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THE MARGINS OF TRADE

Jonathan Eaton
Ana Cecília Fieler

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ABSTRACT

We introduce quality differentiation and an extensive margin of products into a standard quantitative, general equilibrium model of international trade. Both the quality and the quantity of a product play a role in its contribution both to consumption and to production. The framework allows bilateral trade to vary at the extensive and intensive margins and the intensive margin of trade to vary at the quantity and unit-value margins. We estimate the parameters of the model using bilateral data on trade flows and on unit values in trade. The model captures (i) the well-documented increasing relation between unit values and both importer and exporter per capita income and (ii) how the extensive margin rises with importer and exporter size. But, unlike other contributions to the literature confronting these margins in international trade, our framework delivers a standard gravity formulation for trade flows and standard measures of the gains from trade apply.

Jonathan Eaton
Department of Economics
Pennsylvania State University
303 Kern Graduate Building
University Park, PA 16801
and NBER
jxe22@psu.edu

Ana Cecília Fieler
Yale University
Department of Economics
27 Hillhouse Ave
New Haven, CT 06511
and NBER
ana.fieler@yale.edu

A data appendix is available at http://www.nber.org/data-appendix/w26124

1 Introduction

Quantitative work in international trade has advanced on several fronts in the last two decades. One line of research has developed global general equilibrium models to understand the determinants of bilateral trade flows and their implications for welfare.[1] Another literature has delved into trade data to ask how total bilateral exports decompose into various margins, such as that between number of products (the extensive margin) and sales per product (the intensive margin), and how sales per product decompose into quantity and unit value.[2] These studies have revealed several robust and intriguing regularities.

While both lines of research have been extremely fruitful, they remain somewhat at odds with each other. Capturing a complex world with a general equilibrium system has required assumptions inconsistent with richer countries paying more for the same product and richer countries charging more for the same product, two of the most robust regularities to emerge from this second line of inquiry. Incorporating how trade volumes break down into their extensive and intensive margins has also proved challenging for general equilibrium modeling.

This paper seeks to reconcile these two research fronts by developing a general equilibrium framework consistent with observed regularities in the margins of trade. The model delivers the same aggregate relationships governing bilateral trade that emerge in a standard general equilibrium framework, in particular the Ricardian formulation of Eaton and Kortum (2002) (henceforth EK), with a continuum of varieties, CES aggregation, and perfect competition. Hence, at the level of total spending, our model delivers the same observations as EK's.

In line with previous work, we associate differences in unit values of a variety with differences in its quality.[3] We allow for two dimensions of quality, which we call vertical and horizontal. Horizontal quality substitutes perfectly for quantity, and is valued equally by all users of the variety, whether a household using the variety for final consumption or a producer using the variety as an intermediate input. Vertical quality complements quantity. As a buyer chooses to spend more on a variety, the increased spending is divided between more effective quantity and higher vertical quality. The framework implies that

[1] Early examples are Anderson and Van Wincoop (2003), using an Armington approach, Eaton and Kortum (2002), whose approach is Ricardian, and quantitative papers building on Melitz (2003), such as Chaney (2008) and Eaton et al. (2011).

[2] Early contributions here are by Hummels and Klenow (2005), which we build on very directly, Schott (2004), and Hallak (2006).

[3] Aside from Hummels and Klenow (2005) and Hallak (2006), other authors making this connection are Schott (2004), Khandelwal (2010), Hallak (2010), Hallak and Schott (2011), Baldwin and Harrigan (2011), Hummels and Skiba (2004), Choi et al. (2009), Bekkers et al. (2012), and Atrianfar (2019).

a higher wage is associated with higher quality in both dimensions.

First, a final consumer receiving a higher wage chooses to spend more on any variety, and this higher spending divides into both a larger physical amount and a higher vertical quality. A producer having to pay a higher wage seeks to equip her worker with more intermediates. As she spends more on each variety of intermediate she also will seek both more quantity and quality. Our model thus predicts that a buyer in a higher wage country will spend more per unit and buy more units both for intermediate and for final uses.

Second, we posit that a better equipped worker produces higher horizontal quality. Hence our model implies that vertical quality rises with the wage of the buyer while horizontal quality rises with the wage of the seller. Our model captures these relationships very parsimoniously with two parameters that relate closely to the observed elasticities of unit value with respect to importer and to exporter per capita income.

A standard observation is that the ranges of products a country imports and exports grows with its overall size. But the relationship between the extensive margin and GDP is a nonlinear one. In particular, it dies out with importer size at quite a small level. Our framework captures these features by introducing stochastic minimum shipping sizes.[4]

We estimate the parameters of the model using data on trade flows, product varieties, unit values, and country characteristics. We then simulate the model to show how it can deliver decompositions of trade into the margins identified by Hummels and Klenow (2005) (henceforth HK).

Our conceptual framework applies to a range of situations beyond international trade. It has implications, for example, for quantifying the role of quality improvement in economic growth. In this paper we choose a trade context in order to exploit the United Nations COMTRADE data. COMTRADE reports annual bilateral trade between most countries, in terms of both value and physical quantity, using a harmonized and detailed product classification. It thus provides unique insight into how countries across all sizes and income levels are producing (as exporters) and absorbing (as importers) a vast array of products. We know of no other dataset that delivers such a thorough picture.

Our framework builds on the theoretical literature on quality differentiation in international trade. Early on, Flam and Helpman (1987) developed a two-country, two-good general equilibrium framework that explained why a rich country might both produce and demand a good of higher quality. More recently Fajgelbaum et al. (2011) provided a much richer framework that allowed for many goods and countries.

Applying these approaches to the problem at hand poses two challenges. These models

[4] Here we build on Armenter and Koren (2014), who introduce granularity into a model of international trade.

employ a discrete-choice framework in which the buyer is contemplating buying only a single unit of the good. They thus don't allow for increased per capita spending on a good to reflect a combination of more quantity and higher quality. Second, with only a single dimension of quality, if rich countries both prefer higher quality goods and are better at making them, rich countries should have larger market shares in other rich countries than in poor ones, and vice versa. This pattern isn't one we observe in the data. Our framework can deal with each issue.

Other investigators have pursued different general equilibrium approaches to understanding the role of unit values in trade. In contrast to what we do here, these alternatives depart from perfect competition in various directions.

Feenstra and Romalis (2014) build on the Melitz (2003) model. Their framework is less in keeping with standard general equilibrium modeling in that they introduce a specific trade cost as well as the iceberg costs commonly used in the literature. It consequently doesn't deliver the standard homothetic gravity specification for aggregate trade implied by our approach here. While Feenstra and Romalis (2014)'s framework provides an explanation for why unit values rise with importer per capita GDP it doesn't speak to the effect of exporter per capita GDP.

Atrianfar (2019) incorporates quality as well as price competition into a model of Bertrand competition, building on Bernard et al. (2003). An intriguing implication of his analysis is that rich and poor countries compete in different dimensions. His framework can explain why a low-wage country might respond to increased competition from a third party (e.g., China) by lowering price and maintaining market share, while a high-wage country might respond by raising quality and price, allowing market share to fall. These rich interactions preclude his framework from delivering the standard gravity specification implied by the approach we take here.

These two papers, like ours and much of the other literature, interpret higher unit values to reflect higher quality. Another explanation is that variation in unit values represent different markups. Lashkaripour (2019b) develops a general equilibrium multicountry version of the Krugman (1979) model in which different classes of goods have different elasticities of substitution, so their producers charge different Dixit-Stiglitz markups. This framework can also explain some of the empirical regularities we address here. Again the approach maps less directly than ours into the standard gravity specification.

We also build on a large literature on the intensive and extensive margins of trade. The distinction goes back at least to Vernon (1966)'s product cycle model and the literature that followed. More recent contributions are Evenett and Venables (2002), Besedeš and Prusa (2006) (for U.S. imports), Besedeš and Prusa (2011) (for exports), Amiti and Freund

(2010) (for Chinese exports), Debaere and Mostashari (2010) (looking at the effect of tariffs on the two margins), Kehoe and Ruhl (2013) (looking at the role of the extensive margin in growth), Baier et al. (2014) (looking at the effect of economic integration on the two margins), and Silva et al. (2014) (who consider the role of the two margins for gravity estimation).

We proceed as follows. Section 2 presents our data and revisits the empirical regularities pursued before. Section 3 presents our model and Section 4 our estimation of it. In Section 5 we evaluate our model's ability to capture the margins of trade. We pursue our analysis in Sections 2 through 5 at the level of aggregate merchandise trade. In Section 6 we probe the extent to which our results survive disaggregation into finer classes of products. Section 7 concludes.

2 Overview of the Data

Our analysis applies to overall merchandise trade and its decomposition into various margins. Our trade data are from the United Nations COMTRADE data set. We work with the most disaggregated product category in these data, which is HS6. We refer to an HS6 product category as a product. We restrict our analysis to trade among the fifty largest countries in terms of GDP in the 2007 cross section. We ignore small countries to avoid zero bilateral trade flows and to ensure sufficient overlap in HS6 products across importer-exporter pairs.[5] We take data on GDP and population from the World Development Indicators and data on geographical characteristics from CEPII. Appendix A provides a list of the countries and further detail on the construction of our data set.[6]

We follow HK in decomposing the total value X_{ni} of exports to each destination n from each source i into an extensive, a quantity, and a price margin. We define the extensive margin E_{ni} as the fraction of HS6 products that n imports from i. We construct the price margin P_{ni} as

$$\log P_{ni} = \frac{1}{|K_{ni}|} \sum_{k \in K_{ni}} [\log(p_{nik}) - \log(p_{\text{world},k})]$$

where K_{ni} is the set of HS6 products n imports from i, p_{nik} is the unit value of product k imported by n from i, and $p_{\text{world},k}$ is the average unit value of product k across all

[5]Among these 50 countries COMTRADE reports total merchandise trade of US $11.1 trillion consisting of 3,239,484 importer-exporter-HS6 triads. For various reasons described in Appendix A, we pare these data down to to 2,611,700 triads constituting US $9.62 trillion.

[6]CEPII provides a very user friendly version of the COMTRADE data which, among other things, reconciles potentially conflicting reports from importing and exporting countries. Because of concerns that CEPII's procedures for processing the data might influence some of the regularities we explore here, we decided to use the raw data downloaded directly from the COMTRADE website.

Table 1: Decomposition of trade flows

dependent variable →	value	extensive margin	quantity	price
Panel A				
exporter GDP	1.16	0.76	0.36	0.04
importer GDP	1.11	0.34	0.73	0.05
distance	-0.81	-0.43	-0.39	0.02
Panel B				
exporter GDP per capita	1.18	0.84	0.19	0.15
exporter population	1.15	0.71	0.46	-0.02
importer GDP per capita	1.10	0.41	0.56	0.13
importer population	1.12	0.30	0.82	0.00
distance	-0.81	-0.37	-0.53	0.10
number of observations	2448	2448	2448	2448

All variables are in logs. We report standard errors in Appendix B.

importer-exporter pairs in our sample.[7] We define the quantity margin as the residual $X_{ni}/(E_{ni}P_{ni})$.[8]

Following HK we apply a standard gravity analysis to relate our margins of trade to geographical indicators and to importer and exporter characteristics. Table 1 reports the results. The first column of Panel A shows the coefficients of the regression of total exports to each destination from each source against distance and importer and exporter GDP. The subsequent three columns repeat the regression for each of the three margins. By construction, for each independent variable, the coefficients on these last three columns sum to the coefficient in the first column.

