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Lawrence J. Christiano
Martin S. Eichenbaum
Mathias Trabandt

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Why is Unemployment so Countercyclical?

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ABSTRACT

We argue that wage inertia plays a pivotal role in allowing empirically plausible variants of the standard search and matching model to account for the large countercyclical response of unemployment to shocks.

Lawrence J. Christiano
Department of Economics
Northwestern University
2001 Sheridan Road
Evanston, IL 60208
and NBER
l-christiano@northwestern.edu

Mathias Trabandt
Freie Universität Berlin
School of Business and Economics
Boltzmannstrasse 20
14195 Berlin
Germany
and DIW and IWH
mathias.trabandt@gmail.com

Martin S. Eichenbaum
Department of Economics
Northwestern University
2003 Sheridan Road
Evanston, IL 60208
and NBER
eich@northwestern.edu

1 Introduction

Wage rigidities play a critical role in many quantitative business cycle models. Estimated New Keynesian (NK) models consistently feature important nominal wage rigidities.¹ The newest generation of heterogeneous agent (HANK) models also assigns prominent roles to wage rigidities.² There is also a long tradition of wage rigidities in open-economy models of aggregate fluctuations.³ Wage rigidities enable all of these models to generate employment fluctuations that are comparable to those observed in the data.

But do wage rigidities play an important role in understanding why *un*employment is so countercyclical? The standard framework used to study unemployment features search and matching frictions of the sort emphasized by Diamond (1982), Mortensen (1982) and Pissarides (1985) (henceforth, DMP). Shimer (2005) showed that standard versions of these models cannot - at least with plausible parameter values - explain the countercyclical behavior of unemployment. This ‘Shimer critique’ has led to a very large literature whose goal is to account quantitatively for the dynamics of unemployment. There is intense disagreement within this literature about the role of wage inertia in the cyclical behavior of unemployment.

Authors like Hall (2005) and Rogerson and Shimer (2011) argue that wage inertia greatly increases the cyclical volatility of unemployment in search and matching models. Similarly, Gertler and Trigari (2009) and Christiano, Eichenbaum and Trabandt (2016, CET) stress the importance of wage rigidities in estimated New-Keynesian search and matching models. In sharp contrast, Hagedorn and Manovskii (2008, HM) and Ljungqvist and Sargent (2017, LS) argue that the emphasis on wage rigidity is misplaced. Building on HM’s analysis, LS stress the usefulness of what they call the ‘fundamental surplus fraction’ for understanding the response of unemployment to shocks. By the fundamental surplus fraction they mean the “upper bound on the fraction of a job’s output that the invisible hand can allocate

¹See for example Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007). See also Del Negro, Eusepi, Giannoni, Sbordone, Tambalotti, Cocci, Hasegawa and Linder (2013) for an example of DSGE models used for policy analysis.

²See for example Broer, Hansen, Krusell and Oberg (2018) and Auclert, Rognlie and Straub (2019).

³See for example Kollman (2001), Schmitt-Grohé and Uribe (2016) and Eichenbaum, Johannsen and Rebelo (2019).

to vacancy creation”.⁴ LS argue that the fundamental surplus fraction must be small to account for the Shimer puzzle and that wage rigidities don’t play any special role in this connection. We argue that wage rigidities do in fact play a pivotal role in allowing variants of the standard search and matching models to account for the large countercyclical response of unemployment to shocks. In fact wage rigidities are necessary in empirically plausible versions of those models for explaining the cyclical volatility of unemployment.

In section two we present a simple labor market search and matching model. In section three we proceed as in much of the literature and use comparative steady state analysis as a short-cut for analyzing model dynamics. We characterize the implications of the search and matching model for the volatility of unemployment by a standard statistic: the elasticity of steady state labor market tightness with respect to a shift in the steady state marginal revenue product of labor. We decompose that elasticity into a wage inertia component and a profit rate component. Our decomposition makes clear that wage inertia is a necessary condition for generating a large countercyclical response of unemployment to a shift in the marginal revenue product of labor. If the wage rate responds one-to-one to the marginal revenue product across steady states, then the unemployment rate remains exactly the same across those steady states. Our decomposition also makes clear that, other things equal, the less the wage rate responds to the marginal revenue product, the more responsive is the unemployment rate. We show that similar conclusions hold for changes in the discount rate of the sort considered in Hall (2017).

We apply our decomposition to analyze the role of wage inertia in different wage-setting models estimated by CET. Specifically, we consider versions of their DSGE model where wages are set according to Nash bargaining and alternating offer bargaining (AOB). Both models solve the Shimer puzzle because they have more wage inertia than does the model considered by Shimer. A sticky wage variant of the model, in which wages are constant across steady states, generates an enormous response of unemployment to a permanent shift in firms’ marginal revenue product. As a by-product of our analysis, we provide a counter-

⁴See LS, abstract.

example to the claim in LS that search and matching models can only successfully address the Shimer critique by incorporating features that make the inverse of the fundamental surplus fraction large. In that section we also address the HM critique of wage inertia as a solution to the Shimer puzzle.

The comparative steady state approach can yield important insights about the role of wage rigidity in dynamics. However, it can also be very misleading. The approach assumes the underlying shock is close to a random walk and the economy does not have quantitatively important state variables. These two assumptions are satisfied in the simple search and matching model, e.g., Shimer (2005). But they are not satisfied in generalizations of that model which include transitory shocks and a rich assortment of state variables. Moreover, model features like adjustment costs and nominal rigidities give rise to additional sources of dynamics while leaving no trace in the steady state. A steady-state analysis could assign no role at all to wage rigidity even if in fact it plays an very important role.

In light of these considerations, section four focuses on the dynamic response of unemployment to shocks. We do so using variants of the Nash and AOB models estimated in CET. Our main results can be summarized as follows. First, wage inertia greatly magnifies the response of unemployment to shocks. That is, models which do well at matching the data like the estimated AOB model do poorly if one replaces the wage determination mechanism by one in which wages are less inertial. Models that do badly at matching the data, like the Nash model with plausible replacement ratios, do much better if we impose on them wage processes that are more inertial. Second, we show that steady-state based measures of the fundamental surplus are uninformative about the dynamic response of unemployment to shocks. Models which have identical steady states exhibit dynamic response functions of unemployment to shocks that are very different from each other. These differences are driven by different degrees of wage inertia.

Section five contains concluding remarks.

2 A Simple Labor Market Model

In this section we consider a simple discrete time search and matching model developed in CET. There is a continuum of identical workers and firms. To map into the medium-sized DSGE model in section 4, we refer to these firms as wholesaler firms. Firms produce a good using labor as the sole input. A wholesaler firm that wishes to meet a worker in period t must post a vacancy at a cost s_t , expressed in units of the consumption good. The vacancy is filled with probability Q_t . Following Pissarides (2009), the firm must pay a fixed real cost, κ_t , before bargaining with the newly-found worker.

Let J_t denote the value to the firm of a worker, expressed in units of the final consumption good:

$$J_t = \vartheta_t^p - w_t^p. \quad (1)$$

Here, ϑ_t^p denotes the expected present discounted value, over the duration of the worker/firm match, of the marginal revenue product of the worker. The latter could be stochastic because technology shocks affect the productivity of workers or because there are shocks to the price of the wholesale firm good. The variable w_t^p denotes the discounted value of the real wage. The latter is determined by worker-firm bargaining and is discussed below. In recursive form:

$$\vartheta_t^p = \vartheta_t + \rho E_t m_{t+1} \vartheta_{t+1}^p, \quad w_t^p = w_t + \rho E_t m_{t+1} w_{t+1}^p. \quad (2)$$

Here ρ is the probability that a given firm/worker match continues from one period to the next. In equation (2), m_{t+1} is a discount factor which firms and workers view as an exogenous stochastic process with a well defined steady-state value. We allow m_t to be stochastic in anticipation of the medium-sized DSGE model considered in section 4.

The law of motion for aggregate employment, l_t , is given by:

$$l_t = \rho l_{t-1} + x_t l_{t-1}.$$

The term ρl_{t-1} denotes the number of workers that were attached to firms in period $t-1$ and remain attached at the start of period t . The variable x_t denotes the hiring rate so that $x_t l_{t-1}$ represents the number of new firm/worker meetings at the start of period t . The number of workers searching for work at the start of period t is the sum of the number of unemployed workers in period $t-1$, $1 - l_{t-1}$, plus the number of workers that separate from firms at the end of $t-1$, $(1 - \rho)l_{t-1}$. Consequently, the probability, f_t , that a searching worker meets a firm is given by:

$$f_t = \frac{x_t l_{t-1}}{1 - \rho l_{t-1}}.$$

Free entry by wholesalers implies that, in equilibrium, the expected benefit of a vacancy equals the cost:

$$Q_t(J_t - \kappa_t) = s_t. \quad (3)$$

Let V_t denote the value to a worker of being matched with a firm. We write V_t as the sum of the expected present discounted value of wages earned while the firm-worker match endures and the continuation value, A_t , when the match terminates:

$$V_t = w_t^p + A_t. \quad (4)$$

Here

$$A_t = (1 - \rho)E_t m_{t+1} [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}] + \rho E_t m_{t+1} A_{t+1} \quad (5)$$

and

$$U_t = D_t + \tilde{U}_t \quad (6)$$

where D_t denotes unemployment benefits. In addition, \tilde{U}_t denotes the continuation value of unemployment:

$$\tilde{U}_t \equiv E_t m_{t+1} [f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1}]. \quad (7)$$

Labor market tightness, Γ_t , is the ratio of aggregate vacancies to the number of workers searching for work. Assuming a standard constant returns to scale matching function, the

vacancy filling rate, Q_t , and the job finding rate for workers, f_t , are related to Γ_t , as follows:

$$f_t = \sigma_m \Gamma_t^{1-\sigma}, \quad Q_t = \sigma_m \Gamma_t^{-\sigma} \quad (8)$$

where $\sigma_m > 0$, $0 < \sigma < 1$ and

$$\Gamma_t = \frac{v_t l_{t-1}}{1 - \rho l_{t-1}}. \quad (9)$$

Here, $v_t l_{t-1}$ denotes the total number of vacancies posted by firms at the start of period t .

As soon as the l_t matches are determined at the start of period t , each worker in l_t engages in bilateral bargaining over the current wage rate, w_t , with a wholesaler firm. Each worker-firm bargaining pair takes the outcome of all other period t bargains as given. In addition, they take as given the outcome of future wage agreements as long as the worker and firm remain matched. Because bargaining in period t applies only to the current wage rate, we refer to it as period-by-period bargaining. The bargaining problem of all worker-firm pairs is the same, regardless of how long they have been matched.⁵

In the basic search and matching model, the match surplus $J_t + V_t - U_t$ is split between a matched firm and worker according to Nash bargaining. The Nash-sharing rule is given by:

$$J_t = \frac{1 - \eta}{\eta} (V_t - U_t) \quad (10)$$

where η is the share of total surplus going to the worker.

Following Hall and Milgrom (2008) and CET, we also consider a version of the model in which real wages are determined by alternating offer bargaining (AOB). We suppose that bargaining proceeds across M sub-periods within the period, where M is even.⁶The firm

⁵This result follows from our assumptions that hiring costs, i.e. s_t and κ_t , are sunk when bargaining occurs and the expected duration of a match is independent of how long a match has already been in place.

⁶ Our model differs from Hall and Milgrom's in two ways. First, they assume alternating offers are made in successive periods, $t, t + 1$, etc., and can potentially continue indefinitely. With this assumption, they must specify the time period of the model to be shorter than the quarterly or monthly rate over which many macroeconomic variables are measured. Our approach, which assumes that bargaining proceeds within a period, means that when we use standard time series estimation methods which use quarterly data, as in CET, we can avoid having to explicitly take into account temporal aggregation effects. Second, we assume that a worker can go from one job to another without passing through unemployment.

makes a wage offer at the start of the first sub-period. It also makes an offer at the start of a subsequent odd sub-period in the event that all previous offers have been rejected. The cost to a firm of making an offer is γ_t . Similarly, the worker makes a wage offer at the start of an even sub-period in case all previous offers have been rejected. The worker makes the last offer, which is take-it-or-leave-it. In sub-periods $j = 1, \dots, M - 1$, the recipient of an offer has the option to accept or reject it. If the offer is rejected, the recipient may declare an end to the negotiations or she may plan to make a counteroffer at the start of the next sub-period. In the latter case, with probability δ bargaining breaks down.

