NBER WORKING PAPER SERIES

BUYBACKS, EXIT BONDS, AND THE OPTIMALITY OF DEBT AND LIQUIDITY RELIEF

Kenneth A. Froot

Working Paper No. 2675

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 1988

This paper is part of a more ambitious project on forgiveness, joint with Jeff Sachs. I thank Rudi Dornbusch, Paul Krugman, and Jeffs Sachs, for sharing many of their insights on these issues, Mick Dooley and Jonathan Eaton for comments, and the Alfred P. Sloan and Ford Foundations for financial support. This research is part of NBER's research program in International Studies. Any opinions expressed are those of the author not those of the National Bureau of Economic Research.

NBER Working Paper #2675
August 1988

BUYBACKS, EXIT BONDS, AND THE OPTIMALITY OF DEBT AND LIQUIDITY RELIEF

ABSTRACT

We compare various forms of market-based debt relief with coordinated debt forgiveness on the part of creditors. These schemes lead to different allocations of resources and levels of debtor and creditor welfare, but all attempt to stimulate debtor investment through reductions in the level of debt. If investment-incentive effects are present, then investment in liquidity-constrained debtors will respond by enough to make a reduction in debt profitable, but not by enough to make the reduction in debt optimal. For these countries the optimal debt-relief package (from the creditors' perspective) will include an infusion of new lending.

Kenneth A. Froot
Sloan School of Management
E52-436
Massachusetts Institute of Technology
Cambridge, MA 02139

Buybacks, Exit Bonds, and the Optimality of Debt and Liquidity Relief

Kenneth A. Froot
Sloan School of Management,
Massachusetts Institute of Technology
and
NBER

Most observers agree that the muddling-through strategy of dealing with problem debtors is at a crossroads. The debtor economies have suffered through reform and severe contraction, yet their prospects are little improved. For their part, creditors have been unable to provide the new lending needed to sustain investment and growth. Exposure has even been reduced slightly, but at a cost of a steady deterioration in the quality of outstanding loans.

This has prompted calls by some observers for muddling-out: partially writing down creditor claims to make way for business as usual. Their argument is that high debt levels act like a tax on investment incentives. Partial forgiveness would provide more stimulus to growth and adjustment, and to the return of capital flight, and therefore could increase debt service. To use Krugman's (1987) terminology, the debt is so high that countries are on the wrong side of the "debt-relief Laffer curve."[1] Few debts, however, have thus far been forgiven. One reason may be that it is not in creditors' interest to give up their chance for full repayment. But it is hard to be sure because the same free-rider problem that has crippled new lending will also block a coordinated write down.

A different group of observers has sought instead to fill the debt-reduction void through market-based schemes, such as buybacks, buyouts, and exit bonds. This unlikely group includes advocates of the debtors, who are frustrated by the free-rider problem and are attracted by the voluntary nature of these schemes, creditors, who believe they would be better off under these schemes than under a write down, and investment bankers, for whom a market made is a penny earned.

Yet these market-based schemes are not well understood. Important papers by Helpman

[1] This view is originally described in Sachs (1988a, b).

(1987), Dooley (1988) and Krugman (1988) have clarified the analytics of some of the market-based proposals. Nevertheless, general conclusions about the similarities and differences between buybacks, buyouts, exit bonds and pure forgiveness have not yet emerged.[2] The first part of this paper, which is similar in spirit to Krugman (1988), seeks to compare systematically the equilibria implied by these market-based schemes and pure debt relief (i.e., a coordinated write down by creditors). We also compare the pricing of buybacks, buyouts, and exit bonds.

To summarize broadly, our findings in this part are that there is a great deal of similarity between these market-based schemes and pure debt relief. The common rationale for these plans is that investment may be powerfully stimulated through a reduction in the debt overhang. But we also identify important differences across plans. The differences come from two sources. First, the incentive effects of debt forgiveness, the distribution of welfare gains and losses, and the price of old debt all depend critically on the source of the resources used for debt repurchase. Like the homeowner who sells his windows to pay for the furnace, a country that relinquishes *current* resources to achieve debt reduction may make itself (and possibly its creditors) worse off, and can even reduce investment. Second, the agent making the take-it-or-leave-it offer differs across these plans. The country basically chooses the amount of buyback or exit bonds it will offer (within limits), and the creditors' collective chooses the amount of pure debt relief. This has an important effect on the equilibria these plans imply.

Despite their differences, unilateral debt forgiveness and market-based debt relief schemes rely heavily on one common feature: the negative impact of a large debt overhang on investment incentives. How important is this "incentive constraint" likely to be in practice? Since 1982, investment has fallen on average by over 5 percent of GNP, exactly equal to the increase in the noninterest external surplus (which roughly measures the reduction in liquidity).[3] In the meantime, the debt itself has grown only slowly. Liquidity constraints, not incentive constraints, are probably most responsible for the low levels of investment in the problem debtors. It would therefore be surprising if debt reduction alone would be the optimal stimulus to investment.

The second part of this paper studies the role of liquidity in the design of an optimal relief plan.

[2] Helpman (1987) provides a very general analysis of debt/equity swaps and debt forgiveness. Dooley (1988) discusses the pricing of buybacks and simulates their welfare effects. Krugman (1988) incorporates incentive effects and shows that marginal buybacks and exit bond offerings are equivalent to unilateral debt relief. See also Williamson (1988), which analyzes how differences in preferences across creditors may strengthen the argument in favor of market-based debt relief schemes.

[3] See the discussion in Dornbusch (1988), particularly table 3.10.

We find that countries that are severely liquidity constrained are the best candidates for a debt reduction that will benefit all. That is, these countries are more likely to be on the wrong side of the debt-relief Laffer curve. But they are also the countries that can benefit least from a write down (since current resources are already so dear). We then show that by offering some current liquidity, creditors can induce a greater investment response and yet forgive less. In liquidity-constrained countries, pure debt relief alone will raise, but not maximize, the value of creditors' claims. Thus, relative to pure debt relief, creditors' optimal arrangement will supply less forgiveness, but more liquidity, and in doing so will also make the debtor better off.

Taken together, the two parts of the paper suggest that in many cases market-based debt relief schemes are in *no one's* interest. Debtors stand to lose to the extent that debt relief depletes currently available resources. In dealing with a liquidity-constrained debtor, creditors stand to lose by providing too much forgiveness on any relief package that reduces the level of outstanding debt without providing new lending. This is not to say that market-based schemes will never be desirable from at least one agent's point of view. But in order to evaluate their benefits, proponents will have to pay more attention to the source of debt-relief resources and to the severity of liquidity constraints. Even if potent investment-incentive effects are present in LDCs, they are not enough to make any manner of debt reduction best.

The paper is structured as follows. For readers who are unfamiliar with market-based debt relief schemes, the first appendix contains a primer on how buybacks and exit bonds work. Section 1 in the text presents a formal model which incorporates the investment incentive effects we wish to study. The equilibria associated with several debt-relief schemes are then derived in section 2. Section 3 considers the impact of liquidity relief on creditors' optimal choice of debt reduction. Section 4 concludes.

1. A model with investment incentive effects

Several authors, most notably Sachs (1988a, b) and Krugman (1987, 1988), have argued that the disincentive effects of an inherited debt may make partial forgiveness beneficial to both the debtor and its creditors. In this section, we build a more formal model that can trace out the incentive effects which are the critical element behind these debt relief schemes. The approach is deliberately simple, but our basic conclusions are far more general.

We consider a two-period model similar to that in Froot, Scharfstein and Stein (1988). The debtor country derives welfare from the discounted sum of the utility of consumption in periods 1 and 2:

$$W = U_1(C_1) + \beta C_2, \tag{1}$$

where U_1 satisfies the Inada conditions, and $U_1' > 0$ and $U_1'' \leq 0$. The world discount factor is 1, and $\beta < 1$. We choose this special formulation for welfare in order to separate clearly the effects of risk aversion and intertemporal substitutability. Welfare is linear in period-two consumption in order to abstract from the risk-sharing issues considered by Helpman (1987). Naturally, these issues are important, but they complicate the algebra without adding to the intuitions below.[4] A major disadvantage of linear welfare, however, is that it implies an infinite elasticity of intertemporal substitution. By allowing for concavity in period-one utility, we can explore the implications of finite intertemporal substitutability without forcing preferences to be risk-averse.

