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CONSUMPTION AND INVESTMENT

Andrew B. Abel

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ABSTRACT

This paper presents an overview of current models of consumption and investment behavior. First, the stochastic implications of the permanent income model and empirical tests of these implications are discussed. Then the simple theoretical model is extended to include expenditure on consumer durables. In addition, the implications of liquidity constraints and the unpredictability of the rate of return on wealth are discussed. The overview of consumption behavior closes with a critical discussion of the Ricardian Equivalence Theorem.

Investment behavior is analyzed using a dynamic optimization model of a firm facing costs of adjustment. This framework integrates the accelerator model, the neoclassical model and the q theory. The model is then used to analyze the interaction of corporate taxes, inflation and investment and also to analyze the effects of uncertainty on investment. The overview of investment concludes with a discussion of inventory investment.

Andrew B. Abel
Department of Finance
Wharton School
3620 Locust Walk
Philadelphia, PA 19104-6367
(215) 898-4801

Consumption and investment expenditure together account for 80 percent of GNP in the United States and for a similarly large percentage of GNP in other major economies.[1] This chapter analyzes the behavior of consumption and investment focusing on the response of these components of spending to changes in income and to changes in assets markets. I have tried to present the material in this chapter so that it will be useful both to Keynesian macroeconomists and to new classical macroeconomists. To a Keynesian economist, the organizing principle of the chapter can be viewed as the development of private domestic behavioral relations underlying the IS schedule. In particular, I have stressed the effects of income and interest rates on consumption and investment. Although a new classical economist would not find it helpful to think of this chapter in terms of the IS curve, he or she could view the separate treatments of consumption and investment as developing, within an intertemporal optimization framework, the behavior of different economic actors.

This chapter is, by design, partial equilibrium in nature. What is missing is the endogenous determination of income and interest rates. A Keynesian economist would close the model and determine income and interest rates by adding an LM schedule, but the LM schedule is covered elsewhere in this handbook. A new classical economist would specify a production function and then would allow prices and interest rates to adjust to clear all markets. With the exception of a brief discussion of the implications of general equilibrium for testing the permanent income hypothesis, this chapter does not touch upon general equilibrium considerations.

In keeping with the partial equilibrium focus of this chapter, I will first discuss the determinants of consumption and then I will discuss investment. Since the 1950's, economic models of consumption behavior have explicitly recognized that in making consumption decisions, consumers take account of their lifetime resources rather than simply their current income. Both the life-cycle model of Modigliani and Brumberg (1954) and Ando and Modigliani (1963)

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[1]The ratio of private consumption to GNP and the ratio of gross fixed investment to GNP for the period 1980-85 are: U.S.: 64.3% and 15.7%; U.K.: 60.1% and 16.8%; Germany: 57.0% and 20.8%; Japan (1980-84): 59.0% and 27.8%.

and the permanent income model of Friedman (1957) are based on the notion that consumers prefer smooth streams of consumption over time. Access to capital markets allows consumers to choose a sequence of consumption over time that is smoother than the sequence of income. In Section I.A, I show that if income in every period is deterministic, then for a consumer with access to perfect capital markets, there would be no relation between income and contemporaneous consumption. However, if income follows a stochastic process, then there is, in general, a positive contemporaneous correlation between consumption and income. Section I.B. analyzes the relation between consumption and income and discusses several empirical tests of the permanent income hypothesis.

The simple permanent income model discussed in Section I is applicable to the consumption of nondurable goods and services. However, the model is not applicable to consumers' expenditures on durable goods. Because durable goods produce services to consumers over several periods, these goods are consumed over several periods. Because the expenditure on a durable good usually takes place in one period, it is important to distinguish the consumption of durable goods from the expenditure on these goods. From the viewpoint of the individual consumer, what matters is the flow of consumption services from durable goods. From the viewpoint of the macroeconomic determination of aggregate income, expenditure on durable goods is important. Section II enriches the permanent income model to incorporate durable goods as well as non-durable goods and services into the decision problems of individual consumers.

The theoretical analysis of consumer expenditure in Sections I and II is based on the assumption that consumers have access to perfect capital markets and can borrow or lend at an exogenous rate of interest. However, a substantial fraction of consumers is unable to consume as much as predicted by the permanent income model because they cannot borrow as much as they would like at the prevailing interest rate. Consumers who would like to increase their current borrowing in order to increase current consumption are said to be liquidity constrained. The importance of liquidity constraints from the viewpoint of macroeconomics is that the relation between consumption and contemporaneous income is generally different for liquidity constrained consumers than it is for consumers who do not face binding liquidity constraints. The implication of liquidity constraints for the relation between consumption and income is discussed in Section III.

In analyzing the relation between income and consumption in the first three sections of the paper, the rate of return on wealth is assumed to be constant. However, there are important links between asset markets and consumption behavior. In particular, the level of consumption depends on the consumer's wealth, and the intertemporal pattern of consumption depends on the rate of return on assets. Section IV presents a formal model of consumption by an infinitely-lived consumer who faces a stochastic rate of return on wealth. This model produces a simple relation between consumption and wealth and allows us to distinguish the effects of ex post changes in the rate of return from changes in the ex ante rate of return.

Although the statement that consumption depends on the consumer's level of wealth is not controversial, there is still wide-ranging disagreement about what constitutes the wealth of a consumer. In particular, should a consumer's holding of government bonds be counted as net wealth? An equivalent question in a different guise is whether a bond-financed cut in lump-sum taxes has an effect on consumption. At first glance, it would appear that consumers who receive a tax cut would view themselves as having an increase in lifetime disposable resources and would increase their consumption accordingly. However, because the government must eventually pay interest on the newly issued bonds and repay the principal, the bond-financed tax cut implies that future taxes will be increased. Indeed, the increase in future taxes will have a present value equal to the current tax cut, and thus, it is argued by some economists, there will be no response of consumption to a change in tax policy. In Section V, this argument, which is known as the Ricardian Equivalence Theorem, will be presented and critically evaluated.

The discussion of capital investment begins in Section VI with the Jorgensonian neoclassical theory of investment. This theory explicitly treats the demand for capital as a derived demand by starting with the firm's production function and demand curve. The demand curve and production function are used to obtain a relation between a firm's cash flow and its contemporaneous stock of fixed capital (plant and equipment). The firm's demand for fixed capital is set at a level that equates the marginal profit of capital with the user cost of capital. The user cost is a concept that captures the cost of using a unit of capital in production over a certain period of time. The neoclassical theory of investment predicts that a firm's demand for capital will be positively related to the firm's level of output and will be negatively related to the user cost of capital. A more restricted model which corresponds to a special case of the neoclassical model is the accelerator model, in which the demand for capital is proportional to the level of output but is independent of the user cost. The accelerator model and the more general neoclassical model are discussed in Section V.A.

An alternative theory of investment behavior by firms is the q theory. Tobin (1969) defined q to be the ratio of the market value of a firm to the replacement cost of the firm. This ratio is meant to measure the value of fixed capital relative to its cost. The greater is this ratio, the greater would be the incentive to acquire the capital and hence the greater would be the rate of investment. Because the value of the firm is measured using data from equity and bond markets, the link between asset markets and investment expenditure is quite explicit. Although Tobin's presentation of the q theory did not explicitly model the firms' production function and demand curve, it is possible to start with the demand curve and the production function and then derive the q theory as the result of intertemporal maximization by firms. A formal derivation of the q theory, and the link between the formal model and Tobin's q, is presented in Section VI.B.

The corporate tax environment--in particular, the corporate tax rate, the investment tax credit, and the schedule of depreciation allowances--has a potentially important impact on capital investment decisions. Although the effects of these aspects of the tax code on investment are important in their own right, from the viewpoint of monetary economics the most interesting feature of the taxation of capital income and expenditure is the interaction of inflation, taxes and investment. This interaction is briefly discussed in Section VII.

The models of investment analyzed in Sections VI and VII do not take explicit account of uncertainty facing firms. The decision to present deterministic models in these sections reflects two considerations: First, as a matter of expositional clarity, the deterministic models are much simpler than the stochastic models. Second, and more importantly, is that, in contrast to models of consumption, state-of-the-art models of investment behavior do not rely critically on the stochastic nature of the decision problem facing firms. Nevertheless, a brief discussion of the impact of uncertainty is presented in Section VIII.

In addition to investment in plant and equipment, firms also invest in inventories. Inventory behavior has been a particularly puzzling component of aggregate demand. It would appear that just as consumers with concave utility functions would want to have smooth time profiles of consumption, firms with convex cost functions would want to have smooth time profiles of production. Inventories provide firms with a means to have smooth production in the face of fluctuating sales. However, it does not appear that firms actually take advantage of inventory accumulation and decumulation to smooth out production relative to sales. Section IX discusses this apparent contradiction in the simple production smoothing model as well as possible explanations.

I. Consumption

The life cycle and permanent income hypotheses, which are the major theories of consumption behavior, each relate the consumption of a consumer to his lifetime income rather than to his contemporaneous income. The underlying choice-theoretic framework is that a consumer has an intertemporal utility function that depends on consumption in every period of life. The consumer maximizes utility subject to single lifetime budget constraint. There is no static, or period by period, budget constraint that requires consumption in a period to equal the income in that period.[2] Indeed, in the absence of uncertainty, the life cycle and permanent income hypotheses both predict that there will be no relation between consumption and contemporaneous income. However, the introduction of uncertainty will generally induce a positive relation between consumption and contemporaneous income.

To develop the implications of the permanent income model, consider the decision problem facing an individual consumer at time t. Let y_{t+j} denote the consumer's after-tax labor

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[2] In the presence of binding liquidity constraints, which are discussed in Section III, the consumer will face a sequence of period by period budget constraints.

income at time t+j, for j = 0, 1, 2, . . . It is convenient to assume that the consumer lives forever. Strictly speaking, this assumption is consistent with the permanent income hypothesis but is inconsistent with the life-cycle hypothesis. One of the major implications of the life-cycle hypothesis is that saving is done by consumers when they are working to provide for consumption when they are retired. This implication will not be captured in a model in which the consumer lives, and earns income, forever. However, for the purpose of examining the cyclical relation between consumption and contemporaneous income, it is simply not important whether the consumer has a finite horizon.[3] Let c_{t+j} denote the consumption of the consumer in period t+j and let W_t denote the wealth of the consumer at the beginning of period t before earning interest. The rate of return on wealth carried from period t-1 to period t is r_t. The accumulation of wealth is described by

$$W_{t+1} = (1 + r_t)W_t + y_t - c_t \,. \tag{1}$$

Equation (1) describes the evolution of the consumer's wealth over time but, by itself, does not constrain behavior. There is nothing in equation (1) that prevents the consumer from borrowing to finance arbitrarily large consumption. An additional constraint is needed. If the consumer has a finite lifetime, with period T being the last period of his life, then one could impose the constraint $W_{T+1} \geq 0$, which states that the consumer cannot die in debt. Under an infinite horizon, the appropriate constraint is

$$\lim_{j \to \infty}\{(1 + r_t)(1 + r_{t+1})(1 + r_{t+2}) \ldots (1 + r_{t+j})\}^{-1} W_{t+j+1} \geq 0 \,.$$

The intertemporal utility function of the consumer is assumed to be additively separable over time.[4] Let $u(c_{t+j})$ denote the utility of consumption in period t+j. The period utility function u( ) is assumed to be strictly increasing and strictly concave. As of the beginning of period t the

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[3]The wealth effects associated changes in the timing of lump-sum taxes and the validity of the Ricardian Equivalence Theorem discussed below depend critically on whether the horizon of an individual consumer is finite or infinite. Poterba and Summers (1987) argue that empirically the distinction between infinite horizons and finite horizons has a small effect on the impact of tax policy.

[4]For examples of non-time-separable utility functions, see Hayashi (1985b) and Eichenbaum, Hansen and Singleton (1986).

consumer maximizes the intertemporal utility function

$$U_t = E_t\left\{ \sum_{j=0}^{\infty} (1 + \rho)^{-j}\, u(c_{t+j}) \right\} \tag{2}$$

where ρ is the rate of time preference and where $E_t\{\ \}$ denotes the expectation conditional on information available at the beginning of period t. This available information includes the realization of current income y_t and the current rate of return on wealth r_t.

It is now straightforward to derive the first-order condition characterizing optimal consumption behavior

$$u'(c_t) = (1 + \rho)^{-1} E_t\{(1 + r_{t+1})\, u'(c_{t+1})\} \tag{3}$$

To interpret (3) consider a reduction in c_t of one unit accompanied by a one unit increase in the wealth carried into period t+1. The additional unit of wealth carried into period t+1 produces an additional $1 + r_{t+1}$ units of disposable resources in period t+1 which can be consumed in period t+1 without affecting any future opportunities of the consumer. In evaluating whether this potential intertemporal rearrangement of consumption is a good idea, the consumer compares $u'(c_t)$, which is the loss in utility from the unit reduction in c_t, with

$$(1 + \rho)^{-1} E_t\{(1 + r_{t+1})\, u'(c_{t+1})\}\ ,$$

which is the expected discounted gain in utility from the increase of $(1 + r_{t+1})$ in c_{t+1}. If the utility loss associated with a unit reduction in c_t is smaller than the expected discounted utility gain from the increase in period t+1 consumption, then the consumer can increase expected utility by reducing c_t. Alternatively, if the utility loss associated with a unit reduction in c_t is greater than the expected discounted utility gain from the increase in period t+1 consumption, then the consumer can increase expected utility by increasing c_t. Optimality requires that neither an increase nor a decrease in c_t can lead to higher expected utility, which is implied by equation (3).