As shown in CET, the solution to the AOB problem is given by:

$$w_t^p = \frac{1}{\alpha_1 + \alpha_2} (\alpha_1 \vartheta_t^p + \alpha_2 (U_t - A_t) + \alpha_3 \gamma_t - \alpha_4 (\vartheta_t - D_t)). \quad (11)$$

where $\alpha_1 = 1 - \delta + (1 - \delta)^M$, $\alpha_2 = 1 - (1 - \delta)^M$, $\alpha_3 = \alpha_2(1 - \delta)/\delta - \alpha_1$ and $\alpha_4 = (1 - \delta)(\alpha_2/M)/(2 - \delta) + 1 - \alpha_2$. Relations (1), (4) and (11) can be combined and written in the form of the AOB-sharing rule:

$$J_t = \beta_1 (V_t - U_t) - \beta_2 \gamma_t + \beta_3 (\vartheta_t - D_t) \quad (12)$$

with $\beta_i = \alpha_{i+1}/\alpha_1$, for $i = 1, 2, 3$. The Nash-sharing rule can be viewed as a special case of the AOB-sharing rule, in which $\beta_1 = (1 - \eta)/\eta$ and $\beta_2 = \beta_3 = 0$.

We use (2) to solve for the real wage: $w_t = w_t^p - \rho E_t m_{t+1} w_{t+1}^p$. In principle w_t^p is consistent with a wide variety of wage payments over the periods in which the worker and firm remain matched. We resolve this potential non-uniqueness in w_t by assuming that each period's wage rate is the same time-invariant function of variables that are exogenous to the worker and firm.

3 Steady-State Analysis of Wage Inertia and Labor Market Volatility

This section analyzes the role of wage inertia in generating labor market volatility using the type of comparative steady-state methods commonly used in the literature. We consider three models of wage determination: Nash bargaining, AOB, and sticky wages. In the latter case, w is simply a constant, \tilde{w} , like in Hall (2005).

The analysis in this section focuses on the elasticity, $\eta_{\Gamma,\vartheta}$, of market tightness (Γ) with respect to the marginal revenue product of labor (ϑ). The first subsection develops a decomposition of this elasticity that highlights the role of wage inertia. The second subsection studies the fundamental surplus fraction decomposition developed in LS. In the examples studied in LS, that decomposition can be used in a simple and transparent way to determine the impact on $\eta_{\Gamma,\vartheta}$ of a change in the value of a model parameter. We show that for the Nash bargaining case with a positive fixed cost, κ , the LS decomposition is not sufficient to determine the impact of a parameter change on $\eta_{\Gamma,\vartheta}$. In that case it is necessary to also recompute the steady state finding rate, using the relevant steady state equilibrium conditions. The same observations hold for the AOB case when $\kappa \geq 0$. The third subsection below presents a quantitative analysis of the decompositions considered in this section. The version of the AOB model estimated in CET has an elasticity, $\eta_{\Gamma,\vartheta}$, that is about six times bigger than the value implied by a standard version of the Nash bargaining model. We show that the inverse fundamental surplus fraction plays only a small role in this substantial increase. Using our wage inertia-based decomposition, we show that wage inertia is the main reason for the jump in $\eta_{\Gamma,\vartheta}$. The fourth subsection extends the analysis to disturbances in the discount rate. Hall (2017) has recently argued that discount rate shocks are important in labor market fluctuations, and we show that wage inertia is important for amplifying the effects of those shocks.

3.1 A Wage-Inertia Decomposition

The free-entry condition and the bargaining equation play a central role in the steady-state equilibrium conditions of the model. Equation (8) implies that in steady state, the vacancy filling probability, Q , and market tightness, Γ , are related by:

$$Q = \sigma_m \Gamma^{-\sigma},$$

where a time series variable without a time index denotes its steady-state value. The steady-state version of the free entry condition, (3) is:

$$\frac{s}{\sigma_m} \Gamma^\sigma + \kappa = \frac{\vartheta - w}{1 - \rho\beta}. \quad (13)$$

Here, β is the steady-state value of the representative household's discount factor, m_t . The right hand side of equation (13) corresponds to the steady-state expected profits from a filled vacancy.

Denote the elasticity of market tightness with respect to the marginal revenue product ϑ , by $\eta_{\Gamma, \vartheta}$:

$$\eta_{\Gamma, \vartheta} \equiv \frac{d \log \Gamma}{d \log \vartheta}. \quad (14)$$

Shimer (2005) and much of the related literature use $\eta_{\Gamma, \vartheta}$ as a measure of the labor market volatility implied by a model.

Totally differentiating (13) and rewriting implies:

$$\eta_{\Gamma, \vartheta} = \underbrace{\frac{1}{\sigma} \frac{\vartheta}{\vartheta - w - \kappa(1 - \rho\beta)}}_{1/(\text{Profit rate})} \underbrace{\left[1 - \frac{dw}{d\vartheta} \right]}_{\text{Wage inertia term}}. \quad (15)$$

Expression (15) decomposes labor market volatility $\eta_{\Gamma, \vartheta}$ into a term that reflects the inverse profit rate and a term that is a function of $dw/d\vartheta$. Note that equation (15) uses only the free entry condition, so that it holds regardless of how wages are determined. Other things equal,

a wage determination mechanism that implies greater wage inertia, i.e. a smaller value of $dw/d\vartheta$, implies a larger value of $\eta_{\Gamma,\vartheta}$. The intuition is simple. When the wage rate is more inertial, firms receive a greater share of the rent associated with vacancies after a rise in the marginal revenue product (or technology) ϑ . So the more inertial is the wage, the greater is the incentive of the firm to post vacancies in the wake of an increase in ϑ . This increased incentive leads to a greater increase in market tightness and a larger drop in unemployment after an increase in ϑ .

Expression (15) makes clear that *some* wage inertia is necessary for the model to generate a larger value of $\eta_{\Gamma,\vartheta}$. If, across steady states, a change in ϑ is fully reflected in the real wage rate ($dw/d\vartheta = 1$), then $\eta_{\Gamma,\vartheta}$ must be zero.

3.2 Fundamental Surplus-Based Decompositions

We now study the steady-state decomposition of $\eta_{\Gamma,\vartheta}$ developed in LS. This decomposition is based on what they call the *fundamental surplus fraction*, which we denote by FS . In contrast to our decomposition, an FS -based decomposition is derived using both the free entry condition and the details of the wage-setting mechanism. That decomposition takes the following form,

$$\eta_{\Gamma,\vartheta} = \frac{\mathcal{Y}}{FS},$$

where $\mathcal{Y} > 0$. We show that this type of decomposition is in general not unique. In what follows we discuss two such decompositions for each of the Nash and AOB models. In the first decomposition FS is an analytic function of model structural parameters. In contrast, \mathcal{Y} generally involves steady-state variables. We refer to this type of decomposition as a *structural decomposition*. With the possible exception of the section on the financial accelerator model, LS focus on such decompositions. In the second decomposition FS depends on steady-state variables. We refer to this type of decomposition as a *non-structural decomposition*.

3.2.1 Nash Bargaining

In Appendix A.1.1 we derive the following structural decomposition for $\eta_{\Gamma, \vartheta}$ in the Nash bargaining model,

$$\eta_{\Gamma, \vartheta}^{Nash} = \frac{\Upsilon^{Nash}}{FS^{Nash}} \quad (16)$$

where

$$\Upsilon^{Nash} = \frac{1 - \rho\beta + \eta\rho\beta f \left(1 + \frac{\kappa\sigma_m}{s} \left(\frac{f}{\sigma_m}\right)^{\frac{-\sigma}{1-\sigma}}\right)}{\sigma(1 - \rho\beta) + \eta\rho\beta f \left(1 + (1 - \sigma) \frac{\kappa\sigma_m}{s} \left(\frac{f}{\sigma_m}\right)^{\frac{-\sigma}{1-\sigma}}\right)}, \quad \tau_{\kappa}^{Nash} = \frac{1 - \rho\beta}{1 - \eta}, \quad (17)$$

$$FS^{Nash} = \frac{\vartheta - D - \tau_{\kappa}^{Nash} \kappa}{\vartheta}. \quad (18)$$

This decomposition effectively coincides with the decomposition reported in section A.5 of the LS online technical appendix.⁷ There is a slight difference between our structural decomposition and the one in LS, reflecting timing differences in job-to-job transitions. These timing differences do not affect the substance of our analysis.⁸

A distinguishing feature of FS in equation (16) is that it does not involve endogenous variables like f . In contrast, Υ^{Nash} involves f .

In Appendix A.1.2 we derive a non-structural decomposition for $\eta_{\Gamma, \vartheta}$. It takes the form of (16) with FS^{Nash} given in (18). The difference is that Υ^{Nash} and τ_{κ}^{Nash} in (17) are then given by:

$$\Upsilon^{Nash} = \frac{1 - \rho\beta + \eta\rho\beta f}{\sigma(1 - \rho\beta) + \eta\rho\beta f}, \quad \tau_{\kappa}^{Nash} = \frac{(1 - \beta\rho(1 - f\eta))^2}{(1 - \eta)(1 - \beta\rho(1 - f\eta/\sigma))}. \quad (19)$$

Notice that τ_{κ}^{Nash} and, hence, FS^{Nash} , depend on the steady-state value of f . In the special

⁷The equivalence between the two decompositions is easier to see using the fact that $Q = \sigma_m \left(\frac{f_t}{\sigma_m}\right)^{\frac{1}{1-\sigma}}$.

⁸LS use the same timing assumptions adopted by Hall and Milgrom (see footnote 6). In their main text LS assume, as in Mortensen and Nagypal (2007), that κ is paid after bargaining has occurred. In section A.5 of their online technical appendix, LS assume, as in Pissarides (2009), that κ is paid before workers and firms bargain. We adopt the latter assumption.

case, $\kappa = 0$, the structural and non-structural decompositions coincide.⁹

We now show that when $\kappa \neq 0$ there is an important difference between the structural and non-structural decompositions in terms of their usefulness for analyzing how changes in parameter values affect $\eta_{\Gamma, \vartheta}$. To understand the difference, it is useful to distinguish between two types of model parameters. First, some parameters enter the decomposition explicitly. Second, there may be other parameters that only enter via their impact f . In the case of our non-structural decomposition there is such a background parameter, σ_m . In the presence of such a background parameter, the non-structural decomposition provides a simple and transparent way to perform a particular experiment: evaluate the impact on $\eta_{\Gamma, \vartheta}$ of a change in a model parameter holding f and the non-background parameters constant. This type of experiment is of interest to the extent that the analyst has flat priors about the value of σ_m .¹⁰ In the case of the structural decomposition stressed in LS, when $\kappa \neq 0$ there are no background parameters that can implicitly be adjusted to keep f fixed. As a consequence the formula loses its transparency and simplicity. This loss is easy to see for the experiment described above: the analyst is now forced to solve a non-trivial problem to determine the required change in σ_m .

3.2.2 Alternating Offer Bargaining

We now consider the alternating offer bargaining case. In Appendix A.2.1 we derive the following structural decomposition for $\eta_{\Gamma, \vartheta}$,

$$\eta_{\Gamma, \vartheta}^{AOB} = \frac{\Upsilon^{AOB}}{FS^{AOB}} \quad (20)$$

⁹In this special case the two formulas also coincide with the one studied in HM.

¹⁰The proposed change in the structural parameter must be sufficiently small so that it does not require inadmissible values for the background parameters, e.g., $\sigma_m < 0$.

where

$$\Upsilon^{AOB} = \frac{\beta_1 + (1 - \rho\beta)\beta_3 + \rho\beta\beta_3 f}{\beta_1 + (1 - \rho\beta)\beta_3} \Xi \quad (21)$$

$$\Xi = \frac{(1 - \rho\beta)(1 + \beta_1) + \rho\beta f \left(1 + (\kappa + \beta_2\gamma - \beta_3(\vartheta - D)) \frac{\sigma_m}{s} \left(\frac{f}{\sigma_m}\right)^{\frac{-\sigma}{1-\sigma}}\right)}{\sigma(1 - \rho\beta)(1 + \beta_1) + \rho\beta f \left(1 + (1 - \sigma)(\kappa + \beta_2\gamma - \beta_3(\vartheta - D)) \frac{\sigma_m}{s} \left(\frac{f}{\sigma_m}\right)^{\frac{-\sigma}{1-\sigma}}\right)} \quad (22)$$

$$\tau_\kappa^{AOB} = \frac{(1 + \beta_1)(1 - \rho\beta)}{\beta_1 + (1 - \rho\beta)\beta_3}, \quad \tau_\gamma^{AOB} = \frac{\beta_2(1 - \rho\beta)}{\beta_1 + (1 - \rho\beta)\beta_3} \quad (23)$$

$$FS^{AOB} = \frac{\vartheta - D - \tau_\kappa^{AOB}\kappa - \tau_\gamma^{AOB}\gamma}{\vartheta}. \quad (24)$$

The coefficients β_1 , β_2 and β_3 are defined after equation (11). Notice that FS^{AOB} is not a function of endogenous variables like f .