The results in the first column of Panel A are consistent with standard gravity results: The coefficients on importer and exporter GDP are around one and the distance elasticity

[7]We construct unit values from the COMTRADE data by dividing, for each importer-exporter-HS6 triad, the reported value by the reported quantity. Values are always in terms of current U.S. dollars. The absence of quantity data forces us to drop a small number of observations. Of the remaining ones, eighty percent of the triads report quantities in terms of weight (corresponding to 72 percent of the total value of trade in our analysis). The remaining ones are nearly all in terms of counts. See Appendix A for details. Lashkaripour (2019a) provides an analysis of alternative quantity measures in trade data.

[8]These definitions differ from HK, who use a weighted definition of the extensive margin and construct the price margin using the price index introduced by Sato (1976) and Vartia (1976) and discussed extensively by Feenstra (1994). Appendix B reports results using their methodology. Our definition of the extensive margin is simpler, and the correlation with their measure is 0.93. The correlation between our price index and theirs is weaker, 0.76. But the simpler, unweighted price index is more tightly linked to our product-level analysis below.

is around minus one. How GDP relates to the extensive margin, however, differs between importer and exporter. Larger countries export many more products than smaller countries, but they don't import so many more. At this level of aggregation, the intensive margin (price $\times$ quantity) is almost all dominated by quantity.

Panel B repeats the analysis breaking GDP down into GDP per capita and population. The first column of Panel B shows that breaking down GDP into GDP per capita and population has no significant effect on trade values: The elasticity with respect to income per capita and population is close to one for both importer and exporter. But for both importers and exporters, the elasticity of the extensive margin is greater for GDP per capita than for population, but the population effect is not far from the elasticity with respect to total GDP. For the price margin, however, both importer and exporter GDP per capita have distinctly positive elasticities, while population does not.[9]

2.1 Price Relationships

To probe further into the price margin of trade, we turn from the bilateral price index to prices at the level of individual HS6 product categories. The aggregate results on the elasticity of price with respect to exporter GDP per capita in the bottom panel of Table 1, for example, could arise from selection. Say, for example, that countries charge the same price to all destinations for a given product. The results in Table 1 could still arise if rich countries sell their more expensive products disproportionately to rich destinations and their relatively cheaper products to poor destinations. Table 2 shows that forces other than selection are at work.

In Column (1), we report the results from a regression of unit values, for each importer-exporter-product triad, against distance, importer GDP per capita, and exporter-product fixed effects. The coefficient on importer per capita income is 0.12, nearly as large as in Table 1. The implication is that individual exporters sell the same product to richer countries at systematically higher prices.

Column (2) reports the mirror regression of unit values against distance, exporter GDP per capita, and importer-product fixed effects. The coefficient on exporter per capita GDP is 0.22, even larger than in Table 1. Countries systematically pay higher prices for the same products from richer countries.[10]

[9]Note that distance has a positive effect on unit value which becomes large and significant once GDP is broken down into GDP per capita and population, the Alchian-Allen effect analysed by Hummels and Skiba (2004).

[10]Schott (2004) reports similar results for imports into the United States at the level of 10-digit product categories.

Table 2: Descriptive price regressions

Dependent variable is the price for each importer, exporter, and product.

independent variable ↓	pooled by exporter-product (1)	pooled by importer-product (2)	pooled by product (3)	income interaction (4)	Rauch (1999) differentiated products[b] (5)	manufacturing only (6)
importer GDP per capita	0.117		0.119	0.157	0.128	0.118
	(0.022)		(0.022)	(0.093)	(0.024)	(0.024)
exporter GDP per capita		0.219	0.219	0.257	0.236	0.242
		(0.034)	(0.033)	(0.118)	(0.038)	(0.038)
distance	0.123	0.113	0.100	0.098	0.093	0.089
	(0.015)	(0.015)	(0.014)	(0.015)	(0.016)	(0.016)
interaction[a]				-0.0039		
				(0.0097)		
product-exporter fixed effect	yes	no	no	no	no	no
product-importer fixed effect	no	yes	no	no	no	no
product fixed effect	no	no	yes	yes	yes	yes
R-squared	0.84	0.85	0.81	0.81	0.80	0.81
number of observations	2,611,700	2,611,700	2,611,700	2,611,700	1,751,913	1,447,277

Notes: All variables are in logs. [a]The interaction term equals log (importer GDP per capita) × log (exporter GDP per capita). [b]We use Rauch's liberal definition of products, which contains the smallest set of differentiated products. Results with Rauch's conservative definition are similar.

Column (3) reports what happens if we use only product fixed effects with both exporter and importer per capita income. The coefficients on these variables do not change from columns (1) and (2).

Column (4) includes a term that interacts exporter and importer GDP per capita. The coefficient is negative and statistically insignificant. Hence we find no evidence that rich countries disproportionately pay more for goods from other rich countries.

Columns (5) and (6) consider the sensitivity of the results in column (3) to the set of products we consider. In column (5) we restrict the sample to products classified by Rauch (1999) as differentiated (using his liberal definition of referenced price and organized exchange products). In column (6) we look only at manufactures. In neither case are the results notably different.

Going back to Flam and Helpman (1987), the literature on quality and trade has provided an explanation for why unit values rise with both exporter and importer per capita income: Rich countries have a comparative advantage in producing high quality, and hence charge higher prices, and, because of nonhomotheticity in preferences, rich countries have a greater taste for quality, so pay higher prices.[11]

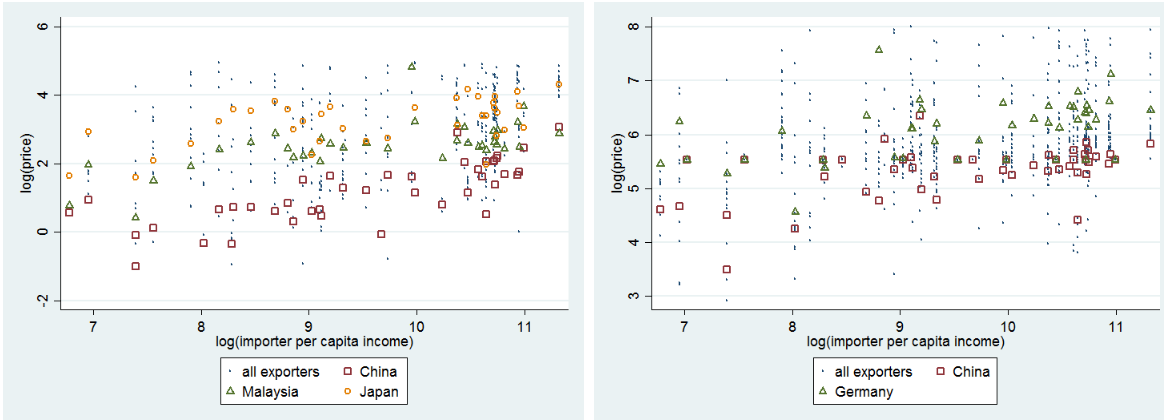
The assumption that rich countries have a comparative advantage in high quality implies that, as long as quality is one dimensional, there should be no overlap in the prices charged for a given product by a rich country and a poor country. Even if Japan sells to Pakistan at a lower price than it sells to Norway, the price it charges in Pakistan should still exceed the price Malaysia charges in Norway. Otherwise, why would Norwegians prefer the high-priced Malaysian product to the low-priced product that Japan is selling in Pakistan?

A back-of-the-envelope calculation based on the regression coefficients in Column (3) of Table 2 suggests, however, systematic overlap in predicted prices. We calculate, for example, that a Malaysian product should sell in Norway at 0.3 log points more than a Japanese product in Pakistan.

Overlaps aren't just what's predicted by the regression. They are common in the raw data. Figure 1 illustrates price patterns for HS6 categories HS871493 and HS845011. Code HS871493 corresponds to hubs for motorcycles, bicycles, and vehicles for the disabled. Code HS845011 corresponds to washing machines with capacity less than 10kg.[12] The figures plot unit values against importer per capita income for all importer-exporter pairs. For hubs, we highlight the three major Asian exporters: China (GDP per capita US$2,708) with a square, Malaysia (GDP per capita US$11,358) with a triangle, and Japan (GDP per

[11]Subsequent papers in this tradition are Stokey (1991) and Fajgelbaum et al. (2011).

[12]See hts.usitc.gov for a more complete definition.



(a) HS871493

(b) HS845011

Figure 1: Examples of Products

capita US$34,313) with a circle. Across destinations, Japan's unit values are higher than Malaysia's, which are higher than China's. For all three exporters, unit values rise with the importer's GDP per capita so much that Japan is selling in the poorest destination at a price lower than China sells in the richest destination.

For washing machines, Figure 1(b) highlights the two largest exporters, China with squares and Germany with triangles (GDP per capita US$40,324). Note how China sells to the richest country, Norway (GDP per capita US$82,480), at a price above that at which Germany sells to the poorest country, Pakistan (GDP per capita US$879).

2.2 Trade Values

The literature on quality and trade discussed above also has implications for trade values between countries of different income levels. In these models, rich countries tend to sell to rich households in all countries while poor countries tend to sell to poor households in all countries. Since poorer countries have a larger share of poor households, exports from rich countries to poor would systematically decline with differences in income. At the extreme, in Flam and Helpman's model, internal income inequality is the only reason for international trade. Fajgelbaum et al. (2011), by introducing an idiosyncratic component to demand, relax this strong prediction, but their model nevertheless predicts that the average consumer in a poor country has lower demand for goods produced in rich countries. The large coefficient on exporter per capita income in the price regression implies, through the lens of this literature, a strong degree of specialization in income elastic quality on the part of rich countries.

Figure 2 shows the limited scope for internal income inequality to generate substantial

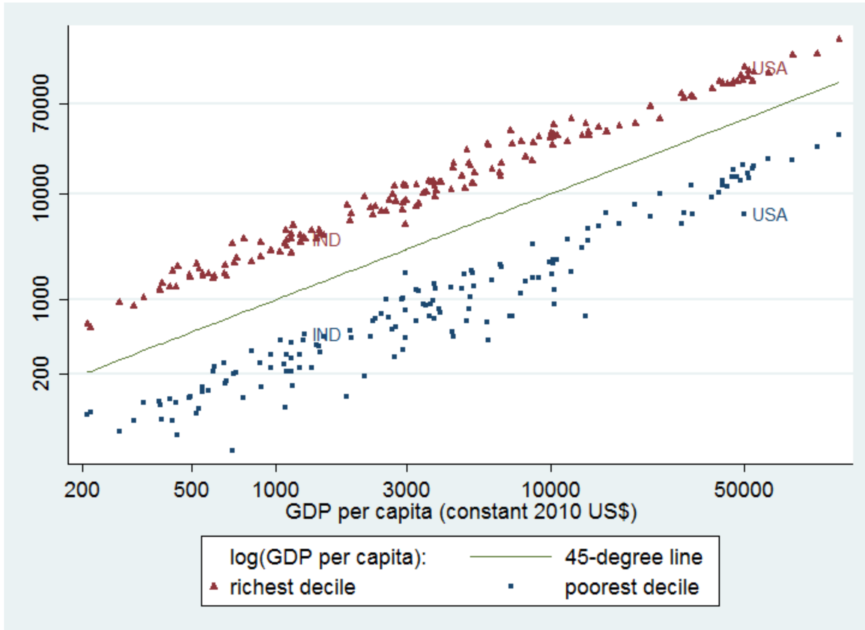


Figure 2: World Income Distribution

trade between rich and poor countries. The figure plots, for 149 countries, GDP per capita at the top and bottom deciles (on the y-axis) against average GDP per capita (on the x-axis).[13] Note how cross-country differences in GDP per capita swamp internal ones. The poorest decile in the United States is slightly richer than the richest in India. An implication of the literature on quality in trade is that the only buyers of U.S. goods in India are the narrow sliver of Indians with incomes high enough to appreciate goods that appeal to U.S. consumers.

