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RISKS TO HUMAN CAPITAL

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ABSTRACT

What is the connection between financing constraints and the equity premium? To answer this question, we build a model with inalienable human capital, in which investors finance individuals who can potentially become skilled. Though investment in skill is always optimal, it does not take place in some states of the world, due to moral hazard. In other states of the world, individuals acquire skill; however outside investors and individuals inefficiently share risk. We show that this simple moral hazard problem and the resultant financing friction leads to a realistic equity premium, a low riskfree rate, and severe negative consequences for distribution of wealth and for welfare. When investment fails to take place, the economy enters an endogenous disaster state. We show that the possibility of these disaster states distorts risk prices, even under calibrations in which they never occur in equilibrium.

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1 Introduction

The 2008–2009 financial crisis directed the attention of economists and policymakers to links between intermediary balance sheets, asset pricing fluctuations and economic variability (He and Krishnamurthy, 2012; Brunnermeier and Sannikov, 2014). One hypothesis is that deterioration in intermediary balance sheets was the cause of these fluctuations. However, for intermediary balance sheets to matter quantitatively, there must be some underlying friction that prevents capital from flowing, either directly or through intermediaries, in times of financial distress. Given the innovations and technological advances over the last century, the question remains: what is the friction that can survive the powerful incentives to move capital to where it is most productive? The answer seems likely to be independent of specific institutional arrangements.

A second hypothesis is that institutional factors are of minor significance, and the fluctuations in balance sheets are a symptom and not a cause. According to this view, underlying fluctuations in productivity drive asset prices in a nearly frictionless way and an economic crisis is simply a large productivity decline. A line of literature focuses on the quantitative implications of such rare disasters for asset prices (Barro, 2006; Gabaix, 2012; Gourio, 2012; Wachter, 2013) assuming complete markets. However, other than world-wide conflicts, the source of these large, rare macroeconomic fluctuations is unknown.

In this paper, we build a model in which financial frictions arise from risk to human capital. In so doing, we provide a foundation for rare output-related disasters. We consider an individual's decision to finance a non-convex human capital investment. A substantial empirical literature shows, for example, that the decision to go to college is non-convex. A little bit of college does not produce a little bit of the benefits of going to college; rather it produces nothing.[1] We assume markets are incomplete: we take as

[1] See e.g. Hungerford and Solon (1987); Altonji (1993); Jaeger and Page (1996); Card (1999); Heckman et al. (2006)

given individuals cannot sell claims to their human capital. However, individuals can pledge some of the resultant cash flows. Because pledgeability is limited, individuals cannot receive outside funding in every state of the world. This friction amplifies negative shocks to equityholders, reducing the riskfree rate to zero and generating an equity premium that is close to the historical average, even under log utility.

To summarize, the model explains the following empirical results:

1. A high equity premium under low risk aversion

2. A low riskfree rate (in fact, a riskfree rate that is at the zero lower bound).

3. Non-participation in the stock market.

4. Procyclical dividends (and wages that are relatively unresponsive to the business cycle).

We explain these facts through a methodological advance: we extend the basic moral hazard framework of Holmstrom and Tirole (1997) and Aghion and Bolton (1997) to the case of aggregate risk. The solution has quantitatively realistic implications, but does not rely on complicated dynamics and has an analytical characterization.

Our paper relates to a literature using nonconvexities to explain variations in employment over the business cycle. For example, Hansen (1985), Rogerson (1988), and Christiano and Eichenbaum (1992) argue that a nonconvex employed/unemployed distinction lies behind the volatility of unemployment relative to output, whereas Prescott (1992) argues that nonconvexities explain differential rates of unemployment among developed countries. The debate on the significance and form of non-convexity continues: Chetty et al. (2011) argue that indivisibilities are insufficient to reconcile the macro and micro evidence, while Kehoe et al. (2019) derive non-convexities from on-the-job learning, and show that the resultant model explains unemployment volatility given variation in discount rates. This literature assumes that unemployment risk is perfectly shared and thus that markets are complete. We depart from this literature in that markets are realistically incomplete in our model, and imperfect risk sharing plays an important role in our results.

While the above literature focuses on explaining unemployment assuming complete markets, a second literature uses market incompleteness generated by unemployment to explain asset prices. Early work focused on how countercyclical labor income risk together with incomplete markets offered a solution to the equity premium puzzle (Constantinides and Duffie, 1996; Storesletten et al., 2007). More recent work has focused on the role of negative skewness (Schmidt, 2016; Constantinides and Ghosh, 2017; Catherine, 2019). Guvenen et al. (2014) show that labor income of those on the low end of the income scale displays greater countercyclicality relative to the labor income of those at the top. Our model endogenously generates this outcome through financial contracting frictions, while also generating an increased equity premium as in Constantinides and Duffie (1996).

Our paper also relates to a literature on the consequences of imperfect financial markets for economic development. In this literature, financing frictions hinder some individuals from making positive net present value investments in human capital. Individuals remain poor, and continue to face the same financing problem; thus the economy stagnates into a "poverty trap" (Banerjee and Newman, 1993; Ljungqvist, 1993; Levine, 2005). To this framework, we add aggregate risk. We show how aggregate risk makes these frictions more severe, and can lead, endogenously, to economic disasters.

Finally, our paper relates to a substantial and growing literature on financing frictions and the macroeconomy. Classic references include Kiyotaki and Moore (1997) and Bernanke et al. (1999). These papers assume a class of risk neutral individuals ("experts") who have special access to investment technologies. More recent work (Gertler and Kiyotaki, 2010) allows for risk aversion, but assumes, in effect, complete markets. Given cash flows, asset prices are the same as in a standard endowment economy without experts and financing frictions. Our model departs from these along several dimensions. At the core of our model is an investment decision in human capital—after which individuals become analogous to experts. Also, in our model, in which markets are endogenously incomplete, financing frictions deepen negative shocks as they occur (as opposed to responding to a sequence of negative shocks; an effect not observed in the data).[2] Non-convexities further amplify the effects of incomplete contracts, leading to risky aggregate cash flows, volatile state prices, and a greater equity premium.

Our work is closer to the more recent papers of He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014) in that we focus on non-participation and market incompleteness. However, our moral hazard problem is of a quite different nature, leading to very different results. As discussed above, in our model financing frictions deepen negative shocks as they occur. The reason is that our agent can (realistically) divert cash flows subsequent to the aggregate shock. This gives rise to three regions, determined by the outcome of an aggregate productivity shock. When productivity is sufficiently high, risk is perfectly shared, and outcomes are as in the frictionless case. There are two inefficient regions. In the lowest region, financing constraints are locally non-binding. However, investment is inefficient. For intermediate values of productivity, investment is efficient, but risk sharing is not. The existence of the lowest region distorts savings (because agents attempt to stay out of it), leading to low interest rates, whereas the second region is the main driver of the equity premium. The lowest region corresponds to disaster states. Intriguingly even when this region (endogenously) disappears, the potential occurrence of this region off-equilibrium drives results. In our model, unlike others, disasters can exert a substantial influence over asset prices, even when they *never* occur in equilibrium.

[2] In this sense, the model is similar to the fire-sales model of Shleifer and Vishny (2011).

2 Model

We present a three-period model in which individuals and investors face a financial contracting friction due to moral hazard and limited liability, similar to Holmstrom and Tirole (1997) and Aghion and Bolton (1997). As in Hart and Moore (1994), the individual cannot sell claims to his or her human capital. To this standard framework, we add aggregate risk and risk averse agents.

2.1 Setup

Environment. There is one non-perishable investment/consumption good and there are three types of agents: i) a unit mass of *outside investors* with aggregate initial endowment $e^{\mathcal{I}}$; ii) a mass m^h of *high-cash-on-hand individuals*, henceforth h-individuals, with initial endowment e^h; iii) a mass m^ℓ of *low-cash-on-hand individuals*, henceforth ℓ-individuals, with initial endowment $e^\ell \ll e^h$.[3] There are three periods in the model. In period 0 agents write financial contracts and trade Arrow-Debreu securities. The state of the world—aggregate total factor productivity (TFP) —is realized in period 1. Let $\delta \in \Delta$ denote TFP; it is drawn from cumulative distribution function $F : \Delta \to [0, 1]$. Afterwards an individual chooses her occupation and (potentially) calls for funds from outside investors based on securities traded in period 0 to pay for the investment costs. Production takes place in period 2. An individual either shirks or puts forth effort. Output depends on the choice of effort, occupational choice, and aggregate TFP. Finally, all payments are settled, everyone consumes her endowment.

Technology. The occupational choice set is nonconvex. An individual may either work in the traditional sector to generate $H_0 \geq 0$ units of the consumption good, where H_0 is independent of the state of the world; or invest in her human capital,

[3] We use the terminology individual rather than the more common entrepreneur to emphasize the fact that the model is one of human rather than physical capital.

which returns δH units of goods in the production period (period 2) if the individual puts forth effort, where H is a constant parameter governing the return to investment in human capital. If the individual shirks, the return to investment is zero. Let f denote the investment cost. We assume investment in human capital is first-best optimal in any state of the world:

$$\forall \delta : \ \delta H - f > H_0 \geq 0. \tag{1}$$

Finally, we assume everyone is endowed with an inventory technology which delivers a zero net return across periods.

Preferences. All agents have log utility with time discount factor β:

$$u(c_0, c_2) = (1 - \beta) \log c_0 + \beta \log c_2$$

Here, c_0 and c_2 denote consumption in period 0 and period 2, respectively. For simplicity, we abstract from consumption in the intermediate period. Given the weights β and $1 - \beta$, we can interpret utility at time 2 as representing continuation value into the infinite future. Note that, in our model, all agents have the same risk aversion. This distinguishes our setting from those such as Kihlstrom and Laffont (1979), and, more recently Berk et al. (2010) in which outside investors have lower risk aversion, or are risk neutral. In our model, both agents have the same patience, differentiating it from settings such as those of DeMarzo et al. (2012) and Kiyotaki and Moore (1997) in which a patient investor funds an impatient entrepreneur.

Financing Friction. If an individual shirks, she receives a nonpecuniary private benefit equal to αf in units of the consumption good. Namely, α is the fraction of investment funds than an individual can divert. The action of an individual is not observable, and we assume all agents are protected by limited liability. A contract that prevents shirking will take the form that, should production be zero, the individual receives zero (the individual cannot receive less than zero because of limited liability), and should production be positive, the individual receives something at least as large as αf. That is, the payoff to an individual should meet the following incentive-compatibility constraint:

$$c_2 \geq \alpha f. \tag{IC}$$

This lack of commitment on the part of the individual limits the payoff that can be pledged to the outside investor. The outside investor receives the payoff $\delta H - c_2 \leq \delta H - \alpha f$. The outside investor cannot receive any more than $\delta H - \alpha f$ without violating the incentive compatibility constraint.

Note that α governs the severity of the financing friction. We assume $\alpha \leq 1$, hence, shirking is socially inefficient. In equilibrium, individuals will not shirk; shirking is an off-equilibrium-path credible threat. In a frictionless world $\alpha = 0$. The greater returns to human capital imply that, if the individual puts forth effort, there are sufficient funds to meet the IC constraint: $\delta H - \alpha f \geq \delta H - f > H_0 \geq 0$.

It is useful to contrast the nature of the financing friction in this model with the standard financing friction in macro-finance. In macro-finance, the standard friction is a collateral constraint: the entrepreneur can finance operations with debt up to a fraction of physical capital stock (Kiyotaki and Moore, 1997). Should the entrepreneur default, the lenders can seize part of the capital and sell it at market value. Often in these models, default can occur because the investment is sub-optimal in certain states of the world.

In this setting, however, we tie our hands by assuming that the investment is always first-best optimal. Default is endogenous in that the agent cannot commit to put forth effort. In the case where shirking occurs, there is no collateral to be seized and sold on the market (given modern institutions). The incentivizing device is weaker (because no penalty is paid for default) and thus the financing friction is more severe. Rather than being able to borrow up to a specific fraction of human capital, the agent must commit to a specific cash flow pattern. This is what we mean by inalienability.

This concept of inalienability is essentially the same as the concept introduced by Hart and Moore (1994). In our model as well as in theirs, the agent will no longer choose the efficient action if his or her payoff falls below a threshold. The "capital stock" H, which is what the agent brings to the production process, cannot be transfered or seized. Therefore, the production opportunity is gone when the agent decides not to participate. In the absence of an incentive to shirk ($\alpha = 0$), we would still have inalienable human capital; however, the outcome would be the same as if capital could be sold, because as a practical matter, the agent would choose the efficient action.

Table 1 summarizes the timing in the model.

Financial Contracting We assume that, at $t = 0$, investors and individuals can write all forms of long-term contingent contracts, subject to (IC). We assume investors trade contingent claims in a competitive financial market.

Note that we assume full commitment on the part of outside investors. Appendix C solves for the case with limited commitment from outside investors.

2.2 Agents' Optimization

Let $\pi : \Delta \to \mathbb{R}^+$ denote the state-price density. In the following sections, we specify the optimization problem of investors and individuals.

2.2.1 Outside Investors

We assume a perfectly competitive market for investment in human capital.[4] Investors thus solve a standard consumption and portfolio choice problem in which they trade Arrow securities indexed by δ:

$$\begin{aligned} V^{\mathcal{I}}(e^{\mathcal{I}}) = & \max_{\{c_0^{\mathcal{I}}, c_2^{\mathcal{I}}(.)\}} (1-\beta)\log c_0^{\mathcal{I}} + \beta \int \log c_2^{\mathcal{I}}(\delta)\ dF(\delta) \\ & \text{such that:} \quad c_0^{\mathcal{I}} + \int \pi(\delta) c_2^{\mathcal{I}}(\delta)\ dF(\delta)\ \leq e^{\mathcal{I}}. \end{aligned}$$

The optimal consumption plan is simply

$$\begin{aligned} c_0^{\mathcal{I}} &= (1-\beta)\ e^{\mathcal{I}} \\ c_2^{\mathcal{I}}(\delta) &= \frac{\beta}{\pi(\delta)}\ e^{\mathcal{I}}. \end{aligned}$$

2.2.2 Individuals

Individuals face perfectly competitive markets. They solve a consumption, portfolio choice, and occupational choice problem. Let $o : \Delta \to \{0, 1\}$ denote the optimal state-contingent occupational plan, where $o = 1$ implies investment in human capital. Namely, individuals solve the following long-term financial contracting problem:

$$V(e) = \max_{\{c_0, c_2(\cdot)>0, o(\cdot)\in\{0,1\}\}} (1-\beta)\log c_0 + \beta \int \log c_2(\delta)\ dF(\delta),$$

subject to

$$c_0 + f - e \leq \int \pi(\delta)\{o(\delta)\ \delta H + [1-o(\delta)](H_0 + f) - c_2(\delta)\}\ dF(\delta) \qquad \text{(BC)}$$

[4]Namely, for investors, the returns to investing in human capital equal to the returns to investing in the contingent claim market.

and

$$\forall\delta \;\mid\; o(\delta) = 1: \quad c_2(\delta) \geq \alpha f \tag{IC}$$

where $e \in \{e^\ell, e^h\}$ represents the initial endowment of an individual.

Included in the right-hand side of (BC) is the individual's payoff from either investing or not investing in human capital. If $o(\delta) = 1$, the individual produces δH. If $o(\delta) = 0$, the individual pays back the fixed investment cost f, and works in the traditional sector, producing H_0. The individual's budget constraint implies that the market value of the individual's state-contingent consumption plan must not exceed her endowment plus the value of her human capital. However, (BC) is not the only constraint that the individual faces: for the individual's plan to be incentive-compatible, the plan must meet (IC). The incentive-compatibility constraint is the source of market incompleteness.

Note that (BC) has a flow-of-funds interpretation. The left-hand side of (BC) is the (net) amount an individual raises at time 0 in the capital market; she needs funds to cover the investment cost f and to pay for early consumption. The right-hand side of (BC) is the time-0 value of the resources that she can promise to repay in period 2; if she invests in human capital ($o = 1$) she produces δH and if not ($o = 0$) she would return the investment fund and also the labor income of traditional sector; period 2 consumption is deducted from what she can produce.

Ultimately (BC) and (IC) form the optimal contract between the investor and the individual. The investor commits to provide funds f in states of the world in which (IC) is satisfied. Equation BC is a participation constraint for the outside investor. By definition, the individual consumes c_0 and $c_2(\cdot)$. Thus the outside investor pays $f - e - c_0$ and receives $o(\delta)\,\delta H + [1 - o(\delta)](H_0 + f) - c_2(\delta)$ in state of the world δ. The budget constraint (BC) implies that the net present value to the investor is greater than or equal to zero.

Equation BC implies that setting aside f at time 0 is a prerequisite to investment at time 1. An alternative modeling choice would be to allow f to be paid out of production. If this were the case, f would not appear on the left-hand side of (BC), but rather it would be subtracted from δH on the right hand side. The requirement that f be set aside at time 0 implies, realistically, that there is a gap between investment and production, namely that time to build is required.

Solution to the individual's problem. Let λ be the shadow price of (BC). Appendix A.1 shows that, given an occupational choice plan $o(\cdot)$, the optimal consumption plan satisfies

$$\begin{aligned} c_0 &= \frac{1-\beta}{\lambda} \\ c_2(\delta) &= \begin{cases} c_2^\star(\delta) \equiv \frac{\beta}{\lambda}\frac{1}{\pi(\delta)} & \text{if} \quad o(\delta)=0 \\ \max\{c_2^\star(\delta)\ ,\ \alpha f\} & \text{if} \quad o(\delta)=1 \end{cases} \end{aligned}$$

Here, $c_2^\star(\delta)$ is the optimal consumption plan in the absence of (IC).[5] The second-best allocation requires the individual to consume at least αf, assuming $o(\delta)=1$, in order to keep incentives aligned. For h-individuals, endowment e is high, and thus λ is relatively low. For these individuals, $c_2^\star(\delta) \geq \alpha f$ for all δ. The incentive compatibility constraint (IC) does not bind, and the first-best allocation holds: $o(\delta)=1$ for all δ.

For ℓ-individuals, e is low and λ is relatively high. There are states of the world in which, in the absence of (IC), the individual would prefer to consume less than αf: $c_2^\star(\delta) < \alpha f$. Supposing that $o(\delta)=1$ (we discuss $o(\delta)$ below), (IC) requires that the individual consumes more than she would under the first-best allocation. Utility in these states is higher than it would be under the first-best, but the budget constraint

[5] That is, c_2^* is the optimal consumption plan for the agent, taking λ and π as given. This is not to be confused with this agents' consumption in a frictionless economy, which will take the same form, but for which the numerical values of λ and π will change.

tightens (it is more difficult to meet the participation constraint of outside investors).