In Appendix A.2.2 we derive a non-structural decomposition for $\eta_{\Gamma, \vartheta}$ in which:

$$\Upsilon^{AOB} = \frac{\beta_1 + \beta_3(1 - \rho\beta(1 - f))}{\psi a},$$

where

$$\psi \equiv \frac{\rho\beta f + \sigma(1 - \rho\beta)(1 + \beta_1)}{\rho\beta f + (1 - \rho\beta)(1 + \beta_1)}, \quad a = \beta_1 + \left(1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right)\beta_3.$$

Also,

$$\tau_\kappa^{AOB} = \frac{(1 + \beta_1)(1 - \rho\beta) + \beta\rho f + \frac{\rho\beta f(\sigma - 1)}{\psi}}{a}, \quad \tau_\gamma^{AOB} = \frac{\left[1 - \beta\rho(1 - f) + \frac{\rho\beta f(\sigma - 1)}{\psi}\right]\beta_2}{a}. \quad (25)$$

Here β_1, β_2 and β_3 are the same as above. Equations (20) and (24) continue to hold. Note that f now appears in FS^{AOB} via τ_κ^{AOB} and τ_γ^{AOB} (see (25)).

Recall that in discussing Nash bargaining, we discussed a particular experiment in which the analyst wants to study the effect on $\eta_{\Gamma, \vartheta}^{Nash}$ of a structural parameter, keeping f fixed. We argued that it is easy to do so using the non-structural decomposition and is more com-

plicated using the structural decomposition. The same result holds in the case of AOB bargaining. One interesting difference is that in the Nash case the structural and non-structural decompositions are the same when $\kappa = 0$. In the AOB case, the two decompositions are different, even when $\kappa = 0$.

3.2.3 Sticky Wages

When wages are constant ($dw/d\vartheta = 0$) (15) implies:

$$\eta_{\Gamma, \vartheta}^{Sticky} = \Upsilon^{Sticky} \frac{\vartheta}{\vartheta - \tilde{w} - (1 - \rho\beta)\kappa}, \quad \Upsilon^{Sticky} = 1/\sigma.$$

Here, \tilde{w} denotes the constant wage. LS interpret this expression as an *FS*-based decomposition.

3.2.4 Using *FS*-based Decompositions for Cross-model Comparisons

The discussion above focused on the use of the *FS*-based decomposition to analyze how, within a given model, changes in parameter values affect $\eta_{\Gamma, \vartheta}$. LS also argue that *FS*-based decompositions are useful for understanding why different models imply different values for $\eta_{\Gamma, \vartheta}$.

We now briefly summarize their argument. LS consider structural, *FS*-based decompositions for various models, including a version of the AOB model. For those models, Υ has an upper bound of roughly $1/\sigma$. The consensus view in the literature is that σ , the marginal product of unemployment in the matching function, is roughly $1/2$. So, Υ has an upper bound of about 2. Shimer argues that a reasonable data-based target for $\eta_{\Gamma, \vartheta}$ is about 20.¹¹ These observations lead LS to conclude that researchers should look for models in which *FS* is small.

Critically, the version of the AOB model that LS consider differs from ours along three important dimensions. First, the time needed to make an offer or a counteroffer is the same as

¹¹Mortensen and Nagypál (2007) and Pissarides (2009) argue that a better empirical estimate of the target is 12, also a large number.

the time needed to produce the good. Second, the bargaining process can in principle go on forever. Third, to solve their version of the AOB model, LS adopt a particular *approximating assumption*: the probability that a job is destroyed is the same as the probability, δ , that bargaining breaks down. This assumption has the important consequence that workers do not consider the value of their outside option when they decide whether or not to accept a wage offer from a firm.¹²

Ljungqvist and Sargent (forthcoming) consider our version of the AOB model. However, in solving and analyzing the model they adopt the approximating assumption just discussed. Recall that in our AOB model jobs can be destroyed at the beginning of a period, but not within a period. So the approximating assumption requires $\delta \rightarrow 0$. As above, this assumption leads to the extreme implication that workers do not consider the value of their outside option when bargaining.

To see why setting δ to zero in our model has this extreme implication, it is useful to understand how a firm chooses its wage offer in bargaining round, j . Let $w_{j,t}$ denote a firm's wage offer in bargaining round j of period t , where j is an odd number between 1 and $M - 1$. The firm wants to set $w_{j,t}$ as low as possible, subject to not being rejected by the worker. So, $w_{j,t}$ has the property that workers are indifferent between accepting and rejecting the offer:

$$w_{j,t} + \tilde{w}_t^p + A_t = \delta \left[\frac{M - j + 1}{M} D + \tilde{U}_t \right] + (1 - \delta) \left[\frac{D}{M} + w_{j+1,t} + \tilde{w}_t^p + A_t \right]. \quad (26)$$

The term, $w_{j,t} + \tilde{w}_t^p$, represents the present discounted value of the wages associated with accepting the firm's offer, $w_{j,t}$. So, the left-hand side of (26) represents the value to the worker of accepting the firm's offer. Under our period-by-period bargaining assumption, the firm takes \tilde{w}_t^p as given. The right-hand side of the indifference condition is the value to the worker of rejecting an offer and, with probability $1 - \delta$, making a counteroffer in bargaining round, $j + 1$. The firm takes $w_{j+1,t}$ as given and understands that if its current offer is rejected, then

¹²LS also assume $\kappa = 0$ when they work with the AOB model. But, this assumption does not affect our qualitative results for that model.

$w_{j+1,t}$ will be accepted unless bargaining breaks down. The first term on the right-hand side of (26) is δ times the sum of two terms: (i) the pro-rata unemployment benefits received by the worker in the event that bargaining breaks down; and (ii) the continuation value of being unemployed (see (7)). The second term on the right-hand side is $1 - \delta$ times the sum of one sub-period's unemployment benefit plus the value to the worker of an accepted offer.

When $\delta = 0$ the worker indifference condition, (26) reduces to

$$w_{j,t} = \frac{D}{M} + w_{j+1,t}.$$

So, under Ljungqvist and Sargent (forthcoming)'s approximating assumption, workers in our AOB model do not consider the value of their outside option, U_t , when considering a particular wage offer, $w_{j,t}$.

In the special case of $\delta = 0$, the structural and non-structural FS -based decompositions coincide and are given by

$$\eta_{\Gamma,\vartheta} = \frac{1}{\sigma} \frac{\vartheta}{\vartheta - D - (M - 2)\gamma - 2\kappa}. \quad (27)$$

Here, $\Upsilon^{AOB} = 1/\sigma$. So, our AOB model with the approximating assumption, $\delta = 0$, fits the pattern of the models considered by LS in which Υ is bounded above by roughly 2.¹³

While the arguments associated with the LS approximation assumption are elegant, we find the models above that embed this assumption unappealing. First, the implication that workers do not consider their outside option when considering a wage offer seems implausible on a priori grounds. Second, CET find that $\delta = 0$ is empirically implausible. In particular, the 95 percent probability interval associated with CET's estimate of δ easily excludes a value of zero.¹⁴ Third, CET report that the mode of the posterior distribution of δ implies

¹³It is easily verified that $\delta \rightarrow 0$ implies $\beta_1 \rightarrow 0, \beta_2 \rightarrow \frac{M-2}{2}, \beta_3 \rightarrow \frac{1}{2}$. Substitute out for J and Q in (3) using (12) and (8), respectively. Then, take the limit, $\delta \rightarrow 0$. Finally, totally differentiate (3) with respect to Γ and ϑ and rearrange, to obtain (27).

¹⁴CET estimate their model using Bayesian methods. Notably, the lower bound of the posterior probability interval is substantially higher than the lower bound of the prior probability interval for that parameter. So, the data push the distribution of δ away from zero. See Table (4).

that the total job destruction probability, conditional on no resolution to bargaining over a quarter, is meaningfully higher than zero, at roughly 10 percent.¹⁵ Fourth, formula (27), which corresponds to the case $\delta = 0$, provides a strikingly bad approximation to the value of $\eta_{\Gamma,\vartheta}$ implied by CET’s estimated AOB model. The value of $\eta_{\Gamma,\vartheta}$ implied by the mode of the posterior distribution reported by CET is 24.1.¹⁶ In sharp contrast, equation (27) implies a negative value, -28.14 , for $\eta_{\Gamma,\vartheta}$.

3.3 Quantitative Analysis

In this section we provide a quantitative analysis of the role played by the fundamental surplus and wage inertia on labor market volatility in different models.

3.3.1 *FS*–based Decompositions

We begin by discussing the implications of CET’s AOB model for $\eta_{\Gamma,\vartheta}^{AOB}$. Row one of Table 2 displays components of the structural *FS*–based decomposition of $\eta_{\Gamma,\vartheta}^{AOB}$ as well as the wage-inertia decomposition. These are evaluated at the mode of the model parameters’ posterior distribution and steady states (see Tables 1 and 4). Notice that $\eta_{\Gamma,\vartheta}^{AOB}$ is roughly 24, so the model is successful in terms of generating a large elasticity of labor market tightness (and unemployment) to a change in the marginal revenue product of labor. The key factor underlying the model’s success in generating a large value of $\eta_{\Gamma,\vartheta}^{AOB}$ is the high value of Υ^{AOB} , 7.08. The latter exceeds by a factor 3 the upper bound of Υ^{AOB} emphasized by LS for the models that they consider. The inverse fundamental surplus fraction, $1/FS$, plays a smaller role than Υ^{AOB} in generating the large value of $\eta_{\Gamma,\vartheta}^{AOB}$. This finding provides a stark counterexample to the claim in LS that a small surplus fraction is crucial for a model to generate empirically plausible volatility in labor market variables.

Row four of Table 2 displays results for the estimated Nash model (for parameter and

¹⁵Table (4) reports that the mode of δ is 0.0019. CET set $M = 60$, so $\sum_{j=1}^M (1 - \delta)^{j-1} \delta = 1 - (1 - \delta)^M = 0.11$. We thank Lars Ljungqvist for suggesting this way of interpreting the magnitude of δ .

¹⁶The number is generated using equation (20).

Table 1: Steady States and Implied Parameters at Estimated Posterior Mode in Alternating Offer Bargaining and Nash Bargaining Models.

Variable	Alternating Offer Bargaining	Nash Bargaining	Description
ϑ	0.910	0.844	Marginal revenue product
w	0.904	0.837	Real wage
D	0.333	0.734	Unemployment benefits
κ	0.057	0.072	Fixed hiring cost
σ	0.552	0.542	Matching function parameter
ρ	0.900	0.900	Job survival probability
β	0.9968	0.9968	Discount factor
f	0.632	0.632	Job finding rate
η	-	0.674	Worker bargaining power
δ	0.002	-	Prob. of bargaining breakdown
γ	0.009	-	Firm counteroffer costs
Q	0.7	0.7	Vacancy filling probability
M	60	-	Max. bargaining rounds

Notes: For model specifications where particular parameter values are not relevant, the entries in this table are blank. Further calibrated and estimated model parameters are provided in Tables 3 and 4. Source: CET.

steady state values see Tables 1 and 4). The estimated Nash model also generates a value of $\eta_{\Gamma,\vartheta}$ in excess of 20. But, it does so with a low Υ^{Nash} and a high $1/FS^{Nash}$. Taken together, the Nash and AOB model results imply that FS is not a reliable guide to understanding whether a model generates a high value of $\eta_{\Gamma,\vartheta}$.

We now consider the role of the FS -based decomposition in cross-model comparisons of $\eta_{\Gamma,\vartheta}$. As CET point out, the estimated Nash model is able to generate a high elasticity because the estimated value of unemployment benefits D , reported in Table 1, is very high (88% of the steady-state wage). The last row of Table 2 presents results for what we call the *restricted Nash model*. This is a version of the estimated Nash model in which D is a more reasonable 37% of the steady-state wage, a replacement ratio that corresponds to the one in the estimated AOB model.¹⁷ Notice that $\eta_{\Gamma,\vartheta}$ is only about 4 in the restricted Nash model, as opposed to 24 in the estimated AOB model. The lion's share of this six-fold increase in

¹⁷We adjusted η in the restricted Nash model to keep the steady-state job finding rate as in the estimated Nash model.

the elasticity is due to the higher value of Υ in the AOB model. So, the inverse surplus fraction is not a good guide for understanding why one model generates much higher labor market volatility than another.

3.3.2 Wage Inertia

We now turn to the role of wage inertia in generating high values of $\eta_{\Gamma,\vartheta}$. Consider first the estimated AOB and Nash models. According to Table 2, wage inertia fully accounts for the fact that $\eta_{\Gamma,\vartheta}^{AOB}$ is larger than $\eta_{\Gamma,\vartheta}^{Nash}$. The wage inertia effect more than makes up for the fact that the inverse profit rate is lower in the AOB model than in the Nash model.

Although the wage inertia component may appear to be numerically small in the estimated AOB model, it is the percent change in that term that matters for $\eta_{\Gamma,\vartheta}^{AOB}$ when wage inertia changes. This percent change can be large even if the level seems low.

