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SOVEREIGN-DEBT RENEGOTIATIONS: A STRATEGIC ANALYSIS

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ABSTRACT

Abstract: The process of debt-rescheduling between a creditor and a sovereign (LDC) debtor is modeled as a noncooperative game built on a one-sector growth model. The creditor's threat to impose default penalties is ignored here as inherently incredible; instead, the debtor's motivation for repayment is to reap benefits from attaining an improved credit standing in international capital markets. The creditor can forgive portions of the outstanding debt, so that a real-time bargaining process results with concessions being in the form of debt-service payments by the debtor and debt forgiveness by the creditor. Subgame-perfect equilibria of the game are characterized: the main finding is that these all result in Pareto optima in which the creditor extracts all the surplus.

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1. Introduction

Any attempt to shed light on LDC-debt renegotiations must necessarily come to grips with the question of why sovereign nations repay any portion of their debts in the first place (or with its counterpart of why banks ever choose to lend to sovereign nations). Unlike in the case of domestic lending, there is usually little collateral available in the form of seizable public assets held outside the country. Nonetheless, lending to sovereign countries coexists along with the historical possibility of widespread default. As explanations of this phenomenon, it is sometimes argued that countries may attempt to meet their debt obligations in order to gain future benefits, such as improved future access to capital markets, or to avoid future penalties, such as restricted trade credits and limitations on future lending, that debt repudiation may entail. In this paper we explore the strategic ramifications of the former explanation.

The "carrots-versus-sticks" division of nations' motivations to service and repay their debts, while somewhat arbitrary (since into which category any particular measure falls depends on one's perception of the status quo), has proven to be a useful way of organizing thoughts about the LDC-debt crisis. Eaton and Gersowitz (1981) examine the borrowing that can be sustained by a country whose income periodically alternates between high and low levels. Repudiation of the debt in this model causes the country to be excluded from future borrowing with the consequence that the country must use costlier methods to reduce fluctuations in consumption (such as stockpiling). Sachs (1983) investigates a two-period growth model where the default penalty translates into a fixed-proportion reduction in the output that can be produced from any given inputs and exclusion from future borrowing.

While most work on sovereign lending has recognized that the set of loans that a bank expects will be fully repaid is more restricted than the set of loans that are merely feasible for the debtor to repay, this expectational (or strategic) consideration has usually been incorporated by requiring that the loan made by the bank be such that the debtor is at least as well off repaying its debt as repudiating it. Unless this statement is made in an explicitly strategic environment, however, its significance is questionable. The knowledge, common to both parties, that a country prefers to repay some portion of its debt to incurring the costs of default (or to foregoing the benefits associated with repayment) and that the bank prefers some repayment as opposed to nothing merely means that a country may attempt to bargain with the bank to reduce its total debt. Why should the bank hold all the power in this bargaining game? The amount of the loan that will be repaid in the game's equilibrium, the factors that influence the bargaining outcome, and the strategies that sustain it must be explictly determined.

The existing sovereign-debt literature has, for the most part, been content to assume, often implicitly, that the threat of applying the stick or withholding the carrot is completely credible. The consequences of departing from the assumption that the bank can somehow precommit to imposing a penalty or withholding a bonus unless the entire debt is repaid, has been examined previously only by Bulow and Rogoff (1986). Making use of Rubinstein's (1982) bargaining model, they study the subgame-perfect bargaining equilibrium that emerges as a consequence of a country and bank negotiating over how much of an exogenously determined debt shall be repaid. In their model, the country prefers to trade its domestically produced good for a foreign good, but, if declared in default, it is liable to having a certain percentage of its traded output seized. Their game possesses a unique subgame-perfect bargaining equilibrium which depends on the rates of time preference of both parties, the gains from trade, and the bank's ability to impose costs on the country's trade. As in Rubinstein's model, and unlike ours, Bulow and Rogoff assume that no economic actions take place during the periods of negotiation.

Our paper investigates a game-theoretic model of debt renegotiation between a sovereign debtor and a creditor in which the motivation for repayment is only of the carrot variety, so that any threatened default penalties are ignored as incredible by both parties. More specifically, we assume that whenever the (renegotiated) debt is repaid in full, the former debtor receives a bonus (in a generalized sense), not paid by the creditor. (The bonus can be interpreted as improved access to international capital markets.) We are particularly interested in exploring debt renegotiation in a scenario which is able to capture some of the tension that exists between debt repayment and LDCs' short-to-medium-term growth prospects and living standards. To this end the game is built upon the traditional one-sector growth model.

