MACRO RECRUITING INTENSITY FROM MICRO DATA

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ABSTRACT

We merge QCEW and JOLTS microdata to study the recruiting intensity of firms in the cross-section and over time. We show that vast establishment-level heterogeneity in vacancy filling rates is entirely explained by differences in gross hiring rates. We provide theory that supports these empirical facts and, through the lens of this theory, aggregate firm-level decisions and data into an empirical measure of aggregate recruiting intensity (ARI).We find that procyclicality of ARI is primarily due to cutting recruiting effort in slack labor markets. This leads us to formulate an ARI index easily computable from publicly available macroeconomic time series. Declining ARI in the Great Recession accounted for much of the increase in unemployment, but little of its persistence.

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1 Introduction

Aggregate match efficiency is a useful concept in macroeconomics. Its fluctuations expand and contract the hiring possibility frontier of the economy in an analogous manner to the way changes in total factor productivity shift its production possibility frontier. As such, movements in match efficiency are crucial to understanding the volatility of the job finding rate, the chief determinant of unemployment (Shimer, 2012).

The Great Recession offers a striking example. At the onset of the recession, the job finding rate fell sharply, leading to sustained unemployment. However, its decline was much more severe than what would usually be implied by the decrease in labor market tightness—the ratio of available jobs (vacancies) to idle workers (the unemployed)—alone. The reason is precisely that measured productivity, or efficiency, of the matching process broke down significantly over this period (Elsby, Michaels, and Ratner, 2015).

A deterioration in aggregate match efficiency may in principle derive from a number of sources. There may be a reduction in search intensity among the pool of job seekers, or a compositional shift in this pool toward workers with lower job finding rates. In addition, there may be a surge in misallocation across labor markets between the job requirements of open positions and the characteristics of job seekers. The respective role of workers' search effort and mismatch as shifters of the aggregate matching function have been well understood and investigated for almost three decades, as thoroughly discussed in the survey by Petrongolo and Pissarides (2001).[1]

Instead, and somewhat surprisingly, macroeconomists have not focused as much on the role played by firms' recruiting intensity (the counterpart of workers' search effort) in labor market fluctuations. The empirical analysis of Davis, Faberman, and Haltiwanger (2013) (henceforth DFH) has been a game changer in this literature. DFH exploited establishment-level JOLTS data to document a great deal of heterogeneity in recruiting intensity across firms and, in particular, a strong positive relationship between the vacancy yield (the success rate of a vacancy)

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[1]Clearly, the Great Recession has revived both literatures. In the context of the U.S., Hornstein, Kudlyak, and Lange (2016), Hall and Schulhofer-Wohl (2018), and Mukoyama, Patterson, and Şahin (2018) have investigated the jobseekers' search intensity channel. Barlevy (2011), Şahin, Song, Topa, and Violante (2014), Herz and Van Rens (2019), and Kothari, Saporta-Eksten, and Yu (2013) have explored the role of the mismatch hypothesis.

and the hiring rate in the cross section.[2] Their work has spurred new interest on the topic, in terms of both measurement from microdata (see working papers by Mueller, Kettemann, and Zweimuller, 2018; Carrillo-Tudela, Kaas, and Gartner, 2018; Lochner, Merkl, Stuber, and Gurtzgen, 2019) and theoretical equilibrium modelling (Kaas and Kircher, 2015; Gavazza, Mongey, and Violante, 2018; Leduc and Liu, 2019). Our paper contributes to this line of research.

We use U.S. microdata on hires, employment, and vacancies, combined with minimal structure from a dynamic model of firm hiring in a frictional environment, in order to answer a number of questions. What economic forces drive differences in observed recruiting outcomes at the firm level? What explains the dynamics of aggregate recruiting intensity (ARI) over the cycle? And what lessons can we draw from these regarding aggregate labor market dynamics?

To answer these questions our paper systematically addresses heterogeneity in hiring behavior across firms. To take a broad view of this heterogeneity we create a new dataset that links firm-level observations in two existing sources of firm-level microdata: the *Job Openings and Labor Turnover Survey* (JOLTS) and the *Quarterly Census of Employment and Wages* (QCEW) microdata at the U.S. Bureau of Labor Statistics (BLS). We can therefore, for the first time, incorporate heterogeneity in establishment age and per-worker earnings into the analysis, along with size and industry which are available in JOLTS. As a companion to this paper, we are placing online a repository of cross-tabulations from our new linked JOLTS-QCEW microdata that will benefit other researchers interested in firm dynamics, job flows, and worker flows.

Using this new data we answer our questions in four steps. First, to link micro to macro we derive an expression for aggregate recruiting intensity from first principles, using relationships in the microdata to motivate our theory. We theoretically decompose this measure of ARI into three factors: slackness, growth, composition. The first factor summarizes the general equilibrium response to labor market conditions that are common across firms, the second captures the economy-wide hiring rate, the third reflects heterogeneity within and across groups of firms. Second, we show how to build each of these components from the ground up using

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[2]In particular, this key empirical finding represents a rejection of the classical theoretical result in chapter 5 of Pissarides (2000) stating that if the recruiting cost per vacancy is isoelastic in effort (and independent of the vacancy rate), then the optimal search intensity is a constant and we are back to the model without effort choice. This 'neutrality' result was taken as a benchmark reference for a long time and was, perhaps, one of the reasons why macroeconomists appeared uninterested in studying recruiting intensity.

our QCEW-JOLTS microdata. These data enter the measures directly as inputs, and indirectly through parameters that we estimate in a first stage. Third, having constructed these measures we empirically decompose the time-series variance of ARI into its three theoretical components. Finally, we use our decomposition results to motivate a simple empirical index that can be entirely computed using publicly available data, and conduct counterfactual exercises.

Our analysis contains three main findings. First, at the micro level, the bulk of cross-sectional variation in the speed at which firms fill vacancies can be entirely explained by heterogeneity in firm-level hiring rates, even after controlling for firm age, establishment wage, size and industry information, which are available from our merged QCEW-JOLTS data. This stark regularity significantly extends—through the addition of controls—the initial observation of DFH that in the *cross-section of establishment gross hiring rates*, firms with higher gross hiring rates tend to have higher vacancy yields.[3] We conclude that at the firm level the hiring rate is, quantitatively, a 'sufficient statistic' for the vacancy filling rate. This empirical regularity is important. We show in Proposition 1 that, joint with firm optimality, it puts tight restrictions on functional forms in a theory in which firms choose cost-minimizing combinations of recruiting inputs.

Second, at the macro level, we combine our microdata and theory to aggregate our micro-founded recruiting intensity decisions up to the macro level. Decomposing our aggregate measure of recruiting intensity reveals that its procyclicality is chiefly due to firms optimally cutting back on recruiting effort when labor markets are slack and plenty of job seekers are available. This dominant general equilibrium feedback motivates the construction of a proxy-index that explains the bulk of time-series variation in ARI and is easily computable from publicly available data. Proposition 2 shows that a representative firm choosing aggregate vacancies and recruiting effort will in equilibrium yield a measure of recruiting intensity identical to our index. We show that this index is conceptually different from that proposed by DFH: their approach imputes an estimated cross-sectional elasticity to a macro elasticity and so abstracts from general equilibrium.

Third, in order to guide future research on the dynamics of unemployment around the Great

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[3]See their Figure IX, which our Figure 1 extends from their 2001-2006 sample to 2001-2018.

Recession, we conduct a simple counterfactual experiment. We illustrate that the sharp fall in our empirical measure of the recruiting intensity of hiring firms can account for much of the decline in the job finding rate in the aftermath of the Great Recession, but little of its slow recovery. In other words, the high duration of unemployment which lingered well after the end of the recession appears to have other culprits.

Outline. The paper proceeds as follows. Section 2 presents the theoretical framework for our empirics. Section 3 describes JOLTS and QCEW microdata and our empirical approach. Section 4 presents our main empirical results. Section 5 returns to the model to interpret our results through an index of ARI and conduct counterfactuals. Section 6 concludes. An online appendix contains additional figures and tables (Appendix A), mathematical derivations (Appendix B), and additional details on data construction and treatment (Appendix C).

2   Theory

We derive our empirical model of recruiting intensity from a dynamic decision problem of a firm hiring in a frictional labor market. We first specify the partial equilibrium microeconomic environment, and then close this in order to study the aggregate labor market.

2.1   Microeconomic environment

Consider a firm i in period t that has n_{it} employees at its disposal, productivity z_{it} and fixed idiosyncratic match efficiency ϕ_i. Flow profits of the firm π_{it} consist of its value added net of operating costs $f(z_{it}, n_{it})$ minus wage payments w_{it}, and the costs associated with recruiting.

A firm recruits workers by spending resources on two inputs: vacancies v_{it} and recruiting intensity e_{it}. In order to be consistent with microdata, our notion of a vacancy in this paper hues to the tight definition of a vacancy used by the Bureau of Labor Statistics (BLS). In the JOLTS a vacancy is an *"open position ready to be staffed in 30 days, for which the establishment is actively recruiting externally"*.[4] We allow *costs per vacancy* to depend on employment n_{it}, the number

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[4]See Bureau of Labor Statistics, Handbook of Methods, Chapter 18 - *Job Openings and Labor Turnover Sur-*

of open positions v_{it}, and on *recruiting intensity* e_{it}: $\mathcal{C}_i(e_{it}, v_{it}, n_{it})$. Recruiting intensity includes expenditures on all other variable inputs such as advertising, screening, recruiting events, etc.[5] More recruiting intensity increases the effectiveness of a vacancy, such that firm hires are a product of the firm's *effective vacancies* $v_{it}^* = \phi_i e_{it} v_{it}$ and the aggregate meeting rate of effective vacancies Q_t^*, which the firm takes as given.[6] For now we defer closing the model which will require specifying Q_t^* as a function of individual firm decisions and the composition of workers searching for jobs, in a way that is consistent with the firm level hiring technology.

The firm's problem is as follows. Let s^t be the history of firm-level and aggregate shocks until date t, $\mathcal{M}_i(s^t)$ the subjective discount rate associated with history s^t, and $\delta_{it}(s^t)$ the rate of exogenous job destruction. The firm solves the following dynamic problem:

$$\max_{\{e_{it}(s^t), v_{it}(s^t)\}_{\forall t, \forall s^t | s_0}} \sum_{t=0}^{\infty} \sum_{s^t | s_0} \mathcal{M}_{it}(s^t) \, \pi_{it}(s^t) \quad , \quad \text{subject to} \tag{1}$$

$$\pi_{it}(s^t) = f\left(z_{it}(s^t), n_{it}(s^t)\right) - w_{it}(s^t) \, n_{it}(s^t) - \mathcal{C}_i\left(e_{it}(s^t), v_{it}(s^t), n_{it}(s^t)\right) v_{it}(s^t)$$

$$n_{it+1}(s^{t+1}) = \left(1 - \delta_{it+1}(s^{t+1})\right) n_{it}(s^t) + h_{it}(s^t)$$

$$h_{it}(s^t) = Q_t^*(s^t) \, \phi_i \, e_{it}(s^t) \, v_{it}(s^t).$$

The first equality is the definition of profits, the second is the law of motion for employment, and the third is the firm-level hiring technology.

Separability. Naturally, the problem separates into two stages: (1) controlling the dynamics of employment through hiring, which delivers the state-contingent policy $\{h_{it}(s^t)\}_{t, s^t}$, (2) minimizing at every s^t the recruiting costs associated with $h_{it}(s^t)$. Intuitively, since (i) the recruiting inputs are variable and (ii) costs are sunk after hiring a worker, the choice of inputs is irrelevant for future hiring decisions. Therefore, given a path for hires $h_{it}(s^t)$, the firm solves a static recruiting cost-minimization problem at each node s^t.

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vey, for detailed definition of responses. A copy of the short form filled in by hiring managers is available at https://www.bls.gov/jlt/jltc1.pdf

[5]The survey data collected in O'Leonard, Krider, and Erickson (2015) show that these expenditures are sizable and vary by type of firm.

[6]Note that this implies that the hiring technology is constant returns to scale in vacancies. We provide evidence of this later.
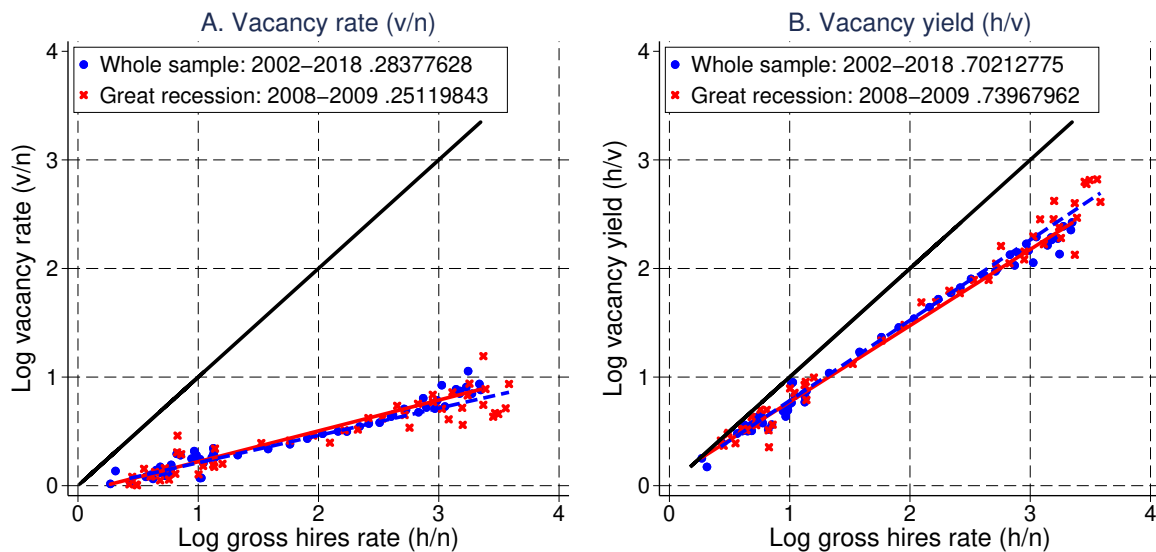
Figure 1: Vacancy rate and vacancy yield by gross hiring rate - JOLTS 2002-2018

<u>Notes</u> Establishment-month observations in JOLTS microdata 2002-2018 (blue circles), or 2008-2009 (red crosses) are pooled in bins, where bins are determined by net monthly growth rate, and have a width of 1 percent. Growth rates computed as in DFH. Within bins b, total hires h_b, total vacancies v_b, total employment n_b are computed. From these, the gross hiring rate h_b/n_b, vacancy yield h_b/v_b and vacancy rate $v_b/(v_b + n_b)$ are computed. Bins with positive gross hiring rates are kept. Points plotted are logs of these variables, differenced about the bin representing a one percent net growth rate.

