

NBER WORKING PAPER SERIES

MARKUPS, LABOR MARKET INEQUALITY AND THE NATURE OF WORK

Greg Kaplan  
Piotr Zoch

Working Paper 26800  
<http://www.nber.org/papers/w26800>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
February 2020

We thank Fernando Alvarez, Susanto Basu, Ariel Burstein, Mark Gertler, Jan De Loecker, John Fernald, Simon Mongey, Emi Nakamura, Brent Neiman, Ezra Oberfield, Esteban Rossi-Hansberg, Rob Shimer, Jon Steinsson and Chad Syverson for valuable comments and discussions. All errors are our own. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

At least one co-author has disclosed a financial relationship of potential relevance for this research. Further information is available online at <http://www.nber.org/papers/w26800.ack>

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2020 by Greg Kaplan and Piotr Zoch. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Markups, Labor Market Inequality and the Nature of Work  
Greg Kaplan and Piotr Zoch  
NBER Working Paper No. 26800  
February 2020  
JEL No. D2,D3,D4,E3,E5,J2,L1

**ABSTRACT**

We demonstrate the importance of distinguishing between the traditional use of labor for production, versus alternative uses of labor for overhead, marketing and other expansionary activities, for studying the distribution of both factor income and labor income. We use our framework to assess the impact of changes in markups on the overall labor share and on labor income inequality across occupations. We identify the production and expansionary content of different occupations from the co-movement of occupational income shares with markup-induced changes in the labor share. We find that around one-fifth of US labor income compensates expansionary activities, and that occupations with larger expansionary content have experienced the fastest wage and employment growth since 1980. Our framework can rationalize a counter-cyclical labor share in the presence of sticky prices and can be used to study the distributional effects of demand shocks, monetary policy and secular changes in competition.

Greg Kaplan  
Department of Economics  
University of Chicago  
1126 E 59th St  
Chicago, IL 60637  
and NBER  
gkaplan@uchicago.edu

Piotr Zoch  
Department of Economics  
University of Chicago  
1126 E 59th St  
Chicago, IL 60637  
pzoch@uchicago.edu

# 1 Introduction

The wedge between the marginal cost paid to factor inputs and the price of final goods paid by consumers, known as the markup, plays a central role in macroeconomics. In the long-run, the markup reflects the nature of competitive forces and is the key channel through which industrial and trade policy affect the economy. In the short-run, movements in the markup are the main channel through which demand shocks and monetary policy affect the economy, when viewed through the lens of New Keynesian models of the business cycle. In this paper, we develop a framework for understanding the effects of a change in the markup on the factor income and labor income distributions. We use our framework to rationalize cyclical movements in the labor share versus the profit share, to interpret differences in the exposure of different occupations to aggregate fluctuations, and to offer a new perspective on the relative wage and employment growth in different occupations.

Our main innovation is to distinguish theoretically and empirically between two uses of labor in a modern economy. We refer to the traditional role of labor as an input to the production of existing goods for sale in existing markets as *production*, or *Y*-type, labor. We contrast this with an alternative role of labor that facilitates extensive-margin replication, which we refer to as *expansionary*, or *N*-type, labor. *N*-type labor encompasses a broad array of corporate activities that include overhead, product design, research and development, logistics and marketing. Incorporating these expansionary uses of labor turns out to have important implications for the dynamics of aggregate labor income and for labor income inequality. We estimate that roughly one-fifth of total US labor income compensates *N*-type activities, and that those occupations whose share of *N*-type activities is largest are the same occupations that have experienced the fastest wage and employment growth over the last forty years.

The first part of the paper is theoretical. In Section 2, we consider a static model of production in which labor is the only input, but is used for both production and expansionary purposes. Our theoretical results are summarized in three simple but powerful theorems that describe how a change in the markup redistributes national income. Theorem 1 shows that whether an increase in the markup leads to an increase or a decrease in the labor share depends on the share of *N*-type labor in the economy. In existing models that abstract from the expansionary role of labor, the markup and the labor share move in opposite directions. This negative relationship is so strongly engrained in macroeconomic thinking that it forms the basis for most empirical efforts to measure the cyclicalities of markups. But when a sufficiently large fraction of labor income compensates *N*-type activities, this co-movement is reversed. Theorem 2 shows that despite the theoretical ambiguity in the effect of the markup on the overall labor share, an increase in the markup unambiguously redistributes

income away from  $Y$ -type labor and towards  $N$ -type labor. Thus when the markup changes, some workers' incomes rise and some workers' incomes fall, depending on the nature of the work that they perform. Theorem 3 then shows that the markup is counter-cyclical in our environment, as it is in standard business cycle models with sticky prices.

Since the labor share is strongly counter-cyclical in US data, together these three theorems imply that the share of labor income compensating expansionary labor is positive, and that in economic downturns labor income falls less for workers performing these expansionary activities than for workers performing traditional production activities. Moreover, since demand shocks and monetary policy transmit through changes in markups in New Keynesian models, our framework offers an explanation for why the labor share increases in response to contractionary demand shocks or monetary policy shocks. And since  $Y$ -type labor is more exposed to these aggregate shocks than is  $N$ -type labor, our framework also offers a mechanism through which aggregate demand and monetary policy affect the labor income distribution.

The second part of the paper is empirical. We attempt to quantify just how much of US labor income compensates  $N$ -type labor, how it has changed over time and which occupations are most  $N$ -intensive. We estimate the structural parameters that govern these quantities in two stages.

In Section 3 we exploit the prediction of Theorem 1 that the sign and strength of the co-movement of the markup with the labor share reveals what fraction of labor income compensates  $Y$ -type versus  $N$ -type labor. We explain how the aggregate structural parameters can be identified given de-trended time-series data on the labor share and the aggregate markup. We estimate these parameters using post-war US data and show that they imply that between 5% and 35% of labor income compensates expansionary activities, depending on the assumption we make about the average markup over this period. We show that this fraction has increased since the 1970's, before which it was declining.

In Section 4 we introduce the notion of an occupation into our framework. We exploit the prediction of Theorem 2 that, given the parameter estimates from the first stage, the sign and strength of the co-movement of occupational labor income shares with markup-induced variation in the overall labor share reveals the relative intensity of each occupation in the two types of activities. Occupations that are more intensive in expansionary activities are those occupations whose share of aggregate labor income increases when the overall labor share increases due to a change in the markup. We estimate the model parameters using three different approaches to isolate markup-induced variation in the labor share: (i) using the de-trended labor share itself; (ii) using the de-trended markup as an instrument; and (iii) using external estimates of lagged monetary policy shocks as a set of instruments. We clarify the orthogonality conditions that are required for each approach.

Regardless of which of these sources of variation that we exploit, we find that the occupations that are the most  $N$ -intensive are those that are typically associated with white-collar jobs (high-tech, service, managerial and admin occupations), while those that are the most  $Y$ -intensive are those that are typically associated with blue-collar jobs (construction, extractive, production, repair and farming occupations). We find both high-wage and low-wage occupations among the  $N$ -intensive occupations, so the correlation of wages with the expansionary content of occupations is weak. But we find a strong positive correlation between the expansionary content of occupations and both wage growth and hours growth over the last 40 years. This suggests that the demand for  $N$ -type labor is growing faster than is the demand for  $Y$ -type labor.

Although our main contribution is to offer new insights into how the aggregate labor share and the labor income distribution are affected by markups, we also shed new light on several issues in the recent macroeconomics and labor economics literatures. First, we offer a simple explanation for the counter-cyclical labor share, conditional on a monetary policy shock, which has been a long-standing puzzle for New Keynesian models (Cantore et al., 2019). Indeed, when we estimate a medium-scale DSGE model as in Smets and Wouters (2007) with our production structure, we obtain similar parameter estimates to what we obtain in our baseline approach, and the estimated model generates the correct co-movement of the labor share with output.

Second, a corollary of the counter-cyclical labor share is that our framework offers a mechanism for generating pro-cyclical profits in response to monetary policy and other aggregate demand shocks in New Keynesian models. This is a problematic feature of existing Heterogeneous Agent New Keynesian (HANK) models in which the counter-cyclical of profits generates counterfactual patterns of wealth redistribution (McKay et al., 2016; Kaplan et al., 2018; Bhandari et al., 2018). Third, our occupational framework suggests a mechanism for endogenizing the labor income distribution in HANK models. Together, these two features pave the way for using HANK models to analyze the effect of monetary policy and aggregate demand on the the joint distribution of labor income and wealth.

Fourth, our measures of the expansionary and production content of different occupations offer an alternative to the task-based framework of Autor et al. (2006) and Acemoglu and Autor (2011) as a lens through which to view changes in the US occupational structure over the last forty years. We correlate our measures of the  $N$ -intensity of occupations with the manual, routine and abstract content of occupations as measured by Autor et al. (2006). We find that  $N$ -intensity is weakly negatively correlated with the manual content of occupations and weakly positively correlated with the abstract content, but is not correlated with the routine content.

Our model and exercises are also related to several other strands of literature that we discuss at appropriate points in the paper.

## 2 Theoretical Framework

### 2.1 Wholesale sector

A representative wholesaler hires labor  $L_Y$  in a competitive labor market at wage  $W_Y$ , which it uses to produce a homogeneous intermediate good  $Y$ . We refer to the labor used in the wholesale sector as  $Y$ -type labor or production labor. The intermediate good  $Y$  is then sold to retailers in a competitive market at price  $P_W$ . The wholesaler thus chooses labor and output to maximize profits  $\Pi_W$ :

$$\begin{aligned} \Pi_W &:= \max_{L_Y, Y} P_W Y - W_Y L_Y \\ &\text{subject to} \\ Y &= Z_Y L_Y^{\theta_Y} \end{aligned}$$

Wholesale profits may be non-zero if there are decreasing returns to scale in production ( $\theta_Y < 1$ ), in which case we interpret these profits as rents arising from a fixed factor in production.

### 2.2 Retail Sector

A unit measure continuum of identical retailers each hire labor  $L_N$  in a competitive labor market at wage  $W_N$ , which they use to manage product lines. We refer to the labor used in the retail sector as  $N$ -type labor, overhead labor, or expansionary uses of labor. Each product line  $j$  generates gross profits  $\Pi_j$ , which the retailer's expansion department takes as given when deciding on the number of lines to operate. The retailer thus chooses labor and product lines to maximize net profits  $\Pi_R$ :

$$\begin{aligned} \Pi_R &:= \max_{L_N, N} \int_0^N \Pi_j dj - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N} \end{aligned}$$

We allow for the possibility of decreasing returns to scale in managing product lines ( $\theta_N \leq 1$ ) to reflect span-of-control considerations. Although we will use the language of “expansion”

and “product lines” for the  $N$  margin, this language should not be interpreted literally.  $N$  might represent any activity by which the retailer can replicate its gross profits – such as developing new products, operating in a new geographic market, marketing to different demographic markets or increasing advertising or sales effort for existing products in existing markets.

The retailer’s pricing department for product line  $j$  purchases homogenous intermediate goods from the wholesale sector, which it costlessly differentiates and sells to consumers at a markup  $\mu \geq 1$  over marginal cost  $P_W$ . Hence the price charged for product line  $j$  is

$$p_j = \mu P_W$$

and the gross profits in each product line are given by

$$\Pi_j := y_j (p_j - P_W)$$

where the quantity  $y_j$  of differentiated goods sold is determined by a demand curve  $(y_j, p_j)$  that the retailer takes as given.

For now, we will treat the markup  $\mu$  as exogenous and we will remain agnostic about the source of variations in the markup  $\mu$ , because the theorems that follow about the effects of changes in the markup do not depend on a particular micro-foundation. In Section 2.6 we describe various market environments and preferences that are all consistent with this structure. However, readers who prefer to have a concrete example in mind can think of a model of monopolistic competition with a Constant Elasticity of Substitution (CES) aggregator over product lines, in which the markup  $\mu$  is equal to the elasticity of substitution across varieties  $\sigma$ , and variation in  $\mu$  arises from exogenous variation in  $\sigma$ .

## 2.3 Factor Shares

We focus on symmetric equilibria in which  $p_j = p \forall j$  and  $y_j = y \forall j$ . Market clearing for intermediate goods then implies that

$$yN = Y$$

and nominal GDP in the economy is given by  $pY$ .

The shares of total income accruing to  $Y$ -type labor and  $N$ -type labor are defined as

$$S_N := \frac{W_N L_N}{pY} \text{ and } S_Y := \frac{W_Y L_Y}{pY},$$

and the overall labor share is defined as  $S_L = S_N + S_Y$ . The overall profit share in the economy is given by the sum of profit shares in the wholesale and retail sectors,  $S_\Pi = S_R + S_W$ , where

$$S_W := \frac{\Pi_W}{pY} \text{ and } S_R := \frac{\Pi_R}{pY}.$$

**Lemma 1.** *In an economy with this production structure, the equilibrium factor shares are given by*

$$\begin{aligned} S_Y &= \frac{1}{\mu} \theta_Y \\ S_N &= \left(1 - \frac{1}{\mu}\right) \theta_N \\ S_W &= \frac{1}{\mu} (1 - \theta_Y) \\ S_R &= \left(1 - \frac{1}{\mu}\right) (1 - \theta_N). \end{aligned}$$

*Proof.* See Appendix [A.1](#). □

Lemma 1 shows that the factor shares in this economy are determined by only three parameters: the level of the markup  $\mu$  and the degrees of decreasing returns to scale in production and expansion,  $(\theta_Y, \theta_N)$ . In particular, neither the demand structure that gives rise to the markup  $\mu$ , nor the relative productivities in the two sectors, matter for the income shares. The latter property is a feature of having assumed iso-elastic production functions, which we relax in Section [2.7](#). The following two theorems about the effect of a change in the markup on the income distribution follow directly from the factor shares in Lemma 1.

## 2.4 Effect of the Markup on Labor Income Shares

The majority of existing macroeconomic models abstract from  $N$ -type labor. These models are a special case of our framework in which  $\theta_N = 0$ , so that the labor share and profit share are given by  $(S_L, S_\Pi) = \left(\frac{\theta_Y}{\mu}, 1 - \frac{\theta_Y}{\mu}\right)$ . In this familiar case, an increase in the markup  $\mu$  unambiguously increases the profit share and lowers the labor share, redistributing income away from workers and towards the owners of claims on profits. Indeed, the tight negative relationship between the markup and labor share is so strongly engrained in macroeconomic thinking that empirical work on measuring movements in the markup often equates the markup with the inverse of the labor share ([Bils and Klenow, 2004](#); [Nekarda and Ramey, 2019](#)), or estimates the markup as the ratio of the labor (or other variable input) share to



the output elasticity of labor (or other variable input) (De Loecker and Warzynski, 2012; De Loecker et al., 2019).

But in economies in which some workers are engaged in  $N$ -type activities ( $\theta_N > 0$ ), this relationship can break down. The following Theorem shows that, away from this special case, the relationship between the markup and the labor share is ambiguous.

**Theorem 1.** *An increase (decrease) in the markup  $\mu$  leads to an increase (decrease) in the overall labor share if and only if the degree of decreasing returns to scale is stronger for  $Y$ -type workers than for  $N$ -type workers, i.e.*

$$\frac{\partial S_L}{\partial \mu} \begin{matrix} \geq \\ < \end{matrix} 0 \text{ if and only if } \theta_N \begin{matrix} \geq \\ < \end{matrix} \theta_Y.$$

*Conversely, an increase (decrease) in the markup  $\mu$  leads to an increase (decrease) in the overall profit share if and only if the degree of decreasing returns to scale is stronger for  $N$ -type workers than for  $Y$ -type workers, i.e.*

$$\frac{\partial S_\Pi}{\partial \mu} \begin{matrix} \geq \\ < \end{matrix} 0 \text{ if and only if } \theta_Y \begin{matrix} \geq \\ < \end{matrix} \theta_N.$$

*Proof.* See Appendix A.2. □

Theorem 1 reveals that a change in the markup has an ambiguous effect on the share of income accruing to labor versus profits. In particular, an increase in the markup leads to a fall in the labor share if and only if  $\theta_Y > \theta_N$ . In the special case when  $\theta_N = \theta_Y$ , a change in the markup has no effect on the labor share relative to the profit share. And when  $\theta_N > \theta_Y$ , which we will argue below is the empirically relevant case, an increase in the markup leads to an increase in the labor share and a decrease in the profit share. Thus the co-movement of the markup with the labor share is informative about the relative sizes of  $\theta_N$  versus  $\theta_Y$ , and hence about the share of total labor income that compensates  $Y$ -type activities versus  $N$ -type activities.

The reason why a change in the markup has an ambiguous effect on the profit share is because economic profits in this economy arise from two different sources. To see the importance of the relative sizes of  $\theta_N$  versus  $\theta_Y$ , it is useful to consider two extreme cases. When  $\theta_Y = 1$  and  $\theta_N < 1$ , all economic profits accrue to the retail sector and profits reflect rents from charging prices above marginal cost. In this case, an increase in the markup unambiguously raises the profit share because it raises the rents that are the source of profits in the economy. When  $\theta_Y < 1$  and  $\theta_N = 1$ , all economic profits accrue to the wholesale sector and reflect rents accruing to the fixed factor in production. Although the

retail sector still charges a markup above marginal cost, those profits are eaten up by entry of new product lines. In this case, an increase in the markup unambiguously lowers the profit share because the additional entry diverts resources away from the wholesale sector, limiting the scope for the wholesale sector to use the implicit fixed factor. In the general case where  $\theta_Y < 1$  and  $\theta_N < 1$ , which of the two effects dominates depends on which source of profits is more responsive to a change in the markup, which depends on the relative sizes of  $\theta_Y$  versus  $\theta_N$ . The co-movement of the profit share with the markup is hence informative about the nature of economic profits in the economy.

Despite this ambiguity in the effect of a change in the markup on the overall labor share, the next theorem shows that a change in the markup unambiguously redistributes labor income between  $Y$ -type and  $N$ -type workers.

**Theorem 2.** *An increase (decrease) in the markup  $\mu$  leads to a decrease (increase) in the income share of  $Y$ -type labor and an increase (decrease) in the income share of  $N$ -type labor.*

$$\frac{\partial S_Y}{\partial \mu} < 0 \text{ and } \frac{\partial S_N}{\partial \mu} > 0$$

*Proof.* See Appendix [A.3](#). □

The theorem states that a change in the markup redistributes national income between the two types of labor in the economy. Whereas the labor income of workers performing traditional activities ( $Y$ -type labor) is negatively exposed to markups, the labor income of workers performing expansionary activities ( $N$ -type labor) is positively exposed to markups.

The intuition behind Theorem 2 is that whether the demand for an input rises or falls with an increase in the markup depends on whether the real marginal value of that input to producers (in terms of final goods) is higher or lower when the markup is higher. For the retail sector, the marginal value of  $N$ -type labor is higher when markups, and hence gross profits, are higher because  $N$ -type workers allow retailers to replicate their existing activities. For the wholesale sector, a higher markup translates into a lower value of the intermediate good in units of the final good, which lowers the marginal value of the  $Y$ -type labor that is used to produce the good.

Theorem 2 suggests a strategy for learning about which types of workers are compensated for traditional versus expansionary activities. The theorem suggests that in response to an increase in the markup, the labor income paid to  $N$ -type labor should increase relative to the income paid to  $Y$ -type labor. In Section 4, we will pursue this strategy in the context of occupations in the US labor market.

## 2.5 Effect of the Markup on Output

The previous theorems emphasized the connection between movements in the markup and movements in the labor share. We now establish a relationship between movements in the markup and movements in output. This will allow us to interpret the cyclicity of the labor share through the lens of our framework, which will help us to learn about the relative size of  $\theta_Y$  and  $\theta_N$ , and help us to resolve some puzzles that have plagued New Keynesian business cycle models. However, in order to make statements about output, we need to impose some additional structure on the household side of the model.

We assume that there is a representative household with preferences

$$U(C, L_Y, L_N) = \log C - v(L_Y, L_N)$$

where  $C = \mathcal{C}(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega)$  is a symmetric homothetic aggregator over distinct varieties  $c_\omega$  and  $\Omega$  is the measure of varieties. This implies that there exists a price index  $P = \mathcal{P}(\{p_\omega\}_{\omega \in [0, \Omega]}, \Omega)$ , and in a symmetric equilibrium in which  $p_\omega = p_j = p \forall j, \omega$ , nominal GDP satisfies  $pY = PC$ . At this point we make no additional assumptions about the measure of unique varieties  $\Omega$ , nor its relationship to the measure of product lines  $N$  operated by the retail sector.

**Theorem 3.** *Suppose that (i) the aggregator  $\mathcal{C}$  does not exhibit love of variety and (ii)  $v$  is convex in  $(L_Y, L_N)$ , i.e.  $v_{L_N L_N} > 0$ ,  $v_{L_Y L_Y} > 0$  and  $v_{L_Y L_Y} v_{L_N L_N} \leq (v_{L_Y L_N})^2$  and (iii)  $v_{L_N L_Y} \geq 0$ . Then an increase in  $\mu$  leads to a decrease in aggregate nominal output  $pY$ .*

*Proof.* See Appendix A.4. □

Theorem 3 states that the markup is always countercyclical in this environment. The convexity condition on preferences is sufficient but not necessary. Two simple examples are perfect substitutes,  $v(L_Y, L_N) = \chi \frac{(L_Y + L_N)^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}}$ , and imperfect substitutes with the same Frisch elasticity,  $v(L_Y, L_N) = \chi_Y \frac{L_Y^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} + \chi_N \frac{L_N^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}}$ . The restriction on love-of-variety is also sufficient but not necessary for the result.

The intuition behind the theorem is that we can write total output  $Y$  as the product of the number of product lines  $N$  and the total sales of each product line  $y$ . The elasticity of output with respect to a change in the markup can then be expressed as

$$\frac{\partial \log Y}{\partial \mu} = \frac{\partial \log y}{\partial \mu} + \frac{\partial \log N}{\partial \mu}.$$

The first term is always negative for the reasons discussed in the previous section: a higher markup lowers the real value to the wholesaler of producing additional output per product line. The second term is positive because a higher markup raises the real return on expanding through replication. As long as the labor supply curves faced by the two sectors are not too different, the first effect always dominates. In Section 2.7, we describe modifications of the environment that can reverse this finding by making the second term more responsive.

**Taking stock** We have established that in our framework, an increase in the markup is associated with a fall in output. We have also established conditions under which an increase in the markup is associated with an increase in the labor share. This means that we can now look at evidence on the cyclicity of the labor share, conditional on changes in the markup, to infer which of  $\theta_N$  and  $\theta_Y$  is larger, and evidence on the co-movement of the labor share and the markup to estimate the absolute size of  $\theta_Y$  and  $\theta_N$ . We will pursue this in Section 3. However, before turning to estimation, we first analyze a series of generalizations to the model and we discuss alternative interpretations of the source of movements in the markup.

## 2.6 Interpretations of Movements in the Markup

We have so far treated the aggregate markup  $\mu$  as an exogenous wedge between the (competitive) wholesale price for intermediate goods produced by  $Y$ -type labor, and the retail price paid by consumers. In this section we describe a number of micro-founded environments in which markup variation arises either as a result of exogenous variation in a structural parameter or for endogenous reasons. Full details on each environment are described in Appendix B. The results of Theorems 1 and 2 hold in each case.

Recall that  $\Omega$  is the measure of unique varieties in the economy and  $N \geq \Omega$  is the number of product lines or establishments operated by firms. This allows for the possibility that more than one establishment or sales unit produces the same variety. We focus on symmetric equilibria and denote the measure of retail sales units operating in each variety market as  $M := N/\Omega$ . We denote the elasticity of demand by

$$\varepsilon \left( \{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega \right) := -\frac{p_\omega}{c_\omega} \frac{\partial c_\omega}{\partial p_\omega}$$

which in a symmetric equilibrium is  $\varepsilon(p, C, \Omega)$  and with a homothetic aggregator can be written as  $\varepsilon(P, \Omega)$  where  $P = \mathcal{P}(p, \Omega)$  is the price index. Below we provide several examples.

### 2.6.1 Monopolistic Competition

Under monopolistic competition, each sales unit has a monopoly over a single unique variety ( $M = 1, N = \Omega$ ) so adding or subtracting product lines  $N$  induces one-for-one changes in the measure of goods  $\Omega$  available for consumption. The markup is given by

$$\mu = \frac{\varepsilon}{\varepsilon - 1}.$$

Theorems 1, 2 and 3 apply to movements in the markup due to exogenous changes in parameters that enter the demand elasticity, due to endogenous changes in variables that enter the demand elasticity ( $C, \Omega, p$ ) or due to frictions in price setting that induce a wedge between the markup  $\mu$  and  $\frac{\varepsilon}{\varepsilon-1}$ . Importantly, our theorems apply in dynamic versions of the model in which the pricing department faces costs of adjusting prices as in Rotemberg (1982) or Calvo (1983), which lead to endogenous movements in the markup.