But do rich countries lose market share as their importing partner's GDP per capita declines? Table 3 reports the result of a gravity regression of total bilateral trade value against importer and exporter fixed effects, distance, and an interaction term between exporter and importer GDP per capita. A positive coefficient on the interaction term would confirm, in line with the Linder (1961) hypothesis, that rich countries do indeed have a larger client base in other rich countries. The coefficient is in fact small and statistically insignificant.[14]

[13] The data are from the World Bank's World Development Indicators. GDP per capita is from 2007. Income per capita at the top and bottom deciles is calculated from the share of income at these deciles for the closest year to 2007 within a 10-year window.

[14] Hallak (2010) reports the same result looking at aggregate bilateral trade. He argues that the aggregate data mask a positive interaction effect at the sectoral level, showing that the effect is significantly positive in half of 116 sectors and significantly the opposite in only 20 percent of them. We classified our data into 97 two-digit HS product categories, performing the regression in Table 3 separately for each category. In contrast to Hallak, we find a significantly positive interaction effect for only 19 categories and a significantly negative interaction for 33 categories. Running these sectoral regressions as well as

Table 3: Gravity with Interaction

Dependent variable is the log of bilateral trade flows	
distance	-1.148
	(0.041)
interaction	0.0020
	(0.016)
importer fixed effect	yes
exporter fixed effect	yes
R-squared	0.75
number of observations	2,448

Notes: Distance is in logs. As in table 2, the interaction term equals log (importer GDP per capita) × log (exporter GDP per capita). It captures whether rich countries disproportionately sell more to other rich countries.

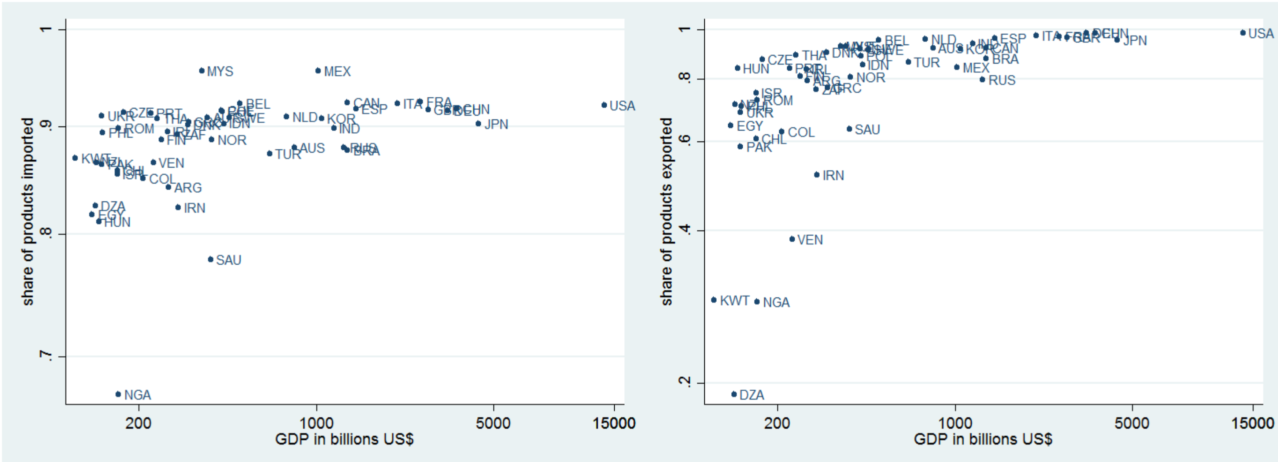
2.3 The Extensive Margin

To probe further into the extensive margin of trade, Figure 3 plots the fraction of HS6 product categories that a country imports (a) and the fraction that it exports (b) against total GDP (both in logs). Confirming the results from Table 1 above, the extensive margin varies much more for exporters than for importers, hence the very different scales for the two y-axes. Not revealed by the regression is the concave relation between the extensive margin of exports and exporter GDP: For the largest countries, the relationship between GDP and extensive margin levels off, both for imports and for exports.

Before turning to our model, it's useful to review what standard models say about the extensive margin of trade for imports and exports. The EK model provides a simple framework for breaking trade values down into the measure of varieties and spending per variety that one country sells to another.

A stark implication of their model is that, for a given destination, all the variation in imports across sources is at the extensive margin. If we interpret varieties in their model as products in the data, then the coefficient on the extensive margin of exporters would equal the coefficient on value in Table 1. In fact, the coefficient is 0.76, substantially

the aggregate one above using pseudo Poisson maximum likelihood (PPML), as in Silva and Tenreyro (2006), or pseudo multinomial maximum likelihood (PMML), as in Eaton et al. (2013), yields similar results.



(a) Importer

(b) Exporter

Figure 3: Extensive Margin and GDP

less than the coefficient on value, 1.16. A modeling challenge, then, is to account for the intensive margin of 0.38.[15]

Another implication of the EK model is that the extensive margin of importers for varieties should be negative: Larger importers should source a greater range of varieties domestically, so import fewer. Again, interpreting varieties in their model as products in the data poses a challenge in explaining the *positive* importer extensive margin elasticity of 0.34 in Table 1.[16]

As panel B of Table 1 shows, the coefficients on GDP per capita and population aren't very different from each other, either for exporters or importers, in both the value and extensive margin regressions.[17] This result is in line with both the EK and Melitz models, for which this breakdown doesn't matter.

3 The Model

Having reviewed regularities in the data that pose challenges for standard trade models, we now turn to a framework that seeks to accommodate these regularities. To explain

[15]The Melitz model breaks trade down into the **firm** dimension of export participation, and sales per firm. If we equate a firm in his model with a product in the data, it, too, predicts that all the action across exporters in a given destination is at the extensive margin.

[16]The Melitz model does predict that larger markets will attract more firms from a given source. An issue with equating a Melitz firm with an HS6 product is that we see many countries exporting the same HS6 product.

[17]Appendix B shows formally that we cannot reject the null that these coefficients are equal in all four cases. This result is in contrast with the role of GDP per capita and population on prices and quantities, for which this null of equality is clearly rejected.

why unit values rise with both importer and exporter per capita income the framework incorporates two dimensions of quality: One captures the difference in the unit values of different exporters across importers reflected in the vertical differences in Figure 1. The other captures the difference in unit values in what is purchased by different importers, reflected in the slopes in Figure 1. In our framework both dimensions of quality rise endogenously with a country's productivity. To explain the interplay of the extensive and intensive margins, the framework introduces granularity in shipments.

Our model begins with basic Ricardian ingredients. The world has N countries, indexed by $i, n = 1, ..., N$, each endowed with a measure L_i of workers who are also the households in the economy. A worker can perform different jobs within a country but can't change countries. A worker in country i earns a wage w_i determined in equilibrium. Competition is perfect, so that unit production costs determine prices in all markets.

Output consists of a measure one continuum Ω of varieties each denoted by ω. A unit of variety ω has two dimensions of quality: One dimension $q(\omega) \in [0, \infty)$ complements quantity $y(\omega) \in [0, \infty)$ while the other $Q(\omega) \in [0, \infty)$ perfectly substitutes for quantity. Examples of the first dimension of quality might be Robert Parker's rating of a wine or the precision of a machine tool. Examples of the second dimension might be the heating value of a ton of coal, the durability of a light bulb, or the caffeine content of a cup of coffee. The same product might differ in both dimensions. For the washing machines, aspects of Q might be the durability or reliability of the machine, while aspects of q might be gentleness to clothing, cycle options, electronic controls, or an automatic bleach dispenser.

While the term has been used differently in different contexts, we refer to Q as "horizontal quality" since, as we show below, all buyers value an increase in Q equivalently, and to q as "vertical quality", since a buyer spending more values an increase in q disproportionately. Nevertheless, all buyers value an increase in either Q or q.

3.1 Aggregation

We now turn to how individual varieties aggregate into the composite output. To simplify notation we temporarily ignore the international dimension of the problem and suppress country subscripts.

Varieties combine to form a composite in amount Y according to the function:

$$Y = \left[\int_{\omega \in \Omega} u(\omega)^{\beta} d\omega \right]^{1/\beta} \tag{1}$$

where the variety-specific benefit is:

$$u(\omega) = \left[(Q(\omega)y(\omega))^{\rho} + q(\omega)^{\rho}\right]^{1/\rho}. \tag{2}$$

Here $\beta \leq 1$ governs the elasticity of substitution between varieties while $\rho \leq 1$ governs the elasticity of substitution between effective quantity and the vertical dimension of quality. This composite provides utility to a final consumer or equips an individual worker with intermediates. These two dimensions of quality allow us to capture features of the price data discussed in Section 2.1.

The cost of producing $y(w)$ physical units of vertical quality $q(\omega)$ of variety ω is

$$x(\omega) = y(\omega)q(\omega)^{\gamma}c(\omega). \tag{3}$$

Here $\gamma > 0$ is a parameter reflecting the cost of producing higher vertical quality and $c(\omega) > 0$ is the cost of creating one unit of variety ω of vertical quality $q(\omega) = 1$, which is determined in equilibrium.[18] An agent with a budget X seeks to maximize (1) subject to:

$$\int_{\omega\in\Omega} x(\omega)d\omega = X. \tag{4}$$

We split the problem into two parts. We first ask, for a particular variety ω with given horizontal quality $Q(\omega)$, how to choose $q(w)$ and $y(w)$ maximize the benefit $u(\omega)$ given spending $x(\omega)$ on this variety. We then ask how the buyer should allocate his budget X across spending on each variety $x(\omega)$ subject to the budget constraint (4).

3.1.1 Quality versus quantity

Since we first focus on a given variety, we temporarily drop the ω argument. If the buyer has chosen to spend x on this variety, the problem is:

$$\max_{y,q} \left[(Qy)^{\rho} + q^{\rho}\right]^{1/\rho}$$

subject to:

$$yq^{\gamma}c \leq x.$$

[18]See Bekkers et al. (2012) for a very similar formulation of preferences and the cost of what we're calling vertical quality, the only dimension of quality in their analysis.

To satisfy the second-order conditions for a minimum we need to impose the condition that $\rho < 0$.[19] Taking the ratio of the two first-order conditions gives:

$$q = \gamma^{1/\rho} Qy,$$

which, upon substitution into the problem above, reduces it to:

$$\max_y (1+\gamma)^{1/\rho} Qy$$

subject to:

$$y^{1+\gamma} Q^{\gamma} c \leq x.$$

Defining the term:

$$A = \gamma^{\gamma/[\rho(1+\gamma)]}$$

the implied quantity is:

$$y = A^{-1} \left(\frac{x}{c}\right)^{1/(1+\gamma)} Q^{-\gamma/(1+\gamma)}$$

with corresponding vertical quality:

$$q = A^{1/\gamma} \left(\frac{Qx}{c}\right)^{1/(1+\gamma)}.$$

The price per unit is then:

$$p = cq^{\gamma} = Ac \left(\frac{Qx}{c}\right)^{\gamma/(1+\gamma)}$$

and the benefit is:

$$u = A^{-1}(1+\gamma)^{1/\rho} \left(\frac{Qx}{c}\right)^{1/(1+\gamma)}$$

Instead of working with the unit cost c of vertical quality $q = 1$ we introduce:

$$v = \frac{Q}{c},$$

[19]Graphically, the budget constraint:

$$x \geq yq^{\gamma}c$$

has a surface that's Cobb-Douglas in q and y. For a tangency to represent a minimum requires that the isobenefit curve:

$$\bar{u} = [(Qy)^{\rho} + q^{\rho}]^{1/\rho}$$

have an elasticity of substitution strictly below 1.

the effective inverse cost of variety ω. We can then write these expressions more compactly as functions of x and v:

$$\begin{aligned} y(x,v) &= A^{-1}Q^{-1}\left(xv\right)^{1/(1+\gamma)} \\ q(x,v) &= A^{1/\gamma}\left(xv\right)^{1/(1+\gamma)} \\ p(x,v) &= Ax^{\gamma/(1+\gamma)}v^{-1/(1+\gamma)}Q \\ u(x,v) &= A^{-1}(1+\gamma)^{1/\rho}\left(xv\right)^{1/(1+\gamma)}. \end{aligned} \tag{5}$$

The parameter γ governs how spending x gets divided into the quantity and price margins, with quantity having an elasticity $1/(1+\gamma)$ and price an elasticity $\gamma/(1+\gamma)$.