Next, we compare the estimated and restricted Nash models. By construction, the inverse profit rate is the same in these two models. But, $\eta_{\Gamma,\vartheta}$ is much larger in the estimated Nash model. All of the difference is due to the rise in wage inertia as we move from the restricted to the estimated Nash model.

We now consider our two sticky wage models. In the first one we fix the wage rate at its steady state value in the estimated AOB model. The non-bargaining parameters are the same as in the estimated AOB model. The results are reported in row two of Table 2. Notice that $\eta_{\Gamma,\vartheta}$ is now 3837 – orders of magnitude larger than in the estimated AOB model. As one might anticipate from LS, the inverse of the fundamental surplus increases dramatically compared to the estimated AOB model. But the interesting economic question is why? By construction the profit rate is the same in this model and in the estimated AOB model. So, *all* of the change in the fundamental surplus is due to the wage inertia component, which is equal to 1 by construction in the sticky wage model.

Results for the second sticky wage model appear in the third row of Table 2. That model has the same parameters and steady state wage as the estimated Nash model. Notice that the results of the two sticky wage models are similar.

Table 2: Numerical Expressions for the Elasticity of Labor Market Tightness with Respect to Technology and Decomposition of Fundamental Surplus into Profit Rate and Wage Inertia in Estimated Nash, Estimated Alternating Offer Bargaining (AOB), Sticky Wage and Restricted Nash Models.

	Elasticity = $\eta_{r,\vartheta}$	$\Upsilon \times$	1/Fundamental = Surplus	$\frac{1}{\sigma} \times$	1/Profit \times Rate	Wage Inertia Term	Wage
Alternating Offer Bargaining (AOB)							
	24.1	7.08	3.41	1.81	2120.7	0.0063	0.90
Sticky Wage							
AOB Params.	3837.4	1.81	2120.7	1.81	2120.7	1	0.90
Nash Params.	7333.1	1.84	3985.4	1.84	3985.4	1	0.84
Nash Bargaining							
	20.4	2.11	9.65	1.84	3985.4	0.0028	0.84
Restricted Nash							
	3.97	2.11	1.88	1.84	3985.4	0.0005	0.84

Notes: The sticky wage model is evaluated using parameters of the estimated alternating offer bargaining (AOB) model and estimated Nash model. The restricted Nash model is the estimated Nash model with the estimated replacement rate of 88% set to 37%.

Our results may appear to contradict some claims in the literature, according to which wage inertia has at best only a marginal impact on labor market volatility. For example HM reach this conclusion using the Nash model. To make wages inertial in that model HM reduce η to zero. Recall that η governs the bargaining power of workers. Reducing η drives the wage down to the workers' outside option, D , so that $dw/d\vartheta = 0$. When HM induce wage inertia in this way, $\eta_{\Gamma,\vartheta}$ rises by only a negligible amount.

HM's finding depends on their assumption that wages are determined by Nash bargaining and their way of inducing wage inertia. To see this, recall our decomposition of $\eta_{\Gamma,\vartheta}$, given by equation (15). That decomposition is derived using only the firms' free entry condition. It holds regardless of how wages are determined. Equation (15) implies that, other things equal, wage inertia drives $\eta_{\Gamma,\vartheta}$ *up*. But other things aren't equal. HM induce wage rigidity in a way that drives the level of the wage down. According to (15), this level effect drives $\eta_{\Gamma,\vartheta}$ *down*. The net effect of these offsetting forces is the negligible rise in $\eta_{\Gamma,\vartheta}$ that HM find. Clearly HM's results do not imply that other ways of inducing wage inertia in other models of wage determination will have small effects on $\eta_{\Gamma,\vartheta}$.

3.4 Alternative Driving Forces: Discount Rate Shocks

A variety of authors argue that variations in discount rates can be an important source of variations in unemployment. Hall (2017) models these variations as exogenous shocks to the stochastic discount factor in a search and matching model. A comparative steady-state analysis implies that wage inertia is a necessary condition for variations in the discount factor to induce movements in labor market tightness and unemployment.

For simplicity, we assume that the fixed hiring cost κ is equal to zero. In steady state the value of a worker to a firm is equal to

$$J = \vartheta - w + \beta\rho J. \tag{28}$$

In addition, the free entry condition is given by:

$$\frac{s}{\sigma_m} \Gamma^\sigma = J.$$

Substituting out J , totally differentiating and re-arranging yields the elasticity of labor market tightness with respect to the discount rate, $\eta_{\Gamma,\beta}$:

$$\eta_{\Gamma,\beta} = \frac{\frac{d\Gamma}{\Gamma}}{\frac{d\beta}{\beta}} = \frac{\beta}{\sigma\vartheta} \underbrace{\frac{\vartheta}{\vartheta - w}}_{1/(\text{Profit rate})} \underbrace{\left[\rho J - \frac{dw}{d\beta} \right]}_{\text{Wage inertia term}}. \quad (29)$$

To interpret this expression suppose that the steady-state value of ϑ is unaffected by the value of β . This is the case in standard search and matching models. If $\frac{dw}{d\beta} = \rho J$, then the right hand side of equation (28) is unaffected by a change in β . In equation (28), the term $\beta\rho J$ is the discounted expected future value of a worker to the firm. If the discount rate change induces a change in w that is exactly equal to $\beta\rho J$, then the current value of a worker to the firm, J , is unchanged. Then the firm has no incentive to post more vacancies and the unemployment rate is not affected by a change in β . So, equation (29) shows that for $\eta_{\Gamma,\beta}$ to be positive, wages must be inertial in the sense that $\frac{dw}{d\beta} < \rho J$. Of course the actual value of $\frac{dw}{d\beta}$ is determined by the wage determination mechanism in the model, e.g. sticky wages, Nash bargaining or AOB.

4 Dynamic Analysis

In this section we consider the dynamic impact of wage inertia in generating labor market volatility in search and matching models. We make two key points. First, comparative steady-state analysis can be deeply misleading about the dynamic behavior of these models. Models which have identical fundamental surpluses can exhibit very different dynamic response functions. In these cases, the fundamental surplus is uninformative about the question of interest. Second, even conditioning on a low inverse fundamental surplus, wage

inertia greatly magnifies labor market fluctuations in empirically plausible versions of search and matching models. In the first subsection we consider a simple dynamic example. In the second subsection, we analyze the impact of wage inertia in estimated DSGE models conditional on a given value of the fundamental surplus.

4.1 Wage Inertia: A Simple Dynamic Example

In this section we consider the role of wage inertia using a simple dynamic example. Suppose that the equilibrium wage rate is given by the following simple inertial wage rule:

$$w_t = \phi \vartheta_t - \gamma(1 - \phi)(\vartheta_t - \vartheta) \quad (30)$$

where $\phi > 0$ and $D \leq w_t \leq \vartheta_t$. In addition, assume that $0 < \gamma(1 - \phi) < \phi$. The latter assumption implies that a rise in ϑ_t generates a positive response in w_t . The dynamics of w_t depend on γ and ϕ . Note that γ has no impact on the steady-state value of w_t . We can think of this wage rule as being a variant of the Hall (2005) wage norm, as long as $D \leq w_t \leq \vartheta_t$ so that the firm and worker each have an incentive to produce. A larger value of γ means more inertia in w_t : a given shock to ϑ_t is associated with a smaller change in w_t . For simplicity, in this subsection, we abstract from the hiring cost, i.e. $\kappa = 0$.

We seek to evaluate the generic formula for the steady-state elasticity of labor market tightness with respect to technology, i.e. equation (15), for the simple wage rule (30). Note that in steady state $\frac{dw}{d\vartheta} = \phi$. Inserting the latter result together with the steady-state version of equation (30) into equation (15) yields the following expression for the steady-state elasticity:

$$\eta_{\Gamma, \vartheta}^{steady} = \frac{1}{\sigma}.$$

Note that the steady-state elasticity is *independent* of the fundamental surplus. Moreover, the elasticity is identical to one implied by the Nash bargaining model with $\eta = D = 0$.

To derive the actual dynamics of the simple model, we must take a stand on the law of

motion for ϑ_t . To this end, we assume:

$$\vartheta_t = (1 - v)\vartheta + v\vartheta_{t-1} + \varepsilon_t. \quad (31)$$

where ε_t is uncorrelated over time and uncorrelated with ϑ_{t-u} for $u > 0$.

In Appendix B we show that the equilibrium solution for the value of a worker to the firm, J_t , is given by:

$$J_t = \delta_0 + \delta_1\vartheta_t \quad (32)$$

where

$$\delta_0 = -\frac{\gamma(1-\phi)\vartheta - \beta\rho\delta_1(1-v)\vartheta}{1-\beta\rho} \text{ and } \delta_1 = \frac{(1-\phi)(1+\gamma)}{1-\beta\rho v}.$$

Combining the free-entry condition (13) with the solution for J_t we obtain:

$$\frac{s}{\sigma_m}\Gamma_t^\sigma = J_t = \delta_0 + \delta_1\vartheta_t.$$

Totally differentiating and rearranging yields the following expression for the dynamic elasticity of labor market tightness with respect to the marginal revenue product:

$$\eta_{\Gamma,\vartheta}^{dynamic} \equiv \frac{\frac{d\Gamma_t}{\Gamma_t}}{\frac{d\vartheta_t}{\vartheta}} = \frac{1}{\sigma} \frac{(1-\phi)(1+\gamma)}{(1-\phi)(1+\gamma) - \frac{1-\beta\rho v}{1-\beta\rho} \left(\gamma(1-\phi) - \frac{\beta\rho(1-\phi)(1+\gamma)(1-v)}{1-\beta\rho v} \right)}. \quad (33)$$

Consider v close to unity, so that the marginal revenue product of a worker is close to a random walk. The limiting case implies that a shock to ϑ_t is permanent which mimics the permanent shift in technology considered in the comparative steady-state analyses in section 3. For $v \rightarrow 1$, the dynamic elasticity (equation 33) becomes:

$$\eta_{\Gamma,\vartheta}^{dynamic} \simeq \frac{1}{\sigma} (1+\gamma) = \eta_{\Gamma,\vartheta}^{steady} (1+\gamma).$$

Note that if $\gamma > 0$ then $\eta_{\Gamma,\vartheta}^{dynamic} > \eta_{\Gamma,\vartheta}^{steady}$. The restriction on the parameters $\phi > \gamma(1-\phi)$ discussed above implies that $\phi > \frac{\gamma}{1+\gamma}$ or $\gamma < \frac{\phi}{(1-\phi)}$. By making γ sufficiently large, i.e. by

making wages sufficiently inertial, $\eta_{\Gamma,\vartheta}^{dynamic}$ can be made *arbitrarily* large, even though $\eta_{\Gamma,\vartheta}^{steady}$ is always simply equal to $1/\sigma$. That is, the more inertia there is in wages, the larger is the impact of a shock to ϑ_t on labor market tightness and unemployment. Clearly, in this example comparative steady-state analysis is very misleading about the dynamic effects of a persistent shock to technology. Also notice that the parameter γ which plays a central role in $\eta_{\Gamma,\vartheta}^{dynamic}$ is completely absent from the fundamental surplus formula. So the latter is not helpful for anticipating the results of a dynamic analysis of a shock to the marginal revenue product of labor.

4.2 Wage Inertia in an Estimated Dynamic Search and Matching Model

In this subsection we consider the model of CET who embed the labor market model of subsection 2 into a medium-sized DSGE NK model.

4.2.1 Households

The economy is populated by a large number of identical households. The representative household has a unit measure of workers which it supplies inelastically to the labor market. We denote the fraction of employed workers in the representative household in period t by l_t . An employed worker earns the nominal wage rate, W_t and an unemployed worker receives D_t goods in government-provided unemployment compensation. Each worker has the same concave preferences over consumption. Households provide perfect consumption insurance to their members, so that each worker receives the same level of consumption, C_t . The preferences of the representative household are the equally-weighted average of the preferences of its workers:

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln(C_t - bC_{t-1})$$

where the parameter $0 \leq b < 1$ controls the degree of habit formation in preferences. The representative household's budget constraint is:

$$P_t C_t + P_{I,t} I_t + B_{t+1} \leq (R_{K,t} u_t^K - a(u_t^K) P_{I,t}) K_t + (1 - l_t) P_t D_t + W_t l_t + R_{t-1} B_t - T_t. \quad (34)$$

Here, T_t denotes lump sum taxes net of profits, P_t denotes the price of consumption goods, $P_{I,t}$ denotes the price of investment goods, B_{t+1} denotes one period risk-free bonds purchased in period t with gross return, R_t and I_t denotes the quantity of investment goods. The object $R_{K,t}$ denotes the rental rate of capital services, K_t denotes the household's beginning of period t stock of capital, $a(u_t^K)$ denotes the cost, in units of investment goods, of the capital utilization rate, u_t^K and $u_t^K K_t$ denotes the household's period t supply of capital services. We discuss details about the capital utilization cost function in subsection 4.2.4. All prices, taxes and profits in equation (34) are in nominal terms. The representative household's stock of capital evolves as follows:

$$K_{t+1} = (1 - \delta_K) K_t + (1 - S(I_t/I_{t-1})) I_t$$

where δ_K denotes the depreciation rate and $S(I_t/I_{t-1})$ are convex investment adjustment costs. We discuss details about the latter in subsection 4.2.4.

