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THE TIME-VARIATION OF RISK AND RETURN IN THE FOREIGN EXCHANGE AND STOCK MARKETS

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ABSTRACT

Recent empirical work indicates that, in a variety of financial markets, both conditional expectations and conditional variances of returns are time-varying. The purpose of this paper is to determine whether these joint fluctuations of conditional first and second moments are consistent with the Sharpe-Lintner-Mossin capital-asset-pricing model. We test the mean-variance model under several different assumptions about the time-variation of conditional second moments of returns, using weekly data from July 1974 to December 1986, that include returns on a portfolio composed of dollar, Deutsche mark, Sterling, and Swiss franc assets, together with the US stock market. The model is estimated constraining risk premia to depend on the time-varying conditional covariance matrix of the residuals of the expected returns equations.

The results indicate that estimated conditional variances cannot explain the observed time-variation of risk premia. Furthermore, the constraints imposed by the static CAPM are always rejected.

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Introduction

Rates of return on international financial assets are characterized by statistical properties that are quite common to all financial markets: they are highly volatile and largely unpredictable. These properties make it very difficult to extract statistically reliable estimates of systematic exchange-rate and asset-price movements, and are at the root of the generally poor empirical performance of international asset pricing models. Nevertheless, two important results have been uncovered by empirical researchers and can be considered a fair characterization of the data: (a) expected returns on foreign assets vary over time (Cumby and Obstfeld [1981] and the numerous articles that followed, recently surveyed by Frankel and Meese [1987]); (b) the volatility of returns on foreign assets also changes over time (Cumby and Obstfeld [1984], Hodrick and Srivastava [1984], Hsieh [1985], among others). The purpose of this paper is to determine whether the observed fluctuations of conditional variances and conditional expectations of returns in international financial markets are consistent with a family of asset pricing models.[1]

We test the mean-variance capital-asset-pricing model (CAPM) under several different assumptions about the time-variation of conditional second moments of returns, using weekly data from July 1974 to December 1986. The model is estimated constraining risk premia to depend on the time-varying conditional covariance matrix of the residuals of the expected returns equations. Unlike all formal tests of capital asset pricing models we are aware of, we pool data

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[1] Giovannini and Jorion [1987] argue that the time variation of conditional second moments might have important implications for the empirical performance of various asset pricing models.

on the foreign exchange market and on the US stock market. This strategy is justified by the sheer size of the stock market in international financial portfolios: in our sample, the average share of the US stock market is .55, versus .31 for dollar-denominated external assets, and only .06 for pound sterling and Deutsche mark assets, respectively. Furthermore, we can explore whether some puzzling aspects of the behavior of risk premia, which have been noted in the stock market by Mehra and Prescott [1985], and in the foreign exchange market by Fama [1984] and Frankel [1986], have common characteristics across these different assets.

Section I briefly summarizes the issues in the recent empirical applications of the static asset pricing model to international financial data. Section II describes the empirical methodology we follow. Section III reports our results, while section IV discusses the implications of our estimates for the predicted variations of risk premia. Some concluding comments appear in section V.

I. The Issues

We postulate a representative investor, maximizing a utility function defined over the (conditional) expectation and the (conditional) variance of end-of-period wealth:

$$\text{MAX} \quad U[E_t(W_{t+1}), \sigma_t^2(W_{t+1})] \tag{1}$$

where

$$E_t(W_{t+1}) = W_t x_t' E_t(R_{t+1}) + W_t(1 - x_t'\underline{1})R_t^f \tag{2}$$

$$\sigma_t^2(W_{t+1}) = W_t^2 x_t' \Omega_{t+1} x_t \tag{3}$$

and where W represents the investor's wealth, x_t the vector of investment shares in risky assets, whose rates of return have conditional means and covariances denoted by $E_t(R_{t+1})$ and Ω_{t+1}, respectively. R_t^f (a scalar) is the rate of return on the riskless asset, and $\underline{1}$ is a unit vector. Equation (1) is the starting point for the Sharpe-Litner-Mossin static capital-asset-pricing model, but can also be obtained from an alternative, explicitly dynamic, framework, as we show in appendix A. Indeed the model we estimate is "static" only because it imposes unit elasticity of intertemporal substitution, but is consistent with time-variation of the distribution of returns.

The first order conditions for problem (1) imply the following relation between asset shares and the conditional moments of returns:

$$x_t = (\rho\Omega_{t+1})^{-1}(E_t(R_{t+1}) - R_t^f) \tag{4}$$

where ρ stands for the relative risk-aversion coefficient, defined as $-2W_tU_2/U_1$, and assumed to be constant. U_1 and U_2 are the partial derivatives of the utility function with respect to its first and second explanatory variable. Note that equation (4) is both a first-order necessary condition for individual optimization and a market equilibrium condition, when x is substituted with the actual value-shares of risky assets available at time t. Thus equation (4) can be solved to obtain equilibrium expected returns:

$$E_t(R_{t+1}) - R_t^f = \rho\Omega_{t+1} x_t \qquad (5)$$

Since the expectation of R_{t+1} equals its realization minus a forecast error, we have:

$$R_{t+1} = R_t^f + \rho\Omega_{t+1} x_t + \xi_{t+1} \qquad (6)$$

where ξ is the rate-of-return "surprise," orthogonal--under rational expectations--to all variables in agents' information sets, and therefore orthogonal to the variables on the right-hand side of equation (5).

Equation (6) was estimated by Frankel [1982]. Frankel first recognized that the CAPM imposes the restriction that the covariance of returns Ω be equal to the covariance matrix of the disturbance vector ξ. In addition, Frankel assumed a constant conditional covariance matrix Ω. He could not estimate with any precision the coefficient of risk aversion ρ, and in particular could not reject the hypothesis that $\rho=0$.[2]

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[2] Frankel did not test the overidentifying restriction imposing the equality of

Some intuition for the failure to estimate the coefficient of risk aversion with any precision using this model can be obtained from the literature on the volatility of the risk premium. Fama [1984] shows that, if the assumption of rational expectations is true, the variance of risk premia in the foreign exchange market should be of the same order of magnitude as the variance of forward premia. Frankel [1986] argues that the variation of asset supplies, if the model of equation (6) with he assumption of constant Ω is true, cannot possibly explain the numbers reported by Fama. For example, in the case of the Deutsche mark, and assuming that $\rho=2$, observed fluctuation of asset supplies can only predict a standard error of the risk premium that is 1/200 of the standard error of the risk premium estimated with unrestricted projection equations. Thus the difficulties encountered in estimating the coefficient of risk aversion are due to the exceedingly low variation of ρx_t which cannot statistically be distinguished from a constant.

