NBER WORKING PAPER SERIES

# TRADE WARS, TECHNOLOGY AND PRODUCTIVITY

Ching-mu Chen
Wan-Jung Cheng
Shin-Kun Peng
Raymond Riezman
Ping Wang

Working Paper 26468
http://www.nber.org/papers/w26468

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2019

We are grateful for comments by Eric Bond, Matt Cole, Ronald Jones, Wen-Jung Liang, Costas Syropoulos, Stephen Yeaple, and participants at the Econometric Society Meeting, the Midwest Economic Theory and International Trade Meeting, the Public Economic Theory Conference, the Society for Advanced Economic Theory Conference, CES-ifo Area Conference on the Global Economy, the Japan-Hong Kong-Taiwan Conference on Trade and Industrial Organization, Workshop on Variable Markups and seminar participants at Academia Sinica, Donghwa University, National Taiwan University, Syracuse University, West Virginia University and UC-Riverside. Financial support from Academia Sinica (AS-IA-105-H04) and National Science Council (NSC 99-2410-H-001-013-MY3) to enable this international collaboration is gratefully acknowledged. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Trade Wars, Technology and Productivity
Ching-mu Chen, Wan-Jung Cheng, Shin-Kun Peng, Raymond Riezman, and Ping Wang
NBER Working Paper No. 26468
November 2019
JEL No. D92,F12,O33

## ABSTRACT

If international trade is strictly trade in intermediate goods, would the common presumption, that small, less developed economies (the South) lose from trade wars still be true? We address this question by constructing a dynamic general equilibrium model in which the North and the South trade technology-embodied intermediate goods. We show that the detrimental effects of the trade war are mitigated by the fact that producers in the South can adjust their choice of imported intermediate goods and their investment in domestic technologies. We establish sufficient conditions under which the steady-state trade equilibrium length of the production line and the range of domestic production in the South both expand in response to a tariff war. It thereby creates a novel channel of scale-scope trade-off: The South counters the losses from trade protection in the volume and value of trade (scale) with an upward movement along the value chain (scope). As a result, average productivity in the South and aggregate technology used by the South both turn out to be higher.

Ching-mu Chen
School of Business Administration
Zhongnan University of Economics and Law
Wuhan, Hubei
China
b79208014@gmail.com

Wan-Jung Cheng
Institute of Economics
Academia Sinica
Taipei City, Taiwan
Republic of China
wjcheng@econ.sinica.edu.tw

Shin-Kun Peng
Institute of Economics
Academia Sinica
Taipei City, Taiwan
Republic of China
speng@econ.sinica.edu.tw

Raymond Riezman
Department of Economics
Henry B. Tippie College of Business
W360 PBB
University of Iowa
Iowa City, IA 52242-1000
raymond-riezman@uiowa.edu

Ping Wang
Department of Economics
Washington University in St. Louis
Campus Box 1208
One Brookings Drive
St. Louis, MO 63130-4899
and NBER
pingwang@wustl.edu

# 1 Introduction

Since the Great Recession, we have seen increasing trade protectionism, from Brexit, to the renegotiation NAFTA, the U.S.-China trade war, the ongoing Japan-Korea and the possible U.S.-EU trade wars. Surprisingly, this bout of protectionism has focused on tariffs imposed on intermediate products.[1] Taxing intermediate goods violates the Diamond and Mirrlees (1971) principle of optimal taxation. Equally surprising, much of this increased protection originates from high income countries (the North). This suggests an obvious question: If international trade is strictly trade in intermediate goods, would the common presumption, that small, less developed economies (the South) lose from trade wars still be true, as predicted in the strategic final product trade literature (cf. Kennan and Riezman 1988)?[2] In particular, if the South is able to adjust its technology via the composition of intermediate goods trade, would the South necessarily suffer income and productivity losses as a result of a trade war with the North?

We address these important questions by constructing a dynamic North-South model of *technology-embodied intermediate goods trade*.[3] We stress two channels of technology advancement in the South: one via technology-embodied intermediate goods trade with the North and another via investment in domestic technologies. Our paper examines the interplay of these two forces allowing the *length of the production line* that transforms intermediate goods into final product and the *ranges of exports and imports* to be endogenously determined. In response to a tariff war, the South optimally adjusts their choice of imports from the North that are embodied with better technology in addition to adjusting its own investment in domestic technology. We will show that *the effect of the trade war is dampened* by the fact that intermediate producers in the South adjust their investment in technology and that final producers adjust the mix of intermediate goods by importing new varieties embodied with superior technology, which helps the South move up along the value chain by taking advantage of the North technologies that are available for trade.

Consider a developing economy, the South, that produces final and intermediate goods. The final good is produced according to a general quadratic production function with a continuum of intermediate goods (the production line (cf. Shubik 1959; Peng, Thisse and Wang 2006)). Intermediate goods may be domestically produced or imported from the North. While intermediate

[1] For example, in the U.S., nearly 90% of intermediate imports from China face increased tariffs (cf. Bown 2019).

[2] This literature goes back to the original contribution by Johnson (1953), later generalizated with more complicated strategies by Mayer (1981) and Riezman (1982) and with political economy by Grossman and Helpman (1995). It serves as a foundation for the formation of custom unions. In our paper, we are abstracting from such strategic or lobbying behaviors, but focusing on internationally integrated middle product markets between the North and the South.

[3] As documented by Hummels, Ishii and Yi (2001), the intensity of intermediate goods trade measured by the VS index has risen from below 2% in the 1960s to over 15% in the 1990s.

production technologies in the South are inferior to those in the North, intermediate producers in the South may invest to improve its own technology. Because we use the general quadratic production function, we can order all intermediate goods used by the South according to their levels of embodied technology and solve the model *without imposing ex post symmetry*. We show that in equilibrium the South will export intermediate goods with the lowest technology while importing higher technology intermediate goods from the North.

We establish sufficient conditions for the following results. Foreign trade protection hurts domestic intermediate producers' exports to the North on the *intensive margin* while lowering domestic prices of these exportables. Under our general quadratic production function, a longer production line tends to improve productivity due to the usage of more sophisticated intermediate goods embodying more advanced technologies. Thus, in response to foreign trade protection final producers in the South can counter it by choosing to *expand their production line* at the expense of reduced demand of lower-end intermediate varieties. This thereby creates a of *extensive margin* effect and establishes a novel channel of *scale-scope trade-off* in response to protectionism. Similarly, domestic trade protection decreases imported intermediate inputs on the intensive margin, but final goods producers can react to the tariffs by replacing the less sophisticated intermediate varieties with higher-end intermediate goods, again by expanding the production line as discussed in the foreign trade protection above. That is, there is an effect on the extensive margin and a scale-scope trade-off as well. Furthermore, when importing from the North brings in more advanced technologies, the incentive to invest in lower-end technologies is reduced.

These conflicting intensive and extensive margin effects mean that the net effects of a trade war are ambiguous. What we can say is that the potential damage of a trade war is dampened by the fact that final goods producers adjust their choice of intermediate goods both at the intensive and extensive margins, and intermediate good producers adjust by investing directly in technology improvements in their intermediate goods. Since trade acts as a mechanism to transfer technology from the North to the South, final producers contributes to technology upgrading in the South via shifting toward importing intermediates embodied with higher end of technology, while intermediate producers have less incentive to invest in domestic technology upgrading. Whether a trade war hurts the South is then a quantitative question which requires calibration.

For the calibration, we use the data from Socio-Economic Accounts and World Input-Output Tables for the period 1995-2009. We divide all countries (40 countries plus the rest of world) into two groups: high-income countries (the North) and middle or low-income countries (the South). We then calibrate our model to fit various data moments of this constructed North-South trade dataset. We measure the size of the trade war by using information from the current U.S.-China tariff negotiations.

Our quantitative results show that both exports and imports fall and the production line expands. This is consistent with our theoretical results. Although imports decrease, an expanded production line implies that marginal imports embody better technology, thus enhancing aggregate technology. The trade war turns out to induce a modest increase in average technology and average productivity in the South. As a result, both the value-added ratio (per unit of domestic aggregate intermediate demand) and the consumption ratio in the South also increase modestly despite the detrimental effects of a trade war on the volumes and values of trade.

To better understand these results we decompose the tariff effects on exports and imports. We find that the reduction in imports is exclusively due to the extensive margin effects. The pure extensive margin effect on exports is negligible. While there is a large negative intensive margin effect, this effect is partially offset by changes in net domestic aggregate intermediate good demand.

Next, we look into the role of domestic technology upgrading versus imported technology embodied in intermediate goods trade. We conduct counterfactual analysis by eliminating domestic technology upgrading and import-induced technological advancement one at a time. Our results indicate that the intermediate importing channel is the dominant effect. Overall, the response of increasing high technology intermediate imports as a response to a trade war, in conjunction with a dominant extensive margin effect, constitutes a key *scale-scope* trade-off driving the main findings. By performing sensitivity analysis with respect to various pre-set parameters, we find that our main results remain valid.

Related Literature

The idea that technology can be transferred by means of intermediate goods trade is based on the argument by Keller (2000). This argument is recently renewed in a study of multinational enterprise by Alviarez, Cravino and Ramondo (2019), identifying that firm-embedded technologies transferable globally account for 20% of cross-country TFP differences. Ramanarayanan (2014) uses Chilean plant-level data to establish that importing more implies higher productivity, whereas Halpern, Koren, and Szeidl (2015) also establish a counterfactual result, using Hungarian microdata, that should all input varieties be imported, a firm's revenue productivity would be higher by 22%. That is, all of these empirical studies verify the importance of imported intermediate goods for productivity enhancement.

There is another literature linking the use of intermediate goods with final good productivity. In his pivotal work, Ethier (1982) argues that the expansion of the use of intermediate goods is crucial for improving the productivity of final goods production. While Ethier (1982) determines the endogenous range of intermediate products with embodied technologies, there is no trade in intermediate goods. Yi (2003) and Peng, Thisse and Wang (2006) examine the pattern of intermediate goods trade, the range of intermediate products with exogenous embodied technology. In

Flam and Helpman (1987), a North-South model of final goods trade is constructed in which the North produces an endogenous range of high quality goods and South produces an endogenous range of low quality goods. Although their methodology is similar to ours, their focus is again on final goods trade. Impullitti and Licandro (2018) have a model of final goods trade in which trade liberalization leads to higher productivity through increased firm competition, lower markups, and higher R&D investment. In contrast with all these papers, our paper determines endogenously both the pattern and the extent of intermediate goods trade with endogenous technology choice. Thus, our framework focuses on the trade-off between importing technology embodied in intermediate goods and advancing domestic technology. Furthermore, we characterize the effects of trade protection on the volume and value of trade as well as on average technology and average productivity by accounting for both the intensive and extensive margins.

# 2 The Model

We consider a small open economy as the domestic country, which is less advanced (South) technically than the foreign country (North). There are two sectors: (i) an intermediate good sector in which goods may be domestically produced or imported from the North, and (ii) a final sector that manufactures a single nontraded good using a basket of traded intermediate goods as inputs. All foreign variables are labelled with the superscript $^*$.

This economy represents a global value chain featuring an endogenously determined production line along which a single final product can be manufactured using a basket of technology-embodied and internationally traded intermediate goods.

## 2.1 The Basic Environment

We focus on efficient production of the final good using intermediate goods. In addition to the nontraded final manufactured good, there is a nontraded service produced using a Ricardian technology with unit labor requirement equal to $1/\Omega$ where $\Omega > 0$.

The domestic economy is populated with a continuum of identical workers of mass one, each is endowed with a unit of time supplied to the labor market inelastically and allocated to either manufacture-related activities ($\overline{N}$) and the service sector ($1-\overline{N}$). Individual consumers value both final good consumption ($c$) and service consumption ($s$), which are fractional substitutes to yield utility $u(c+\chi s)$, with $\chi > 0$ and $u$ strictly increasing and concave. Individual income $I$ is the sum of wage income ($w \cdot 1$), capital incomes ($KI$) and redistributed profits from final and intermediate firms ($PI$), i.e., $I = w + KI + PI$. Further assume that capital installed the beginning of a period is fully depreciated at the end of the period and is rented out to intermediate producers every period. With rental income and investment exactly offsetting each other and given the relative price of service $p^s$, the remainder of individual income is used for consumption purposes, i.e., $c + p^s s = I - KI = w + PI$.

With this simple structure in the absence of capital or asset accumulation, the consumer's optimization becomes *atemporal*, so the final good can be chosen as the numeraire whose price is normalized to one at all time. When the consumption bundle is interior (consuming both final good and service), we have $p^s = \chi$. In this case, there is indeterminacy from the demand side and final good consumption is entirely pinned down by the supply $Y$. When $p^s < \chi$, we have a corner solution with agents consuming the final good only. We rule out the other corner case when the final good is not produced. In the interior case, wage equalization between the two manufactured sectors and the service sector subsequently implies: $w = \chi\Omega$. The budget constraint then becomes: $c = \chi(\Omega - s) + PI = Y$, from which we can solve for service consumption. This enables us to restrict our attention purely to the manufactured final and intermediate goods sectors to which we now turn.

## 2.2 The Final Sector

The output of the single final good at time $t$ is produced using a basket of intermediate goods of measure $M_t$. The endogenous determination of the overall length of the production line $M_t$ plays a crucial role in assessing the "extensive margin" effects of trade protection on the respective ranges of export, import and domestic production.

Each variety requires $\phi$ units of labor and each unit of labor is paid at a market wage $w > 0$. The more varieties used in producing the final good the more labor is required to coordinate production. This follows Becker and Murphy (1992). Denoting the mass of labor for production-line coordination at time $t$ as $D_t$, we have:

$$M_t = \frac{1}{\phi} D_t \tag{1}$$

In the absence of coordination cost ($\phi \to 0$), the length of the production line $M_t$ becomes trivial, depending on the choke price and the price gradient. In Melitz and Ottaviano (2008), there is a choke price which sets an upper bound on the number of varieties. In our model, there is no choke price because higher $M_t$ is associated with better technology and a lower price. Thus, in order to have an interior solution for $M_t$, we introduce a coordination cost associated with final good production.

The final good at time $t$ is produced with a generalized quadratic production technology:

$$Y_t = \int_0^{M_t} \left[\alpha - \frac{\beta - \gamma}{2} x_t(i)\right] x_t(i) di - \frac{\gamma}{2} \left[\int_0^{M_t} x_t(i) di\right]^2 \tag{2}$$

where $x_t(i)$ measures the amount of intermediate good $i$ that is used by the final good producers and $\alpha > 0$, $\beta > \gamma$. The first term is typical quadratic production that leads to linear demand. $\alpha$ measures final good productivity, whereas $\beta > \gamma$ means that the level of production is higher when the production process is more sophisticated. We thus refer to $\beta - \gamma > 0$ as the *production sophistication effect*, which measures the positive effect of the sophistication of the production process on the productivity of the final good. To consider the product differentiation effect, the second term is added – it is thus referred to as general quadratic production a la Shubik (1959) and Shubik and Levitan (1980). In particular, $\gamma$ measures the complementarity or substitutability between different varieties of the intermediate goods, where $\gamma >$ (resp., $<$) 0 means that intermediate good inputs are Pareto substitutes (resp., complements). This generalizes the typical linear demand models in the industrial organization literature by considering firms' competition not only in the quantity but quality (cf. Singh and Vives 1984; Vives 1985).

It is important to note that, with the conventional Spence-Dixit-Stiglitz-Ethier setup, *ex post* symmetry is imposed to get closed form solutions. For our purposes, we must allow different intermediate goods to have different technologies. Using this generalized quadratic production technology, we can solve the model analytically without imposing *ex post* symmetry. Moreover, under this production technology, intermediate producer markups are endogenous, varying across different firms.

### 2.3 The Intermediate Sector

Each variety of intermediate good is produced by a single intermediate firm that has local monopoly power *domestically* as long as varieties are not perfect substitutes. Consider a Ricardian technology in which production of one unit of each intermediate good $y_t(i)$ requires $\eta$ units of nontraded capital (e.g., building and infrastructure) in unit of the numeraire final good:

$$k_t(i) = \eta y_t(i) \tag{3}$$

where $i \in I$ that represents the domestic production range (to be endogenously determined).

In addition to capital inputs, each intermediate firm $i \in I$ also employs labor, both for manufacturing and for R&D purposes. Denote its production labor as $L_t(i)$ and R&D labor as $H_t(i)$. Thus, an intermediate firm $i$'s total demand for labor is given by

$$N_t(i) = L_t(i) + H_t(i). \tag{4}$$

With the required capital, each intermediate firm's production function is specified as:

$$y_t(i) = A_t(i)L_t(i)^{\theta} \tag{5}$$

where $A_t(i)$ measures the level of technology and $\theta \in (0, 1)$. By employing R&D labor, the intermediate firm can improve the production technology according to,

$$A_{t+1}(i) = (1 - \nu) A_t(i) + \psi_t(i) H_t(i)^{\mu} \quad (6)$$

where $\psi_t(i)$ measures the efficacy of investment in technological improvement, $\nu$ represents the technology obsolescence rate, and $\mu \in (0, 1)$. To ensure an interior solution, we impose: $\theta + \mu < 1$.

**Remark 1:** Note that we have technology choice, not technology adoption or technology spillovers. These concepts are sometimes confused. Technology adoption permits the use of foreign technologies to produce goods domestically by paying licensing fees. Technology spillovers are uncompensated positive effects of foreign technologies on domestic technologies. What we mean by technology choice, is that domestic producers of final goods implicitly choose the level of technology they use through their choice of intermediate goods used in the production process. They can use lower technology by using domestically produced intermediate goods; they can also employ higher technology by using imported intermediate goods that are produced using foreign technologies. The trade-off these firms face is that adopting higher technology production means a larger range of intermediate goods and higher coordination costs.

One may easily extend our setup to incorporate technology spillovers. In particular, consider the case in which foreign technologies embodied in imported intermediate goods also contribute to domestic technology improvements via reverse engineering. We can modify equation (6) to allow for spillovers

$$A_{t+1}(i) = (1 - \nu) A_t(i) + [(1 - \varsigma) \psi_t(i) + \varsigma \psi_t^*(i)] H_t(i)^{\mu}$$

where $\psi_t^*(i)$ measures the efficacy of investment in technological improvement for the foreign country and $\varsigma \geq 0$ indicates the strength of international technology spillovers. While we will discuss the implication of this modification in Section 5 below, it is clear that such an extension would not affect our main findings so long as $\varsigma$ is not too large.

## 3 Optimization

When a particular intermediate good is produced domestically but not exported to the world market, such an intermediate producer has local monopoly power. Thus, we will first solve for the final sector's demand for intermediate goods, and then we solve for each intermediate firm's supply and pricing decisions for the given demand schedule. Throughout the paper, we assume the final good sector and the intermediate good sector are a small enough part of the entire economy to take all factor prices as given.

The dynamic optimization problem across two adjacent periods can be divided into two sub-stages:

**(i)** Determine the length of production line $M$ and ranges of trade and production $[0, n^E]$, $[0, n^P]$: this problem is atemporal, solved by final producers, taking intermediate producers' prices as given.

**(ii)** Given the length of production line and ranges of trade and production, determine:

- **a.** intermediate demands, supplies and producer prices: this problem is atemporal, solved by final and intermediate producers, taking technology as given;
- **b.** investment (R&D labor) in domestic technology and hence labor allocated to production: this problem is intertemporal, solved by intermediate producers.

Backward solving is applied to solving these sub-stages.

## 3.1 Stage 2

We first solve for final producers' intermediate demands, followed by intermediate producers' supplies and producer prices (Stage ii-a) as well as labor allocation to production and R&D (Stage ii-b).

### 3.1.1 The Final Good Sector

Assume that the South produces all intermediate goods in the range $\left[0, n_t^P\right]$ and exports intermediates in the range $\left[0, n_t^E\right]$, where $n_t^E \leq n_t^P$, while intermediates in the range $\left[n_t^P, M_t\right]$ are imported (see Figure 1 for a graphical illustration). We later solve for $n_t^E$, $n_t^P$, and $M_t$.

The final good producers have the following first-order condition with respect to the demand for intermediate goods $x_t(i)$:

$$\frac{dY_t}{dx_t(i)} = \alpha - (\beta - \gamma)x_t(i) - \gamma \left[\int_0^{M_t} x_t(i^{'})di^{'}\right] = p_t(i), \;\; \forall \; i \in [0, M_t]$$

which can be rewritten as the intermediate goods demand function:

$$\alpha - (\beta - \gamma)x_t(i) - \gamma\widetilde{X}_t = p_t(i), \; \forall \;\; i \in [0, M_t] \tag{7}$$

where $\widetilde{X}_t$ measures the aggregate demand for intermediate goods by domestic firms:

$$\widetilde{X}_t \equiv \int_0^{M_t} x_t(i^{'})di^{'}.$$

Solving from (7), we can derive the price elasticity of intermediate good demand (in absolute value) as $\xi_t(i) \equiv \frac{p_t(i)}{\alpha - \gamma \widetilde{X}_t - p_t(i)}$.[4] We now have the following Lemma.

**Lemma 1:** (Demand for Intermediate Goods) *The demand for intermediate good is downward sloping. If intermediate goods are Pareto substitutes (* $\gamma > 0$ *), a larger aggregate intermediate good demand by domestic firms (higher* $\widetilde{X}_t$ *) reduces individual intermediate good demand but raises the price elasticity for all varieties.*

From (7), it can be seen that the generalized production function yields a linear relative demand for intermediate goods:

$$p_t(i) - p_t(i') = -(\beta - \gamma)[x_t(i) - x_t(i')]. \tag{8}$$

**Lemma 2:** (Relative Demand for Intermediate Goods) *The relative demand for intermediate goods is downward sloping. Additionally, the stronger the production sophistication effect is (higher* $\beta - \gamma$ *), the less elastic the relative demand will be.*

We can then derive the final good producer's first-order condition with respect to the length of the production line $M_t$ (see the Appendix):[5]

$$\left[\alpha - \frac{\beta - \gamma}{2} x_t(M_t) - \gamma \widetilde{X}_t\right] x_t(M_t) = w\phi + (1 + \tau) p_t^*(M_t) x_t(M_t) \tag{9}$$

where the left-hand side represents the marginal benefit from and the right-hand side the marginal cost of expanding the production line. Given $\beta > \gamma$, the solution to relative demand exists if $[\alpha - \gamma \widetilde{X}_t - (1 + \tau) p_t^*(M_t)]^2 > 2(\beta - \gamma) w\phi$.

Next, we analyze the intermediate good sector.

### 3.1.2 The Intermediate Sector

With local monopoly power, each intermediate firm $i$, given its own production function (5) and final producers' intermediate goods demand functions (7), can jointly determine the quantity of intermediate good to supply $y_t(i)$ and the associated price $p_t(i)$, as well as its labor allocation to production $L_t(i)$ and technology advancement $H_t(i)$. By utilizing capital requirement expression

[4] From (7), the intermediate demand can be expressed as $x_t(i) = \frac{\alpha - \gamma \widetilde{X}_t - p_t(i)}{\beta - \gamma}$. Thus, the price elasticity of intermediate good demand in absolute value is derived as

$$\xi_t(i) \equiv -\frac{p_t(i) dx_t(i)}{x_t(i) dp_t(i)} = \frac{p_t(i)}{x_t(i)} \frac{1}{\beta - \gamma} = \frac{p_t(i)}{\alpha - \gamma \widetilde{X}_t - p_t(i)}.$$

[5] It is assumed there is a very large $M^*$ being produced in the world so that any local demand for $M$ can be met with imports from the rest of the world.

