_{cp}, μ_y, μ_k and μ_g denote the unconditional expected growth rates of c_t^p, y_t k_t, and g_t, so that

(3.10) $E[\mu_{cp} - \ln(c_t^p/c_{t-1}^p)] = 0,$

(3.11) $E[\mu_y - \ln(y_t/y_{t-1})] = 0,$

(3.12) $E[\mu_k - \ln(k_{t+1}/k_t)] = 0.$

and

(3.13) $E[\mu_g - \ln(g_t/g_{t-1})].$

All of our models imply the testable restriction that

(3.14) $\lambda = \mu_{cp} = \mu_y = \mu_k = \mu_g.$

Given our assumptions regarding the stochastic process generating government

[11]In section 3, we specified λ_t to be iid, whereas an empirical estimate of this quantity seems to display negative first order autocorrelation. (This is also reported in Prescott [1986].) We nevertheless set the theoretical first order autocorrelation of λ_t to zero because, as documented in Christiano (1987d,1988b), our models predict that the autocorrelation of $\Delta\log(y_t)$ closely matches that of λ_t. However, empirically $\Delta\log(y_t)$ has lag one autocorrelation roughly equal to .36. Given that our models cannot accommodate at the same time both the serial correlation properties of λ_t and y_t, we thought it a reasonable compromise to go half–way in matching both, by setting the theoretical autocorrelation of λ_t to zero.

expenditures we have the unconditional moment restrictions,

$$(3.15)\quad \begin{aligned} &E[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})] = 0,\\ &E[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]\bar{g}_{t-1} = 0,\\ &E\{[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]^2 - \sigma_g^2\} = 0. \end{aligned}$$

Since c_t/c_{t-1} is contained in agents' time t information set relation (3.4)′ implies that

$$(3.16)\quad E\{\beta^{-1}c_{t+1}/c_t - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]\}(c_t/c_{t-1}) = 0.$$

This unconditional moment restriction can be exploited to estimate α.

In order to discuss our estimation procedure it is convenient to define the vector valued function

$$(3.17)\quad X_{t+1}=[c_{t+1}/c_t,c_t/c_{t-1},k_{t+1}/k_t,y_{t+1}/k_{t+1},y_t/y_{t-1},y_t/c_t,n_t,dk_t/y_t,g_t/g_{t-1}],$$

and the parameter vector,

$$(3.18)\quad \Psi = [\delta^*,\theta,\gamma^*,\lambda,\sigma_\lambda,\mu_{cp},\mu_y,\mu_k,\mu_g,\bar{g},\rho,\sigma_g,\alpha].$$

With this notation we can summarize (3.3), (3.4), (3.7) – (3.13), (3.15) and (3.16) as

$$(3.19)\quad EH[X_{t+1},\Psi] = 0 \quad \forall\, t \geq 0,$$

for $\Psi = \Psi_0$, the true parameter vector. Here, $H(\cdot,\cdot)$ is the 13 x 1 vector valued function whose 13 elements are:

$$
\begin{aligned}
(3.20)\quad & H_1 = [\delta^* - (1 + dk_t/k_t - k_{t+1}/k_t)] \\
& H_2 = [\beta^{-1}c_{t+1}/c_t - \{\theta(y_{t+1}/k_{t+1}) + 1 - \delta\}] \\
& H_3 = [\gamma^* - (1-\theta)(y_t n_t^{-1})/(c_t V'(T-n_t))] \\
& H_4 = [\lambda_t - \lambda] \\
& H_5 = \{\lambda_t^2 - 2\lambda_t\lambda + \lambda^2 - \sigma_\lambda^2\} \\
& H_6 = \{\mu_{c^p} - \ln(c_t^p/c_{t-1}^p)\} \\
& H_7 = [\mu_y - \ln(y_t/y_{t-1})] \\
& H_8 = [\mu_k - \ln(k_{t+1}/k_t)]. \\
& H_9 = [\mu_g - \ln(g_t/g_{t-1})] \\
& H_{10} = [\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})], \\
& H_{11} = [\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]\bar{g}_{t-1}, \\
& H_{12} = \{[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]^2 - \sigma_g^2\} \\
& H_{13} = \{\beta^{-1}c_{t+1}/c_t - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]\}(c_t/c_{t-1}).
\end{aligned}
$$

The 13 dimensional function g_T

$$(3.21)\quad g_T(\Psi) = (1/T)\sum_{t=0}^{T} H(X_{t+1},\Psi),$$

can be calculated given a sample on $\{X_t\colon t=1,2,\ldots T+1\}$. Both our models imply that X_{t+1} is a stationary and ergodic stochastic process. It follows from results in Hansen (1982) that Ψ_0 can be estimated by choosing that value of Ψ, say Ψ_T, that minimizes the quadratic criterion

$$(3.22)\quad J_T = g_T(\Psi)W_T^{-1}g_T(\Psi)'.$$

where W_T is a positive definite matrix that can depend on sample information.

Hansen (1982) also shows that the estimator which results in the minimum asymptotic covariance matrix of Ψ_T is obtained by choosing W_T^{-1} to be a consistent estimator of

$$(3.23) \quad R_0 = \sum_{k=-\infty}^{\infty} E[H(X_{t+k+1},\Psi)][H(X_{t+1},\Psi)]'.$$

Proceeding as in Hansen (1982) we can estimate R_0 by replacing the population moments in (3.23) by their sample counterparts evaluated at Ψ_T. In order to guarantee that our estimate of R_0 is positive definite we use the damped truncated covariance estimator discussed in Eichenbaum and Hansen (1988). The results we report were calculated by truncating (3.23) after 6 lags. Since we have exactly thirteen parameters and thirteen unconditional moment restrictions, the minimized value of the criterion function J_T will be exactly equal to zero. This simply reflects the fact that we are not imposing any overidentifying restrictions on this version of the model.

The restrictions on the growth rates summarized by (3.14) can be tested by taking the difference between the minimized value of the criterion (3.22) when the restrictions are imposed and the minimized value of the criterion when the restrictions are not imposed. The latter value is equal to zero in our case. The same distance matrix should be used for both runs and should be a consistent estimate of R_0 even when the restrictions are not satisfied. The resulting test statistic which we denote S_T is distributed asymptotically as a Chi–square with degrees of freedom equal to the number of restrictions being tested.

In practice we found it very difficult to estimate α in conjunction with the other parameters of the model, in the sense that our estimate of α depended sensitively on the initial starting values for α. Consequently we estimated the remaining 12 elements of Ψ_0 under two alternative assumptions: private and publicly provided consumption goods are completely nonsubstitutable ($\alpha = 0$) and perfect substitutes ($\alpha = 1.0$), respectively. In both cases we simply simply deleted H_{13} from the moment conditions being investigated.