The country enters the model with an endowment E, and an inherited debt, D. In period zero, the country announces its plans for a buyback. In period one, there is a competitive auction among creditors in which they exchange old debt for the new securities. Also in period one the country chooses a level of investment, I, which yields period-two output of $\tilde{y} = f(I) + \epsilon$, where f also satisfies the Inada conditions, $f' > 0$, $f'' < 0$, and ϵ is a random variable with support $[\underline{\epsilon}, \bar{\epsilon}]$.[5] In period two, the country must make a payment on its outstanding obligations, $D - x$, where x is the amount of old debt retired less any securities issued, that is, x is the amount of effective debt relief. The investment incentives we wish to study are sharpest if we make the "gunboat technology" assumption that the entire output, $\tilde{y}$, can be confiscated by creditors in the event of default.[6] Period-two payments are then:

$$R = \min(D - x, \tilde{y}). \tag{2}$$

Under these assumptions, the country chooses investment to maximize its objective function, taking x as given:

$$W^* = \max_I \quad U_1(\mathrm{E} - I) + \beta E\Big(\max(0, \tilde{y} - D + x)\Big), \tag{3}$$

[4] Indeed, some of the propositions below go through with trivial modification for concave period-two subutility.

[5] If the price of output is uncertain, then the randomness would enter multiplicatively, rather than additively. The analysis below goes through in either case.

[6] Qualitatively, our analysis relies on the assumption that the country sacrifices an amount that increases with the value of output when default occurs. The results would also hold if we were to assume that creditors cannot actually confiscate output, but can impose penalties on the debtor in proportion to the value of output.

where E is the expectations operator.[7] The last term in equation (3) follows directly from the repayment assumption in (2). In good states, the country pays off its debt and gets to consume whatever is left. In all other states, the country cannot fully meet its debt service requirements, so that the investment project's output is confiscated.

The country's first-order condition for investment is given by:

$$f'(I^*) = \frac{U_1'}{\beta G}, \tag{4}$$

where $G = G(I^*, x) = \int_{\epsilon^*}^{\bar{\epsilon}} g(\epsilon)d\epsilon$ is the probability that the country will reap some surplus from the project, and $\epsilon^* = D - x - f(I^*)$ defines the level of output that exactly pays off the outstanding obligations. In some states a marginal increase in output is confiscated, which is a disincentive to invest. The factor $1/G > 1$ measures the investment distortion, the extent to which the marginal product of investment is greater than at the country's first-best level.

Equation (4) defines implicitly the optimal level of investment as an increasing function of x, $I^* = I^*(x)$.[8] As the overall debt payment is reduced, additional investment raises period-two consumption in more states of the world. The debtor perceives this as a higher return on investment, and it therefore invests more.

2. A menu of debt-relief schemes

Where do the initial resources needed to generate the effective relief, x, come from? We consider four sources: 1) partial forgiveness from creditors, 2) aid from foreign governments, 3) output from the debtor's investment project, and 4) the debtor's endowment.

2.1. Pure debt relief

Suppose that creditors agree to write down their collective claims on the country, an action we call pure debt relief. We take their choice of x as exogenous, and assume that the debtor sets investment optimally ($I = I^*(x)$). We return to how x is determined under pure debt relief after discussing the following Proposition:

[7] We assume the endowment E is small enough so that the country would be a borrower at the world interest rate if it were not credit rationed. This assumption is critical for the incentive effects to have an impact on investment. See the discussion in section 2.4 and footnote 25 below.

[8] We assume that $f(I)$ is sufficiently concave so that this statement is true. Applying the implicit function theorem to (4), and using (3) yields

$$\frac{dI^*}{dx} = \frac{-f'\beta g(\epsilon^*)}{\beta G f'' + U'' + \beta G(f')^2} > 0,$$

where the denominator is the second-order condition for the problem in (3).

Proposition 1. Under pure debt relief, the welfare of the debtor, the welfare of creditors (taken together), and the price of a dollar's worth of the remaining debt, can be completely described by the amount of effective debt relief, x:

(i) Debtor welfare is given by

$$W^*(x) = U_1(\mathrm{E} - I^*(x)) + U_2(I^*(x), x), \tag{5}$$

where $U_2(I^*(x), x) = E(\max(0, f(I^*) + \epsilon - D + x))$ and $\frac{dW^*}{dx} > 0$;

(ii) Creditors' collective welfare is the market value of their claims:

$$V^*(x) = E\Big(\min(f(I^*(x)) + \epsilon, D - x)\Big), \tag{6}$$

where

$$\frac{dV^*}{dx} = (1 - G)f'\Big(\frac{dI^*}{dx}\Big) - G, \qquad \frac{dV^*}{dx} \in [\infty, -1]. \tag{7}$$

(iii) The price of a marginal unit of debt after the debt relief takes effect, given by $\frac{1}{\theta^*}$, is the average market value of the debt:

$$\frac{1}{\theta^*(x)} = \frac{V^*(x)}{D}. \tag{8}$$

Part (i) of the proposition shows that debtor welfare is an increasing function of the amount forgiven: pure debt relief always makes debtors better off. Part (ii) shows that creditors are better off only when an increase in forgiveness increases expected payments, that is, when $\frac{dV^*}{dx} > 0$.[9] (Notice that we have assumed that creditors are risk neutral and that they know the optimal investment schedule of the country.) V^* is increasing when (7) is dominated by the first term, which represents the increase in expected payments due to a higher level of investment. The second term in (7), which is negative, measures the loss in states with full repayment as the face value of the debt is reduced. When the probability of full repayment is small, (7) is positive. Creditors gain from a reduction in contracted payments, with the size of the gain proportional to the impact on investment of the change in incentives. On the other hand, when the probability of complete repayment is high, (7) is negative. $V^*(x)$ is therefore hump-shaped, as drawn in figure

[9] The debt sells at a discount in the secondary market as long as the probability of full repayment is less than one, $G < 1$. In the neighborhood of $\frac{dV^*}{dx} = 0$, G is necessarily less than one.

1. This, of course, is Krugman's (1987) debt-relief Laffer curve. The value of creditors' claims is maximized at the top of the curve, where $\frac{dV^*}{dx} = 0$. Pure debt relief is in the interest of *both* the debtor and its creditors when the country is on the wrong side of the Laffer curve.

As long as the country is on the right side of the Laffer curve, pure debt relief is not in creditors' collective interest. But, even worse, it is *never* in an individual's interest. A given creditor's claims can have only a small impa on V^*. Thus, conditional on other creditors ripping up their claims, an individual creditor would prefer to hold on to his. Pure debt relief will therefore be hindered by the free rider problem. Even a country on the wrong side of the Laffer curve should not expect individual creditors of their own volition to set x such that $\frac{dV^*}{dx} = 0$.

The difficulty of getting creditors to act as a collective entity has spawned the market-based proposals we consider next. There are two important features that distinguish the equilibria envisaged in these proposals from that in pure debt relief. First, the country, and not the creditors' collective, acts by making a take-it-or-leave it buyback offer. Second, individual creditors must voluntarily participate in a market-based scheme. To be successful such schemes must therefore circumvent the free rider problem.

2.2. Buybacks out of aid from foreign governments

Suppose that resources for buying back debt are donated specifically for that purpose by another country.[10] In period zero, the country informs creditors that in period one it will auction off these resources in return for old debt. We assume that the debtor and its creditors rationally anticipate the optimal period-one investment response, $I = I^*(x)$, and that the auction is competitive. Let b represent the donated funds, given exogenously, and let $x = x(b) = \theta^b(b)b$ denote the face amount of old debt repurchased in the auction, i.e., the *effective* amount of relief generated by the buyback. The buyback equilibrium is summarized in the following proposition (and proven in the second appendix):

Proposition 2. When resources for a competitive buyback are donated, the equilibrium is

[10] A country's ability to buy back debt on the secondary market is a matter of some controversy, although Bolivia recently completed a buyback in which about 1/2 of its debt was retired. Syndication agreements in commercial bank loans make buybacks problematic. These agreements contain a mandatory prepayment clause, which stipulates that any prepayment must be distributed among creditors according to exposure, and a sharing clause which requires that any payment received by a creditor in excess of its exposure must be shared among all banks according to exposure. See the discussion in section 2.5 below.

characterized by:[11]

(i) Debtor welfare is the same as under pure debt relief:

$$W^b(x) = W^*(x), \tag{9}$$

and is strictly increasing in amount of effective relief, $\frac{dW}{dx} > 0$.

(ii) Creditors' collective welfare is greater than under pure debt relief by the amount of aid:

$$V^b(x, b) = V^*(x) + b, \tag{10}$$

which is increasing in the amount of the buyback, $\frac{dV^b}{db} \geq 0$.