(3), labor identities (4), and technology evolution equation (6), its optimization problem is described by the following Bellman equation:

$$\begin{aligned} \underset{\forall\ i\in[0,n_t^P]}{V(A_t(i))} &= \max_{p_t(i),y_t(i),L_t(i),H_t(i)} [(p_t(i)-\eta)y_t(i)] - w_t\,[L_t(i)+H_t(i)] + \frac{1}{1+\rho}V(A_{t+1}(i)) \quad (10) \\ & \text{s.t.} \quad (5),\ (6) \text{ and } (7). \end{aligned}$$

Define $p_t^*(i)$ as the North intermediate goods producer price, which is the delivered price (CNF price) in the North. Let $\tau$ be the South tariff and $\tau^*$ the North tariff. Next, consider pricing decisions. Given unlimited demand in the North, South intermediate producers would not be able to sell above the North tariff adjusted price, so the exporting price is given by $P_t^E(i) = \frac{p_t^*(i)}{1+\tau^*}$ over the exporting range $[0, n_t^E]$. South importers are paying the South tariff adjusted North price $P_t^M(i) = (1+\tau)p_t^*(i)$ over the import range $[n_t^P, M_t]$. Over the nontraded range $[n_t^E, n_t^P]$, we solve (10) to derive intermediate good producer prices $P_t^P(i)$. Hence,

$$p_t(i) = \begin{cases} P_t^E(i) \equiv \frac{p_t^*(i)}{1+\tau^*}, & i \in \left[0, n_t^E\right] \\ P_t^P(i) \equiv \alpha - (\beta-\gamma)x_t(i) - \gamma\widetilde{X}_t, & i \in \left[n_t^E, n_t^P\right] \\ P_t^M(i) \equiv (1+\tau)p_t^*(i), & i \in \left[n_t^P, M_t\right] \end{cases} \quad (11)$$

Notice that $p_t(i)$ is decreasing in $A_t(i)$, which implies that better technology corresponds to lower costs and hence lower intermediate good prices. As a result, it is expected that $\frac{dp_t(i)}{di} < 0$; that is, the intermediate good price function is downward-sloping in ordered varieties $(i)$.

Given that intermediate goods are Pareto substitutes, a higher aggregate demand for intermediate goods by domestic firms lowers the marginal product of each variety, thereby lowering the local monopoly pricing of each variety, implying a downward shift in the $P_t^P(i)$ locus. Thus, we have the following Lemma.

**Lemma 3:** (Producer Price Schedule) *Within the nontraded range $[n_t^E, n_t^P]$, the steady-state intermediate good price schedule is downward sloping in ordered varieties $(i)$, shifting downward in response to a larger aggregate intermediate goods demand by domestic firms $(\widetilde{X}_t)$.*

Denoting $z_t^*(i)$ as South's exports of intermediate good $i$ and $z_t(i)$ as South's imports of intermediate good $i$, we can express the supply of intermediate good $i$ as:

$$y_t(i) = \begin{cases} y_t^E(i) \equiv x_t(i) + z_t^*(i) > x_t(i), & i \in \left[0, n_t^E\right] \\ y_t^P(i) \equiv x_t(i), & i \in \left[n_t^E, n_t^P\right] \\ y_t^M(i) \equiv x_t(i) = z_t(i) > 0, & i \in \left[n_t^P, M_t\right] \end{cases} \quad (12)$$

where final producers' demand for intermediate goods can be solved from (7) and (11):

$$x_t(i) = \begin{cases} x_t^E(i) \equiv \frac{\alpha - \gamma \widetilde{X}_t - \frac{p_t^*(i)}{1+\tau^*}}{\beta - \gamma}, & i \in \left[0, n_t^E\right] \\ x_t^P(i) \equiv A_t(i) L_t(i)^\theta, & i \in \left[n_t^E, n_t^P\right] \\ x_t^M(i) \equiv \frac{\alpha - \gamma \widetilde{X}_t - (1+\tau) p_t^*(i)}{\beta - \gamma}, & i \in \left[n_t^P, M_t\right] \end{cases} \tag{13}$$

We next solve for labor allocation for both nontraded intermediate goods $i \in [n_t^E, n_t^P]$ and exported intermediate goods $i \in [0, n_t^E]$. As shown in the Appendix, the first-order conditions with respect to the two labor demand variables $L_t(i)$ and $H_t(i)$ over $[n_t^E, n_t^P]$ can be derived as:

$$[p_t(i) - \eta - \beta A_t(i) L_t(i)^\theta] \theta A_t(i) L_t(i)^{\theta - 1} = w_t, \quad \forall\ i \in [n_t^E, n_t^P] \tag{14}$$

$$\frac{\mu}{1+\rho} V_{A_{t+1}}(i) \psi_t(i) H_t(i)^{\mu - 1} = w_t, \quad \forall\ i \in [n_t^E, n_t^P] \tag{15}$$

The Benveniste-Scheinkman condition with respect to $A_t(i)$ is given by,

$$V_{A_t}(i) = [p_t(i) - \eta - \beta A_t(i) L_t(i)^\theta] L_t(i)^\theta + \frac{1-\nu}{1+\rho} V_{A_{t+1}}(i), \quad \forall\ i \in [n_t^E, n_t^P] \tag{16}$$

Similarly, we can also obtain the first-order conditions with respect to $L_t(i)$ and $H_t(i)$ over $[0, n_t^E]$, respectively, as follows:

$$\theta \left[\frac{p_t^*(i)}{1+\tau^*} - \eta\right] A_t(i) L_t(i)^{\theta - 1} = w_t, \quad \forall\ i \in [0, n_t^E] \tag{17}$$

$$\frac{\mu}{1+\rho} V_{A_{t+1}}(i) \psi_t(i) H_t(i)^{\mu - 1} = w_t, \quad \forall\ i \in [0, n_t^E] \tag{18}$$

Finally, the Benveniste-Scheinkman condition is given by,

$$V_{A_t}(i) = \left[\frac{p_t^*(i)}{1+\tau^*} - \eta\right]] L_t(i)^\theta + \frac{1-\nu}{1+\rho} V_{A_{t+1}}(i), \quad \forall\ i \in [0, n_t^E] \tag{19}$$

We now turn to solving the system for a steady state.

# 4 Steady-State Trade Equilibrium

Recall that the North-South trade follows a trade-cost-augmented pricing rule, that is, (i) (*willingness to export*) $P_t^P(i) \leq P_t^E(i) = \frac{p_t^*(i)}{1+\tau^*}$ over the export range $i \in \left[0, n_t^E\right]$ and (ii) (*international competition*) $P_t^P(i) > P_t^M(i) = (1+\tau) p_t^*(i)$ over the import range $i \in \left[n_t^P, M_t\right]$. We are now prepared to define the dynamic trade equilibrium:

**Definition:** (Dynamic Trade Equilibrium) *A* ***dynamic trade equilibrium*** *(DTE) consists of consumption choice* $\{c_t, s_t\}$*, final producer's intermediate demand* $\{x_t(i)\}_{i \in [0, M_t]}$ *and production-line length* $M_t$*, intermediate producers' goods supply* $\{y_t(i)\}_{i \in [0, n^P]}$ *and labor demands* $\{L_t(i), H_t(i)\}_{i \in [0, n^P]}$*, a pair of trade cutoffs* $\left\{n_t^E, n_t^P\right\}$*, and an array of producer prices, relative price of service and wage* $\left\{\{p_t(i)\}_{i \in [n_t^E, n_t^P]}, p_t^s, w_t\right\}$*, such that*

**(i)** *(Optimization) consumers, final producer and intermediate producers all optimize;*

**(ii)** *(Trade) intermediate goods are traded according to trade-cost-augmented pricing rule;*

**(iii)** *(Identities) capital requirement (3), production-line coordination requirement (1), and labor identities (4);*

**(iv)** *(Technology advancement) technologies evolved according to (6);*

**(v)** *(Market clearing) labor and final good market clear domestically whereas each intermediate good supply net of demand equals net export.*

A ***steady-state trade equilibrium*** (SSTE) is a DTE such that technologies and all other quantities and values cease to grow.

### 4.1 Labor Allocation and Technology

In steady-state equilibrium, all endogenous variables are constant over time. Thus, (6) implies:

$$H(i) = \left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1}{\mu}}, \quad \forall\, i \in [0, n^P]. \tag{20}$$

This expression implies a positive relationship between the investment in domestic technology in forms of $H(i)$. By manipulation (see the Appendix), we obtain the steady-state level of domestic technology $A(i)$ over the range $i \in [0, n^P]$:

$$A(i) = \overline{A}\psi(i)L(i)^{\mu}, \quad \forall\, i \in [0, n^P] \tag{21}$$

where

$$\overline{A} \equiv \frac{1}{\nu^{1-\mu}}\left[\frac{\mu}{\theta(\rho+\nu)}\right]^{\mu} > 0.$$

One can think of $\overline{A}$ as the technology scaling factor and $\psi(i)$ as the technology gradient that measures how quickly technology improves as $i$ increases.

Next, we substitute (21) into (17), yielding the following expression in $L(i)$ alone:

$$\theta\left[\frac{p_t^*(i)}{1+\tau^*} - \eta\right]\overline{A}\psi(i)L(i)^{\theta+\mu-1} = w, \quad \forall\, i \in [0, n^E] \tag{22}$$

which can be used to derive labor demand for $i \in [0, n^E]$, denoted as $L^E(i)$:

$$L^E(i) = \left\{\frac{\theta}{w}\left[\frac{p_t^*(i)}{1+\tau^*} - \eta\right]\overline{A}\psi(i)\right\}^{\frac{1}{1-\theta-\mu}}.$$

For $i \in [n^E, n^P]$, the following is used to derive the labor demand:

$$MPL(i) = \theta\overline{A}\psi(i)L(i)^{-(1-\mu-\theta)}[\alpha - \eta - \gamma\widetilde{X} - (2\beta-\gamma)\overline{A}\psi(i)L(i)^{\mu+\theta}] = w \tag{23}$$

and the labor demand for $i \in [n^E, n^P]$ is denoted as $L^P(i, \widetilde{X})$. The marginal product of labor $MPL(i)$ is strictly decreasing in $L^P(i)$ with $\lim_{L^P(i)\longrightarrow 0} MPL(i) \longrightarrow \infty$ and $\lim_{L^P(i)\longrightarrow L_{\max}} MPL(i) = 0$, where

$$L_{\max} \equiv \left[\frac{\alpha - \eta - \gamma \widetilde{X}^{-i}}{2\beta \overline{A}\psi(i)}\right]^{\frac{1}{\theta+\mu}}$$

where $\widetilde{X}^{-i} \equiv \int_{i' \neq i} x(i')di' = \widetilde{X} - x(i)$. Figure 2 depicts the $MPL(i)$ locus, which intersects $w$ to pin down labor demand in steady-state equilibrium (point $E$).

That is, an increase in the degree of production sophistication $(\beta - \gamma)$, the degree of substitutability between intermediate good varieties $(\gamma)$, or the aggregate intermediate goods demand by domestic firms $(\widetilde{X})$ all shifts the $MPL$ schedule down. As a consequence, intermediate firms' demand for labor falls.

**Lemma 4:** (Labor Demand for Intermediate Goods Production) *Within the nontraded range $[n^E, n^P]$, labor demand is decreasing in the aggregate intermediate goods demand by domestic firms $(\widetilde{X})$, the degree of production sophistication $(\beta - \gamma)$, and the degree of substitutability between intermediate good varieties $(\gamma)$ in the steady state.*

Next, we can use (4), (20) and (21) to derive R&D labor demand and total labor demand by each intermediate firm as follows:

$$H(i) = \left(\nu \overline{A}\right)^{\frac{1}{\mu}} L(i), \quad \forall\, i \in [0, n^P], \tag{24}$$

$$N(i) = L(i) + H(i) = \left[1 + \left(\nu \overline{A}\right)^{\frac{1}{\mu}}\right] L(i), \quad \forall\, i \in [0, n^P]. \tag{25}$$

The aggregate labor demand in the manufacture sectors is given by,

$$\overline{N} = \phi M + \left[1 + \left(\nu \overline{A}\right)^{\frac{1}{\mu}}\right] \left[\int_0^{n^P} L(i)di\right] \tag{26}$$

where the residual $1 - \overline{N}$ is allocated to the service sector.

### 4.2 Intermediate Goods Supply, Exports, and Profits

Substituting (21) into (5) yields the steady-state supply of intermediate goods:

$$y(i) = \overline{A}\psi(i) L(i)^{\theta+\mu}, \quad \forall\, i \in [0, n^P]. \tag{27}$$

From (27) and (13), we obtain the steady-state exports:

$$z^*(i) = y(i) - x(i) = \overline{A}\psi(i) \left[L^E(i)\right]^{\theta+\mu} - \frac{\alpha - \gamma \widetilde{X} - \frac{p_t^*(i)}{1+\tau^*}}{\beta - \gamma}, \quad \forall\, i \in [0, n^E]. \tag{28}$$

To ensure nonnegative profit, we impose $\frac{p(i)-\eta}{p(i)-\eta-\beta x(i)} > \theta[1+(\nu\overline{A})^{\frac{1}{\mu}}]$ for $i \in [n^E, n^P]$ and $\theta[1+(\nu\overline{A})^{\frac{1}{\mu}}] < 1$ for $i \in [0, n^E]$. Since the latter condition always implies the former, we can use the definition of $\overline{A}$ to specify the following condition to ensure positive profitability:

**Condition N:** (Nonnegative Profit) $\frac{\mu\nu}{\rho+\nu} < 1-\theta$.

This condition requires that the technology obsolescence rate be small enough.

## 4.3 Production Line and Pattern of Production and Trade

The local country's technology choice with regards to intermediate goods production depends crucially on whether local production of a particular variety is cheaper than importing it. For convenience, we arrange the varieties of intermediate goods from the lowest technology to highest technology. Consider

$$\psi(i) = \overline{\psi}(1+\delta \cdot i), \quad \psi^*(i) = \overline{\psi}^*(1+\delta^* \cdot i). \tag{29}$$

It is natural to assume that the advanced country has weakly better basic technology $\overline{\psi}^* \geq \overline{\psi}$ and strictly better advanced technologies, implying a steeper technology gradient $\delta^* > \delta$.

Since technology embodied in intermediated goods is upward-sloping in ordered varieties ($i$), a longer production line is associated with a new composition of varieties with higher end of technologies. Through intermediate goods trade, an expansion of production line thus constitutes a form of technology upgrading via an upward movement along the global value chain.

### 4.3.1 Determination of the Production Line

We can now derive an expression for aggregate intermediate goods demand by domestic firms (see the Appendix):

$$\begin{aligned}
\widetilde{X} \equiv & \int_0^M x(i)di = \overline{A}\psi(i)\left[\int_0^{n^E} L^E(i)^{\theta+\mu}di + \int_0^{n^P} L^P(i,\widetilde{X})^{\theta+\mu}di\right] + \int_{n^P}^M z(i)di - \int_0^{n^E} z^*(i)di \\
= & \overbrace{\overline{A}\int_{n^E}^{n^P}\psi(i)\left[L^P(i,\widetilde{X})\right]^{\theta+\mu}di}^{\text{nontraded intermediate demand}} + \overbrace{\frac{\alpha-\gamma\widetilde{X}}{\beta-\gamma}(M-n^P) - \frac{(1+\tau)}{\beta-\gamma}\int_{n^P}^M p^*(i)di}^{\text{imports}} \\
& + \overbrace{\frac{\alpha-\gamma\widetilde{X}}{\beta-\gamma}n^E - \frac{1}{(\beta-\gamma)(1+\tau^*)}\int_0^{n^E} p^*(i)di}^{\text{domestic demand for exportables}}
\end{aligned}$$

which can be rewritten as:

$$\widetilde{X} = \frac{\overline{A}\int_{n^E}^{n^P}\psi(i)\left[L^P(i,\widetilde{X})\right]^{\theta+\mu}di + \frac{\alpha}{\beta-\gamma}(M+n^E-n^P) - \frac{1}{\beta-\gamma}\left[(1+\tau)\int_{n^P}^M p^*(i)di + \frac{1}{1+\tau^*}\int_0^{n^E} p^*(i)di\right]}{1+\frac{\gamma}{\beta-\gamma}(M+n^E-n^P)}. \tag{30}$$

This is called the *domestic aggregate intermediate demand* locus. In addition, by substituting (13) into (9), we can get the boundary condition at $M$:

$$\alpha - \gamma\widetilde{X} - (1+\tau)p^*(M) = \sqrt{2(\beta-\gamma)w\phi} \tag{31}$$

which will be referred to as the *production-line trade-off* $(MM)$ locus.

Before characterizing the relationship between $M$ and $\widetilde{X}$, it is important to check the second-order condition with respect to the length of the production line. From (9), and (30), we can derive the second-order condition as:

$$\frac{\gamma M x(M)}{(1+\tau)p^*(M)} > -\frac{M}{p^*(M)}\frac{dp^*(M)}{dM}$$

For tractability, world price is specified by:

$$p^*(i) = \overline{p} - b \cdot i$$

The second-order condition for an interior solution of $M$ becomes:

**Condition S:** (Second-Order Condition) $(1+\tau)\,b < \gamma\sqrt{\frac{2w\phi}{\beta-\gamma}}$.

To obtain an interior solution of the length of production line, it is necessary to assume that intermediate goods are Pareto substitutes in producing the final good ($\gamma > 0$), which we shall impose throughout the remainder of the paper. This condition requires that the gradient of the tariff augmented imported intermediate goods prices be properly flat.

**Remark 2:** Under the North-South setting, assume that $M \leq M^*$, that is, the North owns better technology in producing intermediate goods. If Condition S fails to hold, we end up with a corner solution $M = M^*$.

We next turn to the determination of the length of the production line for a given $\left(n^E, n^P\right)$.[6] To do this, we rewrite (30) as an implicit function: $\widetilde{X} = \tilde{G}(\widetilde{X}, M\left|_{n^E,n^P}\right.)$ where the function $\tilde{G}$ is defined as the right-hand side of (30). From (23), the effect of an increase in aggregate intermediate goods $\widetilde{X}$ is to lower labor demand, As a consequence, the implicit function above gives rise to a unique fixed point relationship: $\widetilde{X} = G(M\left|_{n^E,n^P}\right.)$. This is plotted as the $XX$ locus in Figure 3(a) in which the $MM$ locus is given by (31). Differentiation of both loci with respect to $M$ indicates both are positively sloped.[7]

[6] We determine $\left(n^E, n^P\right)$ in subsection 4.3.2 below.

[7] Intuion for the $MM$ locus result is that since intermediate goods are Pareto substitutes, the direct effect of an increase in aggregate intermediate goods, $\widetilde{X}$, reduces the demand for each intermediate good. As $M$ increases, the price of the intermediate good at the boundary, $p^*(M)$, falls, as does the cost of using this intermediate good. This encourages the demand for $x(M)$ and, to restore equilibrium in (31), one must adjust $\widetilde{X}$ upward, implying that the $MM$ locus is upward sloping. The intuition for the $XX$ locus is more complicated. For illustrative purposes, let us

Since the $MM$ locus is the boundary condition pinning down the overall length of the production line, it is expected to be more responsive to changes in $M$ compared to the $XX$ locus. As a result, the $XX$ locus is flatter than the $MM$ locus. This slope requirement is formally specified as:

**Condition C:** (Correspondence Principle) $\left.\frac{d\widetilde{X}}{dM}\right|_{XX\ locus} < \left.\frac{d\widetilde{X}}{dM}\right|_{MM\ locus}$

This condition is particularly important for producing reasonable comparative statics in accordance with Samuelson's Correspondence Principle.[8] Specifically, consider an improvement in technology (higher $\overline{\psi}$ or $\delta$, or lower $\nu$). While the $MM$ locus is unaffected, the $XX$ locus will shift upward. Should the $XX$ locus be steeper than the $MM$ locus, better technology would cause the aggregate demand of intermediate goods ($\widetilde{X}$) to fall, which is counter-intuitive. Thus, based on Samuelson's Correspondence Principle, one may rule out this type of equilibrium. The equilibrium satisfying Samuelson's Correspondence Principle is illustrated in Figure 3(a) by point $E$. In Section 5, we verify these results with our calibration.

Defining the expression in (30) as $\widetilde{X}(M)$, we can substitute it into (9) to obtain:

$$\Gamma(M) \equiv \gamma\widetilde{X}(M) + (1+\tau)p^*(M) = \alpha - \sqrt{2(\beta-\gamma)w\phi} \tag{32}$$

By examining $\Gamma(M)$, it is seen that $M$ has two conflicting effects: a positive effect via the aggregate intermediate goods input $\widetilde{X}(M)$ and a negative effect via the import price $p^*(M)$. Specifically, an increase in the overall length of the production line raises the aggregate intermediate goods input but lowers the import price. Since the $XX$ locus is flatter than the $MM$ locus, the negative effect via the import price dominates the positive effect via the aggregate intermediate goods input. We summarize this result below.

**Lemma 5:** (The Length of the Production Line) *Under Conditions S, N, and C the steady-state overall length of the production line is uniquely determined by the $XX$ and $MM$ loci.*

### 4.3.2 Determination of Production and Trade Ranges

We next turn to determining the pattern of domestic production and export. From (11) and (13), we can obtain the following two key relationships that determine the cutoff values, $n^E$ and $n^P$,

focus on the direct effects. As indicated by (30), the direct effect of a more sophisticated production line (higher $M$) is to raise the productivity of manufacturing the final good as well as the cost of intermediate inputs. While the productivity effect increases aggregate demand for intermediate goods, the input cost effect reduces it. On balance, it is not surprising that the positive effect dominates as long as such an operation is profitable. Nonetheless, due to the conflicting effects, the positive response of $\widetilde{X}$ to $M$ is not too large.

[8] Samuelson (1947) highlights the purpose of Correspondence Principle as: "to probe more deeply into its analytical character, and also to show its two-way nature: not only can the investigation of the dynamic stability of a system yield fruitful theorems in statical analysis, but also known properties of a (comparative) statical system can be utilized to derive information concerning the dynamic properties of a system."

respectively:

$$P^P(n^E) \;=\; \alpha - \gamma\widetilde{X} - (\beta-\gamma)\overline{A}\psi(n^E)\left[L^P\left(n^E,\widetilde{X}\right)\right]^{\theta+\mu} = \frac{p^*(n^E)}{1+\tau^*} = P^E(n^E) \tag{33}$$

$$P^P(n^P) \;=\; \alpha - \gamma\widetilde{X} - (\beta-\gamma)\overline{A}\psi(n^P)\left[L^P\left(n^P,\widetilde{X}\right)\right]^{\theta+\mu} = (1+\tau)p^*(n^P) = P^M(n^P) \tag{34}$$

The two loci are plotted in Figure 4 along with the exogenously determined locus for $P^M(i)$ given by equation (11).

The equilibrium price locus is captured by $\widetilde{ABCD}$ in Figure 4. To see this, we note that, in order for domestic intermediate producers' to be willing to export, their exporting prices must be higher than producer prices over the export range $i \in \left[0, n^E\right]$: $P^P(i) \le P^E(i) \equiv \frac{p^*(i)}{1+\tau^*}$. That is, the equilibrium price schedule takes the upper envelope of $P^P(i)$ and $P^E(i)$ in the export range. On the contrary, to compete with the North, domestic intermediate producers cannot charge higher than trade-cost-augmented import prices. That is, the equilibrium price schedule takes the lower envelope of $P^P(i)$ and $P^M(i)$ for $\left[n^E, M\right]$. Within this range, domestic intermediate producers can produce when $P^P(i) \le P^M(i) \equiv (1+\tau)p^*(i)$, whereas they are out-competed over the import range $i \in \left[n^P, M\right]$.

### 4.3.3 Extensive versus Intensive Margins

From Figure 3(a) and Lemma 5, the $XX$ and the $MM$ loci define a unique association between $M$ and $\widetilde{X}$, which immediately leads to two novel effects.

There is a *boundary effect*: in response to a shift in technology or a policy parameter, the length of the production line $M$ changes, as do the ranges of production and trade captured by the cutoffs $n^E$ and $n^P$. These changes are thus called *extensive margin effects.* These extensive margin effects differ from those in the new trade literature where extensive margin effects are the result of firm entry. In our model, extensive margin effects arise from *changes in the length of the production line* and from the subsequent changes in the domestic production, export and import ranges.

There is also a *domestic aggregate intermediate demand effect*: the corresponding change in $\widetilde{X}$ in response to a shift in technology or a policy parameter will directly affect intermediate goods demand as given in (13), intermediate producers' variety supply (27), and labor demand (22) and (23) for producing a particular variety. We call this an aggregate effect because the effects induced by this channel work through the aggregate demand for intermediate goods and contains both *extensive margin* and *intensive margin* effects. In our analysis on trade protection below, this key scale-scope trade-off will play a central role.

