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ABSTRACT

We propose a new solution concept for a game among oligopolists that simultaneously compete in several segmented markets. The motivation for this solution comes from international trade, but it also has applications in other areas. It is based on a three-stage extension of the two-stage Kreps-Scheinkman game. We show that two-way trade is not an equilibrium outcome and that there exist bounds on possible cross-market priced differentials that are defined by transport costs. Prices are the same when transport costs are zero. In fact, in the limiting case of zero transport costs the equilibrium coincides with a Cournot equilibrium in a single integrated market. In the presence of transport costs there may exist multiple equilibria.

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OLIGOPOLY IN SEGMENTED MARKETS

by

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1. Introduction

The theory of oligopoly has focused on single markets. There exist, however, many interesting circumstances in which oligopolistic firms interact in several markets, with international trade forming a prime example. Indeed, international trade theory has been concerned with this phenomenon for a number of years (see, for example, Brander (1981) and Helpman and Krugman (1985, chap. 5)), and it has also been recently addressed (albeit in a different contex) by Bulow, Geanakopolos and Klemperer (1985).

Whenever there exist identifiable markets, there often also exist specific costs of servicing them and these costs differ across firms. For example, in a domestic economy context one may distinguish markets according to cities. Firms located in different cities face different transport costs to various markets. In the context of international trade firms are located in countries and they may find it cheaper to service the domestic market than to export to other countries. These costs are also affected by tariffs, export subsidies, and other trade policies. All such costs generate some degree of market segmentation and introduce room for market specific sales

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decisions rather then overall production decisions. In addition, the availability of strategic moves may bring about segmentation under various forms of conduct even in the absence of market related servicing costs.

These considerations raise a fundamental question: What is a suitable description of such markets? Namely, how do oligopolistic firms behave when faced with segmented markets and what are the consequences of this behavior? This is a broad question that cannot possibly be answered in the framework of a single article. A complete answer requires treatment of products with different characteristics, various forms of conduct, entry considerations, and the like. In this paper we deal with a limited problem concerning homogeneous products. We propose a new approach and examine its implications. We argue that our approach is more appealing than the existing alternative, both in principle and because it yields more sensible results.

A detailed motivation and justification of our formulation is provided in the next section. In Section 3 we describe the formal model. Then, in Section 4, we characterize its solution and discuss economic implications. This is followed by the development of an example in Section 5. Concluding comments are provided in Section 6.

2. Motivation

In order to justify our formulation of oligopolistic competition in segmented markets, we first describe the accepted formulation. We rely on an explicit example in order to bring out as clearly as possible some of its features. Consider, therefore, a homogeneous product that is traded in two separate markets, identified by $i=1,2$. The demand functions are $x_i=A_i-p_i$, where x_i is quantity and p_i is price. There exist two firms, each one located in a different market. Both have the same unit production costs $c<A_i$

for $i=1,2$; zero transport costs to the local market; and transport costs $t<(A_i-c)/2$, $i=1,2$, to the other market. What is a reasonable structure of competition between these firms?

The accepted formulation assumes that each firm j chooses a sales vector $x^j=(x_1^j,x_2^j)$, where x_i^j are sales of j in market i, taking as given the sales vector of its rival. The outcome is identified with a Nash equilibrium of this game. This is suggested to be a natural extension of Cournot competition to a multi-market setting (see, for example, Brander (1981) and the monopoly-duopoly case in Bulow, Geanakopolos and Klemperer (1985)). It is easy to show that the unique equilibrium of this game is $x_i^j=(A_i-c-2t)/3$ for $i\neq j$, $x_i^i=(A_i-c+t)/3$ for $i=1,2$, and $p_i=(A_i+2c+t)/3$ for $i=1,2$.

In this example every firm sells in both markets despite the existence of cross market transport costs. In the context of international trade it implies intra-industry (two-way trade) in identical products, which is wasteful. Clearly, the profit level of every firm is higher if they are restricted to sell only in their own markets. However, given the postulated strategy (conduct), no firm can credibly precommit to stay away from its rival's market. The limit of the equilibrium allocation when transport costs t go to zero is equal sharing of each market. Hence, even in the absence of transport costs the distinction of sales in different markets remains meaningful from strategic considerations.

It is also clear from this example that in the absence of transport costs prices differ across markets whenever the intercepts of the demand functions differ. In this case there exist arbitrage opportunities. It is therefore necessary to exclude cross-market resell possibilities on other grounds in order to sustain this equilibrium. It is, however, difficult to find a good justification for such an exclusion in the absence of transport costs and other impediments to trade. This model yields results that are very different from other international trade models. Some of them, such as the result on intra-industry trade, do not change when the model is extended to allow for economies of scale in production and free entry of firms. In the latter case a country has also an incentive to impose a tariff. The tariff improves the terms of trade to an extent that outweighs the negative welfare effect caused by additional entry (see Venables (1985)). It has also been used to derive an argument about import protection as export promotion (see Krugman (1984)).

Evidently, the accepted model of multiple-markets oligopolies has strong and unusual implications. It is therefore desirable to examine its reasonableness in view of accepted ways of thinking about Cournot competition. It is widely believed that oligopolistic firms do not dump quantities on markets, but rather choose prices (see, for example, Scherer (1980, p. 152)). For this reason Bertrand competition is often regarded as a closer approximation to reality. Nevertheless, the Cournot paradigm has been recently resurrected by Kreps and Scheinkman (1983), who have shown that it describes the outcome of a two stage game in which firms choose capacities in the first stage and compete in prices in the second (see Tirole (1987, chp. 5) for a discussion of the role of rationing rules and capacity buildup costs). This interpretation is appealing, because it separates in a reasonable way price from quantity decisions. Firms have to build up capacity first. Later on, when they are precommited to a capacity level, they still have the flexibility to choose prices. Naturally, the choice of capacity is done in anticipation of the outcome of the second stage game (the solution is subgame perfect). An important distinction between price and quantity in this setup is that a firm can precommit to a capacity level but cannot precommit to a price.

In view of this description one may ask how reasonable is the accepted formulation of segmented markets. The answer is not transparent and requires a careful analysis. What our discussion reveals, however, is that following the lead of Kreps and Scheinkman there exists a natural way in which this problem can be approached. Let firms choose productive capacity in the first stage. This capacity can be used to serve either market. In the next stage they choose selling prices for different markets. Given that these markets are segregated a firm can in principle set different prices for each market. Naturally, its ability to discriminate depends on how markets operate. After setting prices, firms can allocate sales across markets in the most desirable way. The last stage does not exist in the Kreps-Scheinkman formulation, which deals with a single market. However, it becomes essential in a multi-market setting. A comparison of this approach to the accepted formulation reveals a major difference: here prices are set before the allocation of sales while in the accepted formulation sales are allocated before the determination of prices. Hence, the accepted approach is suitable for situations in which, say, shipments of goods are committed to various destinations before the arrival of concrete orders, while our formulation is suitable to situations in which firms set prices first, receive orders later, and ship commodities afterwards. We believe that the latter is a better description of most transactions.

In the following section we formalize this idea. Then we ask: What are the properties of equilibria that result from the proposed three stage game and how do they compare to equilibria in the accepted formulation? The answers prove to be rather interesting. For one thing, in our formulation two-way trade in identical products is not an equilibrium outcome when cross-market unit sales costs (such as transport costs) are higher (on average) than in local markets. In addition, cross-market price differentials are bounded by differentials in unit sales costs, and they vanish when the difference in unit sales costs vanishes. Hence, there exist no cross-market arbitrage opportunities. Moreover, in the absence of differences in unit sales costs the equilibria collapse to Cournot equilibria in a single integrated market. This result provides a firm basis to a class of models of international trade that was developed in Helpman and Krugman (1985, chap. 5).