3.1.2 How much of a variety?

Having solved for the benefit $u[x(\omega), v(\omega)]$ of spending an amount $x(\omega)$ on variety ω we turn to the problem of how much to spend on each variety. Specifically, we solve the problem:

$$\max_{x(\omega)}\left[\int_{\omega\in\Omega} u\left[x(\omega), v(\omega)\right]^{\beta} d\omega\right]^{1/\beta}$$

where, from the fourth equation of (5):

$$u\left[x(\omega), v(\omega)\right] = A^{-1}(1+\gamma)^{1/\rho}\left[x(\omega)v(\omega)\right]^{1/(1+\gamma)}$$

subject to (4).

The solution gives us:

$$x(\omega) = \left(\frac{v(\omega)}{V}\right)^{\beta/(1+\gamma-\beta)} X \tag{6}$$

where:

$$V = \left[\int_{\omega'\in\Omega} v(\omega')^{\beta/(1+\gamma-\beta)} d\omega'\right]^{(1+\gamma-\beta)/\beta}. \tag{7}$$

From (6) and its substitution into (5), we can write:

$$\begin{aligned} y(\omega) &= A^{-1}v(\omega)^{1/(1+\gamma-\beta)}\left[XV^{-\beta/(1+\gamma-\beta)}\right]^{1/(1+\gamma)}Q(\omega)^{-1} \\ q(\omega) &= A^{1/\gamma}v(\omega)^{1/(1+\gamma-\beta)}\left[XV^{-\beta/(1+\gamma-\beta)}\right]^{1/(1+\gamma)} \\ p(\omega) &= Av(\omega)^{-(1-\beta)/(1+\gamma-\beta)}\left[XV^{-\beta/(1+\gamma-\beta)}\right]^{\gamma/(1+\gamma)}Q(\omega) \\ u(\omega) &= A^{-1}(1+\gamma)^{1/\rho}v(\omega)^{1/(1+\gamma-\beta)}\left[XV^{-\beta/(1+\gamma-\beta)}\right]^{1/(1+\gamma)} \end{aligned} \tag{8}$$

where we continue to take horizontal quality $Q(\omega)$ as given.

We can then solve for Y as a function of X and V:

$$\begin{aligned} Y &= \left[\int_{\omega\in\Omega} u(\omega)^{\beta} d\omega\right]^{1/\beta} \\ &= A^{-1}(1+\gamma)^{1/\rho}\left[XV^{-\beta/(1+\gamma-\beta)}\right]^{1/(1+\gamma)}\left[\int_{\omega\in\Omega} v(\omega)^{\beta/(1+\gamma-\beta)} d\omega\right]^{1/\beta} \\ &= A^{-1}(1+\gamma)^{1/\rho}(XV)^{1/(1+\gamma)} \end{aligned} \tag{9}$$

To obtain a closed-form solution, we have to take a stand on the distributions of the inverse unit costs $v(\omega)$.

3.1.3 The distribution of efficiency

We now make our multicountry setting explicit by denoting a destination country by n and an origin country by i. We assume that vertical quality $Q(\omega)$, determined below, depends only on origin. Thus, if country n buys variety ω from country i, $Q(\omega) = Q_i$. As in EK, if country n buys variety ω from country i then

$$c(\omega) = \frac{d_{ni}C_i}{Z_i(\omega)}$$

where C_i is the unit cost of a bundle of inputs in country i, d_{ni} is the iceberg cost of shipping a unit from country i to country n, and $Z_i(\omega)$ is country i's efficiency producing variety ω. The probability that country i's efficiency $Z_i(\omega) \leq z$ is

$$F_i(z) = \exp(-T_i z^{-\theta}).$$

with the $Z_i(\omega)$ drawn independently across source countries i for each variety ω.

We define

$$\tilde{C}_i = \frac{C_i}{Q_i}, \tag{10}$$

the cost of inputs in source i adjusted for source i's horizontal quality. Then we can write effective inverse cost in destination n, taking into account iceberg transport costs:

$$v_{ni}(\omega) = \frac{Z_i(\omega)}{d_{ni}\tilde{C}_i}.$$

An agent in country n sources variety ω from country i if

$$i = \arg\max_{i'=1,\ldots,N} \left\{v_{ni'}(\omega)\right\}.$$

The corresponding effective inverse cost is

$$v_n(\omega) = \max_{i'=1,...,N} \{v_{ni'}(\omega)\}.$$

Using the distribution of z, the share of varieties that country n sources from country i is

$$\pi_{ni} = \frac{T_i(d_{ni}\tilde{C}_i)^{-\theta}}{\Phi_n} \tag{11}$$

where

$$\Phi_n = \sum_{i'} T_{i'}(d_{ni'}\tilde{C}_{i'})^{-\theta} \quad n = 1, ..., N. \tag{12}$$

The distribution of $v_{ni}(\omega)$ conditional on i being the lowest cost supplier to country n is

$$\begin{aligned} G_n(v) &= \Pr\left(V_{ni} \leq v | \, i = \arg\max_{k \leq N} \{v_{ni}\}\right) \\ &= \exp(-\Phi_n v^{-\theta}). \end{aligned} \tag{13}$$

As in EK, the distribution G_n is independent of source i. Hence the unconditional distribution $v_n(\omega)$ in equation (7) has the same cumulative distribution $G_n(v)$, so that π_{ni} given in (11) is also country i's share in absorption by n. Despite the nonhomothetic intricacies introduced by the quality dimensions of our model, it delivers the same trade-share equation as the homothetic EK model.

We can use (13) to solve:

$$V_n = \left[\int_{\omega \in \Omega} v_n(\omega)^{\beta/(1+\gamma-\beta)} d\omega\right]^{(1+\gamma-\beta)/\beta} = \left[\int_0^\infty v^{\beta/(1+\gamma-\beta)} dG_n(v)\right]^{(1+\gamma-\beta)/\beta} = \Gamma_0 \Phi_n^{1/\theta}, \tag{14}$$

which corresponds to the inverse of the price index in EK. Here:

$$\Gamma_0 = \left[\Gamma\left(1 - \frac{\beta}{\theta(1+\gamma-\beta)}\right)\right]^{(1+\gamma-\beta)/\beta}$$

and Γ is the gamma function. For reasons similar to those in EK and Melitz, for the price index to be well-defined we require that:

$$\theta > \frac{\beta}{1+\gamma-\beta}.$$

We can rearrange (9) to solve for the expenditure X_n required to achieve an aggregate Y:

$$X_n(Y) = \Gamma_1 \frac{Y^{1+\gamma}}{V_n} \tag{15}$$

where

$$\Gamma_1 = \left[\gamma^\gamma (1+\gamma)^{-(1+\gamma)}\right]^{1/\rho}.$$

We introduce the term:

$$\epsilon(\omega) = v_n(\omega)/V_n \tag{16}$$

which has the distribution:

$$\begin{aligned} J(\epsilon) &\equiv \Pr[E \leq \epsilon] = \Pr[v_n(\omega) \leq \epsilon V_n] \\ &= \exp\left[-(\epsilon \Gamma_0)^{-\theta}\right] \end{aligned} \tag{17}$$

independent of both n and i. By introducing ϵ we can now write (6) and (8) in terms of features X_n and V_n of the importer n, feature Q_i of the exporter i, and the realization of ϵ, which is our structural error:

$$\begin{aligned} x_{ni}(\epsilon) &= X_n \epsilon^{\beta/(1+\gamma-\beta)} \\ y_{ni}(\epsilon) &= A^{-1}\left(V_n X_n\right)^{1/(1+\gamma)} Q_i^{-1} \epsilon^{1/(1+\gamma-\beta)} \\ q_{ni}(\epsilon) &= A^{1/\gamma}\left(V_n X_n\right)^{1/(1+\gamma)} \epsilon^{1/(1+\gamma-\beta)} \\ p_{ni}(\epsilon) &= A\left(X_n\right)^{\gamma/(1+\gamma)} V_n^{-1/(1+\gamma)} Q_i \epsilon^{-(1-\beta)/(1+\gamma-\beta)} \\ u_{ni}(\epsilon) &= A^{-1}(1+\gamma)^{1/\rho}\left(V_n X_n\right)^{1/(1+\gamma)} \epsilon^{1/(1+\gamma-\beta)}. \end{aligned} \tag{18}$$

Since spending across varieties has to integrate to X_n, from the first line of (18), $\epsilon^{\beta/(1+\gamma-\beta)}$ has mean one.

We now incorporate this demand system into both production with intermediates and final consumption.

3.2 Production

We start with the determination of production costs, $\tilde{C}_i$ in equation (10), and of horizontal quality Q_i. Physical output is produced with a Cobb-Douglas combination of labor and intermediates at constant returns to scale, with intermediates combining varieties according to (1). The horizontal-quality adjusted output o of a single worker equipped with an amount m of intermediates is

$$o = Qm^{1-\alpha} \tag{19}$$

where $1 - \alpha$ is the elasticity of output per worker with respect to intermediate inputs, given Q, where $\alpha \in (0, 1)$.

A producer's problem, then, can be stated as hiring labor in amount l and intermediates per worker m to minimize the cost of producing one unit of horizontal-quality adjusted composite output. Dropping the country subscript i, the cost of hiring a worker is the wage w and the cost of equipping her is $X(m)$, where the function X is given in equation (15). The producer's problem is thus:

$$\tilde{C} = \min_{l,m} \{l(w + X(m))\}$$

subject to providing one efficiency unit of the composite output:

$$Qlm^{1-\alpha} = 1. \tag{20}$$

We posit that the horizontal quality Q a worker produces increases with the extent to which she is equipped with intermediates according to:

$$Q = m^{\nu}, \tag{21}$$

where $\nu > 0$ is a parameter relating intermediate use per worker to horizontal quality. For concavity we require that $\nu < \alpha$.[20] Substituting (21) into the constraint (20):

$$lm^{1-\alpha+\nu} = 1. \tag{22}$$

As we show below, the share of labor in quality-adjusted production is:

$$\tilde{\alpha} = \frac{\alpha + \gamma - \nu}{1 + \gamma}$$

[20] Denoting effective output by O we can write the quality-adjusted production function as:

$$O = QLm^{1-\alpha} = Lm^{1-\alpha+\nu}$$

where:

$$m = \left[\int_{\omega \in \Omega} u(\omega)^{\beta} d\omega \right]^{1/\beta}$$

and where $u(\omega)$ is given by (2).

while the share of materials is:

$$1 - \tilde{\alpha} = \frac{1 - \alpha + \nu}{1 + \gamma}$$

Whether the labor share in quality-adjusted production is larger or smaller than α depends on whether γ exceeds or is exceeded by $\nu/(1-\alpha)$. In the first case the increased cost of higher vertical quality intermediates dominates the effect of intermediates in enhancing horizontal quality, and vice-versa. Our parameter estimates below put us in the range where the second effect dominates the first, so that the labor share in quality-adjusted production is less than α.