The optimal occupational choice $o(\delta)$ is based on the tradeoff between the added gains from efficient investment, and inefficient over-consumption. As Appendix A.1 shows, the optimal occupational choice satisfies:

i) $\alpha f \leq c_2^\star(\delta) \Longrightarrow o(\delta) = 1$

ii) $\alpha f > c_2^\star(\delta) : o(\delta) = 1, \iff$

$$\underbrace{\lambda\pi(\delta)[\delta H - f - H_0]}_{\text{efficiency gain}} - \underbrace{\left(\lambda\pi(\delta)[\alpha f - c_2^\star(\delta)] - \beta \log\left(\frac{\alpha f}{c_2^\star(\delta)}\right)\right)}_{\text{cost of over-consumption}} \geq 0. \quad (2)$$

If the unconstrained solution (what the agent would consume in the absence of incentive compatibility) lies above αf, the individual always invests in human capital, because it is more efficient then the alternative. This is statement (i). If the unconstrained solution lies below αf, the individual faces a tradeoff, captured by (2). Equation 2 shows the difference in indirect utility between investing ($o(\delta) = 1$), and not investing ($o(\delta) = 0$), in the region where the agent is constrained (states δ where $c_2^\star > \alpha f$).[6] Thus (ii) simply states that $o(\delta) = 1$ when the utility from investment is higher.

We see that the decision to invest trades off between the gain in efficiency from investing (the first term) and the cost of inefficient consumption (the second). Efficiency gain is $(\delta H - f - H_0)$, multiplied by its value today ($\pi(\delta)$), and the benefit of relaxing (BC), λ. When the individual invests she must consume more to meet (IC). The cost of over-consumption incorporates the utility difference between constrained consumption αf and unconstrained consumption $c_2^\star$, as well as the cost of greater-consumption, state-by-state, multiplied by the cost of tightening (BC).

We conclude this subsection by discussing the role of commitment on the part of

[6]In this exercise we consider a deviation from optimal $o(\delta)$ in each state of the world, so that λ remains fixed. See Appendix A for detail.

outside investors. Consider states of the world δ such that

$$\delta H - c_2(\delta) < f \tag{3}$$

(the left hand side is the payoff to the investor and the right hand side denotes the funds she provides). In these states, the outside investor would prefer not to invest in human capital.[7] Thus the outside investor has an incentive to violate the contract when (3) holds. Note that (3) also implies

$$\delta H - \alpha f \leq f \Longrightarrow \delta \leq \frac{(1+\alpha)f}{H}$$

because $c_2(\delta) \geq \alpha f$. Thus $\delta \leq \frac{(1+\alpha)f}{H}$, is a sufficient condition for the outside investor to wish to renege on her investment.

Under the benchmark calibration, there is a region of outcomes δ for which investment is part of the optimal contract but the outside investor would prefer to renege. We assume a legal environment in which there is always such commitment on the investor's side, but we also solve for the limited commitment case in Appendix C.

2.3 Equilibrium

We define equilibrium as

- a state-price density: $\pi(\cdot)$,
- a set of consumption plans and occupational choices for investors, ℓ- and h-individuals: $c_0^{\mathcal{I}}$, $c_2^{\mathcal{I}}(\cdot)$, $o^{\ell}(\cdot)$, c_0^{ℓ}, $c_2^{\ell}(\cdot)$, $o^h(\cdot)$, c_0^h, $c_2^h(\cdot)$,

such that

1. given prices, allocations maximize optimization problems specified above, and

[7] This argument assumes that if the investors reneges, she does not get to claim H_0. However, if she does get to claim H_0, the same argument goes through, as we show in the Appendix.

2. markets clear at time 2:

$$c_2^{\mathcal{I}}(\delta)+m^\ell c_2^\ell(\delta)+m^h c_2^h(\delta)=\sum_{j=\ell,h} m^j\{o^j(\delta)\,\delta H+(1-o^j(\delta))(H_0+f)\}+I, \quad \text{(MC)}$$

where $I \geq 0$ stands for the amount of resources that agents might keep in inventory.

3. Storage is costless: if $\int \pi(\delta)\ dF(\delta) < 1$, then $I = 0$. Otherwise, I is such that $\int \pi(\delta)\ dF(\delta) = 1$.

Appendix A.2 shows that the cut-off rule described in the previous section is an equilibrium result. We jointly solve for δ^c and $\pi(\delta)$ as a function of λ^ℓ and I. We then solve for λ^ℓ and λ^h.

3 Frictionless Case: $\alpha = 0$

The case without frictions offers a useful point of comparison. In this case, all individuals set $o(\cdot) = 1$ as the (IC) constraint does not bind. Optimal consumption equals

$$c_0^j = \frac{1-\beta}{\lambda^j} \tag{4}$$

$$c_2^j(\delta) = \frac{\beta}{\pi(\delta)\lambda^j} \tag{5}$$

for all types of agents: $j \in \{\ell, h, \mathcal{I}\}$.

The Lagrange multipliers λ^j satisfy the budget constraints. For investors,

$$\lambda^{\mathcal{I}} = 1/e^{\mathcal{I}}, \tag{6}$$

while for individuals,

$$\lambda^j = \left(e^j + \int \pi(\delta)\, \delta H \, dF(\delta) - f \right)^{-1}, \qquad j \in \{\ell, h\}. \tag{7}$$

We can interpret $\int \pi(\delta)\, \delta H \, dF(\delta) - f$ as the time-0 value of the individual's human capital. In the frictionless case, human capital becomes a traded asset. Because human capital is the same across individuals, only the initial endowment determines differences in consumption. This is in sharp contrast to the case with frictions, in which the payoffs to human capital positively correlates with initial cash-on-hand.

Market clearing (MC) determines the state-price density. Assume parameters are such that the equilibrium riskfree rate is non-negative, i.e. $\int \pi(\cdot)\, dF(\delta) \leq 1$ (this will be the case in our calibration, assuming no frictions). Then inventory $I = 0$. Noting that all individuals invest in human capital ($o(\cdot) = 1$), and substituting in from (5), we have the following simplified version of the market clearing condition (MC):

$$\frac{\beta}{\pi(\delta)} \left(\frac{1}{\lambda^{\mathcal{I}}} + \frac{m^\ell}{\lambda^\ell} + \frac{m^h}{\lambda^h} \right) = \delta H.$$

Define

$$w_H \equiv \int \pi(\delta)\, \delta H \, dF(\delta) - f$$

as the time-0 value of total human capital in the economy (note that $m^\ell + m^h = 1$). Substituting in from (6) and (7), we have

$$\beta[e^{\mathcal{I}} + m^\ell e^\ell + m^h e^h + w_H] = \pi(\delta)\, \delta H. \tag{8}$$

Integrating over δ implies that w_H must satisfy

$$\beta[e^{\mathcal{I}} + m^\ell e^\ell + m^h e^h + w_H] = w_H + f. \tag{9}$$

Therefore,

$$w_H = \frac{\beta[e^{\mathcal{I}} + m^\ell e^\ell + m^h e^h] - f}{1-\beta}$$

Substituting into (8) and solving for $\pi(\cdot)$ implies:

$$\pi(\delta) = \frac{\beta[e^{\mathcal{I}} + m^\ell e^\ell + m^h e^h - f]}{(1-\beta)\ \delta H}. \tag{10}$$

Thus $\pi(\delta) \propto \delta^{-1}$. This is the standard representative agent result with log utility. Market clearing implies that aggregate consumption at time 0 equals $e^{\mathcal{I}} + m^\ell e^\ell + m^h e^h - f$ and aggregate consumption at time 2 equals δH, pinning down the constant of proportionality.

4 Results

4.1 Calibration

As an illustrative calibration, we assume a time-discount rate of 0.95 and zero payoff in the traditional sector ($H_0 = 0$). We normalize f to 1. We calibrate H so that 5% of the time, low cash-on-hand (ℓ) individuals do not invest in human capital. We interpret this lowest 5% of the the TFP distribution as corresponding (endogenously) to economic disaster states. We assume that 50% of individuals have low cash on hand and 50% have high cash on hand. We assume aggregate TFP δ is approximately lognormal such that $\log \delta \sim N(0, 0.10)$. We censor draws of δ so that (1) obtains.[8] We set endowments so that the riskfree rate is equal to $1/\beta$ in the frictionless economy, in effect calibrating the economy so that the final period represents a stream of consumption flow into the future.

[8]Censoring occurs with probability less than 0.0001 in our benchmark calibration. Thus δ is very close to lognormal.

4.2 Occupational choice

Under our assumptions, the individual with high cash-on-hand will choose to invest in human capital in any state of the world. However, the low-cash-on-hand individual's occupation choice displays state-dependence. For values of δ sufficiently low, it is not optimal for the investor to pay the individual a sufficiently high wage to satisfy incentive compatibility (IC). Without incentive compatibility, the individual cannot commit to not shirk. This problem, endogenously, does not exist for the high cash-on-hand individuals, who invest using their own funds and receive both the dividend and the wage.

Figure 1 shows the occupation choice of the low cash-on-hand individual as a function of the state of the world δ.[9] This choice is always 1 for δ sufficiently high. There is a region in which equilibrium determines the fraction of agents who invest and who do not. The region for which this fraction is not 1 or zero is very small. For δ sufficiently small, occupational choice is zero. The case with a convex technology (which we solve for in Appendix B) also features state-dependence, but is a smooth function at lower levels of δ.

4.3 Consumption, dividends, and asset prices

Figure 2 shows aggregate consumption as a function of δ. In the frictionless case, all agents consume in proportion to TFP. In the case with frictions, aggregate output (and hence consumption) is equal to that of the frictionless case, as long as individuals make the efficient occupational choice. . In the lowest states, agents no longer make efficient occupational choices, and aggregate consumption is sharply lower at the point in the distribution where agents stop investing in human capital. In the case of a convex occupational choice, aggregate consumption is lower, but the effect is smooth over the

[9] In this and in subsequent figures, policy functions and equilibrium outcomes are shows as functions of the cumulative distribution function of δ rather than δ itself.

outcomes of δ. Namely, disasters are more rare in the non-convex case, but they are also more severe.

The similarity between consumption in the frictionless case and in the case with frictions masks sharp distributional differences. These distributional differences turn out to be important for asset pricing. Figure 3, Panel A shows the dividend to investors:

$$D_2(\delta) = ((1 - o^\ell(\delta))[H_0 + f] + o^\ell(\delta)\delta H - c_2^\ell(\delta))m^\ell, \tag{11}$$

where o^ℓ is the fraction of ℓ-individuals who invest in human capital. Figure 3, Panel B shows the payoff to low cash-on-hand individuals. These two payoffs show how investors and low cash-on-hand individuals share aggregate risk. For most of the range of TFP, investors bear nearly all of the risk. Again, this effect is present for a convex opportunity set, but is amplified in the non-convex case. The consumption of low cash-on-hand investors is flat over this range, whereas, relative to the frictionless case, the dividend displays excess sensitivity to the aggregate state. Note that excess dividend sensitivity is a endogenous outcome of the contracting problem. The investor "overcompensates" the individual in certain states of the world because otherwise the individual cannot commit to work.

The state-price density reflects imperfect risk sharing, as Figure 4 shows. In the frictionless case, the state-price density is proportional to the inverse of consumption, which in turn is proportional to the inverse of TFP. Bad states receive higher prices, as in the standard Lucas (1978) economy. As compared to the case with no frictions, frictions imply a state-price density that is weakly higher at every level of δ. At very high values of δ, the state-price densities are the same (the economy is close to the frictionless case, because all individuals consume enough to make effort optimal). At the lowest values, they are also nearly the same (investment in human capital is not optimal in the world with frictions and barely efficient in the frictionless world). For most of the state space, state prices are quite different.

The increase in the state-price density relative to the frictionless case reflects an intertemporal distortion. Consider the decision of the low cash-on-hand investors. In the frictionless case, the value of human capital becomes a traded asset, and the individual's consumption and savings decision is determined only by the standard Euler equation. In the case with frictions, however, human capital is not traded, because the individual cannot commit to provide effort unless he or she receives sufficient compensation.[10] This compensation amounts to a claim on the project. By self-financing (consuming less today, in return for earnings from the project in the future), the agent shrinks the region in which efficient investment fails to take place. Self-financing the project, is, of course, a form of savings, in that it shifts consumption from the time 0 to time 2. This savings raises the prices of all future states relative to the current state (because investors need to buy the claims to these states), lowering the equilibrium interest rate.

There is a second channel leading to a lower interest rate. In equilibrium, outside investors must bear more of the risk relative to the frictionless case. Besides leading to a more volatile state-price density, it also leads to a greater precautionary motive, and thus higher state-price densities overall.

It is also of interest to compare the state-price density under the convex and non-convex technologies. One might think that the convex case would be a smoothed version of the non-convex case. For many outcomes, this is correct: it is a smoothed version of occupational choice (Figure 1), aggregate consumption (Figure 2), and even ℓ-individual per-capita consumption in period 2 (Figure 2).[11] However, the state-price density under the convex case is qualitatively different than that under the non-convex

[10]The higher the consumption in the second period, the smaller the distortion from having to over-consume in low-δ states. See (2) and the subsequent discussion.

[11]Note that even here, the welfare consequences for ℓ-individuals are quite different, as will be discussed below.

case. When the technology is not convex, the state-price density is non-monotonic in TFP: there is a level of δ at which it spikes upward and then declines. In contrast, the convex technology (like the frictionless case) implies state-price density that monotonically declines in the state. This upward spike is purely due to imperfect risk sharing. In Figures 1–4, the convex and non-convex case differ when δ falls below 20th percentile value. At this value, all individuals invest in human capital when opportunities are non-convex. Under the case of convex technology, all individuals invest, but they do not invest the full amount. While aggregate consumption is higher for some outcomes in the nonconvex case, risk sharing is less efficient, as seen in the sharply higher state-price density and in the lower dividend to outside investors.[12] For aggregate consumption, it does not matter much if some investors are partially employed, versus some fully employed and some not employed. However, for individual outcomes, and for the equity premium, it clearly matters.

These figures highlight the differences between our economy and complete-market endowment and production economies, and even economies with financial frictions such as Gertler and Kiyotaki (2010). First, aggregate consumption exhibits a disaster state relative to TFP (Figure 2). Second, Individuals of differing levels of endowments have quite different consumption profiles. Specifically, the low cash-on-hand individuals is a non-participant in the equity market, explaining the non-participation puzzle (Guvenen, 2009) among individuals with low financial wealth —Campbell (2016) shows that these agents are much more likely to be non-participants.[13] Third, dividends are procyclical, a fact that matches the data, but which endowment economies take as a given. Production economies, on the other hand, struggle to match this result (Kaltenbrunner and Lochstoer, 2010). In a standard production setting with time-varying investment

[12] On average, individuals need to be paid more under the non-convex case because they can divert the full amount.

[13] Berk and Walden (2013) also link non-participation to the optimal wage contract between investors and workers.

opportunities, dividends counterfactually fall in good economic states. In our model, however, dividends rise more than consumption in good economic states. Finally, relative to Gertler and Kiyotaki (2010), markets are incomplete, leading to a more volatile state price density, relative to what would be implied by aggregate consumption.

The dividend and consumption dynamics also speak to puzzles in macroeconomics. The flat consumption profile of the low-cash-on-hand investor indicates a wage that is insensitive to the economic state, without the need to assume complicated wage-setting procedures.[14] The model speaks to why economic growth (multiple realizations of a good economic state) might lead to a declining labor share, a well-known empirical finding (Karabarbounis and Neiman, 2013). Figure 3 shows that, relative to TFP, ℓ-individuals give up some consumption (wages) in high states, in order to meet the pledgeability constraint (combined with investors' participation constraint) in less good states of the world. Thus, when times are good, investors benefit more than proportionally (Figure 3).

Figure 3 also highlights an important tradeoff in matching the data that will be ubiquitous in models with a moral hazard friction. The high and volatile state-price density occurs because of imperfect risk-sharing, a problem that grows worse as productivity falls. However, for very low values of TFP δ (the states agents attempt to avoid), it is the low cash-on-hand individuals, not the investors who bear all of the risk. In these states, investors cease supplying external capital. Their dividends *rise*. Because consumption for investors is higher in these states, the state-price density jumps downward – a non-monotonicity reflected in Figure 4. This effect will hurt the model's ability to match the equity premium puzzle. That said, the existence of this

[14] A large literature in macroeconomics focuses on wage setting mechanisms that might lead wages to be sticky (Christiano et al., 2016). Favilukis and Lin (2014) show that a model with exogenous sticky wage, together with long run risk and recursive utility can explain the equity premium. Kilic and Wachter (2018) show that partially sticky wages can account for excess volatility and the equity premium in a model with rare disasters. Ai and Bhanderi (2019) endogenize sticky wages through a contracting problem. However, unlike us, they assume that workers cannot invest in the stock market.

state determines the equity premium in the model in the first place.

4.4 Quantitative implications

Table 3 describes the quantitative implications of our model. As we assume log utility and normally distributed log TFP in a static setting, the model does not have the mechanisms that explain the equity premium in the existing literature (high risk aversion, exogenous disasters, long run risk). Nonetheless, the model generates some quantitatively significant results. In the frictionless case, the equity premium is 1%, with an unrealistically high riskfree rate of 5%. In the cases with frictions, the riskfree rate falls to 0%, and, in the case of the non-convex technology, the equity premium doubles to more than 2%. Meanwhile, the volatility of log dividends (equal in this case to the volatility of log returns), rises from 10% per annum to almost 20%. As Figure 3 shows, this higher volatility reflects greater sensitivity to the economic state. The higher volatility of dividends (endogenous leverage) is only part of the reason for the higher equity premium. The state-price density is also more volatile than in the frictionless case, as Figure 4 shows. This greater volatility, which comes from endogenous market incompleteness, generates a higher equity premium.[15]

Does our assumption of a non-convex choice set matter for our results? Quantitative results for aggregate consumption, the state-price density, and the equity premium are indeed smaller in the case of the convex occupational choice, as compared with our benchmark case. Non-convexities amplify the effects of financial frictions on these variables. There are, moreover, some results that are qualitatively different when the occupational choice is non-convex. These are the results pertaining to the ℓ-individual. Consumption for the ℓ-individual has a bimodal distribution in the non-convex case, but not in the convex case. The ℓ-individual is more likely to find herself in a very low

[15] The aggregate consumption claim also has a higher equity premium than in the frictionless case. This arises from higher volatility of the state-price density (from imperfect risk sharing) and the fall in consumption in poor economic states. By definition there is no leverage on the consumption claim.

state in this case than when occupational choices are convex. These states represent large shifts to the left relative to better economic times. This is reflected in consumption that is more than twice as volatile in the non-convex case than the convex case (see Table 3). In our model, consumption and wages are the same for ℓ-individuals. The extreme sensitivity of wages at the low end to aggregate market conditions (Guvenen et al., 2014) suggests that non-convexities are an important aspect of the return to human capital.

Table 3 also shows the effect of raising the return to human capital H, and the effect of increasing the mass of low-cash-on-hand individuals to one (so that there are no high cash-on-hand individuals from the economy). Increasing H increases the equity premium from 2.2% to 2.8%.[16] Both increasing H and setting $m^\ell = 1$ increases the equity premium to 4%, explaining the majority of the equity premium in the data, even despite the assumption of log utility.

4.5 Comparative statics

In this section, we explore the effects of changing the mass of low-cash-on-hand individuals (higher m^ℓ) and the return to human capital investment (H). In each case we recalibrate endowments so that, in the frictionless economy, the riskfree rate stays the same and is equal to β^{-1}.

