NBER WORKING PAPER SERIES

IMPERFECT COMPETITION, COMPENSATING DIFFERENTIALS AND RENT SHARING IN THE U.S. LABOR MARKET

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Working Paper 25954
http://www.nber.org/papers/w25954

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2019

The opinions expressed in this paper are those of the authors alone and do not reflect the views of the Internal Revenue Service, the U.S. Treasury Department, or the National Bureau of Economic Research. This work is a component of a larger project on income risk in the United States, conducted through the SOI Joint Statistical Research Program. We are grateful to Raj Chetty, Nathan Hendren, Danny Yagan and Owen Zidar for help and guidance in using the IRS data. We also appreciate the constructive comments and suggestions from Thomas Lemieux, three anonymous referees, Neele Balke, Stephane Bonhomme, Derek Neal, and discussants and participants at various conferences and seminars. Mogstad and Setzler acknowledge funding from NSF Grant SES-1851808, the Washington Center for Equitable Growth, and the Norwegian Research Council.

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NBER Working Paper No. 25954
June 2019
JEL No. J20,J30,J42

ABSTRACT

We quantify the importance of imperfect competition in the U.S. labor market by estimating the size of rents earned by American firms and workers from ongoing employment relationships. To this end, we construct a matched employer-employee panel data set by combining the universe of U.S. business and worker tax records for the period 2001-2015. Using this panel data, we identify and estimate an equilibrium model of the labor market with two-sided heterogeneity where workers view firms as imperfect substitutes because of heterogeneous preferences over non-wage job characteristics. The model allows us to draw inference about imperfect competition, compensating differentials and rent sharing. We also use the model to quantify the relevance of non-wage job characteristics and imperfect competition for inequality and tax policy, to assess the economic determinants of worker sorting, and to offer a unifying explanation of key empirical features of the U.S. labor market.

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A data appendix is available at http://www.nber.org/data-appendix/w25954

1 Introduction

How pervasive is imperfect competition in the labor market? Arguably, this question is really about the size of rents earned by employers and workers from ongoing employment relationships (Manning, 2011). In the textbook model of a competitive labor market, the law of one price holds and there should exist a single market compensation for a given quality of a worker, no matter which employer she works for. If labor markets are imperfectly competitive, however, the employer or worker or both may also earn rents from an employment relationship. If a worker gets rents, the loss of the current job makes the worker worse off—an identical job cannot be found at zero cost. If an employer gets rents, the employer will be worse off if a worker leaves—the marginal product is above the wage and worker replacement is costly.

To draw inference about imperfect competition in the labor market, it therefore seems natural to measure the size of rents earned by employers and workers. However, these rents are not directly observed, and recovering them from data has proven difficult for several reasons. One challenge is that observationally equivalent workers could be paid differentially because of unobserved skill differences, not imperfect competition (see, e.g., Abowd et al., 1999; Gibbons et al., 2005). Another challenge is that observed wages may not necessarily reflect the full compensation that individuals receive from working in a given firm. Indeed, both survey data (e.g., Hamermesh, 1999; Pierce, 2001; Maestas et al., 2018) and experimental studies (e.g., Mas and Pallais, 2017; Wiswall and Zafar, 2017; Chen et al., 2020) suggest that workers may be willing to sacrifice higher wages for better non-wage job characteristics or amenities when choosing an employer. Thus, firm-specific wage premiums could reflect unfavorable amenities, not imperfect competition.

The primary goal of our paper is to address these challenges and quantify the importance of imperfect competition in the U.S. labor market by estimating the size of rents earned by American firms and workers from ongoing employment relationships. To this end, we construct a matched employer-employee panel data set by combining the universe of U.S. business and worker tax records for the period 2001-2015. Using this panel data, we identify and estimate a model of the labor market that allows us to draw inference about imperfect competition, compensating differentials and rent sharing. We also use the model to quantify the relevance of non-wage job characteristics and imperfect competition for inequality and tax policy, to assess the economic determinants of worker sorting, and to offer a unifying explanation of key empirical features of the U.S. labor market.

In Section 2, we develop the equilibrium model of the labor market. This model builds on work by Rosen (1986), Boal and Ransom (1997), Bhaskar et al. (2002), Manning (2003), and Card et al. (2018). Competitive labor market theory requires firms to be wage takers so that labor supply to the individual firm is perfectly elastic. The evidence that idiosyncratic productivity shocks to a firm transmit to the earnings of its workers is at odds with this theory (see, e.g., Guiso et al. 2005). To allow labor supply to be imperfectly elastic, we let employers compete with one another for workers who have heterogeneous preferences over amenities. Since we allow these amenities to be unobserved to the analyst, they can include a wide range of characteristics, such as distance of the firm from the worker's home, flexibility in the work schedules, the type of tasks performed, the effort required to perform these tasks, the social environment in the workplace, and so on.[1]

The importance of workplace amenities has long been recognized in the theory of compensating differentials (Rosen, 1986). This is a theory of vertical differentiation: some employers offer better amenities than others. Employers that offer favorable amenities can attract labor at lower than average wages, whereas employers offering unfavorable amenities need to pay premiums as offsetting compensation in order to attract labor. Our model combines this vertical differentiation with horizontal employer differentiation: workers have different preferences over the same workplace amenities. As a result of this preference heterogeneity, the employer faces an upward sloping supply curve for labor, implying wages are an increasing function of firm size. We assume employers do not observe the idiosyncratic taste for amenities of any given worker. This information asymmetry implies employers cannot price discriminate with respect to workers' reservation values. Instead, if a firm becomes more productive and thus wants to increase its size, the employer needs to offer higher wages to all workers of a given type. As a result, the equilibrium allocation of workers to firms creates surplus or rents to inframarginal workers.

The size of rents depends on the slope of the labor supply curve facing the firm. The steeper the labor supply curve, the more important amenities are for workers' choices of firms as compared to wages. Therefore, imperfect competition as measured by rents increases in the progressivity of labor income taxes and in the variability of the idiosyncratic taste for amenities. However, the existence of rents does not imply the equilibrium allocation of workers to firms is inefficient. In our model, the market allocation will be inefficient if the firms differ in wage-setting power, and, as a result, differ in the extent to which they mark down wages relative to the marginal product. To allow for such differences, we let workers view firms as closer substitutes in some markets than others. This structure on the workers' preferences captures that workplace characteristics are likely to vary systematically across firms depending on location and industry.

In Section 3, we describe the business and worker tax records, which provides us with panel data on the outcomes and characteristics of U.S. firms and workers. The firm data contain information on revenues and expenditures on intermediate inputs as well as industry codes and geographical identifiers. We merge the firm data set with worker tax records, creating the matched employer-employee panel data. The key variables we draw from worker tax returns are the number of employees and their annual earnings at each employer.

In Section 4, we demonstrate how the model is identified from the data. To increase our confidence in the empirical findings from the model, we allow for rich unobserved heterogeneity across workers with respect to preferences and productivity and between firms in terms of technology and amenities. Even so, it is possible to prove identification of the parameters of

[1] There is limited empirical evidence on which non-wage characteristics matter the most. However, survey data from Maestas et al. (2018) point to the importance of flexibility in work schedules, the type of tasks performed, and the amount of effort required. The analysis of Marinescu and Rathelot (2018) suggests distance of the firm from the workers' home may be important. Chen et al. (2020) use field experiments to estimate high willingness to pay for flexibility in work schedules.

interest given the panel data of workers and firms. For example, the rents earned by workers can be measured given data on earnings and the elasticity of the labor supply curve specific to the firm. These elasticities can be recovered from estimates of the pass-through of firm shocks to incumbent workers' earnings. As another example, the correlation structure in a worker's tastes for the amenities of firms in the same market can be identified by comparing estimates of the pass-through rates of shocks specific to the firm versus common to the market. Estimates of worker effects, firm effects and worker sorting allow us to recover the productivity of workers, the compensating differentials due to the vertical differentiation of firms, and the extent to which preferences for amenities vary by worker productivity. To determine whether productive workers and firms are complements in production, we take advantage of the estimated interaction coefficients between worker and firm effects recovered from changes in earnings when workers move between employers.

The model yields four key findings that we discuss in Section 6. First, there is a significant amount of rents and imperfect competition in the U.S. labor market due to horizontal employer differentiation. Workers are, on average, willing to pay 13 percent of their wages to stay in the current jobs. Comparing these worker rents to those earned by employers suggests that total rents are divided relatively equally between firms and workers. Second, the evidence of small firm effects does not imply that labor markets are competitive or that rents are negligible. Instead, firm effects are small because productive firms tend to have good amenities, which pushes down the wages that these firms have to pay. As a result of these compensating differentials, firms contribute much less to earnings inequality than what is predicted by the variance of firm productivity only. Third, a key reason why better workers are sorting into better firms is production complementarities, not heterogeneous tastes for workplace amenities. These complementarities are important to explain the significant inequality contribution from worker sorting. Fourth, the monopsonistic labor market creates significant misallocation of workers to firms. We estimate that a tax reform which would eliminate labor and tax wedges would increase total welfare by 5 percent and total output by 3 percent.

The insights from our paper contribute to a large and growing literature on firms and labor market inequality, reviewed by Card et al. (2018). A number of studies show that trends in wage dispersion closely track trends in productivity dispersion across industries and workplaces (Faggio et al., 2010; Dunne et al., 2004; Barth et al., 2016). While this correlation might reflect that some of the productivity differences across firms spill over to wages, it could also be driven by changes in the degree to which workers of different quality sort into different firms (see, e.g., Murphy and Topel, 1990; Gibbons and Katz, 1992; Abowd et al., 1999; Gibbons et al., 2005). To address the sorting issue, a growing body of work has taken advantage of matched employer-employee data. Some studies use this data to estimate the pass-through of changes in the value added of a firm to the wages of its workers, while controlling for time-invariant firm and worker heterogeneity.[2] These studies typically report estimates of pass-through rates

[2]See, e.g., Guiso et al. (2005), Card et al. (2013), Card et al., 2018, Carlsson et al. (2016), Balke and Lamadon (2020), and Friedrich et al. (2019). A concern with this approach is that measures of firm productivity may reflect a number of factors. Some studies have therefore examined the pass-through of specific, observable changes. For example, Van Reenen (1996) studies how innovation affects firms' profit and workers' wages. He also investigates

in the range of 0.05-0.20. We complement this work by providing evidence of the pass-through rates for a broad set of firms in the U.S. with a variety of empirical approaches, and by showing how the estimated pass-through of firm and market level shocks can be used to draw inferences about imperfect competition, rents, and allocative inefficiency.

Another set of studies use the matched employer-employee data to estimate the additive worker and firm effects wage model proposed by Abowd et al. (1999).[3] We complement this work by extending the Abowd et al. (1999) model to allow for both firm-worker interactions and time-varying firm effects, which enable us to economically interpret the firm effects in terms of rents and compensating differentials, understand the sources of worker sorting, and clarify the contribution of firm productivity shocks to earnings inequality.

Our paper also relates to a literature that tries to measure the role of compensating differentials for wage-setting and earnings inequality. This literature is reviewed in Taber and Vejlin (2020) and Sorkin (2018). Much of the existing evidence comes from hedonic regressions of earnings on one or more observable non-wage characteristics of jobs, employers, or industries, interpreting the regression coefficients as the market prices of those amenities. Typical estimates of these coefficients are small in magnitude and sometimes of the wrong sign (see the discussion by Bonhomme and Jolivet, 2009). These estimates could be severely biased, either due to correlations between observed amenities and unobserved firm characteristics or because of assortative matching (on unobservables) between workers and firms (see, e.g., the discussion by Ekeland et al., 2004). Several recent studies have used panel data in an attempt to address these concerns. Like us, Taber and Vejlin (2020), Lavetti and Schmutte (2017), and Sorkin (2018) take advantage of matched longitudinal employer-employee data to allow for unobserved heterogeneity across firms.

Our paper differs from the existing literature on compensating differentials in several ways. One important difference is that amenities, in our model, create both vertical and horizontal employer differentiation. The latter generates imperfect competition, wage-setting power and rents; the former acts as standard compensating differentials. By comparison, compensating differentials have typically been analyzed in models with perfect competition or search frictions (see, e.g., Mortensen, 2003). Our paper also allows for ex-ante worker heterogeneity in productivity and preferences which generates sorting between firms and workers, in contrast to, for example, Sorkin (2018). Our estimates suggest that worker heterogeneity and sorting are empirically important features of the U.S. labor market which are necessary to take into account to understand the determinants of earnings inequality. By taking our model to the data, we are

patents as a source of variation, but finds them to be weakly correlated with profits. Building on this insight, Kline et al. (2019) studies the incidence of patents that are predicted to be valuable. See also their correction of the reported findings (Kline et al., 2021). A related literature has examined the wage and productivity effects of adoption of new technology in firms (see Akerman et al., 2015, and the references therein).

[3] Song et al. (2018) and Sorkin (2018) provide estimates using the approach of Abowd et al. (1999) for the U.S. A recent literature addresses the concern that estimates of firm effects will be biased upward and estimates of worker sorting will be biased downward when using the approach of Abowd et al. (1999) due to limited worker mobility across firms. Our main estimates use the bias-correction approach of Bonhomme et al. (2019) while alternative bias-correction approaches by Andrews et al. (2008) and Kline et al. (2020) are considered in our Online Supplement. See Bonhomme et al. (2020) for a comparison of bias-correction procedures using data from several countries.

able to quantify the relative importance of amenities versus production complementarities for worker sorting and earnings inequality. Lastly, our paper differs in that we move beyond the impact of amenities on wages and worker sorting, examining also the implications for tax policy and allocative efficiency. In our model, wages are taxed but amenities are not. Thus, progressive taxation on labor income may distort the worker's decision of which firm and market to work in. We analyze, theoretically and empirically, the consequences of this distortion and how changes in the tax system may help improve the allocation of workers to firms.[4]

2 Model of the labor market

This section develops an equilibrium model of the labor market. We begin by describing the primitives of the model, including the heterogeneous preferences and productivity of the workers and the heterogeneous technology and non-wage characteristics of the firms. Once the primitives are described, we define the environment, derive the labor supply and demand functions, and show that there exists a unique equilibrium. Next, we discuss the sorting of workers to firms, before deriving the key structural equations to be taken to the data. Lastly, we show the mapping between these equations and the key economic quantities of interest, including rents, compensating differentials, and sources of allocative inefficiency.

2.1 Agents, preferences and technology

The economy is composed of a large number of workers indexed by i and a large set of firms indexed by $j = 1, ..., J$. Each firm belongs to a market $r(j)$. Let J_r denote the set of firms in market r. We will rely on the approximation that firms employ many workers and that each market has many firms. For tractability, we assume that workers, firms and markets face exogenous birth-death processes which ensure stationarity in the productivity distributions of workers, firms and markets.

Worker productivity and preferences

Workers are heterogeneous both in preferences and productivity. Workers are characterized by a permanent skill level X_i. In period t, worker i with skill X_i has the following preferences over alternative firms j and earnings W:

$$u_{it}(j, W) = \log \tau W^{\lambda} + \log G_j(X_i) + \beta^{-1}\epsilon_{ijt}$$

where $G_j(X)$ denotes the value that workers of quality X are expected to get from the amenities that firm j offers, and ϵ_{ijt} denotes worker i's idiosyncratic taste for the amenities of firm j. The parameters (τ, λ) describe the tax function that maps wages to income available for consumption. Subection 5.3 shows that this parsimonious tax function well-approximates the US tax system.

[4]Tax theory in the Mirrlees (1971) tradition generally assumes the labor markets are perfectly competitive. A notable exception is Cahuc and Laroque (2014) who develop a model for optimal taxation under monopsonistic markets. See also Powell and Shan (2012) and Powell (2012) who argue that marginal tax rates distort the relative value of amenities to wages. There is also a literature that considers tax design in situations with search frictions. See Yazici and Sleet (2017) and the references therein.

This specification of preferences allows for the possibility that workers view firms as imperfect substitutes. Fixing worker quality X, the preference term $G_j(X)$ gives rise to *vertical* employer differentiation: some employers offer good amenities while other employers have bad amenities. Our preference specification combines this vertical differentiation with *horizontal* employer differentiation: workers are heterogeneous in their preferences over the same firm. This horizontal differentiation has two distinct sources. The first is that $G_j(X)$ varies freely across values of X. Thus, we permit systematic heterogeneity in the preferences for a given firm depending on the permanent component of worker productivity. The second is the idiosyncratic taste component $\beta^{-1}\epsilon_{ijt}$. The importance of this second source of horizontal differentiation is governed by the parameter β. As β becomes smaller, $\beta^{-1}\epsilon_{ijt}$ becomes more dispersed and thus horizontal differentiation becomes more important in determining the worker's preferred firm.

We assume that $(\epsilon_{i1t}, ..., \epsilon_{iJt}) \equiv \vec{\epsilon}_{it} \sim \Xi(\vec{\epsilon}|\vec{\epsilon}_{it-1}, X_i)$ follows a Markov process with independent innovations across individuals. This assumption does not imply strong restrictions on the copula of workers' skills and preferences over time (and, by extension, the patterns of mobility across firms by worker quality). We assume, however, that the (cross-sectional) distribution of $\vec{\epsilon}_{it}$ has a nested logit structure in each period:

$$F(\vec{\epsilon}_{it}) = \exp\left[-\sum_r \left(\sum_{j\in J_r} e^{-\frac{\epsilon_{ijt}}{\rho_r}}\right)^{\rho_r}\right].$$

This structure allows the preferences of a given worker to be correlated across alternatives within each nest. In the empirical analysis, we specify the nest as the combination of industry and region, and refer to it as a market. The parameter ρ_r measures the degree of independence in a worker's taste for the alternative firms within market r, i.e. $\rho_r = \sqrt{1 - corr(\epsilon_{ijt}, \epsilon_{ij't})}$ if $r(j) = r(j') = r$. Thus, $\rho_r = 0$ if each worker views firms within the same market as perfect substitutes, while $\rho_r = 1$ if the worker views these firms as completely independent alternatives.

Firm productivity and technology

We let firms differ not only in workplace amenities but also in terms of productivity and technology. We start by introducing the total efficiency units of labor at the firm:

$$L_{jt} = \int X^{\theta_j} \cdot D_{jt}(X)\mathrm{d}X,$$

where X^{θ_j} tells us the efficiency of a worker of quality X in firm j. The component $D_{jt}(X)$ is the mass of workers with productivity X demanded by the firm.

The value added (revenues minus expenditure on intermediate inputs) Y_{jt} generated by firm j in period t is determined by the production function

$$Y_{jt} = A_{jt}L_{jt}^{1-\alpha_{r(j)}}$$

where A_{jt} is the firm's productivity (TFP) and $1 - \alpha_{r(j)}$ is the firm's returns to scale. The returns to scale depends on the total efficiency units of labor (reflecting both the quality and quantity of labor), and we let it vary freely across markets to allow for differences in technology.

Our specification of the value added production function abstracts from capital, or equivalently, assumes that capital can be rented at some fixed price. However, the specification does not require the product market to be competitive. As shown in Online Appendix A.6, it is possible to derive the same specification of the value added production function (and, by extension, labor demand) if firms have price-setting power in the product market.

It is useful to express the productivity component A_{jt} as:

$$A_{jt} \quad = \quad \overline{A}_{r(j)t}\tilde{A}_{jt} = \overline{P}_{r(j)}\overline{Z}_{r(j)t}\tilde{P}_j\tilde{Z}_{jt}$$

where $\overline{A}_{r(j)t}$, $\overline{P}_{r(j)}$, and $\overline{Z}_{r(j)t}$ represent the overall, the permanent and the time-varying components of productivity that are shared by all firms in market r, while $\tilde{A}_{jt}$, $\tilde{P}_j$ and $\tilde{Z}_{jt}$ denote the overall, the permanent and the time-varying components that are specific to firm j. Let $W_{jt}(X)$ denote the wage that firm j offers to workers of quality X in period t and $B_{jt} = \int W_{jt}(X)D_{jt}(X)\mathrm{d}X$ denote the wage bill of the firm, i.e. the total sum of wages paid to its workers. The profit of the firm is then given by $\Pi_{jt} = Y_{jt} - B_{jt}$.

2.2 Information, wages and equilibrium

We consider an environment where all labor is hired in a spot market and ϵ_{ijt} is private information to the worker. Hence, the wage may depend on the worker's attributes X, but not her value of ϵ_{ijt}. Given the set of offered wages $\mathbf{W}_t = \{W_{jt}(X)\}_{j=1,\ldots,J}$ by all firms, worker i chooses a firm j to maximize her utility u_{it} in each period:

$$j(i,t) \equiv \arg\max_j u_{it}(j, W_{jt}(X_i)). \tag{1}$$

We introduce a wage index at the level of the market r defined by:

$$I_{rt}(X) \equiv \left(\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} W_{j't}(X) \right)^{\frac{\lambda\beta}{\rho_r}} \right)^{\frac{\rho_r}{\lambda\beta}} \tag{2}$$

from which we can derive the probability that an individual of type X chooses to work at firm j given all offered wages in the economy:

$$Pr[j(i,t){=}j|X_i{=}X, \mathbf{W}_t] = \frac{I_{r(j)t}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}} \left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \frac{W_{jt}(X)}{I_{r(j)t}(X)} \right)^{\frac{\lambda\beta}{\rho_{r(j)}}}.$$

We consider an equilibrium where the firm views itself as infinitesimal within the market.[5] Thus, given the total mass of workers N and the stationary cross-sectional distributions of X, $M(X)$, employer j considers the following firm-specific labor supply curve when setting wages $W_{jt}(X)$:

$$S_{jt}(X, W) \equiv NM(X) \frac{I_{r(j)t}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}} \left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \frac{W}{I_{r(j)t}(X)} \right)^{\frac{\lambda\beta}{\rho_{r(j)}}}.$$

[5]See Berger et al. (2019) for an analysis of strategic interactions in the firms' wage setting. See also Jarosch et al. (2019), who develop a search framework with large firms. However, identification is difficult in models with strategic behavior in the wage-setting.

This means the firm ignores the negligible effect of changing its own wages on the market level wage index $I_{rt}(X)$. Then each firm chooses labor demand $D_{jt}(X)$ by setting wages $W_{jt}(X)$ for each type of worker X to maximize profits subject to labor supply $S_{jt}(X, W)$:

$$\Pi_{jt} = \max_{\{W_{jt}(X)\}_X} A_{jt} \left(\int X^{\theta_j} D_{jt}(X) \mathrm{d}X \right)^{1-\alpha_{r(j)}} - \int W_{jt}(X) D_{jt}(X) \mathrm{d}X$$
$$\text{s.t. } D_{jt}(X) = S_{jt}(X, W_{jt}(X)) \quad \text{for all } t, j, X \tag{3}$$

From this environment, the definition of equilibrium naturally follows:

Definition 1. Given firm characteristics $(\alpha_{r(j)}, A_{jt}, \theta_j)_{j,t}$, worker distributions $N, M(\cdot)$, preference parameters $(\beta, \rho_r, G_j(\cdot))$ and tax parameters (λ, τ), we define the equilibrium as the worker decisions $j(i,t)$, market level wage indices $I_{rt}(X)$, firm-specific labor supply curves $S_{jt}(X, W)$, wages $W_{jt}(X)$ and labor demand $D_{jt}(X)$ such that:

i. Workers choose firms that maximize their utility, as defined in equation (1).
ii. Firms choose labor demand $D_{jt}(X)$ by setting wages $W_{jt}(X)$ for each worker quality X to maximize profits subject to the labor supply constraint $S_{jt}(X, W)$, as described in equation (3).
iii. The market level wage indices $I_{rt}(X)$ are generated from the workers' optimal decisions $j(i,t)$, as described in equation (2).

In Lemma 2 in Online Appendix A.1, we show the uniqueness of the equilibrium which proves useful in the estimation of the model and is needed for the counterfactual analyses.

2.3 Sorting in equilibrium

To understand how workers may sort in our model, it is important to note that we do not restrict the relationship between amenities $G_j(X)$, permanent productivity components $(\overline{P}_{r(j)}, \tilde{P}_j)$, and technology $(\theta_j, \alpha_{r(j)})$. As a result, our model permits multiple sources of systematic sorting of worker quality and firm productivity in equilibrium.

One source of sorting is that we allow workers of different quality X to be differentially productive across different firms j. For example, if more productive firms have greater θ in the production function, the marginal product of high quality workers is relatively high at more productive firms, so that worker quality and firm productivity are strong complements in production (i.e. strict log supermodularity, as in Shimer and Smith, 2000 and Eeckhout and Kircher, 2011). Empirically, we will find evidence that more productive firms have greater θ and, therefore, conclude that worker quality is strongly complementary with firm productivity. Thus, firms with high productivity offer relatively high (log) wages to workers with high X, which contributes to a disproportionate employment of high ability workers in productive firms.

A second source of systematic worker sorting is captured by the amenity term $G_j(X)$ in the preference specification. This specification allows the valuation of the amenities of a given firm to vary freely across worker quality X, and it allows the valuation of amenities for a given worker quality X to vary freely across firms. Empirically, we will find that productive firms tend to have better amenities, and that high ability workers tend to value amenities more than low ability workers. This contributes to a disproportionate employment of high quality workers in productive firms.

When assessing the sorting patterns, it is important to observe that our model does not imply that the most productive firms (either in terms of A or θ) hire all workers (in total or of a given quality X) in the economy. One reason for this is we find that the labor supply curve is upward-sloping ($\beta < \infty$), so the marginal cost of labor is increasing in the number of workers. Another reason is that we find that firms face diminishing returns to scale in labor ($1-\alpha_r < 1$), which implies that the marginal product of labor is decreasing in the number of workers.

2.4 Structural equations

As shown in Proposition 1 in Online Appendix A.1, our model delivers the following structural equations for (log of) wages, value added and wage bill of firm $j \in J_r$:

$$w_j(x, \overline{a}, \tilde{a}) = \theta_j x + c_r - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta}\overline{a} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} \tag{4}$$

$$y_j(\overline{a}, \tilde{a}) = (1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} \tag{5}$$

$$b_j(\overline{a}, \tilde{a}) = c_r + (1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} \tag{6}$$

where we use lower case letters to denote logs (e.g., $x \equiv \log X$), c_r is a market-specific constant that is equal to $\log \frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}$, and h_j is the solution to a fixed point equation. As shown in Lemma 3 in Online Appendix A.1, h_j depends on the firm's amenity terms but does not depend on $\tilde{a}$ or $\overline{a}$. These equations describe how the *potential* outcomes of workers and firms are determined, that is, they tell us the realizations of $w_j(x)$, y_j, and b_j that would have been experienced had worker productivity x, firm TFP $\tilde{a}$ and market TFP $\overline{a}$ been exogenously set.

The equations in (4)-(6) show that $w_j(x, \overline{a}, \tilde{a})$, $y_j(\overline{a}, \tilde{a})$, and $b_j(\overline{a}, \tilde{a})$ depend on the same three components: the component of productivity that is specific to the firm $\tilde{a}$, the component of productivity that is common to firms in the same market $\overline{a}$, and an amenity component h_j. In addition, $w_j(x, \overline{a}, \tilde{a})$ depends on the worker's own productivity x. Moreover, workers with the same x who work in different firms can be paid differentially depending on the firm-specific parameter θ_j. As expected, if a firm j becomes more productive ($\tilde{a}$ or $\overline{a}$ increase) then $y_j(\overline{a}, \tilde{a})$ increases. Because firm j has become more productive, it will demand more labor, raising $w_j(x, \overline{a}, \tilde{a})$ and $b_j(\overline{a}, \tilde{a})$.

Combining equations (4)-(6), we obtain a structural equation for the log efficiency units of labor of firm $j \in J_r$:

$$\ell_j(\overline{a}, \tilde{a}) = h_j + \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a} + \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} \tag{7}$$

where h_j (see definition in Lemma 3 in Online Appendix A.1) can be interpreted as the efficiency units of labor the firm would have if $\tilde{a}$ and $\overline{a}$ were exogenously set to zero. The key component of h_j is the vertical differentiation of firms due to the amenities. All else equal, better amenities raise the size of the firm, thus increasing its wage bill and value added. Furthermore, h_j also reflects worker composition, which depends both on the horizontal amenity differentiation of firms, as captured by $G_j(X)$, and on the complementarity in production, as captured by θ_j.

Another important feature of the structural equations (4)-(6) is that they are additive in the arguments $\theta_j x$, h_j, $\overline{a}$, and $\tilde{a}$. This additivity is useful for several reasons. First, it makes it straightforward to quantify the relative importance of the determinants of worker and firm outcomes. Second, it forges a direct link between the structural log wage equation and the log-additive fixed effect models discussed in Section 5.4. This link will help interpret the sources of variation in log earnings through the lens of the model. Third, it facilitates identification of the parameters of the model, as shown in Section 4.

2.5 Rents, compensating differentials, and allocative inefficiencies

We conclude the presentation of the model by showing the mapping between the structural equations and the key economic quantities of interest, including rents, compensating differentials, and sources of allocative inefficiency.

Worker rents

In our model, rents are due to the idiosyncratic taste component ϵ_{ijt} that gives rise to horizontal differentiation of firms, upward sloping labor supply curves, and employer wage-setting power. We assume that employers do not observe the idiosyncratic taste for amenities of any given worker. This information asymmetry implies that firms cannot price-discriminate with respect to workers' reservation wages. As a result, the equilibrium allocation of workers to firms creates surpluses or rents for inframarginal workers, defined as the excess return over that required to change a decision, as in Rosen (1986). In our model, worker rents may exist at both the firm and the market level:

Result 1. *We define the **firm level rents** of worker i, R^w_{it}, as the surplus she derives from being inframarginal at her current choice of firm. Given her equilibrium choice $j(i,t)$, R^w_{it} is implicitly defined by:*

$$u_{it}(j(i,t), W_{j(i,t),t}(X_i) - R^w_{it}) = \max_{j' \neq j(i,t)} u_{it}(j', W_{j',t}(X_i)).$$

As shown in Lemma 4 in Online Appendix A.2, expected worker rents at the firm level are:

$$\mathbb{E}\left[R^w_{it} | j(i,t)=j\right] = \frac{1}{1+\lambda\beta/\rho_{r(j)}} \mathbb{E}\left[W_{jt}(X_i) | j(i,t)=j\right].$$

Result 2. *We define the **market level rents** of worker i, R^{wm}_{it}, as the surplus derived from being inframarginal at her current choice of market. Given her equilibrium choice of market $r(j(i,t))$, R^{wm}_{it} is implicitly defined by:*

$$u_{it}(j(i,t), W_{j(i,t),t}(X_i) - R^{wm}_{it}) = \max_{j' \,|\, r(j') \neq r(j(i,t))} u_{it}(j', W_{j',t}(X_i)).$$

As shown in Lemma 4 in Online Appendix A.2, expected worker rents at the market level are:

$$\mathbb{E}\left[R_{it}^{wm}|j(i,t)=j\right] = \frac{1}{1+\lambda\beta}\mathbb{E}\left[W_{jt}(X_i)|j(i,t)=j\right]$$

Market level rents exceed firm level rents whenever the next best firm is in the same market as the current choice of firm. If the preferences of a given worker are independent across firms within each market, then the next best firm will almost surely be in a different market. If, on the other hand, these preferences are correlated then there could well exist other firms within the same market that are close substitutes to the current firm. The next best firm may then be in the same market as the current choice of firm, in which case R_{it}^{wm} will exceed R_{it}^{w}.

To interpret the measure of firm level rents and link it to compensating differentials, it is useful to express R_{it}^{w} in terms of reservation wages. The worker's reservation wage for her current choice of firm is defined as the lowest wage at which she would be willing to continue working in this firm. Substituting in preferences in the above definition of R_{it}^{w} for a worker whose current firm is j and next best option is j', it follows that:

$$\begin{aligned}\underbrace{\log W_{j(i,t),t}(X_i)}_{\text{current wage}} - \underbrace{\log\left(W_{j(i,t),t}(X_i) - R_{it}^{w}\right)}_{\text{reservation wage}} = \underbrace{\log W_{j(i,t),t}(X_i)}_{\text{current wage}} - \underbrace{\log W_{j'(i,t),t}(X_i)}_{\text{wage at best outside option}} \\ + \underbrace{\log G_{j(i,t)}^{1/\lambda}(X_i)e^{\frac{1}{\lambda\beta}\epsilon_{ij(i,t)t}}}_{\text{current amenities}} - \underbrace{\log G_{j'(i,t)}^{1/\lambda}(X_i)e^{\frac{1}{\lambda\beta}\epsilon_{ij'(i,t)t}}}_{\text{amenities at best outside option}}\end{aligned}$$

The average worker choosing firm j may be far from the margin of indifference and would maintain the same choice even if her current firm offered significantly lower wages.

