PRECAUTIONARY SAVING AND THE TIMING OF TAXES

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ABSTRACT

This paper analyzes the effects of government debt and income taxes on

consumption and saving in a world of infinitely-lived households having

uncertain and heterogeneous incomes. The special structure of the model

allows exact aggregation across households despite incomplete markets. The

effects of government debt are shown to be substantial, roughly comparable

to those resulting from finite horizons, and crucially dependent on the length

of time until the debt is repaid. Also, anticipated changes in taxes are

shown to cause anticipated changes in consumption. Finally, an index of

fiscal stance is derived.

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I.   Introduction

Economists are increasingly aware of the importance of heterogeneity among individuals for issues in macroeconomics. One such issue is the interaction between individual income uncertainty and tax policy. Since an individual's personal tax liability is typically contingent on his income, and since future income is uncertain, future taxes provide a form of insurance. This insurance effect of income taxes has normative implications regarding the desirability of the taxes (Varian, 1980) and positive implications regarding their impact on consumption and saving (Chan, 1983; Barsky, Mankiw, Zeldes, 1986).[1]

The purpose of this paper is to study the response of consumption to the timing of labor income taxes. We assume that individuals are infinitely lived, so that the taxes do not redistribute across generations. We also assume that labor supply is inelastic, so that the taxes are not distortionary. The failure of Ricardian equivalence in our model is fully attributable to the insurance effect of the income tax system. This failure of Ricardian equivalence, which was discussed by Barro (1974, p. 1115) and Tobin (1980, p. 59-60), was first analyzed explicitly by Chan using a two-period model. Barsky, Mankiw, and Zeldes argued that this insurance effect is likely to be quantitatively important; they examined multi-period examples but only through the use of computer simulations and under the assumption that income is independently distributed in each period. Here we allow individual income to follow a Markov process. Under the assumption that the utility function is exponential, we are able to examine analytically the response of consumption over time to various policy interventions.

After describing the model and its solution in Section II, we examine in Section III the impact of changes in the timing of income taxes. All the policy interventions satisfy an intertemporal government budget constraint. If contingent claims markets were complete or if utility were quadratic, one would obtain the Ricardian result that these interventions have no impact on consumption. We assume, however, that individuals face idiosyncratic income risk and that, since tax liabilities are contingent on individual income, changes in the timing of these liabilities change perceived risk. This change in risk interacts with the precautionary motive for saving (Leland, 1968; Sandmo, 1970; Dreze and Modigliani, 1972). As in Barsky, Mankiw, and Zeldes, the implied behavior appears in some ways more Keynesian than Ricardian.

First, we examine a current tax cut, coupled with a tax increase in the future to repay the additional debt and accumulated interest. We show that the horizon over which the debt is repaid is crucial to the effect of the tax cut. Tax reschedulings over short periods of time have little impact on consumption, while tax reschedulings over long periods of time have substantial impact.

Second, we consider the empirically plausible case in which a tax cut is coupled with a permanently higher level of debt. In this case, all future tax rates are raised just enough to service the debt. We obtain partially Ricardian results. A one dollar tax cut increases consumption, but by less than does a one dollar increment to wealth. For reasonable parameter values, the marginal propensity to consume out of such a tax cut is about half the marginal propensity to consume out of wealth.

Third, we examine the impact of an announced future tax cut. We show that this announcement causes an immediate increase in the level of

consumption, followed by further increases in consumption as the tax cut approaches. Hence, while news about future taxes has an immediate impact on consumption, anticipated changes in taxes are also associated with anticipated changes in consumption.

Fourth, we derive an index of fiscal stance analogous to that suggested by Blanchard (1985). The index implies that fiscal policy has similar effects on aggregate demand in precautionary-saving models and finite-horizon models, even though the mechanisms are very different.

We share with much recent work the strategy of examining the implications of capital market imperfections without deriving the imperfections from the economic environment. (See, for example, Scheinkman and Weiss (1986) and Hubbard and Judd (1986).) A crucial assumption of our model is that individual human capital risk cannot be diversified. It would of course be better to derive this feature from the more primitive informational considerations of moral hazard and adverse selection. We hope that our model can provide a prelude to a more complete analysis of the interactions between precautionary saving and the timing of taxes.