**Example 1.** With a **Constant Elasticity of Substitution (CES)** demand system as in Dixit and Stiglitz (1977) and Blanchard and Kiyotaki (1987), the elasticity takes the form  $\varepsilon = \sigma$  so with flexible price setting, exogenous changes in  $\sigma$  are the only source of markup variation.<sup>1</sup>

**Example 2.** With a **Translog** demand system as in Feenstra (2003), Bilbiie et al. (2012) and Maggi and Félix (2019), the elasticity takes the form  $\varepsilon = \sigma\Omega + 1 = \sigma N + 1$ . Any changes (other than to  $\theta_Y$  or  $\theta_N$ ) that lead to a decrease in the number of product lines in the economy, such as a fall in  $Z_N$ , will induce a rise in markups and the distributional effects in Theorems 1 and 2 will apply. Changes in  $\theta_Y$  or  $\theta_N$  also have direct effects on factor shares, but the indirect effects that arise through the resulting change in the markup also satisfy Theorems 1 and 2.

**Example 3.** With a **Linear Demand** system as in Melitz and Ottaviano (2008), the elasticity takes the form  $\varepsilon = \sigma \frac{\Omega}{C} = \sigma \frac{N}{C}$ . Since preferences are not homothetic, the elasticity of demand depends on the level of consumption. Any shock that affects aggregate consumption without directly impacting factor shares (examples of such shocks include technology ( $Z_Y, Z_N$ ) or labor supply  $v(\cdot)$ ) leads to a change in the markup and the results of Theorems 1 and 2 hold.

### 2.6.2 Cournot Competition

Under Cournot competition, there are a large number of sales units  $M \gg 1$  producing each unique variety. If  $M$  is sufficiently large so that individual retailers do not internalize

---

<sup>1</sup>In a symmetric equilibrium with homogenous goods the Kimball (1995) demand as used in Klenow and Willis (2016) and Edmond et al. (2018) has a constant markup as in CES.

the effect of their own price on the aggregate price index, then the equilibrium markup is

$$\mu = \frac{\varepsilon}{\varepsilon - \frac{1}{M}}.$$

Consider first the case in which  $M$  is a primitive technological or policy parameter describing the nature of competition. In this case the creation of new product lines  $N$  by retailers generates proportionately more varieties  $\Omega$ . A change in the markup then arises as a result of exogenous shifts in  $M$ , and the distributional and aggregate effects of such changes satisfy Theorems 1, 2 and 3.

Alternatively, one could consider the measure of unique varieties  $\Omega$  as a primitive. In this case, a shock that leads to the creation of new product lines by retailers generates a proportionate increase in  $M$ , the measure of sales units competing in each market, and hence leads to a fall in the markup. Theorems 1 and 2 apply to this change in the markup. An example of such a shock would be a shock to the relative productivities in the two sectors  $\frac{Z_N}{Z_Y}$ .

### 2.6.3 Oligopoly

Oligopoly refers to the case in which  $M > 1$  retail sales units produce each variety, but  $M$  is sufficiently small that retailers take strategic considerations into account when setting prices. As in the previous case we can treat either the number of firms in each market  $M$  or the total measure of unique varieties  $\Omega$  as a primitive. We focus on the nested CES case as in [Atkeson and Burstein \(2008\)](#), [Jaimovich and Floetotto \(2008\)](#) and [Mongey \(2019\)](#), in which the elasticity of substitution across the same variety sold by different retailers is  $\eta > \sigma$ . The previously considered case in which the same varieties sold by different retailers are perfect substitutes corresponds to  $\eta \rightarrow \infty$ .

**Example 4.** Under **Bertrand competition** the markup is given by

$$\mu = \frac{\eta + \frac{\sigma - \eta}{M}}{\eta - 1 + \frac{\sigma - \eta}{M}}$$

which gives  $\mu = 1$  when  $\eta \rightarrow \infty$ .

**Example 5.** Under Cournot competition the markup is given by

$$\mu = \frac{\sigma \eta}{\sigma (\eta - 1) + \frac{\sigma - \eta}{M}}$$

which gives  $\mu = \frac{\sigma}{\sigma - \frac{1}{M}}$  when  $\eta \rightarrow \infty$ .

In either case a change in the markup arising from a change in either demand elasticity, or from a change in the degree of concentration  $M$ , induces the distributional effects implied by Theorems 1 and 2.

#### 2.6.4 Limit Pricing

As in Barro and Tenreyro (2006), assume that there exists an alternative technology for retailers to operate a product line that does not require hiring any  $N$ -type labor. Instead the retailer incurs additional input costs so that their effective marginal cost of undifferentiated goods is  $\kappa P_W$ , with  $\kappa > 1$ . This captures, for example, the costs of licensing an existing product to sell in a new market, or the costs of trying to compete in a product market without setting up the necessary sales infrastructure or overhead. If  $\kappa < \mu$  in any of the aforementioned market structures, then the markup is  $\kappa$  and any change in  $\kappa$  will generate the redistributive effects described in Theorems 1, 2 and 3.

## 2.7 Generalizations of Production Structure

Our assumed production structure contains several special features that are not strictly necessary for a change in the markup to have the redistributive effects described in Theorems 1 and 2. Relaxing these features is useful for understanding the economic forces at work. For simplicity we describe these generalizations in the context of monopolistic competition with  $N = \Omega$ .

### 2.7.1 Variety-specific DRS in production

Our baseline model features a homogenous intermediate good that is then differentiated. This implies that DRS in the use of  $Y$ -type labor operates at the economy-wide level. In the absence of love-of-variety in preferences, the production of new product lines is a socially wasteful activity and a planner would choose to set  $N \rightarrow 0$ . Some readers may find this feature of our environment unappealing. However, this assumption is not important for Lemma 1 or for Theorems 1 or 2. In Appendix C we describe an alternative version of the model, in which each variety is produced with a separate DRS production function in the wholesale sector. We show that in this alternative model, the factor shares are identical to those in the baseline model. With this alternative formulation, there is indeed a social benefit to introducing new product lines, and a planner would choose a value of  $N > 0$  as in Bilbiie et al. (2016). But the distributional effects of a change in the markup are not affected.

The main difference with variety-specific DRS is in the effects of the markup on aggregate output, i.e. the cyclicity of markups. Unlike in the baseline model the cyclicity of markups is theoretically ambiguous. Aggregate output is still given by the product of the number of product lines  $N$  and output per product line, so

$$\frac{d \log Y}{d\mu} = \frac{d \log y}{d\mu} + \frac{d \log N}{d\mu}.$$

As before, the first term is always negative and the second term is always positive. However, unlike in the baseline model, the fall in output in each product line  $y$  may not be large enough to overcome the increase in the number of product lines  $N$ , and for economies in which the degree of DRS is strong (low  $\theta_Y$ ) the second term may dominate, leading to pro-cyclical markups.

For example when  $\theta_Y = 0$  so that all labor is expansionary, as in [Kaplan and Menzio \(2016\)](#), this is indeed what happens. However for calibrations of  $(\theta_Y, \theta_N, \sigma)$  in which an intermediate fraction of labor income goes to  $N$ -type workers it is typically the case that markups remain counter-cyclical.

### 2.7.2 Integrated wholesale and retail sectors as single firms

The model described above assumes that wholesalers' decisions about how much  $Y$ -type labor to hire are independent of retailers' decisions about how much  $N$ -type labor to hire. This means that when a retailer is deciding about whether to expand its number of product lines, it does not internalize the effect that this will have on the marginal cost of production.

In [Appendix C](#), we analyze a version of the model in which a single firm decides jointly on production and expansion. In this model, [Theorem 2](#) is unaffected. It remains the case that an increase in the markup re-distributes labor income away from  $Y$ -type labor towards  $N$ -type labor. However, [Theorem 1](#) no longer holds. Rather, one can show that when the production and expansion decisions are integrated, the labor share is always inversely related to the markup, and so is pro-cyclical.

Thus, in order for the model to generate a counter-cyclical labor share, it is important that the production and expansion decisions are separated. The reason why the baseline model can generate a positive co-movement between the labor share and the markup is because when markups rise, the marginal value of expansion rises, and expansion requires  $N$ -type labor. But in the integrated model, retailers internalize the fact that expanding  $N$  requires more goods to be produced, which raises the marginal cost of production, and lowers the marginal benefit of expansion. This latter force is strong enough so that total demand for labor of both types of workers always falls when the markup rises.



In Section 3, we will re-confirm the well-known fact that the labor share is strongly counter-cyclical in post-war US data. This is what leads us to prefer our baseline production structure over this alternative.

### 2.7.3 Capital

So far we have considered a production structure in which labor is the only factor of production. Embedding our framework in a general equilibrium macroeconomic model that we can take to the data requires adding capital. We add capital by assuming that producing output and operating product lines both require labor and capital, with possibly different factor intensities. We thus extend the production functions as

$$Y = Z_Y (L_Y^{1-\alpha_Y} K_Y^{\alpha_Y})^{\theta_Y} \quad \text{and} \quad N = Z_N (L_N^{1-\alpha_N} K_N^{\alpha_N})^{\theta_Y} .$$

Including capital in the model has only a minimal impact on the equilibrium factor shares in Lemma 1. The only difference is that  $S_Y$  and  $S_N$  are now defined as the share of national income going to all inputs used in the  $Y$  and  $N$  sectors, respectively. With this definition, the shares are given by those in Lemma 1. Within the  $Y$  sector, the split between capital and labor is given by  $S_{L_Y} = (1 - \alpha_Y) S_Y$  and  $S_{K_Y} = \alpha_Y S_Y$ , and within the  $N$ -sector, the split between capital and labor is given by  $S_{N_Y} = (1 - \alpha_N) S_N$  and  $S_{K_N} = \alpha_N S_N$ .

The result in Theorem 1 concerning the relationship between the markup and the profit share is unaffected. However, the result concerning the relationship between the markup and the labor share must be modified when the capital intensities within each sector are different. In Appendix C, we show that the condition for the labor share to increase when the markup increases becomes  $\theta_N (1 - \alpha_N) > \theta_Y (1 - \alpha_Y)$ . In words, this says that the degree of decreasing returns to scale in labor must be stronger in the  $Y$  sector than in the  $N$  sector. We also show in Appendix C that adding capital in this way does not change the redistributive effects of a change in the markup (Theorem 2), nor the relationship between the markup and output, if capital is in fixed supply (Theorem 3).

### 2.7.4 Other Generalizations

We also generalize the model to allow for CES production functions, for entry in the wholesale sector, and for markups in both the labor market and the market for wholesale goods. Details of these generalizations, along with the modifications to the conditions of Theorems 1 and 2 that they require, are contained in Appendix C.

## 2.8 Interpretation of the $Y$ -type vs $N$ -type Distinction

Before moving to estimation, some notes about the distinction between the two types of labor are in order. We have thus far referred to  $Y$ -type labor as production labor and  $N$ -type labor as overhead or expansionary labor; but the words we use are less important than the economic distinction. We think that the key distinction between the two types of labor is that  $N$ -type labor facilitates extensive margin replication, i.e. the possibility of replicating a profit-generating activity without lowering gross profits, whereas  $Y$ -type labor does not.

This distinction is easiest to appreciate in the versions of the model described in Section 2.7.1 and Section 2.7.2, in which each product line requires its own wholesale sector, or when both sectors are integrated within a single firm. In these cases, expansion through using more  $Y$ -type labor requires producing and selling more in existing product lines - which incurs either higher input prices or lower sales prices, and hence lowers gross profits  $\Pi_j$ . But expansion through using more  $N$ -type workers does not lower gross profits. Since a change in the markup changes gross profits, it changes the relative attractiveness of expanding through  $N$ -type or  $Y$ -type labor. This is why we think of  $N$ -type labor as performing any activities that expand real output without lowering gross profits. Various activities fit this description: (i) innovation activities like R&D that literally invent new goods; (ii) marketing activities that inform or persuade new customers to purchase existing goods at existing prices; (iii) management activities that allow firms to operate in more geographic markets or manage more product lines; (iv) expansion activities that open additional establishments; (v) any other activities that are complementary with these activities.

In the baseline two-sector version of the model, a similar force is at work, but it occurs only in general equilibrium. Hiring additional  $Y$ -type labor raises the wholesale price  $p_W$  in equilibrium, and thus lowers gross profits, whereas hiring additional  $N$ -type labor does not.

For practical purposes, the most useful description of the distinction, is probably the result rather than the definition:  $N$ -type ( $Y$ -type) labor is labor whose income share falls (rises) when the markup falls. Indeed, this is the distinction we will exploit in the remainder of the paper. Hence an alternative approach might have been to define  $Y$ -type and  $N$ -type labor in this way, and then to think of our theory as providing an explanation for how the labor income distribution covaries with the markup.

Finally, one element that is *not* an important part of the distinction between the two types of labor is adjustment costs. We have purposely focused on a static model to emphasize that the distinction between the two types of labor is not fundamentally about differences

in the nature of adjustment costs they incur. It may turn out that adjustment costs do in fact differ, but this is not important for the forces driving the effects of a change in the markup in Theorems 1 or 2.

### 3 Estimation of Aggregate Parameters

Our goal in this and the following section is to estimate the parameters of the model that determine what share of aggregate US labor income compensates  $N$ -type versus  $Y$ -type activities, how this share has changed over time, and which workers in the economy perform  $N$ -type activities. However, measuring  $N$ -type workers poses several challenges. First, as discussed in the previous section, the notion of  $N$ -type workers is abstract. It encompasses a broad range of activities and refers to any workers whose real marginal value to firms rises when markups rise. Second, firms do not hire workers directly into  $N$ -type jobs and  $Y$ -type jobs. Rather, firms hire workers into occupations, and most occupations perform a variety of different activities, some of which are better characterized as  $N$ -type and some of which are better characterized as  $Y$ -type.

One possible approach would be to try to identify  $N$ -type and  $Y$ -type occupations by examining their daily activities (using data from a source such as O\*NET) or by measuring their contribution to specific corporate activities (such as R&D, product design, overhead, sales and marketing). We choose not to take this route because we do not presume to know ex-ante the mapping from specific tasks to  $N$ -type versus  $Y$ -type activities. Instead, we prefer to infer the aggregate share of  $N$ -type labor income, as well as the  $N$ -type and  $Y$ -type content of different occupations, by exploiting the model predictions underlying Theorems 1 and 2. According to the model,  $N$ -type workers are workers whose share of total labor income increases in response to a markup-induced increase in the overall labor share, whereas  $Y$ -type workers are those whose share of total labor income falls in response to a markup-induced increase in the overall labor share.

We pursue this approach in two stages. In this section, we start by explaining how to identify the two key elasticities  $(\theta_Y, \theta_N)$  given data on the markup  $\mu$  and the overall labor share  $S_L$ . Knowledge of these parameters is then sufficient to infer the aggregate fraction of US labor income that compensates  $N$ -type activities. Then, in Section 4, we introduce occupations into our framework and show how to identify the parameters that govern the  $N$ -type and  $Y$ -type content of each occupation, given estimates of  $(\theta_Y, \theta_N)$  and data on the occupational labor income shares, under various assumptions about the availability of data on the markup or instruments for the markup.

### 3.1 Identification of Overall Shares $(\theta_Y, \theta_N)$

Our starting point is the expression for factor shares in the version of the model with capital, described in Section 2.7.3. Throughout our empirical analysis we restrict attention to the case where all capital is used in the wholesale sector for  $Y$ -type activities and capital is not used for  $N$ -type activities ( $\alpha_N = 0$ ).<sup>2</sup> Under this assumption, the factor share equation can be re-arranged to give the following expression relating the markup to the labor share,

$$S_L = \theta_N + (\theta_Y (1 - \alpha_Y) - \theta_N) \frac{1}{\mu}. \quad (1)$$

Given data on the markup  $\mu$ , in which variation arises from any of the sources described in Section 2.6 that do not involve a change in  $(\theta_N, \theta_Y)$ , equation (1) implies that we can recover  $(\theta_N, (1 - \alpha_Y) \theta_Y)$  from the average levels of the markup and the labor share, and the co-movement of the labor share with the markup. Intuitively, Lemma 1 showed that the factor shares are determined by  $(\theta_N, \theta_Y, \mu)$  and thus three moments are required. The levels of the labor share and the markup provide two of these. The third moment exploits the insight of Theorem 2: the sign and strength of co-movement between the labor share and the markup identifies the gap between  $\theta_N$  and  $\theta_Y (1 - \alpha_Y)$ . In order to separately identify  $\theta_Y$  from  $\alpha_Y$  we also need to make an assumption about the average capital share relative to the average profit share.

According to equation (1), time-variation in the measured labor share could arise because of variation in the markup due to one of the forces described in Section 2.6, because of variation in the production parameters or because of measurement error. Allowing for non-linear deterministic trends in the production parameters and the markup, we thus obtain the following estimating equation,

$$\begin{aligned} S_{L,t} &= \theta_{N,t} + (\theta_{Y,t} (1 - \alpha_Y) - \theta_{N,t}) \frac{1}{\mu_t} + \epsilon_{L,t} \\ \theta_{N,t} &= g_{\theta_N}(\beta_{\theta_N}, t) \\ \theta_{Y,t} &= g_{\theta_Y}(\beta_{\theta_Y}, t) \\ \frac{1}{\mu_t} &= g_{\mu}(\beta_{\mu}, t) + \epsilon_{\mu,t}. \end{aligned} \quad (2)$$

---

<sup>2</sup>Identifying the capital share parameters  $(\alpha_Y, \alpha_N)$  would require us to observe capital income and economic profits separately at a quarterly frequency. If this were possible, we would be able to identify both parameters from the level of the capital share and its co-movement with the markup, analogously to how we use the co-movement of the labor share with the markup to identify  $(\theta_Y, \theta_N)$ . However, splitting accounting profits into these components is a challenging task. As it stands, we need to make an assumption about the *average* profit share and capital share. Separating these components at a high frequency is beyond the scope of this paper. See Karabarbounis and Neiman (2019) for further discussion of this issue.

Measurement error in the labor share is captured by  $\epsilon_{L,t}$  and flexible deterministic trends are captured by the parametric functions  $g_i(\beta_i, t) \forall i \in \{\theta_N, \theta_Y, \mu\}$ . Hence the labor share could exhibit a trend, because of either a trend in the production parameters or the markup.

The assumption required for identification of  $\{\beta_{\theta_N}, \beta_{\theta_Y}, \beta_{\mu}\}$  is that de-trended variation in the inverse markup  $\epsilon_{\mu,t}$  is independent of other stochastic variation in the labor share  $\epsilon_{L,t}$

$$E[\epsilon_{L,t}] = 0 \quad \forall t \tag{3}$$

$$E[\epsilon_{L,\tau} | \epsilon_{\mu,t}] = 0 \quad \forall (t, \tau). \tag{4}$$

These moment conditions form the basis of our estimation strategy.

An important assumption is that the model parameters other than the markup ( $\alpha_Y, \theta_Y, \theta_N$ ) vary at a lower frequency than the de-trended variation in the markup ( $\epsilon_{\mu,t}$ ). In our estimation, we impose this by removing the trend in the markup in a first stage, and then using a de-trended series for the markup that isolates variation at business cycle frequencies. Thus our implicit assumption is that the technological parameters ( $\alpha_Y, \theta_Y, \theta_N$ ) do not vary at business cycle frequencies. This assumption is pervasive in the business cycle literature - it is imposed, either explicitly or implicitly, in almost all existing applications that use Real Business Cycle and New Keynesian models. We also use a de-trended series for the labor share in our estimation, which we remove in a first stage. Hence we implicitly treat the trends in  $\theta_N$  and  $\theta_Y$  as nuisance parameters. Equation (2) implies that we can interpret the trend that we remove from the labor share as a combination of the trends in  $\theta_N$  and  $\theta_Y$ , the trend in the markup and the structural parameters.

### 3.2 Labor Share and Markup Data

In order to implement this estimation approach, we require data on the labor share  $S_{L,t}$  and the markup  $\mu_t$ .

**Labor share data.** We construct our baseline measure of the labor share using quarterly data from the National Economic Statistics produced by the Bureau of Economic Analysis, following the procedures in [Gomme and Rupert \(2004\)](#) to adjust for ambiguous components of income. We use data for 1947:Q1 to 2019:Q2 and de-trend using an HP-filter with a smoothing parameter of 1600. All of our estimates are robust to using alternative measures of the labor share and alternative methods for de-trending. [Appendix D.1](#) contains full details of the construction of the series and [Appendix D.2](#) reports estimates using alternative data series for the labor share.

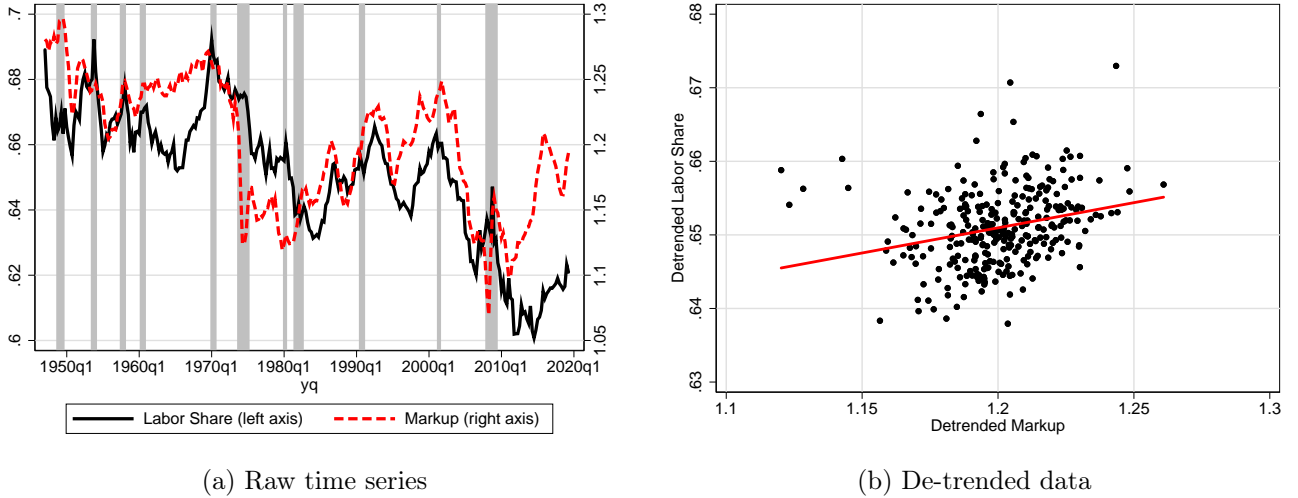


Figure 1: Panel (a): Raw time series for labor share and markup. Labor share (left axis) is defined is computed from BEA data following [Gomme and Rupert \(2004\)](#). Markup (right axis) is defined as ratio of PPI series WPSFD49207 to series WPSID61. Shaded areas are NBER recessions. Panel (b): De-trended labor share and de-trended markup. Labor share and inverse markup are de-trended with a HP-filter with smoothing parameter 1600.

The raw series for the labor share is displayed in Figure 1a (black solid line, left axis). The mean of this series over the sample is 65.1%. The series displays the well-documented downward trend in the labor share, from an average of 67.0% pre-1960 to 61.2% post-2010. The labor share is also counter-cyclical, which can be seen in Figure 1a, by noting that the NBER recessions (shaded grey areas) typically correspond to local maxima of the series. The correlation of the de-trended labor share series with de-trended log per-capita real output is  $-0.13$ . The counter-cyclicity of the labor share is consistent with a large body of evidence, and is one of the key reasons that we will estimate  $\theta_N > \theta_Y (1 - \alpha_Y)$  and conclude that part of the US labor force is engaged in  $N$ -type activities.

We do not attempt to split the remaining 34.9% of national income between capital income and economic profits. For our baseline estimates, we simply assume that the overall profit share is 10%, and we report the sensitivity of our estimates to assuming a profit share between 5% and 15%. The assumption about the mean profit share affects only the estimates of  $\alpha_Y$  and  $\theta_Y$ , and does not affect the estimate of the overall fraction of labor income compensating  $N$ -type activities,  $\frac{S_{LN}}{S_L}$ .