We first consider the effect of raising m^ℓ. Comparing the left and right panels of Figure 5 shows that raising m^ℓ leads fewer low cash-on-hand individuals to choose to invest in human capital (the red line shifts to the right). It is not obvious that this should be the case, as this figure shows occupational choice per individual (it does not simply reflect the fact that the total number of individuals investing in human capital is smaller). The larger number of individuals failing to make the efficient investment leads

[16] As we explain below, we recalibrate endowments so that the riskfree rate in the frictionless benchmark remains the same.

to more severe consumption disasters, that occur slightly more frequently (Figure 6). Changing m^ℓ affects per-capita consumption of the ℓ-individual (Figure 8), as well as the per-share dividend (Figure 7), in a second-order way. Mainly, it affects the state-price density (Figure 9).

We can conclude from Figures 5–9 that the most important affect of a change in m^ℓ is on risk sharing (through the state-price density). The state-price density, is both higher and more volatile (except in the very worst states) if more individuals require outside financing. When m^ℓ equals to 1, these individuals are the sole producers of the time-2 consumption good. Moderately low states, in which they consume too much, relative to their marginal product, are thus extremely costly.

It may seem surprising that increasing the return to human capital increases the equity premium and the quantity of risk in the economy. To further investigate this effect, we show display various quantities as a function of the excess return to human capital $(H - H_0 - f)/f$. Note that these are comparative statics results, and in each case we recalibrate the economy so that, in the frictionless case, the riskfree rate equals β^{-1} (thus eliminating any effects stemming purely from a higher growth rate of consumption). The benchmark case in Table 3 corresponds to $(H - H_0 - f)/f = 0.965$.

Figure 10, Panel A shows the disaster probability, where a disaster state is one in which individuals do not fully invest in human capital. The higher is H, the lower is the disaster probability. This makes sense: a higher value of H makes it easier to persuade outside investors to participate, given the need to compensate individuals. It is therefore not surprising that expected aggregate consumption in the second period is also, for the most part, increasing in H (Panel B). In both cases, there is a value of H sufficiently large for which all individuals invest in human capital across all states (so that the disaster probability equals zero), and for which aggregate consumption is equal in expectation to the frictionless case. Note that these values are slightly different. For H implying a probability equal to zero, but close to the boundary at which the probability exceeds zero, second period aggregate consumption is above the frictionless benchmark. In this economy, an econometrician would not see any disasters. This is an endogenous outcome; the disasters still matter, and agents save in order to (successfully) avoid them.

Panel C shows the riskfree rate as a function of $(H - H_0 - f)/f$. The riskfree rate figure has a very different pattern than the disaster probability and aggregate consumption. For values of H far above those that generate observed disasters, and for which aggregate consumption appears identical to the frictionless case, distortions caused by the financial friction still matter. Agents still have a greater precautionary motive, and the interest rate is still depressed toward its zero lower bound, for virtually the entire region we consider. This figure emphasizes the result that the disaster probability of zero in Panel A and well-behaved aggregate consumption, are endogenous outcomes. Avoiding disasters has a strong effect on asset prices.

Panel D shows the equity premium. The equity premium is non-monotonic in $(H - H_0 - f)/f$, illustrating the tradeoff we described in Section 4.3. Lower values of $(H - H_0 - f)/f$ clearly worsen the friction. They raise the probability that some individuals fail to invest (the disaster probability in Panel A), and they lower the riskfree rate in Panel B. However, they lead to a *lower* equity premium because outside investors offload the risk onto the low cash-on-hand individuals. In low states of the world, investment is suboptimal, leading to higher values of dividends. To summarize, the lower is $(H - H_0 - f)/f$, the greater the friction, and the less risky the dividend. Thus, the equity premium increases in $(H-H_0-f)/f$, even as the probability of disaster falls, and the riskfree rate rises. However, for sufficiently high values of $(H-H_0-f)/f$, the friction becomes sufficiently unimportant and risk comes closer to being perfectly shared. The equity premium still is far above the frictionless benchmark, but it declines in H, because, over this range, higher H implies that there is sufficient funds to meet the incentive compatibility constraint without too much distortion in risk sharing.

4.6 Wealth distribution and welfare

Our model has surprising implications for the wealth distribution, and for the welfare of agents.

Table 4 reports the marginal utility of wealth (the Lagrange multiplier on the budget constraint λ), total wealth, and human capital for the three types of agents, for the economy with non-convex choice set, convex choice set, and with no frictions. We define human capital wealth for individual j, $j = \ell, h$ as the time-0 value of cash flows generated, less the investment cost:

$$\int \pi(\delta) \left(o^j(\delta)\ \delta H + (1 - o^j(\delta))(H_0 + f) \right)\ dF(\delta) - f.$$

Total wealth for individual j is simply human capital plus the time-0 endowment e^j. Because of the log utility assumption, the marginal utility of wealth for the investor is simply equal to the inverse of the initial endowment, and does not vary across economies. This is not the case for individuals endowed with human capital. For the ℓ-individual, marginal utility of wealth (λ^ℓ) is sharply higher in the non-convex case, as compared with the convex case, and is sharply higher in the convex case as compared with the frictionless case. Because marginal utility of wealth is proportional to marginal utility of time-0 consumption at the optimum, this indicates that the individual cuts her time-0 consumption to rely on internal funds to cover investment cost as much as possible.[17] Interestingly, the marginal utility of the h-individual's wealth is slightly *lower* in the economies with friction (though the magnitude is not large). Below, we explain why this occurs.

Panel B of Table 4 reports total wealth (in units of time-0 consumption), which includes both cash-on-hand and human capital. For investors, this does not vary across

[17] Note that reduced time-0 consumption helps the individual meet the participation constraint for investors, given incentive compatibility. This agent saves more than in the case without frictions.

economies, as the value of total wealth is simply cash-on-hand (the time-0 endowment). Wealth is very slightly higher for the ℓ-individual, and moderately higher for the h-individual. This implies aggregate wealth that is higher in the case of the economies with frictions.

This result seems surprising. Why is aggregate wealth higher, when, as Table 3 reports, the equity premium attached to wealth rises, while aggregate cash flows fall? Moreover, why is aggregate wealth higher, even for the ℓ-individual when the marginal utility of wealth is also higher? The reason is that the value of wealth masks the state-dependence of the payoffs at time 2, which is what matters for utility. This wealth is not something that the agent is capable of selling, and turning into consumption in any manner that he or she pleases. For the ℓ-individual, moral hazard constraints dictate that wealth must be consumed in a largely suboptimal state-independent way, except in the worst economic states, in which case consumption falls sharply. Moreover, the fact that state prices are higher overall hurts this investor, whose wealth has not risen with the state prices to the same degree that the h-individual's wealth has. One can understand this result in terms of a Campbell and Shiller (1988) decomposition: higher state prices overall imply a lower riskfree rate: the lower riskfree rate overwhelms the equity premium and cash flow effects.

The h-individuals are unconstrained, and in their case, wealth is proportional to the inverse of the marginal utility of wealth. The fact that their wealth is higher, and the marginal utility lower, in cases with frictions, indicates that they are able to sell their human capital at a higher value because state prices are higher. On the other hand, consuming is more expensive, which implies (due to log utility) that their certainty equivalent is the same. Note however, that the financial interests of h-individuals are to some extent opposed to those of the ℓ-individuals. Type h-individuals are not hurt by the presence of financial frictions, and in fact, their wealth is substantially higher. By comparison, ℓ-individuals suffer a substantial utility loss due to financial frictions.

Figure 11 considers comparative statics for welfare. We show welfare (in consumption-equivalent terms) as a function of $(H - H_0 - f)/f$. In the frictionless economy, welfare for all individuals increases linearly in the quantity of goods (solid black line). For the high cash-on-hand individual, welfare equals the value in the frictionless case. The higher value of their human capital, but the requirement to consume at higher state prices cancel out, and their welfare exactly equals the frictionless value. This is not so for either the investor or for the low cash-on-hand individual. As long as there is some probability of failure to invest in human capital, the ℓ-individual suffers a welfare loss relative to the frictionless case. The magnitude of this loss increases sharply with the disaster probability. When the probability of a disaster is zero, the ℓ-individual enjoys a small welfare *gain*, due to the fact that she must be compensated at a greater wage. Investors, on the other hand, suffer a (small) welfare loss that is positive at any point. For these investors, decreased risk sharing means that they are worse off. Unlike the h-individuals, they have no human capital to sell at a higher price to compensate.

Importantly, the inalienability of human capital and the financing friction is not per-se consequential for welfare. What matters is the interaction between the financial friction and aggregate risk. Table 5 reports welfare with pre-determined TFP ($\sigma(\log \delta) = 0$), and compares this with our benchmark case. In the economy without risk, the presence of the friction slightly raises the welfare of the ℓ-individuals (compare the second and the fourth column, Panel D). Frictions lead the ℓ-individual to require a high reservation wage; without uncertainty, this reservation wage is guaranteed. There is a pure redistribution of consumption goods from investors to ℓ-individuals in the case with frictions as compared to the case without, assuming no risk.

Moreover, risk per-se is also not consequential for welfare. In a model without frictions, welfare loss due to uncertain productivity is negligible (compare the third and fourth columns, Panel D). This is the standard business-cycle cost result (Lucas, 2003).[18] However, risk in TFP considerably lowers the welfare of a ℓ-individual in a model with frictions (see columns one and two). As explained before this is due to the endogenous left-tail risk and imperfect risk sharing in the economy.

5 Conclusion

We have introduced a model of inalienable human capital that we show can explain a number of macroeconomic and asset pricing facts. First, inalienable human capital exacerbates downturns, as it is no longer possible to compensate individuals for putting forth unobservable effort. This leads to a thick left tail in consumption, even if productivity is normally distributed. It leads to countercyclical labor income risk, as some agents no longer can finance human capital investments in low economic states. It also leads to dividends that are more volatile than consumption, and wages that are fixed. All of these are consistent with the data. Most importantly, financing frictions imply a low riskfree rate, and an equity premium that is between 2 and 4 times what it would be in the frictionless case.

While not explored here, our model is also consistent with a dynamic role of finance in asset prices. We have kept various elements of the economic environment fixed, such as financing frictions and the ability of outside investors to make commitments. Comparative statics in our model show that an increase in financial frictions leads to a greater equity premium, lower cash flows, and a "flight to quality" – a much lower riskfree rate. A change in the ability to form commitments also has similar affects. A generalized model has the potential to explain financial market fluctuations in terms of changes in the institutional environment.

[18] Unlike in earlier comparative statics, we do not change endowments so that the risk/no risk case has the same riskfree rate. This is appropriate given that our aim is welfare comparison.

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A Non-Convex Technology

This Appendix solves the model for the benchmark case of a non-convex technology. Section A.1 characterizes the solution to the individual's optimization problem. Section A.2 solves for equilibrium for a given level of Lagrange multipliers and inventory. Section A.3 then describes the algorithm for computing the full solution.

A.1 Individual's Problem

In this section, we take the state price density $\pi(\cdot)$ as given and solve the problem of an individual with access to a non-convex investment technology. We suppress the superscript l, h. For a given time-0 endowment e, define the value function

$$V(e) = \max_{\{0\leq c_0,\ 0\leq c_2(\cdot),\ o(\cdot)\in\{0,1\}\}} (1-\beta)\log(c_0) + \beta \int \log(c_2(\delta))\ dF(\delta)$$

$$\text{such that: } c_0 + f - e \leq \int \pi(\delta)\{o(\delta)\ \delta H + [1-o(\delta)](H_0+f) - c_2(\delta)\}\ dF(\delta) \quad \text{(BC)}$$

$$\forall\delta \ \mid\ o(\delta) = 1: \quad c_2(\delta) \geq \alpha f \quad \text{(IC)}$$

Let λ and $\mu(\cdot)$ denote the Lagrange multipliers for (BC) and (IC) respectively. For now, we take these, and $o(\cdot)$ as given, and solve for for optimal consumption plans $c_0(\lambda, \mu(\cdot), o(\cdot))$ and $c_2(\delta; \lambda, \mu(\cdot), o(\cdot))$.

Because log is an increasing function, $\lambda > 0$. Furthermore, $\mu(\cdot) \geq 0$, and $\mu > 0$ if and only if $c_2 = \alpha f$. Differentiating the Lagrangian associated with the individual's problem above leads to the following first-order conditions:

$$c_0 = \frac{1-\beta}{\lambda} \quad \text{(A.1)}$$

$$c_2(\delta) = \frac{\beta}{\lambda\pi(\delta) - \mu(\delta)} \quad \text{(A.2)}$$

If $o(\delta) = 0$, the IC constraint does not bind, and thus $\mu(\delta) = 0$. It follows that

$c_2(\delta) = \beta/(\lambda\pi(\delta))$. If $o(\delta) = 1$, there are two cases:

1. $\mu(\delta) > 0$ and $c_2(\delta) = \alpha f$.

2. $\mu(\delta) = 0$ and $c_2(\delta) = \beta/(\lambda\pi) \geq \alpha f$

Therefore, $c_2(\delta) = \max\{\beta/(\lambda\pi), \alpha f\}$ and $\mu(\delta) = \max\{\lambda\pi - \beta/(\alpha f), 0\}$. We have shown the following Lemma:

Lemma 1. *Given a value for the Lagrange multiplier on (BC) λ and a state-price density π, the following equations summarize the optimal consumption plan:*

$$c_0 = \frac{1-\beta}{\lambda} \tag{A.3}$$

$$c_2(\delta) = \begin{cases} c_2^*(\delta) \equiv \frac{\beta}{\lambda\pi(\delta)} & \textit{if } o(\delta) = 0 \\ \max\left\{\frac{\beta}{\lambda\pi(\delta)}, \alpha f\right\} & \textit{if } o(\delta) = 1 \end{cases} \tag{A.4}$$

Having characterized the optimal consumption plans and the shadow price of (IC), we now characterize the optimal occupational choice $o(\delta; \lambda)$. In each state of the world δ we consider a deviation in optimal $o(\delta)$. A *necessary* condition for an occupational plan to be optimal is that the alternative occupation in each δ delivers a lower indirect utility. Assuming a continuum of states, a deviation in a single $o(\delta)$ should not affect λ. In what follows, we treat λ as fixed.

The objective function in terms of o is

$$U(o(\cdot)) = \beta \log c_2(o(\cdot)) + \lambda\pi[o(\cdot)(\delta H - f - H_0) - c_2(o(\cdot))] + \text{const.} \tag{A.5}$$

where the constant term is not a function of $o(\cdot)$. Define

$$x \equiv \frac{\alpha f \lambda \pi}{\beta}$$

Note that x is the ratio of αf to c_2^*, the optimal frictionless consumption (taking λ and π as given). This dimensionless quantity, which summarizes the effects of the agent's choices on the budget constraint through λ, and the effect of prices π, is the focus of our analysis.

First consider $x \leq 1$, corresponding to $c_2^* > \alpha f$. Optimal consumption simply equals $c_2 = c_2^* = \beta/(\lambda\pi)$ and is independent of occupational choice o. Because $\delta H > f + H_0$, the individual loses nothing by investment, and in fact gains because of the greater productivity. To summarize, when $x \leq 1$, $U(o(\cdot))$ is a strictly increasing function of $o(\cdot)$, and thus $o(\cdot) = 1$.

The interesting case is $x > 1$. Define scaled gain to investment:

$$\gamma(\delta) \equiv \frac{\delta H - f - H_0}{\alpha f}$$

Define

$$u(x;\gamma) \equiv \log x + (\gamma - 1)x + 1, \qquad x > 1. \tag{A.6}$$

It follows from the definition of u that

$$\beta u = U|_{o=1} - U|_{o=0} \qquad x > 1, \tag{A.7}$$

where we note that the utility difference on the right-hand side depends only on x, holding γ constant.

Recall the economic tradeoffs that (A.6 –A.7) represent: when the individual invests in human capital, she consumes a greater amount αf ($x > 1$ implies $\alpha f > c_2^*$). That is better in terms of utility (as represented by the log term). The choice to invest also leads to more resources, as indicated by γx. However, consumption is expensive, as captured by $-x$.

Observation 1. *Suppose $\gamma \geq 1$. Then $o = 1$.*

For $x \leq 1$, $o = 1$ as we observe previously. For $x > 1$, Observation 1 follows from the fact that $u > 0$ when $\gamma \geq 1$. Intuitively, when production is very successful, there are plenty of resources to satisfy the IC constraint.

Observation 2. *Suppose $\gamma = 0$. Then (for $x > 1$), $o(0) = 0$.*

Observation 2 follows from the fact that $u(1) = \gamma$. Because $u' < 0$ over the relevant range, $u < 0$ when $x > 1$. When production is unsuccessful, the extra compensation required by agents when $x > 1$ is a waste of resources.

Observations 1 and 2 serve as boundary conditions. The main case of interest is the following:

Observation 3. *Suppose $\gamma \in (0, 1)$. Then u takes its unique global maximum at $x^* = 1/(1 - \gamma)$.*

Observation 3 follows from the fact that $u'(x^*) = 0$, and that $u'' < 0$.

Observation 4. *Suppose $\gamma \in (0, 1)$. Then there is a unique value $x^c > x^*$ such that $u(x^c) = 0$. Moreover, $o = 0$ if and only if $x > x^c$.*

For x^* defined in Observation 3, $u(x^*) > 0$. Moreover, $\lim_{x\to\infty} u(x) < 0$. The existence of x^c such that $u(x^c) = 0$ follows from the continuity of u. Note also that $u' < 0$ for $x > x^*$; hence, x^c is unique. Thus $u > 0$ for $x < x^c$ and $u < 0$ for $x > x^c$. Observation 4 follows.

Intuitively, the log of consumption grows more slowly than the linear term in consumption. When the difference between optimal frictionless consumption and consumption required by the IC constraint grows sufficiently large, the cost of supporting this consumption (linear in x) must dominate the utility gain (log in x). The cut-off point x^c is the point at which the linear term starts to dominate the log term.

We combine these results into the following proposition:

Proposition 1. *The optimal labor choice $o = 1$ if $x < x^c$, $o = 0$ if $x > x^c$. If $x = x^c$, the agent is indifferent between $o = 0, 1$. The cut-off point x^c satisfies*

$$x^c = \begin{cases} \infty & \gamma \geq 1 \\ > 1 & \gamma \in (0,1) \\ 1 & \gamma = 0 \end{cases}$$

For $\gamma \in (0,1)$, x^c uniquely solves the fixed point problem:

$$x = 1 + \gamma x + \log x. \tag{A.8}$$

Finally, x^c is increasing in γ.

Proof. It suffices to consider the case $\gamma \in (0,1)$ Define

$$F(x;\gamma) \equiv 1 + \gamma x + \log x.$$

First note that for

$$x^c = F(x^c;\gamma),$$

$u(x^c) = 0$. Thus, if x^c solves the fixed-point problem, it is the cut-off point defined in Observation 4. Moreover, manipulating (A.8) implies $x^c > 1/(1-\gamma) > 1$. The result that x^c is increasing in γ follows from taking a total derivative of u with respect to γ. □

A.2 Market Clearing

Proposition 1 of the previous section characterized $o(\cdot)$ in terms of x (ratio of αf to c_2^*), for a given γ (gain from investment). Note that x is a positive scalar multiple of the state-price density π, whereas γ is a linear function of TFP δ. In equilibrium, of course, there will be a one-to-one mapping between π and δ, but for now, it is useful to think of them separately. The economic content of Proposition 1 is that (provided productivity is in an intermediate range), there are state prices that are sufficiently high for which it no longer is optimal to supply human capital.