Substituting $X(m)$ from equation (15) and (22) into the objective function, the problem becomes:

$$\min_{l} \left\{ wl + \frac{\Gamma_1}{V} l^{-\tilde{\alpha}/(1-\tilde{\alpha})} \right\}.$$

The solution is:

$$l = \left(\frac{\tilde{\alpha}}{1 - \tilde{\alpha}} \cdot \frac{\Gamma_1}{wV} \right)^{1-\tilde{\alpha}}.$$

From the constraint $lm^{1-\alpha+\nu} = 1$:

$$m = \left(\frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \cdot \frac{wV}{\Gamma_1} \right)^{1/(1+\gamma)}$$

so that horizontal quality is:

$$Q = \left(\frac{1 - \tilde{\alpha}}{\tilde{\alpha}} \cdot \frac{wV}{\Gamma_1} \right)^{\nu/(1+\gamma)} \tag{23}$$

which is increasing in w and V. From (15), spending on intermediates per worker is:

$$\begin{aligned} X(m) &= \Gamma_1 \frac{m^{1+\gamma}}{V} \\ &= \frac{1 - \tilde{\alpha}}{\tilde{\alpha}} w. \end{aligned} \tag{24}$$

The cost of producing an effective unit of the composite output is

$$\begin{aligned} \tilde{C} &= l(w + X(m)) \\ &= \tilde{A} \left(\frac{\Gamma_1}{V} \right)^{1-\tilde{\alpha}} w^{\tilde{\alpha}} \end{aligned} \tag{25}$$

where:

$$\tilde{A} = \tilde{\alpha}^{-\tilde{\alpha}}(1-\tilde{\alpha})^{-(1-\tilde{\alpha})}$$

with labor share:

$$\frac{lw}{\tilde{C}} = \tilde{\alpha}$$

and materials share:

$$\frac{lX(m)}{\tilde{C}} = 1-\tilde{\alpha}.$$

We can insert $X(m)$ from (24) and Q from (23) into (18) to derive spending, quantity, quality, price, and benefit of variety ω when used as an intermediate in n purchased from i:

$$\begin{aligned}
x^M_{ni}(\omega) &= \left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)\epsilon(\omega)^{\beta/(1+\gamma-\beta)}w_nL_n \\
y^M_{ni}(\omega) &= A^{-1}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{(1+\nu)/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\left(V_nw_n\right)^{1/(1+\gamma)}L_n\left(\frac{V_iw_i}{\Gamma_1}\right)^{-\nu/(1+\gamma)} \\
q^M_{ni}(\omega) &= A^{1/\gamma}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\left(V_nw_n\right)^{1/(1+\gamma)} \\
p^M_{ni}(\omega) &= A\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{(\gamma-\nu)/(1+\gamma)}\epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)}V_n^{-1/(1+\gamma)}w_n^{\gamma/(1+\gamma)}\left(\frac{V_iw_i}{\Gamma_1}\right)^{\nu/(1+\gamma)} \\
u^M_{ni}(\omega) &= A^{-1}(1+\gamma)^{1/\rho}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{1/(1+\gamma)}\epsilon(\omega)^{1/(1+\gamma-\beta)}\left(V_nw_n\right)^{1/(1+\gamma)} \qquad (26)
\end{aligned}$$

Note that source i matters only for quantity and unit value.

3.3 Consumption

Total income in country n is w_nL_n. Since we assume balanced trade and income equality within countries, spending per worker is $X_n = w_n$. Household utility is given by (1).

We can then use (18) to get expressions, for variety ω sourced from i, of total household spending, total quantity demanded, vertical quality, unit value, and benefit in destination

n:

$$\begin{aligned}
x^C_{ni}(\omega) &= \epsilon(\omega)^{\beta/(1+\gamma-\beta)} w_n L_n \\
y^C_{ni}(\omega) &= A^{-1}\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{\nu/(1+\gamma)} \epsilon(\omega)^{1/(1+\gamma-\beta)} \left(V_n w_n\right)^{1/(1+\gamma)} L_n \left(\frac{V_i w_i}{\Gamma_1}\right)^{-\nu/(1+\gamma)} \\
q^C_{ni}(\omega) &= A^{1/\gamma}\epsilon(\omega)^{1/(1+\gamma-\beta)} \left(V_n w_n\right)^{1/(1+\gamma)} \\
p^C_{ni}(\omega) &= A\left(\frac{1-\tilde{\alpha}}{\tilde{\alpha}}\right)^{-\nu/(1+\gamma)} \epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)} V_n^{-1/(1+\gamma)} w_n^{\gamma/(1+\gamma)} \left(\frac{V_i w_i}{\Gamma_1}\right)^{\nu/(1+\gamma)} \\
u^C_{ni}(\omega) &= A^{-1}(1+\gamma)^{1/\rho}\epsilon(\omega)^{1/(1+\gamma-\beta)} \left(V_n w_n\right)^{1/(1+\gamma)} \qquad (27)
\end{aligned}$$

Note again that source i matters only for quantity and unit value.

3.4 Unit Values in Bilateral Trade

Since our data don't distinguish between imports for final and for intermediate use, we define the value, quantity, and unit value of a variety as

$$\begin{aligned}
x_{ni}(\omega) &= x^C_{ni}(\omega) + x^M_{ni}(\omega) \\
y_{ni}(\omega) &= y^C_{ni}(\omega) + y^M_{ni}(\omega) \\
p_{ni}(\omega) &= \frac{x_{ni}(\omega)}{y_{ni}(\omega)}.
\end{aligned}$$

From equations in (26) and (27) we can write the value as:

$$x_{ni}(\omega) = \frac{1}{\tilde{\alpha}} w_n L_n \epsilon(\omega)^{\beta/(1+\gamma-\beta)} \qquad (28)$$

and the unit value as

$$p_{ni}(\omega) = \Gamma_2 w_n^{\delta_{w,M}} \Phi_n^{\delta_{\Phi,M}} w_i^{\delta_{w,X}} \Phi_i^{\delta_{\Phi,X}} \epsilon(\omega)^{-(1-\beta)/(1+\gamma-\beta)} \qquad (29)$$

where

$$\begin{aligned}
\delta_{w,M} &= \frac{\gamma}{1+\gamma}, \\
\delta_{\Phi,M} &= -\frac{1}{\theta(1+\gamma)}, \\
\delta_{w,X} &= \frac{\nu}{1+\gamma}, \\
\delta_{\Phi,X} &= \frac{\nu}{\theta(1+\gamma)}, \qquad (30)
\end{aligned}$$

where

$$\Gamma_2 = \Gamma_0^{(\nu-1)/(1+\gamma)} \left(\tilde{\alpha}\left[1+((1-\tilde{\alpha})/\tilde{\alpha})^{1/(1+\gamma)}\right]\right)^{-1} \left(\frac{\tilde{\alpha}}{(1-\tilde{\alpha})\Gamma_1}\right)^{\nu/(1+\gamma)} A,$$

and where we have used (14) to replace V with Φ. The model thus implies that the unit value varies with (i) the importer wage with an elasticity $\delta_{w,M}$, (ii) the importer Φ with an elasticity $\delta_{\Phi,M}$, (iii) the exporter wage with an elasticity $\delta_{w,X}$, and (iv) the exporter Φ with an elasticity $\delta_{\Phi,X}$. Buyers in a destination with a high wage or a high price index (low Φ) pay more because they demand higher vertical quality and because competition is less intense (allowing on average a higher-priced variety to compete). Producers in a source with a high wage or low price index (high Φ) equip their workers with more intermediates, so produce goods with higher horizontal quality.

Expression (29) provides the basis of how we use our model to connect data on unit values in bilateral trade to importer and exporter characteristics reflecting their wages and price indices. Before quantifying the model we show what it says about the gains from trade.

3.5 The Gains from Trade

We can use expressions (14) and (15) to get an expression for the aggregate bundle that a worker in country n can achieve with a wage w_n and price index $\Phi_n^{-1/\theta}$:

$$Y_n = \left(\frac{\Gamma_0}{\Gamma_1} \cdot \frac{w_n}{\Phi_n^{-1/\theta}}\right)^{1/(1+\gamma)}. \tag{31}$$

A monotonic transformation gives us an expression for the worker's utility U_n that's linear in the wage:

$$U_n = \frac{\Gamma_0}{\Gamma_1} \cdot \frac{w_n}{\Phi_n^{-1/\theta}}. \tag{32}$$

We can substitute equation (11), with $i = n$, into (32), using (25), to get:

$$U_n = \left(\frac{\Gamma_0}{\tilde{A}\Gamma_1}\right)^{1/\tilde{\alpha}} \cdot \left(\frac{T_n d_{nn}^{-\theta}}{\pi_{nn}}\right)^{1/\tilde{\alpha}\theta} \tag{33}$$

which is the standard ACR formula (Arkolakis et al. (2012)), taking into account intermediates and domestic trade costs. The elasticity of real income with respect to the home share is $-1/\tilde{\alpha}\theta$.

4 Quantification

We estimate the Φ's from bilateral trade flows as we describe in Section 4.1. In Section 4.2 we use our estimates of the Φ's to estimate the parameters γ, ν, and θ. In Section 4.3 we use product level prices and volumes to estimate the parameter β which governs the distribution of the structural error ϵ. We turn to how we model and quantify the extensive margin in 4.4.

4.1 Trade Flows and Multilateral Resistance

We estimate the Φ's exploiting equation (11) using data on trade flows, GDP, and distance. We parameterize the effect of iceberg costs on trade share as

$$d_{ni}^{-\theta} = \delta^0 dist_{ni}^{\delta^g} \tag{34}$$

for $i \neq n$, where $dist_{ni}$ is the distance between i and n. Here δ^0 is a constant and δ^g is a parameter that relates trade share to distance, taking into account both the effect of distance on trade costs and the role of θ in relating trade costs to trade share. We estimate the d_{nn} individually as country fixed effects.

We construct trade shares as:

$$\pi_{ni} = \frac{X_{ni}}{X_n}$$

for $i \neq n$ and:

$$\pi_{nn} = \frac{X_n - \sum_{i' \neq n} X_{ni'}}{X_n},$$

where X_n is country n's total absorption.[21] For all $i \neq n$, we regress:

$$\log\left(\frac{\pi_{ni}}{\pi_{nn}}\right) = A_n + B_i + \delta^g \log dist_{ni} + \varepsilon_{ni}^X, \tag{35}$$

where A_n is an importer fixed effect, B_i is an exporter fixed effect, and ε_{ni}^X is the residual.[22] Equivalent to Waugh (2010), and in contrast to EK, we attribute country-level differences in openness to differences in internal trade costs (d_{nn}). Under this interpretation, equation

[21] Our absorption measure is:

$$X_n = \frac{GDP_n}{\tilde{\alpha}} + D_n,$$

where GDP_n is country n's GDP (corresponding to $w_n L_n$ in our model) and D_n is country n's trade deficit. While we've assumed balanced trade elsewhere our trade share measures takes deficits into account. The term $\tilde{\alpha}$, set equal to 0.5 for all countries, is to account for intermediate demand.

[22] The discussion in Anderson and Van Wincoop (2004) on assumptions on the residual term holds here.

(11) implies that fixed effects correspond to:

$$A_n = -\log\left(T_n \tilde{C}_n^{-\theta}\right) - \log(d_{nn}^{-\theta})$$
$$B_i = \log\left(T_i \tilde{C}_i^{-\theta}\right).$$

A consistent estimate of Φ_n is then

$$\hat{\Phi}_n = \exp(-\hat{A}_n) + \sum_{i \neq n} \exp(\hat{B}_i + \hat{\delta}^g \log dist_{ni}), \tag{36}$$

where $\hat{x}$ denotes the estimate of x.