Now suppose that the rate of return on wealth is perfectly predictable one period in advance; more precisely, suppose that r_{t+1} is in the information set at time t. This assumption holds, for example, if the real interest rate r_t is constant over time. Empirically, if the length of a period is taken to be a calendar quarter and if r_t is the real return on 90-day Treasury Bills, then the assumption that r_t is perfectly predictable one period in advance may be a reasonable approximation.[5] Alternatively, if r_t is the one-period holding return on common stocks, then the assumption that r_t is perfectly predictable one period in advance is clearly inappropriate. Nevertheless, I make this assumption to understand some of the implications of the first-order condition in equation (3). Observe that equation (3) can be rewritten as

$$u'(c_{t+1}) = [(1+\rho)/(1+r_{t+1})]\, u'(c_t) + e_{t+1} \tag{4}$$

where e_{t+1} is an unpredictable random variable. More precisely, $E_t\{e_{t+1}\} = 0$. Equation (4) is particularly useful for understanding the stochastic implications of the permanent income hypothesis. Before examining the stochastic implications of (4), I will first discuss the implications of intertemporal utility maximization in the absence of uncertainty.

A. Deterministic Income

In the absence of uncertainty the random disturbance e_{t+1} in (4) is identically equal to zero. In this case, equation (4) implies that the marginal utility of consumption grows (or falls) at a rate equal to $(\rho - r_{t+1})/(1 + r_{t+1})$. Thus, if the rate of return r_{t+1} exceeds the discount rate ρ, then the marginal utility falls over time which implies that consumption rises over time. That is, if the reward to postponing consumption (r_{t+1}) exceeds the impatience cost of waiting (ρ), then the consumer will choose to have lower consumption today than in the future. Alternatively, if the rate of return on saving is less than the rate of time preference, then the consumer will choose to have higher consumption today than in the future.

Now make the stronger assumption that the rate of return r_t is a constant, and furthermore that r_t is equal to the rate of time preference ρ. It follows immediately from (4) that if $r_t = \rho$ and if e_{t+1} is identically zero, then consumption is constant over time. The level of consumption will be the maximum permanently sustainable flow of consumption, which Friedman (1957) has

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[5] Although the nominal rate of return on 90-day T-bills is perfectly predictable, the rate of inflation cannot be predicted perfectly, so the real rate of return cannot be predicted perfectly.

called permanent income. Note that if the consumer always consumes an amount equal to the net return on his or her wealth (appropriately defined, as below, to include human as well as non-human wealth), then his or her total wealth will remain constant over time. Any attempt to permanently consume more than the return on wealth will not be sustainable. Thus permanent income is equal to the real rate of return on total wealth multiplied by total wealth.

To calculate the level of permanent income, it is necessary to calculate human wealth. In the absence of uncertainty, and in the presence of a real interest rate which is constant and equal to ρ, human wealth is simply the present value of current and future labor income,[6] which I will denote as H_t^*. More precisely,

$$H_t^* = \frac{1}{1+r} \sum_{j=0}^{\infty} (1+r)^{-j} y_{t+j} \; . \qquad (5)$$

The factor $1/(1 + r)$ appears in front of the summation because, consistent with the definition of non-human wealth, I am defining human wealth in period t to be calculated prior to earning the rate of return r in that period. The implication of this definition is that if income is always equal to some constant, say y_0, then human wealth would be equal to y_0/r. In this case, the return to human wealth would be y_0 so that in the absence of nonhuman wealth, permanent income would be equal to y_0.

In the presence of nonhuman wealth, permanent income, y_t^P* is equal to the return on human plus nonhuman wealth so that

$$y_t^P* = r\left(W_t + H_t^*\right) \; . \qquad (6)$$

Recall that with $r = \rho$, consumption is constant over time. The invariance of consumption over time holds even if labor income is (deterministically) time-varying. Thus, for an individual living in a world without uncertainty, there would be no relation between consumption and contemporaneous income over time. However, in a cross-section of individuals with different

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[6]Flavin (1981) pointed out that Sargent (1978) erroneously defined permanent income as the present value of disposable income. However, because disposable income includes the return on wealth, this concept involves double counting.

levels of permanent income, there would be a positive cross-sectional relation between consumption and permanent income. From the viewpoint of macroeconomics and stabilization policy, it is the time-series co-movement of consumption and contemporaneous income which is of interest. Since there would be no systematic co-movement of consumption and contemporaneous income for a consumer in a deterministic environment, it is necessary to shift attention to a stochastic environment.

B. Stochastic Income

In the presence of income uncertainty the definition of permanent income needs to be modified somewhat. Although current nonhuman wealth W_t and current after-tax labor income y_t are each known at the beginning of period t, future labor income is uncertain at the beginning of period t. Therefore, human wealth as defined in (5) is not observable to the individual consumer at time t. In the presence of uncertainty, the expression on the right hand side of (5) will be called the ex post human wealth and the expression on the right hand side of (6) will be called ex post permanent income at time t. An individual consumer in period t must choose consumption in period t prior to observing the ex post permanent income.

Let $H_t \equiv E_t\left\{H_t^*\right\}$ denote ex ante human wealth in period t and let $y_t^P \equiv E_t\left\{y_t^P*\right\}$ denote ex ante permanent income in period t. Taking the conditional expectation of each side of (6) yields

$$y_t^P = r(W_t + H_t) \ . \tag{7}$$

Suppose that the consumer sets consumption in period t equal to ex ante permanent income y_t^P so that

$$c_t = y_t^P = r(W_t + H_t) \ . \tag{8}$$

Strictly speaking, it is not generally optimal to set consumption equal to permanent income as in (8). The uncertainty associated with future income flows may generate precautionary saving which would imply that an intertemporally optimizing consumer would choose to consume less than permanent income as defined here.[7] However, if the utility function u( ) is quadratic, which implies that the third derivative of u( ) is identically equal to zero, then the certainty equivalence principle implies that it is indeed optimal to set consumption equal to permanent income as defined in (6). I will ignore the complications associated with a nonzero third derivative of u( ), and proceed as if optimal consumption is equal to permanent income in (6).

Before proceeding to study the response of consumption to income for a fairly general stochastic process for income, I first derive a consumption function for a simple special case. Suppose that y_t evolves according to the first-order autoregressive process

$$y_t - \bar{y} = a_1 (y_{t-1} - \bar{y}) + u_t \tag{9}$$

where $0 \leq a_1 < 1$, $E_{t-1}\{u_t\} = 0$, and $\bar{y}$ is the unconditional expected value of y_t. In this case, $E_t\{y_{t+j}\} = \bar{y} + a_1^j (y_t - \bar{y})$ so that using the definition of permanent income in (5) and (6) it can be shown that consumption is

$$c_t = rW_t + [r/(1 + r - a_1)]\, y_t + [(1 - a_1)/(1 + r - a_1)]\, \bar{y} \; . \tag{10}$$

Equation (10) relates consumption to wealth and contemporaneous income and thus resembles a traditional aggregate consumption function.[8] Note that the coefficients on y_t and $\bar{y}$ are each positive and they sum to one. Thus, ignoring wealth W_t, consumption would be a weighted average of current income and the unconditional average value of income. The weight on current income is an increasing function of a_1 which measures the persistence of deviations in income. Although (10) may not appear at first glance to be a forward-looking consumption function, it

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[7]See Dreze and Modigliani (1972), Kimball (1986), and Zeldes (1986) for discussions of precautionary saving. Recently, Caballero (1987) has derived the solution to the consumer's optimization problem under uncertainty with a constant absolute risk aversion utility function. He has argued that precautionary saving behavior can explain the excess sensitivity and excess smoothness phenomena discussed below.

[8]See, for example, Ando and Modigliani (1963) and Modigliani (1975). In the formulation presented in equation (10), a one dollar increase in current wealth leads to an r dollar increase in current consumption. This result depends on the assumption that the consumer has an infinite horizon. Alternatively, under the life cycle model, which assumes that the consumer has a finite horizon, the consumer consumes some of the principal in addition to the interest on his wealth. In this case, the coefficient on wealth is larger than the real interest rate r. Empirically, Ando and Modigliani (1963) estimated this coefficient to be in the range from 0.04 to 0.10 for a sample of U.S. data; in examining Italian data, Modigliani (1975) estimated the coefficient on wealth to be roughly in the range from 0.06 to 0.09.

does take account of forecasts of future income. It turns out that for a first-order autoregressive process, y_t contains all information that is known about future deviations of income from $\bar{y}$.

For a more general stochastic process on y_t, I will not derive a consumption function relating consumption to wealth and current and past income. Instead I will focus on the relation between fluctuations in consumption and fluctuations in income.

To study the fluctuations in consumption, recall that consumption, c_t, is equal to contemporaneous (ex ante) permanent income y_t^P. Therefore, fluctuations in consumption will be identical to fluctuations in permanent income. If the rate of return on wealth is constant, then all fluctuations in permanent income are due to fluctuations in human wealth; specifically, fluctuations in permanent income are due to revisions in expectations about future labor income. It follows immediately from the definition of human wealth (5) and the fact that $E_t\{E_{t+1}\{y_{t+j}\}\} = E_t\{y_{t+j}\}, j = 1, 2, 3, \ldots$, that

$$H_t = \frac{1}{1+r}\left[y_t + E_t\{H_{t+1}\}\right] . \tag{11}$$

Adding nonhuman wealth to both sides of (11) yields

$$W_t + H_t = \frac{1}{1+r}\left[(1+r)W_t + y_t + E_t\{H_{t+1}\}\right] . \tag{12}$$

Now multiply both sides of (12) by (1 + r) and use the wealth accumulation equation (1) to replace $(1 + r)W_t + y_t$ by $c_t + W_{t+1}$ to obtain

$$(1+r)\left[W_t + H_t\right] = c_t + W_{t+1} + E_t\{H_{t+1}\} . \tag{13}$$

Equation (13) was derived simply by manipulating the definition of human wealth and using the wealth accumulation equation; it does not embody any behavioral assumptions. Now suppose that consumption is equal to permanent income and use (8) and (13) to obtain

$$r(W_t + H_t) = r(W_{t+1} + E_t\{H_{t+1}\}) . \tag{14}$$

Equation (14) indicates that if consumption is equal to permanent income, then permanent income is not expected to change. Equivalently, any change in permanent income and consumption between period t and period t+1 must be unanticipated from the viewpoint of period t. The underlying economic reason for this result, of course, is that if the return on wealth is equal to the rate of time preference, the individual optimally plans to have constant consumption over his life. Indeed, using the definition of permanent income in (8), and using the fact that $E_t\{H_{t+1}\} = E_t\{E_{t+1}\{H_{t+1}\}\}$ yields

$$c_t = y_t^P = E_t\left\{y_{t+1}^P\right\} = E_t\{c_{t+1}\} . \tag{15}$$

Thus, the conditional forecast of c_{t+1} based on information available in period t is equal to c_t.[9] Therefore, any deviation from constant consumption must be the result of unanticipated factors.

It follows immediately from (15) and the permanent income hypothesis in (8) that

$$c_{t+1} - c_t = y_{t+1}^P - E_t\left\{y_{t+1}^P\right\} . \tag{16}$$

The change in consumption from one period to the next is equal to the innovation, i.e., the unanticipated change, in permanent income.

In order to calculate the changes in permanent income and consumption, it is necessary to specify the stochastic process for after-tax labor income. The simplest stochastic environment to analyze is one in which after-tax labor income is stochastic but the rate of return on wealth r_t is constant. Let r denote the constant value of r_t. Suppose that y_t evolves according to a univariate autoregressive process

$$y_t - \bar{y} = a(L)(y_t - \bar{y}) + u_t \tag{17}$$

where a(L) is a polynomial in the positive powers of the lag operator L, and the innovation u_t is a random disturbance with the property that $E_t\{u_{t+1}\} = 0$. For example, if y_t follows the first-order autoregressive process $y_t - \bar{y} = a_1(y_{t-1} - \bar{y}) + u_t$, then the polynomial a(L) is simply a_1L. It is sometimes more convenient to work with the moving average representation of the income process

$$y_t - \bar{y} = b(L)\,u_t \tag{18}$$

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[9]Hall (1978) first observed that consumption should follow a random walk. This observation is based on the first-order condition (3). If the utility function $u(c_t)$ is quadratic and if the interest rate is equal to ρ, then (3) can be written as $\phi_0 - \phi_1 c_t = E_t\{\phi_0 - \phi_1 c_{t+1}\}$, which implies that $E_t\{c_{t+1}\} = c_t$.

where b(L) is a polynomial in the non-negative powers of the lag operator L. It follows from inspection of (17) and (18) that $b(L) = (1 - a(L))^{-1}$. The effect on future income y_{t+j} of a one unit innovation in y_t is b_j so that

$$E_{t+1}\{y_{t+1+j}\} - E_t\{y_{t+1+j}\} = b_j u_{t+1} . \tag{19}$$

Using the definitions of ex post and ex ante permanent income yields an expression that relates the innovation in permanent income to the innovation in after-tax labor income

$$y^P_{t+1} - E_t\{y^P_{t+1}\} = r/(1 + r) \sum_{j=0}^{\infty} (1 + r)^{-j} [E_{t+1}\{y_{t+1+j}\} - E_t\{y_{t+1+j}\}] . \tag{20}$$

Equation (20) states that the revision in expected permanent income is equal to the present value of revisions in expectations of y_{t+j}, j = 1, 2, 3, ... Substituting (19) into (20) and recalling that b(L) is the polynomial in the non-negative powers of L, the expression for the innovation in permanent income in (20) can be written more succinctly as

$$y^P_{t+1} - E_t\{y^P_{t+1}\} = (r/(1 + r))\, b(1/(1 + r))\, u_{t+1} . \tag{21}$$

Equation (21) relates the innovation in permanent income to the innovation in current after-tax income.[10] It is perhaps easiest to interpret (21) and its implications in the special case of a first-order autoregressive process. In this case the coefficients b_i are equal to a^i for $i = 0, 1, 2, \ldots$ so that equation (21) and equation (16) together imply that

$$c_{t+1} - c_t = r/(1 + r - a_1)\, u_{t+1} . \tag{22}$$

Equation (22) relates consumption to the contemporaneous innovation in income. Interpreting the response of consumption to the contemporaneous innovation in income as the marginal propensity to consume (MPC), equation (20) implies that the MPC is equal to $r/(1 + r - a_1)$. The size of the marginal propensity to consume plays a crucial role in Keynesian models of aggregate demand. Equation (22) illustrates that the value of the MPC depends on the nature of the

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[10] The factor $(r/(1 + r))b(1/(1 + r))$ on the right hand side of (21) is equal to $(r/(1 + r))(1 - a(1/(1 + r))^{-1}$. Note that if y_t is stationary, then this factor is positive. However, it is not necessarily less than one, even if y_t is stationary.

stochastic process of income as stressed by Friedman (1957). If income is serially uncorrelated, then $a_1 = 0$ and the MPC is equal to $r/(1 + r)$. The average annual real rate of return in the U.S. is, depending on the asset, somewhere between zero and perhaps seven percent.[11] This suggests that if annual income is serially uncorrelated, then the MPC is quite small, ranging roughly from zero to 0.07.