4.2.2 Final Goods Producers

A final homogeneous good, Y_t , is produced by competitive and identical firms using the following technology:

$$Y_t = \left(\int_0^1 (Y_{j,t})^{\frac{1}{\lambda}} dj \right)^\lambda \quad (35)$$

where $\lambda > 1$. The representative firm chooses specialized inputs, $Y_{j,t}$, to maximize profits:

$$P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj,$$

subject to the production function (35). Output, Y_t can be used to produce either consumption goods or investment goods. The production of the latter uses a linear technology in which one unit of Y_t is transformed into Ψ_t units of I_t .

4.2.3 Retailers and Wholesalers

The j^{th} input good in (35) is produced by a retailer, with production function:

$$Y_{j,t} = k_{j,t}^\alpha (z_t h_{j,t})^{1-\alpha} - \varphi_t.$$

Here $k_{j,t}$ denotes the total amount of capital services purchased by firm j and φ_t represents a fixed cost of production which evolves according to an exogenous stochastic process that is consistent with balanced growth. We discuss details about the latter in subsection 4.2.4. The variable z_t is a technology shock and $h_{j,t}$ is the quantity of an intermediate good purchased by the j^{th} retailer. This good is purchased in competitive markets at the price P_t^h from a wholesaler discussed in subsection 2. To produce in period t , the retailer must borrow $P_t^h h_{j,t}$ at the gross nominal interest rate, R_t . The retailer repays the loan at the end of period t after receiving sales revenues. The j^{th} retailer sets its price, $P_{j,t}$, subject to the demand curve for its good, and a Calvo sticky price friction. With probability $1 - \xi$, the retailer can re-optimize his price and with probability ξ , $P_{j,t} = P_{j,t-1}$.¹⁸

Wholesaler firms produce the intermediate good using labor which has a fixed marginal productivity of unity. The real price of the intermediate good is P_t^h/P_t where P_t^h and P_t are the nominal prices of the intermediate and final good, respectively. Let $\vartheta_t \equiv P_t^h/P_t$ which is the marginal revenue of wholesalers. Then the analysis of the labor market discussed in sections 2 and 3 obtains directly with the understanding that all of the shocks to the economy, including monetary policy, impact ϑ_t , thereby affecting firms incentives to post vacancies.

¹⁸We assume that producers make their price setting decision before observing the current period realization of the monetary policy shock, but after the time t technology shocks.

4.2.4 Monetary Policy and Functional Forms

We adopt the following specification for monetary policy:

$$\ln(R_t/R) = \rho_R \ln(R_{t-1}/R) + (1 - \rho_R)[r_\pi \ln(\pi_t/\pi) + r_y \ln(GDP_t/GDP_t^*)] + \sigma_R \varepsilon_{R,t}.$$

Here, π denotes the monetary authority's target inflation rate. The monetary policy shock, $\varepsilon_{R,t}$, has unit variance and zero mean. Also, R is the steady-state value of R_t . The variable, GDP_t , denotes Gross Domestic Product (GDP), which equals $C_t + I_t/\Psi_t + G_t$ and GDP_t^* denotes the value of GDP_t along the non-stochastic steady-state growth path. We assume that the growth rate of neutral technological progress, $\ln\mu_{z,t} \equiv \ln(z_t/z_{t-1})$, is i.i.d. and that the growth rate of investment-specific technological progress, $\ln\mu_{\Psi,t} \equiv \ln(\Psi_t/\Psi_{t-1})$, follows a stochastic first order autoregressive process.

The sources of growth in the model are neutral and investment-specific technological progress because Ψ_t and z_t grow over time. Let $\Phi_t = \Psi_t^{(\alpha/(1-\alpha))} z_t$ denote the composite level of technology. To guarantee balanced growth in the non-stochastic steady state, we require that each element in $[\varphi_t, s_t, \kappa_t, \gamma_t, G_t, D_t]$ grows at the same rate as Φ_t in steady state. To this end, we adopt the following specification:

$$[\varphi_t, s_t, \kappa_t, \gamma_t, G_t, D_t]' = [\varphi, s, \kappa, \gamma, G, D]' \Omega_t.$$

Here, Ω_t is a diagonal matrix with the i^{th} diagonal element, Ω_t^i , where $i \in \varphi, s, \kappa, \gamma, G, D$ and $\Omega_t^i = \Phi_{t-1}^{\theta_i} (\Omega_{t-1}^i)^{1-\theta_i}$.¹⁹

The cost of adjusting investment takes the form:

$$S(I_t/I_{t-1}) = 0.5(\exp[\sqrt{S''}(I_t/I_{t-1} - \mu \times \mu_\Psi)] + \exp[-\sqrt{S''}(I_t/I_{t-1} - \mu \times \mu_\Psi)]) - 1.$$

¹⁹With this specification, Ω_t^i/Φ_{t-1} converges to a constant in non-stochastic steady state, for each i . When θ_i is close to zero, Ω_t^i is virtually unresponsive in the short-run to an innovation in either of the two technology shocks, a feature that we find attractive on a priori grounds.

Here, μ and μ_Ψ denote the unconditional growth rates of Φ_t and Ψ_t . Also, S'' denotes the second derivative of $S(\cdot)$, evaluated in steady state. The cost associated with setting capacity utilization is given by:

$$a(u_t^K) = \sigma_a \sigma_b (u_t^K)^2 / 2 + \sigma_b (1 - \sigma_a) u_t^K + \sigma_b (\sigma_a / 2 - 1)$$

where σ_a and σ_b are positive scalars. For a given value of σ_a , we select σ_b so that the steady-state value of u_t^K is unity.

Finally, we refer the reader to CET and CET's technical appendix for the market clearing conditions, the definition of equilibrium and the set of dynamic and steady-state equilibrium equations.

4.2.5 The Role of Wage Inertia in the Estimated Model

CET estimate the medium-sized DSGE NK model discussed above for various wage bargaining environments. Their estimation strategy is a Bayesian variant of the strategy in Christiano, Eichenbaum and Evans (2005) that minimizes the distance between the dynamic responses to three shocks in the model and the analog objects in the data. The shocks used by CET include a shock to monetary policy, a neutral technology shock, and an investment-specific technology shock. The dynamic responses to those shocks are obtained using an identified VAR for post-war quarterly U.S. times series that include key labor market variables, see CET for further details.

Here we focus on the versions of the model in which wages are determined by Nash and AOB bargaining. Table 3 reports the values of parameters that CET set a priori. Table 4 reports the mean and 95 percent probability intervals for the priors and posteriors of the estimated parameters in the Nash and AOB bargaining models. Table 1 reports the implied steady states.

Note that the estimated values of the replacement ratio D/w in the Nash and AOB models are 0.88 and 0.37, respectively. CET argue that the estimated value of the replace-

Table 3: Non-Estimated Parameters and Calibrated Variables.

Parameter	Value	Description
<i>Panel A: Parameters</i>		
δ_K	0.025	Depreciation rate of physical capital
β	0.9968	Discount factor
ρ	0.9	Job survival probability
M	60	Max. bargaining rounds per quarter (AOB)
$400\ln(\mu)$	1.7	Annual output per capita growth rate
$400\ln(\mu \cdot \mu_\Psi)$	2.9	Annual investment per capita growth rate
<i>Panel B: Steady State Values</i>		
$400(\pi - 1)$	2.5	Annual net inflation rate
<i>profits</i>	0	Intermediate goods producers profits
Q	0.7	Vacancy filling rate
u	0.055	Unemployment rate
G/Y	0.2	Government cons. to gross output ratio

Notes: Table based on CET.

ment ratio of 0.88 in the Nash model, i.e. steady-state unemployment benefits amount to 88% percent of the steady-state wage, is strongly at odds with the micro evidence. Given that in the Nash model, the posterior mode of D/w is in the tail of the prior distribution, the marginal likelihood of the Nash model is about 14 log points lower than the AOB model. We also find it useful to consider the restricted Nash model defined in section 3, i.e. we set D/w to 0.37, keeping all other estimated parameters in the estimated Nash model unchanged.

The solid thin back lines in rows 1 and 2 of Figure 1 display VAR-based estimates of the dynamic responses of the unemployment rate, the real wage rate and the inflation rate to a monetary policy shock and a neutral technology shock.²⁰ The grey bands correspond to 95% confidence intervals. The blue lines correspond to the dynamic response functions of the Nash model, evaluated at the mode of the parameter estimates. Notice that the model does a good job of matching the dynamic response of unemployment to both shocks. There is no Shimer puzzle here.

This result depends critically on the fact that the model does a reasonably good job of

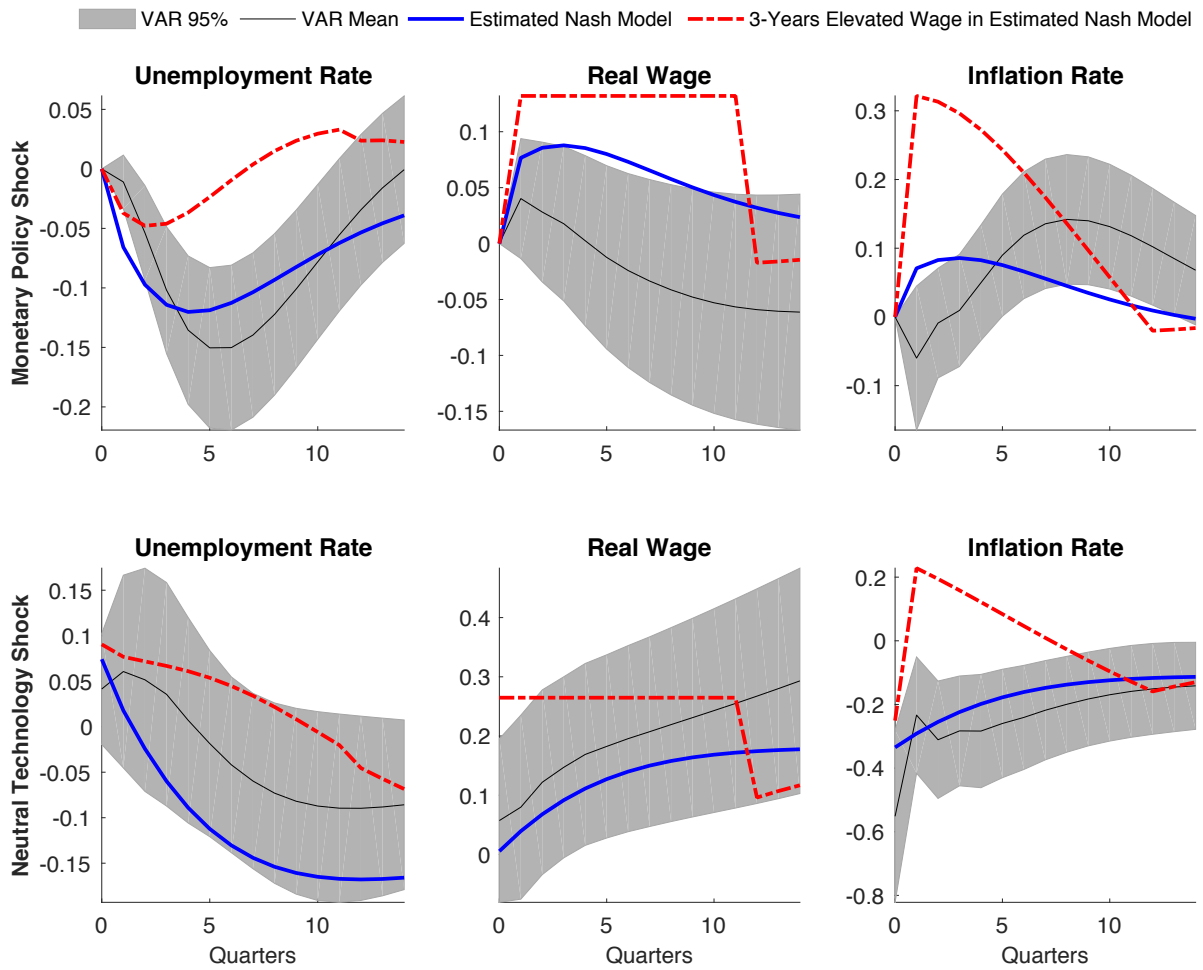
²⁰For brevity and without loss of generality, we exclude the responses to the investment-specific technology shock.

Table 4: Priors and Posteriors of Parameters in Estimated Bargaining Models.

