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DEBT NEUTRALITY, REDISTRIBUTION AND CONSUMER HETEROGENEITY A SURVEY AND SOME EXTENSTIONS

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ABSTRACT

For an economic system not to exhibit debt neutrality it must be true that changes in the time profile of lump-sum taxes redistributes resources between heterogeneous consumers. OLG models have age heterogeneity because of a positive birth rate. Unless a bequest motive or child-to-parent gift motive is operative, a positive birth rate is sufficient for absence of debt neutrality. Uncertain lifetimes are neither necessary nor sufficient for absence of debt neutrality, with or without efficient life insurance markets. Heterogeneous survival probabilities are a sufficient condition. Heterogeneous time preference rates or elasticities of marginal utility does not destroy debt neutrality, since with common survival rates, changes in the pattern over time of lump-sum taxes do not redistribute resources. Any representative agent model, regardless of the scope and severity of capital market imperfections, will exhibit debt neutrality.

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DEBT NEUTRALITY, REDISTRIBUTION AND CONSUMER HETEROGENEITY
A SURVEY AND SOME EXTENSIONS

Willem H Buiter

1. Introduction

The teacher-pupil relationship between Jim Tobin and me by no means came to an end after I obtained my Ph.D in 1975. Like so many who experienced his influence, I have tried to internalise his insistence that we practice economics as if it mattered beyond the narrow confines of the profession. No matter how formal and abstract our analyses may have to be in order to answer certain complex substantive questions, our subject is not an intellectual game or a branch of pure logic. It is a potentially powerful tool for understanding and influencing the real world and the lives of many who may not even be aware of the existence of an academic discipline called economics and its practitioners.

At the methodological level, I have become convinced more and more of the correctness of his view that representative agent models make for uninteresting economics. Robinson Crusoe didn't need much economic theory before Friday arrived. After that he needed game theory. No economic policy issue of any significance can be addressed satisfactorily without introducing some measure of heterogeneity among (depending on the issue) consumers, producers, workers, employers or investors. This poses a serious problem for macroeconomics, which approaches economic policy issues using highly agggregative sequential general equilibrium models. How much disaggregation and heterogeneity is possible before the virtues of simplicity, transparency and analytical tractability are lost completely?

Many potentially important kinds of heterogeneity come to mind. Consumers can have heterogeneous endowments (including abilities), opportunities, ages, life expectancies, tastes (risk aversion, impatience, etc.) or information sets. Producers can have different technologies, tastes or information sets. In this paper I shall consider the consequences of four kinds of consumer heterogeneity for debt neutrality. An economic system exhibits debt neutrality if, given a program for public spending on goods and services over time, the equilibrium of the economy is not affected by a change in the pattern over time of lump-sum taxes. If there is debt neutrality, e.g. the substitution of government borrowing today for lump-sum taxation today (followed by such further changes in the path of future lump-sum taxes as may be required to maintain the solvency of the public sector) does not affect current and future private consumption, capital formation and interest rates. The four kinds of consumer heterogeneity are age, life expectancy, time preference and elasticity of intertemporal substitution. The overlapping generations (OLG) model is the natural vehicle for this kind of modeling as it is designed specifically to handle the "entry" and "exit" of consumers.

The issue of debt neutrality is central to an understanding both of the short-run cyclical stabilization role of fiscal policy and of the long-run effect of fiscal and financial policy on the path of the capital stock. (See e.g. the contributions in Ferguson (1964) and Modigliani (1961).) It therefore comes as no surprise that Jim Tobin studied this subject early in his career (Tobin (1952)) and returned to it time and again (e.g. Tobin (1976, 1979,1980)). I was fortunate to be involved in two collaborations with him on this subject matter (Buiter and Tobin (1979), Tobin and Buiter (1980)).

There is no better way to introduce the key issue than by quoting from one of Jim's key writings on the subject.

"How is it possible that society merely by the device of incurring a debt to itself can deceive itself into believing that it is wealthier? Do not the additional taxes which are necessary to carry the interest charges reduce the value of other components of private wealth? There certainly must be effects in this direction." (Tobin (1952), p.117).

The central issue can be phrased as follows: when does, (at given prices and interest rates), postponing lump-sum taxation while maintaining public sector solvency change binding constraints faced by consumers[1] alive today in such a way that aggregate consumption changes? The answer is that postponing lump-sum taxation must achieve one or both of the following. First, it redistributes (lifetime) resources among "isolated" heterogeneous survivors, i.e. among households alive in the period when the taxes are cut. Second, it redistributes (lifetime) resources between survivors (who may be homogeneous) and overlapping new entrants from whom they are "isolated" (and who may also be homogeneous), i.e. households that are born after the period during which taxes are cut but whose lifespan overlaps with that of households alive when the taxes are cut. "Isolation" here means a situation without interior solutions for gifts or bequests, intertemporal or a-temporal. This can either be the result of egoistic utility functions (only own liftime consumption yields utility) or of zero gift or bequest corner solutions despite altruistic utility functions.

Absence of debt neutrality therefore requires that postponing lump-sum taxation causes ***redistribution*** among ***heterogeneous*** households.

The plan of the remainder of the paper is as follows. Section 2 reviews some important features of the 2-period OLG model with intergenerational gift and bequest motives. It draws heavily on the recent work of Kimball (1987a, b), which contains the first (to my knowledge) complete solution of the two-sided intergenerational caring problem with population growth and parthogenesis. This model has a positive birth rate (the representative household born in any given period is assumed to have at least one child) and a finite (in this case a 2-period) lifetime, i.e. a zero probability of death at the end of the first period and a 100 per cent probability of death at the end of the second period. Debt neutrality occurs for (small) changes in the pattern of borrowing and lump-sum taxation when the equilibrium is one with an operative intergenerational gift or bequest motive (i.e. with positive bequest or child-to-parent gift). If the intergenerational gift and bequest motives are non-operative there is no debt neutrality as long as there is a positive birth rate. If there is a zero birth rate, we are of course back in the representative consumer model. The representative consumer has a finite horizon, but this doesn't mean she'll benefit from postponing taxes as there are no "new entrants" (succeeding generations) to whom (part of) the tax burden can be shifted.

If there is a positive birth rate, the presence of debt neutrality despite heterogeneity when there is an operative intergenerational gift or bequest motive can be attributed to the failure to achieve intergenerational redistribution by postponing lump-sum taxes. Changes in official involuntary intergenerational transfers are offset by changes in private voluntary intergenerational transfers in the opposite direction, as long as the legal constraints that gifts and bequests cannot go negative do not become binding. Alternatively, the sequence of altruistically linked successive generations can be interpreted as a single dynastic representative consumer. Absence of heterogeneity is the reason for debt neutrality in this view.

The key references for this section are Barro (1974), Carmichael (1979, 1982), Buiter (1979, 1980), Buiter and Carmichael (1984), Burbridge (1983), Abel (1985), Weil (1987) and especially Kimball (1987a,b).

Section 3 considers an OLG model without intergenerational gift and bequest motives but with potentially infinite-lived consumers. The birth rate is non-negative and there is a common age and time-independent probability of death which can be zero. When there is a positive probability of death, an efficient competitive life insurance or annuities market is assumed to exist. When the utility function is time additive and the single period utility function has constant elasticity of marginal utility, it can be shown that a positive birth rate is necessary and sufficient for absence of debt neutrality. Uncertain lifetimes (or productivity growth) do not destroy debt neutrality when there is zero birth rate. Note that in this model with its uniform death rate, and productivity growth rate,[2] age is the only form of household heterogeneity. A zero birth rate destroys this one form of heterogeneity. This section draws on the work of Yaari (1965), Blanchard (1985), Weil (1985), Frenkel and Razin (1986), Abel (1987) and Buiter (1988a,b).

In Section 4 the perfect capital market assumption is relaxed. I first consider the case of a complete absence of life insurance markets. As long as there is no consumer heterogeneity, however, this capital market imperfection is no independent source of absence of debt neutrality. (In quite a different context a similar point has been made by Yotsuzuka (1987)).

When there is heterogeneity in death rates, there will be absence of debt neutrality even with a zero birth rate and perfect annuities markets. Postponing taxes will redistribute lifetime resources towards the households with the higher death rate (assuming the current tax cuts and later tax increases fall equally on all households alive at the time, independently of their death rates). These households have a higher marginal propensity to spend out of lifetime resources. Postponing lump-sum taxes therefore redistributes wealth from high savers to low savers, boosting aggregate consumption. Note that heterogeneity through different time preference rates does not cause absence of debt neutrality when there is a common death rate and a zero birth rate. The reason is of course that postponing uniform lump-sum taxes (i.e. taxes falling equally on all alive, regardless of time preference rates) does not redistribute income between high and low time preference households as both kinds have the same life expectancy. Redistribution and heterogeneity are both necessary for absence of debt neutrality.

2. An OLG Model with Finite Lifetimes and Intergenerational Gift and Bequest Motives

2.1 The consumer's problem

The utility function of a representative member of the generation born in period t is given by equation (1). Utility is additively separable intergenerationally.

$$W_t = u(c_t^1, c_t^2) + (1+\rho)^{-1}W_{t-1} + (1+\delta)^{-1}W_{t+1} \qquad \delta, \rho > 0 \tag{1}$$

A member of generation t derives utility directly from his own lifetime consumption. This is captured by $u_{\delta,o}^t = u(c_t^1, c_t^2)$. I shall refer to $u_{\delta,o}^t$ as the egoistic utility of a member of generation t and to W_t as her total utility. Where there is no ambiguity the superscript and subscripts will be omitted. Each consumer lives for 2 periods. Labour-leisure choice is omitted.[3] u is strictly concave, increasing, and twice continuously differentiable. It satisfies the Inada conditions. Note that equation (1) exhibits *direct* two-sided intergenerational altruism: a member of generation t cares directly both about his parent[4] and about his 1+n children. For most of this section we consider the case of one or more children, i.e. $n \geq 0$. ρ is the discount rate applied to parental utility and δ that applied to the utility of one's children. There are no crucial modifications to the model if the consumer lives for $N>2$ periods and cares directly about the $2(N-1)$ generations with whom he overlaps.[5] All members of all generations have identical egoistic and total utility functions.

In the case of one-sided intergenerational caring, $\delta>0$ is required for boundedness of the utility functional when the parent-to-child bequest motive is the only one ($(1+\rho)^{-1} = 0$) and there is no "last generation" in finite time'; $\rho>0$ is required for boundedness of the utility functional when the child-to-parent gift motive is the only one ($(1+\delta)^{-1} = 0$) and there is no "first generation" a finite number of periods in the past. As shown in Carmichael (1979) and Buiter (1980) and recently in Kimball (1987a,b), stronger conditions that $\rho>0$ and $\delta>0$ are required to obtain a sensible objective functional with two-sided caring.

W_{t+1} is to be interpreted as the average total utility of the n+1 children of the member of generation t, i.e.

$$W_{t+1} = \frac{1}{1+n} \sum_{i=1}^{1+n} W_{t+1,i},$$

where i indexes the children of the member of generation t. Equation (1) is, however, consistent with a "the more, the merrier" view of intergenerational caring by reinterpreting $1+\delta$ in the way suggested below:

$$1 + \delta = \frac{(1+\delta')}{1+n}$$

Here δ' is the true discount rate applied to the sum of the utilities of the 1+n children each of which is weighted equally in the parent's objective functional. We continue to express our algebra in terms of δ rather than δ'.

A member of generation t is assumed not to care directly about her n siblings. She will of course care indirectly about her siblings (and about more distant relatives of the same generation) to the extent that her parent (and through them more remote ancestors) do.

Kimball (1987a, Appendix D), in an argument that is both ingenious and involved for the case of more than one child (n>0), shows how the total utility of a member of generation t, W_t can be expressed as a function of the egoistic utilities of all relatives (contemporaries, ancestors and descendants). $u^t_{j,k,i}$ is the egoistic utility of the i^{th} relative of type (j,k) of a member of generation t. The index j measures "vertical" or generational distance and the index k measures "horizontal" or lateral distance. $\gamma_{j,k}$ is the weight attached to the egoistic utility of any relative of type j,k[6], i.e.

$$W_t = \sum_{j,k,i} \gamma_{j,k} u^t_{j,k,i} \tag{2}$$

j ranges from $-\infty$ to $+\infty$; k ranges from 0 to $+\infty$, i ranges from 1 to N(j,k), the number of relatives of type (j,k).