The MPC is an increasing function of the parameter a_1, which is the first-order serial correlation coefficient of a first-order univariate autoregressive process for income. In fact, income--more precisely aggregate income--tends to be very highly serially correlated. Note that if income follows a random walk, then $a_1 = 1$ and the MPC is equal to one. The reason for a unitary MPC in the case of a random walk is that any innovation in income is expected to be permanent. That is, a one dollar innovation in income at time t raises the forecast of income at all future dates by one dollar and hence raises the expectation of permanently sustainable consumption by one dollar.

The relation between consumption and income that is predicted by the permanent income hypothesis serves as a basis for econometrically testing this hypothesis. Flavin (1981) examined the joint behavior of consumption and income and concluded that consumption displays excessive sensitivity to the anticipated change in contemporaneous income. Of course, this conclusion depends on the estimated stochastic process for income. More recently, Deaton (1986) and Campbell and Mankiw (1986) have suggested that income has a random walk component so that, for example, a positive innovation to income raises the forecasts of future income into the indefinite future. Deaton (1986) and Campbell and Deaton (1987) have estimated the stochastic process for income including a random walk component and have argued that permanent income is more variable than current income because changes in labor income are positively serially correlated. Therefore, if consumption is equal to permanent income, then consumption should be more variable than current income. Deaton and Campbell and Deaton calculate, based on the

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[11] Mehra and Prescott (1985) report that in the U.S. over the period from 1889 to 1978 the average real rate of return on short-term bonds is 0.80% per year and the average real rate of return on equity is 6.98% per year.

estimated time-series process for income, the degree to which the variance of consumption should exceed the variance of current income. They conclude that consumption is "too smooth."[12] At first glance Flavin's finding of excess sensitivity of consumption to income appears to be contradicted by Campbell and Deaton's finding that consumption responds too little to innovations in income. Campbell and Deaton resolve this apparent contradiction by observing that Flavin's result concerns the relation between consumption and the anticipated change in income, whereas their excess smoothness result concerns the relation between consumption and the contemporaneous innovation to the income process. When Campbell and Deaton examine the relation between consumption and anticipated changes in income, they also find excess sensitivity. In addition, they present an analytic argument that "there is no contradiction between excess sensitivity and excess smoothness; they are the same phenomenon." (p. 33)

The tests of the permanent income hypothesis based on the time-series properties of income and consumption maintain the assumption that the rate of interest used to discount future cash flows is constant. This seemingly innocuous assumption has important implications for the interpretation of tests of permanent income hypothesis. Michener (1984) developed a simple stochastic general equilibrium model in which the interest rate is endogenously determined. He showed that even if consumers maximize the expected value of a standard time-separable utility function, the stochastic process for aggregate consumption can fail to satisfy the properties discussed above. Although Michener's model includes production and capital accumulation, his point can be made more simply, and more starkly, by considering an endowment economy in which each (identical) consumer receives an endowment y_t of the homogeneous perishable good. In equilibrium, aggregate consumption (per capita), c_t, will be equal to aggregate income, y_t, and hence aggregate consumption would inherit the time-series properties of aggregate income. In this situation, consumption and income would have equal variances so that comparisons of the variances of these series would be uninformative. Also, if the change in income were

[12]West (1987) models the income process with a random walk component and develops a variance bounds test of the permanent income model. He also finds that consumption is too smooth relative to income.

forecastable, the change in consumption would be forecastable which violates one of the implications of the permanent income hypothesis. The lesson from Michener's analysis is that the tests of the permanent income hypothesis discussed above maintain several auxiliary assumptions in addition to the hypothesis that consumers maximize an intertemporal utility function subject to a budget constraint and subject to available information about future income. Therefore, rejections of the permanent income hypothesis based on the time series properties of consumption and income can be interpreted as rejecting specific formulations of the permanent income hypothesis but do not necessarily reject the hypothesis of intertemporal utility maximization under uncertainty.

II. Consumer Durables

The discussion so far has proceeded under the assumption that there is a homogeneous consumption good. While this assumption is intended to be only a simplifying abstraction, one must ask what sorts of important or interesting differences among goods are masked by this assumption. The major heterogeneity among goods that is recognized in the literature on consumption is the distinction between durable goods and non-durable goods. In fact, expenditure on durable goods and consumption of nondurable goods (and services) have quite different cyclical behavior. Durable goods expenditures display much more volatility over the business cycle than do nondurable goods and services. More precisely, the percentage variation in durables expenditures is much greater than the percentage variation in nondurables consumption. However, the level of expenditure on durables is much smaller than the level of expenditures on nondurables. In fact, it is this difference in the average level of expenditures on durables and nondurables that accounts for the difference in the percentage variation. The variation in the absolute level of durables expenditures is smaller than the variation in the absolute level of nondurable expenditures.[13]

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[13]Startz (1987) reports that "the standard deviation of deviations from trend for durables is large (60 dollars), about one-half the size of that for nondurables." (p. 2) Mankiw (1982) uses lagged information to forecast durables expenditure and nondurables consumption. He finds that the standard deviations of the forecast errors are roughly equal (13.1 for durables vs. 13.2 for nondurables).

In analyzing the behavior of consumer durables, it is important to distinguish between expenditure on consumer durables which I will denote by x_t, and the consumption of the services of durables which I will denote by d_t. I will assume that the flow of consumption services from durables during period t, d_t, is proportional to the stock of durables held at the beginning of period t, D_t, plus the durables acquired during period t, x_t. In particular, suppose that

$$d_t = \psi(D_t + x_t) \ . \tag{23}$$

Although the concept of the consumption of the flow of services from the durable good is important for some purposes, it is the level of expenditure on durable goods which is important for the determination of aggregate demand.

It is useful to introduce the concept of the user cost of durables. To simplify the analysis, suppose that the relative price of durables and nondurables remains fixed over time. Let μ denote the price of durables in terms of nondurables. In the absence of relative price changes, there are two components to the user cost of the durable: foregone interest and depreciation. By holding a unit of a durable rather than interest-earning wealth, the consumer foregoes interest of $r\mu$ per period, where r is the real interest rate. In addition, if the durable depreciates at a rate δ per period, then depreciation imposes a cost of $\delta\mu$ per period to the owner of the durable. Therefore the user cost of a durable is $(r + \delta)\mu$.

The introduction of durables implies that the consumer holds two assets: interest-earning wealth and durables. Previously, W_t was defined to be the nonhuman wealth of a consumer at the beginning of period t. Now W_t is to be interpreted as the interest-earning wealth, or equivalently, as the nonhuman wealth of the consumer minus the value of the consumer's stock of durable goods. The budget constraint in (1) must be amended to

$$W_{t+1} = (1 + r_t)\, W_t + y_t - c_t - x_t \tag{24}$$

where c_t is now interpreted as the consumption of nondurables. Recalling that δ is the depreciation rate per period of the durable leads to the following relation between the stock of the durable and expenditures on the durable

$$D_{t+1} = (1 - \delta)(D_t + x_t) \ . \tag{25}$$

In order to motivate expenditure on durables as well as nondurables, the utility function must be augmented to include services from durables as well as from nondurables. Let $u(c_t, d_t)$ be the utility function in period t. The consumer will allocate spending in period t between the purchase of nondurables and the rental of durables. The optimal allocation will equate the marginal rate of substitution with the rental price. Writing the utility function as $u(c_t, \psi(D_t + x_t))$ makes clear that the marginal rate of substitution between nondurables and durable goods is $\psi u_d/u_c$ where u_d is the derivative of u(c, d) with respect to durable services and u_c is the derivative of u(c, d) with respect to nondurables. Setting this marginal rate of substitution equal to the rental price of durables yields

$$\psi u_d/u_c = (r + \delta)\mu \ . \tag{26}$$

For simplicity, suppose that the period utility function has the following Cobb-Douglas specification

$$u(c_t, d_t) = c_t^{1-\alpha} d_t^{\alpha} \ . \tag{27}$$

In this case, the consumer will allocate a fraction 1-α of his or her consumption basket to nondurables, c_t, and a fraction α to the rental of services of durables, $(r + \delta)\mu(D_t + x_t)$. Therefore,

$$D_t + x_t = \frac{1}{(r+\delta)\mu} \cdot \frac{\alpha}{1-\alpha} \cdot c_t \ . \tag{28}$$

Substituting (28) into (25) yields

$$D_{t+1} = (1 - \delta) \frac{1}{(r+\delta)\mu} \frac{\alpha}{1-\alpha} c_t \ . \tag{29}$$

To obtain the relation between expenditures on durables and expenditures on nondurables, substitute (29) into (25) to obtain

$$x_t = \kappa\{c_t - (1 - \delta) c_{t-1}\} \tag{30a}$$

where

$$\kappa = [1/(r + \delta)\mu]\,[\alpha/(1 - \alpha)] \ . \tag{30b}$$

Equation (30a) can be used to determine the response of expenditures on durables to an innovation in income. It follows immediately from (30a) that in response to an innovation in income, the marginal propensity to spend on durables is equal to κ times the marginal propensity to consume nondurables. Let $\sigma_t(x)$ denote the standard deviation of the one-period forecast error of x_t, conditional on information known at the end of period t-1; similarly, let $\sigma_t(c)$ denote the standard deviation of the one-period forecast error of c_t, conditional on information known at the end of period t-1. It follows immediately from (30a) that

$$\sigma_t(x) = \kappa\sigma_t(c) . \tag{31}$$

The parameter κ measures the cyclical volatility of expenditures on durables relative to the cyclical volatility of nondurables consumption. In principle, κ can be either greater or less than one so that the cyclical variability of durables expenditures can exceed or fall short of the cyclical variability of nondurables consumption. However, the theory predicts that the relative variability of durables expenditure must exceed the relative variability of nondurables consumption. To derive this implication of the theory, first observe from (30a) that if x_t and c_t are stationary, then

$$\bar{x} = \kappa\delta\bar{c} \tag{32}$$

where $\bar{x}$ is the average value of x and $\bar{c}$ is the average value of c. Then divide (31) by (32) to obtain $\sigma_t(x)/\bar{x} = (1/\delta)\,\sigma_t(c)/\bar{c}$. Therefore, since δ is less than one, this simple extension of the permanent income model to include consumer durables as well as nondurables explains the fact that the percentage volatility of durables expenditures exceeds that of nondurable consumption.

Equation (32) can be used to get an estimate of the parameter κ using data on the rate of depreciation and the average levels of expenditures on durables and nondurables. Using the figures for average durables expenditure and average nondurables consumption reported in Startz (1987, p. 2) yields a value of $\bar{x}/\bar{c}$ equal to 12.2%. Therefore, the factor κ is equal to $.122/\delta$. Bernanke (1985, p. 53) reports a depreciation rate for consumer durables of 0.0506 per quarter. Therefore, for quarterly data, the value of κ is 2.41. This value of κ appears to be substantially larger than is reflected in consumer spending. It implies, counterfactually, that $\sigma_t(x)$ should be larger than $\sigma_t(c)$. Also recall from (30) that κ is equal to the ratio of the effect of an income innovation on durables expenditure to the effect of an income innovation on nondurables consumption. Bernanke (1985, p. 57) estimates this ratio to be .775. The fact that the calculated value of κ appears to overstate the cyclical variability of durables expenditures may reflect that the model derived above has ignored costs of adjusting the stock of durables (see Bernanke (1985)) and has ignored implications of irreversibility discussed below.

Equation (30a) can be used to analyze the serial correlation of expenditure on consumer durables. The contrast between the predicted serial correlation of durables expenditure and nondurables consumption is particularly striking in the case in which durables are perfectly durable. Formally, durables are perfectly durable when the rate of depreciation, δ, is equal to zero. In this case, equation (30a) implies that x_t is proportional to $c_t - c_{t-1}$, the change in nondurables consumption. Under the permanent income hypothesis the change in nondurables consumption is completely unpredictable, and thus expenditure on durables cannot be predicted. Equivalently, $E_{t-1}\{x_t\} = 0$. Therefore, in the absence of depreciation, expenditure on durables follows a white noise process but expenditure on nondurables follows a random walk.

It is also worth noting that if the rate of depreciation is equal to one, so that durables are completely nondurable, then equation (30a) states that x_t is proportional to c_t. That is, x_t and c_t both follow a random walk, as should be expected because in this case "durables" are nondurable.

The analysis above suggests that the serial correlation of expenditures on durables is an increasing function of the rate of depreciation. In fact, expenditures on durables are highly serially correlated.[14] If the large degree of serial correlation is to be consistent with the model outlined above, then the rate of depreciation would probably have to be implausibly large. Alternative explanations for the high degree of serial correlation would point to departures from

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[14] Lam (1986) reports that, except for motor vehicles, most categories of consumer durables expenditures "are well characterized by a first-order process of coefficient around 0.95." (p. 12) The first-order autocorrelation of new cars is 0.770.

the simple model. Two such departures are liquidity constraints, to be discussed later, and irreversibility of durables expenditures.

