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THE ROLE OF WOMEN'S EMPLOYMENT

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Changing Business Cycles: The Role of Women's Employment

Stefania Albanesi

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ABSTRACT

This paper studies the impact of changing trends in female labor supply on productivity, TFP growth and aggregate business cycles. We find that the growth in women's labor supply and relative productivity added substantially to TFP growth from the early 1980s, even if it depressed average labor productivity growth, contributing to the 1970s productivity slowdown. We also show that the lower cyclical of female hours and their growing share can account for a large fraction of the reduced cyclical of aggregate hours during the great moderation, as well as the decline in the correlation between average labor productivity and hours. Finally, we show that the discontinued growth in female labor supply starting in the 1990s played a substantial role in the jobless recoveries following the 1990-1991, 2001 and 2007-2009 recessions. Moreover, it depressed aggregate hours, output growth and male wages during the late 1990s and mid 2000s expansions. These results suggest that continued growth in female employment since the early 1990s would have significantly improved economic performance in the United States.

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1 Introduction

The rise in women’s market work is one of the most notable economic developments in the post-war period in the United States. Female participation rose from 37% in 1960 to a peak of 61% in 1997, and then flattened out since then, as shown in figure 1. This phenomenon contributed substantially to the rise in aggregate hours per person in the US in the 1970s and 1980s. Additionally, the composition of the female labor force also changed substantially over time, as female workers increased their labor market experience and their education level, eventually surpassing male workers in educational attainment. While a large literature has studied the determinants of the rise in women’s employment, the implications of this phenomenon for the aggregate performance of the U.S economy have been left largely unexplored.

This paper proposes that the steep rise in women’s labor force participation throughout the 1970s and 1980s, and its flattening out since the early 1990s can contribute an explanation for three puzzling phenomena that have changed the behavior of the US economy: i) the *slowdown in productivity growth* in the 1970s and the decline in the cyclical correlation between aggregate per capita hours and productivity; ii) the decline in the cyclical correlation between output and aggregate hours starting in the early 1980s, known as the *great moderation*; and iii) the sluggish growth in employment in the aftermath of recessions starting in the early 1990s, often referred to as *jobless recoveries*.

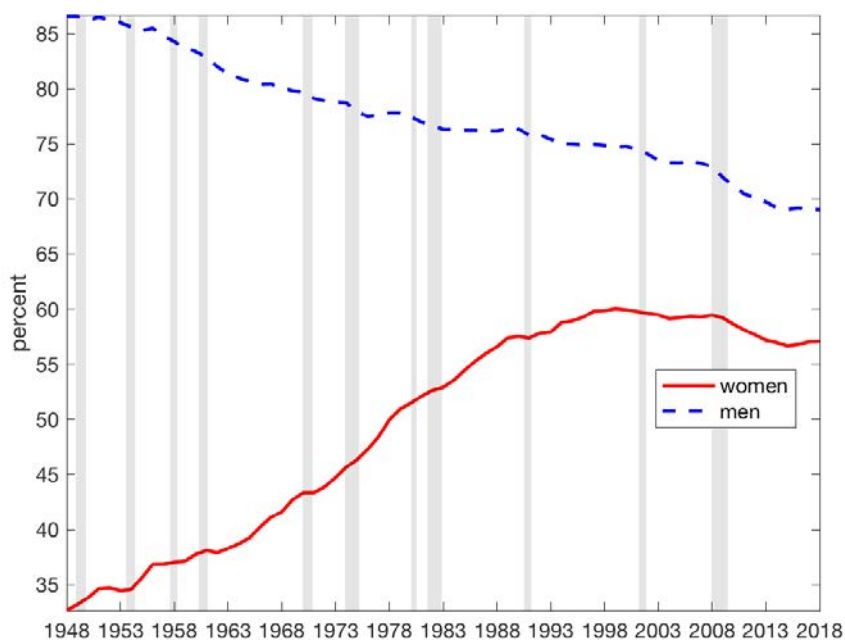


FIGURE 1: Civilian labor force participation rate by gender. Source: Current Population Survey.

The first part of the paper makes the empirical case for this argument. To do so, we use micro data to construct aggregate time series for hours and wages by gender similar to the corresponding

aggregates used in growth accounting and business cycle analysis. Using a series of decompositions and counterfactuals, we empirically assess the role of the changing trend in female employment. We show that the strong growth in female hours induced a decline in the growth rate of average labor productivity between the mid 1970s and 2003, though this pattern reversed in subsequent years. Despite this, we show that growth in female hours and relative productivity had a sizable positive effect on TFP growth starting in the early 1980s. Additionally, we show that female hours display substantially lower cyclical behavior than male hours, and the growth in the female share of aggregate hours plays a large role in the decline in the cyclical behavior of aggregate hours during the great moderation. We also argue that the rise in female employment contributed to a decline in the correlation between average labor productivity, aggregate hours and output over this period. Finally, we show the cyclical behavior of male employment and hours has remained stable over time. By contrast, there has been a stark change in the cyclical behavior of female hours and employment since female labor force participation stopped growing in the early 1990s. As long as female participation was rising, women did not experience sizable declines in hours and employment during recessions and exhibited very strong growth in hours and employment during recoveries. Starting with the 1991 recession, there was convergence in the cyclical behavior of female and male hours and employment. Even though female hours remain less cyclical than male hours, women now experience sizable declines in employment and hours during recessions and their hours recover at a rate similar to men's. We show that this change in the behavior of female hours can account for most of the joblessness of the recovery from the 1991 and 2001 recession, and about half of the missing job growth after the 2007-2009 recession.

The second part of the paper develops and estimates a dynamic stochastic general equilibrium (DSGE) model that allows for gender differences in labor supply and productivity to assess the implications of the changing trend in female participation and wages on the behavior of aggregate variables. This model with gender differences allows us to decompose the observed patterns in hours worked and the relative wage into components due to demand and supply factors, both in the trend and at the cyclical frequency. This decomposition can provide a useful reference point for the more detailed modeling of the underlying drivers of the increased labor market participation and relative wages of women, and, perhaps most importantly, it allows us to better understand and predict aggregate responses of hours, employment and wages to macroeconomic shocks and government policies.

The key feature of the model is that, unlike in most of the business cycle literature, men and women's hours are not perfect substitutes, and instead enter the production function as a CES aggregate with different gender specific productivities, which fluctuate over time. Moreover, we assume that men and women differ in their disutility of labor, both because of different Frisch elasticities of labor supply—a well documented empirical fact—as well as because of a different “taste” for market work, which we model as an exogenous stochastic process. The model can be used to assess the contribution of the growth in female hours and their relative productivity to changes in total factor and average productivity, as well as GDP. Using a series of counterfactuals, we show that the rise

in female relative productivity contributes positively to TFP and output growth, though the strong rise in female hours depresses average labor productivity growth, especially in the 1970s and 1980s. Additionally, we show that female labor supply displays a strong countercyclical component, consistent with an *added worker effect* on women’s labor supply, and that this substantially contributed to reduce the business cycle volatility of aggregate hours per capita as the share of female hours grew in the 1980s. Both the trend growth in female hours and their lower cyclicity contribute to the reduction in the correlation between output, aggregate per capita hours and productivity in this period, which according to [23] is an important component of the great moderation.

We estimate the model with Bayesian methods using yearly data from 1969 to 2017 and we compare its performance to that of a standard real business cycle model with no gender differentiation, estimated over the same period.¹ We find that the gender specific shocks account for a larger fraction of variance of output, hours and investment than the technology shock at medium and long horizons. We show that low frequency variation in the demand shocks, specifically, the preference and government consumption shock, absorb the trend in aggregate hours and wages not captured by the missing gender specific shocks in the standard model. We also show that in the model without gender specific shocks the estimated variance of the technology shock declines starting in the mid 1980s, leading to the conclusion that this change is a key factor in the decline in business cycle volatility of hours and GDP for that time period. However, in the model with the gender specific shocks, there is a decline in the estimated volatility of the cyclical component of these shocks and an *increase* in the volatility of the technology shock. This combination is due to the strong countercyclical behavior of the female labor supply shock. This suggests that the decline in the volatility of the gender specific shocks plays a large role in the decline in output and aggregate hours volatility observed starting in the mid 1980s. Taken together, these findings suggest that a DSGE model of the U.S. economy for the post-war period that does not include gender specific shocks to labor supply and productivity is misspecified, and may lead to faulty inference on the source of economic fluctuations.

We also estimate the model in two separate periods, 1969-1992, when female participation was rising and women’s wages were converging rapidly to men’s, and 1993-2017, when these two phenomena stopped or slowed down considerably. We find that the gender specific shocks account for smaller fraction of the variance of endogenous variables and hours in 1993-2017, and the technology and government spending shock play a larger role. Moreover, these aggregate shocks display a higher estimated persistence in the second period. To quantify the role of the change in trend female labor supply on the slow recovery of hours after a recession since the early 1990s, we run a counterfactual in the version of the model estimated with 1993-2017 data. In this exercise, we simulate the model for this period forcing female hours to growth at the same average rate as they did in 1969-1992. We find that the trend growth in female hours over this period would have reduced

¹Since female hours exhibit a strong positive trend up until the early 1990s, when they become stationary, in our estimation we take the period between 1995 and 2004 to correspond to a balanced growth path where only aggregate variables display a trend component, while we allow for gender specific trends in prior years. Even if the gender specific shocks are highly persistent, they are stationary, and the only non-stationary shock in the model is TFP.

the decline in aggregate hours during the 2001 and 2007-2009 recessions by about one third and would have substantially increased the growth in aggregate hours in the recovery. Moreover, both output and aggregate hours would have experienced substantial growth in the late 1990s and mid 2000s expansions, whereas they were stable in the data. Finally, continued growth in female hours would have increased the level and growth rate of male wages, as higher female hours determine a rise in the marginal product of male wages. Taken together, these findings suggest that the flattening of female participation since the early 1990s negatively impacted U.S. economic performance both in the trend and in at business cycle frequencies.

1.1 Contributions and Relation to the Existing Literature

There is an extensive literature on the rise in women's market work and the increase in their wages relative to men. Many of these explanations have focussed on the role of technological advances in increasing both the supply and the demand of female labor. On the supply side, Goldin and Katz (2002) [25] show that the diffusion of oral contraceptives reduced the costs and increased the returns to women's education, contributing to a rise in their participation and wages. Greenwood, Seshadri and Yorugoklu (2005) [27] argue that advances in home appliances increased female participation by reducing the time required for home production. Albanesi and Olivetti (2016) [3] show that improvements in maternal health and the introduction of infant formula are key to explaining the rise in participation of married women with children, and the rise in women's education and wages relative to men.²

On the demand side, Galor and Weil (1996) [24] attribute the rise in women's market work and the growth in their relative wages to technological innovations that increase the returns to intellectual rather than physical skills, in which women have a biological comparative advantage. Empirical evidence also supports this notion. Black and Juhn (2000) [8] argue that the rising demand for skilled workers may have contributed to the rise in participation of skilled women, and to the increase in the fraction of women in professional and managerial occupations, which were traditionally male. Additionally, Black and Spitz-Oener (2007) [9], find that women have witnessed relative increases in non-routine analytic tasks and non-routine interactive tasks, associated with higher skill levels. Rendall (2010) [36] shows that, as job requirements have shifted from more physical to more intellectual attributes, women, who always worked in occupations with relatively low physical requirements, also shifted from occupations with low to high intellectual requirements.

Compared to the macroeconomic literature on the rise in female labor supply and the decline in the gender wage gap, which attempts to spell out the detailed mechanisms that fueled the shift of female labor supply from the home to the market and the technological determinants of the rise

²Little work has been done on the flattening out of female participation in the 1990s. Albanesi and Prados (2014) [4] show that the flattening out is driven by a decline in participation of prime age women with college degrees, and those married to high earning husbands. They identify the acceleration of the rise in the skill premium in the 1990s, and the corresponding sharp rise in top incomes for men, as the main contributor to the decline in the growth of female participation. Fernandez (2012) [18] and Fogli and Veldkamp (2012) [19] explore learning models in which women's participation rises as the perceived costs of women's work fall over time. These learning models also predict a flattening out of the rise in participation at the end of the learning process.

in female relative wages, our approach treats the differences in productivity and disutility of labor between men and women and their evolution over time as exogenous and focuses on quantifying the link between these phenomena and the changing behavior of aggregate labor market outcomes in the trend and at the cyclical frequency. Fukui, Nakamura, Steinsson (2018) [22] discuss the role of negative wealth effects on male labor supply stemming from the rise in women’s participation and argue that the flattening of female participation can partially account for jobless recoveries.

1.2 Organization

The motivating empirical evidence is presented in Section 2. We describe the model and discuss its main qualitative and quantitative properties in Section 3. In Section 4, we present estimation results for the model estimated for yearly data from 1969 to 2017. Section 5 concludes.

2 Evidence

We begin our analysis with descriptive evidence relating the changing trends in female labor force participation to the behavior of average labor productivity and TFP, aggregate per capita hours, the great moderation and the emergence of jobless recoveries. The analysis is based on the construction of time series for hours per capita and hourly wages by gender from micro data. These are not directly available from the standard sources, such as the Current Population Survey and Current Establishment Survey, for most of the period of interest. So we construct these measures from micro data from the CPS starting in 1968 for hours and 1969 for wages. The goal in the construction of these series is that they be as close as possible as the corresponding aggregates used in business cycle research and DSGE estimation.