Thus the previous section solves for $o(\cdot)$ as a function of $\pi(\cdot)$. Now we solve for $\pi(\cdot)$ as a function of occupational choice $o(\cdot)$ using the market clearing condition (MC) and the relative population of ℓ- and h-individuals, taking as given the resources stored in inventory I and marginal utility of wealth for ℓ- and for h-individuals λ^ℓ and λ^h. We make the following assumptions on the parameters.

Assumption 1. *The endowment of the h-individual, e^h is sufficiently large that $o^h(\delta) = 1$ for all δ.*

Assumption 1 requires that e^h be high enough so that, in equilibrium, λ^h is small, and thus $c_2^*(\delta)$ is sufficiently high across all states so that (IC) does not bind.

Assumption 2. *There exists a δ such that, for all ℓ-individuals, $o(\delta) = 1$.*

Assumption 2 states that all low cash-on-hand individuals invest in human capital in some states of the world.

We make an assumption on the type of equilibrium permitted.

Assumption 3. *Equilibrium does not permit endogenous inequality. Namely, λ is the same across otherwise homogenous agents.*

Assumption 3 allows us to restrict attention to symmetric equilibria.

At the optimum, market clearing conditions are (piecewise) linear functions of the inverses of Lagrange multipliers and of state prices. It is convenient to define the following notation for these inverses:

$$\begin{aligned}\hat{\lambda}^j &\equiv 1/\lambda^j \qquad j = \ell, h, \mathcal{I} \\ \hat{\pi}(\cdot) &\equiv 1/\pi(\cdot) \\ \hat{x}^c(\gamma) &\equiv 1/x^c(\gamma)\end{aligned}$$

Also define:

$$\begin{aligned}e &\equiv e^{\mathcal{I}} + m^h e^h + m^\ell e^\ell, \quad \text{total endowment} \\ \hat{e} &\equiv \frac{\beta(e - I - f)}{(1-\beta)\alpha f} \\ y &\equiv \frac{I + m^h \delta H + m^\ell (f + H_0)}{\alpha f} \\ z &\equiv \frac{\hat{\lambda}^\ell \beta}{\alpha f}.\end{aligned}$$

Here, z is the per-capita consumption of ℓ-individuals, scaled by αf. Note that

$$z\hat{\pi} = x$$

from the previous section. Finally, y equals "base" output (namely, output if no ℓ-individuals invest), scaled by αf. Like γ (scaled gain from investment), y is increasing in TFP δ.

With this new notation, we can restate Proposition 1:

$$o^\ell(\delta) = \begin{cases} 1 & \text{if} \quad z\hat{\pi}(\delta) > \hat{x}^c \\ 0 \leq \cdot \leq 1 & \text{if} \quad z\hat{\pi}(\delta) = \hat{x}^c \\ 0 & \text{if} \quad z\hat{\pi}(\delta) < \hat{x}^c \end{cases} \tag{A.9}$$

Recall that $\hat{x}^c(\gamma) = 1/x^c(\gamma)$ is strictly decreasing with $\gamma \in [0, 1]$, $\hat{x}^c = 0$ if $\gamma \geq 1$ and $\hat{x}^c = 1$ if $\gamma = 0$. Also, $\hat{x}^c \leq 1 - \gamma$.

We now use the market-clearing conditions to characterize π as a function of optimal occupational choice o. We will show that π and o solve a system of two equations in two unknowns. Proposition 2 describes the solutions as functions of δ:

Proposition 2. *Given λ^ℓ and I, there exist three cut-off points, δ_0, δ_c, and δ_f such that the equilibrium occupational plan and state-price density satisfy*

$$\delta \leq \delta_0 \quad : \quad \begin{cases} o^\ell = 0 \\ \hat{\pi} = y/\hat{e} \end{cases} \tag{A.10}$$

$$\delta_0 \leq \delta \leq \delta_c \quad : \quad \begin{cases} o^\ell = (y - \hat{x}^c \hat{e}/z)/[m^\ell(1 - \gamma - \hat{x}^c)] \\ \hat{\pi} = \hat{x}^c/z \end{cases} \tag{A.11}$$

$$\delta_c \leq \delta \leq \delta_f \quad : \quad \begin{cases} o^\ell = 1 \\ \hat{\pi} = [y + m^\ell(\gamma - 1)]/(\hat{e} - m^\ell z) \end{cases} \tag{A.12}$$

$$\delta_f \leq \delta \quad : \quad \begin{cases} o^\ell = 1 \\ \hat{\pi} = (y + m^\ell \gamma)/\hat{e} \end{cases} \tag{A.13}$$

Proposition 2 describes four regions of the state space. For δ sufficiently low (below δ_0), no ℓ-individual invests in human capital. The state-price density equals its value in a frictionless economy (given a values for λ^ℓ and I). For $\delta \in [\delta_0, \delta_c]$, some ℓ-individuals invest. Those that invest consume at a constrained level. In this region, an additional unit of investment leads to *lower* resources and higher state prices, because individuals must be compensated at rates greater than their productivity. For $\delta \in [\delta_c, \delta_f]$, all ℓ-individuals invest, and consume at the constrained level. At some point in this region, state prices become independent of occupational choice, as ℓ-types consume exactly what they produce. Finally, for $\delta \geq \delta_f$, all individuals invest, and consumption is once again unconstrained. State prices are what they would be in a frictionless economy.

Proof. We normalize $m^\ell + m^h = 1$. Recall that $m^{\mathcal{I}} = 1$. The time-0 market clearing condition is:

$$(1-\beta)[e^{\mathcal{I}} + m^h\hat{\lambda}^h + m^\ell\hat{\lambda}^\ell] = \underbrace{e^{\mathcal{I}} + m^h e^h + m^\ell e^\ell}_{\equiv e} - f - I \tag{MC-0}$$

where we replace the optimal consumption choices by the outcome of agents' maximization problems. Assumptions 1 and 2 imply that the economy must set aside the full cost f at time 0 for potential use at time 1, because there are some states of the world in which all individuals will invest in human capital.

The time-2 market clearing condition is

$$\beta[e^{\mathcal{I}}\hat{\pi} + m^h\hat{\lambda}^h\hat{\pi} + m^\ell(1-o^\ell)\hat{\lambda}^\ell\hat{\pi}] + m^\ell o^\ell \max\{\beta\hat{\lambda}^\ell\hat{\pi}, \alpha f\} =$$
$$I + (m^h + m^\ell o^\ell)\delta H + m^\ell(1-o^\ell)(f + H_0). \tag{MC-2}$$

where $o^\ell \in [0,1]$ is the fraction of ℓ-individuals investing in human capital at time 1. Because we are interested in $\hat{\pi}$ as a function of o^ℓ, for now, we treat this value as fixed.

The left-hand side of (MC-2) is a piecewise linear function of $\hat{\pi}$. It follows that:

$$\hat{\pi} = \min\{\hat{\pi}^f, \hat{\pi}^*\}, \tag{A.14}$$

where $\hat{\pi}^f$ and $\hat{\pi}^*$ solve (MC-2) over the relevant range:

$$\hat{\pi}^* \equiv \frac{I + m^h\delta H + m^\ell(f + H_0) + m^\ell o^\ell(\delta H - f - H_0)}{\beta[e^{\mathcal{I}} + m^h\hat{\lambda}^h + m^\ell\hat{\lambda}^\ell]} \tag{A.15}$$

$$\hat{\pi}^f \equiv \frac{I + m^h\delta H + m^\ell(f + H_0) + m^\ell o^\ell(\delta H - f - H_0) - m^\ell o^\ell\alpha f}{\beta[e^{\mathcal{I}} + m^h\hat{\lambda}^h + m^\ell(1-o^\ell)\hat{\lambda}^\ell]} \tag{A.16}$$

(see Figure A.1).

We use (MC-0) and the notation defined above to substitute out $\hat{\lambda}^h$ and to simplify (A.15) and (A.16):

$$\hat{\pi}^* = \frac{y + m^\ell \gamma o^\ell}{\hat{e}} \tag{A.17}$$

$$\hat{\pi}^f = \frac{y + m^\ell(\gamma - 1)o^\ell}{\hat{e} - m^\ell z o^\ell} \tag{A.18}$$

Note that $\hat{\pi}^*$ is the state-price density in the frictionless economy. Equations A.17 and A.18 imply that $\hat{\pi}^* = \hat{\pi}^f$ when $m^\ell o^\ell = (\hat{e} - yz)/\gamma z$. The left-hand side of (MC-2) indicates that this must occur when exactly when the frictionless consumption equals αf.

We now consider two cases, $zy < \hat{e}(1-\gamma)$ and $zy \geq \hat{e}(1-\gamma)$.

Case 1: $zy < \hat{e}(1-\gamma)$

It follows from (A.18) that $\hat{\pi}^f$ is decreasing in o^ℓ, and also $\hat{\pi}^f < \hat{\pi}^*$, for $o^\ell > 0$, with $\hat{\pi}^f = \hat{\pi}^*$ for $o^\ell = 0$.

Define δ_0 such that

$$\frac{y(\delta_0)}{\hat{e}} = \frac{\hat{x}^c(\delta_0)}{z} \tag{A.19}$$

The left-hand side of (A.19) is $\hat{\pi}$ from (A.18), assuming $o^\ell = 0$. The right-hand side is $\hat{\pi}$ from (A.9), also assuming $o^\ell = 0$. Therefore, $o^\ell = 0$, together with state prices defined by (A.19) represents an equilibrium. Moreover, for all values of $\delta < \delta_0$, $\hat{\pi} < \frac{\hat{x}^c(\delta)}{z}$. Thus $o^\ell = 0$, again combined with $\hat{\pi}$ given by $\frac{y(\delta_0)}{\hat{e}}$ represents an equilibrium.

Figure A.2 Panel A illustrates the equilibrium. The blue line is $\hat{\pi} = \hat{\pi}^f$ as a function of o^ℓ and δ, from (A.18). The red line is o^ℓ as a function of $\hat{\pi}$ and δ, from (A.9). Equilibrium is at the intersection point. Increasing δ shifts the $\hat{\pi}$ curve to the right and the o^ℓ curve to the left. At $\delta = \delta_0$ is the maximal value of δ at which $o^\ell = 0$

represents an equilibrium.

The economic content is as follows: we know from Proposition 1 that there are state prices high enough at which ℓ-individuals cannot invest. Provided that δ is sufficiently low, this is consistent with equilibrium, because δ and state prices are inversely related (this is in spite of the fact that, for low δ, investment on the part of ℓ-individuals raises state prices).

Now define δ_c as

$$\frac{y(\delta_c) - m^\ell(1 - \gamma(\delta_c))}{\hat{e} - m^\ell z} = \frac{\hat{x}^c(\delta_c)}{z}$$

Because the left hand side is again $\hat{\pi}^f$ (for $o^\ell = 1$), this represents an equilibrium. For $\delta \in (\delta_0, \delta_c)$, the unique equilibrium (for a given λ^ℓ and I) occurs when (A.18) and $\hat{\pi}^f = \hat{x}^c/z$ is satisfied – this pins down $o^\ell \in (0, 1)$.

Case 2: $zy \geq \hat{e}(1 - \gamma)$

It follows from (A.18) that $\hat{\pi}^f$ is increasing in o^ℓ. By assumption, in this case

$$\frac{y}{\hat{e}} \geq \frac{1 - \gamma}{z} \geq \frac{\hat{x}^c}{z},$$

where the last inequality follows from Proposition 1. Note that $y/\hat{e}$ equals $\hat{\pi}$ when $o^\ell = 0$. Because $\hat{\pi}^f$ is increasing in o^ℓ over this range, it follows that equilibrium is reached when $o^\ell = 1$ (see Panel B of figure Figure A.2, Panel B).

The question of whether $\hat{\pi} = \pi^*$ or π^f depends on the value of δ. Note that for finite δ_f satisfying

$$\frac{\hat{e}/z - y(\delta_f)}{m^\ell \gamma(\delta_f)} = 1 \tag{A.20}$$

$\hat{\pi}_f = \hat{\pi}^*$ from (A.17) and (A.18) (note that we have used $o^\ell = 1$). For $\delta < \delta^f$, $\hat{\pi} = \hat{\pi}_f < \hat{\pi}^*$: state prices are high enough that the individual consumes at the constrained level.

For $\delta > \delta^f$ the (IC) is satisfied even for unconstrained consumption. To summarize:

$$\hat{\pi} = \begin{cases} \hat{\pi}^* = \dfrac{y + m^\ell \gamma}{\hat{e}} & \text{if} \quad \delta \geq \delta_f, \\ \hat{\pi}^f = \dfrac{y - m^\ell(1-\gamma)}{\hat{e} - m^\ell z} & \text{if} \quad \delta \leq \delta_f. \end{cases}$$

□

We then need to solve for λ^ℓ and I, using budget constraint for a ℓ-individual, and the optimal decision by the investors to invest in the inventory: $I = 0$ if $\int \pi dF < 1$; otherwise, $I > 0$ and $\int \pi dF = 1$.

A.3 Numerical Algorithm and Equilibrium Selection

In principle, we have not ruled out multiple equilibria. We select the equilibrium with the lowest level of inventory I (as it is the inferior technology), and the lowest value of λ^ℓ.[19]

We conjecture inventory $I = 0$. Given this I, we solve for λ^ℓ; to do so, we start with the lowest possible value of λ^ℓ, solved from frictionless-world with $\alpha = 0$, then we solve for o^ℓ and π using analytical expressions above and see if budget constraint for a ℓ-individual holds; if not, we incrementally increase λ^ℓ and solve for a new partial equilibrium at time 2. After solving for a λ^ℓ, given $I = 0$, we check if $\int \pi dF \leq 1$. If yes, we are done; otherwise, we incrementally increase I, solving for λ^ℓ as just described. We iterate on these two loops (the inner loop in which we solve for λ^ℓ and the outer loop in which we solve for I) until $\int \pi dF = 1$.

[19]Could equilibria exist with higher λ^ℓ? Conjecture that such an equilibrium existed. Because frictionless time-2 consumption would be lower, the financing constraint might start to bind at a lower value of the state-prices. This would confirm the lower wealth consistent with higher λ^ℓ. For simplicity, we rule out these (potential) equilibria.

B Convex technology

B.1 Individual's Problem

In this section, we take the state price density $\pi(\cdot)$ as given and solve the problem of an individual with access to a convex technology. We suppress the superscript ℓ, h. For a given time-0 endowment e, define the value function

$$\begin{aligned} V^c(e) = & \max_{\{0 \leq c_0,\ 0 \leq c_2(\cdot), 0 \leq o(\cdot) \leq 1\}} (1-\beta)\log(c_0) + \beta \int \log(c_2(\delta))\ dF(\delta) \\ \text{such that: } & c_0 + f - e \leq \int \pi(\delta)\{o(\delta)\ \delta H + [1 - o(\delta)](H_0 + f) - c_2(\delta)\}\ dF(\delta) \\ & \forall \delta: \quad c_2(\delta) \geq o(\delta)\alpha f \qquad \text{(IC')} \end{aligned}$$

Proposition 3. *Given a set of equilibrium prices, $V^c(e) \geq V(e)$.*

Proof. The optimal solution to the contracting problem in the benchmark model with nonconvex technology meets the feasibility and incentive compatibility constraint in the program with convex technology. $\square$

Define the shadow price of budget constraint by λ and the shadow price of incentive compatibility constraint (IC') after scaling by risk neutral probabilities with $\mu(\cdot) \geq 0$. Given λ and $\mu(\cdot)$, the optimal consumption plan follows

$$c_0 = \frac{(1-\beta)}{\lambda} \tag{B.1}$$

$$c_2 = \frac{\beta}{\lambda - \mu(\delta)} \frac{1}{\pi(\delta)} \tag{B.2}$$

We now jointly solve for $\mu(\cdot)$ and $o(\cdot)$. Suppose $\frac{\beta}{\lambda}\frac{1}{\pi(\delta)} \geq \alpha f$. In this case the scale of operation would be the maximal first-best level: $o = 1$ as (IC') does not bind. Also $\mu = 0$.

Now consider the case $\frac{\beta}{\lambda}\frac{1}{\pi(\delta)} < \alpha f$. The Lagrangian is linear with o. Therefore, the

FOC for occupational choice is independent of o:

$$\lambda[\delta H - H_0 - f] = \mu(\delta)\alpha f \Leftrightarrow \mu(\delta) = \lambda\gamma(\delta) \tag{B.3}$$

where as defined before $\gamma(\delta) := (\delta H - f - H_0)/\alpha f$. The optimal o follows

$$o(\delta) = \begin{cases} 0 & \text{if } \mu(\delta) > \lambda\gamma(\delta) \\ 0 \leq . \leq 1 & \text{if } \mu(\delta) = \lambda\gamma(\delta) \\ 1 & \text{if } \mu(\delta) < \lambda\gamma(\delta) \end{cases} \tag{B.4}$$

Note that $\mu > \lambda\gamma$ is impossible, as it implies $o = 0$ for which eq. (IC') doesn't bind; a contradiction to $\mu > 0$. In the intermediate case $\mu(\delta) = \lambda\gamma(\delta)$, however, the optimal scale $o(\delta)$ is pinned down from (IC') which holds with equality (since $\mu > 0$) jointly with eq (B.2). This is the case if such solution to $o(\delta)$ indeed satisfies $o(\delta) \leq 1$. In case the individual reaches to the full capacity $o = 1$, $\mu(\delta) < \lambda\gamma(\delta)$ and (IC') with $o = 1$ together with eq (B.2) solves $\mu(\delta)$. This is the case if such solution to $\mu(\delta)$ indeed satisfies $\mu(\delta) < \lambda\gamma(\delta)$. Straightforward calculation delivers the condition for which each of these cases happens.

Here is the compact form of expressing optimal consumption and occupational choice followed from different cases above.

$$\mu(\delta) = \min \left\{ \max \left\{ \lambda - \frac{\beta}{\alpha f \pi} , 0 \right\} , \lambda\gamma \right\} \tag{B.5}$$

$$o = \begin{cases} 1 & \text{if} \quad \lambda(1-\gamma) \leq \dfrac{\beta}{\alpha f \pi} \\ \dfrac{\beta}{\alpha f \pi \lambda (1-\gamma)} & \text{otherwise} \end{cases} \tag{B.6}$$

$$c_2 = \begin{cases} \dfrac{\beta}{\lambda \pi} & \text{if} \quad \lambda \leq \dfrac{\beta}{\alpha f \pi} \\ \alpha f & \text{if} \quad \dfrac{\beta}{\alpha f \pi} \leq \lambda \ , \ \lambda(1-\gamma) \leq \dfrac{\beta}{\alpha f \pi} \\ \dfrac{\beta}{\lambda(1-\gamma)\pi} & \text{if} \quad \dfrac{\beta}{\alpha f \pi} \leq \lambda(1-\gamma) \end{cases} \tag{B.7}$$

$$c_0 = \frac{(1-\beta)}{\lambda} \tag{B.8}$$

Observation 5. *If* $\gamma(\delta) \geq 1$ *Then* $o = 1$.

This result is similar to *Observation 1* in the non-convex technology case.