Compensating differentials

By definition, marginal workers are indifferent between the current choice of firm and the next best option. They earn no rents as their reservation wages equal the actual wages paid by their current firms. The equilibrium allocation of workers to firms is such that utility gains (or losses) of marginal workers due to the amenities of their firms are exactly offset by wage differentials. Thus, wage differentials across firms for the same worker define the equalizing or compensating differentials:

Result 3. *Consider worker i of type X whose current firm is j and best outside option is j' and who is marginal at the current firm (that is, $R_{it}^{w}=0$). The* ***compensating differential*** *between j and j' for a worker of type X is then defined as,*

$$\begin{aligned} CD_{jj't}(X) &= u_{it}(j', W_{jt}(X)) - u_{it}(j, W_{jt}(X)) \\ &= \log W_{j't}(X) - \log W_{jt}(X) \\ &= (\theta_{j'} - \theta_j)x + \psi_{j't} - \psi_{jt} \end{aligned}$$

where the second equality comes the fact that worker i is marginal, and the last equality follows *from equation (4) and defining the firm effect ψ_{jt} as,*

$$\psi_{jt} \equiv c_r - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta}\overline{a}_{r(j),t} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} \tag{8}$$

For any two firms j and j', there exists a distribution of compensating differentials. This distribution arises because of differences in technology across firms. If θ_j does not vary across firms, there is only one compensating differential per employer, ψ_{jt}, which is paid to all workers independently of their productivity.

Employer rents

The equilibrium allocation of workers to firms may also create surpluses or rents for employers. The employer rents arise because of the additional profit the firm can extract by taking advantage of its wage-setting power. To measure employer rents, we therefore compare the profit Π_{jt} the firm actually earns to what it would have earned if the employer solved the firm's problem under the assumption that the labor supply it faced was perfectly elastic. In other words, wages, profits and employment are such that $D^{pt}_{jt}(X)$ solves the firm's profit maximization given $W^{\text{pt}}_{jt}(X)$:

$$\Pi^{\text{pt}}_{jt} = \max_{\left\{D^{\text{pt}}_{jt}(X)\right\}_X} A_{jt}\left(\int X^{\theta_j}\cdot D^{\text{pt}}_{jt}(X)\mathrm{d}X\right)^{1-\alpha_{r(j)}} - \int D^{\text{pt}}_{jt}(X)\cdot W^{\text{pt}}_{jt}(X)\mathrm{d}X,$$

The only difference in the firm's problem in this counterfactual environment is that the firm does not take into account its wage-setting power through the upward-sloping labor supply curve. In other words, the firm behaves as if it faces a perfectly elastic labor supply curve, i.e. as if it was a "price taker"; thus the superscript pt. Similarly we define $W^{\text{ptm}}_{jt}(X)$, $D^{\text{ptm}}_{jt}(X)$, and Π^{ptm}_{jt} as the equilibrium outcome when all firms in a market act as price takers.

Result 4. *We define the **employer rents** at the firm level R^f_{jt} and at market level R^{fm}_{jt} as the additional profit that firm j in market r derives by taking advantage of its wage-setting power:*

$$R^f_{jt} = \Pi_{jt} - \Pi^{pt}_{jt} = \left(1 - \frac{\alpha_r\left(\rho_r+\lambda\beta\right)}{\rho_r+\alpha_r\lambda\beta}\left(\frac{\lambda\beta}{\rho_r+\lambda\beta}\right)^{-\frac{(1-\alpha_r)\lambda\beta}{\rho_r+\alpha_r\lambda\beta}}\right)\Pi_{jt}$$

$$R^{fm}_{jt} = \Pi_{jt} - \Pi^{ptm}_{jt} = \left(1 - \frac{\alpha_r\left(\rho_r+\lambda\beta\right)}{\rho_r+\alpha_r\lambda\beta}\left(\frac{\lambda\beta}{\rho_r+\lambda\beta}\right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}\right)\Pi_{jt}$$

where the latter equality in each equation is shown in Lemmas 5 and 6 in Online Appendix A.3.

To understand how and why employer rents may differ at the firm and the market level, recall that ρ_r measures the degree of independence in a worker's taste for the alternative firms within market r. If $\rho_r = 1$, the worker views these firms as completely independent alternatives, and the rents at the firm level equal the rents at the market level. In contrast, if $\rho_r = 0$ then each worker views firms within the same market as perfect substitutes. In this case, firms do not get any rents from imperfect competition at either the firm or the market level. For values of ρ between 0 and 1, the rents at the market level will strictly exceed the rents at the firm level.

It is important to observe that R_{jt}^{f} and R_{jt}^{fm} do not necessarily represent ex-ante rents. Suppose, for example, that each employer initially chooses the amenities offered to the workers by deciding on the firm's location, the working conditions, or both. Next, the employers compete with one another for the workers who have heterogeneous preferences over the chosen amenities. These heterogeneous preferences give rise to wage-setting power which employers can use to extract additional profits or rents. Of course, the existence of such ex-post rents could simply be returns to costly choices of amenities.

Empirically, it is difficult to credibly distinguish between ex-ante and ex-post employer rents. It would require information (or assumptions) about how firms choose and pay for the amenities offered to workers. Given our data, we are severely limited in the ability to distinguish between ex-ante and ex-post rents. Instead, we assume firms are endowed with a fixed set of amenities, or, more precisely, we restrict amenities to be fixed over the estimation window. It is important to note what is *not* restricted under this assumption. First, it does not restrict whether or how amenities $G_j(X)$ relate to the technology parameters $\alpha_{r(j)}$, θ_j or the productivity components $\tilde{P}_j, \overline{P}_{r(j)}$. Second, it neither imposes nor precludes that employers initially choose amenities to maximize profits. Indeed, it is straightforward to show that permitting firms to initially choose amenities would not affect any of our estimates. Nor would it matter for the interpretation of any result other than whether R_{jt}^{f} and R_{jt}^{fm} should be viewed as ex-ante or ex-post rents.

Wedges and allocative inefficiencies

We conclude the model section by investigating the questions of whether and in what situations the equilibrium allocation of workers to firms will be inefficient. We present here the key results, and refer to Online Appendix A.4 for details and derivations. To draw conclusions about allocative inefficiencies, we compare the allocation and outcomes in the monopsonistic labor market to those that would arise in a competitive (Walrasian) labor market. By a competitive market, we mean that there are no taxes ($\lambda = \tau = 1$) and that all firms act as price takers, as if they faced perfectly elastic labor supply curves. This comparison allows us to draw inferences about allocative inefficiencies within and between markets.

Within each market, there is a tax wedge that arises because $\lambda < 1$. It is the only source of allocative inefficiency at this level, distorting the worker's ranking of firms in favor of those with better amenities. As λ decreases and thereby the wage tax becomes more progressive, amenities become more valuable relative to (pre-tax) wages. Thus, with progressive taxation, firms with better amenities can hire workers at relatively low wages, and, therefore, get too many workers as compared to the allocation in the competitive labor market. Between markets, allocative inefficiencies may arise not only because of the tax wedge but also due to differences in labor wedges across markets. To understand the latter source of inefficiencies, consider the special case when $\lambda = 1, \beta > 0$ and ρ_r is non-zero but the same across all markets. In this case, taxes are proportional but there are still labor wedges and rents in the economy. However, the labor wedges will be the same across all markets. As a consequence, the monopsonistic market allocation of workers to firms is identical to the allocation one would obtain in the competitive equilibrium. A corollary of this result is that tax wedges are the only source of allocative inefficiencies if one assumes a standard logit structure on the distribution of $\vec{\epsilon}_{it}$ (as in, for example, Card et al., 2018).

With the nested logit structure on the distribution of $\vec{\epsilon}_{it}$, allocative inefficiencies across markets may arise because ρ_r can vary across markets, implying that workers may view firms as closer substitutes in some markets than others. This will create differences across markets in the wage-setting power of firms, and so in their abilities to mark down wages. Markets facing an elastic labor supply curve (i.e. low value of ρ_r) will have relatively high wages and, as a result, attract too many workers compared to the allocation in the competitive equilibrium. Progressive taxation will amplify any differences in ρ_r across markets, leading to an even larger misallocation of workers to firms.

To improve the allocation of workers to firms, the government can change the tax system in two ways. First, a less progressive tax system (i.e. increase λ) may reduce the misallocation that arises from the tax wedge. Second, letting τ vary across markets may improve the allocation of workers by counteracting differences in the wage-setting power of firms. For example, τ could vary across markets (defined as the combination of geographical area and industry) due to state income taxes or because of subsidies to certain industries or regions.[6] After estimating the parameters of the model, we perform, in Section 6, counterfactual analyses that quantify the impacts of such tax reforms on the equilibrium allocation and outcomes, including earnings, output and welfare. In interpreting these results, it is important to note that we assume firms initially choose amenities $G_j(X)$, but do not change $G_j(X)$ in response to counterfactuals. With better data on, and an instrument for, amenities, it would be interesting to extend this analysis to allow for firms to adjust amenities in response to these counterfactuals.

3 Data sources and sample selection

3.1 Data sources

Our empirical analyses are based on a matched employer-employee panel data set with information on the characteristics and outcomes of U.S. workers and firms. This data is constructed by linking U.S. Treasury business tax filings with worker-level filings for the years 2001-2015. Below, we briefly describe data sources, sample selection, and key variables, while details about data construction and the definition of each of the variables are given in Online Appendix B.

Business tax returns include balance sheet and other information from Forms 1120 (C-corporations), 1120S (S-corporations), and 1065 (partnerships). The key variables that we draw on from the business tax filings are the firm's employer identification number (EIN) and its value added, commuting zone, and industry code. Value added is the difference between receipts and

[6] Income taxes vary considerably across geographic regions. For example, the 2015 state income tax rates were 0 percent in Florida and Texas, between 3 and 4 percent in Illinois and Pennsylvania, and above 5 percent in Massachusetts and North Carolina (Tax Foundation, 2015). Moreover, the U.S. Empowerment Zone Program provides a 20 percent wage subsidy (up to a cap) to firms located in a designated disadvantaged location (IRS, 2004). Furthermore, minimum wages vary considerably across regions.

the cost of goods sold. Commuting zone is constructed using the ZIP code of the firm's business filing address. Industry is defined as the first two digits of the firm's NAICS code. In our baseline specification, we define a market as the combination of an industry and a commuting zone, with alternative market definitions provided in sensitivity checks. We will occasionally aggregate these markets into "broad markets" according to the combination of Census regions (Midwest, Northeast, South, and West) and broad sectors (Goods and Services).

Earnings data are based on taxable remuneration for labor services reported on Form W-2 for direct employees and on Form 1099 for independent contractors. Earnings include wages and salaries, bonuses, tips, exercised stock options, and other sources of income deemed taxable by the IRS. These forms are filed by the firm on behalf of the worker and provide the firm-worker link. All monetary variables are expressed in 2015 dollars, adjusting for inflation using the CPI.

3.2 Sample selection

In each year, we start with all individuals aged 25-60 who are linked to at least one employer. Next, we define the worker's firm as the EIN that pays her the greatest direct (W-2) earnings in that year. This definition of a firm conforms to previous research using the U.S. business tax records (see, e.g., Song et al., 2018). The EIN defines a corporate unit for tax and accounting purposes. It is a more aggregated concept than an establishment, which is the level of analysis considered in recent research on U.S. Census data (see, e.g., Barth et al., 2016), but a less aggregated concept than a parent corporation. As a robustness check, we investigate the sensitivity of the estimated firm wage premiums to restricting the sample to EINs that appear to have a single primary establishment. These are EINs for which the majority of workers live in the same commuting zone. It is reassuring to find that the estimated firm wage premiums do not materially change when we use this restricted sample.

Since we do not observe hours worked or a direct measure of full-time employment, we follow the literature by including only workers for whom annual earnings are above a minimum threshold (see, e.g., Song et al., 2018). In the baseline specification, this threshold is equal to $15,000 per year (in 2015 dollars), which is approximately what people would earn if they worked full-time at the federal minimum wage. As a robustness check presented in our Online Supplement, we investigate the sensitivity of our results to other choices of a minimum earnings threshold. We further restrict the sample to firms with non-missing value added, commuting zone, and industry. The full sample includes 447.5 (39.2) million annual observations on 89.6 (6.5) million unique workers (firms).

In parts of the analysis, we consider two distinct subsamples. The first subsample, which we refer to as the *stayers sample*, restricts the full sample to workers observed with the same employer for eight consecutive years. This restriction is needed to allow for a flexible specification of how the worker's earnings evolve over time. Specifically, we omit the first and last years of these spells (to avoid concerns over workers exiting and entering employment during the year, confounding the measure of annual earnings) and analyze the remaining six-year spells. Furthermore, the stayers sample is restricted to employers that do not change commuting zone or industry during those eight years. Lastly, we restrict the stayers sample to firms with at least

10 such stayers and markets with at least 10 such firms, which helps to ensure sufficient sample size to perform the analyses at both the firm and the market level. The stayers sample includes 35.1 (6.5) million spells on 10.3 (1.5) million unique workers (firms).

The second subsample, which we refer to as the *movers sample*, restricts the full sample to workers observed at multiple firms.[7] That is, it is not the same EIN that pays the worker the greatest direct (W-2) earnings in all years. Following previous work, we also restrict the movers sample to firms with multiple movers. This restriction might help reduce limited mobility bias and makes it easier to compare the estimates of firm effects across methods (as the approach of Kline et al. 2020 requires at least two movers per firm).[8] The movers sample includes 32.1 (3.6) million unique workers (firms).

Online Appendix Table A.1 compares the size of the baseline, the stayers, and the movers samples. Detailed summary statistics of these samples of linked firms and worker are given in Online Appendix Table A.2. The samples are broadly similar, both in the distribution of earnings but also in firm level variables such as value added, wage bill, size, and the distribution across regions and sectors. The most noticeable differences are that the stayers have, on average, somewhat higher earnings and tend to work in firms with higher value added.

4 Identification

We now describe how to take our model to the data, providing a formal identification argument while summarizing, in Table 1, the parameters needed to recover a given quantity of interest and the moments used to identify these parameters. Our results reveal that many of these quantities do not require knowledge of all the structural parameters. Thus, some of our findings may be considered more reliable than others.

4.1 Rents of workers and employers

It follows from Results 1, 2 and 4 that the expected rents of workers and employers depend on the parameters $(\beta, \rho_r, \alpha_r)$ and the data $(Y_{it}, W_{it}, j_{it}, r_{it})$. Our identification argument therefore proceeds by showing how these parameters can be identified from the panel data on workers and firms. However, before we present the formal identification argument, it is useful to consider what one can and cannot identify directly from an ideal experiment. This consideration clarifies the necessary assumptions even with an ideal experiment and the additional ones needed in the absence of such an experiment.

Ideal experiment

To see how one may recover $(\beta, \rho_r, \alpha_r)$, consider the structural equations (4) and (5) that express wages $w_j(x, \overline{a}, \tilde{a})$ and value added $y_j(\overline{a}, \tilde{a})$as functions of model primitives $\Gamma = (\overline{p}_r, \tilde{p}_j, g_j(x), x_i)$

[7] Note that, since workers outside the movers sample are not necessarily stayers for 8 consecutive years (e.g., due to a year in which earnings at the primary employer are below the full-time equivalence threshold, or aging in or out of the sample), the stayers sample is a subset of the non-movers sample.

[8] See our Online Supplement for such a comparison and an analysis of limited mobility bias.

and potential firm and market level productivity outcomes $(\overline{a}, \tilde{a})$. Suppose we were able to independently and exogenously change $\tilde{a}$, the component of productivity that is specific to a firm, and $\overline{a}$, the component of productivity that is common to all firms in a market. As evident from equations (4) and (5), exogenous changes in $\tilde{a}$ and $\overline{a}$ affect both the wages a firm offers to its workers of a given quality, $w_j(x, \overline{a}, \tilde{a})$, and the firm's value added, $y_j(\overline{a}, \tilde{a})$. We can express the ratio of these effects as

$$\frac{\partial w_j(x, \overline{a}, \tilde{a})}{\partial \tilde{a}} \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \tilde{a}} \right)^{-1} = \frac{1}{1 + \lambda\beta/\rho_r} \equiv \gamma_r$$
$$\frac{\partial w_j(x, \overline{a}, \tilde{a})}{\partial \overline{a}} \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \overline{a}} \right)^{-1} = \frac{1}{1 + \lambda\beta} \equiv \Upsilon$$

where we refer to γ_r and Υ as the firm level and market level pass-through rates.

Since λ is a known (or pre-estimated) tax parameter, β and ρ_r can be identified from these two equations. In this ideal experiment, the pass-through of value added $y_j(\overline{a}, \tilde{a})$ to wages $w_j(x, \overline{a}, \tilde{a})$ of an $\overline{a}$ induced change would identify β. Similarly, given this parameter, the pass-through of $y_j(\overline{a}, \tilde{a})$ to $w_j(x, \overline{a}, \tilde{a})$ of an $\tilde{a}$ induced change would identify ρ_r. Importantly, in this framework, we only need to be able to induce a change in productivity then observe how value added and wages change; we do not need to observe productivity directly.

Next, equations (5)-(6) imply,

$$\mathbb{E}\left[y_{jt} - b_{jt} | j \in J_r\right] \quad = \quad -c_r = -\log(1 - \alpha_r) - \log\left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r}\right). \tag{9}$$

Since $\mathbb{E}\left[y_{jt} - b_{jt} | j \in J_r\right]$ can be estimated directly from the data, and λ is known, it follows that α_r is identified given (β, ρ_r), which are in turn identified from (γ_r, Υ). Thus, the key challenge for identifying $(\beta, \rho_r, \alpha_r)$ is to identify (γ_r, Υ).

While it is not feasible to perform such an ideal experiment, it is possible to achieve identification of $(\beta, \rho_r, \alpha_r)$ either by using the panel data to construct internal instruments (i.e., instruments implied by model restrictions) or by finding external instruments (instruments based on data other than or external to the data generating process of our model). We now discuss identification with these two types of instruments in turn.

Difference-in-differences illustration of internal instruments

Before presenting the formal identification argument behind the internal instruments, we graphically illustrate how such instruments can be constructed through difference-in-differences (DiD) strategies.

Consider first how to recover the market level pass-through rate, Υ. Let $\overline{y}_{rt}$ denote market level average log value added and $\overline{w}_{rt}$ denote market level average log earnings for the sample of stayers in market r. Suppose for simplicity that workers can be assigned to two groups of firms in year t: one half has $\Delta\overline{y}_{r(i)t} = +\delta$ (treatment group) and the other half has $\Delta\overline{y}_{r(i)t} = -\delta$ (control group). Implicitly conditioning on stayers ($S_i = 1$) at firms in region r ($j(i,t) = j \in J_r$), we construct the following estimand:

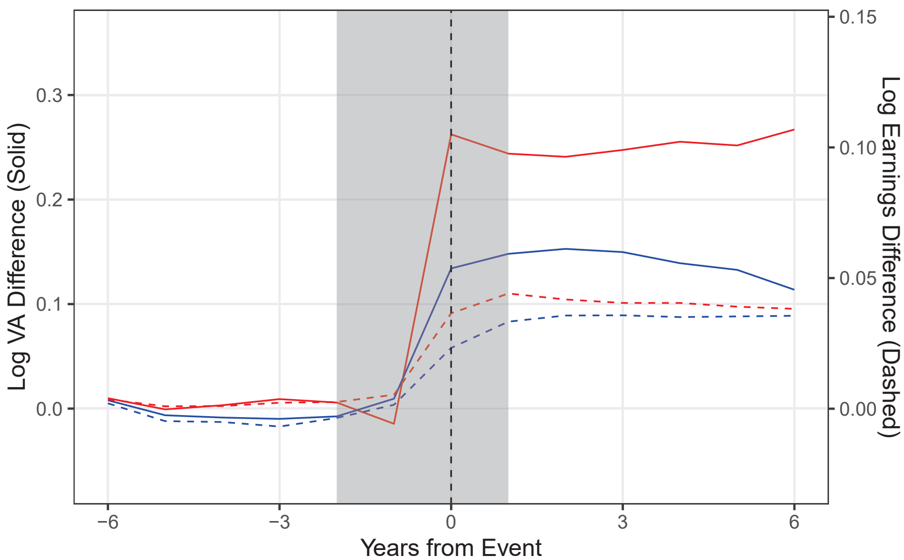


Figure 1: Difference-in-differences representation of the estimation procedure

Notes: This figure displays the mean differences in log value added (solid lines) and log earnings (dotted lines) between firms that receive an above-median versus below-median log value added change at event time zero. Results are presented for the measures of log value added and log earnings net of market interacted with year effects (red lines) and for the averages of log value added and log earnings by market and year (blue lines). The shaded area denotes the time periods during which the orthogonality condition need not hold in the identification of the permanent pass-through rate.

$$\frac{\mathbb{E}\left[\overline{w}_{rt+e} - \overline{w}_{rt-e'}|+\delta\right] - \mathbb{E}\left[\overline{w}_{rt+e} - \overline{w}_{rt-e'}|-\delta\right]}{\mathbb{E}\left[\overline{y}_{rt+e} - \overline{y}_{rt-e'}|+\delta\right] - \mathbb{E}\left[\overline{y}_{rt+e} - \overline{y}_{rt-e'}|-\delta\right]}$$

where $e + t$ is a post-period e years after t and $t - e'$ is a pre-period e' years before t. The numerator is a DiD estimand for market level changes in log earnings while the denominator is DiD estimand for market level changes in log value added. As shown formally below, the ratio of these DiD estimands recover Υ if amenities are fixed over time (at least within the estimation window) and the measurement error in value added, if any, is transitory. Under these assumptions, the observed market level changes in value added and log earnings (within firms and workers) surrogate for the ideal experiment.

In Figure 1, we visualize and assess this DiD strategy at the market level. The blue line in this figure is constructed as follows: In any given calendar year t, we i) order markets according to the increase $\Delta\overline{y}_{rt}$; ii) separate the firms at the median in the worker-weighted distribution of $\Delta\overline{y}_{rt}$, letting the upper half constitute the treatment markets and the lower half the control markets; and iii) plot the differences in $\overline{y}_{rt+e}$ between these two groups in period $e = 0$ as well as in the years before ($e < 0$) and after ($e > 0$). We perform these steps separately for various calendar years, weighting each market by the number of workers. The solid (dashed) blue line represents the difference in log value added (earnings) for the treatment and control markets.

By construction, the treatment and control groups differ in the value added growth from period $t-1$ to period t. On average, markets in the treatment group experience about 13 percentage points larger growth in value added as compared to markets in the control group. Furthermore, we find a similar trend in both log value added and log earnings between the treatment and control group before $e = -2$ and after $e = 2$. In other words, markets that experienced large growth in value added and earnings in period 0 are no more or less likely to experience growth in value added or earnings in periods -6 to -3 or in periods 3 to 6. This observation of common trends between the treatment and control groups at the market level supports our assumption that the measurement error is transitory.

To recover the market level pass-through rate γ_r, we apply the same logic as above, taking the ratio of a DiD estimand for firm level changes in log earnings to a DiD estimand for firm level changes in log value added. This ratio recovers γ_r under the same assumptions as above, except now applied to the firm level. To visualize and assess this DiD strategy, consider the red lines of Figure 1. These lines are constructed using firm level deviations from market level averages. We plot value added deviations $\tilde{y}_{jt} \equiv y_{jt} - \overline{y}_{rt}$ (solid line) and earnings deviations $\tilde{w}_{it} \equiv w_{it} - \overline{w}_{rt}$ (dashed line), splitting firms into the treatment and control groups at the median in the distribution of $\Delta\tilde{y}_{jt}$ and weighting each firm by the number of workers. We find that firms that experienced large growth in value added in period 0 are no more or less likely to experience growth in value added or earnings in periods -6 to -3 or in periods 3 to 6. This observation of common trends between the treatment and control groups at the market level supports our assumption that the measurement error is transitory.

Formal identification using internal panel instruments

We now turn to the formal identification argument for the internal instruments to identify (γ_r, Υ). To this end, we specify a process for the productivity shocks to firms. Suppose that firm productivity evolves as a unit root process at both the firm level and market level:[9]

$$\tilde{a}_{jt} = \tilde{p}_j + \tilde{z}_{jt}, \quad \textit{where} \quad \tilde{z}_{jt} = \tilde{z}_{jt-1} + \tilde{u}_{jt} \tag{10}$$

$$\overline{a}_{rt} = \overline{p}_r + \overline{z}_{rt}, \quad \textit{where} \quad \overline{z}_{rt} = \overline{z}_{rt-1} + \overline{u}_{rt} \tag{11}$$

To ensure relevance of the internal instrument, we first assume that productivity shocks exist. Denoting the variance of $\tilde{u}$ by $\sigma^2_{\tilde{u}}$ and the variance of $\overline{u}$ by $\sigma^2_{\overline{u}}$, we require the following:

Assumption 1.a. *The variances of productivity shocks at the firm and market levels are strictly positive, i.e.,* $\sigma^2_{\tilde{u}} > 0$ *and* $\sigma^2_{\overline{u}} > 0$.

We also allow for measurement error ν_{jt} in the observed value added in the form of a transitory component with finite time dependence, i.e., $y_{jt} = y_j(\overline{a}_{r(j)t}, \tilde{a}_{jt}) + \nu_{jt}$. It is necessary to invoke some restrictions on the relationships between the primitives. Denoting the history of time-varying unobservables at time t by $\Omega_t \equiv \{\tilde{u}_{jt'}, \overline{u}_{rt'}, \epsilon_{ijt'}\}_{i,j,r,t' \leq t}$, we assume the following:

[9] The assumption of a unit root process for productivity can be replaced by any process with persistence beyond the persistence of the measurement error in value added.

Assumption 1.b. *The value added measurement error ν_{jt} is i) mean independent of Ω_T, i.e, $\mathbb{E}[\nu_{jt}|\Omega_T] = 0$, and ii) have finite time dependence, i.e., $\mathbb{E}[\nu_{jt}\nu_{jt'}|\Omega_T] = 0$ if $|t - t'| \geq 2$.*

We also allow for measurement errors v_{it} in earnings, i.e., $w_{it} = w_{j(i,t)}(x_i, \overline{a}_{r(j(i,t))t}, \tilde{a}_{j(i,t)t}) + v_{it}$. We then make the following assumption:

Assumption 1.c. *The wage measurement error v_{it} is mean independent of value added measurement error and Ω_T, i.e., $\mathbb{E}[v_{it}|\nu_{j1}, ..., \nu_{jT}, \Omega_T] = 0$.*

Under assumptions 1.b and 1.c, we derive in Online Appendix C.1 the following moment conditions which identify (γ_r, Υ):

$$\mathbb{E}\left[\Delta\tilde{y}_{jt}\left(\tilde{w}_{it+e} - \tilde{w}_{it-e'} - \gamma_r\left(\tilde{y}_{jt+e} - \tilde{y}_{jt-e'}\right)\right)|S_i{=}1, j(i){=}j \in J_r\right] = 0 \tag{12}$$

$$\mathbb{E}\left[\Delta\overline{y}_{rt}\left(\overline{w}_{rt+e} - \overline{w}_{rt-e'} - \Upsilon\left(\overline{y}_{rt+e} - \overline{y}_{rt-e'}\right)\right)|S_i{=}1, j(i){=}j \in J_r\right] = 0 \tag{13}$$

for $e \geq 2, e' \geq 3$, where $\overline{y}_{rt} \equiv \mathbb{E}\left[y_{jt}|S_i{=}1, j(i,t){=}j \in J_r\right]$ and $\overline{w}_{rt} \equiv \mathbb{E}\left[w_{it}|S_i{=}1, j(i){=}j \in J_r\right]$ are market level means, $\tilde{w}_{it} = w_{it} - \overline{w}_{rt}$ and $\tilde{y}_{jt} = y_{jt} - \overline{y}_{rt}$ are deviations from market level means, and $S_i{=}1$ denotes a worker who does not change firms between $t - e'$ and $t + e$. These moment conditions are equivalent to regressions of long-differences in log earnings on long-differences in log value added, instrumented by short-differences in log value added. In addition, assumption 1.a ensures the rank condition and consequently the identifiability of these parameters.

To interpret these assumptions, it is useful to return to Figure 1. From assumption 1.b, the growth in value added should be the sum of a permanent component and a transitory, mean-reverting component. Due the transitory component, $\Delta\overline{y}_{rt}$ could be correlated with $\Delta\overline{y}_{rt+e}$ at $e = -2, ..., 2$. However, $\Delta\overline{y}_{rt}$ should be orthogonal to $\Delta\overline{y}_{rt+e}$ in the periods before $e = -2$ and after $e = 2$. Consistent with this orthogonality condition, Figure 1 shows a very similar trend in log value added between the treatment and control group at these periods. By similar reasoning in assumption 1.c, $\Delta\overline{y}_{rt}$ should be orthogonal to $\Delta\overline{w}_{rt+e}$ in the periods before $e = -2$ and after $e = 2$. Consistent with this orthogonality condition, Figure 1 shows a very similar trend in log earnings between the treatment and control group at these periods.

It is useful to observe what is and is not being restricted by assumptions 1.b and 1.c that deliver the internal instruments. Importantly, these assumptions permit arbitrary correlation between the components of Γ, that is $(\overline{p}_r, \tilde{p}_j, g_j(x), x_i)$. As a result, our model allows for rich heterogeneity of both firms and workers, and systematic sorting of different workers into different firms. However, assumption 1.b implies that worker-specific innovations to productivity are independent across coworkers and orthogonal both to innovations to firm productivity and to idiosyncratic taste realizations. Moreover, worker-specific wage measurement error is independent of the choice of firm, and, thus, does not matter for worker mobility. This is key to identifying the pass-through rates of firm shocks by looking at changes over time in the earnings of incumbent workers.

Identification using external instruments

To complement the analyses based on internal instruments, we also use external instruments that allow us to relax assumptions on the joint process of amenities, firm productivity, and measurement error in value added. In particular, we can allow both firm-specific and market-specific amenities to vary over time as well as unrestricted dependence over time in the value added measurement error. The key limitation of the external instruments is that we only have a firm-specific shock for a single industry, not all industries in the economy.

To see why external instruments can achieve identification under weaker assumptions, we derive the wage equation in the presence time varying firm ($\tilde{g}_{jt}$) and market ($\overline{g}_{rt}$) level amenities. As shown in Lemma 8 in Appendix A.5, the structural wage equation is the same as in (6) except for the amenity term h_j which is now time-varying and given by:

$$h_{jt} = \check{h}_{j(i,t)} + \frac{\alpha_{r(i,t)}\beta}{1+\alpha_{r(i,t)}\lambda\beta}\overline{g}_{r(i,t)t} + \frac{\alpha_{(i,t)}\beta/\rho_{(i,t)}}{1+\alpha_{r(i,t)}\lambda\beta/\rho_r}\tilde{g}_{j(i,t)t},$$

and can be aggregated at the market level to $\overline{h}_{rt} \equiv \mathbb{E}\left[h_{jt}|j \in J_r\right]$.

Suppose we observe an instrument for firm level TFP $\tilde{a}$, denoted $\tilde{\Lambda}_{jt}$, satisfying the following firm level condition:

Assumption 1.d. *The firm level instrument $\tilde{\Lambda}_{jt}$ is relevant for firm level productivity changes, $\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right)|S_i{=}1, j(i){=}j \in J_r\right] \neq 0$, and exogenous of changes in firm level amenities h_{jt}, $\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e} - h_{j(i)t-e'}\right)|S_i{=}1, j(i){=}j \in J_r\right] = 0$.*

Furthermore, suppose we observe a market level instrument for market level TFP $\overline{a}$, denoted $\overline{\Lambda}_{rt}$, satisfying the following market level condition:

Assumption 1.e. *The market level instrument $\overline{\Lambda}_{rt}$ is relevant for market level productivity changes, $\mathbb{E}\left[\overline{\Lambda}_{rt}\left(\overline{a}_{rt+e} - \overline{a}_{rt-e}\right)|S_i{=}1, j(i){=}j \in J_r\right] \neq 0$, and exogenous of changes in market level amenities $\overline{h}_{rt}$, $\mathbb{E}\left[\overline{\Lambda}_{rt}\left(\overline{h}_{rt+e} - \overline{h}_{rt-e}\right)|S_i{=}1, j(i){=}j \in J_r\right] = 0$.*

Impose assumptions 1.d and 1.e and invoke the restrictions on the measurement errors 1.b part i) and 1.c. Then it follows directly that equation (12) recovers γ_r using $\tilde{\Lambda}_{jt}$ instead of $\Delta\tilde{y}_{jt}$, and equation (13) recovers Υ using $\overline{\Lambda}_{rt}$ instead of $\Delta\overline{y}_{rt}$. See Appendix C.3 for additional details.