Our game begins with the debt in place and growing according to a given rate of interest. There is no link to any previous time to explain how this debt was incurred, although it and the resulting game can be made consistent with a larger game in which there is uncertainty about future shocks at the time the loan is made. The game is one of alternating moves: in each period, the creditor first decides how much, if any, of the debt to forgive, and the debtor then makes output-allocation decisions for the period. The country begins the game with an exogenously given capital stock. At the beginning of each period production takes place determining the amount of output available that period. The country then decides how to allocate its output among investment, consumption, and debt service. The greater the debt serviced, the smaller the output available for consumption and investment. These decisions carried out, time advances, returns to investment are realized, interest accrues, and the moves are repeated. The game ends whenever the outstanding debt is repaid, whereupon the country automatically receives the bonus; if this never happens, the game simply continues indefinitely. There is no uncertainty in the model, and both players are fully aware of everything at all times.

We study the subgame-perfect equilibria of the game. This equilibrium concept is the natural refinement of Nash equilibrium for extensive-form games like ours with perfect information. It possesses the property of ruling out those threats which an agent would not be willing to carry out if called upon to do so. We find that the behavior produced along the equilibrium path by the subgame-perfect Nash equilibria of our game is easy to describe. We now do so for one of them. At the beginning of the game, the creditor forgives exactly the amount of debt that makes the debtor just indifferent between two plans: ignoring the debt and optimizing in the growth model on the one hand, and, on the other, proceeding along the optimal program leading to ultimate repayment assuming no further forgivenesses will be forthcoming. The debtor then follows the optimal repayment program, and the creditor never forgives any additional debt. (If at the beginning of the game the optimal program leading to full repayment of the debt yields the debtor greater utility than the optimal program ignoring the debt, the creditor does not forgive any portion of the debt.) Furthermore, all other subgame-perfect equilibria are similar in the sense that all generate the same payoffs as this one to both parties. We find this result surprising: it implies that the bank indeed possesses all the bargaining power in the game, though why this should be so is not apparent *a priori*.

The rest of the paper is organized as follows. In Section 2 the model is presented and the main result stated. Section 3 is dedicated to a proof of the main result, and Section 4 examines the issue of uniqueness. Section 5 contains remarks and conclusions.

2. The Model

The game $G(K_0, D_0)$ is played by two players: the creditor, named A, and the debtor, B, over discrete time periods 0,1,... . There is a single commodity (best thought of as creditor-country's currency), in the units of which everything is measured. The debtor's capital stock at the beginning of each period t is denoted K_t, and the level of debt at the beginning of each period t is denoted D_t. Initial nonnegative values for these variables, K_0 and D_0, are specified exogenously. The game continues until the first time T at which D_t falls to zero; if this never occurs, $T = \infty$. At the beginning of each period $t \leq T$, the output from investing K_t in production is realized. The first move at each t ($\leq T$) then belongs to A, who selects f_t, the part of D_t to be forgiven currently. Next, B, knowing A's choice of f_t, selects current levels of consumption, c_t, and debt-service payment, p_t. Thus, the following restrictions on the players' moves apply:

$$0 \leq f_t \leq D_t, \quad \text{and} \tag{1}$$

$$0 \leq c_t, \quad 0 \leq p_t \leq D_t - f_t, \quad \text{and } c_t + p_t \leq g(K_t), \tag{2}$$

where $g:R_+ \to R_+$ is the debtor's production function. If $p_t = D_t - f_t$, the game ends; otherwise, the next period is entered with

$$K_{t+1} = g(K_t) - c_t - p_t \quad \text{and} \quad D_{t+1} = (1+r)(D_t - p_t - f_t), \tag{3}$$

where $r(>0)$ is the interest rate on the debt. Both players are assumed to know and remember all past moves in the game.

The creditor wishes to maximize the discounted sum of debtor's payments $\Sigma_t \alpha^t p_t$, where α is the creditor's discount factor; similarly, the debtor wishes to maximize

$$\sum_{t=0}^{T} \beta^t u(c_t) + \beta^{T+1} Z(K_{T+1}) \tag{4}$$

where $\beta<1$ is the debtor's discount factor, u is B's one-period utility-from-consumption function, and Z(K) gives the value of the future to the debtor when ending the game with no debt and capital stock K. We assume that $Z:R_+ \to R$ is increasing, continuous, and bounded below by the function v, with $Z(0)=v(0)$, where v(K) is the value to B of following the optimal plan for the one-sector growth model defined by β, g, and u, with initial capital stock K (hereafter GM(K)). (If the game never ends, the last term in (4) is identically zero.) We assume that u and g are C^2, increasing, strictly-concave functions with $g'(0) = \infty$ and $g'(\infty) = 0$ (see e.g. Cass (1965)); this insures that GM has a unique solution for all initial conditions and that v is strictly concave. The game $G(K_0,D_0)$ is now completely specified, and its extensive form is expressed schematically in Figure 1. Note that $G(K_0,D_0)$ is a game of perfect information with no moves by nature.