Tedious calculation shows that

$$N(j,k) = \begin{cases} 1 & ; j\leq 0,\ k=0 \\ (1+n)^j & ; j\geq 0,\ k=0 \\ n(1+n)^{k-1} & ; j\leq 0,\ k\geq 1 \\ n(1+n)^{k-1}(1+n)^j & ; j\geq 0,\ k\geq 1 \end{cases} \tag{3}$$

Let $u^t_{j,k}$ be the average egoistic utility of all relatives of type j,k i.e.

$$u^t_{j,k} = \frac{1}{N(j,k)} \sum_{i=1}^{N(j,k)} u^t_{j,k,i}$$

This permits us to rewrite equation (2) as:

$$W_t = \sum_{j,k} N(j,k)\gamma_{j,k} u^t_{j,k} \tag{2'}$$

Kimball imposes the following reasonable restrictions on the $\gamma_{j,k}$.

a) $\gamma_{j,k} \geq 0$ for all j,k (i.e. no ill-will towards relatives and no self-hatred).

b) for j<0, $\gamma_{j,k}$ is a geometric series in j for every k.

c) $\gamma_{j,0}$ is a geometric series in j for j≥0.

d) $\lim_{j \to -\infty} \gamma_{j,k} = 0$ for all k.

e) $\lim_{j \to +\infty} (1+n)^j \gamma_{j,0} = 0$

Restriction (c) is a necessary condition for dynamic consistency of choice across generations. Restriction (b) is necessary e.g. to rule out forms of dynamic inconsistency in models with more than two overlapping generations in which grandparents overlap with their grandchildren. Restrictions (d) and (e) assert that the indirect concern for very distant ancestors and descendants should vanish.

These restrictions imply that

$$\frac{1}{1+\delta} + \frac{1}{1+\rho} < 1 \tag{4}$$

If $\delta = \rho$, this implies the need for an intergenerational discount rate of over 100 per cent! Equation (4) is equivalent to $\delta\rho > 1$, the conditions given in Buiter (1980) for well-behaved steady-state utility.

Given the five restrictions (a)-(e), Kimball (1987a) shows that the weights $\gamma_{j,k}$ are given by:

$$\gamma_{0,0} = \frac{1}{\sqrt{1-4(1+\delta)^{-1}(1+\rho)^{-1}} + \frac{n}{1+n}\mu(1+\rho)^{-1}} \tag{5a}$$

$$\gamma_{j,k} = \begin{cases} \gamma_{0,0} \left[\frac{\mu}{1+n}\right]^j \left[\frac{\mu}{(1+n)\lambda}\right]^k & ;\ j\geq 0 \quad k\geq 0 \\ \gamma_{0,0}\ \lambda^j \left[\frac{\mu}{(1+n)\lambda}\right]^k & ;\ j\leq 0 \quad k\geq 0 \end{cases} \tag{5b}$$

$$\mu = \left[\frac{1+\rho}{2}\right]\left[1-\sqrt{1-4(1+\delta)^{-1}(1+\rho)^{-1}}\ \right] \qquad ;\ 0<\mu<1 \tag{5c}$$

$$\lambda = \left[\frac{1+\rho}{2}\right]\left[1+\sqrt{1-4(1+\delta)^{-1}(1+\rho)^{-1}}\ \right] \qquad ;\ \lambda>1 \tag{5d}$$

Substituting equations (3) and (5a,b,c,d) into (4) and rearranging yields:

$$W_t = \gamma_{0,0} \left\{ \left[u^t_{0,0} + \frac{n}{1+n} \sum_{k=1}^{\infty} \left[\frac{\mu}{\lambda}\right]^k u^t_{0,k}\right] + \sum_{j=-\infty}^{-1} \lambda^j \left[u^t_{j,0} + \frac{n}{1+n} \sum_{k=1}^{\infty} \left[\frac{\mu}{\lambda}\right]^k u^t_{j,k}\right] + \sum_{j=1}^{\infty} \mu^j \left[u^t_{j,0} + \frac{n}{1+n} \sum_{k=1}^{\infty} \left[\frac{\mu}{\lambda}\right]^k u^t_{j,k}\right] \right\} \tag{6}$$

Having expressed the utility function (1) in terms of equation (6), with $\gamma_{0,0}$, μ and λ given by equations (5a) and (5c,d), I now turn to the lifetime budget constraint of the representative i^{th} member of generation t: where there is no ambiguity, the superscript i is omitted.

$$\left[\frac{B_{t-1}}{1+n} - G^i_t + w_t - c^1_t - \tau^1_t\right](1+r_{t+1}) \geq c^2_t + B_t - \sum_{j=1}^{1+n} G^j_{t+1} + \tau^2_t \tag{7}$$

B_t is the total bequest left in the second period of his life by the i^{th} member of generation t to his 1+n children. The bequest is assumed to be shared equally among the children. G^j_{t+1} is the gift given by the j^{th} child (j=0,1,...,1+n) born in period t+1 to his parent. w_t is the real wage earned while young. Each worker-consumer only works during the first period of his life and supplies labour inelastically during that period. A lump sum per capita tax or transfer is paid (received) during one's youth, τ^1_t, and during old age, τ^2_t. r_{t+1} is the one-period real interest rate established in period t. Equation (7) will hold as a strict equality.

Note that equation (7) does not include gifts to siblings, to more distant lateral relatives (cousins, etc.) or to more distant (non-lineal) relatives in generations t-1 and t+1. Kimball (1987a) shows (see equation (5b)) that while with n>1 one will always care *indirectly* for one's siblings, etc. (because one's parent does) one will always (when all agents of a given generation have the same egoistic utility levels) care less about a sibling than about oneself. Similarly, siblings will carry more weight than more distant lateral relatives and non-lineal relatives will carry less weight than linear relatives of the same age cohort. No-one will, when all agents of a given generation have the same egoistic utility, ever give anything to a sibling or to a non-lineal relative.

The consumer maximizes (1) by optimally choosing c^1_t, c^2_t, B_t and G_t, subject to the constraint (7) and

$$c^1_t,\ c^2_t \geq 0 \tag{8a}$$

$$B_t \geq 0 \tag{8b}$$

$$G_t \geq 0 \tag{8c}$$

The Inada conditions ensure that (8a) is satisfied as long as the consumer has positive lifetime resources. Equations (8b) and (8c) reflect legal restrictions that rule out he possibility of a private individual "taxing" his parents or children.

The consumer is competitive in the labour and capital markets and takes taxes to be exogenous. It is also assumed that all relatives of a given type (j,k) have the same egoistic utilities and behave in the same manner.

To obtain a well-defined unique solution, many further restrictions must be imposed on the "games" the household plays with members of other generations. The following assumptions are made.

(A1) Intergenerational Nash behaviour

A member of generation t takes B_{t-1} and G^j_{t+1}, j=0,1,...,1+n, as given (i.e. as independent of his choices of c^1_t, c^2_t, B_t and G_t). Note that this is not trivial, as the bequest B_{t-1} is left in period t simultaneously with c^1_t and G_t, while the gifts G^j_{t+1}, j=0,...,1+n, are given in period t+1 after c^1_t and G_t and simultaneously with c^2_t and B_t. This intergenerational Nash assumption is by no means overwhelmingly plausible, but simplifies the analysis greatly.

Further strategic conjectures are required as regards the behaviour of one's siblings, if there is more than one child (see Abel (1985) and Kimball (1987a,b)), (n>0). I'll consider the following three.

(A2) Sibling Nash gift behaviour

This means that the siblings of the i^{th} child born in generation t are assumed not to change their gift behaviour when the i^{th} child changes its consumption, gift or bequest behaviour:

$$\frac{\partial G^j_t}{\partial G^i_t} = \frac{\partial G^j_t}{\partial c^1_t} = \frac{\partial G^j_t}{\partial c^2_t} = \frac{\partial G^j_t}{\partial B_t} = 0;\ j=1,\dots,1+n;\ j\neq i \quad 7$$

Abel (1985) favours this assumption.

(A2') Co-operative sibling gift behaviour

Kimball (1987b, p.316) proposes a co-operative solution among siblings in which each sibling agrees to give exactly the same amount that each of the others gives while one of them decides the total amount to be given. The agent who decides the total amount to be given to the common parent simply maximises her own *total* utility and therefore effectively values the egoistic utility loss of each of his n siblings only $\mu/[(1+n)\lambda]$ as much as her own egoistic utility loss (see equation (5b) with $j=0$ and $k=0$ (own utility) vs. $j=0$ and $k=1$ (sibling utility)). This kind of behaviour is probably better characterised as ***imitative*** rather than as ***co-operative***.

Let i be the "leader", then:

$$\frac{\partial G_t^j}{\partial c_t^1} = \frac{\partial G_t^j}{\partial c_t^2} = \frac{\partial G_t^j}{\partial B_t} = 0; \; j=1,\ldots,1+n; \; j\neq i$$

and $$\frac{\partial G_t^j}{\partial G_t^i} = 1; \quad j=1,\ldots,1+n; \; j\neq i.$$

(A3) The consumption choices of relatives of type (j,k) other than siblings are affected by the choices of the current generation only if the latter directly affect their budget constraints. Formally we assume that:

$$\frac{\partial u_{j,k}^t}{\partial c_t^1} = \frac{\partial u_{j,k}^t}{\partial c_t^2} = 0 \quad ; j\leq -1 \text{ and } k\geq 0;\; j\geq 1 \text{ and } k\geq 0;\; k\geq 1 \text{ and } j=0. \qquad (9a)$$

$$\frac{\partial u_{j,k}^t}{\partial B_t} = 0 \quad ; j\leq -1 \text{ and } k\geq 0;\; j>1 \text{ and } k\geq 0;\; k\geq 1 \text{ and } j=0;\; k>1 \text{ and } j=1. \qquad (9b)$$

$$\frac{\partial u^t_{j,k}}{\partial G_t} = 0 \quad ;j<-1 \text{ and } k\geq 0;\ j\geq 1 \text{ and } k\geq 0;\ k\geq 1 \text{ and } j=0;\ k>1 \text{ and } j=-1 \quad (9c)$$

This assumption that changes in c^1_t, c^2_t, B_t and G_t only affect the consumption of relatives (other than siblings) if these relatives' lifetime budget constraints are directly affected[8] is implicit in Kimball (1987a,b). It is discussed at greater length in Carmichael (1979) and Buiter (1980).

Given all this, the maximization of (6) subject to (7) (holding as a strict equality) and (8a,b,c) yields:

$$\frac{\partial}{\partial c^1_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] = (1+r_{t+1})\frac{\partial}{\partial c^2_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] \quad (10a)$$

$$\frac{\partial}{\partial c^2_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] \geq \frac{\mu}{1+n}\frac{\partial}{\partial c^1_t}\left[u^t_{1,0}(c^1_{t+1},c^2_{t+1})\right] \quad (10b)$$

If $B_t>0$ then (10b) holds with equality. If (10b) holds as a strict inequality, then $B_t=0$.

With (A2) (Nash sibling gift behaviour) we also have

$$\frac{\partial}{\partial c^1_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] \geq \lambda^{-1}\frac{\partial}{\partial c^2_{t-1}}\left[u^t_{-1,0}(c^1_{t+1},c^2_{t+1})\right] \quad (10c)$$

If $G_t>0$ then (10c) holds with equality. If (10c) holds as a strict inequality, then $G_t=0$.

With (A2') (Co-operative sibling gift behaviour) we have instead (see Kimball (1987a,b)):

$$\frac{\partial}{\partial c^1_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] + \frac{n}{1+n}\frac{\mu}{\lambda}\frac{\partial}{\partial c^1_t}\left[u^t_{0,1}(c^1_t,c^2_t)\right]$$

$$\geq (1+n)\lambda^{-1}\frac{\partial}{\partial c^2_{t-1}}\left[u^t_{-1,0}(c^1_{t-1},c^2_{t-1})\right] \quad (10c')$$

If $G_t>0$ then (10c') holds with equality. If (10c') holds a strict inequality, then $G_t=0$.