(iii) The buyback takes place at a price where a marginal unit of debt after the buyback, $\frac{1}{\theta^b}$, is equal to the average market value of the debt remaining:

$$\frac{1}{\theta^b(x)} = \frac{V^*(x)}{D - x}. \tag{11}$$

The effective amount of relief is strictly increasing in b: $\frac{dx}{db} = \frac{d(\theta^b b)}{db} > 0$.

Notice the similarity between parts (i) and (ii) of propositions 1 and 2. Buybacks funded by a third party reduce future debt payments, and therefore have an effect on future payments equivalent to that of pure debt relief. Up to the value of the transfer, b, *buybacks out of aid are equivalent to pure debt relief, for any given level of effective relief.* It is as if the aid is given directly to creditors in return for a write down of size x. The auction merely serves to translate a fixed amount of buyback resources, b, into effective debt relief, x. The larger the transfer, the more creditors gain. From an efficiency point of view, nothing is different from proposition 1: once the transfer is netted out, Pareto improvements are possible only if the country is on the wrong side of the debt-relief Laffer curve. Figure 1 graphs expected creditor payments under pure debt relief and a buyback out of aid, V^* and V^b, respectively. While V^* is hump-shaped, V^b is concave and increasing everywhere, which reflects the added value of the transfer, b.

Part (iii) of the proposition says that the buyback price is the inverse of the expected value of the last unit of old debt repurchased. If the auction is competitive, the price, $\frac{1}{\theta^b}$, must be such

[11] Some of these results are discussed by Dooley (1988).

that individual creditors are indifferent between holding onto their old debt, and trading in their old debt for cash. Thus in equilibrium, the expected payoff from holding θ^b units of old debt must be one: $\theta^b\left(\frac{V^*(x)}{D-x}\right) = 1$, which is just equation (11).

The price of a unit of the remaining debt, $1/\theta^b(x)$, is graphed in figure 2. Before the buyback is announced, the price is as in proposition 1, $1/\theta^b(0) = 1/\theta^*(0)$. The price of the remaining debt rises with the size of the buyback for two reasons. First, as debt is bought back, the quality of the remaining obligations improves. Second, as the country gains surplus in the better states of nature, it invests more, further improving the quality of the remaining debt. The concavity (or convexity) of the curve is determined by the interaction of these two factors. The frequency distribution, $g(\epsilon)$, determines how much more likely complete repayment becomes for a marginal increase in b. When $g(\epsilon)$ is increasing, the price curve tends to be convex. On the other hand, the country's investment response, $f'\frac{dI^*}{dx}$, is decreasing (due to the concavity of f), which tends to make the price curve concave. If ϵ is uniformly distributed and f is concave, then the path of the price will resemble the concave curve shown in figure 2.[12] Finally, when the buyback is large enough to retire completely the outstanding debt, $\lim_{x \to D} \theta^b(x) = 1$, provided the last unit of debt is riskless: the entire debt can be repurchased only at its full face value.[13]

Finally, part (iii) shows that the amount of effective relief increases with the size of the buyback, even though the rate at which old debt is exchanged, θ^b, falls. An increase in the size of the buyback, b, therefore raises the welfare of both the debtor and its creditors.

2.3. Buybacks out of future cash flows, or exit bonds

Next we consider the case in which old debt is repurchased by issuing senior claims to future cash flows. We call these claims exit bonds. If these bonds are to be senior to the existing debt, every creditor must agree to honor their seniority before the auction takes place.[14] For the moment we assume the seniority of these bonds, but we return to whether creditors would in fact grant it. We do not require, however, that the exit bonds are riskless.

In period zero, the country announces the face amount of exit bonds it plans to issue, given

[12] See Dooley (1988), who discusses in detail the impact of alternative frequency distributions on buyback pricing.

[13] This will be the case as long as output is positive in all states, $f(I^*) + \underline{\epsilon} > 0$.

[14] New securities can be treated as senior only if the sharing clause, mentioned in footnote 10, is waived. If the sharing clause is not waived, any creditor is entitled to sue for its share of payments made by the country. Thus a single "holdout" creditor undermines the assurance of other creditors that they will be able to keep their exit-bond repayments.

by k, and asks creditors to make the bonds senior. As in the previous section, we assume that the debtor and its creditors rationally anticipate the optimal period-one investment response, $I = I^*(x)$, and that the auction is competitive. The period-one auction retires $\theta^k k$ in face value of the old debt. The amount of effective debt relief – the reduction in the total face value of old debt less the value of the exit bonds – is then $x = x(k) = (\theta^k(k) - 1)k$. For the same face value, exit bonds are strictly preferred to old debt because they are senior. Thus $\theta^k(k) > 1$, and exit bonds generate effective relief, $x(k) > 0, \forall k > 0$.

The exit-bond equilibrium is summarized in the following proposition, with proofs in the second appendix:

Proposition 3. When resources for a competitive buyback come from future cash flows, the equilibrium must satisfy:

(i) Debtor welfare is the same as under pure debt relief:

$$W^k(x) = W^*(x) \quad , \forall x. \tag{12}$$

(ii) Creditors' collective welfare is the same as under pure debt relief:

$$V^k(x) = V^*(x) \quad , \forall x. \tag{13}$$

(iii) If the buyback is small enough to be riskless – that is, if $f(I^*) + \underline{\epsilon} > k$ – then the buyback price, θ^k, is equal to the price under pure debt relief:

$$\theta^k(x) = \theta^*(x), \quad \forall x. \tag{14}$$

The greater the exit bond offering, the greater the level of effective relief, $\frac{dx}{dk} > 0$.

(iv) If the buyback is not riskless, then the equilibrium price solves:

$$\theta^k(k) = \Big(\frac{D - \theta^k k}{E(\min(f(I^*) + \epsilon, D - \theta^k k))}\Big)\Big(\frac{E(\min(f(I^*) + \epsilon, k))}{k}\Big), \tag{15}$$

where $\theta^k(k) < \theta^*(x(k)), \ \forall k$.

Note that in parts (i) and (ii) of the proposition debtor and creditor welfare can be written exclusively as functions of x – regardless of the size of the exit bond offering. By distinguishing between senior and subordinate claims, the auction generates effective debt relief, with no other effects on the debtor or its creditors. Thus, *for any given amount of effective relief, an issue of exit bonds is equivalent to pure debt relief.*[15]

Part (iii) of the proposition – which applies to riskless exit bonds – shows that the price at which the old debt is retired, $1/\theta^k$, is purely a function of the level of effective relief. Indeed, the price is exactly equal to the ; ice that would prevail after an equivalent amount of pure debt relief is granted. The price of bond issues that are large enough to be risky cannot be written simply as a function as the level of effective relief, as in part (iv). Once the bond issue is risky, the *relative* riskiness of the original debt improves, so that the price must rise above what it would have been if the bonds were riskless $(\frac{1}{\theta^*(x(k))})$. The price is drawn if figure 2. The $\frac{1}{\theta^*(x)}$ and $\frac{1}{\theta^k(x)}$ curves separate at the point when the bond offering becomes risky.

It is worth dwelling for a moment on how the swap rate, $\theta^k(k)$ evolves. Consider the impact on the value of creditors' claims of an increase in the size of the bond offering. Using (7) and (13):

$$\frac{d\theta^k}{dk} = \Big(\frac{-D}{(V^k)^2}\Big)\Big((1-G)f'\Big(\frac{dI^*}{dx}\Big) - G\Big)\frac{dx}{dk}. \tag{16}$$

Suppose for a moment that investment is fixed, $\frac{dI^*}{dx} = 0$, so that the first term in (16) is zero. Then larger exit bond offerings *lower* total expected payments. How is it that a strictly positive exit bond offering reduces expected payments without any change in the total resources available for debt service? Because creditors are competitive, the seniority of the exit bonds creates an externality: as some creditors swap in their old debt for senior exit bonds, they degrade the value of the old debt remaining. At the price of the first increment of the buyback, $\frac{1}{\theta^k(0)} = \frac{1}{\theta^*(0)}$, each creditor would strictly prefer to swap in his old debt rather than to hold on: conditional on no other creditors swapping, each creditor is indifferent between swapping and not swapping, and conditional on other creditors swapping, each creditor is strictly better off by swapping in old debt. The resulting excess supply of old debt drives up the price of the exit bond in terms of old debt (θ^k rises).

Now if we allow investment to respond to the amount of effective relief, the first term in (16) becomes positive. The excess supply of old debt at $\theta^k(0)$ is smaller. When the country is

[15] Krugman (1988) discusses this equivalence and presents results for small buybacks.

on the wrong side of the Laffer curve, the investment response is strong enough to overcome the subordination effect, and θ^k actually falls with k.