The term "irreversibility" of durables expenditures is meant to capture the notion that an individual who tries to sell a used consumer durable generally receives a price that is lower than the value of the remaining durables services evaluated at the market price for new durables. In the extreme case of complete irreversibility, the consumer cannot obtain any resources by selling a used durable good. To see the effects of irreversibility, consider a small unanticipated decrease in the consumer's income. Under perfect reversibility, the consumer should reduce consumption of both durables services and nondurables. However, if the resale price of the durable is low, then the consumer may choose not to sell any of the durable, but instead may reduce nondurable consumption or interest-earning wealth by more than in the optimal plan under complete reversibility. If income continues to be unexpectedly low for a few periods, then the consumer may have to sell off some of the durable in order to avoid a large decline in nondurables consumption. The date at which it becomes optimal to sell some of the durable depends on the level of the consumer's other wealth. With a higher level of wealth, the consumer can wait longer before selling some of the nondurable. Although this discussion has focused on the response to a decrease in income, the consumer will display a conservative response to an increase in income because of the possibility of a future decline in income. At the level of the individual consumer, the effect of the introduction of irreversibility is to reduce the marginal propensity to purchase durables in response to an increase in income. As for the behavior of aggregate expenditure on durables, Lam (1986) has used simulation techniques to show that if there is cross-sectional variation in household wealth, then irreversibility will induce a high degree of serial correlation in aggregate durables expenditures.

III. Liquidity Constraints

The permanent income hypothesis presented above is based on the assumption that an individual consumer can borrow and lend at the same interest rate, and furthermore that the consumer can borrow or lend any amount subject to the lifetime budget constraint described above. An important departure from this assumption is the possibility that the consumer may face a liquidity constraint. Broadly interpreted, the term liquidity constraint is meant to capture the notion that an individual is not able to borrow any amount he or she chooses at an interest rate equal to the rate he or she earns on financial wealth. The departure from the assumption of perfect capital markets may take any of several forms. For instance, the individual may be able to borrow any amount he or she chooses at a fixed interest rate but this rate exceeds the rate of return on financial assets. In this case the intertemporal budget constraint of the individual is piecewise linear, with a kink occurring at a point where current consumption is equal to current income plus liquid financial wealth. An extreme example of this type of liquidity constraint, which corresponds to an infinite borrowing rate, is the case in which the consumer is simply unable to borrow. Alternatively, the capital market imperfection may manifest itself in the form of an interest rate on borrowing that rises with the level of the consumer's borrowing.[15] For the sake of simplicity, I will use the term liquidity constraint to refer to a situation in which the consumer is unable to borrow at all.

Liquidity constraints have important implications for the relation between consumption and contemporaneous income.[16] A consumer who is currently liquidity constrained would like to increase current consumption but is unable to do so because he or she cannot borrow. If the consumer's income turns out to be one dollar higher, then it is both feasible and desirable to increase current consumption by one dollar. Alternatively, if income turns out to be one dollar lower, then the consumer is forced to reduce consumption by one dollar. Thus, for a consumer currently facing a binding liquidity constraint, the marginal propensity to consume out of current disposable income is equal to one. Even if the consumer does not face a binding liquidity

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[15] See Stiglitz and Weiss (1981) for a discussion of credit rationing.

[16] In addition, liquidity constraints have important implications for the effects of tax policy. See Hayashi (1985c) and Yotsuzuka (1987) for a discussion of the implications of liquidity constraints for the Ricardian Equivalence theorem.

constraint in the current period, the prospect of a binding liquidity constraint in the future would affect the current marginal propensity to consume.[17]

Although liquidity constraints have strong implications for the marginal propensity to consume, they may be difficult to detect in aggregate data. The reason for this difficulty is that under the permanent income hypothesis, the marginal propensity to consume depends on the stochastic properties of income. Since aggregate income is highly serially correlated, and indeed may even be a random walk, the permanent income hypothesis predicts an MPC of about one even in the absence of liquidity constraints. Thus an MPC near unity could result from either a binding liquidity constraint or highly serially correlated income.

Evidence of binding liquidity constraints has been found in econometric analyses of panel data. Hall and Mishkin (1982) analyzed expenditures on food in the Panel Study of Income Dynamics and concluded that about 20% of the households in their sample of U.S. households were liquidity constrained. Hayashi(1985a) and Zeldes(1985) used data on individual household wealth and found that households with large amounts of liquid assets appeared to adhere to the permanent income hypothesis but households with small liquid wealth appeared to behave as if liquidity-constrained. The importance of liquidity constraints from the viewpoint of the cyclical relation between consumption and income is that the MPC of constrained households is equal to one. Thus, if the MPC implied by the permanent income hypothesis is less than one and is less than the apparent MPC in the data, one might appeal to liquidity constraints to explain the "excess sensitivity" of consumption. Alternatively, if income has a unit root and if the changes in income are as persistent as estimated by Campbell and Deaton (1987), then one might appeal to liquidity constraints to explain excess smoothness.

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[17] For example, suppose that $\rho = r = 0$ and the consumer does not currently (in period t) face a binding liquidity constraint. If the consumer will face a binding liquidity constraint inperiod $t + N$, then the marginal propensity to consume out of a one-time addition to wealth in period t is $1/(N + 1)$.

IV. Interest Rate and Wealth Effects on Consumption

The response of consumption to changes in after-tax labor income y_t were analyzed above in a model in which consumers take account of stochastic variation in y_t in optimally reaching consumption decisions. Ideally, to analyze the response of consumption to changes in the rate of interest or to changes in the value of wealth, one would like to develop a model of a consumer maximizing an intertemporal utility function subject to random variation in the rate of return as well as random variation in labor income. Unfortunately, it is difficult to develop a simple model with a closed form solution for a consumer facing both labor income uncertainty and rate of return uncertainty. To analyze the response of consumption to changes in the rate of return on wealth, I will present a simple model in which the only source of income is the return on non-human wealth.

Suppose that the consumer has no labor income so that the wealth accumulation equation (1) can be written as

$$W_{t+1} = (1 + r_t)\,W_t - c_t \tag{33}$$

where the real rate of return on wealth is now treated as a random variable. For analytic simplicity suppose that the random rate of return r_t is identically and independently distributed over time. The consumer attempts to maximize the time-separable utility function in (2). The maximum attainable value of U_t depends only on the consumer's available resources in period t, $(1 + r_t)W_t$. Let the function $V((1 + r_t)W_t)$ denote the maximum attainable value of U_t in (2) and note that

$$V((1 + r_t)\,W_t) = \max\, u(c_t) + (1/(1 + \rho))\,E_t\{V((1 + r_{t+1})\,W_{t+1})\}\ . \tag{34}$$

The function V( ) cannot be specified independently. It is a solution to the functional equation in (34).

In general the functional equation in (34) is difficult to solve, but in the case of isoelastic utility a solution can be derived in a straightforward manner.[18] Suppose that the utility function

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[18] Samuelson (1969) derives the solution for the finite-horizon version of this problem using backward induction.

u(c) has the isoelastic form $u(c) = c^{1-\gamma}/(1-\gamma)$ where $\gamma > 0$ is the (constant) coefficient of relative risk aversion. In this case, the intertemporal utility function in (2) is homothetic so that income expansion path relating consumption at various dates is a straight line through the origin. Thus, changes in $(1 + r_t)W_t$ induce an equiproportionate change in c_t. A solution to the functional equation is[19]

$$V((1 + r_t) W_t) = A((1 + r_t) W_t)^{1-\gamma}/(1 - \gamma) \tag{35}$$

where A is a coefficient to be determined later. To solve the consumer's optimization problem, consider a reduction in c_t of one unit and an accompanying increase of one unit in W_{t+1}. The reduction in c_t will reduce current utility on the right hand side of (34) by $u'(c_t)$ and the increase in W_{t+1} will increase the expected present value of next period's utility on the right hand side of (34) by $(1/(1+\rho))E_t\{(1 + r_{t+1}) V'((1 + r_{t+1}) W_{t+1})\}$. At the optimum, the net effect on the consumer's utility will be zero, and hence consumption will be at the optimal level when

$$u'(c_t) = (1/(1 + \rho))E_t\{(1 + r_{t+1}) V'((1 + r_{t+1}) W_{t+1})\}\ . \tag{36}$$

Using the isoelastic specification for u( ) and using (35), equation (36) can be written as

$$(c_t)^{-\gamma} = (A/(1 + \rho))E_t\left\{(1 + r_{t+1})^{1-\gamma}\right\}((1 + r_t) W_t - c_t)^{-\gamma}\ . \tag{37}$$

Now raise both sides of (37) to the $-1/\gamma$ power to obtain

$$c_t = G(1 + r_t)W_t \tag{38a}$$

where

$$G = \left\{1 + \left[(A/(1 + \rho))E_t\left\{(1 + r_{t+1})^{1-\gamma}\right\}\right]^{1/\gamma}\right\}^{-1}\ . \tag{38b}$$

Substituting the optimal consumption rule (38a) into (34) and using the definition of G in (38b)

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[19]The functional equation (34) has a continuum of solutions of the form

$$V((1 + r_t) W_t) = A((1 + r_t) W_t)^{1-\gamma}/(1 - \gamma) + \phi(1 + \rho)^t$$

where ϕ is an arbitrary constant. The solution in (35) sets ϕ equal to zero.

implies, after some manipulation, that

$$A = G^{\gamma} . \tag{39}$$

Finally, substituting (39) into (38b) yields

$$G = 1 - \left[E_t \left\{(1 + r_{t+1})^{1-\gamma}\right\} /(1 + \rho)\right]^{1/\gamma} . \tag{40}$$

Observe from (38a) that G is the marginal (= average) propensity to consume out of available resources. I have now derived the optimal consumption rule of a consumer who faces a stochastic rate of return on wealth. It follows immediately from (38a) that consumption is proportional to the contemporaneous value of wealth including the current return to capital. Note, in particular, that consumption in period t is an increasing function of the ex post rate of return r_t.

This model allows us to distinguish the effects of changes in the ex ante probability distribution of the rate of return from changes in the ex post rate of return. From the point of view of period t, the ex ante information about the stochastic rate of return r_{t+1} is summarized by the factor $\left[E_t \left\{(1 + r_{t+1})^{1-\gamma}\right\}\right]^{1/\gamma}$ in the marginal propensity to consume out of wealth, G. Note first that if γ is equal to one, in which case the utility function u( ) is logarithmic, then $G = 1 - [1/(1 + \rho)]^{1/\gamma}$. Thus, under logarithmic utility, consumption, c_t, is invariant to the ex ante distribution of the rate of return. The reason for this invariance is that the income and substitution effects associated with an increase in the prospective rate of return offset one another exactly. An increase in the prospective interest rate has a positive income effect because the consumer is assumed to be a net lender rather than a net borrower (i.e., $W_t > 0$). The substitution effect of a higher prospective interest rate is to make current consumption more expensive relative to future consumption and thus to reduce current consumption. The income and substitution effects are in opposite directions, and for the case of logarithmic utility, they are of equal magnitude. If the utility function u( ) displays less curvature than the logarithmic function, i.e., if $\gamma < 1$, then the substitution effect is strengthened; if the rate of return, r_{t+1}, is nonstochastic, then consumption, c_t, would be a decreasing function of r_{t+1}. Alternatively, if the utility function is more curved than the logarithmic function, ($\gamma > 1$), then the substitution effect is diminished; if the rate of return, r_{t+1}, is nonstochastic, consumption, c_t, would be an increasing function of r_{t+1}.

I have shown that consumption, c_t, increases in response to an increase in the ex post interest rate r_t, but may rise, fall, or remain unchanged in response to a given change in the ex ante distribution of the rate of return r_{t+1}. The difference in the effects of ex post and ex ante interest rates is that the ex post interest has only an income effect associated with it, whereas a change in the ex ante interest rate has both an income effect and a substitution effect. In particular, if, at the beginning of period t, the consumer sees that the realized value of the interest rate r_t is higher than expected, then the consumer is wealthier than expected and therefore increases consumption. The interest rate r_t represents the consumer's terms of trade between periods t - 1 and period t; because r_t does not affect the terms of trade between period t and any future period, there is no substitution effect on c_t. By contrast, if in period t the value of r_{t+1} is seen to increase, then the terms of trade between period t and period t + 1 are altered, thereby inducing a substitution effect in addition to the income effect.

The magnitude of the response of consumption to changes in the expected rate of interest can be measured by the intertemporal elasticity of substitution. Under the isoelastic utility function $u(c) = c^{1-\gamma}/(1 - \gamma)$, the intertemporal elasticity of substitution is equal to $1/\gamma$. This result can be derived by substituting $u'(c) = c^{-\gamma}$ into the first-order condition (3) and rearranging to obtain

$$\ln c_{t+1}/c_t = -(1/\gamma)\ln(1 + \rho) + (1/\gamma)\ln(1 + r_{t+1}) - (1/\gamma)\ln \eta_{t+1} \tag{41}$$

where η_{t+1} is a positive random variable and $E_t\{\eta_{t+1}\} = 1$. Recalling that $1 + r_{t+1}$ is the relative price of consumption in periods t and t+1, it is clear from (41) that a one percent change in the relative price of c_{t+1} and c_t induces a $1/\gamma$ percent change in c_{t+1}/c_t. Thus, the intertemporal elasticity of substitution is equal to $1/\gamma$. Using monthly data and measuring the rate of return on wealth by the value-weighted aggregate return on stocks on the New York Stock Exchange, Hansen and Singleton (1983) estimated γ to be between zero and two.