Using equation (10a), equations (10b,c,c') can be rewritten as:

$$\frac{\partial}{\partial c_t^2}\left[u^t_{0,0}(c^1_t,c^2_t)\right] \geq \frac{\mu}{1+n}(1+r_{t+2})\frac{\partial}{\partial c_t^2}\left[u^t_{1,0}(c^1_{t+1},c^2_{t+1})\right] \qquad (11a)$$

$= \text{ if } B_t>0$

$\text{if } > \text{ then } B_t=0.$

$$\frac{\partial}{\partial c^1_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] \geq \frac{\lambda^{-1}}{1+r_t}\frac{\partial}{\partial c^1_{t-1}}\left[u^t_{-1,0}(c^1_{t-1},c^2_{t-1})\right] \quad \text{(Nash)} \qquad (11b)$$

$$\frac{\partial}{\partial c^1_t}\left[u^t_{0,0}(c^1_t,c^2_t)\right] + \frac{n}{1+n}\frac{\mu}{\lambda}\frac{\partial}{\partial c^2_t}\left[u^t_{0,1}(c^1_t,c^2_t)\right]$$

$$\geq (1+n)\frac{\lambda^{-1}}{1+r_t}\frac{\partial}{\partial c^1_{t-1}}\left[u^t_{-1,0}(c^1_{t-1},c^2_{t-1})\right] \quad \text{(Co-operative)} \qquad (11b')$$

$= \text{ if } G_t>0$

$\text{if } > \text{ then } G_t=0$

In a stationary equilibrium with an operative intergenerational bequest motive ($B>0$) equation (12) must hold:

$$\frac{1+r}{1+n} = \frac{1}{\mu} \qquad (12)$$

Since $0<\mu<1$, this means that proposition (1) holds, as shown in Carmichael (1979), Buiter (1980) and Kimball (1987a,b).

Proposition 3:

Any stationary state in which a bequest motive is operative is dynamically efficient, i.e. the interest rate, r, exceeds the population growth rate n.

Note that if there is only the bequest motive ($(1-\rho)^{-1}=0$), equation (12) reduces to $(1+r)/(1+n)=1+\delta$.

Weil (1987) shows that dynamic inefficiency of the economy without bequest motive is sufficient to rule out operative bequests in the economy with a bequest motive[9] both for an endowment economy and in the Diamond (1965) production economy.

Similarly, in a stationary equilibrium with an operative intergenerational gift motive ($G>0$), equation (13a) must hold in the case of Nash sibling gift behaviour and equation (13b) must hold in the case of cooperative sibling gift behaviour

$$1+r = \frac{1}{\lambda} \qquad \text{(Nash)} \qquad (13a)$$

$$\frac{1+r}{1+n} = \frac{1}{\lambda\left(1+\frac{n}{1+n}\frac{\mu}{\lambda}\right)} \qquad \text{(Cooperative)} \qquad (13b)$$

Since $\lambda>1$, $\mu>0$ and $n\geq 0$, it follows that Proposition 4 holds as shown in Carmichael (1979), Buiter (1980) and Kimball (1987a,b)

Proposition 4:

Any steady state in which a child-to-parent gift motive is operative is dynamically inefficient ($r<n$).

Note that if there is only the gift motive ($(1+\delta)^{-1}=0$), equation (13a) becomes $1+r=(1+\rho)^{-1}$, equation (13b) becomes $(1+r)/(1+n)=(1+\rho)^{-1}$ and equation (13c) becomes

$$\frac{1+r}{1+n} = \frac{(1+\rho)^{-1}}{1+\frac{n}{1+n}\frac{\mu}{\lambda}} .$$

As shown in Kimball (1987a,b) a constant proportional rate of growth of steady-state per capita income π can be incorporated easily into the analysis if the egoistic utility function assumes the constant elasticity of marginal utility form given in equation (14)

$$u(c_t^1, c_t^2) = \frac{1}{1-\alpha}(c_t^1)^{1-\alpha} + \frac{\beta}{1-\alpha}(c_t^2)^{1-\alpha} \qquad ;\alpha>0;\ \beta>0 \tag{14}$$

In steady state, consumption will grow at the constant rate π, i.e. $c_t^1 = (1+\pi)c_{t-1}^1$ and $c_t^2 = (1+\pi)c_{t-1}^2$. Equation (11a) becomes, with B>0,

$$\frac{1+r}{(1+n)(1+\pi)} = \frac{1}{\mu}\left[\frac{1}{1+\pi}\right]^{(1-\alpha)} \tag{15a}$$

With G>0, equations (11b) and (11b') become respectively (in steady state):

$$\frac{1+r}{(1+n)(1+\pi)} = \frac{1}{\lambda(1+n)}\left[\frac{1}{1+\pi}\right]^{(1-\alpha)} \qquad \text{(Nash)} \tag{15b}$$

$$\frac{1+r}{(1+n)(1+\pi)} = \frac{1}{\lambda(1+\frac{n}{1+n}\ \frac{\mu}{\lambda})}\left[\frac{1}{1+\pi}\right]^{(1-\alpha)} \qquad \text{(Cooperative)} \tag{15b'}$$

Note that with α, the elasticity of marginal utility, greater than 1, a sufficiently large per-capita income growth rate (high value of π) will ensure dynamic efficiency $(1+r>(1+n)(1+\pi))$ in a steady state with operative gifts. $\alpha<1$ will ensure inefficiency of a steady state with operative gifts if $\pi>0$.

It might appear from (15a) that with $\pi>0$ and $\alpha<1$ we might get dynamic inefficiency $(1+r<(1+n)(1+\pi))$ with an operative bequest motive. As pointed out in Kimball (1987a,b), this is not in fact the case, since (15a) no longer characterises a privately optimal plan if $1+r<(1+n)(1+\pi)$. The total utility functional no longer converges and even the overtaking criterion cannot be used to rank feasible paths.

2.2 Production and market equilibrium

The production technology is identical to that in Diamond (1965). A single homogeneous durable commodity is produced by a well-behaved neoclassical production function which is linear homogeneous in capital and labour. Productivity growth is omitted for simplicity.

$$Y_t = F(K_t, L_t) \tag{16}$$

F is increasing, strictly concave, twice continuously differentiable and satisfies the Inada conditions. L_t is the size of the labour force in period t, i.e. the number of (young) members of the generation born in period t; $L_t=(1+n)L_{t-1}$. Let $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$. Equation (16) can be rewritten in intensive form as in (16').

$$y_t = f(k_t) \qquad f'>0;\ f''<0;\ f(0)=0;\ \lim_{k\to 0} f'(k)=+\infty;\ \lim_{k\to\infty} f'(k)=0 \tag{16'}$$

The labour market and the capital rental market clear and are competitive:

$$w_t = f(k_t)-k_t f'(k_t) \tag{17}$$

$$r_t = f'(k_t) \tag{18}$$

Output market equilibrium is given by equation (19) where E_t denotes total public consumption expenditure in period t.

$$c_t^1 L_t + c_{t-1}^2 L_{t-1} + E_t + K_{t+1} - K_t = Y_t \tag{19}$$

From the public sector budget identity given in (20) it follows that equation (19) can be replaced by the equivalent equation (21). D_t denotes the stock of public debt outstanding at the end of period t-1. Debt has a fixed face value of unity and a maturity of one period.

$$E_t+r_tD_t-\tau_t^1L_t-\tau_{t-1}^2L_{t-1} = D_{t+1}-D_t \qquad (20)$$

$$(w-c_t^1-\tau_t^1+\frac{B_{t-1}}{1+n}-G_t)L_t = D_{t+1}+K_{t+1} \qquad (21)$$

Equation (21) states that the savings of the young in period t have to equal the sum of the capital stock and the public debt stock in period t+1. Letting $d_t \equiv D_t/L_t$ and $e_t \equiv E_t/L_t$ equations (20) and (21) can be rewritten as:

$$e_t+r_td_t-\tau_t^1-\frac{\tau_{t-1}^2}{1+n} = (1+n)d_{t+1}-d_t \qquad (20')$$

$$w_t-c_t^1-\tau_t^1+\frac{B_{t-1}}{1+n}-G_t = (d_{t+1}+k_{t+1})(1+n) \qquad (21')$$

2.3 Stationary equilibrium

A stationary equilibrium is characterised by equations (23)-(27). In equation (24) cooperative sibling gift behaviour is assumed.

$$u_1(c^1,c^2) = (1+f'(k))u_2(c^1,c^2) \qquad (22)$$

$$\frac{1+f'(k)}{1+n} \leq \frac{1}{\mu} \qquad (23)$$

$= $ if $B>0$

if $<$ then $B=0$.

$$\frac{1+f'(k)}{1+n} \geq \frac{1}{\lambda\left[1+\frac{n}{1+n}\frac{\mu}{\lambda}\right]} \qquad (24)$$

$=$ if $G>0$

if $>$ then $G=0$

$$(f(k)-kf'(k)-c^1)(1+f'(k)) = c^2+\tau^1(1+f'(k))+\tau^2+(n-f'(k))\left[\frac{B}{1+n}-G\right] \qquad (25)$$

$$w-c^1-\tau^1+\frac{B}{1+n}-G = (d+k)(1+n) \tag{26}$$

$$e-\tau^1-\frac{\tau^2}{1+n} = (n-f'(k))d \tag{27}$$

When there is neither a gift nor a bequest motive ($(1+\delta)^{-1}=(1+\rho)^{-1}=0$ and $B=G=0$) nor a public sector ($\tau^1=\tau^2=e=d=0$), equations (25) and (26) can be solved for c^2 as a function of c^1 as in (28)

$$c^2=\psi(c^1) \tag{28}$$

with

$$\psi' = -(1+f')\left[1+\frac{k(n-f')}{1+f'}f''(1+n+kf'')^{-1}\right] \tag{29a}$$

In the case of a Cobb-Douglas production function with $f(k)=k$ $0<\bar{\alpha}<1$,

$$\psi' = \frac{(1+n)(1+\bar{\alpha}^2k^{(\bar{\alpha}-1)})}{(1-\bar{\alpha})\bar{\alpha}k^{\bar{\alpha}-1}-(1+n)} \tag{29b}$$

The stationary competitive consumption possibility locus without gifts and bequests and without government, is graphed in Figure 1 for the Cobb-Douglas case. At the origin ($k=0$) its slope is $(1+n)\bar{\alpha}/(1-\bar{\alpha})$. The capital-labour ratio increases monotonically as we move up along the locus from 0 to A. At A, when $c_1=0$ again, $1+r=(1+\bar{\alpha}n)/(1-\bar{\alpha})$ and the slope of the locus is $\bar{\alpha}-1-\bar{\alpha}^2(1+n)<0$. For small values of $\bar{\alpha}$, the interest rate at A is therefore below the golden rule value n (if $n>0$). If $\bar{\alpha}>n/(1+2n)$, then even the lowest possible stationary interest rate is always above the golden rule value. It is assumed in what follows that $\bar{\alpha}<n/(1+2n)$. The golden rule capital-labour ratio k^* defined by $f'(k^*)=n$ therefore defines a point

somewhere on the downward portion of locus, such as Ω_1. The locus is strictly concave towards the origin. For more general constant returns to scale production functions than the Cobb-Douglas, ψ' can become positive again for large k. Such a backward and downward-bending locus represents a case of extreme overaccumulation.[10]

Adding gifts and bequests but still omitting government spending, debt and taxes modifies the stationary competitive consumption possibility locus as in Figure 2. It is assumed that $\bar{\alpha}<n/(1+2n)$ and that there exists a feasible value of k, k^G say, such that

$$\frac{1+f'(k^G)}{1+n} = \frac{1}{\lambda\left[1+\frac{n}{1+n}\,\frac{\mu}{\lambda}\right]}$$

(the condition for G>0). In that case there certainly exists a feasible value of k, k^B say, such that B>0, i.e.

$$\frac{1+f'(k^G)}{1+n} = \frac{1}{\mu}.$$

Note that bequests and child-to-parent gifts cannot be positive simultaneously in steady state.

The stationary competitive consumption possibility locus with bequests and gifts is obtained by deleting from the stationary competitive consumption possibility locus without bequests and gifts the segment corresponding to capital-labour ratios above k^B (i.e.the dotted segment $0\Omega_2$) and the segment corresponding to capital-labour ratios above k^G (i.e. the dotted segment $\Omega_4\Omega_1$).

From Ω_2 to A_3 the straight line segment with slope $-(1+n)$ gives the locus where bequests are positive. With k given, $\partial c^1/\partial B=1/(1+n)$ and $\partial c^2/\partial B=-1$. Along the positive bequest locus therefore, $dc^2/dc^1=-(1+n)$.

Larger bequests correspond to movements towards the south-east along $\Omega_2 A_3$.

From Ω_4 to A_2 the straight line segment with slope $-(1+n)$ gives the locus where child-to-parent gifts are positive. With k given, $\partial c^1/\partial G = -1$ and $dc^2/dG = 1+n$ so again $\partial c^2/\partial c^1 = -(1+n)$. Larger gifts correspond to movement to the North-West along $\Omega_4 A_2$.