	Alternating Offer Bargaining		Nash Bargaining
	Prior Distribution $\mathcal{D}, \text{Mode}, [2.5-97.5\%]$	Posterior Distribution Mode , [2.5-97.5%]	
<i>Price Setting Parameters</i>			
Price Stickiness, ξ	$\mathcal{B}, \mathbf{0.68}, [0.45 \ 0.84]$	$\mathbf{0.75}, [0.69 \ 0.78]$	$\mathbf{0.74}, [0.69 \ 0.79]$
Price Markup, λ	$\mathcal{G}, \mathbf{1.19}, [1.11 \ 1.31]$	$\mathbf{1.42}, [1.33 \ 1.51]$	$\mathbf{1.43}, [1.35 \ 1.52]$
<i>Monetary Authority Parameters</i>			
Taylor Rule: Smooth., ρ_R	$\mathcal{B}, \mathbf{0.76}, [0.37 \ 0.94]$	$\mathbf{0.84}, [0.81 \ 0.87]$	$\mathbf{0.84}, [0.82 \ 0.87]$
Taylor Rule: Inflation, r_π	$\mathcal{G}, \mathbf{1.69}, [1.42 \ 2.00]$	$\mathbf{1.38}, [1.21 \ 1.65]$	$\mathbf{1.38}, [1.23 \ 1.69]$
Taylor Rule: GDP, r_y	$\mathcal{G}, \mathbf{0.08}, [0.03 \ 0.22]$	$\mathbf{0.03}, [0.01 \ 0.07]$	$\mathbf{0.04}, [0.02 \ 0.08]$
<i>Preferences and Technology Parameters</i>			
Consumption Habit, b	$\mathcal{B}, \mathbf{0.50}, [0.21 \ 0.79]$	$\mathbf{0.80}, [0.78 \ 0.84]$	$\mathbf{0.81}, [0.78 \ 0.84]$
Cap. Util. Adj. Cost, σ_a	$\mathcal{G}, \mathbf{0.32}, [0.09 \ 1.23]$	$\mathbf{0.11}, [0.04 \ 0.30]$	$\mathbf{0.18}, [0.05 \ 0.32]$
Investment Adj. Cost, S''	$\mathcal{G}, \mathbf{7.50}, [4.57 \ 12.4]$	$\mathbf{15.7}, [11.0 \ 19.6]$	$\mathbf{15.2}, [10.7 \ 19.0]$
Capital Share, α	$\mathcal{B}, \mathbf{0.33}, [0.28 \ 0.38]$	$\mathbf{0.26}, [0.20 \ 0.27]$	$\mathbf{0.23}, [0.21 \ 0.27]$
Technology Diffusion, θ	$\mathcal{B}, \mathbf{0.50}, [0.13 \ 0.87]$	$\mathbf{0.05}, [0.02 \ 0.07]$	$\mathbf{0.03}, [0.01 \ 0.05]$
<i>Labor Market Parameters</i>			
Prob. Barg. Breakup, 100δ	$\mathcal{G}, \mathbf{0.18}, [0.04 \ 1.53]$	$\mathbf{0.19}, [0.09 \ 0.37]$	-
Replacement Ratio, D/w	$\mathcal{B}, \mathbf{0.39}, [0.21 \ 0.60]$	$\mathbf{0.37}, [0.22 \ 0.63]$	$\mathbf{0.88}, [0.85 \ 0.90]$
Hiring Cost/Output, $100\eta_h$	$\mathcal{G}, \mathbf{0.91}, [0.50 \ 1.67]$	$\mathbf{0.46}, [0.24 \ 0.84]$	$\mathbf{0.64}, [0.34 \ 1.07]$
Search Cost/Output, $100\eta_s$	$\mathcal{G}, \mathbf{0.05}, [0.01 \ 0.28]$	$\mathbf{0.03}, [0.00 \ 0.12]$	$\mathbf{0.02}, [0.00 \ 0.09]$
Match. Fun. Parameter, σ	$\mathcal{B}, \mathbf{0.50}, [0.31 \ 0.69]$	$\mathbf{0.55}, [0.47 \ 0.61]$	$\mathbf{0.54}, [0.47 \ 0.61]$
<i>Exogenous Processes Parameters</i>			
Std. Mon. Pol., $400\sigma_R$	$\mathcal{G}, \mathbf{0.65}, [0.56 \ 0.75]$	$\mathbf{0.63}, [0.57 \ 0.70]$	$\mathbf{0.63}, [0.58 \ 0.70]$
Std. Neut. Tech., $100\sigma_{\mu_z}$	$\mathcal{G}, \mathbf{0.08}, [0.03 \ 0.22]$	$\mathbf{0.16}, [0.11 \ 0.19]$	$\mathbf{0.14}, [0.11 \ 0.18]$
Std. Inv. Tech., $100\sigma_\Psi$	$\mathcal{G}, \mathbf{0.08}, [0.03 \ 0.22]$	$\mathbf{0.12}, [0.08 \ 0.15]$	$\mathbf{0.11}, [0.08 \ 0.16]$
AR(1) Invest. Tech., ρ_Ψ	$\mathcal{B}, \mathbf{0.75}, [0.53 \ 0.92]$	$\mathbf{0.72}, [0.60 \ 0.85]$	$\mathbf{0.74}, [0.59 \ 0.83]$
<i>Memo Item</i>			
Log Marginal Likelihood:		286.7	272.9

Notes: Posterior mode and parameter distributions are based on a standard MCMC algorithm with a total of 10 million draws (11 chains, 50 percent of draws used for burn-in, draw acceptance rates about 0.24). \mathcal{B} and \mathcal{G} denote beta and gamma distributions, respectively. Source: CET.

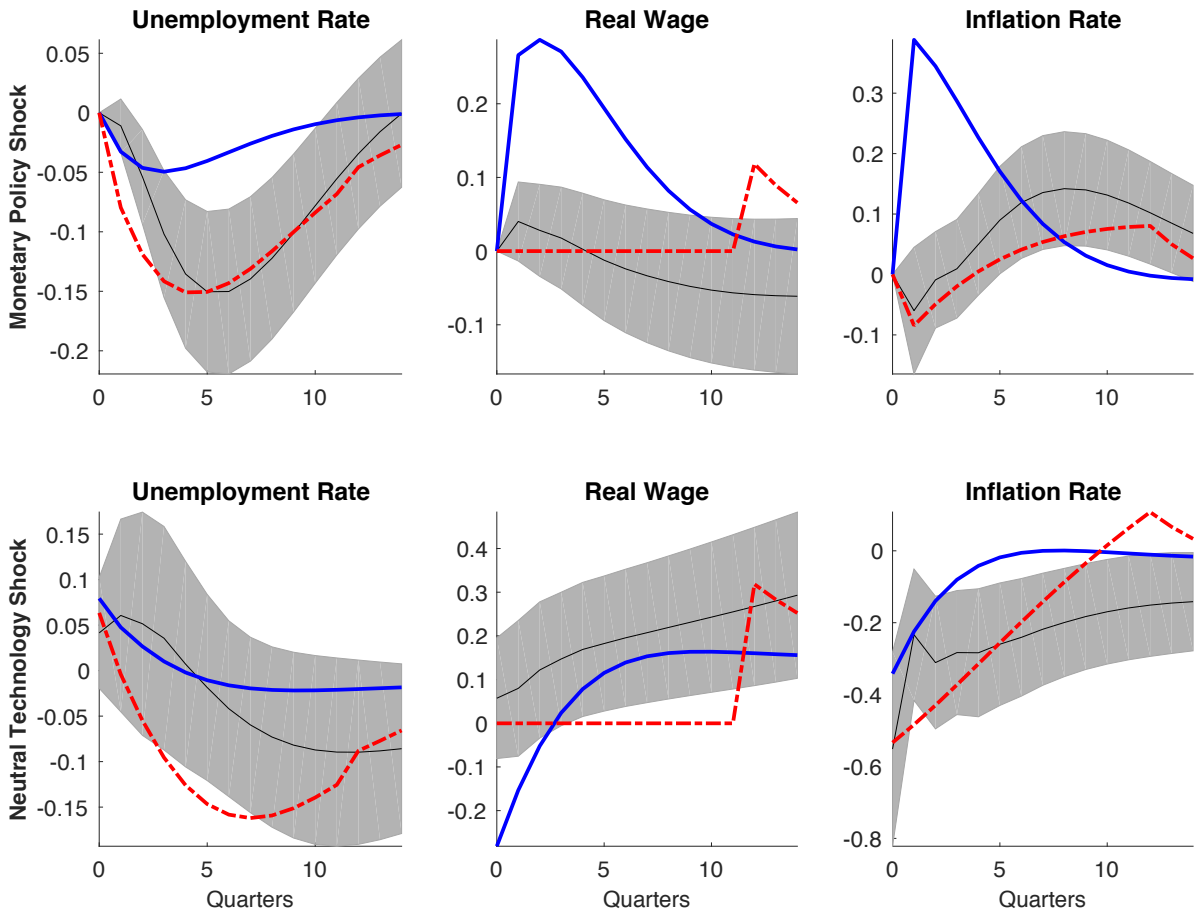
Figure 1: Impulse Responses in Estimated Nash Model.



Notes: unemployment in percentage points, real wage in percent deviations and inflation in annualized percentage points.

Figure 2: Impulse Responses in Restricted Nash Model.

■ VAR 95% — VAR Mean — Restricted Nash Model (D/w=0.37) - - 3-Years Constant Wage in Restricted Nash Model (D/w=0.37)



Notes: unemployment in percentage points, real wage in percent deviations and inflation in annualized percentage points.

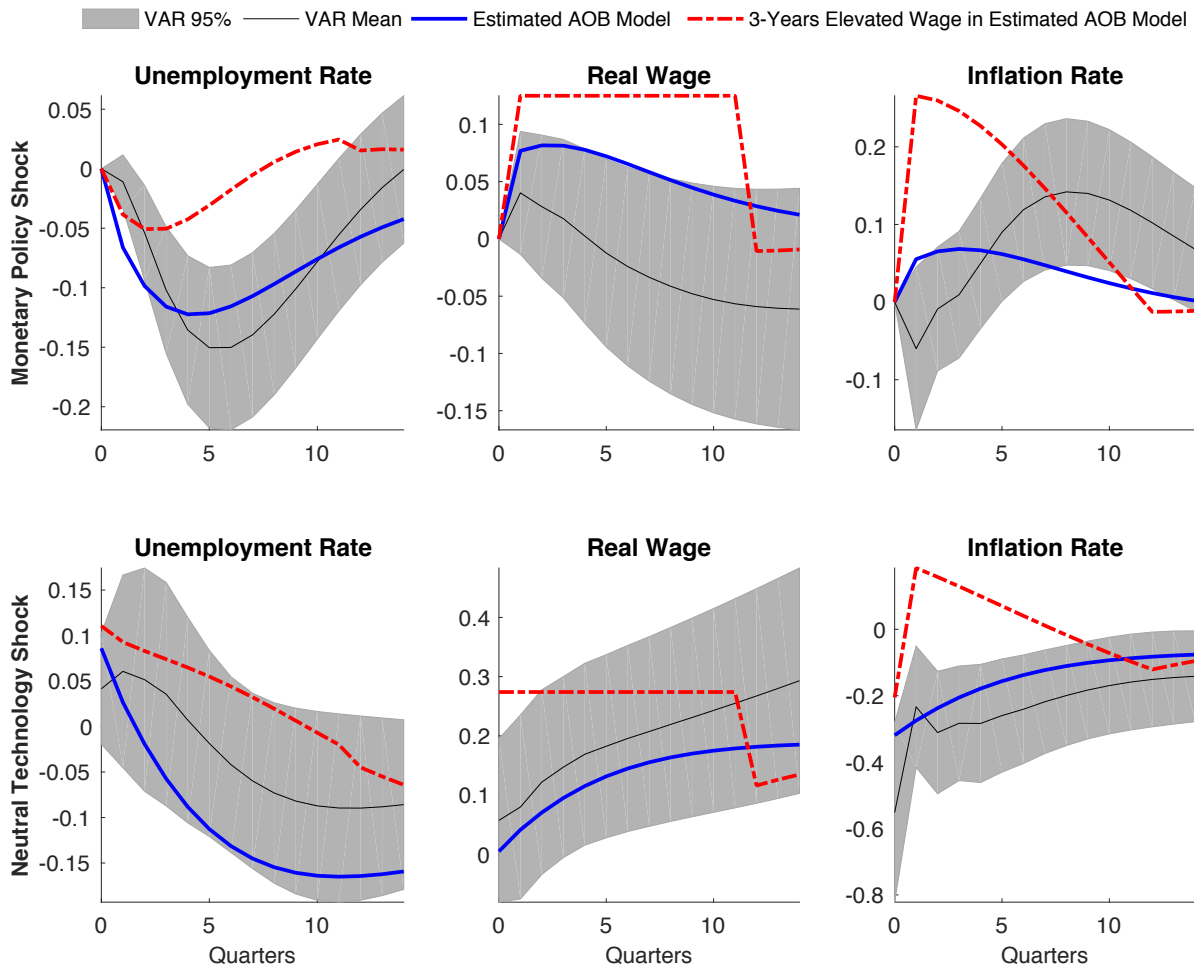
matching the inertial response of real wages to the shocks. To substantiate this claim we consider the following experiment. We impose on the model the assumption that real wages go up, in period 1, by 50% more than their peak response in the estimated Nash model and then stay at that level for three consecutive years. After the three years the economy returns to the wage rule implied by the restricted Nash-sharing rule. The dotted red lines in rows 1 and 2 of Figure 1 display the implied impulse response functions. The response functions are calculated using the Fair and Taylor (1983) perfect foresight simulation method in which households and firms know, at the time of the shock, the assumed wage path in the experiment, see Christiano, Eichenbaum and Trabandt (2015) for a detailed description.