In the empirical implementations below, we consider two external instruments. We estimate the firm level pass-through γ_r in the construction sector using the research design of Kroft et al. (2021). In particular, we instrument for changes in value added using plausibly exogenous product demand shocks at the firm-level generated by government procurement auction outcomes. We estimate the market level pass-through Υ using a shift-share research design in the tradition of Bartik (1991) and Blanchard and Katz (1992). In particular, we instrument for changes in market level value added using industry-wide value added growth shocks interacted with the past concentration of that industry's value added across commuting zones.

4.2 Quality of workers and technology and amenities of firms

To draw inferences about compensating differentials and the sources of wage inequality, we need to recover the quality of workers as well as the technology and amenities of firms. The identification argument consists of three steps. First, we use equations (4) and (8), which show that the variation in log earnings can be decomposed into firm effects (ψ_{jt}), interactions between worker quality (x) and firm complementarities (θ_j), and the pass-through of productivity shocks from firms to workers. We demonstrate how to use the observed changes in earnings for workers moving across firms to separately identify each of these components. Second, we combine these results with equation (7) and the parameters ($\beta, \rho_r, \alpha_r, \lambda$) identified in the previous subsection to decompose the variation in firm effects into the time-varying TFP components at the firm-level ($\tilde{a}_{jt}$) and the market-level ($\overline{a}_{rt}$) as well as the amenity component (h_j). Lastly, we use equations (10) and (11) to recover the permanent components of TFP at the firm-level ($\tilde{p}_j$) and market-level ($\overline{p}_r$), as well as the variances of TFP shocks at the firm-level ($\sigma^2_{\tilde{u}}$) and market-level ($\sigma^2_{\overline{u}}$).

We now go through these three steps, referring to Online Appendix C.4 for derivations and additional details. Consider first how to recover the time-invariant firm-specific earnings premium ψ_j as well as the firm-worker interaction parameters θ_j using the earnings of movers. To do so, we remove time-varying firm and market level components of earnings, which allows us to express the expected earnings of worker i in firm j in terms of only x_i, ψ_j, and θ_j:

$$\mathbb{E}\Big[\underbrace{w_{it}-\Big(\frac{1}{1+\lambda\beta}\left(\overline{y}_{rt}-\overline{y}_{r1}\right)+\frac{\rho_r}{\rho_r+\lambda\beta}\left(\tilde{y}_{jt}-\tilde{y}_{j1}\right)\Big)}_{w^a_{it}}\,\Big|j(i,t){=}j\in J_r\Big]=\theta_j x_i+\psi_j \tag{14}$$

where we refer to w^a_{it} as adjusted log earnings, and for $j \in J_r$ we define the firm fixed effect as:

$$\psi_j \equiv c_r-\alpha_r h_j+\frac{1}{1+\lambda\beta}\tilde{p}_r+\frac{\rho_r}{\rho_r+\lambda\beta}\overline{p}_j. \tag{15}$$

The fixed effect ψ_j is the common wage intercept in the firm that can be attributed to permanent productivity and amenities.

The structure of the adjusted log earnings equation (14) matches the model of earnings of Bonhomme et al. (2019) and implies the following set of moments:

$$\mathbb{E}\left[\left(\frac{w^a_{it+1}}{\theta_{j'}}-\frac{\psi_{j'}}{\theta_{j'}}\right)-\left(\frac{w^a_{it}}{\theta_j}-\frac{\psi_j}{\theta_j}\right)\Big|j(i,t){=}j,j(i,t{+}1){=}j'\right]=0.$$

Bonhomme et al. (2019) show that this set of moments uniquely identifies (ψ_j, θ_j) if a rank condition holds that workers moving to a firm are not of the exact same quality as workers moving from that firm, i.e.,

$$\mathbb{E}\left[x_i|j(i,t){=}j,j(i,t{+}1){=}j'\right]\neq\mathbb{E}\left[x_i|j(i,t){=}j',j(i,t{+}1){=}j\right].$$

We test this rank condition and find that it holds in our data. Given (ψ_j, θ_j), x_i is identified from $\mathbb{E}\left[\frac{w^a_{it}-\psi_{j(i,t)}}{\theta_{j(i,t)}}\middle| i\right]$. The estimates of x_i and θ_j allow us to construct the total efficiency units of labor for each firm, which together with the time varying part of the wage premium at the firm give us a linear system of equations in $h_j, \tilde{a}_{jt}$ and $\overline{a}_{rt}$ for each firm and time. Using the process assumptions on $\tilde{a}_{jt}$ and $\tilde{a}_{rt}$ and the market level normlization of p_j, we can then identify $(\overline{p}_r, \tilde{p}_j, \sigma^2_{\tilde{u}}, \sigma^2_{\overline{u}})$. See Online Appendix C.4 for further details.

4.3 Amenities and worker preferences

To make inference about welfare and to perform counterfactuals, it is necessary to also recover the preference terms $G_j(X)$. This is done through a revealed preference argument: Holding wages fixed, firms with favorable amenities (for a given type of worker) are able to attract more workers (of that type). Conditional on wages, the size and composition of firms and markets should therefore be informative about unobserved amenities.

We formalize this intuition in Lemma 9 in Online Appendix C.5, showing that $G_j(X)$ can be identified from data on the allocation of workers to firms and markets. Using the probability that workers choose to work for firm j conditional on selecting market r, $Pr\,[j(i,t){=}j|X, r{=}r(j)]$, we consider two firms j and j' in the same market r. The differences in size and composition of these firms depend on the gaps in wages and amenities:

$$\underbrace{\lambda\left((\theta_j - \theta_{j'})x_i + \psi_j - \psi_{j'}\right)}_{\text{wage gap}} + \underbrace{\log G_j(X) - \log G_{j'}(X)}_{\text{amenity gap}} \quad = \quad \frac{\rho_r}{\beta} \underbrace{\log \frac{Pr\,[j(i,t){=}j \mid X, r(j){=}r]}{Pr\,[j(i,t){=}j' \mid X, r(j'){=}r]}}_{\text{relative size by worker type}}$$

where ρ_r/β is the inverse (pre-tax) firm-specific labor supply elasticity. Since both the wage gap and the within-market elasticity are already identified, we can recover the value of amenities up to a common market factor by comparing the size and composition of firms. Using a similar argument, we show in Online Appendix C.5 that comparing the size and composition of firms across markets allows us to pin down the common market factor.

Name		Unique Parameters	Mean Estimate	Moments of the Data	
Panel A. Rents and Scale					
Idiosyncratic Taste Parameter	β	1	4.99	Market Passthrough	$\frac{\mathbb{E}[\Delta\bar{y}_{rt}(\bar{w}_{rt+e}-\bar{w}_{rt-e'})\|S_i=1]}{\mathbb{E}[\Delta\bar{y}_{rt}(\bar{y}_{rt+e}-\bar{y}_{rt-e'})\|S_i=1]}$
Taste Correlation Parameter	ρ_r	8	0.70	Net Passthrough	$\frac{\mathbb{E}[\Delta\tilde{y}_{jt}(\tilde{w}_{it+e}-\tilde{w}_{it-e'})\|S_i=1,r(j)=r]}{\mathbb{E}[\Delta\tilde{y}_{jt}(\tilde{y}_{jt+e}-\tilde{y}_{jt-e'})\|S_i=1,r(j)=r]}$
Returns to Scale Parameter	α_r	8	0.21	Labor Share	$\mathbb{E}[b_{j(i,t)} - y_{j(i,t)}\|r(j) = r]$
Name		Unique Parameters	Var. Estimate	Moments of the Data	
Panel B. Firm and Worker Heterogeneity					
Time-varying Firm Premium	ψ_{jt}	10,669,602	0.02	Structural Wage Equation	$\mathbb{E}[w_{it}-\frac{1}{1+\lambda\beta}\bar{y}_{r,t}$
Firm-specific Technology Parameter	θ_j	10	0.04		$-\frac{\rho_r}{\rho_r+\lambda\beta}\tilde{y}_{j,t}\|r(j) = r]$
Worker Quality	x_i	61,670,459	0.31	Wage Changes around Moves	$\mathbb{E}[w_{it+1}\|j \to j'] - \mathbb{E}[w_{it}\|j' \to j]$
Amenity Efficiency Units at Neutral TFP	h_j	1,953,915	0.14		$\mathbb{E}[w_{it}\|j' \to j] - \mathbb{E}[w_{it+1}\|j \to j']$
Time-varying Firm-specific TFP	$\tilde{a}_{jt}$	10,669,602	0.14	Total Labor Input &	$\ell_{jt} = \log \sum X_i^{\theta_j}$ and ψ_{jt}
Time-varying Market-specific TFP	$\overline{a}_{rt}$	111,829	0.12	Time-varying Firm Premium	
Name		Unique Parameters	Var. Estimate	Moments of the Data	
Panel C. Model Counterfactuals					
Preferences for amenities for:	$g_j(X)$	6,974,519	0.20	Firm Size &	$\Pr[j]$
Firm j for workers of quality X				Firm Composition &	$\Pr[x\|k(j) = k]$
Market r for workers of quality X				Market Composition	$\Pr[x\|r(j) = r]$

Table 1: Quantities of Interest, Model Parameters and Targeted Moments

Notes: This table displays the model parameters and the moments targeted in their estimation.

5 Estimation procedure, parameter estimates and fit

We now present the estimates of the key empirical quantities, including the pass-through rates, the worker and firm effects, and the sorting of workers to firms. Armed with these estimates, we empirically recover and discuss the key model parameters, such as the labor supply curve, the firms' technology, TFP, and amenities, as well as the workers' preferences and productivity. The estimation procedure follows closely the identification arguments laid out in Section 4 and summarized in Table 1, mostly replacing the population moments with their sample counterparts. In the estimation, however, we impose a few additional restrictions on the heterogeneity of workers, firms and markets. These restrictions are not necessary for identification, but they help reduce the number of parameters to estimate. We now describe these restrictions before presenting the parameter estimates, assessing the fit of the model, and examining overidentifying restrictions.

5.1 Empirical specification

We begin by restricting the market-specific parameters α_r and ρ_r to be the same within broad markets (as defined in Section 3). The restriction on α_r means the scale parameter can vary freely across (but not within) broad regions and sectors of the economy. The assumption on ρ_r restricts the nested logit structure of the preferences. Recall that the parameter ρ_r measures the degree of independence in a worker's taste for alternative firms within the nest. We specified the nest as the combination of commuting zone and two-digit industry. We now restrict the parameter ρ_r to be the same for all nests within each broad market. As a result, labor wedges may vary across but not within broad regions and sectors. In Online Appendix Table A.5, we demonstrate that the estimates of $(\beta, \rho_r, \alpha_r)$ and rent shares are robust to alternative definitions of nests, such as states instead of commuting zones and three-digit rather than two-digit industries.

A second set of restrictions is that we draw the firm-specific components θ_j and ψ_j from a discrete distribution. We follow Bonhomme et al. (2019) in using a two-step grouped fixed-effects estimation, which consists of a classification and an estimation step. In a first step, firms are classified into groups indexed by k based on the empirical earnings distribution using the k-means clustering algorithm. The k-means classification groups together firms whose earnings distributions are most similar.[10] Then, in a second step, we estimate the parameters $\theta_{k(j)}$ and $\psi_{k(j)}$. In the baseline specification, we assume there exist 10 firm types. We view the assumption of discrete heterogeneity as a technique for dimensionality reduction in the estimation. The estimates of firm effects do not change materially if we instead allow for 20, 30, 40 or 50 firm types (see our Online Supplement).

Lastly, we also make the following discreteness assumption for the systematic components of firm amenities:

$$G_j(X) = \overline{G}_{r(j)} \tilde{G}_j G_{k(j)}(X),$$

[10] Here, we follow Bonhomme et al. (2019). Concretely, we use a weighted k-means algorithm with 100 randomly generated starting values. We use the firms' empirical distributions of log earnings on a grid of 10 percentiles of the overall log-earnings distribution.

where we define the firm class $k(j)$ within market r using the classification discussed above interacted with the market. This multiplicative structure reduces the number of parameters we need to estimate while allowing for systematic differences in amenities across firms and markets $(\tilde{G}_j, \overline{G}_{r(j)})$ and heterogeneous tastes according to the quality of the worker $G_{k(j)}(X)$. As a result, amenities may still generate sorting of better workers to more productive firms, and compensating differentials may still vary across firms, markets and workers. For estimation purposes, we take advantage of the derivations in Online Appendix C.5, which express the preference components $(\overline{G}_{r(j)}, \tilde{G}_j, G_{k(j)}(X))$ as functions of the size and composition of firms and markets. In the estimation of $G_k(X)$, we discretize the distribution of X into 10 points of support by ranking the estimated values of X and evenly grouping workers into 10 bins. In the estimation of $\overline{G}_r$, we also group markets into 10 different market types based on their realized empirical distribution of earnings, using the same k-means algorithm as discussed above.

5.2 Estimates of the pass-through rates

We now present the estimates of the pass-through rates, finding that the internal and the external instruments give very similar results.

Estimates using internal instruments

In Table 2, we use the internal instruments to estimate the pass-through rates and the implied labor supply elasticities at both the firm and market levels. We directly implement the sample counterpart to equation (12) at the firm level under the assumption that measurement errors follow an MA(1) process ($e = 2, e' = 3$). We allow γ_r, and thus ρ_r, to vary by broad market, where a broad market is a set of markets.[11] In practice, we consider eight broad markets defined by a Census region and goods versus services sectors (see Section 3). Similarly, we directly implement the sample counterpart to equation (13) to estimate Υ.

In the first row of Panel A, we estimate that the average firm level pass-through rate γ_r is about 0.13 with a standard error of about 0.01. This suggests that the earnings of an incumbent worker increases by 1.3 percent if her firm experiences a 10 percent permanent increase in value added, controlling for common shocks in the market. The firm level pass-through rate implies a firm level (pre-tax) labor supply elasticity of about 6.5. This estimate implies that, holding all other firms' wage offers fixed, a one percent increase in a firm's wage offer increases that firm's employment by 6.5 percent.[12]

In the first row of Panel B, we estimate that the market level pass-through rate Υ is about 0.18 with a standard error of about 0.03. This suggests that the earnings of incumbent workers increases by 1.8 percent if all firms in their market experience a 10 percent permanent increase

[11] We estimate γ_r and Υ separately for each cohort t and then average across t. By doing so, we avoid the problem pointed out by Callaway and Sant'Anna (2020) that cohorts can be negatively weighted in pooled cohort DiD estimators.

[12] This estimate is at the upper end of the range of estimates found in a recent empirical literature. Card et al. (2018) pick 4 as the preferred value in their calibration exercise. A related literature using experimentally manipulated piece-rates for small tasks typically finds labor supply elasticities in the 2-6 range (Caldwell and Oehlsen, 2018; Dube et al., 2020; Sokolova and Sorensen, 2018).

Panel A.	Firm-level Estimation	
Instrumental Variable	**Passthrough ($\mathbb{E}[\gamma_r]$)**	**Implied LS Elasticity**
Internal instrument:	0.13	6.52
Lagged firm-level value added shock under MA(1) errors	(0.01)	(0.56)
External instrument:	0.14	6.02
Procurement auction shock at firm-level	(0.07)	(3.37)
Panel B.	**Market-level Estimation**	
Instrumental Variable	**Passthrough (Υ)**	**Implied LS Elasticity**
Internal instrument:	0.18	4.57
Lagged market-level value added shock under MA(1) errors	(0.03)	(0.80)
External instrument:	0.19	4.28
Shift-share industry value added shock	(0.04)	(1.13)

Table 2: Estimates of pass-through rates and labor supply elasticities

Notes: This table summarizes estimates of the pass-through rates and pre-tax labor supply (LS) elasticities when using internal or external instrumental variables. Panel A provides these estimates at the firm level, while Panel B provides these estimates at the market level.

in value added. This finding highlights the importance of distinguishing between shocks that are specific to workers in a given firm versus those that are common to workers in a market. The market level pass-through rate implies a market level (pre-tax) labor supply elasticity of about 4.6. This estimate implies that, if all firms in a market increase their wage offers by one percent, each firm's employment in the market increases by 4.6 percent.

In Online Appendix D.1, we provide a number of specification and robustness checks for the pass-through estimates using internal instruments. First, we show that the firm level and market level pass-through rates are not sensitive to using an MA(2) specification rather than an MA(1) specification for the transitory shock process, which is consistent with previous work (see, e.g., Guiso et al. 2005; Friedrich et al. 2019). Second, when allowing for transitory shocks to value added to also pass-through to earnings, we find very small pass-through rates of transitory shocks while the pass-through rates for permanent shocks are not materially affected. Third, in Online Appendix Figure A.1, we explore robustness of the pass-through estimates across subsamples of workers, finding that the pass-through rates do not vary that much by the worker's age, previous wage, gender, or tenure. Fourth, while value added is a natural measure of firm performance (see the discussion by Guiso et al. 2005), it is reassuring to find that the estimates of the pass-through rates are broadly similar if we measure firm performance by operating profits, earnings before interest, tax and depreciation (EBITD), or value added net of reported depreciation of capital. We also show that the estimated pass-through rates are in the same range as our baseline result if we exclude multinational corporations or exclude the largest firms.

Lastly, to compare with existing work (e.g., Guiso et al. 2005), we also consider estimating the restricted specification that imposes $\gamma_r = \Upsilon, \forall r$. In our model, this is equivalent to imposing $\rho_r = 1, \forall r$, so that idiosyncratic worker preferences over firms are uncorrelated within markets.

The estimated pass-through rate is then 0.14, which is broadly similar to the existing literature which ignores the distinction between firm and market level shocks.

Estimates using external instruments

Our analyses so far have relied on statistical processes of earnings and value added. An advantage of our approach is that it provides both a market level and a firm level instrument for each firm, allowing us to draw inference for the entire population. While we have provided a number of diagnostics and sensitivity checks which support our approach, the identifying assumptions remain debatable. To examine the sensitivity of our results to the assumptions on the statistical processes for value added and earnings – and thereby improve the quality and credibility of our analyses – we now provide complementary analyses based on external instruments.

To recover the firm level pass-through and labor supply elasticity, we take advantage of the same research design as Kroft et al. (2021), except we apply it to our estimation sample and parameters of interest.[13] In particular, we examine how firms in the construction sector respond to a plausibly exogenous shift in product demand through a DiD design that compares first-time procurement auction winners to the firms that lose, both before and after the auction. Formally, consider the cohort of firms that received a procurement contract in year t ($\mathcal{D}_{jt} = 1$) and the set of comparison firms that bid for a procurement in year t but lost ($\mathcal{D}_{jt} = 0$). Let e denote an event time relative to t and $\bar{e}$ denote the omitted event time. For each event time $e = -4, ..., 4$, the DiD regression is implemented as

$$w_{jt+e} = \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} \mu_{te'}}_{\textit{event time fixed effect}} + \underbrace{\sum_{j'} 1\left\{j' = j\right\} \psi_{j't}}_{\textit{firm fixed effect}} + \underbrace{\sum_{e' \neq \bar{e}} 1\left\{e' = e\right\} \mathcal{D}_{jt} \vartheta_{te'}}_{\textit{treatment status by event time}} + \underbrace{\nu_{jte}}_{\textit{residual}}$$

We report the average across t of the estimated ϑ_{te} parameters, which can be interpreted as the average treatment effect on the treated for those firms receiving an exogenous demand shock.[14] We use the same regression model to estimate the effects of an exogenous demand shock on log value added. The ratio of the effects on log mean earnings and log value added is the pass-through rate. We cluster standard errors at the firm-level and find a strong first stage coefficient; see Online Appendix C.3 for additional details. Using this external instrument, we find in the second row of Panel A in Table 2 a firm level pass-through rate of 0.14 and labor supply elasticity of about 6, which are very similar to our baseline estimates under assumptions 1.b-1.c.

In order to provide IV estimates of the market level pass-through and labor supply elasticity, we follow Bartik (1991) and Blanchard and Katz (1992) in constructing a shift-share instrument.

[13] The main limitation of the approach using external instruments is that the instrument may only be available for a subsample of firms. The instrument of Kroft et al. (2021) is only defined for the construction industry, which may not be nationally representative. To investigate this possibility, we apply the internal instruments design to the construction industry, finding a firm-level pass-through rate of about 0.15 and a firm-level labor supply elasticity of about 5.5, which are similar to the estimates for the full sample.

[14] We estimate ϑ_{te} for all t and e and then average across t, using the delta method to compute standard errors (which are clustered at the firm level j to account for serial correlation). By doing so, we avoid the problem pointed out by Callaway and Sant'Anna (2020) that cohorts can be negatively weighted in pooled cohort DiD estimators.

Let cz denote a commuting zone and ind denote a 2-digit NAICS industry, and recall that a market is defined by the pair (cz, ind) in our main specification. Let $\overline{Y}_{cz,ind,t}$ denote the total value added in the (cz, ind) at time t, and $\overline{Y}_{ind,t} \equiv \sum_{cz} \overline{Y}_{cz,ind,t}$ denote aggregate industry value added. Then, the shift-share total value added shock to the commuting zone is constructed as $\sum_{ind} S_{cz,ind,t_0} \zeta_{ind,t}$, where $S_{cz,ind,t} \equiv \frac{\overline{Y}_{cz,ind,t}}{\sum_{ind} \overline{Y}_{cz,ind,t}}$ is the exposure of the cz to a particular ind (the "share" component), $\zeta_{ind,t} \equiv \log \overline{Y}_{ind,t} - \log \overline{Y}_{ind,t-\tau}$ is the log change in industry value added (the "shift" component), and we measure the share component at the earliest period in the sample. To estimate the market-level pass-through, we regress the log change in earnings per stayer in the commuting zone on the log change in total value added in the commuting zone, instrumented by the shift-share value added shock. We find a strong first stage; see Online Appendix C.3 for additional details. We find in the second row of Panel B in Table 2 a market level pass-through rate of 0.19 and labor supply elasticity of about 4.3, which are very close to our baseline estimates under assumptions 1.b-1.c.

5.3 Estimates of the parameters needed to recover rents

Once we have estimates of firm level and market level pass-through rates (γ_r, Υ) and tax progressivity λ, we can recover the model parameters $(\beta, \rho_r, \alpha_r)$ needed to identify rents. We begin by estimating the tax progressivity parameter λ as well as the proportional tax parameter τ outside the model. In each year, we regress log net household income (earnings plus other income minus taxes) on log household gross income (earnings plus other income) for our sample. The construction of these income measures is detailed in Online Appendix B. The intercept from this regression gives us τ while λ is identified from the slope coefficient. We estimate τ of around 0.89 whereas λ is estimated to be about 0.92.[15] In a proportional tax-transfer system, λ is equal to one and $(1 - \tau)$ is the proportional effective tax rate. By contrast, if $0 < \lambda < 1$, then the marginal effective tax rate is increasing in earnings. Thus, our estimate indicates modest progressivity in the U.S. tax system. Online Appendix Figure A.2 shows how well our parsimonious tax function approximates the effective tax rates implicit in the complex U.S. tax-transfer system. Comparing predicted log net income from the regression to the observed log net income across the distribution of log gross income, we find this specification provides an excellent fit.

Armed with λ, we can identify $(\beta, \rho_r, \alpha_r)$ using the pre-tax labor supply elasticities at the firm level and market level summarized in Table 2 and the equations in Section 4.1. We estimate the (post-tax) market level labor supply elasticity β to be 4.99. This finding suggests considerable variability across workers in the idiosyncratic tastes for firms. We estimate the average ρ_r across markets to be 0.70. This implies a substantial correlation of about 0.5 in the idiosyncratic tastes of workers across firms within the same industry and location. We estimate the average α_r across markets to be 0.21. This indicates that returns to labor $1 - \alpha_r$ are about 0.79 on average, consistent with modestly diminishing returns.

In Online Appendix Figure A.3(a), we report the estimates of (post-tax) firm level labor supply elasticities from the main specification. On average, this elasticity is about 7.3. Behind

[15] These results mirror closely existing U.S. estimates of τ and λ (Guner et al., 2014, Heathcote et al., 2017).

this average, however, there is important variation. Empirically, labor supply is most inelastic in the goods sector (which has lower rates of unionization) and more elastic in the Northeast (which has lower rates of right-to-work law coverage). These results are consistent with stronger institutions that favor workers being associated with less wage-setting power of firms. However, these are only correlational patterns and may not be given a causal interpretion.

In Online Appendix Table A.5, we demonstrate that the estimates of $(\beta, \rho_r, \alpha_r)$ as well as the rent shares are robust to various alternative market definitions. First, we show that the estimates of β and the average rent shares are robust to shutting down broad market heterogeneity (that is, restricting $\rho_r = \overline{\rho}$ and $\alpha_r = \overline{\alpha}$). Next, we find that the results are materially unchanged when, instead of NAICS two-digit codes, we define the industry to be more aggregated (NAICS supersectors) or less aggregated (NAICS three-digit codes). Lastly, we demonstrate that the results are materially unchanged when, instead of commuting zones, we define the geographic units to be more aggregated (states) or less aggregated (counties).

5.4 Worker heterogeneity, firm wage premiums and worker sorting

We estimate worker effects x_i, firm wage premiums $\psi_{j(i)}$, and firm-worker interaction parameters $\theta_{j(i)}$ following closely Subsection 4.2. To do so, we first construct adjusted log earnings w_{it}^a using equation (14) and the estimates of $(\beta, \rho_r, \alpha_r, \lambda)$ discussed in the previous subsection.[16] Given the classification of firms into groups discussed above, we implement the estimating equations provided in Online Appendix C.4 on w_{it}^a in order to recover $\left(\psi_{k(j)}, \theta_{k(j)}\right)$ for each group k. Then, given (ψ_k, θ_k), we recover x_i from equation (14), as described in Subsection 4.2.[17]

Figure 2 summarizes the estimates (see our Online Supplement for further details). On the y-axis, we plot the predicted log earnings for each firm type using the equation $\psi_k + \theta_k x_q$, where each quantile in the distribution of worker types x_q is presented as a separate line. On the x-axis, firm types are ordered in ascending order of mean log earnings. If $\psi_{k(j)}$ did not vary across firm types k, the typical worker would not experience an upward slope when moving from lower to higher firm types. We find a weakly positive slope, indicating some role for time-invariant firm fixed effects. If $\theta_{k(j)}$ did not vary across firm types, then the lines in this plot would have the same slope for lower and higher worker types. Instead, the results show clear evidence that higher worker types experience a more positive slope across firm types. As shown in Online Appendix C.4, the parameters governing nonlinearities are identified from comparing the gains from moving from a low to a high type of firm for workers of different quality. As evident from Figure 2, the gains from such a move are considerably larger for better workers. For example, moving from the lowest to the highest type of firm increases earnings by 15, 47

[16] In a preliminary step, we regress log-earnings on a full set of indicators for calendar years and a cubic polynomial in age, where we follow Card et al. (2018) in restricting the age profile to be flat at age 40. Thus, w_{it} is log earnings net of age effects and common aggregate time trends. We verify that the two way fixed effect estimates are nearly identical if jointly estimating the age and year effects with the firm and worker fixed effects.

[17] Note that $\left(\psi_{k(j)}, \theta_{k(j)}\right)$ are estimated using the movers in the connected set of firms, while x_i is estimated for both movers and non-movers in this connected set. Since x_i is estimated using an average over time for a given worker, the estimated variance in x_i may be upward-biased due to serial correlation in earnings measurement errors or finite sample bias. In our Online Supplement, we derive and estimate the bias in the estimated variance of x_i for the case in which the error process is unit root plus MA(0), finding a small bias for our panel length.

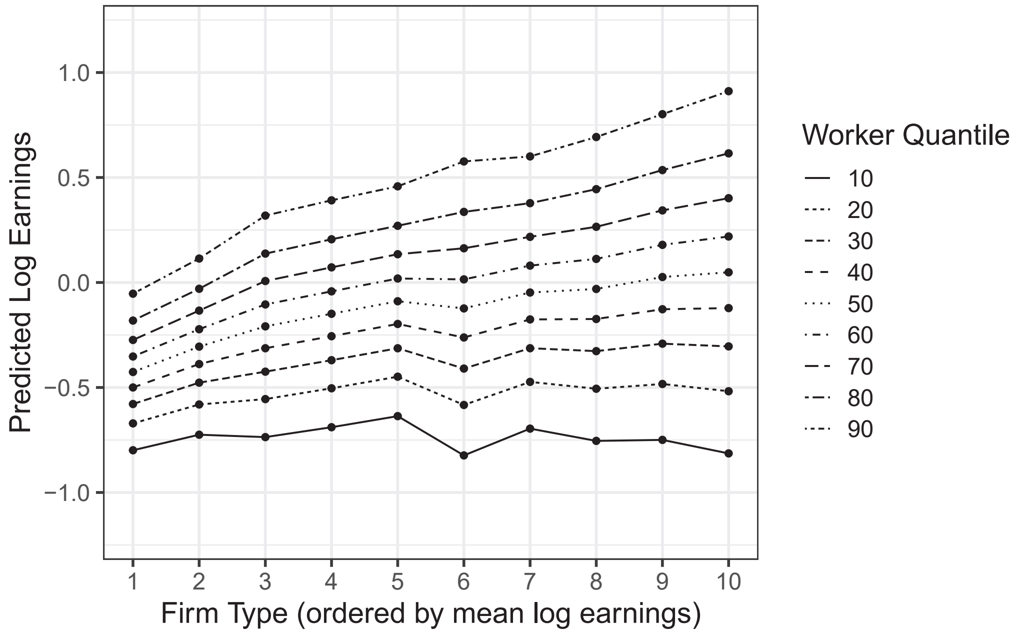


Figure 2: Predicted log earnings from the estimated model

Notes: In this figure, we summarize the estimates of worker ability x_i, time-invariant firm premiums $\psi_{k(j)}$, and firm-worker interactions $\theta_{k(j)}$, for 10 firm groups k. On the y-axis, we plot the predicted log earnings for each firm type using the estimated equation $\psi_k + \theta_k \cdot x_q$, where each quantile in the distribution of worker types x_q is presented as a separate line. On the x-axis, firm types are ordered in ascending order, where "lower" and "higher" types refer to low and high mean log earnings.

and 80 percentage points for individuals at the 20, 50 and 80 percentiles of worker quality.

To compare and interpret the estimates of x_i, ψ_{jt}, and θ_j, we re-arrange equation (14) so that we can decompose log earnings as,

$$w_{it} = \underbrace{\bar{\theta}(x_i - \bar{x})}_{\tilde{x}_i} + \underbrace{\psi_{j(i,t),t} - \psi_{j(i,t)}}_{\tilde{\psi}_{j(i,t),t}} + \underbrace{\left(\psi_{j(i,t)} + \theta_{j(i,t)}\bar{x}\right)}_{\tilde{\psi}_{j(i,t)}} + \underbrace{(\theta_{j(i,t)} - \bar{\theta})(x_i - \bar{x})}_{\varrho_{ij(i,t)}} + v_{it}$$

where $\overline{\theta} \equiv \mathbb{E}\left[\theta_{j(i,t)}\right]$ and $\overline{x} \equiv \mathbb{E}\left[x_i\right]$. This equation decomposes the earnings of worker i in period t into four distinct components: $\tilde{x}_i$ gives the direct effect of the quality of worker i (evaluated at the average firm), $\tilde{\psi}_{j(i,t),t}$ is the time variation in the firm premium due to the pass-through of value added shocks, $\tilde{\psi}_{j(i,t)}$ represents the average effect of firm j (evaluated at the average worker), $\varrho_{ij(i,t)}$ captures the interaction effect between the productivity of firm j and the quality of worker i, and v_{it} is the measurement error.

Using this representation, we obtain a variance decomposition of log earnings:

$$Var\left[w_{it}\right] = \underbrace{Var\left[\tilde{x}_i\right]}_{\text{i) Worker Quality: 71.6\%}} + \underbrace{Var\left[\tilde{\psi}_{j(i,t)}\right]}_{\text{ii) Firm Effects: 4.3\%}} + \underbrace{2Cov\left[\tilde{x}_i, \tilde{\psi}_{j(i,t)}\right]}_{\text{iii) Sorting: 13.0\%}} + \underbrace{Var\left[v_{it}\right]}_{\text{iv) Meas. Error: 10.0\%}}$$
$$+ \underbrace{Var\left[\varrho_{ij(i,t)}\right] + 2Cov\left[\tilde{x}_i + \tilde{\psi}_{j(i,t)}, \varrho_{ij(i,t)}\right]}_{\text{v) Interactions: 0.9\%}} + \underbrace{Var\left[\tilde{\psi}_{j(i,t),t}\right] + 2Cov\left[\tilde{x}_i, \tilde{\psi}_{j(i,t),t}\right]}_{\text{vi) Time-varying Effects: 0.3\%}}$$

The first conclusion is that the most important determinant of earnings inequality is worker quality, which explains about 72 percent of the variation in log earnings. The second conclusion is that firm fixed effects explain around 4 percent of the variation in log earnings, with a standard deviation of firm effects of about 0.12. In order to place the firm effect estimates in context, we compare them to the literature on the effects of job displacement. The majority of these studies focus on the US and find that long-run earnings losses from a job displacement are around 10-20 percent (see the survey by Couch and Placzek 2010). Thus, a job displacement has about the same effect on earnings as moving to a firm that is one standard deviation lower in the bias-corrected firm effects distribution.