The complete stationary competitive consumption possibility locus with gifts and bequests is therefore given by the curve $A_3\Omega_2\Omega_4 A_2$ and consists of 3 segments. The positive bequest locus A_3A_2 where $(1+f'(k^B)) = (1+n)/\mu$; the locus with zero bequest and zero gift, $\Omega_2\Omega_4$ corresponding to the segment of the original no gift or bequest locus with $k^B > k > k^G$, and the positive gift locus $\Omega_4 A_2$ where

$$(1+f'(k^G)) = \frac{1+n}{\lambda\left[1+\frac{n}{1+n}\frac{\mu}{\lambda}\right]} .$$

A typical steady state with positive bequest has been drawn at Ω_3 where the indifference curve $u^{B\ 11}$ has a tangent to an intertemporal budget constraint with slope $-(1+f'(k^B))$. A typical steady state with positive gift has been drawn at Ω_5 where the indifference curve $u^{G\ 12}$ has a tangent to an intertemporal budget constraint with slope $-(1+f'(k^G))$. Stationary equilibria with zero gift and zero bequest on the segment $\Omega_2\Omega_4$ could either have the interest rate above the golden rule level (on $\Omega_2\Omega_1$) or below it (on $\Omega_1\Omega_4$).

For reasons of space, the analysis of fiscal policy will be focussed on the consideration of steady states, with but a brief excursion into non-steady state behaviour.

τ^2, e and d will be treated as steady-state policy parameters. τ^1 adjusts endogenously to satisfy the steady-state government budget identity given in equation (30)

$$\tau^1 = e - \frac{\tau^2}{1+n} + (f'(k)-n)d \tag{30}$$

The substitution of (30) into the steady state private life-time budget constraint and capital market equilibrium condition yields equations (31) and (32)

$$f(k)-kf'(k)-c^1-e = k(1+n) - \left[\frac{\tau^2}{1+n}+\frac{B}{1+n}-G-(1+f'(k))d\right] \tag{31}$$

$$f(k)-kf'(k)-c^1-e=(1+f'(k))^{-1}\left[c^2+(n-f'(k))(\frac{\tau^2}{1+n}+\frac{B}{1+n}-G-(1+f'(k))d)\right] \tag{32}$$

Outside the steady state, the fiscal policy parameters τ^2, d and e can be governed by any rules that are consistent with convergence to the steady state. We consider three policy experiments: (1) an increase in the debt stock, financial with taxes on the young or on the young and the old; (2) a balanced budget increase in unfunded social security payments to the old financed by higher taxes on the young; (3) an increase in exhaustive public spending financed by a tax on the young. The last experiment isn't concerned with debt neutrality, as exhaustive public spending is varied, but is interesting in its own right.

2.4 Steady state comparative statics of debt neutrality

From equations (31) and (32), note that, holding k constant, $\partial c^1/\partial d=-(1+f')$ and $\partial c^2/\partial d=(1+f')(1+n)$. Again therefore, $\partial c^2/\partial c^1=-(1+n)$. A larger stock of public debt financed with taxes on the young (with τ^1 increasing if $f'>n$ and decreasing if $f'<n$) acts just like a reduction in benefits (when B>0) or an increase in child-to-parent gifts (when G>0). In general, it shifts the stationary competitive consumption possibility locus in Figure 3 from $A_3\Omega_2\Omega_1\Omega_4A_2$ up and to the left to $A_3\Omega_2'\Omega_1\Omega_4'A_2$.

A larger stock of public debt financed in steady state with higher taxes on the old, has, at given k, the following effects on consumption while young and while old on the locus: $\partial c^1/\partial d=-(1+n)$; $\partial c^2/\partial d=(1+n)^2$, so again $\partial c^2/\partial c^1=-(1+n)$.

A balanced budget increase in τ_1 and reduction in τ_2, i.e. an increase in the scale of an unfunded social security retirement scheme has the following effect on the locus at given k: $-\partial c^1/\partial \tau^2=-1/(1+n)$ and $-\partial c^2/\partial \tau^2=1$. Like a larger stock of debt, it therefore shifts the locus to the North-West.

At given k, an increase in public consumption financed with taxes on the young simply shifts the consumption possibility locus to the left one-for-one: $\partial c^1/\partial e=-1$ and $\partial c^2/\partial e=0$.

The following results are immediately apparent. For public debt increases financed by taxing the young, propositions (5a,b,c) hold.

Proposition 5a:

When the bequest motive is operative ($B>0$ and $r>n$) a larger stock of public debt financed with higher taxes on the young will be offset by larger bequests; $\Delta B=((1+r)/(1+n))\Delta d$; c^1, c^2, and k will be unaffected. A smaller stock of public debt financed with lower taxes on the young will be offset by smaller bequests as long as $-((1+r)/(1+n))\Delta d$ does not exceed the initial bequest and the $B\geq 0$ constraint does not become binding.

Proposition 5b:

When the gift motive is operative ($G>0$ and $r<n$), a larger stock of public debt "financed" with higher transfer payments to the young[13], will be offset by reduced child-to-parent gifts as long as $(1+r)\Delta d$ does not exceed the initial child-to-parent gift and the $G\geq 0$ constraint does not become binding; $\Delta G=-(1+r)\Delta d$. A smaller stock of public debt financed with higher taxes on the young will be offset by increased child-to-parent gifts.

If neither the bequest motive nor the gift motive is operative ($B=0$ and $G=0$), if we are in the interior of the no gift and no bequest region initially, in the new steady state and during the adjustment process, consumption is the same as in the Diamond (1965) model and can be written as:

$$c_t^1 = c^1(w_t-\tau_t^1, \tau_t^2, r_{t+1}) \tag{33a}$$

$$c_t^2 = (1+r_{t+1})(w_t-c_t^1-\tau_t^1)-\tau_t^2 \tag{33b}$$

Proposition 5c: (Diamond, 1965)

When neither the bequest motive nor the gift motive is operative, the long run effect of a higher public debt stock of (financed with taxes on the young) on the capital-labour ratio is given by

$$\frac{\partial k}{\partial d} = \frac{[1+c_w^1 n+(1-c_w^1)r](1+n)^{-1}}{(c_w^1-1)f''(k+d)-(1+n+f''c_r^1)} \tag{34}$$

If the model is locally stable when d is kept constant throughout, with $\tau_t^{'}$ varying endogenously to keep the budget balanced, if both goods are normal ($0<c_w^1<1$) and if a higher interest rate does not raise consumption in period 1 ($c_r^1\leq 0$), then the denominator of (34) is negative and if $r,n>0$ $\partial k/\partial d<0$: public debt crowds out private capital.

For public debt increases financed by taxing the old, propositions (6a,b,c) hold.

Proposition 6a:

When the bequest motive is operative ($B>0$ and $r>n$) a larger stock of public debt financed with higher taxes on the old will be offset by larger bequests $\Delta B=(1+n)^2\Delta d$. A smaller stock of public debt "financed" with lower taxes on the old will be offset by smaller bequests as long as $-(1+n)^2\Delta d$

does not exceed the initial bequest.

Proposition 6b:

When the child-to-parent gift motive is operative ($G>0$ and $r<n$) a larger stock of public debt financed with higher transfer payments to the old, will be offset by reduced child-to-parent gifts; $\Delta G=-(1+n)\Delta d$ as long as $(1+n)\Delta d$ does not exceed the initial child-to-parent gift.

A smaller stock of public debt financed with higher taxes on the old will be offset by increased child-to-parent gifts.

Proposition 6c:

When neither the bequest motive nor the gift motive are operative, the effect on the long-run capital-labour ratio of an increase in public debt financed with taxes on the old is given by:

$$\frac{\partial k}{\partial d} = \frac{(1+n)(1+c_w^1 n+(1-c_w^1)r)(1+r)^{-1}}{\left[(c_w^1-1)k+\frac{c_w^1}{1+r}(1+n)d\right]f''-(1+n+c_r^1 f'')} \qquad (35)^{14}$$

local stability when d is constant implies

$$\left|\left[(c_w^1-1)k+\frac{c_w^1}{1+r}(1+n)d\right]f''(1+n+c_r^1 f'')^{-1}\right| <1$$

Even with $0<c_w^1<1$ and $c_r^1\leq 0$, this does not suffice to ensure that the denominator of (35) is negative as $(c_w^1-1)k+(c_w^1/((1+r)^{-1}(1+n)d))$ could be positive. It is clear that the smaller c_w^1 and the smaller d, the more likely it is that $\partial k/\partial d<0$ but the opposite outcome cannot be ruled out in the present case.

When equal taxes are levied on the old and the young ($\tau^1=\tau^2$) a higher level of public debt when the bequest motive is operative leads to an increase in bequests given by $\Delta B=((2+r)/(2+n))\Delta d$. When the gift motive is operative gifts are cut (if they are large enough initially) by $\Delta G=-(1+n)(2+r)/(2+n)\Delta d$. When neither altruistic motive is operative, crowding out of private capital by public debt again follows under conditions that are more restrictive than when only the young were taxed and less restrictive than when only the old were taxed.

An increase in taxes on the young (τ^1) with τ^2 lowered to maintain budget balance continuously acts exactly like an increase in public debt with τ^1 endogenously maintaining budget balance. When the bequest motive is operative, bequests are increased by $\Delta B=(1+n)\Delta\tau_1$. When the gift motive is operative, gifts are cut (assuming they are sufficiently large initially) by $\Delta G=-\Delta\tau_1$. When neither motive is operative, the effect of the increase in the scale of the unfunded social security retirement scheme is (assuming for simplicity that $d=0$)

$$\frac{\partial k}{\partial \tau_1} = \frac{[1+c_w^1 n+(1-c_w^1)r](1+r)^{-1}}{(c_w^1-1)kf''-[1+n+c_r^1 f'']} \qquad (36)$$

Again local stability, a non-positive effect of r on c^1, normality of c^1 and $r,n>0$ are sufficient to guarantee a lower capital-labour ratio as a result of higher social security taxes and retirement benefits.

2.5 <u>Debt neutrality outside the steady state</u>

It is easily checked that all the neutrality propositions (such as Propositions (5a,b), (6a,b)) that apply to stationary equilibria extend to non-steady state responses as long as the policy action or other exogenous shock does not alter the "regimes" (the gift or bequest constraints that are binding and which may vary from generation to generation), for current and future generations. Consider an unexpected immediate permanent change in d (financed with an increase in τ^1) in period t. The intertemporal consistency of child-to-parent gift and bequest behaviour (see Burbridge

(1983) and Buiter and Carmichael (1984)) rules out the possibility of any two generations simultaneously wishing to make voluntary transfers to each other, i.e. $G_t>0 \Rightarrow B_{t-1}=0$; $B_{t-1}>0 \Rightarrow G_t=0$; $B_t>0 \Rightarrow G_{t+1}=0$ and $G_{t+1}>0 \Rightarrow B_t=0$. Any given generation, t say, may of course wish both to make a gift to its parent and to leave a bequest to its children if, absent the gift and bequest, the welfare of parent and children would be very different from its own (reflecting say differences in endowments, taxes or factor prices). That is, $G_t>0$ may be consistent with $B_t>0$. In a stationary equilibrium this is ruled out; equations (12) and (13a) (or (13b) or (13c)) cannot hold simultaneously.

When in any given period t, the gift and bequest motives of generation t, the bequest motive of generation t-1 and the gift motive of generation t+1 are non-operative (before and after a policy change or shock) the dynamic analysis of the Diamond (1965) model is applicable for that period. E.g. when d is raised in period t (financed with a tax on the young), the response of the capital stock is given by

$$\frac{\partial k_{t+1}}{\partial d} = \frac{[1+c_w^1 n+(1-c_w^1)r](1+n)^{-1}}{-(1+n+c_r^1 f'')} .$$

This is negative if $0<c_w^1<1$, $c_r^1 \leq 0$ and $r,n>0$ so public debt crowds out the private capital stock in the short run as well.

2.6 Debt neutrality and exhaustive public spending

The presence or absence of debt neutrality has implications for the analysis of fiscal policy experiments other than those involving public debt and lump-sum taxes.

In steady state, when the bequest motive is operative, the successive generations of consumers are in many respects equivalent to the standard model of a single representative infinite-lived consumer with an exogenous pure rate of time preference who discounts his future utility of consumption. One of the key similarities is that the long-run real interest rate is fixed by policy-invariant parameters: $1+r=(1+n)(1/(1+\pi))^{-1}1/u$ in the OLG model with an operative bequest motive; $1+r=(1+\delta^1)(1/(+\pi))^{-\alpha}$ in the representative agent model with a pure rate of time preference δ^1.