The key result here is that for both shocks the response of unemployment is much smaller, in absolute value, than in the estimated model. For the monetary policy shock, the response is counterfactually small, even taking VAR sampling uncertainty into account: without wage inertia, there is a Shimer-like puzzle. Note also that by construction the estimated Nash model and the estimated Nash model with the three-years elevated wage have the same steady state so the fundamental surplus is exactly the same in both models. So, the fundamental surplus contains no information about the dynamic responses of the two economies being compared.

Rows 1 and 2 of Figure 2 reports the results for the restricted Nash model. From the solid blue lines we see that real wages respond by a counterfactually large amount to both shocks. Not surprisingly, the response of unemployment in this model is much smaller than in the estimated Nash model. Again, without wage inertia, there is a Shimer-like puzzle. The red dashed lines display the impulse response functions if we hold real wages fixed for three consecutive years and then let wages be determined by the Nash-sharing rule afterwards. With inertial wages, unemployment now responds by much more. So, wage inertia allows a Nash model with a low replacement ratio to overcome the Shimer-like puzzle. Note again, that the two models being compared have identical steady states and fundamental surpluses.

Finally rows 1 and 2 of Figure 3 repeat the experiment presented in Figure 1 for the estimated AOB model. The results are consistent with those emerging from the estimated

Figure 3: Impulse Responses in Estimated Alternating Offer Bargaining (AOB) Model.



Notes: unemployment in percentage points, real wage in percent deviations and inflation in annualized percentage points.

Nash model. Again, wage inertia plays a pivotal role for the model's ability to account for the dynamic response of unemployment to shocks. And again the fundamental surplus is fundamentally uninformative about the model's dynamic impulse response functions.

The results in this section highlight the potential danger of using comparative steady-state analyses as a substitute for dynamic analyses. This warning is particularly clear in the case of monetary policy shocks which have no impact on the steady state. Nevertheless, different models of wage determination imply very different responses of unemployment to a monetary policy shock.

As we noted in the introduction, there do exist assumptions under which comparative steady-state analyses do a reasonably good job of mimicking dynamics, i.e. shocks are close to a random walk and there are no important state variables. These assumptions are not satisfied in CET. Another recent example of note is the competitive search model developed in Kehoe et al. (2019) which adopts the specification of preferences developed by Campbell and Cochrane (1999). That specification generates important additional sources of dynamics while leaving no trace in steady state. A comparative steady-state analysis sheds no light on the interesting dynamic properties of their model.

We conclude that one should be skeptical about the use of comparative steady-state analysis to understand the dynamics of empirically plausible models.

5 Conclusion

Wage inertia is widely recognized as playing an important role in business cycle fluctuations. An important exception to this view is the search and matching literature where the role of wage inertia is the subject of an ongoing debate. In this paper we argued that wage inertia does in fact play a crucial role in allowing variants of standard search and matching models to account for the large countercyclical response of unemployment to shocks. We made this argument using comparative steady state and dynamic analyses of estimated DSGE models. While the former mode of analysis is widely used and can generate useful insights, it can also

be very misleading in the present context. Specifically, dynamic models with the same steady state and fundamental surplus can exhibit very different dynamic responses of unemployment to shocks. In the models that we investigate, large dynamic responses of unemployment to shocks always coincide with an inertial response of wages. The basic intuition is that if wages increase too much after a change in the marginal revenue product of labor, firms have little incentive to invest in new jobs.

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Appendix

A Steady-State Elasticity Formulae

A.1 Nash Bargaining

In this appendix we derive the structural and non-structural fundamental surplus based decompositions of $\eta_{\Gamma, \vartheta}^{Nash}$.

A.1.1 Structural Fundamental Surplus Based Decomposition

Recall the set of equilibrium labor market equations with the Nash-sharing rule:

$$\begin{aligned} S_t^w &= w_t - D + \rho\beta (1 - \sigma_m \Gamma_{t+1}^{1-\sigma}) E_t S_{t+1}^w \\ J_t &= \vartheta_t - w_t + \rho\beta E_t J_{t+1} \\ \frac{s}{\sigma_m} \Gamma_t^\sigma &= J_t - \kappa \\ \eta J_t &= (1 - \eta) S_t^w \end{aligned}$$

Substitute out for S_t^w and $E_t S_{t+1}^w$ in the first equation using the last equation. Then add the first and second equation to get rid of the wage:

$$\begin{aligned} \frac{1}{1 - \eta} J_t &= \vartheta_t - D + \rho\beta E_t J_{t+1} + \rho\beta \frac{\eta}{1 - \eta} (1 - \sigma_m E_t \Gamma_{t+1}^{1-\sigma}) E_t J_{t+1} \\ \frac{s}{\sigma_m} \Gamma_t^\sigma &= J_t - \kappa \end{aligned}$$

Substitute out for J_t and $E_t J_{t+1}$ in first equation using the last equation:

$$\frac{1}{1 - \eta} \left(\frac{s}{\sigma_m} \Gamma_t^\sigma + \kappa \right) = \vartheta_t - D + \rho\beta E_t \left(\frac{s}{\sigma_m} \Gamma_{t+1}^\sigma + \kappa \right) + \rho\beta \frac{\eta}{1 - \eta} (1 - \sigma_m E_t \Gamma_{t+1}^{1-\sigma}) E_t \left(\frac{s}{\sigma_m} \Gamma_{t+1}^\sigma + \kappa \right)$$

Impose steady state, multiply out the last term on the right hand side, collect terms and re-arrange:

$$(1 - \rho\beta) \frac{s}{\sigma_m} \Gamma^\sigma + \rho\beta\eta s \Gamma + \rho\beta\eta\sigma_m \Gamma^{1-\sigma} \kappa = (1 - \eta) (\vartheta - D) - (1 - \rho\beta) \kappa \quad (36)$$

Totally differentiating:

$$\sigma (1 - \beta\rho) \frac{s\Gamma^\sigma}{\sigma_m} \frac{d\Gamma}{\Gamma} + \eta\rho\beta\Gamma s \frac{d\Gamma}{\Gamma} + (1 - \sigma) \kappa \eta \rho \beta \sigma_m \Gamma^{1-\sigma} \frac{d\Gamma}{\Gamma} = (1 - \eta) \vartheta \frac{d\vartheta}{\vartheta}$$

Rearranging:

$$\eta_{\Gamma, \vartheta}^{Nash} = \frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} = (1 - \eta) \frac{\vartheta}{\left(\sigma (1 - \beta\rho) \frac{\Gamma^\sigma}{\sigma_m} + \eta\rho\beta\Gamma + (1 - \sigma) \frac{\kappa}{s} \eta \rho \beta \sigma_m \Gamma^{1-\sigma} \right) s} \quad (37)$$

Rewrite equation (36) as follows:

$$s = \frac{(1 - \eta) (\vartheta - D) - (1 - \rho\beta) \kappa}{(1 - \rho\beta) \frac{1}{\sigma_m} \Gamma^\sigma + \rho\beta\eta\Gamma + \rho\beta\eta\sigma_m \Gamma^{1-\sigma} \frac{\kappa}{s}}$$

Substitute for s in the elasticity formula (37) and rearrange:

$$\eta_{\Gamma, \vartheta}^{Nash} = \frac{(1 - \rho\beta) \frac{1}{\sigma_m} \Gamma^\sigma + \rho\beta\eta\Gamma + \rho\beta\eta\sigma_m \Gamma^{1-\sigma} \frac{\kappa}{s}}{\sigma (1 - \beta\rho) \frac{\Gamma^\sigma}{\sigma_m} + \eta\rho\beta\Gamma + (1 - \sigma) \frac{\kappa}{s} \eta \rho \beta \sigma_m \Gamma^{1-\sigma}} \frac{\vartheta}{\vartheta - D - \frac{1 - \rho\beta}{1 - \eta} \kappa}$$

Recall that $f = \sigma_m \Gamma^{1-\sigma}$ and $Q = \sigma_m \Gamma^{-\sigma}$ so that:

$$\eta_{\Gamma, \vartheta}^{Nash} = \frac{1 - \rho\beta + \rho\beta\eta f \left(1 + \frac{\kappa}{s} Q\right)}{\sigma (1 - \beta\rho) + \eta\rho\beta f \left(1 + (1 - \sigma) \frac{\kappa}{s} Q\right)} \frac{\vartheta}{\vartheta - D - \frac{1 - \rho\beta}{1 - \eta} \kappa}$$

Defining Υ^{Nash} and τ_κ^{Nash} yields:

$$\eta_{\Gamma, \vartheta}^{Nash} = \Upsilon^{Nash} \frac{\vartheta}{\vartheta - D - \tau_\kappa^{Nash} \kappa}$$

where

$$\Upsilon^{Nash} = \frac{1 - \rho\beta + \eta\rho\beta f (1 + \kappa \frac{Q}{s})}{\sigma(1 - \rho\beta) + \eta\rho\beta f (1 + (1 - \sigma) \kappa \frac{Q}{s})}, \quad \tau_{\kappa}^{Nash} = \frac{1 - \rho\beta}{1 - \eta}$$

which are the expressions for $\eta_{\Gamma, \vartheta}^{Nash}$, Υ^{Nash} and τ_{κ}^{Nash} in the main text. Note that the inverse fundamental surplus fraction does not contain endogenous variables.

A.1.2 Non-structural Fundamental Surplus Based Decomposition

As discussed in the main text, we derive a non-structural fundamental surplus based decomposition of $\eta_{\Gamma, \vartheta}$ for the Nash model in which the inverse fundamental surplus term involves the endogenous variable f . Note that the elasticity formula, equation (37), can be written as:

$$\eta_{\Gamma, \vartheta}^{Nash} = (1 - \eta) \frac{\vartheta}{\left[\sigma(1 - \beta\rho) \frac{\Gamma^{\sigma}}{\sigma_m} + \eta\rho\beta\Gamma \right] s + (1 - \sigma) \kappa\eta\rho\beta\sigma_m\Gamma^{1-\sigma}}$$

Also, equation (36) can be solved for s :

$$s = \frac{(1 - \eta) [\vartheta - D] - (1 - \rho\beta) \kappa - \kappa\eta\rho\beta\sigma_m\Gamma^{1-\sigma}}{(1 - \beta\rho) \frac{\Gamma^{\sigma}}{\sigma_m} + \eta\rho\beta\Gamma}$$

Substituting for s in the elasticity formula and simplifying yields:

$$\eta_{\Gamma, \vartheta}^{Nash} = \Upsilon^{Nash} \frac{\vartheta}{\vartheta - D - \tau_{\kappa}\kappa}$$

where

$$\Upsilon^{Nash} = \frac{1 - \rho\beta + \eta\rho\beta f}{\sigma(1 - \rho\beta) + \eta\rho\beta f}, \quad \tau_{\kappa} = \frac{(1 - \beta\rho(1 - f\eta))^2}{(1 - \eta)(1 - \beta\rho(1 - f\eta/\sigma))}$$

which are the expressions provided in the main text. Notice that τ_{κ} contains the endogenous variable f in this alternative expression for the inverse fundamental surplus.