#### 4.3.4 Markups

Next we derive variable markups for all intermediate good firms. For $i \in [n^E, n^P]$, maximum profit is $\pi(i) = \Lambda(i)wN(i)$, and the markup for the producer of intermediate good $i$ is (see the Appendix):

$$\Lambda(i) \equiv \frac{p(i) - \eta}{\theta[1 + (\nu\overline{A})^{1/\mu}]\,[p(i) - \eta - \beta x\,(i)]} - 1. \tag{35}$$

For $i \in [0, n^E]$, maximum profit is $\pi(i) = \Lambda_0 wN(i)$, where the markup becomes a constant given by (see the Appendix), $\Lambda_0 \equiv \left\{\theta[1 + (\nu\overline{A})^{1/\mu}]\right\}^{-1} - 1$. Note that in this general quadratic setup, as the price $(p(i) - \eta)$ increases, the marginal cost $(\theta[1 + (\nu\overline{A})^{1/\mu}]\,[p(i) - \eta - \beta x(i)])$ increases more than proportionately, leading to lower markups. This differs from the constant markup CES aggregator.

By rearrangement, markup can be rewritten as:

$$\Lambda(i) = \frac{1}{\theta[1 + (\nu\overline{A})^{1/\mu}]\left[1 - \beta\frac{x(i)}{p(i)-\eta}\right]} - 1$$

which is positively related to individual variety demand but negatively related to variety price for $i \in [n^E, n^P]$. It is clear that the intermediate good supply schedule $(x^P(i) = \overline{A}\psi(i)L^P(i)^{\theta+\mu})$ is upward sloping, as is $\Lambda(i)$. Likewise, an increase in the technology scaling factor $(\overline{A})$ or the technology gradient $(\psi(i))$ reduces the marginal cost more than the price of intermediate goods leading to higher markups.

Under Condition N (Nonnegative Profit), we then have:

**Lemma 6:** (Producer Markup Schedule) Under Condition N, the steady-state intermediate good markup schedule possesses the following properties:

**(i)** it is upward sloping in ordered varieties $(i)$ within the nontraded range $[n^E, n^P]$, but is a constant $\Lambda_0$ over the exporting range $[0, n^E]$;

**(ii)** an increase in individual intermediate variety demand or a decrease in the producer price leads to a higher markup function;

**(iii)** an increase in the technology scaling factor or the technology gradient leads to a higher markup function.

### 4.4 Trade Protection

As a preliminary step towards analyzing trade wars we begin by looking at the effect of trade protection on the pattern of production and trade, firm markups, aggregate and average technology as well as overall productivity.

### 4.4.1 Effects on the Production Line

We begin by determining the effect of trade protection on the overall length of the production line. Consider an increase in the North tariff ($\tau^*$). In response to foreign protection (an increase in $\tau^*$), the $XX$ locus shifts up but the $MM$ locus remains unchanged.[9] The absence of an effect of North tariff on the $MM$ locus is seen from (31). A higher North tariff reduces exportable prices, thus leading to higher domestic demand for exportables. From (30), the aggregate intermediate good demand by domestic firms increases, as does the $XX$ locus. As a result, the length of production line expands from $M^0$ to $M'$ and the aggregate intermediate good demand by domestic firms increases, as shown in Figure 3(b). To decompose the net effect into the *intensive* and the *extensive margins*, we first fix the production length at $M^0$, under which the equilibrium point shifts from $E$ to $E''$. This induces an *intensive margin* effect on aggregate demand as shown in Figure 3(b). There is also an *extensive margin* effect, which can been seen from $E''$ to $E'$.

Domestic protection (an increase in South tariff $\tau$) increases the domestic cost of imported intermediate inputs $i$, $(1+\tau)p^*(i)$ and hence decreases demand. This causes the $MM$ locus to shift down (see Figure 3(c)). The increase in the domestic tariff decreases the demand for importables at any given $M$ (and thus $n^E, n^P$). As a result, the $XX$ locus solved by fixed point $\widetilde{X} = G(M\,|_{n^E,n^P})$ shifts downward. The shift of the $XX$ locus is always small compared to the shift in the $MM$ locus as the aggregate channel via $\widetilde{X}$ is expected to dominate any individual change via $x(i)$. Therefore, in this case one expects the net effect of domestic trade protection to increase the length of the production line from $M^0$ to $M'$ and to increase the aggregate intermediate goods $\widetilde{X}$, as seen in Figure 3(c). Mathematically, the above arguments are ensured by Conditions E and R below. Specifically, we can differentiate (32) to obtain:

$$\frac{dM}{d\tau} = \frac{p^*(M)}{(1+\tau)b - \gamma(d\widetilde{X}/dM)}$$

which is positive if $(1+\tau)b > \gamma d\widetilde{X}/dM$. We summarize this condition:

**Condition E:** (Dominant Extensive Margin Boundary Effect) $(1+\tau)b > \gamma d\widetilde{X}/dM$.

Under Condition E, the extensive margin boundary effect holds true even in general equilibrium when prices change. In this case, domestic trade protection leads to a longer production line. This condition is met if the response of $\widetilde{X}$ to $M$ is not too large, the degree of substitution between different varieties of intermediate goods is not too large (low $\gamma$) and the price gradient is sufficiently steep (high $b$). Of course, whether this condition holds is a quantitative issue that we shall address in the calibration analysis. Using Condition E we are able to obtain a number of useful results.

[9] Please refer to the Appendix for detailed derivations on the shifts of $MM$ and $XX$ loci.

**Proposition 1:** (The Length of the Production Line) *Under Conditions S, N, C and E the SSTE overall length of the production line ($M$) is increasing in response to either foreign or domestic trade protection (higher $\tau^*$ or $\tau$).*

While the production line becomes longer (Proposition 1), the effect on the net domestic aggregate intermediate demand is generally ambiguous. To sign aggregate demand we impose an additional sufficient condition (which is consistent with our calibrated economy to be outlined in the next section):

**Condition R:** (Regularity Condition) $\frac{p^*(M)}{(1+\tau)b-\gamma(d\widetilde{X}/dM)} \cdot \overbrace{\left.\frac{d\widetilde{X}}{dM}\right|_{MM \text{ locus}}}^{\text{slope of } MM} > \overbrace{\left.\frac{d\widetilde{X}}{d\tau}\right|_{MM \text{ locus}}}^{\text{shift of } MM}$

This is called a regularity condition because it ensures that the net domestic aggregate intermediate demand is positively related to the length of production line in equilibrium. Intuitively, Condition R requires that the shift in the $XX$ locus is relatively small compared to the shift in the $MM$ locus. Under Condition R, we can repeat the same exercise to disentangle the intensive margin effect by fixing the length of the production line at the original level $M^0$, which is measured by the vertical distance from $E$ to $E''$ (a negative intensive margin effect). Subtracting this from the (positive) net domestic aggregate intermediate demand effect yields the extensive margin effect.

Finally, we decompose the net effect of domestic protection. Fix the production length at $M^0$(see Figure 3(c)), the equilibrium point shifts from $E$ to $E''$, leading to a lower $\widetilde{X}$. Domestic trade protection results in a negative *intensive margin* effect on the aggregate intermediate good demand by domestic firms. However, there is a positive *extensive margin* effect because final producers react to it by shifting from importing intermediate goods at $n^P$ to producing domestically (see the shift from $E''$ to $E'$).

### 4.4.2 Effects on Production and Trade Ranges

We next turn to the effects of trade protection on the two cutoffs, , $n^E$ and $n^P$. Let us begin with the aggregate demand for intermediate goods $\widetilde{X}$. Recall that intermediate goods are Pareto substitutes, so aggregate demand for intermediate goods has a negative effect on the marginal product of each variety and hence tends to reduce the price of each variety. When trade protection induces a higher aggregate demand for intermediate goods as discussed above, this translates into a downward shift in the $P^P(i)$ locus. Other things being equal, both cutoffs expand. Mathematically, we have:

$$\frac{dP^P(i)}{d\tau^*} = \frac{\partial P^P(i)}{\partial M}\frac{dM}{d\tau^*} < 0, \quad \frac{dP^P(i)}{d\tau} = \frac{\partial P^P(i)}{\partial \tau} + \frac{\partial P^P(i)}{\partial L^P(i)}\frac{dL^P(i)}{d\tau} + \frac{\partial P^P(i)}{\partial M}\frac{dM}{d\tau} < 0$$

for all $i \in [n^E, n^P]$.

Thus, an increase in the foreign tariff $\tau^*$ reduces domestic price of exportables $\frac{p^*}{1+\tau^*}$ and hence causes the $P^E(i)$ locus to shift down. That is, for each variety $i \in [0, n^E]$, a higher foreign tariff makes domestic intermediate producers less competitive. The $P^M(i)$ locus is clearly unaffected; while rising foreign tariff has no direct effect on the $P^P(i)$ locus, Figure 3(b) indicates a higher domestic aggregate intermediate demand ($\widetilde{X}$) in response to this tariff increase, thereby inducing the $P^P(i)$ locus to shift down (recall that the aggregate channel via $\widetilde{X}$ is expected to dominate any individual change via $x(i)$). From Figures 5-2(a), we can see higher foreign tariff would lower export cutoff point $n^E$ if the aggregate demand channel on $P^P(i)$ were absent (refer to a shift from $E$ to $E''$); with rising aggregate demand, however, the cutoff point $n^E$ would increase (refer to a shift from $E''$ to $E'$). Combining these effects yields an ambiguous outcome in the export cutoff. From Figures 5-2(b), because the only change is via the $P^P(i)$ locus, one concludes immediately that domestic production cutoff $n^P$ increases unambiguously (refer to a shift from $E$ to $E'$). Note that the production line is also expanding (as shown in Proposition 1). In sum, while the domestic production range $[0, n^P]$ expands in response to foreign protection, the export range $[0, n^E]$ and the import range $[n^P, M]$ are generally ambiguous.

Consider next an increase in domestic tariff $\tau$, as depicted in Figures 5-3(a) and (b). It is easily seen that this leads to higher prices of importables and hence causes the $P^M(i)$ locus to rotate upward (a shift from $E$ to $E''$). Thus, while it does not affect the export cutoff, the import cutoff is higher, implying that domestic intermediate producers become more competitive as a result of trade protection. Similar to foreign trade protection, we also have an effect through domestic aggregate demand for intermediate goods (and again keep in mind that the aggregate channel via $\widetilde{X}$ dominates any individual change via $x(i)$). This in turn shifts the $P^P(i)$ locus downward, thus inducing both the export and the domestic production cutoffs to rise. Combining both channels, we see that both the export and the domestic production ranges expand, though its effects on the nontraded and the import ranges remain ambiguous.

Mathematically, the analysis is governed by (33) and (34), where the trade protection effects on the ranges of exports and domestic production are given by,

$$\frac{dn^E}{d\tau^*} = \frac{\partial n^E}{\partial \tau^*} + \frac{\partial n^E}{\partial M}\frac{dM}{d\tau^*}, \quad \frac{dn^E}{d\tau} = \frac{\partial n^E}{\partial \tau} + \frac{\partial n^E}{\partial M}\frac{dM}{d\tau}$$
$$\frac{dn^P}{d\tau^*} = \frac{\partial n^P}{\partial M}\frac{dM}{d\tau^*}, \quad \frac{dn^P}{d\tau} = \frac{\partial n^P}{\partial \tau} + \frac{\partial n^P}{\partial M}\frac{dM}{d\tau}$$

How the export and the import ranges change in response to trade protection thus depends crucially on the relative magnitudes of the extensive/intensive margin effects and the net aggregate intermediate goods demand effects on the length of the production line and two cutoffs. We summarize the results in Proposition 2.

**Proposition 2:** (The Range of Exports, Domestic Production and Imports) *Under Conditions S, N, C and E, the SSTE pattern of international trade features exporting over the range* $\left[0, n^E\right]$ *and* *importing over the range* $\left[n^P, M\right]$ *with the range* $\left[n^E, n^P\right]$ *being nontraded. Moreover, the SSTE possesses the following properties*

**(i)** *in response to foreign trade protection (higher* $\tau^*$*),*

- **a.** *the export price* $P^E(i)$ *and the domestic producer price* $P^P(i)$ *decrease;*
- **b.** *the range of domestic production* $\left[0, n^P\right]$ *expands, but the effects on export and import ranges,* $\left[0, n^E\right]$ *and* $\left[n^P, M\right]$*, and on nontraded range,* $\left[n^E, n^P\right]$*, are ambiguous;*

**(ii)** *in response to domestic trade protection (higher* $\tau$*),*

- **a.** *the import price* $P^M(i)$ *rises whereas the domestic producer price* $P^P(i)$ *falls;*
- **b.** *both the ranges of exports and domestic production,* $\left[0, n^E\right]$ *and* $\left[0, n^P\right]$*, expand, but the effects on the nontraded and the import ranges,* $\left[n^E, n^P\right]$ *and* $\left[n^P, M\right]$*, remain ambiguous.*

**Remark 3:** (Exogenous Length of the Production Line) When the length of the production line $M$ is fixed, domestic trade protection decreases aggregate intermediate goods whereas foreign trade protection increases it (see Figures A1(a) and A1(b) in the Appendix). Domestic trade protection causes producer prices to increase, thus shrinking the export range (as shown in Figure A2(a)). In contrast, foreign trade protection decreases export prices and expands the range of domestic production (Figure A2(b)). Because the overall length is fixed, the import range ($M - n^P$) must decrease. Domestic trade protection raises the overall length and expands the export range, whereas foreign trade protection causes both the domestic production range and the overall length to increase, leading to an ambiguous effect on the import range.

#### 4.4.3 Markups, Productivity and Technology

We next turn to consideration of the effect of trade protection on markups. In the domestic exporting range $\left[0, n^E\right]$, an intermediate firm's markup is constant over $i$. In the nontraded range $i \in [n^E, n^P]$, we can see from (35) that markups will respond endogenously to domestic trade policy. As shown in Proposition 2, in response to a higher foreign or domestic tariff, the domestic producer price $P^P(i)$ falls when the effect via the extensive margin is strong. Moreover, under Condition R, both types of trade protection tend to raise domestic aggregate intermediate demand $\widetilde{X}$ and hence lower individual variety demand $x(i)$. While the fall in variety price tends to raise the markups received by domestic intermediate good firms, the fall in variety demand tends to reduce them. Thus, we have:

**Proposition 3:** (Markups) *Under Conditions S, N, C, E and R, both foreign and domestic trade protection have ambiguous effect on domestic intermediate firms' markups in the SSTE.*

We now turn to determining how trade protection affects productivity and technology. Consider the benchmark case where Conditions $S$, $N$, $C$, $E$ and $R$ hold. Define the aggregate technology used by domestic producers as $\widetilde{A} = \int_0^{n^P} A(i, M)\, di$. Utilizing (21), we can write:

$$\widetilde{A} = \overline{A} \int_0^{n^P} \psi(i) L(i)^{\mu} di \tag{36}$$

One the one hand, from Proposition 2, we learn that the range of domestic production $\left[0, n^P\right]$ expands. On the other hand, we also know $A(i)$ is lower for all $i$ because both $L^E(i)$ and $L^P(i)$ locus shifts down with higher tariff and $\widetilde{X}$, as can be seen from (22) and (23). With a dominant extensive margin effect, there is an increase in aggregate technology $\widetilde{A}$. The effect on average technology $\frac{\widetilde{A}}{n^P}$, however, is ambiguous. Average productivity, measured by $\frac{Y}{\widetilde{X}}$, will increase due to the use of more advanced imported intermediate inputs as the overall length of the production line expands. These results are summarized in the following proposition.

**Proposition 4:** (Productivity) *Under Conditions S, N, C, E and R, both foreign and domestic trade protection result in average productivity gains and higher aggregate technology of domestic producers in the SSTE, but its effect on average technology is ambiguous.*

This result is interesting because it points out that average productivity and average technology do not always move together. This is again a result due to our novel channel of the scale-scope trade-off: importing higher-end intermediate goods improves aggregate technology, but this is accompanied by reduced incentive for investing lower-end domestic technologies. In our model, although the aggregate technology is higher as a result of a stronger extensive margin effect, the average technology may be lower theoretically.

#### 4.4.4 Taking Stock

In sum, our model features:

**(i)** a pure extensive margin effect operating through the boundary effect, $M$,

**(ii)** a pure intensive margin effect via changes in the three price schedules, namely, $P^E(i)$, $P^P(i)$ and $P^M(i)$ loci,

**(iii)** a net domestic aggregate intermediate demand effect via $\widetilde{X}$ that affects both intensive and extensive margins.

These three channels pin down the net effects of trade protection.

With unilateral trade protection by the South (higher $\tau$), import prices $P^M(i)$ rise, whereas domestic producer prices $P^P(i)$ fall. As a result of an expanded production line, both export and domestic production ranges, $\left[0, n^E\right]$ and $\left[0, n^P\right]$, increase. Under a unilateral tariff increase by the North (higher $\tau^*$), the export and domestic producer prices in the South, $P^E(i)$ and $P^P(i)$, drop. Again, the expansion of the production line induces a wider range of domestic production $\left[0, n^P\right]$, though its effect on the export range $\left[0, n^E\right]$ is ambiguous. Because both the production length the domestic production range expand, the range of imports is ambiguous. Nonetheless, either type of trade protection leads to higher aggregate technology of domestic producers $\widetilde{A}$ and average productivity $Y/\widetilde{X}$.

A trade war thus lengthens the production line and the domestic production range in the South. The effect on the range of exports and imports, its effect on the volumes and the values of exports and imports remain a quantitative question, to which we now turn.

# 5 Quantitative Analysis

In this section we calibrate our model and perform several policy experiments.

## 5.1 Data

For the quantitative analysis, we use the data from Socio-Economic Accounts (SEA) and World Input-Output Tables (WIOT) in Release 2013 (Timmer et al., 2015).

### 5.1.1 Grouping of Countries and Industries

Since we consider the trade between the North and the South, we divide all countries (40 countries and one RoW) into two groups: high-income countries (the North, or N) and low-income countries (the South, or S) according to the World Bank's 2002 country income classifications. The grouping of countries is listed in Table A1 in the Appendix. Moreover, we focus on the manufacturing sector ($M$) in which we include 14 industries as shown in Table A2 in the Appendix. All the other industries that are not included are referred to as the non-manufacturing sector ($O$), which is purely for the purpose of aggregation in the GRAS algorithm.

### 5.1.2 Price Deflators

Data in the WIOT are expressed in current dollars and at constant prices of the previous year (World IO Tables PYP) which are combined to derive the data series from 1995 to 2009 at 1995 constant prices (in 1995 USD). Such data include gross outputs, value added, and total final demand by industry and country. This yields the row totals and column totals for the inputs and outputs of all intermediate goods transactions at 1995 constant prices. To compute each value of intermediate goods transactions at 1995 prices requires additional work – specifically, we use the Generalized RAS (GRAS) algorithm to figure out all cells to match row and column totals (see the Appendix for details). Based on these, we can compute the aggregated WIOT by groups of countries and industries. Tables A3 and A4 in the Appendix display the converted WIOT at 1995 constant prices for the years of 1996 and 2006, respectively.

### 5.1.3 Values and Price Indexes of Intermediate Goods

By aggregating the converted WIOT at 1995 constant prices, we have obtained the real values of intermediate goods from the North to the South, from the South to the North, and within the South. That is, for each year, we have the South's real value of imported intermediate goods from the North, denoted $ImV_t$, the South's real value of non-traded intermediate goods production, denoted $DomV_t$, and the South's real value of exported intermediate goods to the North, denoted $ExV_t$, all at 1995 constant prices. The averages of these variables over the years from 1995 to 2009 are summarized in Table 1.

From the two sets of price series, one at current and another at constant 1995 prices, we obtain all the price deflators of gross outputs and all intermediate goods in the North and the South. We then calculate the price indexes of intermediate goods $ImV_t$, $DomV_t$, and $ExV_t$ by dividing price deflator associated with each cell of intermediate goods by its corresponding gross output deflator for each year (i.e., relative prices). Lastly, we take the average of the years from 1995 to 2009 for the purpose of calibration. Table 2 summarizes these averaged prices.

Table 1: Values of Intermediate Goods

| Variable | Description |
|---|---|
| $ImV_{data}$ | South's average real value of imported intermediate goods from the North |
| $DomV_{data}$ | South's average real value of non-traded intermediate goods production |
| $ExV_{data}$ | South's average real value of exported intermediate goods to the North |

Note: All values are at 1995 constant prices of USD

Table 2: Price Indexes of Intermediate Goods

| Variable | Description | Source |
|---|---|---|
| $p^M_{data}$ | average price of South's imported intermediate goods from the North | $\frac{Im\_deflator}{S\_GO\_deflator}$ |
| $p^P_{data}$ | average price of South's non-traded intermediate goods | $\frac{Dom\_deflator}{S\_GO\_deflator}$ |
| $p^E_{data}$ | average price of South's exported intermediate goods to the North | $\frac{Ex\_deflator}{N\_GO\_deflator}$ |

Note: $Im\_deflator$, $Dom\_deflator$, $Ex\_deflator$ are South's price deflator of imported intermediate goods, of non-traded intermediate goods, of exported intermediate goods, respectively. $S\_GO\_deflator$ stands for South's gross output deflator, and $N\_GO\_deflator$ represents North's gross output deflator.

### 5.2 Calibration

First we pin down the parameter value that can be directly imputed from data. The unit capital requirement $\eta$ is imputed by

$$\eta = \frac{CAP_{cp}}{DomV_{cp} + ExV_{cp}} = 0.9663$$

where $CAP_{cp}$ is South's aggregate nominal value of the capital compensation in the manufacturing sector; $DomV_{cp}$ is South's nominal value of non-traded intermediate goods production, and $ExV_{cp}$ is South's nominal value of exported intermediate goods to the North.

Given that we only have limited number of observed moments, we must assign pre-set values to several model parameters. To begin, we choose standard values in the literature by setting the market discount rate as 2.5% and the tariff rates in the North and the South as 5% and 15%, respectively. In the benchmark, we ignore international technology spillover. To be consistent with Moore's law, we set the technology obsolescence rate at 0.462.[10] In the benchmark, we set the degree of variety substitution to be $\gamma = 0.5$. That is, we consider a general quadratic form of final good production in which intermediate goods are Pareto substitutes. The value of $\gamma$ is so chosen that Condition S is met and that the markup ratio is in line with the literature.[11] Sensitivity analysis will be performed for $\gamma$ to rise or fall by 50%. We further assume that labor share of intermediate goods production is 4 times as much as R&D labor share. To satisfy Condition N under the above preset values of $(\rho, \nu)$, we thus set the intermediate goods production labor share as 0.4 and the R&D labor share 0.1. Moreover, we assume the North's technology scales and gradients are 25% higher than the counterparts in the South. Finally, to yield nondegenerate ranges of export,

[10] The law of motion of $A(i)$ in continous time representation gives $dA(t,i) = [I_A(i) - \nu A(t,i)]\,dt$, where $I_A(i) = [(1-\varsigma)\,\psi_t(i) + \varsigma\psi^*_t(i)]\,H_t(i)^\mu$. Moore's law then implies that under $I_A(i) = 0$, that is, when $A(t,i) = A(0,i)\exp(-\nu t)$, it takes $t = 18$ months such that $A(t,i) = \frac{1}{2}A(0,i)$.

[11] The markup ratio in the aggregate economy is often taken as 1.5 (cf. Hsieh and Klenow 2009), whereas at the industrial level it falls into the range from 1.15 to 2.15 (cf. Blaum, LeLarge and Peters 2018).

domestic production and import, we set the foreign technology gradient to be 0.05 (under which the domestic technology gradient becomes 0.04) and set the intermediate choke price to be twice as much as the average world price of the intermediate goods (the world price $p^*$ can be measured by $(1+\tau^*)\, p^E_{data}$ according to (11)). We summarize these parameters in Table 3.