In steady state the gift motive is operative, the aggregate behaviour of the successive generations is equivalent to that of a single representative infinite-lived consumer with an exogenous rate of negative time preference, i.e. one who ***discounts*** his past utility of consumption. For the positive analysis of changes in exhaustive public spending it is immaterial that with G>0 we have the equivalent of standard representative consumer standing on his head. All that matters is that the real interest rate is again fixed by policy-invariant parameters when sibling gift behaviour is operative, i.e. :

$$1+r = (1+n)\left[\frac{1}{1+\pi}\right]^{-\alpha}\frac{1}{\lambda(1+\frac{n}{1+n}\frac{\mu}{\lambda})}$$

When neither the gift motive nor the bequest motive are operative, a higher level of exhaustive public spending financed, say, with taxes on the young, will lead to a lower long-run capital stock and a higher real interest rate if the model is locally stable.

$$\frac{\partial k}{\partial e} = \frac{1-c_w^1}{(c_w^1-1)f''k-(1+n+c_r^1f'')} < 0 \qquad \text{if } 0<c_w^1<1 \text{ and } c_r^1\leq 0 \qquad (37)^{15}$$

c^1 will fall (since w is down, τ^1 is up and r is up) but the effect on c^2 is ambiguous[16] because of the increase in r.

When either the gift or the bequest motive is operative, changes in exhaustive government spending that leave the gift or bequest motive operative do not alter the long run real interest rate and capital-labour ratio.

In Figure 4, higher exhaustive public spending financed with taxes on the young shifts the stationary competitive consumption possibility locus horizontally to the left by the amount of the increase in e and in τ_1. The old locus is $A_1A_2A_3A_4$. The new locus is $A_1'A_2'A_3'A_4'$. If the initial equilibrium has positive bequests (as at Ω_1 on the private budget constraint B_1B_1) the corresponding point on the new locus with the same bequest is Ω_2 on the private budget constraint B_2B_2. If consumption in both periods is a normal good, Ω_2 cannot be an equilibrium as at an unchanged intertemporal relative price, the lower after-tax income is reflected only in lower period 1 consumption. Bequests will increase and move the private budget constraint to the right. The new equilibrium is at a position such as Ω_3 on the private budget constraint B_3B_3 with both c^1 and c^2 reduced and $u_1(c^1,c^2)=u_2(c^1,c^2)(1+r)$ in Ω_1 and Ω_3.

When the gift motive is operative as at Ω_1' on the private budget line $B_1'B_1'$, higher exhaustive public spending will, following the same reasoning, lead to a new equilibrium such as Ω_3' on the private budget line $B_3'B_3'$ with both c^1 and c^2 lower than at Ω_1'. Child-to-parent gifts will be lower in Ω_3' than in Ω_1'.

When neither the gift motive nor the bequest motive is operative as at Ω_1'' on $B_1''B_1''$, the new long run equilibrium following an increase in exhaustive public spending has a higher real interest rate. Ω_3'' on $B_3''B_3''$ is an example of such an equilibrium.

3. The irrelevance of finite horizons and uncertain lifetimes for debt neutrality

To evaluate the role of finite horizons and uncertain lifetimes for debt neutrality, the OLG model of the previous Section is modified in two ways. First, gift and bequest motives are omitted ($(1+\delta)^{-1}=(1+\rho)^{-1}=0$). Gift and bequest motives could be added to the model to be considered, in which case the analysis that follows can be interpreted as applying only to those equilibria in which gift or bequest motives are non-operative. Second, instead of living for an exogenously given number of periods, N, with certain death at the end of the Nth period, each agent currently alive is given an age- and time-independent probability γ of surviving to the next period. We consider the range $0<\gamma\leq 1$ so the infinite-lived consumer ($\gamma=1$) in included in our specification. This OLG model was developed by Blanchard (1985) from a model of consumer behaviour with uncertain lifetimes due to Yaari (1965). The individual consumer's discrete time version is from Frenkel and Razin (1986).

3.1 The individual consumer

Each individual consumer born at time s and alive at time $t\geq s$ maximises the time-additive objective functional given in equation (38). All consumers of all ages have identical objective functionals.

$$W(t-s,t) = E_t \sum_{i=t}^{\infty} \left[\frac{1}{1+\theta}\right]^{i-t} u(\bar{c}(t-s,i)) \qquad s\leq t \tag{38}$$

$\theta>0$ is the pure rate of time preference. E_t is the expectation operator conditional on information at time t.

Assuming that the uncertainty concerning time of death is the only form of uncertainty, (38) can be rewitten as

$$W(s,t) = \sum_{i=t}^{\infty} \left[\frac{1}{1+\theta}\right]^{i-t} \gamma^{i-t} u(\bar{c}(t-s,i)) \tag{38'}$$

The single period utility function is of the constant elasticity of marginal utility class:

$$u(\bar{c}) = \begin{cases} \frac{1}{1-\alpha}\bar{c}^{1-\alpha} & \alpha>0 \; (\alpha\neq 1) \\ \ln \bar{c} & (\alpha=1) \end{cases} \tag{39}$$

The individual's budget identity is given by:

$$\bar{a}(t-s,t+1) \equiv (1+r(t+1))\gamma^{-1}[\bar{a}(t-s,t)+\bar{w}(t-s,t)-\bar{\tau}(t-s,t)-\bar{c}(t-s,t)] \quad s\leq t \tag{40}$$

Equation (40) reflects the assumption of the existence of an efficient competitive annuity or life insurance market. r is the riskfree single-period real rate of interest. Each agent alive today contracts with a life insurance company to receive a gross rate of return $(1+r(t+1))\psi$ on his non-human wealth $\bar{a}$ as long as he survives. If he dies all of $\bar{a}$ accrues to the insurance company. If $\bar{a}$ is negative, the consumer pays a gross premium rate ψ as long as he survives with the debt cancelled when he dies. The insurance market is competitive with risk-neutral firms and free entry. Each age cohort is assumed to consist of a large number of identical agents. γ is not only the individual's probability of surviving for one period but also the fraction of each cohort (and therefore of the total population) which survives each period. It follows that $\psi = 1/\gamma$ $\bar{w}$ is the individual's wage and $\bar{\tau}$ the lump-sum tax he pays.

We define the market present value factor R as:

$R(t) = \prod_{i=0}^{t} (1+r(i))$ with $R(0) = 1$. Note that $1+r(t+1) = \frac{R(t+1)}{R(t)}$

Solving (40) forward in time and imposing the transversality condition in (41) we obtain the present value budget constraint (42)

$$\lim_{j\to\infty} \gamma^{j-t} \frac{R(t)}{R(j)} \bar{a}(t-s,j) = 0 \tag{41}$$

$$\bar{a}(t-s,t) + \bar{h}(t-s,t) = \sum_{i=t}^{\infty} \bar{c}(t-s,i)\, \gamma^{i-t} \frac{R(t)}{R(i)} \tag{42}$$

Human capital, $\bar{h}$, the present value of after-tax labour income is given by

$$\bar{h}(t-s,t) = \sum_{i=t}^{\infty} [\bar{w}(t-s,i)-\bar{\tau}(t-s,i)]\gamma^{i-t} \frac{R(t)}{R(i)} \tag{43}$$

or, in difference form

$$\bar{h}(t-s,t+1) = \frac{R(t+1)}{R(t)} \gamma^{-1} \{\bar{h}(t-s,t)-[\bar{w}(t-s,t)-\bar{\tau}(t-s,t)]\} \tag{43'}$$

Maximising (38') with respect to current and future choices of consumption yields

$$\bar{c}(t-s,t) = \eta(t)[\bar{a}(t-s,t)+\bar{h}(t-s,t)] \tag{44}$$

$$\eta(t) = \begin{cases} \left[\sum_{i=t}^{\infty} \left[\frac{R(t)}{R(i)} \gamma^{i-t}\right]^{\frac{\alpha-1}{\alpha}} \left[\frac{\gamma}{1+\theta}\right]^{(i-t)\frac{1}{\alpha}}\right]^{-1} & \text{if } \alpha \neq 1 \\ 1-\frac{\gamma}{1+\theta} & \text{if } \alpha = 1 \end{cases} \tag{45}$$

3.2 Aggregation

Without loss of generality, let population size at time 0 be $L(0) = 1$. To every person alive in period t, $\beta > 0$ identical children are born. I shall refer to β as the birth-rate. The size of the surviving cohort at time t which was born at time t-s, $s \geq 0$ is $\beta L(t-s)\gamma^s = \beta(1+\beta)^{t-s}\gamma^t$ as $L(t+1) = (1+\beta)\gamma L(t)$.

Note that total population can, if $\beta > 0$, be expressed as the sum of all survivors of all past cohorts:

$$L(t) = (1+\beta)^t\gamma^t = \begin{cases} \sum_{s=1}^{\infty} \beta(1-\beta)^{t-s}\gamma^t & \text{if } \beta>0 \\ \gamma^t & \text{if } \beta=0 \end{cases}$$

Corresponding to any individual agent's stock or flow variable $\bar{v}(t-s,t)$ we define the population aggregate (t) by:

$$V(t) \equiv \begin{cases} \sum_{s=1}^{\infty} \beta(1+\beta)^{t-s}\gamma^t\bar{v}(t-s,t) & \text{if } \beta>0 \\ \bar{v}(0,t)\gamma^t & \text{if } \beta=0 \end{cases} \tag{46}$$ [17]

Each surviving agent, reglardless of age, earns the same wage income and pays the same taxes i.e.

$$\bar{w}(t-s,t) = \bar{w}(t) \qquad s \geq 0$$

$$\bar{\tau}(t-s,t) = \bar{\tau}(t) \qquad s \geq 0$$

It follows that each surviving agent has the same human capital.

$$\bar{h}(t-s,t) = \bar{h}(t) \qquad s \geq 0.$$

Noting that $\bar{a}(t,t+1)=0$ (people are born with just their human capital) and using the notational convention given in (46) aggregate consumption is, by direct computation, determined by equations (47a, b, c)

$$C(t) = \eta(t)[A(t)+H(t)] \tag{47a}$$

$$A(t+1) = (1+r(t+1))[A(t)+W(t)-T(t)-C(t)] \tag{47b}$$

$$H(t) = \sum_{i=t}^{\infty} [W(i)-T(i)] \left[\frac{1}{1+\beta}\right]^{i-t} \frac{R(t)}{R(i)} \tag{47c}$$

$$H(t+1) = (1+\beta)(1+r(t+1))[H(t)-W(t)+T(t)] \tag{47c'}$$

Comparing (40) and (47b) we notice that the intra-private sector payments associated with the insurance scheme from the point of view of the individual cancel out when the behaviour of aggregate non-human wealth is concerned: in the aggregate, non-financial wealth earns the riskless rate. Comparing (43) or (43') and (47c) or (47c') we notice that $H(t)$ is the human capital of those currently alive. It excludes the present discounted value of the future after-tax labour income of the "new entrants", i.e. those born in period t and later. The practical consequence is that the effective single period discount factors applied by those currently alive to the ***aggregate*** future expected after-tax labour income stream (which includes the disposable wage income of the "new entrants"), is raised from $1/(1+r(t+i))$ to $1/[1+r(t+i)][1+\beta]$.

Let there be labour-augmenting productivity growth at a constant proportional rate π. The level of productivity at $t=0$ equals unity by choice of units. For each population aggregate stock or flow variable V the corresponding quantity "per unit of labour measured in efficiency units" v is defined as

$$v(t) = V(t)[(1+\pi)(1+\beta)\gamma]^{-t} \tag{48}$$

Consumption per unit of efficiency labour is therefore governed by:

$$c(t) = \eta(t)[a(t)+h(t)] \tag{49a}$$

$$a(t+1) = \frac{(1+r(t+1))}{(1+\pi)(1+\beta)\gamma}[a(t)+w(t)-\tau(t)-c(t)] \tag{49b}$$

$$h(t) = \sum_{i=t}^{\infty} [w(i)-\tau(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t} \tag{49c}$$

or

$$h(t) = \frac{(1+r(t+1))}{(1+\pi)\gamma}[h(t)-w(t)+\tau(t)] \tag{49c'}$$

Note that aggregate consumption behaviour is the same as the behaviour of a representative consumer who discounts his future expected labour income using a discount rate in excess of the rate of return on his non-human assets, the excess being equal to the birth rate, β.