A.2 Alternating Offer Bargaining

A.2.1 Structural Decomposition

Recall the set of equilibrium labor market equations with the AOB-sharing rule:

$$\begin{aligned}
 S_t^w &= w_t - D + \rho\beta (1 - \sigma_m E_t \Gamma_{t+1}^{1-\sigma}) E_t S_{t+1}^w \\
 J_t &= \vartheta_t - w_t + \rho\beta E_t J_{t+1} \\
 \frac{s}{\sigma_m} \Gamma_t^\sigma &= J_t - \kappa \\
 J_t &= \beta_1 S_t^w - \beta_2 \gamma + \beta_3 (\vartheta_t - D)
 \end{aligned}$$

Add the first and second equation and use the third equation to substitute out J_t and $E_t J_{t+1}$

$$\begin{aligned}
 S_t^w + \frac{s}{\sigma_m} \Gamma_t^\sigma + \kappa &= \vartheta_t - D + \rho\beta E_t \left(\frac{s}{\sigma_m} \Gamma_{t+1}^\sigma + \kappa \right) + \rho\beta (1 - \sigma_m \Gamma_{t+1}^{1-\sigma}) E_t S_{t+1}^w \\
 \frac{s}{\sigma_m} \Gamma_t^\sigma + \kappa &= \beta_1 S_t^w - \beta_2 \gamma + \beta_3 (\vartheta_t - D)
 \end{aligned}$$

Use the last equation to substitute out for S_t^w and $E_t S_{t+1}^w$ in the first equation and impose steady state:

$$\begin{aligned}
 &(1 + \beta_1) (1 - \rho\beta) \frac{s}{\sigma_m} \Gamma^\sigma + \rho\beta s \Gamma + \rho\beta \sigma_m \Gamma^{1-\sigma} \kappa + \rho\beta \sigma_m \Gamma^{1-\sigma} \beta_2 \gamma - \rho\beta \sigma_m \Gamma^{1-\sigma} \beta_3 (\vartheta - D) \\
 = &(\beta_1 + (1 - \rho\beta) \beta_3) (\vartheta - D) - (1 + \beta_1) (1 - \rho\beta) \kappa - (1 - \rho\beta) \beta_2 \gamma
 \end{aligned} \tag{38}$$

Totally differentiating, using $f = \sigma_m \Gamma^{1-\sigma}$ and $Q = \sigma_m \Gamma^{-\sigma}$ and re-arranging gives:

$$\frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} = \frac{\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3}{\left[\sigma (1 - \rho\beta) (1 + \beta_1) + \rho\beta f + (1 - \sigma) \rho\beta f \kappa \frac{Q}{s} + (1 - \sigma) \rho\beta f \beta_2 \gamma \frac{Q}{s} - (1 - \sigma) \rho\beta f \beta_3 (\vartheta - D) \frac{Q}{s} \right] \frac{s}{Q}} \vartheta$$

Rewrite equation (38):

$$\frac{s}{Q} = \frac{(\beta_1 + (1 - \rho\beta) \beta_3) (\vartheta - D) - (1 + \beta_1) (1 - \rho\beta) \kappa - (1 - \rho\beta) \beta_2 \gamma}{(1 + \beta_1) (1 - \rho\beta) + \rho\beta \Gamma Q + \rho\beta f Q \frac{\kappa}{s} + \rho\beta f Q \beta_2 \frac{\gamma}{s} - \rho\beta f Q \beta_3 \frac{\vartheta - D}{s}}$$

Substituting into the elasticity formula and rewriting yields:

$$\eta_{\Gamma, \vartheta}^{AOB} = \frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} = \Upsilon^{AOB} \frac{\vartheta}{\vartheta - D - \tau_{\kappa}^{AOB} \kappa - \tau_{\gamma}^{AOB} \gamma}$$

where

$$\begin{aligned} \Upsilon^{AOB} &= \frac{\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta\beta_3 f}{\beta_1 + (1 - \rho\beta) \beta_3} \Xi \\ \Xi &= \frac{(1 - \rho\beta) (1 + \beta_1) + \rho\beta f (1 + (\kappa + \beta_2\gamma - \beta_3 (\vartheta - D)) \frac{Q}{s})}{\sigma (1 - \rho\beta) (1 + \beta_1) + \rho\beta f (1 + (1 - \sigma) (\kappa + \beta_2\gamma - \beta_3 (\vartheta - D)) \frac{Q}{s})} \\ \tau_{\kappa}^{AOB} &= \frac{(1 + \beta_1) (1 - \rho\beta)}{\beta_1 + (1 - \rho\beta) \beta_3}, \quad \tau_{\gamma}^{AOB} = \frac{\beta_2 (1 - \rho\beta)}{\beta_1 + (1 - \rho\beta) \beta_3} \end{aligned}$$

which are the expressions provided in the main text for the AOB model where β_1 , β_2 and β_3 are defined in subsection 2. Note that the Nash model corresponds to the special case of $\beta_1 = (1 - \eta)/\eta$ and $\beta_2 = \beta_3 = 0$.

A.2.2 Non-Structural Decomposition

Recall the set of equilibrium equations, with the AOB sharing rule:

$$\begin{aligned} S_t^w &= w_t - D + \rho\beta (1 - \sigma_m E_t \Gamma_{t+1}^{1-\sigma}) E_t S_{t+1}^w \\ J_t &= \vartheta_t - w_t + \rho\beta E_t J_{t+1} \\ \frac{s}{\sigma_m} \Gamma_t^\sigma &= J_t - \kappa \\ J_t &= \beta_1 S_t^w - \beta_2 \gamma + \beta_3 (\vartheta_t - D) \end{aligned}$$

Substitute:

$$\begin{aligned} S_t^w + \frac{c}{\sigma_m} \Gamma_t^\sigma + \kappa &= \vartheta_t - D + \rho\beta E_t \left(\frac{c}{\sigma_m} \Gamma_{t+1}^\sigma + \kappa \right) + \rho\beta (1 - \sigma_m \Gamma_{t+1}^{1-\sigma}) E_t S_{t+1}^w \\ \frac{c}{\sigma_m} \Gamma_t^\sigma + \kappa &= \beta_1 S_t^w - \beta_2 \gamma + \beta_3 (\vartheta_t - D) \end{aligned}$$

Impose steady state:

$$\begin{aligned}
(*) & (1 + \beta_1) (1 - \rho\beta) \frac{c}{\sigma_m} \Gamma^\sigma + \rho\beta c \Gamma + \rho\beta \sigma_m \Gamma^{1-\sigma} \kappa + \rho\beta \sigma_m \Gamma^{1-\sigma} \beta_2 \gamma - \rho\beta \sigma_m \Gamma^{1-\sigma} \beta_3 (\vartheta - D) \\
= & (\beta_1 + (1 - \rho\beta) \beta_3) (\vartheta - D) - (1 + \beta_1) (1 - \rho\beta) \kappa - (1 - \rho\beta) \beta_2 \gamma
\end{aligned}$$

Totally differentiate:

$$\begin{aligned}
& \left[\sigma (1 + \beta_1) (1 - \rho\beta) \frac{c}{\sigma_m} \Gamma^\sigma + \rho\beta c \Gamma + (1 - \sigma) \rho\beta f \kappa + (1 - \sigma) \rho\beta f \beta_2 \gamma - (1 - \sigma) \rho\beta f \beta_3 (\vartheta - D) \right] \frac{d\Gamma}{\Gamma} \\
= & (\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta \sigma_m \Gamma^{1-\sigma} \beta_3) \vartheta \frac{d\vartheta}{\vartheta}
\end{aligned}$$

Re-arrange:

$$\frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} = \frac{\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3}{[\sigma (1 - \rho\beta) (1 + \beta_1) + \rho\beta f] \frac{c}{Q} + (1 - \sigma) \rho\beta f \kappa + (1 - \sigma) \rho\beta f \beta_2 \gamma - (1 - \sigma) \rho\beta f \beta_3 (\vartheta - D)} \vartheta$$

Solve (*) for $\frac{c}{Q}$:

$$\frac{c}{Q} = \frac{\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3}{(1 - \rho\beta) (1 + \beta_1) + \rho\beta f} (\vartheta - D) - \kappa - \frac{1 - \rho\beta + \rho\beta f}{(1 - \rho\beta) (1 + \beta_1) + \rho\beta f} \beta_2 \gamma$$

Substitute into elasticity formula:

$$\begin{aligned}
\frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} &= \frac{\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3}{\frac{\sigma(1-\rho\beta)(1+\beta_1)+\rho\beta f}{(1-\rho\beta)(1+\beta_1)+\rho\beta f} (\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3) - (1 - \sigma) \rho\beta f \beta_3} \times \\
& \frac{\vartheta}{\vartheta - D - \frac{\sigma(1-\rho\beta)(1+\beta_1)+\rho\beta f - (1-\sigma)\rho\beta f}{\frac{\sigma(1-\rho\beta)(1+\beta_1)+\rho\beta f}{(1-\rho\beta)(1+\beta_1)+\rho\beta f} (\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3) - (1 - \sigma) \rho\beta f \beta_3} \kappa - \zeta \beta_2 \gamma}
\end{aligned}$$

$$\text{where } \zeta = \frac{[\sigma(1-\rho\beta)(1+\beta_1)+\rho\beta f] \frac{1-\rho\beta+\rho\beta f}{(1-\rho\beta)(1+\beta_1)+\rho\beta f} - (1-\sigma)\rho\beta f}{\frac{\sigma(1-\rho\beta)(1+\beta_1)+\rho\beta f}{(1-\rho\beta)(1+\beta_1)+\rho\beta f} (\beta_1 + (1 - \rho\beta) \beta_3 + \rho\beta f \beta_3) - (1 - \sigma) \rho\beta f \beta_3}$$

Re-arrange to obtain the elasticity formula for the AOB model:

$$\frac{\frac{d\Gamma}{\Gamma}}{\frac{d\vartheta}{\vartheta}} = \Upsilon^{AOB} \frac{\vartheta}{\vartheta - D - \tau_\kappa \kappa - \tau_\gamma \gamma}$$

where

$$\begin{aligned}
\Upsilon^{AOB} &= \frac{\beta_1 + \beta_3 (1 - \rho\beta (1 - f))}{a\psi} & (39) \\
\tau_\kappa &= \frac{(1 + \beta_1) (1 - \rho\beta) + \rho\beta f + \frac{\rho\beta f(\sigma-1)}{\psi}}{a} \\
\tau_\gamma &= \left(1 - \rho\beta (1 - f) + \frac{\rho\beta f (\sigma - 1)}{\psi}\right) \frac{\beta_2}{a} \\
a &= \beta_1 + \left(1 - \rho\beta (1 - f) + \frac{\rho\beta f (\sigma - 1)}{\psi}\right) \beta_3 \\
\psi &= \frac{\rho\beta f + \sigma (1 - \rho\beta) (1 + \beta_1)}{\rho\beta f + (1 - \rho\beta) (1 + \beta_1)}
\end{aligned}$$

B Dynamic Elasticity Formula in Inertial Wage Rule

Here we derive the equilibrium solution for the value of a worker to a firm, J_t , provided in subsection 4.1.

Note that the value of a worker to a firm is given by:

$$J_t = \vartheta_t - w_t + \beta\rho E_t J_{t+1}. \quad (40)$$

Substituting out for w_t using (30) gives:

$$J_t = \vartheta_t - \phi\vartheta_t + \gamma(1 - \phi) (\vartheta_t - \vartheta) + \beta\rho E_t J_{t+1} \quad (41)$$

Next, we solve for J_t using the method of undetermined coefficients. Guess that the solution takes the following form:

$$J_t = \delta_0 + \delta_1 \vartheta_t \quad (42)$$

where δ_0 and δ_1 are undetermined coefficients that are to be determined as a function of model parameters. Substituting (42) into (41) also making use of (31) gives:

$$0 = [(1 - \phi) (1 + \gamma) - \delta_1 + \beta\rho\delta_1 v] \vartheta_t + [\beta\rho\delta_0 - \gamma(1 - \phi)\vartheta - \delta_0 + \beta\rho\delta_1(1 - v)\vartheta]$$

Setting the two square brackets to zero and solving for δ_0 and δ_1 gives:

$$\delta_0 = -\frac{\gamma(1-\phi)\vartheta - \beta\rho\delta_1(1-v)\vartheta}{1-\beta\rho} \text{ and } \delta_1 = \frac{(1-\phi)(1+\gamma)}{1-\beta\rho v}$$

Next, combine the free-entry condition (13) with the solution for J_t :

$$\frac{s}{\sigma_m}\Gamma_t^\sigma = J_t = \delta_0 + \delta_1\vartheta_t.$$

Totally differentiating:

$$\frac{s}{\sigma_m}\sigma\Gamma_t^\sigma \frac{d\Gamma_t}{\Gamma_t} = \delta_1\vartheta \frac{d\vartheta_t}{\vartheta_t}.$$

Re-arranging and using $\frac{s}{\sigma_m}\Gamma_t^\sigma = \delta_0 + \delta_1\vartheta$ yields:

$$\begin{aligned} \eta_{\Gamma,\vartheta}^{dynamic} &\equiv \frac{\frac{d\Gamma_t}{\Gamma_t}}{\frac{d\vartheta_t}{\vartheta_t}} = \frac{\delta_1\vartheta}{\sigma\frac{s}{\sigma_m}\Gamma_t^\sigma} \\ &= \frac{1}{\sigma} \frac{(1-\phi)(1+\gamma)}{(1-\phi)(1+\gamma) - \frac{1-\beta\rho v}{1-\beta\rho} \left(\gamma(1-\phi) - \frac{\beta\rho(1-\phi)(1+\gamma)(1-v)}{1-\beta\rho v} \right)} \end{aligned}$$

which is the expression for the dynamic elasticity of labor market tightness with respect to the marginal revenue product (or technology) provided in subsection 4.1.