The assumption of constant conditional covariance of returns, however, has been proven wrong by the evidence on conditional heteroskedasticity, both in the stock market and in the foreign exchange market. Evidence on the stock market was reported by Christie [1982], Poterba and Summers [1984], French et al. [1987], among others, while tests of homoskedasticity using foreign exchange data were performed by Cumby and Obstfeld [1984], Hodrick and Srivastava [1984], and Hsieh [1985]. Giovannini and Jorion [1987] find that both in the stock market and in the foreign exchange market nominal interest rates have substantial explanatory power for the variation of conditional (non-central)

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the covariance of disturbances with the matrix Ω.

second moments. They argue that the time variation of conditional second moments could improve the empirical performance of the static CAPM.

Recently, a number of papers have attempted to explicitly account for the variation of conditional second moments in tests of the static CAPM.[3] Bollerslev, Engle and Wooldridge [1985] apply the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to returns on bills, bonds and stocks, and find some empirical support to the static CAPM, although the diagnostic tests they perform do not include the test of overidentifying restrictions. Engel and Rodrigues [1987] and Attanasio and Edey [1987]--independently of this paper--use the data on asset supplies originally constructed by Frankel [1982] to test the international CAPM with different specifications of the conditional covariance of returns. Both studies test a version of the ARCH model, while Engel and Rodrigues also project second moments onto macroeconomic indicators. Both papers find a substantial improvement in the performance of the model once the time variation of conditional second moments is accounted for. However, they still obtain rather imprecise estimates of the coefficient of risk aversion, and Engle and Rodrigues reject in all cases the overidentifying restrictions associated with the CAPM.

The common result of these papers is that the specification of the process for conditional covariances substantially affects the empirical performance of the CAPM. Frankel [1988], surveying the various specifications for conditional

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3 Ferson et al. [1987] use a different approach, and assume that risk aversion can change over time, but that the conditional covariance matrix of returns is constant. Kaminski and Peruga [1987] estimate the intertemporal asset pricing model assuming that forecast errors of rates of return follow a multivariate GARCH process.

second moments used so far in the literature, notes that alternative models appear to imply widely different magnitudes for the predicted volatility of risk premia. This result is especially disturbing, since all of the models that have been used for the process followed by conditional second moments are just projection equations, with no theoretical grounding. Therefore it seems particularly important to explore alternative specifications for the process followed by conditional second moments. This is the task of this paper, which we describe in more detailed in the next section.

II. Specification and Estimation

Equation (6) is the starting point for the estimation of the CAPM. Since we use data on nominal returns, the own-currency interest rate is riskless: we assume that R_t^f represents the dollar interest rate. This implies that the investor's consumption basket is denominated in dollars and is not subject to purchasing-power risk. Given the large variability of nominal asset returns relative to inflation rates, empirical tests do not seem to be affected by the choice of the deflator.[4] Define r_{t+1} as the difference between R_{t+1} and the riskfree rate R_t^f. A general expression for equation (6) is then:

$$r_{t+1} = \mu + f(\theta, t) + \epsilon_{t+1} \tag{7}$$

[4] Frankel [1982] and Engel and Rodrigues [1987] use real rates of return in their tests.

where θ stands for the vector of parameters of the model. The constant term μ is added in order to account for effects--like preferred habitats and differential tax effects--that are not directly captured by the CAPM.

For the maximum likelihood estimation, we assume that the error terms at time t are distributed as normal i.i.d. variables, <u>conditional</u> on information available at time t-1. This information determines the covariance of returns Ω_t. The conditional log-likelihood function for observation t is:

$$\ell_t = \ln L_t = -(N/2)\ln(2\pi) - (1/2)\ln|\Omega_t| - (1/2)\ \epsilon_t'\Omega_t^{-1}\ \epsilon_t, \tag{8}$$

where N is the number of assets in the portfolio, i.e. the dimension of all vectors and square matrices. Since innovations in rate of returns are conditionally independent identically distributed, the log-likelihood of the whole sample is simply

$$\ell(\mu,\theta) = \sum_{t=1}^{T} \ell_t \tag{9}$$

The maximum likelihood estimates are obtained by maximizing the likelihood function over μ and θ.[5] At the maximum, an estimate of the covariance matrix of the estimated parameters is obtained from the inverse of the sum of the outer

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5 The optimization was performed in FORTRAN double-precision by the NAG subroutine E04JBF. The optimization for the largest version of the model took approximately 2 days of CPU time on a VAX 11 computer.

product of the score vectors, as suggested by Berndt et al. (1974).

If the covariance matrix of the error terms is constant, the restrictions imposed by the CAPM on the function f in equation (7) are:

$$f(\theta,t) = \rho\ \Omega\ x_t \tag{10}$$

In this case the conditional and unconditional distributions of ϵ coincide.

If the covariance matrix of the error terms varies over time, the restrictions imposed on the function f in equation (7) are:

$$f(\theta,t) = \rho\ \Omega_{t+1} x_t \tag{11}$$

As we argued in section 2, one important factor in the empirical implementation of the CAPM with time-varying second moments is the specification of the fluctuations of Ω_t. For this reason, we present in this paper a number of alternative specifications of the time-variation of conditional variances and compare their impact on tests of asset pricing. This strategy is necessary since there is no economic model of the fluctuation of variances to rely on. The first general specification is the ARCH process proposed by Engle [1982], which implies that the conditional covariance matrix is a nonstochastic function of the current information set:

$$\Omega_t = \Gamma + A \bullet \epsilon_{t-1} \epsilon_{t-1}' + B \bullet \Omega_{t-1} + \Phi \bullet i_{t-1} i_{t-1}' \tag{12}$$

where $\bullet$ indicates element-by-element matrix multiplication. ϵ_{t-1} is a vector of lagged forecast errors, $i_{t-1} = (i^*_{t-1} - R^f_{t-1})$, and i^* is a vector containing, for each foreign-currency asset, the interest rate of that foreign currency,[6] and for the stock market, a zero. In practice the symmetric matrices Γ, A, B and Φ are constrained to be positive-definite by estimating their Choleski factors: this yields, with 4 assets, a total of 40 parameters to estimate for the conditional covariances.

An alternative, more parsimonious specification constrains the off-diagonal terms of Ω to be the product of a constant correlation coefficient and the corresponding standard errors of returns. The variances are assumed to follow the following processes:

$$\sigma^2_t = \gamma + \alpha\epsilon^2_{t-1} + \beta\sigma^2_{t-1} + \phi i_{t-1} \tag{13}$$

This reduces the number of parameters to estimate for the Ω_t to 27.