The formal analysis of the effect of the interest rate is based on the assumption that all of the consumer's disposable resources come from return on wealth. In particular, after-tax labor income is ignored. To the extent that there will be positive flows of after-tax labor income in the future, an increase in the ex ante interest rate would have a smaller income effect, or possibly even a negative income effect. Intuitively, an increase in the prospective rate of return would reduce the present value of future labor income and thus would reduce the current value of human wealth. Indeed, if the consumer's current nonhuman wealth and current income are sufficiently low compared to his future earnings, the consumer may be a net borrower rather than a net lender (i.e., W_t may be negative). In this case, an increase in the interest rate would have a negative income effect; both the income effect and the substitution effect would tend to reduce consumption in response to an increase in the interest rate.[20]

V. Government Bonds and Ricardian Equivalence

Having shown that consumption is an increasing function of nonhuman wealth, the next task is to examine whether government bonds are to be included in wealth. To see why this is an interesting question consider the effects of a $100 cut in current lump-sum tax revenues that the government finances by issuing $100 of bonds. It was pointed out by Ricardo (1911), and later modeled formally by Barro (1974), that under certain conditions forward-looking consumers would not change their consumption at all in response to this tax change. The reason is that consumers recognize that the government will have to increase taxes in the future to repay the principal and interest on the newly-issued bonds. Because of the need to increase taxes in the future, the opportunity set of the representative consumer is unaltered by this policy. The consumer can achieve exactly the same path of current and future consumption by increasing current saving by $100, and by holding this additional saving in the form of government bonds. The consumer can hold these bonds in his portfolio earning interest until future taxes are increased to pay the interest and principal on the bonds. The future tax increases will be exactly

[20] Using a simulation model, Summers (1981) finds that the reduction in human wealth that accompanies an increase in the interest rate is quantitatively substantial. In addition, he finds that the interest elasticity of savings is quite substantial (roughly in the range from 1 to 3) and that this interest elasticity is not very sensitive to the parameter q. Estimates of this elasticity based on consumption and rate of return data are generally much smaller (Boskin's (1978) estimate of 0.4 for the interest elasticity of saving is at the high end of the range), but Summers attributes much of the difference to the fact that these other studies hold wealth constant, whereas he takes account of the negative impact of higher interest rates on human wealth.

equal in value to the principal and interest earned by the consumer. Therefore, the consumer can support the same path of consumption as initially planned. Furthermore, since the tax change is in a lump-sum tax, there will be no change in relative prices. Therefore, it is both feasible and optimal for the consumer to maintain the same consumption and portfolio decisions (except for increasing the holding of government bonds) as before the tax change. This invariance of private spending to changes in the timing of lump-sum taxes has been dubbed "The Ricardian Equivalence Theorem" by Buchanan (1976). It is worth noting that although Ricardo stated the basic argument, he cautioned against taking the argument seriously as a description of the actual impact of debt financed tax cuts, claiming that such a system tends to discourage saving (1911, pp. 162-163).

To analyze the question of whether government bonds are net wealth, suppose that the interest rate is constant and use the expression for permanent income implied by (5) and (6) to obtain

$$c_t = r\left[W_t + (1/(1+r)) \sum_{j=0}^{\infty} \left\{ (1+r)^{-j} E_t\{y_{t+j}\} \right\} \right]. \tag{42}$$

Now distinguish government bonds, B_t, from the rest of the consumer's nonhuman wealth, K_t, so that

$$W_t = K_t + B_t . \tag{43}$$

It is also convenient to separate after-tax labor income into pre-tax labor income y_{Lt} and taxes T_t so that

$$y_t = y_{Lt} - T_t . \tag{44}$$

Substituting (43) and (44) into the consumption function (42) yields

$$\begin{aligned} c_t = {} & r\left[K_t + (1/(1+r)) \sum_{j=0}^{\infty} \left\{ (1+r)^{-j} E_t\{y_{Lt+j}\} \right\} \right] \\ & - r\left[(1/(1+r)) \sum_{j=0}^{\infty} \left\{ (1+r)^{-j} E_t\{T_{t+j}\} \right\} - B_t \right]. \end{aligned} \tag{45}$$

The Ricardo-Barro insight is that the government's budget constraint implies that the present value of tax revenues must equal the sum of the current government debt outstanding plus the present value of government expenditure on goods and services. Thus, the second line of (45), which is the excess of the present value of current and future tax revenues over the value of currently outstanding government bonds, is equal to the present value of current plus future government spending. Therefore, debt-financed changes in taxes that leave the path of government spending unchanged have no effect on consumption. To make the point starkly, suppose that government spending on goods and services is always equal to zero so that the second line of (45) is equal to zero. In this case, the consumption function is simply

$$c_t = r\left[K_t + (1/(1+r)) \sum_{j=0}^{\infty} \{(1+r)^{-j} y_{L+j}\} \right]. \qquad (46)$$

Inspection of (42) and (46) sheds light on the question of whether government bonds should be treated as part of net wealth in an economy in which the Ricardian Equivalence Theorem holds. It follows from (42) that if the income variable is net of taxes, then the wealth variable should include government bonds. Alternatively, if the income variable is pre-tax labor income, then it follows from (46) that the wealth variable should not include government bonds as net wealth. In addition, if there is government spending, then (45) implies that the present value of government spending should appear as an additional explanatory variable along with pre-tax labor income and K_t.

The above discussion has proceeded under the assumption that the Ricardian Equivalence Theorem holds. There is a large theoretical literature that explores many reasons why the Ricardian Equivalence Theorem may not hold. I will mention only three reasons why the impact of actual tax policy may not be accurately described by the Ricardian Equivalence Theorem. First, the argument underlying the Ricardian Equivalence Theorem requires the taxes to be non-distortionary taxes. However, virtually all taxes are distortionary taxes in that they affect the relative price of some economic activity. In addition to non-distortionary taxes, the Ricardian Equivalence Theorem relies on the assumption of forward-looking intertemporally optimizing consumers who do not face binding constraints on the intertemporal allocation of consumption.

The Ricardian Equivalence Theorem will fail to hold if consumers lack the foresight to take account of the implications for future taxes of current fiscal policy.

A third source of departure from the Ricardian Equivalence Theorem may arise if consumers face binding constraints on the intertemporal allocation of consumption. One such binding constraint is a binding liquidity constraint as discussed above in Section III. Consumers who face currently binding liquidity constraints will reduce their current consumption by an amount equal to the current increase in taxes, if the constraint on their borrowing does not change when taxes are changed. It is possible to construct models in which the borrowing constraint endogenously adjusts with tax changes in a way that leaves current consumption unchanged. However, whether the borrowing constraint endogenously adjusts in a manner to maintain Ricardian Equivalence depends on the rationale for liquidity constraints and on the extent of communication among lenders.[21]

An alternative type of constraint on the intertemporal allocation of consumption that violates the Ricardian Equivalence Theorem is a binding constraint on the intergenerational allocation of consumption. If current taxes are reduced, and if the implied future tax increase is levied on future generations, then the current recipients of the tax reduction would increase their consumption unless they have operative altruistic bequest motives. In particular, current consumption will be increased under any of the following sets of assumptions: (1) consumers do not have bequest motives; (2) the bequest motive is a function of the size of the bequest; (3) the bequest motive is of the altruistic form, but is not strong enough to induce the consumer to leave a positive bequest.[22]

There is also a large empirical literature that attempts to test whether the Ricardian Equivalence Theorem accurately describes the impact of tax policy in actual economies. There are many papers which support each side of this question. This literature can be read as supporting the Ricardian Equivalence Theorem (see, for example, Seater (1985)) or as rejecting it (see,

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[21] See Hayashi (1985c) and Yotsuzuka (1987).

[22] See Weil (1987) and Abel (1987).

for example, Bernheim (1987)). More importantly, the critical question is not whether the Ricardian Equivalence Theorem is literally true; the critical question is whether there are quantitatively substantial departures from Ricardian Equivalence and, if so, what is the magnitude of these departures. This last question remains unanswered.

VI. Investment

In addition to consumption expenditure, the other major component of private spending is business investment, which includes inventory investment as well as fixed investment in plant and equipment. The discussion below will analyze the sources of fluctuations in investment with particular attention to the effects of interest rates, asset prices, inflation, and aggregate demand. Ideally, a theory of investment fluctuations should be developed in a stochastic environment just as the consumption function presented above was based on utility-maximizing consumers facing uncertain streams of exogenous income. Although the literature does contain formal models of investment under uncertainty,[23] and also contains the econometric implementation of models of investment under uncertainty,[24] it has not developed and tested a set of stochastic implications with the sharpness of the stochastic implications of the permanent income hypothesis. Therefore, I will develop the basic models of investment under the assumption of certainty; the effects of uncertainty will be briefly discussed later. Of the several popular models of fixed investment, I will limit attention to three models: the accelerator model, the neoclassical model and the q-theory model.[25]

A. The Neoclassical Model and the Accelerator

The demand for productive capital is a derived demand by firms. I will begin by considering the investment and employment decisions of a firm in a deterministic environment without taxes. For analytic tractability and clarity, the model will be set in continuous time. Let K_t be

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[23] See, for example, Lucas and Prescott (1971), Hartman (1972), and Abel (1985).

[24] See, for example, Pindyck and Rotemberg (1983) and Bernanke (1983).

[25] See Bischoff (1971) and Clark (1979) for comparison of the empirical performance of alternative investment models. Nickell (1978) provides an excellent comprehensive treatment of several models of investment and many issues related to investment behavior.

the stock of capital at time t, let L_t be the amount of labor employed at time t, and let I_t be the firm's gross investment at time t. Let $Y(K_t, L_t)$ be the real revenue function of the firm and assume that it is concave. This function embodies the firm's production function and the demand curve it faces. For a price-taking firm $Y(K_t, L_t)$ is simply the output of the firm multiplied by the real price of its output. At the level of the aggregate economy, $Y(K_t, L_t)$ is to be interpreted as the aggregate real revenue function, which depends on aggregate demand. The net real cash flow at time t, X_t, is

$$X_t = Y(K_t, L_t) - w_t L_t - p_t I_t - c(I_t, K_t) \tag{47}$$

where p_t is the real price of investment goods and w_t is the real wage rate. The final term in the expression for cash flow in (44), $c(I_t, K_t)$, requires further explanation. This function represents the cost of adjusting the capital stock. This cost is in addition to the price of investment goods. The adjustment cost function is meant to capture the notion that if the capital stock is to be increased by a given increment, it is more costly to achieve this increase rapidly rather than slowly. This idea was formalized by Eisner and Strotz (1963) and was used later by Lucas (1967), Gould (1968), Treadway (1969), Mussa (1977), Abel (1980, 1982), Yoshikawa (1980) and Hayashi (1982). The adjustment cost function is non-negative and is convex in the rate of investment I_t. It is convenient to think of the adjustment cost function as representing installation costs. With this interpretation, p_t can be called the price of uninstalled capital, and $p_t + c_I(I_t, K_t)$ is the marginal cost of new installed capital. Although the neoclassical and accelerator models ignore costs of adjustment, I introduce adjustment costs at this point to develop a unifying framework that will include the q theory of investment to be developed in the next section.

The firm attempts to maximize the present value of its net cash flow over an infinite horizon. Let r_t be the instantaneous real rate of interest at time t and define $R(t, s) = \exp\left[-\int_t^s r_v \, dv\right]$ to be the discount factor that discounts real cash flows at date s back to date t. Let V_t be the value of the firm at time t and observe that

$$V_t = \max \int_t^{\infty} X_s R(t, s) \, ds . \tag{48}$$

In the maximization on the right hand side of (48) the firm can choose the path of employment, L_s, and the path of gross investment, I_s, for $s \geq t$. The level of the capital stock at time t, K_t, is treated as an initial condition. The change in the capital stock, i.e., net investment, is equal to gross investment less depreciation. Assuming that capital depreciates at a constant proportional rate h, the evolution of the capital stock is given by

$$\dot{K}_t = I_t - hK_t \tag{49}$$

where a dot over a variable denotes the derivative of that variable with respect to time.

The firm chooses the paths of employment and investment to perform the maximization in (48) subject to the dynamic constraint in (49) and the condition that K_t is given. To solve this maximization problem define

$$H_t = X_t + q_t \dot{K}_t \tag{50}$$

where q_t is the shadow price of a unit of installed capital. The determination of q_t will be discussed further below. Interpreting q_t as the shadow price of capital, the term $q_t \dot{K}_t$ is the value of the net increment to the capital stock, $\dot{K}_t$. Thus, the right hand side of (50) can be viewed as the value accruing to the firm's employment and investment activities at time t. These activities produce real cash flow at the rate X_t and increase the capital stock by an amount worth $q_t \dot{K}_t$. Technically, H_t is the "current value Hamiltonian."

To solve the firm's maximization problem, employment and investment must be chosen to maximize H_t. Substituting (47) and (49) into (50) yields

$$H_t = Y(K_t, L_t) - w_t L_t - p_t I_t - c(I_t, K_t) + q_t (I_t - hK_t) . \tag{51}$$

Differentiating H_t with respect to L_t and I_t, respectively, and setting the derivatives equal to zero, yields

$$Y_L(K_t, L_t) = w_t \tag{52a}$$

$$c_I(I_t, K_t) = q_t - p_t . \tag{52b}$$

Equation (52a) simply states that the firm hires labor to the point at which the marginal revenue product of labor is equal to the wage rate. Equation (52b) states that the firm chooses a rate of investment such that the marginal cost, which is equal to the price of the investment good p_t plus the marginal adjustment cost $c_I(I_t, K_t)$, is equal to the value of an additional unit of installed capital q_t.[26] This equation has important implications to which I will return later.