II. The Model[2]

Consider an infinitely lived consumer who has additively time-separable von Neumann-Morgenstern utility $\int_0^\infty e^{-\rho s} u(c_{t+s}) ds$. The consumer is assumed to face a constant real interest rate, r, and stochastic income following a continuous-time Markov process.[3] Let $\Lambda = [\lambda_{ij}]$ be the Markov transition matrix among the J states with λ_{ij} for $i \neq j$ representing the instantaneous probability of moving from state i to state j and $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$ representing the instantaneous probability of leaving state i.[4] The optimization problem for the consumption and saving decision can then be written as follows:

(2.1) $\quad V(w_t, i, t) = \max_c \int_0^\infty e^{-(\rho + \sum_{j \neq i} \lambda_{ij})s} \left[u(c_{t+s}) + \sum_{j \neq i} \lambda_{ij} V(w_{t+s}, j, t+s) \right] ds$

$\quad\quad\quad$ s.t. $\dot{w}_{t+s} = rw_{t+s} + y^i_{t+s} - c_{t+s}$,

$\quad\quad\quad w_t$ given, and

$\quad\quad\quad \lim_{s \to \infty} e^{-rs} w_{t+s} = 0$ with probability one,

where $V(w_t, i, t)$ is expected utility when starting at time t in state i with wealth w_t, ρ is the subjective rate of time preference, c_{t+s} is the flow of consumption, and y^i_{t+s} the flow of income in state i at time t+s. The continuous time Bellman equation for this problem is[5]

(2.2) $\quad \rho V(w_t, i, t) - V_t(w_t, i, t) - \sum_{j \neq i} \lambda_{ij} [V(w_t, j, t) - V(w_t, i, t)]$

$\quad\quad\quad = \max_{c_t} [u(c_t) + (rw_t + y^i_t - c_t) V_w(w_t, i, t)] \quad\quad\quad i = 1, \ldots, J,$

where subscripts on the value function indicate partial derivatives.

We examine the special case in which the consumer has constant absolute risk aversion, that is,

(2.3) $\quad u(c_t) = \dfrac{e^{-\gamma c_t}}{-\gamma}.$

Given this assumption, (2.2) can be solved explicitly.[6] In particular, there is a solution of the form

(2.4) $\quad V(w_t, i, t) = \dfrac{-1}{\gamma r} e^{-\gamma(rw_t + a^i_t)} \quad\quad\quad\quad i = 1, \ldots, J.$

This can be demonstrated as follows. First, the maximization on the right-hand side of (2.2) implies the first order condition

$$(2.5) \qquad u'(c_t^i) = V_w(w_t, i, t) \qquad\qquad i=1, \ldots, J,$$

where c_t^i is the optimal rate of consumption if in state i at time t. In words, the marginal utility of a dollar's worth of consumption must be set equal to the marginal value of a dollar in the overall program. Given the utility function assumed and the value function we are trying out, (2.5) implies

$$(2.6) \qquad e^{-\gamma c_t^i} = e^{-\gamma(rw_t + a_t^i)} \qquad\qquad i=1, \ldots, J,$$

which means that

$$(2.7) \qquad c_t^i = rw_t + a_t^i \qquad\qquad i=1, \ldots, J.$$

Thus a_t^i is the intercept of the "consumption function" in state i at time t, and r is the marginal propensity to consume out of wealth.

On substituting (2.3), (2.4) and (2.7) into (2.2), and dividing by $\frac{-1}{\gamma r} e^{-\gamma(rw_t + a_t^i)}$, we obtain the equation

$$(2.8) \qquad \rho + \gamma \dot{a}_t^i + \sum_{j \neq i} \lambda_{ij} - \sum_{j \neq i} \lambda_{ij} e^{\gamma(a_t^i - a_t^j)} = r + \gamma r(a_t^i - y_t^i) \qquad i=1, \ldots, J.$$

In (2.8), the consumer's wealth w_t has dropped out, leaving a set of ordinary differential equations in the vector of consumption intercepts. The solution to (2.8) therefore indicates a solution to Bellman equation (2.2) of the proposed form (2.4).

It can be shown that given a fixed vector y of incomes in each state, (2.8) has a unique steady-state vector of consumption intercepts a.[7] Given a steady-state solution for the vector of consumption intercepts a, we can readily find the effects on consumption of marginal changes in the income available in various states at various times. Denoting marginal departures from steady-state values by a tilde $(\sim)$, and the steady-state values themselves by the omission of the time subscript, we can "totally differentiate" (2.8) to obtain

$$(2.9) \qquad r\tilde{y}_t^i = r\tilde{a}_t^i - \dot{\tilde{a}}_t^i + \sum_{j \neq i} \lambda_{ij}(\tilde{a}_t^i - \tilde{a}_t^j)e^{\gamma(a^i - a^j)} \qquad\qquad i=1,\ldots,J.$$