We consider individuals 15 years or older and restrict attention to the non-farm business sector. We include both dependent workers and the self-employed and count hours worked at all jobs. Our measure of aggregate hours by gender are constructed from data on the number of employees, weeks worked per year and usual weekly hours. For wages, we consider yearly earnings and divide them by the number of yearly hours worked to obtain an estimate of hourly wages. Additional details on the construction of these series are discussed in Appendix A, which also displays plots and compares them to other measures of hours and wages by gender available directly from the Bureau of Labor Statistics starting in later periods (1976 and 1979, respectively). We also compare the implied aggregate series for the employment to population ratio, aggregate hours per capita and hourly wages to the available counterparts in the CPS and CES. We find that our constructed measures of hours and wages are comparable to the available series by gender available starting in later periods for the overlapping years, and that the corresponding series for aggregate hours closely matches the aggregate series directly available from the Bureau Labor Statistics.

The resulting series for yearly hours per capita and the corresponding female/male ratio, as well as the female/male ratio of hourly wages are displayed in figure 2. There was a rapid convergence in female and male hours between 1969 and 1993, when the female/male ratio of hours growth

from 0.39 to 0.67. Starting in 1993, the female/male hours ratio continues to grow, despite the flattening of female participation, though at a much slower rate. The female/male wage ratio was stable between 1969 and 1983, at around 0.65, and then grows steadily after that, including after female participation stopped growing, reaching values close to 0.8 by the end of the sample. The female share of aggregate hours and labor income grow in parallel throughout the sample period, and mostly reflects the behavior of female hours.

We now use the information on hours and wages by gender to examine the impact of women’s participation on changing business cycles.

2.1 Productivity Slowdown

We first relate the growth in female labor supply to the slowdown in productivity growth experienced by the United States starting in the early 1970s. Aggregate per capita hours appear to be non-stationary during the 1970s and 1980s. Figure 2 shows that male hours per capita have been stationary throughout the sample period. Figure 30 in Appendix B presents the trend component of the log of female and male hours. Whereas female hours grow by 0.14 log points between 1976 and 1993, male hours decline by 0.06 log points between 1968 and 1983, and then recover that loss by the early 1990s and do not display any trends. The trend components of female and male hours reveal that even in the 1990s there was a continued increase in female hours, but contrary to the prior period, it was very similar to male hours, leading to a constant female hours share over that decade. In light of the growth in female hours, it is not surprising that aggregate hours per capita should display trend like growth in the 1970s and 1980s.

Non-stationary hours are inconsistent with the standard real business cycle model, based on the assumption that time endowments are finite and constant over time.³ However, this is based on the notion that all agents in the model fully participate in the labor market. Since female participation was very low at the beginning of the sample period and grew over time, it is not surprising that female hours and as a consequence aggregate hours should appear non-stationary for most of the post-war period. Yearly aggregate hours per capita can be calculated as:

$$\frac{H_t^j}{P_t^j} = h_t^j \times w_t^j \times E_t^j / P_t^j, \quad (1)$$

where h_t^j denotes average hours worked per week and w_t^j average weeks worked per year for gender $j = f, m$. The evolution over time of each of these components for women and men is displayed in figure 28 in Appendix A, which shows that the growth in the employment to population is the most important factor in the growth of female hours.

The trend growth of aggregate per capita hours resulting from the rise in female hours has implications for the evolution of average labor productivity. Figure 3 plots the growth rate of average labor productivity and of female hours over the sample period. The late 1970s registered a

³Another dimension of the debate on the stationarity of aggregate hours in the empirical macroeconomics literature focusses on the implications for the response to a technology shock in structural VARs. See [14] and [15] and [21].

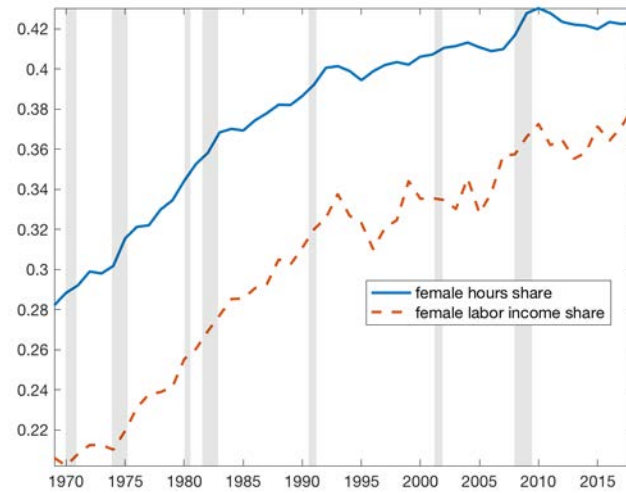
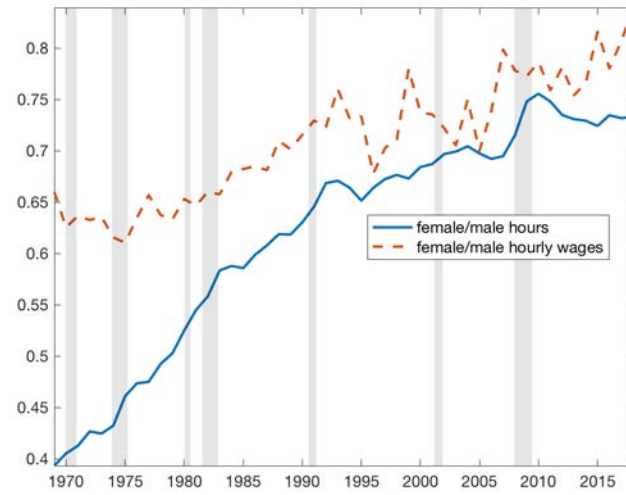
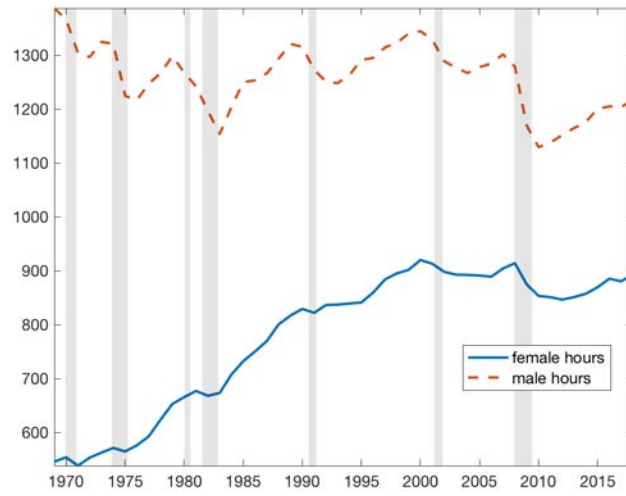


FIGURE 2: Female and male yearly aggregate hours per capita, female/male ratio of aggregate hours and hourly wages, female share of aggregate hours and labor income. Source: Author's calculations based on CPS.

dramatic reduction in the growth rate of average labor productivity, which coincides exactly with the acceleration in the growth of female hours. The causes of this slowdown are still debated, however, the leading explanations point to the acceleration in the growth of computing power, the rise in obsolescence and a number of measurement issues.⁴ However, the coincidence in timing with the acceleration of the growth in female hours offers a potential explanation.

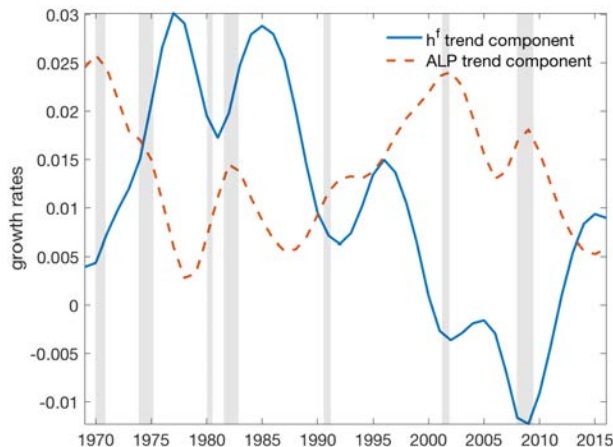


FIGURE 3: Trend component of logarithms of female aggregate per capita hours (h^f) and average labor productivity (ALP), yearly growth rates. Trend component extracted with Hodrick-Prescott filter with $\lambda = 6.5$. Source: Author’s calculations based on CPS March Supplement.

In Section 3.3, we use the model to quantify the role of female hours in productivity growth. We show that between the 1978 and 2003, the growth in female hours reduced average labor productivity growth, even as it contributed positively to TFP growth from 1981 onwards. This discrepancy is explained by the changing selection of women in the labor force. Women who entered the labor force in the early 1970s were negatively selected relative to men, as they were younger, less experienced and had lower levels of educational attainment. Subsequently, working women became older on average, due to increased attachment to the labor market of married women with children, and therefore more experienced, and they also displayed increasingly high levels of educational attainment. The resulting growth in female relative productivity, as reflected in the shrinking gender wage gap, explains why women contribute positively to TFP growth. However, as long as female hours continued to grow very strongly, the negative effect on productivity prevailed. Since female hours stopped growing in the 1990s, the growth in women’s productivity became dominant leading to reverse the impact of women’s effect of average labor productivity from negative to positive.

2.2 Great Moderation

The second phenomenon we consider is the great moderation, that is the decreased cyclical volatility of many key macroeconomic indicators observed starting in 1983.⁵ Focussing on labor market

⁴See for example [26], [7], or [34] for an overview.

⁵See [38] for an excellent review of the evidence.

indicators, the main phenomena associated with the great moderation can be summarized as follows:

1. Decline in the volatility of aggregate hours per capita and GDP;
2. Decline in the correlation between aggregate hours per capita and GDP;
3. Rise in the volatility of aggregate hours per capita relative to GDP;
4. Decline in the correlation between average labor productivity, aggregate hours per capita and GDP.

Figure 4 presents the standard deviation of the cyclical components of log GDP and log aggregate hours per capita in 1969-1982 and 1983-2011 (top panel) and the contemporaneous correlation of aggregate hours and GDP and their relative standard deviation. There is a very modest reduction in the volatility of aggregate hours, and a substantial reduction in the volatility of GDP, from 0.017 to 0.0125, with a reduction in the correlation of hours and GDP from 0.78 to 0.69. Consistent with [23], the standard deviation of per capita hours relative to GDP rose from 0.27 in 1969-1982 to 0.33 in 1983-2011.

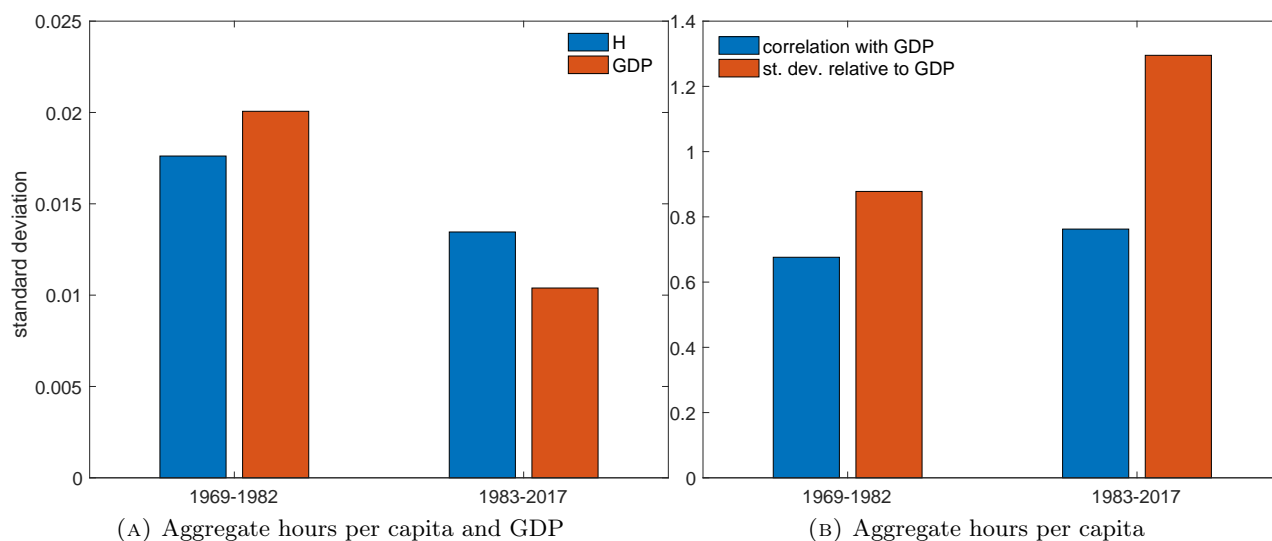


FIGURE 4: A: Cyclical component of log aggregate hours per capita and log GDP, 1969-1982 and 1983-2017 obtained with Hodrick-Prescott filter with $\lambda = 6.5$. B: Aggregate hours per capita, correlation with and standard deviation relative to GDP. Source: Author's calculations based on CPS March Supplement.