To understand the different implications of equations (12) and (13), combine equations (7) and (11):

$$r_{t+1} = \mu + \rho\, \Omega_{t+1} x_t + \epsilon_{t+1} \tag{14}$$

As is well known, non-zero conditional expected returns are frequently found in empirical research, while unconditional expected returns in the foreign exchange market appear to be small. These two pieces of evidence indicate that foreign

6 From interest rate parity, the difference between foreign and US interest rates equals the forward foreign exchange premium.

exchange risk premia change sign over time.[7] How can we get sign reversals in conditional risk premia with the specifications for Ω in equations (12) and (13)? Consider model (14) and (13) first. Changes in sign of risk premia can arise from negative covariance terms in Ω, or when μ and ρ have opposite signs. In model (14) and (12), on the other hand, sign reversals in risk premia can also arise from sign reversals in the estimated conditional covariance terms in Ω. As we argue below, the two specifications end up producing nearly identical estimates of the movements of risk premia, for all assets.

Our equations include as a special case the simple ARCH model adopted by Engel and Rodrigues [1987], where the conditional variance depends on the lagged value of the squared forecast error (thus B and Φ are constrained to equal zero). That specification <u>could</u> generate persistence in fluctuations of Ω, because ϵ_{t-1} is drawn from a distribution with covariance Ω_{t-1}: a large value of Ω_{t-1} makes a large realization of ϵ_{t-1} more likely, which in turn, through the matrix A, makes Ω_t larger. A nonzero B coefficient, by contrast, <u>does always</u> produce persistent fluctuations of Ω. As we show below, the observed persistence in volatility cannot be adequately captured if B is constrained to zero.

Christie [1982] and Giovannini and Jorion [1987], among others, point that nominal interest rates are significantly correlated with variances of returns in the stock market and in the foreign exchange market. Giovannini [1987] shows how these correlations could arise from the joint movements of money demand and

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[7] We thank an anonymous referee for raising these issues. In the stock market unconditional expected returns are positive when significant.

asset demands. Hence we include the interest-rate terms in equations (12) and (13).[8]

To test the restrictions imposed by the CAPM, an alternative hypothesis is needed. We specify the following general model:

$$r_{t+1} = \nu + Q_t x_t + \xi_{t+1} \tag{15}$$

where the elements of the matrix Q are defined similarly to the restricted version, but are of course unrelated to the covariance of the residuals in (15).

[8] The two alternative specifications of the conditional covariance matrices allow for somewhat different interest-rate effects. While in equation (12) we assume that all cross products of interest-rate differentials affect the risk premium on each security, in equation (13) only the own-currency interest rates are assumed to determine conditional variances of each asset's return. The specification of (12) insures that Ω is positive definite.

III. Empirical Results

The models surveyed above were estimated using weekly data. Since the asset supplies data can only be constructed on a monthly basis, the weekly series of asset shares have been computed by interpolating the own-currency values of asset supplies, and by translating them into dollars at the actual weekly exchange rates, to compute the value shares. We believe that this method should not affect our estimates dramatically, because exchange-rate changes account for a large fraction of the variation of dollar values of asset supplies, as shown in table 1. In the case of the stock market we use the actual capitalization data, that are available on a weekly basis. The currencies in the portfolio, together with the US dollar, include the British pound, the Deutsche mark and the Swiss franc. Our sample ranges from July 5, 1974 to December 19, 1986, and includes 651 observations.

Tables 2 and 3 report the maximum-likelihood estimation results, for the homoskedastic model, and the GARCH model of equation (12). The tables report point estimates and t statistics for the coefficient of risk aversion, the constant terms in the regressions, and the parameters of the conditional covariance matrix. We also compute the Lagrange multiplier test for the CAPM restrictions implied by equation (14).[9]

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[9] The Lagrange multiplier is computed as follows. Define n and (n+r) as the number of parameters for the restricted and unrestricted models, respectively. The test statistic is $q'H^{-1}q$, where q is the score vector--defined as the derivative of the log-likelihood function with respect to the parameters--and H the Hessian matrix, both of dimensions (n+r) and evaluated at the restricted point. By the Cramer-Rao inequality, the Hessian matrix is itself computed from the outer product of the score vectors.

To make a comparison with Frankel's [1982] results, in table 2 we report the estimates of the model where we assume a constant covariance matrix of returns (although our model is estimated with weekly data on a longer sample period, and includes a different set of assets from Frankel's). We find that the coefficient of risk aversion is negative and very large. This is a strong rejection of the model, since a negative risk aversion implies a failure of the necessary conditions for optimization. The Lagrange multiplier test also rejects the restrictions imposed by the model against a general (homoskedastic) alternative specification of excess returns. The alternative model is specified as in equation (15), assuming that the matrix of unrestricted coefficients Q is constant. No other restrictions are imposed on Q.

Table 3 contains the estimates of the risk aversion parameter in the heteroskedastic model of equation (12), and of the elements of the matrices Γ, A, B, and Φ. Since--as pointed out above--we actually estimate the Choleski factors of those matrices, the t statistics are obtained by using the invariance property of maximum-likelihood estimators. The table reveals several important facts. First, the hypothesis of constant conditional second moments is strongly rejected, as shown by the χ^2 tests at the bottom of the table: the statistic tests the hypothesis that the elements of the matrices A, B and Φ are all equal to zero. Releasing the constraint that the covariance of returns is constant over time seems also to improve the estimate of the coefficient of risk aversion, which becomes positive and of reasonable magnitude, although insignificantly different from zero. Second, we find that the autoregressive terms--the elements of the matrix B--are highly significant: changes in volatility of returns have a high degree of persistence. Finally, the overidentifying restrictions imposed by the CAPM are strongly rejected. In this case, the alternative hypothesis for the test of overidentifying restrictions assumes that the matrix of time-varying coefficients Q_t of equation (15) evolves as the matrix Ω_t in equation (12). In order to save space, we do not report the results obtained assuming constant conditional correlations (equation (13)): the estimate of the coefficient of risk aversion is in that case .23 (with t statistic .03), and, as above, both the constancy of conditional variances and the restrictions of the CAPM are strongly rejected.

Table 4 contains a number of specification tests on the two models of conditional covariances: constant correlations (equation (13)) and the model of table 2 (equation (12), referred to as "General Model"). The table shows that the explanatory value of the variables we include in the model of time-varying covariances is very similar under the two alternative specifications: in both cases we reject at very high confidence levels the hypothesis that lagged rate-of-return innovations have no marginal explanatory power over a constant, and the hypothesis that movements in conditional variances are not autocorrelated. Interest rates appear to be highly significant in the case where conditional correlations are constant, but are just below the 10 percent significance level in the more general model. This discrepancy between the two models is due to the way interest rates enter equations (12) and (13), as we explain in footnote 8 above.