Table 3: Imputed and Pre-set Parameters

| Category | Parameter | Description | Value |
|---|---|---|---|
| Imputed | $\eta$ | unit capital requirement | 0.9663 |
| Pre-set | $\rho$ | market discount rate | 0.025 |
| | $\tau^*$ | foreign tariff | 0.05 |
| | $\tau$ | tariff | 0.15 |
| | $\varsigma$ | international technology spillover | 0 |
| | $\nu$ | technology obsolescence rate | 0.462 |
| | $\gamma$ | degree of intermediate goods substitution | 0.5 |
| | $\theta$ | intermediate goods production labor share | 0.4 |
| | $\mu$ | R&D labor share | 0.1 |
| | $\overline{\psi}^*/\overline{\psi}$ | ratio of foreign to domestic technology scales | 1.25 |
| | $\delta^*$ | foreign technology gradient | 0.05 |
| | $\delta^*/\delta$ | ratio of foreign to domestic technology gradient | 1.25 |
| | $\overline{p}$ | intermediate goods choke price | $2(1+\tau^*)p^E_{data} = 4.3029$ |

The remaining parameters to be calibrated are the final good productivity $\alpha$, final production sophistication parameter $\beta$, unit labor requirement for coordinating production $\phi$, intermediate goods price gradient $b$, and technology scale parameter $\overline{\psi}$. These 5 parameters are calibrated to fit 4 observed ratios. Specifically, these ratios are obtained from three value variables and three price variables: $ImV$, $DomV$, and $ExV$ (South's average real import value of intermediate goods from the North, average real value of non-traded intermediate goods production, and average real export value of intermediate goods to the North); $p^M$, $p^P$, and $p^E$ (average prices of South's imported intermediate goods from the North, South's non-traded intermediate goods, and South's exported intermediate goods to the North). Given the value of the wage per employee $w$ that is imputed from the SEA data, the corresponding expressions of these 6 key variables are given as follows:

$$
\begin{aligned}
ImV\left(\alpha,\beta,\phi,b,\overline{\psi}\right) &= \int_{n^P}^{M}(1+\tau)p^*(i)x^M(i)di, \\
DomV\left(\alpha,\beta,\phi,b,\overline{\psi}\right) &= \int_0^{n^E}\frac{p^*(i)}{1+\tau^*}x^E(i)di+\int_{n^E}^{n^P}p(i)x^P(i)di \\
ExV\left(\alpha,\beta,\phi,b,\overline{\psi}\right) &= \int_0^{n^E}\frac{p^*(i)}{1+\tau^*}y^E(i)di-\int_0^{n^E}\frac{p^*(i)}{1+\tau^*}x^E(i)di,
\end{aligned}
$$

$$
\begin{aligned}
p^M(\alpha,\beta,\phi,b,\overline{\psi}) &= \frac{ImV\left(\alpha,\beta,\phi,b,\overline{\psi}\right)}{ImQ(\alpha,\beta,\phi,b,\overline{\psi})}, \\
p^P(\alpha,\beta,\phi,b,\overline{\psi}) &= \frac{DomV(\alpha,\beta,\phi,b,\overline{\psi})}{DomQ(\alpha,\beta,\phi,b,\overline{\psi})}, \\
p^E(\alpha,\beta,\phi,b,\overline{\psi}) &= \frac{ExV\left(\alpha,\beta,\phi,b,\overline{\psi}\right)}{ExQ(\alpha,\beta,\phi,b,\overline{\psi})},
\end{aligned}
$$

where the volumes of imports, domestic production and exports are, respectively, $ImQ(\alpha,\beta,\phi,b,\overline{\psi}) = \int_{n^P}^{M}x^M(i)di$, $DomQ(\alpha,\beta,\phi,b,\overline{\psi}) = \int_{n^E}^{n^P}x^P(i)di+\int_0^{n^E}x^E(i)di$, and $ExQ(\alpha,\beta,\phi,b,\overline{\psi}) = \int_0^{n^E}y^E(i)di-\int_0^{n^E}x^E(i)di$. The moments used as the targets for the calibration are two relative values and two relative prices: $\frac{ExV}{DomV}$, $\frac{ImV}{DomV}$, $\frac{p^E}{p^P}$, and $\frac{p^M}{p^P}$, which are all functions of the five parameters $(\alpha,\beta,\phi,b,\overline{\psi})$. Basically, we calibrate $\left(\alpha,\beta,\phi,b,\overline{\psi}\right)$ by minimizing the distances between model moments and data moments using quadratic loss function with equal weight under nonnegativity constraints. The details and the imputed values are reported in the Appendix.

For the data moments, since the values in WIOT are in basic prices for producers and all international flows are expressed in free-on-board prices, we need to make adjustments to let the data moments be consistent with their model counterparts. The details of the adjustments are reported in Table A6 in the Appendix. The calibration results are reported in Table 4 from which we can see that the fit is reasonable. The minimizing distance algorithm delivers calibrated parameter values, which are summarized in Table 5.

Three remarks are in order. First, in our benchmark calibrated economy, regularity conditions, Conditions S, N, and C, are all satisfied and $x^E(0) > 0$. Second, $\beta-\gamma > 0$, so there is a production sophistication effect in which a longer production line is more productive. Third, Condition E is met as well, implying a strong extensive margin effect for the overall length of the production line to play a dominant role.

In this calibrated benchmark economy, the computed ranges of exports, nontraded intermediate goods and imports turn out to be: $[0,n^E] = [0,4.95]$, $[n^E,n^P] = [4.95,16.16]$ and $\left[n^P,M\right] = [16.16,19.53]$, respectively. Thus, all trading and production ranges are nondegenerate. The corresponding quantities and values of intermediate exports and imports are $ExQ = 0.2313$, $ImQ = 0.5668$, $ExV = 0.9158$, $ImV = 1.9188$, respectively. The aggregate intermediate goods

Table 4: Model Fit

| Variable | Data | Model |
|---|---|---|
| $\frac{ImV}{DomV}$ | 0.3622 | 0.3603 |
| $\frac{ExV}{DomV}$ | 0.1728 | 0.1719 |
| $\frac{p^M}{p^P}$ | 1.0113 | 0.9232 |
| $\frac{p^E}{p^P}$ | 1.1001 | 1.0800 |

Table 5: Calibrated Parameters

| Parameter | Description | Value |
|---|---|---|
| $\alpha$ | final good productivity | 5.11185 |
| $\beta$ | final production sophistication | 4.69010 |
| $\phi$ | unit labor requirement for coordinating production | 0.33574 |
| $b$ | intermediate goods price gradient | 0.07567 |
| $\overline{\psi}$ | technology scale parameter | 0.06827 |

demand and production turn out to be $\widetilde{X} = 2.0193$ and $\widetilde{X}^P \equiv \int_0^{n^E} y^E(i)di + \int_{n^E}^{n^P} y^P(i)di = 1.6838$. Thus, total intermediate trade dependence is about half, i.e., $\frac{ExQ+ImQ}{\widetilde{X}^P} = 0.474$. The average markup of domestic non-exporting producers is about 36%, so the average markup ratio $\frac{\tilde{\Lambda}}{n^P} \equiv [n^E\Lambda_0 + \int_{n^E}^{n^P} \Lambda(i)di]/n^P = 1.3604$ falls in the range obtained in the literature (cf. Hsieh and Klenow, 2009; and Blaum, LeLarge and Peters, 2018). While aggregate and average technology used by domestic producers are $\widetilde{A} = 2.4836$ and $\frac{\widetilde{A}}{n^P} = 0.1536$, respectively, the computed final good output is $Y = 11.8612$ and the corresponding productivity measure is $\frac{Y}{\widetilde{X}} = 5.8739$. Finally the total value added in the South is $VA = 10.8582$ and the value-added ratio is $\frac{VA}{\widetilde{X}} = 5.3772$.[12] With population normalized to one, individual consumption $IC$ is thus given by,

$$\begin{aligned} IC &= c + p^s s = I - KI = VA - \eta \int_0^{n^P} y(i)di = VA - \eta\widetilde{X}^P \\ &= 10.8582 - 0.9663 \cdot 1.6838 = 9.2311, \end{aligned}$$

and the consumption ratio is $\frac{IC}{\widetilde{X}} = 4.5714$, which is a unit free measure of consumer welfare in the South.

[12] The computation of value added is described in the Appendix.

### 5.3 Policy Experiments

We conduct several policy experiments to examine the impact of trade policy on trade patterns, output, productivity, and markup. Specifically, we analyze the effects of trade wars by considering the following two scenarios: (i) a trade war in which $\tau$ increases to 20% and $\tau^*$ increases to 12%; (ii) a trade war in which $\tau$ increases to 25% and $\tau^*$ increases to 25%. The first scenario represents the situation of the current trade war between the U.S. and China, and the second one represents an escalation of the trade war.

In the current US-China trade war, under Section 301 of the Trade Act of 1974, the U.S. has imposed import tariffs of 25% on roughly $250 billion of its imports from China until now.[13] China has also immediately retaliated by increasing tariffs on $110 billion of imports from the U.S. (tariffs of 25%, 20%, 10%, and 5%). To calculate the trade-weighted average tariffs, we use the 2018 US-China bilateral trade data from the COMTRADE data set of the United Nations at the 6-digit product level of the Harmonized System (HS). We find that the weighted average import tariff in the US is 11.57% and that in China is 19.05%. Hence, we set $\tau = 20\%$ and $\tau^* = 12\%$ as the first policy experiment of trade wars. These estimates are close to the calculation in Bown (2019) (19.6% and 12.4%, respectively, see Figure 4 in the paper).[14] In addition to the ongoing trade war, the U.S. president has also threatened to escalate the trade war by putting tariffs on the remaining $300 billion (tariffs of 25%) Chinese imports that currently don't have tariffs. If these threats become actions, China will probably increase its existing tariffs on the U.S. merchandise to 25% in response to this escalation of the trade war. Accordingly, we set $\tau = \tau^* = 25\%$ to represent this scenario of an escalated trade war.[15]

The results of the experiments are reported in Table 6(a) and (b). From Propositions 1 and 2, we learn that an increase in either tariff under a trade war would lead to a longer production line $M$ and higher aggregate intermediate goods demand $\widetilde{X}$. Moreover, either also results in an increase in $n^P$. Thus, in our experiments (i) and (ii), the percentage increases in the production length $[0, M]$ and the domestic production range $\left[0, n^P\right]$ are large, exceeding 20%. However, in our trade war experiments, South intermediate firms out-compete North firms, thus resulting in a larger expansion in domestic production range than the production line. This subsequently leads

[13] It is worth noting that nearly 90% of US imports of intermediate inputs from China were subject to special protection in the trade war during 2018-2019 (Bown, 2019). It is strikingly different from the conventional wisdom that governments always avoid applying protection to imports of intermediate inputs, because their domestic firms need access to cheaper inputs to maintain competitiveness advantages (Bown and Crowley, 2016).

[14] Bown (2019) presents two series of average applied tariffs: the simple average applied tariff, which is calculated by weighting equally the applied MFN tariff of each of the roughly 5,000 HS6-digit products, and the trade-weighted average applied tariff, where the weights are the trading partner's product-level exports to the world.

[15] Ossa (2014) estimates that countries would impose tariffs of almost 60% in a fully-escalated trade war. Appling this to the current US-China trade war, Ossa (2019) suggests that there is still substantial room for escalation.

to a narrower import range. (Since the increase in $n^P$ is larger than $M$, the net effect on $M - n^P$ is negative.) From Figures 5-2 and 5-3, the increases in $\tau$ and $\tau^*$ would generate conflicting effects on $n^E$. Our quantitative results suggest that the foreign trade protection effect is stronger than domestic protection in North imports of intermediate goods. That is, the South export range $\left[0, n^E\right]$ shrinks. Quantitatively, both export and import ranges are narrowed by more than 10%. These extensive margin effects combined with intensive margin and extensive-intensive mixed domestic aggregate intermediate demand effects generate sizable negative effects on export volume and value and import volume and value.

It is worth noting that, although imports decrease, a longer production line implies that marginal imports are embedded with better technologies. As a result, in either scenario of the trade war, the South gains in average technology used ($\widetilde{A}/n^P$) and average final good productivity ($Y/\widetilde{X}$), as well as in value added ($VA$) and consumption ($IC$) ratios, $VA/\widetilde{X}$ and $IC/\widetilde{X}$, though these gains are quantitatively small.

Table 6(a): Policy Experiments – (i) Current Trade War

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.9536 | 16.1648 | 19.5215 | 3.3603 | 2.0193 | 1.6838 | 2.4836 | 0.1536 | 1.3604 | 11.8612 | 5.8739 |
| After trade war | 4.3484 | 20.6955 | 23.6092 | 2.9137 | 2.4785 | 2.1479 | 3.3503 | 0.1619 | 1.4547 | 14.8316 | 5.9841 |
| difference | -0.6052 | 4.5307 | 4.0840 | -0.4466 | 0.4592 | 0.4641 | 0.8667 | 0.0082 | 0.0943 | 2.9704 | 0.1102 |
| % change | -12.2% | 28.0% | 20.9% | -13.3% | 22.7% | 27.6% | 34.9% | 5.4% | 6.9% | 25.0% | 1.9% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2313 | 0.5668 | 0.9158 | 1.9188 | 10.8582 | 5.3771 | 9.2311 | 4.5714 |
| After trade war | 0.1705 | 0.5012 | 0.6357 | 1.5755 | 13.8918 | 5.6049 | 11.8163 | 4.7675 |
| difference | -0.0607 | -0.0657 | -0.2801 | -0.3433 | 3.0336 | 0.2278 | 2.5851 | 0.1961 |
| % change | -26.3% | -11.6% | -30.6% | -17.9% | 27.9% | 4.2% | 28.0% | 4.3% |

Next, we see that, in the second scenario of an escalated trade war, all effects are amplified. There are particularly strong negative effects on exports and export range $n^E$, and smaller decreases in imports and the import range $M - n^P$. Nevertheless, there are still positive effects on average technology ($\frac{\widetilde{A}}{n^P}$), average productivity ($\frac{Y}{\widetilde{X}}$), and value added ($VA$) due to the expansion of the production line $M$. Overall, we can see that the escalated trade war leads to the expansion in the production line and domestic production range but the shrinkage in export and import ranges. Thus, as a result of trade war, there is import substitution in intermediate goods with more domestic production replacing imports. We can also see that the escalated trade war may not hurt the South

Table 6(b): Policy Experiments – (ii) Escalated Trade War

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.9536 | 16.1648 | 19.5251 | 3.3603 | 2.0193 | 1.6838 | 2.4836 | 0.1536 | 1.3604 | 11.8612 | 5.8739 |
| After trade war | 0.5903 | 24.5756 | 27.3144 | 2.7388 | 2.9278 | 2.4611 | 4.1218 | 0.1677 | 1.5683 | 17.8293 | 6.0896 |
| difference | -4.3633 | 8.4108 | 7.7893 | -0.6215 | 0.9085 | 0.7773 | 1.6382 | 0.0141 | 0.2079 | 5.9681 | 0.2157 |
| % change | -88.1% | 52.0% | 39.9% | -18.5% | 45.0% | 46.2% | 66.0% | 9.2% | 15.3% | 50.3% | 3.7% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2313 | 0.5668 | 0.9158 | 1.9188 | 10.8582 | 5.3771 | 9.2311 | 4.5714 |
| After trade war | 0.0061 | 0.4729 | 0.0211 | 1.3793 | 16.4711 | 5.6257 | 14.0929 | 4.8134 |
| difference | -0.2251 | -0.0939 | -0.8947 | -0.5396 | 5.6129 | 0.2486 | 4.8618 | 0.2420 |
| % change | -97.3% | -16.6% | -97.7% | -28.1% | 51.7% | 4.6% | 52.7% | 5.3% |

much because of the expansion in the production line and that goods being imported are all with superior technology. Therefore, should one remove the extensive margin effect as in the conventional literature, we would expect a fall in both technology and productivity in the South.

In summary, while trade wars cause both exports and imports to fall, the production line expands and aggregate technology rise accompanied by a modest increase in average technology and average productivity due to a switch from scale to scope in intermediate goods usage. As a result, value-added in the South turns out to be higher despite the detrimental trade effects. Next we decompose the trade war effects on exports and imports, and then we look into the scale-scope components through counterfactual analysis to better understand the respective roles of import-induced technological advancement and domestic technology upgrading.

### 5.4 Decomposition Analysis

We look at the effects of tariffs on trade by decomposing the effects into extensive and intensive margins as well as the mixed margin.[16] The results are reported in Table 7. First, begin with exports. A trade war leads to a decrease in exports on the extensive margin due lower $n^E$. This is the case because the negative effect of $\tau^*$ on $n^E$ dominates the positive effect of $\tau$ in our benchmark calibration in which the variety substitution between intermediate goods $\gamma$ is 0.5. We will demonstrate later that trade wars may lead to higher $n^E$ if the variety substitution is sufficiently high. Second, as expected, the intensive margin effect on exports is negative. The increase in

[16] The mixed margin effects are due to changes in domestic net aggregate demand for intermediate goods, $\widetilde{X}$.

foreign tariff leads to decreases in the exports of each variety with decreases in the supply ($y^E(i)$) and increases the domestic demand ($x^E(i)$). Finally, there is also an effect caused by the change in aggregate intermediate goods demand. Accompanied by a longer production line, the aggregate intermediate goods demand becomes higher. Since an increase in $\widetilde{X}$ will suppress domestic demand for the exportables, it will raise the exports of each variety given that the supply is not affected. Note that this effect is a mixture of both extensive and intensive margins, as the increase in $\widetilde{X}$ here is induced by longer production line, while it in turn raises the exports of each variety. In other words, the detrimental effect of a trade war on exports is dampened by the mixed margin effects via $\widetilde{X}$. Moreover, from Lemma 1, an increase in aggregate intermediate goods demand by domestic firms raises price elasticities of intermediate demands for all varieties. This increased responsiveness therefore magnifies the contribution of the mixed margin in mitigating the trade war damage on exports by about 78% (= 296.4%/381.2%).

Next look at the change in imports. There is no intensive margin effect because the increase in the domestic production range $n^P$ is large enough to induce a full shift of the varieties imported to the South. In other words, the trade war results in a large extensive margin effect so that the South will import an entirely new range of varieties which embody better technology. The decomposition analysis indicates that the decrease in exports is mainly attributed to strong intensive margin effects while modestly offset by the aggregate intermediate goods demand effects. The decrease in imports is solely from the extensive margin effects because of the absence of the intensive margin.

Table 7: Decomposition – (i) Current Trade War

| | | Source | | |
|---|---|---|---|---|
| | Total | Extensive | Mixed ($\widetilde{X}$) | Intensive |
| $\triangle$ $ExV$ | $-0.2801$ | $-0.0425$ | 0.8300 | $-1.0676$ |
| $\triangle$ $ImV$ | $-0.3433$ | $-0.3433$ | 0 | 0 |
| Contribution | | | | |
| Decrease in $ExV$ | 100% | 15.2% | $-296.4$% | 381.2% |
| Decrease in $ImV$ | 100% | 100% | 0% | 0% |

### 5.5 Counterfactual Analysis

Recall that there two channels to improve technology in the South: investing in technology upgrading by domestic intermediate producers and importing technology embodied in intermediate imports from the North. We next determine which of the two channels play a more significant role in mitigating the detrimental effects of a trade war. To do this, we perform two counterfactual exercises: (i) Scenario 1: shut down domestic investment in technology upgrading and (ii) Scenario 2: disable changes in production length and trade ranges, which wipes out technology advancement via intermediate imports.

We first perform counterfactual scenario 1. In this case, the intermediate production technology $A(i)$ is not allowed to be adjusted, while other variables are free to change. That is, $A(i)$ stays as in the benchmark case before a trade war:

$$A^0(i) = \begin{cases} A^{E^0}(i) \equiv \overline{A}\psi(i)L^{E^0}(i)^{\mu} & \forall\ i \in [0, n^E] \\ A^{P^0}(i) \equiv \overline{A}\psi(i)L^{P^0}(i)^{\mu} & \forall\ i \in [n^E, n^P] \end{cases}$$

where the superscript $^0$ denotes values before a trade war. Remaining details of the derivations are described in the Appendix. We then conduct counterfactual scenario 2, where we fix the range variables $\{n^E, n^P, M\}$ as in the benchmark case but allow for domestic technology upgrading. Finally, we shut both channels, which removes all the technological factors.

Results are in Table 8. Note that due to the presence of covariance effects between the two technological factors, the sum of the contributions from counterfactual scenarios 1 and 2 may not equal the overall contributions. This requires rescaling to produce the relative contributions.

First note that with no domestic investment in technology upgrading and intermediate imports from the North, the length of production, the ranges of production and trade, and average technology are all unchanged, remaining at the before-trade war values. However, exports, imports and their values all change. When we allow technology to change it turns out that intermediate importing (scenario 2) has a much stronger effect across the board than does domestic investment in technology upgrading.

Note that, in response to a trade war, domestic intermediate producers have less incentive to invest in their own lower-end technology because of lower demand by the North and less competitiveness for exporting. That is, a trade war induces a shift from domestic upgrading to technology-embedded importing. This adds to the dominance of the extensive margin effects, together constituting a key *scale-scope* trade-off driving the main results. Because domestic upgrading is purely from the scale aspect but intermediate importing contains the scope perspective, our decomposition analysis shows a clear dominance of the contribution of the latter channel toward explaining changes in trade, average productivity, and value added and consumption ratios. These results suggest that although the trade war discourages domestic technology upgrading in the South, it leads to another form of technology upgrading from expanding the production line and climbing upward along the value chain. Moreover, the scope effects dominate the scale effects in mitigating the damage of a trade war, suggesting the important role of intermediate goods trade in technology transfers between the North and the South.

Table 8: Counterfactual Analysis – (i) Current Trade War

| Benchmark: | $n^E$ | $n^P$ | $M$ | $M - n^P$ | $\frac{\widetilde{A}}{n^P}$ |
|---|---|---|---|---|---|
| before trade war | 4.9536 | 16.1648 | 19.5251 | 3.3603 | 0.1536 |
| after trade war | 4.3484 | 20.6955 | 23.6092 | 2.9137 | 0.1619 |
| trade war effect | −0.6052 | 4.5307 | 4.0840 | −0.4466 | 0.0082 |
| After trade war w/o tech changes | 4.9536 | 16.1648 | 19.5251 | 3.3603 | 0.1536 |
| Contributions in mitigating trade war | −0.6052 | 4.5307 | 4.0840 | −0.4466 | 0.0082 |
| 1. domestic investment | −0.2950 | −0.1765 | −0.1857 | −0.0092 | −0.0030 |
| 2. production & trade ranges | −0.6052 | 4.5307 | 4.0840 | −0.4466 | 0.0092 |
| Share of contributions | 100% | 100% | 100% | 100% | 100% |
| 1. domestic investment | 32.8% | −4.1% | −4.8% | 2.0% | −48.9% |
| 2. production & trade ranges | 67.2% | 104.1% | 104.8% | 98.0% | 148.9% |

| Benchmark: | $ExV$ | $ImV$ | $\frac{Y}{\widetilde{X}}$ | $\frac{VA}{\widetilde{X}}$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|
| before trade war | 0.9158 | 1.9188 | 5.8739 | 5.3771 | 4.5714 |
| after trade war | 0.6357 | 1.5755 | 5.9841 | 5.6049 | 4.7675 |
| trade war effect | −0.2801 | −0.3433 | 0.1102 | 0.2278 | 0.1961 |
| After trade war w/o tech changes | 0.0798 | 1.4688 | 5.8702 | 5.2085 | 4.4505 |
| Contributions in mitigating trade war | 0.5559 | 0.1067 | 0.1139 | 0.3964 | 0.3170 |
| 1. domestic investment | −0.1213 | 0.0042 | −0.0116 | −0.0666 | −0.0527 |
| 2. production & trade ranges | 0.5722 | 0.1041 | 0.1146 | 0.4066 | 0.3207 |
| Share of contributions | 100% | 100% | 100% | 100% | 100% |
| 1. domestic investment | −26.9% | 3.9% | −11.2% | −19.6% | −19.7% |
| 2. production & trade ranges | 126.9% | 96.1% | 111.2% | 119.6% | 119.7% |

### 5.6 Sensitivity Analysis

In this section, we examine the effects of a trade war in the presence of international technology spillovers or partial tariff pass through. We then perform a number of robustness checks by changing the pre-set parameter values.

#### 5.6.1 Technology Spillover

Recall that our model allows for international technology spillover captured by parameter $\varsigma$. In the quantitative exercises above, we have set $\varsigma = 0$ as the benchmark. In Table 9 below, we report the results assuming technology spillover at $\varsigma = 0.2$. Overall, the changes in the quantitative results are essentially minor compared to our benchmark.