3.2 Data Description

Private consumption, c_t^p, was measured as quarterly real expenditures on nondurable consumption goods plus services, plus the imputed service flow from the stock of durable goods. The first two measures were obtained from the Survey of Current Business. The third measure was obtained from the data base documented in Brayton and Mauskopf (1985). Government consumption, g_t, was measured by real government purchases of goods and services minus real government (federal, state and local) investment.[12] A measure of government investment was provided to us by John Musgrave of the Bureau of Economic Analysis. This measure is a revised and updated version of the measure discussed in Musgrave (1980). Gross investment, dk_t, was measured as private sector fixed investment plus real expenditures on durable goods plus government fixed investment. The capital stock series, k_t, was chosen to match the investment series. Accordingly, we measured k_t as the stock of consumer durables, producer structures and equipment, plus government and private residential capital plus government nonresidential capital. Gross output, y_t, was measured as c_t^p plus g_t plus dk_t plus time t inventory investment. Given our consumption series, the difference between our measure of gross output and the one reported in the Survey of Current Business is that ours includes the imputed service flow from the stock of consumer durables but excludes net exports. Our measure of hours worked correspond to the one constructed by Hansen (1984). The data

[12]It would be desirable to include in g_t a measure of the service flow from the stock of government owned capital, since government capital is included in our measure of k_t. Unfortunately we know of no existing measures of that service flow. This contrasts with the case of household capital, for which there exist estimates of the service flow from housing and the stock of consumer durables. The first is included in the official measure of consumption of services, and the second is reported in Brayton and Mauskopf (1985).

were converted to per capita terms using an efficiency weighted measure of the population (see section 2). All series cover the period 1955,3 – 1983,4. For further details on the data, see Christiano (1987c).

Several first moment properties of the data are reported in Table 3 under the heading "U.S. Data".[13] Of particular interest are the mean growth rates of per capita private consumption, output, investment and government consumption. According to these point estimates, $\mu_k > \mu_{cp} > \mu_y > \mu_g$ with $\mu_k \approx 1.88\%$, $\mu_{cp} \approx 1.80\%$, $\mu_y = 1.60\%$, and $\mu_g =$ .92% on an annualized basis. The estimated value of the annualized growth rate in per capita hours worked is .08 percent, roughly zero.

While the point estimate of average growth in government consumption seems problematic from the perspective of the model the estimated standard error of μ_g is quite large. Consequently we formally tested the hypothesis:

$$(3.24) \quad \mu_{cp} = \mu_y = \mu_k = \mu_g,$$

using the GMM procedure for testing parameter restrictions described above. The resulting value of S_T which is asymptotically distributed as a Chi–square with 3 degrees of freedom, equaled 2.69, with corresponding probability value .56. Thus this test yields very little evidence against the balanced growth hypothesis. We interpret these results with some caution since our test assumes the growth rates are constant throughout the sample. In fact, the low average growth rate in g_t appears to reflect the fact that, beginning in the early 1970s, government consumption began to occupy a shrinking share of y_t. For example, the annual growth rate of g_t averaged 2.8 and –.7 percent in the periods 1956,3 – 1969,4 and 1970,1 – 1984,1 respectively.

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[13]Standard errors in Table 3 were estimated using an exactly identified version of the GMM procedure described in this section. The analogue to the martix R_0 (defined in [3.23]) which is required to calculate standard errors was estimated in the way described immediately following equation (3.23) in the text.

The first four columns of Table 1 report point estimates and standard errors for the various versions of the model which we consider. The first two columns report results for the case in which private and public consumption are perfect substitutes ($\alpha = 1.0$). The third and fourth columns report results for the case in which private and public consumption expenditures are completely nonsubstitutable. In all cases we imposed restriction (3.14) which was tested using the GMM procedure discussed above. The resulting value of S_T which is asymptotically distributed as a Chi–square with 4 degrees of freedom, equaled 3.24, with corresponding probability value .48. Thus we found very little evidence against the growth rate implications of the model.

Notice that the parameters are estimated with small standard errors. In order to assess implications of our point estimates for the first moments of the data we simulated the models given the values of the structural parameters reported in Table 3 and generated 1000 simulated time series, each of length 113. First moments were calculated on each of the data sets. The numbers reported in Table 4 correspond to the average sample moment across the different data sets. As can be seen all four models do extremely well in matching the subset of first moments investigated.

4. Empirical Results

This section investigates the quantitative properties of our models. We are particularly interested in their implications for the Dunlop–Tarshis observations and the volatility of real wages and hours worked. We document that the RBC models with $\alpha = 1$ are unable to account for the Dunlop–Tarshis observations. In addition they understate the volatility of hours relative to wages as well as the volatility of hours per se. We then allow government spending to play a nontrivial role in labor market dynamics by setting $\alpha = 0$. This change generates a substantial improvement in the models' implications for the volatility of wages and hours. However the implications of the models remain spectacularly at variance with the Dunlop–Tarshis observations.

Our methodology for investigating these issues is as follows. In section 3 we reported estimated values for the structural parameters of our models. Using these we solved for the equilibrium laws of the system exploiting the methods discussed in section 2 and simulated synthetic time series for the endogenous variables using government and technology shocks drawn from a Normal random number generator. Finally we computed selected second moments using the simulated data sets and compared them to analog moments computed using the actual post war US data.

Table 2 reports the coefficients of the equilibrium laws of motion for $\bar{k}_{t+1}$ and n_t for the four versions of the RBC model described in section 2. Since these are used to generate the synthetic time series that are the basis of our quantitative analysis it is useful briefly to discuss their qualitative properties. Recall that the coefficient e_n denotes the response of $\ln(n_t)$ to an innovation in the technology shock. All of the models which we considered imply that n_t depends positively on λ_t. When $\alpha = 1$, e_n equals .36 and .48 in the divisible and indivisible labor models, respectively. When $\alpha = 0$, e_n equals .45 and .59 in the divisible and indivisible models, respectively. Evidently, as explained in Hansen (1985), indivisibilities in labor supply increase the sensitivity of hours worked to movements in the technology shock. Reducing α also increases e_n. This reflects our specification that a positive technology shock drives up government spending (ie., $g_t = z_t\bar{g}_t$.) Other things equal, this induces an increase in the number of hours worked (see section 2.) A larger value of e_n increases the conditional volatility of hours worked and reduces the conditional volatility of real wages. The latter effect arises because of diminishing returns in the production technology. These considerations suggest that indivisibilities in labor and a nontrivial role for g_t in the labor market will be useful in accounting for the unconditional labor market volatility observations.

According to Table 2, when $\alpha = 0$ the coefficient d_n, which represents the elasticity of n_t with respect to $\bar{g}_t$, equals .21 and .28 in the divisible and indivisible labor models, respectively.[14] As suggested by the intuition in section 2.4, the magnitude of these elasticities reflects in part the high degree of persistence in the exogenous government spending shock, μ_t. For example, in the $\alpha = 0$ version of the models, when ρ is set to zero, then $d_n = .018$ and .025 in the divisible and indivisible labor models, respectively.[15] The positive sign on these elasticities implies that increases in government consumption due to an innovation in μ_t generate increases in hours worked. Thus, other things equal, μ_t shocks generate opposing moves in average productivity and hours worked. By increasing the quantitative magnitude of this effect, the high estimated persistence of government shocks ($\rho = .97$) improves the models' chances of matching the Dunlop–Tarshis observations.