The third conclusion is that the US economy is characterized by strong sorting of high quality workers to high paying firms, with a correlation of 0.37 between worker and firm fixed effects. Indeed, sorting explains about three times as much of the variation in log earnings as firm fixed effects on their own. The fourth conclusion is that the dispersion of interaction effects across firms explains about 1 percent of earnings inequality.[18] The final conclusion is that the time-varying component of firm effects due to the pass-through of TFP shocks at the firm level and market level explains less than half of a percent of earnings inequality, indicating a small role for the pass-through of shocks in cross-sectional earnings inequality.

In Online Appendix D.2, we discuss a number of specification checks. First, we consider estimating the model when excluding firm-worker interactions (imposing $\theta_j = \overline{\theta}$) or excluding time-varying effects (imposing $\gamma_r = \Upsilon = 0$). Second, we assess the degree of limited mobility bias in our data. Third, we consider increasing the number of groups in the k-means algorithm from the baseline value of 10 up to 50 in increments of 10, finding that the estimates are not sensitive to the number of groups. Fourth, we compare estimates for two distinct time periods, finding that the variance decomposition estimates change little over time. Fifth, we consider a number of checks on the reliability of the estimates of the interaction parameters θ_j. These include a comparison between our estimates and the interaction effects that arise due to observed worker heterogeneity and a check against data on hourly wages instead of annual earnings.

5.5 Estimates of remaining parameters and overidentification checks

We conclude this section by discussing estimates of the remaining parameters. We recover TFP and amenity components $(\tilde{a}_{j,t}, \overline{a}_{r,t}, h_j)$ from the estimates of (x_i, ψ_j, θ_j) using the approach

[18] Using a random effects approach, Woodcock (2015) also provides a decomposition with firm-worker interactions in the US. He also finds that interactions explain less variation than firm effects. However, the approach of Woodcock (2015) requires that match heterogeneity is purely idiosyncratic. By contrast, we find systematic deviations from the linear model in a way that is structurally related to other sources of heterogeneity, such as worker effects and firm effects.

explained in Subsection 4.2. Given estimated TFP and amenities, we can use them to construct predicted values of firm effects, value added, efficiency units of labor, and wage bill. In Online Appendix Figure A.4, we compare the observed and the predicted values of these variables in order to examine the model fit. We make this comparison separately according to the actual and predicted firm size.[19] It is reassuring that the model fits them well.

As an overidentification check, in Online Appendix Figure A.5, we take advantage of the fact that there are two distinct methods to identify the amenity component h_j. One possibility is the baseline approach discussed in Subsection 4.2, which recovers it from the equation for firm wage premiums. Another possibility is to use the fixed-point definition of h_j as a function of $(\tilde{P}_j, \overline{P}_r, G_j(X))$, as shown in Lemma 3 in Online Appendix A.1. This definition comes from the equilibrium constraint of the model, which we do not directly use in the baseline estimation. Online Appendix Figure A.5 shows that the estimates of h_j we obtain from solving the equilibrium constraint of the model are very similar to the baseline estimates. This finding increases our confidence in the moment conditions implied by our economic model.

As another overidentification check, we combine the earnings equation (4) with the equation for the wage bill (6) (instead of value added equation 5) to estimate the firm-specific labor supply elasticity using our internal instruments. This does not alter the conclusion that each firm is facing an economically and statistically significant upward-sloping labor supply curve. In other words, firms have considerable wage-setting power. In terms of magnitudes, we estimate a firm-specific labor supply elasticity above 6 based on value added changes and around 5 based on wage bill changes. Given the precision we have, however, one may want to be cautious in drawing strong conclusions about meaningful differences between these point estimates.

6 Empirical insights from the model

We now present five sets of empirical insights from the estimated model. These insights require an explicit model of the labor market, and, thus, they may be susceptible to model misspecification. As shown in Section 4, however, many of the insights do not require knowledge of all the structural parameters. Thus, some of our findings may be considered more reliable than others. To make this clear, we first present the findings that rely on the least assumptions and then move to those that require additional restrictions on the functioning of the labor market.

6.1 Rents and and labor wedges

Our first set of insights from the estimated model is about the rents and labor wedges that arise due to imperfect competition in the labor market. Table 3 presents estimates of the size of rents earned by American firms and workers from ongoing employment relationships. We report national averages and refer to Online Appendix Table A.7 for the market-specific results.

We find evidence of a significant amount of rents and imperfect competition in the U.S. labor market due to horizontal employer differentiation. At the firm level, we estimate that workers

[19] Note that firm effects and efficiency units of labor are targeted directly, while the relationship with firm size is not, so subfigures (b-c) in Online Appendix Figure A.4 are only untargeted in the relationship with firm size. The other subfigures are untargeted in both dimensions.

	Rents and Rent Shares			
	Firm level		Market level	
Workers' Rents:				
Per-worker Dollars	5,447	(395)	7,331	(1,234)
Share of Earnings	13%	(1%)	18%	(3%)
Firms' Rents:				
Per-worker Dollars	5,780	(1,547)	7,910	(1,737)
Share of Profits	11%	(3%)	15%	(3%)
Workers' Share of Rents	49%	(4%)	48%	(3%)

Table 3: Estimates of rents and rent sharing (national averages)

Notes: This table displays our main results on rents and rent sharing. Standard errors are in parentheses and are estimated using 40 block bootstrap draws in which the block is taken to be the market.

are, on average, willing to pay 13 percent of their annual earnings to stay in their current jobs. This corresponds to about $5,400 per worker. By comparison, firms earn, on average, 11 percent of profits from rents (with profits being measured as value added minus the wage bill). This amounts to about $5,800 per worker in the firm. Thus, we conclude that firm level rents from imperfect competition in the labor market are split equally between employers and their workers.

At the market level, we estimate that rents are considerably larger than firm level rents. Workers are, on average, willing to pay about $7,300 (18 percent of their annual earnings) to avoid having to work for a firm in a different market, which is almost $1,900 more than they would pay to avoid having to work for a different firm in the same market. The relatively large market level rents reflect that firms within the same market are more likely to be close substitutes than firms in different markets. At the market level, rents are again split almost evenly between firms and their workers.

In Online Appendix Figure A.3, we show that labor wedges are significant and vary substantially across markets. On average, the marginal revenue product of labor is 15 percent higher than the wage. Furthermore, the labor wedges are most pronounced in the goods sector (which have higher values of ρ_r). In the Western region of the U.S., for example, the labor wedge is 6 percentage points larger for firms in the goods sector as compared to those in the service sector.

6.2 Compensating differentials

The estimates of rents suggest the average American worker is far from the margin of indifference in her choice of firm, and would maintain the same choice even if her current firm offered significantly lower wages. In other words, the average worker considers amenities important to her choice of firm. This finding does not, however, imply marginal workers view the amenities of the current firm as much better or much worse than those offered by other firms. The second insight from our estimated model is the quantification of the preferences for amenities of marginal workers, as captured by the compensating differentials.

The estimates of the expected compensating differentials are displayed in Online Appendix Figure A.6. To estimate these quantities, we randomly draw two firms, j and j', from the overall distribution of firms (where each firm is drawn with probability proportional to its size). Using result 3, we compute the compensating differential between j and j' for a worker of given quality x as $\psi_{j'} + x\theta_{j'} - \psi_j - x\theta_j$. We repeat this procedure for many draws of firms.

The solid horizontal line in Online Appendix Figure A.6 shows the mean absolute value of compensating differentials for marginal workers. For two randomly drawn firms, the one with worse amenities can be expected to pay an additional 18 percent in order to convince marginal workers (of average quality) to accept the job. There is, however, considerable heterogeneity in compensating differentials according to worker quality. The upward sloping solid line shows how the expected compensating differential varies with worker quality. For high quality workers (95th percentile in the national distribution), the expected compensating differential is as large as 30 percent. By comparison, marginal workers of low quality (5th percentile in the national distribution) require less than 10 percent additional pay to work in the firm with unfavorable amenities.

The dashed lines of Online Appendix Figure A.6 display the compensating differentials across firms within a market. To compute these quantities, we use the same procedure as above, except we now compare firms within each market. For two randomly drawn firms in the same market, the one with worse amenities can be expected to pay an additional 14 percent in order to convince marginal workers (of average quality) to accept the job. This suggests that three-quarters of compensating differentials reflect differences in amenities within, rather than between, markets.

6.3 Understanding firm effects and their implications for inequality

The third set of insights from our estimated model shed light on why different firms pay identical workers differentially and the implications of firm premiums for inequality in wages versus total compensation (inclusive of amenities). As evident from equation (8), variation in the firm effects ψ_{jt} depends not only on the heterogeneity in firm amenities, but also on the differences in productivity across firms as well as the covariance between productivity and amenities within firms. The reason is that firms have wage-setting power, which generates a positive relationship between the firm's productivity and the wages it pays. To quantify the importance of these sources, consider the decomposition,

$$\begin{aligned} Var(\psi_{j(i,t),t}) = & \underbrace{Var(c_r - \alpha_r h_{j(i,t)})}_{\text{Amenities}} + \underbrace{Var(\frac{1}{1+\alpha_r\lambda\beta}\bar{a}_{rt} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{j(i,t),t})}_{\text{TFP}} \\ & + \underbrace{2Cov(c_r - \alpha_r h_{j(i,t)}, \frac{1}{1+\alpha_r\lambda\beta}\bar{a}_{rt} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{j(i,t),t})}_{\text{Covariance between amenities and TFP}} \end{aligned}$$

These components can be broken down between and within broad markets and, within broad markets, further decomposed within and between markets.[20]

[20]Recall that a broad market is a Census region interacted with a broad sector (goods or services), while a

	Between Broad Markets	Within Broad Markets	
		Between Detailed Markets	Within Detailed Markets
	Preferred Specification		
Total	0.4%	2.0%	3.1%
Decomposition:			
Amenity Differences	16.0%	7.8%	7.1%
TFP Differences	15.5%	11.9%	8.6%
Amenity-TFP Covariance	-31.1%	-17.7%	-12.6%
	Log-additive Fixed Effects Specification		
Total	0.6%	2.8%	6.6%
Decomposition:			
Amenity Differences	15.7%	6.5%	7.2%
TFP Differences	14.6%	13.2%	10.0%
Amenity-TFP Covariance	-29.8%	-16.9%	-10.5%

Table 4: Decomposition of the Variation in Firm Premiums

Notes: This table displays our estimates of the decomposition of time-varying firm premium variation in three levels: variation between broad markets, between detailed markets (within broad markets), and between firms (within detailed markets). Broad markets are defined as the combination of Census regions and broad sectors, and detailed markets are defined as the combination of industries and commuting zones. We decompose the variation in time-varying firm premiums into the contributions from amenity differences, TFP differences, and the covariance between amenity and TFP differences. All components are expressed as shares of log earnings variation. The first panel reports results from our preferred approach described in Section 4.2. The second panel reports results from the standard approach to estimate firm effects, as in Abowd et al. (1999), which may suffer from bias due to limited worker mobility across firms and does not permit firm-worker interactions.

The results from these decompositions are reported in Table 4. The first panel reports results from our preferred approach described in Section 4.2. The second panel reports results from the standard approach of Abowd et al. (1999), which may suffer from bias due limited worker mobility across firms and rules out firm-worker interactions. We find that the shares of the variance in firm effects explained by each component are fairly insensitive across these alternative estimation procedures. Either way, the results suggest substantial variation in amenities and productivity across firms. If one were to ignore the covariance between amenities and productivity, the considerable heterogeneity in amenities and productivity across firms would imply that firm effects should have a large contribution to inequality. However, productive firms tend to have good amenities, which act as compensating differentials and push wages down in productive firms. As a result, firm effects explain only a few percent of the overall variation in log earnings. For example, firm effects within detailed markets explain 3.1 percent of the variation in log earnings, which is much less than predicted by the variances of firm productivity (8.6 percent) and amenities (7.1 percent).

The positive correlation between TFP and amenities gives a negative contribution to earnings inequality, as indicated by the negative terms reported in the last row of Table 4. Since labor supply is upward sloping, more productive firms must offer greater total compensation per worker (inclusive of amenities) than smaller firms to achieve their optimal size. Since TFP and amenities are positively correlated, high TFP firms disproportionately offer compensation

market is a commuting zone interacted with a 2-digit NAICS industry.

through amenities rather than wages. Thus, earnings inequality would be even greater if amenities were uncorrelated with TFP, since high TFP firms would rely more heavily on paying higher wages instead of higher amenities.

6.4 Understanding why different workers sort into different firms, and the implications of this sorting for inequality

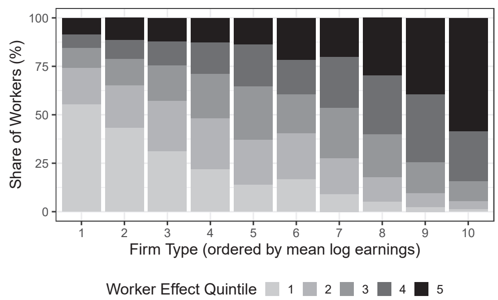
We now present the fourth insight from our estimated model: Production complementarities are important both to understand why better workers are sorting into better firms and to explain the significant inequality contribution from worker sorting.

To understand how we reach these conclusions, recall that the data reveals positive sorting between worker and firm fixed effects, which contributes significantly to inequality in earnings (see the discussion in Section 5.4 and our Online Supplement). In Figure 3(a), we present the sorting of workers to firms in our data. In this figure, firm types are ordered along the x-axis in ascending order of mean log earnings. On the y-axis, we rank workers by their worker effects x_i and divide them into five equally sized quintile groups. The bars present the share of workers within each firm type belonging to each quintile group. Figure 3(a) reveals that the highest quality workers are vastly overrepresented at the highest paying firms. For example, in the lowest firm type, less than 10 percent of workers belong to the top quality quintile group. By contrast, in the highest firm type, about 60 percent of workers belong to the top group.

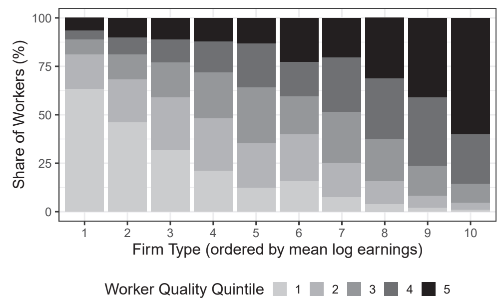
To build confidence in the estimated pattern of sorting, we exploit that there are two distinct methods to estimate sorting. One possibility is the baseline approach discussed in Subsection 4.2, which recovers worker and firm fixed effects from the equation for firm wage premiums (14) and uses the allocation of workers to firms observed in the data. Another possibility is to use the fixed-point definition of h_j as a function of the estimated values of $(\tilde{P}_j, \overline{P}_r, G_j(X))$, as shown in Lemma 3 in Online Appendix A.1, then simulate the allocation of worker quality to firm types using only estimated model parameters. This approach relies on the equilibrium constraint of the model, which we do not directly use in the baseline estimation. The results from this simulation are presented in Figure 3(b). The strong similarity between Figures 3(a) and 3(b) serves as an overidentification check that increases our confidence in the moment conditions implied by our economic model.

As discussed in Subsection 2.3, there are several possible reasons why better workers are overrepresented in higher paying firms. One possible reason is that productive firms have better amenities, and high ability workers may value amenities more than low ability workers. Another possible reason is complementarities in production, which leads productive firms to offer relatively high wages to better workers and thus incentivizes better workers to sort into productive firms. We now perform counterfactuals that help quantify the importance of these distinct reasons for sorting.

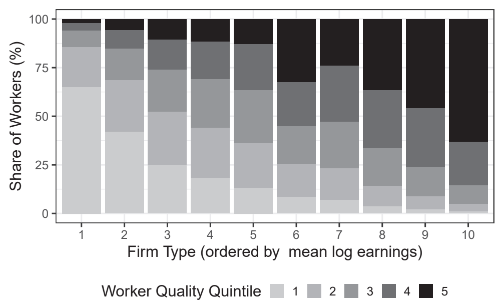
In the counterfactuals we consider, we reduce the heterogeneity across firms in amenities or production complementarities by replacing either $g_j(x)$ with $(1-s)\,g_j(x) + s\overline{g}_j$ or θ_j with $(1-s)\,\theta_j + s\overline{\theta}$, where $\overline{g}_j = \mathbb{E}_x\,[g_j(x)]$ and $\overline{\theta} = \mathbb{E}\,[\theta_j]$. Here, $s \in [0, 1]$ is the shrink rate with $s = 0$



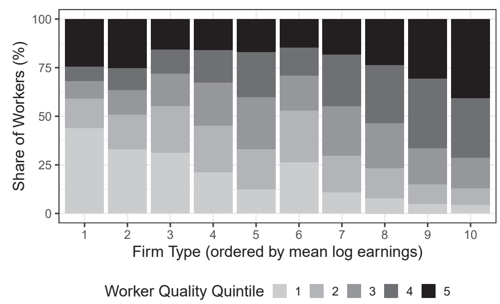
(a) Actual: Baseline Estimates



(b) Actual: Simulating from the Equilibrium Model



(c) Counterfactual: Shrink $g_j(x)$



(d) Counterfactual: Shrink θ_j

Figure 3: Actual and counterfactual composition of the workforce by firm types

Notes: In this figure, we first compare the baseline estimates of the worker quality composition by firm type from the equation for firm wage premiums (15) in subfigure (a) versus those estimated using the equilibrium constraint by solving the fixed-point definition of h_j as a function of $(\tilde{P}_j, \overline{P}_r, G_j(X))$, as shown in Lemma 3 in Online Appendix A.1 then simulating the sorting of workers to firms (subfigure b). Then, we reduce the heterogeneity across firms in amenities or production complementarities by replacing either $g_j(x)$ with $(1-s)\, g_j(x) + s\overline{g}_j$ or θ_j with $(1-s)\, \theta_j + s\overline{\theta}$, where $\overline{g}_j = \mathbb{E}_x\left[g_j(x)\right]$, $\overline{\theta} = \mathbb{E}\left[\theta_j\right]$, then re-simulate the equilibrium. Here, $s \in [0,1]$ is the shrink rate with $s = 0$ corresponding to the baseline model. We report the quality of the workforce by firm type for the counterfactual economies with $s = \frac{1}{2}$ for either amenities (subfigure c) or production complementarities (subfigure d).

corresponding to the baseline model. By reducing the heterogeneity in production complementarities, we are effectively making amenities more important for the allocation of workers to firms (and vice versa). Keeping ψ_{jt} fixed at baseline values ($s = 0$), we solve for the counterfactual allocation of workers to firms given the chosen counterfactual values of $g_j(x)$ or θ_j.

Figures 3(c) and 3(d) illustrate the importance of amenities versus production complementarities for the sorting of workers to firms. Here, we solve the equilibrium counterfactual economies with $s = \frac{1}{2}$ for either amenities (subfigure c) or production complementarities (subfigure d). The results suggest that production complementarities are the key reason why better workers are sorting into higher paying firms. Online Appendix Figure A.7 complements these results by plotting estimates of $Corr(x_i, \psi_{j(i,t)})$ and $2Cov(x_i, \psi_{j(i,t)})$ for counterfactual economies with many values of s. These findings indicate that production complementarities are the driving force of the strong positive correlation between worker and firm effects and the significant inequality contribution from worker sorting.

		(1) Monopsonistic Labor Market	(2) No Labor or Tax Wedges	Difference between (1) and (2)
Log of Expected Output	$\log \mathbb{E}[Y_{jt}]$	11.38	11.41	0.03
Total Welfare (log dollars)		12.16	12.21	0.05
Sorting Correlation	$Cor(\psi_{jt}, x_i)$	0.44	0.47	0.03
Labor Wedges	$1 + \frac{\rho_r}{\beta\lambda}$	1.15	1.00	-0.15
Worker Rents (as share of earnings):				
Firm-level	$\frac{\rho_r}{\rho_r+\beta\lambda}$	13.3%	12.4%	-0.9%
Market-level	$\frac{1}{1+\beta\lambda}$	18.0%	16.7%	-1.3%

Table 5: Consequences of Eliminating Tax and Labor Wedges

Notes: This table compares the monopsonistic labor market to a counterfactual economy which differs in two ways. First, we eliminate the tax wedge in the first order condition by setting the tax progressivity $(1 - \lambda)$ equal to zero. Second, we remove the labor wedges in the first order conditions of the firms by setting τ_r equal to the labor wedge $1 + \frac{\rho_r}{\lambda\beta}$ in each market r. After changing these parameters of the model, we solve for the new equilibrium allocation and outcomes, including wages, output and welfare. Results are displayed for output, welfare, the sorting correlation, the mean labor wedge, and worker rents.

6.5 Implications of imperfect competition for progressive taxation and allocative efficiency

Our final set of insights from the model are to quantify the misallocation of workers to firms that arise because of the monopsonistic labor market, and to empirically illustrate how this misallocation may be corrected through tax policy.

As discussed in Section 2.5, there are two types of wedges. Within each market, there is a tax wedge that arises because there is a progressive tax on wages but not on amenities. As λ decreases and thereby the wage tax becomes more progressive, amenities become more valuable relative to (pre-tax) wages. This distorts the worker's ranking of firms in favor of those with better amenities. Thus, with progressive taxation, firms with better amenities can hire workers at relatively low wages, and, therefore, get too many workers as compared to the allocation in the competitive labor market. Between markets, allocative inefficiencies may arise not only because of the tax wedge but also due to differences in labor wedges across markets. This is because the labor supply curves and, as a result, the wage markdowns vary systematically across markets.

As shown in Section 2.5, the government can improve the allocation of workers to firms in two ways. First, a less progressive tax system may reduce the misallocation that arise from the tax wedge. Second, letting the tax rates vary across markets may improve the allocation by counteracting the differences in the wage-setting power of firms. We now use the estimated model to perform a counterfactual that quantifies the impacts of such a tax reform on the equilibrium allocation and outcomes, including wages, output and welfare.

The counterfactual we consider involves two changes to the monopsonistic labor market. First, we eliminate the tax wedge in the first order condition, which distorts the worker's ranking of firms in favor of those with better amenities. This is done by setting the tax progressivity

$(1-\lambda)$ equal to zero. Second, we remove the labor wedges in the first order conditions of the firms. These wedges cause misallocation of workers across firms with different degrees of wage-setting power. As shown in Lemma 7 in Online Appendix A.4, labor wedges can be eliminated by setting τ_r equal to the labor wedge $1+\frac{\rho_r}{\lambda\beta}$ in each market r. After changing these parameters of the model, we solve for the new equilibrium allocation and outcomes, including wages, output and welfare. For a set of wages $\{W_{jt}(X)\}_{j,t}$ and a tax policy (λ, τ), we define the welfare as:

$$\mathbb{W}_t = \mathbb{E}\left[\max_j u_{it}\left(j, (1+\phi_t)\tau W_{jt}(X_i)^{\lambda}\right)\right]$$

where ϕ_t is the government spending rule set so that the government budget clears and profits and tax revenues are distributed among all the workers in proportion to their earnings:

$$\underbrace{\phi_t \cdot \mathbb{E}\left[\tau W_{jt}(X_i)^{\lambda}\right]}_{\text{redistribution}} = \underbrace{\frac{1}{N}\sum \Pi_{jt}}_{\text{profits}} + \underbrace{\mathbb{E}\left[W_j(X_i) - \tau W_j(X_i)^{\lambda}\right]}_{\text{government revenue}},$$

In other words, we redistribute aggregate profits and government tax revenues to workers in a non-distortionary way.

The results are presented in Table 5. They suggest the monopsonistic labor market creates significant misallocation of workers to firms. Eliminating labor and tax wedges increases total welfare by 5 percent and total output by 3 percent. When we decompose this change by performing the counterfactuals one at a time, we find that 4 percentage points of the welfare gains are due to eliminating the labor wedge while the remaining 1 percentage point is due to eliminating the tax wedge. We also find that removing these wedges would increase the sorting of better workers to higher paying firms and lower the rents that workers earn from ongoing employment relationships. When we decompose this change by performing the counterfactuals one at a time, we find that nearly all of the change in sorting is due to eliminating the tax wedge, with the labor wedge having a small impact on sorting.

In interpreting these results, it is important to recall that we assume firms initially may choose amenities $g_j(x)$, but they do not change $g_j(x)$ in the counterfactuals. With better data on, and an instrument for, amenities, it would be interesting to extend this analysis to allow for firms to adjust amenities in response to these counterfactuals.

7 Conclusion

The goal of our paper was to quantify the importance of imperfect competition in the U.S. labor market by estimating the size of rents earned by American firms and workers from ongoing employment relationships. To this end, we constructed a matched employer-employee panel data set by combining the universe of U.S. business and worker tax records for the period 2001-2015. Using this panel data, we identified and estimated an equilibrium model of the labor market with two-sided heterogeneity where workers view firms as imperfect substitutes because of heterogeneous preferences over non-wage job characteristics. The model allowed us to draw inference about imperfect competition, compensating differentials and rent sharing.

We also used the model to quantify the relevance of non-wage job characteristics and imperfect competition for inequality and tax policy, to assess the economic determinants of worker sorting, and to offer a unifying explanation of key empirical features of the U.S. labor market.

When considering the interpretation and generality of our study, we emphasize a few caveats and extensions. One of these is that we focus on distortions in the allocation of workers to firms and markets. However, tax and labor wedges may also distort the choices of whether and how much to work. Related, we do not consider unemployment, and, as a result, we are reluctant to draw conclusions about how imperfect competition matters for the impact of minimum wages. Doing so is an important but challenging task, as it requires identification of the value of non-employment and a non-linear supply curve. We also assume the labor market is a spot market and, thus, we are unable to analyze the role of long-term contracts and firm insurance against shocks.[21] Furthermore, our structural model makes several simplifying assumptions, partly because of data availability but also to prove identification. For example, we abstract from observed heterogeneity in preferences and skills and, moreover, model individual behavior, and hence do not consider any interdependencies between spouses in the choices of whether and where to work.[22] Moreover, we assume no mobility costs or search frictions, and we do not explicitly model human capital investments or work experience. While incorporating these features would be interesting, it would also present severe challenges to identification, especially if one allows for two-sided heterogeneity. Additionally, we focus on the wage-setting power of firms, and the analyses do not incorporate that firms may have price-setting power in the product market. Extending the model to allow for both forms of imperfect competition and how they interact is an important avenue for future research.[23] Lastly, we consider an equilibrium where each firm views itself as infinitesimal within the market. This assumption is motivated by the fact that very few firms in the U.S. have a large share of the local labor market (as measured by commuting zone). Thus, optimizing firms would essentially ignore the negligible effect of changing their own wages on the overall supply of workers to the market as a whole. However, if labor markets are sufficiently segmented (geographically or by industry), it is possible that strategic interactions can play an important role.[24]

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[21] See Balke and Lamadon (2020) for a model and empirical analysis of long-term contracts and firm insurance.

[22] Autor et al. (2019) and Blundell et al. (2016) estimate a life cycle model with two earners jointly making consumption and labor supply decisions. Their findings suggest an important role for consumption smoothing through household labor supply.

[23] Kroft et al. (2021) analyze imperfect competition in both the labor and the product market in the U.S. construction industry.

[24] See Berger et al. (2019) for an analysis of strategic interactions in the firms' wage setting and Jarosch et al. (2019) for a search framework with large firms. However, identification of such interaction effects is challenging with two-sided heterogeneity.