Let H_t denote the set of possible partial histories of play h_t through the end of period (t-1); i.e., $H_0 = \emptyset$ and, for $t \geq 1$,

$H_t = \{(f_0, c_0, p_0, \ldots, f_{t-1}, c_{t-1}, p_{t-1})$: (1)-(3), defined recursively, hold$\}$. Let a and b be any strategies for A and B, respectively; i.e. a is a sequence of functions $(a_0, a_1, \ldots)$, where, for all t, a_t selects for each $h_t \in H_t$ an f_t between zero and the D_t determined by h_t; and b is a sequence of functions $(b_0, b_1, \ldots)$, where, for each $h_t \in H_t$ and feasible f_t, b_t selects c_t and p_t feasible for $g(K_t)$ and $(D_t - f_t)$. The strategies a and b form an equilibrium of $G(K_0, D_0)$ if a unilateral switch to any other strategy by either player does not yield that player increased utility. As is well-known by now, some of the equilibria in games like $G(K_0, D_0)$ can be based on implausible threats. Subgame-perfection is imposed to rule these equilibria out. For $t \geq 1$, any $h_t \in H_t$ generates a new game $G_A(K_0, D_0, h_t)$ initiated by the K_t and D_t that result from h_t. Similarly, a and b generate strategies a^t and b^t for $G_A(K_0, D_0, h_t)$ as follows: delete the first (t-1) component functions of a and b, then restrict the domains of a_τ and b_τ for all $\tau \geq t$ to begin with h_t. Similarly, for all t, h_t followed by feasible f_t generates another kind of subgame, call it $G_B(K_0, D_0, h_t, f_t)$ in which B moves first after the initial condition $(K_t, D_t - f_t)$ determined by h_t and f_t. Strategies induced by a and b for $G_B(K_0, D_0, h_t, f_t)$ are defined similarly to those for $G_A(K_0, D_0, h_t)$. The strategies a and b form a subgame-perfect equilibrium of the game $G(K_0, D_0)$ if they form an equilibrium of $G(K_0, D_0)$ and, in addition, if they induce equilibria on all subgames $G_A(K_0, D_0, h_t)$ generated by all $h_t \in H_t$ and on all subgames $G_B(K_0, D_0, h_t, f_t)$ generated by each $h_t \in H_t$ followed by each feasible f_t.

Proposition: If $\alpha = (1+r)^{-1}$ there is a subgame-perfect equilibrium for $G(K_0, D_0)$ for any $(K_0, D_0) \geq 0$, the play of which has the following properties: All debt-forgiveness (if any) occurs at time 0. The debt (possibly reduced) together with accrued interest is repaid at some $T<\infty$. The players' utilities at the equilibrium play are Pareto-efficient, with the creditor receiving all the surplus over the debtor's maximin payoff, which is either $v(K_0)$ or the maximum of $v(K_0)$ and whatever the debtor can obtain by repaying all the original debt with appropriate interest in the event that this is a feasible plan. Furthermore, all other subgame-perfect equilibria generate the same payoffs as this one for both players.

3. Proof of Proposition

For each $K\geq 0$, let $R(K)$ denote the set of debts $D>0$ such that eventual repayment of D plus accrued interest is feasible starting from (K,D), assuming no future forgivenesses, and let $W = \{(K,D): K\geq 0 \text{ and } D\in R(K)\}$. For any $(K,D)\in W$, let $w(K,D)$ denote the payoff to B from pursuing an optimal program to repay all of D, together with appropriately accrued interest, starting from (K,D). This growth-with-debt model having objective function w will be termed GDM(K,D).

Lemma 1: For every $K\geq 0$, $R(K)$ is either empty or an open interval. Throughout W, the function w exists, increases in its first argument, and decreases continuously in its second.