Recruiting problem.   Since the aggregate state and history only enters the recruiting problem through $Q_t^*(s^t)$ and $h_{it}(s^t)$, it is redundant once Q_t^* and h_{it} are taken as given by the firm at the recruiting stage. The problem of a firm with employment n_{it} and target hires h_{it} is therefore

$$\min_{e_{it}, v_{it}} \; \mathcal{C}_i\Big(e_{it}, v_{it}, n_{it}\Big) v_{it} \qquad \text{s.t.} \qquad h_{it} = Q_t^* \, \phi_i \, e_{it} \, v_{it}. \qquad (2)$$

In specifying the cost function we ensure that the model is consistent with the empirical observation that, in the microdata, the vacancy yield of firms (h_{it}/v_{it}) is approximately log-linear in the gross hiring rate of the firm (h_{it}/n_{it}). First documented by DFH in JOLTS microdata from 2002 to 2006, we update this relationship and show it to be robust through and after the Great Recession in Figure 1. Our first theoretical contribution is to show that this relationship places tight and precise restrictions on C_i.

Proposition 1.   *If and only if (i) the per-vacancy cost function $\mathcal{C}_i$ is of the following form:*

$$C_i\left(e_{it}, v_{it}, n_{it}\right) = x_i\, G_c\left(G_e(e_{it}) + G_v\left(\frac{v_{it}}{n_{it}}\right)\right), \tag{3}$$

*where the functions G_c, G_e and G_v are all isoelastic (constant elasticity), **then** (ii) firm optimality implies that the firm's job-filling rate $f_{it} = (h_{it}/v_{it})$ and vacancy rate (v_{it}/n_{it}) are log-linear in the hiring rate (h_{it}/n_{it}).*

Proof. *See Appendix B.*

The ***if*** component of Proposition 1 could be viewed as an exercise in structural reverse engineering: it reassuringly demonstrates that *there exists* a cost function that delivers the empirical relationship we observe in the data. The more substantive contribution of Proposition 1 is the ***only if*** component, which requires a more involved proof stating that the empirics places a very strong restriction on the theory. The ***only if*** part provides a family of functions for future work.[7]

We make three points regarding cost functions implied by Proposition 1. First, individual level firm heterogeneity is restricted to entering through the multiplicative shifter x_i. Second, the requirement that the cost function depends on the vacancy *rate* tells us that in the data, firms find it more costly to add a given number of positions v in a small firm than in a larger firm (e.g., in terms of reorganization of production). Third, hiring inputs are *complements* in production: raising recruiting intensity makes each vacancy more productive. Through the cost function the data also reveals to us that they complements in costs. That is, any further microfoundation of behavior will require models in which increasing vacancies must increases the marginal cost of allocating more resources to recruiting intensity.

In order to adhere to the data, in what follows we consider only the class of functions $\mathcal{C}_i$ that satisfy this property. We let the three constant elasticities of the functions $G_c(\cdot)$, $G_e(\cdot)$ and $G_v(\cdot)$ be given by γ_c, γ_e and γ_v, respectively.

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[7]In the only two existing equilibrium macroeconomic models of recruiting intensity, Gavazza, Mongey, and Violante (2018) assume a cost function that is a special case of this class and Leduc and Liu (2019) assume a functional form for recruiting costs that is not included in this class. Proposition 1 should provide some guidance to future literature that models firms' hiring effort decisions.

Optimal recruiting intensity. In Appendix B we show that minimization under (3) subject to the hiring constraint yields the following optimal policy for recruiting intensity which we express in logs

$$\log e_{it} = Const. - \frac{\gamma_v}{\gamma_e + \gamma_v} \log Q_t^* - \frac{\gamma_v}{\gamma_e + \gamma_v} \log \phi_i + \frac{\gamma_v}{\gamma_e + \gamma_v} \log \left(\frac{h_{it}}{n_{it}} \right). \tag{4}$$

The constant includes the elasticity γ_c and other parameters.

To interpret the optimal policy it is useful to think of the firm's hiring technology in (2) as a production function that produces a hiring rate (h_{it}/n_{it}), with inputs of the vacancy rate (v_{it}/n_{it}) and recruiting intensity (e_{it}) and productivity term $(Q_t^* \phi_i)$. The firm's recruiting intensity depends positively on its hiring rate: more output requires more inputs. These inputs are more productive when the rate at which effective vacancies produce hires due to market-wide productivity in matching (Q_t^*) or idiosyncratic productivity in matching (ϕ_i), are high, requiring less inputs. In equilibrium Q_t^* encodes the mass of idle workers U_t, the intensity of worker search which below we denote A_t, the vacancies of competitors V_t and their recruiting intensity decisions. Hence firms' recruiting decisions depend, in general equilibrium, on the recruiting decisions of their competitors. Below we show that these recruiting choices are *strategic complements*.

How is the shape of the policy function determined by the elasticities γ_v and γ_e? When γ_v is large relative to γ_e increasing marginal costs of vacancies set in quickly. This leads the firm to adjust more on the recruiting intensity margin in response to an increase in the demand for inputs due to an increase in output ($\uparrow h_{it}/n_{it}$). Similarly, when productivity falls ($\downarrow Q_t^* \phi_i$), the firm raises the recruiting intensity margin more the larger is γ_v relative to γ_e. The functional form for the hiring technology implies that optimal recruiting intensity respond symmetrically to either shift, which is reflected in equal but oppositely signed coefficients in (4).

Vacancy yield. Using the firm's hiring technology we can use (4) to determine the optimal vacancy yield which, unlike e_{it}, is observable in JOLTS microdata:

$$\log \left(\frac{h_{it}}{v_{it}} \right) = Const. + \frac{\gamma_e}{\gamma_e + \gamma_v} \log Q_t^* + \frac{\gamma_e}{\gamma_e + \gamma_v} \log \phi_i + \frac{\gamma_v}{\gamma_e + \gamma_v} \log \left(\frac{h_{it}}{n_{it}} \right). \tag{5}$$

$\log e_{it}$

$\mathcal{C}\left(\uparrow e_{it}, \uparrow \frac{v_{it}}{n_{it}}, \overline{x}_i\right) \uparrow v_{it}$

$\uparrow \frac{h_{it}}{n_{it}} = \overline{Q}_t^*\{\uparrow e_{it}\}\{\uparrow \frac{v_{it}}{n_{it}}\}$

$\log \frac{v_{it}}{n_{it}}$

(a) Optimal recruiting choice

$\log \frac{h_{it}}{v_{it}}$

$\uparrow \frac{h_{it}}{n_{it}} \rightarrow \uparrow e_{it} \rightarrow \uparrow \frac{h_{it}}{v_{it}}$

$\log \frac{h_{it}}{n_{it}}$
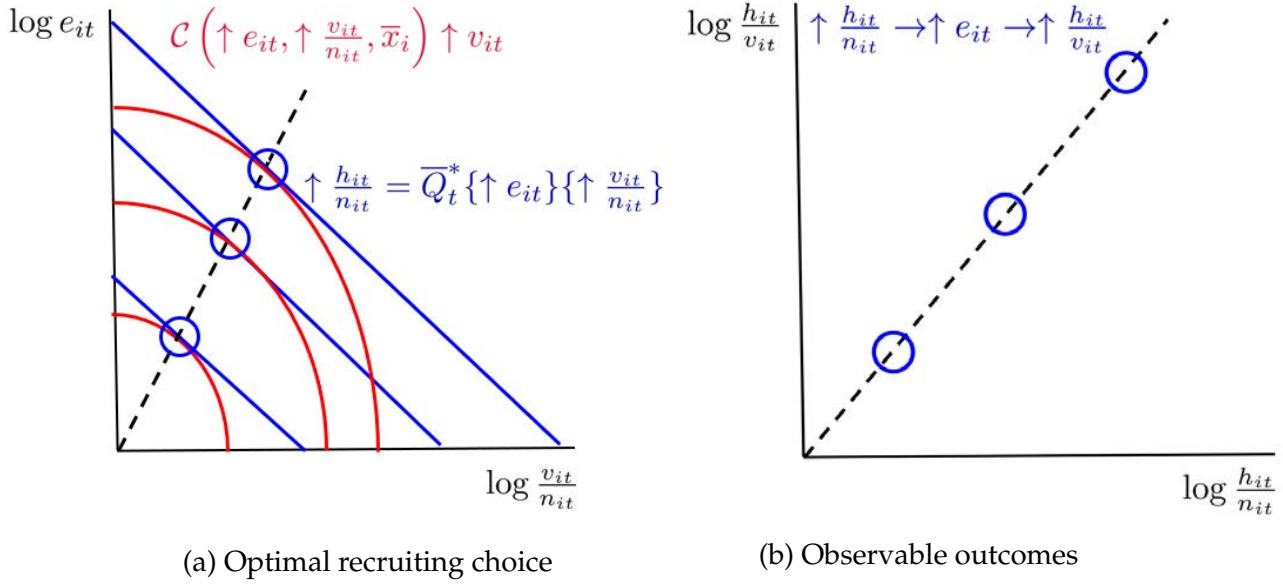
(b) Observable outcomes

Figure 2: Micro of recruiting intensity decisions

<u>Notes</u>: Panels (a) and (b) describe the unobserved recruiting choice and observed recruiting outcomes of a firm. Panel (a) plots isoquants of the hiring production technology in logs (blue), and the isocosts also in logs (red). Keeping $Q_t^* \phi_i$ fixed, an increase in production $\uparrow (h_{it}/n_{it})$ requires an increase in the level of inputs: e_{it} and v_{it}/n_{it}. The parameter γ captures the elasticity of substitution in the average vacancy cost function $\mathcal{C}$ and so determines the slope of the expansion path in logs. Panel (b) shows the observable implications for the relationship between vacancy yield and hiring rate. On the x-axis, the hiring rate, which is our comparative static variable, is increasing. On the y-axis, the log vacancy yield—which is equal to $\log e_{it} + \log Q_t^* + \log \phi_i$—increases linearly (with slope 1) as $\log e_{it}$ increases linearly.

Figure 2(a)-(b) provide a graphical characterization of the cost-minimizing recruiting choice in terms of unobserved intensity and observed vacancy yield, and how these respond to changes in the firm's desired hiring rate. Figure 3(a) shows how an increase in the hiring rate is equivalent to shifting out isoquants of the hiring technology in log vacancy-rate / recruiting intensity space. The firm then chooses the combination of hiring inputs that place it on its lowest isocost curve subject to the technology, where the composite parameter $\gamma = \gamma_v/(\gamma_e + \gamma_v)$ determines the gradient of the isocost curves. Under a cost function that satisfies Proposition 1, an increase in h_{it}/n_{it} leads to a log-linear expansion path of the vacancy-rate and recruiting intensity, with the slope of this path determined by γ. Figure 2(b) shows how this maps into the log-linear relationship between the hiring rate and vacancy yield that we observe in the microdata in Figure 1.

2.2 Macroeconomic environment

We construct our empirical measure of ARI in two steps. First, we aggregate establishment level recruiting decisions into our measure of ARI. Second, since establishment level recruiting decisions depend on the meeting rate Q_t^* which itself depends on aggregate recruiting intensity we solve a general equilibrium fixed point, showing that this remains tractable.

In order to aggregate establishment level recruiting decisions we specify an aggregate matching function that is consistent with the firm level hiring constraint $(h_{it} = Q_t^* \phi_i e_{it} v_{it})$:

$$H_t = V_t^{*\,\alpha} S_t^{*\,1-\alpha} \quad , \quad V_t^* = \int \phi_i e_{it} v_{it}\, di \quad , \quad S_t^* = \sum_{k=1}^K a_{kt} \mathcal{S}_{kt}, \tag{6}$$

This delivers consistency in that $H_t = \int h_{it} di = Q_t^* V_t^*$. In this expression V_t^* is the mass of *effective vacancies*. The mass of *effective worker search effort* S_t^* is determined by the time-varying search intensity a_{kt} of the K different searcher types, and their masses $\{\mathcal{S}_{kt}\}_{k=1}^K$. Let type $k = 1$ denote unemployed workers. We normalize the search intensity of the unemployed to 1 such that in (6) we have $a_{1t} \mathcal{S}_{1t} = U_t$ where U_t is the measure of unemployed workers. If we then multiply and divide the matching function by $U_t^{1-\alpha}$ we obtain $H_t = A_t V_t^{*\,\alpha} U_t^{1-\alpha}$, where A_t is *aggregate worker search intensity*:

$$A_t = \left[\sum_{k=1}^K a_{kt} \frac{S_{kt}}{U_t} \right]^{1-\alpha}. \tag{7}$$

The term in the square bracket reflects the composition of the pool of job seekers and can be estimated from type-k worker flow data. In what follows, however, we show how to estimate A_t as a residual from aggregate data.