Equation (2.9) can be written in matrix form as

$$(2.10) \qquad r\tilde{y}_t = r\tilde{a}_t - \dot{\tilde{a}}_t - \Psi\tilde{a}_t,$$

where $\tilde{y}_t = [\tilde{y}_t^i]$ and $\tilde{a}_t = [\tilde{a}_t^i]$ are J x 1 vectors and Ψ is a J x J matrix with elements

$$(2.11) \qquad \psi_{ij} = \lambda_{ij}e^{\gamma(a^i - a^j)} \qquad\qquad \text{for } i \neq j$$

and

$$(2.12) \qquad \psi_{ii} = -\sum_{j \neq i} \psi_{ij}.$$

Equation (2.10) is a linear matrix differential equation with the standard solution

$$(2.13) \qquad \tilde{a}_t = r \int_0^\infty e^{-rs} e^{\Psi s} \tilde{y}_{t+s} \, ds + h e^{rt} e^{-\Psi t},$$

where h must be zero for the altered path of consumption to continue to satisfy the budget constraint cum nonsatiation condition

$\lim_{t \to \infty} e^{-rt} w_t = 0$ with probability one.[8]

Therefore,

$$(2.14) \quad \tilde{a}_t = r \int_0^\infty e^{-rs} e^{\Psi s} \tilde{y}_{t+s} \, ds.$$

To find the impact of income changes on aggregate consumption, we must know the distribution of consumers across income states. Assuming that there are many consumers in the economy facing independent Markov transitions but with the same transition matrix, and that Markov transitions have been taking place for a long time, the distribution of consumers across states will be described by the stationary distribution Π^* corresponding to the transition matrix Λ. Using C_t, W_t and Y_t for per capita averages, (2.7) and (2.14) imply

$$(2.15) \quad \tilde{C}_t = r\tilde{W}_t + r\Pi^* \int_0^\infty e^{-rs} e^{\Psi s} \tilde{y}_{t+s} \, ds$$

$$= r\{\tilde{W}_t + \int_0^\infty e^{-rs} \tilde{Y}_{t+s} \, ds + \Pi^* \int_0^\infty e^{-rs}(e^{\Psi s} - I)\tilde{y}_{t+s} \, ds\}.$$

The term $\Pi^* \int_0^\infty e^{-rs}(e^{\Psi s} - I)\tilde{y}_{t+s} \, ds$ incorporates all of the precautionary saving effects on consumption resulting from the changes in distribution of income across states. The change in consumption due to the income disturbance is the interest rate r times not only the change in financial wealth $\tilde{W}_t$ and the change in average human wealth $\tilde{H}_t = \int_0^\infty e^{-rs} \tilde{Y}_{t+s} \, ds$, but also the change in what one might call "phantom human wealth," $\tilde{\Phi}_t = \Pi^* \int_0^\infty e^{-rs}(e^{\Psi s} - I)\tilde{y}_{t+s} \, ds$, which describes these precautionary saving effects.

A Special Case

To gain a better understanding of how the precautionary saving motive affects consumption, it is helpful to examine the precautionary saving effect $\tilde{\Phi}_t$ in the special case in which there are only two states -- a high income state (state 1) and a low income state (state 2) -- and the transition matrix Λ is symmetric; that is,

$$(2.16) \qquad \Lambda = \begin{bmatrix} -p & p \\ p & -p \end{bmatrix},$$

where p is the instantaneous probability of a transition from one state to the other. With Λ as in (2.16), Ψ is given by

$$(2.17) \qquad \Psi = \begin{bmatrix} -px & px \\ p/x & -p/x \end{bmatrix},$$

where

$$(2.18) \qquad x = e^{\gamma(a^1 - a^2)} = u'(c^2)/u'(c^1).$$

The quantity x is the ratio of marginal utilities between the high income and low income states (for a given value of non-human wealth) in the initial steady state. Its value can be determined from the following equation, which is derived by subtracting the steady-state version of (2.8) with i=1 from the steady-state version of (2.8) with i=2, and then using the definition (2.18):

$$(2.19) \qquad \ln(x) + \frac{p}{r}(x - \frac{1}{x}) = \gamma(y^1 - y^2).$$

Equation (2.19) is simple enough that it can readily be solved for x with the help of a pocket calculator. Given x, all other calculations we make can be done explicitly.

To find saving per capita in the steady state, we can add together the steady-state versions of (2.8) for i=1,2, obtaining

(2.20) $\quad \frac{\rho-r}{r} - \frac{p}{2r} (x + \frac{1}{x} - 2) = -\gamma \left[\frac{y^1 + y^2}{2} - \frac{a^1 + a^2}{2}\right]$.