Figure 5 presents the contemporaneous correlation with GDP with the cyclical components of the log of female and male hours per capita for 1969-1982 and 1983-2017 and the relative standard deviation.⁶ Female hours are less procyclical than male hours in both periods. The correlation with GDP is 62% in 1969-1982 and 61% in 1983-2017 for female hours, while it is 68% and 79% for male hours. Similarly, the standard deviation relative to GDP is 67% in 1969-1982 and 1 in 1983-2017 for female hours, while it is 107% and 171% for male hours. Albanesi and Sahin (2018) [5]

⁶The cyclical component of hours by gender is reported in figure 30 in Appendix B.

show that the gender difference in the cyclical component of hours and employment can be mostly accounted for by the gender differences in occupational distribution in starting with the 1991 cycle, and by a combination of the occupational distribution and the trend in female participation in earlier cycles. Another factor affecting the cyclical component of female labor supply is the "added worker" effect (Lundberg (1985) [33]). For women who are married or cohabiting, the prospective of a loss in income for their spouse or partner may lead them to increase their own labor supply in response.⁷ At a cyclical frequency, this would tend to increase female labor supply during recessions, reducing the procyclicality of female hours.

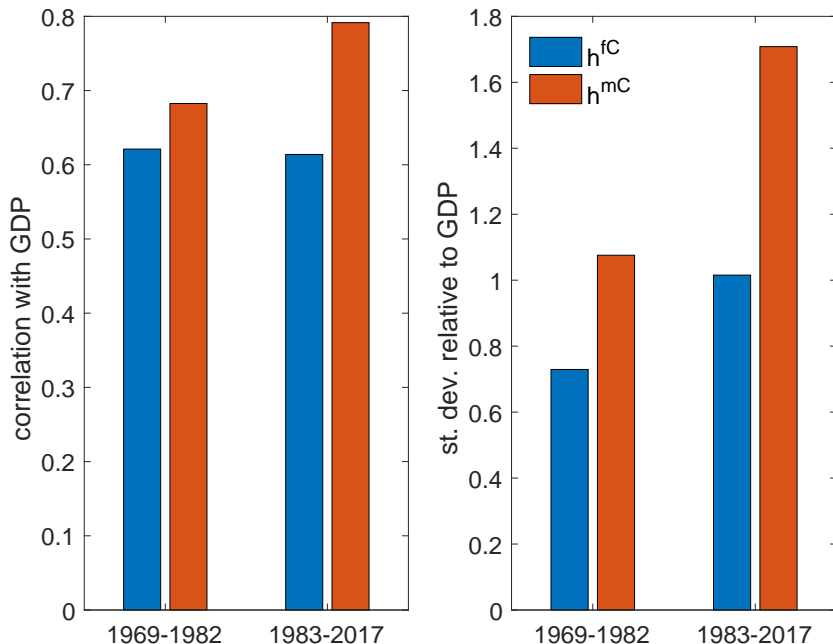


FIGURE 5: Cyclical component of female, male and aggregate hours per capita, obtained with Hodrick-Prescott filter with $\lambda = 6.5$. Contemporaneous correlation with and relative standard deviation to cyclical component of GDP, 1969-1982 and 1983-2017. Source: Author's calculations based on CPS March Supplement.

Figure 6 reports the contemporaneous correlation of aggregate labor productivity (ALP) with aggregate hours per capital and GDP before and during the great moderation. The figure shows a decline in the correlation of aggregate labor productivity with GDP, from 0.52 to 0.03, and a substantial decline in the correlation with hours, from -0.26 to -0.62, consistent with Galí and Gambetti (2009)[23].

This evidence suggests that rising female participation up until the early 1990s and the corresponding rise in the share of female hours may have reduced the cyclical component of aggregate hours over this period. The decline in the cyclical component of female hours the 1983-2011 further contributes this phenomenon. Moreover, the strong growth in female hours contributed positive to the growth in output but negatively to productivity growth, thus reducing the correlation between output, aggregate hours and productivity. In Section 3.3, we will develop a model based counterfactual that is

⁷Stevens (2002) [37] estimates the added worker effect in response to displacement and Juhn and Potter (2007) [29] examine the evolution over time of this effect.

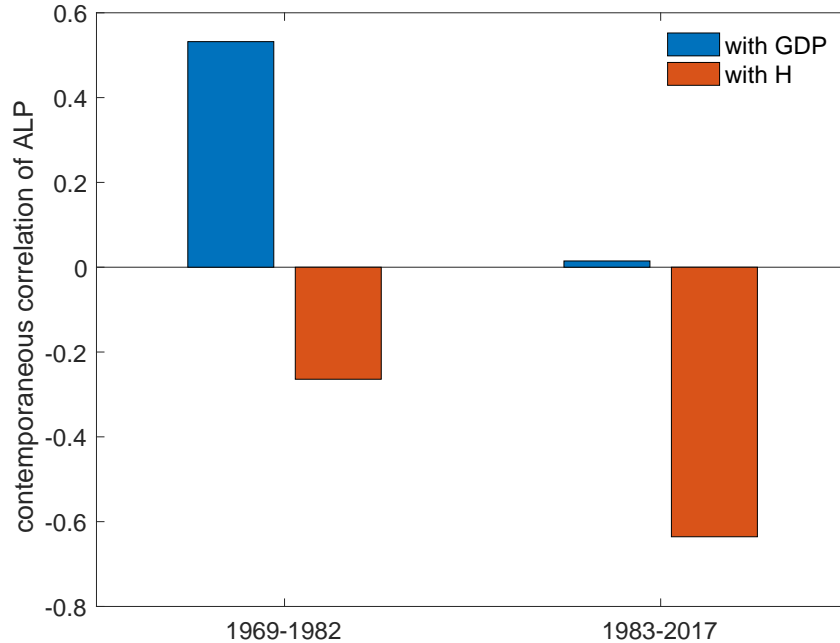


FIGURE 6: Cyclical component of aggregate labor productivity (ALP), aggregate per capita, and GDP obtained with Hodrick-Prescott filter with $\lambda = 6.5$. Contemporaneous correlation for 1969-1982 and 1983-2017. Source: Author’s calculations based on CPS March Supplement.

able to measure the contribution of rising female hours to the patterns associated with the great moderation.

2.3 Jobless Recoveries

As a final step, we consider the emergence of jobless recoveries, that is the slow growth of employment and hours after a recession, even as output bounces back from its cyclical trough (Foote and Ryan (2012) [20] and Jaimovich and Siu (2015) [28]). Jobless recoveries first emerged with the 1991 cycle, in conjunction with the flattening in female labor force participation, and this change in trend accounts for a large fraction of the jobless recoveries. Figure 7 displays evidence supporting this hypothesis. These charts plot the cumulative changes in the *log* of hours per capita from the unemployment through of the preceding expansion, ending three years after the unemployment rate peak. As figure 7 shows, in the 1970, 1974-75 and 1981-82 recessions, as women were experiencing a strong positive trend in labor force participation, hours per capita for women do not drop during the recession, while they experience a strong growth in the recovery. The 1990-91 cycle marks a change in this pattern, as the trend growth in female participation slowed substantially. While hours for women still do not decline during the recession, the recovery is weaker than in previous cycles. In the 2001 and 2007-09 cycles—when the upward trend in female participation had completely stopped—the behavior of female hours per capita is very similar to men’s, except for a more modest amplitude of the fluctuation.⁸ Figure 7 also shows that the behavior of male hours was very similar

⁸Albanesi and Sahin (2018) [5] show that the smaller decline in female hours during the 2007-2009 recession is mostly accounted for by gender differences in the industry distribution over this period.

in all these cycles, with the only variation driven by the severity of the recession. Hours per capita always declined for men during recessions and did not regain pre-recession values except for the 1981-82 cycle, with the recovery particularly sluggish in the most recent cycles. These figures show that the changing trend in female participation is reflected in a change in the cyclical behavior for hours for women starting in early 1990s, while for men both the trend and the cyclical behavior of participation has been similar since 1970.

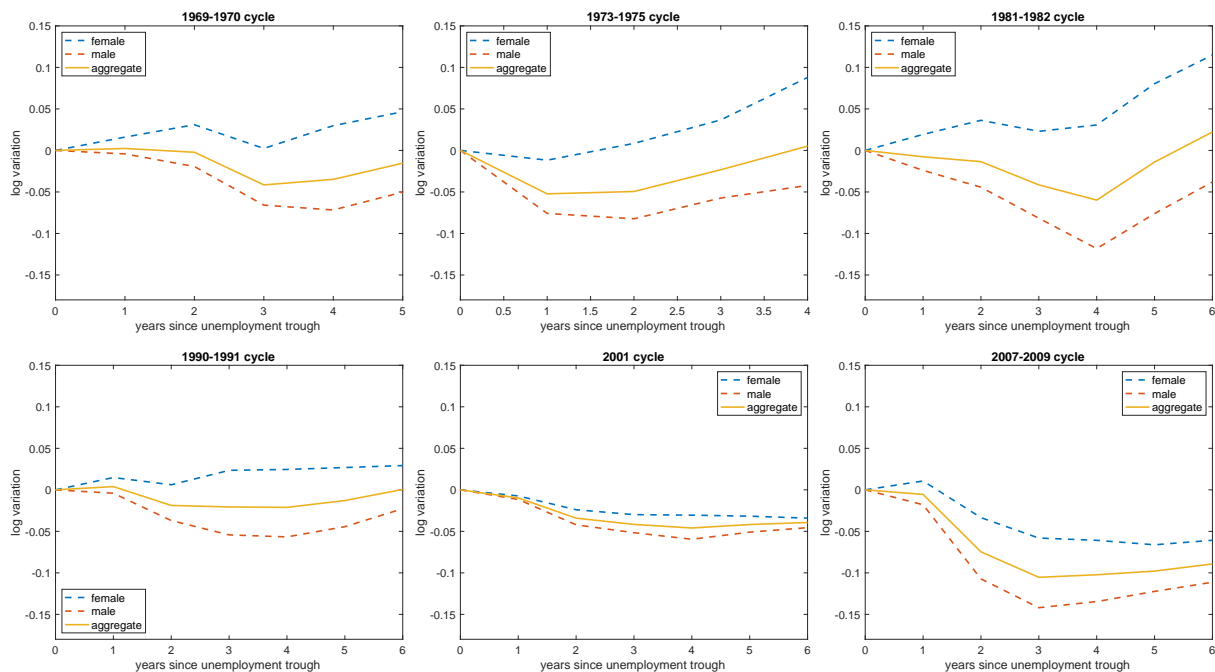
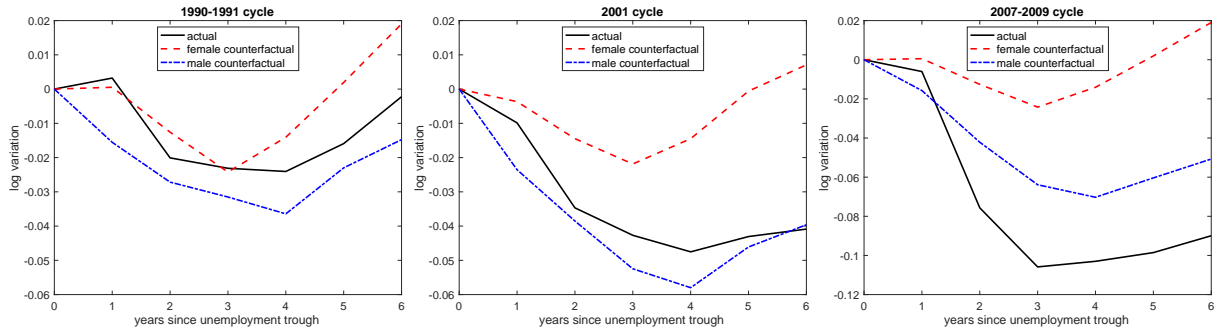


FIGURE 7: Log changes in hours per capita in the aggregate and by gender. Source: Author’s calculations based on CPS March Supplement.

To quantify the effects of the changing behavior of female hours on aggregate hours we now present some simple counterfactuals. Specifically, we compute counterfactual *aggregate* hours per capita for recent cycles, by replacing the *female* growth rate of hours per capita in each of the last three cycles with the average growth rate of female per capita hours in the early cycles at each date. We keep the evolution of male hours per capita as it is in the data, and compute the counterfactual aggregate using period specific population weights. We also run a similar counterfactual for male hours, in which female hours are maintained at their historical values while male hours are replaced by their average log variation in the three early cycles. The results are presented in the top panel of Figure 8.