In summary, our empirical analysis suggests three main results: first, the time variation of conditional second moments is not adequately captured by the simple models which include as explanatory variables only the lagged forecast errors. Second, the risk aversion parameter does not seem to be estimable with any precision. And third, constraining conditional correlations to be constant does not dramatically affect the significance of explanatory variables of the conditional covariance matrix.

IV. The Volatility of Risk Premia

The ability of conventional asset pricing models to reproduce the patterns of unrestricted estimates of foreign exchange risk premia has been questioned by several authors, and in several different contexts. Fama [1984] shows that--under rational expectations--the fluctuations of risk premia in the foreign exchange market are at least as large as those of forward premia. While Frankel concludes that such evidence cannot be reconciled with the static CAPM (in the version with constant conditional covariances), Hodrick and Srivastava [1986] claim that such evidence is not in principle inconsistent with the dynamic general equilibrium model due to Lucas [1982]. Work on the US stock market by Mehra and Prescott [1985] indicates that the conventional general-equilibrium dynamic asset pricing model cannot explain simultaneously the relatively low level of the risk-free rate and the (on average) high risk premia for the stock market.[10]

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10 Mehra and Prescott, while addressing similar questions as Frankel, use quite a different framework of analysis: rates of return on the riskless asset are endogenous in Mehra and Prescott's model, given assumptions about the exogenous distribution of output growth. Hodrick and Srivastava [1986] also carry out a general equilibrium exercise.

Our objective in this section is limited to the comparison between unrestricted estimates of risk premia, and the predictions of the models estimated above. Figures 1 and 2 illustrate unrestricted risk premia for the US stock market and the Deutsche mark.[11] These were obtained by projecting excess returns on a constant, the own-asset portfolio share, the product of the forward premium and the own-asset share, and the product of the lagged value of the squared return and the own-asset share. The hypothesis that excess returns in the two markets are constant was rejected, for all assets, at the 99 percent significance levels, using a Hansen [1982] and White [1980] estimate of the covariance matrix of the parameters. Although these forecasts are quite noisy, the size of the fluctuations is remarkable. The ex-ante excess return of the stock market over dollar deposits fluctuates within plus and minus 1 percent per week, while the ex-ante excess return of DM assets fluctuates within plus and minus 0.6 percent per week: hence the annualized numbers in figures 1 and 2, which are obtained by multiplying the weekly returns by 52.

The ability of the CAPM to reproduce these numbers depends on the volatility of asset supplies, the volatility of conditional second moments, and the size of the risk aversion coefficient. Does the volatility of conditional second moments--which was not taken into account by Frankel's original calculations--make the model's predictions closer to the unrestricted estimates? To answer this question we plot the predicted values of the risk premia obtained from the model whose estimates are reported in table 3. Figures 3 and 4 plot

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[11] Once again, we omit the other currencies to save space. The general conclusions we draw from the discussion of the DM and STK simulations also hold for the other currencies.

the estimated conditional variances of returns and the estimated risk premia for the US stock market and DM assets.[12] Although we do not report predicted values of expected returns and standard errors in all alternative models, we should point out that the exclusion of nominal interest rates from the specification of the conditional covariance matrix does not affect the estimates of means and standard errors of returns in any noticeable way. On the other hand, excluding the lagged conditional variance term does: as expected, the persistence of fluctuations of conditional variances, and, to some extent, of risk premia, decreases dramatically. This is highlighted by figures 5 and 6, which report the estimates of conditional standard errors of the stock market and Deutsche mark assets, obtained when the (significant) autoregressive term is omitted.

The most striking fact appearing from a comparison of figures 1 and 2 with 3 and 4 is that the fluctuations of the estimates of risk premia implied by the CAPM are dramatically different from those of the unrestricted ones. Although this evidence is only of a qualitative nature, it is borne out by the Lagrange multiplier tests discussed above. Excess returns on DM assets conditional on the estimated CAPM model fluctuate between 0 and 2 percent per annum, while the unrestricted estimate fluctuates between plus and minus 30 percent. Similarly, the estimated excess return on the US stock market, conditional on the CAPM, fluctuates between 6 and 10 percent per year, while the unconstrained estimate ranges between plus and minus 40 percent.

We have also found no appreciable difference between our two alternative specification of conditional covariance matrices--represented by equations (12)

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[12] Returns and their standard errors are also in annual terms.

and (13). The estimated conditional variances obtained from the two specifications have very high correlation coefficients.[13] The correlations between risk premia in the two specifications is 0.96 for the pound, 0.91 for the Deutsche mark, 0.95 for the Swiss franc, and 0.99 for the stock market. In other words, assuming constant correlation coefficients does not in any way affect the pattern of fluctuations of risk premia consistent with the estimated capital asset pricing model.[14]

V. Summary and Concluding Remarks

This paper has specified and estimated a static capital asset pricing model to explain the empirical behavior of rates of return in the US stock market and in the foreign exchange market. The purpose of the paper was to explore the role of alternative specifications for the process followed by the conditional second moments of returns.

The empirical findings indicate that the specification of the process followed by conditional second moments of returns affects significantly the estimate of the risk aversion parameter, and as a result, affects the estimates

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13 The correlations are: 0.99 for the pound, 0.97 for the Deutsche mark, 0.99 for the Swiss franc, and 0.99 for the stock market.

14 While the two models predict the same fluctuations of risk premia, the average risk premia differ in the two models, because the estimates of the risk aversion parameters differ.

of the ex-ante risk premium on various assets. Both lagged conditional variances and nominal interest rates have significant predictive ability for second moments of asset returns. For all specifications of conditional variances we estimate, however, the overidentifying restrictions imposed by the CAPM are rejected at very high confidence levels.

Simulations with the estimates of the model show that our estimates of the CAPM fail to reproduce unrestricted estimates of risk premia obtained from projection equations. Furthermore, since the general shapes of estimated conditional variances (their peaks and troughs) do not differ dramatically across the various specifications we adopt, it appears that the empirical failure of the CAPM can be ascribed to the absolute lack of resemblance of the fluctuations of conditional variances and the fluctuations of unrestricted estimates of risk premia. This lack of resemblance is clearly not made up for by fluctuations in asset supplies.

Overall, the results of this paper tend to be discouraging to those who believe that the static CAPM is a fair description of the determination of equilibrium returns in world financial markets. However, the evidence also seems to suggest that a thoroughly satisfactory test of the static CAPM would probably require the inclusion of many more assets than those we use, and a much more complete specification of the process followed by conditional second moments. Both of these extensions involve the construction of very large models, that--given the current computational technology--are quite difficult and expensive to estimate.