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A Details on Model Solutions

A.1 Derivation of equilibrium wages

Given the nested logit preferences and a given set of wages $\mathbf{W}_t = \{W_{jt}(X)\}_{j=1..J}$ we get that

$$Pr[j(i,t){=}j|X_i{=}X, \mathbf{W}_t] = \frac{\left(\sum_{j' \in J_{r(j)}} (\tau G_{j'}(X))^{\beta/\rho_{r(j)}} W_{j't}(X)^{\lambda\beta/\rho_{r(j)}}\right)^{\rho_{r(j)}}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} (\tau G_{j'}(X))^{\beta/\rho_{r'}} W_{j't}(X)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \times \frac{(\tau G_j(X))^{\beta/\rho_{r(j)}} W_{jt}(X)^{\lambda\beta/\rho_{r(j)}}}{\sum_{j' \in J_{r(j)}} (\tau G_{j'}(X))^{\beta/\rho_{r(j)}} W_{j't}(X)^{\lambda\beta/\rho_{r(j)}}}$$

and

$$\mathbb{E}\left[u_{it}|X_i = X, \mathbf{W}_t\right] = \frac{1}{\beta}\left[\log\left(\sum_r \left(\sum_{j \in J_r} (\tau G_j(X))^{\beta/\rho_r} (W_{jt}(X))^{\lambda\beta/\rho_r}\right)^{\rho_r} + \overline{C}\right)\right],$$

where $\overline{C}$ is an unrecoverable constant. It is useful to introduce the following definition before stating the Lemmas:

$$C_r \equiv \frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\lambda\beta/\rho_r}.$$

Lemma 1. *Assume that firms believe they are strategically small. That is, in the firm's first order condition, we impose that*

$$\frac{\partial I_{rt}(X)}{\partial W_{jt}(X)} = 0.$$

We can then show that for firm j in market r

$$Y_{jt} = (A_{jt})^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} (H_{jt})^{1-\alpha_r} \tag{16}$$

$$W_{jt}(X) = C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \tag{17}$$

$$L_{jt} = H_{jt} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}, \tag{18}$$

where H_{jt} is implicitly defined by

$$H_{jt} \equiv \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X) (\tau G_j(X))^{\beta/\rho_r} C_r^{\lambda\beta/\rho_r} dX\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

and we define

$$K_{rt}(X) \equiv NM(X)\frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r},$$

$$I_{rt}(X) \equiv \left(\sum_{j'\in J_r}\left(\tau^{1/\lambda}G_{j'}(X)^{1/\lambda}C_r X^{\theta_{j'}}A_{j't}\left(\frac{Y_{j't}}{A_{j't}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)}.$$

Proof. We start from the firm's problem specified in the main text including the tax parameters. Using shorthand r for $r(j)$, we have

$$\max_{\{W_{jt}(X),D_{jt}(X)\}} A_{jt}\left(\int X^{\theta_j}D_{jt}(X)\mathrm{d}X\right)^{1-\alpha_r} - \int W_{jt}(X)D_{jt}(X)\mathrm{d}X$$

$$\text{s.t. } D_{jt}(X) = NM(X)\frac{I_{rt}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(G_j(X)^{1/\lambda}\tau^{1/\lambda}\frac{W_{jt}(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}$$

and define:

$$K_{rt}(X) \equiv NM(X)\frac{I_{rt}(X)^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(\frac{1}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r}.$$

We substitute in the labor supply function and derive the first order condition with respect to $W_{jt}(X)$:

$$(1-\alpha_r)X^{\theta_j}\left(\frac{\lambda\beta}{\rho_r}W_{jt}(X)^{\lambda\beta/\rho_r-1} + \frac{1}{K_{rt}(X)}\frac{\partial K_{rt}(X)}{\partial W_{jt}(X)}W_{jt}(X)^{\lambda\beta/\rho_r}\right)\tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}$$
$$= \tau^{\beta/\rho_r}G_j(X)^{\beta/\rho_r}\left(\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X)^{\lambda\beta/\rho_r} + \frac{1}{K_{rt}(X)}\frac{\partial K_{rt}(X)}{\partial W_{jt}(X)}W_{jt}(X)^{1+\lambda\beta/\rho_r}\right).$$

Under the assumption that $\frac{\partial I_{rt}(X)}{\partial W_{jt}(X)} = 0$, the first order condition simplifies to

$$\left(1+\frac{\lambda\beta}{\rho_r}\right)W_{jt}(X) = \frac{\lambda\beta}{\rho_r}(1-\alpha_r)X^{\theta_j}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}},$$

or

$$W_{jt}(X) = C_r X^{\theta_j}A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}.$$

Turning to the output of the firm,

$$Y_{jt}/A_{jt} = \left(\int X^{\theta_j}K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r}W_{jt}(X)^{\lambda\beta/\rho_r}\mathrm{d}X\right)^{1-\alpha_r}$$
$$= \left(\int \left(X^{\theta_j}\right)^{1+\lambda\beta/\rho_r}K_{rt}(X)\left(\tau G_j(X)\right)^{\beta/\rho_r}\left(C_rA_{jt}\right)^{\lambda\beta/\rho_r}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r\lambda\beta/\rho_r}{1-\alpha_r}}\mathrm{d}X\right)^{1-\alpha_r}$$

and so:

$$(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} = \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\,(\tau G_j(X))^{\beta/\rho_r}\, C_r^{\lambda\beta/\rho_r}\mathrm{d}X\right)^{1-\alpha_r} (A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r}.$$

Introducing

$$H_{jt} \equiv \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\,(\tau G_j(X))^{\beta/\rho_r}\, C_r^{\lambda\beta/\rho_r}\mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}},$$

we can simplify the previous expression as

$$\begin{aligned}(Y_{jt}/A_{jt})^{1+\alpha_r\lambda\beta/\rho_r} &= (H_{jt})^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)}\,(A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r},\\ Y_{jt} &= (A_{jt})^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\,(H_{jt})^{1-\alpha_r}.\end{aligned}$$

Then, we can write the wage as

$$\begin{aligned}W_{jt}(X) &= C_r X^{\theta_j} A_{jt}\left(\frac{Y_{jt}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\\ &= C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{(1+\alpha_r\lambda\beta/\rho_r)}}.\end{aligned}$$

Finally, we can write the efficiency units of labor as

$$\begin{aligned}L_{jt} &= \int X^{\theta_j} K_{rt}(X)\,(\tau G_j(X))^{\beta/\rho_r}\, W_{jt}(X)^{\lambda\beta/\rho_r}\mathrm{d}X\\ &= \int X^{\theta_j(1+\lambda\beta/\rho_r)} K_{rt}(X)\,(\tau G_j(X))^{\beta/\rho_r}\left(C_r H_{jt}^{-\alpha_r}\right)^{\lambda\beta/\rho_r}(A_{jt})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\,\mathrm{d}X\\ &= H_{jt}^{1+\alpha_r\lambda\beta/\rho_r-\alpha_r\lambda\beta/\rho_r} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\\ &= H_{jt}A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}.\end{aligned}$$

□

Lemma 2 (Uniqueness of H_{jt})**.** *The firm- and time-specific equilibrium constants H_{jt} are uniquely defined.*

Proof. As we have established in Lemma 1, for firm j in market r, H_{jt} solves the following system:

$$H_{jt} = \left[\int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'} \lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1} \right.$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r}^{\lambda} H_{j't}^{-\alpha_{r} \lambda} \right)^{\beta/\rho_{r}} A_{j't}^{\frac{\lambda\beta/\rho_{r}}{1+\alpha_{r}\lambda\beta/\rho_{r}}} \right)^{\rho_r - 1}$$
$$\left. \times X^{\theta_j (1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} NM(X) \mathrm{d}X \right]^{\frac{1}{1+\alpha_r \lambda\beta/\rho_r}},$$

where we have replaced $K_{rt}(X)$ and then $I_{rt}(X)$ and finally Y_{jt} with their expressions in terms of H_{jt}. We will show that $\tilde{H}_{jt} \equiv (H_{jt})^{\alpha_r}$ is unique, which implies that H_{jt} is unique. Defining $\vec{H}_t \equiv (\tilde{H}_{1t}, ..., \tilde{H}_{Jt})$, we will show that $\vec{H}_t$ solves the following fixed point expression:

$$\tilde{H}_{jt} = \left[\int \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1} \right. \tag{19}$$
$$\times \left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r}} A_{j't}^{\frac{\lambda\beta/\rho_{r}}{1+\alpha_{r}\lambda\beta/\rho_{r}}} \right)^{\rho_r - 1}$$
$$\left. \times X^{\theta_j (1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} NM(X) \mathrm{d}X \right]^{\frac{\alpha_r}{1+\alpha_r \lambda\beta/\rho_r}}$$
$$= \Gamma_{jt}(\vec{H}_t).$$

We show that this expression satisfies the two conditions required to apply Theorem 1 of Kennan (2001). We first consider the component that is common to all j given by

$$\bar{\Gamma}_t(X, \vec{H}_t) \equiv \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$

and see that

$$\bar{\Gamma}_t(X, \mu \cdot \vec{H}_t) = \left(\sum_{r'} \mu^{-\lambda\beta} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} \tilde{H}_{j't}^{-\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1}$$
$$= \mu^{\lambda\beta} \bar{\Gamma}_t(X, \vec{H}_t).$$

Hence

$$
\begin{aligned}
\Gamma_{jt}(\mu\cdot\vec{H}_t) =& \Bigg[\int X^{\theta_j(1+\lambda\beta/\rho_r)}\bar{\Gamma}_t(X,\mu\cdot\vec{H}_t)\left(\tau G_j(X)C_r^{\lambda}\right)^{\beta/\rho_r} \\
&\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)\mu^{-\lambda}C_r^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} NM(X)\mathrm{d}X\Bigg]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
=&\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\Bigg[\int X^{\theta_j(1+\beta/\rho_r)}\bar{\Gamma}_t(X,\vec{H}_t)\left(\tau G_j(X)C_r^{\lambda}\right)^{\beta/\rho_r} \\
&\times\left(\sum_{j'\in J_r}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_r^{\lambda}\tilde{H}_{j't}^{-\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r-1} NM(X)\mathrm{d}X\Bigg]^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
=&\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\Gamma_{jt}(\vec{H}_t).
\end{aligned}
$$

Then for any $0<\mu<1$, r and $j\in J_r$, given $\vec{H}_t>0$ such that $\Gamma_t(\vec{H}_t)=\vec{H}_t$, where $\Gamma_t(\cdot)\equiv(\Gamma_{1t}(\cdot),...,\Gamma_{Jt}(\cdot))$, we have

$$
\begin{aligned}
\Gamma_{jt}(\mu\cdot\vec{H}_t)-\mu\cdot\tilde{H}_{jt} &= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\Gamma_{jt}(\vec{H}_t)-\mu\cdot\tilde{H}_{jt} \\
&= \mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\cdot\tilde{H}_{jt}-\mu\cdot\tilde{H}_{jt} \\
&= \mu\underbrace{\left(\mu^{\frac{\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}-1}-1\right)}_{>0}\cdot\tilde{H}_{jt} \\
&> 0,
\end{aligned}
$$

which means that we have shown that $\Gamma_t(\vec{H}_t)-\vec{H}_t$ is strictly "radially quasi-concave". The next step is to show monotonicity. Consider $\vec{H}_{1t}$ and $\vec{H}_{2t}$ such that for a given j we have $\tilde{H}_{1jt}=\tilde{H}_{2jt}$ and $\tilde{H}_{1j't}\leq\tilde{H}_{2j't}$ for all other $j'\neq j$. Then we have that for all j', t, X and $r'=r(j')$,

$$
\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \geq \left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}
$$

and for any r',

$$
\sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{1j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \geq \sum_{j'\in J_{r'}}\left(X^{\lambda\theta_{j'}}\tau G_{j'}(X)C_{r'}^{\lambda}\tilde{H}_{2j't}^{-\lambda}\right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}}.
$$

Hence, summing over r' and taking it to the power of minus one, this implies that $\bar{\Gamma}_t(X,\vec{H}_{1t})\leq$

$\bar{\Gamma}_t(X, \vec{H}_{2t})$. Then, since $\rho_r \leq 1$ we also have that

$$\left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_r^{\lambda} \tilde{H}_{1j't}^{-\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r \lambda\beta/\rho_r}} \right)^{\rho_r - 1} \leq \left(\sum_{j' \in J_r} \left(X^{\lambda \theta_{j'}} \tau G_{j'}(X) C_r^{\lambda} \tilde{H}_{2j't}^{-\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r \lambda\beta/\rho_r}} \right)^{\rho_r - 1}.$$

Combining the last two results and observing that the third term in the expression for $\Gamma_{jt}(\vec{H}_t)$ is the same for $\vec{H}_{1t}$ and $\vec{H}_{2t}$ gives us that:

$$\Gamma_{jt}(\vec{H}_{1t}) \leq \Gamma_{jt}(\vec{H}_{2t}).$$

Then

$$\Gamma_{jt}(\vec{H}_{1t}) - \tilde{H}_{1j't} \leq \Gamma_{jt}(\vec{H}_{2t}) - \tilde{H}_{2j't},$$

and since the last inequality holds for all j, we obtain the quasi-increasing property:

$$\Gamma_{jt}(\vec{H}_{1t}) - \vec{H}_{1t} \leq \Gamma_{jt}(\vec{H}_{2t}) - \vec{H}_{2t}.$$

The fact that the function is "radially quasi-concave" together with monotonicity gives uniqueness of the fixed point by the theorem in Kennan (2001). This means that $\vec{H}_t$ is unique, and hence that $\tilde{H}_{jt}$ is unique and finally that H_{jt} is unique. □

Definition 2. We consider a sequence of increasingly larger economies indexed by an increasing number of regions n^{r} where $n_r^{\mathrm{f}} = \kappa_r n^{\mathrm{r}}$ for some fixed κ_r. In this sequence of economies we assume that the amenities scale according to $G_j(X) = \mathring{G}_j(X) \left(n_{r(j)}^{\mathrm{f}} \right)^{-\rho_{r(j)}/\beta}$ for some fixed $\mathring{G}_j(X)$. We also assume that the mass of workers grows according to $N = n^{\mathrm{r}} \cdot \bar{n}^{\mathrm{f}} \cdot \mathring{N} = n^{\mathrm{r}} \cdot n^{\mathrm{r}} \cdot \bar{\kappa} \cdot \mathring{N}$, where $\bar{n}^{\mathrm{f}}$ is the average of n_r^{f} and $\bar{\kappa}$ is the average of κ_r.

Lemma 3. *The unique solution for H_{jt} in the limit of a sequence of growing economies is given by*

$$H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r - 1)}{(1+\alpha_r\lambda\beta/\rho_r)}},$$

where H_j solves the following fixed point:

$$H_j = \left(\int X^{\theta_j} \left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) dX \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

$$I_{r0}(X)^{\lambda\beta/\rho_r} \equiv \mathbb{E}_j \left[\left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} H_j^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$

$$I_0(X)^{\lambda\beta} \equiv \mathbb{E}_r \left[I_{r0}(X)^{\lambda\beta} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right].$$

Proof. Consider the expression for H_{jt} from the beginning of Lemma 2:

$$\begin{aligned} H_{jt} = \Bigg[\int & \left(\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1} \\ & \times \left(\sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau G_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1} \\ & \times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau G_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} N M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}. \end{aligned}$$

We substitute in n^{r} , n_r^{f}, κ_r, $\mathring{G}_j(X) = \left(n_{r(j)}^{\mathrm{f}} \right)^{\rho_{r(j)}/\beta} G_j(X)$ and $\mathring{N} = (n^{\mathrm{r}}\ n^{\mathrm{r}}\ \bar{\kappa})^{-1} N$:

$$\begin{aligned} H_{jt} = \Bigg[\int & \left(\frac{1}{n^{\mathrm{r}}} \sum_{r'} \left(\frac{1}{n_{r'}^{\mathrm{f}}} \sum_{j' \in J_{r'}} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right)^{\rho_{r'}} \right)^{-1} \\ & \times \left(\frac{1}{n_r^{\mathrm{f}}} \sum_{j' \in J_r} \left(X^{\lambda\theta_{j'}} \tau \mathring{G}_j(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right)^{\rho_r - 1} \\ & \times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}. \end{aligned}$$

As the economy grows large, i.e. as n^{r} grows to infinity, we have

$$\begin{aligned} H_{jt} = \Bigg[\int & \left(\mathbb{E}_{r'} \left[\left(\mathbb{E}_{j' \in J_{r'}} \left[\left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_{r'}^{\lambda} H_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \right] \right)^{\rho_{r'}} \right] \right)^{-1} \\ & \times \left(\mathbb{E}_{j' \in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \right)^{\rho_r - 1} \\ & \times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}. \end{aligned}$$

Next we show that H_{jt} can indeed be expressed as stated in this Lemma. We guess that

$H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$ and verify that it solves the problem. To verify, note that

$$
\begin{aligned}
&\mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_{j'}(X)C_r^{\lambda}H_{j't}^{-\alpha_r\lambda}\right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right] \\
&\quad = \mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_{j'}(X)C_r^{\lambda}H_{j'}^{-\alpha_r\lambda}\right)^{\beta/\rho_r} \times \bar{A}_{rt}^{-\alpha_r\lambda\beta/\rho_r\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right] \\
&\quad = \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}\mathbb{E}_{j'\in J_r}\left[\left(X^{\lambda\theta_{j'}}\tau\mathring{G}_{j'}(X)C_r^{\lambda}H_{j'}^{-\alpha_r\lambda}\right)^{\beta/\rho_r}\tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right] \\
&\quad = \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r},
\end{aligned}
$$

where we used $A_{jt} = \bar{A}_{r(j)t}\tilde{A}_{jt}$. Hence

$$
\begin{aligned}
H_{jt} =& \Bigg[\int\left(\mathbb{E}_{r'}\left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta}\right]\right)^{-1} \times \left(\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r}\right)^{\rho_r-1} \\
&\qquad \times X^{\theta_j(1+\lambda\beta/\rho_r)}\left(\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
=& \left[\int X^{\theta_j}\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&\qquad \times \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}\frac{\rho_r-1}{1+\alpha_r\lambda\beta/\rho_r}} \\
=& H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}},
\end{aligned}
$$

where we used that H_j solves

$$
H_j = \left[\int X^{\theta_j}\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta}\left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r}\left(X^{\lambda\theta_j}\tau\mathring{G}_j(X)C_r^{\lambda}\right)^{\beta/\rho_r}\frac{\bar{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}
$$

with

$$
I_0(X) \equiv \left(\mathbb{E}_{r'}\left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} I_{r'0}(X)^{\lambda\beta}\right]\right)^{1/(\lambda\beta)}.
$$

□

We can then establish the final result.

Proposition 1. *The wage equation is given by*

$$
w_j(x, \bar{a}, \tilde{a}) = c_r + \theta_j x - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha_r\lambda\beta}\bar{a},
$$

where

$$
h_j = \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta}\bar{a}_{rt}.
$$

Proof. Recall $L_{jt} = H_{jt} A_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}$ from Lemma 1 and $H_{jt} = H_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}$ from Lemma 3. Then:

$$\begin{aligned} h_{jt} &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} a_{jt} \\ &= \frac{(\rho_r - 1)\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)(1+\alpha_r\lambda\beta/\rho_r)} \bar{a}_{rt} + h_j. \end{aligned}$$

Hence, we get

$$\begin{aligned} h_j &= \ell_{jt} - \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}_{jt} - \frac{\lambda\beta}{1+\alpha_r\lambda\beta} \bar{a}_{rt} \\ \ell_{jt} &= h_j + \frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}_{jt} + \frac{\lambda\beta}{1+\alpha_r\lambda\beta} \bar{a}_{rt} \\ &\equiv \ell_j(\bar{a}_{rt}, \tilde{a}_{jt}). \end{aligned}$$

Next, we replace H_{jt} and A_{jt} in the expression for the wage from Lemma 1, $W_{jt}(X) = C_r X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$ to get

$$\begin{aligned} w_{jt}(x) &= c_r + \theta_j x - \alpha_r h_j + \frac{1}{1+\alpha_r\lambda\beta/\rho_r} \tilde{a}_{jt} + \frac{1}{1+\alpha_r\lambda\beta} \bar{a}_{rt} \\ &\equiv w_j(x, \bar{a}_{rt}, \tilde{a}_{jt}). \end{aligned}$$

Note that $w_{jt}(x)$ depends on time only through $\bar{a}_{rt}$ and $\tilde{a}_{jt}$.

□

Corollary 1. *The firm's demand for labor is given by:*

$$D_{jt}(X) = \frac{N}{n^r} M(X) \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)} \right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda} W_{jt}(X)}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}} \right)^{\lambda\beta/\rho_r}.$$

Proof. As n^{r} grows to infinity, we first note:

$$\begin{aligned} I_{rt}(X)^{\lambda\beta/\rho_r} &= \sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} C_r X^{\theta_{j'}} A_{j't} \left(\frac{Y_{j't}}{A_{j't}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} \right)^{\lambda\beta/\rho_r} \\ &= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \frac{1}{n_r^{\mathrm{f}}} \sum_{j' \in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} H_{j'}^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\ &= \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} I_{r0}(X)^{\lambda\beta/\rho_r} \\ I_{rt}(X) &= \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}} I_{r0}(X). \end{aligned}$$

The firm's demand can then be written as:

$$
\begin{aligned}
D_{jt}(X) &= NM(X)\frac{(I_{rt}(X))^{\lambda\beta}}{\sum_{r'} I_{r't}(X)^{\lambda\beta}}\left(G_j(X)^{1/\lambda}\tau^{1/\lambda}\frac{W_{jt}(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} \\
&= \frac{N}{n^{\mathrm{r}}}M(X)\left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}W_{jt}(X)}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r}.
\end{aligned}
$$

□

We also derive the other quantities of the model.

Corollary 2. *The firm's value added and wage bill are given by*

$$
\begin{aligned}
y_j(\overline{a},\tilde{a}) =&(1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a} \\
b_j(\overline{a},\tilde{a}) =&c_r + (1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\overline{a} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}.
\end{aligned}
$$

Proof. For the firm's value added, note that

$$
\begin{aligned}
Y_{jt} &= H_{jt}^{1-\alpha_r}A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(H_j\cdot\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}}\right)^{1-\alpha_r}\left(\bar{A}_{rt}\tilde{A}_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
y_{jt} &= (1-\alpha_r)h_j + \left(\frac{1+\lambda\beta}{1+\alpha_r\lambda\beta}\right)\overline{a}_{rt} + \left(\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\right)\tilde{a}_{jt} \\
&\equiv y_j(\overline{a}_{rt},\tilde{a}_{jt})
\end{aligned}
$$

and for the wage bill,

$$
\begin{aligned}
B_{jt} &= \int W_{jt}(X)D_{jt}(X)\mathrm{d}X \\
&= \int W_{jt}(X)\left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r}(W_{jt}(X))^{\lambda\beta/\rho_r}\,\frac{NM(X)}{n^{\mathrm{r}}}\mathrm{d}X \\
&= \int\left(\frac{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta}\left(\frac{\tau^{1/\lambda}G_j(X)^{1/\lambda}}{I_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r} \\
&\qquad\times\left(C_rX^{\theta_j}H_j^{-\alpha_r}\right)^{1+\lambda\beta/\rho_r}\left(\tilde{A}_{jt}\right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\bar{A}_{rt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}}n^{\mathrm{r}}\bar{\kappa}\mathring{N}M(X)\mathrm{d}X \\
b_{jt} &= c_r + (1-\alpha_r)h_j + \frac{1+\lambda\beta}{1+\alpha\beta}\overline{a}_{rt} + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\tilde{a}_{jt} \\
&= b_j(\overline{a}_{rt},\tilde{a}_{jt}).
\end{aligned}
$$

It follows that

$$y_j(\overline{a}, \tilde{a}) - b_j(\overline{a}, \tilde{a}) = c_r.$$

□

Note that the previous expressions deliver the structural pass-through rates of market and firm level shocks (with abuse of notation):

$$\frac{\partial w_j(x, \overline{a}_{rt}, \tilde{a}_{jt})}{\partial \overline{a}} \cdot \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \overline{a}}\right)^{-1} = \frac{1}{1 + \lambda\beta}$$
$$\frac{\partial w_j(x, \overline{a}_{rt}, \tilde{a}_{jt})}{\partial \tilde{a}} \cdot \left(\frac{\partial y_j(\overline{a}, \tilde{a})}{\partial \tilde{a}}\right)^{-1} = \frac{\rho_r}{\rho_r + \lambda\beta}.$$

Corollary 3. *Firm j worker composition does not depend on $\overline{a}$ or $\tilde{a}$.*

Proof. Consider $\Pr[X|j,t]$:

$$\begin{aligned}
\Pr[X|j,t] &= \Pr[X, j|t]/\Pr[j|t] \\
&= \frac{\Pr[j|X,t]\Pr[X]}{\int \Pr[j|X',t] M(X') \mathrm{d}X'} \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r} \left(\tau \mathring{G}_j(X) W_{jt}(X)^{\lambda}\right)^{\beta/\rho_r} M(X)}{\int \left(\frac{I_{r0}(X')}{I_0(X')}\right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X')}\right)^{\lambda\beta/\rho_r} \left(\tau \mathring{G}_j(X') W_{jt}(X')^{\lambda}\right)^{\beta/\rho_r} M(X') \mathrm{d}X'} \\
&= \frac{\left(\frac{I_{r0}(X)}{I_0(X)}\right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)}\right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \mathring{G}_j(X)\right)^{\beta/\rho_r} M(X)}{\int \left(\frac{I_{r0}(X')}{I_0(X')}\right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X')}\right)^{\lambda\beta/\rho_r} \left(X'^{\lambda\theta_j} \mathring{G}(X')\right)^{\beta/\rho_r} M(X') \mathrm{d}X'} \\
&= \Pr[X|j].
\end{aligned}$$

where we used the fact that

$$\begin{aligned}
\Pr[j|X,t] &= D_{jt}(X)/M(X) \\
&= \frac{N}{n^{\mathrm{r}}} \left(\frac{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}{I_0(X)}\right)^{\lambda\beta} \left(\frac{\tau^{1/\lambda} G_j(X)^{1/\lambda} W_{jt}(X)}{I_{r0}(X) \bar{A}_{rt}^{\frac{1}{1+\alpha_r\lambda\beta}}}\right)^{\lambda\beta/\rho_r}.
\end{aligned}$$

□

A.2 Worker rents

Lemma 4. *We establish that for workers of type X working at firm j in market r at time t, the average firm-level rents are given by $\frac{W_{jt}(X)}{1+\lambda\beta/\rho_r}$ and the average market level rents are given by $\frac{W_{jt}(X)}{1+\lambda\beta}$.*

Proof. The average worker rents at the firm are defined as the difference between the worker's willingness to accept W and the wage they actually get at firm j at time t, denoted by $W_{jt}(X)$. The supply curve $S_{jt}(X, W)$ exactly defines the number of people willing to work at firm j at some given wage W. Hence, the density of the willingness to accept among workers in firm j at time t at wage $W_{jt}(X)$ is given by:

$$\frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial S_{jt}(X, W)}{\partial W}.$$

We obtain the average rents by taking the expectation with respect to this density:

$$\begin{aligned}
R^w_{jt}(X) &\equiv \mathbb{E}\left[R^w_{it} \,|\, j(i,t) = j, X_i = X\right] \\
&= \int_0^{W_{jt}(X)} (W_{jt}(X) - W) \frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial S_{jt}(X, W)}{\partial W} \mathrm{d}W \\
&= W_{jt}(X) \int_0^1 (1-\omega) \frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial S_{jt}(X, \omega W_{jt}(X))}{\partial \omega} \mathrm{d}\omega \\
&= W_{jt}(X) \int_0^1 (1-\omega) \frac{\partial \omega^{\lambda\beta/\rho_r}}{\partial \omega} \mathrm{d}\omega \\
&= \frac{W_{jt}(X)}{1 + \lambda\beta/\rho_r},
\end{aligned}$$

where the second to last step relies on the definition of $S_{jt}(X, W)$ and the fact that we assume the presence of many firms in each market to show that $S_{jt}(X, \omega W) = \omega^{\lambda\beta/\rho_r} S_{jt}(X, W)$. We can then take the average over the productivity levels X_i of the workers i in firm $j \in J_r$ at time t to get:

$$\begin{aligned}
\mathbb{E}\left[R^w_{it} \,|\, j(i,t) = j\right] &= \mathbb{E}\left[R^w_{jt}(X_i) \,|\, j(i,t) = j\right] \\
&= \frac{1}{1 + \lambda\beta/\rho_r} \mathbb{E}\left[W_{jt}(X_i) \,|\, j(i,t) = j\right].
\end{aligned}$$

Next we want to compute the integral of the market-level supply curve for each worker of type X. In contrast to the worker rents at the firm level, we want to shift the wages of all firms in a given market for a given individual. This means that we want to shift both the current firm j but also all other firms j' in market r. Given the labor supply curve of firm j, we integrate by scaling all wages in market r by ω in $[0, 1]$. More precisely, we consider the demand realized by the set of wages $\left\{\omega^{\mathbf{1}[j \in J_r]} W_{jt}(X)\right\}_{jt}$ for a given market r. The supply curve of firm j in this market as a function of the scaling factor ω is then

$$
\begin{aligned}
N \cdot M(X) & \frac{\left(\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} W_{j't}(X)\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}} \\
& \times \frac{\left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{jt}(X)\right)^{\lambda\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \omega W_{j't}(X)\right)^{\lambda\beta/\rho_r}} \\
&= \omega^{\lambda\beta} S_{jt}(X, W_{jt}(X)),
\end{aligned}
$$

where we used the assumption that there are many markets in the first denominator. Hence, the market level density of the willingness to accept is given by

$$
\frac{1}{S_{jt}(X, W_{jt}(X))} \frac{\partial}{\partial \omega} \left[\omega^{\lambda\beta} S_{jt}(X, W_{jt}(X))\right].
$$

Using the same logic we used to solve for the firm level rents, we find

$$
\begin{aligned}
R^{wm}_{jt}(X) &\equiv \mathbb{E}\left[R^{wm}_{it} \,|\, j(i,t) = j, X_i = X\right] \\
&= \frac{W_{jt}(X)}{1 + \lambda\beta},
\end{aligned}
$$

and can finally compute the average market level rents across X_i as

$$
\begin{aligned}
\mathbb{E}\left[R^{wm}_{it} \,|\, j(i,t) = j\right] &= \mathbb{E}\left[R^{wm}_{jt}(X_i) \,|\, j(i,t) = j\right] \\
&= \frac{1}{1 + \lambda\beta} \mathbb{E}\left[W_{jt}(X_i) \,|\, j(i,t) = j\right].
\end{aligned}
$$

□

A.3 Employer rents

Lemma 5. *We establish that the firm rents are given by*

$$
R^f_{jt} = \Pi_{jt} - \Pi^{pt}_{jt} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r}\right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right) \Pi_{jt}.
$$

Proof. The firm rents are defined as the difference between the profit that a firm would make if it were a wage taker in the labor market and the profit it actually achieves when taking advantage of its wage setting power. To solve for the wage taker profit, we maximize

$$
\Pi^{\mathrm{pt}}_{jt} = \max_{\{D^{\mathrm{pt}}_{jt}(X)\}} A_{jt} \left(\int X^{\theta_j} \cdot D^{\mathrm{pt}}_{jt}(X) \mathrm{d}X\right)^{1-\alpha_r} - \int W^{\mathrm{pt}}_{jt}(X) \cdot D^{\mathrm{pt}}_{jt}(X) \mathrm{d}X,
$$

taking the wage $W^{\mathrm{pt}}_{jt}(X)$ as given, and then equate demand with the supply equation. The first order condition is

$$\underbrace{(1-\alpha_r)}_{\equiv C_r^{\mathrm{pt}}} A_{jt} X^{\theta_j} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}} = W_{jt}^{\mathrm{pt}}\,(X)$$

and the realized demand is given by

$$D_{jt}^{\mathrm{pt}}(X) = N \cdot M(X) \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} \left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \frac{W_{jt}^{\mathrm{pt}}\,(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r},$$

where we use $I(X)^{\lambda\beta} \equiv \sum_{r'} I_{r't}(X)^{\lambda\beta}$, assumed constant due to the large number of markets. We then get that

$$\begin{aligned}
\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} &= \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{pt}}(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}^{\mathrm{pt}}\,(X)}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\mathrm{pt}} A_{jt} X^{\theta_j}}{I_{rt}(X)} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= (A_{jt})^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C_r^{\mathrm{pt}}}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&\quad \times \left(\int X^{\theta_j} \left(\frac{I_{rt}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{X^{\theta_j} C_r}{I_{rt}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r}
\end{aligned}$$

and

$$\begin{aligned}
\left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}}\right)^{1+\alpha_r\lambda\beta/\rho_r} &= \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt}\right)^{(1-\alpha_r)\lambda\beta/\rho_r} H_{jt}^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \\
Y_{jt}^{\mathrm{pt}} &= \left(\frac{C_r^{\mathrm{pt}}}{C_r}\right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r\lambda\beta/\rho_r}} Y_{jt},
\end{aligned}$$

which we replace to get the wage

$$
\begin{aligned}
W_{jt}^{\mathrm{pt}}(X) &= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{Y_{jt}^{\mathrm{pt}}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} \\
&= C_r^{\mathrm{pt}} A_{jt} X^{\theta_j} \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r \lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot C_r A_{jt} X^{\theta_j} \left(\frac{C_r^{\mathrm{pt}}}{C_r} A_{jt} \right)^{-\alpha_r \frac{\lambda\beta/\rho_r}{(1+\alpha_r \lambda\beta/\rho_r)}} H_{jt}^{-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X).
\end{aligned}
$$

Similarly, we can express demand as

$$
D_{jt}^{\mathrm{pt}}(X) = \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X)
$$

and the wage bill as

$$
\begin{aligned}
B_{jt}^{\mathrm{pt}} &= \int W_{jt}^{\mathrm{pt}}(X) \cdot D_{jt}^{\mathrm{pt}}(X) \mathrm{d}X \\
&= \int \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} W_{jt}(X) \cdot \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} D_{jt}(X) \mathrm{d}X \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} B_{jt}.
\end{aligned}
$$

Next, we recall $Y_{jt} = A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt}^{1-\alpha_r}$ and get that:

$$
\begin{aligned}
B_{jt} &= \int W_{jt}(X) \cdot D_{jt}(X) \mathrm{d}X \\
&= \int X^{\theta_j} C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \cdot D_{jt}(X) \mathrm{d}X \\
&= C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \left(\frac{Y_{jt}}{A_{jt}} \right)^{\frac{1}{1-\alpha_r}} \\
&= C_r H_{jt}^{-\alpha_r} (A_{jt})^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} H_{jt} (A_{jt})^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta/\rho_r)}} \\
&= C_r Y_{jt}.
\end{aligned}
$$

Similarly, we get that $B^{\text{pt}}_{jt} = C^{\text{pt}}_r Y^{\text{pt}}_{jt}$. Finally, we see that

$$\begin{aligned}
\frac{\Pi_{jt} - \Pi^{\text{pt}}_{jt}}{\Pi_{jt}} &= 1 - \frac{Y^{\text{pt}}_{jt} - B^{\text{pt}}_{jt}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C^{\text{pt}}_r}{1 - C_r} \left(\frac{C^{\text{pt}}_r}{C_r} \right)^{\frac{\lambda\beta/\rho_r \cdot (1-\alpha_r)}{1+\alpha_r \lambda\beta/\rho_r}} \\
&= 1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}
\end{aligned}$$

$$\Pi_{jt} - \Pi^{\text{pt}}_{jt} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right) \Pi_{jt}.$$

□

Lemma 6. *We establish that the market level rents for firm $j \in J_r$ are given by*

$$R^{fm}_{jt} = \Pi_{jt} - \Pi^{ptm}_{jt} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r \lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \right) \Pi_{jt}.$$

Proof. Here we consider the case where all firms in a given market are wage takers. In this case we also get that the $I_{rt}(X)$ terms change. The firm's wage is still determined by the following first-order condition:

$$(1-\alpha_r) A_{jt} X^{\theta_j} \left(\frac{Y^{\text{ptm}}_{jt}}{A_{jt}} \right)^{-\frac{\alpha_r}{1-\alpha_r}} = W^{\text{ptm}}_{jt}(X).$$

However, the labor supply curve is no longer the same as in equilibrium since all firms change their labor demands:

$$S^{\text{ptm}}_{jt}(X, W) = NM(X) \left(\frac{I^{\text{ptm}}_{rt}(X)}{I(X)} \right)^{\lambda\beta} \left(G_j(X)^{1/\lambda} \frac{\tau^{1/\lambda} W}{I^{\text{ptm}}_{rt}(X)} \right)^{\lambda\beta/\rho_r},$$

where

$$I^{\text{ptm}}_{rt}(X) \equiv \left(\sum_{j' \in J_r} \left(\tau G_{j'}(X)\right)^{\beta/\rho_r} \left(W^{\text{ptm}}_{j't}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r/(\lambda\beta)}.$$