4.2 Unit Values

We think of a variety ω in our model as a very finely defined product. If a variety in our model corresponded to 6-digit HS categories in the data, our model would incorrectly predict that, for any product, an importer would buy from only one source. We reconcile this discrepancy between theory and data by thinking of a 6-digit product category in the COMTRADE data as corresponding to a finite set of varieties ω in our model, with varieties within a product measured in the same units.

Taking logs of equation (29), for each product category k:

$$\log p_{nik} = \delta_k + \delta_{w,M} \log w_n + \delta_{\Phi,M} \log \Phi_n + \delta_{w,X} \log w_i + \delta_{\Phi,X} \log \Phi_i + \varepsilon_{nik}^P. \tag{37}$$

Here the product fixed effect δ_k incorporates Γ_2 and accounts for the units in which product k is measured and ε_{nik}^P is a residual.[23] We use per capita GDP to measure importer and exporter wages w.

The top panel of Table 4 shows the results of estimating equation (37). Column (1) reports the simple OLS estimates. The estimates satisfy the model's restrictions, from equation (30), that $\delta_{w,M} \in (0,1)$, $\delta_{\Phi,M} < 0$, $\delta_{w,X} \in (0,1)$, and $\delta_{\Phi,X} > 0$.

Column (2) reports the results of replacing w_n, Φ_n, w_i and Φ_i with importer-exporter fixed effects. Column (3) then reports the results of regressing the importer-exporter fixed

[23] If we attribute the residual to variation across realizations of $\epsilon(\omega)$ in equation (29), it corresponds to:

$$\varepsilon_{nik}^P = -\frac{1-\beta}{1+\gamma-\beta} \sum_{\omega \in \Omega^k} s_{nik}(\omega) \ln \epsilon(\omega)$$

where Ω^k is the set of varieties constituting product k and $s_{nik}(\omega)$ is the share of variety ω in i's exports to n of product k.

Table 4: Results from Price Regression

dependent variable → independent variable ↓	p_{nik} (1)	p_{nik} (2)	importer-exporter fixed effect from specification (2) (3)	same as specification (3) restricting $\theta_n = \theta_i$ (4)	same as specification (3) restricting $\theta = 4$ (5)
importer per capita income (w_n)	0.101		0.101	0.105	0.117
	(0.023)		(0.023)	(0.023)	(0.010)
$\hat{\Phi}_n$	-0.073		-0.072	-0.080	-0.163
	(0.044)		(0.045)	(0.045)	(0.002)
exporter per capita income (w_i)	0.182		0.181	0.194	0.190
	(0.027)		(0.027)	(0.031)	(0.029)
$\hat{\Phi}_i$	0.088		0.090	0.017	0.035
	(0.041)		(0.043)	(0.010)	(0.005)
importer-exporter fixed effect	no	yes	no	no	no
product category fixed effect	yes	yes	no	no	no
R-squared	0.81	0.82	0.50	0.49	0.49
number of observations	2,611,700	2,611,700	2,448	2,448	2,448
parameters implied by importer coefficients					
γ	0.11		0.11	0.12	0.13
	(0.03)		(0.03)	(0.03)	(0.013)
θ_n not adjusted	12.3		12.5	11.2	
	(7.4)		(7.8)	(6.4)	
θ_n adjusted for attenuation	9.1		9.2	8.2	4.0
	(5.5)		(5.8)	(4.7)	-
parameters implied by exporter coefficients					
ν	0.20		0.20	0.22	0.22
	(0.03)		(0.03)	(0.04)	(0.03)
θ_i not adjusted	2.1		2.0	11.2	
	(0.94)		(0.92)	(6.36)	
θ_i adjusted for attenuation	1.5		1.5	8.2	4.0
	(0.70)		(0.68)	(4.70)	-
Hypothesis testing (t-statistic)					
$\theta_n = \theta_i$	1.39		1.35		
$\theta_n = \theta_i = 4$				0.90	

All variables are in logs. Observations in the regressions reported in columns (3), (4), and (5) are weighted by the number of products that the country pair trades in column (2). Standard errors are clustered by importer and exporter. See Cameron et al. (2011) for multiway clustering.

effects from the regression in column (2) on the corresponding importer and exporter characteristics in column (1). Note that, comparing columns (1) and (3), the coefficients and their standard errors are almost identical.

Our model implies that three parameters ν, γ, and θ determine the four coefficients $\delta_{w,M}$, $\delta_{\Phi,M}$, $\delta_{w,X}$, and $\delta_{\Phi,X}$. Hence the regression coefficients overdetermine these parameters. Each column in the bottom panel of Table 4 reports the implications of the corresponding coefficients in the top panel for the parameters ν, γ, and θ.

Since the coefficients overdetermine the parameters we report their implications for θ based first on the coefficient $\delta_{\Phi,M}$ on Φ_n and then on the coefficient $\delta_{\Phi,X}$ on Φ_i. Since the variables Φ_n and Φ_i were constructed as described in subsection 4.1, their estimated coefficients suffer from potential attenuation bias. The bottom panel reports the implications of correcting this bias, as described in Appendix C, for the two estimates of θ. Adjusting for attenuation lowers the implied values of θ in each case. Whether we adjust or not, the θ implied by the importer coefficient is much larger than the θ implied by the exporter coefficient. Still, because the importer θ is imprecisely estimated, we cannot reject the model's restriction that the two θ's are the same at the 5 percent confidence level from either the one-stage (column (1)) or two-stage (column (3)) procedures.

Column (4) reports the results of performing the regression reported in column (3) imposing the restriction that the θ's implied by the importer and exporter coefficients are equal. The point estimate of θ of 8.2 remains imprecisely estimated. We conclude that our price data do not nail θ precisely.

A number of authors have pointed to a value of around 4 based on various sources of evidence.[24] This value is not rejected at the 5 percent confidence level by the procedure reported in column (4). Column (5) reports the results of the same regression as column (4) with the additional restriction that $\theta = 4$. The implied values of γ and ν barely change.

4.3 Estimating β

Solving for $\epsilon(\omega)$ in the price equation (29) and substituting it into the expression for value (28), we can write the relationship between value and price in log-linear form:

$$\log x_{ni}(\omega) = \delta_n + \delta_i - \frac{\beta}{1-\beta} \log p_{ni}(\omega)$$

[24]See, for example, Bernard et al. (2003), Costinot et al. (2011), Simonovska and Waugh (2014), and Caliendo and Parro (2015) .

Table 5: Estimate of elasticity of spending with respect to prices

independent var → dependent var ↓	unit price$_{nik}$ OLS	dependent var ↓	value$_{nik}$ OLS	value$_{nik}$ IV
instrument	0.412	unit price$_{nik}$	-0.252	-1.828
	(0.031)		(0.038)	(0.019)
importer fixed effect	yes		yes	yes
exporter fixed effect	yes		yes	yes
product fixed effect	yes		yes	yes
R-squared	0.70		0.25	0.10
number of observations	2,585,111		2,585,111	2,585,111

The table shows the results from estimating the price elasticity of spending on a product. Observations are specific to importer n, exporter i, and product k. The instrument is the average price of exporter i's exports of product k to importers other than n. All variables are in logs. The first column reports the first-stage regression of the price on the instrument. The second column reports the OLS regression of spending on price and the third column reports the second-stage IV regression. All regressions include exporter, importer, and product fixed effects.

Aggregating across varieties within a product k we get a product level expression:

$$\log x_{nik} = \delta_n + \delta_i + \delta_k - \frac{\beta}{1-\beta} \log p_{nik} + \varepsilon^X_{nik}$$

where δ_n, δ_i, and δ_k are, respectively, importer, exporter, and product fixed effects, and ε^X_{nik} is a residual. To account for potential demand shifts in country n for product k, we instrument the price p_{nik} with the average price of exporter i in product k to destinations different from n.

Table 5 shows the results. The first column shows that the instrument has power: It's highly correlated with prices even after controlling for all fixed effects. In the last two columns, the estimated coefficient on price is -0.25 with OLS and -1.83 with IV. The small coefficient in the OLS regression suggests large simultaneity or measurement error in prices. The coefficient on price in the IV regression implies an elasticity of demand with respect to prices of -2.83.[25] The implied β is 0.65.

Our model has allowed us to decompose the intensive margin of trade into unit values and quantities. Estimating the model with data on values and unit values in bilateral trade has given us estimates of the parameter values γ, ν, θ, and β. We now turn to the extensive margin.

[25] This figure compares with the median elasticity of 2.7 reported in Broda and Weinstein (2006) for U.S. imports at the SITC-5 level.

4.4 The Extensive Margin

We interpret equation (28) as determining destination n's annual absorption of variety ω, which is sourced from i. We think of this flow, however, as provided through discrete shipments that come in size $\underline{x}(\omega)$. If $x_{ni}(\omega) \geq \underline{x}(\omega)$, then a shipment is observed every year. Otherwise, it's observed with probability $x_{ni}(\omega)/\underline{x}(\omega)$. Assuming that $\underline{x}$ has a cumulative distribution function H, the probability of observing the shipment of a variety ω with trade flow $x_{ni}(\omega)$ in any given year is

$$H(x_{ni}(\omega)) + x_{ni}(\omega) \int_{x_{ni}(\omega)}^{\infty} (1/\underline{x}) dH(\underline{x}). \tag{38}$$

Trade flow x_{ni} is given by equation (28) with the distribution J of $\epsilon(\omega)$ given in (17). Assuming H and J are independent from each other and across varieties, the share of varieties that country n sources from i in a given year is

$$\begin{aligned}\tilde{\pi}_{ni} =& \pi_{ni} \int_0^{\infty} \Big[H(\tilde{\alpha}^{-1} w_n L_n \epsilon^{\beta/(1+\gamma-\beta)}) \\ &+ \tilde{\alpha}^{-1} w_n L_n \epsilon^{\beta/(1+\gamma-\beta)} \int_{\tilde{\alpha}^{-1} w_n L_n \epsilon^{\beta/(1+\gamma-\beta)}}^{\infty} (1/\underline{x}) dH(\underline{x}) \Big] dJ(\epsilon).\end{aligned}$$

To translate the extensive margin at the variety level into the corresponding margin at the product level, we need to take a stance on the partition of varieties into products. We think of a product as containing an integer number of varieties. Letting $f(M)$ denote the fraction of products with M varieties, the share of products that country n buys from i in a given year is

$$E_{ni} = \sum_{M=1}^{\infty} f(M) \left[1 - (1 - \tilde{\pi}_{ni})^M \right] \tag{39}$$

We map this extensive margin in the model to the data. To do so, we parameterize H as exponential:

$$H(\underline{x}) = 1 - \exp(-\lambda_1 \underline{x}) \tag{40}$$

and the probability mass function f as:

$$f(M) = \exp(-\lambda_2 (M-1)^{\lambda_3}) - \exp(-\lambda_2 M^{\lambda_3}), \tag{41}$$

a discretized Weibull density. We estimate the parameters λ_1, λ_2, and λ_3 to minimize:

$$\sum_{i=1}^{N} \sum_{n \neq i} (E_{ni} - E_{ni}^{\text{data}})^2$$

where E_{ni}^{data} is the share of HS6 product categories n buys from i in the 2007 cross section. The estimated parameters are $\lambda_1 = 2.26e-7$ (standard error 1.21e-7), $\lambda_2 = 0.042$ (0.020), and $\lambda_3 = 0.48$ (0.10). The R-squared is 0.79. Hence these three parameters explain the extensive margin quite parsimoniously.[26]

5 Simulating World Trade

Now that we've quantified the key parameters of our model we can turn to how well it captures the three margins of trade discussed in Section 2. Since the margins of trade in the data are at the HS6 product level while our model is about trade in varieties, our simulation has two stages. The first stage simulates trade among our 50 countries in five million varieties. The second stage aggregates the simulated varieties into products.