The term in brackets on the right-hand side of (2.20) is per capita saving. Thus,

(2.21) $\quad S = rW + Y - C = \frac{r-\rho}{\gamma r} + \frac{p}{2\gamma r} (x + \frac{1}{x} - 2)$.

It is clear that the part of saving due to the interaction of individual income uncertainty with the precautionary saving motive is $\frac{p}{2\gamma r}(x + \frac{1}{x} - 2)$, since this term is zero when there is no income uncertainty (p=0 or x=1), while the other term $\frac{r-\rho}{\gamma r}$ is unaffected by income uncertainty. We use this expression below to calculate the magnitude of precautionary saving.

We now turn to the analysis of marginal departures from the steady state. To evaluate the matrix $(e^{\Psi s} - I)$ in (2.15) for the special case we are considering, we need to find two eigenvectors, which together form a diagonalizing matrix. We find that

(2.22) $\quad \Psi = \frac{1}{\delta}\begin{bmatrix} 1 & -px \\ 1 & p/x \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & -\delta \end{bmatrix}\begin{bmatrix} p/x & px \\ -1 & 1 \end{bmatrix}$

where the central diagonal matrix shows the two eigenvalues of Ψ (0 and $-\delta$), and

(2.23) $\quad \delta = (x + \frac{1}{x})p$.

Then

(2.24) $\quad e^{\Psi s} - I = \dfrac{1}{\delta}\begin{bmatrix}1 & -px \\ 1 & p/x\end{bmatrix}\begin{bmatrix}0 & 0 \\ 0 & e^{-\delta s}-1\end{bmatrix}\begin{bmatrix}p/x & px \\ -1 & 1\end{bmatrix}$

$\qquad\qquad\qquad = \dfrac{1-e^{-\delta s}}{\delta}\begin{bmatrix}-px \\ p/x\end{bmatrix}\begin{bmatrix}1 & -1\end{bmatrix}.$

Finally, since $\Pi^* = [\tfrac{1}{2},\tfrac{1}{2}]$ for a symmetric transition matrix Λ such as in (2.16), the definition of $\tilde{\Phi}$ simplifies to

(2.25) $\quad \tilde{\Phi}_t = e^{-rs}(\dfrac{1-e^{-\delta s}}{\delta})[\tfrac{1}{2},\tfrac{1}{2}]\begin{bmatrix}-px \\ p/x\end{bmatrix}(\tilde{y}^1_{t+s} - \tilde{y}^2_{t+s})$

$\qquad\qquad = -\tfrac{1}{2}\theta \displaystyle\int_0^\infty e^{-rs}(1-e^{-\delta s})(\tilde{y}^1_{t+s} - \tilde{y}^2_{t+s})ds,$

where

(2.26) $\quad \theta = \dfrac{x - \dfrac{1}{x}}{x + \dfrac{1}{x}}.$

Summary

Individuals face idiosyncratic risk, but there is no aggregate uncertainty. In our special case, half of all individuals at time t are in the good state earning income y^1_t and half are in the bad state earning income y^2_t. The probability of leaving a state is p each period. Each individual is infinitely lived and has a time-separable, constant absolute risk aversion utility function.

Given this specification, aggregate consumption locally obeys

(2.27) $\quad \tilde{C}_t = r[\tilde{W}_t + \tilde{H}_t + \tilde{\Phi}_t],$

where a tilde denotes the deviation from the steady state value. In (2.27),

$\tilde{W}_t$ is the deviation of aggregate non-human wealth, $\tilde{H}_t$ is the deviation of aggregate human wealth defined as the present value of aggregate labor income --

$$(2.28) \qquad \tilde{H}_t = \int_0^\infty e^{-rs} \, \tilde{Y}_{t+s} \, ds$$

-- and $\tilde{\Phi}_t$ is the deviation of precautionary saving defined by

$$(2.29) \qquad \tilde{\Phi}_t = -\tfrac{1}{2}\theta \int_0^\infty e^{-rs} \, (1 - e^{-\delta s})(\tilde{y}^1_{t+s} - \tilde{y}^2_{t+s})ds.$$

The parameters θ and δ are between zero and one and depend on the dispersion in income $(y^1 - y^2)$, the coefficient of absolute risk aversion γ, the interest rate r, and the transition probability p.[9]

III. The Timing of Taxes

We can now analyze the effects of a tax rescheduling. Some assumption must be made about the type of tax used. Any component of a tax that falls equally on both high income and low income consumers has no precautionary saving effect, because it does not affect $y^1_{t+s} - y^2_{t+s}$. Thus, Ricardian equivalence holds for lump-sum taxes.