**Markup data.** Estimating markups and their co-movement with aggregate economic activity has a long and controversial history in economics. We refer the reader to the recent paper by [Nekarda and Ramey \(2019\)](#) for an excellent discussion of the relevant literature

and different approaches to estimating the dynamics of the markup at business cycle frequencies.<sup>3</sup> Unfortunately, none of the approaches used in this literature are appropriate in the context of our model. Most existing approaches require the researcher to specify which are the variable factors that are used in the production of goods (as opposed to revenue). In our model that would require distinguishing between  $N$ -type versus  $Y$ -type labor as a pre-requisite to estimating the markup. Instead, we desire to go in the opposite direction and to learn about the split between  $N$ -type versus  $Y$ -type from movements in the markup. We thus we cannot adopt one of these existing methods and use it as an input to our estimation procedure.<sup>4</sup>

Starting with [Bils \(1987\)](#), the most common approach to investigating the cyclicity of markups is to use the mapping between the labor share and the markup to infer movements in the markup from movements in the labor share. This general idea also underlies the approach taken by [Nekarda and Ramey \(2019\)](#). But in our framework, the mapping between the markup and the labor share is ambiguous since it depends on  $(\theta_Y, \theta_N)$ . Indeed, this entire literature has worked with production structures in which the markup and the labor share are inversely related, effectively assuming that  $\theta_N < \theta_Y (1 - \alpha_Y)$  (typically that  $\theta_N = 0$ ). Clearly, we cannot use such estimates of the markup to estimate the relative size of  $((1 - \alpha_Y) \theta_Y, \theta_N)$ .

Since we require a markup series that has been constructed without assumptions on the production function, we instead construct a measure of markups by comparing the prices of goods at different stages of the production process, similarly to [Barro and Tenreyro \(2006\)](#). In our model, the markup is defined as the ratio of the retail price (i.e the price of differentiated final goods that are sold to consumers) to the wholesale price (i.e. the price of undifferentiated good produced from raw materials, capital and labor). The Bureau of Labor Statistics’s (BLS) Producer Price Index (PPI) program produces prices indexes under its Production Flow structure that closely match these definitions and date to 1947. We construct a series for the markup by taking the ratio of the series for “Finished demand” (WPSFD49207), which measures the price changes of goods for (i) personal consumption and (ii) capital investment, and the series for “Processed goods for intermediate demand” (WPSID61), which measures the price changes of (i) partially processed goods that have to undergo further processing before they can be sold to the public and (ii) supplies consumed by businesses.

---

<sup>3</sup>[Nekarda and Ramey \(2019\)](#) classify four approaches that have been used to estimate a time-series for the markup: (i) using direct data on price and average variable costs; (ii) generalizations of the Solow residual as in [Hall \(1986\)](#); (iii) generalized production functions with quasi-fixed factors; (iv) using factor share equations, adjusted for fixed costs.

<sup>4</sup>In a companion note, [Kaplan and Zoch \(2020\)](#), we explain why the methods used in [De Loecker and Warzynski \(2012\)](#) and [De Loecker et al. \(2019\)](#) cannot be used in the context of our model, unless one could separately observe  $N$ -type and  $Y$ -type workers.

Our markup measure is constructed by comparing two indexes. It thus allows us to measure changes in the markup but does not provide an estimates of the *level* of the markup, which is also required for our estimation procedure. So for our baseline estimates, we simply assume that the average markup over the post-war period is 1.2, and we show how our estimates are affected by assuming a value between 1.05 and 1.35. It turns out that the level of the markup has very little effect on the estimates of  $(\theta_N, \theta_Y, \alpha_Y)$ , since these are identified by the co-movement of the markup with the labor share, and the level of the profit share. However, the level of the markup does matter for the implied estimate of  $\frac{S_{LN}}{S_L}$ .

The raw markup series is displayed in Figure 1a (red dashed line, right axis), re-scaled to have a mean of 1.2. The raw series displays a slight downward trend. The markup is also counter-cyclical and co-moves positively with the labor share at business cycle frequencies. The correlation of the de-trended markup with de-trended log per-capita real output is  $-0.28$ , and the correlation with the de-trended labor share is  $0.28$ . This positive co-movement between the markup and the labor share is the key feature of the data suggesting that  $\theta_N \geq (1 - \alpha_Y) \theta_Y$  and that the share of  $N$ -type labor is positive.

It is important to note that the counter-cyclical of our markup series is not at odds with the conclusions in Nekarda and Ramey (2019) that the markup based on labor compensation is pro-cyclical, since this is simply the inverse of the labor share which is also pro-cyclical in our data.

### 3.3 Estimates of Overall Shares $(\theta_Y, \theta_N)$

**Baseline estimates** We estimate equation (2) by OLS, subject to the constraints that  $\theta_N, \theta_Y \leq 1$  (which do not bind at our estimates). The scatter plot corresponding to this regression, which illustrates the positive co-movement of the labor share and the markup, is displayed in Figure 1b. Our baseline estimates are shown in the first column of Table 1. We estimate a value for  $\theta_N$  of 0.73 and a value for  $\theta_Y$  of 0.93, implying a value for  $\theta_Y(1 - \alpha_Y) = 0.64 < \theta_N$ . Given our assumptions of a profit share of 10% and a mean markup of 1.2, our estimates imply that roughly 19% of labor income compensates activities that are better characterized as  $N$ -type than  $Y$ -type.

The second and third columns of Table 1 show that if we assume a higher or lower profit share than in the baseline, neither our estimates of  $\theta_N$  and  $\theta_Y(1 - \alpha_Y)$ , nor the implied share of  $N$ -type labor income, are affected. The only effect is to change the split of non-labor income in the  $Y$  sector between profits and capital income. The fourth and fifth columns show that the estimate of the share of labor income compensating  $N$ -type activities is sensitive to the assumption about the mean markup. If one believes that the mean level



	(1)	(2)	(3)	(4)	(5)
	Baseline	Low Profit Share	High Profit Share	Low Markup	High Markup
$\theta_Y$	0.934 (0.005)	0.994 (0.005)	0.874 (0.005)	0.908 (0.001)	0.963 (0.008)
$\theta_N$	0.730 (0.024)	0.730 (0.024)	0.730 (0.024)	0.741 (0.027)	0.721 (0.021)
Implied value of $\frac{S_{N,L}}{S_L}$	19%	19%	19%	5%	29%
P-val for test $\theta_N = \theta_Y (1 - \alpha_Y)$	0.00	0.00	0.00	0.00	0.00
Assumed mean markup, $\mu$	1.20	1.20	1.20	1.05	1.35
Assumed profit share, $S_{\Pi}$	10%	5%	15%	10%	10%
Capital share parameter, $\alpha_Y$	0.320	0.361	0.273	0.288	0.349

Table 1: First stage estimation results

of the markup is lower (higher) than 1.2, then one would obtain lower (higher) estimates of this share. However, the estimates of the production function parameters  $\theta_Y, \theta_N$  are not sensitive to the assumed level of the markup. Tables 4 and 5 in Appendix D.2 show that none of these results are sensitive to using alternative series for the labor share or alternative assumptions about how we de-trend the data.

**Changes over time** In order to understand whether the share of labor income compensating expansionary activities has changed over time, we repeat the estimation using 23-year rolling windows that span our sample period. The estimates are displayed in Table 2 and Figure 2.

This exercise requires us to make an assumption about how the markup and the labor share has trended over time. Recall that our estimation strategy removes trends from both the markup and labor share series and uses only cyclical variation to identify the structural parameters. Given the broad consensus on the downward trend in the labor share, we incorporate this trend into our estimates (although it matters very little for the results). But given the recent debate over the trend in the markup, we prefer not to take a strong stand on the direction of this trend. Accordingly, in Panel A of Table 2 we report estimates under the assumption that the markup has remain constant at 1.2, and in Panel B we report estimates under the assumption that the markup has followed the broad downward trend in our markup series.

	$\theta_Y$	$\theta_N$	$\theta_N - \theta_Y (1 - \alpha_Y)$	P-val test $\theta_N =$ $\theta_Y (1 - \alpha_Y)$	Implied $\frac{S_{LN}}{S_L}$	Assumptions $\mu$ $S_L$	
<hr/> Panel A: Fixed Markup <hr/>							
1947-1969	0.932	0.853	0.219	0.016	21%	1.20	67.0%
1957-1979	0.874	1.000	0.406	0.000	25%	1.20	66.2%
1967-1989	0.999	0.632	-0.048	0.262	16%	1.20	67.1%
1977-1999	0.947	0.646	0.002	0.977	17%	1.20	64.5%
1987-2009	0.914	0.805	0.183	0.000	21%	1.20	65.2%
1997-2019	0.902	0.712	0.098	0.004	19%	1.20	63.0%
<hr/> Panel B: Changing Markup <hr/>							
1947-1969	0.917	0.842	0.219	0.016	27%	1.27	67.0%
1957-1979	0.845	1.000	0.425	0.000	31%	1.26	66.2%
1967-1989	1.000	0.632	-0.048	0.262	17%	1.22	67.1%
1977-1999	0.947	0.646	0.002	0.977	13%	1.14	64.5%
1987-2009	0.915	0.805	0.183	0.000	20%	1.20	65.2%
1997-2019	0.905	0.714	0.098	0.004	16%	1.16	63.0%

Table 2: First stage estimation results, 23-year rolling windows. Capital share parameter  $\alpha_Y$  assumed equal to 0.32 throughout.

The time series for the full set of rolling window estimates in Figure 2 indicates a broadly U-shaped pattern for both  $\theta_N$  and  $\frac{S_{LN}}{S_N}$ , irrespective of whether the average markup is flat or declining over this period. If one believed that the average markup had risen over this period, then the estimates would imply a steeper increase in the labor income share of  $N$ -type labor since the 1980's. These patterns reflect the changing cyclical co-movement of the markup and the overall labor share during the post-war period. In periods where the cyclical co-movement was more positive, such as before 1970 (correlation = 0.42) and after 1990 (correlation = 0.40), we infer a higher share of  $N$ -type labor. In periods where the cyclical co-movement was weaker, such as from 1970 to 1990 (correlation = -0.01), we infer a lower share.

**Taking Stock** What should we take away from these estimates? Almost all existing models of the business cycle implicitly assume that  $\theta_N = 0$  and that all workers perform  $Y$ -type activities. Our estimates imply that  $\theta_N$  is substantially larger than zero; in fact it is larger even than the labor share in the production sector,  $\theta_Y (1 - \alpha_Y)$ . This means that unless one believes that markups are very small, one must conclude that a non-trivial fraction of

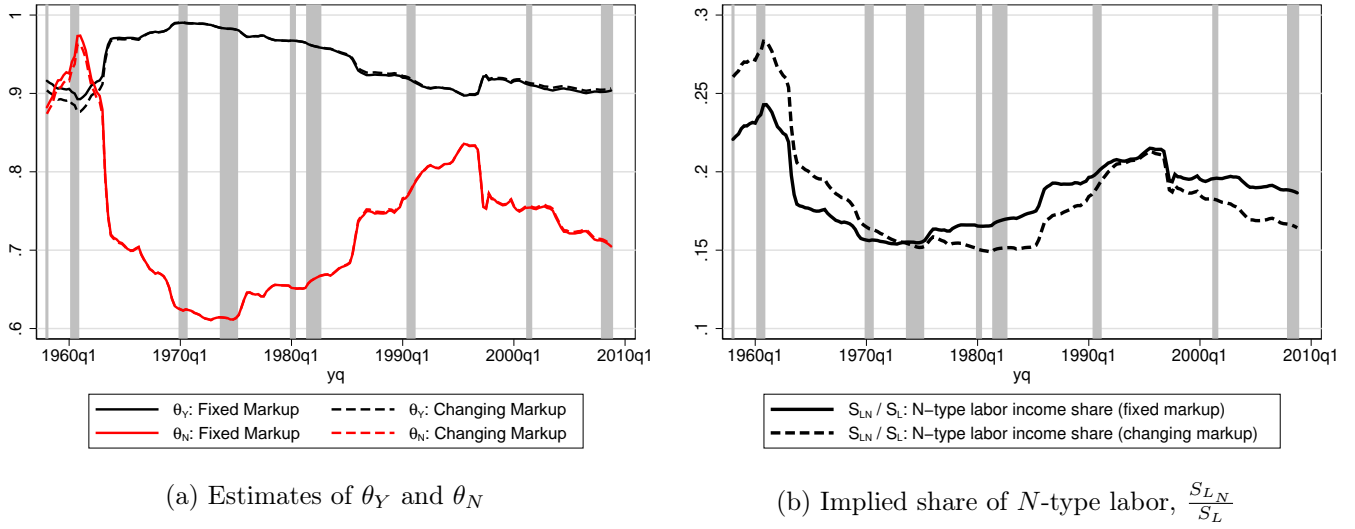


Figure 2: Estimates from 23-year rolling windows. Windows are centered. Capital share parameter  $\alpha_Y$  assumed equal to 0.32 throughout. Panel (a): Estimates of  $\theta_Y$  and  $\theta_N$  assuming a constant markup (solid lines) and markup changing as in Figure 1a (dashed lines). Panel (b): Implied share of labor income compensating  $N$ -type labor assuming a constant markup (solid line) and markup changing as in Figure 1a (dashed line).

the US labor force is engaged in  $N$ -type activities, which has two important implications. First, it explains the positive co-movement of markups with the labor share and, as we will see below, the negative co-movement of monetary shocks with the labor share. Second, it means that some workers stand to gain and some workers stand to lose from changes in markups, whether they be cyclical changes induced by shifts in aggregate demand, or structural changes induced by changes in the competitiveness of product markets. In the next section, we shed light on which workers are which.

## 4 Estimation of Occupation-Specific Parameters

In Section 2, we distinguished between  $Y$ -type labor, whose share of income falls when the markup rises, and  $N$ -type labor, whose share of income rises when the markup rises. In Section 3, we showed that the co-movement of the markup and the labor share implies that some workers in the economy are better characterized as  $N$ -type workers than  $Y$ -type workers. The goal of this section is to learn about which workers in the US economy these are.

## 4.1 Occupational Framework

We assume that there are a fixed set of  $J$  occupations, indexed by  $j = 1 \dots J$ . Each occupation is defined by a pair  $(\eta_{jY}, \eta_{jN})$  which describe the share of each occupation out of the total labor input that is needed to produce output  $Y$  and manage product lines  $N$ , respectively. The production functions are Cobb-Douglas in labor of each occupation and in capital,

$$\begin{aligned} Y &= Z_Y \left[ K_Y^{\alpha_Y} \left( \prod_{j=1}^J L_{jY}^{\eta_{jY}} \right)^{1-\alpha_Y} \right]^{\theta_Y} \\ N &= Z_N \left[ K_N^{\alpha_N} \left( \prod_{j=1}^J L_{jN}^{\eta_{jN}} \right)^{1-\alpha_N} \right]^{\theta_N}. \end{aligned}$$

The occupation factor share parameters are jointly restricted by  $\eta_{jY}, \eta_{jN} \geq 0$  and  $\sum_{j=1}^J \eta_{jY} = \sum_{j=1}^J \eta_{jN} = 1$ . Labor market clearing requires that the total quantity of labor supplied in occupation  $j$ , denoted by  $L_j$  is equal to the sum of the quantities of labor from occupation  $j$  demanded for production and expansion,

$$L_j = L_{jY} + L_{jN} \quad \forall j.$$

We show in Appendix A.5 that the income share of each occupation in total income, which we denote by  $S_j$ , can be expressed as a weighted sum of the labor income shares of the two sectors

$$S_j = \eta_{jY} S_{Y,L} + \eta_{jN} S_{N,L}, \quad (5)$$

where  $S_{Y,L} = (1 - \alpha_Y) S_Y$  is the income share for  $Y$ -type labor and  $S_{N,L} = (1 - \alpha_N) S_N$  is the income share for  $N$ -type labor. If  $S_{Y,L}$  and  $S_{N,L}$  were directly observed, it would be straightforward to estimate the occupation factor share parameters  $\{\eta_{jY}, \eta_{jN}\}_{j=1}^J$ , by projecting occupational income shares onto these variables. Since  $S_{Y,L}$  and  $S_{N,L}$  are not directly observed, we instead infer the occupation factor shares parameters from the co-movement of occupational income shares  $S_j$  with markup-induced variation in the overall labor share  $S_L$ . We explain this identification strategy in the next section.

## 4.2 Identification of Occupational Factor Shares $\{\eta_{jY}, \eta_{jN}\}_{j=1}^J$

Combining equation (5) with the expressions for factor shares yields an expression that relates the share of total labor income going to occupation  $j$  (denoted by  $s_j := \frac{S_j}{S_L}$  with

$\sum_j s_j = 1$ ) to the overall labor share,

$$s_j = \eta_{jY} + (\eta_{jN} - \eta_{jY}) \left(1 - \frac{\theta_Y (1 - \alpha_Y)}{\theta_N}\right)^{-1} \left(1 - \theta_Y (1 - \alpha_Y) \frac{1}{S_L}\right) \quad \forall j. \quad (6)$$

As in Section 3, we assume that  $\alpha_N = 0$ . Equation (6) shows that time variation in occupational labor income shares could arise from time-variation in  $(\eta_{jN}, \eta_{jY})$ , or from the variation in  $(\theta_N, \theta_Y, S_L)$  described in the previous section. Given knowledge of  $(\theta_N, \theta_Y)$ , we can recover  $(\eta_{jN}, \eta_{jY})$  from the average occupational labor income shares and the co-movement of occupational labor income shares with any variation in the labor share  $S_L$  that is independent of variation in the occupational factor share parameters  $(\eta_{jN}, \eta_{jY})$ .

We derive an estimating equation by allowing for shocks and non-linear deterministic trends in the occupational factor share parameters, and for measurement error in the occupational labor income shares. Hence (6) becomes

$$s_{j,t} = \eta_{jY,t} + (\eta_{jN,t} - \eta_{jY,t}) \left(1 - \frac{\theta_Y (1 - \alpha_Y)}{\theta_N}\right)^{-1} \left(1 - \theta_Y (1 - \alpha_Y) \frac{1}{S_{L,t}}\right) + \epsilon_{s_j,t} \quad \forall j \quad (7)$$

$$\eta_{jY,t} = g_{\eta_{jY}}(\beta_{\eta_{jY}}, t) + \epsilon_{jY,t}$$

$$\eta_{jN,t} = g_{\eta_{jN}}(\beta_{\eta_{jN}}, t) + \epsilon_{jN,t},$$

where  $\epsilon_{Y,t} := (\epsilon_{1Y,t}, \dots, \epsilon_{JY,t})'$ ,  $\epsilon_{N,t} := (\epsilon_{1N,t}, \dots, \epsilon_{JN,t})'$  and  $\epsilon_{s,t} := (\epsilon_{s_1,t}, \dots, \epsilon_{s_J,t})'$  are mutually independent  $J \times 1$  random vectors that are IID over time and each sum to zero. We define  $\epsilon_{j,t} := (\epsilon_{jY,t}, \epsilon_{jN,t}, \epsilon_{s_j,t})'$  and assume that  $E[\epsilon_{j,t}] = 0 \quad \forall t, \forall j$ .<sup>5</sup> We now describe three different sets of moment conditions that can be used as the basis for estimation. Each differs in the type of variation in the labor share that it exploits.

**De-trended Markup** From equations (1) and (6), we see that movements in the markup affect occupational labor shares only through their effect on the overall labor share. Thus a valid source of variation in the labor share that can be used for identification is markup-induced variation. To exploit such variation we can use the markup, or lags of the markup, as an instrument for the de-trended labor share in 7. This identification strategy thus imposes the moment condition

$$E[\epsilon_{j,\tau} | \epsilon_{\mu,t}] = 0 \quad \forall (t, \tau), \quad \forall j. \quad (8)$$

---

<sup>5</sup>An example that satisfies these assumptions is that  $\epsilon_{Y,t}$ ,  $\epsilon_{N,t}$  and  $\epsilon_{s,t}$  are each drawn from translated Dirichlet distributions. We also implicitly assume that the trends  $\{g_{\eta_{jY}}(t), g_{\eta_{jN}}(t)\}_{j=1}^J$  are such that the adding-up and non-negativity constraints on  $\{\eta_{jY,t}, \eta_{jN,t}\}_{j=1}^J$  hold for all  $t$ .

In our baseline specification we use the contemporaneous de-trended markup as an instrument for the de-trended inverse labor share. We choose this specification as our baseline because it uses the same variation to estimate the occupational factor share parameters  $\{\eta_{jY}, \eta_{jN}\}_{j=1}^J$  as we used to estimate the overall factor share parameters  $(\theta_Y, \theta_N)$  in Section 3.

**De-trended Labor Share** A simple alternative approach is to assume that the de-trended inverse labor share itself is orthogonal to shocks to the occupational factor share parameters and to measurement error in the occupational income shares  $s_{j,t}$ . Formally, this means assuming that

$$S_{L,t} = g_{S_L}(\beta_{S_L}, t) + \epsilon_{S_L,t}$$

and imposing the moment condition

$$E[\epsilon_{j,\tau} | \epsilon_{S_L,t}] = 0 \quad \forall (t, \tau), \quad \forall j \tag{9}$$

Equation (9) says that any movements in occupational labor shares  $s_{j,t}$  that are correlated with movements in the overall labor share  $S_{L,t}$  must be at lower frequencies than are captured by the trend in the labor share  $g_{S_L}(\beta_{S_L}, t)$ . Recall from equation (6) that the sources of the trend in the labor share  $g_{S_L}(\beta_{S_L}, t)$  are the trends in the production parameters and the markup. Hence this assumption requires that business cycle variation in the labor share does not arise from sources that directly affect the occupational income shares, other than through the channel in equation (6). Through the lens of the model, this means that business cycle variation in the labor share must come from variation in the markup  $\mu$ , rather than from the technology parameters  $(\theta_Y, \theta_N)$ . Assuming that technological parameters are fixed at business cycle frequencies is a common assumption.

We also need to assume that any shocks to individual occupation shares  $(\epsilon_{Y,t}, \epsilon_{N,t})$  are independent of shocks to the overall labor share  $\epsilon_{S_L,t}$ . This is a relatively weak assumption because the random vectors  $(\epsilon_{Y,t}, \epsilon_{N,t})$  each sum to zero - so failure of this assumption would require a re-shuffling of occupations at exactly the same time as a shock to the labor share, without any change in the markup. Finally, we need to assume that measurement error in the occupational labor income shares is independent of measurement error in the overall labor share. This assumption is likely to be satisfied because we use different data sources to measure  $s_{j,t}$  and  $S_{L,t}$ , rather than constructing a measure of  $S_{L,t}$  by summing over  $S_{j,t}$ .

**Monetary Policy Shocks** Given estimates of  $(\theta_Y, \theta_N)$ , it is also possible to estimate the occupational factor shares without data on the markup. Assume that a variable  $Z_t$  is

available, which is related to the inverse markup by

$$\frac{1}{\mu_t} = g_\mu(\beta_\mu, t) + \gamma Z_t + \epsilon_{\mu,t}, \quad (10)$$

where  $\gamma \neq 0$ , so that there is a valid first-stage, and with  $E[Z_t] = 0$ . Then if the moment condition

$$E[\epsilon_{j,\tau} | Z_t] = 0 \quad \forall (t, \tau), \quad \forall j \quad (11)$$

holds, we can estimate  $(\eta_{jN}, \eta_{jY})$  by using  $Z_t$  as instrument for the labor share in Equation (7). This assumption requires that the instrument  $Z_t$  only affects the occupational labor income shares through its effect on the overall labor share, which in our framework can only occur if the instrument causes a change in the markup.