2.3.1. Exit bond equilibria

While it is clear from proposition 3 that exit bonds and pure debt relief have many similarities, it is their differences that explain exit bonds' popularity. First, there are differences between the free-rider and seniority problems. Consider an individual creditor's decision about whether to grant seniority when a country is at a point like O in figure 3. Suppose that the country announces a small issue of exit bonds, and that other creditors agree to treat the bonds as senior. If the individual creditor refuses to grant seniority, then the exit bonds are perfect substitutes for the old debt. In that case, the equilibrium is $\theta^k = 1$ and $x = 0$: no relief is generated, and the individual creditor's claims do not change in value. If, on the other hand, this individual creditor agrees to the subordination of old debt, the value of its claims rise marginally as the country moves up the Laffer curve. Since an individual creditor faces no penalty in granting seniority when others do not, each creditor will find that granting seniority is a dominant strategy when the country is on the wrong side of the Laffer curve. (When the country is on the right side of the Laffer curve – point A in figure 3 – the argument runs in reverse; refusing seniority becomes the dominant strategy.) Because individual creditors are not "small" with respect to seniority, exit bonds break the free-rider barrier to debt relief.

A second difference between pure debt relief and an exit bond offering is the amount of effective relief generated in equilibrium. Consider again a country at point O in figure 3. If creditors were to coordinate and write down their claims, then they would choose x so that $\frac{dV}{dx} = 0$, moving to point L – the top of the Laffer curve. Under exit bonds, however, it is the country that chooses the amount of effective relief. Naturally, the country would like to set x as high as possible, and will choose k accordingly.[16] But if the country announces an offering so large as to lower the value of creditors' claims, moving, say, from point L to I in figure 3, individual creditors will not grant seniority, and the exit bond offering will generate no effective relief. The country will therefore respect the individual creditor's rationality constraint, setting k such that $V^k(x(k)) \geq V^k(0)$. At a point like O, the country will optimally choose x such that $V^k(x^*) = V^k(0)$ – across the Laffer curve

[16] The upper bound on the amount of effective relief a country can obtain by offering riskless exit bonds is given by the point at which all of the old debt is retired: $x_{max} = (\theta^k - 1)k_{max}$, and k_{max} is such that $\theta^k k_{max} = D$.

at point A. Assuming the exit bond offering is small enough to be riskless, equation (14) implies that the price of the remaining debt will be given by $\frac{1}{\theta^k(x^*)} = \frac{1}{\theta^k(0)} = \frac{V^*(0)}{D}$, the pre-existing price of a unit of old debt. The face value of the optimal offering follows directly: $k^* = x^*/(\theta^k(0) - 1)$. Thus in equilibrium *exit bonds can generate more effective relief than pure debt relief.*

It would appear that an exit bond equilibrium provides at least as much effective relief as pure debt relief, and sometimes strictly more. Do exit bonds dominate pure debt relief from the country's point of view? In general the answer is no. While the exit bond equilibrium yields greater effective relief than a pure debt relief equilibrium in the neighborhood to the left of point L, the neighborhood may be small. A country that starts out at point R will *not* be able to generate enough relief to reach point D using exit bonds. There are two important criteria that determine how far along the Laffer curve a country can move.

The first criterion is that the exit bond issue can, at most, retire the entire outstanding debt.[17] The precise level of k that exhausts the old debt has no closed form solution, and is a complex function of the frequency distribution $g(\epsilon)$, the probability distribution $\int g(\epsilon)d\epsilon$ and the production function $f(I^*)$. But the important point is that the further to the left of the Laffer-curve peak the country starts out, the more likely it is to run out of old debt *before* reaching the other side. Indeed, the country may run out of old debt before reaching the top.[18]

The second criterion that determines how far along the Laffer curve an exit bond issue can move a country is the behavior of the auction price, $\frac{1}{\theta^k}$. The marginal utility to the debtor of an increase in the exit bond offering is $\frac{dW}{dx}\frac{dx}{dk}$. We know from proposition 1 (i) that welfare is always increasing in the level of effective relief, $\frac{dW}{dx} > 0$. As proposition 3 (iii) shows, when the bond offering is small enough to be riskless, the level of effective relief is monotonically increasing in the size of the bond offering, $\frac{dx}{dk} > 0$. Small exit bond issues therefore always improve country welfare. However, this need not be true if the exit bond issue is large enough to be risky. A bigger, risky exit bond issue may drive θ^k down so rapidly that the level of effective relief *falls*, i.e. $\frac{dx}{dk} = \theta^k - 1 + \frac{d\theta^k}{dk}k < 0$. Thus, even when there is plenty of old debt outstanding, more exit bonds may not generate more effective relief.[19]

[17] See footnote 16 above.

[18] Note that in figure 2, the $1/\theta^k$ curve stops when all of the old debt is retired, before reaching $x = D$.

[19] A loose intuition for this result is as follows. When the exit bonds are risky, a marginal increase in the offering may increase the riskiness of the exit bonds substantially, without changing much the riskiness of the old debt. (In terms of the model, this depends on the density function evaluated at the point where there is just enough output to service the outstanding exit bonds,

2.4. Buybacks out of the country's endowment

Propositions 1 through 3 have stressed the similarities between buybacks and pure debt relief. The schemes we have considered – pure debt relief, buybacks out of aid, and exit bonds – are all ways of releasing resources to the country in the second period. Their common feature is that the relief funds become available in the same period in which they are used. In this section we turn to a different source of funds for buybacks: the country's current resources. We will see that these buybacks have intertemporal implications, which are the reason they fail to be equivalent to pure debt relief.

We now assume that the country must finance the debt repurchase using its period-one endowment, E. This pertains to a country that purchases the debt with reserves (savings), or raises taxes on current consumption. Of course, in a maximizing model, such a distinction is irrelevant. Regardless of where it comes from initially, a reduction in period-one resources will be spread optimally across consumption, saving, and investment.[20]

In period zero the country announces its buyback, e, out of the initial endowment, E. Let the buyback price be given by θ^e and effective relief by $x = x(e) = \theta^e e$. Once the resources for the buyback are fixed at e, the country's investment problem is given by

$$\max_{I} \quad U_1(\mathrm{E} - e - I) + \beta E\Big(\max(0, y - D + x)\Big), \tag{17}$$

with the first-order condition again given by equation (4). Inspection of (17) and (4) shows that the optimal level of investment is no longer completely summarized by the level of effective relief. We now denote optimal investment by $I^{**} = I^{**}(x(e), e)$. The following proposition is proven in the second appendix:

Proposition 4. For any given level of effective relief, x, the investment incentives associated with a buyback out of current resources are *smaller* than under pure debt relief:

$$I^{**}(0,0) = I^{*}(0), \tag{18}$$

$g(\epsilon')$, where $\epsilon' = k - f(I^*)$.) The relative riskiness of the old debt would then *improve* rapidly, so that θ^k would fall sharply. Bigger buybacks would then lower the amount of effective relief.

[20] If the period-one consumption decision is made *before* reserves are used for the buyback, then the buyback will have no effect on period-one consumption. But this timing would also imply that the buyback cannot have an effect on investment either.

$$I^{**}(x(e), e) < I^{*}(x), \ \forall x, e > 0, \tag{19}$$

$$\frac{\partial I^{**}(x, e)}{\partial e} < 0, \ \forall x. \tag{20}$$

The intuition for proposition 4 is straightforward. A buyback out of current resources must lower the available endowment (by e) in order to generate a positive amount of effective relief. When $E - e$ falls, the marginal utility of period-one consumption must rise. The marginal return on investment then rises above what it otherwise would have been. Investment is therefore lower than if the buyback resources came from elsewhere. Indeed, *these intertemporal considerations can dominate the investment-incentive effects, so that investment falls with an increase in the size of the buyback.*[21]

The buyback is characterized in the following proposition:

Proposition 5. For a given level of effective relief, a buyback out of the period-one endowment implies:

(i) Debtor welfare is lower than under pure debt relief:

$$W^{e}(x, e) = U_1(\mathrm{E} - e - I^{**}(x, e)) + U_2(I^{**}(x, e), x) < W^{*}(x), \quad \forall x > 0, \tag{21}$$

where $\frac{\partial W^e}{\partial e} < 0$.

(ii) Creditors' collective welfare is lower than under an equivalent buyback out of aid:

$$V^{e}(x_1(e), e) = E\Big(\min(f(I^{**}(x_1(e), e)) + \epsilon, D - x_1(e))\Big) + e < V^{b}(x_2(b), b), \quad \forall e = b, \tag{22}$$

where $\frac{\partial V^e}{\partial e} < 0$.