We examine here the polar opposite case in which taxes are levied only on high income individuals. Since at any time half of the population is composed of high-income individuals, a \$1 per capita tax increase overall requires a \$2 per capita tax increase on "the rich". Therefore, if $\tilde{T}_{t+s}$ is the overall per capita tax increase in period t+s and taxes fall entirely on the high-income individuals, then

(3.1) $\tilde{y}^1_{t+s} - \tilde{y}^2_{t+s} = -2\tilde{T}_{t+s}.$

Substituting this expression into (2.30), we find that

(3.2) $\tilde{\Phi}_t = \theta \int\limits_0^\infty e^{-rs}(1 - e^{-\delta s})\tilde{T}_{t+s}ds.$

The parameters θ and δ have an important influence on the effects of various tax changes, as will become clear below.

Equation (3.2) shows the precautionary saving effect of tax changes. A balanced budget tax rescheduling has no immediate impact on the sum W_t+H_t of aggregate human and financial wealth. Thus the sole initial impact of a balanced budget tax change is the precautionary saving effect $r\tilde{\Phi}_t$.

If the tax change is first announced at time 0, then

(3.3) $\tilde{C}_t = r[\tilde{W}_t + \tilde{H}_t + \tilde{\Phi}_t]$

$$= r\left[-\int\limits_0^t e^{r\tau}\tilde{C}_{t-\tau}d\tau + \tilde{\Phi}_t\right]$$

$$= r\left[-r\int\limits_0^t \tilde{\Phi}_{t-\tau}d\tau + \tilde{\Phi}_t\right].$$

At time zero, when the tax change is newly announced, this simplifies to

(3.4) $\tilde{C}_0 = r\tilde{\Phi}_0 = r\theta \int\limits_0^\infty e^{-rs}(1 - e^{-\delta s})\tilde{T}_s ds.$

The immediate precautionary saving effect on consumption, $r\tilde{\Phi}_t$, is the interesting effect. The other term in (3.3) involving lagged consumption changes -- or, equivalently, lagged precautionary saving changes -- is just

what is necessary to insure that consumers do not violate their budget

constraints: if they consume $1 more in one year, they must consume $r less

every year from then on to make up for it. The key insight is that the

insurance effects of an income tax can induce consumers to consume more now

without any immediate change in their aggregate resources $W_t + H_t$.

Policy Experiment 1

There are several interesting special cases. The simplest is a tax

cut repaid k years later. If such a tax cut begins in period 0, then

(3.5) $\tilde{\Phi}_0 = \theta(1 - e^{-\delta k}) \tilde{D}$,

where $\tilde{D}$ is the size of the initial tax cut and the initial addition to the

national debt as a result of that tax cut. Equation (3.5) indicates that an

income tax cut followed by a compensating income tax increase the next year

has very little effect on consumption, but a tax cut followed by a tax

increase many years later has a much larger effect. In other words, the

interval between tax cut and tax increase is crucial to the impact of the tax

cut. Individuals face little uncertainty about their income next year and

there is correspondingly little insurance effect of higher income taxes next

year. But individuals face much more uncertainty about their income ten or

twenty years from now and, as a result, higher income taxes ten or twenty

years in the future have a substantial insurance effect.

The theory presented here is one way to rationalize the intuitive notion

that tax rescheduling within a year or any other short period of time should

not have much effect, but that tax rescheduling over longer periods of time

should have substantial effects on consumption. A common debating point for

Ricardians has been that if tax rescheduling within a year does not matter, then tax rescheduling over the course of 20 years also should not matter. We have identified here a clear distinction between tax rescheduling over short periods of time and tax rescheduling over long periods of time, even for infinitely-lived consumers.

Some Illustrative Calculations

To judge the magnitude of the precautionary saving effects, it is necessary to calibrate the model. There are two key parameters: $\gamma(y^1-y^2)$ and p/r. From these two magnitudes, the other parameters of interest, such as θ and δ/r, can be computed.[10]

The first parameter, $\gamma(y^1-y^2)$, incorporates both the degree of risk aversion γ and the cross-sectional dispersion in income $(y^1 - y^2)$. Note that $\gamma(y^1-y^2)$ can be written as $\gamma y^2[(y^1/y^2)-1]$ and that γy^2 is the coefficient of relative risk aversion evaluated at the level of income in the bad state. If we make the conservative assumption that income in the good state is twice income in the bad state, so that the cross-sectional coefficient of variation in income is only 1/3, then $\gamma(y^1-y^2)$ can be interpreted simply as the coefficient of relative risk aversion. We therefore allow this parameter to range over the region from 0.5 to 10.