Proof: If $K=0$ and $g(0)=0$, then $R(K)$ is empty. Otherwise, $g(K)\in R(K)$ and if $D\in R(K)$, then $(0,D)\subset R(K)$. To see that $\sup(R(K))\notin R(K)$, consider the value $\bar{K}$ such that $g'(\bar{K})=(1+r)$. Above $\bar{K}$, marginal productivity is less than $(1+r)$, so that an efficient payback plan (with no consumption) involves using exactly $\bar{K}$ as input to production. Hence, if $D\geq g(K)\geq\bar{K}$ and if $r(D-g(K)+\bar{K})\geq g(\bar{K})-\bar{K}$, then B can never reduce the debt after time 0. On the other hand, if

$r(D-g(K)+\bar{K}) < g(\bar{K})-\bar{K}$, the debt can be reduced by an increasing amount in each period and hence repaid; therefore R(K) is open when $g(K)\geq\bar{K}$. If $g(K)<\bar{K}$ efficiency requires using all of K in production. Under our assumptions, however, after finitely many iterations (say τ) of this, $g^{\tau+1}(K)\geq\bar{K}$. At this point, a test similar to the one above applies, with the left side of the inequalities replaced by $r((1+r)^{\tau}D-g^{\tau+1}(K)+\bar{K})$, so that the same conclusion, that R(K) is open, follows.

To see that w increases with K, it is sufficient to notice that the additional units of K can simply be consumed immediately, then the optimal program for GDM(K,D) followed as before, resulting in increased utility. Similarly, if D is reduced, the optimal program for GDM(K,D) together with additional units of K reinvested each time and available at T results in increased utility.

For the existence and continuity properties, first fix any values for τ and K. Now note that the set of feasible $(c_0,p_0,\ldots,c_{\tau},p_{\tau})$ that result in repayment by τ is a compact-valued continuous correspondence over R(K). (If repayment occurs at $T<\tau$, set $c_{T+1},p_{T+1},\ldots,c_{\tau},p_{\tau}$ equal to zero.) Since (4) is continuous when viewed as a function of $(D_0,c_0,p_0,\ldots,c_T,p_T)$, the maximum theorem applies, so that w_{τ} (w restricted to programs that repay by τ) exists and is continuous as a function of D. As above, the set of D's feasible for w_{τ} is an interval (though closed on the right now), and w_{τ} decreases on this interval to the value

$$u(0)\sum_{t=0}^{\tau}\beta^{t} + \beta^{\tau+1}Z(0). \qquad (5)$$

Now extend w_{τ} continuously to all of R(K) in steps: first by setting it equal to the expression in (5) until $w_{\tau+1}$ crosses this constant from above, then letting w_τ track $w_{\tau+1}$ (which eventually tracks $w_{\tau+2}$, etc.) thereafter. Finally, w exists and is continuous (in its second argument) on R(K), since w is just the maximum of the (w_τ).||

Lemma 2: If $(K,D)\in W$, let $(\tilde{K},\tilde{D})$ denote next period's capital and debt values, respectively, when following the optimal (repayment) program for GDM(K,D) and let K* be next period's capital when following the optimal (nonrepayment) program for GM(K). Then, $w(\tilde{K},\tilde{D})-v(\tilde{K})\geq\beta^{-1}(w(K,D)-v(K))$; and $v(K^*)-w(K^*,D(1+r))\geq\beta^{-1}(v(K)-w(K,D))$. In particular, $w(K,D)\geq v(K)$ implies $w(\tilde{K},\tilde{D})\geq v(\tilde{K})$; and $v(K)>w(K,D)$ implies $v(K^*)>w(K^*,D(1+r))$.

Proof: Let $\tilde{c}$ denote optimal current consumption when following an optimal program for GDM(K,D). Then $w(K,D)-v(K) \leq u(\tilde{c})+\beta w(\tilde{K},\tilde{D}) - (u(\tilde{c})+\beta v(\tilde{K}))$. Similarly for the other case.||

Lemma 3: For all $K\geq 0$, if either $D\notin R(K)$, or $D\in R(K)$ and $w(K,D)<v(K)$, then there is a unique value $\bar{f}$ for f such that $w(K,D-\bar{f})=v(K)$.

Proof: If K=0 and g(0)=0, then $\bar{f}=D$, since Z(0)=v(0). Otherwise, for δ sufficiently small, $\delta\in R(K)$ and $\lim_{\delta\to 0}w(K,\delta) \geq u(c^*)+\beta Z(K^*) \geq u(c^*)+\beta v(K^*) = v(K)$, where c* and K* are current consumption and next-period's capital, respectively, under the optimal program for GM(K). The result now follows from Lemma 1 and the fact that $\inf_{D\in R(K)}w(K,D) = (1-\beta)^{-1}u(0) < v(K)$.||

We are now ready to specify subgame-perfect-equilibrium strategies a and b for all possible initial conditions. For Player A: at each A-move, if $D\notin R(K)$ or if $D\in R(K)$ and $w(K,D)<v(K)$, set $f=\bar{f}$ defined by $w(K,D-\bar{f})=v(K)$ (see Lemma 3); otherwise set f=0. For Player B, at each B-move if $w(K,D-f)<v(K)$, make no payment and proceed according to the (unique) optimal program for GM(K); otherwise, proceed according to any optimal program for GDM(K,D-f).