Table 4 reports the implications of the different models for various second moments

[14]The intuition underlying the fact that d_n is larger in the indivisible labor economy is discussed in footnote 9.

[15]In addition, d_k was $-.0154$ and $-.0150$ in the divisible and indivisible labor models, respectively. All other parameters in our approximate decision rules are functionally independent of the value of ρ.

of the data.[16] Table 4A reports results obtained using data which have been transformed using the Hodrick/Prescott filter. Table 4B reports the analog results obtained using the Growth 1 and Growth 2 filters. For each of our four models we generated 1000 data sets, each of length 113, using the parameter values reported in Table 1. The data sets were then processed using the Hodrick/Prescott, Growth 1 and Growth 2 filters. Second moments were calculated using each of the transformed synthetic data sets. The numbers in columns 2 – 5 in Tables 4A and 4B correspond to the average second moments across each of the transformed 1000 synthetic data sets. Associated numbers in parentheses are standard deviations, across data sets. The numbers in the last column of Tables 4A and 4B are the indicated empirical second moments. The associated numbers in parentheses are the corresponding empirical standard errors.

First consider the results in Table 4A. We measure the volatility of a variable, say x, by its standard deviation, which we denote σ_x. All of the models do well at matching the volatility of output and the volatility of consumption and investment relative to output. In contrast, all do poorly at matching the volatility of hours worked relative to output. To see this compare the ratio of σ_n/σ_y generated by each of the models with our point estimate of σ_n/σ_y. For the versions of the divisible and indivisible labor models in which α equals 1, this ratio equals .41 and .50 respectively. For the corresponding models

[16]Point estimates and standard errors for the U.S. data reported Tables 4A and 4B were obtained in the following manner. Let μ_i and σ_i denote the mean and standard error of variable i. First, the unconditional moment conditions $E(x_i-\mu_i)=0$ and $E[(x_{it}-\mu_i)^2(\sigma_j/\sigma_i)^2-(x_{jt}-\mu_j)^2]=0$ were used to estimate μ_i and σ_i/σ_j (and their standard errors). This was done using an exactly identified version of the GMM procedure described in section 3. Next, let ρ_{ij} denote the unconditional correlation between variables i and j. Then the unconditional moment restrictions $E(x_{it}-\mu_i)=0$, $E[(x_{it}-\mu_i)^2-\sigma_i^2]=0$ and $E[\rho_{ij}\sigma_i\sigma_j-(x_{it}-\mu_i)(x_{jt}-\mu_j)]=0$ were used to estimate the parameters μ_i, μ_j, σ_i, σ_j, and ρ_{ij}. This was done using an exactly identified version of the GMM procedure described in section 3. In all cases the analogue to the matrix R_0 (defined in [3.23]) which is required to calculate standard errors was estimated in the way described immediately following equation (3.23) in the text.

in which $\alpha = 0$, this ratio equals .54 and .64, respectively. These results indicate that indivisibilities in labor supply generate additional volatility in n_t, as does accounting for random movements in g_t when $\alpha = 0$. Interestingly, the quantitative impact of the latter perturbation to the base model (divisible labor, $\alpha = 1$) is actually larger than the former perturbation. In fact the divisible labor model with $\alpha = 0$ outperforms the indivisible labor model with $\alpha = 1$. Nevertheless all of the models seriously underpredict the volatility of hours worked by over 25%.

Next we investigate the volatility of hours worked relative to the volatility of average productivity. To do this we compare the ratio of $\sigma_n/\sigma_{y/n}$ generated by each of the models to the estimated value of $\sigma_n/\sigma_{y/n}$ which obtains in the data. When α equals 1, the divisible and indivisible labor models imply that this ratio equals .67 and .96 respectively. Thus, when $\alpha = 1$ the model understates $\sigma_n/\sigma_{y/n}$ regardless of whether labor is divisible or not. When $\alpha = 0$ this ratio equals 1.01 and 1.42 respectively. Consequently, letting g_t play a role in labor market market dynamics improves the models' ability to account for the empirical value of $\sigma_n/\sigma_{y/n}$. Notice that the model with $\alpha = 0$ and indivisible labor actually *overstates* $\sigma_n/\sigma_{y/n}$. One obvious way to correct this problem is to increase α. However, this causes the model to understate σ_n/σ_y even more seriously.

At this time we note that our results differ in an important way from Hansen's (1985), which are also based on data processed using the Hodrick/Prescott filter. He reports that the indivisible labor model with $\alpha = 1$ implies a value of $\sigma_n/\sigma_{y/n}$ equal to 2.7 (see Hansen [1985], Table 1.) This exceeds the corresponding empirical quantity by over 220%. In contrast, our version of this model underpredicts $\sigma_n/\sigma_{y/n}$ by over 20% (see the column in Table 4A labelled "Indivisible Labor"). The reason for the discrepancy is that Hansen chooses to model innovations to technology as having a transient effect on z_t, whereas we assume its effect is permanent. One way of viewing these differences lies in their implications for the growth process. According to our model, y_t, y_t/n_t, k_t, c_t and g_t grow on average, but they have no tendency to return to a trend in levels. This reflects our assumption that $\log z_t$ is a random walk with drift. In contrast, Hansen (1985) models z_t as an AR(1) process with a root that is less than one (.95). As stated his model does not accommodate steady state growth.[17] Not surprisingly the intertemporal substitution effect of a shock to technology is considerably magnified in Hansen's version of the model.

We now investigate whether our RBC models are capable of accounting for the Dunlop–Tarshis observations, i.e we look at the models' implications for the correlation between average productivity and per capita hours worked. From Table 4A we see that all of the models which we considered fail dramatically along this dimension. The correlation between average productivity and hours worked in the data equals –.20 whereas when α = 1 both the divisible and indivisible labor models predict a correlation in excess of .90. In