The second parameter is p/r. Note that the transition probability p has the same units as the interest rate r; hence, p/r is a pure number. We allow p/r to vary from 0.10 to 5.0. If r is 2 percent per year, then p is varying from 0.2 percent per year to 10 percent per year. To judge the magnitude of p, note that over a 25-year horizon, the probability that an individual leaves

the state in which he begins is 12 percent if p is 0.5 percent and is 64 percent if p is 4.0 percent.

Tables 1, 2, and 3 present the values of θ, δ/r, and x for these parameter values. Assuming for the moment that the debt and accumulated interest associated with a tax cut are pushed far enough into the future that $e^{-\delta k}$ can be ignored, equation (3.5) shows that the marginal propensity to consume out of a tax cut is θ times the marginal propensity to consume out of wealth. Table 1 shows that the value of θ is usually in excess of ½ and is often close to one. These numbers, together with equation (3.5), imply that the precautionary saving effect can be quite potent.

The numbers for δ/r in Table 2 can be used to see how quickly $e^{-\delta k}$ declines with k. For an interest rate r of 2 percent per year, a value for δ of 4 percent per year is likely. For k = 25, $e^{-\delta k}$ is 0.37. Hence, for a tax liability pushed 25 years into the future, the marginal propensity to consume out of the tax cut is 0.63 θ times the marginal propensity to consume out of wealth.

Table 3 presents x, the ratio of marginal utility in the bad state and good state given equal non-human wealth in both states. These numbers imply that marginal utility in the bad state is about two to five times marginal utility in the good state. This ratio is, of course, larger if the cross-sectional dispersion in income is larger, if individuals are more risk averse, or if the transition probability is smaller relative to the interest rate.

The amount of precautionary saving expressed as a fraction of average labor income, $\left[\dfrac{p}{2\gamma(y^1 - y^2)r} \dfrac{y^1 - y^2}{Y} (x + \dfrac{1}{x} - 2)\right]$, is given in Table 4. It is clear that the amount of precautionary saving can be substantial. It should be remembered, though, that in general equilibrium the precautionary

saving motive might show up as much in a lower interest rate r as in increased saving.[11]

Policy Experiment 2

Another interesting experiment is a permanent increase in government debt, with the interest on the extra debt financed by higher taxes. For this experiment, (3.2) implies that

$$(3.6) \qquad \tilde{\Phi}_0 = \tilde{\Phi}_t = \frac{\delta}{r+\delta} \theta \tilde{D}.$$

In words, consumers act as if a permanent addition to government debt is at least partially net wealth, where the fraction that is treated as net wealth is $\frac{\delta}{r+\delta}\theta$. Table 5 shows this fraction for alternative values of the parameters. It is clear from Table 5 that substantial departures from Ricardian equivalence are likely. Yet government debt is still far from being treated as 100% net wealth. Whether this model gives results closer to full Ricardian neutrality or naive neglect of future tax liabilities depends on the parameters, but the best guess is probably halfway in between.[12]

The numbers in Table 5 are similar in magnitude to figures derived from a life-cycle model of consumption. Poterba and Summers (1987) simulate a realistic model of life-cycle saving; they compute the fraction of government debt that is net wealth assuming that taxes are levied to service the debt. Their Table 1 reports that, depending on the parameters, this fraction varies from 0.53 to 0.85. Hence, the precautionary saving effect highlighted here can potentially provide as great a deviation from the Ricardian benchmark as does the finiteness of life.

Policy Experiment 3

A final experiment of interest is a tax rescheduling announced several periods in advance. Let k be the time at which the tax change is to begin but let that change be announced at the beginning of period 0. Equation (3.2) implies that for $0 \leqslant t \leqslant k$,

$$(3.7) \qquad \tilde{\Phi}_t = \theta \int_0^\infty e^{-r(s-t)}(1-e^{-\delta(s-t)})\tilde{T}_s ds = \theta \int_0^\infty e^{-r(s-t)}(-e^{-\delta(s-t)})\tilde{T}_s ds$$

$$= e^{(r+\delta)(t-k)}\theta \int_k^\infty e^{-r(s-k)}(1 - e^{-\delta(s-k)})\tilde{T}_s ds$$

$$= e^{(r+\delta)(t-k)}\tilde{\Phi}_k,$$

where the key simplification in the first line results from the fact that the tax change is a balanced budget tax change. Then using (3.3), we can find the overall effect on consumption:

$$(3.8) \qquad \tilde{C}_t = re^{-(r+\delta)k}\left[\frac{r}{r+\delta} + \frac{\delta}{r+\delta} e^{(r+\delta)t}\right]$$

for $0 \leqslant t \leqslant k$.