3. The Model

Consider two markets, indexed by $i=1,2$. There is a single firm located in each one of them; firm j is located in market j. The firms compete in three stages. In the first stage they choose capacities $\bar{x}^j$; in the second stage they choose prices $p^j=(p^j_1,p^j_2)$, where p^j_i is the price charged by firm j in market i; and in the third stage they choose sales $x^j=(x^j_1,x^j_2)$, where x^j_i is sales of firm j in market i. There exists a capacity buildup cost that we need not specify at this stage. The unit profit vector of firm j, which does not include capacity buildup costs, is $\pi^j=(p^j_1-t^j_1-c^j,p^j_2-t^j_2-c^j)$, where c^j is unit manufacturing costs and t^j_i is the unit sales cost in market i. The latter may result from transport costs, tariffs, export subsidies, sales taxes, and the like. The typical case is $t^j_k>t^j_j$, $k\neq j$. Namely, it is cheaper to sell in ones own market than in the rival's market. This is necessarily the case if there are no taxes and transport costs are higher to the rival's market. In the context of international trade this inequality is reinforced by the existence of tariffs. On the other hand, export subsidies can reverse it. For these reasons we do not impose a priori restrictions on sales costs.

The demand function in market i is denoted by $D_i(p_i)$, where p_i is the consumer price. The demand functions are continuous and decreasing. In what follows we use the efficient rationing rule (see (P.1) below). However, before we state it formally, we provide it with a rational that is based on an interesting economic structure. Assume that in each market there exists a large number of competitive retailers who buy goods from the producers and sell them to final users, and who operate with zero costs. We assume at this stage that neither retailers nor final users can resell commodities in the other market. We will show, however, that this assumption is not needed in most interesting cases. Since the retailers operate with zero costs, the market clearing price p_i is determined by $D_i(p_i)=x_i^1+x_i^2$. Whenever the prevailing market i clearing price exceeds producer j'th requested price; i.e., $p_i>p_i^j$, retailers are willing to buy additional units from this producer. If the prevailing market i clearing price falls short of producer j'th requested price; i.e., $p_i<p_i^j$, retailers refuse to buy goods from this producer. And if $p_i^1=p_i^2\leq p_i$, retailers are indifferent as to from whom to buy. We assume that in the last case retailers send half the orders to each one of them. If a producer does not satisfy the placed orders, the orders are rechanneled to the other producer. It is easy to see that this procedure produces the efficient rationing rule: If a firm, say firm 1, charges a lower price in market i; i.e., $p_i^1<p_i^2$, it can supply as much as it chooses up to $D_i(p_i^1)$. If it chooses not to supply all this quantity the second firm can choose to supply up to $D_i(p_i^2)-x_i^1$, provided this expression is non-negative. When both firms charge the same price, firm j is free to choose sales up to the limit $\max[D_i(p_i^j)/2,D_i(p_i^j)-x_i^k]$.

All the above specified information is known to all players. We identify the outcome of the game with the subgame perfect equilibrium of this three stage game. In order to derive properties of such equilibria we work backwards in the usual way, starting with the last stage.

Sales Game

In the third stage the capacity vector $\bar{x}=(\bar{x}^1,\bar{x}^2)$ and the price vector $p=(p^1,p^2)$ are given. They imposes restrictions on feasible sales. The decision problem of firm j is:

$$\text{(P.1)} \qquad \max_{x^j} \ \pi^j \cdot x^j$$

$$\text{s.t.} \qquad x_1^j + x_2^j \le \bar{x}^j ,$$

$$x_i^j \le \begin{cases} D_i(p_i^j) & \text{for} \quad p_i^j < p_i^k , \\ \max\ [D_i(p_i^j)/2,\ D_i(p_i^j) - x_i^k\ ] & \text{for} \quad p_i^j = p_i^k , \quad i=1,2. \\ \max\ [\ 0,\ D_i(p_i^j) - x_i^k\ ] & \text{for} \quad p_i^j > p_i^k , \end{cases}$$

This is a linear programming problem whose constraints are described in Figure 1 (except that some of the adjacent points, such as A and B, can coincide). It is clear from the figure that given positive unit profit levels its solution is at point B when unit profits are higher in market 2 and at C when unit profits are higher in market 1. When unit profits are equal the solution set consists of the entire line segment $\overline{BC}$. It is also clear from this representation that whenever the constraints are as described in the figure and the rival increases sales in market i, the firm responds by either not changing its sales in market i or by redirecting sales to the other market.

A third possibility arises when points B and C coincide below the full capacity line (the downward sloping line). Then it responds by cutting back sales in market i without changing sales in the other market. It is also clear that the quantity response of the firm is one for one to the expansion of the rival. These explanations clarify to some extent the nature of the sales game. We now proceed to describe its solution.

Let $X(p,\bar{x})$ be the set of (x^1,x^2) that constitute a pure strategy Nash equilibrium of the sales game. This set may contain more than one element. For this reason it is useful to focus on a particular solution, a focal point, if there exists one with particularly appealing properties. In our context it is natural to assume that the solution is Pareto optimal from the point of view of the firms whenever there exists a single Pareto optimal Nash equilibrium. We will show that this is indeed the case. For this purpose we define:

<u>Definition</u>: (x^1,x^2) is an <u>agreeable</u> <u>sales</u> <u>allocation</u> if $(x^1,x^2)\in X(p,\bar{x})$ and $\pi^j\cdot x^j\geq\pi^j\cdot x^{j\prime}$ for $j=1,2$ and all $(x^{1\prime},x^{2\prime})\in X(p,\bar{x})$.

<u>Definition</u>: $\Pi_s(p,\bar{x})=(\pi^1\cdot x^1,\pi^2\cdot x^2)$ is an <u>agreeable</u> <u>payoff</u> of the sales game if (x^1,x^2) is an agreeable sales allocation for $(p,\bar{x})$.

<u>Remark</u>: It is clear from these definitions that if there exists more than one agreeable sales allocation all of them have the same agreeable payoff. Hence, whenever there exists an agreeable sales allocation, $\Pi_s(p,\bar{x})$ is a vector function.

Proposition 1: For every $(p,\bar{x})$ there exists an agreeable sales allocation.

(See Appendix for proof and description of agreeable allocations.)

Price Game

In the second stage the firms take as given capacity levels (that were chosen in the first stage) and choose prices. Prices are chosen in anticipation of the third stage. We are interested in subgame perfect equilibria. Since in the third stage there is a unique agreeable payoff $\Pi_s(p,\bar{x})=[\Pi_s^1(p,\bar{x}),\Pi_s^2(p,\bar{x})]$, firm j solves in the second stage:

$$\max_{p^j} \Pi_s^j(p,\bar{x}) \quad , \quad j=1,2,$$

and the Nash equilibrium of this price game is the solution to the second stage.

It is known from the work of Kreps and Scheinkman (1983) that in the single market case there may not exist an equilibrium in pure strategies to the price game. This happens when capacities are in some sense too large. However, since capacities are chosen in the first stage they cannot be arbitrary and their level is determined amongst other things by the costs of their buildup (see Tirole (1987, chap. 5). In our multiple market setup this issue is somewhat more complicated. We proceed by assuming that the parameters of the problem ensure existence of a subgame perfect pure strategy equilibrium for the three stage game. Hence, the relevant set of capacities for the price game is the set of capacities which are a component of the overall equilibrium. This assumption avoids explicit treatment of the existence issue and enables us to concentrate on the characterization of equilibria, which is our main concern.