From Lemma 2, it is apparent that if f_t is zero at ($\underline{a}$,$\underline{b}$) for all $t\geq 1$, B's optimal program remains in the GDM regime; hence, there is at most one forgiveness along the play determined by ($\underline{a}$,$\underline{b}$), and that occurs at time 0. It is also clear that at ($\underline{a}$,$\underline{b}$) the debtor receives the payoff $v(K_0)$, unless he can repay all of D_0 with interest and do better than $v(K_0)$ thereby; and the creditor receives as much as possible subject to the constraint that the debtor receive $\max(v(K_0),w(K_0,D_0))$.

Lemma 4: The strategies $\underline{a}$ and $\underline{b}$ form an equilibrium for any nonnegative initial values of K_0 and D_0.

Proof: We must show that the respective strategies are best responses to each other. Given $\underline{b}$, Player A can improve only if he can induce some payment stream with present value higher than $(D_0-\bar{f})$ ($\bar{f}$ defined, as above, relative to D_0,K_0), which can only be possible if $f_0<\bar{f}$ and for some t there is some other f' with the properties that $(1+r)^tD_0-f' > (1+r)^t(D_0-\bar{f})$ and that after operating according to the optimal program for $GM(K_0)$ for the first (t-1) periods, B does at least as well to switch to a repayment strategy at period t after seeing f'. But

$$\sum_{s=0}^{t}\beta^s u(c_s^*)+\beta^{t+1}v(K_{t+1}^*) = v(K_0) = w(K_0,D_0-\bar{f}) \geq \Sigma\beta^s u(c_s^*)+\beta^{t+1}w(K_{t+1}^*,(1+r)^t(D_0-\bar{f}))$$

$$> \Sigma\beta^s u(c_s^*)+\beta^{t+1}w(K_{t+1}^*,(1+r)^tD_0-f'); \text{ hence } w(K_{t+1}^*,(1+r)^tD_0-f')<v(K_{t+1}^*),$$

a contradiction. Given $\underline{a}$, Player B can gain only if, by deviating, a future forgiveness is induced which leaves B better off. This is also impossible, however, since B is made indifferent to some GM program after a deviation; and the total payoff, from the deviation on, cannot exceed what B could have obtained by following the optimal GM program, which is just what is generated by $\underline{a}$ and $\underline{b}$.||

From Lemma 4 we can deduce that the equilibrium ($\underline{a}$,$\underline{b}$) is also subgame perfect. The G_A-subgames are all instances of the game G with various initial conditions; but ($\underline{a}$,$\underline{b}$) forms an equilibrium in all such games. While the G_B-subgames are not instances of the game G, the argument that the strategies form equilibria for G_B-games is exactly the same as in Lemma 4.

We come finally to the payoff-uniqueness issue. Observe that at any subgame-perfect equilibrium, B must receive a payoff of at least max $(\{v(K_t), w(K_t,D_t)\})$ in every A-subgame, where (K_t,D_t) is determined by h_t, since B has a strategy that guarantees this payoff regardless of A's strategy in the subgame. (Technically, this is inaccurate, since A can force a repayment earlier than planned by forgiving all the debt. Obviously, such a move cannot be part of a subgame-perfect equilibrium, however.) Now, suppose that at some subgame-perfect-equilibrium strategy combination $(\hat{a},\hat{b})$, B receives more than this maximum in some A-subgame. Let $q=\sup(w(K,D-f)-v(K))$, where the supremum is taken over all B-subgames which follow immediately after an $f>0$ determined by $\hat{a}$, and where K and D result from the history leading to the A-subgame. Now $q>0$, since under the strategy combination $(\hat{a},\hat{b})$ there is an A-subgame in which A forgives more under $\hat{a}$ than under $\underline{a}$.

Lemma 5: Suppose f, K, and D are such that $(w(K,D-f)-v(K))>\beta q$ at the beginning of some B-subgame. Then, along the equilibrium path of that subgame, $\hat{b}$ is identical to $\underline{b}$ and $\hat{a}$ makes no further forgivenesses.