Equation (3.8) shows that the effect of the tax cut on consumption grows through time as the tax cut approaches. This magnification of the insurance effects of an announced tax change as it draws closer causes a change in consumption predictable in advance, contrary to the proposition that changes in consumption should be unpredictable. This departure from Hall's random walk property of consumption is not surprising theoretically, since the utility functions assumed here are exponential rather than quadratic. But in

tax policy we have identified a factor that can affect expected variances of individual income and therefore expected changes in consumption in a systematic way.

An Index of Fiscal Stance

Blanchard (1985) has recently suggested an index of fiscal stance to summarize the impact of fiscal policy on aggregate demand. His model is non-Ricardian because agents have finite horizons: they die at rate q and are replaced with newly born individuals. Blanchard's index of fiscal stance is

$$(3.9) \qquad g_t = (q+\rho)[D_t - \int_0^\infty e^{-(r+q)s} T_{t+s} ds] + G_t$$

where D_t is the debt, G_t is government purchases, and T_t is taxes. This index includes the direct effect of government purchases and the indirect effects of government debt and taxes on consumption.

The model of precautionary saving presented here suggests an analogous index of fiscal stance. Equations (2.27), (2.28), and (3.2) indicate that the appropriate index is

$$(3.10) \qquad g_t^* = r[D_t - (1-\theta) \int_0^\infty e^{-rs} T_{t+s} ds - \theta \int_0^\infty e^{-(r+\delta)s} T_{t+s} ds] + G_t,$$

This index includes the effect of debt through non-human wealth W_t, the effect of taxes through human wealth H_t, and the effect of taxes through the precautionary saving term Φ_t.

In the limiting case in which $\theta = 1$, the two indices of fiscal stance are almost identical. The marginal propensity to consume is $q + \rho$ in Blanchard's index, while it is r in ours. More important, the discount rate for future

taxes is $r + q$ in Blanchard's index, while it is $r + \delta$ in ours. In both cases, the discounting of tax liabilities at a rate higher than r is the reason for the failure of Ricardian equivalence.[13]

Remember that the index in (3.10) is derived under the assumption that taxes fall only on the high-income individuals. More generally, suppose a fraction f of taxes falls on the poor and $(1-f)$ falls on the rich. Such a tax can be decomposed as $2f$ lump-sum and $(1-2f)$ falling only on the rich. Therefore, the more general index is

$$(3.11) \quad g_t^* = r[D_t - (1-\theta') \int_0^\infty e^{-rs} T_{t+s} ds - \theta' \int_0^\infty e^{-(r+\delta)} T_{t+s} ds] + G_t.$$

where $\theta' = \theta(1-2f)$. A fraction θ' of future taxes is discounted at rate $r + \delta$, while the remainder is discounted at rate r. The index thus readily handles any degree of progressivity.

IV. Conclusion

This paper has analyzed rigorously the role of the timing of taxes in a world in which taxes are contingent on individual income and individual income is subject to non-diversifiable idiosyncratic risk. Casual empiricism, as well as the more formal empirical work discussed by Barsky, Mankiw, and Zeldes, suggests that the sort of heterogeneity examined here is substantial. Such heterogeneity among individuals has potentially important aggregate effects. Under reasonable auxiliary assumptions, these aggregate effects can be explicitly derived.

Previous authors analyzing the interaction between taxes and precautionary saving have typically relied on two-period examples. Our goal

has been to extend the analysis to a more general and more realistic setting. The infinite horizon model presented here is much richer in its implications, is more easily compared with standard dynamic models, and should prove a more useful guide for empirical work.

Footnotes

1.   This insurance effect of the taxes also affects many other decisions, such as the accumulation of human capital (Eaton and Rosen, 1980).

2.   Some readers may wish to skip to the summary at the end of this section.

3.   The assumption of a constant real interest rate is maintained throughout this paper. The model can be viewed as "partial equilibrium;" alternatively, one can posit a small open economy or a linear storage technology.

4.   We will consistently use " $\sum_{j \neq i}$ " as an abbreviation for " $\sum_{\substack{j=1 \\ j \neq i}}^{J}$ ".

5.   See Kamien and Schwartz (1981), pp. 238-242. They call it "the fundamental partial differential equation" of dynamic programming.

6.   One could guess that this would be so from the example of optimal consumption with random income on p. 398 of Merton (1971).