Capacity Game

In the general case there does not exist a single Nash equilibrium or focal point to the price game (see the example in Section 5). For this reason when firms choose capacity they have to form expectations on the outcome of the second stage for every capacity choice. Fortunately, most economic characteristics of the resulting equilibria that we wish to emphasize depend only on the last two stages of the game. Therefore, they do not depend on the structure of expectations in the first stage and apply to all subgame perfect equilibria.

4. Economic Implications

We discuss in this section economic properties of the resulting equilibria. Our analysis is based mainly on the last two stages of the game. For this reason most of the discussion is conducted for given capacity levels, which are suppressed unless needed explicitly. The first result, which is proved in the Appendix, establishes a relationship between consumer and producer prices.

Proposition 2: If (p^1,p^2,x^1,x^2) is an equilibrium of the last two stages (i.e., prices are a Nash equilibrium of the price game and sales are an agreeable sales allocation for these prices) such that $x_i^j>0$, then $p_i^j=p_i$, where p_i is the consumer price in market i.

Namely, if a firm is selling in a market it charges the consumer price. In this case the retailers are making zero profits. In addition, it implies that when both firms sell in a market they charge the same price.

Next we establish that whenever average unit sales costs are higher in rivals' markets than in local markets, at most one firm is selling in a rival's market.

Proposition 3: If (p^1,p^2,x^1,x^2) is an equilibrium of the last two stages and $t_2^1+t_1^2>t_1^1+t_2^2$, then either $x_2^1=0$ or $x_1^2=0$.

Proof: The proof is by contradiction. Suppose to the contrary, that $(x_2^1,x_1^2)\gg 0$. This, we argue, implies

(i) $p_2^1-t_2^1\geq p_1^2-t_1^1$, and

(ii) $p_1^2-t_1^2\geq p_2^1-t_2^2$.

First we prove (i). Suppose to the contrary, that

(a) $p_2^1-t_2^1<p_1^2-t_1^1$.

Then in the second stage firm 1 can choose $p_1^{1\prime}=p_1^2-\epsilon$, $\epsilon>0$, and gain a price advantage over firm 2 in market 1. For ϵ sufficiently small its unit profit $\pi_1^{1\prime}=p_1^{1\prime}-t_1^1-c^1$ is larger than the unit profit in market 2 (which has not changed). Given its price advantage in market 1 and its preference for sales in this market, the solution to the sales game yields (see (P.1))

(b') $$x_1^{1\prime} = \min\,[\bar{x}^1,D(p_1^{1\prime})],$$

which is larger, we argue, than its sales in market 1 in the original agreeable sales allocation; i.e., $x_1^{1\prime}>x_1^1$. This is seen as follows. From

Proposition 2 we know that when both firms sell in a market they charge the same price. Therefore $D_1(p_1^{1\prime})\geq D_1(p_1^2)\geq x_1^1+x_1^2$. Using this inequality and the capacity constraint $\bar{x}^1\geq x_1^1+x_2^1$ condition (b′) yields

$$(b) \quad x_1^{1\prime} \geq x_1^1 + \min(x_1^2, x_2^1) > x_1^1.$$

The next thing to note is that total sales of firm 1 do not decline as a result of the price reduction in market 1; i.e.,

$$(c) \quad x_1^{1\prime}+x_2^{1\prime}\geq x_1^1+x_2^1.$$

This is shown as follows. From (P.1) we know that in response to an increase in sales by firm 1 in market 1 by say Δ firm 2 does not increase its sales in market 2 by more than Δ. Moreover, we know that the upper limit on sales that firm 1 faces in market 2 does not decline by more than the increase in sales in this market by firm 2. Therefore

$$x_1^{1\prime}-x_1^1\geq x_2^2-x_2^{2\prime}\geq x_2^1-x_2^{1\prime},$$

which proves (c).

Now, using condition (c) the change in the agreeable payoff to firm 1 as a result of the proposed price reduction is calculated to satisfy

$$\Delta\Pi_s^1 = (\pi_1^{1\prime}x_1^{1\prime} + \pi_2^{1\prime}x_2^{1\prime}) - (\pi_1^1 x_1^1 + \pi_2^1 x_2^1) \geq x_1^{1\prime}(\pi_1^{1\prime} - \pi_2^1) + x_1^1(\pi_1^1 - \pi_2^1).$$

The first term on the far right hand side is strictly positive for sufficiently small ϵ. Therefore, when $x_1^1=0$ the entire expression on the far right hand side is positive. On the other hand, when $x_1^1>0$ we know from Proposition 2 that both firms charge the same price in market 1. Hence, in this case $\pi_1^1 = \pi_1^{1\prime} + \epsilon$, which together with (b) implies

$$\Delta\Pi_s^1 \geq \min\,(x_1^2, x_2^1)(\pi_1^{1\prime} - \pi_2^1) - \epsilon x_1^1.$$

The right hand side is positive for ϵ sufficiently small, because the first term on the right hand side does not approach zero as ϵ goes to zero (see (a) and the construction of $p_1^{1\prime}$). This shows that when $(x_2^1, x_1^2) \gg 0$ and (a) holds firm 1 can chose prices in the second stage which increase its agreeable payoff. Hence, (a) cannot hold in equilibrium and (i) is satisfied. A symmetrical argument implies that (ii) is satisfied.

Finally, combining (i) and (ii) we obtain $t_1^1 + t_2^2 \geq t_2^1 + t_1^2$, which contradicts the proposition's supposition. Hence, whenever average unit sales costs are lower in the firms' own markets at least one of them does not service the rival's market. ■

This proposition has important implications. Note that under normal circumstances a firm's unit sales costs are higher in a rival's market than in the local market, because transport costs are higher to a rival's market. The proposition implies that in these circumstances we will not observe cross-hauling of identical products. This conclusion is, of course, different from the implication of the accepted formulation in which cross-hauling of identical products is an equilibrium phenomenon. Another implication is that when both firms are selling positive quantities and both markets are served, each firm is selling a positive quantity in its own market.

As far as international trade is concerned, it implies that oligopolistic competition in segmented markets per-se cannot explain intra-industry trade. In addition, the existence of tariffs, which increase unit sales costs of a rival, reinforces the inability of the model to predict intra-industry trade.

Export subsidies reduce a firm's unit sales costs to a rival's market. Nevertheless, the proposition implies that even in their presence (in both countries or in the country that would be importing the product under free trade) there exists no intra-industry trade if international transport costs and tariffs are sufficiently high. Naturally, it is possible to produce examples of two-way exports with sufficiently high export subsidies. But this type of two-way trade is not specific to oligopolistic market structures; it can also be generated in competitive environments.

Our next proposition establishes bounds on cross-market price differentials.

<u>Proposition 4</u>: If (p^1,p^2,x^1,x^2) is an equilibrium of the last two stages and

(a) average cross-market unit sales costs are higher than average local market unit sales costs: $t_2^1+t_1^2>t_1^1+t_2^2$;

(b) both markets are active: $D_i(p_1)>0$ for $i=1,2$;

(c) both firms are active: $x^j+x^j>0$ for $j=1,2$;

then

(d) $t_1^1-t_2^1\le p_1-p_2\le t_1^2-t_2^2$.

<u>Proof</u>: First, observe that the conditions of this proposition satisfy the conditions of Proposition 3. Therefore we conclude that either $x_2^1=0$ or $x_1^2=0$. This conclusion together with suppositions (b) and (c) imply $(x_1^1,x_2^2)\gg 0$. The letter implies

(i) $p_1^1-t_1^1\ge p_2^2-t_2^1$, and

(ii) $p_2^2 - t_2^2 \geq p_1^1 - t_1^2$.