[17]There is an interpretation of Hansen's work according to which he implicitly assumed that growth follows a geometric trend. Under this interpretation, the model he worked with is the stationary version of a model with the following instantaneous preference function and resource constraint: $\log(c_t) + v(n_t)$, and $c_t + k_{t+1} - (1-\delta)k_t = q^t\omega_t n_t^{(1-\theta)} k_t^{\theta} \equiv y_t$. Here ω_t is a stationary shock and the economy's gross quarterly growth rate is $q > 1$. The stationary version of this model has instantaneous preferences and resource constraint: $\log(c_t^*) + v(n_t)$ and $c_t^* + (1-\delta^*)k_t^* = \eta_t n_t^{(1-\theta)}(k_t^*)^{\theta} \equiv y_t^*$. Here, $\delta = 1-(1-\delta^*)q$ and starred and unstarred time series variables are related as follows: $x_t^* = x_t/q^t$. Evidently, thinking of the model Hansen actually wrote down as the stationary representation of an underlying nonstationary model requires reinterpreting his depreciation rate and technology shock, and thinking of consumptioo, capital and output in his model as having been geometrically detrended. The latter is of no operational significance in the context of his paper, since Hansen only studies the Hodrick/Prescott cyclical component of the logs of variables. This is invariant to prior geometric detrending (ie., the cyclical component of $\log x_t^*$ is identical to the cyclical component of $\log x_t$.) For example, had he simulated artificial observations on y_t^* and then looked at the cyclical properties of $\log q^t y_t^*$ his results would have been unchanged. Under our interpretation, he reports the cyclical properties implied by the model for $\log y_t^*$. Regarding the depreciation rate and technology shock: First, Hansen assumes $\delta^* = .025$ so that if—as seems empirically reasonable—we assume $q = 1.004$, then the implied value of δ is .021. Also, the shock in the underlying nonstationary economy is expressed in terms of η_t as follows: $q^{(t+\theta)}\eta_t$. Hansen chose the statistical properties of η_t as follows. First, using data on y_t, n_t, and k_t and setting θ = .36, he computed a time series of $q^t\omega_t$ and decided it is well approximated as a linear AR(1) process with autoregressive parameter .95. Assuming $q = 1.004$, this implies a linear AR(1) representation for η_t with autoregressive parameter .95, after rounding to two digits. The mean of η_t was arbitrarily set to 1 and its innovation variance was chosen to equate the standard deviation of cyclical $\log y_t^*$ with the corresponding empirical quantity. In this way, by suitably reinterpreting variables, one is free to think of Hansen as having analysed an economy with geometric growth in which the rate at which a unit of per capita capital depreciates is .021. The latter depreciation figure is virtually identical to the one that emerged from our empirical analysis (see Table 1.)

Appendix A we argue that this probably overstates the mismatch between model and data for measurement error reasons, and that the true aggregate correlation may well be closer to zero. Either way, there is no doubt that the model substantially overstates the correlation between hours and wages. This is not surprising in light of the fact that the *only* shocks driving the labor market dynamics in these models are shifts to the aggregate technology. Agents work more precisely because the returns to working are higher. In contrast when α equals 0, disturbances to μ_t can induce movements in hours worked. Since the marginal productivity of labor is a declining function of hours worked and the aggregate technology does not shift in response to an increase in public consumption, increases in g_t simultaneously generate an increase in n_t and a decrease in average productivity. Consequently the models which assume $\alpha = 0$ generate correlations between $\ln(n_t)$ and $\ln(y_t/n_t)$ which are smaller than those which obtain when $\alpha = 1$. Nevertheless even here both models predict correlations in excess of .60. Evidently g_t is not sufficiently volatile to overcome the strong positive correlation between hours worked and average productivity induced by the assumed technology shocks.[18] By making α sufficiently negative, (i.e. the marginal utility of c_t^p is increasing in g_t), it is possible to substantially decrease the predicted correlation between y_t/n_t and n_t.[19] Unfortunately doing this results in a private consumption series (c_t^p) whose volatility is grossly counterfactual. In addition,

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[18]Not surprisingly, a lower value of ρ hurts the models' ability to account for the Dunlop—Tarshis observations. When $\rho = 0$ and $\alpha = 0$, the implied correlation between $\ln(n_t)$ and $\ln(y_t/n_t)$ is .94 and .90 for the divisible and indivisible labor models, respectively. With regard to σ_n/σ_y the models imply .48 and .57, respectively. Similarly, with regard to σ_n/σ_{y-n}, they imply .89 and 1.27. Evidently, along these dimensions, reducing the magnitude of ρ has the same effect as keeping ρ large, and raising the value of α toward unity.

[19]To investigate the effect of α negative, we set $\alpha = -2$, $T = 2190$, $\beta = 1.03^{-.25}$, $\rho = .96$, $\sigma_\epsilon = .0176$, $\sigma_\mu = .020$ in the indivisible labor model. In addition, we chose γ, $\bar{g}$, δ, θ, λ so that, in steady state, $n = 320.5$, $g/y = .176$, $c^P/y = .55$, $k/y = 10.59$ and $\text{dlog}(y) = .004$. This resulted in the following decision rule parameters: $r = .94$, $d_k = .011$, $e_k = -.94$, $r_n = -.86$, $d_n = .89$, $e_n = .86$. We simulated the model in the same way as the models in the text and found $\sigma_{c^P}/\sigma_y = .71$ (.04), $\sigma_n/\sigma_y = .97$ (.06), $\sigma_n/\sigma_{y-n} = 2.57$ (.49), and corr(y—n,n) = −.12 (.16) (numbers in parentheses are standard errors.)

this causes $\sigma_n/\sigma_{y/n}$ to substantially overshoot its empirical counterpart.

The results in Table 4B which correspond to the Growth 2 filter are in many ways similar to those reported in Table 4A. First, all of the models substantially understate the volatility of the growth rate of per capita hours worked relative to the growth rate of output. The magnitude of the shortcomings of the models along this dimension is quantitatively much larger when we work with the Growth 2 filter than with the Hodrick/Prescott filter. Second, the base model greatly understates the volatility of the growth rate of hours worked in relation to the growth rate of average productivity. At the same time, the other versions of the model actually *overstate* the volatility of the growth rate of hours to the growth rate of average productivity. As before, the most salient failure of the models is that they generate correlations between the growth rates of average productivity and hours worked that are strikingly counterfactual. In the US data this correlation equals approximately −.72 while all of the models predict that this correlation ought to exceed .65. As with the volatility of hours worked the failure of the models along this dimension is more striking when the data are processed with the Growth 2 filter as opposed to the Hodrick/Prescott filter.

Next we consider the results reported in Table 4B which are obtained using the Growth 1 filter. The entries in Tables 4B which differ because of the filter used are those pertaining to σ_n/σ_y, $\sigma_n/\sigma_{y/n}$ and corr(y/n,n). The corresponding results obtained with the Growth 1 filter are denoted by σ_{n^*}/σ_y $\sigma_{n^*}/\sigma_{y/n}$ and corr(y/n,n*) respectively. Consistent with the Hodrick/Prescott and Growth 2 filters, our results with the Growth 1 filter indicate that the models substantially underpredict the relevant measure of σ_n/σ_y. Interestingly, all of the models overpredict the volatility of the log *level* of hours relative to the *growth rate* of average productivity. As before all of the models fail to reproduce even the sign of the correlation between the relevant measure of average productivity and hours worked.

Viewed as a whole our results are consistent with the view that the most striking empirical shortcoming of existing RBC models lies in their implications for the correlation between average productivity and hours worked. We conclude that the puzzle faced by real business cycle theories is the classic one long faced by business cycle theorists: how can we explain the fact that per capita hours worked display such marked fluctuations when real wages and average productivity do not display a marked positive correlation?