In addition to choosing I_t and L_t to maximize H_t, the solution to the firm's intertemporal maximization problem in (48) requires that the shadow price q_t obey the relation $\dot{q}_t - r_t q_t = -\partial H_t/\partial K_t$. Using (51) this relation can be written as

$$\dot{q}_t = (r_t + h)\, q_t - Y_K(K_t, L_t) + c_K(I_t, K_t) \ . \tag{53}$$

Although equation (53) may appear to be merely a technical condition for optimality, it has important economic interpretations. As a step toward interpreting (53), observe that it is a differential equation and that the stationary solution of this differential equation is

$$q_t = \int_t^{\infty} [Y_K(K_s, L_s) - c_K(I_s, K_s)]\, R(t, s)\, e^{-h(s-t)}\, ds \ . \tag{54}$$

Equation (54) states that the shadow price of capital is equal to the present discounted value of the stream of marginal cash flow attributable to a unit of capital installed at time t. At each future date s, the marginal cash flow consists of two components: (1) $Y_K(K_s, L_s)$ is the extra revenue attributable to an additional unit of capital at time s; (2) $-c_K(I_s, K_s)$ is the reduction in the adjustment cost made possible by an additional unit of installed capital. The (instantaneous) rate at which marginal cash flows at date s are discounted is equal to $r_s + h$ rather than simply r_s because a unit of capital depreciates at rate h. Thus, if a unit of capital is installed at time t, then at some future time s, only a fraction $e^{-h(s-t)}$ of the unit of capital remains.

A second interpretation of (53) is based on viewing the shadow price q_t as if it were the price at which a marginal unit of installed capital could be bought or sold. With this interpretation in mind, consider the decision of whether to invest in an additional unit of installed capital at

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[26] This analysis ignores any non-negativity constraint on gross investment; instead I have assumed that it is possible for individual firms to remove capital goods and sell them subject to an adjustment cost. See Sargent (1980) for an analysis that explicitly incorporates a non-negativity constraint on gross investment in a discrete-time model. Bertola (1987) analyzes the implications of the non-negativity constraint in a continuous-time model.

a cost of q_t or alternatively, to invest the q_t units of numeraire in a financial asset paying a rate of return r_t. If the capital investment decision is optimal, then the firm should be indifferent between these two alternative uses of q_t units of the numeraire, which implies that the rate of return on capital investment is equal to r_t. The return to capital investment consists of four components: (1) a unit of capital increases revenue by $Y_K(K_t, L_t)$; (2) a unit of capital reduces the adjustment cost by $-c_K(I_t, K_t)$; (3) capital depreciates at rate h so that the value of the capital lost to depreciation is hq_t; and (4) the price of capital changes at the rate $\dot{q}_t$ which represents a capital gain if $\dot{q}_t$ is positive and a capital loss if $\dot{q}_t$ is negative. Adding together the four components of the return to capital, and then dividing by the shadow price of capital to express the return as a rate of return, yields $\left[Y_K(K_t, L_t) - c_K(I_t, K_t) - hq_t + \dot{q}_t\right]/q_t$. Equation (53) simply states that this rate of return is equal to r_t.

A third, related, interpretation of equation (53) involves the concept of the user cost (sometimes called the rental cost) of capital derived by Jorgenson (1963). The rental cost interpretation is facilitated by again viewing the shadow price q_t as the price at which a unit of capital can be bought or sold. Consider someone who owns a unit of capital that will be rented to someone else for use. The owner of the capital will charge a rental cost u_t such that the rate of return from renting the capital is equal to r_t, which is the rate of return available on financial assets. The owner's return consists of the rental cost u_t plus the capital gain $\dot{q}_t$ less the value of the physical depreciation $q_t h$. Therefore, the owner's rate of return is $\left[u_t + \dot{q}_t - hq_t\right]/q_t$. Setting this rate of return equal to r_t yields

$$u_t = (r_t + h)\, q_t - \dot{q}_t \, . \tag{55}$$

Equation (55) is the analogue of the Jorgensonian user cost of capital, except that in place of the shadow price q_t, Jorgenson uses the price of the investment good, p_t. The reason for this difference is that Jorgenson ignores the adjustment cost function, or equivalently, assumes that $c(I_t, K_t)$ is identically zero. Under this assumption, equation (52b) indicates that q_t is equal to p_t so that the user cost in (55) is identical to the Jorgensonian user cost in this case.

The definition of the user cost in (55) can be used to rewrite equation (53) as

$$Y_K(K_t, L_t) - c_K(I_t, K_t) = u_t . \tag{56}$$

Equation (56) states that at each instant of time, the marginal cash flow of an additional unit of capital is equal to the user cost u_t. Except for the fact that Jorgenson's formulation does not have an adjustment cost function, so that $c_K(I_t, K_t)$ is identically zero, this relation is the same as Jorgenson's condition which states that the marginal product of capital is equal to the user cost of capital.

For the purpose of expositional clarity, I will assume that the adjustment cost function has the following form

$$c(I_t, K_t) \equiv g(I_t - hK_t) \tag{57}$$

where $g( ) \geq 0$, $g(0) = 0$, $g'(0) = 0$, and $g''( ) > 0$. The adjustment cost function specified in (57) is a non-negative convex function of the rate of net investment. When the rate of net investment is zero, the adjustment cost is assumed to be zero.[27]

Now suppose that the price of investment goods, p_t, the real wage rate w_t, and the real interest rate r_t are constant and consider the steady state in which both the capital stock, K_t, and the shadow price of capital, q_t, are constant. When the capital stock is constant, $I_t = hK_t$, so that it follows from the specification of the adjustment cost function in (57) that the adjustment cost is equal to zero. In addition, the partial derivatives c_I and c_K are each equal to zero. The fact that c_I is equal to zero implies, using (52b), that the shadow price q_t is constant and equal to the price of investment goods, p_t. Because q_t is equal to p_t, it follows from the definition of the user cost in (55) that

$$u_t = (r + h)p_t . \tag{58}$$

Finally, recall that the partial derivative, c_K, is equal to zero in the steady state so that (56)

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[27]Gould (1968), Treadway (1969) and Abel (1985) express the adjustment cost function as a function of gross investment. Expressing the adjustment cost function as a function of net investment implies that in the long run, when $\dot{K} = 0$, the value of capital, q_t, is equal to its replacement cost, p_t, as argued by Tobin (1969).

implies that

$$Y_K(K_t, L_t) = u_t . \tag{59}$$

Equations (58) and (59) correspond to the user cost of capital and the desired capital stock derived by Jorgenson under the condition that the price of investment goods, p_t, is constant. The desired capital stock is determined from (59). To illustrate Jorgenson's derivation, as well as the Eisner-Nadiri criticism of Jorgenson's derivation, suppose that the revenue function is constant returns to scale in K_t and L_t and displays a constant elasticity of substitution between K_t and L_t. In particular, suppose that

$$Y(K, L) = A[mK^{-\phi} + (1 - m) L^{-\phi}]^{-1/\phi} \tag{60}$$

where $A > 0$, $0 < m < 1$, and $\phi > -1$. The revenue function in (60) describes the revenue function of a competitive firm with a constant elasticity of substitution production function. It can be shown that the elasticity of substitution, σ, is equal to $1/(1 + \phi)$ and that the marginal revenue product of capital is

$$Y_K(K, L) = [m/A^{\phi}][Y/K]^{1/\sigma} . \tag{61}$$

Recalling from (59) that the marginal revenue product of capital is set equal to the user cost u_t, (61) can be rearranged to yield

$$K = [m/A^{\phi}]^{\sigma} Yu^{-\sigma} . \tag{62}$$

Equation (62) expresses the steady state capital stock, K, as a function of the real revenue of the firm and the user cost of capital. Of course, the revenue of the firm is a decision variable of the firm, so (62) cannot properly be regarded as a relation expressing the steady state capital stock as a function of exogenous variables. It is more appropriately regarded as a relation among endogenous variables in the steady state.

The neoclassical investment model developed by Jorgenson is based on a special case of (62). In particular, Jorgenson assumed that the revenue function is Cobb-Douglas, which in terms of (62) implies that the elasticity of substitution, σ, is equal to one. In this case, the steady state capital stock, K, is proportional to revenue Y and is inversely proportional to the user cost u.

Jorgenson's derivation of the investment equation proceeded in two steps. The first step was the derivation of a "desired capital stock", which corresponds to the steady state capital stock in (62) with σ equal to one. The second step is the determination of the rate at which the firm's capital stock approaches its desired level. Rather than specifying a particular dynamic adjustment mechanism, such as an adjustment cost function, in the firm's optimization problem, Jorgenson assumed that there is some exogenous mechanism that determines the rate at which the gap between the desired capital stock and the actual capital stock is closed. In particular, Jorgenson specified the investment equation as

$$I_t = \left\{ \sum_{i=0}^{n} \omega_i \left[K^*_{t-i} - K^*_{t-1-i} \right] \right\} + hK_t \qquad (63)$$

where K^*_t is the desired capital stock at time t.

Observe that a strong implication of Jorgenson's assumption of a unitary elasticity of substitution between K and L is that the desired capital stock depends only on the ratio of revenue to the user cost of capital. Jorgenson exploits this fact in his estimation by constraining the response of investment to revenue to be the same (proportionately, except for sign) as the response to the user cost. However, Eisner and Nadiri have claimed that this procedure may overstate the response of investment to cost of capital changes because the elasticity of substitution is less than one. When the elasticity of substitution is not equal to one, then it follows from (62) that the elasticity of the desired capital stock with respect to real revenue Y is still unity but the elasticity of the desired capital stock with respect to the user cost is equal to $-\sigma$. With an elasticity of substitution less than one, the magnitude of the response of investment to the user cost will be smaller than the response to revenue. Therefore, constraining the responses to be of equal (percentage) magnitude, as Jorgenson does, will tend to overstate the response of investment to the user cost. Eisner and Nadiri argue that the elasticity of substitution is nearer zero than unity (1968, p. 381), but Jorgenson and Stephenson (1969) claim that empirical evidence supports a unitary elasticity.

There is another reason to expect the response of investment to the user cost to differ from the response to output or revenue. In discussing the elasticity of substitution between capital and labor, it is important to distinguish the ex ante elasticity of substitution from the ex post elasticity of substitution. More specifically, before a piece of capital is built and put into place, there may be a substantial degree of substitutability between capital and labor. However, after the capital is put in place, there may be very limited, or even zero, substitutability. An extreme version of this notion is the putty-clay hypothesis: ex ante, capital is malleable like putty and the firm can choose the capital labor ratio; ex post, capital is not malleable, like clay, and there is no substitutability between capital and labor. Under the putty-clay hypothesis, we might expect to see larger and more rapid responses to changes in output than to changes in the user cost. For example, an increase in output may lead to an increase in the desired capital stock as described above in equation (62). However, a fall in the user cost would lead to a smaller response of the desired capital stock than in (62). The reason is that a fall in the user cost leads to an increase in the desired capital labor ratio but under the putty-clay hypothesis, the capital labor ratio on existing capital is immutable. Thus newly installed capital will be less labor intensive, but the old capital will not be replaced with labor-saving capital until the old capital becomes uneconomical.

An even more extreme limitation on capital labor substitutability gives rise to the accelerator model of investment. In particular, suppose that there is no substitutability either ex ante or ex post. In terms of the expression for the desired capital stock in (62), suppose that the elasticity of substitution, σ, is equal to zero. In this case, the desired capital stock in (62) is simply proportional to revenue. Investment in this case would be a distributed lag function of changes in the level of revenue

$$I_t = \left\{ \sum_{i=0}^{n} \omega_i \left[Y_{t-i} - Y_{t-1-i} \right] \right\} + hK_t \, . \tag{64}$$

The accelerator model (64) is a special case of the neoclassical investment model (63) in which the user cost of capital is ignored.[28] Although some studies find significant effects of the user cost,[29] it is part of the "folk wisdom" that user cost effects on investment are harder to estimate in the data than are accelerator or output effects. Perhaps one reason for the difficulty of finding user cost effects is the problem of simultaneity. If there is an exogenous increase in the real interest rate, then the user cost would increase and investment would decrease. However, if for some reason there were an upward shift in the investment function, (for example, Keynesian "animal spirits," (Keynes (1936), pp. 161-163)) then investment would increase and would put upward pressure on the real interest rate. Thus, as a consequence of the upward shift in the investment function, both investment and the user cost would increase. If data contain both exogenous increases in the real interest rate and exogenous shifts in the investment function, then the predicted negative relation between user cost and investment might be masked by the positive relation between user cost and investment in response to exogenous shifts in the investment function.

It should be noted that simultaneity of the sort discussed above would tend to exaggerate, rather than diminish or reverse, the estimated accelerator effects. An exogenous upward shift in the investment function would increase investment, which would increase output. This positive relation between investment and output reinforces the positive relation due to the accelerator effect discussed above. Finally, it should be noted that although simultaneity problems can be alleviated by the use of instrumental variables, the resulting estimates are only as good as the instruments.

An additional complication in estimating the effect of the user cost is the question of whether to use a short-term or long-term interest rate in measuring the cost of capital. Traditionally, the long-term interest rate is viewed as the appropriate rate but Hall (1977) argues that "as a matter of theory, what belongs in the service price of capital is a short-run interest rate, though the issue of short versus long rates is unlikely to be resolved empirically" (p. 100). Hall's point

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[28] See Eisner (1978) for an excellent comprehensive treatment of the accelerator model using the McGraw-Hill Publishing Co. capital expenditure surveys from 1956 to 1969.

[29] See Feldstein (1982), for example.

that the user cost depends on the short rate rather than the long rate is illustrated by equation (55). In the absence of adjustment costs, q_t is equal to the price of investment goods p_t, and the expression for the user cost of capital in (55) yields the Jorgensonian user cost $u_t = (r_t + h) p_t + \dot{p}_t$. The interest rate in this expression is the instantaneous real rate of interest.