These counterfactuals reveal that the change in female hours behavior is an important factor in driving aggregate hours during recoveries, especially for the 2001 and 2007-09 cycles. For the 1990-1991 cycle, the growth in counterfactual employment is approximately 2 percentage higher than the actual at the end of the cycle. For the 2001 cycle, the counterfactual with female hours features both a smaller decline in aggregate per capita hours during the recession, by 3 percentage points,

UNSCALED COUNTERFACTUALS



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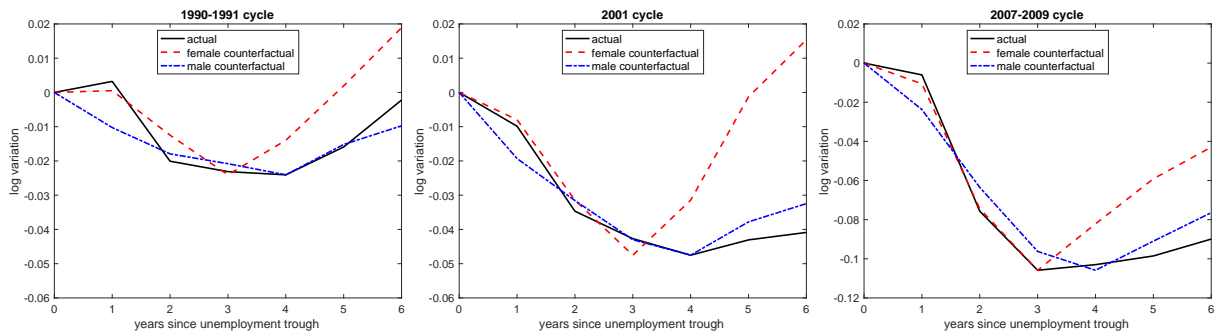


FIGURE 8: Female hours per capita counterfactual: Female hours per capita replaced with average for early recessions. Male hours per capita counterfactual: Female hours per capita replaced with average for early recessions. Scaled counterfactuals are scaled so that the drop in aggregate per capita hours is the same in the actual and counterfactual. Source: Author’s calculation based on CPS March Supplement.

and a 3 percentage points higher growth in hours during the recovery. For the 2007-2009 cycle, counterfactual aggregate hours per capita only drop very modestly and recover quickly ending the window above initial levels. The counterfactuals with male hours suggest changes in the behavior of men played a very small role. For the 1990-1991 and 2001 cycles, counterfactual aggregate hours drop by a similar amount than the actual and recover at a slower or equal rate. For the 2007-2009 cycle, counterfactual hours drop by approximately 3 percentage points less than the actual, but recover at the same rate. To summarize, if female hours growth had continued to exhibit the behavior seen for the early cycles, recoveries from the the last three recessions would not have been jobless.

The bottom panel of figure Figure 8 presents scaled counterfactuals in which the drop in female/male hours per capita in the counterfactual is scaled to be identical to the actual in each of the recessions. This exercise enables use to focus more specifically on the behavior of hours in the recovery. For all three cycles, the counterfactual recovery in aggregate hours for the female counterfactual is much stronger than the actual. Counterfactual hours grow beyond precession levels by 3 and 1 percentage points respectively in the 1990-1991 and 2001 cycles, whereas they still remain 5 percentage points below pre recession levels in the 2007-2009 cycle, though still 5 percentage points

higher than the actual. There is virtually no difference between the male counterfactual and the actual.

To summarize, the strong upward trend in women’s labor force participation through the 1970 and 1980s was mirrored in their employment growth and hours per worker, which masked the relatively weak recoveries in the employment and hours of men for cycles in that period. When female participation stopped growing in the early 1990s, the cyclical variation in female hours started to resemble that of men, resulting in very weak growth in aggregate employment and hours in cyclical recoveries.

3 Model

Our quantitative analysis is based on a variant of the real business cycle model with investment specific productivity shocks, investment adjustment costs and variable capital utilization often used in DSGE estimation.⁹ The economy is populated by a representative household and perfectly competitive representative firms. The household is comprised by a continuum of individuals of different gender. Household members have a joint utility from consumption with an internal habit, but exhibit gender specific disutility for supplying labor. The disutility of labor can change over time according to gender specific persistent shocks. Female and male labor are both used in production, as well as capital. There are time varying gender specific productivity shocks, in addition to an aggregate total factor productivity shock and an investment specific shock.

We now describe each component of the model in detail.

Households The representative household comprises a continuum of unit measure of agents of different gender. A fraction p_t^j of the population is of gender $j = f, m$, where f refers to female and m to male with $\sum_{j=f,m} p_t^j = 1$. All individuals of the same sex are identical. The household utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t b_{t+s} \left[\log(C_t - \eta C_{t-1}) - \sum_j p_t^j \varphi_t^j \frac{(H_t^j)^{1+\nu^j}}{1+\nu^j} \right],$$

where C_t is per capita consumption, η is the degree of habit formation and b_t is a shock to the discount factor, which follows the stochastic process:

$$\log b_t = \rho_b \log b_{t-1} + \varepsilon_{b,t},$$

with $\varepsilon_{b,t} \sim i.i.d. N(0, \sigma_b^2)$. H_t^j denotes per capita hours for agents of gender j and φ_t^j is a shock to the disutility of working, which is gender specific. These gender specific utility shocks follow stochastic processes described in Section ??.

This formulation of household utility reflects the assumption that there is consumption sharing within the household, so that all household members consume the same amount and their utility of

⁹For example, see Justiniano, Primiceri and Tambalotti (2010) [31].

consumption can be aggregated, while the disutility from working accrues to each member individually, and differs by gender. We allow both the scaling factor for the disutility from labor, $\tilde{\varphi}_t^j$, and the Frisch elasticity $1/\nu^j$ to vary by gender, to match systematic differences in average per capita hours by gender and micro evidence on gender variation in wage elasticity of labor supply.

Household members supply labor on competitive markets and the households rent capital. The resulting household budget constraint is:

$$C_t + I_t + T_t \leq \sum_j p_t^j W_t^j H_t^j + r_t^k u_t \bar{K}_{t-1} - a(u_t) \bar{K}_{t-1},$$

where T_t is lump-sum taxes, w_t^j is the real wage for gender j and r_t^k is the rental rate of effective capital. Effective capital is:

$$K_t = u_t \bar{K}_{t-1},$$

where u_t is a utilization rate set by the household, at cost $a(u_t)$ per unit of physical capital \bar{K}_t . In steady state, $u = 1$, $a(1) = 0$ and $\chi \equiv \frac{a''(1)}{a'(1)}$. In the log-linear approximation of the model solution this curvature is the only parameter that matters for the dynamics.

The physical capital accumulation equation is:

$$\bar{K}_t = (1 - \delta) \bar{K}_{t-1} + \mu_t \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t,$$

where δ is the depreciation rate and the investment shock μ_t follows the stochastic process:

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t},$$

with $\varepsilon_{\mu,t} \text{ i.i.d. } N(0, \sigma_\mu^2)$. The function S captures the presence of adjustment costs in investment, as in Christian, Eichenbaum and Evans (2005) [13]. In the steady state, $S = S' = 0$ and $\zeta \equiv S'' > 0$.¹⁰

Firms Production is conducted by competitive firms by using capital and labor rented on competitive factor markets. The production function, expressed in per capita terms, is:

$$Y_t = K_t^\alpha \left(\tilde{A}_t \tilde{L}_t \right)^{1-\alpha}, \quad (2)$$

where Y_t is output and \tilde{L}_t is a total labor input. Total labor input is derived from hours supplied by male and female workers, and combined according to a CES aggregator:

$$\tilde{L}_t = \left[\omega^f \left(\tilde{L}_t^f \right)^\rho + \omega^m \left(\tilde{L}_t^m \right)^\rho \right]^{1/\rho}. \quad (3)$$

The parameter $\rho \in (-\infty, 1]$, determines the substitution elasticity between female and male labor inputs, with $\rho = 1$ corresponding to perfect substitutability, $\rho \rightarrow -\infty$ corresponding to a Leontieff

¹⁰Households are assumed to own firms and collect profits, which are zero in equilibrium, and omitted from the household's budget constraint for brevity.

production function and $\rho \rightarrow 0$ representing the Cobb-Douglas case. The parameters $\omega^j \in [0, 1]$ for $j = f, m$, with $\omega^f + \omega^m = 1$, correspond to weight of the gender specific contribution to total labor input. The labor inputs \tilde{L}_t^j are measured in efficiency units per capita:

$$\tilde{L}_t^j = \frac{a_t^j p_t^j H_t^j}{a^j p^j H^j}, \quad (4)$$

where a_t^j is a gender specific productivity index. All variables are normalized by their steady state value, which is indicated by dropping the time subscript.

This normalization implies that the steady state value of the effective labor input L_t is normalized to $L = 1$, and the distribution parameters in the CES aggregator, ω^j , are equal to the income shares in steady state. To see this, we use the homogeneity of the production structure to normalize the production function as:

$$\begin{aligned} Y_t &= K_t^\alpha (A_t L_t)^{1-\alpha}, \\ L_t &= \left[\omega^f \left(\tilde{a}_t^f \frac{H_t^f}{H^f} \right)^\rho + \omega^m \left(\frac{H_t^m}{H^m} \right)^\rho \right]^{1/\rho}, \end{aligned}$$

with

$$\begin{aligned} L_t &= \frac{\tilde{L}_t}{\frac{a_t^m p_t^m}{a^m p^m}}, \\ A_t &= \tilde{A}_t \frac{a_t^m p_t^m}{a^m p^m}, \\ \tilde{a}_t^f &= \frac{a_t^f / a^f}{a_t^m / a^m} \pi_t^f, \\ \pi_t^f &= \frac{p_t^f / p^f}{p_t^m / p^m}. \end{aligned}$$

Female relative productivity in production then depends on two factors, the ratio of female to male raw productivities, a_t^j / a^j for $j = f, m$, and the relative fraction of women in the population, π_t^f / π^f , which in the steady state are both equal to 1, so that relative female productivity in steady state is also equal to 1. It follows that that $L = 1$ and that, if male productivity is stationary, the growth rate of the augmented TFP factor A_t is the same as the growth rate of \tilde{A}_t .

The relative efficiency of women follows the stationary stochastic process:

$$\begin{aligned} \log \tilde{a}_t^f &= \log \tilde{a}_t^{fT} + \log \tilde{a}_t^{fC}, \\ \log \tilde{a}_t^{fT} &= \rho_{afT} \log \tilde{a}_{t-1}^{fT} + \varepsilon_{afT,t}, \\ \log \tilde{a}_t^{fC} &= \rho_{afC} \log \tilde{a}_{t-1}^{fC} + \varepsilon_{afC,t}, \end{aligned}$$

with $\varepsilon_{axf,t}$ i.i.d. $N(0, \sigma_{axf}^2)$ for $X = T, C$. This decomposition of the relative efficiency into a trend

(T) and a cycle (C) component, allows us to isolate the slow-moving component of this process, without at the same time overly restricting its cyclical frequency behavior.¹¹

The aggregate technology factor A_t , which compounds total factor and male-specific productivity, follows a stationary AR(1) process in its growth rate $z_t \equiv \Delta \log A_t$:

$$z_t = (1 - \rho_z)\gamma + \rho_z z_{t-1} + \varepsilon_{z,t},$$

with $\varepsilon_{z,t}$ distributed *i.i.d.* $N(0, \sigma_z^2)$. This specification implies that the level of technology is non stationary.

Resource Constraint The aggregate resource constraint is:

$$C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t,$$

where government spending is a time-varying fraction of GDP,

$$G_t = \left(1 - \frac{1}{g_t}\right) Y_t,$$

and the government spending shock g_t follows the stochastic process:

$$\log g_t = (1 - \rho_g) \log g + \rho_g \log g_{t-1} + \varepsilon_{g,t},$$

with $\varepsilon_{g,t} \sim i.i.d. N(0, \sigma_g^2)$.

Equilibrium An equilibrium for this economy can be defined in the standard way. In the following section, we describe the main properties of the equilibrium to provide intuition on the role of gender differences in the model.

Before we proceed to characterize the model solution, we derive some properties of gender ratios in the model that will be useful in deriving the steady state conditions and aid in defining processes for the exogenous gender specific shocks that can be identified using data on hours, wages and income shares by gender.

3.1 Implications for Wages and Income Shares

The households' and firms' first order necessary conditions for optimality are derived in Appendix C and they imply a number of equilibrium restrictions that are useful for understanding the workings of the model and aspects of the identification in the estimation.

¹¹A more parsimonious alternative would be to assume that there is no cyclical component in the relative efficiency of the two genders, and that all cyclical fluctuations come from the aggregate component. An intermediate case would be to add to the trend component just an i.i.d. process, rather than an AR(1).

The optimality conditions for labor supply imply:

$$\frac{W_t^f}{W_t^m} = \frac{\varphi_t^f (H_t^f)^{\nu^f}}{\varphi_t^m (H_t^m)^{\nu^m}}, \quad (5)$$

from which we can recover the *relative* disutility of effort $\tilde{\varphi}_t^f \equiv \frac{\varphi_t^f}{\varphi_t^m}$ from observations on wages and hours worked, given the Frisch elasticities ν^f and ν^m :

$$\log \tilde{\varphi}_t^f = \log W_t^f - \log W_t^m - \nu^f \log H_t^f + \nu^m \log H_t^m. \quad (6)$$

From the mens' labor supply function:

$$W_t^m = \frac{\varphi_t^m (H_t^m)^{\nu^m}}{\Lambda_t}, \quad (7)$$

where Λ_t is the marginal utility of consumption, it is then possible to recover the level of φ_t^m conditional on Λ_t .