5. Concluding Remarks

Existing RBC theories assume that the only source of impulses to post war US business cycles are exogenous shocks to technology. We have argued that this feature of these models generates a strong positive correlation between hours worked and average productivity. Unfortunately, this implication is grossly counterfactual, at least for the post war US.

Of course, documenting the empirical shortcomings of existing RBC models on a particular dimension of the data does not constitute evidence in favor of alternative paradigms. In fact, we believe RBC models are useful starting points for business cycle analysis. Nevertheless, our results indicate an important failing which must be remedied before it can be plausibly claimed that theory is ahead of measurement. It simply seems unlikely that better measurement alone will lead us to conclude that the correlation between hours and real wages is above .9, as existing RBC models imply.

To us it seems more likely that the reconciliation of theory and fact will come from identifying other disturbances, in addition to technology shocks, which impact on aggregate labor markets. In this paper, we have explored the potential role for shocks to government spending. Using a specification suggested by the work of Aschauer (1985), Barro (1981,1987) and Kormendi (1983), we find that government shocks, when parameterized in an empirically plausible way, can go only part way in accounting for the Dunlop–Tarshis observations. Either our model of government spending needs to be modified, or additional disturbances need to be incorporated into the model.

One obvious measureable shock is a disturbance to the supply of money. In Lucas (1972), an unperceived increase in the aggregate money supply causes agents to mistakenly believe that they face a temporary increase in the real wage. This induces an increase in aggregate hours worked and, due to diminishing marginal productivity, a decrease in the ex post real wage. In this way, monetary shocks can be expected to act very much like government shocks in our model in counteracting the sources of positive correlation between real wages and hours worked captured by existing real business cycle models. Constructing empirically tractable models of this type will be a challenging task.

Other sources of disturbances have been identified in the literature and also seem promising to us. For example, Greenwood, Hercowitz and Huffman (1988) model the impact of disturbances to the relative price of capital goods, while Christiano and Eichenbaum (1988) investigate the role of human capital shocks.

Finally, we hope that our discussion of the role of government spending constitutes an independent contribution of the paper. One important finding is that the date t impact on output and employment of a date t government spending shock is much larger if it has a lot of persistence than if it is temporary. In particular, given the high empirical degree of persistence in government shocks, we find that the implied elasticity of hours worked with respect to an exogenous shock to government spending is roughly 1/4. When instead the persistence of the shocks is set to zero the elasticity drops to less than 1/40.

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Appendix A: The Cyclical Behavior of Aggregate Productivity.

For the data set described in text, the correlation between aggregate hours worked and productivity is negative (see Tables 4A and 4B.) In this appendix we show that this result holds true for other measures of aggregate hours worked and output. We then argue that the sign of this correlation probably reflects two sources of measurement error. The first source of measurement error is that the aggregate output data cover more sectors than does the aggregate hours data. Another factor that can account for a downward bias in the productivity/hours correlations is measurement error in the hours data. We conclude that, most likely, the true correlation between average productivity and hours worked is weakly positive. Overall we view our results as being consistent with the Dunlop–Tarshis observations.

A.1 The Cyclical Behavior of Productivity in the Aggregate Data

The Aggregate Data

Our first measure of aggregate hours worked, denoted N_1, corresponds to total hours worked by wage and salary workers in non–agricultural establishments as reported by the Bureau of Labor Statistics (BLS).[20] The BLS obtains its information by a mail

[20]The establishment hours data refer to hours of all employees—production workers, nonsupervisory workers, and salaried workers—and are based largely on establishment data. An establishment is defined as an economic unit which produces goods or services, such as a factory, mine, or store. The employment statistics for government refer to civilian employees only. For more details, see Handbook of Methods (1988). The establishment hours data we used are the sum of government (HRSGOV) and private (HRSPST) hours worked, where names in parentheses are the Wharton Econometric Forecasting Associates (WEFA) mnemonics. The data can also be found in the "total" row of Table C–9 of Employment and Earnings (1988,p.112).

questionnaire which solicits information about employment status over the payroll period that includes the 12th day of each month.[21] Since this measure reports total hours paid for by employers, it includes total hours of paid vacation and sick leaves. Our second measure of aggregate hours worked, denoted N_2, corresponds to total hours worked in non–agricultural industries as calculated by the Bureau of the Census for the BLS. This data is based on household interviews obtained from a sample survey of the population 16 years of age and over.[22] Unlike N_1, this measure covers actual hours worked, rather than hours paid for. For more details on these measures of hours worked, see Employment and Earnings (1988,pp.157–184). Our third measure of hours worked, N_3, was computed by Gary Hansen (1984) who converts the household data on aggregate hours worked (N_2) to efficiency units by weighting the different age–sex categories on the basis of the different groups' average wages in the 1970's. Hansen motivates this transformation by a desire to correct for a presumed discrepancy between actual aggregate labor services and aggregate reported hours worked when there is non–trivial labor heterogeneity.

Aggregate hours worked, N_1 and N_2, were divided by the total US population to obtain the per capita measures of hours worked, H_1 and H_2.[23] The latter corresponds to the hours worked measure used in Kydland and Prescott (1982). Hansen's measure of aggregate hours, N_3, was divided by a measure of the quality adjusted working age population which was obtained using the same procedure underlying the construction of

[21]An exception is Federal Government workers, for which the hours data represents the number of hours paid for on the last day of the calender month (see Handbook of Methods [1988].)

[22]The data were obtained by multiplying persons at work (NAWTTTONAG_U) with average hours (NHTTTNAG_U), where names in parentheses are the WEFA data mnemonics. The persons at work data can also be found in the "nonagricultural industries" column of Table A–27 in Employment and Earnings (1988,p.33) and the average hours data can be found in the "total at work" column of Table A–29 in Employment and Earnings (1988,p.34). The product of average hours and persons at work was seasonally adjusted by the Research Department, Federal Reserve Bank of Minneapolis.

[23]Our population series is total U.S. population including armed forces overseas, with WEFA data mnemonic NPT.

N_3. (See Christiano [1987c] for details.) The resulting series on hours worked per quality adjusted working age person is denoted by H_3. This measure of per capita hours worked was used in the text and in Christiano (1988b).

Initially we consider three measures of aggregate real output. The first measure, Y_1, consists of quarterly real GNP divided by the total US population and corresponds to the output measure in Kydland and Prescott (1982). The second measure, Y_2, consists of quarterly real GNP divided by the quality adjusted working age population. Our third measure of output, Y_3, consists of quarterly real GNP minus net exports plus an estimate of the services produced from the stock of durable goods obtained from Brayton and Mauskopf (1985). When divided by the quality adjusted working age population, this measure corresponds to the concept of output used in the text and in Christiano (1988b).