Appendix A: Unit Intertemporal Substitution and the "Static" CAPM

In this appendix, which draws heavily from Giovannini and Weil [1988], we prove that the equations characterizing the "static" Sharpe-Litner-Mossin capital asset pricing model can be derived in a dynamic model where intertemporal substitution and risk aversion are explicitly distinguished, and where the elasticity of intertemporal substitution is constrained to unity. Merton [1971] showed that assuming logarithmic utility (which implies unit intertemporal elasticity of consumption), the dynamic saving and portfolio selection problem collapses to one where the consumer maximizes the expectation of the logarithm of end-of-period wealth. The advantage of the framework outlined here, as stressed by Giovannini and Weil [1988], is that, unlike in Merton's model, no restrictions are imposed on the coefficient of risk aversion.

Consider the problem of a consumer whose preferences are represented by the following functional equation:

$$V_t = \max_{C_t, x_t} \left[(1-\delta)C_t^{1-\eta} + \delta(E_t V_{t+1})^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1-\rho}{1-\eta}} \qquad (a1)$$

$$\text{subject to: } W_{t+1} = (W_t - C_t)x_t' R_{t+1} \qquad (a2)$$

where E_t denotes the expectation operator, conditional on information available at time t, W represents the investor's wealth, C consumption, δ is proportional to the utility discount factor, and $x'\underline{1} = 1$ (we omit, for simplicity but without loss of generality, the riskless asset and assume that only risky investments are available). For notational ease, we denote the maximand in (a1) as $U[C,(EV)]$, where U is referred to as an "aggregator" function.

Equation (a1) has been studied by Weil [1987,1988]. Similar versions were independently developed by Farmer [1987] and Epstein and Zin [1987]. If preferences are as in (a1), the coefficient $1/\eta$ represents the elasticity of intertemporal substitution (as suggested by considering the corresponding problem under certainty) while ρ is the coefficient of relative risk aversion (as suggested by the fact that the risk premium for a lottery on permanent consumption is proportional to ρ, see Weil [1987]).

By application of L'Hopital rule, the following can be established:

$$\lim_{\eta \to 1} U[C_t, (E_t V_{t+1})] = C_t^{(1-\delta)(1-\rho)} (E_t V_{t+1})^{\delta} \tag{a3}$$

As Weil [1988] shows, the first-order necessary condition for the solution of (a1) is for every element R^i of the vector R,

$$E_t \; [\; (U_{2t} U_{1t+1} / U_{1t}) \; R^i_{t+1} \;] = 1 \tag{a4}$$

where U_1 and U_2 are the partial derivatives of the function U with respect to the first and second argument, respectively. The expression for (a4) in terms of the original tastes parameters requires the solution to the functional equation (a1). Given the assumed preferences, it can be verified that V is an power function of wealth, and that optimal consumption, in the case of logarithmic intertemporal preferences, is just $(1-\delta)$ times current wealth. Using these facts, some algebra establishes that (a4) is equivalent to the following:

$$E_t[(x_t'R_{t+1})^{-\rho}R^i_{t+1}] = E_t[(x_t'R_{t+1})^{1-\rho}] \qquad (a5)$$

Equation (a5) is just the first-order condition for the problem of maximizing an exponential utility function, defined over the total return on the portfolio: further standard restrictions on the moments of the joint distribution of R allow to derive equation (1) in the text.

Appendix B: Data Sources

Daily observations on spot exchange rates were obtained from DRI. Daily stock market returns are from the CRSP database, as well for the aggregate capitalization of the market in dollars. We used the value-weighted index constructed by CRSP.

Weekly one-week Eurocurrency rates were collected from the Financial Times.

Exchange rates are recorded at 11:30 am (EST), while the Financial Times data are at the close of the London market, or 12:00 noon (EST). CRSP stock market returns are based on closing trade prices of all securities on the NYSE and on the AMEX, at 4:00 pm (EST).

Aggregate asset supplies data were constructed following the method described by Frankel [1982]. All the data, together with a detailed description of the construction of the asset supplies in dollars, marks, pounds and Swiss francs, are available from the authors on request.

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Table 1:

Statistics on Asset Supplies

Standard Deviation of Percent Changes
5 July 1974 to 19 December 1986

	BP	DM	SF	USD	STK
Monthly Data:					
Asset Supply in Foreign Currency	0.0233	0.0169	0.0647		
Spot Exchange Rate	0.0346	0.0319	0.0391		
Asset Supply in Dollars	0.0426	0.0411	0.0836	0.0121	0.0466
Weekly Data:					
Asset Supply in Dollars	0.027	0.017	0.003	0.023	0.054
Memorandum:					
Average Weight in Portfolio	0.065	0.059	0.011	0.545	0.320

Table 2:

Homoskedastic Model

4 Assets: BP, DM, SF, Stock Market
5 July 1974 to 19 December 1986

Risk Aversion: -142.91*
(-2.97)

	BP	DM	SF	STK
Constant: (x100)	0.58	0.58	0.62	3.84*
	(2.52)	(2.50)	(2.47)	(3.00)
Covariance Matrix: (x10,000)				
	2.03*			
	(28.77)			
	1.40*	2.21*		
	(19.06)	(25.02)		
	1.52*	2.30*	3.05*	
	(17.07)	(22.66)	(22.96)	
	0.34	0.31	0.32	4.71*
	(2.46)	(2.34)	(2.00)	(28.56)

Log-likelihood = -4358.36

Lagrange Multiplier Test of the CAPM restrictions
(against homoskedastic alternative)
$\chi^2(13) = 32.51$, p-value = 0.0002

Note: Asymptotic T-statistics between parentheses. Significance at 1% level denoted by *. 651 observations in the sample.