These equilibrium conditions suggest modeling women's relative disutility and men's absolute disutility of labor as independent processes:

$$\begin{aligned} \log \left(\tilde{\varphi}_t^f / \tilde{\varphi}^f \right) &= \log \tilde{\varphi}_t^{fT} + \log \tilde{\varphi}_t^{fC}, \\ \log \tilde{\varphi}_t^{fT} &= \rho_{\tilde{\varphi}^{fT}}^T \log \tilde{\varphi}_{t-1}^{fT} + \varepsilon_{\tilde{\varphi}^{fT},t}, \\ \log \tilde{\varphi}_t^{fC} &= \rho_{\tilde{\varphi}^{fC}}^C \log \tilde{\varphi}_{t-1}^{fC} + \varepsilon_{\tilde{\varphi}^{fC},t}, \\ \log (\varphi_t^m / \varphi^m) &= \rho_{\varphi^m} \log \varphi_{t-1}^m + \varepsilon_{\varphi^m,t}, \end{aligned}$$

with $\varepsilon_{\tilde{\varphi}^{fX},t}$ is i.i.d. as $N(0, \sigma_{\tilde{\varphi}^{fX}}^2)$ for $X = T, C$ and $\varepsilon_{\varphi^m,t}$ is i.i.d. as $N(0, \sigma_{\varphi^m}^2)$.¹² This formulation of the labor supply shocks parallels the one adopted for the gender specific productivities, and similar considerations hold in terms of the trend/cycle decomposition.¹³

The labor demand equations derived from the firms' optimality conditions determine the labor income shares by gender:

$$\frac{W_t^f H_t^f}{Y_t} = (1 - \alpha) \omega^f \left(\frac{\tilde{a}_t^f (H_t^f / H^f)}{L_t} \right)^\rho, \quad (8)$$

¹²The averages of the φ processes pin down the levels of hours for men and women in steady state, but have no effect on the model's dynamics. We do not exploit this restriction, since the hours used in estimation are in deviation from their steady state value.

¹³One difference is that, unlike for the productivities, whose level is not separately identified from that of the share parameters ω^j , the steady state levels of the disutility of work φ could be identified from information on the level of hours and wages by gender. However, we only retain information on the relative levels of hours and wages to pin down the relative disutility in steady state $\tilde{\varphi}^f$.

$$\frac{W_t^m H_t^m}{Y_t} = (1 - \alpha) \omega^m \left(\frac{(H_t^m / H^m)}{L_t} \right)^\rho, \quad (9)$$

and relative labor incomes:

$$\frac{W_t^f H_t^f}{W_t^m H_t^m} = \frac{\omega^f}{\omega^m} \left(\frac{\tilde{a}_t^f H_t^f}{H_t^m} \right)^\rho. \quad (10)$$

These equations imply that the *relative* paths for gender specific productivity can be easily recovered from observations on hours and wages, given the CES parameter ρ :

$$\log \tilde{a}_t^f = \frac{1}{\rho} \left[\log W_t^f - \log W_t^m - \log \frac{\omega^f}{\omega^m} \right] + \left(\frac{1}{\rho} - 1 \right) \left[\log H_t^f - \log H_t^m \right]. \quad (11)$$

Unitary household utility from consumption and the separability of utility between consumption and labor imply that the marginal utility of income is equalized across genders. It follows that:

$$\frac{(H_t^m)^{\nu^m}}{W_t^m} = \frac{\tilde{\varphi}_t^f (H_t^f)^{\nu^f}}{W_t^f},$$

which combined with the firm's optimality conditions implies:

$$\frac{(H_t^m)^{1+\nu^m-\rho}}{\frac{\omega^m}{(H^m)^\rho}} = \frac{\tilde{\varphi}_t^f (H_t^f)^{1+\nu^f-\rho}}{\frac{\omega^f}{(H^f)^\rho} (\tilde{a}_t^f)^\rho}. \quad (12)$$

This equation clearly shows that both the efficiency and the disutility factor have an effect on hours in equilibrium. This would not be the case under a Cobb-Douglas specification, in which $\rho = 0$, since with that specification income and substitution effects of the increase in gender-specific productivity and wages cancel out.

3.2 Steady State

Total factor productivity A_t is not stationary in the model, therefore, we define normalized stationary variables that are constant relative to GDP in a non-stochastic version of the model, and then characterize the solution in terms of these rescaled variables. This economy also features two additional variables that are stationary but could be highly persistent. These are female labor supply shock $\tilde{\varphi}_t^f$ and the female relative productivity shock \tilde{a}_t^f . As shown in figure 2, the female hours share and the female labor income share are stable in 1995-2004, but outside this period they could exhibit large deviations from their 1995-2004 values.

The derivation of the aggregate variables in steady state is standard and is presented in Appendix

C.5. In addition, equations 8 and 9 pin down the female and male labor share in the steady state:

$$\begin{aligned} w^f &= (1 - \alpha) \frac{y}{H^f} \omega^f, \\ w^m &= (1 - \alpha) \frac{y}{H^m} \omega^m, \end{aligned}$$

for $j = f, m$. which are equal to ω^f and ω^m , respectively.

3.3 Dynamics

The full equilibrium characterization, the log linearization and the derivation of the state equations are presented in detail in Appendix C. Here, we focus on some key dynamic properties of the model that are useful in understanding the role of the changing trends in female hours and wages for aggregate outcomes. Specifically, the differentiation by gender of hours and productivity provide a theory for the Solow residual and aggregate labor productivity, and drive a set of low frequency correlations between these variables and aggregate per capita hours that are different from those that would arise in standard business cycle model.

3.3.1 Gender Specific Shocks

Figure 2 shows some distinct phases in the evolution of gender ratios over the period of interest. For the female/male hours ratio these occur in 1974, 1983, 1993 and 2005, and in 1974, for the female share of labor income these occur in 1993 and 2005. In 1969-1973, we see a rise in female hours relative to men's but no rise in the female share of labor income or in female wages compared to male. This time period was characterized by entry into the labor force of young women with low labor market experience and with lower educational attainment than men (see Olivetti (2006) [35]). Between 1974 and 1993, we see a parallel rise of the female share of aggregate hours and the female labor income share. In 1982-1993, we also observe a rise in the female/male wage ratio, as women in the labor force accumulate experience and their educational attainment grows.¹⁴ Between 1993 and 2005, we still see a very modest growth in the female share of hours and labor income, mostly driven the the decline in male hours, with the female/male ratio of wages remaining stable. The years after 2005 are dominated by the great recession, with men's hours declining much more then female hours, and substantial rise in women's relative hours and wages.

We can use equations (6) and (11) to back out the processes for the female specific shocks from the historical path of the female/male hours ratio and the female share of labor income at the calibrated values of ν^f , ν^m , ω^f , ω^m , and ρ , derived in Section 4. The resulting series for the gender specific shocks are plotted in the left panel of figure 9. Both shocks exhibit substantial trend and cyclical variation.¹⁵ The labor supply shock, $\tilde{\varphi}_t^f$, declines by 12 percentage points between 1969

¹⁴Educational attainment of women in the workforce surpassed men in 1993 and the gap in favor of women has continued to grow at a steady pace since then. See [5] for more details.

¹⁵Figure 31 in Appendix C.1 plots the trend and cyclical components of each of the female shocks.

and 1983 and afterwards shows a strong reversal, with values fluctuating around 0 from 1988 to the end of the sample period. The female relative productivity shock, \tilde{a}_t^f , does not start rising until the early 1980s, and experiences a 25 percentage point uninterrupted rise until the early 1990s, after which it fluctuates around an upward trend. Both the female productivity shock and the female labor supply shock are procyclical. Procyclicality is particularly pronounced for the labor supply shock, which imparts a countercyclical variation to female hours. This is confirmed by the right panel of figure 9, which shows the correlogram of the cyclical component of the labor supply and productivity shocks with the cyclical component of GDP. The labor supply shock and GDP have a contemporaneous correlation above 0.6, while the correlation between the female productivity shock and GDP is just below 0.2 contemporaneously and at a one year lead.

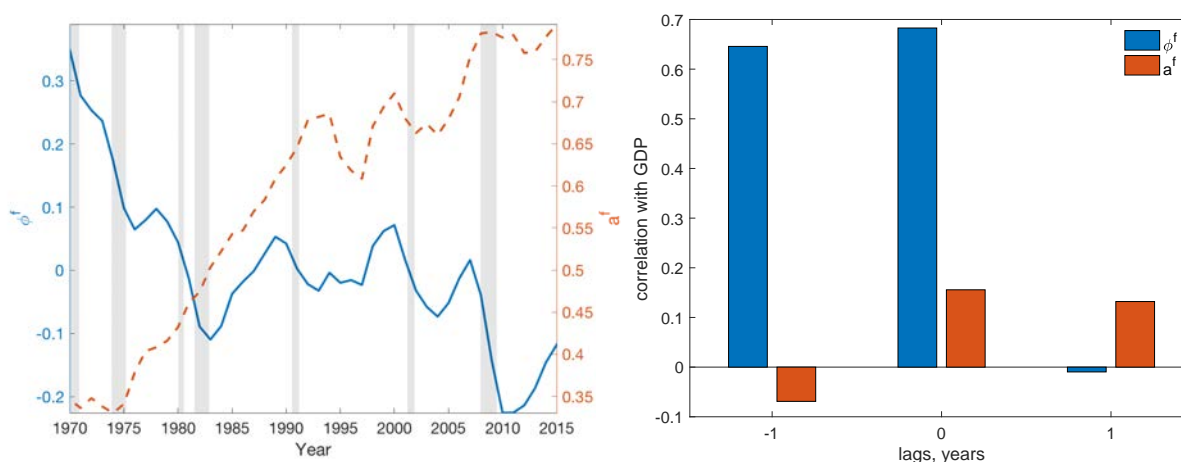
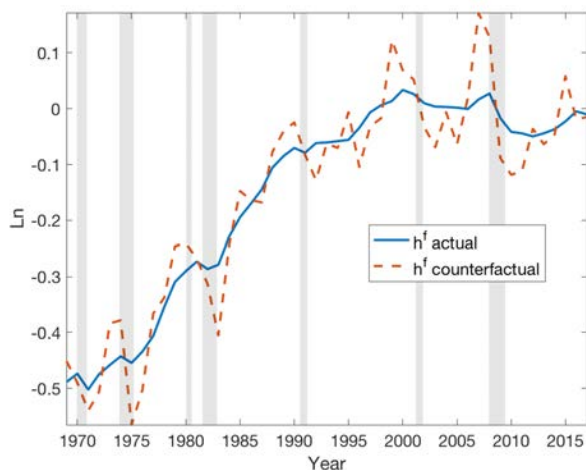
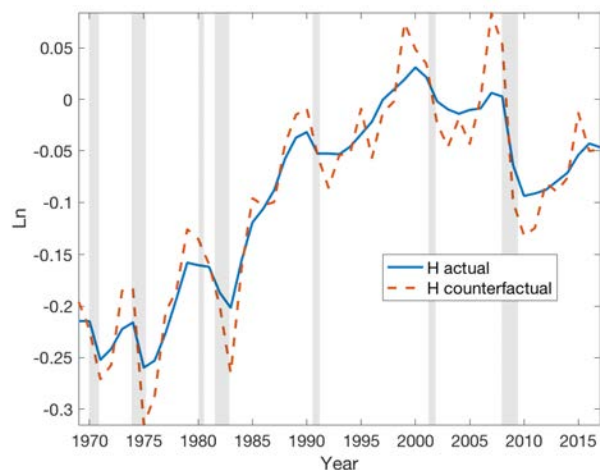


FIGURE 9: Female relative productivity shock and labor supply shock, as defined in equations (6) and (11), 3 year MA. Calibrated model. Cross-correlation of cyclical components of relative productivity shock and labor supply shock with GDP. Source: Author’s calculations based on Current Population Survey.

To illustrate the role of the female labor supply shock in the cyclicity of female and aggregate hours, we use a counterfactual. Specifically, we remove the cyclical component from $\tilde{\varphi}_t^f$, and then compute the model implied h_t^f from equation (6). The results are presented in figure 10. Panel (A) presents actual and counterfactual female hours, and panel (B) the corresponding series for aggregate hours. Counterfactual hours show higher business cycle volatility than actual hours, as confirmed by figure 11, which reports the contemporaneous correlation with and the standard deviation relative to GDP of the cyclical components of actual and counterfactual female hours. Both indicators are much higher for counterfactual female hours, with the gap growing substantially in 1983-2017 compared to 1969-1982, suggesting the countercyclical labor supply shock plays a stronger role in the latter period. Taken together, this evidence suggests that the female labor supply shock reduces the cyclicity of female hours, in a manner that is consistent with the presence of an added worker effect.