Table 9: Sensitivity − Technology Spillover $\varsigma = 0.2$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) =$ (5.03597, 4.60540, 0.33233, 0.08158, 0.06540)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.5275 | 14.8901 | 17.9743 | 3.0842 | 1.8678 | 1.5585 | 2.2898 | 0.1538 | 1.3541 | 10.7505 | 5.7558 |
| After trade war | 3.7307 | 18.9500 | 21.6165 | 2.6665 | 2.2972 | 1.9764 | 3.0634 | 0.1617 | 1.4473 | 13.4586 | 5.8587 |
| difference | -0.7967 | 4.0599 | 3.6422 | -0.4177 | 0.4294 | 0.4179 | 0.7736 | 0.0079 | 0.0932 | 2.7081 | 0.1029 |
| % change | -17.6% | 27.3% | 20.3% | -13.5% | 23.0% | 26.8% | 33.8% | 5.1% | 6.9% | 25.2% | 1.8% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2131 | 0.5224 | 0.8444 | 1.7744 | 9.8205 | 5.2579 | 8.3146 | 4.4516 |
| After trade war | 0.1400 | 0.4608 | 0.5231 | 1.4607 | 12.5210 | 5.4505 | 10.6112 | 4.6192 |
| difference | -0.0731 | -0.0616 | -0.3212 | -0.3136 | 2.7004 | 0.1926 | 2.2966 | 0.1676 |
| % change | -34.3% | -11.8% | -38.0% | -17.7% | 27.5% | 3.7% | 27.6% | 3.8% |

#### 5.6.2 Tariff Pass Through

The benchmark model assumes no pass-through on the producer's price in the North. Now suppose that there is partial pass-through which means that the after-trade-war import price in the North can be specified as

$$P^{M^{*'}}(i) = \varpi(1 + \tau^{*'})P^E(i) \equiv \tilde{p}^*(i)$$

where after-trade-war values are labelled with superscript $'$ and $\varpi$ is given by

$$\varpi = \left(\frac{1+\tau^{*'}}{1+\tau^*}\right)^{-a}, \; 0 < a < 1.$$

When $a = 1$, there is no pass-through effect, which is our benchmark case. When $a = 0$, there is complete pass-through that fully offsets the increase in tariff. In the exercise below, we assume a partial pass-through at the rate of 10%, i.e., $a = 0.9$. Straightforward manipulation implies that with partial pass-through, the North intermediate producer price (which is the delivered price) is:

$$\tilde{p}^*(i) \equiv P^{M^{*'}}(i) = (1+\tau^*)^a(1+\tau^{*'})^{1-a}P^E(i) = \frac{p^*(i)}{(1+\tau^*)^{1-a}(1+\tau^{*'})^{a-1}}.$$

Thus, after the trade war, the export and import prices in the South become:

$$\begin{aligned} P^{E'}(i) &\equiv \frac{\tilde{p}^*(i)}{1+\tau^{*'}} = \frac{\bar{p}-bi}{(1+\tau^*)^{1-a}(1+\tau^{*'})^a}, \\ P^{M'}(i) &\equiv (1+\tau')\tilde{p}^*_t(i) = (1+\tau')\left(\frac{1+\tau^{*'}}{1+\tau^*}\right)^{1-a}(\bar{p}-b\cdot i). \end{aligned}$$

Table 10: Sensitivity – Tariff Pass Through $a = 0.9$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) =$ (5.11185, 4.69010, 0.33574, 0.07567, 0.06827)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.9536 | 16.1648 | 19.5251 | 3.3603 | 2.0193 | 1.6838 | 2.4836 | 0.1536 | 1.3604 | 11.8612 | 5.8739 |
| After trade war | 4.8252 | 20.9890 | 23.8403 | 2.8513 | 2.4816 | 2.1978 | 3.4151 | 0.1627 | 1.4523 | 14.8556 | 5.9862 |
| difference | -0.1284 | 4.8242 | 4.3152 | -0.5090 | 0.4623 | 0.5140 | 0.9316 | 0.0091 | 0.0919 | 2.9944 | 0.1123 |
| % change | -2.6% | 29.8% | 22.1% | -15.1% | 22.9% | 30.5% | 37.5% | 5.9% | 6.8% | 25.2% | 1.9% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2313 | 0.5668 | 0.9158 | 1.9188 | 10.8582 | 5.3771 | 9.2311 | 4.5714 |
| After trade war | 0.2079 | 0.4918 | 0.7776 | 1.5445 | 14.0888 | 5.6772 | 11.9651 | 4.8214 |
| difference | -0.0233 | -0.0750 | -0.1381 | -0.3744 | 3.2306 | 0.3001 | 2.7339 | 0.2500 |
| % change | -10.1% | -13.2% | -15.1% | -19.5% | 29.8% | 5.6% | 29.6% | 5.5% |

With partial tariff pass-through, the $P^E(i)$ schedule rotates upward. This generates a positive effect on exporting range, thereby offsetting part of the detrimental effects of a trade war on exports. From Table 10, we see that the effects on all other indicators are negligible.

### 5.6.3 Robustness Checks

We have conducted robustness checks with respect to reasonable changes in the technology spillover parameter ($\varsigma$), technology returns-to-scale parameters ($\theta$ and $\mu$), technology obsolescence rate ($\nu$), foreign-to-domestic technology gradient ratio ($\delta^*/\delta$), as well as the variety substitution parameter ($\gamma$). Among all, $\gamma$ turns out to be the only pre-set parameter that results in noticeable changes to which we now turn to. The full set of the sensitivity analysis results is reported in the Appendix.

In our benchmark calibration with $\gamma = 0.5$, both scenarios of trade war lead to reductions in export range ($n^E$) and export quantity and value ($ExQ$ and $ExV$), due primarily to the negative effects from the increased $\tau^*$ dominating the positive effects from the increased $\tau$. In this subsection, we will demonstrate that the effects of trade war on exports crucially depend on the variety substitution parameter $\gamma$. Specifically, we recalibrate the model economy using various values of $\gamma$ to demonstrate the properties. Within the range of rise or fall by 50%, i.e., $\gamma \in [0.25, 0.75]$, our main conclusions remain valid qualitatively, as seen in Table 11.

To understand the results, we compute the degree of production sophistication ($\beta - \gamma$), which rises with $\gamma$. Recall from Lemma 2 that a stronger production sophistication effect reduces the price elasticities of intermediate demands. Because demand becomes less responsive, the detrimental effect of a trade war on exports is dampened. Other than this effect, all the other effects are quantitatively small.

Table 11: Effects of Variety Substitution ($\gamma$)

| $\gamma$ | $\beta-\gamma$ | $\triangle\ n^E$ | $\triangle\ n^P$ | $\triangle\ M$ | $\triangle\ (M\text{-}n^P)$ | $\triangle\ ExQ$ | $\triangle\ ImQ$ | $\triangle\ ExV$ | $\triangle\ ImV$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 2.0489 | -100% | 25.5% | 13.4% | -10.5% | -100% | -7.2% | -100% | -9.6% |
| 0.25 | 2.8565 | -22.6% | 25.2% | 18.4% | -14.1% | -41.5% | -12.3% | -44.7% | -17.9% |
| 0.5 | 4.1901 | -12.2% | 28.0% | 20.9% | -13.3% | -26.3% | -11.6% | -30.6% | -17.9% |
| 0.75 | 4.9342 | -0.6% | 30.7% | 23.3% | -12.5% | -7.2% | -10.9% | -13.0% | -18.0% |
| 0.8 | 4.9552 | 2.3% | 31.4% | 23.9% | -12.2% | -2.4% | -10.6% | -8.5% | -18.0% |

| $\gamma$ | $\beta-\gamma$ | $\triangle\ \widetilde{X}$ | $\triangle\ \widetilde{X}^P$ | $\triangle\ \widetilde{A}$ | $\triangle\ \frac{\widetilde{A}}{n^P}$ | $\triangle\ \frac{\widetilde{\Lambda}}{n^P}$ | $\triangle\ Y$ | $\triangle\ \frac{Y}{\widetilde{X}}$ | $\triangle\ VA$ | $\triangle\ \frac{VA}{\widetilde{X}}$ | $\triangle\ IC$ | $\triangle\ \frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 2.0489 | 10.7% | 38.8% | 35.3% | 7.8% | 2.1% | 11.3% | 0.5% | 24.7% | 12.7% | 23.0% | 11.1% |
| 0.25 | 2.8565 | 22.0% | 25.2% | 32.0% | 5.4% | 7.7% | 24.0% | 1.7% | 25.8% | 3.1% | 25.9% | 3.2% |
| 0.5 | 4.1901 | 22.7% | 27.6% | 34.9% | 5.4% | 6.9% | 25.0% | 1.9% | 27.9% | 4.2% | 28.0% | 4.3% |
| 0.75 | 4.9342 | 22.9% | 29.9% | 37.9% | 5.5% | 6.4% | 25.5% | 2.1% | 29.7% | 5.5% | 29.6% | 5.5% |
| 0.8 | 4.9552 | 22.8% | 30.5% | 38.6% | 5.5% | 6.3% | 25.5% | 2.1% | 30.0% | 5.9% | 30.0% | 5.8% |

One may further inquire: what happens if we change the value of $\gamma$ beyond the ±50% range?

We thus report the cases with $\gamma = 0.2$ and $\gamma = 0.8$. Consider the case of $\gamma = 0.2$, which implies $\beta - \gamma = 2.0489$. Because intermediate demands become highly responsive, the detrimental effect of a trade war leads to a corner solution, wiping out South exports entirely. On the contrary, when $\gamma = 0.8$ and $\beta - \gamma = 4.9552$, we have irresponsive intermediate demands. In this case, a trade war results in export range expansion. Despite this, both the quantity and the value of exports decrease.

# 6 Concluding Remarks

We have constructed a dynamic North-South trade model to show that the effects of a trade war on the South are mitigated by importing intermediates goods embedded with more advanced technologies and by upgrading its own technology via domestic R&D. As a result, by shifting from scale to scope despite sizeable detrimental effects on the volume and the value of trade, a trade war actually leads to modest increases in average technology and productivity as well as value-added and consumption ratios. This research points to the importance of considering the dynamic effects of trading intermediate goods that embody technology as well as domestic investment in intermediate good technology when it comes to understanding and quantitatively assessing the effects of trade wars.

## References


[1] Alviarez, Vanessa, Javier Cravino, and Natalia Ramondo (2019), "Accounting for Cross-country Productivity Differences: New Evidence from Multinational Firms," Working Paper.

[2] Becker, Gary S. and Kevin M. Murphy (1992), "The Division of Labor, Coordination Costs, and Knowledge," *Quarterly Journal of Economics* 107, 1137-1160.

[3] Blaum, Joaquin , Claire Lelarge, and Michael Peters (2018), "The Gains from Input Trade with Heterogeneous Importers," *American Economic Journal: Macroeconomics* 10, 77-127.

[4] Bown, Chad P. (2019), "The 2018 US-China Trade Conflict After 40 Years of Special Protection," *Peterson Institute for International Economics Working Paper* 19-7.

[5] Bown, Chad P. and Meredith A. Crowley (2016), "The Empirical Landscape of Trade Policy," In *The Handbook of Commercial Policy, volumn 1A*, ed. Kyle Bagwell and Robert W. Staiger. Amsterdam, Netherlands: Elsevier, North Holland, 3-108.

[6] Diamond, Peter A. and James A. Mirrlees (1971), "Optimal Taxation and Public Production I: Production Efficiency," *American Economic Review* 61, 8-27.

[7] Ethier, Wilfred (1982), "National and International Returns to Scale in the Modern Theory of International Trade," *American Economic Review* 73, 389-405.

[8] Flam, Harry and Elhanan Helpman (1987), "Vertical Product Differentiation and North-South Trade," *American Economic Review* 77, 810-822.

[9] Grossman, Gene M. and Elhanan Helpman (1995), "Trade Wars and Trade Talks," *Journal of Political Economy*, 103, 675-708.

[10] Halpern, László, Miklós Koren, and Adam Szeidl (2015), "Imported Inputs and Productivity," *American Economic Review* 105, 3660–3703.

[11] Hsieh, Chang-Tai and Peter J. Klenow (2009), "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics* 124, 1403–1448.

[12] Hummels, David, Jun Ishii, and Kei-Mu Yi (2001), "The Nature and Growth of Vertical Specialization in World Trade," *Journal of International Economics* 54, 75–96.

[13] Impullitti, G. and O. Licandro (2018), "Trade, Firm Selection, and Innovation: the Competition Channel," *Economic Journal* 128, 189-229.

[14] Johnson, Harry G (1953), "Optimum Tariffs and Retaliation," *Review of Economic Studies* 21, 142-153.

[15] Junius, T. and J. Oosterhaven (2003), "The Solution of Updating or Regionalizing a Matrix with Both Positive and Negative Elements," *Economic Systems Research*, 15, 87–96.

[16] Keller, W. (2000), "Do Trade Patterns and Technology Flows Affect Productivity Growth?" *World Bank Economic Review* 14, 17-47.

[17] Kennan, John and Raymond Riezman (1988), "Do Big Countries Win Tariff Wars?" *International Economic Review* 29, 81-85.

[18] Lenzen, M., R. Wood, and B. Gallego (2007), "Some Comments on the GRAS Method," *Economic Systems Research*, 19, 461–465.

[19] Mayer, Wolfgang (1981), "Theoretical Considerations on Negotiated Tariff Adjustments," *Oxford Economic Papers* 33, 135-153.

[20] Melitz, Marc and I. P. Ottaviano (2008), "Market Size and Productivity," *Review of Economic Studies* 75, 295-360.

[21] Ossa, Ralph (2014), "Trade Wars and Trade Talks with Data," *American Economic Review*, 104, 4104-4146.

[22] Ossa, Ralph (2019), "The Costs of A Trade War," In *Trade War: The Clash of Economic Systems Endangering Global Prosperity*, ed. Meredith A. Crowley. London, UK: CEPR Press, 45-49.

[23] Peng, Shin-Kun, Jacques-F. Thisse, and Ping Wang (2006), "Economic Integration and Agglomeration in a Middle Product Economy," *Journal of Economic Theory* 131, 1-25.

[24] Ramanarayanan, Ananth (2014), "Imported Inputs and the Gains from Trade," Working Paper.

[25] Riezman, Raymond (1982), "Tariff Retaliation from a Strategic Viewpoint," *Southern Economic Journal* 48, 583-593.

[26] Samuelson, Paul A. (1947), *Foundations of Economic Analysis*, Harvard University Press, Cambridge, MA.

[27] Shubik, M. (1959), *Strategy and Market Structure*, Wiley, New York.

[28] Shubik, M. and R. Levitan (1980), *Market Structure and Behavior*, Harvard University Press, Cambridge.

[29] Singh, N. and X. Vives (1984), "Price and Quantity Competition in a Diffrentiated Duopoly," *Rand Journal of Economics* 15, 546-554.

[30] Temurshoev, U., R. E. Miller, and M. C. Bouwmeester (2013), "A Note on the GRAS Method," *Economic Systems Research*, 25, 361–367.

[31] Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer, and G. J. de Vries (2015), "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production," *Review of International Economics*, 23, 575-605.

[32] Vives, X. (1985), "On the effciency of Bertrand and Cournot equilibria with product diffrentation," *Journal of Economic Theory*, 36, 166-175.

[33] Yi, Kei-Mu (2003), "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy* 111, 52-102.

Figure 1. Determination of Intermediate Goods Allocation

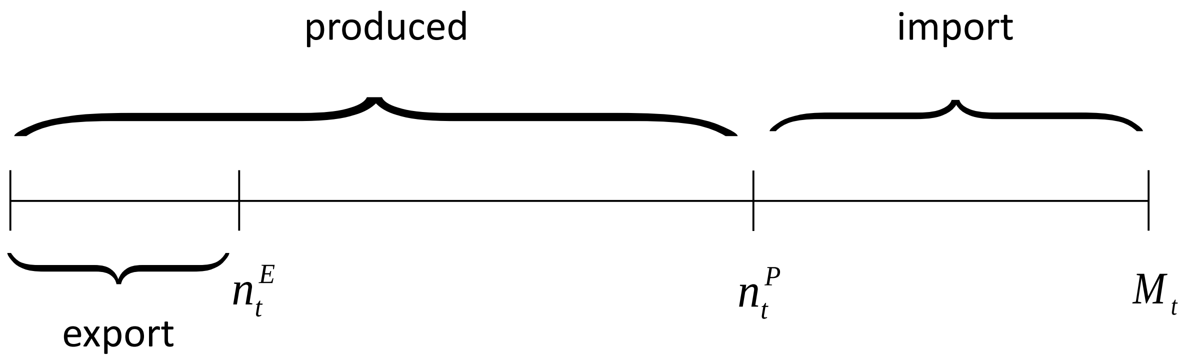



Figure 2. Labor Allocation

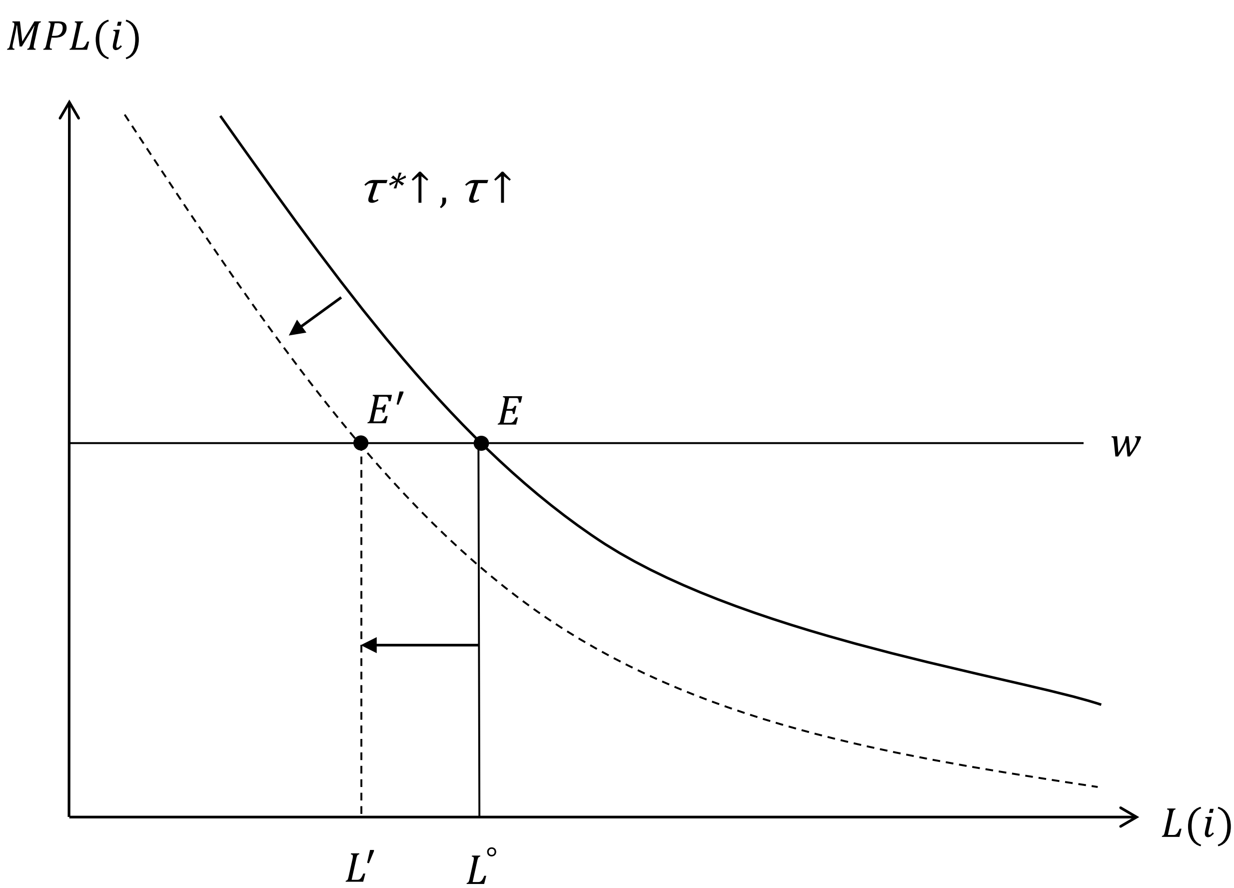


Figure 3. Determination of Length of Production Line

3(a)