(iii) The rate at which old debt is exchanged, θ^e, is greater than the corresponding rate for a buyback out of aid:

$$x, e) = \frac{D - x}{V^{e}(x, e)} > \theta^{b}(x), \quad \forall x > 0, \tag{23}$$

[21] The results in proposition 4 are fairly general. Even though it is doubtful that a country would finance the entire buyback out of period-one resources, the proposition holds as long as a *portion* of the buyback resources comes from the period-one endowment and the remainder comes from one of the sources discussed in sections 2.1 through 2.3. Investment falls as the size of the buyback increases if the subutility, U_1, is sufficiently concave. For small buybacks (i.e., $e = 0$) the condition for this is:

$$\frac{-U_1''}{U_1'} > \frac{\theta^{*}(0)g(\epsilon^{*})}{G},$$

which can be thought of as a condition on the coefficient of absolute risk aversion.

The equivalence in propositions 1 through 3 between pure debt relief, buybacks out of aid, and buybacks out of future cash flows does not carry over to buybacks out of current resources. A smaller investment response to a given amount of effective relief is responsible for the failure of equivalence. *Ceteris paribus*, lower investment implies a lower price of the remaining old debt, $(1/\theta^e)$. Because investment may actually fall with e, there is no guarantee the price will still be increasing in the amount of the buyback. Figure 2 includes $1/\theta^e$ alongside the prices discussed earlier. *Larger buybacks may lower the value of the old debt left outstanding. Even in the presence of potent investment-incentive constraints, the "Laffer curve" for a buyback out of current resources may be flat, or may actually be declining everywhere.*[22]

From the debtor's point of view, buybacks out of current resources are dominated by buybacks either out of aid or out of future cash flows. In fact, we cannot even be sure that debtor welfare rises with e. These buybacks provide effective debt relief, but they may come at too high a cost: *the country's optimal buyback may be zero.*[23] In sum, *both creditors and debtors may be worse off under a buyback out of current resources, even if the country is initially on the wrong side of the debt-relief Laffer curve.*

2.5. Assessing buybacks vs. pure debt relief

Our analysis indicates that market-based schemes and pure debt relief are similar in many respects, but may lead to very different outcomes. These differences are not only a result of the mechanics of each scheme, but also of the conditions needed to make the scheme workable.

Clearly, the free-rider problem will be a substantial barrier to pure debt relief, even when the country is on the wrong side of the deht-relief Laffer curve. The three huyback proposals we looked at could be an alternative when creditors fail to coordinate. Nevertheless, each of these proposals may he practically unworkable. Buybacks out of aid will make both creditors and debtors better off, but at the expense of the donor. This makes large scale buybacks for the major debtors a remote possibility.[24] While none of the buyback proposals is subject to the free-rider problem, all

[22] This requires a condition stronger than that given in footnote 21. Intuitively, period-one subutility must be even more concave: investment must not only fall with e, it must fall rapidly enough for the value of the remaining debt to decline. See the second appendix for technical details.

[23] The condition for a small buyback out of current resources to lower country welfare is given by $U_1' > \beta G \theta^*(0)$, which from the first-order condition (4) is equivalent to $f' > \theta^*(0)$. If f satisfies the Inada conditions, then the above condition will be met for sufficiently low E. Even if investment is zero, the debt will have value as a claim to the random variable ϵ. Thus while $lim_{I=0} f' = \infty$, the price remains bounded, $lim_{I=0}\theta(0) = M < \infty$.

[24] Note from figure 1 that creditors do best under a buyback out of aid. Bulow and Rogoff (1988) point out that as long as there is a chance of such a buyout, creditors have an incentive to block other types of debt-reduction schemes.

nevertheless require a measure of coordination among creditors. A wavier of the sharing clause and mandatory prepayment clause would have to be designed and then agreed to unanimously. This would necessitate negotiation among creditors and the input of legal resources. Since there are so many syndicates with banks from all over the world participating, it is not clear who would enforce the waiver, or how it could be made enforceable at all.

Assuming buybacks could be made workable, market-based schemes may be best for some countries, even in the absence of a large donor. We saw that a successful exit bond offering could conceivably take a country beyond the top of its debt-relief Laffer curve, where it is better off than under pure debt relief. Under other circumstances, however, exit bonds would not allow the country to reach the top. The informational requirements in determining the optimal size of a bond offering and how far along the Laffer curve it would take the country are formidable. As Krugman (1988) has stressed, the investment incentive effects that are responsible for the Laffer curve's upward slope are inherently hard to measure. Essentially, creditors' entire subjective probability distribution of future output, *and* the responsiveness of future output to relief would have to be known.

In practice, the chance is small that market-based schemes would be superior from the countries point of view. Almost inevitably, an exit bond offering would use some current reserves as collateral – as in the recent Mexican case. Then the results of section 2.4 apply, so that the buyback may hurt the debtor. It is important to note that reserves should be thought of as current, and not future resources, even if they are unavailable for current consumption. (In other words, a buyback out of reserves is not equivalent to a buyback out of future cash flows.) When a credit-constrained country holds reserves, the shadow return on foreign exchange is likely to be higher than the marginal return on physical investment. Given that an increase in effective debt relief implies a lower probability that the reserves will be needed for future debt service, a marginal increase in debt relief does *not* generate more investment, it merely increases desired holdings of reserves. Buybacks out of reserves will have smaller incentive effects on investment, just as buybacks out of other current resources.[25]

[25] This point can easily be made in the model above. Consider a case in which the endowment is large enough to allow positive reserves to be held at the world interest rate. Then the choice of reserves and investment is given jointly by equation (4) and the first-order condition that reserves earn the world rate of interest:

$$1 = \frac{U_1'}{\beta G}.$$

As long as reserves are strictly positive, then the first-order conditions together imply $f' = 1$. The optimal level of investment is constant and, therefore, debt relief has no impact on investment.

3. Incentive versus liquidity effects on investment

We have discussed two problems with relying on debt relief to increase investment and growth in problem debtors. First, relief cannot be Pareto improving unless the country is on the wrong side of the debt-relief Laffer curve. Second, if current resources are sacrificed for forgiveness, Pareto improvements may not be possible regardless of where on the Laffer curve the country is.

But it is also clear that future incentives are not the only factor determining investment. In section 2.4, the usual investment response to debt relief is distorted by the use of current resources. Countries may be liquidity constrained in addition to being incentive constrained. This suggests that if creditors maximize the value of their claims, future payments will not be adjusted in isolation. Instead, there will be an optimal adjustment in both the level of debt and of current liquidity.

It is not new to argue that creditors have an interest in providing sufficient liquidity to problem debtors. Sachs (1984) and Krugman (1985) study the role of liquidity in averting default. If an indebted country is prepared to declare default, it makes sense to lend at a loss today in order to retain the chance of collecting the entire debt tomorrow. The incentive-constraint argument for promoting sufficient liquidity is, however, different: by taking advantage of high-return projects that otherwise would have been foregone, additional lending stimulates investment and allows countries to pay more in the future. In this case, there is no choice between either financing or forgiving; there is an optimal combination of the two.

3.1. Optimal liquidity and debt relief

In this section we study a simple optimal liquidity-and-debt contract from creditors' point of view. We then compare the results of this optimal contract with creditors welfare under pure debt relief.

We employ a variant of the model in section 2, with only two changes. First, we leave out the uncertainty in production since it is no longer essential. Output is simply $y = f(I)$. Second, the creditors will make a take-it-or-leave-it offer which consists of a period-two repayment, D, and an injection of liquidity, L, in period one. The initial contractual debt is given by $D_0 \geq D$. In this simple framework, the country must first decide whether to invest. If it invests, the optimal level of investment, $I^* = I^*(L)$, is given by the first-order condition:

$$f'(I^*) = \frac{U_1'(\mathrm{E} + L - I^*)}{\beta}, \tag{24}$$

where, as before, we assume that the country is credit constrained, $f' > 1$. By the implicit function theorem, only a portion of any additional liquidity is invested, the rest is consumed:

$$\frac{dI^*}{dL} = \frac{U_1''}{\beta f'' + U_1''} < 1. \tag{25}$$

Notice that creditors have no control over how the country divides the new liquidity between investment and consumption. If "conditionality" were applied, forcing the country to invest a large-than-desired share of L, then the argument for liquidity relief would be even stronger.