We insert these definitions into Y_{jt}^{ptm} to see that

$$
\begin{aligned}
\frac{Y_{jt}^{\text{ptm}}}{A_{jt}} &= \left(\int X^{\theta_j} \cdot D_{jt}^{\text{ptm}}(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{W_{jt}^{\text{ptm}}(X)}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{(1-\alpha_r)A_{jt}X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\left(\frac{Y_{jt}^{\text{ptm}}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{1-\alpha_r} \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r} \underbrace{\left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)}_{\equiv\left(H_{jt}^{\text{ptm}}\right)^{1+\alpha_r\lambda\beta/\rho_r}}{}^{1-\alpha_r} \left(\frac{Y_{jt}^{\text{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
&= A_{jt}^{(1-\alpha_r)\lambda\beta/\rho_r} \left(H_{jt}^{\text{ptm}}\right)^{(1-\alpha_r)(1+\alpha_r\lambda\beta/\rho_r)} \left(\frac{Y_{jt}^{\text{ptm}}}{A_{jt}}\right)^{-\alpha_r\lambda\beta/\rho_r} \\
\frac{Y_{jt}^{\text{ptm}}}{A_{jt}} &= A_{jt}^{\frac{(1-\alpha_r)\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \left(H_{jt}^{\text{ptm}}\right)^{1-\alpha_r} \\
Y_{jt}^{\text{ptm}} &= A_{jt}^{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \left(H_{jt}^{\text{ptm}}\right)^{1-\alpha_r}
\end{aligned}
$$

This allows us to write the wage equation as

$$
W_{jt}^{\text{ptm}}(X) = C_r^{\text{pt}}\ X^{\theta_j} \left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.
$$

As in the baseline equilibrium, we are left with finding H_{jt}^{ptm} as a function of the market TFP and amenities:

$$
H_{jt}^{\text{ptm}} = \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.
$$

Note that

$$
\begin{aligned}
I_{rt}^{\text{ptm}}(X) &= \left(\sum_{j'\in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(W_{j't}^{\text{ptm}}(X)\right)^{\lambda\beta/\rho_r}\right)^{\rho_r/(\lambda\beta)} \\
&= \left(\sum_{j'\in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r^{\text{pt}}\ X^{\theta_{j'}} \left(H_{j't}^{\text{ptm}}\right)^{-\alpha_r}\right)^{\lambda\beta/\rho_r} (A_{j't})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r/(\lambda\beta)} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right) \left(\sum_{j'\in J_r} (\tau G_{j'}(X))^{\beta/\rho_r} \left(C_r X^{\theta_{j'}} \left(H_{j't}^{\text{ptm}}\right)^{-\alpha_r}\right)^{\lambda\beta/\rho_r} (A_{j't})^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}}\right)^{\rho_r/(\lambda\beta)}
\end{aligned}
$$

We want to show that $H_{jt}^{\text{ptm}} = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}$. To see this we observe that $\tilde{H}_{jt}^{\text{ptm}}$ solves a very similar fixed point to $\tilde{H}_{jt}$. Indeed

$$
\begin{aligned}
\tilde{H}_{jt}^{\text{ptm}} &= \left(\int X^{\theta_j} \cdot \left(\frac{I_{rt}^{\text{ptm}}(X)}{I(X)}\right)^{\lambda\beta} (\tau G_j(X))^{\beta/\rho_r} \left(\frac{C_r^{\text{pt}}\ X^{\theta_j}}{I_{rt}^{\text{ptm}}(X)}\right)^{\lambda\beta/\rho_r} NM(X)\mathrm{d}X\right)^{\frac{\alpha_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \Gamma_{jt}\left[\vec{H}_t^{\text{ptm}}\right],
\end{aligned}
$$

where $\Gamma_{jt}(\cdot)$ is the operator defined in Lemma 2, equation (19) that defines H_{jt} as a fixed point. For this operator, we know that $\Gamma_{jt}(\vec{H}_t) = \tilde{H}_{jt}$ is the unique fixed point. The next step is to check that $\vec{H}_t'$, defined such that its j component, $\tilde{H}_{jt}' = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}}$, is a fixed point of the same operator $\Gamma_{jt}(\cdot)$:

$$
\begin{aligned}
\Gamma_{jt}\left(\vec{H}_t'\right) &= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \Gamma_{jt}\left(\vec{H}_t^{\text{ptm}}\right) \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}\frac{(1-\rho_r)\alpha_r\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta/\rho_r}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} \tilde{H}_{jt}^{\text{ptm}} \\
&= \tilde{H}_{jt}',
\end{aligned}
$$

hence $H_{jt}' = H_{jt}$ for all j and so we get that

$$
H_{jt}^{\text{ptm}} = \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}1[j\in J_r]} H_{jt}.
$$

So, for $j \in J_r$, we find that

$$
\begin{aligned}
W_{jt}^{\text{ptm}}(X) &= C_r^{\text{pt}}\ X^{\theta_j}\left(H_{jt}^{\text{ptm}}\right)^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
&= C_r^{\text{pt}}\ X^{\theta_j} H_{jt}^{-\alpha_r} A_{jt}^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{-\frac{\alpha_r\lambda\beta}{1+\alpha_r\lambda\beta}} \\
&= \left(\frac{C_r^{\text{pt}}}{C_r}\right)^{\frac{1}{1+\alpha_r\lambda\beta}} W_{jt}(X)
\end{aligned}
$$

and then

$$
\begin{aligned}
I_{rt}^{\mathrm{ptm}}(X) &= \left(\sum_{j' \in J_r} \left(\tau G_{j'}(X)\right)^{\beta/\rho_r} \left(W_{j't}^{\mathrm{ptm}}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r/(\lambda\beta)} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{1}{1+\alpha_r\lambda\beta}} I_{rt}(X).
\end{aligned}
$$

Next, let us rewrite the realized demand:

$$
\begin{aligned}
D_{jt}^{\mathrm{ptm}}(X) &= \left(\frac{I_{rt}^{\mathrm{ptm}}(X)}{I(X)} \right)^{\lambda\beta} \left(\tau G_j(X)\right)^{\beta/\rho_r} \left(\frac{W_{jt}^{\mathrm{ptm}}(X)}{I_{rt}^{\mathrm{ptm}}(X)} \right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \left(\frac{I_{rt}(X)}{I(X)} \right)^{\lambda\beta} \left(\tau G_j(X)\right)^{\beta/\rho_r} \left(\frac{W_{jt}(X)}{I_{rt}(X)} \right)^{\lambda\beta/\rho_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} D_{jt}(X).
\end{aligned}
$$

We can then compute the firm's output and wage bill:

$$
\begin{aligned}
Y_{jt}^{\mathrm{ptm}} &= A_{jt} \left(\int X^{\theta_j} \cdot D_{jt}^{\mathrm{ptm}}(X) \mathrm{d}X \right)^{1-\alpha_r} \\
&= \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} Y_{jt} \\
B_{jt}^{\mathrm{ptm}} &= \int W_{jt}^{\mathrm{ptm}}(X) \cdot D_{jt}^{\mathrm{ptm}}(X) \mathrm{d}X \\
&= C_r^{\mathrm{pt}} \, Y_{jt}^{\mathrm{ptm}}.
\end{aligned}
$$

Finally, we establish that:

$$
\begin{aligned}
\frac{\Pi_{jt} - \Pi_{jt}^{\mathrm{ptm}}}{\Pi_{jt}} &= 1 - \frac{Y_{jt}^{\mathrm{ptm}} - B_{jt}^{\mathrm{ptm}}}{Y_{jt} - B_{jt}} \\
&= 1 - \frac{1 - C_r^{\mathrm{pt}}}{1 - C_r} \left(\frac{C_r^{\mathrm{pt}}}{C_r} \right)^{\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \\
&= 1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}}
\end{aligned}
$$

$$
\Pi_{jt} - \Pi_{jt}^{\mathrm{ptm}} = \left(1 - \frac{\alpha_r \left(1 + \lambda\beta/\rho_r\right)}{1 + \alpha_r\lambda\beta/\rho_r} \left(\frac{\lambda\beta/\rho_r}{1 + \lambda\beta/\rho_r} \right)^{-\frac{(1-\alpha_r)\lambda\beta}{1+\alpha_r\lambda\beta}} \right) \Pi_{jt}.
$$

$\square$

A.4 Walrasian equilibrium, wedges, tax policy, and welfare

Walrasian equilibrium

We consider an equilibrium as defined by a set of wages $W_{jt}^{\mathrm{c}}(X)$ such that workers optimally choose where to work given these wages, and firms optimally choose labor demand, also taking these wages as given. In this equilibrium we make the tax system neutral $\lambda = \tau = 1$:

$$\max_{\left\{D_{jt}^{\mathrm{c}}(X)\right\}} A_{jt}\left(\int X^{\theta_j} D_{jt}^{\mathrm{c}}(X)\mathrm{d}X\right)^{1-\alpha_r} - \int W_{jt}^{\mathrm{c}}(X) D_{jt}^{\mathrm{c}}(X)\mathrm{d}X,$$

which gives the first order condition

$$(1-\alpha_r)X^{\theta_j} A_{jt}\left(\int X^{\theta_j}\cdot D_{jt}^{\mathrm{c}}(X)\mathrm{d}X\right)^{-\alpha_r} = W_{jt}^{\mathrm{c}}(X)$$

or

$$W_{jt}^{\mathrm{c}}(X) = \underbrace{(1-\alpha_r)}_{\equiv C_r^{\mathrm{pt}}} X^{\theta_j} A_{jt}\left(\frac{Y_{jt}^{c}}{A_{jt}}\right)^{-\frac{\alpha_r}{1-\alpha_r}}.$$

We then solve for output

$$\begin{aligned}
\left(\frac{Y_{jt}^{\mathrm{c}}}{A_{jt}}\right)^{\frac{1}{1-\alpha_r}} &= \int X^{\theta_j}\cdot D_{jt}^{\mathrm{c}}(X)\mathrm{d}X \\
&= \int X^{\theta_j} NM(X)\cdot \frac{\left(I_{rt}^{\mathrm{c}}(X)\right)^{\beta}}{\sum_{r'}\left(I_{r't}^{\mathrm{c}}(X)\right)^{\beta}}\cdot\left(\frac{W_{jt}^{\mathrm{c}}(X)G_j(X)}{I_{rt}^{\mathrm{c}}(X)}\right)^{\beta/\rho_r}\mathrm{d}X \\
&= \int X^{\theta_j}\cdot \frac{\left(I_{rt}^{\mathrm{c}}(X)\right)^{\beta}}{\sum_{r'}\left(I_{r't}^{\mathrm{c}}(X)\right)^{\beta}}\cdot\left(\frac{C_r^{\mathrm{pt}}X^{\theta_j}G_j(X)}{I_{rt}^{\mathrm{c}}(X)}\right)^{\beta/\rho_r} NM(X)\mathrm{d}X \times A_{jt}^{\beta/\rho_r}\left(\frac{Y_{jt}^{\mathrm{c}}}{A_{jt}}\right)^{-\frac{\alpha_r\beta/\rho_r}{1-\alpha_r}} \\
&= \left(H_{jt}^{\mathrm{c}}\right)^{1+\alpha_r\beta/\rho_r} A_{jt}^{\beta/\rho_r}\left(\frac{Y_{jt}^{\mathrm{c}}}{A_{jt}}\right)^{-\frac{\alpha_r\beta/\rho_r}{1-\alpha_r}} \\
\frac{Y_{jt}^{\mathrm{c}}}{A_{jt}} &= A_{jt}^{\frac{(1-\alpha_r)\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\left(H_{jt}^{\mathrm{c}}\right)^{1-\alpha_r} \\
Y_{jt}^{\mathrm{c}} &= A_{jt}^{\frac{1+\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\left(H_{jt}^{\mathrm{c}}\right)^{1-\alpha_r},
\end{aligned}$$

where we defined

$$\left(H_{jt}^{\mathrm{c}}\right)^{1+\alpha\beta/\rho_r} \equiv \int X^{\theta_j}\cdot \frac{\left(I_{rt}^{\mathrm{c}}(X)\right)^{\beta}}{\sum_{r'}\left(I_{r't}^{\mathrm{c}}(X)\right)^{\beta}}\cdot\left(\frac{C_r^{\mathrm{pt}}X^{\theta_j}G_j(X)}{I_{rt}^{\mathrm{c}}(X)}\right)^{\beta/\rho_r} NM(X)\mathrm{d}X,$$

giving the wage:

$$W_{jt}^{\mathrm{c}}(X) = C_r^{\mathrm{pt}} X^{\theta_j}\left(H_{jt}^{\mathrm{c}}\right)^{-\alpha_r}\left(A_{jt}\right)^{\frac{1}{1+\alpha_r\beta/\rho_r}}.$$

Next, using $H^{\mathrm{c}}_{jt} = H^{\mathrm{c}}_j \bar{A}_{rt}^{\frac{(\rho_r-1)\beta/\rho_r}{(1+\alpha_r\beta)(1+\alpha_r\beta/\rho_r)}}$ and following a similar proof to the main proposition we find that

$$w^{\mathrm{c}}_j(x,\overline{a},\tilde{a}) = c^{\mathrm{pt}} + \theta_j x - \alpha_r h^{\mathrm{c}}_j + \frac{1}{1+\alpha_r\beta/\rho_r}\tilde{a} + \frac{1}{1+\alpha_r\beta}\overline{a},$$

where

$$H^{\mathrm{c}}_j = \left[\int X^{\theta_j(1+\beta/\rho_r)}\left(\frac{I^{\mathrm{c}}_{r0}(X)}{I^{\mathrm{c}}_0(X)}\right)^{\beta}\left(\frac{1}{I^{\mathrm{c}}_{r0}(X)}\right)^{\beta/\rho_r}\left(C^{\mathrm{pt}}_r \mathring{G}_j(X)\right)^{\beta/\rho_r}\frac{\overline{\kappa}}{\kappa_r}\mathring{N}M(X)\mathrm{d}X\right]^{\frac{1}{1+\alpha_r\beta/\rho_r}}$$

$$I^{\mathrm{c}}_{r0}(X) = \left(\mathbb{E}_{j\in J_r}\left[\left(\mathring{G}_j(X)X^{\theta_j}C^{\mathrm{pt}}_r\left(H^{\mathrm{c}}_j\right)^{-\alpha_r}\right)^{\beta/\rho_r}\left(\tilde{A}_{jt}\right)^{\frac{\beta/\rho_r}{1+\alpha_r\beta/\rho_r}}\right]\right)^{\rho_r/\beta}$$

$$I^{\mathrm{c}}_0(X) = \left(\mathbb{E}_r\left[I^{\mathrm{c}}_{r0}(X)^{\beta}\left(\overline{A}_{rt}\right)^{\frac{\beta}{1+\alpha_r\beta}}\right]\right)^{1/\beta}.$$

We can then get the allocation of workers to each firm given by

$$\text{for } j\in J_r \qquad D^{\mathrm{c}}_{jt}(X) = n^{\mathrm{r}}\overline{\kappa}\mathring{N}M(X)\left(\frac{I^{\mathrm{c}}_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}}{I^{\mathrm{c}}_0(X)}\right)^{\beta}\left(\frac{G_j(X)W^{\mathrm{c}}_{jt}(X)}{I^{\mathrm{c}}_{r0}(X)\bar{A}_{rt}^{\frac{1}{1+\alpha_r\beta}}}\right)^{\beta/\rho_r}.$$

Defining wedges

To define wedges, we look at the decisions of firms to set wages, the decisions of workers to choose markets, and the decisions of workers to choose particular firms within a given market. We express each of these decisions in the monopsonistic competition model, clarifying where the sources of wedges are in each equation.

The first wedge is a productivity wedge reflected in the wage equation:

$$W_{jt}(X) = \Big(\underbrace{1+\frac{\rho_r}{\lambda\beta}}_{\text{labor prod. wedge}}\Big)^{-1}\cdot\underbrace{X^{\theta_j}(1-\alpha_r)A_{jt}L_{jt}^{-\alpha_r}}_{\text{marginal product of labor: }\mathcal{M}_{jt}(X)}.$$

We next turn to the expression for the quantity of labor $D_{jt}(X)$. For this we compute the log odds ratio of choosing one firm j versus another firm j' within a market r. We have

$$\log\frac{\Pr[j(i,t)=j|X_i=X,\mathbf{W}_t,j\in J_r]}{\Pr[j(i,t)=j'|X_i=X,\mathbf{W}_t,j'\in J_r]} = \frac{\beta}{\rho_r}\left[\log\frac{G_j(X)}{G_{j'}(X)} + \underbrace{\lambda}_{\text{pref. wedge}}\log\frac{W_{jt}(X)}{W_{j't}(X)}\right],$$

where the allocation is identical in all respects aside from the presence of the tax parameter λ which acts as a preference wedge between amenities and earnings.

We now shift attention to how the worker chooses between two different markets $r\neq r'$. It is useful to express wages using the wage index $I_{rt}(X)$ from equation (2) to see that

$$\log \frac{\Pr[j(i,t) \in J_r | X_i{=}X, \mathbf{W}_t]}{\Pr[j(i,t) \in J_{r'} | X_i{=}X, \mathbf{W}_t]} = \underbrace{\lambda}_{\text{pref. wedge}} \beta \log \frac{I_{rt}(X)}{I_{r't}(X)}.$$

The results clarify two wedges: a productivity wedge equal to $1 + \frac{\rho_r}{\lambda\beta}$ and a preference wedge equal to λ.

Defining tax policy counterfactuals

Lemma 7. *Setting a tax policy with $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ and $\lambda = 1$ achieves the competitive allocation of workers to firms.*

Proof. We substitute $\tau_r = \frac{1+\beta/\rho_r}{\beta/\rho_r}$ into the firm's problem and show that it achieves the planner's solution in this context. Recall from Lemma 3

$$\begin{aligned}
H_j &= \left(\int X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\frac{I_{r0}(X)}{I_0(X)} \right)^{\lambda\beta} \left(\frac{1}{I_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(\mathring{G}_j(X) \tau C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\overline{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
I_{r0}(X)^{\lambda\beta/\rho_r} &= \mathbb{E}_j \left[\left(\tau \mathring{G}_j(X) X^{\lambda\theta_j} C_r^{\lambda} H_j^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\
I_0(X)^{\lambda\beta} &= \mathbb{E}_r \left[I_{r0}(X)^{\lambda\beta} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right],
\end{aligned}$$

where we notice that τC_r^{λ} always appears together and under this particular policy we get that $\tau_r C_r^{\lambda} = (1-\alpha_r) = C_r^{\text{pt}}$. Hence, h_j coincides exactly with h_j^{c} while $I_{r0}(X)$ and $I_0(X)$ coincide with $I_{r0}^{\text{c}}(X)$ and $I_0^{\text{c}}(X)$, respectively. We then see that this implies that $D_{jt}(X) = D_{jt}^{\text{c}}(X)$. In other words such policy achieves exactly the planner's allocation. $\square$

Defining welfare

We start by defining a measure of welfare given a set of wages and tax parameters. Recall that the average utility that a worker enjoys for a given set of wages is given by:

$$\mathbb{E}\left[u_{it} | \mathbf{W}_t\right] = \int \frac{1}{\beta} \left[\log \left(\sum_r \left(\sum_{j \in J_r} \left(\tau G_j(X)\right)^{\beta/\rho_r} \left(W_{jt}(X)\right)^{\lambda\beta/\rho_r} \right)^{\rho_r} + \overline{C} \right) \right] M(X) \mathrm{d}X,$$

where we normalize $\overline{C}$ to zero. The total tax revenue R_t and total firm profits Π_t are given by:

$$\begin{aligned}
R_t &= \int \sum_r \sum_{j \in J_r} D_{jt}(X) \left(W_{jt}(X) - \tau W_{jt}(X)^{\lambda} \right) \mathrm{d}X \\
&= \int \sum_r \sum_{j \in J_r} D_{jt}(X) W_{jt}(X) \mathrm{d}X - \int \sum_r \sum_{j \in J_r} D_{jt}(X) \tau W_{jt}(X)^{\lambda} \mathrm{d}X \\
&= B_t - B_t^{\text{net}} \\
\Pi_t &= \sum_r \sum_{j \in J_r} A_{jt} \left(\int X^{\theta_j} \cdot D_{jt}(X) \mathrm{d}X \right)^{1-\alpha_r} - \int W_{jt}(X) \cdot D_{jt}(X) \mathrm{d}X \\
&= Y_t - B_t.
\end{aligned}$$

To take into account changes in tax revenue and firm profits across counterfactuals, we redistribute Π_t and R_t to workers in the form of a non-distortionary payment proportional to their net wages, governed by ϕ_t. This means that each worker receives $\phi_t \tau W_{jt}(X)^{\lambda}$ in transfers. The total transfer equals $\Pi_t + R_t$ and is given by

$$\begin{aligned}
\int \sum_r \sum_{j \in J_r} \phi_t \tau W_{jt}(X)^{\lambda} \cdot D_{jt}(X) \mathrm{d}X &= \Pi_t + R_t \\
\phi_t B_t^{\text{net}} &= \Pi_t + R_t
\end{aligned}$$

which implies

$$\begin{aligned}
1 + \phi_t &= \frac{\Pi_t + R_t + B_t^{\text{net}}}{B_t^{\text{net}}} \\
&= \frac{\Pi_t + B_t}{B_t^{\text{net}}} \\
&= \frac{Y_t}{B_t^{\text{net}}}.
\end{aligned}$$

Thus, welfare can be decomposed as

$$\begin{aligned}
\mathbb{W}_t &= \int \frac{1}{\beta} \left[\log \sum_r \left(\sum_{j \in J_r} \left((1+\phi_t) \tau G_j(X) \right)^{\beta/\rho_r} \left(W_{jt}(X) \right)^{\lambda\beta/\rho_r} \right)^{\rho_r} \right] M(X) \mathrm{d}X \\
&= \underbrace{\mathbb{E}\left[u_{it} | \mathbf{W}_t \right]}_{\text{utility from net-wages and amenities}} + \underbrace{\log(1+\phi_t).}_{\text{utility from redistributed profits and tax revenue}}
\end{aligned}$$

A.5 An extension with amenity shocks

Lemma 8. *The unique solution for $\check{H}_{jt}$ in the limit of a sequence of growing economies with $G_{jt}(X) = \overline{G}_{rt} \tilde{G}_{jt} G_j(X)$ is given by*

$$\check{H}_{jt} = \check{H}_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r - 1)}{(1+\alpha_r\lambda\beta/\rho_r)}} \cdot \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \cdot \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}},$$

where $\check{H}_j$ solves the following fixed point:

$$\check{H}_j = \left(\int X^{\theta_j} \left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)} \right)^{\lambda\beta} \left(\frac{1}{\check{I}_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) dX \right)^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}$$

$$\check{I}_{r0}(X)^{\lambda\beta/\rho_r} \equiv \mathbb{E}_j \left[\left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \check{H}_j^{-\lambda\alpha_r} \right)^{\beta/\rho_r} \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \tilde{A}_{jt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right]$$

$$\check{I}_0(X)^{\lambda\beta} \equiv \mathbb{E}_r \left[\check{I}_{r0}(X)^{\lambda\beta} \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}} \bar{A}_{rt}^{\frac{\lambda\beta}{1+\alpha_r\lambda\beta}} \right].$$

Proof. Consider the expression for H_{jt} from Lemma 3. Substitute in n^{r} , n_r^{f}, κ_r, $G_{jt}(X) = \overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X) \left(n_{r(j)}^{\mathrm{f}} \right)^{-\rho_{r(j)}/\beta}$ and $\mathring{N} = (n^{\mathrm{r}}\ n^{\mathrm{r}}\ \bar{\kappa})^{-1} N$. As the economy grows large, i.e. as n^{r} grows to infinity, we have the following expression:

$$\begin{aligned}
\check{H}_{jt} = & \Bigg[\int \Bigg(\mathbb{E}_{r'} \Bigg[\Bigg(\mathbb{E}_{j'\in J_{r'}} \Bigg[\left(X^{\lambda\theta_{j'}} \tau \overline{G}_{r't} \tilde{G}_{j't} \mathring{G}_{j'}(X) C_{r'}^{\lambda} \check{H}_{j't}^{-\alpha_{r'}\lambda} \right)^{\beta/\rho_{r'}} A_{j't}^{\frac{\lambda\beta/\rho_{r'}}{1+\alpha_{r'}\lambda\beta/\rho_{r'}}} \Bigg] \Bigg)^{\rho_{r'}} \Bigg] \Bigg)^{-1} \\
& \times \left(\mathbb{E}_{j'\in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \overline{G}_{rt} \tilde{G}_{j't} \mathring{G}_{j'}(X) C_r^{\lambda} \check{H}_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \right)^{\rho_r-1} \\
& \times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \overline{G}_{rt} \tilde{G}_{jt} \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.
\end{aligned}$$

Next we show that $\check{H}_{jt}$ can indeed be expressed as stated in this Lemma. Let's assume that $\check{H}_{jt} = \check{H}_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)}\frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} \cdot \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \cdot \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}}$ and show that it solves the problem. Note that

$$\begin{aligned}
& \mathbb{E}_{j'\in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \overline{G}_{rt} \tilde{G}_{j't} \mathring{G}_{j'}(X) C_r^{\lambda} \check{H}_{j't}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} A_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\
& = \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \overline{G}_{rt}^{(1-\frac{\alpha\lambda\beta}{1+\alpha_r\lambda\beta})\beta/\rho_r} \mathbb{E}_{j'\in J_r} \left[\left(X^{\lambda\theta_{j'}} \tau \mathring{G}_{j'}(X) C_r^{\lambda} \check{H}_{j'}^{-\alpha_r\lambda} \right)^{\beta/\rho_r} \tilde{G}_{j't}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \tilde{A}_{j't}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \right] \\
& = \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \overline{G}_{rt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta}} \check{I}_{r0}(X)^{\lambda\beta/\rho_r}.
\end{aligned}$$

Then,

$$
\begin{aligned}
\check{H}_{jt} =& \Bigg[\int \left(\mathbb{E}_{r'} \left[\bar{A}_{r't}^{\frac{\lambda\beta}{1+\alpha_{r'}\lambda\beta}} \overline{G}_{r't}^{\frac{\beta}{1+\alpha_r\lambda\beta}} \check{I}_{r'0}(X)^{\lambda\beta} \right] \right)^{-1} \times \left(\bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta}} \overline{G}_{rt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta}} \check{I}_{r0}(X)^{\lambda\beta/\rho_r} \right)^{\rho_r - 1} \\
& \quad \times X^{\theta_j(1+\lambda\beta/\rho_r)} \left(\tau \overline{G}_{rt} \tilde{G}_{jt} \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \Bigg]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
=& \left[\int X^{\theta_j} \left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)} \right)^{\lambda\beta} \left(\frac{1}{\check{I}_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}} \\
& \quad \times \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta} \frac{\rho_r - 1}{1+\alpha_r\lambda\beta/\rho_r}} \times \overline{G}_{rt}^{\frac{\beta/\rho_r + \frac{(\rho_r-1)\beta/\rho_r}{1+\alpha_r\lambda\beta}}{1+\alpha_r\lambda\beta/\rho_r}} \times \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} \\
=& \check{H}_j \cdot \bar{A}_{rt}^{\frac{\lambda\beta/\rho_r}{(1+\alpha_r\lambda\beta)} \frac{(\rho_r-1)}{(1+\alpha_r\lambda\beta/\rho_r)}} \cdot \overline{G}_{rt}^{\frac{\beta}{1+\alpha_r\lambda\beta}} \cdot \tilde{G}_{jt}^{\frac{\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}},
\end{aligned}
$$

where we used that $\check{H}_j$ solves

$$
\check{H}_j = \left[\int X^{\theta_j} \left(\frac{\check{I}_{r0}(X)}{\check{I}_0(X)} \right)^{\lambda\beta} \left(\frac{1}{\check{I}_{r0}(X)} \right)^{\lambda\beta/\rho_r} \left(X^{\lambda\theta_j} \tau \mathring{G}_j(X) C_r^{\lambda} \right)^{\beta/\rho_r} \frac{\bar{\kappa}}{\kappa_r} \mathring{N} M(X) \mathrm{d}X \right]^{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}.
$$

□

Corollary 4. *Allowing for time-varying amenities $G_{jt}(X) = \overline{G}_{rt}\tilde{G}_{jt}\mathring{G}_j(X)$ we get the following wage equation:*

$$
w_{jt}(x) = c_r + \theta_j x - \alpha_r \check{h}_j + \frac{\tilde{a}_{jt} - \alpha_r\beta/\rho_r \cdot \tilde{g}_{jt}}{1 + \alpha_r\lambda\beta/\rho_r} + \frac{\bar{a}_{rt} - \alpha_r\beta \cdot \overline{g}_{rt}}{1 + \alpha_r\lambda\beta}.
$$

A.6 An extension with capital and monopolistic competition in the product market

We develop here a simple extension of the model with capital and monopolistic competition in the product market. Without loss of generality, we derive the results here in the case of homogeneous labor.

Consider a firm with production function $Q = AK^{\rho}L^{1-\alpha}$, access to a local monopolistic market with revenue curve $Y = Q^{1-\epsilon}$, hiring labor from a local labor supply curve $L(W) = W^{\beta}$ and renting capital at price r. Profit is given by

$$
Q^{1-\epsilon} - LW - rK.
$$

We first note that we can replace Q with the production function and get

$$
\left(AK^{\rho}L^{1-\alpha}\right)^{1-\epsilon} - LW - rK.
$$

Now we will show that considering perfect or monopolistic competition in the product market gives rise to the same revenue function. We will focus directly on the value added function parameterized as

$$Y = AK^{\tilde{\rho}} L^{1-\tilde{\alpha}},$$

where $\tilde{\rho} \equiv \rho(1-\epsilon)$ and $\tilde{\alpha} \equiv \alpha + \epsilon - \alpha\epsilon$. We then have the following Lagrangian for our problem:

$$AK^{\tilde{\rho}} L^{1-\tilde{\alpha}} - LW - rK - \mu(L - W^{\beta}).$$

We take the first order condition for K and get

$$K = \left(\frac{r}{\tilde{\rho} A L^{1-\tilde{\alpha}}} \right)^{\frac{1}{\tilde{\rho}-1}},$$

which we then replace in

$$\begin{aligned}
AK^{\tilde{\rho}} L^{1-\tilde{\alpha}} - LW - rK &= A \left(\frac{r}{\tilde{\rho} A L^{1-\tilde{\alpha}}} \right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\tilde{\alpha}} - LW - r \left(\frac{r}{\tilde{\rho} A L^{1-\tilde{\alpha}}} \right)^{\frac{1}{\tilde{\rho}-1}} \\
&= (1-\tilde{\rho}) A \left(\frac{r}{\tilde{\rho} A L^{1-\tilde{\alpha}}} \right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\tilde{\alpha}} - LW \\
&= (1-\tilde{\rho}) A \left(\frac{r}{\tilde{\rho} A} \right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}} L^{1-\frac{\tilde{\alpha}+\tilde{\rho}}{1-\tilde{\rho}}} - LW \\
&= \hat{A} L^{1-\hat{\alpha}} - LW,
\end{aligned}$$

which is just a reinterpretation of the original problem with $\hat{A} \equiv (1-\tilde{\rho}) A \left(\frac{r}{\tilde{\rho} A} \right)^{\frac{\tilde{\rho}}{\tilde{\rho}-1}}, \hat{\alpha} \equiv \frac{\tilde{\alpha}+\tilde{\rho}}{1-\tilde{\rho}}$.

B Details on Data Sources and Sample Selection

All firm level variables are constructed from annual business tax returns over the years 2001-2015: C-Corporations (Form 1120), S-Corporations (Form 1120-S), and Partnerships (Form 1065). Worker-level variables are constructed from annual tax returns over the years 2001-2015: Direct employees (Form W-2), independent contractors (Form 1099), and household income and taxation (Form 1040).

Variable Definitions:

- **Earnings:** Reported on W-2 box 1 for each Taxpayer Identification Number (TIN). Each TIN is de-identified in our data.
- **Gross Household Income:** We define gross household income as the sum of taxable wages and other income (line 22 on Form 1040) minus unemployment benefits (line 19 on Form 1040) minus taxable Social Security benefits (line 20a on Form 1040) plus tax-exempt interest income (line 8b on Form 1040). We at times also consider this measure when subtracting off Schedule D capital gains (line 13 on Form 1040).

- **Federal Taxes on Household Income:** This is given by the sum of two components. The first component is the sum of FICA Social Security taxes (given by 0.0620 times the minimum of the Social Security taxable earnings threshold, which varies by year, and taxable FICA earnings, which are reported on Box 3 of Form W-2) and FICA Medicare taxes (given by 0.0145 times Medicare earnings, which are reported on Box 5 of Form W-2). The second component is the sum of the amount of taxes owed (the difference between line 63 and line 74 on Form 1040, which is negative to indicate a refund) and the taxes already paid or withheld (the sum of lines 64, 65, 70, and 71 on Form 1040).