Table 3:

Heteroskedastic Model

$$\Omega_t = \Gamma + A \bullet \epsilon_{t-1}\epsilon_{t-1}' + B \bullet \Omega_{t-1} + \Phi \bullet i_t i_t'$$

4 Assets: BP, DM, SF, Stock Market
5 July 1974 to 19 December 1986

Risk Aversion: 1.70
(0.19)

	BP	DM	SF	STK
Constant: (x100)	0.0311	0.0149	-0.0049	0.0989
	(0.61)	(0.29)	(-0.19)	(0.48)

Covariance Matrix:
(x10,000)

Matrix Γ (symmetric)

	BP	DM	SF	STK
BP	0.074*			
	(3.2)			
DM	0.061*	0.153*		
	(2.7)	(2.7)		
SF	0.062*	0.154*	0.183*	
	(2.6)	(2.6)	(2.6)	
STK	0.143	0.129	0.095	0.325
	(2.5)	(1.1)	(0.7)	(2.4)

Matrix A (symmetric)

	BP	DM	SF	STK
BP	0.148*			
	(5.0)			
DM	0.150*	0.162*		
	(7.1)	(5.6)		
SF	0.162*	0.170*	0.180*	
	(7.4)	(6.2)	(6.3)	
STK	0.010*	-0.022	-0.005	0.095*
	(5.5)	(-1.0)	(-0.2)	(3.6)

Matrix B (symmetric)

	BP	DM	SF	STK
BP	0.826*			
	(31.2)			
DM	0.787*	0.755*		
	(29.6)	(16.3)		
SF	0.782*	0.750*	0.744*	
	(30.4)	(17.2)	(17.6)	
STK	-0.805*	-0.772*	-0.767*	0.790*
	(-19.3)	(-23.1)	(-23.6)	(16.5)

Matrix Φ (symmetric)

	BP	DM	SF	STK
BP	0.069			
	(0.2)			
DM	-0.332	3.117		
	(-0.3)	(1.7)		
SF	-0.359	2.154	2.003	
	(-0.4)	(1.7)	(2.0)	
STK	-0.569	2.196	2.852	-5.005
	(-0.4)	(0.6)	(1.7)	(-2.1)

Log-likelihood = -4107.80

Chi-square test of heteroskedastic process:
$\chi^2(30) = 501.12$, p-value = 0.

Lagrange Multiplier test of the CAPM restrictions
(against heteroskedastic alternative)
$\chi^2(39) = 4444.1$, p-value = 0.

Table 4:

Tests of Alternative Specifications for the Conditional Covariance Matrix

Model	Degrees of Freedom	Log-Likelihood	Chi-square Test: Added parameters: Statistic	N	P-value
Homoskedastic:					
$\Omega = g(\Gamma)$	15	-4358.36			
Heteroskedastic, Constant Correlations:					
$\Omega(t) = g(\Gamma, A)$	19	-4295.67	125.4	4	0.
$\Omega(t) = g(\Gamma, A, B)$	23	-4195.23	200.8	4	0.
$\Omega(t) = g(\Gamma, A, B, \Phi)$	27	-4185.58	23.3	4	0.0001
Heteroskedastic, General Model:					
$\Omega(t) = g(\Gamma, A)$	25	-4246.64	223.4	10	0.
$\Omega(t) = g(\Gamma, A, B)$	35	-4115.74	261.8	10	0.
$\Omega(t) = g(\Gamma, A, B, \Phi)$	45	-4107.80	15.9	10	0.103

The notation $g(\bullet)$ is used for the various restrictions on the models of equations (12) (Heteroskedastic, General Model) and (13) (Heteroskedastic, Constant Correlations). $g(\Gamma)$ indicates that only constant terms are included; $g(\Gamma,A)$ indicates that constant terms and lagged rate-of-return innovations are included; $g(\Gamma,A,B)$ includes all of the above, plus lagged conditional variances, while $g(\Gamma,A,B,\Phi)$ stands for the general case, which includes all of the above plus nominal interest rates. See section II for details on the specification of conditional covariance matrices. The chi-square statistics test the incremental contribution of the last term in each $g(\bullet)$ function.