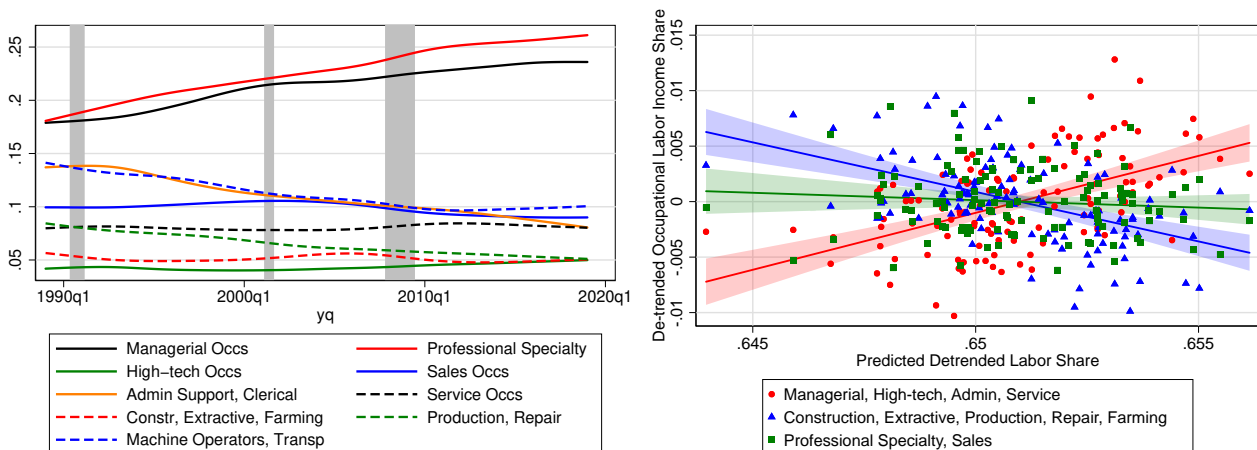
Two types of variables that are likely to satisfy these assumptions are monetary and fiscal policy shocks. In general equilibrium models with sticky prices, such as New Keynesian DSGE models, contractionary monetary and fiscal policy shocks (as well as other contractionary demand shocks) generate a rise in the markup (so  $\gamma \neq 0$ ), and do not affect the labor share except through their effect on the markup.<sup>6</sup>

Monetary policy shocks in particular are a good candidate instrument for the labor share. [Cantore et al. \(2019\)](#) undertake a comprehensive empirical investigation of the dynamic effects of a monetary policy shock on the labor share. Using various different strategies for identifying monetary policy shocks, they document robust evidence that a contractionary monetary shock leads to an increase in the labor share, with a peak response after 1-2 years. They also show that standard New Keynesian models (with  $\theta_N = 0$ ) cannot re-produce those dynamics, exactly because a contractionary monetary shock is associated with an increase in the markup, which is incompatible with a fall in the labor share in standard models. However, in a New Keynesian model with our production structure, and the parameters estimated in Section 3, a contractionary monetary shock does lead to rise in the labor share. Thus, given  $\theta_N > \theta_Y(1 - \alpha_Y)$ , monetary policy shocks can be used as an instrument for the labor share.

Based on the dynamic responses in [Cantore et al. \(2019\)](#) we use three monetary shock series produced with three different strategies: (i) [Romer and Romer \(2004\)](#), extended by [Coibion et al. \(2017\)](#), (ii) [Miranda-Agrippino and Ricco \(2018\)](#), and (iii) [Gertler and Karadi \(2015\)](#). For each series we include lags at horizons of 4 to 8 quarters, for a total of 15 instruments, and estimate the parameters using optimally-weighted GMM. The instrument

---

<sup>6</sup>This is true in a broad class of New Keynesian models. See for example [Christiano et al. \(2005\)](#), [Smets and Wouters \(2007\)](#) and [Galí et al. \(2015\)](#).



(a) Trend components of occupational share  $s_j$ . (b) Cyclical components of occupational shares  $s_j$ .

Figure 3: Panel (a) Trend components of occupational labor income shares. Trends extracted using an HP-filter with smoothing parameter = 1600. Panel (b): Horizontal axis is predicted value of overall labor share from OLS regression of labor share on markup. Vertical axis is cyclical component of occupational labor income shares. Occupations grouped into three broad categories based on estimates of  $N$ -intensity in table 3.

set has a strong first stage and we fail to reject the test of over-identification restrictions.

### 4.3 Data on Occupational Income Shares

We use data from the monthly Outgoing Rotation Groups of the Current Population Survey (CPS-ORG) to construct quarterly series for occupational income shares  $s_j$ . We restrict attention to employed individuals aged 16 and over and measure labor income with the IPUMS variable ‘earnweek’, which records gross weekly earnings on the respondent’s main job. We compute labor income for individuals in each of 9 broad occupation categories, which we construct from the 389 OCC1990 occupation codes. We then aggregate monthly earnings in each occupation to the quarterly level and compute the occupation shares  $s_j$  in each quarter from 1989 to 2018. We de-seasonalize the quarterly series and then detrend using an HP-Filter. The trend components for each of the 9 occupation groups are displayed in Figure 3a. Managerial and professional specialty occupations display the strongest growth in income shares in our sample, while machine operators, transportation, administrative and clerical occupations show the steepest decline.

Figure 3b shows a scatter plot of the cyclical components of the occupational income shares that illustrates our identification strategy. To construct this figure we split the occupations into three broad groupings, based on our baseline estimates of the occupational



factor share parameters. For each of the three groups, we plot the de-trended labor income share of those occupations against the predicted value of the de-trended overall labor share, from an OLS regression of the labor share on the markup. Thus the fitted line reflects the IV estimate of equation (7) using the markup as an instrument, and the slope of the relationship reveals the sign of  $\eta_{jN} - \eta_{jY}$ .

The group represented by the red circles in Figure 3b consists of the most  $N$ -intensive occupations and comprises approximately 45% of total labor income. The relatively large  $N$ -component for these occupations is revealed by the fact that markup-induced variation in the overall labor share is associated with an increase in the share of labor income going to these occupations. In contrast, the group represented by the blue triangles consists of the least  $N$ -intensive occupations and comprises approximately 23% of total labor income. The relatively small  $N$ -component for these occupations is revealed by the fact that markup-induced variation in the overall labor share is associated with a decrease in the share of labor income going to these occupations. The remaining occupations represented by the green squares are intermediate occupations whose share of labor income is roughly unaffected by markup-induced variation in the overall labor share. Figure 6 in Appendix D.3 shows very similar patterns for the OLS relationship between the de-trended labor share and occupational income shares, and for the reduced-form relationship between the markup and occupational income shares.

#### 4.4 Estimates of Occupational Factor Shares

We estimate  $\{\eta_{jY}, \eta_{jN}\}_{j=1}^J$  using a GMM estimator, based on the moment conditions outlined in Section 4.2. Table 3 displays our baseline estimates using each of the three types of variation described in Section 4.2. In each specification we set  $(\theta_Y, \theta_N, \alpha_Y, \mu) = (0.934, 0.730, 0.320, 1.2)$ , based on the estimates in Table 1. Appendix D.3 contains estimates from additional specifications.

Our estimates using the de-trended markup as an instrument for the de-trended inverse labor share are shown in Panel A of Table 3. The occupations are ordered from the most  $N$ -intensive to the least  $N$ -intensive, as measured by the share of occupational labor income that compensates  $N$ -type activities,  $\frac{S_{jN}}{S_j} = \frac{\eta_{jN}S_N}{S_j}$ . As anticipated by Figure 3b, there is heterogeneity across occupations, with  $N$ -content shares ranging from 24% for high-tech occupations to 13% for machine operators and transportation. It is striking that the ranking of occupations lines up with traditional notions of white-collar versus blue-collar occupations, yet these estimates were obtained entirely from the relative co-movement of occupational income shares with markups, and no prior knowledge of the tasks that these occupations actually do was used in constructing this ranking.

	$\eta_Y$	$\eta_N$	P-val $\eta_Y = \eta_N$	Elasticity $\varepsilon_{S_j, S_L}$	Share $\frac{S_{jN}}{S_j}$	P-val overid
Panel A: Instrument: De-trended Markup (IV)						
High-tech Occs	0.041	0.055	0.027	3.38	24%	
Service Occs	0.078	0.094	0.050	2.54	22%	
Admin, Clerical	0.105	0.127	0.014	2.49	22%	
Managerial Occs	0.206	0.243	0.007	2.30	21%	
Prof. Specialty	0.227	0.226	0.909	0.96	19%	
Sales Occs	0.100	0.090	0.083	0.21	17%	
Production, Repair	0.068	0.051	0.022	-0.92	15%	
Constr., Extract., Farm	0.054	0.038	0.014	-1.38	14%	
Machinists, Transp.	0.121	0.076	0.002	-1.98	13%	
First stage: R2	0.16					
First stage F	11.2					
Panel B: Instrument: De-trended Labor Share (OLS)						
High-tech Occs	0.043	0.046	0.194	1.60	20%	
Service Occs	0.080	0.082	0.398	1.19	19%	
Admin, Clerical	0.108	0.117	0.025	1.65	20%	
Managerial Occs	0.211	0.220	0.068	1.30	19%	
Prof. Specialty	0.227	0.228	0.731	1.05	19%	
Sales Occs	0.099	0.096	0.374	0.79	18%	
Production, Repair	0.065	0.062	0.122	0.58	18%	
Constr., Extract., Farm	0.052	0.047	0.021	0.17	17%	
Machinists, Transp.	0.115	0.102	0.000	0.13	17%	
Panel C: Instrument: Lagged Monetary Shocks (GMM)						
High-tech Occs	0.043	0.047	0.287	1.68	20%	0.214
Service Occs	0.079	0.088	0.022	1.80	20%	0.556
Admin, Clerical	0.105	0.126	0.000	2.39	22%	0.287
Managerial Occs	0.210	0.224	0.060	1.48	20%	0.650
Prof. Specialty	0.224	0.239	0.067	1.51	20%	0.341
Sales Occs	0.101	0.088	0.006	0.04	17%	0.222
Production, Repair	0.066	0.061	0.096	0.43	18%	0.670
Constr., Extract., Farm	0.054	0.039	0.001	-1.19	14%	0.437
Machinists, Transp.	0.117	0.092	0.000	-0.68	15%	0.244
First stage: R2	0.16					
First stage F	3.14					

Table 3: Stage 2 estimates of occupational factor share parameters. Estimates in Panel A use de-trended markup as an instrument for de-trended inverse labor share. Estimates in Panel B use OLS. Estimates in Panel C use optimally-weighted GMM with lagged monetary policy shocks at horizons of four to eight quarters, from three series. See text for details of monetary policy shock series.

The relatively small, but statistically significant, differences between  $\eta_{jN}$  and  $\eta_{jY}$ , manifest as large differences across occupations in their exposure to fluctuations in the overall labor share. This can be seen in the column labelled  $\varepsilon_{S_j, S_L}$ , which reports the elasticity of occupational income shares to the overall labor share, implied by the estimates of  $(\eta_{jY}, \eta_{jN})$ . The share-weighted average of these elasticities sums to one. High  $N$ -content occupations have an elasticity above one, whereas low  $N$ -content occupations have an elasticity below one (and even negative for some occupations).

The remaining panels of Table 3 report estimates without an instrument (Panel B) and using monetary shocks as an instrument (Panel C). The occupations are reported in the same order as in Panel A. In both cases, the results are very similar. The main difference is that when all of the cyclical variation in the labor share is used (Panel B), the additional variation leads to less precise estimates and less heterogeneity across occupations.

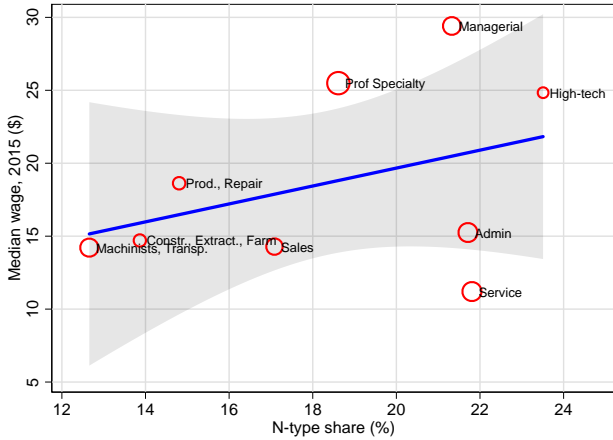
## 4.5 Characteristics of $N$ -intensive Occupation

Having estimated the extent to which workers in different occupations are engaged in production activities versus expansionary activities, in this section we explore the characteristics of these different occupations.

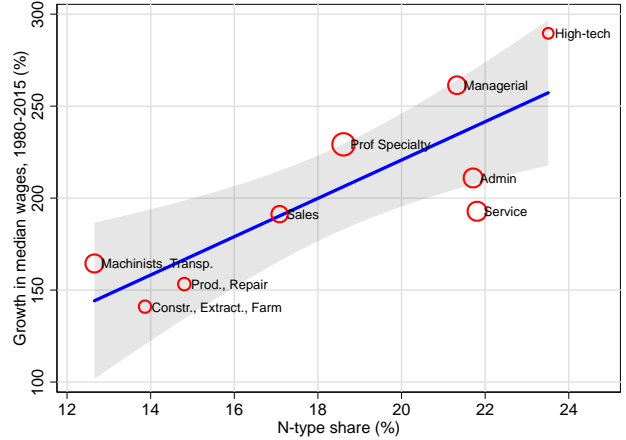
We start by using the 1980 Census and 2015 American Community Survey (ACS) to measure total hours and median hourly wages for each of the nine occupation groups. Figure 4a shows a scatter plot of median hourly wages in 2015 against the  $N$ -content share of each occupation group. There is only a weak relationship between the level of wages and  $N$ -content. Although the high-wage occupation groups are mostly high  $N$ -content occupations (managerial, high-tech), there are also low-wage occupations with high  $N$ -content (service, admin), and the low  $N$ -content occupations are in the middle of the distribution. (Viewed on its side, Figure 4a suggests a U-shaped relationship between  $N$ -content and wages).

Wage growth, on the other hand, is strongly correlated with  $N$ -content. This can be seen in Figure 4b, which plots the cumulative nominal growth in median wages from 1980 to 2015 in each occupation against the  $N$ -share. Figure 4c also shows a positive correlation between  $N$ -content and the growth in the share of total hours from 1980 to 2015. The fact that growth has been strongest in both the quantity and price of occupations with high  $N$ -content, suggest that labor demand for expansionary activities has increased faster than labor demand for traditional production activities. This is also consistent with the rolling-window estimation in Figure 2a over the corresponding period.

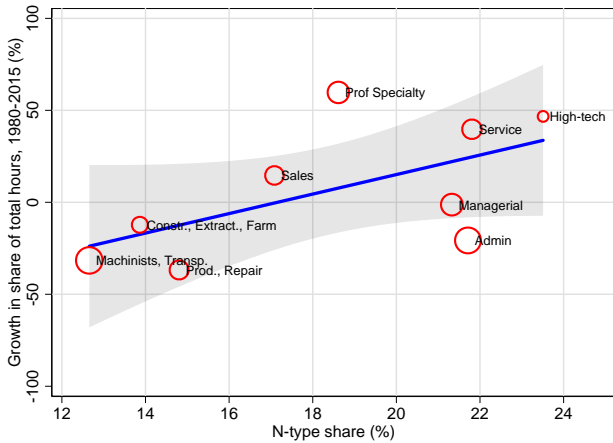
The remaining three panels of Figure 4 show how the estimated  $N$ -content of each occupation correlates with the three broad task measures constructed by Autor et al. (2006)



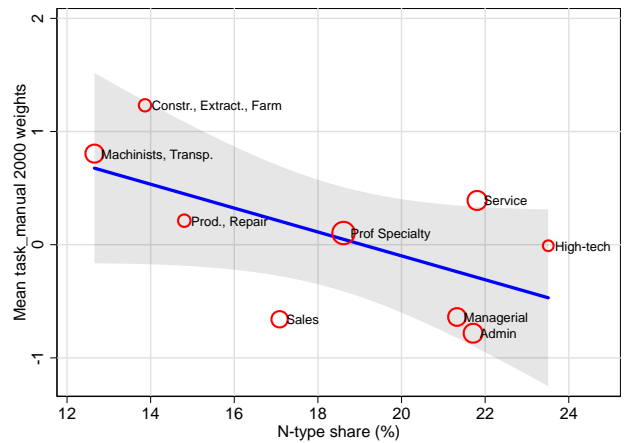
(a) Median wages 2015



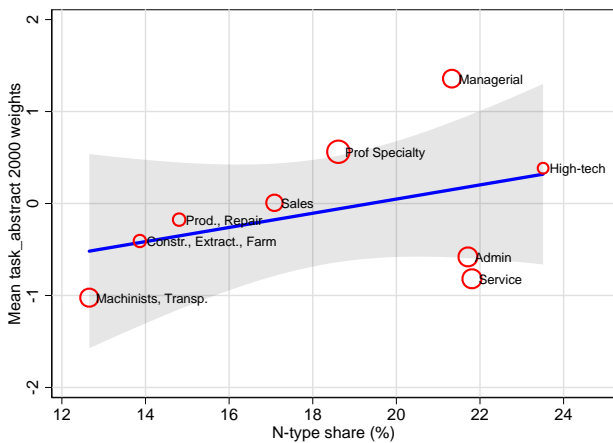
(b) Median wage growth, 1980-2015



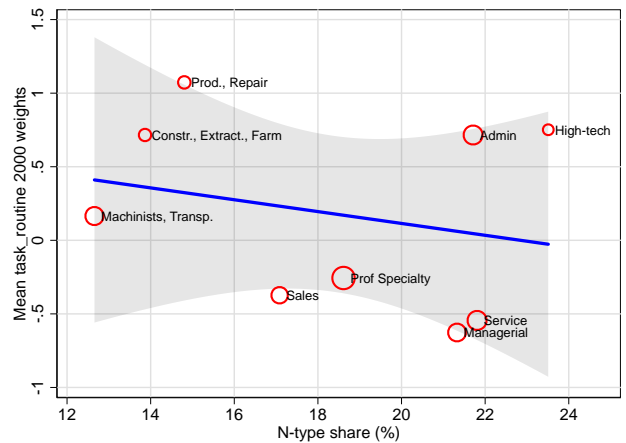
(c) Growth in share of total hours, 1980-2015



(d) Manual content, 2000 weights

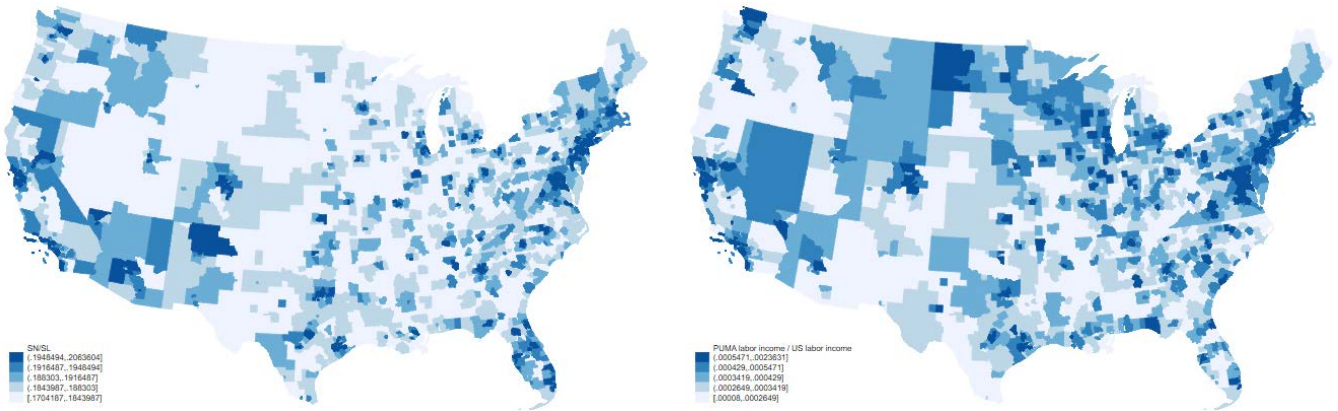


(e) Abstract content, 2000 weights



(f) Routine content, 2000 weights

Figure 4: Correlation of  $N$ -content of occupations with other occupation characteristics. Wage and hours data from 1980 Census and 2015 American Community Survey. Routine corresponds to average of DOT measures: “set limits, tolerances and standards,” and “finger dexterity.” Manual corresponds to DOT measure “eye-hand-foot coordination”. Abstract is average of DOT measures: “direction, control and planning” and “GED math.” See Autor et al. (2006) for details.



(a) Share of labor income compensating  $N$ -type labor in each region. (b) Share of total labor income going to each region.

Figure 5: Geography of  $N$ -intensive occupations. Left panel is constructed by taking a weighted average of the estimates of the  $N$ -type shares  $\frac{S_{jN}}{S_j}$  of each occupation  $j$ , using the labor income of occupation  $j$  in each region as weights for that region. Right panel is constructed as

from the US Labor Department’s Dictionary of Occupational Titles (DOT).<sup>7</sup> These figures suggest that  $N$ -content is negative correlated with the manual content of occupations (as reflected in the DOT measure “eye-hand-foot coordination”), and weakly positively correlated with the abstract content of occupations (as reflected in the DOT measures “direction, control and planning” and “GED math”). But the estimated  $N$ -content bares little relationship to the routine content of occupations (as reflected in the DOT measures “set limits, tolerances and standards,” and “finger dexterity”).

The strong relationship between wage growth, hours growth and the  $N$ -content of occupations, together with the relatively weak relationship between  $N$ -content and the task-based categorizations of occupations used in previous work, suggest that the distinction we emphasize between production activities and expansionary activities is related, but distinct, from the occupational categorizations that this previous work has emphasized.

## 4.6 Geography of $N$ -intensive Occupation

Figure 5a offers a visual representation of the geography of  $N$ -type labor in the USA. To construct this figure we first compute the distribution of labor income across the nine broad

<sup>7</sup>These measures are standardized to have a mean of 0 and standard deviation of 1. They are aggregated from detailed occupation groups using 2000 Census weights. In Figure 7 in Appendix D.3 we present analogous figures using the six Work Context and Work Activity measures from O\*NET as defined in Acemglu and Autor (2011).

occupation groups in each of 2,336 Public Use Microdata Areas (PUMA) for the lower 50 states. We then use these PUMA-specific occupation distributions to form weighted averages of the estimated  $N$ -content of occupations  $\frac{S_{jN}}{S_j}$  in each PUMA. The figure thus shows spatial heterogeneity in the occupational distribution as measured by the extent to which some occupations perform relatively more expansionary activities, while others perform relatively more production activities. For comparison, Figure 5b shows the spatial distribution of total labor income.

The figures show that the spatial distribution of compensation for  $N$ -type labor activities looks different from the spatial distribution of total labor compensation. In particular, relative to their share in overall labor compensation, compensation for  $N$ -type activities is over-represented in northern California, Florida, Arizona and New Mexico, and is under-represented in the Mid-West.

## 5 Conclusion

We have differentiated between two uses of labor in modern economies: expansionary, or  $N$ -type, activities, versus traditional production, or  $Y$ -type, activities. We have demonstrated that some occupations are more  $N$ -intensive and less  $Y$ -intensive than are other occupations, and we have offered a strategy for detecting which are the more  $N$ -intensive occupations. More  $N$ -intensive occupations are those whose relative income shares rises in response to a markup-induced rise in the overall labor share. Applying our strategy to US data reveals that  $N$ -intensive occupations are those we typically associate with white-collar occupations and  $Y$ -intensive occupations are those that we typically associate with blue-collar occupations. Since recessions are associated with an increase in the labor share in post-war US data, this provides an explanation for why the labor income of white collar workers falls less than that of blue collar workers in recessions,

Our work can be extended in several directions. First, with high-frequency data on payments to aggregate capital and disaggregated types of capital, it would be possible to measure the relative capital intensity in expansionary activities versus production activities. Second, one could use a dynamic version of our framework to explore differences across the two activities in the cost of adjusting input usage over time. Third, our production framework can be introduced into New Keynesian DSGE models as a mechanism for generating pro-cyclical profits and a counter-cyclical labor share. This is particularly useful in heterogeneous agent (HANK) versions of these models, where the cyclicity of profits matters for the distributional effect of policies and shocks. Fourth, our structure can be used to study quantitatively the distributional effects of both secular trends in markups from changes in

the nature of competition and production, as well as cyclical variation in markups from changes in aggregate demand and monetary policy in the presence of sticky prices.

## References

- Acemoglu, Daron and David Autor**, “Skills, tasks and technologies: Implications for employment and earnings,” in “Handbook of labor economics,” Vol. 4, Elsevier, 2011, pp. 1043–1171.
- Atkeson, Andrew and Ariel Burstein**, “Pricing-to-market, trade costs, and international relative prices,” *American Economic Review*, 2008, *98* (5), 1998–2031.
- Autor, David H, Lawrence F Katz, and Melissa S Kearney**, “The polarization of the US labor market,” *The American economic review*, 2006, *96* (2), 189–194.
- Barro, Robert J and Silvana Tenreyro**, “Closed and open economy models of business cycles with marked up and sticky prices,” *The Economic Journal*, 2006, *116* (511), 434–456.
- Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent**, “Inequality, business cycles, and monetary-fiscal policy,” Technical Report, National Bureau of Economic Research 2018.
- Bilbiie, Florin O, Fabio Ghironi, and Marc J Melitz**, “Endogenous entry, product variety, and business cycles,” *Journal of Political Economy*, 2012, *120* (2), 304–345.
- , – , and – , “Monopoly Power and Endogenous Product Variety: Distortions and Remedies,” *American Economic Journal: Macroeconomics*, 2016.
- Bils, Mark**, “The cyclical behavior of marginal cost and price,” *The American Economic Review*, 1987, pp. 838–855.
- and **Peter J Klenow**, “Some evidence on the importance of sticky prices,” *Journal of political economy*, 2004, *112* (5), 947–985.
- Blanchard, Olivier Jean and Nobuhiro Kiyotaki**, “Monopolistic competition and the effects of aggregate demand,” *The American Economic Review*, 1987, pp. 647–666.
- Calvo, Guillermo A**, “Staggered prices in a utility-maximizing framework,” *Journal of monetary Economics*, 1983, *12* (3), 383–398.
- Cantore, Cristiano, Filippo Ferroni, and Miguel A León-Ledesma**, “The missing link: monetary policy and the labor share,” *Journal of the European Economic Association*, 2019, *forthcoming*.