The empirical matching function $H_t = A_t V_t^{*\,\alpha} U_t^{1-\alpha}$ contains unobserved effective vacancies, but can be expressed in terms of JOLTS vacancies, unemployment and *aggregate recruiting intensity* Φ_t, which we define:

$$H_t = \Phi_t A_t V_t^{\alpha} U_t^{1-\alpha} \quad , \quad \Phi_t = \left(\frac{V_t^*}{V_t} \right)^\alpha = \left[\int \phi_i e_{it} \frac{v_{it}}{V_t}\, di \right]^\alpha. \tag{8}$$

However, this is only a *partial equilibrium* definition since the establishment-level choice of e_{it}

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itself depends on Φ_t through the slackness of the labor market Q_t^*.

General equilibrium. To solve for Φ_t in general equilibrium, first let $\theta_t = (V_t/U_t)$ denote *measured market tightness*. The matching function implies the aggregate meeting rate Q_t^* depends on $\{\Phi_t, A_t, \theta_t\}$:

$$Q^* (\Phi_t, A_t, \theta_t) = \frac{H_t}{V_t^*} = A_t \left(\frac{V_t^*}{U_t} \right)^{-(1-\alpha)} = \Phi_t^{-\frac{1-\alpha}{\alpha}} \underbrace{A_t \theta_t^{-(1-\alpha)}}_{:=Q(A_t,\theta_t)} \tag{9}$$

where we define the *measured meeting rate* $Q(A_t, \theta_t) := Q^*(1, A_t, \theta_t)$ which we use extensively below.

We measure aggregate recruiting intensity in general equilibrium by substituting the microeconomic optimal policy (4) into the macroeconomic aggregate (8):

$$\Phi_t = \left[\int Q^* (\Phi_t, A_t, \theta_t)^{-\gamma} \phi_i^{1-\gamma} \left(\frac{h_{it}}{n_{it}} \right)^\gamma \frac{v_{it}}{V_t} \, di \right]^\alpha \quad , \quad \gamma := \frac{\gamma_v}{\gamma_e + \gamma_v} \in (0, 1). \tag{10}$$

Along with (9), this describes a mapping $\Phi' = \varphi(\Phi, Q(A_t, \theta_t))$, for which general equilibrium ARI is the fixed point. A key property of this mapping is that recruiting intensity decisions across firms are *strategic complements*: φ given by (10), is such that $\varphi_\Phi > 0$. But despite this, the solution is stable since $\varphi_\Phi < 1$.

Intuitively the reason for stability is as follows. An increase in Φ tightens the labor market. This decreases the meeting rate Q^* with elasticity $\frac{1-\alpha}{\alpha}$ (equation 9). Firms respond to this decline in effective meeting rates by increasing recruiting intensity with elasticity γ (equation 4). When aggregated, this microeconomic response increases Φ' with elasticity α (equation 8). On net, therefore, a one percent increase in Φ increases Φ' by $\gamma(1 - \alpha) \in (0, 1)$ percent. Since $\gamma(1 - \alpha) < 1$, this recursive system is stable, and features a multiplier of $1/(1 - \gamma(1 - \alpha))$. If γ is large, as we will find it is in the data, then these general equilibrium effects can also be large.

A key byproduct of this result is the general equilibrium multiplier associated with a change in the effective meeting rate Q_t^*. The *partial equilibrium direct effect* of an increase in Q_t^* on Φ_t is $-\gamma\alpha$. This is measured directly through the firm level policy function as the product of the
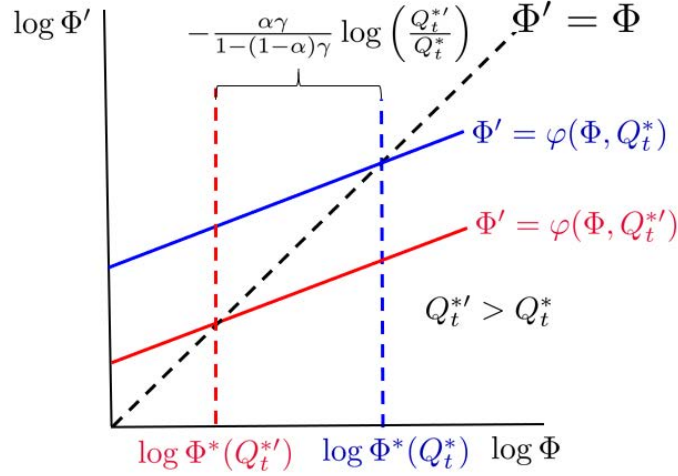
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Figure 3: Macro of recruiting intensity decisions

Notes: This figure describes the general equilibrium of the model. Given Φ on the x-axis, the angled lines plot the equilibrium response $\Phi' = \varphi(\Phi, Q^*)$. An increase in $Q_t^{*'} > Q_t^*$ shifts this function down, leading to a greater than one-for-one decrease in $\Phi_t' < \Phi_t$.

effect of Q_t on e_{it} $(-\gamma)$ times the effect of e_{it} on Φ_t (α). To get to the *general equilibrium indirect effect* we apply the multiplier and conclude that a one percent increase in Q_t^* decreases Φ_t by $-\gamma\alpha/(1 - \gamma(1 - \alpha)) < -\gamma\alpha$.

Figure 3(c) illustrates this argument graphically for an increase in Q_t^* to $Q_t^{*'}$ and the resulting decline in Φ_t.

Theoretical decomposition of ARI.   We obtain our main decomposition by consolidating (10). First, we substitute our expression for the meeting rate (9) and collect Φ_t terms. Second, we write the firm hiring rate in deviations from the aggregate hiring rate. Third, we extract from the integral terms that do not depend on i. We obtain the following expression for ARI which includes the general equilibrium multiplier:

$$\Phi_t = \underbrace{Q(A_t, \theta_t)^{-\frac{\gamma\alpha}{1-\gamma(1-\alpha)}}}_{Slack_t} \underbrace{\left(\frac{H_t}{N_t}\right)^{\frac{\gamma\alpha}{1-\gamma(1-\alpha)}}}_{Growth_t} \underbrace{\left[\int \phi_i^{1-\gamma} \left(\frac{h_{it}/n_{it}}{H_t/N_t}\right)^{\gamma} \frac{v_{it}}{V_t} di\right]^{\frac{\alpha}{1-\gamma(1-\alpha)}}}_{Comp_t}. \tag{11}$$

The first term is the *slackness component*. The labor market slackens in response to increasing worker search effort or a compositional shift toward higher search intensity types, both encoded

in residual match efficiency A_t. It also slackens due to changes in the ratio of vacancies to unemployed workers in the economy, which is encoded in *measured market tightness* $\theta_t = (V_t/U_t)$. When the labor market slackens, a firm's desired hires can be attained with less costly inputs: recruiting intensity and vacancies. The elasticity at which firms cut-back on e_{it} relative to v_{it} is γ. Therefore, the closer is γ to one (i.e., the smaller γ_e is relative to γ_v) the stronger is the response of Φ_t to changes in labor market tightness.

The second term is the *growth component*. When a firm grows it requires more recruiting intensity to realize hires. The common hiring rate of firms in the economy therefore effects aggregate recruiting intensity. Symmetric exponents on the two components stem from the hiring technology:

$$
h_{it} = Q_t^* \, \phi_i \, e_{it} \, v_{it} \qquad \Longrightarrow \qquad e_{it} \left(\frac{v_{it}}{n_{it}} \right) = \left(\frac{1}{Q_t^*} \right) \left(\frac{1}{\phi_i} \right) \left(\frac{h_{it}}{n_{it}} \right).
$$

The response of e_{it} is the same in magnitude, but opposite in sign, following an increase in input productivity (Q_t^*), or a increase in input demand (h_{it}/n_{it}).

The final term is the *composition factor* which reflects the contribution of firm heterogeneity. This term increases when the distribution of vacancies shifts toward (i) firms that are highly efficient in recruiting, i.e. that have high ϕ_i's, and (ii) firms that hire a lot relative to the aggregate. By construction, absent heterogeneity, if hiring firms are identical then Φ_t exactly equals $(Slack_t \times Growth_t)$.

Implementation and A_t. Equation (11) can be used exactly to answer our question: what drives aggregate recruiting intensity over the business cycle, and what are the roles of micro decisions and macro propogation. Our approach is to implement (11) empirically by constructing each term entirely from microdata.[8]

In doing so we face the issue that A_t is not observed. Even though we observe H_t, N_t and θ_t from aggregate data and $\{h_{it}, n_{it}, v_{it}\}$ from JOLTS microdata, we cannot fully construct Φ_t from (11) because we do not observe A_t. Rather than try to construct A_t from worker search data and

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[8]This exercise is therefore distinct from the simulations in Gavazza, Mongey, and Violante (2018), where ARI was inferred within the model and what was studied was an impulse response function of ARI to a financial shock.

(7), which we do not seek to model here, we approach this in a model-consistent way by using an additional equation from the model. We note that the matching function itself provides an additional equation in observables $\{H_t, V_t\}$ and the same two unknowns $\{\Phi_t, A_t\}$ as (11):

$$H_t = \Phi_t A_t V_t^\alpha U_t^{1-\alpha} \tag{12}$$

Our empirical implementation therefore proceeds in three steps. First, we estimate ϕ_i and γ using the firm's first order condition (5). Second, we combine these estimates with a choice of α and microdata $\{h_{it}, n_{it}, v_{it}\}$, to construct $Comp_t$. Third, we use $Comp_t$ along with aggregate data $\{H_t, N_t, V_t, U_t\}$ to simultaneously solve for Φ_t and A_t from equations (11) and (12).[9]

3 Data, identification, and estimation

Data. Our primary data sources are the restricted-use BLS microdata underlying JOLTS and the QCEW. JOLTS data are monthly *establishment level* responses of hiring managers with respect to employment on the twelfth of the month, hires over the calendar month, and open positions (vacancies) at the end of the month.[10] Apart from a permanent sample of firms that have remained in the JOLTS since inception, most establishments are present in the survey for 12 months, giving a short panel dimension. Our sample runs from 2002 to 2018. We drop 2001 due to reliability of JOLTS data in the year in which the initial panels of the survey were being rolled in.[11]

QCEW data are obtained through the UI system and provide month-end payroll and employment observations for the universe of establishments.[12] From the QCEW we compute es-

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[9]Specifically, equation (12) can be written $(H_t/V_t) = \Phi_t Q_t$, and equation (11) can be written $\Phi_t = Q_t^{-\mu}(H_t/N_t)^\mu Comp_t^{\mu/\gamma}$ where $\mu = \gamma\alpha/(1 - \gamma(1-\alpha))$. These can be solved for Φ_t and Q_t. We then use $Q_t = A_t(V_t/U_t)^{-(1-\alpha)}$ to back out A_t. The data requirements for this inversion are the hiring rate (H_t/N_t), vacancy yield (H_t/V_t) and market tightness (V_t/U_t), as well as $Comp_t$ which we construct from microdata.

[10]Since only some or one of a firm's establishments may be surveyed in a given month, one cannot construct firm level measures for multi-establishment firms.

[11]In Appendix A, from the JOLTS micro data, we plot the classic 'hockey stick' figures for hiring rate, separation rate, vacancy yield, vacancy fill rate, and vacancy flow as a function of the establishment growth rate. The estimated patterns conform with those previously documented in the literature for different time periods.

[12]We check monthly employment in the QCEW against the establishment reported employment in the JOLTS and find them to have a correlation coefficient close to one.

tablishment *wage* as payroll divided by employment. We obtain establishment *age* using a BLS produced measure of entry into QCEW sample. The data are merged using BLS identifiers. To the best of our knowledge, few previous papers have used the JOLTS microdata, and this is the first to combine them with the QCEW to construct age and average wage for JOLTS establishments.[13]

Summary statistics.   The decomposition (11) shows that, theoretically through $Comp_t$, heterogeneity can play an important role in shaping ARI. Figure 4 motivates careful measurement of $Comp_t$ by depicting the vast heterogeneity in hiring in the cross-section. There is systematic heterogeneity across industries, ages, sizes and wages in recruiting outcomes of firms. The vacancy rate and gross hiring rates (panels A, B, C, D) and the number of hires relative to open vacancies (panels E, F, G, H), all vary systematically.[14] This evidence rejects the standard random matching model where all firms face the same vacancy filling rates. Our model interprets these differences as systematically different recruiting intensities, and aggregates them into the $Comp_t$ term.

Specification.   Using data at the month t, establishment i level, we would ideally estimate the following specification, which is the empirical counterpart of (5):

$$\log \left(\frac{h_{it}}{v_{it}} \right) = \delta_t + \xi_i + \beta \log \left(\frac{h_{it}}{n_{it}} \right) + \varepsilon_{ijt}. \tag{13}$$

The time effect δ_t absorbs other unobserved aggregates beyond Q_t^*, so we do not use it to infer Q_t^* in our construction of (10). Instead, as we have shown, we construct $Q(A_t, \theta_t)$ directly. However we do use estimates of fixed effects ξ_i to infer recruiting efficiencies ϕ_i, and β to infer γ. With a choice of α, and microdata on $\{h_{it}, n_{it}\}$ we can then construct all terms in $Comp_t$ (11).