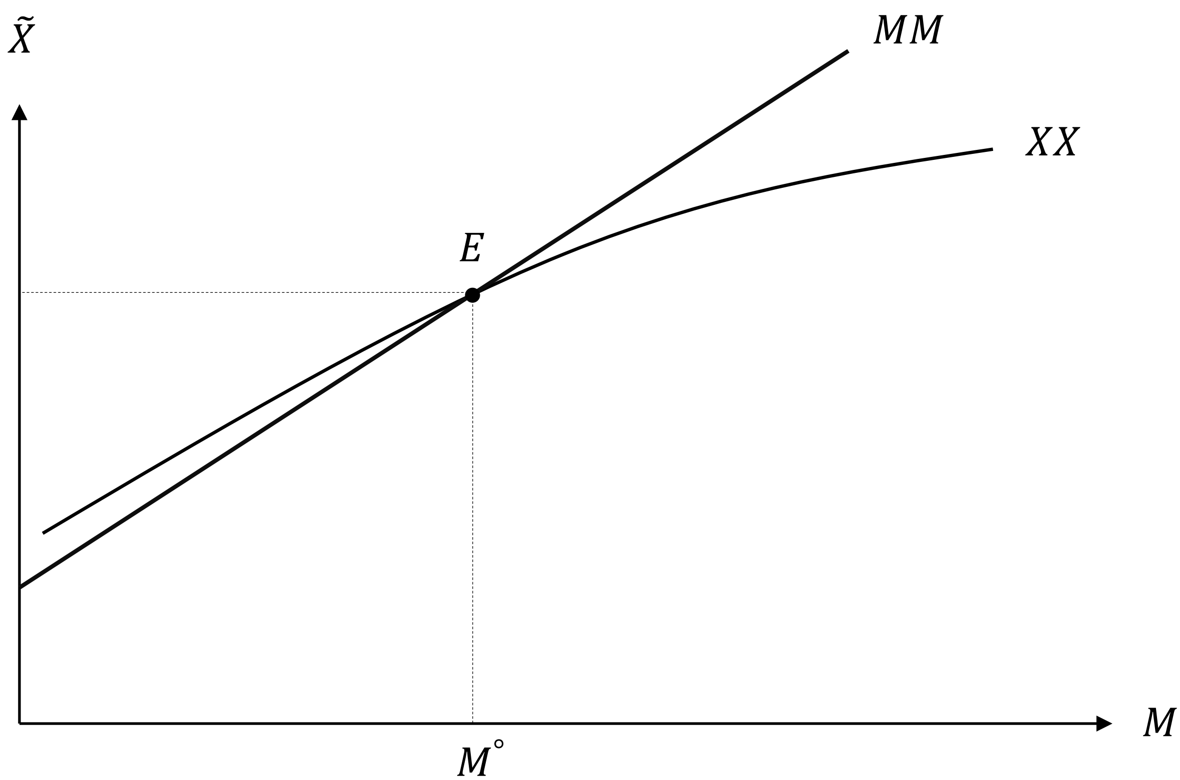


Figure 3. Determination of Length of Production Line

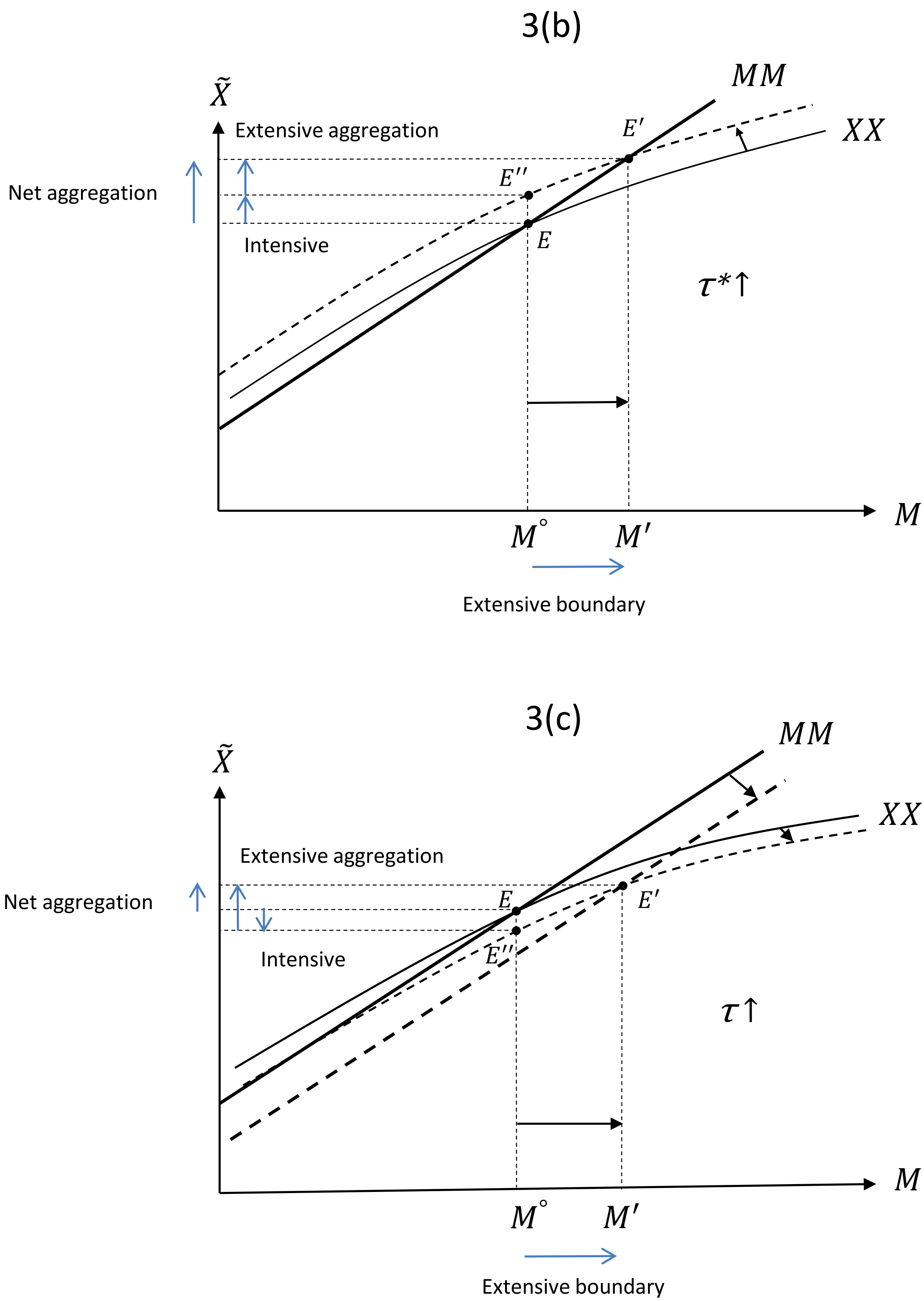
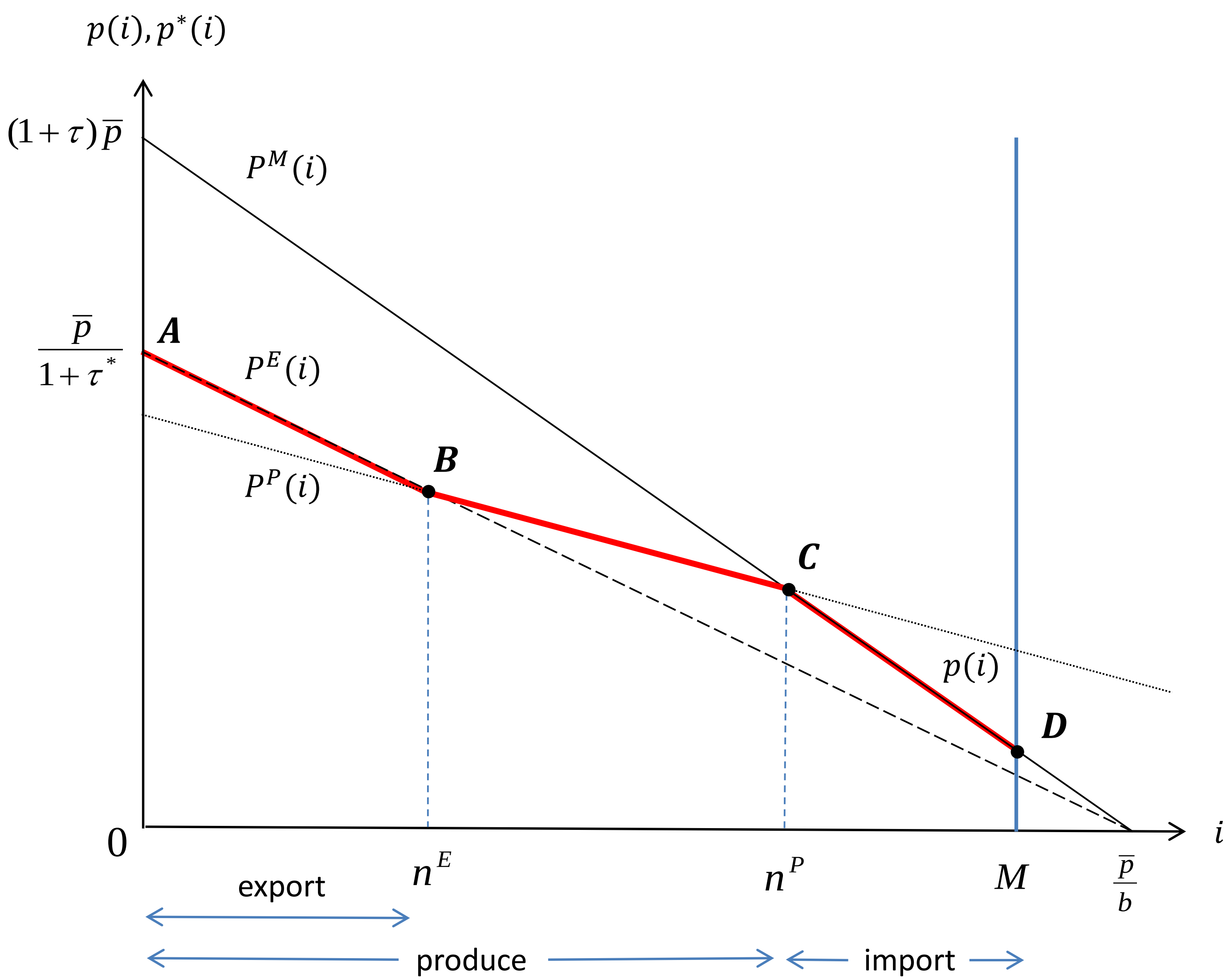
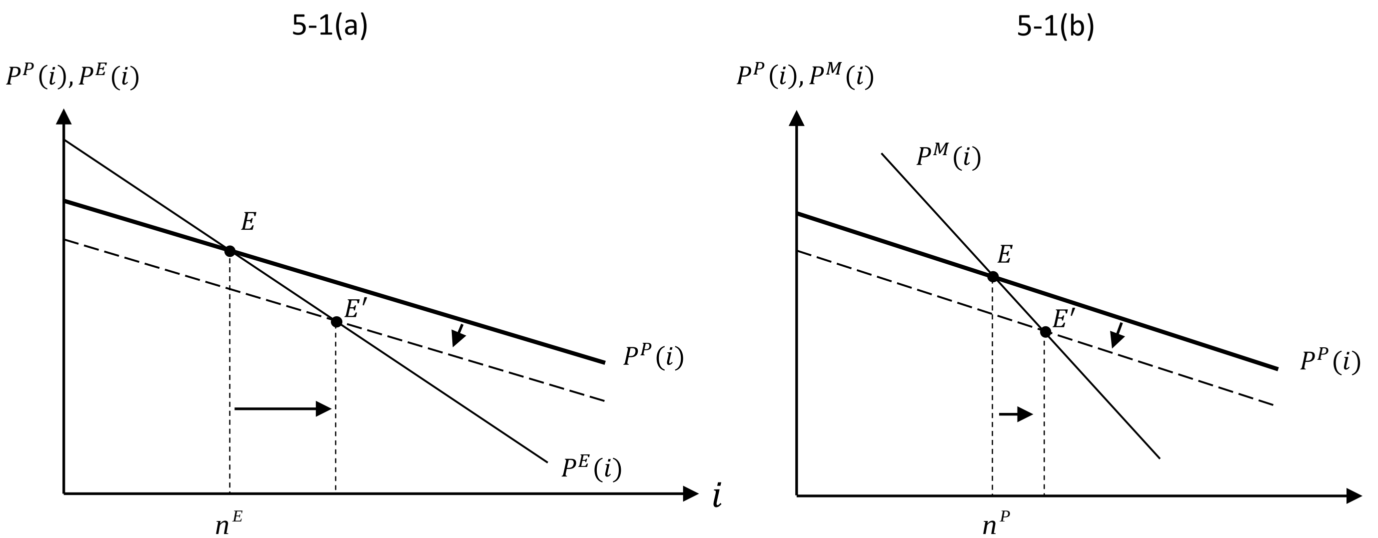


Figure 4. Technology Choice and Trade in Intermediate Goods



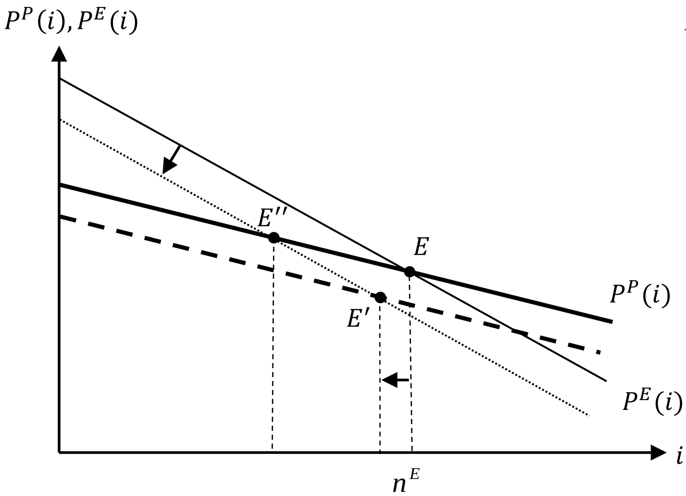
# Figure 5. Determination of Technology and Trade Pattern



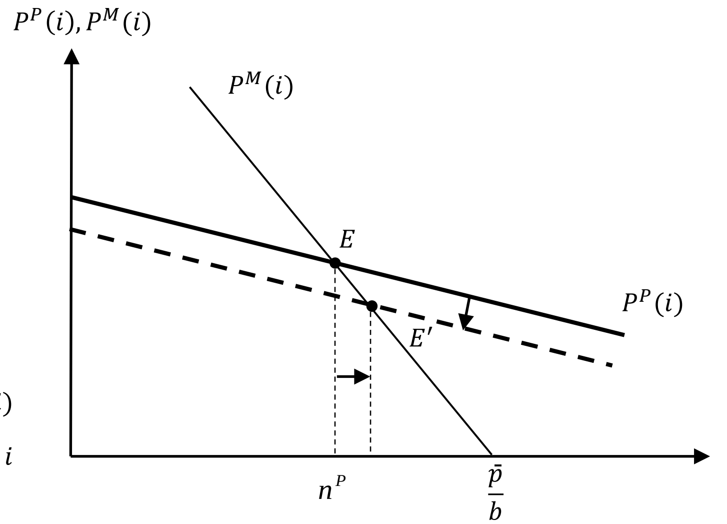

$M \uparrow$
$\Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^E \uparrow$

$M \uparrow$
$\Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^P \uparrow$

5-2(a)




5-2(b)




$\tau^{*}\uparrow$

◎ $P^E(i) \downarrow \Rightarrow n^E \downarrow$ ($E$ to $E''$)

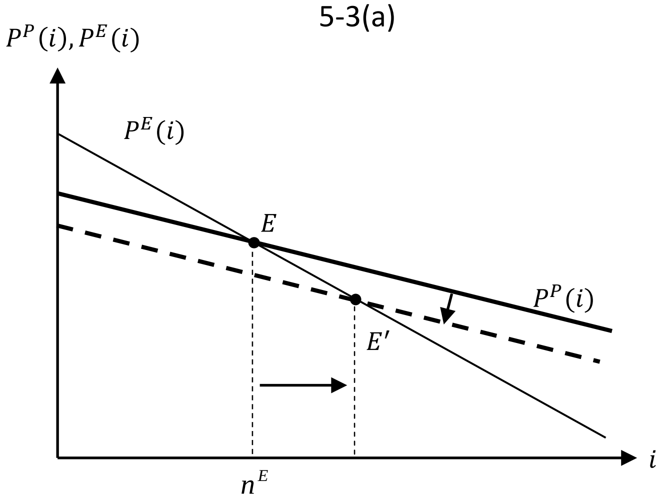
◎ $M \uparrow \Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^E \uparrow$ ($E''$ to $E'$)

$\tau^{*}\uparrow$

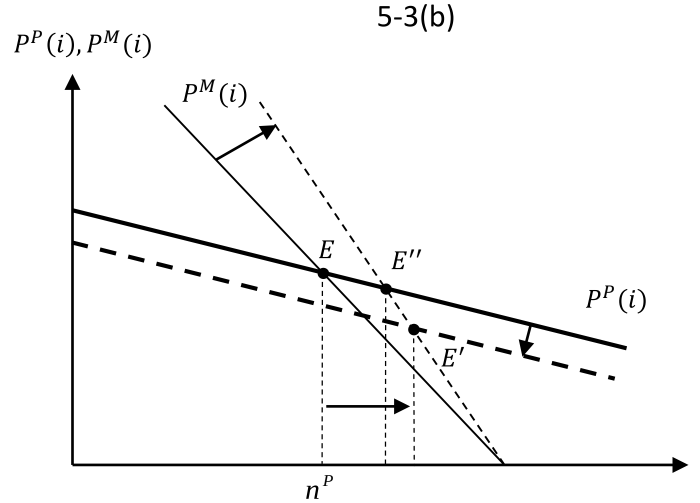
◎ $P^M(i)$ unchanged

◎ $M \uparrow \Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^P \uparrow$

5-3(a)




5-3(b)




$\tau\uparrow$

◎ $P^E(i)$ unchanged

◎ $M \uparrow \Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^E \uparrow$

$\tau\uparrow$

◎ $P^M(i)$ rotates upward $\Rightarrow n^P \uparrow$ ($E$ to $E''$)

◎ $M \uparrow \Rightarrow \tilde{X} \uparrow \Rightarrow P^P(i) \downarrow \Rightarrow n^P \uparrow$ ($E''$ to $E'$)

# Appendix

**(A Major Portion of the Appendix is Not Intended for Publication)**

In the appendix, we provide detailed mathematical derivations, data documentation, as well as detailed calibration strategies and quantitative results.

## Mathematical Appendix

In this Appendix, we provide detailed mathematical derivations of some expressions in the main text.

*Derivation of the first-order condition with respect to the production line* (9): Using Leibniz's rule, the final good producing firm's first-order condition with respect to $M_t$ can be derived as:

$$\frac{dY_t}{dM_t} = \left[\alpha - \frac{\beta - \gamma}{2} x_t(M_t) - \gamma \widetilde{X}_t\right] x_t(M_t) = w\phi + p_t(M_t)x_t(M_t)$$

which can then be combined with the last expression of relative demand function to yield (9).

*Derivation of the first-order conditions with respect to the two labor demand* (14) and (15): The intermediate firm's marginal revenue can be derived as:

$$\begin{aligned}\frac{d[(p_t(i) - \eta)y_t(i)]}{dy_t(i)} &= p_t(i) - \eta + y_t(i)\frac{dp_t(i)}{dy_t(i)} \\ &= p_t(i) - \eta - \beta y_t(i) \\ &= p_t(i) - \eta - \beta A_t(i)L_t(i)^{\theta}\end{aligned}$$

where $p_t(i)$ can be substituted out with (11). Using this expression, we can obtain (14) and (15).

*Derivation of the steady-state level of domestic technology* (21): Since $V_{A_t}(i) = V_{A_{t+1}}(i)$, we can also use (20) to rewrite (18) as:

$$V_A = \frac{(1+\rho)wH(i)^{1-\mu}}{\mu\psi(i)}, \quad i \in [0, n^P]$$

which can then be plugged into (19) to obtain:

$$\frac{(\rho+\nu)w}{\mu\psi(i)}\left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1-\mu}{\mu}} = \left[\frac{p^*(i)}{1+\tau^*} - \eta\right] L(i)^{\theta}, \quad \forall \ \ i \in [0, n^E]$$

Using (17) to simplify the above expression, we have:

$$\frac{(\rho+\nu)w}{\mu\psi(i)}\left[\frac{\nu A(i)}{\psi(i)}\right]^{\frac{1-\mu}{\mu}}=\frac{wL(i)}{\theta A(i)}$$

Manipulating this last expression gives (21).

*Derivation of aggregate intermediate goods* (30): Using (27)-(28) and (13), we derive:

$$\begin{aligned}
\widetilde{X} &= \int_0^{n^P}\overline{A}\psi(i)L(i)^{\theta+\mu}di+\int_{n^P}^{M}z(i)di-\int_0^{n^E}z^*(i)di \\
&= \int_0^{n^P}\overline{A}\psi(i)L(i)^{\theta+\mu}di+\int_{n^P}^{M}\frac{\alpha-\gamma\widetilde{X}-(1+\tau)p^*(i)}{\beta-\gamma}di \\
&\quad -\int_0^{n^E}\left[\overline{A}\psi(i)\left[L^E(i)\right]^{\theta+\mu}-\frac{\alpha-\gamma\widetilde{X}-\frac{p^*(i)}{1+\tau^*}}{\beta-\gamma}\right]di \\
&= \overline{A}\int_{n^E}^{n^P}\psi(i)\left[L^P(i)\right]^{\theta+\mu}di-\frac{1}{\beta-\gamma}\left[(1+\tau)\int_{n^P}^{M}p^*(i)di+\frac{1}{1+\tau^*}\int_0^{n^E}p^*(i)di\right] \\
&\quad +\frac{\alpha-\gamma\widetilde{X}}{\beta-\gamma}(M+n^E-n^P),
\end{aligned}$$

which can be rearranged to yield the $\widetilde{X}$ expression (30).

*Derivation of the intermediate good firms' markups*: By using (23) and (25), the maximized profit function for the intermediate-good firms $i\in[n^E,n^P]$ can be derived as:

$$\pi(i)=[\alpha-\gamma\widetilde{X}^{-i}-\eta-\beta x(i)]\overline{A}\psi(i)\left[L^P(i)\right]^{\theta+\mu}-wL^P(i)[1+(\nu\overline{A})^{\frac{1}{\mu}}]=\Lambda(i)wN(i)$$

where the intermediate producer $i$'s markup is given by (35). For $i\in[0,n^E]$, we can use (22) and (25) to obtain:

$$\begin{aligned}
\pi(i) &= \left[\frac{p^*(i)}{1+\tau^*}-\eta\right]\overline{A}\psi(i)\left[L^E(i)\right]^{\theta+\mu}-wL^E(i)[1+(\nu\overline{A})^{\frac{1}{\mu}}] \\
&= \overline{A}\psi(i)\left[L^E(i)\right]^{\theta+\mu}\left[\frac{p^*(i)}{1+\tau^*}-\eta\right]\{1-\theta[1+(\nu\overline{A})^{\frac{1}{\mu}}]\} \\
&= \Lambda_0 wN(i)
\end{aligned}$$

*Shifts of $XX$ and $MM$ loci*:

$$\widetilde{X}=\frac{\overline{A}\int_{n^E}^{n^P}\psi(i)\left[L^P\left(\widetilde{X},i\right)\right]^{\theta+\mu}di+\frac{\alpha}{\beta-\gamma}(M+n^E-n^P)-\frac{1}{\beta-\gamma}\left[(1+\tau)\int_{n^P}^{M}p^*(i)di+\frac{1}{1+\tau^*}\int_0^{n^E}p^*(i)di\right]}{1+\frac{\gamma}{\beta-\gamma}(M+n^E-n^P)}$$

where $\frac{\partial L^P(\widetilde{X},i)}{\partial \widetilde{X}} < 0$. Totally differentiating, we obtain:

$$\left.\frac{d\widetilde{X}}{d\tau^*}\right|_{XX} = \frac{\partial \widetilde{X}}{\partial \tau^*} + \frac{\partial \widetilde{X}}{\partial M}\frac{\partial M}{\partial \tau^*} + \frac{\partial \widetilde{X}}{\partial n^E}\frac{\partial n^E}{\partial \tau^*} + \frac{\partial \widetilde{X}}{\partial n^P}\frac{\partial n^P}{\partial \tau^*}$$

where

$$\frac{\partial \widetilde{X}}{\partial \tau^*} > 0, \quad \frac{\partial \widetilde{X}}{\partial M} > 0, \quad \frac{\partial \widetilde{X}}{\partial n^E} \lesseqgtr 0, \quad \frac{\partial \widetilde{X}}{\partial n^P} \lesseqgtr 0$$

indicating that the $XX$ locus shifts upward ($\frac{\partial \widetilde{X}}{\partial \tau^*} > 0$).

$$\alpha - \gamma\widetilde{X} - (1+\tau)p^*(M) = \sqrt{2(\beta-\gamma)w\phi}$$

Total differentiation implies

$$\left.\frac{d\widetilde{X}}{d\tau^*}\right|_{MM} = \frac{\partial \widetilde{X}}{\partial \tau^*} + \frac{\partial \widetilde{X}}{\partial M}\frac{\partial M}{\partial \tau^*} > 0$$

where

$$\frac{\partial \widetilde{X}}{\partial \tau^*} = 0, \quad \frac{\partial \widetilde{X}}{\partial M} > 0, \quad \frac{\partial M}{\partial \tau^*} > 0$$

indicating that the $MM$ locus would not shift ($\frac{\partial \widetilde{X}}{\partial \tau^*} = 0$).

$$\widetilde{X} = \frac{\overline{A}\int_{n^E}^{n^P} \psi(i)\left[L^P\left(\widetilde{X},i\right)\right]^{\theta+\mu} di + \frac{\alpha}{\beta-\gamma}(M+n^E-n^P) - \frac{1}{\beta-\gamma}\left[(1+\tau)\int_{n^P}^{M} p^*(i)di + \frac{1}{1+\tau^*}\int_0^{n^E} p^*(i)di\right]}{1+\frac{\gamma}{\beta-\gamma}(M+n^E-n^P)}$$

where $\frac{\partial L^P(\widetilde{X},i)}{\partial \widetilde{X}} < 0$. Totally differentiating, we obtain:

$$\left.\frac{d\widetilde{X}}{d\tau}\right|_{XX} = \frac{\partial \widetilde{X}}{\partial \tau} + \frac{\partial \widetilde{X}}{\partial M}\frac{\partial M}{\partial \tau} + \frac{\partial \widetilde{X}}{\partial n^E}\frac{\partial n^E}{\partial \tau} + \frac{\partial \widetilde{X}}{\partial n^P}\frac{\partial n^P}{\partial \tau}$$

where

$$\frac{\partial \widetilde{X}}{\partial \tau} < 0, \quad \frac{\partial \widetilde{X}}{\partial M} > 0, \quad \frac{\partial \widetilde{X}}{\partial n^E} \lesseqgtr 0, \quad \frac{\partial \widetilde{X}}{\partial n^P} \lesseqgtr 0$$

$$\alpha - \gamma\widetilde{X} - (1+\tau)p^*(M) = \sqrt{2(\beta-\gamma)w\phi}$$

Total differentiation implies

$$\left.\frac{d\widetilde{X}}{d\tau}\right|_{MM} = \frac{\partial \widetilde{X}}{\partial \tau} + \frac{\partial \widetilde{X}}{\partial M}\frac{\partial M}{\partial \tau}$$

where

$$\frac{\partial \widetilde{X}}{\partial \tau} < 0, \quad \frac{\partial \widetilde{X}}{\partial M} > 0$$

Thus for $\tau$ to have a positive effect on $\widetilde{X}$ (as in our quantitative model), it is required that the slope of the $XX$ locus is sufficiently steep (though still flatter than the $MM$ locus) and the shift in the $XX$ locus is relatively small compared to the shift in the $MM$ locus.

# Data, Calibration and Quantitative Results

## Grouping of Countries and Industries

See Table A1 for the grouping of countries and Table A2 for the grouping of industries.