- Christiano, Lawrence J, Martin Eichenbaum, and Charles L Evans**, “Nominal rigidities and the dynamic effects of a shock to monetary policy,” *Journal of political Economy*, 2005, *113* (1), 1–45.
- Coibion, Olivier, Yuriy Gorodnichenko, Lorenz Kueng, and John Silvia**, “Innocent Bystanders? Monetary policy and inequality,” *Journal of Monetary Economics*, 2017, *88*, 70–89.
- Dixit, Avinash K and Joseph E Stiglitz**, “Monopolistic competition and optimum product diversity,” *The American economic review*, 1977, *67* (3), 297–308.
- Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu**, “How costly are markups?,” Technical Report, National Bureau of Economic Research 2018.
- Feenstra, Robert C**, “A homothetic utility function for monopolistic competition models, without constant price elasticity,” *Economics Letters*, 2003, *78* (1), 79–86.
- Fernald, John G.**, “A quarterly, utilization-adjusted series on total factor productivity,” Technical Report, Federal Reserve Bank of San Francisco 2014.
- Galí, Jordi et al.**, “Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications Second edition,” *Economics Books*, 2015.
- Gertler, Mark and Peter Karadi**, “Monetary policy surprises, credit costs, and economic activity,” *American Economic Journal: Macroeconomics*, 2015, *7* (1), 44–76.
- Gomme, Paul and Peter Rupert**, “Measuring labor’s share of income,” *FRB of Cleveland Policy Discussion Paper*, 2004, (7).
- Hall, Robert E**, “Market Structure and Macroeconomic Fluctuations,” *Brookings Papers on Economic Activity*, 1986, *17* (2), 285–338.
- Jaimovich, Nir and Max Floetotto**, “Firm dynamics, markup variations, and the business cycle,” *Journal of Monetary Economics*, 2008, *55* (7), 1238–1252.
- Kaplan, Greg and Guido Menzio**, “Shopping externalities and self-fulfilling unemployment fluctuations,” *Journal of Political Economy*, 2016, *124* (3), 771–825.
- **and Piotr Zoch**, “Some unpleasant markup arithmetic,” Technical Report, University of Chicago 2020.
- **, Benjamin Moll, and Giovanni L Violante**, “Monetary policy according to HANK,” *American Economic Review*, 2018, *108* (3), 697–743.

- Karabarbounis, Loukas and Brent Neiman**, “Accounting for factorless income,” *NBER Macroeconomics Annual*, 2019, 33 (1), 167–228.
- Kimball, Miles S**, “The Quantitative Analytics of the Basic Neomonetarist Model,” *Journal of Money, Credit and Banking*, 1995, 27 (4), 1241–1277.
- Klenow, Peter J and Jonathan L Willis**, “Real rigidities and nominal price changes,” *Economica*, 2016, 83 (331), 443–472.
- Loecker, Jan De and Frederic Warzynski**, “Markups and firm-level export status,” *American economic review*, 2012, 102 (6), 2437–71.
- , **Jan Eeckhout, and Gabriel Unger**, “The Rise of Market Power and the Macroeconomic Implications,” Technical Report, Mimeo 2019.
- Maggi, Chiara and Sónia Félix**, “What is the Impact of Increased Business Competition?,” 2019.
- McKay, Alisdair, Emi Nakamura, and Jón Steinsson**, “The power of forward guidance revisited,” *American Economic Review*, 2016, 106 (10), 3133–58.
- Melitz, Marc J and Gianmarco IP Ottaviano**, “Market size, trade, and productivity,” *The review of economic studies*, 2008, 75 (1), 295–316.
- Miranda-Agrippino, Silvia and Giovanni Ricco**, “The transmission of monetary policy shocks,” 2018.
- Mongey, Simon**, “Market Structure and Monetary Non-Neutrality,” Technical Report, University of Chicago 2019.
- Nekarda, Christopher J and Valerie A Ramey**, “The cyclical behavior of the price-cost markup,” Technical Report, National Bureau of Economic Research 2019.
- Romer, Christina D and David H Romer**, “A new measure of monetary shocks: Derivation and implications,” *American Economic Review*, 2004, 94 (4), 1055–1084.
- Rotemberg, Julio J**, “Monopolistic price adjustment and aggregate output,” *The Review of Economic Studies*, 1982, 49 (4), 517–531.
- Smets, Frank and Rafael Wouters**, “Shocks and frictions in US business cycles: A Bayesian DSGE approach,” *American economic review*, 2007, 97 (3), 586–606.

# A Proofs and Derivations

## A.1 Proof of Lemma 1

*Proof.* The wholesaler solves

$$\begin{aligned}\Pi_W &:= \max_{L_Y, Y} P_W Y - W_Y L_Y \\ &\text{subject to} \\ Y &= Z_Y L_Y^{\theta_Y}\end{aligned}$$

and the first order condition with respect to  $L_Y$  is

$$W_Y = P_W \theta_Y Z_Y L_Y^{\theta_Y - 1}.$$

Use  $Y = Z_Y L_Y^{\theta_Y}$  to write it as

$$W_Y = P_W \theta_Y \frac{Y}{L_Y}$$

and use the fact that in symmetric equilibrium  $p = \mu P_W$  to rewrite it as

$$W_Y = \frac{p}{\mu} \theta_Y \frac{Y}{L_Y} \tag{12}$$

which can be rearranged as

$$\frac{W_Y L_Y}{p Y} = \theta_Y \frac{1}{\mu}.$$

This shows that  $S_Y = \theta_Y \frac{1}{\mu}$ .

The retailer solves

$$\begin{aligned}\Pi_R &:= \max_{N, L_N} \int_0^N \Pi_j dj - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N}\end{aligned}$$

which, in a symmetric equilibrium with  $\Pi_j = \Pi$  for all  $j$ , can be written as

$$\begin{aligned}\Pi_R &:= \max_{N, L_N} N \Pi - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N}\end{aligned}$$

and the first order condition with respect to  $L_N$  is

$$W_N = \theta_N Z_N L_N^{\theta_N - 1} \Pi.$$

and in a symmetric equilibrium

$$\Pi = p y \left( 1 - \frac{1}{\mu} \right).$$

Use this fact in the first order condition to get

$$W_N = \theta_N Z_N L_N^{\theta_N - 1} p y \left(1 - \frac{1}{\mu}\right)$$

and recall market clearing  $Y = N y$  to write

$$W_N = \theta_N Z_N L_N^{\theta_N - 1} p \frac{Y}{N} \left(1 - \frac{1}{\mu}\right) \quad (13)$$

which can be rearranged (using  $N = Z_N L_N^{\theta_N}$ ) as

$$\frac{W_N L_N}{p Y} = \theta_N \left(1 - \frac{1}{\mu}\right).$$

This shows that  $S_N = \theta_N \left(1 - \frac{1}{\mu}\right)$ . Finally,

$$\begin{aligned} S_L &= S_N + S_Y \\ &= \theta_Y \frac{1}{\mu} + \theta_N \left(1 - \frac{1}{\mu}\right) \end{aligned}$$

To obtain profit shares note that

$$\begin{aligned} S_R &:= \frac{\Pi_R}{p Y} \\ &= \frac{p N y \left(1 - \frac{1}{\mu}\right) - W_N L_N}{p Y} \\ &= \left(1 - \frac{1}{\mu}\right) - \theta_N \left(1 - \frac{1}{\mu}\right) \\ &= (1 - \theta_N) \left(1 - \frac{1}{\mu}\right) \end{aligned}$$

and

$$\begin{aligned} S_W &:= \frac{\Pi_W}{p Y} \\ &= \frac{P_W Y - W_Y L_Y}{p Y} \\ &= \frac{1}{\mu} - S_Y \\ &= (1 - \theta_Y) \frac{1}{\mu} \end{aligned}$$

so

$$\begin{aligned} S_{\Pi} &= S_W + S_R \\ &= (1 - \theta_Y) \frac{1}{\mu} + (1 - \theta_N) \left(1 - \frac{1}{\mu}\right) \end{aligned}$$

□

## A.2 Proof of Theorem 1

*Proof.* As shown in Lemma 1

$$S_L = \theta_Y \frac{1}{\mu} + \theta_N \left(1 - \frac{1}{\mu}\right)$$

with

$$\frac{\partial S_L}{\partial \mu} = \frac{1}{\mu^2} (\theta_N - \theta_Y)$$

which is positive if and only if

$$\theta_N > \theta_Y.$$

Similarly, since

$$S_{\Pi} = 1 - S_L$$

we have

$$\frac{\partial S_{\Pi}}{\partial \mu} = -\frac{1}{\mu^2} (\theta_N - \theta_Y)$$

which is positive if and only if

$$\theta_N < \theta_Y.$$

□

## A.3 Proof of Theorem 2

*Proof.* As shown in Lemma 1

$$S_Y = \theta_Y \frac{1}{\mu}$$

$$S_N = \theta_N \left(1 - \frac{1}{\mu}\right)$$

so

$$\frac{\partial S_Y}{\partial \mu} = -\theta_Y \frac{1}{\mu^2} < 0$$

$$\frac{\partial S_N}{\partial \mu} = \theta_N \frac{1}{\mu^2} > 0.$$

□

## A.4 Proof of Theorem 3

*Proof.* The representative household solves

$$\begin{aligned} & \max_{C, L_Y, L_N, \{c_\omega\}_{\omega \in [0, \Omega]}} \log C - v(L_Y, L_N) \\ & \text{subject to} \\ & \int_0^\Omega p_\omega c_\omega d\omega = W_Y L_Y + W_N L_N + \Pi \\ & C = \mathcal{C}(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega) \end{aligned}$$

where  $\Pi$  denotes profits from wholesale and retail. We assumed homothetic preferences and no love of variety (see the definition of love of variety in Appendix B.1) which allows us to rewrite the household's problem in symmetric equilibrium as

$$\begin{aligned} & \max_{C, L_Y, L_N} \log C - v(L_Y, L_N) \\ & \text{subject to} \\ & pC = W_Y L_Y + W_N L_N + \Pi \end{aligned}$$

with first order conditions

$$\begin{aligned} \frac{W_Y}{p} &= C v_{L_Y}(L_Y, L_N) \\ \frac{W_N}{p} &= C v_{L_N}(L_Y, L_N). \end{aligned}$$

Use equations 12 and 13 to rewrite the above first order conditions)

$$\begin{aligned} v_{L_Y}(L_Y, L_N)C &= \theta_Y \frac{Y}{L_Y} \frac{1}{\mu} \\ v_{L_N}(L_Y, L_N)C &= \theta_N \frac{N}{L_N} \left(1 - \frac{1}{\mu}\right) \frac{Y}{N} \end{aligned}$$

which by market clearing,  $pC = pY$ , become

$$\begin{aligned} v_{L_Y}(L_Y, L_N) &= \theta_Y \frac{1}{L_Y} \frac{1}{\mu} \\ v_{L_N}(L_Y, L_N) &= \theta_N \frac{1}{L_N} \left(1 - \frac{1}{\mu}\right). \end{aligned}$$

Total differentiation of the above equations results in

$$\begin{aligned} \theta_Y \frac{1}{L_Y} \frac{1}{\mu^2} d\mu &= - \left( \theta_Y \frac{1}{L_Y^2} \frac{1}{\mu} + v_{L_Y L_Y}(L_Y, L_N) \right) dL_Y - v_{L_N L_Y}(L_Y, L_N) dL_N \\ -\theta_N \frac{1}{L_N} \frac{1}{\mu^2} d\mu &= -v_{L_N L_Y}(L_Y, L_N) dL_Y - \left( \theta_N \frac{1}{L_N^2} \left(1 - \frac{1}{\mu}\right) + v_{L_N L_N}(L_Y, L_N) \right) dL_N \end{aligned}$$

which can be solved for  $\frac{dL_Y}{d\mu}$  and  $\frac{dL_N}{d\mu}$

$$\begin{aligned}\frac{dL_Y}{d\mu} &= T \left[ - \left( \theta_N \frac{1}{L_N^2} \left( 1 - \frac{1}{\mu} \right) + v_{L_N L_Y} (L_Y, L_N) \right) \theta_Y \frac{1}{L_Y} \frac{1}{\mu^2} - \theta_N \frac{1}{L_N} \frac{1}{\mu^2} v_{L_N L_Y} (L_Y, L_N) \right] \\ \frac{dL_N}{d\mu} &= T \left[ v_{L_N L_Y} (L_Y, L_N) \theta_Y \frac{1}{L_Y} \frac{1}{\mu^2} + \left( \theta_Y \frac{1}{L_Y^2} \frac{1}{\mu} + v_{L_Y L_Y} (L_Y, L_N) \right) \theta_N \frac{1}{L_N} \frac{1}{\mu^2} \right]\end{aligned}$$

with

$$T := \frac{1}{\left( \theta_Y \frac{1}{L_Y^2} \frac{1}{\mu} + v_{L_Y L_Y} (L_Y, L_N) \right) \left( \theta_N \frac{1}{L_N^2} \left( 1 - \frac{1}{\mu} \right) + v_{L_N L_Y} (L_Y, L_N) \right) - (v_{L_N L_Y} (L_Y, L_N))^2}$$

Recall that we assumed

$$\begin{aligned}v_{L_N L_N} &> 0 \\ v_{L_Y L_Y} &> 0 \\ v_{L_N L_Y} &\geq 0 \\ v_{L_Y L_Y} v_{L_N L_N} &\geq v_{L_N L_Y} v_{L_Y L_N}\end{aligned}$$

so

$$\begin{aligned}\left( \theta_Y \frac{1}{L_Y^2} \frac{1}{\mu} + v_{L_Y L_Y} (L_Y, L_N) \right) \left( \theta_N \frac{1}{L_N^2} \left( 1 - \frac{1}{\mu} \right) + v_{L_N L_Y} (L_Y, L_N) \right) - (v_{L_N L_Y} (L_Y, L_N))^2 &> 0 \\ - \left( \theta_N \frac{1}{L_N^2} \left( 1 - \frac{1}{\mu} \right) + v_{L_N L_Y} (L_Y, L_N) \right) \theta_Y \frac{1}{L_Y} \frac{1}{\mu^2} - \theta_N \frac{1}{L_N} \frac{1}{\mu^2} v_{L_N L_Y} (L_Y, L_N) &< 0 \\ v_{L_N L_Y} (L_Y, L_N) \theta_Y \frac{1}{L_Y} \frac{1}{\mu^2} + \left( \theta_Y \frac{1}{L_Y^2} \frac{1}{\mu} + v_{L_Y L_Y} (L_Y, L_N) \right) \theta_N \frac{1}{L_N} \frac{1}{\mu^2} &> 0\end{aligned}$$

and therefore

$$\begin{aligned}\frac{dL_Y}{d\mu} &< 0 \\ \frac{dL_N}{d\mu} &> 0\end{aligned}$$

Since  $Y = Z_Y L_Y^{\theta_Y}$  we have  $\frac{dY}{d\mu} < 0$  and since we can normalize  $p = 1$  we have  $\frac{dp_Y}{d\mu} < 0$ . Since  $N = Z_N L_N^{\theta_N}$  we have  $\frac{dN}{d\mu} > 0$ . If there is no love of variety we have

$$C = Y$$

and

$$\frac{dC}{d\mu} < 0.$$

□

## A.5 Factor shares in model with occupations

The wholesaler solves

$$\begin{aligned} \Pi_W &:= \max_{\{L_{jY}\}, K_Y, Y} P_W Y - \sum_{j=1}^J W_{jY} L_{jY} - R_Y K_Y \\ &\text{subject to} \\ Y &= Z_Y \left[ K_Y^{\alpha_Y} \left( \prod_{j=1}^J L_{jY}^{\eta_{jY}} \right)^{1-\alpha_Y} \right]^{\theta_Y} \end{aligned}$$

where  $R_Y$  is the rental rate and  $W_{jY}$  is the occupation's  $j$  wage in the wholesale sector. First order condition with respect to  $L_{jY}$  is

$$W_{jY} = P_W \theta_Y \eta_{jY} (1 - \alpha_Y) \frac{Y}{L_{jY}}. \quad (14)$$

The retailer solves

$$\begin{aligned} \Pi_R &:= \max_{\{L_{jN}\}, K_N, N} \int_0^N \Pi_j dj - \sum_{j=1}^J W_{jN} L_{jN} - R_N K_N \\ &\text{subject to} \\ N &= Z_N \left[ K_N^{\alpha_N} \left( \prod_{j=1}^J L_{jN}^{\eta_{jN}} \right)^{1-\alpha_N} \right]^{\theta_N} \end{aligned}$$

where  $R_N$  is the rental rate and  $W_{jN}$  is the occupation's  $j$  wage in the retail sector. First order condition with respect to  $L_{jN}$  is

$$W_{jN} = \theta_N \eta_{jN} (1 - \alpha_N) \frac{N}{L_{jN}} \Pi_N. \quad (15)$$

Each occupation has its own wage and labor market clearing condition. In symmetric equilibrium equations 14 and 15 can be written as

$$\begin{aligned} \frac{W_j}{P} &= \frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y) \frac{Y}{L_{jY}} \\ \frac{W_j}{P} &= \left( 1 - \frac{1}{\mu} \right) \theta_N (1 - \alpha_N) \eta_{jN} \frac{Y}{L_{jN}} \end{aligned}$$

so the factor shares for the occupations are

$$\begin{aligned} S_{jY} &= \frac{1}{\mu} \theta_Y (1 - \alpha_Y) \eta_{jY} \\ S_{jN} &= \left( 1 - \frac{1}{\mu} \right) \theta_N (1 - \alpha_N) \eta_{jN} \end{aligned}$$



Define

$$L_Y := \prod_{j=1}^J L_j^{\eta_{jY}}$$

$$L_N := \prod_{j=1}^J L_j^{\eta_{jN}}$$

and the wage indices  $W_Y, W_N$

$$W_Y L_Y := \sum_j W_j L_{jY}$$

$$W_N L_N := \sum_j W_j L_{jN}$$

Use equations 14 and 15 to solve for

$$L_j := L_{jY} + L_{jN}$$

$$= \frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y) \frac{PY}{W_j} + \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \eta_{jN} \frac{PY}{W_j}$$

and express  $L_{jY}$  and  $L_{jN}$  as

$$L_{jY} = \frac{\frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y)}{\frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y) + \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \eta_{jN}} L_j$$

$$L_{jN} = \frac{\left(1 - \frac{1}{\mu}\right) \theta_N \eta_{jN} (1 - \alpha_N)}{\frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y) + \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \eta_{jN}} L_j$$

and

$$W_j = \left[ \frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y) + \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \eta_{jN} \right] \frac{PY}{L_j}$$

Using the fact that  $\sum_{j=1}^J \eta_{jY} = 1$  and  $\sum_{j=1}^J \eta_{jN} = 1$  total labor income in each sector can be expressed as

$$W_Y L_Y = \frac{1}{\mu} \theta_Y (1 - \alpha_Y) PY$$

$$W_N L_N = \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) PY$$

and labor share in each sector is

$$S_Y = \frac{1}{\mu} \theta_Y \eta_{jY} (1 - \alpha_Y)$$

$$S_N = \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N).$$

Labor share of occupation  $j$  is defined as

$$S_j := \frac{W_j L_j}{PY}$$

thus

$$\begin{aligned} S_j &= \frac{1}{\mu} \theta_Y (1 - \alpha_Y) \eta_{jY} + \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \eta_{jN} \\ &= \eta_{jY} S_Y + \eta_{jN} S_N \end{aligned}$$

## B Details of Alternative Preference and Market Structures

This Appendix provides further details of the preference and market structures referred to in Section 2.6.

### B.1 Demand System Preliminaries

**Definitions** We define the following objects:

- $\Omega$  is the measure of unique varieties being produced in the economies.  $\omega \in [0, \Omega]$  are individual varieties.  $p_\omega$  is the price faced by consumers for variety  $\omega$ .
- $N$  the measure of establishments or retail sales units. Some varieties might be produced by more than one retail sales unit but each sales unit produces only one variety.  $j \in [0, N]$  are individual retail sales units.  $p_j$  is the price charged by sales unit  $j$ .  $Y_j$  is the quantity sold by sales unit  $j$ .  $\mu_j$  is the markup over marginal charged by sales unit  $j$ .
- $M := \frac{N}{\Omega}$  is the measures of sales units producing each variety. We assume that when a sales unit produces a new variety it is chosen randomly, so that the same measure of firms operate in each variety.

**Households** Households choose  $c_\omega$  given prices  $p_\omega$ . Households have utility defined over an aggregator of varieties  $C(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega)$ . The household solves the following problem:

$$\begin{aligned} V(\{p_\omega\}_{\omega \in [0, \Omega]}, I, \Omega) &= \max_{c_\omega} U(C(\{c_\omega\}_{\omega \in \Omega}, \Omega)) \\ &\text{subject to} \\ I &\geq \int_0^\Omega p_\omega c_\omega d\omega \end{aligned}$$

where  $V(\bullet)$  is the indirect utility function and  $U(\bullet)$  is the direct utility function which is assumed to be strictly increasing. The household as income  $I$ .

**Definition of love-of-variety** Consider a household with income  $I$ . Assume that  $p_\omega = p \forall \omega$  and that a household purchases the same quantity  $c_\omega = c$  of each good, meaning they allocate expenditure equally across the goods. Define the indirect utility associated with this pattern of expenditure as  $V(p, I, \Omega)$ . We say that the demand system features love-of-variety if  $\frac{\partial V(p, I, \Omega)}{\partial \Omega} > 0$  and no love-of-variety if  $\frac{\partial V(p, I, \Omega)}{\partial \Omega} = 0$ . Note that this in this symmetric case

$$I = \Omega pc$$

and the indirect utility function is a monotonic function of  $C(c, \Omega) = C\left(\frac{I}{\Omega^p}, \Omega\right)$ . So the condition for no-love-of-variety is equivalent to

$$-\frac{c}{\Omega} \frac{\partial C}{\partial c} + \frac{\partial C}{\partial \Omega} = 0$$

$$\varepsilon_{c, \Omega} = 1$$

i.e. the elasticity of substitution between  $c$  and  $\Omega$  is equal to 1. To a first-order this implies that

$$C = c\Omega$$

**Definition of price index** Recall that definition of an expenditure function

$$E\left(\{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega\right) = \min_{c_\omega} \int_0^\Omega p_\omega c_\omega d\omega$$

subject to

$$C\left(\{c_\omega\}_{\omega \in \Omega}, \Omega\right) \geq C$$

For homothetic preferences, meaning that the aggregator is homogenous of degree 1, the price index  $P$  is defined as the minimum cost of obtaining one unit of the bundle  $C$ :

$$P\left(\{p_\omega\}_{\omega \in [0, \Omega]}, \Omega\right) = E\left(\{p_\omega\}_{\omega \in [0, \Omega]}, 1, \Omega\right)$$

and the expenditure function takes the form

$$E\left(\{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega\right) = P\left(\{p_\omega\}_{\omega \in [0, \Omega]}, \Omega\right) C$$

If preferences are not homothetic, then we can define a price index that depends on the level of the consumption bundle as

$$P\left(\{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega\right) = \frac{E\left(\{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega\right)}{C}$$

**Market clearing** The total measure of variety  $\omega$  sold must equal the measure of variety  $\omega$  consumed

$$c_\omega = \frac{N}{\Omega} Y_j = M Y_j$$

$$c\Omega = NY$$

$$C = NY$$

where the last line follows from no love- of variety. In the more general case, we would have

$$C^{-1}(C, \Omega) \Omega = NY$$

Typically this takes the form

$$Cf(\Omega) = c\Omega$$

So the market clearing condition gives

$$\begin{aligned}c_\omega &= \frac{N}{\Omega} Y_j = KY_j \\c\Omega &= NY \\Cf(\Omega) &= NY\end{aligned}$$

**Symmetric equilibria** We will focus on symmetric demand systems and symmetric equilibria. This means that the price index takes the form  $P(p, C, \Omega)$  or  $P(p, \Omega)$  in the case of homothetic preferences. In the case of homothetic preferences, we can express the love of variety condition in terms of the price index. With  $p_\omega = p \forall \omega$  and  $c_\omega = c$ , the expenditure function and definition of the price index imply

$$\Omega pc = P(p, \Omega) C$$

the condition for no-love-of-variety is then

$$\begin{aligned}\Omega pc &= P(p, \Omega) c\Omega \\p &= P(p, \Omega)\end{aligned}$$

which implies that the price index satisfies  $P = p$  and does not depend on  $\Omega$ .