5.1 Simulating Varieties

Continuing to index a variety by ω, our simulation for each ω has three components:

1. **Trade in varieties with gravity** For this component of the simulation we use the model's prediction that the probability that country i is the cheapest (horizontal-quality-adjusted) source of variety ω in country n is:

$$\hat{\pi}_{ni} = \begin{cases} \exp(\hat{B}_i)\ \ \hat{\Phi}_n^{-1}\ \ dist_{ni}^{\hat{\delta}^g} & n \neq i \\ \\ \exp(\hat{A}_n)\ \ \hat{\Phi}_n^{-1} & n = i \end{cases} \tag{42}$$

where $dist_{ni}$ is the distance between destination n and source i and $\hat{B}_i$, $\hat{A}_n$, $\hat{\delta}^g$, and $\hat{\Phi}_n$ are taken from the estimation of the bilateral resistance terms reported in Subsection 4.1.

[26]The point estimates imply a mean shipment size of \$4.42 million (median \$3.07 million) and a mean number of varities per product of 1597 (median 344). The frequency of zeros requires a large shipment size while the frequency of multiple sources per product-destination requires the large number of varieties per product.

(a) For each ω we draw $\upsilon_i(\omega)$ from the unit Fréchet distribution:

$$H(\upsilon) = \exp(-\upsilon^{-1})$$

for each source i.

(b) For each bilateral trade pair we calculate:

$$\upsilon_{ni}(\omega) = \hat{\pi}_{ni}\upsilon_i(\omega) \tag{43}$$

which is proportional to the cheapest (horizontal-quality-adjusted) cost of variety ω in destination n from source i.

(c) For each destination n we determine the best source $i^*_n(\omega)$ for variety ω:

$$i^*_n(\omega) = \arg\max_i \upsilon_{ni}(\omega) \tag{44}$$

establishing the source of variety ω for each destination n. The combinations of n and $i^*_n(\omega)$ constitute the set of bilateral trading pairs for variety ω. Since we are modeling only international trade we drop observations for which $i^*_n(\omega) = n$.

2. **Bilateral trade values and prices** For this component of the simulation we calibrate, as above, $\theta = 4$ and $\tilde{\alpha} = 0.5$. Based on the estimation of unit values in Subsection 4.2, we set $\gamma = 0.13$ and $\nu = 0.22$ and, based on the results in Subsection 4.3, we set $\beta = 0.65$.

   (a) For each nontrading pair (for which $i \neq i^*_n(\omega)$) we set $x_{ni}(\omega) = 0$.

   (b) For each bilateral trading pair, using (17) and (43), we calculate:

$$\epsilon_{ni}(\omega) = \left[\upsilon_{ni^*_n(\omega)}(\omega)\right]^{1/\theta} \Gamma_0^{-1} \tag{45}$$

   (c) For each bilateral trading pair we substitute (45) into equation (28) to solve for $x_{ni}(\omega)$ and into equation (29) to solve for $p_{ni}(\omega)$.

   (d) For each bilateral trading pair we set quantity $y_{ni}(\omega) = x_{ni}(\omega)/p_{ni}(\omega)$.

3. **Censoring due to shipment sizes** We draw a shipment size $\underline{x}(\omega)$ from the distribution (40) using the value of λ_1 reported in Section 4.4. For any $x_{ni}(\omega)$ from the previous component of the simulation we set the *reported* trade flow $\tilde{x}_{ni}(\omega)$ from

exporter $i_n^*(\omega)$ to importer n in variety ω as:

$$\tilde{x}_{ni}(\omega) = x_{ni}(\omega) \qquad \text{if } x_{ni}(\omega) > \underline{x}(\omega)$$

Otherwise, if $x_{ni}(\omega) \leq \underline{x}(\omega)$ then the reported trade flow $\tilde{x}_{ni}(\omega)$ is randomly drawn as

$$\tilde{x}_{ni}(\omega) = \begin{cases} \underline{x}(\omega) & \text{with probability } x_{ni}/\underline{x} \\ 0 & \text{with probability } 1 - x_{ni}/\underline{x} \end{cases}$$

We now have, for each variety ω and for each destination n and foreign source $i = i_n^*(\omega) \neq n$, a reported purchase $\tilde{x}_{ni}(\omega)$ and unit value $p_{ni}(\omega)$. The simulated quantity is $\tilde{y}_{ni}(\omega) = \tilde{x}_{ni}(\omega)/p_{ni}(\omega)$. We may not report destination n importing variety ω either because it purchases it domestically or because its simulated purchase from a foreign source is less than the shipment size for that variety.

5.2 Simulating Products

Having now simulated varieties $\omega = 1, 2, ..., 5,000,000$ we simulate K products indexed by $k = 1, 2, ..., K$. We partition varieties into products as follows:

1. For product $k = 1$ we draw its number of varieties M_1 from the probability mass function $f(M)$ given in (41) and assign this product varieties 1 through M_1.

2. For product $k > 1$ we draw its number of varieties M_k from the probability mass function $f(M)$ and assign it varieties ω_k through $\omega_k + M_k$ where

$$\omega_k = \omega_{k-1} + M_{k-1}.$$

   with $\omega_1 = 1$.

3. Sequentially repeating step 2 we continue until we arrive at product K such that $w_K + M_K \geq 5,000,000$ and assign this product varieties ω_K through 5,000,000.

This procedure yields 3842 simulated products. Of these, 35 products contain only varieties that are not traded between any importer-exporter pair, either because they are sourced domestically or because the trade value falls below the shipment size. The remaining 3807 traded products in our simulated dataset compares with 4973 in the COMTRADE data.

For each importer-exporter pair ni with positive trade in product k we construct value, quantity, and price as:

$$x_{ni}^k = \sum_{\omega=\omega_k}^{\omega_k+M_k} \tilde{x}_{ni}(\omega)$$

$$y_{ni}^k = \sum_{\omega=\omega_k}^{\omega_k+M_k} \tilde{y}_{ni}(\omega)$$

$$p_{ni}^k = \frac{x_{ni}^k}{y_{ni}^k}.$$

The results deliver our model's analog to COMTRADE's HS6 bilateral trade data. We now ask how well our model captures the margins of trade in the actual data described in Section 2.

5.3 Capturing the Margins of Trade

Table 6 compares the results of regressing bilateral trade value, extensive margin, quantity, and price on exporter and importer characteristics and distance using the simulated trade data, in the right panel, compared with the results using the actual data (repeating the results from Section 2), in the left.

The coefficients based on the simulated data generally mimic those from the actual data with a couple of exceptions. The model understates the effects of both importer and exporter per capita income on the extensive margin, shifting their effects toward quantity. The model also understates the effect of distance on price, the Allen-Archian effect. The second discrepancy is not surprising given that the model doesn't incorporate any reason for such an effect. We repeat, however, that the effect is significant only when exporter and importer GDP are broken down into per capita GDP and population.

As we pointed out in our discussion of the extensive margin in Section 2.3, the effect of total GDP on the extensive margin appears to be nonlinear: The range of products both exported and imported expands rapidly with GDP for small countries, but then appears to die out as countries get large.

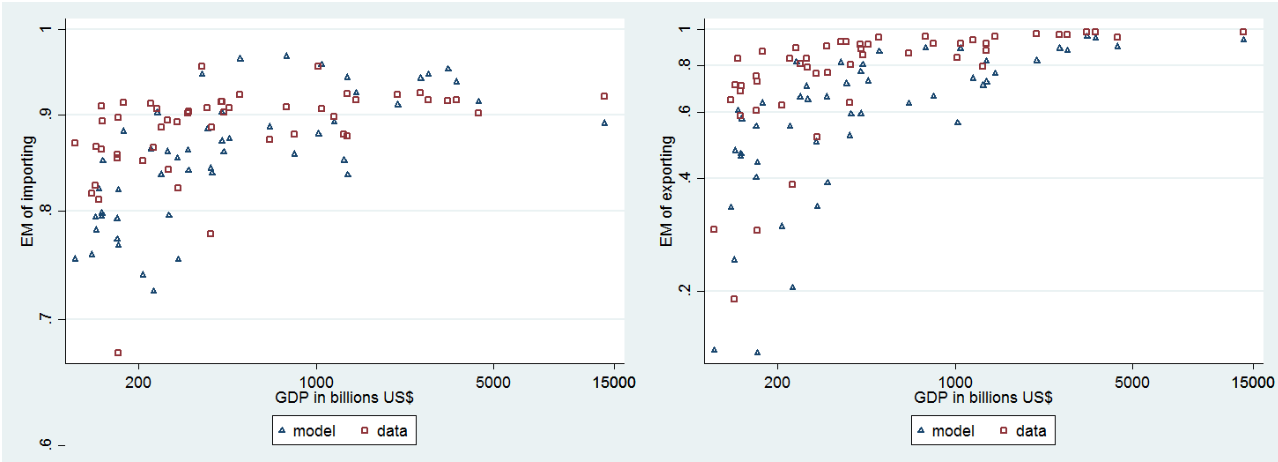
Figure 4 adds observations from the simulated data to those from the actual data reported in Figure 3. Note how the model picks up the concavity of the relationship between GDP and the extensive margin.

To summerize, our model, taken to data, captures essential features of the extensive and intensive margins of trade, and of how the intensive margin in turn breaks down into quantity and unit value. It does so quite parsimoniously, with just seven parameters: γ,

Table 6: Decomposition of trade flows in the data and model

	data				**model**			
dependent variable →	value	extensive margin	quantity	price	value	extensive margin	quantity	price
Panel A								
exporter GDP	1.16	0.76	0.36	0.04	1.16	0.67	0.45	0.04
importer GDP	1.11	0.34	0.73	0.05	1.07	0.28	0.73	0.06
distance	-0.81	-0.43	-0.39	0.02	-0.80	-0.42	-0.35	-0.03
Panel B								
exporter GDP per capita	1.18	0.84	0.19	0.15	1.18	0.70	0.31	0.17
exporter population	1.15	0.71	0.46	-0.02	1.15	0.65	0.53	-0.03
importer GDP per capita	1.10	0.41	0.56	0.13	1.04	0.27	0.64	0.13
importer population	1.12	0.30	0.82	0.00	1.08	0.28	0.77	0.03
distance	-0.81	-0.37	-0.53	0.10	-0.80	-0.42	-0.43	0.05
num. observations	2448	2448	2448	2448	2448	2448	2448	2448

All variables are in logs. We report standard errors in appendix B.



(a) Importer (b) Exporter

Figure 4: Extensive Margin and GDP in the Data and in the Model

ν, θ, β, λ_1, λ_2, and λ_3.

Our specifications of the price equation in Section 4.2 and demand equation in Section 4.3 are at the level of variety in our model. Our estimation uses data at the level of HS6 products in the COMTRADE data. We've reconciled the two levels by treating HS6 products as collections of varieties. To what extent does aggregation of varieties into products impede identification of the model's underlying seven parameters? To address this question we performed a Monte Carlo analysis, applying our estimation procedure to the simulated data described in this section to see if we can recover parameter values close to those used to generate the data. The two sets of parameters are, with the exception of β when product fixed effects are included, close. Appendix D reports the details.

6 Disaggregation

Our analysis so far, both descriptive and analytic, has been at the level of total merchandise trade: In estimating the effects of importer and exporter per capita income on unit values we pooled observations across all importer-exporter-HS6-product triads in the COMTRADE data. In estimating our model we imposed common elasticities γ and ν across all merchandise. We now assess how much damage this (audacious?) level of aggregation inflicts.