Observation 6. *The optimal scale of option is always positive:* $o > 0$.

As long as either e or H_0 is positive, the individual would have a finite λ and positive time-2 consumption. Then there would exist small enough, but positive scale of operation to satisfy eq. (IC'). This contradict the results of non-convex technology, for which the optimal choice might be to not invest in human capital for positive measure states of the world.

Observation 7. *There are some states of the world, for which the individual with access to non-convex technology would invest:* $o = 1$*, but the with convex technology would invest not at the full scale:* $o < 1$.

To see why, consider some $\gamma < 1$. The condition to operate at scales less than one from above is if $z\hat{\pi}$ is less than $(1 - \gamma)$. However, from our discussion of non-convex technology we know individual invests as long as $z\hat{\pi}$ is greater than $\hat{x}^c$, where $\hat{x}^c < 1-\gamma$ as is discussed before. Therefore, in this non-empty range: $\hat{x}^c < z\hat{\pi} < 1-\gamma$ the scale of operation with convex technology is less than the one with non-convex technology. Intuitively, with convex technology the individual is not forced to go all the way up to the full scale and pay a high shadow price of meeting incentive compatibility constraint.

B.2 Market Clearing

Having characterized the optimal scale of investment $o(\cdot)$ given $\pi(\cdot)$ at each state, we now characterize $\pi(\delta)$ given $o(\delta)$ using the market clearing equation. This will determine equilibrium outcomes for a given the Lagrange multiplier λ^ℓ and inventory I,

The market clearing condition for time 0 is still (MC-0). However, there is an important difference in market clearing at time 2 between the convex and non-convex cases. Instead of having a fraction o^ℓ of individuals investing at full scale, now all the ℓ-individuals run at potentially less than full scale. This is reflected in time-2 aggregate consumption.

$$\begin{aligned}\beta e^{\mathcal{I}}\hat{\pi} + m^h\beta\hat{\lambda}^h\hat{\pi} + m^\ell \max\{\beta\hat{\lambda}^\ell\hat{\pi}, o^\ell\alpha f\} = \\ I + (m^h + m^\ell o^\ell)\delta H + m^\ell(1-o^\ell)(f+H_0) \qquad \text{(MC-2)}\end{aligned}$$

We can solve for π as a function of δ, at a given o^ℓ. Using the notations we defined before: $\hat{e} := \beta(e-I-f)/[(1-\beta)\alpha f]$, $y := [I + m^h\delta H + m^\ell(f+H_0)]/(\alpha f)$ and

$z := \hat{\lambda}^\ell \beta / \alpha f$, we have

$$\hat{\pi}^* = \frac{y + m^\ell o^\ell \gamma}{\hat{e}} \tag{B.9}$$

$$\hat{\pi}^F = \frac{y + m^\ell o^\ell (\gamma - 1)}{\hat{e} - m^\ell z} \tag{B.10}$$

$$\hat{\pi} = \min\{\hat{\pi}^*, \hat{\pi}^F\} \tag{B.11}$$

Note that o^ℓ is no longer in the denominator of the "distorted" state-price density ($\hat{\pi}^F$). As a result $\hat{\pi}^F > \hat{\pi}^f$ for any given $o^\ell < 1$ (namely the distorted state price is higher in the economy with non-convex investment).

We now characterize equilibrium outcomes.

Proposition 4. *Time 2 equilibrium objects π and o as functions of δ, given I and λ^ℓ, are specified by*

$$\delta \leq \delta_C \quad : \quad \begin{cases} o^\ell = yz/[\hat{e}(1-\gamma)] \\ \hat{\pi} = y/\hat{e} \end{cases} \tag{B.12}$$

$$\delta_C \leq \delta \leq \delta_F \quad : \quad \begin{cases} o^\ell = 1 \\ \hat{\pi} = [y + m^\ell(\gamma - 1)]/(\hat{e} - m^\ell z) \end{cases} \tag{B.13}$$

$$\delta_F \leq \delta \quad : \quad \begin{cases} o^\ell = 1 \\ \hat{\pi} = (y + m^\ell \gamma)/\hat{e} \end{cases} \tag{B.14}$$

where the cutoff thresholds δ_C and δ_F solve

$$\frac{1 - \gamma(\delta_C)}{z} = \frac{y(\delta_C)}{\hat{e}} \tag{B.15}$$

$$1 = \frac{y(\delta_F) z}{\hat{e} - m^\ell \gamma(\delta_F) z} \tag{B.16}$$

Proof. Recall from the individual's problem that if $o^\ell < 1$, the ℓ-individual consumes at the constrained level $c_2 = o\alpha f$; hence, the market clearing condition delivers $\hat{\pi} = \hat{\pi}^F$. This is a key property, as we can claim that by increasing δ, first, the operation scale becomes one, and then the state price switches from constrained to unconstrained version. We can define two cut-off thresholds for δ, called δ_C and δ_F, similar to what we did with non-convex technology, such that the equilibrium at time 2 features $o^\ell < 1$ and $\hat{\pi} = \hat{\pi}^F$ if δ is small enough: $\delta < \delta_C$, $o^\ell = 1$ and $\hat{\pi} = \hat{\pi}^F$ in an intermediate range of δ: $\delta_C \leq \delta < \delta_F$, and finally $o^\ell = 1$ and $\hat{\pi} = \hat{\pi}^*$ if δ is large enough: $\delta_F \leq \delta$. □

C Non-Convex Technology, No Commitment

The optimal contract between a ℓ-individual and investors now should satisfy ex-post incentive of investors to fund an individual, in any state of the world with ex-ante planned investment in human capital. Given $\pi(\cdot)$, define the value function as a function of e by

$$
\begin{aligned}
V^m(e) = & \max_{\{0\leq c_0,\ 0\leq c_2(\cdot),\ o(\cdot)\in\{0,1\}\}} (1-\beta)\log(c_0) + \beta \int \log(c_2(\delta))\ dF(\delta) \\
\text{such that: } & c_0 + f - e \leq \int \pi(\delta)\{o(\delta)\ \delta H + [1-o(\delta)](H_0+f) - c_2(\delta)\}\ dF(\delta) && \text{(BC)} \\
& \forall\delta \ \mid\ o(\delta) = 1: \quad c_2(\delta) \geq \alpha f && \text{(IC-1)} \\
& \forall\delta \ \mid\ o(\delta) = 1: \quad \delta H - c_2(\delta) \geq f && \text{(IC-2)}
\end{aligned}
$$

The left hand side of (IC-2) shows the gross return to investors which is project's return minus consumption of the individual, and the left hand side shows the investment cost.

First, we take the shadow price of budget constraint λ and occupational choice $o(\cdot)$ as given and solve for the optimal consumption. Note that $o(\delta) = 1$ requires

$\delta H - f \geq \alpha f$ in order to have a nonempty feasible set for $c_2(\delta)$.

$$c_0 = \tfrac{(1-\beta)}{\lambda}$$

$$c_2(\delta) = \begin{cases} c_2^\star(\delta) := \frac{\beta}{\lambda}\frac{1}{\pi(\delta)} & \text{if} \quad o(\delta) = 0 \\ \min\{\ \max\{\ c_2^\star(\delta)\ ,\ \alpha f\}\ ,\ \delta H - f\} & \text{if} \quad o(\delta) = 1 \end{cases}$$

Now we take one step back to solve for the optimal $o(\delta)$, and the market clearing price $\pi(\delta)$. Two cases should be considered separately.

Case 1: $\delta H - f < \alpha f$**.**

We have $o^\ell(\delta) = 0$ for sure as the feasible set for $c_2^\ell(\delta)$ given $o^\ell = 1$ is empty. Total output in this state of the world is $I + m^h \delta H + m^\ell(f + H_0)$ and just obtained in the benchmark economy with commitment and $\delta \leq \delta_0$ the state price is derived by $\pi = \frac{(1-\beta)[I + m^h \delta H + m^\ell(f + H_0)]}{\beta(e - I - f)}$.

Case 2: $\delta H - f \geq \alpha f$**.**

In this case, $o^\ell = 1$ could be the case; however, by switching from $o^\ell = 0$ to $o^\ell = 1$ the constrained consumption $c_2(\delta)$ may be distorted from the frictionless choice $c_2^\star(\delta)$. We need to consider three possible outcomes separately.

– ***Case 2.a.*** $\alpha f \leq c_2^\star(\delta) \leq \delta H - f$**.** The optimal occupation is $o^\ell = 1$, as $c_2 = c_2^\star$ is not distorted by switching from $o^\ell = 0$ to $o^\ell = 1$; meanwhile, $o^\ell = 1$ expands the budget constraint; hence, it is preferred.

– ***Case 2.b.*** $c_2^\star(\delta) < \alpha f$**.** In this case $c_2 = c_2^\star = \beta/(\lambda\pi)$ if $o^\ell = 0$ and $c_2 = \alpha f > \beta/(\lambda\pi)$ if $o^\ell = 1$. Similar to the benchmark environment with commitment, $o^\ell = 1$ if and only if $u \geq 0$, where

$$u(x) = \log x + (\gamma - 1)x + 1$$

Recall that $x := \alpha f \lambda \pi(\delta)/\beta$ and $\gamma(\delta) := (\delta H - f - H_0)/\alpha f$. This results in the same condition $o^\ell = 1$ if and only if $x \leq x_c$ where x_c as in the benchmark case is the solution to $u(x) = 0$.

– ***Case 2.c.*** $c_2^\star(\delta) > \delta H - f$**.** In this case $c_2|_{o=1} = \delta H - f < c_2^\star$. Therefore, if the individual decides to invest: $o^\ell = 1$ then (IC-1) would not bind. However, investing is not necessary the optimal choice. If the individual invests, she will expand her budget, but at the same time she is forced to consume less $(\delta H - f)$ than the amount she would be allowed to consume at $o^\ell = 0$ (i.e. $c_2^\star$).

Define $\eta := (\delta H - f)/\alpha f$. We can simplify trade-offs to conclude that the optimal occupation follows $o^\ell = 1$ if and only if $v \geq 0$, where

$$v(x) := \log x - (\eta - \gamma)x + 1 + \log \eta$$

Proposition 5. *There exits a unique $x_i < \eta^{-1}$ as the solution to $v = 0$; we have $v > 0$ for any $x > x_i$ and $v < 0$ for any $x < x_i$. We know x_i is decreasing with γ and η. The optimal occupational choice follows $o^\ell = 1$ if and only if $x_i \leq x$.* □

Proof. As by assumption $\alpha f \leq \delta H - f < \beta/(\lambda\pi)$, we have $1 \leq \eta < x^{-1}$; therefore, $x < \eta^{-1}$. Also, as $H_0 \geq 0$, we have $\eta \geq \gamma$. In the limit $x \to \eta^{-1}$, v converges to a positive number, which is $v \to \gamma/\eta$. In the limit $x \to 0$, v converges to minus infinity. v takes its maximum at $x_m^{-1} = \eta - \gamma$ which is outside of the permissible range for x (recall that $\eta < x^{-1}$); for any x below x_m, v is strictly increasing with x. Therefore, there is a unique solution to $v = 0$. □

Summary

We can summarize the result of all cases specified above as follows in order to solve for the optimal $o^\ell(\delta)$ for a given $\pi(\delta)$.

If $\delta H - f < \alpha f$, i.e. $\eta < 1$, then $o^\ell = 0$, for any π. The market clearing condition solves π as in the case $\delta < \delta_c$ in the benchmark case with commitment.

Now consider the interesting cases in which $\delta H - f \geq \alpha f$. As before, we define $z := \frac{\beta}{\alpha f \lambda^\ell}$, $x := \pi/z$, $\hat{\pi} := \pi^{-1}$, $\hat{x}^c := x_c^{-1}$ and $\hat{x}^i := x_i^{-1}$. Note that $\hat{x}^c < 1 - \gamma < \eta < \hat{x}^i$ based on the proposition above and the arguments of the benchmark section.

The blue curves show market clearing relationship between $\hat{\pi} = \min\{\hat{\pi}^f, \hat{\pi}^*\}$ and o^ℓ in the two cases *2.a* and *2.b* analyzed above as it is derived in the benchmark section. Arrows show the shift in curves and cut-offs as a result of an increase in δ.

The right vertical section of $o^\ell(\cdot)$ is the impact of no-commitment friction. As δ increases the time-2 equilibrium shifts from $o^\ell = 0$ toward $o^\ell = 1$ since the blue curve $\hat{\pi}^f$ starts to cross the left vertical part of o^ℓ. Once δ passes δ_c (see definition in previous section) the blue curve $\hat{\pi}^f$ crosses the left-top corner of the red curve $o^\ell(\cdot)$ and the economy sets at $o^\ell = 1$; further increase in δ increases $\hat{\pi}$ just as in the benchmark economy. However, since both the blue curves $\hat{\pi}^f$ and $\hat{\pi}^*$ are increasing with δ the equilibrium may switch back to $o^\ell < 1$ as the blue curves might cross the right vertical part of $o^\ell(\cdot)$. I show below that this never happens; because, the right vertical part of $o^\ell(\cdot)$ also shift to the right by increasing δ and it shifts at a faster pace. See the technical details at the end of this section. We have established the following proposition.

Proposition 6. *The time-2 equilibrium of the economy $(\pi(\delta), o^\ell(\delta))$ with no commitment on the investors side is the same as the benchmark economy with commitment, as long as $\alpha f \leq \delta H - f$. Otherwise, the equilibrium simply features $o^\ell = 0$ and $\hat{\pi} = \frac{y}{\hat{e}}$.*

Having characterized time-2 equilibrium outcomes: $o^\ell(\cdot), \pi(\cdot)$ at each δ, we turn to numerically solve for λ^ℓ and I. We follow the same algorithm and equilibrium selection approach as in the benchmark economy with commitment.

We solve for this economy and report results in Table C.1 and Figure A.3). The overall economy is much poorer and less stable in this case, but the outside investors take less risk (reflected in a lower volatility of dividends). As a result, the equity premium is slightly lower in this case. This is not to say that investors are insulated from the negative affects of inability to enforce financial contracts. They still bear part of the cost of lower aggregate consumption, in the form of lower dividends. Thus a decline in financial sophistication would, through a cash flow channel, be expected to lead to a decline in stock prices.

Table 1: Model Summary

$t = 0$	$t = 1$	$t = 2$
- Individuals trade Arrow securities with investors. - investors may put resources into inventory.	- State of the world is realized. - Individuals choose an occupation. - Investment costs are financed.	- Individuals choose to work or shirk. - Returns are realized and security payments are settled. - Everyone consumes her wealth and dies.

Table 2: Parameter values

Panel A: Technology, Preferences	
Time-discount rate β	0.95
Return on investment in the traditional sector H_0	0
Fixed cost of investment in the non-traditional sector f	1
Average return on investment in the non-traditional sector H	1.965
Average TFP, $E(\delta)$	1
Volatility of the log of TFP, $\sigma(\log(\delta))$	0.1
Panel B: Population, Endowments	
Mass of ℓ-individuals m^ℓ	0.5
Mass of h-individuals m^h	0.5
Mass of outside investors $m^{\mathcal{I}}$	1
Endowment of ℓ-individuals e^ℓ	0.049
Endowment of h-individuals e^h	1.049
Endowment of outside investors $e^{\mathcal{I}}$	0.549
Panel C: Financing friction	
Private benefit (as fraction of f), α	1

Notes: The table reports parameter values for the benchmark calibration. We set the initial endowments to $e^h = f + \epsilon$, $e^\ell = \epsilon$ and $e^{\mathcal{I}} = \epsilon + \frac{m^\ell}{m^{\mathcal{I}}} f$, and calibrate the single free parameter ϵ so that the risk-free rate in the frictionless economy equals β^{-1}. This implies $\epsilon = .049$. In effect we assume that the high cash-on-hand individuals are able to pay the investment cost (f) out-of-pocket, whereas low cash-on-hand individuals are not. We calibrate H so that economic disasters (periods when low-cash-on-hand individuals are not fully employed) occur 5% of the time.

Table 3: Asset prices and cash flow volatility

Economic Environment	Non-Convex	Convex	Frictionless
Panel A: Benchmark Specification			
Risk free rate (%)	0.00	0.00	5.26
Equity premium, Dividend claim (%)	2.17	2.02	1.06
Equity premium, Aggregate consumption claim (%)	1.32	1.35	1.06
$\sigma(R)$, Dividend claim	0.19	0.18	0.11
$\sigma(R)$, Aggregate consumption claim	0.12	0.12	0.11
$\sigma(\log D)$, Dividend	0.18	0.17	0.10
$\sigma(\log C)$, Aggregate consumption	0.12	0.12	0.10
$\sigma(\log C)$, ℓ-individual consumption	0.44	0.18	0.10
Panel B: High H			
Risk free rate (%)	0.00	0.00	5.26
Equity premium, Dividend claim (%)	2.72	2.61	1.06
Equity premium, Aggregate consumption claim (%)	1.33	1.33	1.06
$\sigma(R)$, Dividend claim	0.21	0.20	0.11
$\sigma(R)$, Aggregate consumption claim	0.10	0.10	0.11
$\sigma(\log D)$, Dividend	0.20	0.20	0.10
$\sigma(\log C)$, Aggregate consumption	0.10	0.10	0.10
$\sigma(\log C)$, ℓ-individual consumption	0.00	0.03	0.10
Panel C: High H and m^ℓ			
Risk free rate (%)	0.00	0.00	5.26
Equity premium, Dividend claim (%)	3.91	3.51	1.06
Equity premium, Aggregate consumption claim (%)	1.94	1.93	1.06
$\sigma(R)$, Dividend claim	0.21	0.20	0.11
$\sigma(R)$, Aggregate consumption claim	0.10	0.11	0.11
$\sigma(\log D)$, Dividend	0.20	0.19	0.10
$\sigma(\log C)$, Aggregate consumption	0.10	0.11	0.10
$\sigma(\log C)$, ℓ-individual consumption	0.07	0.07	0.10

Notes: We consider three types of economies (non-convex technology, convex technology, frictionless), and three parameterizations. In the non-convex economy, individuals' occupational choice is in the set $\{0,1\}$, as Section 2 describes. Appendix B describes the economy with the convex technology. Section 3 describes the economy without frictions. Table 2 describes the parameters for the benchmark case. For the case with a higher payout of the technology (high H), $H = 2$. For the case with a greater percentage of low cash-on-hand individuals high m^ℓ, $m^\ell = 1$ and $m^h = 0$. In each panel, endowments are adjusted so that the riskfree rate in the frictionless case equals β^{-1} (see Table 2). The dividend claim equals the payout to outside investors of investing in the low cash-on-hand individuals.