Proof: If $\hat{b}$ imitates $\underline{b}$ at the beginning of the subgame, then at $(\tilde{K},\tilde{D})$, by Lemma 2, $w(\tilde{K},\tilde{D})-v(\tilde{K})>\beta^{-1}\beta q$; so a positive forgiveness by A at the next move contradicts the definition of q. If B continues according to $\hat{b}=\underline{b}$, the same reasoning leads to the same no-forgiveness conclusion. On the other hand, if $\hat{b}$ is not identical to $\underline{b}$, either $\hat{b}$ repays D immediately (in which case $\hat{b}$ is inferior to $\underline{b}$), or B's payoff cannot exceed $v(K)+\beta q$ (from Lemma 2 and the definition of q), which is less than B's worst payoff from following $\underline{b}$ (another contradiction).||

Now select any A-subgame at which strategy $\hat{a}$ selects $f>0$ and at which $w(K,D-f)-v(K)>\beta q$ (there must be at least one such subgame). Let A deviate from $\hat{a}$ by reducing f by an amount small enough that the last inequality continues to hold, and then by following $\hat{a}$ thereafter. By Lemma 5, $\hat{b}$ repays with no further forgivenesses; so the deviation is profitable, and $\hat{a}$ is not therefore a best response to $\hat{b}$ in this subgame. This contradiction establishes that every subgame-perfect equilibrium generates the same payoffs as $(\underline{a},\underline{b})$ in every subgame.

4. <u>About Uniqueness</u>

In this section, we indicate why, even with the additional assumption that Z is smooth and strictly concave, the uniqueness of the subgame-perfect-equilibrium strategies, as opposed to just uniqueness of the corresponding payoffs, is too much to expect without still more strong assumptions. First, a partial example to illustrate the point that at a subgame where w equals v it is possible that $\tilde{c}=c^*$ and $\tilde{p}=0$ (with the obvious notation), so that if a forgiveness were required to bring about the equality between w and v, then delaying that forgiveness by one period (adjusting for interest) would make no difference to either player yet still result in a different equilibrium. To produce this effect consider a utility function that is nearly linear, a production function that is steep enough (for small K) and a K small enough that $\beta g'(g(K))>>1$, $g'(g(K))>(1+r)$, $D>g(K)$, and $Z(\cdot)\tilde{=}v(\cdot)$. The first inequality guarantees that all consumption will be postponed under v (and therefore under w given the last condition), and the second and third inequalities guarantee that no payments will take place in the initial period under w.

Next, observe that even with Z strictly concave, there is no reason to expect w to be concave in K; hence, B may have multiple (subgame-perfect) equilibrium best responses to A's strategy $\underline{a}$. For a concrete example of this, it is easiest if we first relax some of the assumptions. Suppose $D_0=1$, $K_0=.5$, $r=.5$, and $\beta=.1$. Suppose $u(c)=.1(1-e^{-c})$, $g(K)=2K$, and

$$Z(K) = \begin{cases} 10 + 180K & \text{if } K\le .5 \\ 100 + u(K) - u(.5) & \text{if } K>.5. \end{cases}$$

Now, for every K, $v(K) \le (1-\beta)^{-1}u(\infty) = 1/9$. Also, $v'(0)=u'(0)=.1$, and $Z'(0)=180$. Clearly, $Z(K)>v(K)\ \forall K$. Repaying D_0 at t=0, B receives $.1Z(0)=1$. In order to repay optimally at time 1, p_0 must be 0, since $g'>(1+r)$. If $c_0=0$ as well, B's payoff is

$$.1 \max_{0\le c_1\le .5} (u(c_1) + .1Z(.5-c_1))$$

It is straightforward to check that the maximum is attained at $c_1=0$, with payoff $.01Z(.5)=1$. Furthermore, for any $c_0\in[0,1]$, the analogous maximum is <1. Repaying at time $t\ge 2$ generates $[u(c_0)+.1u(c_1)+\ldots+.1^t u(c_t)]+.1^{t+1}Z(K_{t+1})$. The expression in brackets is no larger than 1/9, while the last term is no larger than .1001, the sum being therefore less than unity. Hence $\underline{b}$ may call for either $c_0=0$ or 1 at such a position in the game.

To modify the example so that it satisfies all our assumptions: first replace Z with an increasing, strictly-concave function that is zero at K=0 and 100 at K=.5, is very steep initially, and has slope 180 at K=.5. Reduce D_0 slightly so that $Z(1-D_0)=10$. Now approximate g by a C^2-strictly-concave function that agrees with g at K=.5, satisfies $g'(0)=\infty$ and $g'(\infty)=0$, and has slope close to 2 on the interval [.5,1]. Finally, adjust r so that the product $D_0(1+r)$ is as before.