3.3 The government

As before, E(t) denotes total government consumption spending, T_t total lump-sum taxes net of transfers, and D_t the stock of public debt outstanding at the end of period t-1. The government budget identity is:

$$D(t+1) = [1+r(t+1)][D(t)+E(t)-T(t)]$$

Letting d, e and τ denote public debt, public spending and taxes per unit of efficiency labour we have

$$d(t+1) = \frac{(1+r(t+1))}{(1+\pi)(1+\beta)\gamma}[d(t)+e(t)-\tau(t)] \tag{50}$$

With the terminal condition

$$\lim_{v\to\infty} d(t+v)\frac{R(t)}{R(v)}[(1+\pi)(1+\beta)\gamma]^{v-t} = 0,$$

the budget intensity generates the government 's present value budget constraint or solvency constraint:

$$d(t) = \sum_{i=t}^{\infty} [\tau(i)-e(i)]\frac{R(t)}{R(i)}[(1+\pi)(1+\beta)\gamma]^{i-t} \qquad (51)$$

The government's future tax revenue stream consists of taxes levied not only on the survivors among those currently alive, but also on the survivors among cohorts yet to be born.

Debt neutrality in the Yaari-Blanchard-Weil model

There will be no debt neutrality if, holding constant the path of exhaustive public spending, varying the future time path of lump sum taxes (subject to the constraint that the government remains solvent) will alter private consumption at given current and future wages and interest rates and given the stock of private non-human assets other than government debt. Equivalently (subject to the same conditions) if a change in the initial stock of public debt (with future changes in lump sum taxes to satisfy (51)) alters private consumption, there is no debt neutrality.

For simplicity, let non-human wealth consist of the real capital stock and government debt, k denotes the capital stock per unit of efficiency labour.

$$a \equiv k+d \qquad (52)$$

Substitute for a(t) and h(t) in the consumption function (49a) using (52) and (49b) respectively. Then add and subtract the term

$$\eta(t)\sum_{i=t}^{\infty} e(i)\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t}$$

and rearrange.

This yields

$$c(t) = \eta(t)\Big[k(t)+\sum_{i=t}^{\infty}[w(i)-e(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t}\Big]$$

$$+\ \eta(t)\Big[d(t)+\sum_{i=t}^{\infty}[\tau(i)-e(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t}\Big] \qquad (53)$$

The first term on the RHS of (53) represents what consumption would be given debt neutrality. Current consumption is affected by fiscal policy only through current and anticipated future exhaustive public consumption spending

$$\Big[(-\eta(t)\sum_{i=t}^{\infty} e(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t}\Big].$$

The second term represents the influence of debt and lump-sum taxes. It is identically zero if and only if the birth rate is zero, i.e. $\beta=0$, in which case

$$d(t) = \sum_{i=t}^{\infty}[\tau(i)-e(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t},$$

from the government solvency constraint (51).

How does consumption differ when $\eta(t)$, $k(t)$, $R(t)$, $R(i)$, $w(i)$ and $e(i)$ ($i\geq t$) are the same and only debt and current and future lump-sum taxes differ (say $d^{I}(t)$ and $\tau^{I}(i)$, $i\geq t$ in the first case and $d^{II}(t)$ and $\tau^{II}(i)$,

$i>t$ in the second)? It is easily checked that

$$c^{I}(t)-c^{II}(t)=\eta(t)\Big[\sum_{i=t}^{\infty}[\tau^{I}(i)-\tau^{II}(i)]\frac{R(t)}{R(i)}[(1+\pi)\gamma]^{i-t}((1+\beta)^{i-t}-1)\Big] \quad (54)$$

$c^{I}(t)=c^{II}(t)$ for all possible $\tau^{I}(i)$ and $\tau^{II}(i)$ ($i>t$) if and only if the birth rate, β, equals zero. Weil (1985) showed that a positive birth rate was *sufficient* for absence of debt neutrality in OLG models without an operative bequest motive. In Buiter (1988a) I show that in the Yaari-Blanchard-Weil OLG model, a positive birth rate is ***necessary*** for absence of debt neutrality as well. In the original Blanchard model, a constant population was assumed, i.e. $(1+\beta)=\gamma^{-1}$. Only the death rate appeared explicitly in the model, doing "double duty". When the birth rate and death rate are disentangled, it is clear that a positive probability of death in a Yaari-Blanchard model with a zero birth rate does not cause absence of debt neutrality. With $\beta=0$, all surviving agents are identical. Postponing lump-sum taxes therefore does not redistribute income or wealth between heterogeneous consumers. While the probability of surviving to pay the future taxes declines exponentially as γ^{i-t}, $i>t$; the per capita tax burden of the survivors increases exponentially as γ^{t-i}, $i>t$. The two effects cancel each other out exactly.

This suggests the following proposition:

Proposition 7:

In the Yaari-Blanchard-Weil model, a zero birth rate is both necessary and sufficient for debt neutrality.

Corollary 1: When the birth rate is zero, uncertain lifetimes or productivity growth do not generate absence of debt neutrality.

Corollary 2: When the birth rate is positive, infinite individual lifetimes do not generate debt neutrality.

It is easy, once we specify the initial value of the public debt and the behaviour of taxes, to be more precise about the nature of the non-neutrality. In the example discussed earlier, let $d^{I}(t)>d^{II}(t)$. From the government's solvency constraint (51) it is apparent that the higher initial debt in scenario I could be serviced by a strictly higher path of future taxes in scenario I, i.e. $\tau^{I}(i)\geq\tau^{II}(i)$ for all $i\geq t$ and $\tau^{I}(i)>\tau^{II}(i)$ for some $i\geq t$. From (54) this implies that $c^{I}(t)>c^{II}(t)$: higher government debt "crowds out" private saving.

Note that in the OLG model of Section 2, a zero birth rate gives us a representative consumer model with a finite (two period horizon). Debt neutrality obviously prevails again.

4. Debt neutrality and capital market imperfections, heterogeneous survival rates and discount rates

4.1 The complete absence of life insurance or annuity markets

Now consider the case where there are no markets for insuring against the risks associated with an unexpected death. The individual's budget constraint (40) is affected in two ways. First he now earns the riskless rate $1+r(t+1)$ rather than $[1+r(t+1)]\gamma^{-1}$ on his savings. Second, he receives (or pays) an amount $\bar{\lambda}(t-s,t)$, reflecting the fact that other consumers will be dying (unexpectedly) in debt or with positive non-human assets. Without bequest or gift motive, these "involuntary bequests" are assumed to accrue to the state and to be returned by it to the surviving agents. For the moment all that matters is that the individual agent takes $\bar{\lambda}(t-s,t)$ as exogenous. Specifically, he does not see it as an additional return on his non-human assets $\bar{a}(t-s,t)$.

The individual consumer's budget identity now is:

$$\bar{a}(t-s,t+1) = (1+r(t+1))[\bar{a}(t-s,t)+\bar{w}(t-s,t)-\bar{\tau}(t-s,t)$$

$$-\bar{\lambda}(t-s,t)-\bar{c}(t-s,t)] \tag{55}$$

$\bar{\tau}(t-s,t)$ is lump-sum taxes net of transfers excluding payments or receipts associated with involuntary bequests. The terminal condition is

$$\lim_{j\to\infty} \frac{R(t)}{R(j)} \bar{a}(t-s,j) = 0$$

Optimal consumption is given by:

$$\bar{c}(t-s,t) = \hat{\eta}(t)[\bar{a}(t-s,t)+\bar{h}(t-s,t)] \tag{56a}$$

where

$$\bar{h}(t-s,t+1) = (1+r(t+1))[\bar{h}(t-s,t)-\bar{w}(t-s,t)+\bar{\tau}(t-s,t)-\bar{\lambda}(t-s,t)] \tag{56b}$$

or

$$\bar{h}(t-s,t) = \sum_{i=t}^{\infty} [\bar{w}(t-s,i)-\bar{\tau}(t-s,t)+\bar{\lambda}(t-s,t)]\frac{R(t)}{R(i)} \tag{56b'}$$

$$\hat{\eta}(t) \begin{cases} \left[\sum_{i=t}^{\infty} \left[\frac{R(t)}{R(i)}\right]^{\frac{\alpha-1}{\alpha}} \left[\frac{\gamma}{1+\theta}\right]^{(i-t)\frac{1}{\alpha}} \right]^{-1} & \alpha\neq 1 \\ 1-\frac{\gamma}{1+\theta} & \text{if } \alpha=1 \end{cases} \tag{56c}$$

If we assume that not only $\bar{w}$ and $\bar{\tau}$ but also $\bar{\lambda}$ are independent of age we have the following aggregate consumption function

$$C(t) = \hat{\eta}(t)[A(t)+H(t)]$$

where

$$A(t+1) = [1+r(t+1)]\gamma[A(t)+W(t)-T(t)+\Lambda(t)-C(t)]$$

and

$$H(t+1) = [1+r(t+1)]\gamma(1+\beta)[H(t)-W(t)+T(t)-\Lambda(t)]$$

Per unit of efficiency labour, aggregate consumption is given by

$$c(t) = \hat{\eta}(t)[a(t)+h(t)] \tag{57a}$$

$$a(t+1) = \frac{(1+r(t+1))}{(1+\pi)(1+\beta)}[a(t)+w(t)-\tau(t)+\lambda(t)-c(t)] \tag{57b}$$

$$h(t+1) = \frac{(1+r(t+1))}{(1+\pi)}[h(t)-w(t)+\tau(t)-\lambda(t)] \tag{57c}$$

$\lambda(t)$ is the sum of the involuntary bequests per consumer measured in efficiency units. It follows that:

$$\lambda(t) = \left[\frac{1-\gamma}{\gamma}\right][a(t)+w(t)-\tau(t)-c(t)] \tag{58}$$

Note that we have assumed that $\bar{\lambda}(s,t)=\lambda(t)(1+\pi)^{t}$ for all s, i.e. each consumer gets the same share of the aggregate involuntary bequests.

Aggregate non-human wealth evolves in the same way as with perfect life insurance markets

$$a(t+1) = \frac{(1+r(t+1))}{(1+\pi)(1+\beta)\gamma}[a(t)+w(t)-\tau(t)-c(t)] \tag{59a}$$

$$h(t+1) = \frac{(1+r(t+1))}{(1+\pi)}\left[h(t)-\frac{1}{\gamma}[w(t)-\tau(t)]+\left[\frac{\gamma-1}{\gamma}\right][a(t)-c(t)]\right] \tag{59b}$$

While optimal individual consumption satisfies the simple autoregressive process in (60), aggregate consumption cannot be written as a function of lagged aggregate consumption only, but depends on non-human wealth as well, as shown in (61)

$$\bar{c}(t-s,t+1) = \begin{cases} \dfrac{[1+r(t+1)-\hat{\eta}(t)]}{\hat{\eta}(t)}\hat{\eta}(t+1)\bar{c}(t-s,t) & \\ \left(r(t+1)+\dfrac{\gamma}{1+\theta}\right)\bar{c}(t-s,t) & \text{if } \alpha=1 \end{cases} \qquad (60)$$ [18]

$$c(t+1) = \begin{cases} \left[\dfrac{1}{1+\pi}\right]\dfrac{[1+r(t+1)-\hat{\eta}(t)]}{\hat{\eta}(t)}\hat{\eta}(t+1)c(t)-\hat{\eta}(t+1)\beta a(t+1) & \\ \left[\dfrac{1}{1+\pi}\right]\left[r(t+1)+\dfrac{\gamma}{1+\theta}\right]c(t)-\left[\dfrac{\theta+1-\gamma}{1+\theta}\right]\beta a(t+1) & \text{if } \alpha=1 \end{cases} \qquad (61)$$

There clearly is absence of debt neutrality if $\beta>0$, as the authorities can, by varying lump-sum taxation over time, influence $a(t+1)$. Solving (59a) forward in time, imposing the usual terminal condition,[19] substituting in the government's solvency constraint (51) and using the definition $a(t)\equiv k(t)+b(t)$ we obtain (62)

$$k(t)+\sum_{i=t}^{\infty}\frac{R(t)}{R(i)}[1+\pi)\gamma(1+\beta)]^{i-t}[w(i)-e(i)] = \sum_{i=t}^{\infty}\frac{R(t)}{R(i)}[(1+\pi)\gamma(1+\beta)]^{i-t}c(i) \qquad (62)$$

When the birth rate equals zero ($\beta=0$), the "Euler equation" for aggregate consumption (equation (61)) is independent of government debt and lump-sum taxes. So is the aggregate private sector solvency constraint (62).