Table 4: Wealth across agents

Economic Environment	Non-Convex	Convex	Frictionless
Panel A: Marginal Utility of Wealth			
λ^{ℓ}	7.37	3.96	1.12
λ^{h}	0.50	0.50	0.53
$\lambda^{\mathcal{I}}$	1.82	1.82	1.82
Panel B: Wealth			
ℓ-individuals	0.96	0.95	0.90
h-individuals	1.99	1.99	1.90
Investors	0.55	0.55	0.55
Aggregate	3.50	3.49	3.34
Panel C: Value of Human Capital			
ℓ-individuals	0.91	0.90	0.85
h-individuals	0.94	0.94	0.85
Investors	0.00	0.00	0.00
Aggregate	1.85	1.84	1.70
Panel D: Utility, Consumption Equivalent Value			
ℓ-individuals	0.71	0.75	0.78
h-individuals	1.64	1.64	1.64
Investors	0.45	0.45	0.47

Notes: We report the Lagrange multipliers λ, total wealth (both cash-on-hand and human capital), human capital wealth, and the consumption equivalent of indirect utility (the consumption level for which the log equals indirect utility V). Values are reported for the three types of agents: ℓ and h-individuals, and investors ($\mathcal{I}$). We report wealth in units of time-0 consumption in each economy. Wealth includes time-0 consumption. Non-convex refers to the benchmark case of a non-convex choice set for human capital investment. Convex refers to the case of the convex choice set. Both cases imply financial frictions. The last column reports values for the frictionless case.

Table 5: Welfare implications of risk and frictions

	Non-convex technology		Frictionless economy	
	With risk	Without risk	With risk	Without risk
Panel A: Marginal Utility of Wealth				
λ^{ℓ}	7.37	3.67	1.12	1.12
λ^{h}	0.50	0.50	0.53	0.53
$\lambda^{\mathcal{I}}$	1.82	1.82	1.82	1.82
Panel B: Wealth				
ℓ-individuals	0.96	1.01	0.90	0.90
h-individuals	1.99	2.01	1.90	1.90
Investors	0.55	0.55	0.55	0.55
Aggregate	3.50	3.58	3.34	3.34
Panel C: Value of Human Capital				
ℓ-individuals	0.91	0.96	0.85	0.85
h-individuals	0.94	0.96	0.85	0.85
Investors	0.00	0.00	0.00	0.00
Aggregate	1.85	1.93	1.70	1.70
Panel D: Utility, Consumption Equivalent Value				
ℓ-individuals	0.71	0.81	0.78	0.78
h-individuals	1.64	1.65	1.64	1.65
Investors	0.45	0.45	0.47	0.48

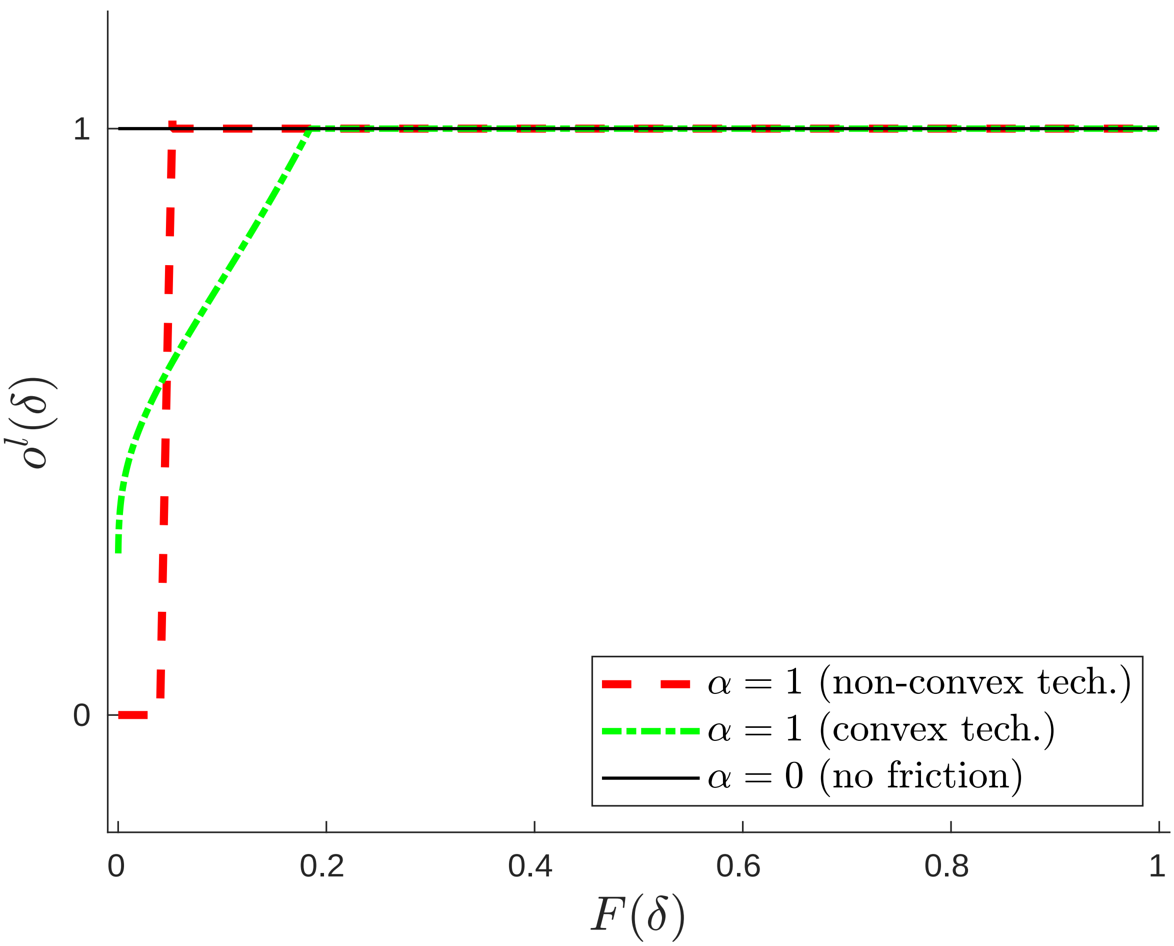
Notes: We report economic outcomes as in Table 4 for the economy with a nonconvex technology in absence of risk: $\sigma(\log(\delta)) = 0$ (second and fourth columns) and compare it to the benchmark case with risk: $\sigma(\log(\delta)) = 0.1$ (first and third columns). The third and fourth columns assume no frictions ($\alpha = 1$). The first column corresponds to the benchmark case (and column 1 in Table 4).

Table C.1: Asset prices and cash flow volatility: The role of commitment

Economic Environment	Non-Convex	Convex	No Commit.	Frictionless
Risk free rate (%)	0.00	0.00	0.00	5.26
Equity premium, Dividend claim (%)	2.17	2.02	1.79	1.06
Equity premium, Aggregate consumption claim (%)	1.32	1.35	2.34	1.06
$\sigma(R)$, Dividend claim	0.19	0.18	0.17	0.11
$\sigma(R)$, Aggregate consumption claim	0.12	0.12	0.22	0.11
$\sigma(\log D)$, Dividend	0.18	0.17	0.16	0.10
$\sigma(\log C)$, Aggregate consumption	0.12	0.12	0.21	0.10
$\sigma(\log C)$, ℓ-individual consumption	0.44	0.18	0.95	0.10

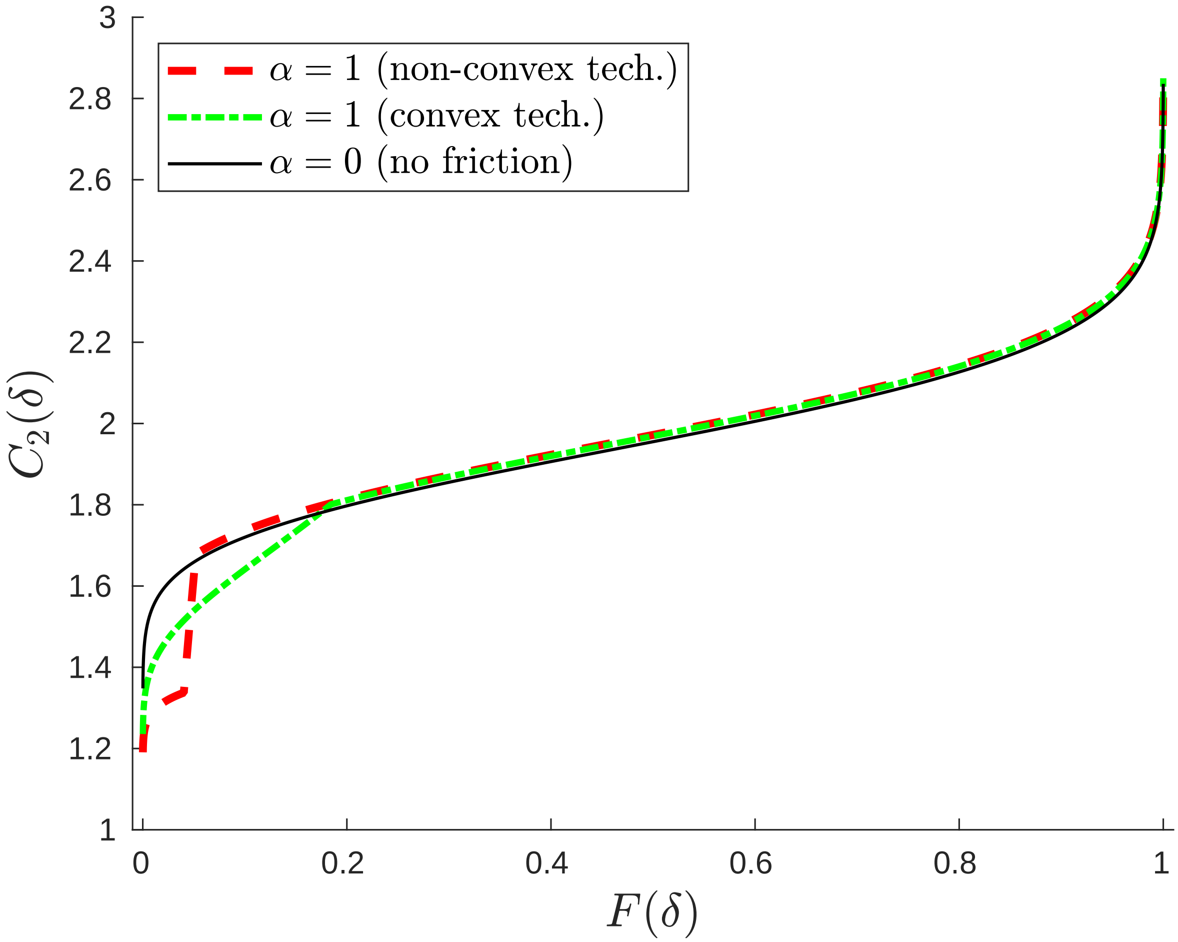
Notes: We compare the economy with no-commitment, which we describe in Appendix C, with the three other types of economies, under the benchmark calibration. See Table 3 for further detail. We report the riskfree rate, the average return on various risky assets less the riskfree rate (Equity premium), and the standard deviation of various quantities and log quantities.

Figure 1: Occupational choice for the low-cash-on hand agent



Notes: The figure shows the occupational choice for the constrained agent as a function of TFP δ, where the x-axis reports the value of the cumulative distribution function CDF(δ). For example, an x-value of 0.4 represents δ at the bottom 40% of the distribution. $o^{\ell}(\delta)$ is the occupational choice for the low cash-on-hand individual. The solid line represents the frictionless case, the dashed line the case with the non-convex technology (with frictions), and the dashed-dotted line the case with the convex technology.

Figure 2: Aggregate consumption

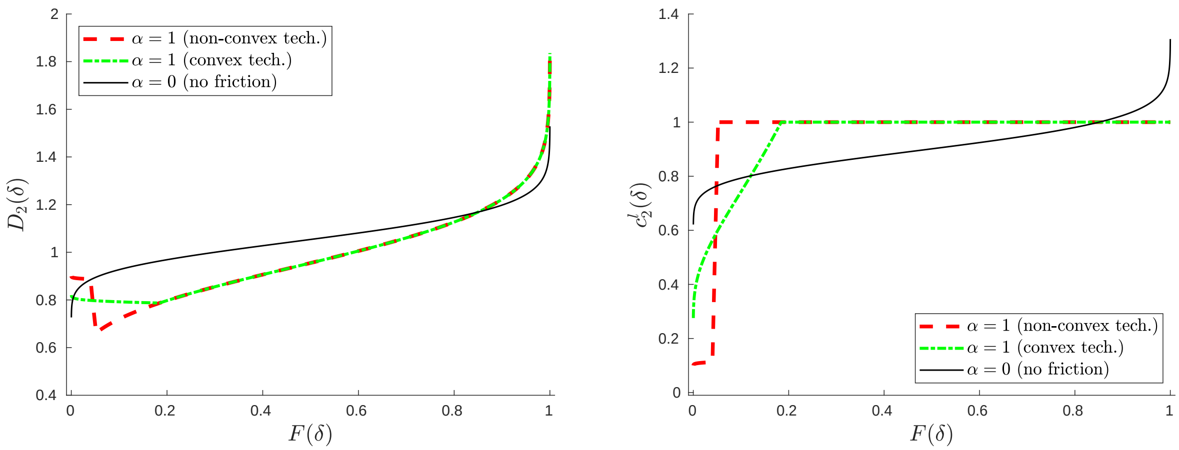


Notes: The figure shows aggregate consumption in the final period as a function of TFP δ, where the x-axis shows CDF(δ). For example, an x-value of 0.4 represents δ at the bottom 40% of the distribution. $C_2(\delta)$ is aggregate consumption in the second period. The solid line represents the frictionless case, the dashed line the case with the non-convex technology (with frictions), and the dashed-dotted line the case with the convex technology.

Figure 3: Payouts to investors and to individuals

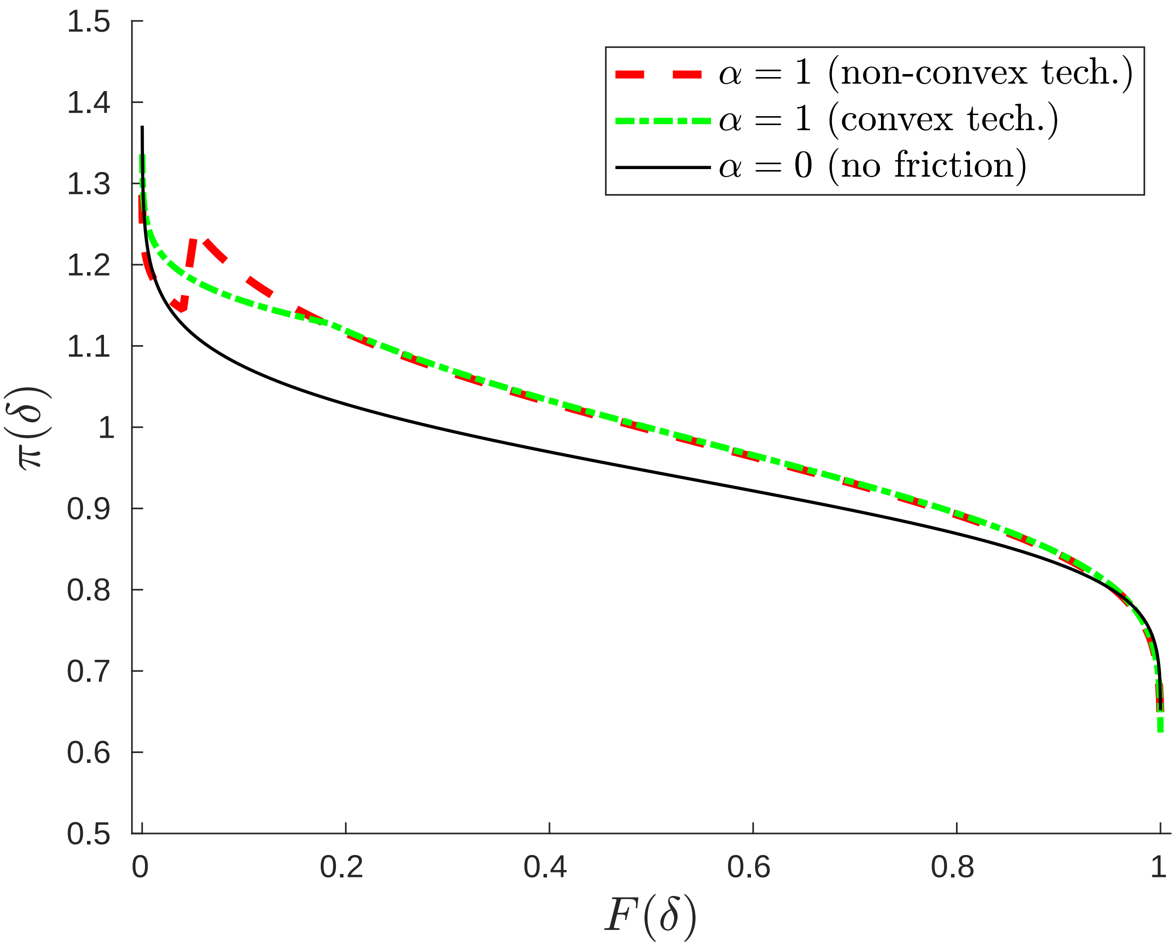
Panel A: Dividend to investors

Panel B: Consumption of ℓ-individuals



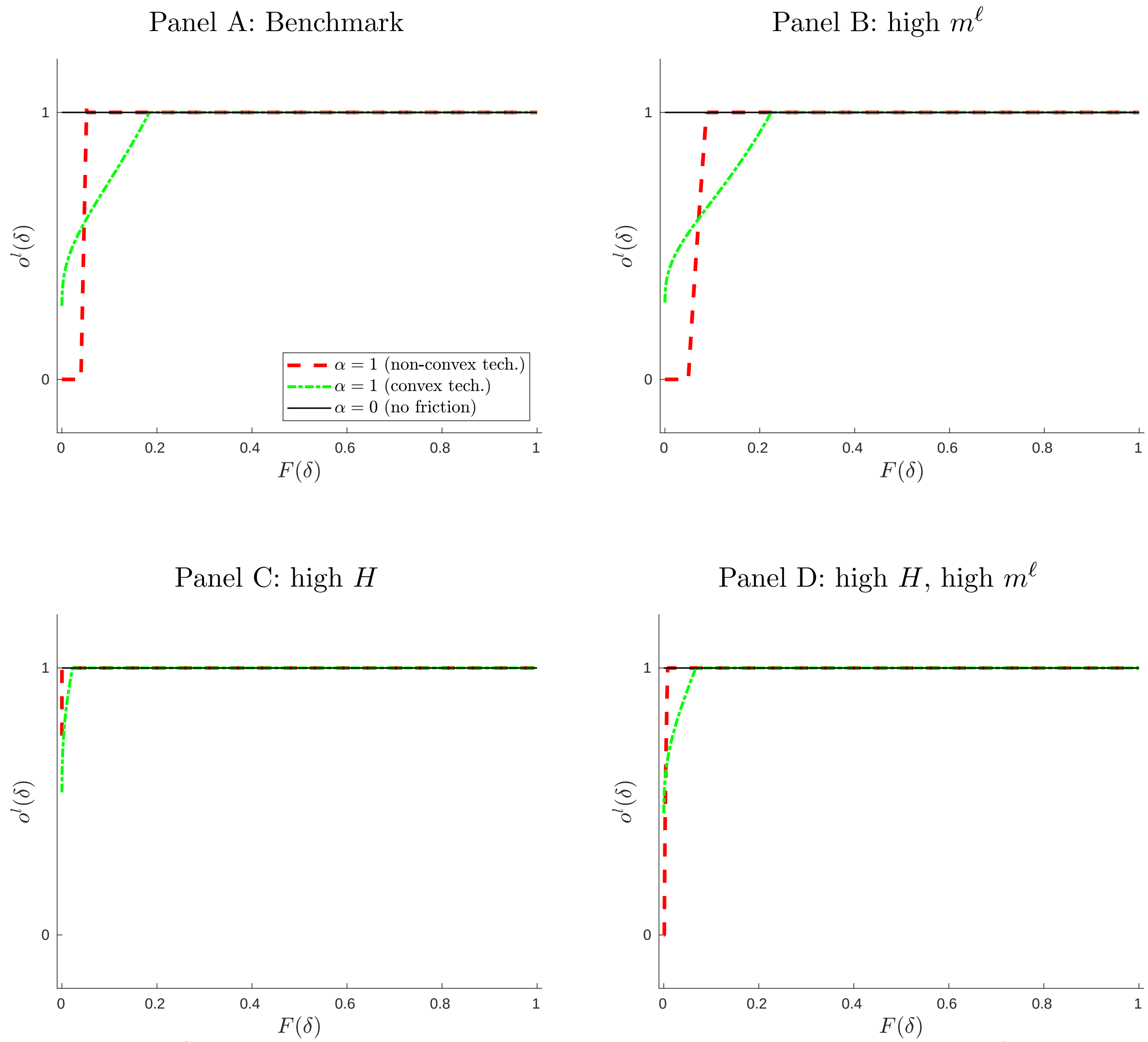
Notes: The figure shows the dividend payout to outside investors (Panel A) and the per-capita consumption of the low cash-on-hand individual (Panel B) as a function of TFP δ, where the x-axis shows CDF(δ). For example, an x-value of 0.4 represents δ at the bottom 40% of the distribution. The solid line represents the frictionless case, the dashed line the case with the non-convex technology (with frictions), and the dashed-dotted line the case with the convex technology.