5. Remarks and Conclusions

The driving forces behind our results, as in other bargaining models requiring subgame-perfect equilibria, are difficult to identify. A possible explanation may be thought to lie in the argument that the debtor cannot credibly refuse to repay a debt if in so doing it is not made worse off than by repudiating it. A symmetric argument, however, should then convince one that the bank must forgive the entire debt since, by similar reasoning, the bank cannot credibly refuse to accept repayment of any positive amount. If the situation is likened to a bilateral monopoly modeled as a noncooperative game in which a sole seller (the creditor) faces a sole buyer (the debtor) and bargains over the price of a good (the bonus), the outcome is generally sensitive to the particular specifications of the institutional structure in which it is embedded. As demonstrated by Rubinstein in a pure iterated bargaining model, the existence of fixed bargaining costs per period borne by each player yields results on the division of the surplus that depend crucially on the relative magnitudes of each player's respective bargaining costs and on the order of the opportunities to make offers. When the assumption of fixed costs is replaced by fixed discount factors, the conclusions as to the way in which the surplus is divided are different and less extreme, but still give a relative advantage to the player who moves first. In our model, on the other hand, where economic actions take place alongside the negotiations, the order in which the players move is not particularly important. (If the order is reversed, B's initial move is determined by $v(K_0)$ and A grants a forgiveness, $\overline{f}$, such that $w(K_1,(1+r)(D_0-\overline{f}))=v(K_1)$.) It seems to be the real-time nature of the bargaining process that is at least partly responsible for the extreme nature of our conclusions, in contrast with Rubinstein's.

The equilibrium behavior described in Section 3 seems, at first glance, to be in conflict with events surrounding the current LDC-debt crisis, where there have apparently been no forgivenesses. It is possible, however, to interpret the widespread granting of new loans, extended to permit countries to keep interest payments on the debt current, as forgivenesses, since the interest rates on the new loans are often lower than on the old. The occurrence of repeated rescheduling of debts, however, most probably testifies to the fact that we have omitted some features that are important in the current crisis, especially default penalties and uncertainty. When incorporated into our model, the latter could account for repeated rescheduling, since uncertainty, say, as to the productivity of next period's capital stock, might create an incentive for the bank to reduce the amount of a forgiveness, with the intention of readjusting it upward later if need be. Unlucky outcomes stemming from this uncertainty at the same time provide an expanation of the existence of a debt too large to be incentive-compatible with full repayment. The role that a default penalty might play is more complex. The qualitative differences between a final declaration of default on a debt and simply never paying over an infinite horizon bars the possibility of adapting our results in a straightforward way to a model with default penalties. Moreover, the historical evidence on default penalties appears mixed, ranging from countries that have suffered invasion and temporary loss of sovereignty (e.g. Egypt 1879 and Mexico 1859-61) to others that have suffered no apparent cost (see Lindert and Morton (1987)).

The existence of multiple subgame-perfect equilibria provides yet another avenue by which to reconcile our results with observed reality, as indicated in Section 4. If the debtor's moves under both $v(K_0)$ and $W(K_0,D_0-\bar{f})$ coincide for some number of periods commencing with the initial period, forgivenesses (the present discounted value of which must equal the value of $\bar{f}$ determined in the initial period) may occur at any one or a combination of those periods, thus permitting negotiations to extend beyond one period.

It is of interest to speculate on the roles that various of our assumptions play in the analysis. If the creditor's discount factor α were less than both $(1+r)^{-1}$ and β, we would expect the debtor to be able to capture some of the surplus, since the creditor might increase f beyond $\bar{f}$ if doing so enabled it to receive debt repayments which possessed a more favorable repayment schedule. The results for other values of α seem less obvious. Allowing Z(0) to exceed v(0) introduces the possibility that the country could attempt "suicide" (if, in addition, g(0)=0) by consuming all its output; thus credibly committing to no future payments. To forestall this, $\underline{a}$ must be modified to require $\bar{f}$ to satisfy $W(K,D-\bar{f})=\max(v(K),u(g(K))+\beta Z(0))$; otherwise our results remain unaffected by this modification. It should be noted in addition that the qualitative nature of our results does not depend on the particular version of the growth model considered; variants of it, including an extension to a two-sector model and the inclusion of international trade, do not alter our main findings. Lastly, the assumption of an infinite horizon is not crucial—our conclusions can also be obtained in a finite-horizon version of the game (in which a bonus is received only if the debt is repaid by some predetermined period) as long as the debtor cannot avoid moving last.