Our first two measures of average labor productivity, P_1 and P_2, were obtained by dividing quarterly real GNP by N_1 and N_2 respectively. Our third measure of average labor productivity, P_3, was obtained by dividing quarterly real GNP by Hansen's measure of aggregate hours worked, N_3. Our fourth measure of average productivity, P_4, was obtained by dividing Christiano's measure of output by Hansen's measure of aggregate hours worked and is the measure used in the text.

Not surprisingly, all of our measures of per capita average productivity display marked trends, as do all of our measures of per capita hours worked, except the one compiled by Hansen. Accordingly some stationary inducing transformation of these data must be adopted. We report results for three such transformations here. The first transformation (Growth 1) is motivated by the fact that all of the structural models considered in this paper imply that the first difference of the logarithm of average labor productivity, the logarithm of per capita output and the logarithm of per capita hours worked are stationary stochastic processes. Second, we report results using the first differences of the logarithms of per capita hours worked, output and average productivity (Growth 2). Third, we report various correlations for data which have been transformed using the HP detrending procedure discussed in Hodrick and Prescott (1980) and Prescott (1986). Our use of this transformation is motivated by the fact that many authors, including most prominently Kydland and Prescott (1982,1988), Hansen (1985) and Prescott (1986) have investigated RBC models using data which have been filtered in this manner. Moreover, King and Rebelo (1988) show that the HP filter involves first differencing, so that the structural models in this paper imply that—apart from endpoint effects—HP filtered data are covariance stationary.

The Correlations

Table A1 summarizes our results for certain combinations of the different measures of output and hours worked. Column 1 reports results for the Growth 1 transformation, column 2 reports results for the Growth 2 transformation of the average productivity and employment data. Finally column 3 reports the analog correlations computed using the output of the HP filter applied to the logarithmic levels of the raw data. The portions of Table A1 marked "Establishment", "Household", "Hansen" and "Christiano" reflect calculations based on $\{H_1,Y_1,P_1\}$, $\{H_2,Y_1,P_2\}$, $\{H_3,Y_2,P_3\}$ and $\{H_3,Y_3,P_4\}$ respectively. The key feature of the results is that, for all measures of hour worked and output, the correlation between per capita hours worked and average productivity is *negative*. This is true regardless of which stationary inducing transformation is applied to the raw data or which sample period is investigated.

Productivity Is, and Is Not, Procyclical

A notable feature of the results in Table A1 is that productivity appears to be countercyclical when the cycle is measured by hours worked, and strongly procyclical when the state of the cycle is measured by output. This is striking in view of the conventional belief that hours worked and output are interchangeable as measures of the state of the cycle. This belief is based on the fact that output and hours are strongly positively correlated. For example, for the period 50,1–87,4, the correlation between Y_1 growth and H_1 growth (Growth2) is .75. The apparent inconsistency can be accounted for algebraically by the fact that hours worked are very volatile relative to output. To see how high volatility of hours could account for corr(y/n,n) < 0 and corr(y/n,y) > 0 even though corr(y,n) > 0, it is useful to express the first two correlations in terms of the third and the relative volatility of hours worked. This is done by exploiting the definition of a correlation and rearranging terms:

(A.1) $$\text{corr}(y-n,n) = [\frac{\text{corr}(y,n)}{\sigma_n/\sigma_y} - 1]\frac{\sigma_n}{\sigma_{y-n}}$$

(A.2) $$\text{corr}(y-n,y) = [\frac{1}{\sigma_n/\sigma_y} - \text{corr}(y,n)]\frac{\sigma_n}{\sigma_{y-n}}$$

where σ_i denotes the standard deviation of (log detrended) variable i.[24] Log detrended average productivity is represented as y–n because the log transformation converts the ratio of output to hours to the difference between the log of output and the log of hours. From these formulas it is evident that if corr(y,n) = 1, then—not surprisingly—the correlation of productivity with the cycle is the same whether the state of the cycle is measured by hours worked or output. However, in the empirically relevant case corr(y,n) < 1, one can have corr(y–n,y) > 0 and corr(y–n,n) < 0 if, and only if,

(A.3) $$\text{corr}(y,n) < \frac{\sigma_n}{\sigma_y}$$

[24]Relation (A.1) is just the relation $\rho(y-n,n) = [\eta-1](\sigma_n/\sigma_{y-n})$ discussed in footnote 5.

and

$$\text{(A.4)} \qquad \text{corr}(y,n) < \left(\frac{\sigma_n}{\sigma_y}\right)^{-1}.$$

Conditions (A.3) and (A.4) are equivalent with corr(y–n,n) < 0 and corr(y–n,y) > 0, respectively. In the case of Y_I and H_I and the Growth2 transformation, $\sigma_n/\sigma_y = .83$, so that (A.3) and (A.4) are satisfied. It is clear from (A.3) that high relative volatility of hours is required for corr(y–n,n) < 0. Condition (A.4) indicates that that volatility cannot be too high if corr(y–n,y) > 0 is to occur.

A.2 Measurement Error and the Aggregate Productivity/Hours Correlation.

There are at least two reasons to believe that the negative correlation between productivity and hours in the aggregate data reflects measurement error and that the actual correlation is closer to zero. One potential source of distortion lies in the fact that the output data covers more sectors than does the hours data. Another possibility is that the results reflect measurement errors in the hours data. We consider these two possibilities in turn.

Misalignment in Hours and Output Coverage

That misalignment considerations may account for the negative productivity–hours correlation in aggregate data is suggested by results in Table A2. That table presents results for the same statistics and sample periods as in Table A1. The first panel, labelled "non–farm business productivity", reports results based on the output and hours series underlying the BLS's productivity data. The BLS's output series cover about 3/4 of GNP, and omit value–added originating in agriculture (1.7%), government (10.7%), non–profit institutions (3.4%), and owner–occupied housing (5.8%).[25] (Numbers in parentheses are the ratio to GNP in 1983, and were computed from the numbers in Tables 1.7 and 1.23 in Survey of Current Business [1987].) The BLS's hours series are the establishment hours worked data which corresponds to their output measure.

The results in the Growth 2 and HP columns of Table A2 differ notably from the corresponding results in Table A1. For both detrending procedures and for all but one sample periods the correlation between productivity and hours worked is nonnegative in Table A2. A distinguishing feature of the results in the first panel of Table A2 is that care has been taken to assure that the underlying output and labor input measures correspond to the same sectors. This suggests the possibility that the results in Table A1 reflect misalignment in the underlying output and hours series. In an effort to improve the alignment in the data underlying the results in the first panel of Table A1, we adjusted the output measure used there by subtracting value–added in farming and non farm housing from GNP.[26] Let Y_4 denote that measure of output after dividing by the total US population. Also, let P_5 denote the ratio of GNP minus value–added in farming and non farm housing to establishment hours worked, N_1. The calculations in the second panel of

[25]For details of the BLS's definition of non–farm business output, see Handbook of Methods (1988). The 3/4 estimate in the text approximates the BLS measure of output by the Bureau of Economic Analysis' measure of value added in the non–farm less non farm housing business sector of the GNP accounts (see, for example, Table 1.7, line 5 in Survey of Current Business [1987].) There is a slight upward trend in the ratio of non–farm less housing business output to GNP. In the early 1950s it was around 72%, whereas by 1987 it had reached 77%. The data used to produce the results in the first panel of Table A2 were taken from the Federal Reserve Board of Governors' database. The hours index has data mnemonic JHNFB and the output index data mnemonic is JQNFB. These data can also be taken from the output and hours rows in the Nonfarm Business Sector panel in Table C—10 of Employment and Earnings (1988,p.113).