Figure 4: State-price density



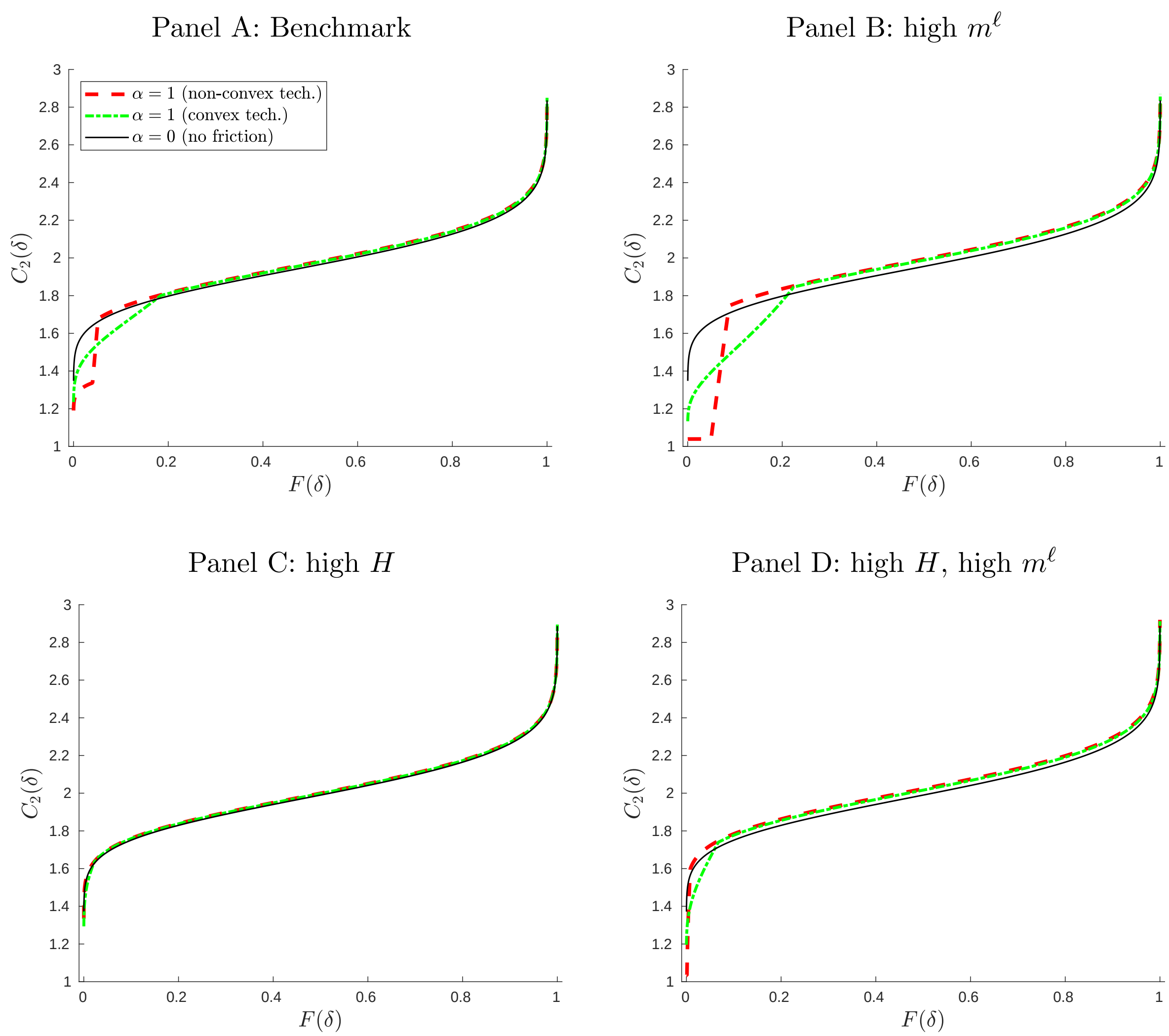
Notes: The figure shows the state-price density as a function of TFP δ, where the x-axis shows CDF(δ). For example, an x-value of 0.4 represents δ at the bottom 40% of the distribution. $\pi(\delta)$ is the state-price density. The solid line represents the frictionless case, the dashed line the case with the non-convex technology (with frictions), and the dashed-dotted line the case with the convex technology.

Figure 5: Comparative statics for occupational choice



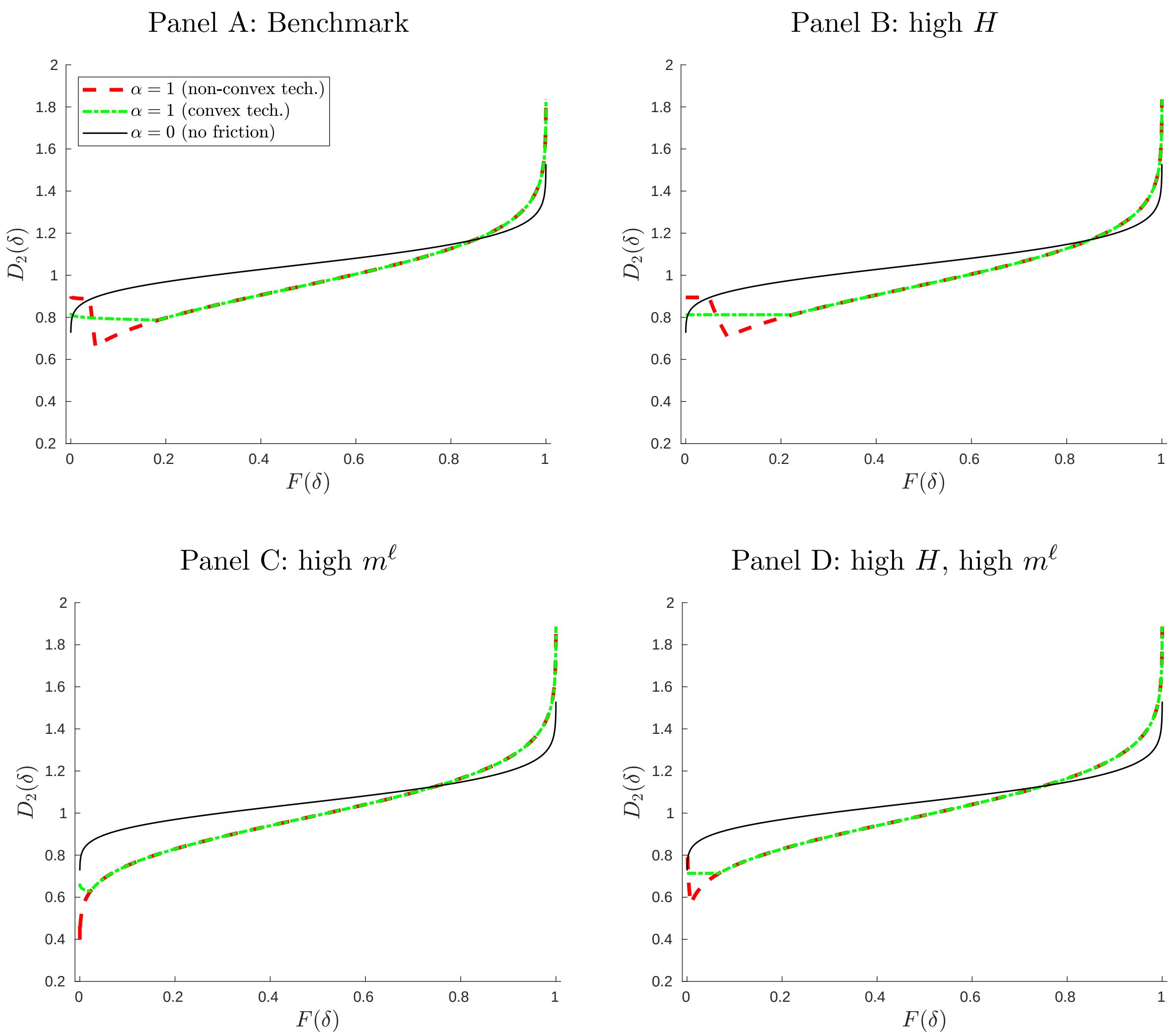
Notes: The figure shows comparative statics on the occupational choice for the low cash-on-hand individual. See Figure 1 and Table 3 for more detail.

Figure 6: Comparative statics for second-period aggregate consumption



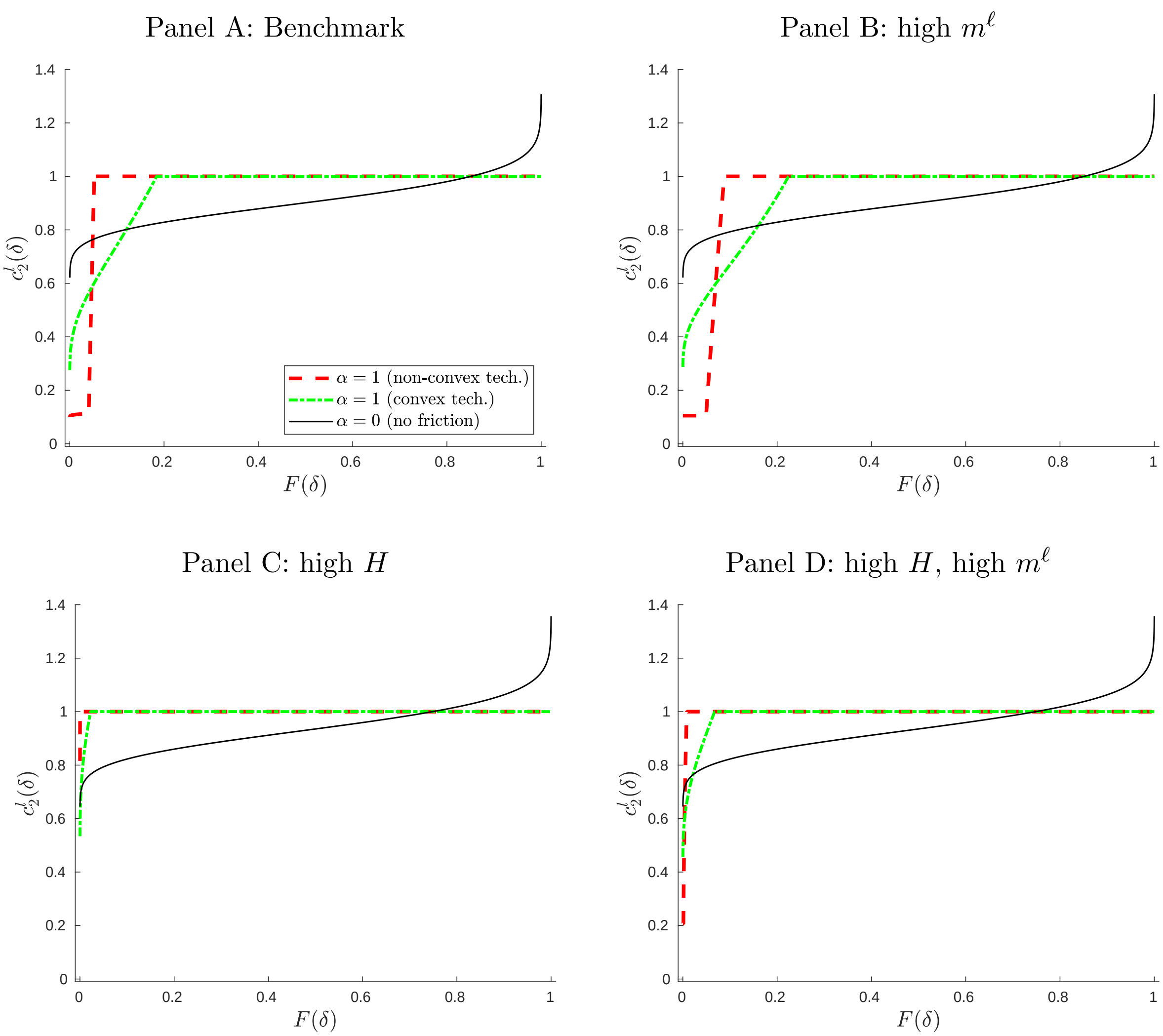
Notes: The figure shows comparative statics on the state-price density See Figure 2 and Table 3 for more detail.

Figure 7: Comparative statics for dividends



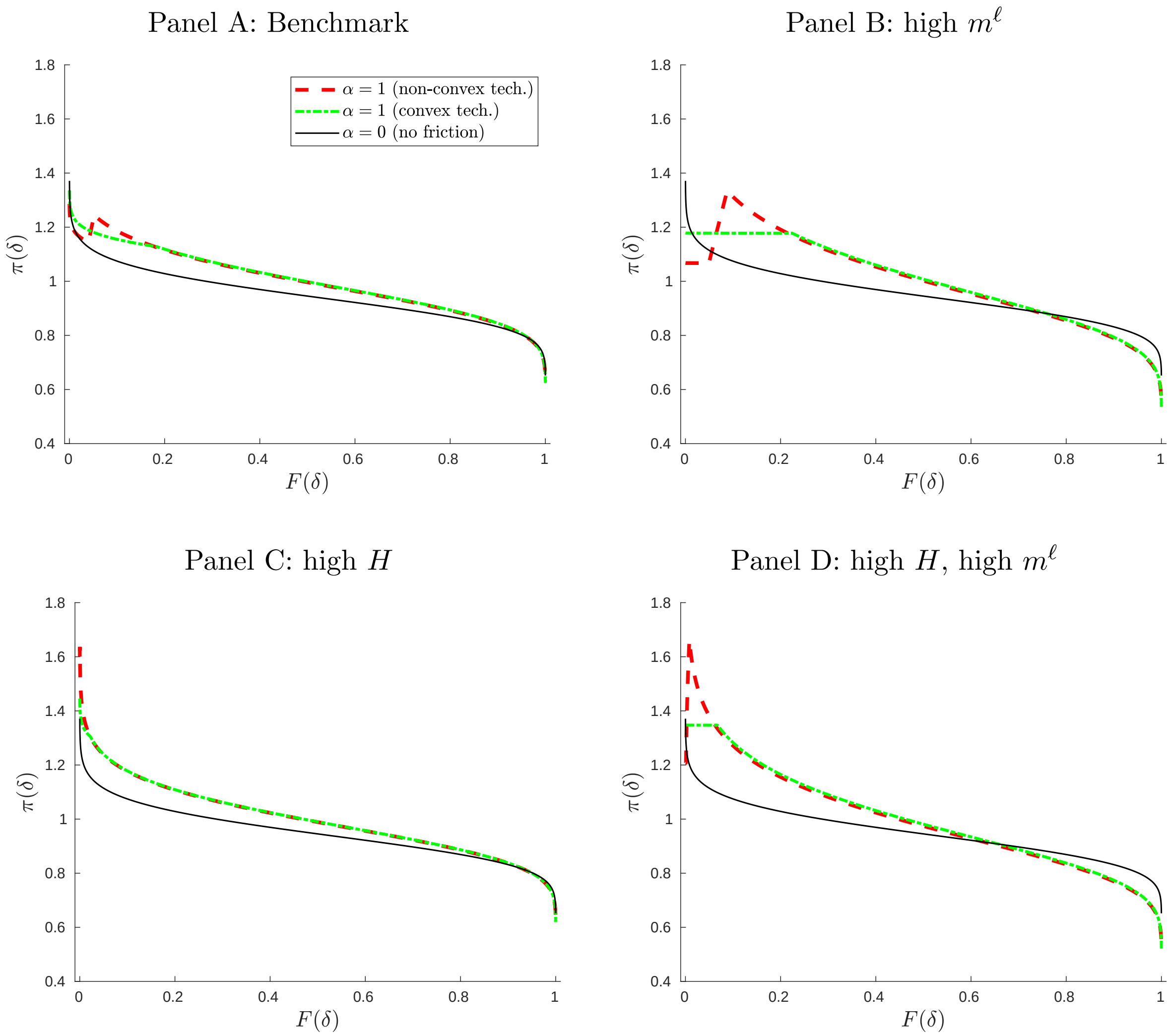
Notes: The figure shows comparative statics on the dividend to investors. See Figure 3 and Table 3 for more detail.