In addition to uncertainty and default penalties as mentioned earlier, inform tional asymmetries, free-rider problems among the banks, and the relationships among the creditor banks and their governments are other important factors influencing the outcome of LDC-debt negotiations that we have ignored. It seems possible that future research on their effects could be undertaken by elaborating on the basic structure we have utilized.

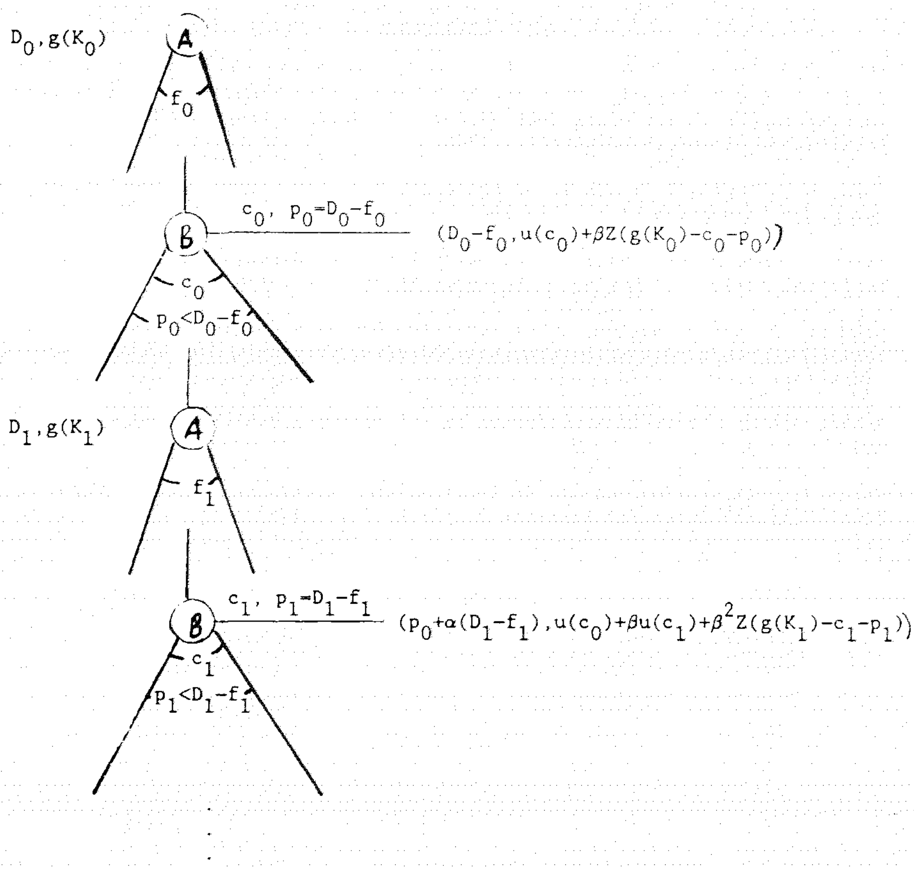
$D_0, g(K_0)$
A
f_0
$c_0,\ p_0 = D_0 - f_0$
B
$(D_0 - f_0, u(c_0) + \beta Z(g(K_0) - c_0 - p_0))$
c_0
$p_0 < D_0 - f_0$
$D_1, g(K_1)$
A
f_1
$c_1,\ p_1 = D_1 - f_1$
B
$(p_0 + \alpha(D_1 - f_1), u(c_0) + \beta u(c_1) + \beta^2 Z(g(K_1) - c_1 - p_1))$
c_1
$p_1 < D_1 - f_1$

FIGURE 1

References

Bulow, J. and K. Rogoff, "A Constant Recontracting Model of Sovereign Debt." Working Papers in Economics E-86-69 Domestic Studies Program, Hoover Institution, Stanford University, 1986.

Cass, D., "Optimal Growth in an Aggregate Model of Capital Accumulation," Review of Economic Studies, *32*, 1965.

Eaton, J. and M. Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," Review of Economic Studies, *48*, 1981.

Lindert, P. and P. Morton, "How Sovereign Debt has Worked," Working Paper Series #45, Institute of Governmental Affairs, University of California-Davis, 1987.

Rubinstein, A. "Perfect Equilibrium in a Bargaining Model," Econometrica, *50*, 1981.

Sachs, J. "Theoretical Issues in International Borrowing," Working Paper No. 1189, National Bureau of Economic Research, Inc., 1983.