The proof of conditions (i) and (ii) proceeds in the same way as the proof of the similar conditions (i) and (ii) in the proof of Proposition 3 (all we have done is replace the indexes of the markets). By Proposition 2 $p_1^1 = p_1$ and $p_2^2 = p_2$. These together with (i) and (ii) prove the postulated bounds on cross-market price differentials. ■

The proposition identifies bounds on possible equilibrium price differences whenever both firms are active and both markets are served. For concreteness consider the realistic case in which unit sales costs are non-negative (i.e., subsidies to sales in the rival's market are not too large). Then the bounds imply

$$p_1 + t_2^1 \geq p_2 + t_1^1 \geq p_2 \quad \text{and} \quad p_2 + t_1^2 \geq p_1 + t_2^2 \geq p_1.$$

Hence, if cross-market transport costs of retailers and other agents are not lower than the firms', then there do not exist arbitrage opportunities across markets.

Some observations on this proposition are in order; we bring them without proof in order to save space. First, when the inequality in (a) is reversed (i.e., average unit sales costs across markets are lower than average unit sales costs in local markets), then there exists a similar inequality to (d) with an appropriate switch of market indexes. Hence, the conclusion that there do not exist profitable arbitrage opportunities remains valid. Next, if market 1 is not active the right hand side inequality in (d) still applies, and if market 2 is not active the left hand side still applies. In these cases arbitrage opportunities do not exist either. Obviously, if one firm is not active the other is a monopoly facing a threat of entry. In this case (d) does not apply. For example, if unit manufacturing costs of one firm are sufficiently high and the other has zero unit sales costs it may nevertheless choose to price discriminate across markets. In this case there exist arbitrage opportunities and our no-resell assumption becomes significant.

Next, observe that when cross-market transport costs are the same in both directions, say t, while local transport costs are zero, we obtain

$$|p_1-p_2| \le t.$$

Namely, the absolute value of the price differential is bounded above by transport costs. Therefore, when transport costs go to zero the price differential vanishes.

<u>Proposition 5</u>: If both markets are served, both firms are active, and unit sales costs are zero, $p_1=p_2$.

<u>Proof</u>: First observe that from the preceding discussion it is obvious that the price differential can be made as small as desirable by a choice of sufficiently small unit sales costs. What prevents this argument to be directly applied to the limiting case of zero unit sales costs is the fact that when unit sales costs are zero supposition (a) of Proposition 4 is not satisfied. For this reason we provide a direct proof of Proposition 5.

Suppose to the contrary, that prices differ across markets. For concreteness let $p_1<p_2$. In this case there must exist a firm, say firm j, that serves market 1 and is not the sole supplier of market 2 (from the fact that both markets are served and both firms are active). This firm can charge a price $p_2-\epsilon$, $\epsilon>0$, in market 2 and gain a price advantage over its rival who charges p_2 (from Proposition 2). It is easy to see that this brings about a profit increase to firm j. ■

This result is also different from the accepted formulation. Here the absence of transport costs leads to equal prices in both markets, independently of demand and cost structures. In the accepted formulation they may differ. The introduction of price competition brings about price integration across markets despite the existence of an a priori identification of separate markets. In fact, we prove the following stronger result:

Proposition 6: When unit sales costs are zero and there exist positive costs to buildup capacity, the equilibria of the three stage game coincide with the equilibria of a one-shot Cournot game in a single market facing the demand function $D(p)=D_1(p)+D_2(p)$.

Proof: Assume that there exists a pure strategy equilibrium to the three stage game. Then, given the supposition that capacity buildup costs are positive an active firm makes positive profits in the last to stages which are at least as high as the capacity costs. In the absence of sales costs this implies that an active firm charges a price that exceeds its marginal manufacturing costs c^j. In addition, from Proposition 2 both firms charge the same price in every market that they share and from Proposition 5 there is no price differential across markets. Hence, an active firm sells its entire capacity, for otherwise it gains by slightly reducing price. Consequently, in every equilibrium of the last two stages

(i) $D(p)=\bar{x}^1+\bar{x}^2$.

Hence, there exists a unique equilibrium to the price game. Now, the first stage game is conducted under constraint (i). Therefore it is a one-shot Cournot game. ■

This proposition demonstrates that the identification of a priori separate markets in which our sequential game is played results in an equilibrium of the Cournot type in a single integrated market whenever sales costs are nil. Consequently, our model provides a foundation for the treatment of oligopolistic firms in a multiple market setup in the manner proposed in the first four sections of Helpman and Krugman (1985, chap. 5).

5. Example

In this section we present an example with linear demand functions whose purpose is to demonstrate some concrete equilibria and the possibility of multiple equilibria. In particular, we show that an equilibrium with trade and without trade may coexist. The demand functions are

(1) $$D_i(p_i)=1-p_i.$$

Marginal manufacturing costs are zero; i.e., $c^j=0$, $j=1,2$. Transport costs are zero in local markets;

(2) $$0 \le t_2^1 = t_1^2 = t < 1/2 \quad \text{and} \quad t_1^1 = t_2^2 = 0.$$

Capacities are fixed and the same for both firms;

(3) $$0 < \bar{x}^j = \bar{x} \le 1/2.$$

This specification focuses on the last two stages of the game: the choice of prices and sales allocation. Observe that Proposition 3 implies that there are only two possible types of equilibria: Type N, in which there is no trade and every firm sells only in its own market; and Type T in which there is trade and one firm sells also in the rival's market. First we establish that in both types of equilibria there is no underutilized capacity.

Claim 1: Both firms sell all their capacity.

Proof: Suppose to the contrary that, say, firm 1 sells less than all its capacity. We show that in this case it can increase its equilibrium profit level. First, observe that Proposition 3 implies that firm 1 sells in market 1. Consider the following two possibilities: (a) Only firm 1 serves market 1; (b) Both firms serve market 1. In the former case it will be better off reducing price in market 1 and selling more, because with sales smaller than 1/2 marginal revenue is positive in market 1. In the latter case both charge the same price in market 1, which is equal to the consumer price (see Proposition 2). An infinitesimal price reduction by firm 1 in this market enables it to increase sales by a finite amount, thereby raising profits. The same arguments apply to firm 2 and market 2. ■

Claim 2: There exists a Type N equilibrium.

Proof: We prove existence by construction. Let

(i) $x_j^j = \bar{x}^j = \bar{x}$ for all j and $x_i^j = 0$ for $i \neq j$;

(ii) $p_i^j = p_j = 1 - \bar{x}$ for all i,j.