**Demand functions** The demand functions solve

$$\begin{aligned}c_\omega \left( \{p_\omega\}_{\omega \in [0, \Omega]}, I, \Omega \right) &= \max_{c_\omega} U \left( C \left( \{c_\omega\}_{\omega \in \Omega}, \Omega \right) \right) \\&\text{subject to} \\I &\geq \int_0^\Omega p_\omega c_\omega d\omega\end{aligned}$$

using the definition of the expenditure function, i.e that  $I = P \left( \{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega \right) C$  we can write these as  $c_\omega \left( \{p_\omega\}_{\omega \in [0, \Omega]}, C, \Omega \right)$ , With homothetic preferences, these take the form  $c_\omega = f \left( \{p_\omega\}_{\omega \in [0, \Omega]}, P, \Omega \right) C$ . The elasticity of demand is denoted by

$$\varepsilon = -\frac{p_\omega}{c_\omega} \frac{\partial c_\omega}{\partial p_\omega}$$

## B.2 Demand Systems

### B.2.1 CES

The aggregator function is

$$C = \left[ \Omega^{-\frac{\rho}{\sigma}} \int_0^\Omega c_\omega^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

In a symmetric equilibrium this gives

$$C = \Omega^{\frac{\sigma-\rho}{\sigma-1}} c$$

so the preferences feature no love of variety if  $\rho = 1$ . The preferences are homothetic because the aggregator is homogenous of degree 1 in  $c$ .

The price index is

$$P = \left[ \Omega^{-\rho} \int_0^\Omega p_\omega^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

which gives in a symmetric equilibrium

$$P = \Omega^{\frac{1-\rho}{1-\sigma}} p$$

The demand functions are

$$c_\omega = \left( \frac{p_\omega}{P} \right)^{-\sigma} C \Omega^{-\rho}$$

so the elasticity of demand is

$$\varepsilon = \sigma$$

## B.2.2 Translog

There is no closed form expression for the aggregator but the preferences are homothetic.

The price index is given by

$$\log P = \frac{1}{2\sigma\Omega} + \frac{1}{\Omega} \int_0^\Omega \log p_\omega d\omega + \frac{1}{2} \int_0^\Omega \int_0^\Omega \frac{\sigma}{\Omega} \log p_\omega (\log p_{\omega'} - \log p_\omega) d\omega d\omega'$$

In a symmetric equilibrium this gives

$$\begin{aligned} \log P &= \frac{1}{2\sigma\Omega} + \log p \\ \log \left( \frac{P}{p} \right) &= \frac{1}{2\sigma\Omega} \\ P &= p e^{\frac{1}{2\sigma\Omega}} \end{aligned}$$

so this features love of variety.

The demand function is

$$\begin{aligned} c_\omega &= \left[ \frac{1}{\Omega} - \sigma \left( \log p_\omega - \frac{1}{\Omega} \int_0^\Omega \log p_{\omega'} d\omega' \right) \right] \frac{I}{p_\omega} \\ &= \left[ \frac{1}{\Omega} - \sigma (\log p_\omega - \log p) \right] \frac{I}{p_\omega} \\ &= \left[ \frac{1}{\Omega} - \sigma \left( \log p_\omega - \log P + \frac{1}{2\sigma\Omega} \right) \right] \frac{PC}{p_\omega} \\ \log c_\omega &= \log \left[ \frac{1}{\Omega} - \sigma \left( \log p_\omega - \log P + \frac{1}{2\sigma\Omega} \right) \right] + \log P + \log C - \log p_\omega \end{aligned}$$

where the second line follows from symmetry and the third line follow from the fact that preferences are homothetic. In a symmetric equilibrium this implies

$$\begin{aligned} \log c &= \log \frac{1}{\Omega} + \frac{1}{2\sigma\Omega} + \log C \\ C &= c \Omega e^{-\frac{1}{2\sigma\Omega}} \end{aligned}$$

which does not give  $C = c\Omega$  because of the love of variety.

The elasticity of demand is

$$\begin{aligned}
\varepsilon &= \frac{\sigma}{\frac{1}{\Omega} - \sigma \left( \log p_\omega - \log P + \frac{1}{2\sigma\Omega} \right)} + 1 \\
&= \frac{\sigma}{\frac{1}{\Omega} - \sigma \left( \log p - \log P + \frac{1}{2\sigma\Omega} \right)} + 1 \\
&= \frac{\sigma}{\frac{1}{\Omega}} + 1 \\
&= \sigma\Omega + 1
\end{aligned}$$

### B.2.3 Linear Demand

The aggregator is

$$C = \int_0^\Omega c_\omega d\omega - \frac{1}{2\sigma} \int_0^\Omega c_\omega^2 d\omega + \frac{1}{\Omega 2\sigma} \left[ \int_0^\Omega c_\omega d\omega \right]^2$$

which gives in a symmetric equilibrium

$$C = \Omega c$$

So in a symmetric equilibrium they do not feature love of variety.

Are preferences homothetic?

$$\int_0^\Omega t c_\omega d\omega - \frac{1}{2\sigma} \int_0^\Omega (t c_\omega)^2 d\omega + \frac{1}{\Omega^{1+\rho} 2\sigma} \left[ \int_0^\Omega (t c_\omega) d\omega \right]^2 = t \int_0^\Omega c_\omega d\omega - \frac{t^2}{2\sigma} \int_0^\Omega c_\omega^2 d\omega + \frac{t^2}{\Omega 2\sigma} \left[ t \int_0^\Omega c_\omega d\omega \right]^2$$

which equals  $tC$  only in the symmetric case, so no, because homotheticity requires this to equal  $tC$  even in the non-symmetric case.

We can derive the price index as

$$\begin{aligned}
P &= \min_{c_\omega} \int_0^\Omega c_\omega p_\omega d\omega \\
&\text{subject to} \\
1 &\leq \int_0^\Omega c_\omega d\omega - \frac{1}{2\sigma} \int_0^\Omega c_\omega^2 d\omega + \frac{1}{2\sigma} \left[ \int_0^\Omega c_\omega d\omega \right]^2
\end{aligned}$$

which gives

$$\begin{aligned}
p_\omega &= \lambda \left[ -1 + \frac{1}{\sigma} c_\omega - \frac{1}{\Omega\sigma} \int_0^\Omega c_{\omega'} d\omega' \right] \\
c_\omega &= \sigma \left( \frac{p_\omega}{\lambda} + 1 \right) + \frac{1}{\Omega} \int_0^\Omega c_{\omega'} d\omega'
\end{aligned}$$

In a symmetric equilibrium

$$c_\omega = \sigma \left( \frac{p}{\lambda} + 1 \right) + c$$

and substituting into the constraint at equality

$$1 = \int_0^{\Omega} \left[ \sigma \left( \frac{p}{\lambda} + 1 \right) + c \right] d\omega$$

$$\frac{p}{\lambda} + 1 = \frac{1 - \Omega c}{\Omega \sigma}$$

so

$$c_{\omega} = \frac{1}{\Omega}$$

$$P = \frac{1}{\Omega} \int_0^{\Omega} p_{\omega} d\omega$$

which gives

$$P = p$$

in a symmetric equilibrium

The demand function is given by

$$\max_{c_{\omega}} \int_0^{\Omega} c_{\omega} d\omega - \frac{1}{2\sigma} \int_0^{\Omega} c_{\omega}^2 d\omega + \frac{1}{\Omega 2\sigma} \left[ \int_0^{\Omega} c_{\omega} d\omega \right]^2$$

subject to

$$\int_0^{\Omega} p_{\omega} c_{\omega} d\omega = I$$

which gives

$$\lambda p_{\omega} = 1 - \frac{1}{\sigma} c_{\omega} + \frac{1}{\Omega \sigma} \int_0^{\Omega} c_{\omega} d\omega$$

$$\lambda \int_0^{\Omega} p_{\omega} d\omega = \int_0^{\Omega} \left[ 1 - \frac{1}{\sigma} c_{\omega} + \frac{1}{\Omega \sigma} \int_0^{\Omega} c_{\omega} d\omega \right] d\omega$$

$$\lambda \Omega p = \Omega - \frac{1}{\sigma} \Omega c + \frac{1}{\sigma} \int_0^{\Omega} c_{\omega} d\omega$$

Dividing

$$\frac{p_{\omega}}{\Omega p} = \frac{1 - \frac{1}{\sigma} c_{\omega} + \frac{1}{\Omega \sigma} \int_0^{\Omega} c_{\omega} d\omega}{\Omega - \frac{1}{\sigma} \Omega c + \frac{1}{\sigma} \int_0^{\Omega} c_{\omega} d\omega}$$

$$= \frac{1 - \frac{1}{\sigma} c_{\omega} + \frac{1}{\sigma} c}{\Omega}$$

$$c_{\omega} = \frac{C}{\Omega} + \sigma \left( 1 - \frac{p_{\omega}}{P} \right)$$

The elasticity of demand is

$$\varepsilon = \frac{\sigma p_{\omega}}{P c_{\omega}}$$

$$= \sigma \frac{\Omega}{C}$$

## B.2.4 Kimball

The aggregator  $C$  is defined implicitly by

$$\frac{1}{\Omega} \int_0^\Omega \Upsilon \left( \frac{\Omega c_\omega}{C} \right) d\omega = 1$$

where  $\Upsilon$  satisfies  $\Upsilon(1) = 1$ . In a symmetric equilibrium this implies  $C = c\Omega$

The demand function is given by

$$c_\omega = \frac{C}{\Omega} \Upsilon'^{-1} \left( \frac{p_\omega}{P} D \right)$$

where  $P$  is a price index defined by

$$PC = \int_0^\Omega p_\omega c_\omega d\omega$$

and  $D$  is a demand index defined by

$$D = \int_0^\Omega \frac{c_\omega}{C} \Upsilon' \left( \frac{\Omega c_\omega}{C} \right) d\omega$$

In a symmetric equilibrium, the demand index is just

$$D = \frac{\Omega c}{C} \Upsilon' \left( \frac{\Omega c}{C} \right)$$

They propose the following functional forms

$$\Upsilon'(x) = \frac{\sigma - 1}{\sigma} \exp \left\{ \frac{1 - x^{\frac{\eta}{\sigma}}}{\eta} \right\}$$

which elasticity of demand

$$\varepsilon = \sigma \left( \frac{\Omega c_\omega}{C} \right)^{-\frac{\eta}{\sigma}}$$

which equals  $\sigma$  in a symmetric equilibrium.

## B.3 Market Structures

Under each of the following market structures, the factor shares take the same form as in Lemma 1, where the (possibly endogenous) markup  $\mu$  is given as follows.

### B.3.1 Monopolistic Competition

There is one firm producing each variety:  $M = 1$ ,  $N = \Omega$ . So changes in  $N$  coincide with change in  $\Omega$ . The markup is given by

$$\mu = \frac{\varepsilon}{\varepsilon - 1}$$



### B.3.2 Cournot competition

There are  $M \gg 1$  firms producing each variety. When  $M$  is large so that firms do not internalize effect on price index  $P$ , the markup is

$$\mu = \frac{\varepsilon}{\varepsilon - \frac{1}{M}}$$

### B.3.3 Oligopoly.

There are  $M > 1$  firms producing each variety, but  $M$  small, so that firms do internalize effect on price index  $P$ . Can think either.

With nested CES preferences, the demand elasticity under Bertrand competition is given by

$$\varepsilon = \eta \frac{M-1}{M} + \frac{1}{M} \sigma$$

where  $\eta > \sigma$  is elasticity of substitution across firms producing the same good. This implies a markup

$$\mu = \frac{\eta + \frac{\sigma - \eta}{M}}{\eta - 1 + \frac{\sigma - \eta}{M}}$$

which is decreasing in  $M$ .

With nested CES preferences, the residual demand elasticity under Cournot competition is given by

$$\varepsilon = \left[ \frac{1}{\eta} \left( \frac{M-1}{M} \right) + \frac{1}{\sigma} \frac{1}{M} \right]^{-1}$$

so that the markup is

$$\mu = \frac{\sigma \eta}{\sigma (\eta - 1) + \frac{\sigma - \eta}{M}}$$

which is also decreasing in  $M$ .

## C Details of Generalizations of Production Structure

### C.1 Entry in the wholesale sector

We generalize the model to allow entry in the wholesale sector. The wholesaler operates  $E$  plants. For each plant the problem is

$$\Pi_W := \max_{L_Y, Y} P_W Y - W_Y L_Y$$

subject to

$$Y = Z_Y L_Y^{\theta_Y}$$

and the first order condition with respect to  $L_Y$  is the same as in the baseline model (see equation 12). To operate plants the wholesaler needs to hire N-type labor. The problem is

$$\begin{aligned} & \max_{L_E, E} E\Pi_W - W_N L_E \\ & \text{subject to} \\ & E = Z_E L_E \end{aligned}$$

with the first order condition

$$W_N = \frac{E}{L_E} \Pi_W$$

The retailer's problem remains unchanged

$$\begin{aligned} \Pi_R & := \max_{L_N, N} \int_0^N \Pi_j dj - W_N L_N \\ & \text{subject to} \\ & N = Z_N L_N^{\theta_Y} \end{aligned}$$

with the first order condition (in a symmetric equilibrium with  $\Pi_j = \Pi$  for all  $j$ )

$$W_N = \theta_N \frac{N}{L_N} \Pi.$$

Market clearing for intermediate goods implies that

$$yN = EY$$

and in a symmetric equilibrium

$$\Pi = p \frac{EY}{N} \left(1 - \frac{1}{\mu}\right).$$

Factor shares are

$$\begin{aligned} S_Y & := \frac{EW_Y L_Y}{pEY} \\ S_N & := \frac{W_N (L_N + L_E)}{pEY} \end{aligned}$$

and in a symmetric equilibrium

$$\begin{aligned} S_N & = \left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu} (1 - \theta_Y) \\ S_Y & = \frac{1}{\mu} \theta_Y \end{aligned}$$

and the overall labor share is

$$\left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu}.$$

Theorem 1 does not hold. The reason is that all profits are retail and it is the nature of profits that matters for Theorem 1. In this case an increase in markups always leads to a fall in the labor share. For Theorem

2, we have

$$S_N = \left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu} (1 - \theta_Y)$$

and

$$S_Y = \frac{1}{\mu} \theta_Y.$$

So  $S_Y$  always falls after an increase in  $\mu$  but for  $S_N$  to rise we need  $\theta_N > 1 - \theta_Y$ . For redistribution, i.e.  $\frac{S_N}{S_L}$  to rise:

$$\begin{aligned} \frac{S_N}{S_L} &= \frac{\left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu} (1 - \theta_Y)}{\left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu}} \\ \frac{\partial \log\left(\frac{S_N}{S_L}\right)}{\partial \frac{1}{\mu}} &= \frac{-\theta_N + 1 - \theta_Y}{\left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu} (1 - \theta_Y)} - \frac{1 - \theta_N}{\left(1 - \frac{1}{\mu}\right) \theta_N + \frac{1}{\mu}} \\ \frac{\partial \log\left(\frac{S_N}{S_L}\right)}{\partial \frac{1}{\mu}} < 0 &\iff \theta_N > 0 \end{aligned}$$

i.e.  $\frac{\partial \log\left(\frac{S_N}{S_L}\right)}{\partial \mu} > 0$  as long as  $\theta_N > 0$ . This means that the share of *labor income* accruing to  $N$ -type workers always increases when markups go up.

## C.2 Markups In Labor and Wholesale Markets

We generalize the model to allow for markups in the wholesale good market and labor markets. We assume that the wholesaler is a monopsonist in the labor market and a monopolist in the product market. First order condition of the wholesaler is

$$\frac{\mu_Y}{\mu_{L_Y}} W_Y = P_W \theta_Y \frac{Y}{L_Y} \quad (16)$$

where  $\mu_Y \geq 1$  and  $\mu_{L_Y} \leq 1$ . Similarly, we assume some degree of monopsonistic power ( $\mu_{L_N} \leq 1$ ) in the retail sector and so

$$\frac{1}{\mu_{L_N}} \frac{W_N}{p} = \theta_N \frac{N}{L_N} \Pi_N \quad (17)$$

and in symmetric equilibrium factor shares become

$$\begin{aligned} S_Y &= \frac{\mu_{L_Y}}{\mu_Y} \frac{1}{\mu} \theta_Y \\ S_N &= \mu_{L_N} \left(1 - \frac{1}{\mu}\right) \theta_N \end{aligned}$$

where  $\mu_{L_Y}$  and  $\mu_{L_N}$  are markups in the labor markets for  $Y$ -type labor and  $N$ -type labor, and  $\mu_Y$  is the markup in the market for intermediate goods.

We assume that markups are independent of each other and exogenous. The presence of markups in these other markets does not change Theorem 2; an increase in the markup still redistributes factor income away from  $Y$ -type labor and toward  $N$ -type labor. However, the condition for Theorem 1 is modified. Positive co-movement between the markup and the labor share requires  $\theta_N > \frac{\mu_{L_Y}}{\mu_{L_N} \mu_Y} \theta_Y$ . When  $\mu_{L_N} = \mu_{L_Y}$ , the presence of a markup in the wholesale sector ( $\mu_Y > 1$ ) thus expands the set of  $(\theta_N, \theta_Y)$

which are consistent with co-movement observed in US data. We also have

$$\begin{aligned}\frac{\partial S_Y}{\partial \mu_Y} &< 0 \\ \frac{\partial S_N}{\partial \mu_Y} &= 0 \\ \frac{\partial S_L}{\partial \mu_Y} &< 0\end{aligned}$$

meaning that an increase in wholesale markup reduces the  $Y$ -type share and the overall labor share. It has no effect on the  $N$ -type share. In addition

$$\begin{aligned}\frac{\partial S_Y}{\partial \mu_{L_Y}} &> 0 \\ \frac{\partial S_N}{\partial \mu_{L_Y}} &= 0 \\ \frac{\partial S_L}{\partial \mu_{L_Y}} &> 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial S_Y}{\partial \mu_{L_N}} &= 0 \\ \frac{\partial S_N}{\partial \mu_{L_N}} &> 0 \\ \frac{\partial S_L}{\partial \mu_{L_N}} &> 0\end{aligned}$$

so a *decrease* in the degree of monoposonistic power always increases the overall labor share in the economy and that happens through an increase in the share of one type of labor. Any type of markup or markdown that appears as a wedge between the real wage and the marginal rate of substitution of households (as it is often the case in New Keynesian models with wage rigidities) would not affect our results as long as  $\mu_{L_Y}$  and  $\mu_{L_N}$  would not be affected by it.

So far we have simply assumed the presence of exogenous wedges  $\mu_Y, \mu_{L_Y}, \mu_{L_N}$  in equations 16 and 17. Below we show an example in which these wedges are functions of structural parameters of the model. Suppose there is a representative household with preferences

$$\log C - \chi_Y \frac{L_Y^{1+\frac{1}{\varphi_Y}}}{1+\frac{1}{\varphi_Y}} - \chi_N \frac{L_N^{1+\frac{1}{\varphi_N}}}{1+\frac{1}{\varphi_N}}$$

where  $C = \mathcal{C}(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega)$  is a symmetric homothetic aggregator over distinct varieties  $c_\omega$  and  $\Omega$  is the measure of varieties. We assume this aggregator features no love of variety. The representative household solves

$$\begin{aligned}\max_{C, L_Y, L_N} \quad & \log C - \chi_Y \frac{L_Y^{1+\frac{1}{\varphi_Y}}}{1+\frac{1}{\varphi_Y}} - \chi_N \frac{L_N^{1+\frac{1}{\varphi_N}}}{1+\frac{1}{\varphi_N}} \\ \text{subject to} \quad & \\ pC = & W_Y L_Y + W_N L_N + \Pi\end{aligned}$$

First order conditions are

$$\begin{aligned}\chi_Y C L_Y^{\frac{1}{\varphi_Y}} &= W_Y \\ \chi_N C L_N^{\frac{1}{\varphi_N}} &= W_N\end{aligned}$$

Belowe we describe an example of micro-founded environment in which markup variation arises as a result of exogenous variation in a structural parameter. There is a wholesaler takes household's labor supply schedule as given and solves

$$\begin{aligned}\Pi_W &:= \max_{L_Y, Y} (1 - \tau) P_W Y - W_Y L_Y \\ &\text{subject to} \\ Y &= Z_Y L_Y^{\theta_Y} \\ L_Y &= \left( \frac{W_Y}{\chi_Y C} \right)^{\varphi_Y}\end{aligned}$$

where  $\tau$  is a tax rate on wholesaler's revenue.<sup>8</sup> Ther rst order condition is

$$\left( 1 + \frac{1}{\varphi_Y} \right) W_Y = (1 - \tau_W) P_W \theta_Y \frac{Y}{L_Y}.$$

The retailer takes household's labor supply schedule as given and solves

$$\begin{aligned}\Pi_R &:= \max_{L_N, N} \int_0^N \Pi_j dj - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N} \\ L_N &= \left( \frac{W_N}{\chi_N C} \right)^{\varphi_N}\end{aligned}$$

and rhe first order condition with respect to  $L_N$  is (in a symmetric equilibrium with  $\Pi_j = \Pi$  for all  $j$ )

$$\left( 1 + \frac{1}{\varphi_N} \right) W_N = \theta_N Z_N L_N^{\theta_N - 1} \Pi.$$

Define

$$\begin{aligned}\mu_{L_Y} &:= \left( 1 + \frac{1}{\varphi_Y} \right)^{-1} \\ \mu_Y &:= (1 - \tau)^{-1} \\ \mu_{L_N} &:= \left( 1 + \frac{1}{\varphi_N} \right)^{-1}\end{aligned}$$

to obtain equations 16 and 17. In this framework shifts in  $\mu_{L_Y}$  and  $\mu_{L_N}$  can be interpreted as changes in Frisch elasticities  $\varphi_Y, \varphi_N$  whole shifts in  $\mu_Y$  result from changes in the tax rate  $\tau$ .

---

<sup>8</sup>Alternatively we could assume that there is a continuum of monopolistically competitive wholesalers and the retailer's pricing department has to purchase a CES bundle of them.