---

[13]Examples of previous articles to use the JOLTS microdata are Faberman and Nagypal (2008), Davis, Faberman, Haltiwanger, and Rucker (2010), Davis, Faberman, and Haltiwanger (2012), Faberman (2014), DFH, Elsby, Michaels, and Ratner (2018).

[14]The *daily filling rate* in these last four panels is computed from the daily recruiting model of DFH. Details and closed forms are found in Appendix B.
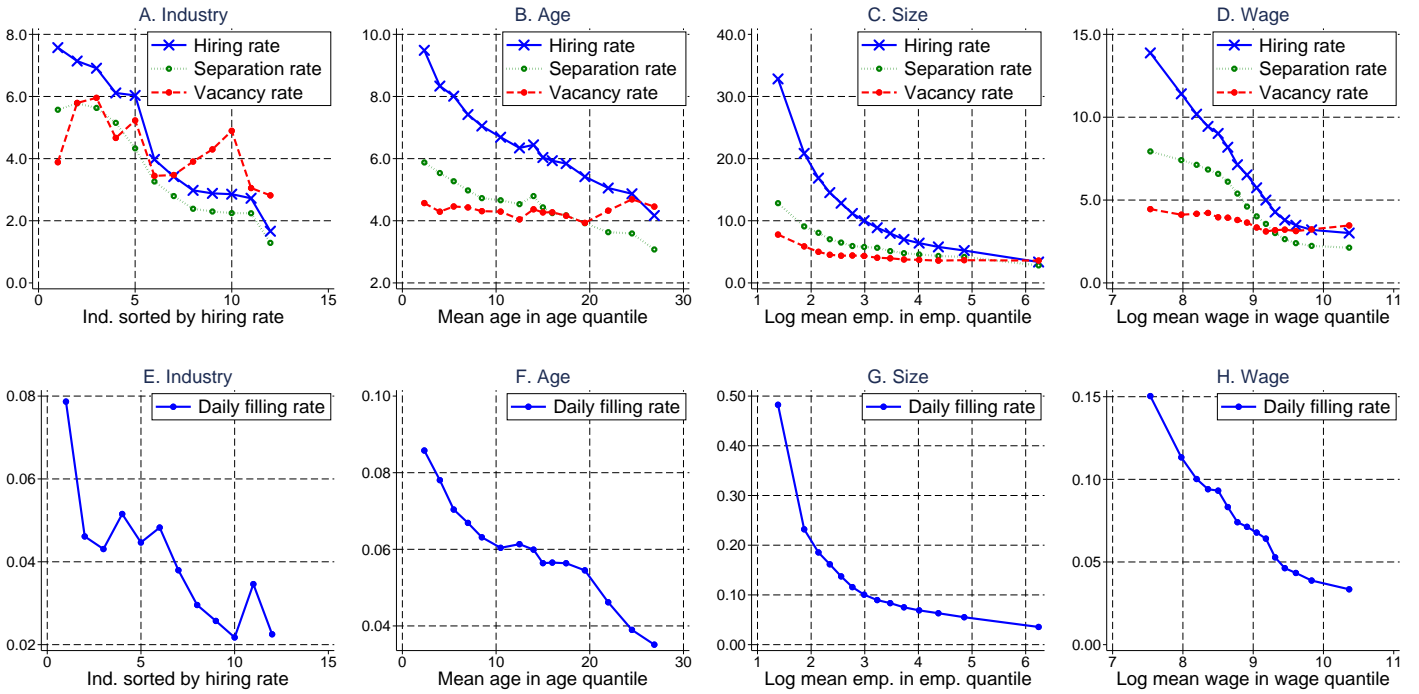
Figure 4: Summary statistics of heterogeneity in recruiting

Notes: Consider a point in panel B, for example. Establishment-month observations are first categorized by 15 quantiles of firm age. When constructing quantiles we pool all data from 2002-2018. Within an age quantile, we then pool across time and compute total hires, vacancies, employment, separations. We use these to construct the hiring rate, separation rate and vacancy rate. For Panel A, establishments are categorized into industries according to groupings of NAICS codes defined in Table C1, we then sort industries by hiring rate to construct the x-axis. In Panel C, they are categorized into size-groups measured as total employment. In panel D, they are categorized by average firm-wage computed as total payroll divided by employment. Panels E to H plot the daily filling rate computed from the daily recruiting model of DFH outlined in Appendix B.

Implementation. First, due to short panels of only twelve months at the firm level, we estimate fixed effects ξ for groups of firms $j = 1, \ldots, J$. We seek to be robust to the particular grouping of firms used, so consider different approaches, allowing j to determine quintiles of either (i) age, (ii) employment, or (iii) wage, as well as (iv) 11 industry categories. Second, *within* these groups j, we aggregate firms within narrow industries. For example, one specification uses age quintiles for j and then within each age quintile we aggregate firms within $NAICS$ 4-digit industries to construct h_{ijt}, n_{ijt} and v_{ijt}. Thus, in this case, i indexes all firms of a specific 4-digit industry within an age quantile.[15] This also addresses issues of time aggregation. We

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[15]When constructing h_{ijt}, v_{ijt} and n_{ijt} we aggregate within ijt using the same weights that the BLS applies to compute published aggregates. These account for systematic biases for non-response, as well as generating a representative sample.

observe hires for all firms during the month h_{it} but some firms will hire with no vacancies v_{it-1}. As noted by DFH, around 40 percent of hires in month t ($h_{it} > 0$) occur in establishments with no vacancies at the end of month $t-1$ ($v_{it-1} = 0$).[16]

The above strategy delivers 15 alternative approaches to estimating the following modification of (13):

$$\log\left(\frac{h_{ijt}}{v_{ijt}}\right) = \delta_t + \xi_j + \beta \log\left(\frac{h_{ijt}}{n_{ijt}}\right) + \varepsilon_{ijt}. \tag{14}$$

We have 4 different variables {age, size, wage, 1-digit industry} whose quintiles are used to define the groups j. Then, within the first three groups we have 4 different levels of aggregation at either the NAICS-{1,2,3, or 4} digit level ($3 \times 4 = 12$) that determine how we aggregate n_{ijt}, h_{ijt} and v_{ijt}. For the fourth group in which we let j denote the NAICS 1-digit level industry (11 in total), we only consider NAICS-{2,3, or 4} digit aggregation leading to $12 + 3 = 15$ specifications in total.

We are confident in the specification of the log-linear model and constant coefficients. Recall that Figure 1 showed (i) non-parameterically that imposing a log-linear relationship is without loss of generality, and (ii) that the relationship is stable in and out of the Great Recession.

Estimates. Table 1 provides estimates of $\gamma = \widehat{\beta}$ for these 15 different specifications of (14). From left to right, aggregating at finer industry levels within groups lowers the coefficient estimate, which is suggestive of measurement error.

Holding the level of aggregation fixed, and going from top to bottom, the estimate of γ is robust to the categorization of j. The overall inter-quartile range of estimates is only 0.07. By comparison, DFH group firms by net employment growth, aggregate all observations within these groups over six years, and obtain an estimate of 0.82 (cf: Figure IX). Our estimates are remarkably similar given that we aggregate at the far finer {month, NAICS4, employment quintile}-level.[17]

---

[16]They pool data across time (2001-2006) within a growth rate bin in constructing their key Figure IX. Our approach is to pool at a far finer level: for example, within an {age-quantile}-{4 digit industry code}-{month} cell.

[17]Two points regarding disclosure of our BLS microdata results. First, we cannot disclose estimates of the ξ_j terms that are used to infer ϕ_j coefficients, or the $h_{ijt}, v_{ijt}, n_{ijt}$ terms constructed for the estimation of (14). Second, allowing these terms to vary by time (ξ_{jt}) is found to have no effect on coefficient estimates in Table 1. Using time-varying estimates to infer $\widehat{\phi}_{jt}$ and then also using these in the decomposition (11) also has no effect on the

Categories j	NAICS			
	1	2	3	4
Industry	-	0.76	0.77	0.73
Age	0.84	0.77	0.76	0.73
Size	0.83	0.76	0.65	0.64
Wage	0.73	0.70	0.74	0.70

Table 1: Recruiting intensity in the cross-section

Notes: Point estimates of the coefficient on the hiring rate from regression (13). In all cases the coefficient is statistically significant at the one percent level. The estimation uses JOLTS microdata from 2002:1 to 2018:12. Rows give the manner in which establishments are grouped in order to estimate ϕ_j terms. Columns give the industry level at which hires, vacancies and employment are aggregated within these groups.

One may be concerned that 'luck' drives these results. Given some measure of vacancies, some firms get lucky and hire many workers, which increases both their vacancy-yield and hiring-rate. These appear on the left and right side of (13), and would lead to a positive β while the underlying parameter γ is potentially zero. This possibility is comprehensively covered by DFH through Monte-Carlo exercises.[18] They conclude that *"the luck effect accounts for one-tenth of the observed positive relationship"*, which would moderate our estimates in Table 1 by $0.06 - 0.08$. In addition, based on Figure 5 which we discuss below, for 'luck' to be driving the relationship between hiring rate and vacancy yield, this luck would have to be perfectly correlated with establishment age, size, wage, and industry.
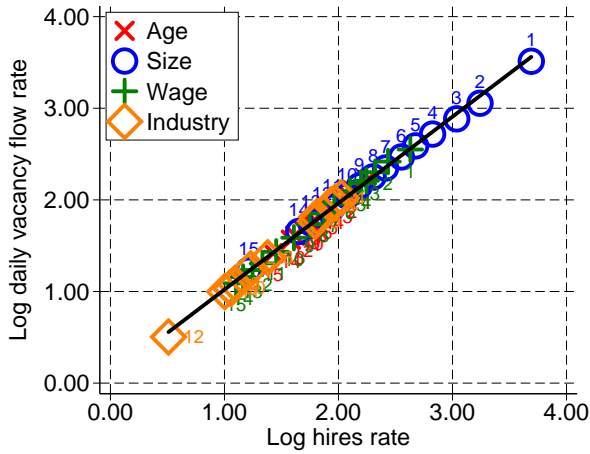
Discussion. Figure 5 sheds light on the robustness of these results across different categorizations of firms. For a given categorization—size, age, wage, industry—we split firms into 15 quantiles. Within each group we pool employment, hires and vacancies to compute the average hiring rate and vacancy yield.[19] Remarkably, across-group differences in vacancy yields are revealed entirely through differences in hiring rates. If there was something special about

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decomposition. This is consistent with our results that common micro-responses to labor market conditions rather than heterogeneity drive fluctuations in ARI. To accelerate disclosure from the BLS, results under time varying (ξ_{jt}) were not part of the data that we disclosed for this version of the paper. These results may be disclosed in the future for purposes of robustness. The specification makes clear identification of β in (14): ξ_{jt} pulls out the time series of group (e.g. age-quintile) means, while β is identified using *within-group-month*, *across-industry* variation, where this variation is at as fine as a NAICS4 level.
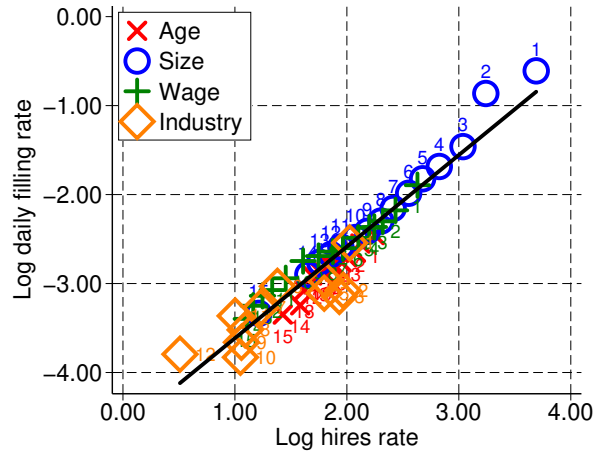
[18] See their page 601 and Figure VIII.

[19] Note that computing total quantile hires $H_q = \sum_{it \in q} h_{it}$, and total employment $N_q = \sum_{it \in q} n_{it}$, and then computing the hiring rate as H_q / N_q, is equivalent to computing the employment weighted hiring rate within quantile q.
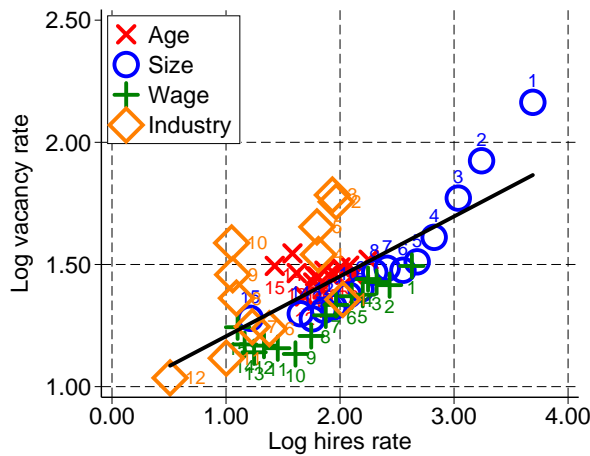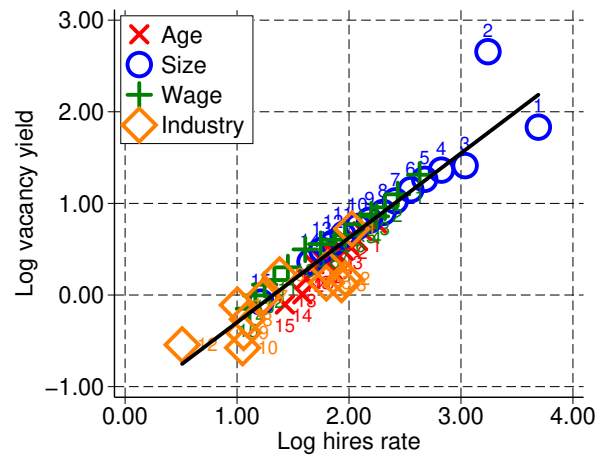
Figure 5: Recruiting intensity in the cross-section

the efficiency of young (or small, high-wage, etc.) firms in attaining higher vacancy yields, we would expect them to deviate from the systematic relationship between hiring rate and vacancy yield displayed in the data.