The fact that (25) is positive implies that the most severely liquidity-constrained debtors will have the lowest chosen levels of investment. Debt relief increases investment from 0 to $I^*(L)$; liquidity-constrained countries will therefore gain less from pure debt relief than countries with more liquidity.

The country will invest only if it gains from doing so. Its rationality constraint requires that welfare with investment is greater than welfare with no investment:

$$U_1(\mathrm{E} + L - I^*) + \beta(f(I^*) - D) \geq U_1(\mathrm{E} + L), \tag{26}$$

where we again assume that the period-two repayment is $\min(y, D)$. Equation (26) implies that for any given amount of liquidity, creditors will maximize the value of their claims by lowering the debt payment to:

$$D(L) = \frac{U_1(\mathrm{E} + L - I^*)}{\beta} - \frac{U_1(\mathrm{E} + L)}{\beta} + f(I^*). \tag{27}$$

Equation (27) says that if creditors write down the debt, they will do so to be at the top of the debt-relief Laffer curve. Given L, lower values of D imply a one-for-one reduction in expected payments, while higher values imply expected payments fall to zero. The function $D(L)$ defines a family of debt-relief Laffer curves, one for each L.

It is easy to show that the debt payment is an increasing function of liquidity, $D'(L) > 0$.[26] Greater liquidity raises the optimal level of investment and, therefore, increases the payment

[26] The envelope theorem implies that $\frac{dD(I^*(L),L)}{dL} = \frac{\partial D(I^*,L)}{\partial L} = \frac{U_1'(\mathrm{E}+L-I^*)}{\beta} - \frac{U_1'(\mathrm{E}+L)}{\beta} > 0$, because marginal utility is higher when investment crowds out current consumption.

creditors can extract. It follows that *countries that are more liquidity constrained are more likely to be on the wrong side of the debt-relief Laffer curve.* Figure 3 demonstrates, showing three Laffer curves with different underlying levels of liquidity, $L_2 > L_1 > L_0$. As the country is more illiquid, the Laffer curve shifts down (since from (25), $\frac{dI^*}{dL} > 0$), and the peak shifts toward the left (since $\mathcal{D}' > 0$).[27] Suppose the debt is initially at D_0. Then it is clear that if the country has liquidity equal to L_2, pure debt relief will not be in creditors' interest. On the other hand, if the country is severely liquidity constrained, at $L = L_0$, then there is scope for pure debt relief. The irony is that countries with weak investment-incentive effects are also the most likely recipients of pure debt relief.

Fortunately for all, creditors may gain by adjusting the level of liquidity. They will not, however, choose the L that gives the highest Laffer curve. They will instead set the pair $\{D, L\}$ to maximize the discounted value of cash flows, $D - L$. Since creditors can collectively choose to set $L = 0$ and still receive a period-two repayment (by setting $D = \mathcal{D}(0)$), any new lending must be profitable in itself. Notice, however, that as long as the initial debt, D_0, is high enough, the free-rider problem remains: an individual creditor would prefer not to write down his portion of the debt in the first place, even when others do.[28] We then have the following proposition, proven in the second appendix:

Proposition 6. The optimal contract, $\{D^*, L^*\}$, solves:[29]

$$f'(I^*) = 1 + \frac{U'(\mathrm{E} + L^*)}{\beta}, \tag{28}$$

$$D^*(L^*) = \frac{U_1(\mathrm{E} + L^* - I^*)}{\beta} - \frac{U_1(\mathrm{E} + L^*)}{\beta} + f(I^*), \tag{29}$$

where I^* is given by equation (24).

The intuition for this contract can be seen in figure 4. Suppose the country has an initial

[27] Note that the horizontal axis in figure 1 is D.

[28] Some debt relief is required before profitable lending can be undertaken. If debt relief were not needed, then there would be no free rider problem; individual creditors would find it in their interest to lend, regardless of the behavior of others.

[29] We assume that the country is sufficiently liquidity constrained to satisfy the second-order condition for this problem,

$$\frac{f'' U_1''(\mathrm{E} + L^* - I^*)}{\beta f'' + U_1''(\mathrm{E} + L^* - I^*)} - U_1''(\mathrm{E} + L^*) < 0.$$

This condition holds, for example, for isoelastic utility and production functions at sufficiently low levels of the endowment, E.

obligation D_0 and liquidity $L_0 = 0$. The expected value of the debt payment is shown by point A. Pure debt relief (or one of the buyback scenarios discussed in section 2) can move the country to the top of the L_0 Laffer curve, point B. The improvement in incentives raises debtor welfare and investment and reduces current consumption. But since marginal utility rises (see equation (4)), the return on investment will not fall as much as the improved incentives merit. The country will therefore be unwilling to undertake all of the investment projects that become profitable at world interest rates. For the liquidity constrained country, we would have such a high marginal return on investment that $f'(I^*) > 1 + U'(\mathrm{E} + L_0)$.[30] Creditors can capture a surplus above the world interest rate on additional investment by providing liquidity while *reducing* (by more) the amount of debt relief. This shifts the value of the claims from point B to C. Note that creditors would be strictly worse off if debt relief and new lending were negotiated separately, because then the new lending would be competitive. Creditors obtain the surplus by offering to provide simultaneously new lending and debt relief. Provided the second-order conditions above hold we have:

Proposition 7. The more liquidity-constrained the country is, the more creditors sacrifice with simple debt-reduction schemes in comparison with the optimal liquidity and debt relief given in proposition 6.

[30] This condition is equivalent to:

$$\frac{U_1'(\mathrm{E} + L^* - I^*)}{\beta} - \frac{U_1'(\mathrm{E} + L^*)}{\beta} > 1,$$

which will be satisfied for low enough E and, for example, isoelastic production and utility functions.

4. Conclusions

Our four main conclusions can be stated as follows:

1) Market-based debt relief schemes are similar to pure debt relief in the sense that they reduce the debt overhang. These plans can therefore be Pareto improving only if investment-incentive effects are sufficiently important.

2) Market-based plans differ from pure debt relief, and from one another, according to the source of resources used to retire old debt. In particular, debtors that finance buybacks with current resources can substantially worsen the investment-incentive effects on which these plans critically rely.

3) If investment-incentive effects are important enough to make debt reduction profitable for creditors, then debt reduction alone will not generally be optimal from the creditor's perspective. Thus neither market-based schemes nor pure debt relief will generally maximize the value of creditor claims.

4) In general, countries that are liquidity-constrained are the best candidates for an optimal relief package which includes new lending as well as partial debt forgiveness.

These conclusions are relatively general, and are likely to come out of more realistic, and more omplicated models of the investment process. We have abstracted from such issues as capital flight, the debtor's internal financing constraints, and how creditors impose penalties in instances of default. Nevertheless, we believe that our general conclusions will remain when these issues are considered explicitly. We have also ignored the moral hazard and adverse selection problems which naturally arise once debt relief is on the table (Froot, Scharfstein and Stein (1988) study these problems).

Finally, our analysis takes as given the presence of investment-incentive effects. For these effects to give the Laffer curve its bowed shape, there must be sufficiently many marginal investment projects with sufficiently high returns. We have assumed the existence of these effects because in their absence there is no scope for Pareto improvements under any debt-reduction plan (given risk neutrality). We have therefore given the benefit of the doubt to the advocates of one or more of these schemes. Nevertheless, there is thus far no empirical evidence that suggests incentive effects are important, or present at all.

5. Appendix 1: A primer on buybacks and exit bonds

In this appendix, we present simple examples to show how market-based debt-reduction schemes work.

By "buyback" we mean a cash repurchase of existing debt on a competitive secondary market. The resources for the repurchase may come from a variety of sources: exogenous aid from outside the country, the country's current resources, or its future cash flows. An "exit bond" is actually a buyback using resources from future cash flows. Exit bonds can be used for debt relief if they are treated as *senior* claims to the future cash flows.

In our simple examples we assume that cash flows are fixed and out of the country's control. (In the text we add the incentive effects which are crucial to the logic of these plans.) Consider a two-period economy with two equiprobable states in period two, and corresponding cash flows of 1 and 2 dollars. We denote the cash flow by $\tilde{y}$. Also in period two, the country's inherited debt of $D = 2$ dollars comes due. To keep things simple, we let the interest rate be zero, and we assume that first-period consumption is positive.

If creditors are to induce the country to service its debt, they must have access to some credible punishment mechanism. We make the "gunboat-technology" assumption that the creditors can seize the entire output if the country cannot pay in full. Thus repayments will be given by

$$R = \min(\tilde{y}, 2) = \tilde{y}. \tag{A1}$$

The expected payment is therefore $E(\tilde{y}) = 1.5$.