We argue that (ii) represents equilibrium prices and (i) is the unique agreeable allocation corresponding to these prices. The second part of the argument is covered by case 9-H in the appendix. In order to prove the first part, observe that there are eight possibilities for a firm, say 1, to deviate from the prices given in (ii), which are summarized in the following table:

Table 1

	$p_1^{1\prime} > p_1$	$p_1^{1\prime} = p_1$	$p_1^{1\prime} < p_1$
$p_2^{1\prime} > p_2$	A	B	C
$p_2^{1\prime} = p_2$	D		E
$p_2^{1\prime} < p_2$	F	G	H

Deviation A reduces sales in market 1 and does not increase sales in market 2. Since marginal revenue is positive (due to (3)), it reduces profits. Deviations B and G do not change the agreeable allocation and do not change profits. Deviation C does not change the agreeable allocation and reduces profits. Deviations E and H lead to lower unit profits and reduce profits. It remains, therefore, to deal with D and F. We only give the argument for F, which corresponds to case 9-B in the appendix, where it is shown that the agreeable allocation satisfies (see (A.1) and (A.2))

(4a) $$x_1^1 = \min\left\{\bar{x}, \max[0, 1 - p_1^{1\prime} - x_1^2]\right\},$$

(4b) $$x_2^1 = \min[\bar{x} - x_1^1, 1 - p_2^{1\prime}],$$

(5a) $$x_2^2 = \min\left\{\bar{x}, \max[0, \bar{x} - x_2^1]\right\},$$

(5b) $$x_1^2 = \min[\bar{x} - x_2^2, \bar{x}].$$

The last two equations imply $x_1^2 = x_2^1$ while (4b) implies $x_2^1 = \bar{x} - x_1^1$. Together they imply $x_1^2 = \bar{x} - x_1^1$. Substituting this result into (4a) yields $x_1^1 = 0$. Hence,

$x_2^2=0$ as well and $x_1^2=x_2^1=\bar{x}$. The conclusion is that deviation F leads each firm to sell all its capacity in the rival's market. This reduces firm 1's profits. ■

Claim 3: There exists no Type T equilibrium for $\bar{x} \leq t$.

Proof: Suppose to the contrary, that a Type T equilibrium exists. Let firm 1 be selling in market 2. Then the agreeable allocation is $x_2^1>0$, $x_1^1=\bar{x}-x_2^1$, $x_1^2=0$, and $x_2^2=\bar{x}$, and prices are $p_1^1=p_1=1-\bar{x}+x_2^1$, $p_2^j=p_2=1-\bar{x}-x_2^1$ for all j. We do not specify p_1^2. Now suppose that firm 1 reduces price in market 1 by $\epsilon>0$ and increases price in market 2 by the same ϵ. In the new agreeable allocation firm 1 loses ϵ sales in market 2 and gains ϵ sales in market 1. For ϵ sufficiently small its net increase in profits per unit price change is given by the difference in marginal revenues $\Delta\Pi=MR_1-MR_2$, which is given by (the demand function in market 1 is $1-x_1^1$ and in market 2 it is $1-t-\bar{x}-x_2^1$):

(6) $$\Delta\Pi = [1 - 2x_1^1] - [1 - t - \bar{x} - 2x_2^1] = (t - \bar{x}) + 4x_2^1.$$

The second equality took advantage of the fact that $x_1^1+x_2^1=\bar{x}$. It is now clear that for $\bar{x}\leq t$ firm 1 gains from the proposed price deviation. Therefore the proposed prices and agreeable allocation are not an equilibrium. ■

Claim 4: There exists a Type T equilibrium for $\bar{x} > t > 0$.

Proof: The proof is by construction. Let

(i) $$0 < x_2^1 \leq \min\ [t/2\ ,\ (\bar{x} - t)/4\ ,\ 1 - 2\bar{x}];$$

(ii) $x_1^1 = \bar{x} - x_2^1$;

(iii) $x_1^2 = 0$; $x_2^2 = \bar{x}$;

(iv) $p_1^1 = p_1^2 = p_1 = 1 - \bar{x} + x_2^1$;

(v) $p_2^1 = p_2^2 = p_2 = 1 - \bar{x} - x_2^1$.

It is straightforward to inspect that (i)-(iii) is an agreeable allocation for the prices given in (iv)-(v) (see case 9-D in the appendix). In order to prove that this is an equilibrium it is necessary to inspect all possible price deviations by firms 1 and 2 and verify that they do not increase profits of the deviating firm. We consider only two interesting deviations, one for each firm, in order to save space. First, consider the possibility that firm 1 reduces price in market 1 and increase it in market 2, with the price reduction being equal to the price increase. The change in profits per unit price change as a result of a small change is given by $\Delta\Pi$ in (6). Clearly, (i) implies $\Delta\Pi<0$, so that this deviation is not profitable. Next, consider the possibility that firm 2 increases price in markets 1 and 2. Its change in revenue takes place along the demand curve $1-x_2^1-x_2^2$, which has a marginal revenue (evaluated at the initial point) of $1-x_2^1-2\bar{x}$. This marginal revenue is non-negative under (i), and therefore this price change does not increase profits. Other price deviations can be similarly analyzed. ∎

Claims 2-4 show that the capacity interval [0,1/2] can be divided into two subsets, $N=[0,t]$ and $T=(t,1/2]$, such that on N there exists a unique equilibrium in which there is no trade and on T there exists a continuum of equilibria with the trade volume being anywhere between zero and the right hand side of (i) in the proof of Claim 4 (trade is, however, unidirectional). In order to compare these equilibria to the equilibria that obtain under the accepted formulation, observe that with fixed identical capacities in the interval [0,1/2] the accepted formulation yields the following unique equilibrium:

$$x_i^j = \max\ [0\ ,\ (\bar{x} - t)/2]; \quad x_j^j = \bar{x} - x_i^j; \quad i \neq j.$$

Hence, on N both solution concepts yield the same equilibrium. On T, however, the accepted formulation yields two-way trade while our solution yields an equilibrium without trade as well as a continuum of equilibria with one-way trade. Naturally, due to the symmetry, either firm 1 or firm 2 can be selling in the rival's market. In our case the volume of trade does not exceed $(\bar{x}-t)/4$, while in the accepted formulation it is $(\bar{x}-t)$. Hence, in our case the volume of trade is at most a quarter of the volume of trade in the accepted formulation.

Next, observe that the measure of the set of equilibria at a point on T can be represented by the upper bound on x_2^1 (or x_1^2). As transport costs t decline towards zero this measure also declines towards zero. Hence, the extent of multiplicity of equilibria is bounded by transport costs whenever they are sufficiently small. In the limit, when t=0, Type N is the only existing equilibrium. For t=0 Proposition 5 ensures that even when capacities are not the same equilibrium prices are the same in both markets. Moreover, Proposition 6 ensures that if we were to endogenize capacity choice, then for t=0 the resulting unique equilibrium would have been the Cournot outcome in a single integrated market.

There are two interesting points concerning Type T equilibria (with positive transport costs). First, the firm that sells in both markets charges a lower price in the rival's market. Since it also incurs transport costs to the rival's market, its unit profit differential is even larger than the price differential. In the context of international trade this price structure represents dumping under some definitions. Observe, however, that the diversified firm cannot gain from a shift of sales from the rival's market to its own, despite the unit profit differential. The reason is that a comparison of marginal revenues across markets makes this shift unprofitable on T. Second, note that the diversified firm makes higher profits than the firm that sells only in its own market. This is seen as follows. Let the diversified firm reduce price in its own market to the level that enables it to sell the entire capacity in this market and let it raise price in the other market. Then the resulting agreeable allocation enables it to sell all its output in its own market at a price that exceeds the price charged by the rival in the rival's market. Its profits under this allocation, which are the profits obtained in the corresponding Type N equilibrium, are higher than the rival's but smaller than in the original Type T equilibrium. Therefore in a trading equilibrium the exporting firm has higher profits than in a no-trade equilibrium and the non-exporting firm has lower profits than in a non-trade equilibrium.

6. Conclusions

The solution to the problem of oligopolists that interact in several segmented markets that has been proposed in this paper has a number of appealing features. First, the structure of the game seems to resembles actual trading practices. Second, in the resulting equilibrium there do not exist arbitrage opportunities across markets. Third, there is no wasteful two-way trade in identical products. Fourth, market segmentation is possible only when cross market unit sales costs are positive. When these costs approach zero, the equilibium approaches the Cournot outcome in a single integrated market.