- **Net Household Income:** We construct a measure of net household income as Gross Household Income minus Federal Taxes on Household Income plus two types of benefits: unemployment benefits (line 19 of Form 1040) and Social Security benefits (line 20a of Form 1040).

- **Employer:** The Employer Identification Number (EIN) reported on W-2 for a given TIN. Each EIN is de-identified in our data.

- **Wage Bill:** Sum of Earnings for a given EIN plus the sum of 1099-MISC, box 7 nonemployee compensation for a given EIN in year t.

- **Size:** Number of FTE workers matched to an EIN in year t.

- **NAICS Code:** The NAICS code is reported on line 21 on Schedule K of Form 1120 for C-corporations, line 2a Schedule B of Form 1120S for S-corporations, and Box A of form 1065 for partnerships. We consider the first two digits to be the industry. We code invalid industries as missing.

- **Commuting Zone:** This is formed by mapping the ZIP code from the business filing address of the EIN on Form 1120, 1120S, or 1065 to its commuting zone.

- **Value Added:** Line 3 of Form 1120 for C-Corporations, Form 1120S for S-Corporations, and Form 1065 for partnerships. Line 3 is the difference between Revenues, reported on Line 1c, and the Cost of Goods Sold, reported on Line 2. We replace non-positive value added with missing values.

  - For manufacturers (NAICS Codes beginning 31, 32, or 33) and miners (NAICS Codes beginning 212), Line 3 is equal to Value Added minus Production Wages, defined as wage compensation for workers directly involved in the production process, per Schedule A, Line 3 instructions. If we had access to data from Form 1125-A, Line 3, we could directly add back in these production wages to recover value added. Without 1125-A, Line 3, we construct a measure of Production Wages as the difference between the Wage Bill and the Firm-reported Taxable Labor Compensation, defined below, as these differ conceptually only due to the inclusion of production wages in the Wage Bill.

- **Value Added Net of Depreciation:** Value Added minus Depreciation, where Depreciation is reported on Line 20 on Form 1120 for C-corporations, Line 14 on Form 1120S for S-corporations, and Line 16c on Form 1065 for partnerships.

- **EBITD:** We follow Kline et al. (2019) in defining Earnings Before Interest, Taxes, and Depreciation (EBITD) as the difference between total income and total deductions other than interest and depreciation. Total income is reported on Line 11 on Form 1120 for C-corporations, Line 1c on Form 1120S for S-corporations, and Line 1c on Form 1065 for Partnerships. Total deductions other than interest and depreciation are computed as Line 27 minus Lines 18 and 20 on Form 1120 for C-corporations, Line 20 minus Lines 13 and 14 on Firm 1120S for S-corporations, and Line 21 minus Lines 15 and 16c on Form 1065 for partnerships.

- **Operating Profits:** We follow Kline et al. (2019), who use a similar approach to Yagan (2015), in defining Operating Profits as the sum of Lines 1c, 18, and 20, minus the sum of Lines 2 and 27 on Form 1120 for C-corporations,, the sum of Lines 1c, 13, and 15, minus the sum of Lines 2 and 20 on Form 1120S for S-corporations, and the sum of Lines 1c, 16, and 16c, minus the sum of Lines 2 and 21 on Form 1065 for partnerships.

- **Firm-reported Taxable Labor Compensation:** This is the sum of compensation of officers and salaries and wages, reported on Lines 12 and 13 on Form 1120 for C-corporations, Lines 7 and 8 on Form 1120S for S-corporations, and Lines 9 and 10 on Form 1065 for Partnerships.

- **Firm-reported Non-taxable Labor Compensation:** This is the sum of employer pension and employee benefit program contributions, reported on Lines 17 and 18 on Form 1120 for C-corporations, Lines 17 and 18 on form 1120S for S-corporations, and Lines 18 and 19 on Form 1065 for Partnerships.

- **Multinational Firm:** We define an EIN as a multinational in year t if it reports a non-zero foreign tax credit on Schedule J, Part I, Line 5a of Form 1120 or Form 1118, Schedule B, Part III, Line 6 of Form 1118 for a C-corporation in year t, or if it reports a positive Total Foreign Taxes Amount on Schedule K, Line 16l of of Form 1065 for a partnership in year t.

- **Tenure:** For a given TIN, we define tenure at the EIN as the number of consecutive prior years in which the EIN was the highest-paying.

- **Age and Sex:** Age at t is the difference between t and birth year reported on Data Master-1 (DM-1) from the Social Security Administration, and sex is the gender reported on DM-1 (see for further details on the DM-1 link).

C Details on Identification, Estimation, and Robustness

C.1 Moment condition for internal panel instruments

In this appendix, we prove that equation (12) holds. Using equations (4), (5), and (10), we can write for the stayers (S_i=1) that

$$\begin{aligned}\tilde{y}_{it+\tau} - \tilde{y}_{j,t-\tau'} =& \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'} + \nu_{j,t+\tau} - \nu_{j,t-\tau'}\\ \tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} =& v_{it+\tau} - v_{i,t-\tau'} + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\sum_{t'=t-\tau'+1}^{t+\tau}\tilde{u}_{jt'}\end{aligned}$$

Combining these equations, it follows that

$$\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right) = -\frac{1}{1+\lambda\beta/\rho_r}\left(\nu_{j,t+\tau} - \nu_{j,t-\tau'}\right) + v_{it+\tau} - v_{i,t-\tau'}$$

Furthermore, the short-difference in log value added can be written

$$\Delta\tilde{y}_{j(i),t} = \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{u}_{j(i),t} + \nu_{j,t} - \nu_{j,t-1}$$

Combining these expressions and taking the expectation,

$$\begin{aligned}&\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \gamma\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right]\\ &= \mathbb{E}\left[\left(\frac{1}{1+\alpha_r\lambda\beta/\rho_r}\tilde{u}_{j(i),t} + \nu_{j,t} - \nu_{j,t-1}\right)\left(-\frac{1}{1+\lambda\beta/\rho_r}\left(\nu_{j,t+\tau} - \nu_{j,t-\tau'}\right) + v_{it+\tau} - v_{i,t-\tau'}\right)|S_i{=}1\right]\end{aligned}$$

Given Assumption 1.b that $\mathbb{E}\left[\nu_{jt}\nu_{j't}|\Omega_T\right]=0$ whenever $|t-t'|\geq 2$, it follows that whenever $\tau \geq 2$ and $\tau' \geq 3$, all cross-products between ν_{jt} terms will be mean zero. Furthermore, $\mathbb{E}\left[\nu_{jt}|\Omega_T\right]=0$ ensures that cross-product terms between $\tilde{u}_{jt}$ and ν_{jt} are also mean zero. Finally the assumption that the measurement error on wages is independent of all firm level variables, Assumption 1.c, implies that all terms involving v_{it} are also mean zero. Thus, provided that $\tau \geq 2$ or $\tau' \geq 3$,

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{w}_{it+\tau} - \tilde{w}_{it-\tau'} - \frac{1}{1+\lambda\beta/\rho_r}\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right] = 0.$$

As a result, $\frac{1}{1+\lambda\beta/\rho_r} \equiv \gamma_r$ is identified as long as,

$$\mathbb{E}\left[\Delta\tilde{y}_{j(i),t}\left(\tilde{y}_{j(i),t+\tau} - \tilde{y}_{j(i),t-\tau'}\right)|S_i{=}1\right] > 0,$$

which is guaranteed by Assumption 1.a.

A similar argument can be used to establish that equation (13) holds. Briefly, among stayers, market level changes in log wages and log value added are given by

$$
\begin{aligned}
\bar{w}_{it+\tau} - \bar{w}_{it-\tau} =& \frac{1}{1+\lambda\alpha_{r(j(i))}\beta} \sum_{d=t-\tau'+1}^{t+\tau} \overline{u}_{r(j(i)),d} \\
\bar{y}_{jt+\tau} - \bar{y}_{jt-\tau'} =& \frac{1+\lambda\beta}{1+\lambda\alpha_{r(j)}\beta} \sum_{d=t-\tau'+1}^{t+\tau} \overline{u}_{r(j),d}
\end{aligned}
$$

which cancel out differences to imply the moment condition

$$
\mathbb{E}\left[\Delta\bar{y}_{j(i),t}\left(\bar{w}_{it+\tau} - \bar{w}_{it-\tau'} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{j(i),t+\tau} - \bar{y}_{j(i),t-\tau'}\right)\right)|S_i{=}1\right] = 0.
$$

Similarly, the rank condition is guaranteed by Assumption 1.a, so $\frac{1}{1+\lambda\beta} \equiv \Upsilon$ is identified.

C.2 Estimating the rest of the process parameters

In this appendix, we describe the estimation procedure for recovering the joint process for log earnings and value added. We rely on the assumed structure that each evolves according to a unit root process plus a moving average process, where both the transitory and permanent shocks to value added pass-through to log earnings. We estimate the pass-through process in two steps. First, we estimate the parameters for the value added process. Second, we jointly estimate the pass-through rates at the firm and market level and the parameters of the wage process.

To estimate the value added process, we consider the variance-covariance matrix of one-year differences over time in a stacked panel of 8-year stayer spells. We index the 8-year spells by event times $e = 1, ..., 8$. The variance-covariance matrix uses the growth at event times $e = 3, ..., 7$.[1] For example, the growth in log value added at event time e means the log value added at e minus log value added at $e-1$. We do not use data from the first ($e = 1$) or last ($e = 8$) year of the spell. We do this because first and last event years can be partial employment spells due to beginning or ending the job spell mid-year. Thus, focusing on the intermediate event years alleviates the issue that we do not observe the exact date at which a job spell begins or ends in our data.

Using our data, we estimate the 5×5 variance-covariance matrix of one-year changes in log value added, denoted M_y, where the (p, q) element is $M_y(p, q) = Cov(\Delta y_{ip}, \Delta y_{iq})$. We construct the analogous population variance-covariance matrix implied by the model as a function of only the parameters $\{\delta^y, \sigma_u, \sigma_\xi\}$; we denote the model-implied variance-covariance matrix by $M_y^*(\delta^y, \sigma_u, \sigma_\xi)$. Given these moments, our GMM estimator solves the minimum distance problem defined by

[1] In the case of MA(1), one can also use $t = 2$, however we wanted to test for MA(2) as a robustness.

$$\min_{\delta^y,\sigma_u,\sigma_\xi} \sum_{p=3}^{7}\sum_{q=3}^{7} W_y(p,q)\left(M_y^*(p,q;\delta^y,\sigma_u,\sigma_\xi) - M_y(p,q)\right)^2$$

where we use diagonal weighting, i.e., $W_y(p,q) = Cov(\Delta y_{ip}, \Delta y_{iq})^2 + Var(\Delta y_{ip})Var(\Delta y_{iq})$.

Next, we construct two matrices each of size 5×5. The first, M_w, is the variance-covariance matrix for one-year changes in log wages; a typical element is $M_w(p,q) = Cov(\Delta w_{ip}, \Delta w_{iq})$. The second, M_{wy}, is the variance-covariance matrix for one-year changes in log wages and log value added; a typical element is $M_{wy}(p,q) = Cov(\Delta w_{ip}, \Delta y_{iq})$. The corresponding model-implied population variance-covariance matrices are $M_w^*(\delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta)$ and $M_{wy}^*(\delta^w, \sigma_\mu, \sigma_\nu)$, respectively. These matrices also depend on $(\delta^y, \sigma_u, \sigma_\xi)$, which were estimated in the first step, so we substitute in to $M_w^*(\delta^w, \sigma_\mu, \sigma_\nu, \gamma, \zeta)$ and $M_{wy}^*(\delta^w, \sigma_\mu, \sigma_\nu)$ the estimated values of $(\delta^y, \sigma_u, \sigma_\xi)$.Then, our GMM estimator in the second step solves the minimum distance problem defined by

$$\begin{aligned}\min_{p,q;\delta^w,\sigma_\mu,\sigma_\nu,\gamma,\zeta} \sum_{p=3}^{7}\sum_{q=3}^{7} W_w(p,q)\left(M_w^*(p,q;\delta^w,\sigma_\mu,\sigma_\nu,\gamma,\zeta) - M_w(p,q)\right)^2 + \\ W_{wy}(p,q)\left(M_{wy}^*(p,q;\delta^w,\sigma_\mu,\sigma_\nu,\gamma,\zeta) - M_{wy}(p,q)\right)^2\end{aligned}$$

where we again use diagonal weighting, i.e., $W_w(p,q) = Cov(\Delta w_{ip}, \Delta w_{iq})^2 + Var(\Delta w_{ip})Var(\Delta w_{iq})$ and $W_{wy}(p,q) = Cov(\Delta w_{ip}, \Delta y_{iq})^2 + Var(\Delta w_{ip})Var(\Delta y_{iq})$.

In practice, the GMM minimum distance problems in the first and second steps are polynomials in the parameters of interest. We solve the minimization problems using global polynomial optimization following Lasserre (2001). This allows us to formally certify the global optimality of the solution.

For inference, we use a joint bootstrap of M_y, M_w, M_{yw}. We conduct inference using a block bootstrap that resamples markets, where a market is definedas the combination ofa commuting zone an an industry. In practice, thereare about 2000 blocks. The GMM estimates and bootstrap standard errors are displayed in Online Appendix Table A.3.

C.3 Pass-through estimation based on external instruments

Identification details

Implicitly conditioning on firms in region r ($j(i,t) = j \in J_r$), we prove that this claim from Section 4.1 holds:

$$\mathbb{E}\left[\Delta\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e} - \tilde{w}_{it-e'} - \gamma_r\left(\tilde{y}_{jt+e} - \tilde{y}_{jt-e'}\right)\right)|S_i{=}1\right] = 0$$

From equations (4), (5), and the expression for h_{jt} in Appendix A.5, we have that

$$\tilde{w}_{it+e} - \tilde{w}_{it-e'} = -\alpha_r \left(h_{j(i)t+e} - h_{j(i)t-e}\right) + \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right) + \left(v_{it+e} - v_{it-e}\right)$$
$$\tilde{y}_{j(i)t+e} - \tilde{y}_{j(i)t-e'} = (1-\alpha_r)\left(h_{j(i)t+e} - h_{j(i)t-e}\right) + \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right) + \left(\tilde{\nu}_{j(i)t+e} - \tilde{\nu}_{j(i)t-e}\right)$$

From assumption 1.d,

$$\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right)|S_i{=}1\right] \neq 0$$
$$\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e} - h_{j(i)t-e'}\right)|S_i{=}1\right] = 0$$

It follows that

$$\begin{aligned}
\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e} - \tilde{w}_{it-e'}\right)|S_i{=}1\right] &= -\alpha_r \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e} - h_{j(i)t-e}\right)|S_i{=}1\right]}_{=0} \\
&+ \frac{1}{1+\alpha_r\lambda\beta/\rho_r}\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right)|S_i{=}1\right]}_{\neq 0} \\
&+ \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(v_{it+e} - v_{it-e}\right)|S_i{=}1\right]}_{=0} \\
\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{y}_{j(i)t+e} - \tilde{y}_{j(i)t-e'}\right)|S_i{=}1\right] &= (1-\alpha_r)\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(h_{j(i)t+e} - h_{j(i)t-e}\right)|S_i{=}1\right]}_{=0} \\
&+ \frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}\underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{a}_{j(i)t+e} - \tilde{a}_{j(i)t-e}\right)|S_i{=}1\right]}_{\neq 0} \\
&+ \underbrace{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\nu_{j(i)t+e} - \nu_{j(i)t-e}\right)|S_i{=}1\right]}_{=0}
\end{aligned}$$

where we imposed the restrictions (1.b part i) and 1.c) to eliminate the terms involving the measurement errors v_{it} and ν_{jt}. Thus, we can write

$$\frac{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{w}_{it+e} - \tilde{w}_{it-e'}\right)|S_i{=}1\right]}{\mathbb{E}\left[\tilde{\Lambda}_{jt}\left(\tilde{y}_{j(i)t+e} - \tilde{y}_{j(i)t-e'}\right)|S_i{=}1\right]} = \frac{\frac{1}{1+\alpha_r\lambda\beta/\rho_r}}{\frac{1+\lambda\beta/\rho_r}{1+\alpha_r\lambda\beta/\rho_r}} = \frac{1}{1+\lambda\beta/\rho_r} \equiv \gamma_r$$

which can be rearranged as the claim above. The same reasoning demonstrates the claim that Υ can be identified using $\overline{\Lambda}_{rt}$.

Procurement auction shocks at firm-level

Our goal is to recover the pass-through regression at the firm-level. Following the research design of Kroft et al. (2021), consider the cohort of firms that received a procurement contract in year t ($D_{jt} = 1$) and the set of comparison firms that bid for a procurement in year t but lost ($D_{jt} = 0$). Let e denote an event time relative to t and $\bar{e}$ denote the omitted event time. For each event time $e = -4, ..., 4$, the DiD regression is implemented as

$$w_{jt+e} = \underbrace{\sum_{e' \neq \bar{e}} 1\{e' = e\}\,\mu_{te'}}_{\textit{event time fixed effect}} + \underbrace{\sum_{j'} 1\{j' = j\}\,\psi_{j't}}_{\textit{firm fixed effect}} + \underbrace{\sum_{e' \neq \bar{e}} 1\{e' = e\}\,D_{jt}\vartheta_{te'}}_{\textit{treatment status by event time}} + \underbrace{\nu_{jte}}_{\textit{residual}}$$

We report the average across t of the estimated ϑ_{te} parameters, which can be interpreted as the reduced form effect on log earnings of receiving an exogenous demand shock, that is, $\vartheta_{te} = \mathbb{E}\left[w_{jt+e} - w_{t-\bar{e}}|D_{jt} = 1\right] - \mathbb{E}\left[w_{jt+e} - w_{t-\bar{e}}|D_{jt} = 0\right]$. We estimate ϑ_{te} for all t and e and then average across t, using the delta method to compute standard errors (which are clustered at the firm level j to account for serial correlation). By doing so, we avoid the problem that cohorts can be negatively weighted in pooled cohort DiD estimators. The analogous regression in which y_{jt+e} is the outcome recovers the first stage effect on log value added, $\mathbb{E}\left[y_{jt+e} - y_{t-\bar{e}}|D_{jt} = 1\right] - \mathbb{E}\left[y_{jt+e} - y_{t-\bar{e}}|D_{jt} = 0\right]$. The ratio of the reduced form effect and the first stage effect yields the second stage effect, which is the pass-through coefficient γ. In the first panel in Appendix Table A.4, we apply this research design to the sample of 8,667 unique firms that bid in the sample of procurement auctions administered by the departments of transportation in 28 states during 2001-2015. We refer to Kroft et al. (2021) for details on how the procurement auction data were collected and linked to IRS tax records as well as institutional details and descriptive statistics. We find a statistically significant first stage coefficient of 0.143, indicating that winners of procurement auctions experience about 14 percent more growth in value added than losers of procurement contracts. We find a statistically significant reduced form coefficient of 0.020, indicating that workers employed by firms that win procurement auctions experience about 2 percent more growth in earnings than workers employed by losers of procurement contracts. The ratio of the reduced form and first stage effects yields a statistically significant firm-level pass-through coefficient γ of 0.142.

Shift share industry value added shock

In order to provide IV estimates of the market level pass-through and labor supply elasticity, we follow Bartik (1991) and Blanchard and Katz (1992) in constructing a shift-share instrument. Let cz denote a commuting zone and ind denote a 2-digit NAICS industry, and recall that a market is defined by the pair (cz, ind) in our main specification. Let $\overline{Y}_{cz,ind,t}$ and $\overline{W}_{cz,ind,t}$ denote the total value added and total earnings per worker of stayers in the (cz, ind) at time t, and $\overline{Y}_{ind,t} \equiv \sum_{cz} \overline{Y}_{cz,ind,t}$ denote aggregate industry value added. Let $\overline{Y}_{cz,t} \equiv \sum_{ind} \overline{Y}_{cz,ind,t}$ and $\overline{W}_{cz,t} \equiv \sum_{ind} \overline{W}_{cz,ind,t}$ denote aggregate commuting zone value added and earnings per stayer, respectively.

Then, the shift-share value added shock to the commuting zone is constructed as $\sum_{ind} S_{cz,ind,t_0}\zeta_{ind,t}$, where $S_{cz,ind,t} \equiv \overline{Y}_{cz,ind,t}/\overline{Y}_{cz,t}$ is the exposure of the cz to a particular ind (the "share" component), $\zeta_{ind,t} \equiv \log \overline{Y}_{ind,t} - \log \overline{Y}_{ind,t-\tau}$ is the log change in industry value added (the "shift" component), and we measure the share component at the earliest period in the sample (i.e., t_0 =2001). To estimate the market level pass-through, we regress the log change in earnings per stayer $\log \overline{W}_{cz,t} - \log \overline{W}_{cz,t-\tau}$ in the commuting zone on the log change in total value added in the commuting zone $\log \overline{Y}_{cz,t} - \log \overline{Y}_{cz,t-\tau}$, instrumented by the shift-share value added shock.

In order to draw statistical inference, we cluster standard errors at the industry-level using the approach of Borusyak et al. (Forthcoming). To do so, we transform the outcome variable $\log \overline{W}_{cz,t} - \log \overline{W}_{cz,t-\tau}$ and the endogenous regressor $\log \overline{Y}_{cz,t} - \log \overline{Y}_{cz,t-\tau}$ into industry-level variables using the equivalence result in Proposition 1 of Borusyak et al. (Forthcoming). Then, we regress the industry-level transformed outcome variable on the industry-level transformed endogenous regressor, instrumented by the industry-level shock $\zeta_{ind,t}$, and calculate heteroskedasticity-robust standard errors. In the second panel in Appendix Table A.4, we apply this research design to the sample of 667 unique commuting zones during 2001-2015. The ratio of the reduced form and first stage effects yields a statistically significant market-level pass-through coefficient Υ of 0.189. The first stage F-statistic using only industry-level variation is about 11.

C.4 Interacted fixed effect equation, firm specific TFP a_{jt} and amenities h_j

Identification details

We consider the equation in the text,

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{rt} - \bar{y}_{r1}\right) - \frac{\rho_r}{\rho_r + \lambda\beta}\left(\tilde{y}_{jt} - \tilde{y}_{j1}\right) \middle| \begin{array}{c} j(i,t)=j \\ j \in J_r \end{array} \right].$$

We assume that the initial conditions for the permanent productivity shocks at the firm and market level satisfy $\tilde{a}_{j1} = \tilde{p}_j$ and $\overline{a}_{r(j)1} = \overline{p}_r$. Then, we can write

$$\begin{aligned} w_{it} &= \theta_j x_i + c_r - \alpha_r h_{j(i,t)} + \frac{1}{1+\lambda\alpha_r\beta/\rho_r}\tilde{a}_{j(i,t)t} + \frac{1}{1+\lambda\alpha_r\beta}\overline{a}_{r(j(i,t))t} + v_{it} \\ \tilde{y}^*_{j,t} - \tilde{y}^*_{j1} &= \frac{1+\lambda\beta/\rho_r}{1+\lambda\alpha_r\beta/\rho_r}\left(\tilde{a}_{jt} - \tilde{p}_j\right) \\ \bar{y}^*_{rt} - \bar{y}^*_{r1} &= \frac{1+\lambda\beta}{1+\lambda\alpha_r\beta}\left(\overline{a}_{rt} - \overline{p}_r\right) \end{aligned}$$

where $\tilde{y}^*_{j,t}$ and $\bar{y}^*_{rt}$ denote $\tilde{y}_{j,t}$ and $\bar{y}_{rt}$ net of measurement error. Given that the measurement error in y_{jt}, ν_{jt}, is mean zero and the same applies to the measurement error in w_{it}, v_{it}, even conditional on mobility (as given by assumptions 1.b and 1.c), we have that

$$\mathbb{E}\left[w_{it} - \frac{1}{1+\lambda\beta}\left(\bar{y}_{rt} - \bar{y}_{r1}\right) - \frac{\rho_r}{\rho_r + \lambda\beta}\left(\tilde{y}_{jt} - \tilde{y}_{j1}\right) \middle| \begin{array}{c} j(i,t)=j \\ j \in J_r \end{array} \right] = \theta_j x_i + \psi_j,$$

where we define

$$\psi_j \equiv c_r - \alpha_r h_j + \frac{1}{1+\lambda\beta}\overline{p}_r + \frac{\rho_r}{\rho_r + \lambda\beta}\tilde{p}_j.$$

Next, we can identify θ_j from data on the changes in earnings associated with these moves:

$$\frac{\mathbb{E}[w^a_{it+1}|j(i,t){=}j', j(i,t{+}1){=}j] - \mathbb{E}[w^a_{it}|j(i,t){=}j, j(i,t{+}1){=}j']}{\mathbb{E}[w^a_{it}|j(i,t){=}j', j(i,t{+}1){=}j] - \mathbb{E}[w^a_{it+1}|j(i,t){=}j, j(i,t{+}1){=}j']} = \frac{\theta_j}{\theta_{j'}} \tag{20}$$

as long as the denominator is non zero, which is ensured by the following assumption:

$$\mathbb{E}\left[x_i|j(i,t){=}j, j(i,t{+}1){=}j'\right] \neq \mathbb{E}\left[x_i|j(i,t){=}j', j(i,t{+}1){=}j\right].$$

Individual types x_i are then also idenfitied from Assumption 1.c since

$$x_i = \mathbb{E}\left[\frac{w^a_{it} - \psi_{j(i,t)}}{\theta_{j(i,t)}}|i\right].$$

Given x_i, we can construct the firm's log efficiency units of labor as

$$l_{jt} = \log\int X^{\theta_j} D_{jt}(X)\mathrm{d}X.$$

Since the production function paramters $\alpha_{r(j)}$ is already known, we get the following expression for a_{jt}:

$$\mathbb{E}\left[y_{jt} - \alpha_{r(j)} l_{jt}|j\right] = a_{jt}.$$

We can use this to construct $\overline{a}_{rt} = \mathbb{E}[a_{jt}|j \in J_r]$ and $\tilde{a}_{jt} = a_{jt} - \overline{a}_{rt}$. This then identifies the permanent components $\tilde{p}_j$ and $\tilde{p}_r$ as well as the inovation variances $\sigma^2_{\tilde{u}}$ and $\sigma^2_{\overline{u}}$. The final step is to rearrange the expression for ψ_j to back out h_j.

Estimation details

Equation (14) and (20) make clear that (ψ_j, θ_j) can be identified from comparing the gains from moving from a low to a high type of firm for workers of different quality. In practice, we simultaneously recover (ψ_j, θ_j) from the following moment condition:

$$\mathbb{E}\left[\left(\frac{w^a_{it+1}}{\theta_{j'}} - \frac{\psi_{j'}}{\theta_{j'}}\right) - \left(\frac{w^a_{it}}{\theta_j} - \frac{\psi_j}{\theta_j}\right)\Big|j(i,t){=}j, j(i,t{+}1){=}j'\right] = 0. \tag{21}$$

This moment condition provides an instrumental variables representation where the interactions between indicators for firm before the move and firm after the move can be interpreted as the instruments and the parameters are $(\frac{1}{\theta_1}, ..., \frac{1}{\theta_K}, \frac{\psi_1}{\theta_1}, ..., \frac{\psi_K}{\theta_K})$. In the general case in which the number of firm types is unrestricted, $\left(\hat{\theta}_j, \hat{\psi}_j\right)$ would suffer from incidental parameter bias, even under the assumption that $\theta_j = 1$ (see the discussion by Bonhomme et al. 2020). As discussed in the text and further explored in our Online Supplement, we alleviate this concern using the grouped fixed effect estimation with 10 firm types proposed by Bonhomme et al. (2019). With 10 firm types, equation (21) provides 100 moments and 20 unknown parameters. As a result, this can be interpreted as an over-identified model. Following Bonhomme et al. (2019), estimation is implemented using LIML on these moment conditions where the θ_j are concentrated on the post-move time period (in theory they can be estimated without imposing stationarity). To check the relevance of these instruments, we compute the F-statistic corresponding to the first-stage regression, which is 9288 with an R-squared of about 0.30.

Regaring the estimation of x_i, we use a sample analog and compute $\hat{x_i} = \frac{1}{T}\sum_t \frac{w^a_{it} - \psi_{k(j(i,t))}}{\theta_{k(j(i,t))}}$. Given (θ_j, ψ_j), this is an unbiased estimate of x_i under Assumption 1.c and the structure of the wage equation. Yet, the plug-in estimator for the variance of x_i can be biased and inconsistent even asymptotically as the number of workers within each firm type grows large. In our Online Supplement, we consider the additional assumption that the measurement error in log earning is the sum of unit root and an MA(0) term. This allows us to compute the implied bias in the plug in estimator of the variance of x_i in finite T. Under this assumption, we find that the bias in the estimated variance of x_i is very small in our context.

C.5 Identification and estimation of $G_j(X)$

Lemma 9. *We show that for all $t, j \in J_r, r, X$ we have:*

$$\tau \exp(\lambda \psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \mid X]\right)^{1/\beta} \left(Pr[j(i,t) = j \mid X, j(i,t) \in J_r]\right)^{\rho_r/\beta}.$$

Proof. We have that:

$$Pr[j(i,t) = j \mid X, j(i,t) \in J_r] = \frac{\left(\tau^{1/\lambda} G_j(X)^{1/\lambda} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}}$$

$$Pr[j(i,t) \in J_r \mid X] = \frac{\left(\sum_{j' \in J_r} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau^{1/\lambda} G_{j'}(X)^{1/\lambda} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_{r'}}\right)^{\rho_{r'}}}$$

We can fix a given t and write $G_j(X) = \overline{G}_r(X)\tilde{G}_j(X)$, imposing the normalization that

$$\sum_{j' \in J_r} \left(\tau^{1/\lambda} \tilde{G}_{j'}(X)^{\frac{1}{\lambda}} \exp(\psi_{j't}) X^{\theta_{j'}}\right)^{\lambda\beta/\rho_r} = 1$$

$$\sum_r \overline{G}_r(X)^{\beta} = 1$$

Substituting, we have

$$Pr[j(i,t) = j \mid X, j(i,t) \in J_r] = \left(\tau^{1/\lambda} \tilde{G}_j(X)^{\frac{1}{\lambda}} \exp(\psi_{jt}) X^{\theta_j}\right)^{\lambda\beta/\rho_r}$$
$$Pr[j(i,t) \in J_r \mid X] = \left(\overline{G}_r(X)\right)^{\beta}$$

Thus,

$$\tau \exp(\lambda\psi_{jt}) X^{\lambda\theta_j} G_j(X) = \left(Pr[j(i,t) \in J_r \mid X]\right)^{1/\beta} \left(Pr[j(i,t) = j \mid X, j(i,t) \in J_r]\right)^{\rho_r/\beta}$$

Since this result does not depend on the normalization, it is true for all t.

$\square$

Next, we explain the estimation procedure that relies on the expression that we just derived. For estimation, we use grouped structure both at the firm and at the market level. We group firms using the classification described in the text based on the firm-specific empirical distribution of earnings; we denote the firm groups by $k(j)$. We follow a similar approach at the market level and group markets based on the market level empirical distribution of earnings; we denote the market groups by $m(r)$. At this point we think of a firm class $k(j)$ as being within market type m, so when using the classification of Section 5, we interact the firm group k with the market group m.

Using these two classifications, we rely on the fact that worker composition can be estimated at the group level instead of trying to estimate a distribution for each individual firm and market. Indeed, in the model we have that:

$$Pr[X|j] = Pr[X|k(j)]$$
$$Pr[X|r] = Pr[X|m(r)].$$

Similarly to the Lemma above, we can define $G_j(X) = \overline{G}_r \tilde{G}_j G_{k(j)}(X)$. Following the lemma we impose the following constraints on $\overline{G}_r$ and $\tilde{G}_j$:

$$\sum_{j' \in J_r} \left(\tau^{1/\lambda} \left(\tilde{G}_{j'} G_{k(j')}(X)\right)^{\frac{1}{\lambda}} \exp(\lambda\psi_{j't}) X^{\lambda\theta_{j'}}\right)^{\lambda\beta/\rho_r} = 1$$
$$\sum_r \overline{G}_r^{\beta} = 1$$

We then directly apply the formula for $G_j(X)$ at the firm group level $k(j)$ within market $m(r(j)))$:

$$G_k(X) = X^{-\lambda\theta_k} \left(\frac{Pr[X|m]}{Pr[X]}\right)^{1/\beta} \left(\frac{Pr[X|k]}{Pr[X|m]}\right)^{\rho_r/\beta}.$$

Next we recover the j-specific component by matching the size of each firm within its market:

$$Pr[j(i,t) = j \mid j(i,t) \in J_r] = \tilde{G}_j^{\beta/\rho_r} \int \frac{\left(\tau G_{k(j)}(X) \exp(\lambda \psi_{jt}) X^{\lambda \theta_j}\right)^{\beta/\rho_r}}{\sum_{j' \in J_r} \left(\tau \tilde{G}_{j'} G_{k(j')}(X) \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}}\right)^{\beta/\rho_r}} Pr[X|m(r)] \mathrm{d}X$$

Similarly, we recover the market level constant by matching the market level size:

$$Pr[j(i,t) \in J_r \mid X] = \overline{G}_r^{\beta} \int \frac{\left(\sum_{j' \in J_r} \left(\tau \tilde{G}_j G_{k(j)}(X) \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}}\right)^{\beta/\rho_r}\right)^{\rho_r}}{\sum_{r'} \left(\sum_{j' \in J_{r'}} \left(\tau \overline{G}_{r'} \tilde{G}_{j'} G_{k(j')}(X) \exp(\lambda \psi_{j't}) X^{\lambda \theta_{j'}}\right)^{\beta/\rho_{r'}}\right)^{\rho_{r'}}} NM(X) \mathrm{d}X$$

D Additional Robustness Checks

D.1 Pass-through estimation

The main results are displayed in Online Appendix Table A.3. Additional heterogeneity and robustness analyses are presented in Online Appendix Figure A.1.