[26]According to Table 1.7 of Survey of Current Business (1987), value–added in nonfarm housing was 7.8% of GNP in 1983. According to Table 1.23 in the same source, in 1983 75% of nonfarm housing was imputed value–added from owner–occupied housing, with the rest deriving from tenant–occupied housing. We obtained our farm (XAF82) and non–farm housing (XEAFH82) output from the Board of Governors' data base, where the expressions in parentheses are the data mnemonics.

Table A2 are based on $\{H_1, Y_4, P_5\}$. Note how much closer the Growth 2 and HP correlations between productivity and hours are to zero than the corresponding numbers in the first panel of Table A1. To us, this evidence suggests that the strong negative correlations in Table A1 reflect the absence of hours worked in farming and owner–occupied housing from N_1 and N_2.[27]

Measurement Errors In Hours Worked

Prescott (1986) has argued that, to a first approximation, the establishment (N_1) and household (N_2) hours data can be viewed as independent measures of aggregate hours worked. Suppose we assume, as does Prescott (1986), that the measurement error in these two time series are orthogonal to each other and to the logarithm of the underlying true process. Then a consistent estimate of the variance in actual hours worked is the covariance between the establishment and household measures of hours worked. In addition we can estimate the variance of the change in true productivity by the covariance between any two measures of average productivity which are constructed using different measures of hours worked. Finally we can estimate the covariance between true average productivity and per capita hours worked by calculating the covariance between any two measures of these objects which are assembled using different measures of per capita hours worked.

In Table A3 we report the results of calculating the correlation between average productivity and hours worked using this alternative procedure as applied to our different data sets and our three stationary inducing transformations. In all cases our output measure is GNP minus value–added in farm and non farm housing. First, note that all the correlations based on HP detrending are now strongly positive. For example, when the

[27]Hours worked in the tenant–occupied housing sector are included in the real estate component of establishment hours.

Gary Hansen and Establishment measures of hours are crossed, the estimated correlation between productivity and hours is .44 on the long sample period. The hours/productivity correlation is somewhat smaller, though still close to zero, when the Growth 2 transformation is used. Overall we conclude that Table A3 does provide some evidence in favor of the hypothesis that part of the negative correlations reported in Table A1 can be attributed to measurement error of the type discussed by Prescott (1986).

Table 1

Model Parameters (Standard Errors)[1]

	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't
T	2190	2190	2190	2190
δ	0.0207 (0.0003)	0.0207 (0.0003)	0.0207 (0.0003)	0.0207 (0.0003)
β	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$
θ	0.347 (0.003)	0.347 (0.003)	0.347 (0.003)	0.347 (0.003)
γ	5.26 (0.04)	0.00281 (0.00002)	7.00 (0.07)	0.00373 (0.00004)
λ	0.0046 (0.0004)	0.0047 (0.0005)	0.0047 (0.0004)	0.0047 (0.0004)
σ_ε	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)
$\bar{g}$	199.5 (2.97)	200.2 (4.24)	199.0 (3.25)	198.8 (3.14)
ρ	0.97 (0.025)	0.98 (0.026)	0.97 (0.03)	0.97 (0.026)
σ_μ	0.021 (0.001)	0.021 (0.0012)	0.020 (0.001)	0.020 (0.001)
J[2]	3.24 (0.48)	3.24 (0.48)	3.32 (0.49)	3.32 (0.49)
d. of f.[3]	4.0	4.0	4.0	4.0

[1]Standard errors are reported only for estimated parameters. Other parameters were set a priori. Apart from γ, point estimates and standard errors are not sensitive to the value of α.

[2]Hansen's J statistic for testing the null hypothesis that the growth rates of z_t, y_t, c_t^p, k_t, and g_t are identical. The numbers in parenthesis are the probability, under the null hypothesis, of getting a J statistic larger than the realized empirical value.

[3]Under the null hypothesis, Hansen's J statistic is a realization from a chi square distribution with the indicated number of degrees of freedom (d. of f.).

Table 2

Decision Rule Parameters

	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't
$\bar{k}$	11771	11754	11709	11694
r	0.95	0.94	0.95	0.94
$\bar{g}$	199.5	200.2	198.9	198.8
d_k	0.0	0.0	0.0020	0.0054
e_k	-0.95	-0.94	-0.95	-0.94
λ	0.0047	0.0047	0.0047	0.0047
n	317.8	317.4	317.5	317.1
r_n	-0.36	-0.48	-0.45	-0.59
d_n	0.0	0.0	0.21	0.28
e_n	0.36	0.48	0.45	0.59

Table 3

Selected First Moment Properties

	MODELS[1]				
	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't	U.S. Data[2] (1955.4-1983.4)
c_t^p/y_t	0.55 (0.013)	0.55 (0.015)	0.55 (0.011)	0.55 (0.012)	0.55 (0.003)
g_t/y_t	0.181 (0.009)	0.182 (0.012)	0.181 (0.009)	0.181 (0.009)	0.177 (0.003)
dk_t/y_t	0.267 (0.009)	0.267 (0.009)	0.267 (0.009)	0.267 0.009)	0.269 (0.002)
k_{t+1}/y_t	10.60 (0.263)	10.60 (0.258)	10.58 (0.293)	10.59 (0.284)	10.62 (0.09)
n_t	317.9 (3.39)	317.6 (4.08)	317.9 (5.59)	317.6 (6.59)	320.2 (1.51)
dlog c_t^p	0.0048 (0.0016)	0.0048 (0.0016)	0.0048 (0.0016)	0.0048 (0.0016)	0.0045 (0.0007)
dlog y_t	0.0048 (0.0016)	0.0048 (0.0017)	0.0048 (0.0017)	0.0048 (0.0017)	0.0040 (0.0014)
dlog k_t	0.0048 (0.0015)	0.0048 (0.0015)	0.0048 (0.0015)	0.0048 (0.0015)	0.0047 (0.0005)
dlog g_t	0.0047 (0.0019)	0.0047 (0.0020)	0.0047 (0.0020)	0.0047 (0.0019)	0.0023 (0.0017)
dlog n_t	0.4E-05 (0.0003)	0.6E-05 (0.0003)	-0.6E-05 (0.0004)	-0.7E-05 (0.0005)	0.0002 (0.0013)

[1] Numbers are averages, across 1,000 simulated data sets of length 113 observations each, of the sample average of the corresponding variable in the first column. Numbers in parenthesis are the standard deviation, across data sets, of the associated statistic.