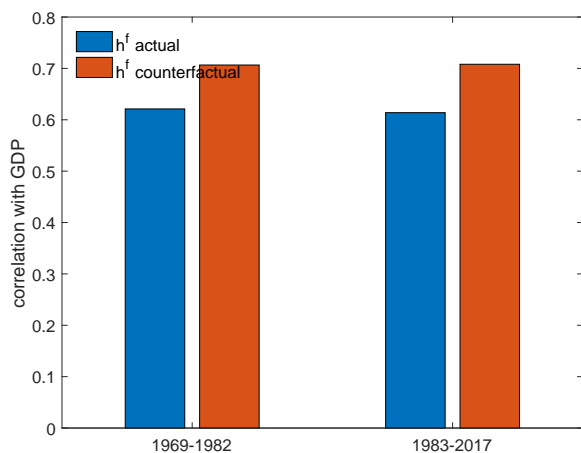


(A) Actual and counterfactual female hours

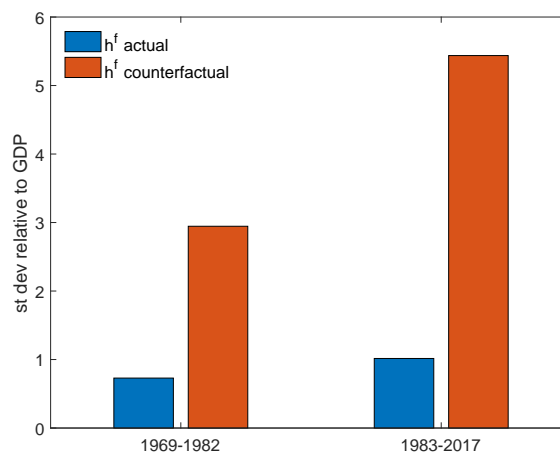


(B) Actual and counterfactual aggregate hours

FIGURE 10: (A): Actual and counterfactual female hours. (B): Actual and counterfactual aggregate hours. Counterfactuals are obtained by removing the cyclical component of the female labor supply shocks and computing the corresponding model generated hours. Source: Author's calculations based on CPS.



(A) Contemporaneous correlation with GDP



(B) Standard deviation relative to GDP

FIGURE 11: Actual and counterfactual female hours. (A): Contemporaneous correlations with GDP per capita. (B): Standard deviation relative to GDP. Counterfactuals are obtained by removing the cyclical component of the female labor supply shocks and computing the corresponding model generated hours. Source: Author's calculations based on CPS.

3.3.2 Output, Productivity and Hours

We now turn to the relation between hours, output and productivity. The log variation of output around the steady state is:

$$\hat{y}_t = (1 - \alpha)\hat{z}_t + \alpha\hat{k}_t + (1 - \alpha)\hat{L}_t, \quad (13)$$

where k_t denotes effective capital and includes utilization and the last term corresponds to the log variation from steady state of the labor supply aggregator:

$$\hat{L}_t = \left[\omega^f \left(\hat{a}_t^f + \hat{h}_t^f \right) + \omega^m \hat{H}_t^m \right]. \quad (14)$$

We can express aggregate hours per capita as:

$$H_t = \pi_t^f \frac{H_t^f}{H^f} + \frac{H_t^m}{H^m}. \quad (15)$$

Here, we use actual hours, not hours in efficiency units, as these correspond to measured hours in the data and would be the relevant concept of hours for deriving the Solow residual and aggregate labor productivity. Moreover, this definition of aggregate hours per capita implies that their state stave value H is equal to 1, consistent with the scaling adopted for the labor input aggregator in equation (3). Then, the log variation in aggregate per capita hours from the steady state is:

$$\hat{H}_t = \hat{h}_t^f + \hat{\pi}_t^f + \hat{H}_t^m. \quad (16)$$

Based on the production function in (2), (14) and (16), the log variation of the Solow residual around the steady state, denoted with \hat{s}_t can be written as:

$$\hat{s}_t = (1 - \alpha)\hat{z}_t + \alpha\hat{u}_t + (1 - \alpha) \left(\omega^f \hat{a}_t^f - \hat{\pi}_t^f \right). \quad (17)$$

Then, if female relative productivity is growing faster than the female population share, that is $\omega^f \hat{a}_t^f > \hat{\pi}_t^f$, the growth in female relative productivity contributes positively in variations of the Solow residual. This condition is likely satisfied most of the time as the variation in the female share of the population is negligible.

We can derive a similar expression for the log variation from steady state of average labor productivity, which we will denote with P_t :

$$\hat{P}_t = (1 - \alpha)\hat{z}_t + \alpha \left(\hat{u}_t + \hat{\bar{k}}_t \right) - \alpha \left(\hat{H}_t^m + \hat{\pi}_t^f \right) + \left[(1 - \alpha)\omega^f - 1 \right] \hat{h}_t^f + (1 - \alpha) \left(\omega^f \hat{a}_t^f - \hat{\pi}_t^f \right), \quad (18)$$

where \bar{k}_t is measured capital. As usual, growth in male hours contributes negatively to the growth in average labor productivity. The growth in female hours contributes positively to the growth in average productivity only if the the growth in female labor productivity is sufficiently larger than the growth in female hours, since $(1 - \alpha)\omega^f < 1$. Even as the growth in female hours boosts output and the growth in female relative productivity contributes to the growth in the Solow residual,

average labor productivity may fall if the growth in female hours is very strong.

Figure 12 plots the female contribution to average labor productivity and Solow residual growth. The female contribution to average labor productivity growth is mostly negative between the early 1980s and 1997 and then it turns mostly positive, as the growth in female hours declines and the growth in female relative productivity accelerates. By contrast, the female contribution to Solow residual growth is mostly positive after 1975.

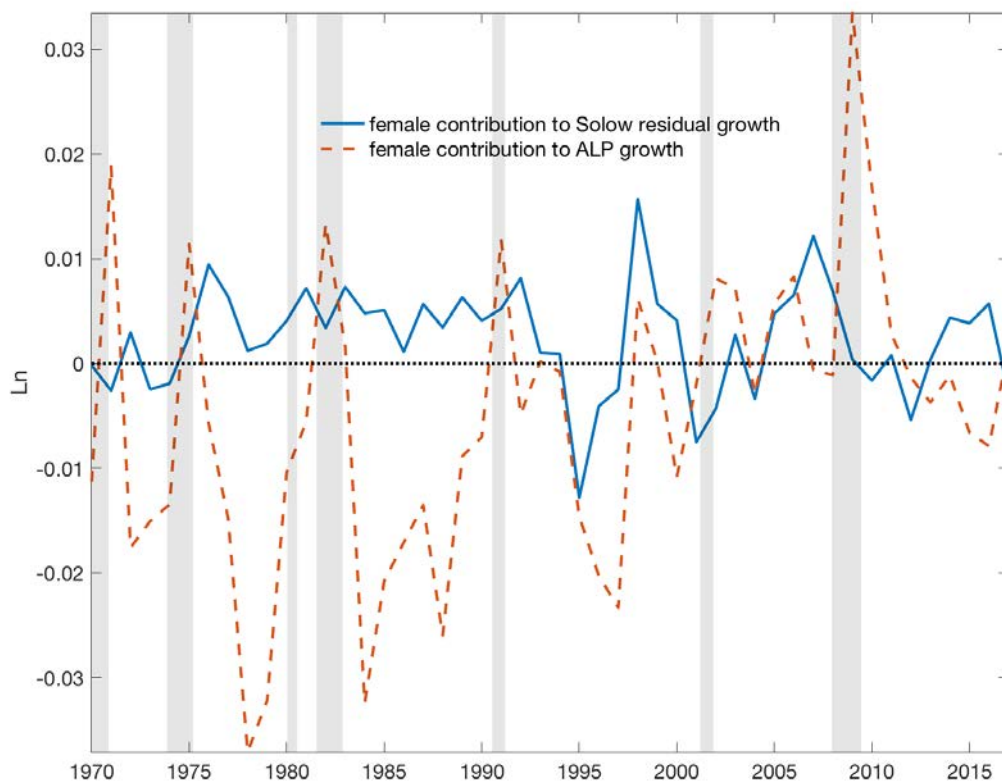


FIGURE 12: Average labor productivity (ALP) and Solow residual growth rates, female contributions as derived in equations (17) and (18). Model implied from calibrated parameters.

These derivations suggest that the the long run behavior of average labor productivity and the Solow residual depends on the relative strength of the growth in female hours and relative productivity, which also affect the cyclical correlation between productivity, output and aggregate hours. We now investigate these forces in more detail.

Trends Given the sizable magnitude of the female contribution to average labor productivity and Solow residual growth shown in figure 12, it is likely that they had an affect on the trend behavior of these variables over the sample period.¹⁶ To quantify this impact, we derive counterfactual values of

¹⁶Figure 32 in Appendix C.2 shows the trend components of average labor productivity and the Solow residual, with the corresponding female contributions.

the trend component of these variables obtained from the actual by subtracting the trend component of the respective female contributions. We report the corresponding growth rates in figure 13.

We find that starting in 1978, the growth rate of average labor productivity would have been approximately 0.5 percentage points higher without the female contribution between 1969 and the mid 1990s. However, since 2002 ALP growth would have been about half a percentage point lower without the female contribution, reflecting the sharp rise of the female contribution to ALP over that period. By contrast, the growth rate of the Solow residual would have been about 0.5 percentage points lower in 1975-1990 without the female contribution and about 0.25 percentage points lower in the mid 2000s. The gap between the growth rate of the actual and counterfactual trend component of the Solow residual tends to increase in expansions, and is at its highest in 1987 and 2006.

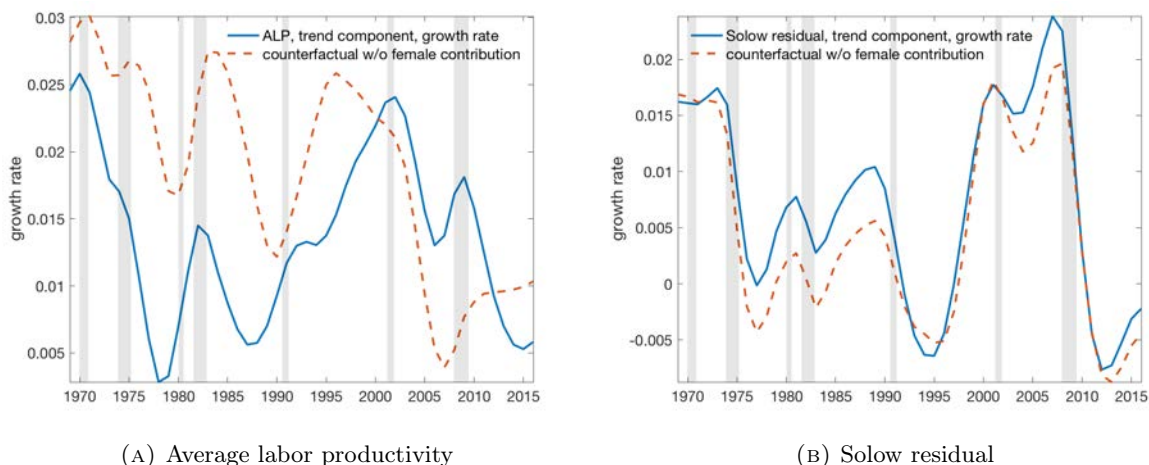


FIGURE 13: Average labor productivity (ALP) and Solow residual trend component growth rates, actual and counterfactual obtained by subtracting the female contribution.

These results have important implications for growth accounting in the United States. First, the rise in female participation may have contributed to the productivity slowdown in the mid-1970s and reduced average labor productivity growth until the mid-1990s. In the early period, this was mainly driven by the low levels of experience and educational attainment of women entering the workforce, while in the late 1980s and early 1990s, the strong growth in female hours played the largest role. Second, the average labor productivity and TFP growth declines in the 2000s (see Fernald (2015) [17]) would have been amplified without the contribution of female employment. By this stage, female hours stopped growing, so it is the growth in female relative productivity that drives this outcome. The results suggest that neglecting the specific contribution of female labor to these phenomena clearly has implications for the interpretation of their origins.

Cyclical Variation We now turn to the cyclical variation in average labor productivity. Figure 14 shows the cyclical components of average labor productivity and its corresponding female con-

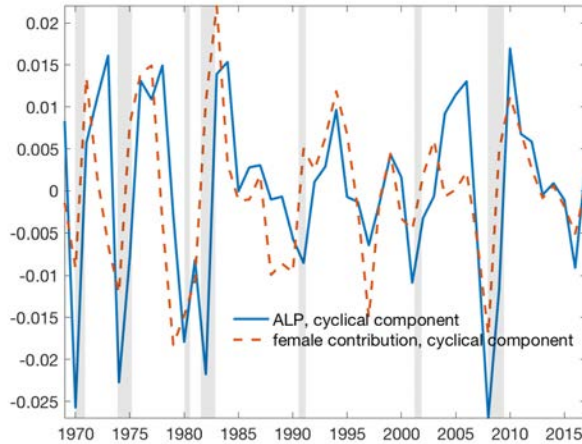


FIGURE 14: Average labor productivity (ALP) and female contribution, cyclical components.

tribution. The cyclical component of the female contribution exhibits smaller amplitude of cyclical fluctuations than average labor productivity. Additionally, it appears to be countercyclical between 1969 and 1985, and then turns pro cyclical, whereas average labor productivity is mostly pro cyclical, though the amplitude of its fluctuations declines markedly starting in 1984.

To explore the role of the female contribution in the cyclicity of average labor productivity, we calculate the contemporaneous correlation of the actual and counterfactual series for each variable as well as the corresponding female contributions, with the cyclical components of GDP and aggregate hours per capita. Figure 15 presents the results. The female contribution is negatively correlated with both GDP and aggregate per capita hours in 1969-1992, with correlation values of -0.56 and -0.95, respectively. In 1993-2017, these correlations are mostly stable. The correlation between actual ALP and GDP drops from 0.53 in 1969-1982 to virtually zero in 1983-2017, whereas its correlation with aggregate hours per capita drops from -0.24 in 1969-1982 to -0.62 in 1983-2017. Counterfactual ALP without the female contribution displays much higher correlation with both GDP and aggregate hours per capita. The correlation with GDP is 0.97 in 1969-1982 and drops to 0.63 in 1983-2017, while the correlation with aggregate hours per capita is 0.45 in 1969-1982 and drops to approximately 0.12.