Now suppose that in period one an outside source agrees to donate funds to be used in buying back some of the outstanding debt. Table 1 shows the cash flows associated with a buyback of 1 dollar. Let θ^b be the dollar amount of original debt retired in the buyback. Then the country will have a second-period liability of $2 - \theta^b$ dollars. The payments on the remaining original debt are now given by $\min(2 - \theta^b, \tilde{y})$.

How much original debt will be retired? If the buyback is competitive and preannounced (so that the resources available for the buyback are known by the creditors), then θ^b must be such that individual creditors are indifferent between holding onto their old debt and trading in their old debt for cash. Then with risk neutral creditors, in equilibrium θ^{b*} units of the old debt must yield one dollar in expectation:

$$\theta^{b*}\Big(\frac{E(\min(2-\theta^{b*},\tilde{y}))}{2-\theta^{b*}}\Big)=1. \qquad (A2)$$

Equation (A2) is highly nonlinear, but in this case it is easy to verify that $\theta^{b*}=1$ is the equilibrium. Notice that just enough of the old debt is retired to make the remaining amount, $2-\theta^{b*}=1$, riskless. Anytime the buyback is big enough to make the remaining old debt pay its face value with certainty, the last increment of old debt retired must be exchanged one-for-one for cash, $\theta^{b*}=1$.

Expected creditor receipts are now $1+E(\min(2-\theta^{b*},\tilde{y}))=2$, higher than the expected payment of 1.5 before the buyback. The intuition is that the buyback makes new resources – a dollar's worth of debtor consumption – available for debt service. The creditors now get the full face value of their original claims. The debtor is also better off. It must pay only 1 dollar in each state, keeping the second dollar in state 2. In this example, there is no particular reason that the buyback resources must come from outside the country (as they did in the Bolivian buyback). As long as the resources are fully additional – previously unavailable for debt service – total creditor receipts are the same. The country could, for example, reduce period 1 consumption in order to obtain the buyback resources. It would lose a dollar's worth of first-period consumption, and gain an equivalent amount of consumption in state 2. As long as the country prefers a certain dollar today to a less-than-certain dollar tomorrow, it is worse off by going ahead with the buyback.

To see why the source of the resources used in the buyback is critical, consider a variation on our example. Suppose that 1 dollar of the future cash flow is already available in period one. We can think of this 1 dollar as resources that are earmarked for current savings, such as a debtor's central bank reserves. All that is important is that the debtor will not consume these resources in period one, and that they are available for consumption or debt service – or confiscation in the case of default – in period 2. If the debtor auctions off these resources in return for some old debt, we refer to it as a buyback out of saving.

Notice that the buyback out of saving is financed directly out of the period-two cash flow. By assumption, there is no effect on current consumption, and no no new resources become available for debt service. Table 2 shows the payments. The country must service in period two the 1 dollar exit bond obligation as well as $2-\theta^{k}$ of remaining old debt. Payments on the old debt are now $\min(2-\theta^{k},\tilde{y}-1)$.

The equilibrium price of the buyback is determined competitively, as in (A2):

$$\theta^{k*}\Big(\frac{E(\min(2-\theta^{k*},\tilde{y}-1))}{2-\theta^{k*}}\Big)=1. \tag{A3}$$

It is easy to verify that $\theta^{k*}=2$ solves equation (A3). The last increment of old debt exchanged pays zero in state 1 and its face value in state 2, an expected return of $1/\theta^{k*}=.5$. Notice that a 1 dollar buyback out of saving retires the entire stock of old debt. Thus the country's expected payments are $1+E(\min(2-\theta^{k},\tilde{y}-1))=1$, *less* than the original expected value of the debt. The country clearly benefits at the expense of the creditors: through the buyback the country is entitled to 1 additional dollar of period-two consumption in state 2, without any reduction in period-one consumption.

How is it that a buyback out of saving reduces the expected payments to the creditors, without any change in the resources available for debt service? Such a buyback allows the country to take advantage of the lack of coordination among creditors. Each creditor is concerned only with the marginal value of the old debt. Thus an externality arises: as some creditors swap in their old debt for cash, they degrade the value of the old debt remaining. At the price of the first increment of buyback, $\theta_0=2/E(\tilde{y})=1.33$, each creditor would strictly prefer to swap in his old debt than to hold on: conditional on no other creditors swapping, each creditor is indifferent between swapping and not swapping, and conditional on other creditors swapping, each creditor is strictly better off by swapping in old debt. The resulting excess supply of old debt drives up the price of the cash in terms of old debt from 1.33 to 2.

Finally, we consider an issue of exit bonds which is treated by both creditors and the debtor as senior to the old debt. Here an exit bond is exactly equivalent to a buyback out of saving: holders of old debt now own claims to the same, residual future cash flows. (In the text, where investment incentives are included in the analysis, this equivalence breaks down.) Thus either type of swap leads to the equilibrium given by equation (A3). The country benefits at the expense of the creditors as the expected value of payments falls from 1.5 to 1.

Our examples therefore demonstrate that when the new security is not backed by additional funds, creditors lose from a market-based debt relief plan.

6. Appendix 2: Proofs of the propositions in the text

Proof of proposition 2. (i) Recall that effective relief is equal to the amount of old debt retired in the buyback, $x = \theta^b b$. The debtor welfare is given by:

$$U_1(E - I^*(x)) + U_2(I^*(x), x) = W^*(x).$$

By the envelope theorem,

$$\frac{dW^*(I^*(x), x)}{dx} = \frac{\partial W^*(I^*(x), x)}{\partial x} = \frac{\partial U_2(I^*(x), x)}{\partial x} = \beta \int_{\epsilon^*}^{\bar{\epsilon}} g(\epsilon) d\epsilon = \beta G > 0.$$

Thus debtor welfare increases with the amount of effective relief.

Next we show that the amount of effective relief increases with the size of the buyback, $\frac{dx}{db} > 0$:

$$\frac{dx}{db} = \theta^b + \frac{d\theta^b}{dx}\frac{dx}{db} b = \frac{\theta^b}{1 - b(d\theta^b/dx)}.$$

From proposition 1 (ii) and proposition 2 (iii):

$$\frac{d\theta^b}{dx} = \frac{-\theta^b\left(\frac{dV^*}{dx}\right) - 1}{V^*}.$$

Since for all x, $\theta^b = \frac{D-x}{E(\min(D-x, f(I^*)+\epsilon))} \geq 1$, and $\frac{dV^*}{dx} \in [\infty, -1]$ then $\frac{d\theta^b}{dx} < 0$, and it follows that $\frac{dx}{db} \geq 1$.

(ii). To see that creditor welfare increases in the size of the buyback, note that:

$$\frac{dV^b}{db} = \left(\frac{dV^*}{dx}\right)\left(\frac{dx}{db}\right) + 1.$$

Since $\frac{dx}{db} > 1$ and $\frac{dV^*}{dx} \in [\infty, -1]$, it follows that $\frac{dV^b}{db} > 0$.

Proof of proposition 3.

(i). Debtor welfare under an exit bond offering is given by:

$$W^k(x) = U_1 + E(\max(0, f(I^*) + \epsilon - k) + \max(0, f(I^*) + \epsilon - D + \theta^k k))$$

$$= U_1 + E(\max(0, f(I^*) + \epsilon - D + (\theta^k - 1)k) = W^*(x).$$

(ii). The value of creditors' claims under an exit bond offering is given by:

$$V^k = E(\max(0, \min(f(I^*) + \epsilon - K, D - \theta^k k)) + k) = E(\min(f(I^*) + \epsilon, D - x)) = V^*,$$

where the first equality follows from the definitions of exit bonds and limited liability, and the second equality is by algebra.

(iii) and (iv). The competitive auction requires:

$$\theta^k\left(\frac{E(\min(f(I^*)+\epsilon, D-\theta^k k))}{D-\theta^k k}\right)=\frac{E(\min(f(I^*)+\epsilon, k))}{k},$$

where the right-hand side is the expected return on a one-dollar exit bond. If the exit bond is riskless, then this expected return is one, and proposition 3 (iii) follows after a little algebra.

Proof of proposition 4. Follows from the debtor's first-order condition, and application of the implicit function theorem.

Proof of proposition 5. Follows directly from proposition 4.

Proof of proposition 6. The creditors' collective maximizes $D(L)-L$. Taking the first-order condition and using equation (24) yields (28). Equation (29) follows from (27) directly.

7. References

Bulow, J. and K. Rogoff, "Sovereign Debt Restructuring: Panacea or Pangloss?," University of Wisconsin, June 1988.