This means there is debt neutrality when $\beta=0$, even if there are uncertain lifetimes ($\gamma<1$).[20] The complete absence of annuities or life insurance markets does not in itself constitute another sufficient condition for absence of debt neutrality. When all surviving agents are identical (except in non-human wealth) and when the involuntary bequests are distributed in lump-sum fashion to the survivors,[21] the non-existence of life insurance markets does not mean that postponing lump-sum taxation permits the government to redistribute income between heterogeneous agents. If every agent alive today or tomorrow is affected in the same manner by a capital market imperfection, the imperfection does not generate absence of debt neutrality. We summarise this as Proposition 8.

Proposition 8:

In the Yaari-Blanchard-Weil model with uncertain lifetimes and no life insurance markets, a zero birth rate remains necessary and sufficient for debt neutrality if involuntary bequests are distributed in a lump-sum manner among the survivors.

Corollary: Under the conditions of Proposition 8, when the birth rate is zero, the absence of life insurance markets does not cause absence of debt neutrality.

4.2 Heterogeneous survival rates, time preference rates and elasticities of marginal utility

Since a zero birth rate is necessary and sufficient for debt neutrality in the Yaari-Blanchard-Weil model with identical agents (except for age), we only consider the zero birth rate ($\beta=0$) case when other kinds of heterogeneity are introduced. For simplicity (and because nothing hinges on it) productivity growth, π, will also be set equal to zero.

There are two kinds of consumers, labelled with subscripts 1 and 2 who may have different survival rates, γ_1 and γ_2, different time preference rates θ_1 and θ_2 and different elasticities of marginal utility α_1 and α_2. The number of consumer of type i in period t is γ_i^t, $i = 1,2$. Total population is $\gamma_1^t + \gamma_2^t$.

Perfect life insurance markets are again assumed. A consumer of type j earns a gross rate of return $(1+r(t+1))\gamma_j^{-1}$ on his non-human assets, i.e. the insurance company can identify the survival probability of each consumer. Each consumer of both types earns the same wage w and pays the same lump sum tax τ.

Aggregate consumption is given by equation (63). d_j is per capita real capital government debt for consumers of type j, k_j is per capita real capital for consumers of type j etc.

$$C(t) = \gamma_1^t \eta_1(t) \left[k_1(t) + \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_1^{(i-t)} w(i) \right]$$

$$+ \gamma_2^t \eta_2(t) \left[k_2(t) + \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_2^{i-t} w(i) \right]$$

$$+ \eta_1(t) \gamma_1^t \left[d_1(t) - \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_1^{i-t} \tau(i) \right]$$

$$+ \eta_2(t) \gamma_2^t \left[d_2(t) - \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_2^{i-t} \tau(i) \right] \tag{63}$$

$$\eta_j(t) = \left[\sum_{i=t}^{\infty} \left[\frac{R(t)}{R(i)}\right]^{\left[\frac{\alpha_j - 1}{\alpha_j}\right]} \left[\frac{1}{1+\theta_j}\right]^{(i-t)\frac{1}{\alpha_j}} \gamma_j^{(i-t)} \right]^{-1} \qquad j = 1,2 \tag{64}$$

$$= 1 - \frac{\gamma_j}{1+\theta_j} \qquad \text{if } \alpha_j = 1$$

The government's solvency constraint is:

$$d(t) = \frac{\gamma_1^t d_1(t) + \gamma_2^t d_2(t)}{\gamma_1^t + \gamma_2^t} = \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \frac{(\gamma_1^i + \gamma_2^i)}{(\gamma_1^t + \gamma_2^t)} [\tau(i) - e(i)] \tag{65}$$

Substituting (65) into (63) and rearranging yields:

$$C(t) = \gamma_1^t \eta_1(t) \left[k_1(t) + \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_1^{(i-t)} [w(i) - e(i)] \right]$$

$$+ \gamma_2^t \eta_2(t) \left[k_2(t) + \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_2^{(i-t)} [w(i)-e(i)] \right]$$

$$+ [\eta_2(t)-\eta_1(t)]\gamma_2^t \left[d_2(t) - \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_2^{(i-t)} [\tau(i)-e(i)] \right] \qquad (66)$$

For there to be debt neutrality, the third term on the R.H.S. of (66) should be identically equal to zero. This requires either

$$\eta_2(t) = \eta_1(t) \quad \text{(Identical marginal propensities to consume out of comprehensive wealth)} \qquad (67a)$$

or

$$d_j(t) = \sum_{i=t}^{\infty} \frac{R(t)}{R(i)} \gamma_j^{i-t} [\tau(i)-e(i)] \qquad j=1,2;\ t\geq 0 \ \text{(No redistribution)} \qquad (67b)$$

Note that the "marginal consumption propensities out of comprehensive wealth", η_1 and η_2 will in general be different when $\gamma_1 \neq \gamma_2$, $\theta_1 \neq \theta_2$ or $\alpha_1 \neq \alpha_2$ since, as can be checked easily,

$$\frac{\partial \eta_j(t)}{\partial \gamma_j} < 0; \quad \frac{\partial \eta_j(t)}{\partial \theta_j} > 0 \quad \text{and} \quad \frac{d\eta_j(t)}{\partial \alpha_j} > 0; \quad j = 1,2.$$

A redistribution of comprehensive wealth between the two types of consumers will, if $\eta_1 \neq \eta_2$, affect aggregate consumption. Postponing lump-sum taxation will redistribute wealth when survival probabilities differ unless (67b) holds.

From the government solvency constraint (65) it follows that, without the ability to levy different per capita taxes on the different types of consumers, (67b) will hold only if $\gamma_1 = \gamma_2$. In that case (65) and (67b) can both hold if $d_1(t) = d_2(t)$.

Consider a one-period postponement of one unit of lump-sum taxation per capita in period t. From the government solvency constraint:

$$\Delta\tau(t+1) = -\frac{[\gamma_1^t+\gamma_2^t]}{\gamma_1^{t+1}+\gamma_2^{t+1}}(1+r(t+1))\Delta\tau(t). \tag{68}$$

with $\Delta\tau(t) = -1$

From equation (66), the effect on aggregate consumption is:

$$\Delta C(t) = [\eta_2(t)-\eta_1(t)]\gamma_2^t\gamma_1^t\frac{(\gamma_2-\gamma_1)}{\gamma_2^{t+1}+\gamma_1^{t+1}}\Delta\tau(t) \tag{69}$$

Let $\theta_1 = \theta_2$ and $\alpha_1 = \alpha_2$. It follows that $\gamma_2<\gamma_1$ implies $\eta_2>\eta_1$ and $\gamma_2>\gamma_1$ implies $\eta_2<\eta_1$. Postponing lump-sum taxes ($\Delta\tau(t) = -1$) therefore raises aggregate consumption whenever $\gamma_1\neq\gamma_2$. It redistributes income from those with a low death rate (high probability of survival) to those with a high death rate (low probability of survival). The higher death rate consumers have the higher marginal propensity to consume out of wealth.

This suggests the following proposition.

Proposition 9:

Heterogeneous survival probabilities cause absence of debt neutrality even with perfect life insurance markets. Cet.par. postponing lump-sum taxation will raise aggregate consumption.

It follows immediately from equation (69) that:

Proposition 10:

When survival probabilities are the same, heterogeneous time preference rates or heterogeneous elasticities of marginal utility do not not cause absence of debt neutrality.

The intuition behind Proposition (10) is that while $\theta_1 \neq \theta_2$ or $\alpha_1 \neq \alpha_2$ imply (cet.par.) $\eta_1 \neq \eta_2$, postponing lump-sum taxation does not redistribute income when $\gamma_1 = \gamma_2$.

Finally, a little tedious algebra establishes the following.

Proposition 11:

Propositions (9) and (10) hold also when there is a complete absence of life insurance markets.

It can be shown the Propositions (9) and (10) go through even when the following more realistic insurance market imperfection exists. The insurance companies cannot identify the survival probability of individual agents but know the two possible values of γ_j and their frequency in the population. In a pooling equilibrium they consequently charge the same insurance premium (pay the same annuity rate of return) to all consumers. The competitive gross rate of return on non-human assets is therefore

$$\frac{1+r(t+1)}{\gamma_1\sigma_1+\gamma_2(1-\sigma_1)},$$

where σ_1 is the fraction of consumers with survival probability γ_1. The length of this paper is, however, adequately excessive without working through this example.

5. Conclusion

Heterogeneity and ***redistribution*** are necessary and sufficient for absence of debt neutrality. Capital market imperfections are neither necessary nor sufficient, although differential incidence of capital market imperfections may well, empirically, be an important source of heterogeneity and a further reason why intertemporal redistributions of lump-sum taxes may not be neutral.

The analysis of the consequences of "deficit financing", i.e. of the intertemporal redistribution of lump-sum taxes by the government, requires the abandonment of the representative consumer model if it is not to beg all the important questions. More generally, virtually every important issue in fiscal, financial and monetary policy involves government actions that alter binding constraints faced by heterogeneous consumers, investors, workers or firms. In order to be policy-relevant, the profession will have to invest in macroeconomics without a representative agent.

FOOTNOTES

1 Or by producers. The latter possibility is not considered here.

2 And with age-independent wage income and taxes.

3 The exogenous labour endowment when young is scaled to unity. The labour endowment when old is zero.

4 Each consumer is assumed to have a single parent. Parthogenesis (a-sexual reproduction) is a key simplifying assumption.

5 The analysis goes through also if the consumer cares directly about generations with whom he does not overlap.

6 E.g. $\gamma_{0,0}$ is the weight attached to my own egoistic utility; $\gamma_{0,1}$ is the weight attached to each of my n siblings; $\gamma_{-2,0}$ is the weight attached to my grandmother's egoistic utility $\gamma_{2,1}$ is the weight attached to my sibling's grandchildren etc.

7 $\frac{\partial G_t^j}{\partial c_t^2} = \frac{\partial G_t^j}{\partial B_t} = 0$ are plausible as c_t^2 and B_t are chosen in period $t + 1$.

8 Obvious modifications would be required if there were more than 2 overlapping generations.

9 He only considers the one-sided altruism case with $(1+\rho)^{-1} = 0$.

10 With the Cobb-Douglas production function, as $k \to \infty$ (in the infeasible region beyond A) the slope of the locus tends to -1.

11 U^B is $[1-(1+\rho)^{-1}+(1+\delta)^{-1}]^{-1}$ times the representative egoistic utility function.

12 U^G is $[1-(1+\rho)^{-1}+(1+\delta)^{-1}]^{-1}$ times the representative egoistic utility function.

13 Since $r<n$.

14 Note that $c_w^1 = -(1+r)c^1{}_\tau 2$.

15 For simplicity it is assumed that d = 0.

16 $\frac{dc^2}{de} = \frac{(1-c_w^1)[f''c^2/(1+r)+(1+r)(1+n)]}{(c_w^1-1)f''k-(1+n+c_r^1 f'')}$

17 When the birth rate is zero we assume, without loss of generality that the initial population arrived in one batch out of the blue at t = 0.

18 Note that: $[\hat{\eta}(t+1)]^{-1} = [(\hat{\eta}(t))^{-1}-1][1+r(t+1)]^{\left[\frac{\alpha-1}{\alpha}\right]} \left[\frac{\gamma}{1+\theta}\right]^{-\frac{1}{\alpha}}$.

19 $\lim_{i\to\infty} a(i) \frac{R(t)}{R(i)} [(1+\pi)(1+\beta)\gamma]^{i-t} = 0$.

20 Equations (60) and (62) imply together with the government solvency constraint (51) that, when $\beta=\pi=0$,

$$c(t) = \Omega(t)\left[k(t)+\sum_{i=t}^{\infty}\frac{R(t)}{R(i)}\gamma^{(i-t)}[w(i)-e(i)]\right]$$

$$\Omega(t) = \left[\sum_{i=t}^{\infty}\frac{R(t)}{R(i)}\gamma^{(i-t)}\prod_{\ell=0}^{i-t-1}\left[\frac{1+r(i-\ell)-\hat{\eta}(i-1-\ell)}{\hat{\eta}(i-1-\ell)}\right]\hat{\eta}(i-\ell)\right]^{-1}$$

21 If all individuals have identical $\hat{\eta}$'s (i.e. identical θ's, α's and γ's) as we assume, the involuntary bequests need not even be distributed equally among the survivors for debt neutrality to hold despite the absence of life insurance markets.