### C.3 Capital

In this section we introduce capital in both sectors of the economy. Production functions are

$$\begin{aligned} Y &= Z_Y (L_Y^{1-\alpha_Y} K_Y^{\alpha_Y})^{\theta_Y} \\ N &= Z_N (L_N^{1-\alpha_N} K_N^{\alpha_N})^{\theta_Y}. \end{aligned}$$

To obtain expressions for factor shares do not need to assume anything about supply of capital. Other than that, everything else remains the same as in Section 2. The wholesaler solves

$$\begin{aligned} \Pi_W &:= \max_{L_Y, K_Y, Y} P_W Y - W_Y L_Y - R_Y K_Y \\ &\text{subject to} \\ Y &= Z_Y (L_Y^{1-\alpha_Y} K_Y^{\alpha_Y})^{\theta_Y} \end{aligned}$$

and the retailer solves

$$\begin{aligned} \Pi_R &:= \max_{L_N, K_N, N} \int_0^N \Pi_j dj - W_N L_N - R_N K_N \\ &\text{subject to} \\ N &= Z_N (L_N^{1-\alpha_N} K_N^{\alpha_N})^{\theta_Y} \end{aligned}$$

where  $R_Y$  and  $R_N$  are capital rental rates in  $Y$  and  $N$  sector respectively. First order conditions in the wholesale sector in symmetric equilibrium (i.e. with  $\Pi_J = \Pi = p \frac{Y}{N} \left(1 - \frac{1}{\mu}\right)$ ) are

$$\frac{W_Y}{p} = \frac{1}{\mu} \theta_Y (1 - \alpha_Y) \frac{Y}{L_Y} \quad (18)$$

$$\frac{R_Y}{p} = \frac{1}{\mu} \theta_Y \alpha_Y \frac{Y}{K_Y} \quad (19)$$

and in the retail sector

$$\frac{W_N}{p} = \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \frac{Y}{L_N} \quad (20)$$

$$\frac{R_N}{p} = \left(1 - \frac{1}{\mu}\right) \theta_N \alpha_N \frac{Y}{K_N} \quad (21)$$

We define

$$S_{L_Y} := \frac{W_Y L_Y}{pY}$$

$$S_{L_N} := \frac{W_N L_N}{pY}$$

$$S_{K_Y} := \frac{R_Y K_Y}{pY}$$

$$S_{K_N} := \frac{R_N K_N}{pY}$$

By rearranging the first order conditions we have

$$\begin{aligned} S_{L_Y} &= \frac{1}{\mu} \theta_Y (1 - \alpha_Y) \\ S_{L_N} &= \left(1 - \frac{1}{\mu}\right) \theta_N (1 - \alpha_N) \\ S_{K_Y} &= \frac{1}{\mu} \theta_Y \alpha_Y \\ S_{K_N} &= \left(1 - \frac{1}{\mu}\right) \theta_N \alpha_N \end{aligned}$$

Total labor share  $S_L := S_{L_Y} + S_{L_N}$  is increasing in  $\mu$  if and only if

$$\theta_N (1 - \alpha_N) > \theta_Y (1 - \alpha_Y).$$

As in Section 2

$$\begin{aligned} \frac{\partial S_{L_Y}}{\partial \mu} &< 0 \\ \frac{\partial S_{L_N}}{\partial \mu} &> 0. \end{aligned}$$

To study response of consumption to a change in the markup we need to make assumptions about capital supply. For example, if capital is sector-specific and its supply is perfectly inelastic, comparative statics will be qualitatively the same as in the baseline economy. We assume that there is a representative household with preferences

$$U(C, L_Y, L_N) = \log C - v(L_Y, L_N)$$

where  $C = \mathcal{C}(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega)$  is a symmetric homothetic aggregator over distinct varieties  $c_\omega$  and  $\Omega$  is the measure of varieties. This implies that there exists a price index  $P = \mathcal{P}(\{p_\omega\}_{\omega \in [0, \Omega]}, \Omega)$ , and in a symmetric equilibrium in which  $p_\omega = p_j = p \forall j, \omega$ , nominal GDP satisfies  $pY = PC$ . At this point we make no additional assumptions about the measure of unique varieties  $\Omega$ , nor its relationship to the measure of product lines  $N$  operated by the retail sector.

**Theorem 4.** *Suppose that (i) the aggregator  $\mathcal{C}$  does not exhibit love of variety and (ii)  $v$  is convex in  $(L_Y, L_N)$ , i.e.  $v_{L_N L_N} > 0$ ,  $v_{L_Y L_Y} > 0$  and  $v_{L_Y L_Y} v_{L_N L_N} \leq (v_{L_Y L_N})^2$ , (iii)  $v_{L_N L_Y} \geq 0$  and (iv) capital can be moved costlessly between sectors and there is a fixed stock of capital in the economy. Then an increase in  $\mu$  leads to a decrease in aggregate nominal output  $pY$ .*

*Proof.* We assumed assume that capital can be used with both sectors (it can be costlessly moved between sectors) and that there is a fixed stock of capital in the economy (which implies that rental rates must be equalized) i.e.

$$\begin{aligned} K &= K_Y + K_N \\ R &= R_Y + R_N \end{aligned}$$

The representative household solves

$$\begin{aligned} & \max_{C, L_Y, L_N, \{c_\omega\}_{\omega \in [0, \Omega]}} \log C - v(L_Y, L_N) \\ & \text{subject to} \\ & \int_0^\Omega p_\omega c_\omega d\omega = W_Y L_Y + W_N L_N + RK + \Pi_W + \Pi_R \\ & C = \mathcal{C}(\{c_\omega\}_{\omega \in [0, \Omega]}, \Omega) \end{aligned}$$

where  $\Pi_W$  and  $\Pi_R$  denote profits from the wholesale and the retail. We assumed no love of variety (see the definition of love of variety in Appendix B.1) which allows us to rewrite the household's problem as

$$\begin{aligned} & \max_{C, L_Y, L_N} \log C - v(L_Y, L_N) \\ & \text{subject to} \\ & pC = W_Y L_Y + W_N L_N + RK + \Pi_W + \Pi_R \end{aligned}$$

which results in the following first order conditions

$$\begin{aligned} \frac{W_Y}{p} &= \chi C L^{\frac{1}{\varphi}} \\ \frac{W_N}{p} &= \chi C L^{\frac{1}{\varphi}}. \end{aligned}$$

In symmetric equilibrium with first order conditions

$$\begin{aligned} \frac{W_Y}{p} &= C v_{L_Y}(L_Y, L_N) \\ \frac{W_N}{p} &= C v_{L_N}(L_Y, L_N). \end{aligned}$$

In symmetric equilibrium (use equations 18 and 20 to rewrite the above first order conditions)

$$\begin{aligned} v_{L_Y}(L_Y, L_N) C &= \theta_Y \frac{Y}{L_Y} \frac{1}{\mu} \\ v_{L_N}(L_Y, L_N) C &= \theta_N \frac{N}{L_N} \left(1 - \frac{1}{\mu}\right) \frac{Y}{N} \end{aligned}$$

which by market clearing,  $pC = pY$ , become

$$\begin{aligned} v_{L_Y}(L_Y, L_N) &= \theta_Y \frac{1}{L_Y} \frac{1}{\mu} \\ v_{L_N}(L_Y, L_N) &= \theta_N \frac{1}{L_N} \left(1 - \frac{1}{\mu}\right). \end{aligned}$$

These are exactly the same as in the baseline version of the model and therefore (see Appendix A.4 for the proof)

$$\begin{aligned} \frac{dL_Y}{d\mu} &< 0 \\ \frac{dL_N}{d\mu} &> 0 \end{aligned}$$



Use 19, and 19 to write

$$\frac{K_Y}{K_N} = \frac{\theta_Y \alpha_Y}{\theta_N \alpha_N} \frac{1}{\mu - 1}$$

so, because capital is in fixed supply,

$$\begin{aligned} \frac{dK_Y}{d\mu} &< 0 \\ \frac{dK_N}{d\mu} &> 0 \end{aligned}$$

Since  $Y = Z_Y (L_Y^{1-\alpha_Y} K_Y^{\alpha_Y})^{\theta_Y}$  we have  $\frac{dY}{d\mu} < 0$  and since we can normalize  $p = 1$  we have  $\frac{dp_Y}{d\mu} < 0$ . Since  $N = Z_N (L_N^{1-\alpha_N} K_N^{\alpha_N})^{\theta_N}$  we have  $\frac{dN}{d\mu} > 0$ . If there is no love of variety we have

$$C = Y$$

and

$$\frac{dC}{d\mu} < 0.$$

□

## C.4 CES Production Function

We explore the effects of extending the production production functions in each sector to Constant Elasticity of Substitution functions:

$$\begin{aligned} Y &= Z_Y [(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma]^{\frac{\theta_Y}{\sigma}} \\ N &= Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}} \end{aligned}$$

We will first show the conditions under which Theorems 1, 2 and 3 still hold in a special case with  $\alpha_N = \alpha_Y = \alpha$  and  $\sigma$  close to 0. Then we will analyze a second special case with  $\alpha_Y > 0$  and  $\alpha_N = 0$ . Throughout this section we will assume

$$U(C, L_Y, L_N) = \log C - \chi \frac{(L_Y + L_N)^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}},$$

that capital and labor are perfectly mobile between sectors and that capital is supplied inelastically.

**Lemma 2.** *In an economy with this production structure, the equilibrium factor shares are given by*

$$\begin{aligned} S_{L_Y} &= \frac{1}{\mu} \theta_Y \frac{(1 - \alpha_Y) L_Y^\sigma}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ S_{L_N} &= \left(1 - \frac{1}{\mu}\right) \theta_N \frac{(1 - \alpha_N) L_N^\sigma}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma} \\ S_{K_Y} &= \frac{1}{\mu} \theta_Y \frac{\alpha_Y K_Y^\sigma}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ S_{K_N} &= \left(1 - \frac{1}{\mu}\right) \theta_N \frac{\alpha_N K_N^\sigma}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma} \end{aligned}$$

*Proof.* The wholesaler solves

$$\begin{aligned}\Pi_W &:= \max_{L_Y, Y} P_W Y - W_Y L_Y - R_Y K_Y \\ &\text{subject to} \\ Y &= Z_Y [(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma]^{\frac{\theta_Y}{\sigma}}\end{aligned}$$

and the retailer solves

$$\begin{aligned}\Pi_R &:= \max_{L_N, K_N, N} \int_0^N \Pi_j dj - W_N L_N - R_N K_N \\ &\text{subject to} \\ N &= Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}}\end{aligned}$$

First order conditions in wholesale and retail sector are, in symmetric equilibrium with  $\Pi_j = \Pi = p \frac{Y}{N} \left(1 - \frac{1}{\mu}\right)$ ,

$$\begin{aligned}\frac{W}{p} &= \theta_Y \frac{1}{\mu} Z_Y [(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma]^{\frac{\theta_Y}{\sigma}} \frac{(1 - \alpha_Y) L_Y^{\sigma-1}}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{R}{p} &= \theta_Y \frac{1}{\mu} Z_Y [(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma]^{\frac{\theta_Y}{\sigma}} \frac{\alpha_Y K_Y^{\sigma-1}}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{W}{p} &= \theta_N \left(1 - \frac{1}{\mu}\right) Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}} \frac{Y}{N} \frac{(1 - \alpha_N) L_N^{\sigma-1}}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma} \\ \frac{R}{p} &= \theta_N \left(1 - \frac{1}{\mu}\right) Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}} \frac{Y}{N} \frac{\alpha_N K_N^{\sigma-1}}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma}\end{aligned}$$

where  $R$  is rental rate of capital. Divide by

$$Y = Z_Y [(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma]^{\frac{\theta_Y}{\sigma}}$$

to rewrite them as

$$\begin{aligned}\frac{W}{pY} &= \theta_Y \frac{1}{\mu} \frac{(1 - \alpha_Y) L_Y^{\sigma-1}}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{R}{pY} &= \theta_Y \frac{1}{\mu} \frac{\alpha_Y K_Y^{\sigma-1}}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{W}{pY} &= \theta_N \left(1 - \frac{1}{\mu}\right) Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}} \frac{1}{N} \frac{(1 - \alpha_N) L_N^{\sigma-1}}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma} \\ \frac{R}{pY} &= \theta_N \left(1 - \frac{1}{\mu}\right) Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}} \frac{1}{N} \frac{\alpha_N K_N^{\sigma-1}}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma}.\end{aligned}$$

which can be rewritten (using  $N = Z_N [(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma]^{\frac{\theta_N}{\sigma}}$ ) as

$$\begin{aligned}\frac{WL_Y}{pY} &= \theta_Y \frac{1}{\mu} \frac{(1 - \alpha_Y) L_Y^\sigma}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{RK_Y}{pY} &= \theta_Y \frac{1}{\mu} \frac{\alpha_Y K_Y^\sigma}{(1 - \alpha_Y) L_Y^\sigma + \alpha_Y K_Y^\sigma} \\ \frac{WL_N}{pY} &= \theta_N \left(1 - \frac{1}{\mu}\right) \frac{(1 - \alpha_N) L_N^\sigma}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma} \\ \frac{RK_Y}{pY} &= \theta_N \left(1 - \frac{1}{\mu}\right) \frac{\alpha_N K_N^\sigma}{(1 - \alpha_N) L_N^\sigma + \alpha_N K_N^\sigma}.\end{aligned}$$

□

#### C.4.1 $\alpha_Y = \alpha_N = \alpha$

**Theorem 5.** *In an economy with preferences given by*

$$U(C, L_Y, L_N) = \log C - \chi \frac{(L_Y + L_N)^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}},$$

*CES production functions with  $\alpha_Y = \alpha_N = \alpha$ , perfect mobility of capital and labor between sectors and with inelastic supply of capital an increase (decrease) in the markup  $\mu$  leads to an increase (decrease) in the overall labor share if and only if the degree of decreasing returns to scale is stronger for Y sector than for N sector, i.e.*

$$\frac{dS_L}{d\mu} \geq 0 \text{ if and only if } \theta_N \geq \theta_Y.$$

*Conversely, an increase (decrease) in the markup  $\mu$  leads to a increase (decrease) in the overall profit share if and only if the degree of decreasing returns to scale is stronger for Y sector than for N sectors, i.e.*

$$\frac{dS_\Pi}{d\mu} \geq 0 \text{ if and only if } \theta_Y \geq \theta_N.$$

*Proof.* In this case

$$\frac{K_Y}{L_Y} = \frac{K_N}{L_N}$$

and we can use Lemma 2 to write

$$S_L = \frac{\theta_Y \frac{1}{\mu} + \theta_N \left(1 - \frac{1}{\mu}\right)}{1 + \frac{\alpha}{1 - \alpha} \left(\frac{K_N}{L_N}\right)^\sigma}.$$

Total differentiation of the above expression with respect to  $\left(\frac{1}{\mu}\right)$  gives

$$\frac{dS_L}{d\frac{1}{\mu}} = \frac{\theta_Y - \theta_N}{1 + \frac{\alpha}{1 - \alpha} \left(\frac{K_N}{L_N}\right)^\sigma} + \left[\theta_Y \frac{1}{\mu} + \theta_N \left(1 - \frac{1}{\mu}\right)\right] \frac{d}{d\frac{1}{\mu}} \left[ \frac{1}{1 + \frac{\alpha}{1 - \alpha} \left(\frac{K_N}{L_N}\right)^\sigma} \right].$$

Now notice that

$$\frac{d}{d\frac{1}{\mu}} \left[ \frac{1}{1 + \frac{\alpha}{1-\alpha} \left(\frac{K_N}{L_N}\right)^\sigma} \right] = -\sigma \frac{\frac{\alpha}{1-\alpha} \left(\frac{K_N}{L_N}\right)^\sigma}{\left(1 + \frac{\alpha}{1-\alpha} \left(\frac{K_N}{L_N}\right)^\sigma\right)^2} \left(\frac{K_N}{L_N}\right)^{-1} \frac{d\left(\frac{K_N}{L_N}\right)}{d\frac{1}{\mu}}$$

and since  $\frac{K_Y}{L_Y} = \frac{K_N}{L_N}$  we have  $\frac{K_N}{L_N} = \frac{K}{L}$ . Since we assume  $K$  is supplied inelastically, the sign of  $\frac{d\left(\frac{K}{L}\right)}{d\frac{1}{\mu}}$  will depend only on the comovement of labor supply and (inverse) markup.

$$\chi C L^{\frac{1}{\varphi}} = \frac{W}{p}$$

so

$$\chi L^{1+\frac{1}{\varphi}} = S_L$$

and thus

$$\frac{d\left(\frac{K_N}{L_N}\right)}{d\frac{1}{\mu}} = -\frac{K}{L} \frac{1}{1 + \frac{1}{\varphi}} \frac{1}{S_L} \frac{dS_L}{d\frac{1}{\mu}}$$

which results in

$$\frac{dS_L}{d\frac{1}{\mu}} = \frac{\theta_Y - \theta_N}{1 + \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{K_N}{L_N}\right)^\sigma \left(1 - \frac{\sigma}{1+\frac{1}{\varphi}}\right)}.$$

We restrict  $\sigma \leq 1$ , the sign of  $1 - \frac{\sigma}{1+\frac{1}{\varphi}}$  is thus always positive. Therefore

$$\frac{dS_L}{d\mu} \gtrless 0 \text{ if and only if } \theta_N \gtrless \theta_Y.$$

Since

$$S_\Pi = \frac{1}{\mu} (1 - \theta_Y) + \left(1 - \frac{1}{\mu}\right) (1 - \theta_N)$$

$$\frac{dS_\Pi}{d\mu} \gtrless 0 \text{ if and only if } \theta_Y \gtrless \theta_N.$$

□

We can also show that an increase in markup always increases *relative* labor share of N-type labor,  $\frac{S_{L_N}}{S_L}$  and always reduces *relative* labor share of Y-type labor  $\frac{S_{L_Y}}{S_L}$ . We have

$$\frac{S_{L_Y}}{S_L} = \frac{\frac{1}{\mu} \theta_Y}{\frac{1}{\mu} \theta_Y + \left(1 - \frac{1}{\mu}\right) \theta_N}$$

$$\frac{S_{L_N}}{S_L} = \frac{\left(1 - \frac{1}{\mu}\right) \theta_N}{\frac{1}{\mu} \theta_Y + \left(1 - \frac{1}{\mu}\right) \theta_N}$$

and so

$$\frac{d \log \left( \frac{S_{LY}}{S_L} \right)}{d \frac{1}{\mu}} = \frac{\mu \theta_N}{\frac{1}{\mu} \theta_Y + \left( 1 - \frac{1}{\mu} \right) \theta_N} > 0$$

$$\frac{d \log \left( \frac{S_{LN}}{S_L} \right)}{d \frac{1}{\mu}} = - \frac{\mu \theta_Y}{(\mu - 1) \left( \frac{1}{\mu} \theta_Y + \left( 1 - \frac{1}{\mu} \right) \theta_N \right)} < 0.$$

#### C.4.2 $\alpha_N = 0$

**Theorem 6.** *In an economy with preferences given by*

$$U(C, L_Y, L_N) = \log C + \chi \frac{(L_Y + L_N)^{1 + \frac{1}{\varphi}}}{1 + \frac{1}{\varphi}},$$

*CES production functions with  $\alpha_Y > 0$  and  $\alpha_N = 0$ , perfect mobility of labor between sectors and with inelastic supply of capital*

$$\frac{dS_L}{d\mu} \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ if and only if } \theta_N \begin{matrix} \geq \\ \leq \end{matrix} \frac{1 - \Phi_Y}{1 - \sigma \Phi_Y} \theta_Y$$

where

$$\Phi_Y := \frac{1}{1 + \frac{\alpha_Y}{1 - \alpha_Y} \left( \frac{K_Y}{L_Y} \right)^\sigma}.$$

Conversely,

$$\frac{dS_\Pi}{d\mu} \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ if and only if } \frac{1 - \Phi_Y}{1 - \sigma \Phi_Y} \theta_Y \begin{matrix} \geq \\ \leq \end{matrix} \theta_N.$$

*Proof.* It is easier to work with loglinearized equilibrium conditions in order to prove this results.

$$l = \frac{L_Y}{L} l_Y + \frac{L_N}{L} l_N$$

$$\hat{\mu} + \frac{1}{\varphi} l = l_Y (\sigma \Phi_Y - 1)$$

$$-\frac{1}{\mu - 1} \hat{\mu} + \frac{1}{\varphi} l = -l_N$$

We can solve for

$$l = \frac{\frac{L_Y}{L} \frac{1}{\sigma \Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1}}{1 + \frac{1}{\varphi} \left( \frac{L_N}{L} - \frac{1}{\sigma \Phi_Y - 1} \frac{L_Y}{L} \right)} \hat{\mu}$$

and since

$$\chi L^{1 + \frac{1}{\varphi}} = S_L$$

$S_L$  is increasing in markup if and only if

$$\frac{\frac{L_Y}{L} \frac{1}{\sigma \Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1}}{1 + \frac{1}{\varphi} \left( \frac{L_N}{L} - \frac{1}{\sigma \Phi_Y - 1} \frac{L_Y}{L} \right)} > 0$$

so we need

$$\begin{aligned} \frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1} &> 0 \\ 1 + \varphi \left( \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} \right) &> 0 \end{aligned}$$

or

$$\begin{aligned} \frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1} &< 0 \\ 1 + \varphi \left( \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} \right) &< 0 \end{aligned}$$

Observe that

$$\begin{aligned} \frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1} &= \frac{1}{(1 - \Phi_Y)\theta_Y + (\mu - 1)\theta_N} \left[ \theta_N - \frac{1 - \Phi_Y}{1 - \sigma\Phi_Y} \theta_Y \right] \\ \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} &= \frac{1}{(1 - \Phi_Y)\theta_Y + (\mu - 1)\theta_N} \left[ (\mu - 1)\theta_N + \frac{1 - \Phi_Y}{1 - \sigma\Phi_Y} \theta_Y \right] \end{aligned}$$

so

$$\frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1} > 0 \iff \theta_N > \frac{1 - \Phi_Y}{1 - \sigma\Phi_Y} \theta_Y$$

and, since  $1 - \sigma\Phi_Y > 0$

$$\frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} > 0$$

which gives

$$\frac{dL}{d\mu} > 0 \iff \theta_N > \frac{1 - \Phi_Y}{1 - \sigma\Phi_Y} \theta_Y$$

□

We can also show that an increase in markup always increases *relative* labor share of N-type labor,  $\frac{S_{L_N}}{S_L}$  and always reduces *relative* labor share of Y-type labor  $\frac{S_{L_Y}}{S_L}$ . Once again we will use loglinearized equilibrium conditions. Normalize  $\hat{\mu} = 1$ . We have

$$\begin{aligned} s_{L_Y} &= \frac{1}{\varphi} l + \frac{1 + \frac{1}{\varphi} l}{\sigma\Phi_Y - 1} \\ s_L &= \left( 1 + \frac{1}{\varphi} \right) l \\ s_{L_Y} - s_L &= \frac{1 + \frac{1}{\varphi} l}{\sigma\Phi_Y - 1} - l \end{aligned}$$

and since

$$l = \frac{\frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1}}{1 + \frac{1}{\varphi} \left( \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} \right)}$$

$$s_{L_Y} - s_L < 0 \iff -\frac{1}{1 - \sigma\Phi_Y} < \frac{\frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1}}{1 + \frac{1}{\varphi} \left( \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} \right)} \left( 1 + \frac{1}{\varphi} \frac{1}{1 - \sigma\Phi_Y} \right)$$

which can be simplified to

$$s_{L_Y} - s_L < 0 \iff -(1 - \Phi_Y)\theta_Y < \left[ \left( 1 + \frac{1}{\varphi} \right) \mu - \sigma\Phi_Y \right] \theta_N.$$

Since  $1 - \Phi_Y > 0$  and  $\sigma\Phi_Y < 1$  it is always satisfied. To show that relative labor share of N-type labor increases notice that

$$\begin{aligned} s_{L_N} &= \frac{1}{\mu - 1} \\ s_L &= \left( 1 + \frac{1}{\varphi} \right) l \\ s_{L_Y} - s_L &= \frac{1}{\mu - 1} - \left( 1 + \frac{1}{\varphi} \right) l \end{aligned}$$

so

$$s_{L_Y} - s_L > 0 \iff \frac{1}{\mu - 1} > \left( 1 + \frac{1}{\varphi} \right) \frac{\frac{L_Y}{L} \frac{1}{\sigma\Phi_Y - 1} + \frac{L_N}{L} \frac{1}{\mu - 1}}{1 + \frac{1}{\varphi} \left( \frac{L_N}{L} - \frac{1}{\sigma\Phi_Y - 1} \frac{L_Y}{L} \right)}$$

which can be simplified to

$$\left( 1 - \sigma\Phi_Y + \frac{1}{\varphi} \right) > -(\mu - 1) \left( 1 + \frac{1}{\varphi} \right)$$

which is always satisfied.