Figure 1: The Excess Return on the US Stock Market

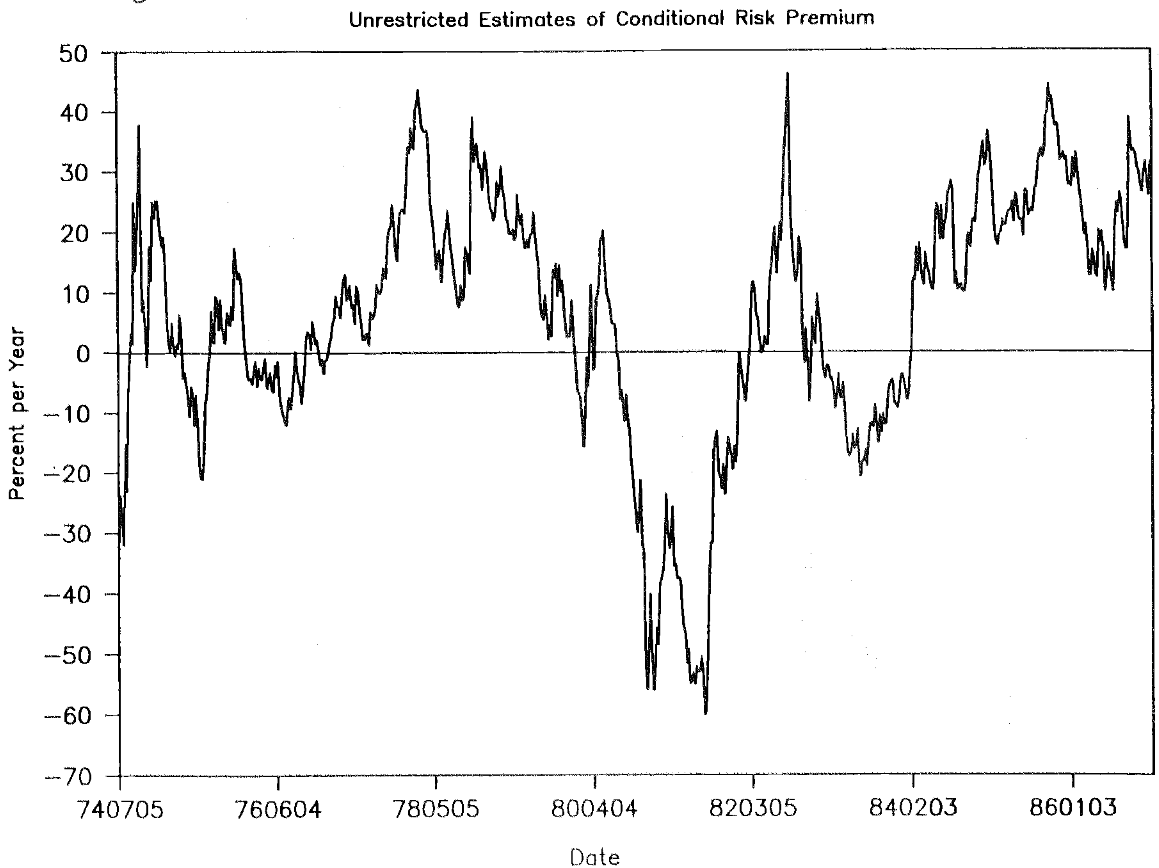

Figure 2: The Excess Return on Deutsche–Mark Assets

Unrestricted Estimates of Conditional Risk Premium

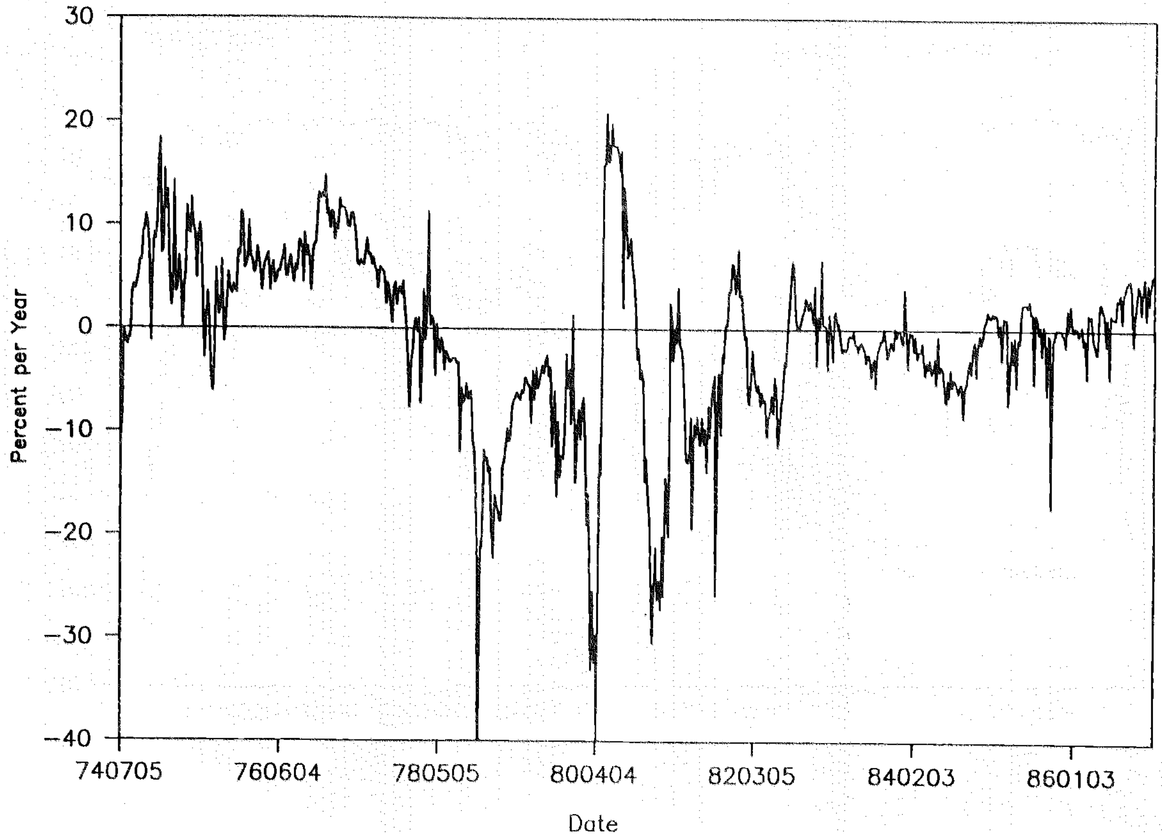

Figure 3: Risk and Return on the US Stock Market

Conditional Expectation and Standard Error from CAPM

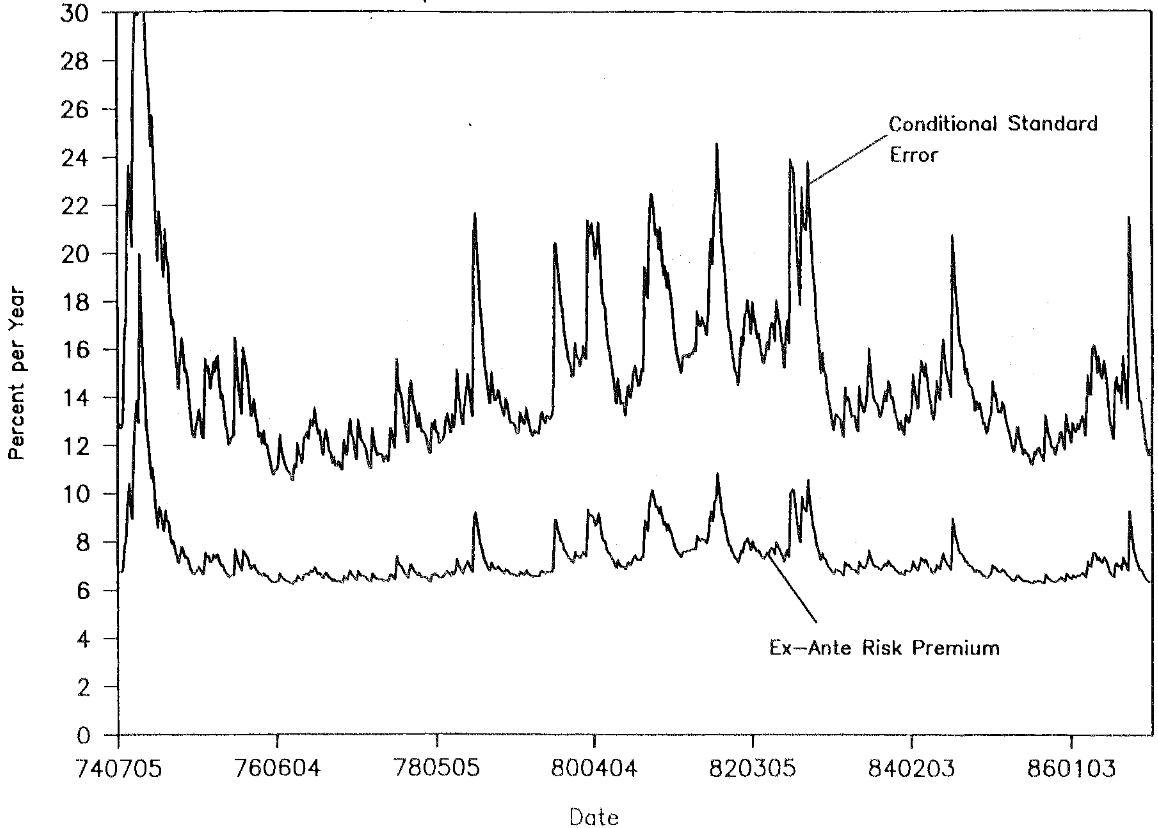