FIGURE 1

Stationary competitive consumption possibility locus without gifts and bequests and without government

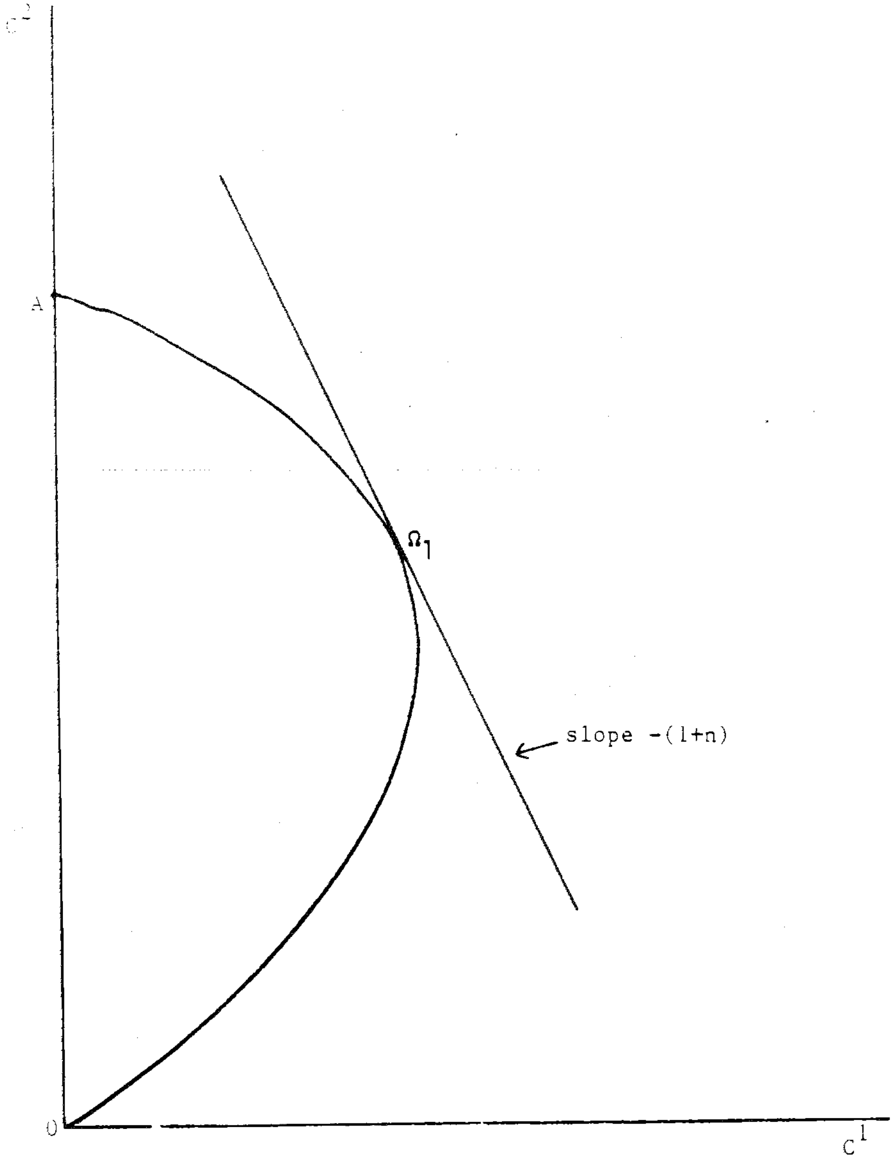


FIGURE 2

Stationary competitive consumption possibility locus with gifts and bequests but without government

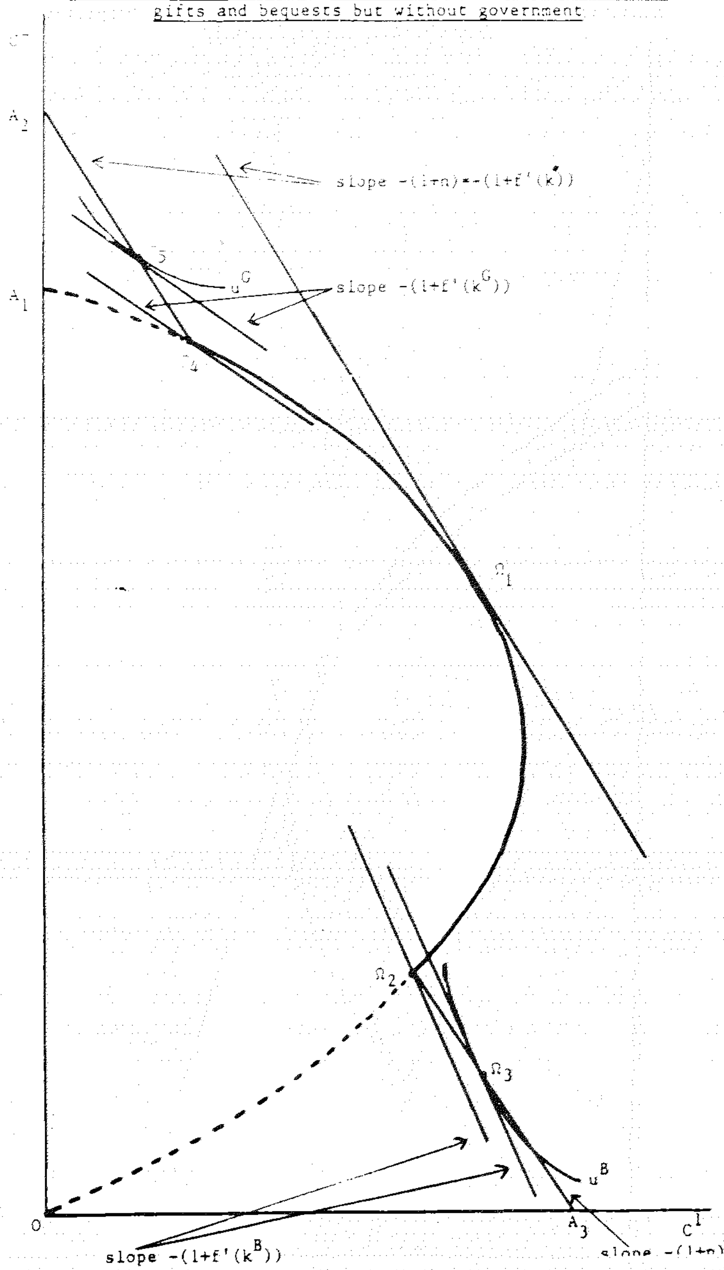


FIGURE 3

The effect of higher public debt on the stationary competitive consumption possibility locus

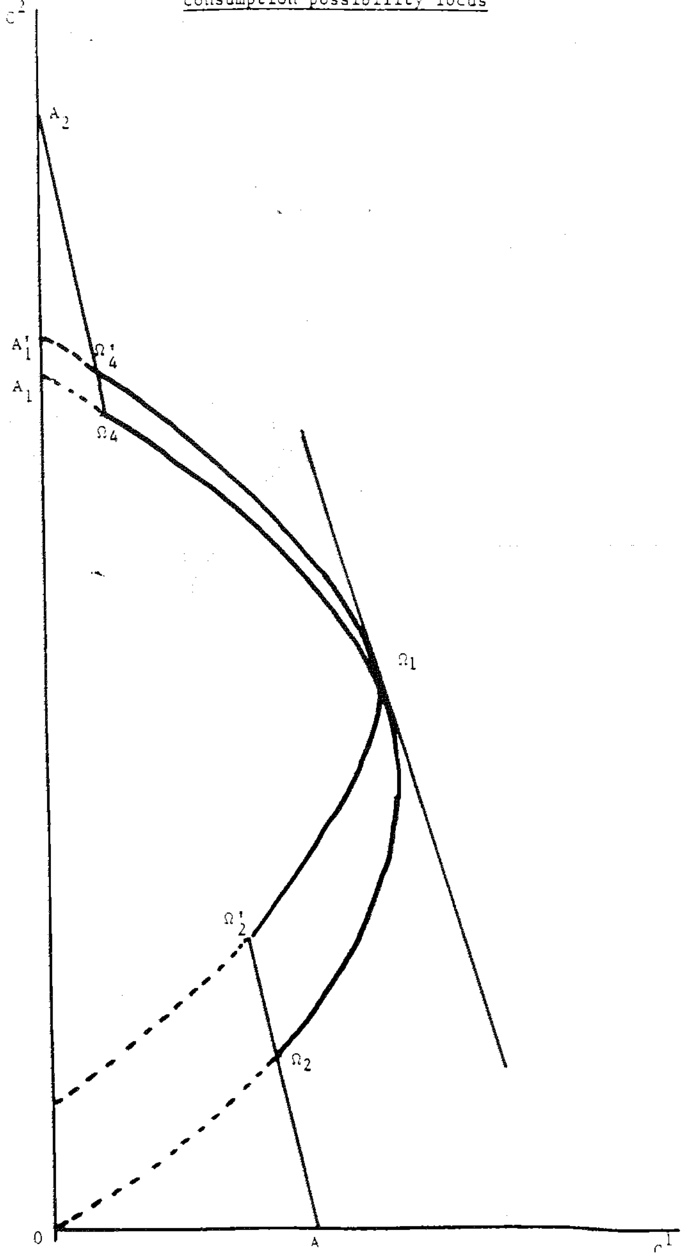
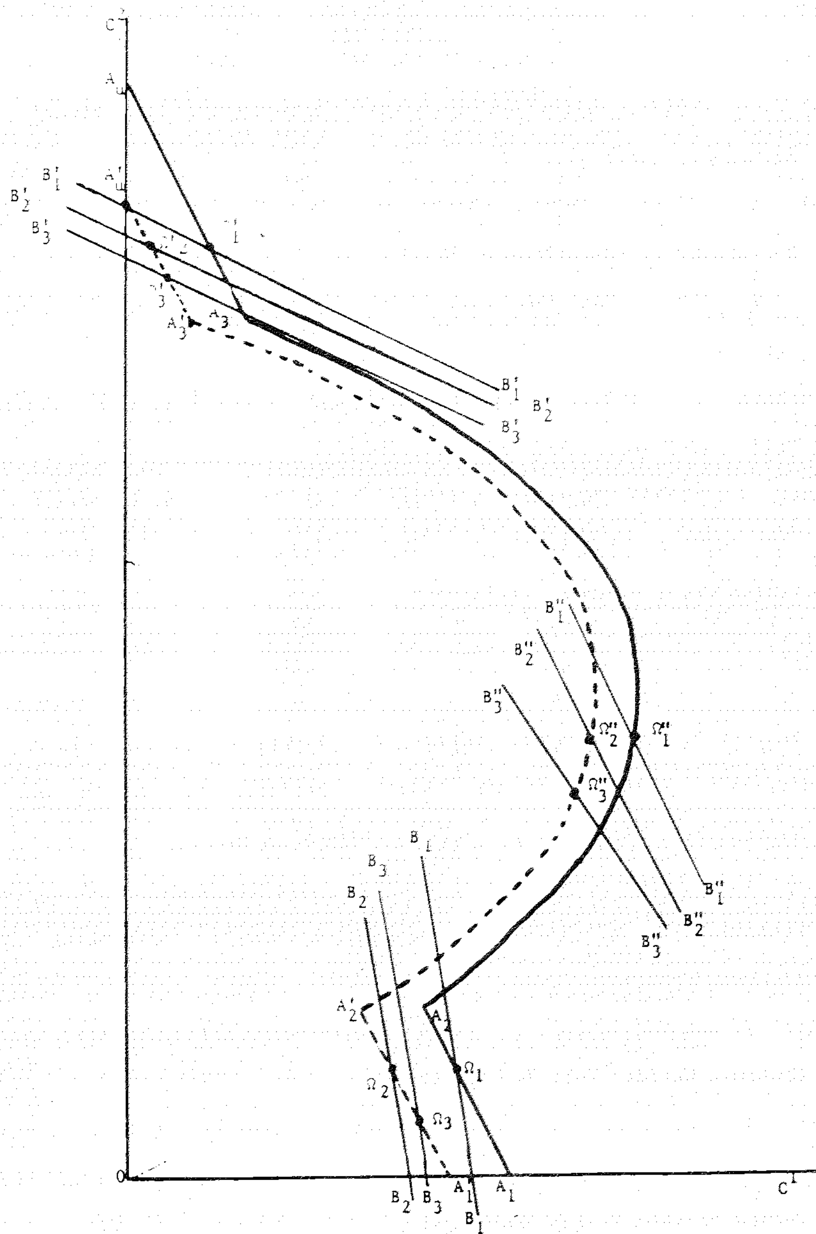


FIGURE 4

An increase in exhaustive public spending financed by taxes on the young



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