Dooley M., "Buybacks and the Market Valuation of External Debt," IMF, forthcoming, *International Monetary Fund Staff Papers,* 1988.

Dornbusch, R., "Our LDC Debts," in M. Feldstein ed., *The United States in the World Economy,* University of Chicago Press, 1988.

Froot, K., Scharfstein, D. and J. Stein, "LDC Debt: Forgiveness, Indexation and Investment Incentives," NBER Working Paper no. 2541, March 1988.

Helpman, E., "The Simple Analytics of Debt-Equity Swaps and Debt Forgiveness," MIT, September 1987.

Krugman, P., "International Debt Strategies in an Uncertain World," in G. Smith and J. Cuddington, eds., *International Debt and the Developing Countries,* Washington D.C., World Bank, 1985.

Krugman, P., "Financing vs. Forgiving a Debt: Some Analytical Notes," NBER Working Paper no. , forthcoming *Journal of Development Economics* 1988.

Krugman, P., "Market-based Debt-Reduction Schemes," NBER Working Paper no. , forthcoming *International Monetary Fund Staff Papers,* 1988.

Sachs, J., "Theoretical Issues in International Borrowing," *Princeton Papers in International Finance,* no. 53, 1984.

Sachs, J., "Conditionality, Debt Relief, and the Developing Country Debt Crisis," Harvard University, 1988.

Sachs, J., "The Debt Overhang of Developing Countries," in De Macedo and Findlay eds., Diaz Memorial Volume, Wider Institute, Helsinki, 1988.

Williamson, J., *Voluntary Approaches to Debt Relief,* Policy Analysis in International Economics, Institute for International Economics, forthcoming 1988.

TABLE 1: Baybank out of Aid

	Total	State 1	State 2
Cash Flow		1	2
Expected payments before buyback, D=2	1.5	1	2
Expected payments w/buyback:			
Donated buyback repurchase	1	1	1
Old debt remaining	$2-\theta^*$	$\min(2-\theta^*,1)$	$\min(2-\theta^*,2)$
In equil. $\theta^*=1$ Total payments including payback	2	2	2

TABLE 2 - Buyback out of Reserves

	Total	State	
		1	2
Cash flow		1	2
Expected pyaments before buyback, $\theta=2$	1.5	1	2
Expected payments w/payback:			
Reserves used for buyback purchase	1	1	1
Remaining cash flows	0.5	0	1
Old debt remaining	$2-\theta^*$	$\min(2-\theta^*,0)$	$\min(2-\theta^*,1)$
In equilibrium, $\theta^*=2$			
Total payment including buyback	1	1	1

Figure 1
Total Value of Creditor Claims

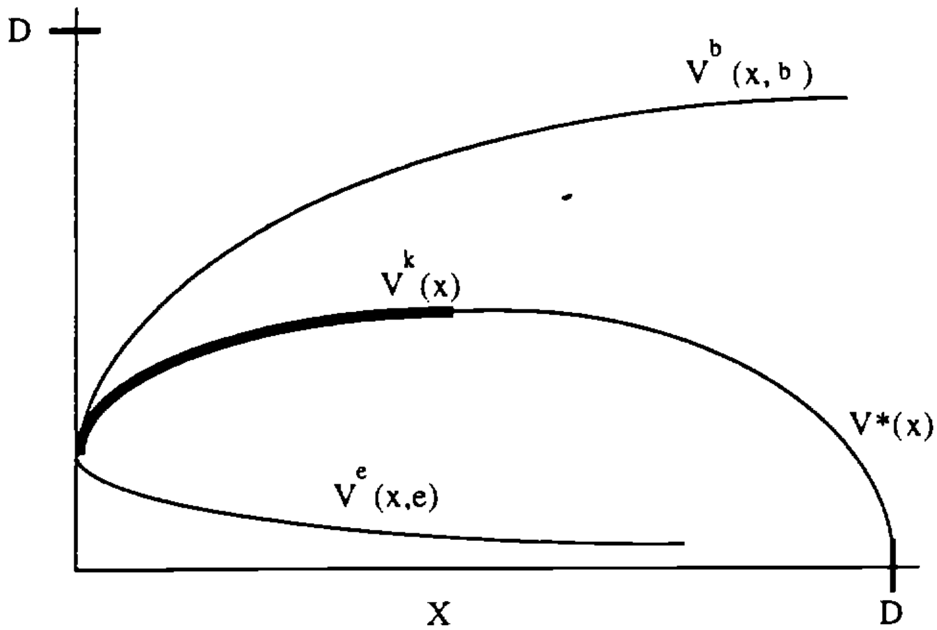

Figure 2
Price of Remaining Debt

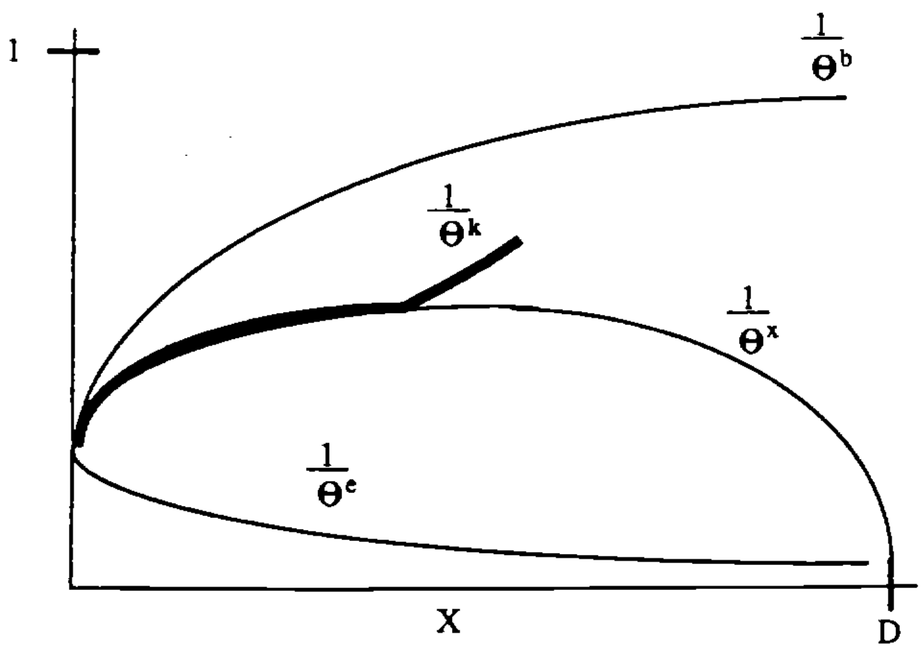

Figure 3

Exit bond offerings

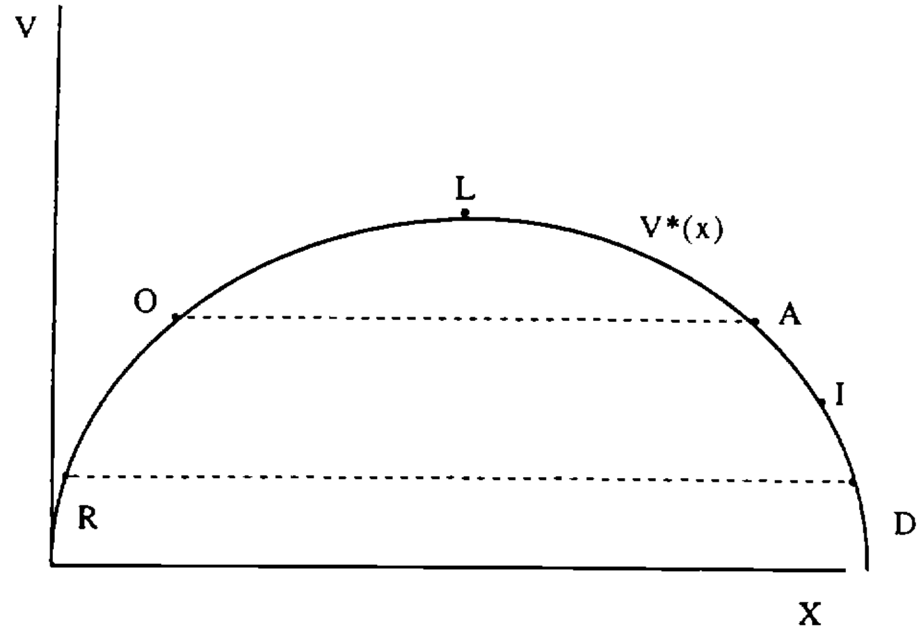

Figure 4

Liquidity and
the Debt-Relief Laffer curve