While it is true that the user cost is related to the contemporaneous instantaneous interest rate, one must avoid the temptation to say that investment depends on the short-term interest rate rather than the long-term interest rate. Halls' argument that the short-term rate is the appropriate interest rate was based on a model without adjustment costs in which the "firm faces an open choice about the scheduling of investment" (p. 74). However, the essence of adjustment costs is to interfere with "the open choice about the scheduling of investment". In the presence of costs of adjustment, the scheduling of investment affects the cost of investment. In this case, investment is necessarily forward-looking and depends on the present value of the stream of marginal products accruing to a newly-installed unit of capital. Observing that the real interest rate prevailing from date t to date s is $[R(t, s)]^{-1} - 1$, equation (54) implies that q_t, and a fortiori investment, depends on the entire term structure of real interest rates. To make this point more sharply, consider the response of investment to an instantaneously-lived increase in r_t and, alternatively, the response of investment to a permanent increase in the interest rate. An instantaneous increase in r_t will have no effect on R(t, s) and will have no effect on q_t or investment at time t. By contrast, a permanent increase in the instantaneous interest rate would reduce the stream of discount factors, R(t, s), and would reduce q_t and investment. In the presence of adjustment costs, investment depends on the entire term structure of interest rates.

Although there is no consensus about the magnitude of the response of investment to changes in interest rates, the analysis above offers some guidance on the size of this effect. Suppose that the relevant real rate of interest is equal to 4% per year and the rate of depreciation is equal to 6% per year, which is an appropriate depreciation rate for structures. Now consider the effect of a 1 percentage point decrease in the real interest rate (from 4% to 3% per year). It follows immediately from the expression for the user cost in (58) that this decrease in the real interest rate decreases the user cost of capital by 10%. Under a Cobb-Douglas production function, this 10% decrease in the user cost increases the desired stock of capital by 10%. In the long run, the rate of investment would rise by 10% in order to maintain the capital stock at its higher level. In the short run, the rate of investment would increase by even more than 10% in order to increase the capital stock to its new desired level. The magnitude of the increase in the rate of investment in the short run depends, of course, on how rapidly the new desired capital stock is achieved.

The 10% increase in the desired capital stock in response to a 1 percentage point decrease in the real interest rate may overstate the response of the desired aggregate capital stock for three reasons. First, as emphasized by Eisner and Nadiri (1968), the elasticity of substitution between capital and labor may be substantially less than one. Recalling that this elasticity of substitution is denoted by σ, it follows immediately from (62) that the response of the desired capital stock to this one percentage point decrease in the real interest rate is equal to (10σ)%. Thus, if $\sigma = 0.1$, the desired capital stock rises by only 1% in response to a one percentage point fall in the real interest rate. Second, the depreciation rate of 6% per year may be a reasonable rate for structures, but the depreciation rate for equipment is about 16% per year. Thus, for equipment a decrease in the real interest rate from 4% to 3% reduces the user cost of capital by only 5% rather than the 10% calculated for structures.[30] Third, the real rate of interest used by firms in capital budgeting decisions is generally a risk-adjusted rate of return such as a weighted average of the after-tax interest rate on debt and the expected rate of return on equity.[31] Feldstein (1982) calculates the cost of funds annually for the period 1954-1977. Although this real cost of funds is about 4% for the first half of this sample, it is higher than 4% throughout the second half of the sample and reached a value of 7.2% in 1977. A one percentage point increase in the real cost of funds has a smaller impact on the user cost of capital if the cost of capital starts from a higher value.

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[30] Hall and Jorgenson (1967) estimate the depreciation rates for broadly defined capital aggregates using the so-called Bulletin F lifetimes from the Department of the Treasury. They present the following (annual) depreciation rates: manufacturing equipment: 0.1471; manufacturing structures: 0.0625; non-farm non-manufacturing equipment: 0.1923; non-farm non-manufacturing structures: 0.0694.

[31] See Auerbach (1979a) for a derivation of the weighted-average cost of Capital.

B. The q Theory

The neoclassical model and the accelerator model were each derived above by using the steady state capital stock as the desired level of the capital stock and then positing some sort of adjustment mechanism of the actual capital stock toward its desired level. An alternative approach is provided by a model that incorporates adjustment costs as well as the price of investment goods directly into the maximization problem and then derives the optimal rate of investment at each point of time. In addition to determining the optimal rate of investment, this model makes explicit the dynamic response of investment to permanent and temporary changes in the firm's economic environment and to anticipated as well as unanticipated changes. Furthermore, the adjustment cost model can be used to provide formal underpinnings to the q theory of investment introduced by Tobin (1969).

Tobins' q theory of investment formalizes a notion of Keynes (1936, p. 151) that the incentive to build new capital depends on the market value of the capital relative to the cost of constructing the capital. If an additional unit of installed capital would raise the market value of the firm by more than the cost of acquiring the capital and putting it in place, then a value maximizing firm should acquire it and put it in place.[32] The greater the amount by which the value of the capital exceeds its cost the greater is the incentive to invest. To capture this notion in an observable quantitative measure, Tobin defined the variable q to be the ratio of the market value of a firm to the replacement cost of its capital stock. He then argued that investment is an increasing function of q. A major advantage of Tobin's q is that it relies on securities markets to value the prospects of the firm.

Before discussing the q theory of investment more formally, it is worthwhile to digress briefly to discuss a related model in which the rate of investment depends on the market valuation of capital. Foley and Sidrauski (1970) developed a two-sector model of the economy in which there is a concave production possibilities frontier relating the aggregate output of the

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[32] This argument depends on the assumption that capital investment is reversible. If investment is irreversible, then firms may optimally forego some projects whose present value exceeds their cost. See McDonald and Siegel (1986).

consumption good and the aggregate output of the capital good. In a competitive economy, resources are allocated to these two sectors depending on the relative prices of these two goods. More precisely, the production of new capital goods is an increasing function of the price of capital goods relative to the price of consumption goods. In the Foley-Sidrauski model, the price of capital is determined endogenously in a securities markets in which three assets--money, bonds, and capital--are traded. The price of capital is determined to equilibrate the demand for capital with the existing supply of capital. This price then determines the flow of new capital goods production. Although the formal model is different from the q theory, in both the Foley-Sidrauski model and the q theory, the rate of investment is an increasing function of the price of capital goods which is determined in asset markets.

A version of the q theory of investment can be derived from the adjustment cost model of investment presented above. For the sake of continuity of exposition, suppose that the adjustment cost function is as specified in (57) so that the marginal cost of investment is $c_I(I_t, K_t) = g'(I_t - hK_t)$. Using this form of the marginal adjustment cost function in the first-order condition for the optimal rate of investment, (52b), yields

$$I_t = G(q_t - p_t) + hK_t \tag{65}$$

where $G(\ ) = g'^{-1}(\ )$ so that $G' > 0$ and $G(0) = 0$. Equation (65) expresses the rate of investment as an increasing function of the shadow price of installed capital, q_t. Note that in the steady state, with $\dot{K}_t = 0$, the rate of gross investment, I_t, is equal to depreciation hK_t; the shadow price of a unit of capital, q_t, is equal to the price of investment goods, p_t. The latter result, ($q_t = p_t$ in the steady state) is a consequence of specifying the adjustment cost function such that the marginal adjustment cost $c_I(I, K)$ is equal to zero when net investment is equal to zero. Alternatively, if the adjustment cost function is specified so that the marginal cost of investment is equal to zero when gross investment is equal to zero, then in the steady state q_t would exceed p_t by the marginal cost of replacement investment, hK_t.

The investment equation in (65) is related to Tobin's q theory of investment. To understand the relation between (65) and Tobin's q theory of investment, it is important to distinguish "average q," which will be denoted as q^A, from "marginal q," which will be denoted as q^M. Tobin defines q to be the ratio of the average value of the capital stock, V_t/K_t, to the price of a unit of capital, p_t. Thus, Tobin's q is q^A where

$$q_t^A = V_t/(p_t K_t) \tag{66}$$

and where V_t is the value of the firm at time t. Alternatively, marginal q, q^M, is the ratio of the marginal value of an additional unit of installed capital, dV_t/dK_t, to the price of a unit of capital, p_t. Therefore,

$$q_t^M = (dV_t/dK_t)/p_t\ . \tag{67}$$

Observe that the numerator of q_t^M, dV_t/dK_t, is equal to the shadow price q_t. Therefore, (65) and (67) imply that

$$I_t = G\left(\left(q_t^M - 1\right)p_t\right) + hK_t\ . \tag{68}$$

A natural question is whether there are conditions under which average q and marginal q are equal to each other. The answer can be obtained using the following proposition, which is a generalization of a result due to Hayashi (1982): Suppose that the revenue function Y(K, L) is linearly homogeneous in K and L, the adjustment cost function is linearly homogeneous in I and K, and that p_s, w_s and R(t, s) are exogenous to the firm. Then the value of the firm, V_t, is proportional to the stock of capital, K_t. Under these conditions, it follows that dV_t/dK_t is equal to V_t/K_t; thus, (66) and (67) imply that average q is equal to marginal q in this case.

The equality of average q and marginal q holds more generally than under the conditions stated above. It holds even if the interest rate is stochastic. It also holds if the cash flow, X_t, is subject to random multiplicative shocks. The key assumption about the behavior of cash flow is that it is a linearly homogeneous function of the three variables K, L, and I.

The use of Tobin's q to explain investment provides an attractive link between asset markets and investment activity. In particular, stock and bond markets are relied upon to value the firm's capital stock, thereby relieving the economist from having to calculate the relevant expected present value of future cash flows. Unfortunately, investment equations based on

Tobin's q are not free of difficulty. Typically, estimated equations relating investment to Tobin's q leave a large unexplained serially correlated residual.[33] In addition, lagged values of Tobin's q often enter significantly as explanators of investment, which contradicts the simple adjustment cost model described above. Finally, other variables such as output and capacity utilization have additional explanatory power in investment equations with q. This finding contradicts the notion that all information that is relevant to the valuation of capital and to the investment decision is captured by the market value of the firm.

There are several possible explanations for the departures of empirically estimated investment equations from the simple predictions of the theory. First, average q and marginal q may display different movements. For example, consider a firm that has a large amount of energy-intensive capital. If the price of energy rises dramatically, then the value of the firm would fall as the quasi-rents available on existing energy-intensive capital would fall. However, the firm may undertake substantial investment in energy-saving capital. Therefore, an observer of this firm would see a drop in average q coinciding with an increase in investment. This example makes clear that heterogeneity of capital can potentially destroy the relation between average q and investment. As for marginal q, it is important to distinguish the marginal q, or shadow price, for the different types of capital. In the example above, the marginal q of energy-intensive capital is reduced and the marginal q of energy-saving capital is increased by the rise in the price of energy.

The fact that lagged values of q are found to be significant explanators of investment is perhaps suggestive of the importance of delivery lags in the investment process. For many types of capital, especially structures, there may be a substantial delay between the date on which it is decided to the acquire and install new capital and the date at which the capital expenditures are

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[33]See von Furstenberg (1977), Summers (1981) and Blanchard and Wyplosz (1981).

actually made.[34] The existence of delivery lags complicates the relation between investment and q. If, for example, there is a two-period delay between the decision to invest and the capital expenditure, then capital expenditure in period t should be related to the forecast in period t-2 of the value of capital in period t, i.e., $E_{t-2}\{q_t\}$. However, the variable which appears significantly in investment equations is lagged q, i.e., q_{t-2}, rather than the lagged expectation of q. To the extent that q_{t-2} is a predictor of q_t, it may serve as a proxy for $E_{t-2}\{q_t\}$.

The q theory of investment is based on the notion that all relevant information is captured in the market valuation of the firm and therefore, other variables such as cash flow, profit or capacity utilization should have no additional predictive power for investment. The fact that cash flow or profit often have significant additional predictive power is consistent with there being different costs of internal and external funds or with firm's having limited ability to finance investment by raising funds in capital markets. The underlying economic reasons for, and implications of, these capital market imperfections remain an open question.[35]

VII. Corporate Taxes and Inflation

The incentive to invest is influenced by the corporate tax environment in general, and by the interaction of corporate taxes and inflation in particular. The three aspects of the corporate tax code that have been analyzed most widely in the context of investment are the corporate tax rate, the depreciation allowance and the investment tax credit.[36] Let τ be the tax rate assessed on corporate profits. Taxable corporate profits are calculated as revenues less wages, depreciation allowances and adjustment costs. For simplicity, I will assume that adjustment costs are expensed, which is consistent with treating adjustment costs as foregone output or revenue. Let D(x) be the depreciation allowance for an asset of age x that cost one dollar when new. Then, following Hall and Jorgenson (1967), let $z = \int_0^\infty D(x)\, e^{-ix}\, dx$ be the present value of depreciation

34 Abel and Blanchard (1986) have have found that for nonelectrical machinery and fabricated metals there is a an average delivery lag of 2 quarters, and for electrical machinery the average lag is 3 quarters. For structures, the average lags range from 3 to 8 quarters.

35 See Fazzari, Hubbard and Petersen (1987).

36 See Hall and Jorgenson (1967), Feldstein (1982), Abel (1982), Taylor (1982), and Auerbach (1983).

deductions over the life of the asset, where i is the nominal interest rate. Finally, let k be the rate of the investment tax credit so that for each dollar spent on investment goods, the firm receives a rebate of k dollars.[37] Thus the net cost to the firm of a dollar of investment goods is $(1 - k - \tau z)$.[38]

Now define X_t^* to be the excess of after-tax real revenues over real wages, adjustment costs and the real net price of investment goods at time t

$$X_t^* = (1 - \tau)\left[Y(K_t, L_t) - c(I_t, K_t) - w_t L_t\right] - (1 - k - \tau z) p_t I_t \ . \tag{69}$$

In the absence of taxes, X_t^* would be equal to the cash flow X_t in (47). The maximization problem of the firm is equivalent to maximizing the present value of the stream of X_t^*.[39] This maximization problem can be solved using the same procedure as presented earlier in the absence of taxes. The current value Hamiltonian, which is analogous to (51), is

$$H_t^* = (1 - \tau)\left[Y(K_t, L_t) - c(I_t, K_t) - w_t L_t\right] - (1 - k - \tau z) p_t I_t + q_t^* (I_t - hK_t) \ . \tag{70}$$

Differentiating the current value Hamiltonian with respect to L_t and setting the derivative equal to zero yields the condition that labor is hired until the marginal revenue product of labor is equal to the wage rate (equation (52a)). Differentiating (70) with respect to the rate of investment, I_t, yields the analogue of (52b)

$$(1 - \tau) c_I(I_t, K_t) = q_t^* - (1 - k - \tau z) p_t \ . \tag{71}$$

The shadow price q_t^* must obey the relation $\dot{q}_t^* - r_t q_t^* = -\partial H_t^* / \partial K_t$ which implies the following analogue of (53)

$$\dot{q}_t^* = (r_t + h) q_t^* - (1 - \tau)\left[Y_K(K_t, L_t) - c_K(I_t, K_t)\right] \ . \tag{72}$$

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[37]The Tax Reform Act of 1986 eliminated the investment tax credit in the United States.