These patterns are broadly consistent with important facts about the behavior of productivity and hours during the great moderation emphasized in [23]. In particular, there was a sizable decline in the correlation between hours and productivity, and the correlation between output and labor productivity also declined. These results suggest that since the correlation between the female contribution and productivity is negative, a rising share of female hours contributed to these changes. However, female behavior also changed, as correlation of the female contribution to productivity with output and hours grew substantially after 1983. Had it remained the same, the growth in female hours would have resulted in an even larger reduction of the correlation of average labor productivity with GDP and hours.

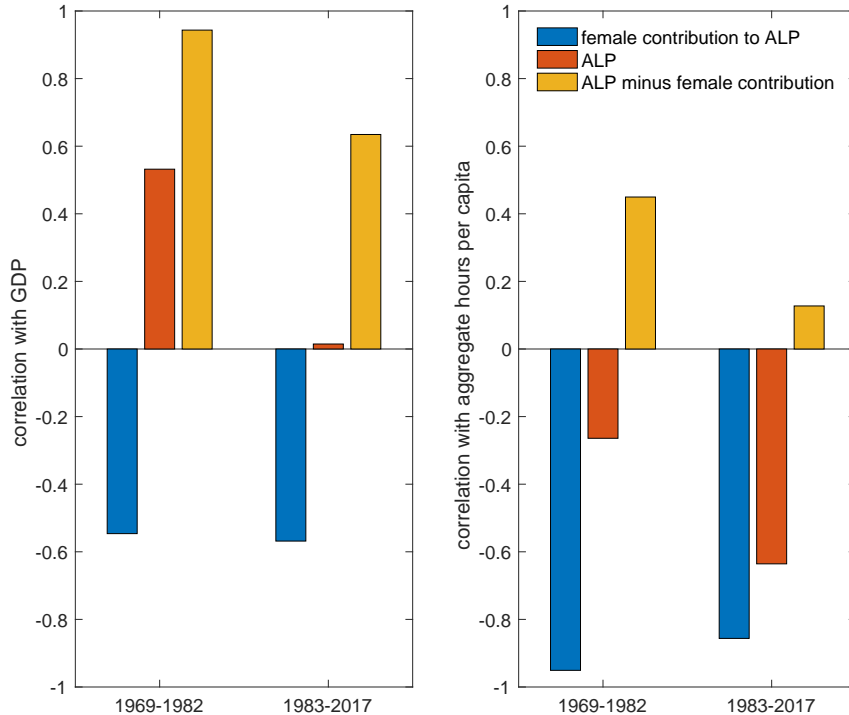


FIGURE 15: Average labor productivity (ALP): actual, counterfactual without female contribution and female contribution, cyclical components. Contemporaneous correlations with GDP (left panel) and aggregate hours per capita (right panel), cyclical components.

4 Estimation

We log linearize the model around the non-stochastic steady state and then estimate it with Bayesian methods (see for example [6]). We use the steady state equations described in Section to calibrate certain parameters of the model for which independent evidence is available, and we estimate all remaining structural parameters. As shown in figure 2, the growth of female hours relative to men’s and of the female labor income share is very close to zero between 1993 and 2006. Thus, we take the 1995-2004 decade to correspond to the steady state in the model, and set the calibrated parameters to match statistics in this 10 year period. The state equations for the 13 endogenous variables are derived in Appendix C.6 and the processes for the exogenous shocks used in the estimation is reported in Appendix C.7, Table 4.

We describe the calibration strategy and the structural parameters in detail below.

Calibrated Parameters We assume that the steady state of the normalized model corresponds to the time period 1995-2004, when female hours relative to male hours and female income share relative to male income share are stable and choose parameters accordingly. Standard estimates (e.g. [10]) suggest that women’s Frisch elasticity of labor supply is approximately three times as large as men’s. Assuming an aggregate Frisch elasticity of 0.75 (see for example [12]), and given

TABLE 1: Calibrated parameters

Parameter	Description	Value
α	capital share	0.33
β	discount factor	0.99
δ	depreciation rate	0.025
g	output share of government spending	$\frac{1}{1-0.15}$
$1/\nu^f$	Frisch elasticity, female	1.25
$1/\nu^m$	Frisch elasticity, male	0.4167
$\frac{1}{1-\rho}$	elasticity of substitution between female&male hours	4.33
ω^f	steady state labor share, female	0.375
ω^m	steady state labor share, male	0.625

TABLE 2: Estimated parameters

Parameter	Description
γ	TFP growth rate
ξ	curvature of capital utilization cost
η	consumption habit parameter
ζ	curvature of investment adjustment cost
ρ_x	autocorrelation coefficient for shock x
σ_x	standard deviation for the error term for shock x
Shocks	$z, \mu, b, g, \tilde{a}^{fT}, \tilde{a}^{fC}, \tilde{\phi}^{fT}, \tilde{\phi}^{fC}, \phi^m$

women’s share in total hours of 0.4 for 1995-2004, we have:

$$0.375 \frac{3}{\nu^m} + 0.625 \frac{1}{\nu^m} = 0.75,$$

which yields the values in Table 1. The substitution elasticity between female and male labor is $1/(1 - \rho)$, and we set it to 4.33 based on the estimates in [?]. The observed average relative labor share of women over the period 1995 to 2004 is 0.375, which will correspond to the calibrated value for ω^f . Finally, the steady state share of government spending in GDP ($1 - 1/g$) is calibrated to 0.15, which roughly corresponds to the average share of G+NX in GDP over the period 1995 to 2004. Table 1 summarizes the calibrated parameters.

Estimated Parameters The structural model parameters that are estimated include γ , the trend growth rate in aggregate technology, ξ , the curvature of the capital utilization cost at full utilization, η , the habit parameter, and ζ , the second derivative of the investment adjustment cost in a non-stochastic steady state. They are summarized in Table 2.

The priors for all the aggregate shocks follow [32]. For the persistence of the trend components of the labor supply shocks preferences and the female relative productivity factor, that is $\rho_{\tilde{\varphi}^{fT}}$ and $\rho_{\tilde{a}^{fT}}$, we center the prior tightly around 0.98, given the observed persistence of the female/male hours and wage ratios. For the cyclical components, we center the prior on the autocorrelation

around 0.4, which is the same as for the growth rate of the technology shock, reflecting the view that the persistence of the cyclical component should be significantly smaller than that of the trend.

Observation Equations We use log GDP growth, log consumption growth, log investment growth, log male hours, log of the female/male hours ratio and the log of the male income share as observables. Since we are not interested in average levels of variables over the entire sample, we will compute the observables in deviation from their steady state value and omit the constant in the observation equations.¹⁷ The observation equation are presented in Table 5 in Appendix C.

Strategy Since we are interested in assessing the impact of the changing trends in female hours and wages on aggregate outcomes, we run the estimation over different sample periods, corresponding to the evolution of the growth in relative female hours and wages. We use annual data for 1969-2017, since this allows us to go back further in time.

For comparison, we also estimate a real version of the model in [32], which is identical to our model, except for the fact that there are no gender specific shocks and no differentiation between male and female hours, so that the disutility of labor and the production function are defined over aggregate hours per capita. We consider two versions of this model. The first has an aggregate labor supply shock that scales the disutility of hours in the household utility function, similar to the female labor supply shock in the full model. The second version reduces this shock to a fixed parameter, which is estimated. The priors for the aggregate shocks for both versions of the simple model are set the same as for the full model, and the prior for the aggregate labor supply shock in the first version of the simple model is set the same as the prior for $\tilde{\phi}^{fT}$ in the full model. We will refer to the model without gender differentiation as the *simple* model, and to our model with gender differences as *full* model when presenting our results.

4.1 Full Model

Table 3 presents the parameter estimates for the period 1969-2011. The gender specific shocks display high degrees of autocorrelation, whereas the aggregate shocks display much lower estimated autocorrelation parameters compared to estimates in the literature. The mode of the estimated autocorrelation of the trend component for the female labor supply and relative productivity shocks are both equal to 0.99. The modal estimate for the autocorrelation coefficient of the male labor supply is shock is 0.86, whereas the modal estimate for the autocorrelation coefficient for the technology shock is 0.20. These results suggest that the persistence in the behavior of key variables is captured by the gender specific shocks, which, as we will show below, account for a relative large fraction of the variation of aggregate variables at long horizons. The modal estimates of the standard deviation of the cyclical components of the gender specific shocks are the same order of magnitude than those

¹⁷In most DSGE applications, it would be of interest to estimate the steady state level of hours for the entire sample period. The system of equations characterizing the steady state could be used for this purpose, so that the 1995-2004 level of a value for the φ^j and a^j would be pinned down from the steady state equations using the level of hours and wages.

TABLE 3: Estimated Parameters, 1969-2011

	Dist	Prior			Posterior						
		10%	Median	90%	Mode	Mean	SE	10%	Median	90%	
γ	N	3.0000	0.4616	0.5000	0.5384	0.4990	0.4990	0.0287	0.4622	0.5014	0.5353
χ	G	2.0000	3.7689	4.9335	6.3167	5.3028	5.3028	1.0042	4.4139	5.5509	6.9182
η	B	1.0000	0.3701	0.5000	0.6299	0.3525	0.3525	0.0641	0.2431	0.3251	0.4092
ζ	G	2.0000	1.5236	2.5776	4.0345	4.0450	4.0450	0.8126	3.5913	4.5039	5.7023
ρ_b	B	1.0000	0.3205	0.6143	0.8574	0.1697	0.1697	0.0844	0.1062	0.2223	0.3303
ρ_μ	B	1.0000	0.3205	0.6143	0.8574	0.2039	0.2039	0.1039	0.1148	0.2301	0.3811
ρ_z	B	1.0000	0.1426	0.3857	0.6795	0.1305	0.1305	0.1028	0.0577	0.1629	0.3191
ρ_g	B	1.0000	0.3205	0.6143	0.8574	0.8444	0.8444	0.2147	0.3402	0.7369	0.9157
$\rho_{\tilde{a}^f, T}$	B	1.0000	0.9599	0.9836	0.9954	0.9837	0.9837	0.0158	0.9532	0.9774	0.9923
$\rho_{\tilde{a}^f, C}$	B	1.0000	0.2306	0.5000	0.7694	0.7743	0.7743	0.1908	0.3633	0.6753	0.8707
$\rho_{\phi^f, T}$	B	1.0000	0.9599	0.9836	0.9954	0.9969	0.9969	0.0061	0.9811	0.9936	0.9980
$\rho_{\phi^f, C}$	B	1.0000	0.2306	0.5000	0.7694	0.9812	0.9812	0.2213	0.4420	0.8930	0.9879
ρ_{ϕ^m}	B	1.0000	0.3205	0.6143	0.8574	0.8798	0.8798	0.0726	0.7593	0.8655	0.9332
σ_b	IG1	4.0000	0.0260	0.0600	0.1880	0.0192	0.0192	0.0029	0.0165	0.0198	0.0242
σ_μ	IG1	4.0000	0.1470	0.3250	0.9390	0.1496	0.1496	0.0344	0.1269	0.1653	0.2145
σ_z	IG1	4.0000	0.3760	0.7480	0.9990	0.1564	0.1564	0.0091	0.1576	0.1639	0.1798
σ_g	IG1	4.0000	0.1470	0.3250	0.9390	0.0645	0.0645	0.0038	0.0653	0.0685	0.0743
$\sigma_{\tilde{a}^f, T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.0401	0.0401	0.0063	0.0347	0.0425	0.0509
$\sigma_{\tilde{a}^f, C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.0606	0.0606	0.0082	0.0552	0.0632	0.0760
$\sigma_{\phi^f, T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.1029	0.1029	0.0382	0.0803	0.1472	0.1847
$\sigma_{\phi^f, C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.1405	0.1405	0.0331	0.0909	0.1202	0.1805
σ_{ϕ^m}	IG1	4.0000	0.1470	0.3250	0.9390	0.1004	0.1004	0.0119	0.0918	0.1059	0.1217

Full model, 1969-2017. Posterior log density at mode: 544.5376.

of the technology shock. The cyclical components of the gender specific shocks also display a fair degree of persistence. The modal estimate for the autocorrelation coefficient for the cyclical component of the female relative productivity and labor supply shock are 0.77 and 0.98, respectively. The magnitude for the modal estimate for the standard deviation of the error term for these variables are 1/3-1/2 of the modal estimates of the standard deviation the technology shocks, which is consistent with their relative small contribution to the explanation of the variability of aggregate variables at higher frequency.