It is also possible to show that a weaker version of Theorem 2 holds in this economy, i.e:

$$\begin{aligned} \frac{dS_{L_Y}}{d\mu} < 0 &\iff \sigma > -\frac{1}{\theta_N\Phi_Y} \left( 1 + \frac{1}{\varphi} \right) [(\mu - 1)\theta_N + (1 - \Phi_Y)\theta_Y] \\ \frac{dS_{L_N}}{d\mu} &> 0 \end{aligned}$$

## C.5 Variety-specific DRS in production

**Wholesale Sector** Each variety  $j \in [0, N]$  in this economy is produced by a variety-specific representative wholesaler. Wholesaler  $j$  hires labor  $L_{Y_j}$  in a competitive labor market at wage  $W_Y$ , which it uses to produce variety intermediate good  $y_j$ . The intermediate good  $Y_j$  is then sold to retailers at price  $P_{Wj}$ . The wholesaler thus chooses labor and output to maximize profits  $\Pi_{Wj}$ :

$$\begin{aligned} \Pi_{Wj} &:= \max_{L_{Y_j}, Y_j} P_{Wj} Y_j - W_{Y_j} L_{Y_j} \\ &\text{subject to} \\ y_j &= Z_Y L_{Y_j}^{\theta_Y} \end{aligned}$$

**Retail Sector** A unit measure continuum of identical retailers each hire labor  $L_N$  in a competitive labor market at wage  $W_N$ , which they use to manage product lines. We refer to the labor used in the retail sector as N-type labor, overhead labor, or expansionary uses of labor. Each product line  $j$  generates gross

profits  $\Pi_j$ , which the retailer's expansion department takes as given when deciding on the number of lines to operate. The retailer thus chooses labor and product lines to maximize net profits  $\Pi_R$ :

$$\begin{aligned}\Pi_R &:= \max \int_0^N \Pi_j dj - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N}\end{aligned}$$

The retailer's pricing department for product line  $j$  purchases  $y_j$  units of good from the representative wholesaler  $j$ , which it costlessly differentiates and sells to consumers at a markup  $\mu \geq 1$  over marginal cost  $P_{Wj}$ . Hence the price charged for product line  $j$  is

$$p_j = \mu P_{Wj} \quad (22)$$

**Factor shares** We focus on symmetric equilibria in which  $p_j = p \forall j$  and  $y_j = y \forall j$ . Market clearing for intermediate goods then implies that

$$\begin{aligned}\Pi_j &= \Pi \\ \Pi_{Wj} &= \Pi_W \\ L_{Yj} &= L_Y \\ P_{Wj} &= P_W \\ C &= Ny \\ L &= NL_Y + L_N\end{aligned}$$

The shares of total income accruing to  $Y$ -type labor and  $N$ -type are defined as

$$S_N := \frac{W_N L_N}{pY} \text{ and } S_Y := \frac{W_Y N L_Y}{pY},$$

and the overall labor share is defined as  $S_L = S_N + S_Y$ . The overall profit share in the economy is given by the sum of profit shares in the wholesale and retail sectors,  $S_\Pi = S_R + S_W$ , where

$$S_W := \frac{N \Pi_W}{pNy} \text{ and } S_R := \frac{\Pi_R}{pNy}.$$

To obtain expressions for factor shares notice that first order conditions in the wholesale and retail sector are

$$\begin{aligned}W_Y &= P_W \theta_Y \frac{y}{L_Y} \\ W_N &= \theta_N \frac{N}{L_N} y (p - P_W)\end{aligned}$$

which in equilibrium (using equation 22) can be rewritten as

$$\begin{aligned}\frac{W_Y N L_Y}{p(Ny)} &= \theta_Y \frac{1}{\mu} \\ \frac{W_N L_N}{pNy} &= \theta_N \left(1 - \frac{1}{\mu}\right).\end{aligned}$$

This shows that expressions for labor share remain unchanged. Lemma 1 still holds. Since the formulas for factor shares do not change Theorems 1 and 2 do not need any modification.



**Cyclicalilty of markups** We assume no love of variety. The representative household solves

$$\begin{aligned} \max_{C,L} \log C - \chi \frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \\ \text{subject to} \\ pC = WL + \Pi_W + \Pi_R \end{aligned}$$

where  $\Pi_W, \Pi_R$  are profits from wholesale and retail. First order conditions are as before. Equilibrium conditions are

$$\begin{aligned} \chi CL^{\frac{1}{\varphi}} &= \frac{1}{\mu} \theta_Y \frac{y}{L_Y} \\ \chi CL^{\frac{1}{\varphi}} &= \left(1 - \frac{1}{\mu}\right) \theta_N \frac{Ny}{L_N} \\ C &= Ny \\ L &= NL_Y + L_N. \end{aligned}$$

We follow the same approach as in Appendix ?? and loglinearize equilibrium conditions.

$$\begin{aligned} l &= \frac{L_N}{L} l_N + \frac{NL_Y}{L} (\theta_N l_N + l_Y) \\ \frac{1}{\varphi} l &= -l_Y - \hat{\mu} - \theta_N l_N \\ \frac{1}{\varphi} l &= -l_N + \frac{1}{\mu-1} \hat{\mu} \end{aligned}$$

and we solve for total labor

$$l = \frac{1}{1 + \frac{1}{\varphi} \theta_Y + \theta_N (\mu-1)} \hat{\mu}$$

and labor in each sector

$$\begin{aligned} l_Y &= -(1 - \theta_N) \frac{1}{\varphi} l - \left(1 + \theta_N \frac{1}{\mu-1}\right) \hat{\mu} \\ l_N &= \frac{1}{\mu-1} \hat{\mu} - \frac{1}{\varphi} l \end{aligned}$$

Aggregate output in this economy is given by

$$\begin{aligned} c &= \theta_N l_N + \theta_Y l_Y \\ &= \left( \theta_N \frac{1 - \theta_Y}{\mu-1} - \theta_Y + \left( \theta_N \frac{1}{\varphi} (1 - \theta_Y) - \theta_Y \right) \frac{1}{1 + \frac{1}{\varphi} \theta_Y + \theta_N (\mu-1)} \right) \hat{\mu} \end{aligned}$$

For example, when  $\theta_Y \rightarrow 0$

$$c = \theta_N \frac{1}{\mu-1} \left( 1 + \frac{\frac{1}{\varphi}}{1 + \frac{1}{\varphi}} \right) \hat{\mu}$$

so consumption increases after an increase in the markup. On the other hand, when  $\theta_N \rightarrow 0$

$$c = -\theta_Y \left( 1 + \frac{\frac{1}{\varphi}}{1 + \frac{1}{\varphi}} \right) \hat{\mu}$$

and the opposite happens. Importantly, it is not the case that the sign of  $\frac{\partial C}{\partial \mu}$  is the same as the sign of  $\frac{\partial S_L}{\partial \mu}$ . When  $\theta_N = \theta_Y = \theta$

$$c = \theta \left( \frac{1 - \theta}{\mu - 1} - 1 \right) \hat{\mu}$$

and it can be positive or negative, depending on the parameters, while

$$s_L = 0.$$

## C.6 Integrated wholesale and retail sectors as single firms

In this section we study a version of the model in which retailers have to produce goods themselves. We assume variety-specific production function

$$y_j = Z_Y L_{Yj}^{\theta_Y}$$

Cost minimization problem in each product line (or variety) is

$$\begin{aligned} TC_j &:= \min_{y_j, L_{Yj}} W_Y L_{Yj} \\ &\text{subject to} \\ y_j &= Z_Y L_{Yj}^{\theta_Y} \end{aligned}$$

which results in expression for marginal cost

$$\frac{1}{\theta_Y} W_Y \left( \frac{y_j}{Z_Y} \right)^{\frac{1}{\theta_Y} - 1} Z_Y$$

We assume price setting behavior is such that

$$p_j = \mu \times \frac{1}{\theta_Y} W_Y \left( \frac{y_j}{Z_Y} \right)^{\frac{1}{\theta_Y} - 1} Z_Y$$

Profits per product line are

$$\Pi_j = p_j y_j - W_Y L_{Yj}$$

so

$$\Pi_j = p_j y_j \left( 1 - \theta_Y \frac{1}{\mu} \right).$$

The retailer chooses labor and product lines to maximize net profits  $\Pi_R$  taking profits per product line as given

$$\begin{aligned} \Pi_R &:= \max_{L_N, N} \int_0^N \Pi_j dj - W_N L_N \\ &\text{subject to} \\ N &= Z_N L_N^{\theta_N} \end{aligned}$$

In a symmetric equilibrium the first order condition can be rewritten as

$$\frac{W_N}{p} = \theta_N \left( 1 - \theta_Y \frac{1}{\mu} \right) \frac{N y}{L_N}$$

so

$$S_N := \frac{W_N L_N}{pNy} = \theta_N \left( 1 - \theta_Y \frac{1}{\mu} \right).$$

We also have

$$S_Y := \frac{W_Y L_Y}{pNy} = \theta_Y \frac{1}{\mu}$$

and labor share in this economy is

$$S_L = \theta_Y \frac{1}{\mu} + \theta_N \left( 1 - \theta_Y \frac{1}{\mu} \right)$$

and is higher than labor share in economy in Section 2 as long as  $\theta_Y < 1$ . In this economy  $N$ -type workers get part of profits resulting from decreasing returns to scale in production. Since

$$\frac{\partial S_L}{\partial \frac{1}{\mu}} = \theta_Y (1 - \theta_N) > 0$$

labor share in this economy always falls when markups increase.

## D Details of Data and Additional Estimation Results

### D.1 Detailed Data Description

#### Labor share

- **Baseline Gomme and Rupert:** The measure excludes the household and government sectors and uses NIPA tables 1.12 and 1.7.5 and corresponds to the second alternative measure of the labor share proposed in [Gomme and Rupert \(2004\)](#). They define unambiguous labor income as compensation of employees, and unambiguous capital income (as corporate profits, rental income, net interest income, and depreciation). The remaining (ambiguous) components are then proprietors' income plus indirect taxes net of subsidies (NIPA Table 1.12). These are apportioned to capital and labor in the same proportion as the unambiguous components. Here  $CE_t$  is compensation of employees (line 2 in NIPA table 1.12),  $RI_t$  rental income (line 12 in NIPA table 1.12),  $CP_t$  corporate profits before tax (line 13 in NIPA table 1.12),  $NI_t$  net interest income (line 18 in NIPA table 1.12) and  $\delta_t$  depreciation (line 5 in table 1.7.5).

$$LS_t = \frac{CE_t}{CE_t + RI_t + CP_t + NI_t + \delta_t}$$

- **BLS Nonfarm Business:** U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Labor Share (FRED id: PRS85006173)
- **BLS Nonfinancial:** U.S. Bureau of Labor Statistics, Nonfinancial Corporations Sector: Labor Share (FRED id: PRS88003173)
- **Cooley and Prescott:** Follows Cooley and Prescott (1995). The labor share of income is defined as one minus capital income divided by output. Cooley and Prescott assume that the proportion of ambiguous capital income  $ACI_t$  to ambiguous income  $AI_t$  is the same as the proportion of unambiguous capital income to unambiguous income. Ambiguous income,  $AI_t$  is the sum of proprietors income (line 9, NIPA table 1.12), taxes on production less subsidies (lines 19 and 20, NIPA Table 1.12), business current transfer payments (line 21, NIPA Table 1.12). Unambiguous income  $UI_t$  consists of compensation of employees (line 2 in NIPA table 1.12) and unambiguous capital income  $UCI_t$  which in turn consists of rental income (line 12, NIPA Table 1.12), net interests (line 13, Table

1.12), corporate profits (line 18, NIPA Table 1.12) and current surplus of government enterprises (line 25, NIPA Table 1.12). Formally

$$\begin{aligned}
 CS_t^U &= \frac{UCI_t + \delta_t}{UI_t} \\
 ACI_t &= CS_t^U AI_t \\
 LS_t &= 1 - \frac{UCI_t + \delta_t + ACI_t}{GNP_t}
 \end{aligned}$$

where  $\delta_t$  is depreciation (line 5 in table 1.7.5 )

- **Fernald:** It is taken from [Fernald \(2014\)](#). It is utilization adjusted quarterly series.

## Markup

- Data from 1947 Q1 to 2019 Q2 from U.S. Bureau of Labor Statistics. We use the following series:
  - **WPSFD49207**, Producer Price Index by Commodity for Final Demand: Finished Goods, Seasonally Adjusted
  - **WPSID61**, Producer Price Index by Commodity for Intermediate Demand by Commodity Type: Processed Goods for Intermediate Demand Seasonally Adjusted

## Occupational income shares

- We use data from the monthly Outgoing Rotation Groups of the Current Population Survey (CPS-ORG) to construct quarterly series for occupational income shares  $s_j$  . We restrict attention to employed individuals aged 16 and over not living in group quarters, and measure labor income with the IPUMS variable ‘earnweek’. This variable reports the amount (in dollars) a given individual earned from their job each week before deductions.
- We compute quarterly labor income by summing weekly labor income for individuals in each of 9 broad occupation categories, which we construct from the 389 OCC1990 occupation codes in a following way:
  - Managerial occupations: OCC1990 codes 3 - 37
  - Professional specialty occupations: OCC1990 codes 43 - 200
  - High-tech occupations: OCC1990 codes 203 - 235
  - Sales occupations: OCC1990 codes 243 - 283
  - Administrative support and clerical occupations: OCC1990 codes 303 - 389
  - Service occupations OCC1990 codes 405 - 469
  - Farming, forestry, and fishing occupations, construction and extractive occupations: OCC1990 codes 473-498, 558 - 599 and 614 - 617
  - Precision production occupations and repair: OCC1990 codes 503 - 549 and 628 - 699
  - Machine operators, assemblers, inspectors, transportation and material moving occupations: OCC1990 codes 703 - 799 and 803 - 889
- We remove all observations with OCC1990 codes that do not belong to any of the above 9 broad categories, for example observations with missing OCC1990 codes or military occupations. We then divide the sum of labor income of individuals in each broad category by the sum of labor income of all individuals in any given quarter to obtain  $s_j$ .

- Because there was a change in occupational codes used in CPS-ORG in 2002 we make the following adjustment: we calculate a difference between  $s_j$  in 2002Q4 and  $s_j$  in 2003Q1 and then we adjust  $s_j$  in 2003Q1 and later by adding this difference. This assumes that all changes in  $s_j$  between 2002 and 2003 were due to the change in occupational codes.
- We then use X-12-ARIMA to do seasonal adjustment of  $s_j$  and renormalize  $s_j$  so that their sum in each quarter is always equal to 1.

## Occupational total hours and median hourly wages

- We use data from the 1980 Census and 2015 American Community Survey (ACS) to calculate occupational total hours and median hourly wages. We restrict attention to employed individuals aged 18 to 65 and not living in group quarters, and measure labor income with the IPUMS variable 'incwage'. This variable reports the amount (in dollars) reports each respondent's total pre-tax wage and salary income - that is, money received as an employee - for the previous 12 months. Sources of income in 'incwage' include wages, salaries, commissions, cash bonuses, tips, and other money income received from an employer. Payments-in-kind or reimbursements for business expenses are not included.
- We keep only individuals with positive and known wage income and hours and positive weights
- To calculate the number of hours per year that the respondent usually worked we use variables 'uhrwork' and 'wkwork2'. 'uhrwork' reports the number of hours per week that the respondent usually worked, if the person worked during the previous 12 months. 'wkwork2' reports the number of weeks that the respondent worked for profit, pay, or as an unpaid family worker during the previous 12 months. Because 'wkwork2' is reported in intervals (1-13 weeks, 14-26 weeks, and so on), instead of the precise number of weeks, we associate each value of 'wkwork2' with the midpoint of the corresponding interval (for example if 'wkwork2'=1 we treat it as 7 weeks). We multiply 'uhrwork' by our measure of weeks based on 'wkwork2'. Hourly wages are then computed by dividing 'incwage' by the number of hours per year.
- We calculate occupational total hours by summing yearly hours for individuals in each of 9 broad occupation categories.

## D.2 Stage 1 Additional Tables and Figures

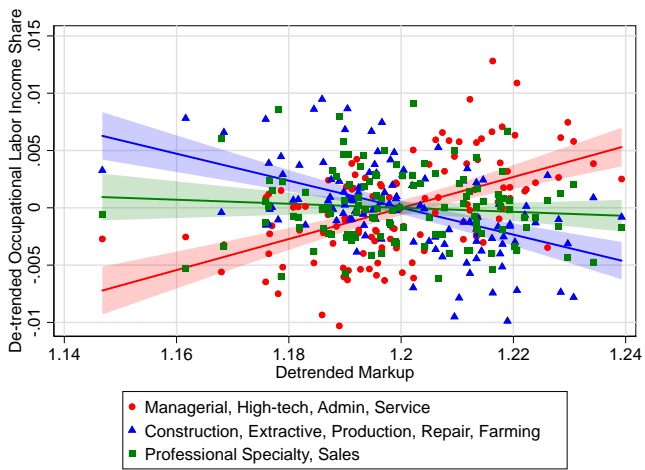
## D.3 Stage 2 Additional Tables and Figures

	(1)	(2)	(3)	(4)	(5)
	Baseline Gomme- Rupert	BLS Non-farm Business	BLS Non-financial Corporate	Cooley- Prescott	Fernald
$\theta_Y$	0.934	0.953	0.940	0.930	0.934
	0.005	0.005	0.007	0.006	0.008
$\theta_N$	0.730	0.637	0.700	0.750	0.731
	0.024	0.027	0.035	0.029	0.037
Implied value of $\frac{S_{N,L}}{S_L}$	19%	17%	19%	19%	18%
P-val for test $\theta_N = \theta_Y(1 - \alpha_Y)$	0.00	0.00	0.00	0.00	0.00
Assumed mean markup, $\mu$	1.2	1.2	1.200	1.2	1.2
Assumed overall profit share, $S_{\Pi}$	10%	10%	10%	10%	10%
Capital share parameter, $\alpha$	0.320	0.354	0.351	0.290	0.297

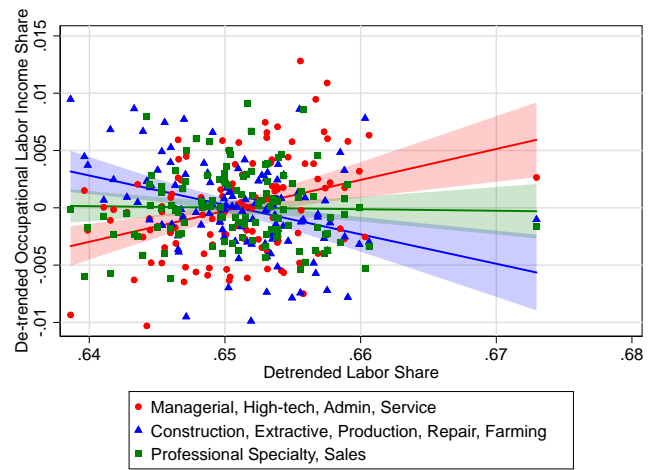
Table 4: First stage estimation results: alternate labor share series

	(1)	(2)	(3)	(4)	(5)
	Baseline Hodrick- Prescott	Baxter- King	Christiano- Fitzgerald	Linear Trend	Quadratic Trend
$\theta_Y$	0.934	0.934	0.937	0.928	0.922
	0.005	0.005	0.004	0.004	0.003
$\theta_N$	0.730	0.731	0.716	0.758	0.789
	0.024	0.023	0.020	0.018	0.016
Implied value of $\frac{S_{N,L}}{S_L}$	19%	19%	18%	19%	20%
P-val for test $\theta_N = \theta_Y(1 - \alpha_Y)$	0.00	0.00	0.00	0.00	0.00
Assumed mean markup, $\mu$	1.2	1.2	1.2	1.2	1.2
Assumed overall profit share, $S_{\Pi}$	10%	10%	10%	10%	10%
Capital share parameter, $\alpha$	0.320	0.320	0.319	0.322	0.324

Table 5: First stage estimation results: alternate methods of de-trending

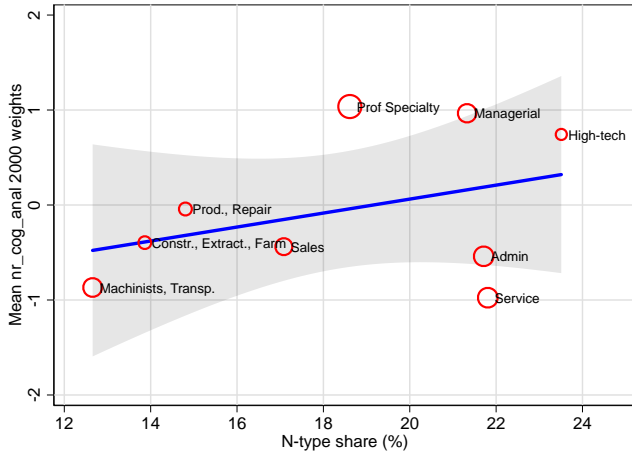


(a) Reduced form scatter plot

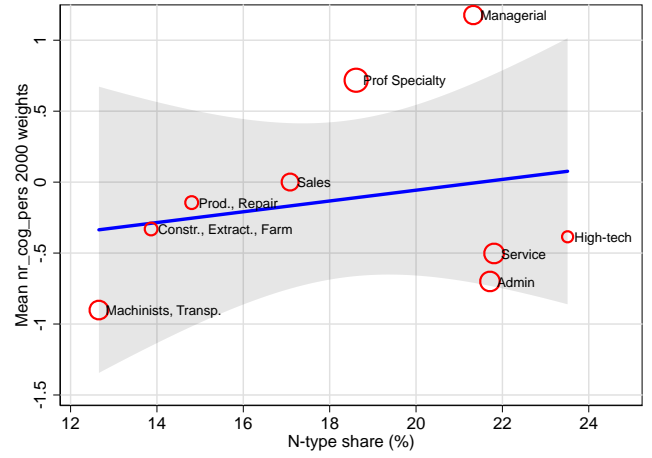


(b) Raw scatter plot, without instrument

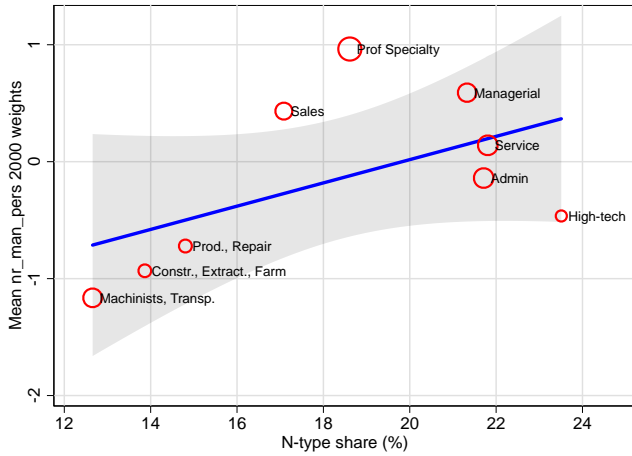
Figure 6: Cyclical components of occupational income shares



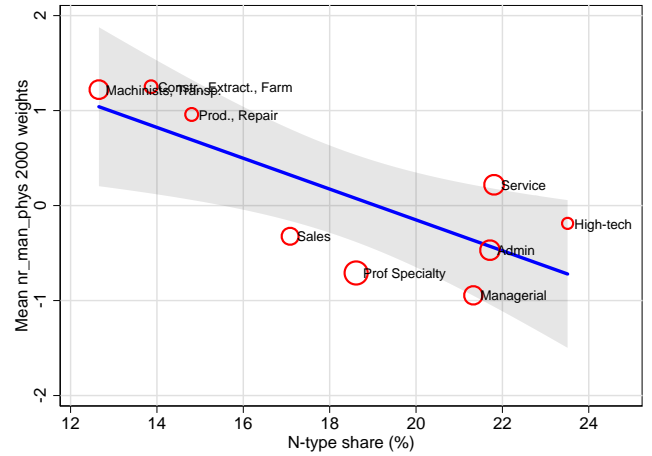
(a) Non-routine, cognitive, analytical



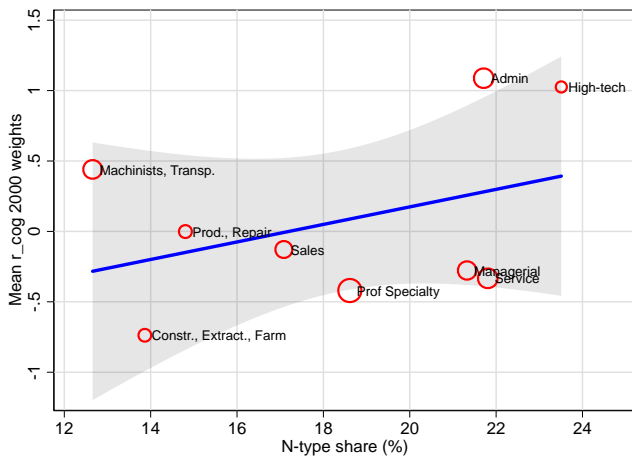
(b) Non-routine, cognitive, inter-personal



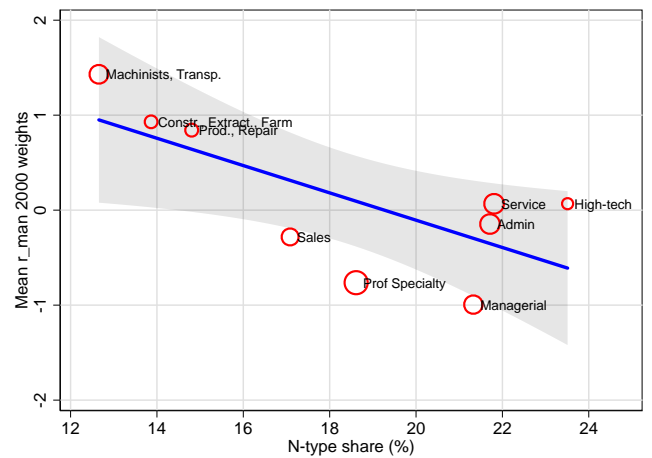
(c) Non-routine, inter-personal, manual



(d) Non-routine, physical, manual



(e) Routine, cognitive



(f) Routine, manual

Figure 7: Correlation of  $N$ -content of occupations with other occupation characteristics