We now provide evidence that the main results are not sensitive to alternative specifications. First, we allow for greater persistence in the transitory shock process by considering a MA(2) specification. This is accounted for by choosing $e = 3, e' = 4$ in the empirical counterparts to equations (12)-(13). Results are provided in the fourth column of Panel B in Online Appendix Table A.3. Under an MA(2) specification of the transitory shock process, we estimate that the average firm level pass-through rate γ_r is 0.13 and the market level pass-through rate Υ is 0.18, which are the same as our main findings from the MA(1) specification.

Second, our specification of the earnings process allows permanent shocks to value added to be transmitted to workers' earnings, whereas transitory firm shocks are not. As a specification check, we allow transitory innovations to value added to transmit to workers' earnings. Results are provided in the fourth column of Panel A in Online Appendix Table A.3. We find little if any pass-through of transitory shocks. As a result, transitory shocks explain as little as 0.1 percent of the variation in log earnings. This finding is consistent with previous work (see, e.g., Guiso et al. 2005; Friedrich et al. 2019). A possible interpretation of this finding is that transitory changes in value added reflect measurement error that do not give rise to economic responses. In the remainder of the paper, we will treat the transitory changes in value added as measurement error and focus on the pass-through of the permanent shocks.

Third, to compare with existing work, we also consider estimating the restricted specification that imposes $\gamma_r = \Upsilon, \forall r$. This is equivalent to imposing $\rho_r = 1, \forall r$, so that idiosyncratic worker preferences over firms are uncorrelated within markets. These results are reported in the first two columns of Panel A in Online Appendix Table A.3. The estimated pass-through rate is then 0.14, which is between our estimates of 0.13 at the firm level and 0.18 at the market level.

Fourth, in Online Appendix Figure A.1, we explore robustness of the pass-through estimates across subsamples of workers. Conditional on a full set of year times market fixed effects, we find in subfigure (a) that the pass-through rates do not vary that much by the worker's age, previous wage, or gender. Moreover, the pass-through rates do not change materially if we restrict the sample to new workers who were first hired at the firm in the beginning of the eight year employment spell versus those that have stayed in the firm for a longer time.

Fifth, in subfigure (b) of in Online Appendix Figure A.1, we present results from several specification checks on firms. Following Guiso et al. (2005), our main measure of firm performance is value added. They offer two reasons for using value added as a measure of firm performance: value added is the variable that is directly subject to stochastic fluctuations, and firms have discretionary power over the reporting of profits in balance sheets, which makes profits a less reliable objective to assess. Nevertheless, it is reassuring to find that the estimates of the pass-through rates are broadly similar if we measure firm performance by operating profits, earnings before interest, tax and depreciation (EBITD), or value added net of reported depreciation of capital. We also show that the estimated pass-through rate is in the same range as our baseline result if we exclude multinational corporations (for which it can be difficult to accurately measure value added) or exclude the largest firms (that are more likely to have multiple plants, which may not necessarily have the same wage setting).

D.2 Firm and worker effect estimation

In Online Appendix Table A.6, we provide a number of specification checks. First, we consider estimating the model when ignoring firm-worker interactions by imposing $\theta_j = \bar{\theta}$. The results are presented in the second column of Table A.6. When interactions are ignored, the share of earnings variation explained by worker quality increases by about two percentage points while that explained by firm effects decreases from 4.3 percent to 3.0 percent. Sorting and time-varying effects are little changed. We conclude that the estimated variance of firm effects is downward-biased when ignoring firm-worker production complementarities.

Second, we consider estimating the model when ignoring time-varying effects by imposing $\gamma_r = \Gamma = 0$. The results are presented in the third column of Table A.6. When time-varying effects are ignored, the share of earnings variation explained by worker quality decreases by about one percentage point while that explained by interactions increases by about half a percentage point. The variance of firm effects and sorting are little changed. We conclude that there is little bias in the other terms in the variance decomposition when ignoring production complementarities.

Third, we consider estimating the model when ignoring both firm-worker interactions and time-varying effects by imposing $\theta_j = \bar{\theta}$ and $\gamma_r = \Gamma = 0$. The results are presented in the fourth column of Table A.6. The estimates for worker quality, firm effects, and sorting are similar to the results when only ignoring firm-worker interactions. Note that specification is the same as the model of Abowd et al. (1999) that has been estimated in a recent literature except that we use a bias-corrected estimate, so we can compare this specification directly to other papers to learn about limited mobility bias. An extensive discussion of limited mobility bias and comparison to the literature is available in our Online Supplement.

In our Online Supplement, we provide additional robustness checks. We consider increasing the number of groups k in the k-means algorithm from the baseline value of 10 up to 50 in increments of 10, finding that the estimates are nearly identical across k. We also present estimates for two different time periods (2001-2008 and 2008-2015), finding that the worker quality, firm effects, and sorting components change little over time.

E Online Appendix: Additional Tables and Figures

	Workers		**Firms**	
Panel A.	**Baseline Sample**			
	Unique	**Observation-Years**	**Unique**	**Observation-Years**
Full Sample:	89,570,480	447,519,609	6,478,231	39,163,975
Panel B.	**Movers Sample**			
	Unique	**Observation-Years**	**Unique**	**Observation-Years**
Movers Only:	32,070,390	207,990,422	3,559,678	23,321,807
Panel C.	**Stayers Sample**			
	Unique	**6 Year Spells**	**Unique**	**6 Year Spells**
Complete Stayer Spells:	10,311,339	35,123,330	1,549,190	6,533,912
10 Stayers per Firm:	6,297,042	20,354,024	144,412	597,912
10 Firms per Market:	5,217,960	16,506,865	117,698	476,878

Table A.1: Overview of the Sample

Notes: This table provides an overview of the full sample, movers sample, and stayers sample, including the steps involved in defining the stayers sample.

	Goods				Services				All
	Midwest	Northeast	South	West	Midwest	Northeast	South	West	All
Panel A.					**Full Sample**				
Observation Counts:									
Number of FTE Worker-Years	42,908,008	26,699,951	40,312,311	31,585,748	69,044,540	62,386,621	103,227,384	71,355,046	447,519,609
Number of Unique FTE Workers	9,318,707	6,088,530	10,215,128	7,712,759	17,314,497	15,167,028	26,519,284	17,949,625	89,570,480
Number of Unique Firms with FTE Workers	294,879	232,717	439,641	329,566	1,051,548	1,054,944	1,908,178	1,314,168	6,478,231
Number of Unique Markets with FTE Workers	1,508	264	1,774	910	4,092	744	4,909	2,492	16,141
Group Counts:									
Mean Number of FTE Workers per Firm	22.1	17.8	16.1	16.3	10.4	9.7	9.5	9.6	11.4
Mean Number of FTE Workers per Market	2,012.9	6,856.7	1,586.3	2,539.3	1,221.0	5,723.0	1,492.8	2,097.7	1,915.1
Mean Number of Firms per Market with FTE Workers	91.3	384.9	98.3	156.0	117.4	588.2	156.6	217.7	167.6
Outcome Variables in Log $:									
Mean Log Wage for FTE Workers	10.76	10.81	10.70	10.81	10.61	10.74	10.62	10.70	10.69
Mean Value Added for FTE Workers	17.36	16.80	16.68	16.64	16.18	16.04	15.94	16.07	16.31
Firm Aggregates in $1,000:									
Wage Bill per Worker	43.6	50.7	42.2	52.9	34.1	44.2	35.8	40.3	40.8
Value Added per Worker	91.2	107.5	85.2	91.7	90.5	111.1	94.2	92.3	95.2
Panel B.					**Movers Sample**				
Observation Counts:									
Number of FTE Mover-Years	17,455,849	11,543,303	18,066,928	15,513,020	31,643,497	28,390,782	50,052,742	35,324,301	207,990,422
Number of Unique FTE Movers	4,124,895	2,829,881	4,819,645	3,876,182	7,723,804	6,662,132	11,904,098	8,321,469	32,070,390
Number of Unique Firms with FTE Movers	188,376	144,268	265,374	215,092	571,360	549,064	1,018,957	700,618	3,559,678
Number of Unique Markets with FTE Movers	1,457	261	1,747	872	3,899	739	4,766	2,342	15,586
Group Counts:									
Mean Number of FTE Movers per Firm with FTE Movers	13.5	11.9	11.2	11.6	8.2	7.9	7.9	8.2	8.9
Mean Number of Movers per Market with FTE Movers	864.8	2,991.3	732.4	1,318.1	599.3	2,655.3	761.5	1,123.7	940.6
Mean Number of Firms per Market with FTE Movers	64.1	251.1	65.5	113.4	72.7	337.1	96.4	137.7	105.5
Outcome Variables in Log $:									
Mean Log Wage for FTE Movers	10.68	10.77	10.64	10.78	10.59	10.72	10.61	10.70	10.67
Mean Value Added for FTE Movers	16.72	16.52	16.28	16.36	16.04	16.02	15.88	16.01	16.12
Panel C.					**Stayers Sample**				
Sample Counts:									
Number of 8-year Worker-Firm Stayer Spells	2,588,628	1,777,928	1,237,821	1,150,115	2,315,238	2,527,212	2,609,997	2,207,552	16,506,865
Number of Unique FTE Stayers in Firms with 10 FTE Stayers	798,575	532,507	416,549	354,518	740,091	764,699	865,629	724,155	5,217,960
Number of Unique Firms with 10 FTE Stayers	13,884	10,896	9,409	9,767	18,083	19,475	19,626	16,185	117,698
Number of Unique Markets with 10 Firms with 10 FTE Stayers	197	111	216	104	335	213	438	219	1,826
Outcome Variables in Log $:									
Mean Log Wage for FTE Stayers	10.95	10.99	10.97	10.99	10.90	11.01	10.96	11.05	10.97
Mean Log Value Added for FTE Stayers	18.04	17.56	17.46	16.56	17.45	17.23	17.89	17.93	17.61

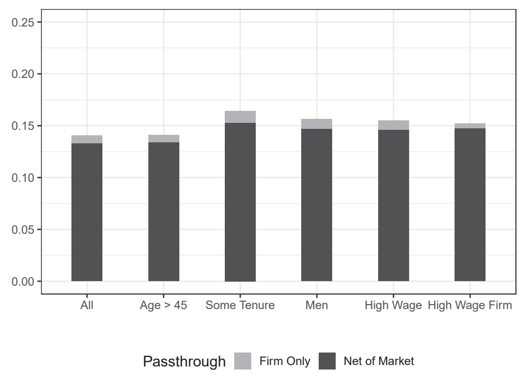
Table A.2: Detailed sample characteristics

Notes: This table provides detailed sample characteristics for the full sample, movers sample, and stayers sample.

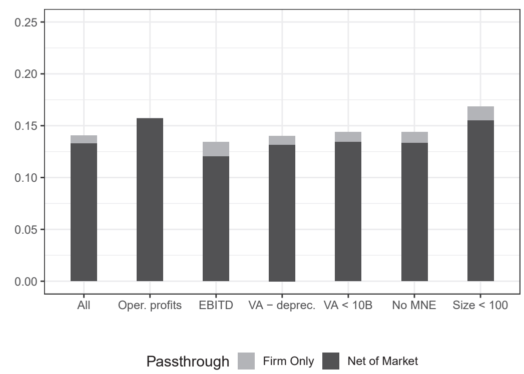
	GMM Estimates of Joint Process			
	Firm Only		Accounting for Markets	
	Log Value Added	Log Earnings	Log Value Added	Log Earnings
Panel A.	**Process: MA(1)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
Transitory Shock (Std. Dev.)	0.18	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.09	0.15	0.09	0.15
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 2	0.00	0.00	0.00	0.00
	(0.00)	(0.00)	(0.00)	(0.00)
Permanent Passthrough Coefficient		0.14		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.01		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.02)
Panel B.	**Process: MA(2)**			
Total Growth (Std. Dev.)	0.31	0.17	0.29	0.16
	(0.01)	(0.00)	(0.01)	(0.00)
Permanent Shock (Std. Dev.)	0.20	0.10	0.17	0.10
	(0.01)	(0.00)	(0.00)	(0.00)
Transitory Shock (Std. Dev.)	0.17	0.10	0.17	0.10
	(0.01)	(0.00)	(0.01)	(0.00)
MA Coefficient, Lag 1	0.05	0.21	0.07	0.21
	(0.05)	(0.01)	(0.04)	(0.01)
MA Coefficient, Lag 2	-0.03	0.04	-0.01	0.04
	(0.03)	(0.00)	(0.02)	(0.00)
Permanent Passthrough Coefficient		0.15		0.13
		(0.01)		(0.01)
Transitory Passthrough Coefficient		-0.02		0.00
		(0.01)		(0.00)
Market Passthrough Coefficient				0.18
				(0.03)

Table A.3: GMM estimates of the earnings and value added processes

Notes: This table displays the parameters of the joint processes of log value added and log earnings. These results come from joint estimation of the earnings and value added processes under assumptions 1.a-1.c using GMM. Columns 1-2 report results from the specification which imposes $\gamma_r = \Upsilon$ ("Firm only"), while columns 3-4 report results from the specification which allows Υ to differ from γ_r and γ_r to vary across broad markets ("Accounting for Markets"). The top panel assumes the transitory components follow an MA(1) process. The bottom panel permits the transitory components to follow an MA(2) process. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.



(a) Heterogeneity across Worker Samples



(b) Heterogeneity across Firm Samples and Value Added Measures

Figure A.1: Sample heterogeneity in pass-through rates of firm shocks

Notes: This figure displays heterogeneity in the GMM estimates of the pass-through rates of a firm shock, both for the firm only model (imposing $\Upsilon = \gamma$) and when removing market by year means (permitting $\Upsilon \neq \gamma$).

Outcome Sample	First Stage (Std. Error)	Reduced Form (Std. Error)	Second Stage (Std. Error)
Procurement auction shock at firm-level			
8,677 unique auction bidders	0.143	0.020	0.142
	(0.039)	(0.006)	(0.068)
Shift-share industry value added shock			
667 unique commuting zones	0.708	0.134	0.189
	(0.216)	(0.061)	(0.041)

Table A.4: Additional details regarding pass-through estimation using external instruments

Notes: This table provides additional details on the pass-through estimation using external instruments.

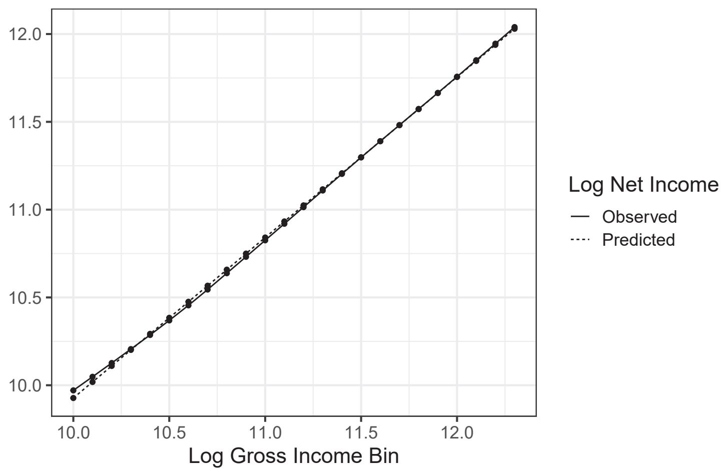
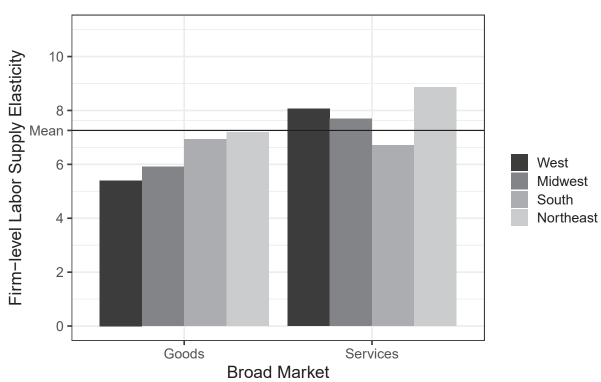
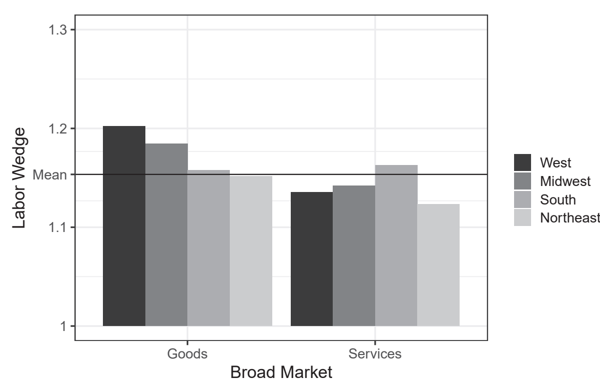


Figure A.2: Fit of the Tax Function

Notes: In this figure, we display the log net income predicted by the tax function compared to the log net income observed in the data.



(a) Labor Supply Elasticity $\frac{\beta}{\rho_r \lambda}$



(b) Labor Wedge $1 + \frac{\rho_r \lambda}{\beta}$

Figure A.3: Broad Market Heterogeneity in Labor Supply Elasticities and Labor Wedges

Notes: In this figure, we display the estimated (post-tax) firm level labor supply elasticity and labor wedge for each of the 8 broad markets. The overall worker-weighted means are represented by horizontal lines.

	Market Count (in 1,000)		Passthrough Rate		Average of the Model Parameters			Workers' Share of Rents Firm-level	Market-level
	Workers	Firms	Market	Firm	β	$1-\rho_r^2$	$1-\alpha_r$	$\frac{R^w}{R^w+R^f}$	$\frac{R^{wm}}{R^{wm}+R^{fm}}$
Baseline (NAICS 2-digit, commuting zone)	1.90	0.17	0.18	0.13	4.99	0.51	0.79	0.52	0.50
Shutdown broad market heterogeneity ($\rho_r=\overline{\rho},\alpha_r=\overline{\alpha}$)	1.97	0.17	0.18	0.13	5.06	0.48	0.79	0.52	0.51
Alternative detailed markets:									
Finer geography (county)	0.54	0.05	0.19	0.14	4.61	0.54	0.79	0.51	0.49
Finer industry (NAICS 3-digit)	0.65	0.06	0.19	0.13	4.60	0.59	0.79	0.52	0.50
Coarser geography (state)	25.44	2.23	0.18	0.13	5.00	0.52	0.79	0.53	0.50
Coarser industry (NAICS supersector)	4.42	0.39	0.20	0.13	4.28	0.66	0.79	0.53	0.51

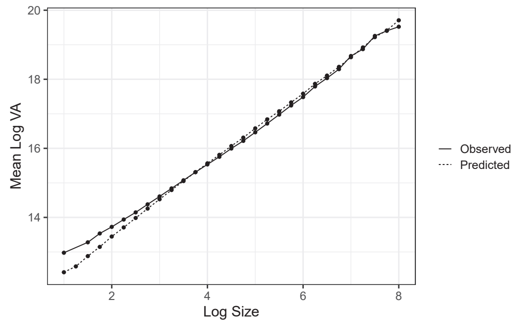
Table A.5: Robustness of the Model Parameters and Rent Sharing Estimates to Alternative Market Definitions

Notes: This table displays robustness of the estimated model parameters and rents to alternative definitions of detailed markets.

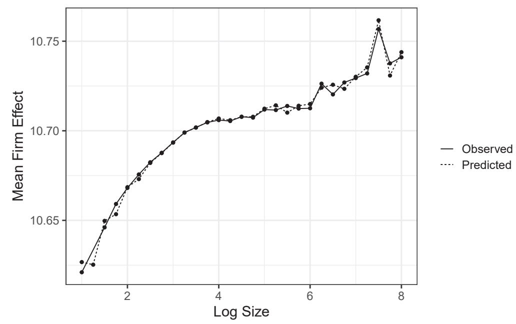
		Model Specifications			
		Main	**Alternatives**		
			$\theta_j=\bar{\theta}$	$\gamma_r=\Upsilon=0$	$\theta_j=\bar{\theta}$ and $\gamma_r=\Upsilon=0$
Share explained by:					
i) Worker Quality	$Var(\tilde{x}_i)$	71.6%	73.5%	70.4%	72.4%
ii) Firm Effects	$Var(\tilde{\psi}_{j(i)})$	4.3%	3.0%	4.3%	3.2%
iii) Sorting	$2Cov(\tilde{x}_i,\tilde{\psi}_{j(i)})$	13.0%	12.8%	13.1%	12.9%
iv) Interactions	$Var(\varrho_{ij})+2Cov(x_i+\psi_{j(i)},\varrho_{ij})$	0.9%		1.2%	
v) Time-varying Effects	$Var(\psi^a_{j(i),t})+2Cov(x_i,\psi^a_{j(i),t})$	0.3%	0.3%		
Sorting Correlation:	$Cor(x_i,\psi_{j(i)})$	0.37	0.43	0.38	0.43
Variance Explained:	R^2	0.90	0.90	0.89	0.89
Specification:					
Firm-Worker Interactions		✓	✗	✓	✗
Time-varying Firm Effects		✓	✓	✗	✗

Table A.6: Decomposition of earnings inequality

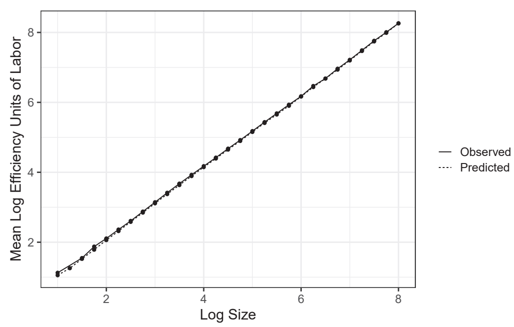
Notes: This table presents the decomposition of log earnings variation into firm and worker effects using the main specification described in the text, as well as alternative specifications that ignore firm-worker interactions ($\theta_j=\bar{\theta}$), ignore time-varying effects ($\gamma_r=\Upsilon=0$), and ignore both ($\theta_j=\bar{\theta}$ and $\gamma_r=\Upsilon=0$). The analysis uses both workers who move between firms and non-movers. All estimates are corrected for limited mobility bias using the grouped fixed-effect method of Bonhomme et al. (2019).



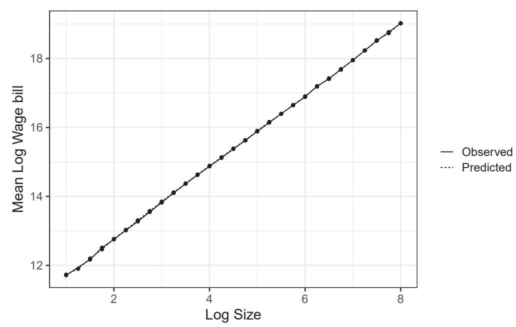
(a) Value Added



(b) Firm Effects



(c) Efficiency Units of Labor



(d) Wage Bill

Figure A.4: Fit of the Model for Untargeted Moments

Notes: In this figure, we compare the observed and the predicted values of firm effects, value added, efficiency units of labor, and wage bill. We make this comparison separately according to actual and predicted firm size.

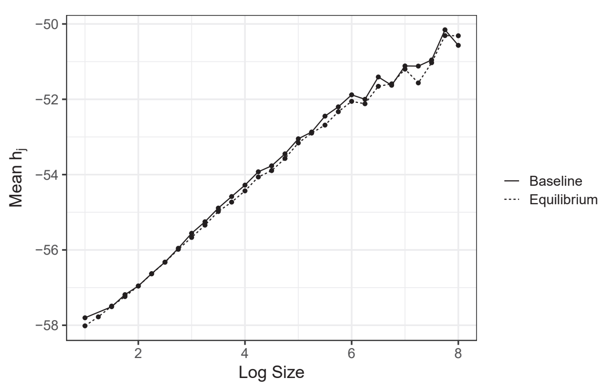


Figure A.5: Estimates of the Amenity Components h_j from the Wage Equation versus the Equilibrium Constraint

Notes: In this figure, we plot the mean of h_j across log size bins. We compare the baseline estimates of h_j from the equation for firm wage premiums (15), versus those estimated using the equilibrium constraint by solving the fixed-point definition of h_j as a function of $(\tilde{P}_j, \bar{P}_r, G_j(X))$, as shown in Lemma 3.

	Goods				Services			
	Midwest	Northeast	South	West	Midwest	Northeast	South	West
Panel A.	**Model Parameters**							
Idyosinctratic taste parameter (β^{-1})	0.200							
	(0.044)							
Taste correlation parameter (ρ)	0.844	0.694	0.719	0.924	0.649	0.563	0.744	0.619
	(0.179)	(0.153)	(0.160)	(0.182)	(0.141)	(0.109)	(0.246)	(0.117)
Returns to scale ($1-\alpha$)	0.746	0.764	0.863	0.949	0.753	0.740	0.814	0.752
	(0.016)	(0.013)	(0.017)	(0.019)	(0.013)	(0.015)	(0.036)	(0.015)
Panel B.	**Firm-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	6,802	6,681	5,737	8,906	4,234	4,847	5,009	4,805
	(770)	(723)	(720)	(867)	(502)	(803)	(1,295)	(684)
Share of Earnings	16%	13%	14%	17%	12%	11%	14%	12%
	(2%)	(1%)	(2%)	(2%)	(1%)	(2%)	(4%)	(2%)
Firms' Rents:								
Per-worker Dollars	4,041	4,198	7,465	20,069	3,531	3,097	6,915	3,018
	(1,243)	(1,130)	(2,681)	(6,323)	(1,004)	(1,305)	(5,650)	(1,060)
Share of Profits	8%	7%	17%	52%	6%	5%	12%	6%
	(3%)	(2%)	(6%)	(16%)	(2%)	(2%)	(10%)	(2%)
Workers' Share of Rents	63%	61%	43%	31%	55%	61%	42%	61%
	(4%)	(4%)	(5%)	(4%)	(4%)	(5%)	(9%)	(5%)
Panel C.	**Market-level Rents and Rent Shares**							
Workers' Rents:								
Per-worker Dollars	7,837	9,102	7,572	9,506	6,115	7,935	6,422	7,230
	(1,319)	(1,532)	(1,274)	(1,600)	(1,029)	(1,335)	(1,081)	(1,217)
Share of Earnings	18%	18%	18%	18%	18%	18%	18%	18%
	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)	(3%)
Firms' Rents:								
Per-worker Dollars	4,940	6,311	10,000	20,846	5,734	5,897	9,363	5,153
	(1,140)	(1,350)	(2,267)	(5,787)	(1,351)	(1,786)	(4,218)	(1,433)
Share of Profits	10%	11%	23%	54%	10%	9%	16%	10%
	(2%)	(2%)	(5%)	(15%)	(2%)	(3%)	(7%)	(3%)
Workers' Share of Rents	61%	59%	43%	31%	52%	57%	41%	58%
	(3%)	(3%)	(4%)	(5%)	(3%)	(4%)	(8%)	(4%)

Table A.7: Market Heterogeneity in Model Parameters and Rent Sharing Estimates

Notes: This table displays heterogeneity in the estimated model parameters and rents. These results correspond to the specification which allows Υ to differ from γ, and for ρ_r and α_r to vary across broad markets. Standard errors are estimated using 40 block bootstrap draws in which the block is taken to be the market.

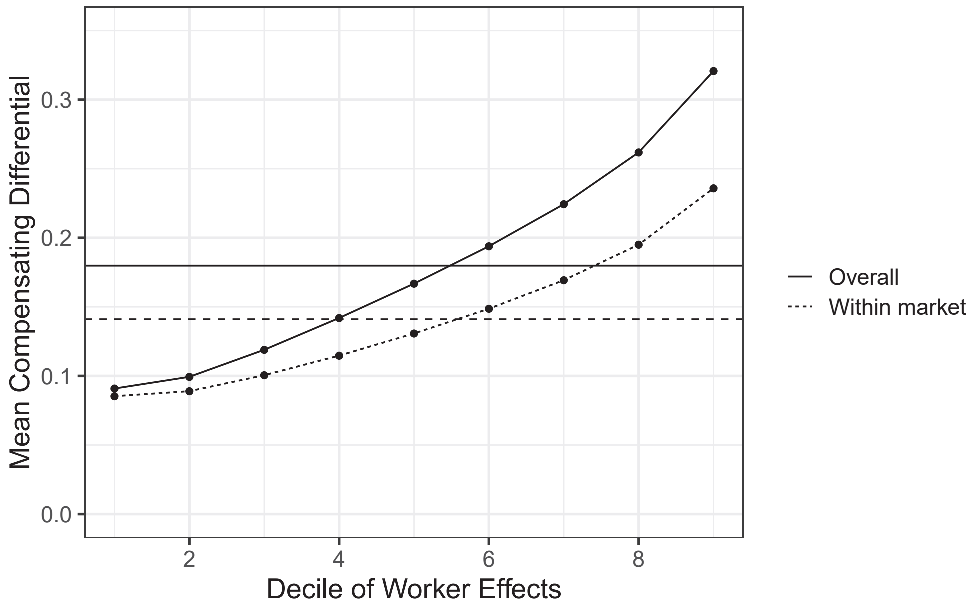
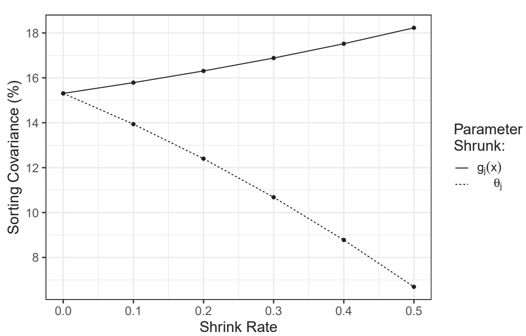
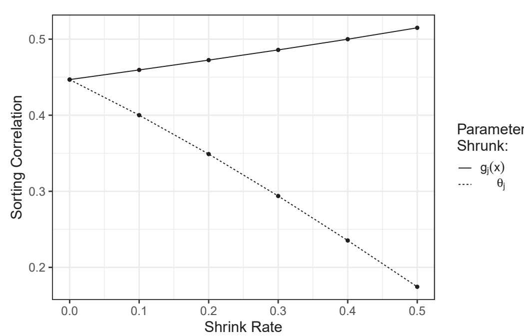


Figure A.6: Compensating differentials

Notes: In this figure, we plot mean compensating differentials overall and within market. To do so, we randomly draw a pair of firms (j, j') with probability proportional to size. Each j' is drawn from the full set of firms when estimating overall compensating differentials and from the set of firms in the same market as j when estimating within-market compensating differentials. Then, we estimate the compensating differential between j and j' for a worker of given quality $x_i = x$ by $\psi_{j'} + x\theta_{j'} - \psi_j - x\theta_j$. This figure plots the mean absolute value of the compensating differentials across deciles of the x_i distribution, where the horizontal lines denote means across the distribution of x_i.



(a) Sorting



(b) Sorting Correlation

Figure A.7: Worker sorting with counterfactual values of $g_j(x)$ and θ_j

Notes: In this figure, we reduce the heterogeneity across firms in amenities or production complementarities by replacing either $g_j(x)$ with $(1-s)\,g_j(x) + s\overline{g}_j$ or θ_j with $(1-s)\,\theta_j + s\bar{\theta}$, where $\overline{g}_j = \mathbb{E}_x\left[g_j(x)\right]$, $\bar{\theta} = \mathbb{E}\left[\theta_j\right]$. Here, $s \in [0,1]$ is the shrink rate with $s = 0$ corresponding to the baseline model. We report the share of log earnings variance explained by sorting (subfigure a) and the sorting correlation (subfigure b).