As discussed in Section 2, COMTRADE's finest level of product categorization is the 6-digit HS6 classification. COMTRADE also provides three courser partition tiers: the 4-digit HS4 level, the 2-digit HS2 level, and the partition of HS2 categories into 15

Table 7: Summary of Sections

Section	Section Name[1]	HS2 categories	importer-exporter -product triads	importer-exporter dyads
1	Animal and Animal Products	01-05	49,819	2,062
2	Vegetable Products	06-15	111,340	2,296
3	Food Items	16-24	97,394	2,296
4	Mineral Products	25-27	35,813	2,177
5	Chemicals and Allied Industries	28-38	325,045	2,374
6	Plastics, Rubbers	39-40	157,993	2,382
7	Raw Hides, Skins, Leather, Furs	41-43	34,028	2,173
8	Wood and Wood Products	44-49	126,612	2,362
9	Textiles	50-63	431,606	2,354
10	Footwear, Headgear	64-67	37,225	2,100
11	Stone, Glass	68-71	102,628	2,302
12	Metals	72-83	335,950	2,375
13	Machinery, Electrical	84-85	471,941	2,409
14	Transportation	86-89	67,927	2,305
15	Miscellaneous	90-97	226,379	2,358
total			**2,611,700**	**2,448**

[1]Section names on the table are the authors' own abbreviations of the official names, listed on the UNCOMTRADE website.

sections. Table 7 lists the sections along with their component HS2 categories, the number of importer-exporter-HS6 product triads in each, and the number of importer-exporter dyads in each.

6.1 Heterogeneity in the Effects of Income per Capita on Prices

Our first exercise examines variation in the effects of importer and exporter per capita income across product categories at the HS6 level. For each of the 4,786 HS6 products with more than 20 importer-exporter pairs we run the regression:

$$\log p_{nik} = \delta_{0k} + \delta_{1k} \log w_n + \delta_{2k} \log w_i + \epsilon_{nik} \tag{46}$$

where p_{nik} is the unit value of the imports of country n from country i of product k, w_n is the income per capita of importer n, w_i is the income per capita of exporter i, and δ_{0k}, δ_{1k}, and δ_{2k} are parameters estimated for each product k. The products represented account for 96% of the total number of HS6 products and nearly all of international trade flows in terms of value.

Overall, 80 percent of the coefficients δ_{1k} on importer per capita income and 94 percent of the coefficients δ_{2k} on exporter per capita income are positive. To summarize the results further, Table 8 reports, for each section, the mean coefficient for that section, the fraction that are positive, and the fraction that are significantly positive. With the exception of Mineral Products (Section 4) on the importer side, positive coefficients on importer and exporter per capita income are pervasive within individual sections. Textiles (Section 9) and Footwear, Headgear (Section 10), sections where we might expect a high degree of quality differentiation, display particularly large shares of positive coefficients.

How much of the heterogeneity in the coefficients on income per capita at the HS6 level can we attribute to courser levels of classification? To answer this question we decompose the variances in our estimates $\hat{\delta}_{1k}$ and $\hat{\delta}_{2k}$ at the HS6 level into within and between industry classifications for the three courser tiers of classification. Table 9 reports the share of the variance that is between industry categories for each of the three. The fifteen sections account for only 10% of the variance across estimates $\hat{\delta}_{1k}$ and 13% of the variance across estimates $\hat{\delta}_{2k}$. Although the number of HS4 product categories, 1,231, is not much smaller than the 4,786 HS6 categories, HS4 categories account for less than half of the variance. In sum, broader industry categories account for relatively small variation in the income elasticities across HS6 product categories. Analysis that focuses on broader industry classifications leaves a lot of within-industry heterogeneity on the table.

6.2 The Model with Sectional Heterogeneity

The model we develop in Section 3 admits a classification of varieties into different categories s with individual elasticities γ^s and ν^s, while maintaining trade barriers d_{ni}, technology parameters T_i, and Fréchet parameter θ that are common across all categories. This extension of the model allows us to reestimate equation (37) as:

$$\widehat{\log p_{nik}^{s(k)}} = \delta_0^{s(k)} + \delta_{w,M}^{s(k)} \log w_n + \delta_{\Phi,M}^{s(k)} \widehat{\log \Phi_n} + \delta_{w,X}^{s(k)} \log w_i + \delta_{\Phi,X}^{s(k)} \widehat{\log \Phi_i} + \varepsilon_{nik}^{P} \tag{47}$$

category by category, where $s(k)$ is product k's category, and:

$$\begin{aligned}
\delta_{w,M}^{s} &= \frac{\gamma^s}{1+\gamma^s}, \\
\delta_{\Phi,M}^{s} &= -\frac{1}{\theta(1+\gamma^s)}, \\
\delta_{w,X}^{s} &= \frac{\nu^s}{1+\gamma^s}, \\
\delta_{\Phi,X}^{s} &= \frac{\nu^s}{\theta(1+\gamma^s)}.
\end{aligned} \tag{48}$$

Table 8: Summary of Coefficients of Regressions (46)

			importer per capita GDP (δ_{1k})			exporter per capita GDP (δ_{2k})		
		number of		percentage			percentage	
	section	products	mean	positive	sig 10%	mean	positive	sig 10%
1	Animal, Animal Products	189	0.11	75	60	0.12	87	66
2	Vegetable Products	294	0.11	84	70	0.12	90	78
3	Food Items	176	0.09	88	73	0.17	97	89
4	Mineral Products	142	-0.001	46	22	0.12	83	65
5	Chemicals, Allied Industries	728	0.04	65	43	0.15	92	77
6	Plastics, Rubbers	189	0.09	90	81	0.17	97	94
7	Raw Hides, Skins, Leather, Furs	61	0.13	93	82	0.11	77	67
8	Wood, Wood Products	212	0.08	83	68	0.11	88	70
9	Textiles	777	0.14	95	86	0.23	98	95
10	Footwear, Headgear	53	0.19	96	91	0.20	100	96
11	Stone, Glass	182	0.11	80	62	0.20	92	84
12	Metals	541	0.07	78	58	0.19	97	91
13	Machinery, Electrical	747	0.06	73	55	0.24	97	90
14	Transportation	124	0.09	79	56	0.19	88	79
15	Miscellaneous	371	0.15	88	78	0.24	98	91
	All	**4786**	**0.090**	**80**	**64**	**0.186**	**94**	**85**

Table 9: Share of the variance in $\hat{\delta}_{1k}$ and in $\hat{\delta}_{2k}$ that is between industry categories

	section	HS2	HS4	HS6
importer per capita income ($\hat{\delta}_{1k}$)	0.10	0.18	0.39	1
exporter per capita income ($\hat{\delta}_{2k}$)	0.13	0.25	0.45	1
number of categories	15	96	1,231	4,786

Here the $\widehat{\log \Phi_i}$'s are those from expression (36) above.[27]

We implement this disaggregation using COMTRADE's sections as our categories. Table 10 reports what results from estimating equation (47) separately by section, imposing (48) and $\theta = 4$. The conditions $\hat{\gamma}^s > 0$ and $\hat{\nu}^s > 0$ are satisfied by all sections (significantly so except for $\hat{\gamma}^{Mineral\,Products}$). For many sections, estimates $\hat{\gamma}^s$ and $\hat{\nu}^s$ are not far from the pooled regression estimates $\hat{\gamma} = 0.13$ and $\hat{\nu} = 0.22$.[28]

We conduct a quasi-likelihood ratio test of whether the γ^s and ν^s are equal across sections as follows. We first run an unrestricted version of (47) imposing only that $\theta = 4$ but with γ^s and ν^s estimated separately for each section. We denote the resulting sum of squared residuals as RSS_U and calculate the average squared residual $\hat{\sigma}_U = RSS_U/Obs$, where $Obs = 34,325$ is the number of observations (one for each importer-exporter-section triad). We then run a restricted version of (47) imposing $\theta = 4$ and restricting $\gamma^s = \gamma$ and $\nu^s = \nu$. We denote the resulting sum of squared residuals as RSS_R and calculate the average squared residual $\hat{\sigma}_R = RSS_R/Obs$. We then calculate:

$$\chi = Obs \frac{\hat{\sigma}_U - \hat{\sigma}_R}{\hat{\sigma}_U}.$$

Under the null hypothesis, χ is distributed chi-squared with 28 degrees of freedom, where 28 is the number of restrictions $\gamma^s = \gamma$ and $\nu^s = \nu$ for $s = 1, ..., 15$. Our estimated test statistic is 1249, well above the critical cutoff 41 for a 5% significance level. The formal rejection of the null is not surprising given the large number of observations, $Obs = 34,325$. But the change in squared residuals, $(\hat{\sigma}_U - \hat{\sigma}_R)/\hat{\sigma}_U$ is only 3.6 percent. After extracting the sector fixed effects, the R-squared increases from 0.353 to only 0.376.

[27] Treating the parameters d_{ni}, T_i, and θ as common across sections justifies our using these same estimates of the $\widehat{\log \Phi_i}$'s. Otherwise we would have to reestimate the gravity equation (35) category by category to obtain category-specific estimates $\widehat{\log \Phi_i^s}$. The paucity of nonzero trade flows at more disaggregate levels discouraged us from pursuing this alternative approach.

[28] We also considered the cases (not consistent with our employing the price equation (36)) (i) in which the θ backed out from (48) can vary by section, importer, and exporter and (ii) in which θ can vary by section but is restricted to be the same for importer and exporter. The story is much as at the aggregate level: θ is poorly identified and we can't reject $\theta = 4$ at the 5% significance level for any section.

Table 10: Results of Price Regression by Section (47) with $\theta = 4$

Section		γ par	γ se	ν par	ν se
1	Animal and Animal Products	0.23	0.05	0.12	0.02
2	Vegetable Products	0.18	0.04	0.16	0.02
3	Food Items	0.16	0.04	0.19	0.02
4	Mineral Products	0.03	0.03	0.13	0.02
5	Chemicals and Allied Industries	0.10	0.03	0.16	0.02
6	Plastics, Rubbers	0.12	0.03	0.20	0.02
7	Raw Hides, Skins, Leather, Furs	0.24	0.04	0.20	0.03
8	Wood and Wood Products	0.16	0.03	0.14	0.02
9	Textiles	0.21	0.04	0.28	0.03
10	Footwear, Headgear	0.29	0.06	0.25	0.04
11	Stone, Glass	0.16	0.04	0.24	0.04
12	Metals	0.12	0.04	0.23	0.03
13	Machinery, Electrical	0.11	0.03	0.24	0.04
14	Transportation	0.15	0.04	0.24	0.03
15	Miscellaneous	0.21	0.03	0.27	0.04
	Pooled (from Table 4)	**0.13**	**0.01**	**0.22**	**0.03**

While the data do imply some statistically significant variation across sections, we find the similarities so pronounced as to vindicate our aggregate approach, leaving further exploration of heterogeneity across industries for future research.

7 Conclusion

The COMTRADE data on bilateral trade reveal striking patterns about the range of products that countries buy and sell as well as about the quantities and prices at which these products are exchanged. Because the data report these magnitudes only for merchandise that crosses borders, we've applied our analysis to international trade. But the framework has implications for a wide range of additional issues, such as the roles of different margins in economic growth. Without a domestic equivalent of the COMTRADE data we have only a much cloudier picture of how these different margins operate. We leave this issue for future research.

Our approach has accommodated the HK facts, as they apply both to intermediate and to final goods, into the perfectly competitive EK framework. As discussed in the introduction, several studies have interpreted these facts using very different approaches that identify different mechanisms, most notably Feenstra and Romalis (2014), Lashkaripour

(2019b), and Atrianfar (2019). A challenge for future research is to assessing the relative quantitative contributions of these different mechanisms.

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