Table A1: Country Grouping

| Group | Country Name | Code | Group | Country Name | Code |
|---|---|---|---|---|---|
| | Australia | AUS | | Brazil | BRA |
| | Austria | AUT | | Bulgaria | BGR |
| | Belgium | BEL | | China | CHN |
| | Canada | CAN | | Czech Republic | CZE |
| | Cyprus | CYP | | Estonia | EST |
| | Denmark | DNK | | Hungary | HUN |
| | Finland | FIN | | India | IND |
| | France | FRA | | Indonesia | IDN |
| | Germany | DEU | | Latvia | LVA |
| | Greece | GRC | | Lithuania | LTU |
| | Ireland | IRL | | Mexico | MEX |
| N | Italy | ITA | S | Poland | POL |
| | Japan | JAN | | Romania | ROU |
| | Korea, Republic of | KOR | | Russia | RUS |
| | Luxembourg | LUX | | Slovak Republic | SVK |
| | Malta | MLT | | Turkey | TUR |
| | Netherlands | NLD | | Rest of World | RoW |
| | Portugal | PRT | | | |
| | Slovenia | SVN | | | |
| | Spain | ESP | | | |
| | Sweden | SWE | | | |
| | Taiwan | TWN | | | |
| | United Kingdom | GBR | | | |
| | United States | USA | | | |

Table A2: Manufacturing Sector ($M$)

| | |
|---|---|
| 1. | Food, Beverages and Tobacco |
| 2. | Textiles and Textile Products |
| 3. | Leather, Leather and Footwear |
| 4. | Wood and Products of Wood and Cork |
| 5. | Pulp, Paper, Printing and Publishing |
| 6. | Coke, Refined Petroleum and Nuclear Fuel |
| 7. | Chemicals and Chemical Products |
| 8. | Rubber and Plastics |
| 9. | Other Non-Metallic Mineral |
| 10. | Basic Metals and Fabricated Metal |
| 11. | Machinery, Nec |
| 12. | Electrical and Optical Equipment |
| 13. | Transport Equipment |
| 14. | Manufacturing, Nec; Recycling |

**Price Deflators**

The well-known iterative RAS-procedure can be used to derive each value of intermediate goods transactions in 1995 prices. However, to deal with negative values frequently shown in the columns with change in inventories, the Generalized RAS (GRAS) algorithm is used to produce the WIOTs in pyp.[17] The GRAS procedure keeps running until the sums over cells in each row are very close to the exogenously given row totals and the same applies to cells in columns. In other words, both rows and columns have been scaled up or down by row- and column- specific factors upon completion of the procedure, and these cell-specific factors can be considered as cell-specific deflators. We therefore use Temurshoev, Miller, and Bouwmeester's (2013) GRAS program in MATLAB to produce the WIOTs in 1995 prices. Tables A3 and A4 below display the converted WIOTs at 1995 constant prices for the years of 1996 and 2006, respectively.

[17] The method is originally proposed by Junius and Oosterhaven (2003) and modified by Lenzen, Wood, and Gallego (2007).

#### Imputation of Exogenous Variable

For exogenous variable $w$, we impute the value from data by

$$w = \frac{L\_rcomp + H\_rcomp}{L\_norm + H\_norm}$$

where $L\_rcomp$ is the sum of the real values of low-skilled and middle-skilled labor compensation, and $H\_rcomp$ represents the real value of high-skilled labor compensation in the WIOD-SEA dataset. Other details are reported in Table A5.

Table A5: Imputation of Exogenous Variable

| Variable | Description | Value | Note |
|---|---|---|---|
| $w$ | Real wage rate at 1995 gross output price level | $0.1293 \cdot p_{1995}$ | trillion 1995 USD |
| $*\_rcomp$ | Real labor compensation of employees at 1995 gross output price level | | |
| $*\_norm$ | Normalized number of employees | | |

We have made adjustments for the purpose of normalization. For $*\_norm$ and $w$, we set the North's average number of employees (which is 62.95 millions people) to be 1. According to this, the South's average number of employees is normalized. Then, the South's real wage rate $w$ is calculated based on its normalized number of employees.

#### Adjustments of Data Moments

Table A6: Adjustments of Data Moments

| Variable | Imputation | Value | Note |
|---|---|---|---|
| $p^M_{data}$ | $(1+\tau)\,\frac{1}{15}\sum_{t=1995}^{2009}\left(\frac{ImV\_deflator_t}{S\_GO\_deflator_t}\right)$ | $(1+\tau)\cdot 0.8190 \cdot p_{1995}$ | |
| $p^P_{data}$ | $\frac{1}{15}\sum_{t=1995}^{2009}\left(\frac{DomV\_deflator_t}{S\_GO\_deflator_t}\right)$ | $0.9313 \cdot p_{1995}$ | |
| $p^E_{data}$ | $\frac{1}{15}\sum_{t=1995}^{2009}\left(\frac{ExV\_deflator_t}{N\_GO\_deflator_t}\right)$ | $1.0245 \cdot p_{1995}$ | |
| $DomV_{data}$ | $\frac{1}{15}\sum_{t=1995}^{2009}(DomV_t)$ | $2.6295 \cdot p_{1995}$ | trillion 1995 USD |
| $ImV_{data}$ | $(1+\tau)\,\frac{1}{15}\sum_{t=1995}^{2009}(ImV_t)$ | $(1+\tau)\cdot 0.8282 \cdot p_{1995}$ | trillion 1995 USD |
| $ExV_{data}$ | $\frac{1}{15}\sum_{t=1995}^{2009}(ExV_t)$ | $0.4544 \cdot p_{1995}$ | trillion 1995 USD |

Note: to ensure all endogenous variables to be positive values, we set 1995 price to be 2 (that is, $p_{1995} = 2$).

Table A3: Converted WIOT in 1995 prices for the year 1996

| in billions of 1995 USD | O sector in S | M sector in S | O sector in N | M sector in N | Final Demand in S | Final Demand in N |
|---|---|---|---|---|---|---|
| O sector in S group | 1,882.3862 | 1,338.4882 | 144.9894 | 179.8326 | 4,718.2512 | 48.5124 |
| M sector in S group | 979.0414 | 1,329.1775 | 112.4443 | 239.8340 | 1,638.3957 | 353.4203 |
| O sector in N group | 205.1002 | 144.4521 | 8,706.1898 | 3,485.4580 | 95.5290 | 18,088.3483 |
| M sector in N group | 214.1143 | 412.6163 | 2,838.9674 | 4,667.8541 | 437.2608 | 4,620.7593 |
| Total Intermaediates Input | 3,280.6421 | 3,224.7340 | 11,802.5910 | 8,572.9786 | 6,889.4367 | 23,111.0406 |
| Value Added | 5,031.8180 | 1,427.5792 | 18,922.4864 | 4,618.5935 | 147.2722 | 703.4063 |
| Gross Output | 8,312.4601 | 4,652.3132 | 30,725.0773 | 13,191.5721 | 0 | 0 |

Table A4: Converted WIOT in 1995 prices for the year 2006

| in billions of 1995 USD | O sector in S | M sector in S | O sector in N | M sector in N | Final Demand in S | Final Demand in N |
|---|---|---|---|---|---|---|
| O sector in S group | 3,226.8744 | 2,300.4255 | 301.7431 | 373.4782 | 7,287.2374 | 105.3818 |
| M sector in S group | 1,744.3221 | 3,787.4357 | 300.2138 | 672.7644 | 2,573.4512 | 1,054.5744 |
| O sector in N group | 382.2413 | 303.1711 | 12,516.7645 | 4,069.6596 | 180.5531 | 23,447.8001 |
| M sector in N group | 432.5384 | 1,272.8903 | 3,298.5719 | 5,849.8013 | 747.9967 | 5,722.2840 |
| Total Intermaediates Input | 5,785.9761 | 7,663.9226 | 16,417.2934 | 10,965.7035 | 10,789.2387 | 30,330.0405 |
| Value Added | 7,809.1643 | 2,468.8390 | 24,482.8962 | 6,358.3792 | 0 | 0 |
| Gross Output | 13,595.1404 | 10,132.7617 | 40,900.1896 | 17,324.0828 | 0 | 0 |

**Value Added**

The total value added $VA$ consists of the value added of final good production $VF$, value added of intermediate goods production $VI$, total wage income, and total capital income, which can be computed in the following ways, respectively.

$$
\begin{aligned}
VF &= \text{value of final output} \\
&\quad - \text{value of intermediate goods that are } \textit{domestically produced and used} - \text{value of imports} \\
&\quad - \text{value of coordination cost (final sector wage)} \\
VI &= \text{value of intermediate goods that are } \textit{domestically produced and used} + \text{value of exports} \\
&\quad - \text{wage cost (intermediate sector wage)} - \text{capital cost}
\end{aligned}
$$

Thus, the total value added $VA$ can be computed as follows:

$$
\begin{aligned}
VA &= VF + VI + \text{total wage income} + \text{total capital income} \\
&= \text{value of final output} + \text{value of exports} - \text{value of imports} \\
&= Y + ExV - ImV
\end{aligned}
$$

Three remarks are in order. First, $Y$ is both the value and the quantity, as final good is the numeraire with its price normalized. Second, this computation is by regarding capital cost as capital income, so total value added is equal to total profit income (from final and intermediate sector) plus total factor income (labor and capital). Third, the capital cost $\eta y(i)$ is the cost incurred in intermediate goods production, and the cost has been reflected in the price $p(i)$ – see equation (11) in the main text.

**Extensive and intensive margins**

- Extensive margin effects: through $M, n^P, n^E$ (boundary)
- Intensive margin effects: through $\tau, \tau^*$ on $x^E, x^M, y^E$ (quantity) and direct effects on prices
- Mixed margin effects: through $\widetilde{X}$ (domestic aggregate intermediate demand) on $x^E, x^M, y^E$

Recall that

$$\begin{aligned} ExV &= \int_0^{n^E} \frac{p^*(i)}{1+\tau^*}\, y^E(i) - \frac{p^*(i)}{1+\tau^*}\, x^E(i)\, di = \int_0^{n^E} \frac{p^*(i)}{1+\tau^*}\, \overline{A}\psi(i) L^E(i)^{\theta+\mu} di - \int_0^{n^E} \frac{p^*(i)}{1+\tau^*}\, x^E(i)\, di \\ ImV &= \int_{n^P}^{M} (1+\tau) p^*(i)\, x^M(i)\, di \end{aligned}$$

Also note that $\frac{\partial \widetilde{X}(\tau,\tau^*)}{\partial \tau} > 0$ and $\frac{\partial \widetilde{X}(\tau,\tau^*)}{\partial \tau^*} > 0$. We can thus decompose the pure extensive margin and the pure intensive margin, in addition to the mixed effect via domestic aggregate intermediate demand $\widetilde{X}$. Let superscripts $^0$ and $'$ denote the values before and after a trade war, respectively.

(i) Exports $\underline{ExV}$

Note that from the policy experiment results (Table 6(a) for the current trade war), export range shrinks: $n^{E'} < n^{E^0}$, and there is a decrease in exports of each variety: $z^{*'}(i) \equiv y^{E'}(i) - x^{E'}(i) < y^{E^0}(i) - x^{E^0}(i) \equiv z^{*^0}(i)$. The decomposition of the trade war effects on value of exports is thus:

$$\begin{aligned} \text{Extensive margin} &= -\int_{n^{E'}}^{n^{E^0}} \frac{p^*(i)}{1+\tau^{*0}} [y^{E^0}(i) - x^{E^0}(i)] di \\ \text{Mixed margin} &= -\int_0^{n^{E'}} \frac{p^*(i)}{1+\tau^{*'}} [x^{E'}(i) - x^{E''}(i)] di \end{aligned}$$

where $x^{E''}(i)$ is defined as $\frac{\alpha - \gamma \widetilde{X}(M^0) - \frac{p^*(i)}{1+\tau^{*'}}}{\beta - \gamma}$

and $\widetilde{X}(M^0)$ is the expression of $XX$ locus (30) evaluated at $M^0$

$$\begin{aligned} \text{Intensive margin} &= \int_0^{n^{E'}} \frac{p^*(i)}{1+\tau^{*'}} [y^{E'}(i) - x^{E'}(i)] - \frac{p^*(i)}{1+\tau^{*0}} [y^{E^0}(i) - x^{E^0}(i)] di - \text{mixed margin} \\ &= \int_0^{n^{E'}} \left[ \frac{p^*(i)}{1+\tau^{*'}}\, y^{E'}(i) - \frac{p^*(i)}{1+\tau^{*0}}\, y^{E^0}(i) \right] - \left[ \frac{p^*(i)}{1+\tau^{*'}}\, x^{E''}(i) - \frac{p^*(i)}{1+\tau^{*0}}\, x^{E^0}(i) \right] di \end{aligned}$$

(ii) Imports $\underline{ImV}$

According to the policy experiment results, there is a full shift of the import range: $M' > M^0$ and $n^{P'} > M^0$. Therefore, there is neither intensive margin nor mixed margin effect on imports. Thus, the trade war effects on value of imports is entirely attributed to extensive margin effects.

The general form of the decomposition of $ExV$ and $ImV$ is given as follows:

$$
\begin{aligned}
ExV &: \\
\text{Extensive margin} &= \begin{cases} \int_{n^{E^0}}^{n^{E'}} \frac{p^*(i)}{1+\tau^{*'}}[y^{E'}(i) - x^{E'}(i)]di & \text{if } n^{E^0} < n^{E'} \\ \int_{n^{E^0}}^{n^{E'}} \frac{p^*(i)}{1+\tau^{*0}}[y^{E^0}(i) - x^{E^0}(i)]di & \text{if } n^{E^0} > n^{E'} \end{cases} \\
\text{Mixed margin} &= -\int_0^{\min\{n^{E^0}, n^{E'}\}} \frac{p^*(i)}{1+\tau^{*'}} \left[x^{E'}(i) - x^{E''}(i)\right] \text{ where } x^{E''}(i) \text{ is defined as } \frac{\alpha - \gamma \widetilde{X}(M^0) - \frac{p^*(i)}{1+\tau^{*'}}}{\beta - \gamma} \\
& \text{and } \widetilde{X}(M^0) \text{ is the expression of } XX \text{ locus (30) evaluated at } M^0 \\
\text{Intensive margin} &= \int_0^{\min\{n^{E^0}, n^{E'}\}} \frac{p^*(i)}{1+\tau^{*'}}[y^{E'}(i) - x^{E'}(i)] - \frac{p^*(i)}{1+\tau^{*0}}[y^{E^0}(i) - x^{E^0}(i)]di - \text{mixed margin} \\
&= \int_0^{\min\{n^{E^0}, n^{E'}\}} \left[\frac{p^*(i)}{1+\tau^{*'}} y^{E'}(i) - \frac{p^*(i)}{1+\tau^{*0}} y^{E^0}(i)\right] - \left[\frac{p^*(i)}{1+\tau^{*'}} x^{E''}(i) - \frac{p^*(i)}{1+\tau^{*0}} x^{E^0}(i)\right] di
\end{aligned}
$$

$$
\begin{aligned}
ImV &: \\
\text{Extensive margin} &= \int_{M^0}^{M'} (1+\tau')p^*(i)\, x^{M'}(i)di - \int_{n^{P^0}}^{n^{P'}} (1+\tau^0)p^*(i)\, x^{M^0}(i)di \\
\text{Mixed margin} &= \begin{cases} \int_{n^{P'}}^{M^0} (1+\tau')p^*(i) \left[x^{M'}(i) - x^{M''}(i)\right] di & \text{if } n^{P'} < M^0 \\ 0 & \text{o.w.} \end{cases} \\
& \text{where } x^{M''}(i) \text{ is defined as } \frac{\alpha - \gamma \widetilde{X}(M^0) - (1+\tau')p^*(i)}{\beta - \gamma} \\
\text{Intensive margin} &= \begin{cases} \int_{n^{P'}}^{M^0} (1+\tau')p^*(i)\, x^{M'}(i) - (1+\tau^0)p^*(i)\, x^{M^0}(i)\, di & \text{if } n^{P'} < M^0 \\ 0 & \text{o.w.} \end{cases} - \text{mixed margin} \\
&= \begin{cases} \int_{n^{P'}}^{M^0} (1+\tau')p^*(i) x^{M''}(i) - (1+\tau^0)p^*(i)\, x^{M^0}(i)\, di & \text{if } n^{P'} < M^0 \\ 0 & \text{o.w.} \end{cases}
\end{aligned}
$$

**Counterfactual Analysis**

This section demonstrates the derivation details of the scenario in which the domestic technology investment channel is shut down. The other scenario of disabling changes in production length and trade ranges is straightforward.

When we shut down the domestic investment in technology upgrading channel, intermediate production technology is not allowed to be adjusted and hence stays in previous steady state level:

$$A^0(i) = \begin{cases} A^{E^0}(i) \equiv \overline{A}\psi(i)L^{E^0}(i)^{\mu} & \forall\ i \in [0, n^E] \\ A^{P^0}(i) \equiv \overline{A}\psi(i)L^{P^0}(i)^{\mu} & \forall\ i \in [n^E, n^P] \end{cases}$$

To solve for relevant variables, we need to rewrite the following equations. The subscript 0 denotes the solutions in the previous steady state, and subscript $noA$ denotes variables *without domestic technology investment*. For $i \in [n^E, n^P]$,

$$MPL_{noA}(i) = \theta A^{P^0}(i)\left[L^P(i)\right]^{\theta-1}\left\{\alpha - \eta - \gamma\widetilde{X} - (2\beta-\gamma)A^{P^0}(i)\left[L^P(i)\right]^{\theta}\right\}$$

and the labor demand $L^P_{noA}(i)$ is solved by $MPL_{noA}(i) = w$:

$$\overline{A}\psi(i)\left[L^{P^0}(i)\right]^{\mu}\left[L^P_{noA}(i)\right]^{\theta-1}\left\{\alpha - \eta - \gamma\widetilde{X} - (2\beta-\gamma)\overline{A}\psi(i)\left[L^{P^0}(i)\right]^{\mu}\left[L^P_{noA}(i)\right]^{\theta}\right\} = w.$$

Thus, we can derive the producer price $P^P_{noA}(i)$ as

$$P^P_{noA}(i) = \alpha - \gamma\widetilde{X} - (\beta-\gamma)A^{P^0}(i)\left[L^P_{noA}(i)\right]^{\theta},$$

and the export range $n^E_{noA}$ is solved by $P^P_{noA}(n^E_{noA}) = P^E(n^E_{noA})$:

$$\alpha - \gamma\widetilde{X} - (\beta-\gamma)\overline{A}\psi\left(n^E_{noA}\right)\left[L^{P^0}\left(n^E_{noA}\right)\right]^{\mu}\left[L^P_{noA}(n^E_{noA})\right]^{\theta} = \frac{p^*(n^E_{noA})}{1+\tau^*}.$$

Also, the domestic production range $n^P_{noA}$ is solved by $P^P_{noA}(n^P_{noA}) = P^M(n^P_{noA})$:

$$\alpha - \gamma\widetilde{X} - (\beta-\gamma)\overline{A}\psi\left(n^P_{noA}\right)\left[L^{P^0}\left(n^P_{noA}\right)\right]^{\mu}\left[L^P_{noA}(n^P_{noA})\right]^{\theta} = (1+\tau)p^*(n^P_{noA}).$$

The $XX$ locus in this scenario is

$$\widetilde{X}_{noA} = \frac{\begin{array}{c}\displaystyle\int_{n^E_{noA}}^{n^P_{noA}} A^{P^0}(i)\left[L^P_{noA}(i)\right]^{\theta} di + \tfrac{\alpha}{\beta-\gamma}(M + n^E_{noA} - n^P_{noA}) \\ \displaystyle -\tfrac{1}{\beta-\gamma}\left[(1+\tau)\int_{n^P_{noA}}^{M_{noA}} p^*(i)di + \tfrac{1}{1+\tau^*}\int_0^{n^E_{noA}} p^*(i)di\right]\end{array}}{1 + \tfrac{\gamma}{\beta-\gamma}(M + n^E_{noA} - n^P_{noA})}$$

and the production length $M_{noA}$ is solved from the intersection of $XX$ and $MM$ loci:

$$\frac{1}{\gamma}\left[\alpha-(1+\tau)p^*(M_{noA})-\sqrt{2(\beta-\gamma)w\phi}\right]$$

$$= \frac{\int_{n_{noA}^E}^{n_{noA}^P} A^{P^0}(i)\left[L_{noA}^P(i)\right]^\theta di + \frac{\alpha}{\beta-\gamma}(M_{noA}+n_{noA}^E-n_{noA}^P) - \frac{1}{\beta-\gamma}\left[(1+\tau)\int_{n_{noA}^P}^{M_{noA}} p^*(i)di + \frac{1}{1+\tau^*}\int_0^{n_{noA}^E} p^*(i)di\right]}{1+\frac{\gamma}{\beta-\gamma}(M_{noA}+n_{noA}^E-n_{noA}^P)},$$

from which we also solve for domestic aggregate intermediate demand $\widetilde{X}_{noA}$. From the above, we can derive $L_{noA}^E(i)$, $y_{noA}^E(i)$, $x_{noA}^E(i)$, $x_{noA}^M(i)$ as follows:

$$\begin{aligned}
L_{noA}^E(i) &= \left\{A^{E^0}(i)\frac{\theta}{w}\left[\frac{p^*(i)}{1+\tau^*}-\eta\right]\right\}^{\frac{1}{1-\theta}}, \\
y_{noA}^E(i) &= A^{E^0}(i)L_{noA}^E(i)^\theta, \\
x_{noA}^E(i) &= \frac{\alpha-\gamma\widetilde{X}_{noA}-\frac{p^*(i)}{1+\tau^*}}{\beta-\gamma}, \\
x_{noA}^M(i) &= \frac{\alpha-\gamma\widetilde{X}_{noA}-(1+\tau)p^*(i)}{\beta-\gamma}.
\end{aligned}$$

Finally, we obtain

$$\begin{aligned}
ExQ_{noA} &= \int_0^{n_{noA}^E} y_{noA}^E(i)di - \int_0^{n_{noA}^E} x_{noA}^E(i)di \\
ImQ_{noA} &= \int_{n_{noA}^P}^{M_{noA}} x_{noA}^M(i)di \\
ExV_{noA} &= \frac{1}{1+\tau^*}\left(\int_0^{n_{noA}^E} p^*(i)\, y_{noA}^E(i)di - \int_0^{n_{noA}^E} p^*(i)\, x_{noA}^E(i)di\right) \\
ImV_{noA} &= (1+\tau)\int_{n_{noA}^P}^{M_{noA}} p^*(i)\, x_{noA}^M(i)di
\end{aligned}$$

**Sensitivity Analysis**

Table A8(a): Sensitivity – $\theta = 0.3$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi})$ = (4.95677, 4.64342, 0.30315, 0.08358, 0.06764)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.4631 | 14.2326 | 17.1905 | 2.9579 | 1.7093 | 1.4321 | 2.1000 | 0.1475 | 1.9255 | 9.6182 | 5.6269 |
| After trade war | 3.7830 | 18.2274 | 20.7305 | 2.5031 | 2.1328 | 1.8580 | 2.8230 | 0.1549 | 2.0441 | 12.2211 | 5.7301 |
| difference | -0.6800 | 3.9948 | 3.5400 | -0.4548 | 0.4235 | 0.4259 | 0.7230 | 0.0073 | 0.1186 | 2.6029 | 0.1032 |
| % change | -15.2% | 28.1% | 20.6% | -15.4% | 24.8% | 29.7% | 34.4% | 5.0% | 6.2% | 27.1% | 1.8% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.1967 | 0.4739 | 0.7803 | 1.6246 | 8.7739 | 5.1329 | 7.3900 | 4.3233 |
| After trade war | 0.1363 | 0.4111 | 0.5093 | 1.3164 | 11.4141 | 5.3517 | 9.6187 | 4.5099 |
| difference | -0.0604 | -0.0628 | -0.2710 | -0.3082 | 2.6402 | 0.2188 | 2.2287 | 0.1866 |
| % change | -30.7% | -13.3% | -34.7% | -19.0% | 30.1% | 4.3% | 30.2% | 4.3% |

Table A8(b): Sensitivity – $\theta = 0.5$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) = (5.06749,\ 4.71304,\ 0.32615,\ 0.07692,\ 0.06686)$