4 Results

Given our estimates of γ and ϕ_j and microdata $\{h_{ijt}, n_{ijt}, v_{ijt}\}$, we can compute our measure of aggregate recruiting intensity Φ_t from (10), and decompose it using (11). The only additional object we require is the matching function elasticity of meetings to effective market tightness. We set $\alpha = 0.50$, a value common in the literature.

4.1 Variance decomposition

Table 2 decomposes the time-series variance of aggregate recruiting intensity using (11). We take logs of (11), first difference to remove the constant, smooth the series using the X13-ARIMA-SEATS filter, then compute time-series variances of each term.[20] Regardless of how we group establishments to compute ϕ_j or coarseness of aggregation within groups, the dominant component is $Slack_t$, on average accounting for nearly 60 percent of the time-series variance of Φ_t.[21] The $Growth_t$ and $Comp_t$ terms, combined, account for less than 10 percent. Unsurprisingly the covariance term is positive and large, and driven mostly by the covariance between the slackness and growth factors.

Empirically, one key result is the small role of heterogeneity. To further understand this

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[20]X13-ARIMA-SEATS is processed in the *R* package *'seasonal'* and chooses the appropriate transformation of the raw series. The X13-ARIMA model assumes the first difference of nonseasonal and seasonal components follow $MA(1)$ processes. This approach was chosen to most closely match the relationship between published (i) non-seasonally-adjusted and (ii) seasonally adjusted BLS data for aggregate hires, unemployment, employment and vacancies.

[21]This macro finding has a micro counterpart. There is a tradition in labor economics of designing small-scale ad-hoc surveys to investigate recruitment methods of firms. Some papers in the literature document that firms respond to aggregate conditions. A recent example is Forsythe and Weinstein (2018) which finds that when campus recruiters expect the labor market to be slack, they cut recruiting intensity through on-campus career fairs, job postings and advertising. A classic article in this literature is Malm (1954). On page 519, the author writes: *During a tightening of the labor market [...] employers react to the increasing difficulty of finding job applicants by using more intensive (usually more expensive, both in terms of time and in cash outlay) recruiting methods.*

j	NAICS	1. Aggregate recruiting intensity				2. Composition		
		Slack	Growth	Comp.	Cov.	Between	Within	Cov.
Industry	2	0.57	0.028	0.020	0.38	0.13	0.47	0.40
	3	0.57	0.025	0.026	0.38	0.09	0.57	0.34
	4	0.54	0.033	0.050	0.38	0.08	0.68	0.24
Age	1	0.67	0.015	0.058	0.26	0.05	0.64	0.31
	2	0.61	0.028	0.045	0.32	0.05	0.67	0.28
	3	0.61	0.030	0.042	0.32	0.03	0.78	0.19
	4	0.55	0.035	0.138	0.28	0.05	0.68	0.27
Size	1	0.72	0.018	0.017	0.24	0.02	0.84	0.14
	2	0.63	0.031	0.028	0.31	0.03	0.85	0.12
	3	0.52	0.051	0.045	0.38	0.02	0.84	0.14
	4	0.50	0.054	0.081	0.36	0.02	0.88	0.10
Wage	1	0.44	0.033	0.074	0.45	0.12	0.47	0.41
	2	0.47	0.042	0.050	0.44	0.11	0.58	0.31
	3	0.54	0.037	0.046	0.38	0.06	0.75	0.19
	4	0.55	0.047	0.078	0.32	0.04	0.98	-0.02
Average		0.57	0.034	0.051	0.35	0.07	0.69	0.24

Table 2: Decomposing aggregate recruiting intensity

<u>Notes:</u> This table presents the time-series variance decomposition of equation (11) (**1. Aggregate recruiting intensity**) and (15) (**2. Composition**). The decomposition in each case is computed as follows. First, logs of the equation are taken. Second, the time-series variance of each term is computed. Third, the entry in the table gives the fraction of the time-series variance of the left-hand side variable attributable to the different right-hand side variables. The contribution due to covariance terms are grouped together under *Cov.*. The different rows represent the alternative groupings used to estimate (13). For example for $j =$"Age" and $NAICS = 3$, the categorical variable used to construct the ϕ_j match efficiency terms are quintiles of establishment age. Within these quintiles firms are split into 3-digit NAICS subsectors. Within these sub-groups we then aggregate establishment-month hires, employment, and vacancies to compute $\{h_{ijt}, n_{ijt}, v_{ijt}\}$ which are used as inputs into the regression and for the computation of the terms in the variance decompositions.

finding we split the composition term $Comp_t$ between and within groups:

$$
\underbrace{\left[\int \phi_j^{1-\gamma} \left(\frac{h_{ijt}/n_{ijt}}{H_t/N_t} \right)^\gamma \frac{v_{ijt}}{V_t} \right]^{\frac{\alpha}{1-\gamma(1-\alpha)}}}_{\text{Composition from (11): } Comp_t} = \tag{15}
$$

$$
\underbrace{\left[\sum_{j=1}^{J} \phi_j^{1-\gamma} \left(\frac{h_{jt}/n_{jt}}{H_t/N_t} \right)^\gamma \frac{v_{jt}}{V_t} \right]^{\frac{\alpha}{1-\gamma(1-\alpha)}}}_{\text{Between groups } j: \, Between_t} \underbrace{\left[\sum_{j=1}^{J} \left[\int_{i \in j} \left(\frac{h_{ijt}/n_{ijt}}{h_{jt}/n_{jt}} \right)^\gamma \frac{v_{ijt}}{v_{jt}} \right] \left\{ \frac{\phi_j^{1-\gamma}(h_{jt}/n_{jt})^\gamma \frac{v_{jt}}{V_t}}{\sum_{j=1}^{J} \phi_j^{1-\gamma}(h_{jt}/n_{jt})^\gamma \frac{v_{jt}}{V_t}} \right\} di \right]^{\frac{\alpha}{1-\gamma(1-\alpha)}}}_{\text{Within groups } j: \, Within_t}
$$

where h_{jt}, v_{jt} and n_{jt} are aggregates at the group-j, month-t level. The final columns of Table

2 show that in general the within group-j term dominates. That is, $Comp_t$ is not driven by the cyclical reallocation of vacancy-shares across high or low ϕ_j groups, which would be captured by $Between_t$. A single sector model, that captures both aggregate $(Slack_t, Growth_t)$ and firm-level behavior $(Within_t)$ could therefore approximate well the cyclical behavior of Φ_t.

Figure 6 presents these results graphically. We fix a particular case in order to plot results, choosing the case where j denotes quintiles of firm size, and within j we aggregate at the $NAICS4$ level (corresponding to line 11 in Table 2). Panel A shows the slackness component closely following Φ_t. A steep drop in the growth component also contributes to the decline in ARI over the Great Recession. In this case, almost 100 percent of the composition term is driven by within size-quantile, across-NAICS4 variation in recruiting intensity (panel B).[22]

In summary, we find that empirically, the components of aggregate recruiting intensity that do not reflect firm heterogeneity per se are the dominant forces that shape aggregate recruiting intensity. This result is robust to the manner in which permanent heterogeneity in recruiting efficiency is handled.

5 Applications

We use the above empirical results to first construct an easily computable index of ARI. We test this index against the exact time-series for ARI that we have constructed, and show that it corresponds to true ARI in a representative firm model. We then use this formulation to conduct a simple counterfactual exercise to understand the role that ARI played in unemployment in the Great Recession.

5.1 An easily computable index of ARI

We build on the results of Section 4 to produce an easy to measure, microfounded, *index* of aggregate recruiting intensity which we denote Φ_t^{Index}. Our microdata exercise has taught us that we can capture the true empirical measure of Φ_t with only $Slack_t$ and $Growth_t$. Abstracting

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[22]Figure A3 in Appendix A plots the implied residual match efficiency A_t. Our estimates suggest that this residual is countercyclical, but less volatile than ARI. This would be consistent with countercyclical worker job search effort as documented by (Mukoyama, Patterson, and Şahin, 2018).
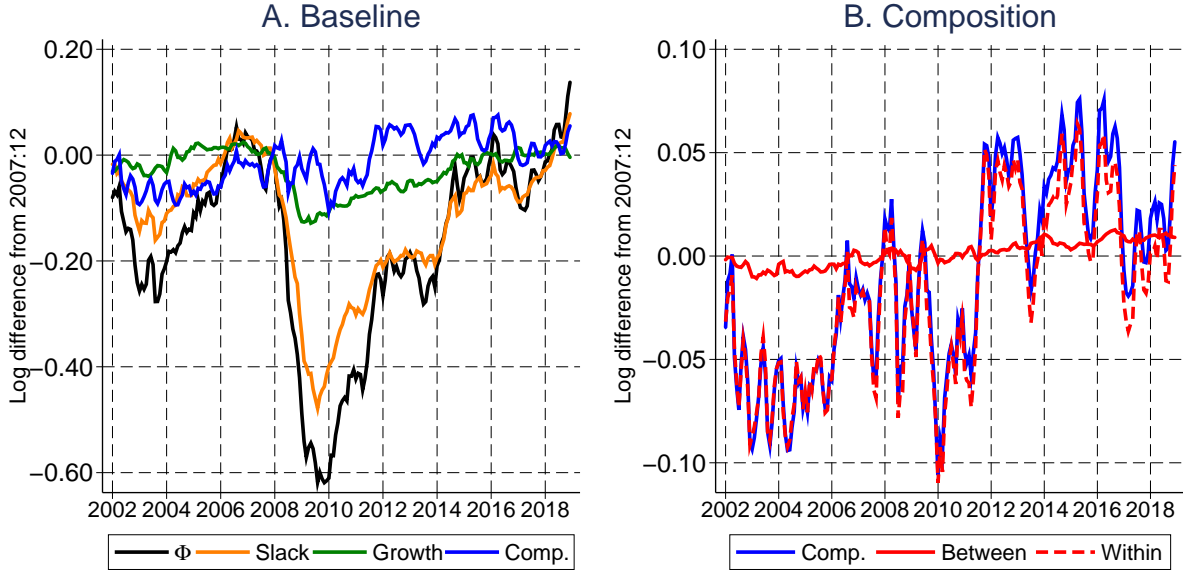
Figure 6: Decomposing aggregate recruiting intensity

Notes: Panels A and B present an example of the components of equations (11) and (15). In this case we have grouped firms by quintiles of employment for estimating ϕ_j, and within these quintiles aggregated hires, vacancies and unemployment within 4 digit industries. Time-series are first deseasonalised using X13-ARIMA-SEATS. For presentation only we also apply a three month centered moving average to each series.

from the composition factor in (11), we obtain

$$\Phi_t^{Index} = \underbrace{Q\left(A_t, \theta_t\right)^{-\frac{\gamma\alpha}{1-\gamma(1-\alpha)}}}_{Slack_t} \underbrace{\left(\frac{H_t}{N_t}\right)^{\frac{\gamma\alpha}{1-\gamma(1-\alpha)}}}_{Growth_t} \tag{16}$$

Since it contains A_t, this expression cannot be computed on publicly available data. However, we can use the aggregate vacancy yield from the matching function to substitute out $Q\left(A_t, \theta_t\right)$:

$$\frac{H_t}{V_t} = \Phi_t^{Index} Q\left(A_t, \theta_t\right). \tag{17}$$

Substituting (17) into (16) via $Q(A_t, \theta_t)$, it is clear that H_t drops out and we are left with a convenient expression that depends only on the aggregate *vacancy rate*. This expression is indexed by the elasticity of the matching function (α) and the micro-elasticity of recruiting intensity (γ), and is consistent with how firm behavior (16) depends on aggregates, and how aggregates
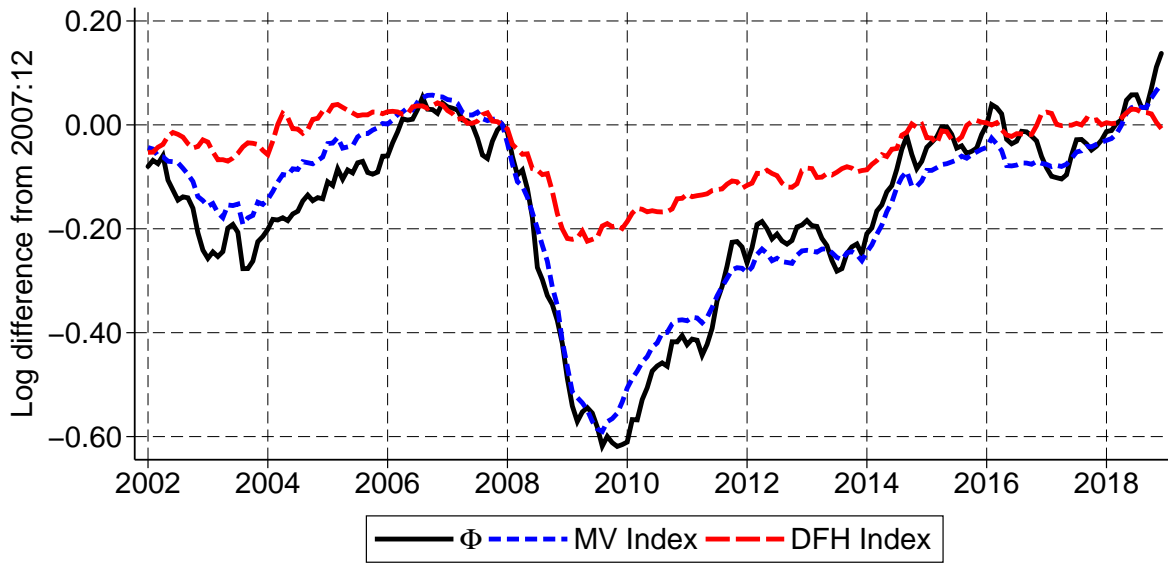
Figure 7: Indexes of aggregate recruiting intensity

depend on firm behavior (17):

$$\Phi_t^{Index} = \left(\frac{V_t}{N_t} \right)^{\frac{\gamma \alpha}{1-\gamma}} . \tag{18}$$

Figure 7 plots Φ_t^{Index}, alongside our empirical measure Φ_t. The index closely tracks Φ_t. On average across our 15 specifications Φ_t^{Index} accounts for 98 percent of the time-series variance of Φ_t. We therefore conclude that this index delivers an excellent approximation of estimated aggregate recruiting intensity.