Figure 8: Comparative statics for consumption of the low-cash-on-hand agent



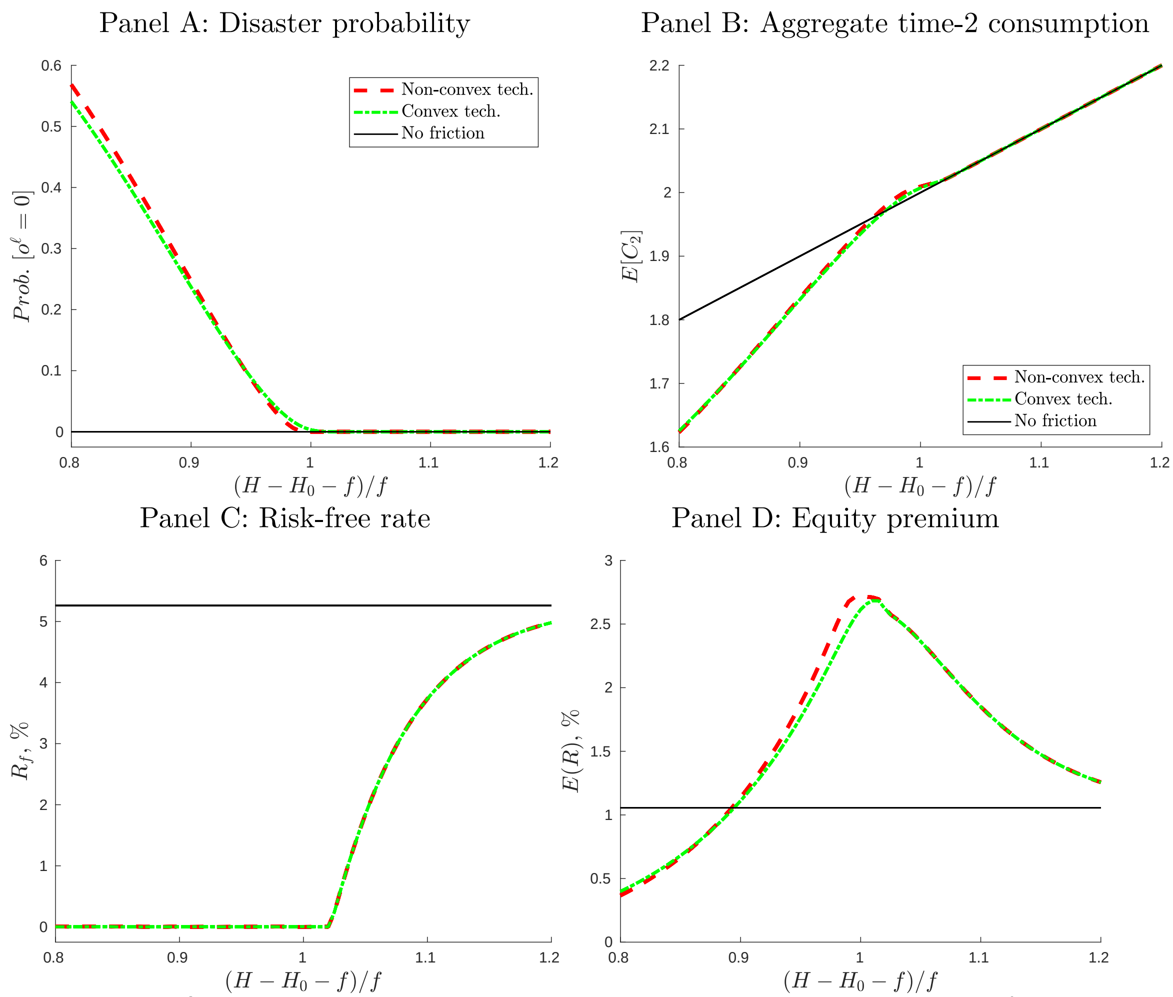
Notes: The figure shows comparative statics for consumption of the ℓ-individual in the second period. See Figure 3 and Table 3 for more detail.

Figure 9: Comparative statics for the state-price density



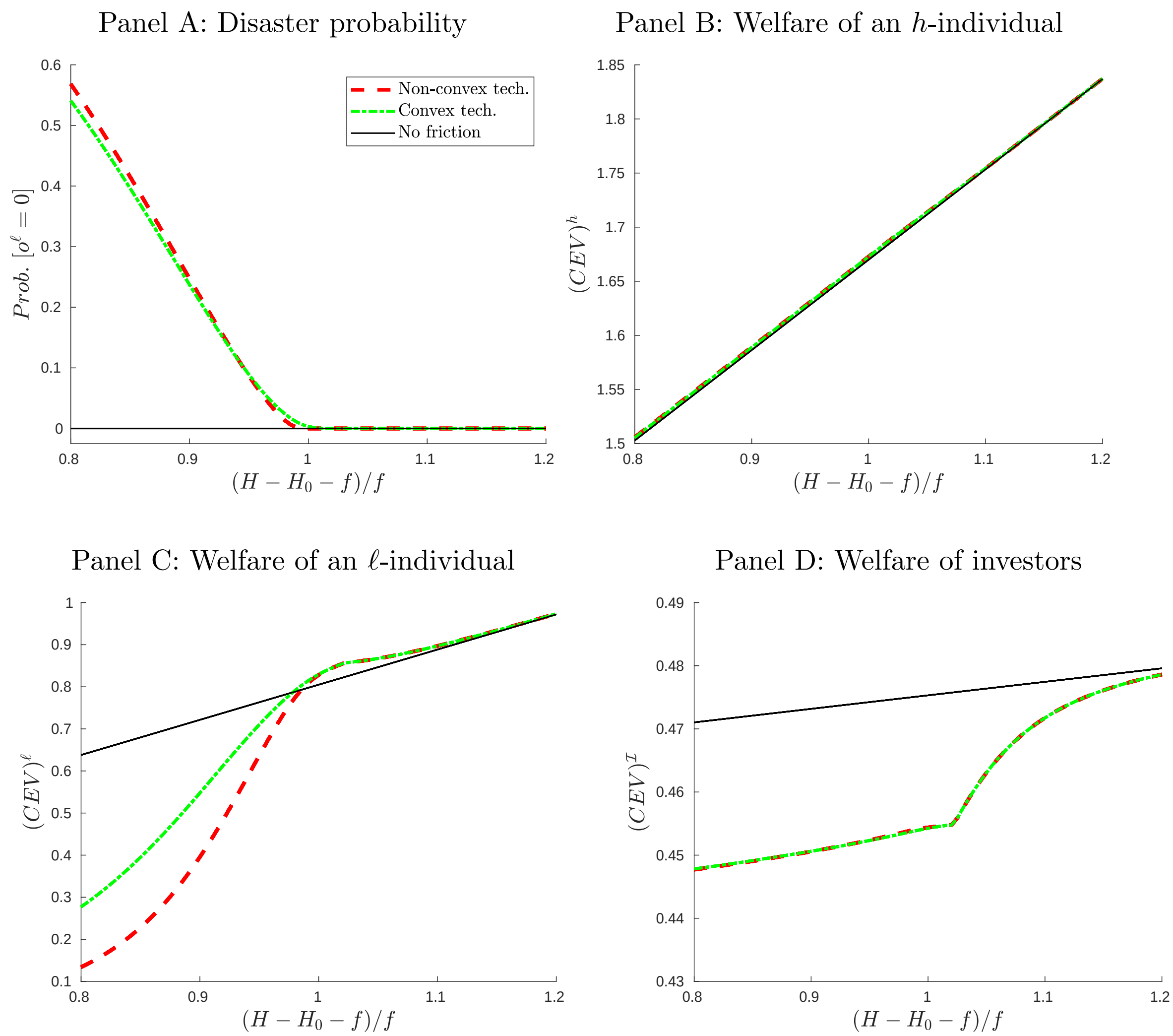
Notes: The figure shows comparative statics on the state-price density See Figure 4 and Table 3 for more detail.

Figure 10: Comparative statics for cash flows and asset prices over a wide range of H



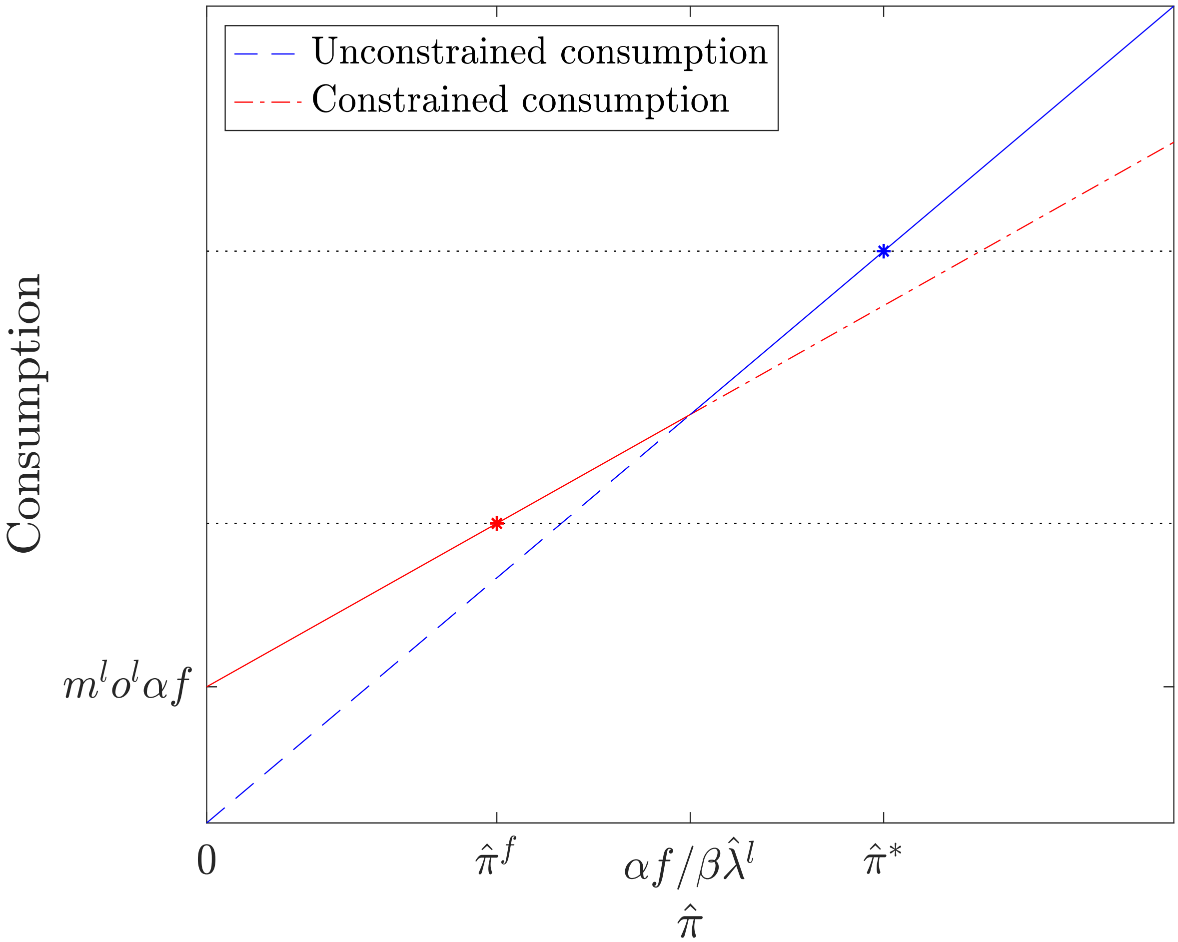
Notes: The figure reports comparative statics on asset prices resulting from varying the productivity H. As H varies, we change the endowment parameter ϵ so that the risk-free rate in the frictionless economy equals β^{-1}. All other parameters are as in the benchmark specification (Table 2). The equity premium and return volatility are calculated based on the dividend claim.

Figure 11: Comparative statics for welfare over a wide range of H



Notes: The figure reports comparative statics on consumption and on welfare resulting from varying the productivity H. As H varies, we recalibrate the endowment parameter ϵ so that the risk-free rate in the frictionless economy equals β^{-1}. All other parameters are as in the benchmark specification (Table 2). Welfare is measured in consumption terms (see Panel D of Table 4).

Figure A.1: Market clearing at time 2 (non-convex economy)



Notes: The figure illustrates market clearing. Along the x-axis is $\hat{\pi}$, the inverse of the state-price density. The solid line illustrates the left-hand side of the market clearing condition:

$$\beta[e^{\mathcal{I}}\hat{\pi} + m^h\hat{\lambda}^h\hat{\pi} + m^\ell(1-o^\ell)\hat{\lambda}^\ell\hat{\pi}] + m^\ell o^\ell \max\{\beta\hat{\lambda}^\ell\hat{\pi}, \alpha f\} = I + (m^h + m^\ell o^\ell)\delta H + m^\ell(1-o^\ell)(f + H_0).$$

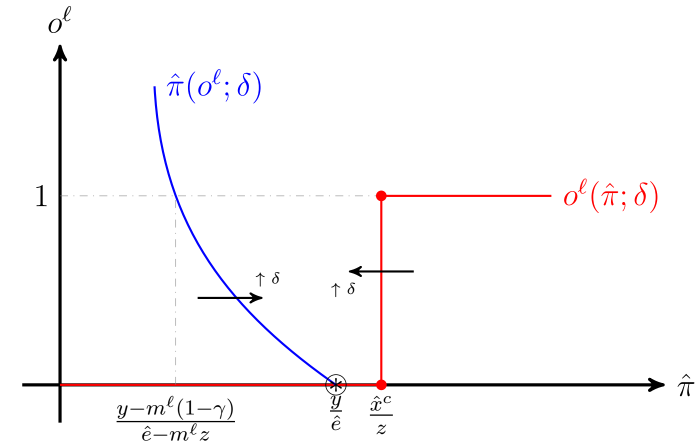
(where $o^\ell \in [0,1]$ is the fraction of ℓ-individuals investing in human capital at time 1) as a function of $\hat{\pi}$. The dashed and dashed-dotted lines represent the left hand side when the maximization operator is replaced by its first argument and its second argument respectively. The dotted lines represent the right-hand side, namely output (determined by TFP δ), for two potential values of δ. The figure illustrates that

$$\hat{\pi} = \min\{\hat{\pi}^f, \hat{\pi}^*\},$$

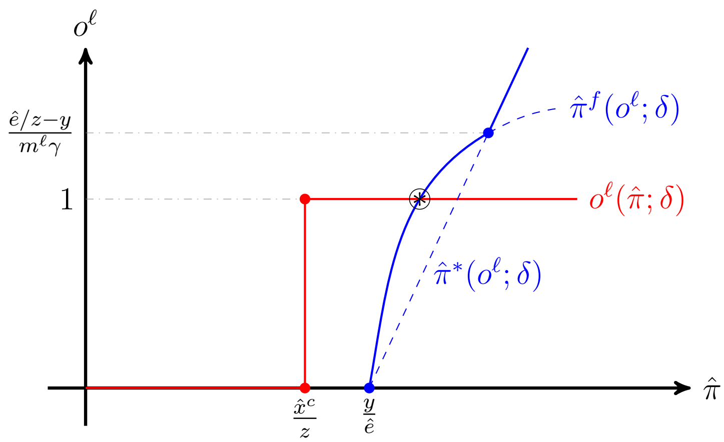
where $\hat{\pi}^f$ is the solution when $\alpha f < \beta\hat{\lambda}^\ell\hat{\pi}$ and $\hat{\pi}^*$ is the solution when the opposite is true.

Figure A.2: Equilibrium in the non-convex economy

Panel A: Equilibrium when no ℓ-individuals invest



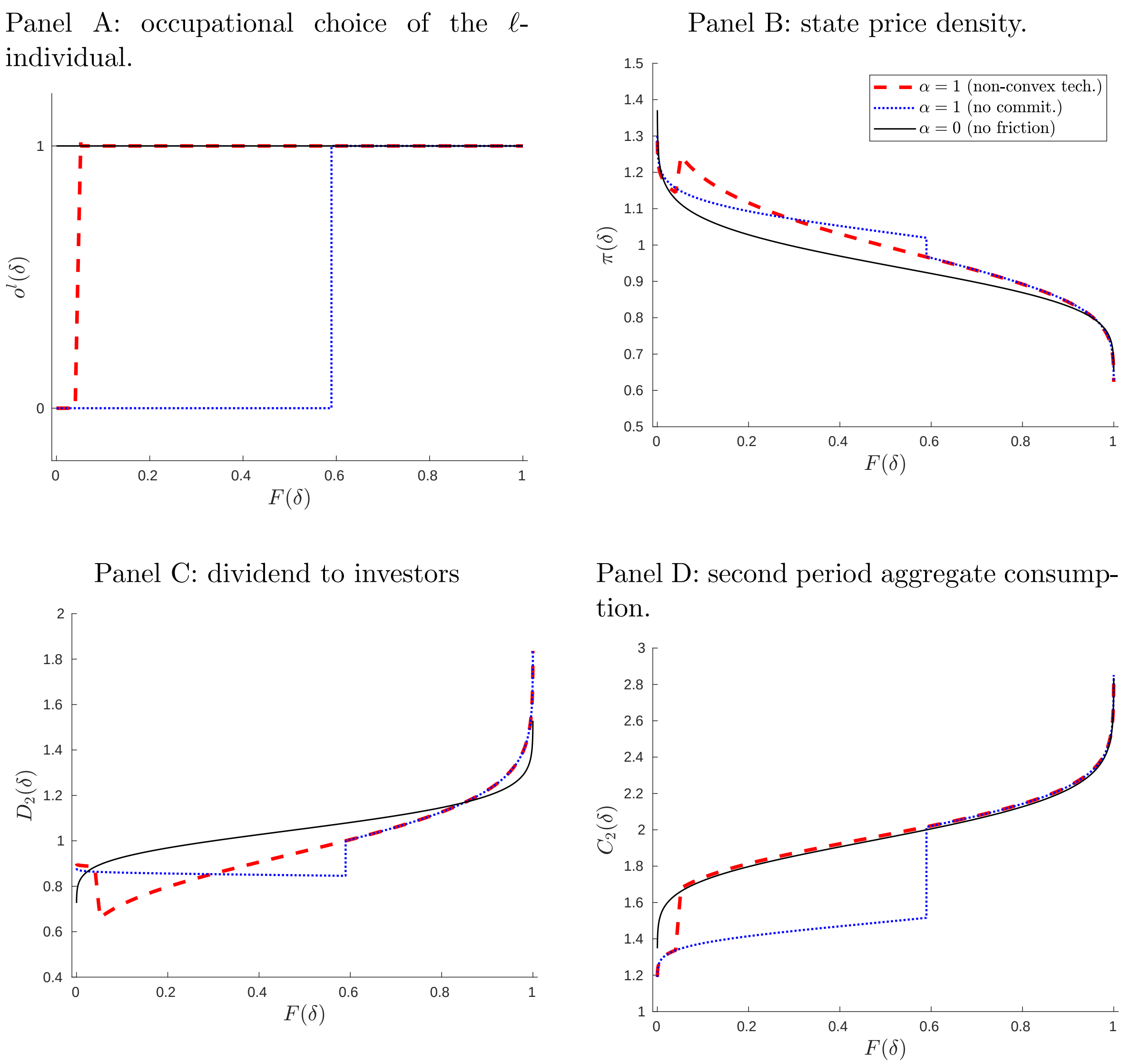
Panel B: Equilibrium when ℓ-individuals all invest



Notes: The figure illustrates the equilibrium in the economy with non-convex investment. The blue curve shows the inverse state-price density $\hat{\pi}$ as a function of o^ℓ and δ; the red curve is the fraction of ℓ-type individuals investing, o^ℓ, as a function of $\hat{\pi}$ and δ. Equilibrium, shown as $\circledast$, is the point at which the curves intersect.

Panel A illustrates the the low δ case (Case 1 in Appendix A.2), at which $o^\ell = 0$ represents an equilibrium. Panel B illustrates the high δ case (Case 2) in which $o^\ell = 1$ represents an equilibrium.

Figure A.3: Real outcomes and state price density; the role of commitment



Notes: The figures shows asset prices and real outcomes in the case of no commitment. These are shows a function of TFP δ, where the x-axis reports the value of the cumulative distribution function CDF(δ). For example, an x-value of 0.4 represents δ at the bottom 40% of the distribution. $o^{\ell}(\delta)$ is the occupational choice of the low cash-on-hand individual, $\pi(\delta)$ is the state-price density, $C_2(\delta)$ is final-period aggregate consumption, and $D_2(\delta)$ is the dividend (output of low cash-on-hand investors, minus consumption).