In deriving these results we have used the efficient rationing rule. In addition, we have provided a new justification for its use in terms of an institutional structure which assings a role to competitive retailers. We belive that our main results remain valid for other institutional structures that produce different rationing rules. For our purpose these rules need only ensure the existence of an agreeable allocation and that consumer prices equal in equilibrium to the prices charged by active firms (Propositions 1 and 2).

The proposed framework can be applied to a number of conventional probles. For example, it is possible to interpret the markets as markets for differentiated products. In this case unit sales costs can be interpreted as the additional costs that a firm has to bear in order to adjust a unit of the basic product to the specified variety. The model can also be extended to spatial problems. Under this interpretation a choice of location in physical or characteristics space involves choosing a tradeoff among different unit sales costs. It can also be applied to the analysis of trade structure and trade policy when each market is interpreted to be a different country. In this case (as well as in others) it is possible to deal with short-run effects, for which the capacity levels are fixed, and long-run effects that take account of capacity adjustments.

A final application which we plan to explore concerns the formation of multinational corporations. The accepted formulation of the game in segmented markets assumes that firms can precommit quantities to particular markets. This, we have argued, is unreasonable when discussing arms length trade with production concentrated in a single location. Under these circumstances a firm can precommit total output (via a capacity buildup) but not its distribution across markets. It is, however, possible to think about foreign direct investment as a precommitment to quantities at particular locations (although in the third stage, after the choice of prices, it is necessary to decide how to allocate outputs across markets). It might, for example, be more expensive to build two plants, each one in a different market, than a single plant with their joint capacity. Nevertheless, the strategic value of these separate plants--that draws from the fact that they change the conditions of the subsequent stages of the game--may make it worthwhile to incur the additional costs. These considerations can be developed into a strategic theory of multinational corporations.

APPENDIX

In what follows we provide a joint proof of Propositions 1 and 2 for the case $(\pi^1,\pi^2)\gg 0$. The arguments do not change much when some unit profit levels are non positive. Our proof outlines a way for the construction of equilibria. In order to prove the second proposition it is sufficient to prove

<u>Proposition 2'</u>: Let (x^1,x^2) be an agreeable sales allocation for $(p,\bar{x})$, such that $x_i^j>0$ and $p_i^j<p_i$ for some j. Then there exists a firm k which can raise its agreeable payoff by deviating from p^k.

For the purpose of this proposition it is sufficient to consider cases in which a firm's price is below the consumer price, because if it is above it retailers do not buy from this firm and its sales are zero, thereby violating the supposition of Proposition 2 that requires positive sales.

We have already shown that (P.1) is a linear programming problem whose constraints are described in Figure 1. Recall that when the firm has a higher unit profit in market 1 it chooses point C, because it desires to sell as much as possible in the market with higher unit profits. The residual capacity is sold in the other market (if possible). If unit profits are the same in both markets it cares only about total sales, and is therefore indifferent between all points on the line segment $\overline{BC}$. In many of the situations to be discussed it proves useful to consider a particular market in isolation. The next lemma provides conditions under which this is possible, and establishes the uniqueness of equilibrium sales in the isolated market. The proof of the lemma is by construction, which helps to understand subsequent arguments.

<u>Lemma</u>: If for every j (i) $\pi^j_k < \pi^j_i$; or (ii) an equilibrium value of x^j_k is known, then

(a) there exist unique values of sales in market i that are part of the equilibrium sales vector; and

(b) if the solution implies that $x^j_i > 0$ and $p^j_i < p_i$, then as long as the above specified conditions remain valid (x^1_i, x^2_i) are also equilibrium sales when p^j_i is replaced with $p^{j\prime}_i = p^j_i + \epsilon$, for $\epsilon > 0$ sufficiently small.

<u>Proof</u>: The known limit of firm j's sales in market i is denoted by y^j_i and it is defined by

$$y^j_i = \begin{cases} \bar{x}^j - x^j_k & \text{for } x^j_k \text{ known,} \\ \bar{x}^j & \text{otherwise.} \end{cases}$$

Hence, if sales in market k are known the limit on sales in market i is given by residual capacity, and if sales in market k are not known (but under the conditions of the lemma profit margins are higher in market i) the limit on sales in market i is given by capacity. In both cases each firm would like to sell as much as possible, up to its sales limit, in market i. If one of the firms, say firm 1, has a strictly lower price in market i, then in the third stage equilibrium $x^1_i = \min[y^1_i, D_i(p^1_i)]$ and $x^2_i = \min\left[y^2_i, \max[0, D_i(p^2_i) - x^1_i]\right]$ (because firm 1, which has the lower price, is free to sell in market i as much as it wishes under the demand constraint). This proves part (a) for the case of unequal prices.

To prove part (b) consider first firm 1. If its price is lower than the consumer price $x^1_i = y^1_i$. Neither this solution nor the sales of firm 2 change when firm 1 slightly raises its price. Next, consider firm 2. If its price is below the consumer price $x_i^1 = y_i^1$ and $x_i^2 = y_i^2$. This solution does not change either when firm 2 slightly raises its price.

Next, consider the case in which both firms charge the same price in market i, say p_i. If for one of them, say firm 1, $y_i^1 < D_i(p_i)/2$, then in equilibrium $x_i^1 = y_i^1$ and $x_i^2 = \min[y_i^2, D_i(p_i) - y_i^1]$. If, on the other hand, the known limits on sales in market i are larger than half the demand for both firms, each one supplies half the demand. This proves part (a) for the equal price case. To prove part (b) note that $p_i^1 = p_i^2 < p_i$ implies $x_i^1 = y_i^1$ and $x_i^2 = y_i^2$. Clearly, this solution does not change when one of the firms slightly raises its price. It is also clear from this proof that the equilibrium in market i corresponds to the efficient rationing rule in a single market (see Tirole (1987, chap. 5)). ■

Now, in order to prove the propositions we have to deal with nine cases that are described in Table A1. The columns describe firm 1's possibilities of unit profit differentials across markets, while rows describe these possibilities for firm 2. It is clear from the table that a proof of case 9 also covers case 1; a proof of case 2 also covers 4, 6, and 8; and a proof of 3 also covers 7. In fact, it will become clear from the proof of 9 that it applies to all even cases. Hence, we proceed to prove cases 3, 5, and 9.

Table A1

	$\pi_1^1 < \pi_2^1$	$\pi_1^1 = \pi_2^1$	$\pi_1^1 > \pi_2^1$
$\pi_2^2 < \pi_1^2$	1	2	3
$\pi_2^2 = \pi_1^2$	4	5	6
$\pi_2^2 > \pi_1^2$	7	8	9

Case 3: In this case both firms have higher unit profits in market 1. Hence, we can apply part (a) of the lemma to establish unique equilibrium sales levels in market 1. Using these quantities we can again use part (a) of the lemma to establish unique sales levels in market 2. These sales levels constitute the unique agreeable sales allocation. This completes the proof of Proposition 1. In this case part (b) of the lemma is applicable, which proves Proposition 2'.

Case 9: Table A2 presents a subdivision of this case on the basis of whoever charges a lower price in the market; columns represent price advantages in market 1 while rows represent price advantages in market 2. In all these cases firm j has a higher unit profit level in market j (the critical distinction is that each firm has a higher unit profit level in a different market). The resulting nine possibilities are grouped into four categories. In category A at least one firm has a price advantage (lower price) in the market in which it prefers to sell (i.e., in which it has a higher unit profit level). In B every firm has a price advantage in the less preferred market. In C one firm has a price advantage in the less preferred market while there is no price difference in the other market. Finally, in D there are no price advantages in either market. We now proceed to prove the propositions for each category.