Comparison. We now compare our index to that computed by DFH. Their index is computed as $\Phi_t^{DFH} = (H_t/N_t)^{0.82}$. The foundation for their index is as follows. As discussed earlier, from JOLTS microdata they estimate equation (5) and obtain a micro-elasticity of the job filling rate to the gross hiring rate of 0.82. They then set the macro elasticity equal to this micro-elasticity. Our measure differs. We find that the main contributing factor to the variation of job filling rates is the response of firm recruiting choices to equilibrium aggregate market tightness, which their transition from cross-sectional to time-series does not capture.

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Representative firm. To further ground our index in theory, we show that in general equilibrium, a representative firm model delivers Φ_t^{Index} as the exact measure of aggregate recruiting intensity.

Consider an economy populated by a unit measure of identical firms. Given initial employment n_t, each firm chooses its hires for the period h_t. They then choose vacancies v_t and recruiting intensity e_t to minimize total hiring costs. Firms are competitive in that they take the meeting rate for effective vacancies Q_t^* as given. For any given pair (h_t, n_t) the firm solves:

$$\min_{e_t, v_t} \; \mathcal{C}\left(e_t, v_t, n_t\right) v_t \quad \text{s.t.} \quad h_t = Q_t^* e_t v_t.$$

Under the assumptions on $\mathcal{C}$ in Proposition 1, and the definition $\gamma := \gamma_v / (\gamma_e + \gamma_v)$, the first order conditions of this problem deliver the same policies for recruiting intensity we derived in our model with heterogeneous firms:

$$e_t = Const. \times \left(Q_t^*\right)^{-\gamma} \left(\frac{h_t}{n_t}\right)^{\gamma}. \tag{19}$$

In equilibrium, $x_t = X_t$ for all variables. Since $V_t^* = \int_0^1 e_t v_t di = E_t V_t$, then $E_t = (V_t^* / V_t) = \Phi_t^{1/\alpha}$. As before, the matching function implies $Q_t^* = \Phi_t^{-(1-\alpha)/\alpha} Q(A_t, \theta_t)$. In equilibrium, these properties and the first order condition (19) imply:

$$\Phi_t = Q_t^{*-\gamma\alpha} \left(\frac{H_t}{N_t}\right)^{\gamma\alpha} = Q(A_t, \theta_t)^{-\frac{\gamma\alpha}{1-\gamma(1-\alpha)}} \left(\frac{H_t}{N_t}\right)^{\frac{\gamma\alpha}{1-\gamma(1-\alpha)}}. \tag{20}$$

This expression contains the slackness and growth components of our general model, and corresponds exactly to Φ_t^{Index}. This formulation may be used by future researchers to represent firm recruiting choices in arbitrarily rich DSGE environments with frictional labor markets.

5.2 Counterfactual

As a final exercise we seek to isolate the role of aggregate recruiting intensity for the dynamics of the job finding rate and the unemployment rate in the Great Recession. Besides unemployment, there are three other inputs into the evolution of job finding rate: vacancies V_t, aggregate

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recruiting intensity Φ_t, and residual match efficiency A_t.

Formally, we ask the following question: Over the Great Recession, how would the job finding rate F_t and unemployment U_t have evolved if aggregate recruiting intensity Φ_t fell, but vacancies V_t and residual match efficiency A_t remained unchanged at their pre-recession level?

To answer this question we consider the following dynamic system:

$$H_t = \Phi_t A_t U_t^\alpha V_t^{1-\alpha} \tag{21}$$

$$U_{t+1} = (1 - F_t) U_t + S_t \quad , \quad \text{where} \quad F_t := H_t / U_t. \tag{22}$$

First, with our series for Φ_t, and data on $\{H_t, U_t, V_t\}$ we construct residual match efficiency A_t from the matching function (21). We also construct a consistent series for separations S_t from observed unemployment dynamics (22). Second, we construct our counterfactual series for match efficiency and vacancies, by fixing values at their pre-recession levels: $\widetilde{A}_t = A_{2008:1}$ and $\widetilde{V}_t = V_{2008:1}$ for all t. Hence, the only time-varying input into hiring is Φ_t. Third, we construct our counterfactual series for our variables of interest by starting from $\widetilde{U}_{2008:1}$, and using $\left\{\widetilde{A}_t, \widetilde{V}_t, \Phi_t, S_t\right\}$ along with (21) and (22) to construct counterfactual $\left\{\widetilde{F}_t, \widetilde{U}_t\right\}_{t=2008:1}^{2018:12}$ (for more details see the footnote of Figure 8).

Figure 8 shows that the decline in the recruiting intensity of hiring firms alone accounts for a large part of the decline in the job finding rate, and for about half the increase in unemployment at the onset of the recession. However, as the vacancy rate recovers quickly and the labor market starts tightening again, recruiting intensity rebounds as firms once more increase their recruiting inputs to realize hires. Thus under our counterfactual, unemployment returns to near its pre-recession level by 2012. In the data 2012 unemployment is still 60 percent above its pre-recession level. We conclude that the decline in ARI is important in explaining unemployment dynamics at the onset of the recession, but not its slow recovery

Further proof on the role of ARI is offered by the dynamics of the aggregate vacancy yield $H_t / V_t = \Phi_t A_t \theta_t^{-(1-\alpha)}$. The sharp drop in ARI is key to the moderate rise of vacancy yield in the Great Recession (by approximately 50% relative to its pre-recession level). In its absence, the vacancy yield would display a wildly counterfactual fourfold increase relative to the data. This finding represents a challenge for models of the last recession which claim to succeed in
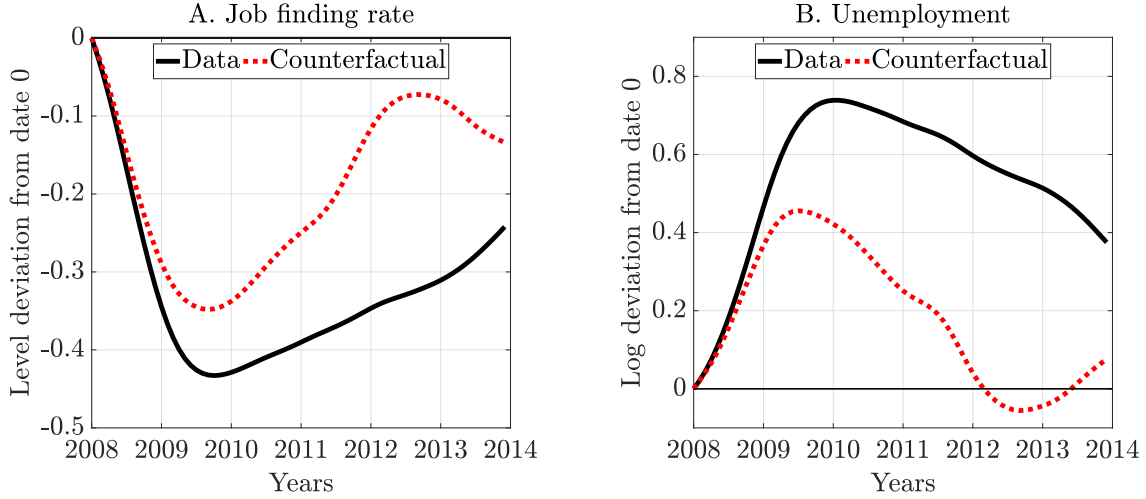
26

Figure 8: Counterfactual job finding rate and unemployment due only to the decline in Φ_t

Notes: The counterfactual series in this figure are constructed as follows. First take the aggregate matching function $H_t = \Phi_t A_t U_t^\alpha V_t^{1-\alpha}$. The job finding rate is $f(A_t, \Phi_t, U_t, V_t) = H_t/U_t$. We combine this with the empirical law of motion $U_{t+1} = (1 - f(A_t, \Phi_t, U_t, V_t)) U_t + S_t$. Given data on $\{H_t, \Phi_t, U_t, V_t\}$ we use the matching function to construct A_t, and the law of motion for unemployment to construct S_t. We then freeze non-recruiting intensity inputs, setting $A_t = A_0$, $V_t = V_0$. We then use $\{A_0, V_0, \Phi_t, S_t\}$ to construct a counterfactual path for unemployment $\widetilde{U}_t$ starting at U_0 as in the data. That is, $\widetilde{U}_1 = (1 - f(A_0, \Phi_0, U_0, V_0)) U_0 + S_0$, and then $\widetilde{U}_2 = (1 - f(A_0, \Phi_1, \widetilde{U}_1, V_0))\widetilde{U}_1 + S_1$. Panel A plots $\widetilde{f}_t = f(A_0, \Phi_t, \widetilde{U}_t, V_0)$. Panel B plots $\widetilde{U}_t$. The red lines therefore measure the drop in the job finding rate and the consequent rise in unemployment due only to the decrease in aggregate recruiting intensity Φ_t, holding all other determinants of the job finding rate fixed, i.e. residual match efficiency A_0 and aggregate vacancies V_0. Note that, by construction, by feeding in also the observed series for V_t and our estimated series for A_t, we would match exactly the data for both job finding rate and unemployment.

explaining unemployment dynamics with constant aggregate match efficiency and without any role for cyclical firm recruiting intensity (e.g. Christiano, Eichenbaum, and Trabandt, 2015).

6   Conclusion

We conclude by highlighting two natural directions for further research.

First, motivated by empirical evidence (O'Leonard, Krider, and Erickson, 2015; Forsythe and Weinstein, 2018), we emphasized expenditures on recruiting activities as the key instrument firms use to modulate their search effort. Other margins, such as varying compensation packages and screening standards, may be important too. There is currently no representative microdata for the U.S. that allows researchers to disentangle these different mechanisms, but progress is being made for other countries, such as Austria and Germany (Mueller, Kette-

mann, and Zweimuller, 2018; Carrillo-Tudela, Kaas, and Gartner, 2018). This promising line of research that digs deeper into the black box of firm-level recruiting decisions could lead to a comprehensive model of firm recruiting which can be embedded into the canonical frameworks used by macroeconomists to study labor market dynamics.

Second, firms' recruiting intensity is only one of the factors that moves aggregate match efficiency and, at the end of the day, that is what matters for the volatility of the unemployment rate. The literature linking micro to aggregate recruiting intensity, effectively initiated by Davis, Faberman, and Haltiwanger (2013), is still in its infancy. A more established literature has studied two other sources of match efficiency dynamics over the business cycle: variation in worker's search effort and variation in misallocation ('mismatch') between vacant jobs and job seekers across sectors (occupations, industries, regions) of the economy. Research on these three factors has been, so far, disjoint. A unified framework to coherently estimate these various forces and theoretically understand how they interact with each other—in producing amplification and complementarities—would be another welcome advancement in the literature (see Crump, Eusepi, Giannoni, and Şahin (2019) and Leduc and Liu (2019) for first steps in this direction).

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This Appendix is organized as follows. Section A provides additional figures and tables. Section B provides details on math. Section C provides additional data on variable construction.

A   Additional figures and tables

This appendix section contains additional figures and tables referenced in the main text.

Figure A1: Hockey stick plots - Hiring rate, separation rate, vacancy yield
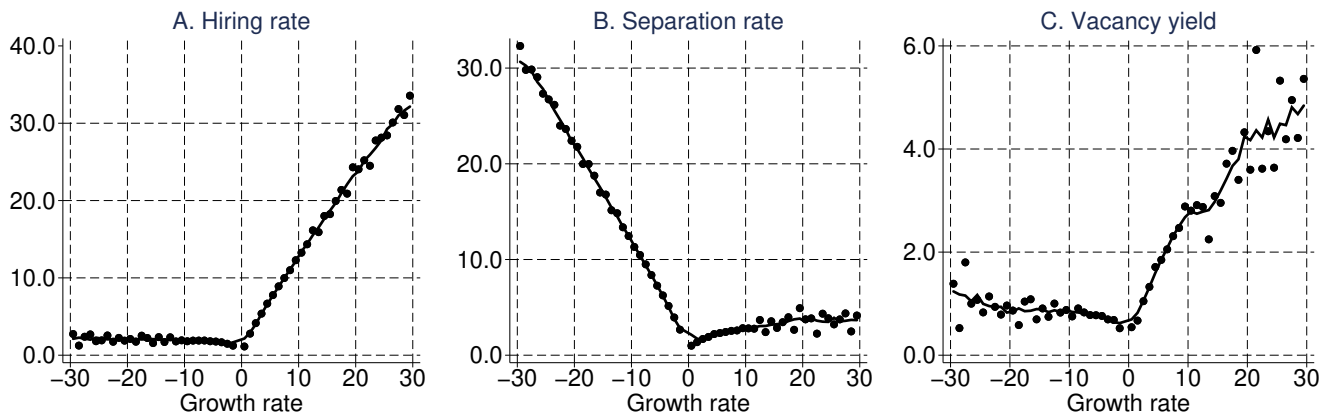
<u>Notes</u> Establishment-month observations in JOLTS microdata 2002-2018 are pooled in bins, where bins are determined by net monthly growth rate, and have a width of 1 percent. Growth rates computed as in DFH. Within bins b, total hires h_b, separations s_b, employment n_b, vacancies v_b are computed. From these, the gross hiring rate h_b/n_b (panel A), separation rate s_b/n_b (panel B), and vacancy yield h_b/v_b (panel C) are computed. Bins with positive gross hiring rates are kept.