[2] Empirical averages, with standard errors. See footnote 13 for details.

Table 4A

Second Moment Properties,
U.S. Data and Exogenous Growth Models,
Using HP detrending

Statistic[1]	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't	U.S. Data[3] 1955.4-1983.4
	Models[2]				
σ_{c^p}/σ_y	0.58 (0.09)	0.56 (0.08)	0.48 (0.03)	0.46 (0.05)	0.44 (0.027)
σ_{dk}/σ_y	2.32 (0.16)	2.40 (0.17)	2.11 (0.15)	2.20 (0.16)	2.24 (0.064)
σ_n/σ_y	0.41 (0.005)	0.50 (0.006)	0.54 (0.03)	0.64 (0.04)	0.86 (0.064)
$\sigma_n/\sigma_{(y/n)}$	0.67 (0.02)	0.96 (0.03)	1.01 (0.12)	1.42 (0.18)	1.21 (0.11)
σ_g/σ_y	1.77 (0.24)	1.62 (0.22)	1.63 (0.19)	1.47 (0.16)	1.15 (0.22)
σ_y	0.019 (0.003)	0.021 (0.003)	0.021 (0.003)	0.023 (0.003)	0.019 (0.002)
corr(y/n,n)	0.94 (0.016)	0.92 (0.022)	0.72 (0.080)	0.65 (0.093)	-0.20 (0.11)

[1]All of the statistics in this table are computed after first logging and then detrending the data using the Hodrick-Prescott (HP) method. σ_i is the standard deviation of variable i detrended in this way. corr (x,w) is the correlation between detrended x and detrended w.

[2]Average of corresponding statistics in column 1, across 1,000 simulated data sets each of length 113. Number in parentheses is the associated standard deviation.

[3]Results for U.S. data. See footnote 16 for details about the standard errors, which appear in parentheses.

Table 4B

Second Moment Properties,
U.S. Data and Exogenous Growth Models,
Using Log First-Difference Detrending

	Models[2]				
Statistic[1]	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't	U.S. Data[3] 1955.4–1983.4
σ_{c^p}/σ_y	0.57 (0.05)	0.54 (0.05)	0.47 (0.04)	0.45 (0.04)	0.47 (0.037)
σ_{dk}/σ_y	2.34 (0.15)	2.41 (0.16)	2.12 (0.14)	2.22 (0.14)	1.96 (0.097)
σ_n/σ_y	0.41 (0.003)	0.51 (0.004)	0.54 (0.017)	0.65 (0.018)	1.32 (0.14)
σ_{n*}/σ_y	1.07 (0.26)	1.27 (0.30)	1.47 (0.38)	1.67 (0.41)	2.11 (0.236)
$\sigma_n/\sigma_{(y/n)}$	0.69 (0.013)	1.00 (0.024)	1.03 (0.063)	1.47 (0.099)	0.97 (0.05)
$\sigma_{n*}/\sigma_{(y/n)}$	1.79 (0.41)	2.50 (0.54)	2.78 (0.70)	3.78 (0.09)	1.56 (0.216)
σ_g/σ_y	1.75 (0.13)	1.61 (0.11)	1.61 (0.10)	1.46 (0.08)	1.30 (0.15)
σ_y	0.016 (0.001)	0.017 (0.001)	0.017 (0.001)	0.019 (0.001)	0.011 (0.001)
corr(y/n,n)	0.97 (0.017)	0.95 (0.022)	0.74 (0.041)	0.68 (0.049)	-0.72 (0.07)
corr (y/n,n*)	0.44 (0.029)	0.49 (0.030)	0.36 (0.074)	0.040 (0.079)	-0.30 (0.060)

[1]In this table, c, dk, y, y/n, n refer to the first difference of the log of the indicated variable. n* refers to the log of hours. Then, σ_i is the standard deviation of variable i and corr (ℓ,P) is the correlation between ℓ and P.

[2]Average of corresponding statistics in column 1, across 1,000 simulated data sets each of length 113. Number in parenthesis is the associated standard deviation.

[3]Results for U.S. data. See footnote 16 for details about the standard errors.

Table A1

Correlations Using Aggregate Data[1]

Hours Worked Measure, Sampling Period	y/n vs. y Growth	y/n vs. y HP	y/n vs. n Growth1	y/n vs. n Growth2	y/n vs. n HP
Establishment, Total					
50,1 - 87,4	.50	.32	-.22	-.19	-.16
50,1 - 79,4	.47	.30	-.27	-.25	-.20
50,1 - 69,4	.36	.20	-.29	-.30	-.32
Household					
55,3 - 87,4	.39	.40	-.19	-.65	-.19
55,3 - 79,4	.40	.40	-.25	-.70	-.25
55,3 - 69,4	.34	.45	-.28	-.68	-.38
Hansen					
55,3 - 84,1	.37	.40	-.26	-.74	-.29
55,3 - 79,4	.36	.37	-.29	-.78	-.36
55,3 - 69,4	.29	.42	-.23	-.77	-.45
Christiano					
55,4 - 83,4	.40	.54	-.30	-.72	-.20
55,4 - 79,1	.38	.51	-.32	-.77	-.23
55,4 - 69,1	.29	.49	-.23	-.77	-.42

[1]Sample correlations between output per hour (y/n) and per-capita output (y) and per-capita hours (n). For a discussion of the detrending procedures and data sources, see the text.

Table A2

Correlations Using Sectoral Data[1]

Data Measure, Sampling Period	-----y/n vs. y----- Growth	HP	---------y/n vs. n--------- Growth1	Growth2	HP
Non-farm Business Sector					
50,1 - 87,4	.63	.59	-.29	.05	.15
50,1 - 79,4	.61	.55	-.35	.00	.09
50,1 - 69,4	.57	.42	-.23	.00	-.05
GNP-farming-housing					
50,1 - 87,4	.61	.51	-.22	-.04	.05
50,1 - 79,4	.59	.49	-.26	-.10	.00
50,1 - 69,4	.51	.40	-.28	-.13	-.10

[1]Sample correlations between output per hour (y/n) and per-capita output (y) and per-capita hours (n). For a discussion of the detrending procedures and data sources, see the text.

Table A3

Measurement Error Adjusted Correlations, y/n vs. n[1]

y/n	n	Sample Period	Growth1	Growth2	HP
Household	Establish, total	55,3 - 87,4	.26	.03	.33
Establish, total	Household	55,3 - 87,4	-.18	-.10	.14
Gary Hansen	Establish, total	55,3 - 84,1	-.57	0.06	.44
Establish, total	Gary Hansen	55,3 - 84,1	-.19	-0.10	.23

[1]Correlations between y/n and n, where each is computed using a different hours measure, as indicated in the first two columns. In all cases, the measure of output used is GNP minus value-added in farming and non-farm housing. For details about the data and detrending procedures, see the text.