[38]Under the Long Amendment, which was in effect in 1962 and 1963, the basis for depreciation allowances was reduced by the investment tax credit, and the net price of investment goods was $(1 - k)(1 - \tau z)$. The expression in the text is appropriate for the period after the repeal of the Long Amendment.

[39]The present value of X_s^* is not equal to the present value of cash flow because it ignores the depreciation allowances on capital installed before date t. Because the cash flows associated with these deductions are predetermined at time t, they can be ignored in the maximization problem at time t.

The stationary solution to the differential equation in (72) is

$$q_t^* = \int_t^\infty \left\{(1-\tau)\left[Y_K(K_s, L_s) - c_K(I_s, K_s)\right] R(t,s) e^{-h(s-t)} ds\right\} . \qquad (73)$$

Equation (73) states that the shadow price q_t^* is equal to the present value of the stream of after-tax marginal products of capital.

Before deriving the investment equation, I will first describe the steady state in which $I_t = hK_t$ and $\dot{q}_t^* = 0$. It follows immediately from (72) that if $\dot{q}_t^*$ is equal to zero, then the shadow price q_t^* is equal to the present value of the stream of constant after-tax marginal cash flows accruing to capital

$$q_t^* = (1-\tau)(Y_K - c_K)/(r+h) . \qquad (74)$$

Now suppose that the adjustment cost function has the specification in (57) so that the marginal adjustment cost, c_I, is equal to zero in the steady state. In this case, the first-order condition in (71) implies that, in the steady state, the shadow price of capital is equal to the tax-adjusted price of investment goods

$$q_t^* = (1 - k - \tau z) p_t . \qquad (75)$$

Next set the right hand side of (74), which is the present value of after-tax marginal cash flow, equal to the right hand side of (75), which is the tax-adjusted price of capital, to obtain

$$Y_K - c_K = T(r+h)p_t \qquad (76a)$$

where $$T \equiv (1 - k - \tau z)/(1-\tau) . \qquad (76b)$$

The right hand side of (76a) is equal to the tax-adjusted user cost of capital derived by Hall and Jorgenson (1967). The factor T is a tax-adjustment factor; when T is equal to one, as it would be in the absence of taxes, then the user cost is identical to the steady state user cost presented above in (58).

To obtain a simple investment equation in the presence of corporate taxes, suppose that the adjustment cost function is independent of the capital stock, i.e., $c_K \equiv 0$. In this case, the first-order condition in (71) can be rewritten as

$$I_t = H\left(\frac{q_t^*}{1-\tau} - Tp_t\right) \tag{77}$$

where $H(\ ) = c_I^{-1}(\ ,\ )$ is an increasing function. The behavior of the investment equation (77) can be easily examined under the assumption that the revenue function is linearly homogeneous in K and L. The linear homogeneity of Y(K, L) implies that when cash flow is maximized with respect to labor, L_t, the maximized value of $Y(K_t, L_t) - w_t L_t$ is equal to $\theta(w_t)K_t$ where $\theta(w_t)$ is a positive but decreasing function of the real wage rate w_t. Therefore, when the firm follows an optimal employment policy, we obtain

$$Y_K(K, L) = \theta(w) \tag{78}$$

so that the marginal revenue product of capital is positive and decreasing in the real wage rate. Recalling that c_K is identically equal to zero, the shadow price of capital can be easily calculated from (73) to be

$$q_t^* = \int_t^\infty \left\{(1-\tau)\,\theta(w_s)\,R(t, s)\,e^{-h(s-t)}\right\} ds\,. \tag{79}$$

It follows immediately from (79) that $q_t^*/(1-\tau)$ is independent of the tax rate τ. The effect of the tax code is captured by the tax adjustment factor T, and inspection of (77) reveals immediately that the rate of investment is a decreasing function of T. Therefore, investment is an increasing function of the investment tax credit, k, and is also an increasing function of the present value of depreciation deductions, z.

Now consider the effects on investment of changes in real and nominal interest rates. In the neoclassical model, an increase in the real rate of interest raises the user cost of capital and hence reduces the desired capital stock and investment. It would appear that changes in the nominal interest rate would not affect the user cost unless they were accompanied by changes in the real interest rate. However, the U.S. tax code contains an important inflation non-neutrality, which gives nominal interest rates an effect on investment over and above the effect of real interest rates. Depreciation allowances are based on the nominal historical cost of a piece of capital rather than on its replacement cost. Thus, inflation reduces the real value of future depreciation deductions so that an increase in inflation reduces z, the present value of real depreciation deductions. An alternative, but equivalent, explanation for the negative relation between inflation and z is that the depreciation deductions represent a stream of nominal flows and can be discounted by a nominal interest rate. If the (expected and actual) rate of inflation rises without any change in the real interest rate, then the nominal interest rate also rises, so that the present value of the unchanged stream of nominal flows is reduced. The reduction in z increases the tax adjustment parameter T and thus tends to reduce the rate of investment. In addition, inflation in the presence of historical cost depreciation may distort the choice among different types of capital with different useful lives and different depreciation allowance schedules.[40] An increase in the rate of inflation can either increase or decrease the degree of durability of capital chosen by firms, depending on the nominal interest rate and the rate of depreciation.[41]

VIII. Uncertainty

The investment behavior of firms has been derived above in the absence of uncertainty. It seems intuitively plausible that the desirability of investment projects would depend on the risk associated with the project and, furthermore, one might suspect that an increase in risk would reduce the rate of investment. However, much of the existing analytic work on investment under uncertainty does not support the notion that greater uncertainty inhibits investment. Hartman (1972) and Abel (1985) have shown that an increase in the variance of the output price or in the variance of the price of variable factors will induce a competitive firm to increase its rate of investment. Pazner and Razin (1974) have shown that an increase in interest rate uncertainty also induces the firm to increase its rate of investment. The argument underlying these results can be illustrated using the expression for q_t^* in (79) which was derived under the assumption that the revenue function is linearly homogeneous in K and L. This equation would apply to a competitive firm with a constant returns to scale production function. It can be shown that the

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[40] See Auerbach (1979b), Kopcke (1981), and Abel (1981).

[41] Feldstein (1982) analyzes other inflation non-neutralities such as the fact that nominal interest payments, rather than real interest payments, are tax deductible.

marginal revenue product of capital, $\theta(w_s)$, is a convex function of w_s. Therefore, if the variance of w_s is increased while its expected value is held constant, then Jensen's inequality implies that the expected value of $\theta(w_s)$ increases. This increase in the expected value of $\theta(w_s)$ implies that the expected present value of marginal revenue products of capital increases and thus the optimal rate of investment increases. Similarly, it can be shown that R(t, s) is a convex function of future instantaneous rates of interest, r_v for $v > t$, and hence an increase in the variance of interest rates will also increase investment.

Recently, Zeira (1987) has developed a model of a monopolistic firm that is uncertain both about its own capacity and about the demand curve it faces. In this particular model, increased price uncertainty will reduce investment.

It should be noted that in all of the above-mentioned works on investment under uncertainty, the firm is modelled as risk-neutral. More precisely, the firm is assumed to maximize the expected present value of cash flow. It seems that future work could usefully model risk-averse managers and/or could model the covariance of the firm's returns with the market portfolio.

IX. Inventories

Up to this point, the discussion of investment has focused on fixed investment. In addition to fixed investment, firms invest in inventories. Although the average value of inventory investment, i.e., the average change in the stock of inventories, is quite small relative to the average level of fixed investment, the volatility of inventory investment is quite large.[42] Rather than develop a formal analytic model of inventory behavior, I will simply discuss some of the major issues. The first step is to explain why firms hold inventories. Two reasons that have been studied are: (a) for technological reasons, there is a lag between the beginning of production and the sale of a good. To the extent that the production process takes time, there will be an inventory of goods in process. To the extent that there is a delay between the completion of produc-

[42] Blinder (1981) reports that declines in inventory investment account for 70% of the peak-to-trough decline in real GNP during recessions. During the period 1959:1 to 1979:4 changes in inventory investment accounted for 37% of the variance of changes in GNP.

tion and the sale of the good, there is a finished goods inventory, (b) even if it were possible to make production always equal to the contemporaneous value of sales, so that there might be no need to hold inventories, cost-minimizing firms may choose to hold inventories as a means of avoiding large fluctuations in production in the face of large fluctuations in sales. This "production smoothing" motive for holding inventories would arise if the marginal cost of production were an increasing function of the level of production. In this case, the cost-minimizing scheduling of any level of average production requires minimizing fluctuations in production.

The production smoothing model of inventories has a striking resemblance to the permanent income model of consumption, which could be described as a model of consumption smoothing. Indeed, some of the lessons from the permanent income model could be carried over to the production smoothing model of inventories. For example, if all changes in a firm's sales were perfectly forecastable, then the production smoothing model would predict that the firm would maintain a smooth profile of production in the face of variations in its sales. Only unanticipated changes in sales would lead firms to alter production. The macroeconomic implication of this observation is that an anticipated increase in final demand, arising from (say) government spending, would not affect GNP because the firm would meet the extra demand by selling out of inventory. The increase in government spending would be exactly offset by negative inventory investment. Alternatively, if the increase in government spending were unanticipated, then the firm would presumably revise its production plans and raise production somewhat.

The production smoothing model of inventories has the implication that the variance of production should be less than or equal to the variance of sales. However, Blinder (1986) and West (1986) argue convincingly that the data on production and sales contradicts this implication of the theory. There are a few potential explanations of the apparent production "counter-smoothing" in the data. A simple but unsatisfying explanation is that shocks to the production function or to the cost of inputs lead firms to vary their production even in the face of unchanging demand. An alternative explanation is that an unanticipated increase in sales implies that future sales will be even higher. If the average level of expected future sales increases by more than the current increase in sales, then a firm facing increasing marginal costs of production would respond by increasing production by more than the current increase in sales. Hence, the variance of production responses to sales shocks would exceed the variance of sales shocks.[43] Thus, for example, an unanticipated increase in government purchases of goods would lead firms to increase production by an even greater amount, thereby increasing inventory investment. Thus the initial sales innovation has a magnified effect on GNP.

A third explanation of production counter-smoothing is that firms have a desired level of inventories that depends on the stochastic distribution of sales.[44] An unanticipated increase in sales would deplete inventories by an equal amount. In order to restore inventories to the originally desired level, production would have to increase by an amount equal to the unanticipated increase in sales. If, in addition, the unanticipated increase in sales leads the firm to revise upward its forecast of future sales, the desired level of inventories would increase. In order to reach the new, higher, desired level of inventories, the firm would have to increase production by even more than the increase in sales. Again, the production response to an innovation in inventories magnifies the effect on GNP.

In addition to depending on sales expectations, inventory investment may depend on the behavior of interest rates. The reason for the dependence of inventory investment on interest rates is similar to the reason that fixed investment should depend on interest rates. Specifically, the interest rate measures the opportunity cost of holding inventories rather than interest-earning assets. An increase in the real interest rate should lead to a decrease in the desired holding of inventories. However, as in the case of business fixed investment, it has been difficult to detect the effect of interest rates on inventory investment econometrically. Recently, Irvine (1981) and Akhtar (1983) have reported statistically significant negative responses of inventory investment to increases in short-term interest rates. Specifically, Akhtar finds that a one percentage point rise in the short-term nominal interest rate would reduce aggregate inventory investment by about

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[43]See Blinder (1986) and Kahn (1987).

[44]See Feldstein and Auerbach (1976). Kahn (1987) motivates the desired level of inventories by explicitly considering stockout costs.

$2 billion; a one percentage point increase in the expected rate of inflation leads to an increase in inventory investment of about $0.8 billion.

X. Concluding Remarks

Although the last decade has seen tremendous progress in understanding the stochastic behavior of consumption, many questions still remain. As mentioned earlier in this chapter, recent evidence suggests that labor income is characterized by a unit root, and the presence of a unit root has important implications for consumption behavior. Whether it is ultimately determined that the trend in labor income is stochastic, as suggested by the evidence on a unit root, or is deterministic, there still remains the question of whether consumers think of the trend as being stochastic or deterministic in making consumption decisions. The formal analysis of the permanent income model employs the assumption of rational expectations which implies that consumers know whether the trend is stochastic or deterministic; however, it must be recognized that the assumption of rational expectations is simply an assumption. Whether it will prove fruitful to explore alternative assumptions about the expectations of consumers is an open question.

Other important questions about consumption behavior remain for policy makers. For instance, are there quantitatively important departures from the Ricardian Equivalence Theorem? If so, what is the source of these departures and is there scope for tax policy to achieve alternative allocations of consumption that might be preferred according to some criterion? Another unresolved question relevant for policy involves the interest elasticity of saving. If this elasticity could be reliably estimated, then there would be scope for fiscal policy to increase the level of saving by somehow subsidizing the rate of return on saving.

The empirical performance of investment equations could also benefit from future advances. Many capital goods are indivisible and take a long time to build. The indivisibility of these goods and the delivery lags associated with capital continue to pose challenges to the theory of investment and its empirical implementation.

Another area of open research questions involves the cost of capital and its relation to investment. The theory of corporate finance is still working toward an understanding of the financing decisions of firms and an appropriate concept of the cost of capital. Further developments in this theory may help to clarify the role of risk in affecting the cost of capital and the investment decisions of firms.

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