Figure A2: Hockey stick plots - Filling and vacancy flow

A. Daily filling rate

B. Vacancy flow rate

Growth rate
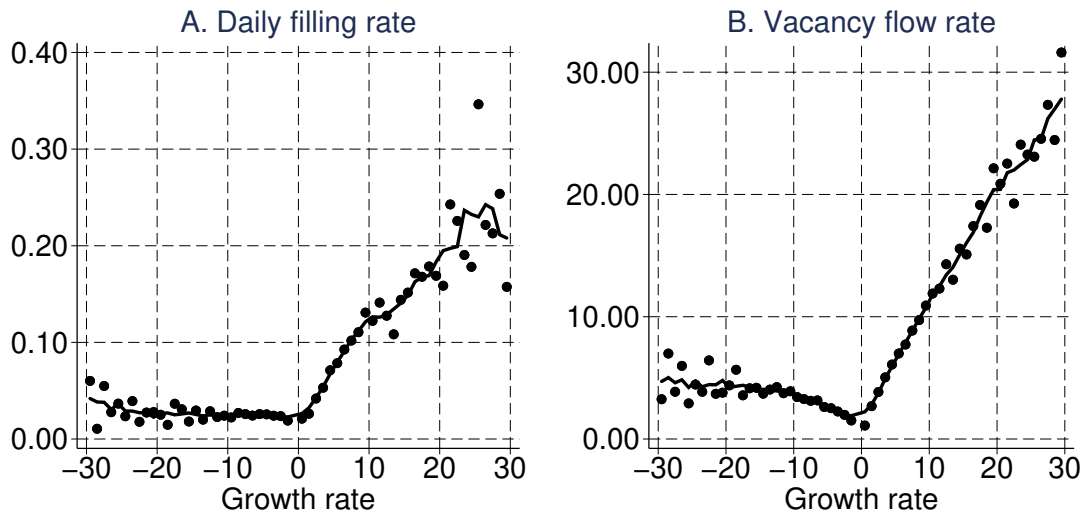
Notes: See note to Figure A1. Daily filling rate and vacancy flow rates are computed using our simplifications of the algebra of DFH daily hiring model. See Appendix B for details.

Figure A3: Residual match efficiency

Notes: This Figure plots the residual match efficiency term A_t implied by our estimation. This time-series is first deseasonalised using X13-ARIMA-SEATS. For presentation only we also apply a three month centered moving average to each series.

B Mathematical details

This section contains (1) the proof of Proposition 1 and (2) the derivation of the daily filling rate and vacancy flow rate used in the text.

B.1 Proof of Proposition 1

We begin by working explicitly with a cost function in the form of $C_i(e_{it}, v_{it}, n_{it}) = x_i C(e_{it}, v_{it}/n_{it})$, and in the necessity part of the proof show that this is the only way in which v and n can enter. Let $\tilde{v} = (v/n)$ denote the vacancy rate, and $\tilde{h} = (h/n)$ denote the hiring rate. The hiring problem can be written as follows:

$$\min_{e_{it}, v_{it}} x_i C\left(e_{it}, \frac{v_{it}}{n_{it}}\right) v_{it} \qquad \text{s.t.} \qquad h_{it} = Q_t^* \phi_i e_{it} v_{it}$$

which, removing it subscripts for convenience, and setting $\phi_i = 1$ without loss of generality, we write as:

$$\min_{e, \tilde{v}} x C(e, \tilde{v}) \tilde{v} n \qquad \text{s.t.} \qquad \tilde{h} = Q^* e \tilde{v} \tag{B1}$$

Sufficiency. We first show the following. If C is an isoelastic function $m(\cdot)$ of two, additive, isoelastic functions $g(e)$ and $f(\tilde{v})$, then the solution to (B1) delivers a vacancy yield $h/v = \tilde{h}/\tilde{v}$ and vacancy rate $\tilde{v}$ that are log-linear in the hiring rate $\tilde{h}$.

The first order conditions of the problem imply the following optimality condition, which along with the hiring constraint can be solved for $e\left(Q^*, \tilde{h}\right)$ and $\tilde{v}\left(Q^*, \tilde{h}\right)$:

$$C_e(e, \tilde{v}) e = C_v(e, \tilde{v}) \tilde{v} + C(e, \tilde{v}). \tag{B2}$$

Note that since x scales the cost function, it does not appear in the optimality condition. Despite affecting the firms' dynamic decision that controls $\tilde{h}$, x does not affect the recruiting input decision. If $C(e, \tilde{v})$ has the form just described:

$$C(e, \tilde{v}) = m\left(g(e) + f(\tilde{v})\right),$$

then the optimality condition ([B2](#)) can be written:

$$g\left(e\right)\left[\left(\frac{m'(g(e)+f(\widetilde{v}))(g(e)+f(\widetilde{v}))}{m(g(e)+f(\widetilde{v}))}\right)\left(\frac{g'(e)e}{g(e)}\right)-1\right]=f\left(\widetilde{v}\right)\left[\left(\frac{m'(g(e)+f(\widetilde{v}))(g(e)+f(\widetilde{v}))}{m(g(e)+f(\widetilde{v}))}\right)\frac{f'(\widetilde{v})\widetilde{v}}{f(\widetilde{v})}+1\right]$$

Since m, g and f are constant elasticity, this reduces to

$$g(e)\left[\gamma_m\gamma_e-1\right]=f(\widetilde{v})\left[\gamma_m\gamma_v+1\right]. \tag{B3}$$

Given that g and f are isoelastic, the solution to ([B3](#)) is of the form $\widetilde{v}=\Omega e^{\omega}$. Substituting this into the hiring technology $\widetilde{h}=Q^*e\widetilde{v}$ gives

$$\widetilde{h}=\Omega Q^*e^{1+\omega} \quad\Longrightarrow\quad e=\Omega^{-\frac{1}{1+\omega}}Q^{*-\frac{1}{1+\omega}}\widetilde{h}^{\frac{1}{1+\omega}}. \quad\overset{\widetilde{h}=Q^*e\widetilde{v}}{\Longrightarrow}\quad \frac{\widetilde{h}}{\widetilde{v}}=\Omega^{-\frac{1}{1+\omega}}Q^{\frac{\omega}{1+\omega}}\widetilde{h}^{\frac{1}{1+\omega}}.$$

Since $\widetilde{h}=Q^*e\widetilde{v}$ then it is immediate that $\widetilde{v}$ is also isoelastic in $\widetilde{h}$. Since γ_m only appears in the constant Ω, it can be normalized to one (i.e. $m(x)=x$) as we do in the paper without any impact on the key properties of the recruiting policies.

Necessity. We want to show the following. Suppose that under optimality the vacancy yield and vacancy rate are isoelastic in the hiring rate. Then the cost function takes the following form, where g and f are isoelastic: $C\left(e,v,n\right)=\left[g\left(e\right)+f\left(\frac{v}{n}\right)\right]$. Given our previous result that constant elasticity m only affects policy function constants we ignore it here. We proceed in five steps.

Step 1. We begin by simplifying the statement that we wish to prove. First, we show that if the supposition is true, then $\widetilde{v}$ and recruiting intensity must be isoelastic with respect to each other, i.e. have a constant elasticity relationship, as in $\widetilde{v}=\Psi e^{\psi}$. By the supposition $(\widetilde{h}/\widetilde{v})$ is log-linear in $\widetilde{h}$. From the hiring constraint $(\widetilde{h}/\widetilde{v})=Q^*e$. Therefore e is log-linear in $\widetilde{h}$: $e=\Omega\widetilde{h}^{\omega}$, which implies that $\widetilde{h}$ is an isoelastic function of e. Substituting this isoelastic function of e into the hiring constraint for $\widetilde{h}$ gives

$$\Omega^{-\frac{1}{\omega}}e^{\frac{1}{\omega}}=Q^*e\widetilde{v}.$$

The relationship between e and $\widetilde{v}$ is therefore constant elasticity: $\widetilde{v}=\Psi e^{\psi}$ for some Ψ and ψ.

Second, the supposition requires that the first order conditions hold. These give the optimality condition (B2).

Combining these two points allows us to simplify the statement that we wish to prove:

Suppose the optimality condition $C_e(e, \widetilde{v}) e = C_v(e, \widetilde{v}) \widetilde{v} + C(e, \widetilde{v})$ implies that $\widetilde{v} = \Psi e^{\psi}$, for some Ψ, ψ. Then $C(e, \widetilde{v}) = m(g(e) + f(\widetilde{v}))$, with isoelastic $m(x)$, $g(e)$ and $f(\widetilde{v})$.

We construct the proof by contradiction. Under the assumption that the cost function is not isoleastic, obtaining an optimal relation between e and $\widetilde{v}$ that features constant elasticity leads to a contradiction.

Step 2. We establish a particular implication in the case that $C(e, \widetilde{v})$ is **not** additively separable. Taking (B2), and rearranging:

$$e = \left[\frac{C_v(e, \widetilde{v})}{C_e(e, \widetilde{v})} \widetilde{v} \right] + \left[\frac{C(e, \widetilde{v})}{C_e(e, \widetilde{v})} \right]. \tag{B4}$$

In order for the supposition to hold, this must imply that $e = \Omega \widetilde{v}^{\omega}$. If C is **not** additively separable, then this requires that $e^{\frac{\omega-1}{\omega}}$ can be factored out of both terms on the right side of (B4), leaving only terms involving $\widetilde{v}$:

$$\frac{C_v(e, \widetilde{v}) \widetilde{v}}{C_e(e, \widetilde{v})} = \Gamma_1(\widetilde{v}) e^{\frac{\omega-1}{\omega}} \qquad , \qquad \frac{C(e, \widetilde{v})}{C_e(e, \widetilde{v})} = \Gamma_2(\widetilde{v}) e^{\frac{\omega-1}{\omega}}.$$

Moreover, to obtain $e = \Omega \widetilde{v}^{\omega}$ we require that $\Gamma_1(\widetilde{v}) = \Gamma_1 \widetilde{v}^{\gamma}$ and $\Gamma_2(\widetilde{v}) = \Gamma_2 \widetilde{v}^{\gamma}$, so that we can add the terms on the right side of (B4). Imposing this condition and then dividing the above two expressions gives

$$\frac{C_v(e, \widetilde{v}) \widetilde{v}}{C(e, \widetilde{v})} = \frac{\Gamma_1}{\Gamma_2}.$$

For this condition to hold, then it must be the case that $C(e, \widetilde{v}) = \Theta g(e) v^{\theta}$. We prove this last step at the end of the proof in **Lemma 1**.

Step 3. We show that if $C(e, \tilde{v}) = \Theta g(e) v^{\theta}$, then there is no way for the supposition to hold. Under this functional form the optimality condition (B2) becomes:

$$C_e(e, \tilde{v}) e = C_v(e, \tilde{v}) \tilde{v} + C(e, \tilde{v}),$$

$$\left[\Theta g'(e) \tilde{v}^{\theta}\right] e = \left[\theta \Theta g(e) \tilde{v}^{\theta-1}\right] v + \Theta g(e) v^{\theta}.$$

Since $\tilde{v}^{\theta}$ can be factored out of both sides, the optimality condition implies that e is independent of $\tilde{v}$ which violates the supposition.

Step 4. From steps 2 and 3 above we have established by contradiction that C must be additively separable for the supposition to hold. Now we show that if C is separable, then g and e must be isoelastic for the supposition to hold. If $C(e, \tilde{v}) = m(g(e) + f(v))$, then the optimality condition can be written

$$\frac{m'(g(e) + f(\tilde{v}))(g(e) + f(\tilde{v}))}{m(g(e) + f(\tilde{v}))} g_e(e) e - g(e) = \frac{m'(g(e) + f(\tilde{v}))(g(e) + f(\tilde{v}))}{m(g(e) + f(\tilde{v}))} f_v(v) v - f(v).$$

The supposition requires that the addition of functions on both left and right sides are isoelastic in e and $\tilde{v}$. This requires that m, g and f are themselves isoelastic.[23]

Step 5. Finally, note that the dependence of $C(e, v, n)$ on $\tilde{v}$ and not v and n separately can be shown. In terms of sufficiency we have already covered this. In terms of necessity, if (v, n) entered not as $\tilde{v} = (v/n)$, then the first order conditions would produce an extra term involving n's which would violate the requirement imposed by the data of an isoelastic relationship between $\tilde{v}$ and e.