7.   Using the notation defined below, since Ψ has row sums that add to zero, it is a continuous-time Markov matrix and has all nonpositive eigenvalues. Thus, $\Psi - rI$ has strictly negative eigenvalues, which proves by the implicit function theorem that (2.8) has a unique steady-state solution for the vector a given the vector y, since with a set to zero, totally differentiating the steady state version of (2.8) yields $r\tilde{y} = (rI - \Psi)\tilde{a}$.

8.   If h were not equal to zero, then as $\tilde{a}_t$ became large, the linearization around the steady-state would become inappropriate, but these nonlinearities would not prevent the violation of the budget constraint.

9.   The marginal propensity to consume out of wealth, r, is the same as under certainty equivalence when $r=\rho$. The "excess sensitivity" discussed by Barsky, Mankiw, and Zeldes arises with constant relative risk aversion but not with

constant absolute risk aversion. The effect of uncertainty on the marginal

propensity to consume is discussed in Kimball (1988). If uncertainty raises

the marginal propensity to consume, any departure from Ricardian equivalence

is magnified.

10. The interest rate r can be viewed as fixing the time unit; all other

rates are given relative to the interest rate.

11. In (2.21), a reduction in r may not at first reduce saving, since a lower

interest rate can increase the precautionary component of saving (see Table

4), but if r falls low enough, saving will begin to decline.

12. It should be remembered that these numbers are based on the assumption

that all taxes fall on the high-labor-income half of the population. Less

progressive taxation would lead to smaller effects, as discussed below.

13. The parameter δ should not be interpreted as simply a risk premium. The

higher discount rate arises from a precautionary saving effect (a positive

third derivative) rather than from risk aversion (a negative second

derivative). For instance, in the case of quadratic utility, certainty

equivalence obtains: the precautionary saving effects are absent, despite the

presence of risk aversion.

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Table 1

Value of θ for Various Parameter Values

				p/r		
		0.10	0.5	1.0	2.0	5.0
	0.5	0.39	0.24	0.16	0.10	0.05
	1.0	0.67	0.45	0.32	0.20	0.09
$\gamma(y^1-y^2)$	2.0	0.91	0.73	0.56	0.37	0.18
	5.0	1.00	0.95	0.88	0.72	0.41
	10.0	1.00	0.99	0.97	0.90	0.68

Table 2

Value of δ/r for Various Parameter Values

p/r

		0.10	0.5	1.0	2.0	5.0
	0.5	0.22	1.03	2.03	4.02	10.01
	1.0	0.27	1.12	2.11	4.08	10.05
$\gamma(y^1-y^2)$	2.0	0.49	1.47	2.42	4.31	10.17
	5.0	2.01	3.29	4.15	5.73	10.98
	10.0	5.92	7.38	8.16	9.40	13.57

Table 3

Value of x for Various Parameter Values

		p/r			
$\gamma(y^1-y^2)$		0.10	0.5	1.0	2.0	5.0
0.5	1.51	1.28	1.18	1.11	1.05
1.0	2.27	1.63	1.39	1.22	1.10
2.0	4.71	2.54	1.89	1.48	1.20
5.0	20.06	6.43	3.90	2.46	1.55
10.0	59.21	14.69	8.04	4.47	2.28

Table 4

Precautionary Saving as a Percentage of Average Labor Income

$$[\frac{p}{2\gamma(y^1 - y^2)r} \frac{y^1 - y^2}{Y}(x + \frac{1}{x} - 2)] \cdot 100\%$$

p/r

	0.10	0.5	1.0	2.0	5.0
0.5	1.1%	2.1%	1.8%	1.3%	0.7%
1.0	2.3	4.1	3.8	2.6	1.4
$\gamma(y^1-y^2)$ 2.0	4.9	7.7	7.0	5.2	2.7
5.0	12.0	15.3	14.3	11.5	6.6
10.0	19.1	21.2	20.5	18.0	11.9

Note: The figures in this table are based on the assumption that $y^1 = 2y^2$ or, equivalently, that $\frac{y^1 - y^2}{Y} = \frac{2}{3}$.

Table 5

Value of $[\delta/(r+\delta)]\theta$ for Various Parameter Values

				p/r		
		0.10	0.5	1.0	2.0	5.0
	0.5	0.07	0.12	0.11	0.08	0.04
	1.0	0.14	0.24	0.22	0.16	0.09
$\gamma(y^1-y^2)$	2.0	0.30	0.43	0.40	0.30	0.16
	5.0	0.66	0.73	0.71	0.61	0.38
	10.0	0.86	0.87	0.86	0.82	0.63