Table A2

	$P_1^1 < P_1^2$	$P_1^1 = P_1^2$	$P_1^1 > P_1^2$
$P_2^2 < P_2^1$	A	A	A
$P_2^2 = P_2^1$	A	D	C
$P_2^2 > P_2^1$	A	C	B

(A) Without loss of generality, consider the case in which firm 1 has a price advantage in market 1 (the first column). Since it also prefers to sell in market 1, it will use the price advantage to sell there as much as possible. Hence, $x_1^1 = \min[\bar{x}^1, D_1(p_1^1)]$. Now consider market 2. Firm 2 prefers to sell there, and firm 1's sales in market 1 are known. Hence, we may apply part (a) of the lemma to obtain the unique equilibrium sales in market 2. Having obtained these sales we may now compute the sales of firm 2 in market 1 as $x^2 = \min\left(\bar{x}^2 - x_2^2, \max[0, D_1(p_1^2) - x_1^1]\right)$. This proves the first proposition. In order to prove the second proposition, consider first market 1, in which firm 1 has a price advantage. Since in this case $x_1^1 = \min[\bar{x}^1, D_1(p_1^1)]$, then if its price is lower than the consumer price $x_1^1 = \bar{x}^1 < D_1(p_1^1)$. In this case it can slightly raise its price and still be able to sell its entire capacity in this market without affecting other components of the agreeable sales allocation. The result is higher profits. Next, consider market 2. We have used part (a) of the lemma to prove uniqueness of the equilibrium allocation. Part (b) can now be used to prove the second proposition.

(B) In this case every firm has a price advantage in the market with lower unit profits. Hence, as is clear from (P.1), its sales in the market with the higher unit profit--in which it wants to sell as much as possible--are constrained by either its capacity or by residual demand. Therefore in the solution to (P.1) for firm 1

$$\text{(A.1)} \qquad x_1^1 = \min \left[\bar{x}^1 , \max [0 , D_1(p_1^1) - x_1^2] \right].$$

Naturally, the firm wants to sell the residual capacity in market 2, but it may face there insufficient demand. Hence, in the solution to (P.1)

$$\text{(A.2)} \qquad x_2^1 = \min [\bar{x}^1 - x_1^1 , D_2(p_2^1)],$$

where x_1^1 is taken from (A.1). It is now clear from (A.1) and (A.2) that the firm's best response in each market depends on x_1^2 but not on x_2^2. We will characterize equilibria by considering the best response functions $x_2^1=R^1(x_1^2)$ and $x_1^2=R^2(x_2^1)$ (the derivation of the best response function for the second firm follows the same steps as the derivation of the best response function for firm 1). There exist only two types of response functions that are implicit in (A.1) and (A.2); they are described by reaction curves in the two panels of Figure 1A. Type I reaction curve arises when $D_1(p_1^1)\geq\bar{x}^1$ and Type II arises when $D_1(p_1^1)\leq\bar{x}^1$. The two types coincide when demand equals capacity.

It is evident from the shapes of these curves that when both firms' reaction curves are combined in the same figure they intersect only once, unless one curve is of Type I and the other of Type II. In the latter case there also exists a single intersection, unless $\bar{x}^1+\bar{x}^2=D_1(p_1^1)+D_2(p_2^2)$. When this condition is met the reaction curves coincide on the upward sloping portion, as demonstrated in Figure 2A. In this case there exists a continuum of Nash equilibria to the sales game. However, the closer an equilibrium point is to the origin, the higher the profit level of every firm (because by moving towards the origin firms swap sales from lower unit profits to higher unit profits markets). Hence, there exists a unique agreeable sales allocation (point A in the figure). This completes the proof of the first proposition.

In order to prove the second proposition we first consider the case in which $\bar{x}^1+\bar{x}^2 \neq D_1(p_1^1)+D_2(p_2^2)$. We have shown that in this case there exists a unique equilibrium of the sales game. It is straightforward to inspect that (i) if both firms sell in a market and one has a lower price the equilibrium does not change when the lower price firm slightly raises its price; and (ii) if a firm sells in a market and its price is lower than the consumer price a slight increase in its price does not change equilibrium sales. Next, consider the case $\bar{x}^1+\bar{x}^2 = D_1(p_1^1)+D_2(p_2^2)$. If in addition $\bar{x}^1 = D_1(p_1^1)$, the unique agreeable sales allocation is $x_i^i = \bar{x}^i$, and therefore also $p_i^i = p_i$ for $i=1,2$. In this case the conditions of the proposition are not satisfied. When $\bar{x}^1 \neq D_1(p_1^1)$ there exists a firm, say firm 2, for whom $\bar{x}^2 > D_2(p_2^2)$. From Figure 2A, which describes this case, one can see that (i) $x_2^2 = D_2(p_2^2)$, $x_2^1 = 0$, and therefore $p_2^2 = p_2$, so that the suppositions of the proposition are not satisfied by market 2; (ii) $x_1^2 > 0$ and $p_1^1 = p_1 > p_1^2$, a case in which point A is the agreeable sales allocation; this point does not change when firm 2 slightly raises its price and therefore profits.

(C) First consider the market in which no firm has a price advantage. Clearly, if the firm that has a higher unit profit in this market has a capacity level that falls short of half the demand, it will sell all its capacity in this market. It is then straightforward to calculate the resulting unique equilibrium allocation. In addition, in these circumstances there is no market in which both sell and one has a price advantage. This covers both propositions for this case.

Next, consider the case in which the firm that has a higher unit profit in the market with equal prices, say firm 1, has a capacity level that exceeds half the demand. Clearly, it will supply at least half the demand. Define, therefore, its pseudo capacity to be $\bar{\bar{x}}^1=\bar{x}^1-D_1(p_1^1)/2$ and the pseudo demand in market 1 to be half the original demand. Using these pseudo quantities in conjunction with the remaining original data we can redefine the problem giving firm 2 as if a price advantage in market 1. This brings us to case (B), and its arguments can now be applied to prove both propositions.

(D) If there exists a firm with a capacity that is lower than half the demand in the market in which it has a higher unit profit, apply the first half of the argument in (C). If not, apply the argument in the second half of (C), defining pseudo capacities for both firms.

<u>Case 5</u>: In this case a firm cares only about its total sales; their division across markets does not affect profits. However, constraints on sales in particular markets depend on prices. For this reason it is possible to categorize the relevant sub cases on the basis of price advantage in different markets, as we have done in Case 9. This classification is presented in Table A3. Instead of discussing each category we demonstrate the arguments for (F) and (H) only.

Table A3

	$p_1^1 < p_1^2$	$p_1^1 = p_1^2$	$p_1^1 > p_1^2$
$p_2^2 < p_2^1$	E	G	F
$p_2^2 = p_2^1$	G	H	G
$p_2^2 > p_2^1$	F	G	E

(F) Here one firm has a price advantage in both markets, say firm 1. Hence, if aggregate demand $D_1(p_1^1)+D_2(p_2^1)$ falls short of its capacity, it supplies both markets and firm 2 sells nothing. This is the unique equilibrium, and it does not satisfy the conditions of Proposition 2'. If, on the other hand, the above aggregate demand exceeds its capacity, it sells its entire capacity and firm 2 picks up the residual demand. Since firm 1 is indifferent in which market it sells, the agreeable sales allocation is achieved when sales of firm 2 are maximized subject to the constraint that firm 1 sells its entire capacity. This problem has a unique solution, which proves the first proposition.