Lemma 1. *If a function $f(x, y)$ has the property that*

$$\frac{f_x(x, y) x}{f(x, y)} = c,$$

---

[23]It is immediate that the terms involving m must both be constants, and hence m is isoelastic. The terms are the same and if they involve both or either of $\tilde{v}$ and $\tilde{e}$ will not result in an isoelastic relationship between e and $\tilde{v}$. To observe that f and g are isoelastic consider the following. We require that $F_x(x)x - F(x) = ax^b$. The left side can be written $F(x) [F_x(x)x/F(x) - 1]$. Therefore we require the term in the bracket to be a constant. This will only be the case if $F(x)$ is a constant elasticity function. We then require that the term outside the bracket is isoelastic. Therefore $F(x)$ must be isoelastic.

where c is a constant, then $f(x,y) = h(y)x^c$ for some function $h(y)$.

Proof. *Rearrange the above expression:*

$$\frac{f_x(x,y)}{f(x,y)} = \frac{c}{x}.$$

Integrating both sides and, without loss of generality, writing the constants of integration $\log h_1(y)$, and $\log h_2(y)$:

$$\log h_1(y) + \log f(x,y) = \log h_2(y) + c \log x.$$

Exponentiating delivers our the functional form we wished to establish:

$$f(x,y) = \underbrace{\frac{h_2(y)}{h_1(y)}}_{:=h(y)} x^c.$$

Policies. We now derive the policy functions in the text. Without loss of generality we let $C(e, \tilde{v}) = c_m \left(c_e e^{\gamma_e} + c_v \tilde{v}^{\gamma_v} \right)^{\gamma_m}$. Recalling equation (B3), the first order conditions implied

$$g(e) \left[\gamma_m \gamma_e - 1 \right] = f(\tilde{v}) \left[\gamma_m \gamma_v + 1 \right] \qquad \rightarrow \qquad \tilde{v}(e) = \underbrace{\left[\frac{c_e}{c_v} \frac{\gamma_m \gamma_e - 1}{\gamma_m \gamma_v + 1} \right]^{\frac{1}{\gamma_v}}}_{:=\kappa} e^{\frac{\gamma_e}{\gamma_v}}$$

which is of the form $\tilde{v}(e) = \Psi e^{\psi}$ as required. Proceeding as above, (i) substituting in for $\tilde{v}$ in the hiring function $\tilde{h}_{it} = Q^* e_{it} \tilde{v}(e_{it})$, (ii) solving for e_{it} as a function of $\tilde{h}_{it}$ and Q_t^*, (iii) multiplying by Q_t^* to convert e_{it} into the vacancy yield, (iv) taking logs:

$$\log \left(\frac{h_{it}}{v_{it}} \right) = -\frac{1}{\gamma_e + \gamma_v} \log \kappa + \frac{\gamma_e}{\gamma_e + \gamma_v} \log Q_t^* + \frac{\gamma_e}{\gamma_e + \gamma_v} \log \phi_i + \frac{\gamma_v}{\gamma_e + \gamma_v} \log \left(\frac{h_{it}}{n_{it}} \right).$$

The vacancy rate can then be obtained from $\tilde{v}(e)$:

$$\log \left(\frac{v_{it}}{n_{it}} \right) = \frac{1}{\gamma_e + \gamma_v} \log \kappa - \frac{\gamma_e}{\gamma_e + \gamma_v} \log Q_t^* - \frac{\gamma_e}{\gamma_e + \gamma_v} \log \phi_i + \frac{\gamma_e}{\gamma_e + \gamma_v} \log \left(\frac{h_{it}}{n_{it}} \right).$$

One can observe immediately that summing the two equations delivers $\log(h_{it}/n_{it})$, which verifies that the hiring constraint holds.

B.2 Daily hiring model of DFH

Here we present the model and computations that underlie the estimates of the (i) daily job filling rate, (ii) daily vacancy flow rate referenced in the text and figures. We progress the results of their paper to arrive at a simple set of equations that can be solved numerically.

Define the following variables. Hires at firm i on day s of month t are h_{ist}. Vacancies at the end of the day are v_{ist}. Let f_{it} be the *daily job filling rate*, such that $h_{ist} = f_{it}v_{is-1t}$, assumed to be constant over the month t. Let θ_{it} be the *daily vacancy in-flow rate* and δ_{it} be the daily exogenous vacancy out-flow rate such that

$$v_{ist} = (1 - f_{it})(1 - \delta_{it})v_{is-1t} + \theta_{it}.$$

Let there be τ days in a month. We observe the following in the JOLTS microdata: (i) *monthly hires* $h_{it} = \sum_{s=1}^{\tau} h_{ist}$, (ii) beginning of month vacancies $v_{it-1} = v_{i0t}$, (iii) end of month vacancies $v_{it} = v_{i\tau t-1}$.

Our aim is to use these data and the above equations to estimate $f_{it}, \theta_{it}, \delta_{it}$. Iterating on the vacancy equation, vacancies at any day s can be written in terms of $f_{it}, \theta_{it}, \delta_{it}$ and v_{it-1}:

$$v_{is-1t} = [1 - f_{it} - \delta_{it} + \delta_{it}f_{it}]^{s-1} v_{it-1} + \theta_{it} \sum_{j=1}^{s-1} [1 - f_{it} - \delta_{it} + \delta_{it}f_{it}]^{j-1}.$$

Using $h_{it} = \sum_{s=1}^{\tau} h_{ist} = \sum_{s=1}^{\tau} f_{it}v_{is-1t}$ and this expression:

$$h_{it} = f_{it}v_{it-1} \sum_{s=1}^{\tau} [1 - f_{it} - \delta_{it} + \delta_{it}f_{it}]^{s-1} + f_{it}\theta_{it} \sum_{s=1}^{\tau} (\tau - s)[1 - f_{it} - \delta_{it} + \delta_{it}f_{it}]^{s-1}. \quad \text{(B5)}$$

Evaluating the vacancy equation at the end of the month, we also have

$$v_{it} = [(1 - f_{it})(1 - \delta_{it})]^{\tau} v_{it-1} + \theta_{it} \sum_{j=1}^{\tau} [(1 - f_{it})(1 - \delta_{it})]^{j-1}. \quad \text{(B6)}$$

Equations (B5) and (B6) are two equations in three unknowns $\{f_{it}, \theta_{it}, \delta_{it}\}$. As in DFH we simplify this by assuming that δ_{it} is equal to the daily layoff rate $\tilde{\varsigma}_{it}$. The daily layoff rate is computed by taking month layoffs s_{it} divided by employment n_{it} and then dividing by τ: $\tilde{\varsigma}_{it} = (s_{it}/\tau n_{it})$. Setting $\delta_{it} = \tilde{\varsigma}_{it}$ makes (B5) and (B6) two equations in two unknowns $\{f_{it}, \theta_{it}\}$.

We can make some progress beyond DFH by applying results in algebra for finite sums. Let

$x_{it} = 1 - f_{it} - \delta_{it} + \delta_{it}f_{it}$. Plugging this in:

$$
\begin{aligned}
v_{it} &= x_{it}^\tau v_{it-1} + \theta_{it} \sum_{j=1}^\tau x_{it}^{j-1}, \\
h_{it} &= f_{it} \left[\sum_{s=1}^\tau x_{it}^{s-1} \right] v_{it-1} + f_{it}\theta_{it} \left[\sum_{s=1}^\tau (\tau - s) x_{it}^{s-1} \right].
\end{aligned}
$$

Manipulating these obtains two expressions that can be computed sequentially given x_{it}:

$$
\theta_{it} = \frac{v_{it} - x_{it}^\tau v_{it-1}}{g_0(x_{it})} \tag{B7}
$$

$$
f_{it} = \frac{h_{it}}{g_0(x_{it})v_{it-1} + \theta_{it}g_1(x_{it})} \tag{B8}
$$

where the functions g_0 and g_1 are given by

$$
g_0(x) = \frac{1 - x^\tau}{1 - x} \quad , \quad g_1(x) = \frac{\tau - g_0(x)}{1 - x}.
$$

This implies a simple algorithm:

1. Guess $f_{it}^{(0)}$ and use this to compute $x_{it}^{(0)} = (1 - \delta_{it})(1 - f_{it}^{(0)})$.

2. Use equation (B7) to compute $\theta_{it}^{(0)}$, then equation (B8) to compute $f_{it}^{(1)}$.

   - Iterate until $\left| f_{it}^{(k+1)} - f_{it}^{(k)} \right| < \varepsilon$.

In practice this converges after a very few iterations. In the figures and text instead of plotting θ_{it} directly, we transform θ_{it} into a monthly rate as a fraction of employment: $\theta_{it}\tau/n_{it}$.

C Empirical details

This section contains additional details about the data used in our estimation.
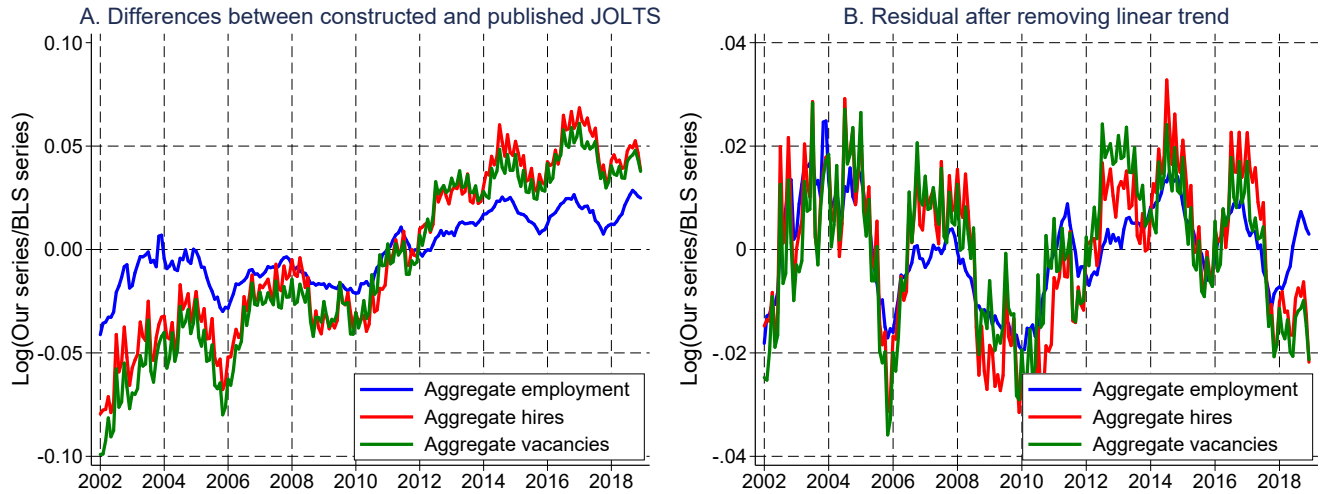
C.1 Trends in data

Figure C1: Trends in our data relative to published JOLTS aggregates

Figure C1A compares our construction of aggregate hires, employment and vacancies to officially published BLS data. For a given series X_t we first adjust our series for mean differences from published series in logs. Figure C1A then plots the ratio of the log of our adjusted series to the published series. As can be observed for all three series there is a trend in the bias, with our series being slightly less than the published data in the early part of the sample, and slightly larger in the latter part. This may be due to differences in compilation of published data or imputation in either data set. To account for these differences we take a linear trend out of both our data and the published data—both in logs—saving the residuals from the regression using our data. We then put the trend of the published data back into our residualized data. Figure C1B, plots the log difference between our final data and published data. There is now no longer any trend in bias between the two series, and differences are small, everywhere less than 3 percent in magnitude. There is some cyclicality but this is small. Importantly as our main measures in the paper consist of various ratios of H_t, N_t and V_t, we find that the difference

NAICS categories	Industry categories from DFH
21	Mining, Quarrying, and Oil and Gas Extraction
23	Construction
31,32,33	Manufacturing
22, 42, 48, 49	Utilities; Wholesale Trade; Transportation and Warehousing
44, 45	Retail Trade
51	Information
52, 53	Finance and Insurance; Real Estate and Rental and Leasing
54, 55, 56	Professional Services, Management, Administrative Services
62	Health Care and Social Assistance
71, 72	Arts, Entertainment, and Recreation; Accommodation and Food Services
81	Accommodation and Food Services
>90	Government

Table C1: Categorization of industries used in analysis

relative to published series move in step across the three variables. Finally, and separately, we take a linear time trend out of each of these series.

C.2 Microdata details

- All data are at the *establishment* level

- Age is defined as the number of years since the establishment first reported having more than one employee.

- QCEW data are reported quarterly but contain monthly payroll and employment at the establishment. These were checked for consistency against the JOLTS.

- Industry categorizations are given in Table C1. We drop Agriculture (11) and Educational Services (61) due to data collection issues that we were informed of by BLS staff.

- Participation in external researcher programs using employment and wage microdata are at the discretion of the states, which run the unemployment insurance programs report data used in the QCEW. Accessibility varies from project to project. Our project was granted access to data from 37 states: AL, AR, AZ, CA, CT, DE, GA, HI, IA, IN, KS, MD, ME MN, MO, MT, NJ, NM, NV, OH, OK, SC, SD, TN, TX, UT, VA, WA, WI, and WV. These represent over 70 percent of the population. The 5 largest states not included are FL, MI, NC, NY, and PA. Throughout we restrict our sample to the states made available

to us. This avoids changing samples when only using JOLTS data, versus when also using establishment age or wage, for which we require the QCEW.

- Wages w_{it} are computed as total payroll divided by total employment in a given month t at establishment i. Nominal wages are deflated to 2016 values using the CPI. These real wages are then detrended using annual fixed effects, and deseasonalized using month fixed effects.

- All aggregation is performed using weights provided by the BLS that adjust for systematic bias in survey non-response rates, and generate a representative sample.

- For further details on data definitions and statistical methods see the *BLS Handbook of Methods - Chapter 18 - Job Openings and Labor Turnover Survey*.