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.5915 | 15.8584 | 19.0964 | 3.2380 | 1.9305 | 1.6168 | 2.3707 | 0.1495 | 0.9603 | 11.2035 | 5.8033 |
| After trade war | 3.4367 | 19.9891 | 22.8720 | 2.8829 | 2.3441 | 1.9808 | 3.1286 | 0.1565 | 1.0296 | 13.8302 | 5.9000 |
| difference | -1.1548 | 4.1307 | 3.7756 | -0.3551 | 0.4136 | 0.3640 | 0.7579 | 0.0070 | 0.0693 | 2.6267 | 0.0967 |
| % change | -25.2% | 26.0% | 19.8% | -11.0% | 21.4% | 22.5% | 32.0% | 4.7% | 7.2% | 23.4% | 1.7% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2241 | 0.5379 | 0.8881 | 1.8247 | 10.2669 | 5.3181 | 8.7046 | 4.5089 |
| After trade war | 0.1226 | 0.4858 | 0.4592 | 1.5435 | 12.7459 | 5.4374 | 10.8318 | 4.6209 |
| difference | -0.1016 | -0.0520 | -0.4289 | -0.2812 | 2.4790 | 0.1193 | 2.1272 | 0.1120 |
| % change | -45.3% | -9.7% | -48.3% | -15.4% | 24.1% | 2.2% | 24.4% | 2.5% |

Table A9(a): Sensitivity – $\mu = 0.05$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) = (5.09481,\ 4.60452,\ 0.33484,\ 0.07749,\ 0.06354)$

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.9093 | 15.6924 | 18.9568 | 3.2644 | 1.9854 | 1.6591 | 2.4300 | 0.1548 | 1.6054 | 11.6062 | 5.8457 |
| After trade war | 4.4733 | 20.2066 | 22.9972 | 2.7905 | 2.4534 | 2.1496 | 3.3217 | 0.1644 | 1.7133 | 14.6209 | 5.9593 |
| difference | -0.4360 | 4.5142 | 4.0403 | -0.4739 | 0.4680 | 0.4905 | 0.8917 | 0.0095 | 0.1078 | 3.0146 | 0.1136 |
| % change | -8.9% | 28.8% | 21.3% | -14.5% | 23.6% | 29.6% | 36.7% | 6.2% | 6.7% | 26.0% | 1.9% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2286 | 0.5549 | 0.9053 | 1.8834 | 10.6281 | 5.3531 | 9.0249 | 4.5456 |
| After trade war | 0.1811 | 0.4850 | 0.6746 | 1.5262 | 13.7692 | 5.6122 | 11.6921 | 4.7656 |
| difference | -0.0474 | -0.0699 | -0.2307 | -0.3572 | 3.1411 | 0.2591 | 2.6672 | 0.2200 |
| % change | -20.7% | -12.6% | -25.5% | -19.0% | 29.6% | 4.8% | 29.6% | 4.8% |

Table A9(b): Sensitivity – $\mu = 0.15$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi})$ = (4.96117, 4.73591, 0.30524, 0.08212, 0.07166)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.2925 | 14.5778 | 17.6184 | 3.0406 | 1.7179 | 1.4326 | 2.1480 | 0.1473 | 1.1337 | 9.6805 | 5.6353 |
| After trade war | 3.1025 | 18.3577 | 21.0263 | 2.6686 | 2.1039 | 1.7745 | 2.8064 | 0.1529 | 1.2105 | 12.0473 | 5.7261 |
| difference | -1.1900 | 3.7799 | 3.4080 | -0.3720 | 0.3861 | 0.3420 | 0.6584 | 0.0055 | 0.0768 | 2.3668 | 0.0909 |
| % change | -27.7% | 25.9% | 19.3% | -12.2% | 22.5% | 23.9% | 30.7% | 3.8% | 6.8% | 24.4% | 1.6% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.1987 | 0.4839 | 0.7881 | 1.6540 | 8.8146 | 5.1312 | 7.4303 | 4.3253 |
| After trade war | 0.1030 | 0.4323 | 0.3863 | 1.3898 | 11.0439 | 5.2492 | 9.3291 | 4.4342 |
| difference | -0.0957 | -0.0516 | -0.4018 | -0.2642 | 2.2293 | 0.1180 | 1.8988 | 0.1088 |
| % change | -48.2% | -10.7% | -51.0% | -16.0% | 25.3% | 2.3% | 25.6% | 2.5% |

Table A10(a): Sensitivity – $\nu = 0.396$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi})$ = (5.02570, 4.47822, 0.32292, 0.08129, 0.05962)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.4319 | 14.7208 | 17.7736 | 3.0528 | 1.8475 | 1.5437 | 2.2570 | 0.1533 | 1.3445 | 10.5902 | 5.7321 |
| After trade war | 3.6549 | 18.7947 | 21.4441 | 2.6494 | 2.2778 | 1.9607 | 3.0227 | 0.1608 | 1.4323 | 13.2911 | 5.8351 |
| difference | -0.7770 | 4.0739 | 3.6705 | -0.4034 | 0.4303 | 0.4170 | 0.7656 | 0.0075 | 0.0878 | 2.7009 | 0.1029 |
| % change | -17.5% | 27.7% | 20.7% | -13.2% | 23.3% | 27.0% | 33.9% | 4.9% | 6.5% | 25.5% | 1.8% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2122 | 0.5160 | 0.8418 | 1.7644 | 9.6676 | 5.2327 | 8.1758 | 4.4253 |
| After trade war | 0.1397 | 0.4568 | 0.5224 | 1.4584 | 12.3551 | 5.4241 | 10.4604 | 4.5924 |
| difference | -0.0725 | -0.0592 | -0.3194 | -0.3060 | 2.6875 | 0.1914 | 2.2846 | 0.1670 |
| % change | -34.2% | -11.5% | -37.9% | -17.3% | 27.8% | 3.7% | 27.9% | 3.8% |

Note: under $\nu = 0.396$, the half life of technology is 21 months (1.75 years).

Table A10(b): Sensitivity – $\nu = 0.555$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi})$ = (5.01113, 4.45101, 0.31964, 0.08208, 0.08347)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.3507 | 14.5212 | 17.5283 | 3.0070 | 1.8184 | 1.5194 | 2.2210 | 0.1530 | 1.3333 | 10.3792 | 5.7080 |
| After trade war | 3.5466 | 18.5331 | 21.1419 | 2.6089 | 2.2438 | 1.9292 | 2.9717 | 0.1603 | 1.4197 | 13.0361 | 5.8097 |
| difference | -0.8041 | 4.0119 | 3.6137 | -0.3982 | 0.4255 | 0.4098 | 0.7506 | 0.0074 | 0.0864 | 2.6570 | 0.1018 |
| % change | -18.5% | 27.6% | 20.6% | -13.2% | 23.4% | 27.0% | 33.8% | 4.8% | 6.5% | 25.6% | 1.8% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2082 | 0.5071 | 0.8259 | 1.7371 | 9.4680 | 5.2069 | 7.9998 | 4.3994 |
| After trade war | 0.1342 | 0.4488 | 0.5020 | 1.4368 | 12.1013 | 5.3931 | 10.2371 | 4.5623 |
| difference | -0.0740 | -0.0583 | -0.3239 | -0.3002 | 2.6333 | 0.1862 | 2.2373 | 0.1629 |
| % change | -35.5% | -11.5% | -39.2% | -17.3% | 27.8% | 3.6% | 28.0% | 3.7% |

Note: under $\nu = 0.555$, the half life of technology is 15 months (1.25 years).

Table A11(a): Sensitivity – $\frac{\delta^*}{\delta} = 1.2$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi})$ = (5.05495, 4.67824, 0.33116, 0.07946, 0.06823)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.6847 | 15.3441 | 18.5168 | 3.1727 | 1.9056 | 1.5913 | 2.3490 | 0.1531 | 1.3572 | 11.0263 | 5.7863 |
| After trade war | 3.9226 | 19.5469 | 22.2907 | 2.7438 | 2.3422 | 2.0200 | 3.1480 | 0.1610 | 1.4513 | 13.7976 | 5.8910 |
| difference | -0.7621 | 4.2029 | 3.7739 | -0.4289 | 0.4366 | 0.4287 | 0.7990 | 0.0080 | 0.0941 | 2.7713 | 0.1046 |
| % change | -16.3% | 27.4% | 20.4% | -13.5% | 22.9% | 26.9% | 34.0% | 5.2% | 6.9% | 25.1% | 1.8% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2180 | 0.5323 | 0.8636 | 1.8051 | 10.0848 | 5.2923 | 8.5471 | 4.4853 |
| After trade war | 0.1475 | 0.4696 | 0.5506 | 1.4843 | 12.8639 | 5.4923 | 10.9120 | 4.6589 |
| difference | -0.0705 | -0.0627 | -0.3130 | -0.3207 | 2.7791 | 0.2001 | 2.3648 | 0.1736 |
| % change | -32.4% | -11.8% | -36.2% | -17.8% | 27.6% | 3.8% | 27.7% | 3.9% |

Table A11(b): Sensitivity – $\frac{\delta^*}{\delta} = 1.3$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) =$ (5.10411, 4.66779, 0.32697, 0.07486, 0.06836)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.9463 | 16.2162 | 19.5817 | 3.3655 | 2.0039 | 1.6736 | 2.4722 | 0.1524 | 1.3529 | 11.7384 | 5.8579 |
| After trade war | 4.3407 | 20.7939 | 23.7162 | 2.9222 | 2.4630 | 2.1363 | 3.3361 | 0.1604 | 1.4441 | 14.6990 | 5.9680 |
| difference | -0.6057 | 4.5777 | 4.1345 | -0.4432 | 0.4591 | 0.4628 | 0.8639 | 0.0080 | 0.0912 | 2.9606 | 0.1102 |
| % change | -12.2% | 28.2% | 21.1% | -13.2% | 22.9% | 27.7% | 34.9% | 5.2% | 6.7% | 25.2% | 1.9% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.2306 | 0.5609 | 0.9137 | 1.9058 | 10.7464 | 5.3628 | 9.1292 | 4.5558 |
| After trade war | 0.1700 | 0.4966 | 0.6340 | 1.5674 | 13.7656 | 5.5891 | 11.7013 | 4.7509 |
| difference | -0.0606 | -0.0643 | -0.2798 | -0.3384 | 3.0192 | 0.2263 | 2.5720 | 0.1951 |
| % change | -26.3% | -11.5% | -30.6% | -17.8% | 28.1% | 4.2% | 28.2% | 4.3% |

Table A12(a): Sensitivity – $\gamma = 0.25$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) =$ (5.00641, 3.10647, 0.56187, 0.06834, 0.08310)

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 5.8084 | 18.5014 | 22.3775 | 3.8762 | 3.6223 | 3.0010 | 3.7369 | 0.2020 | 1.3812 | 20.7755 | 5.7355 |
| After trade war | 4.4978 | 23.1709 | 26.5004 | 3.3295 | 4.4199 | 3.7561 | 4.9330 | 0.2129 | 1.4877 | 25.7707 | 5.8306 |
| difference | -1.3105 | 4.6695 | 4.1229 | -0.5467 | 0.7976 | 0.7551 | 1.1960 | 0.0109 | 0.1065 | 4.9953 | 0.0951 |
| % change | -22.6% | 25.2% | 18.4% | -14.1% | 22.0% | 25.2% | 32.0% | 5.4% | 7.7% | 24.0% | 1.7% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.4083 | 1.0296 | 1.6132 | 3.4306 | 18.9580 | 5.2337 | 16.0582 | 4.4332 |
| After trade war | 0.2390 | 0.9028 | 0.8925 | 2.8158 | 23.8474 | 5.3954 | 20.2179 | 4.5743 |
| difference | -0.1693 | -0.1268 | -0.7207 | -0.6148 | 4.8894 | 0.1617 | 4.1597 | 0.1411 |
| % change | -41.5% | -12.3% | -44.7% | -17.9% | 25.8% | 3.1% | 25.9% | 3.2% |

Table A12(b): Sensitivity – $\gamma = 0.75$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) = (5.24959, 5.68424, 0.26303, 0.07998, 0.06265)$

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.5310 | 15.0093 | 18.0987 | 3.0894 | 1.5289 | 1.2818 | 2.0401 | 0.1359 | 1.3541 | 9.2795 | 6.0693 |
| After trade war | 4.5054 | 19.6160 | 22.3204 | 2.7043 | 1.8788 | 1.6651 | 2.8129 | 0.1434 | 1.4413 | 11.6416 | 6.1962 |
| difference | -0.0256 | 4.6067 | 4.2217 | -0.3850 | 0.3499 | 0.3832 | 0.7728 | 0.0075 | 0.0872 | 2.3621 | 0.1269 |
| % change | -0.6% | 30.7% | 23.3% | -12.5% | 22.9% | 29.9% | 37.9% | 5.5% | 6.4% | 25.5% | 2.1% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.1769 | 0.4240 | 0.7015 | 1.4483 | 8.5327 | 5.5809 | 7.2940 | 4.7707 |
| After trade war | 0.1641 | 0.3779 | 0.6103 | 1.1877 | 11.0642 | 5.8889 | 9.4553 | 5.0325 |
| difference | -0.0128 | -0.0461 | -0.0912 | -0.2606 | 2.5315 | 0.3080 | 2.1612 | 0.2618 |
| % change | -7.2% | -10.9% | -13.0% | -18.0% | 29.7% | 5.5% | 29.6% | 5.5% |

Table A12(c): Sensitivity – $\gamma = 0.2$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) = (4.84405, 2.24887, 0.54850, 0.04736, 0.04801)$

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 1.9265 | 19.6106 | 29.5974 | 9.9868 | 3.7258 | 1.3739 | 2.1845 | 0.1114 | 1.1280 | 20.1867 | 5.4181 |
| After trade war | 0 | 24.6172 | 33.5528 | 8.9355 | 4.1243 | 1.9067 | 2.9562 | 0.1201 | 1.1521 | 22.4580 | 5.4452 |
| difference | -1.9265 | 5.0067 | 3.9554 | -1.0512 | 0.3985 | 0.5328 | 0.7717 | 0.0087 | 0.0242 | 2.2713 | 0.0271 |
| % change | -100% | 25.5% | 13.4% | -10.5% | 10.7% | 38.8% | 35.3% | 7.8% | 2.1% | 11.3% | 0.5% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.0387 | 2.3906 | 0.1574 | 8.5062 | 11.8380 | 3.1773 | 10.5104 | 2.8210 |
| After trade war | 0 | 2.2177 | 0 | 7.6918 | 14.7662 | 3.5803 | 12.9238 | 3.1335 |
| difference | -0.0387 | -0.1729 | -0.1574 | -0.8144 | 2.9282 | 0.4030 | 2.4134 | 0.3126 |
| % change | -100% | -7.2% | -100% | -9.6% | 24.7% | 12.7% | 23.0% | 11.1% |

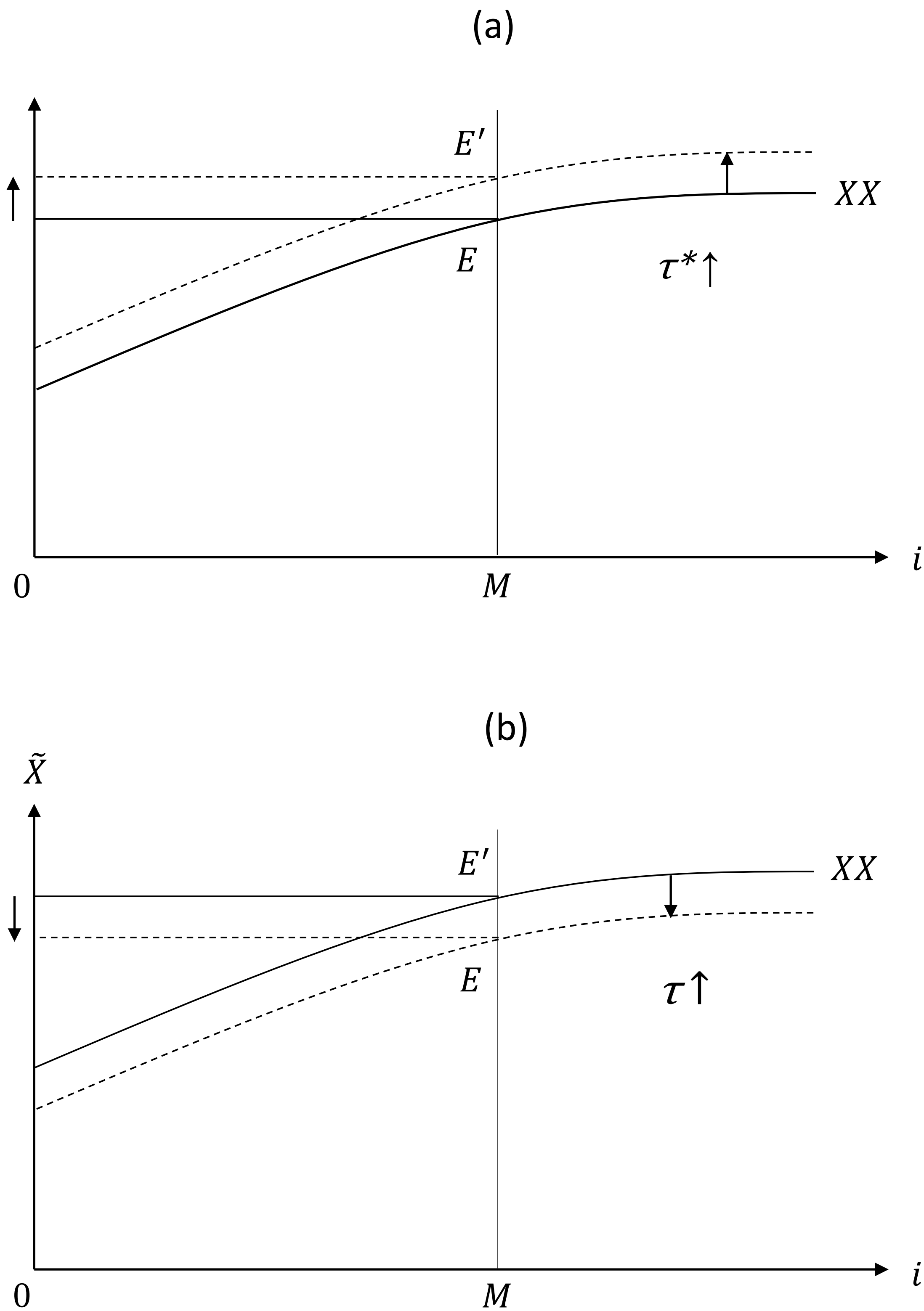
Table A12(d): Sensitivity – $\gamma = 0.8$

Calibrated parameters: $(\alpha, \beta, \phi, b, \overline{\psi}) = (5.28611, 5.75515, 0.25719, 0.08115, 0.06241)$

| | $n^E$ | $n^P$ | $M$ | $M$-$n^P$ | $\widetilde{X}$ | $\widetilde{X}^P$ | $\widetilde{A}$ | $\frac{\widetilde{A}}{n^P}$ | $\frac{\widetilde{\Lambda}}{n^P}$ | $Y$ | $\frac{Y}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | 4.4235 | 14.7187 | 17.7584 | 3.0397 | 1.4789 | 1.2391 | 1.9816 | 0.1346 | 1.3525 | 9.0532 | 6.1214 |
| After trade war | 4.5232 | 19.3340 | 22.0027 | 2.6688 | 1.8167 | 1.6166 | 2.7467 | 0.1421 | 1.4378 | 11.3593 | 6.2526 |
| difference | 0.0997 | 4.6153 | 4.2444 | -0.3709 | 0.3378 | 0.3776 | 0.7651 | 0.0074 | 0.0853 | 2.3061 | 0.1312 |
| % change | 2.3% | 31.4% | 23.9% | -12.2% | 22.8% | 30.5% | 38.6% | 5.5% | 6.3% | 25.5% | 2.1% |

| | $ExQ$ | $ImQ$ | $ExV$ | $ImV$ | $VA$ | $\frac{VA}{\widetilde{X}}$ | $IC$ | $\frac{IC}{\widetilde{X}}$ |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 0.1712 | 0.4110 | 0.6789 | 1.4069 | 8.3252 | 5.6291 | 7.1278 | 4.8196 |
| After trade war | 0.1671 | 0.3673 | 0.6209 | 1.1542 | 10.8260 | 5.9591 | 9.2638 | 5.0992 |
| difference | -0.0041 | -0.0437 | -0.0580 | -0.2527 | 2.5008 | 0.3299 | 2.1360 | 0.2796 |
| % change | -2.4% | -10.6% | -8.5% | -18.0% | 30.0% | 5.9% | 30.0% | 5.8% |

Figure A1. Determination of aggregate intermediate good usage under exogenous M



# Figure A2. Technology Choice and Trade in Intermediate Goods under exogenous M

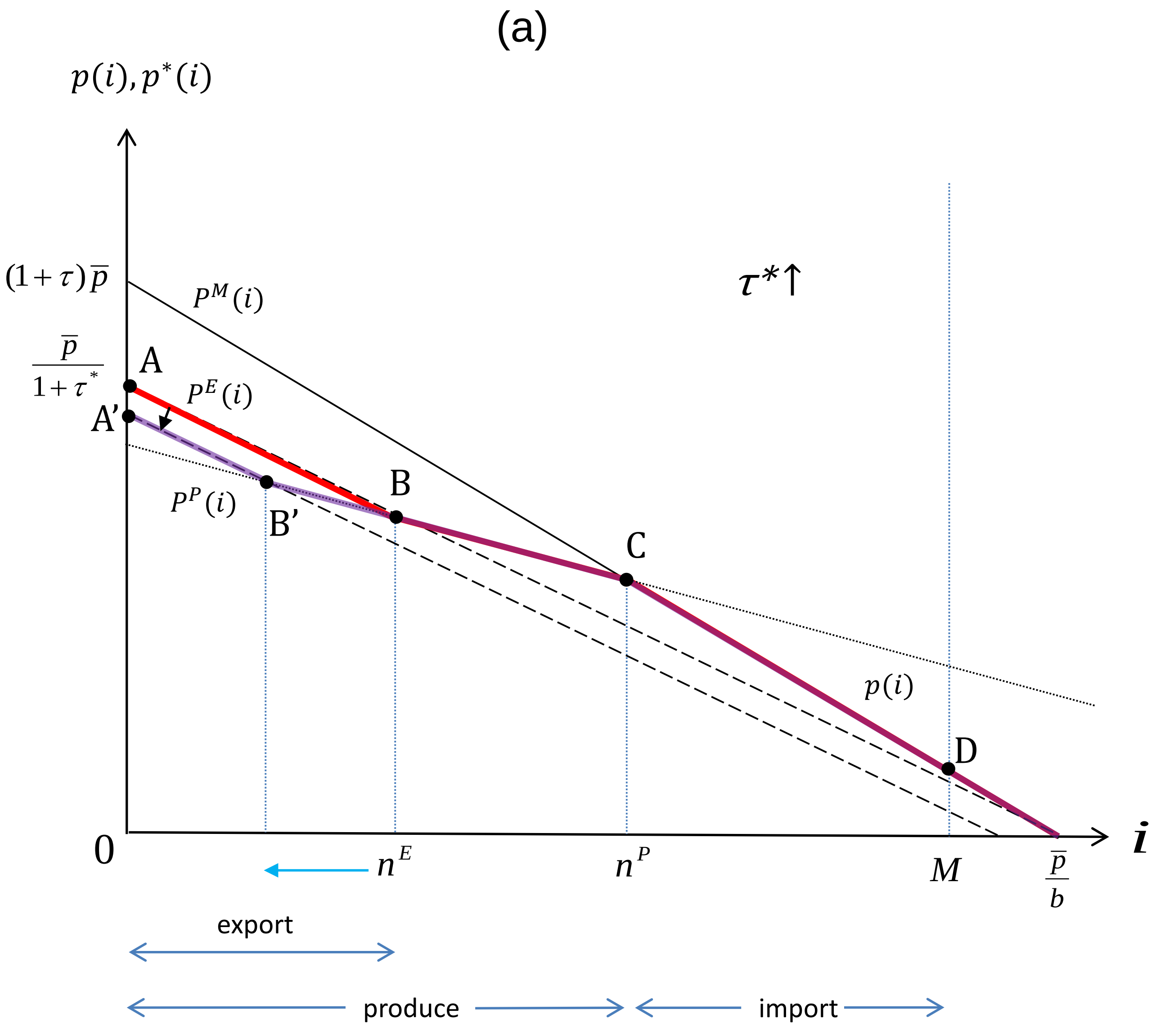


Figure A2. Technology Choice and Trade in Intermediate Goods under exogenous M