Figure 4: Risk and Return on Deutsche Mark Assets

Conditional Expectation and Standard Error from CAPM

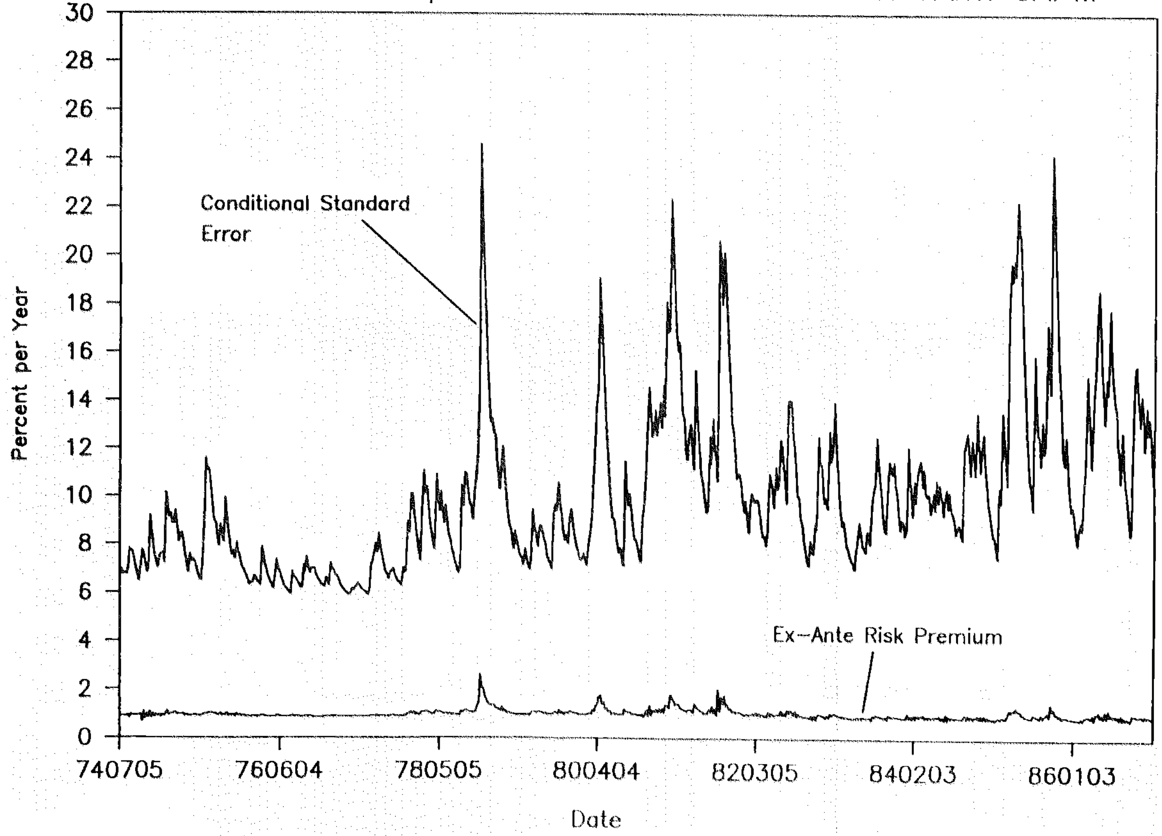

Figure 5: Conditional Standard Error of Return on US Stock Market
The Effect of Omitting Lagged Conditional Covariance

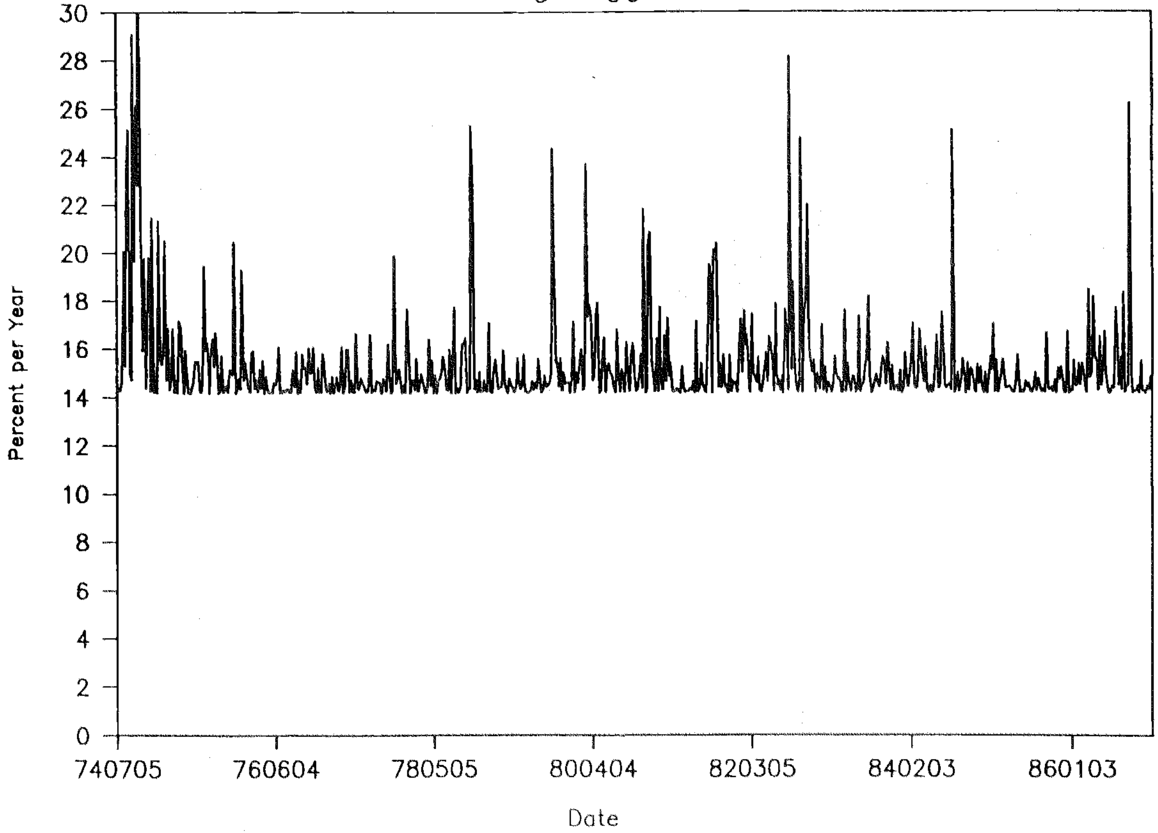

Figure 6: Conditional Standard Error of Return on Deutsche Mark Assets
The Effect of Omitting Lagged Conditional Covariance