To prove the second proposition, observe that as long as firm 1 has a price advantage in both markets and aggregate demand (evaluated at its prices) exceeds its capacity, it is able to sell the entire capacity. Hence, a slight price increase in a market in which its price is lower than the consumer price does not eliminate its price advantage and raises its profits. If, on the other hand, aggregate demand is lower or equal to its capacity, it sells the demanded quantities and the consumer price in each market is equal to its price. In this case the suppositions of the proposition are not satisfied, neither are they satisfied for firm 2 whose price is higher than the consumer price and it has no sales.

(H) In this case both firms are indifferent towards sales in alternative markets (they care only about total sales) and none of them has a price advantage in either market. Therefore, if there exists a firm whose capacity is smaller or equal to $[D_1(p_1^J)+D_2(p_2^J)]/2$, it sells its entire capacity and the other firm picks up the residual demand up to its own capacity level. Since they are indifferent as to which market they are serving, there exists typically a continuum of agreeable sales allocations, all yielding the same agreeable payoff. If the capacity of every firm is larger than half aggregate demand each firm supplies half of every market. This is the unique Nash equilibrium and the agreeable allocation. As far as the second proposition is concerned observe that if the firms' price in a market is below the consumer price, both sell their entire capacity. Let a firm slightly raise its prices in both markets. This shifts us to case (F), but it is quite clear that for sufficiently small price increases it is able to sell the entire capacity after the price increase and thereby increase profits. This proves the second proposition. ■

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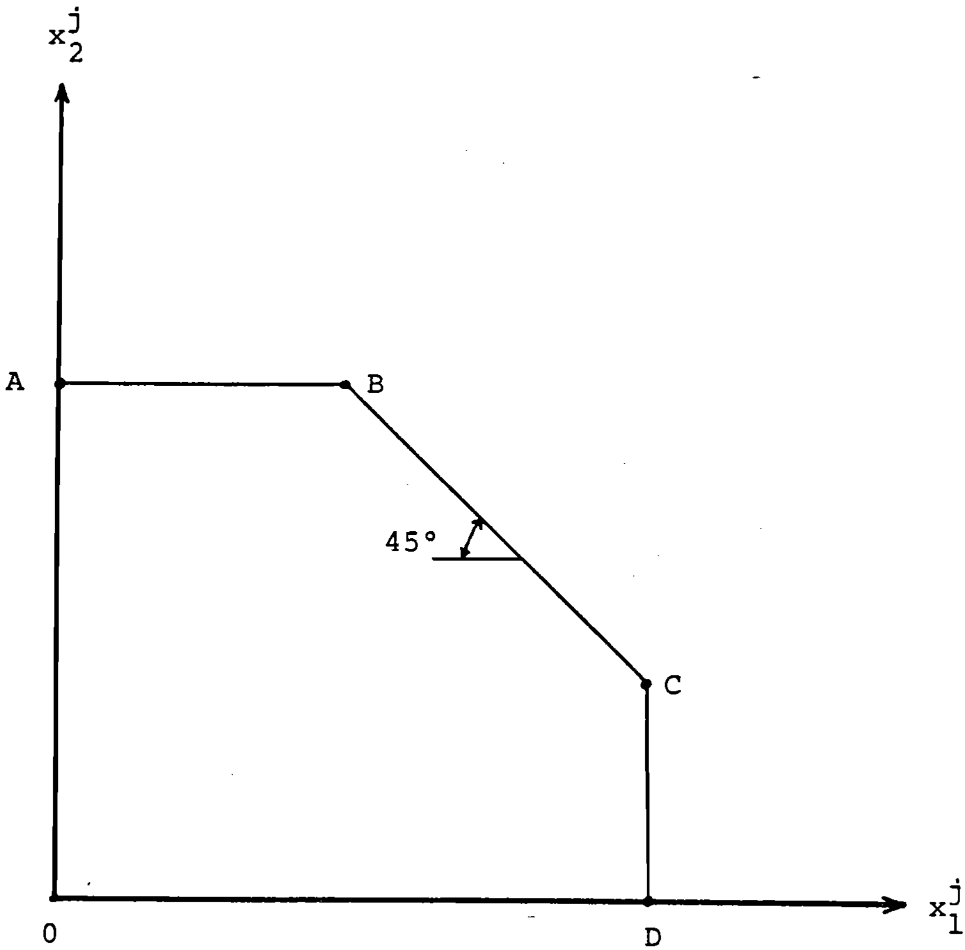

Figure 1

Type I: $\bar{x}^1 \leq D_1(p_1^1)$ Type II: $\bar{x}^1 \geq D_1(p_1^1)$

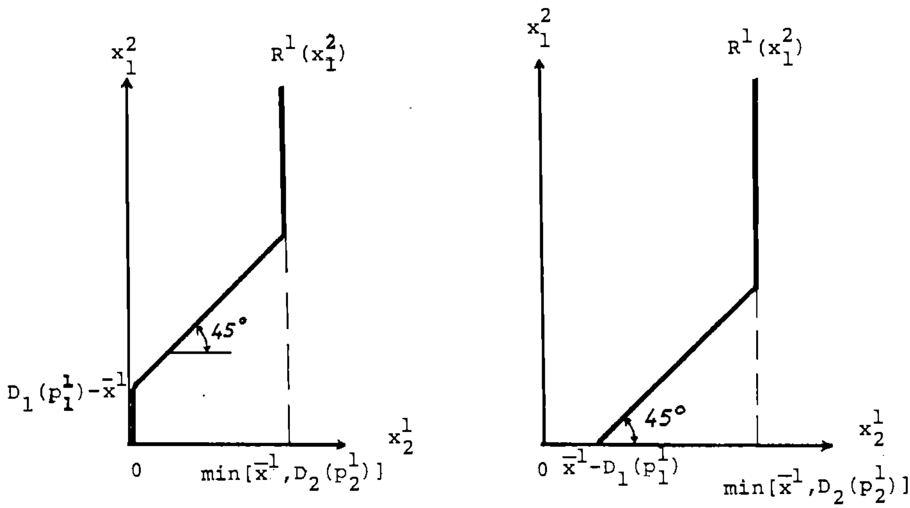

Figure 1A

$$\bar{x}^1+\bar{x}^2 = D_1(p_1^1)+D_2(p_2^2)$$

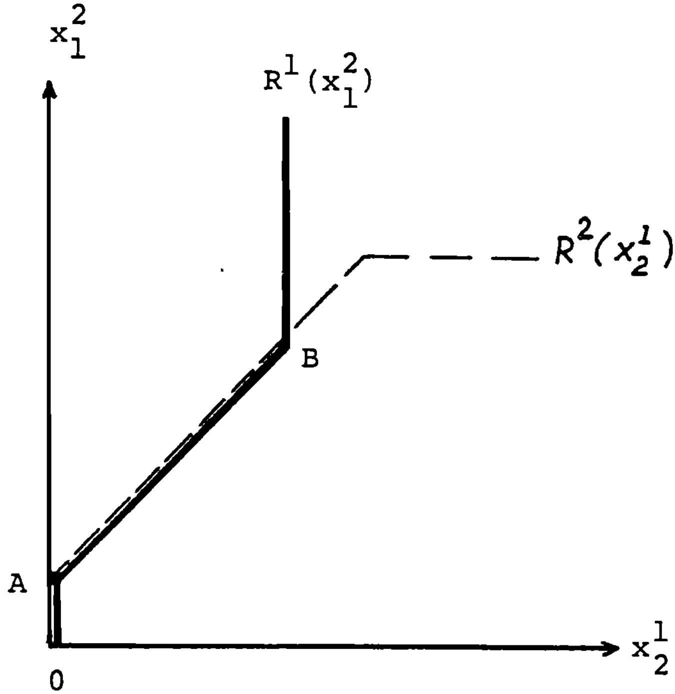

Figure 2A