Estimated Gender Specific Shocks It is interesting to examine the estimated path of the gender specific processes introduced in the model, \tilde{a}_t^f , $\tilde{\phi}_t^f$ and ϕ_t^m . Figure 16 displays the trend components of these variables. There is a clear and precisely estimated trend for all these processes. The path of \tilde{a}^{fT} and $\tilde{\phi}^{fT}$ closely matches the values backed out from the calibrated model and reported in figure 9, reflecting the changes in women’s education attainment and experience. The male labor supply shock, ϕ^m , also declines between the early 1980s and the mid 1990s, then starts rising in the early 2000s and spikes in 2007-2009, consistent with the decline in male employment over that period. The initial decline in ϕ^m is modest compared to ϕ^{fT} , and can be attributed to demographics as baby boomers enter the labor force in the 1970s. Figure 17 presents the estimated path of the cyclical components of the gender specific shocks. Both series display lower high frequency volatility and higher medium frequency volatility of female relative productivity cyclical shock starting in mid 1980s, when the corresponding trend component slows. We will discuss the estimated path of the aggregate shock when we compare the full model with a standard real business

cycle model without gender differentiation in Section 4.2.

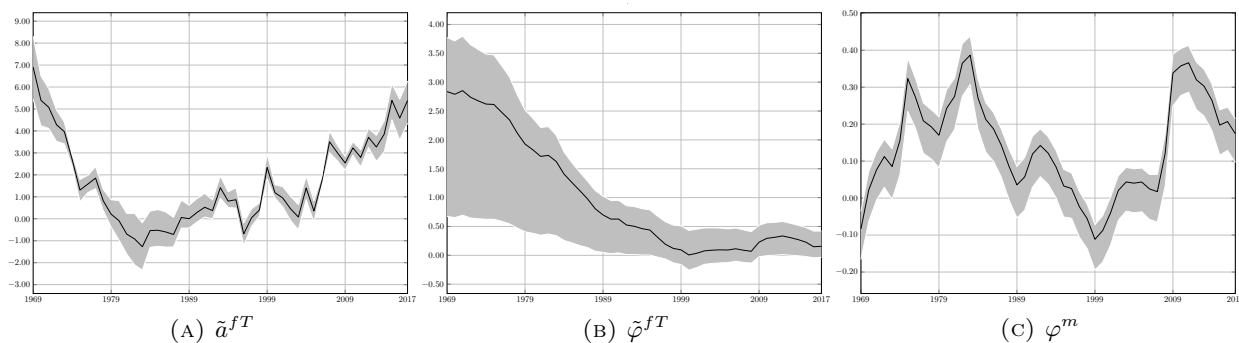


FIGURE 16: Estimated paths of gender specific shocks, trend components. Full model, 1969-2017.

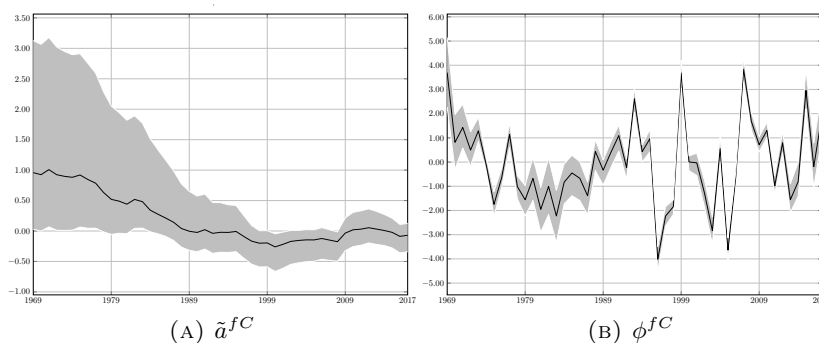


FIGURE 17: Estimated paths of gender specific shocks, cyclical components. Full model, 1969-2017.

Variance Decomposition The gender specific shocks account for a sizable fraction of the variance of female and male hours and of female wages, as displayed in figure 18, which charts the variance decomposition at various horizons marked on the horizontal axis. By year 5, the female labor supply shock explains more than 50% of the variation in female hours. The cyclical component $\tilde{\varphi}^{fC}$ 20-25% of the variation, while the trend component, $\tilde{\varphi}^{fT}$, explains more of the variance of hours as the horizon increases, reaching 30% at year 10. The female relative productivity shocks explains less than 5% of the variance of female hours at any horizon. The male labor supply shock explains 35-50% of the variance of male hours, depending on the horizon. The female labor supply and productivity shocks play a negligible role in accounting for the variation in male hours.

The female relative productivity shock accounts for 40-60% of the variation of female wages, with its contribution decreasing with horizon. The aggregate technology shocks does not explain a sizable fraction of female or male hours though it explains 40-60% of the variance of female wages and 90-95% of the variance of male wages.

The gender specific shocks are introduced to account for the heterogeneity across genders in the trend in wages and hours. They also matter for aggregate variables, as can be clearly seen from the

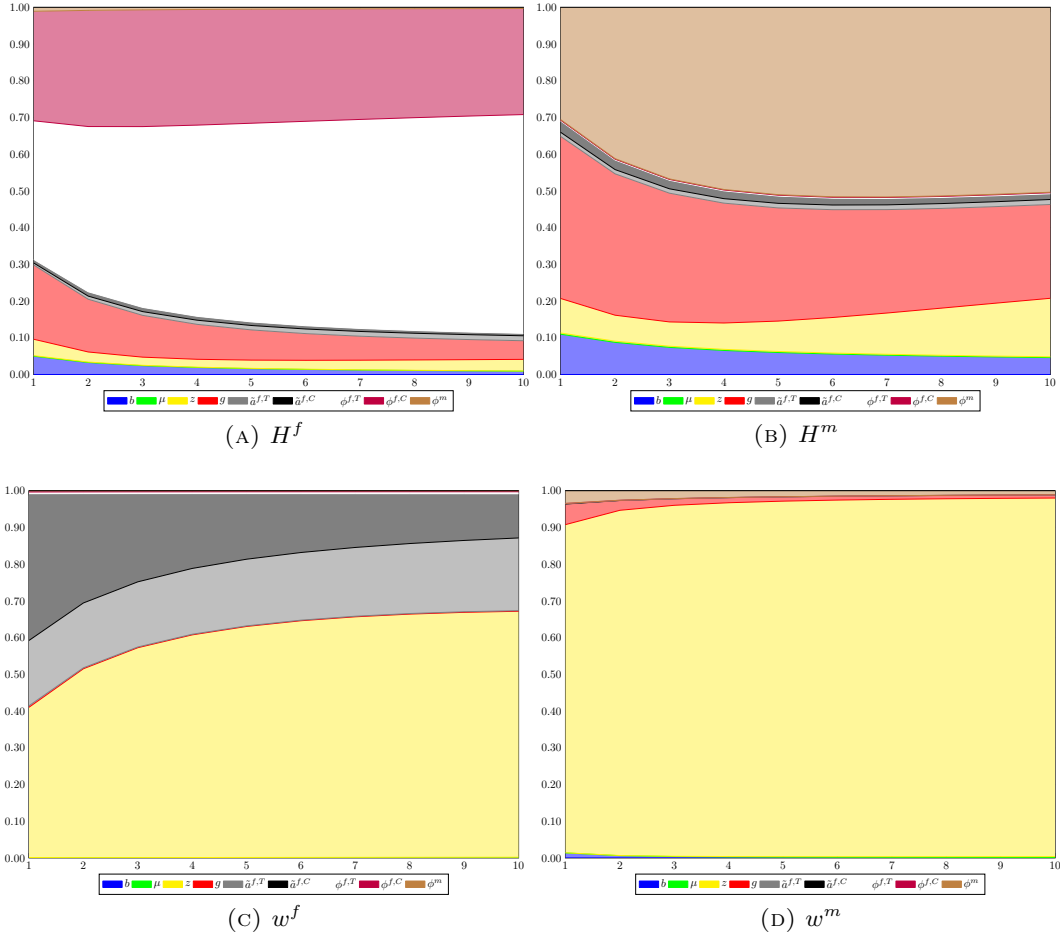


FIGURE 18: Variance decomposition, hours and wages by gender. Full model, 1969-2017.

variance decomposition displayed in figure 33. The female productivity shock accounts for 10-20% of the variance of output and 5-12% of the variance of investment, with the contribution rising in horizon, consistent with the role of the female productivity shock on the Solow residual discussed in Section 3.3. The male labor supply shock dominates the variance of aggregate hours per capita, but together with the female labor supply shock, plays a negligible role in accounting for the variance of the other aggregate variables. At horizons greater than 3 years, the three gender specific shocks combined explain more of the variance of output and investment than the government consumption shock, which is calibrated to average 15% of GDP. The variance of consumption is mostly accounted for by the technology shock at all horizons.

Impulse Responses Figure 19 plots the impulse responses of female and male hours to the aggregate productivity shock and the government consumption shock.¹⁸ Male hours are more responsive to both shocks with the magnitude of the response at the peak twice as large for men than for women. Hours initially fall in response to the z shock, due to the negative wealth effects on labor

¹⁸The response to the preference shock b is very similar to the response to the government consumption shock.

supply, but eventually rise, with the peak occurring 5-6 years after the impulse. Figure 20 plots the response of female and male hours to the gender specific shocks. All these shocks have opposing effects on female and male hours. The response of female hours to the female productivity shock is positive and peaks at 0.28 percentage points 2-3 years after the impulse. The effect on male hours is negative, -0.7 percentage points on impact, and stabilizes at -0.2 percentage points by year 5. The female labor supply shock reduces female hours by about 2 percentage points on impact, with the effect slowly fading over time, while it increase male hours by 0.3 percentage points on impact and only 0.1 percentage points in years 5-19. The male labor supply shock induces responses of female and male hours that are the mirror image of those to the female labor supply shock, except for the fact that the responses are a lot less persistent.

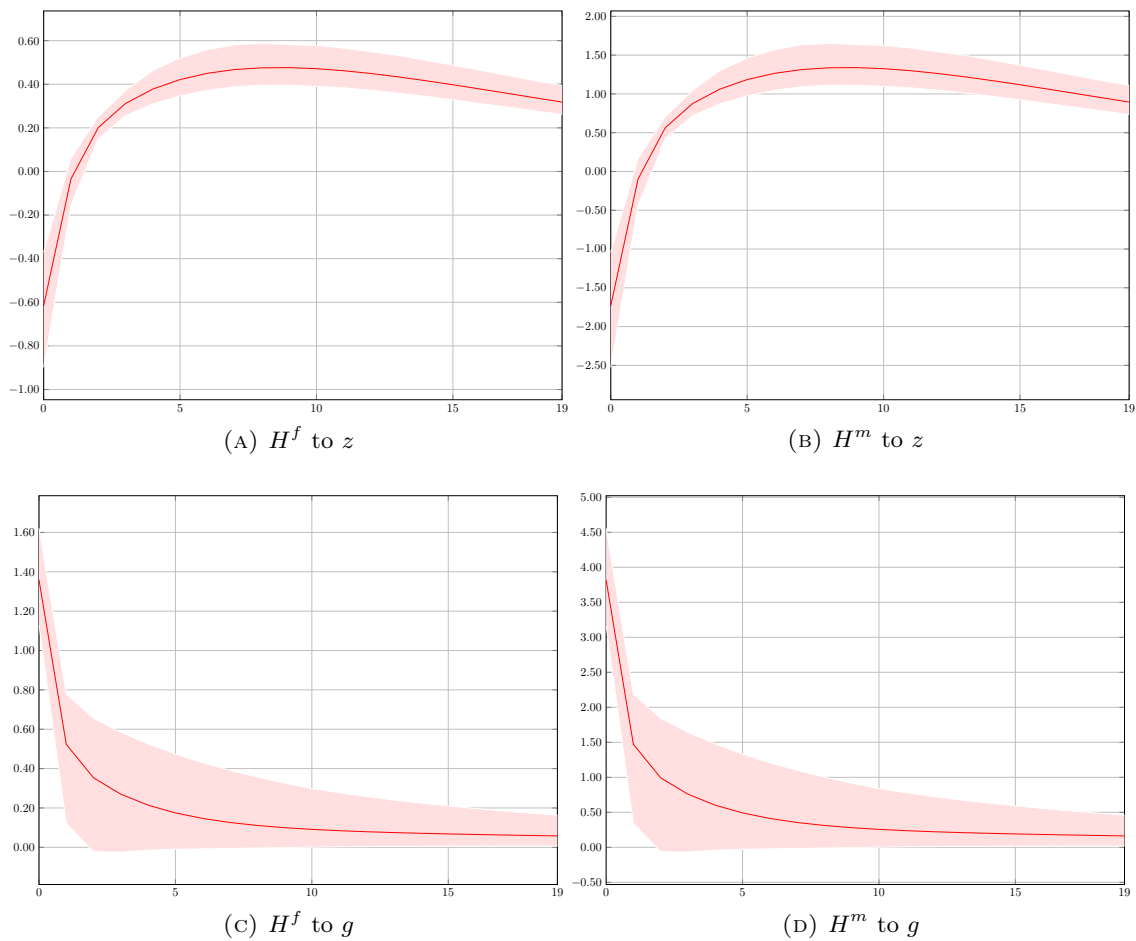


FIGURE 19: Impulse response functions with 5% confidence intervals. Full Model. Positive 1 percent shocks, percent log deviations from steady state. Sample period: 1969-2017.

Shock Decompositions Figure 22 reports the shock decompositions for output, and female and male wages.¹⁹ The trend components of both the female labor supply and relative productivity

¹⁹Shock decompositions for additional variables are reported in figures 34 and 35.

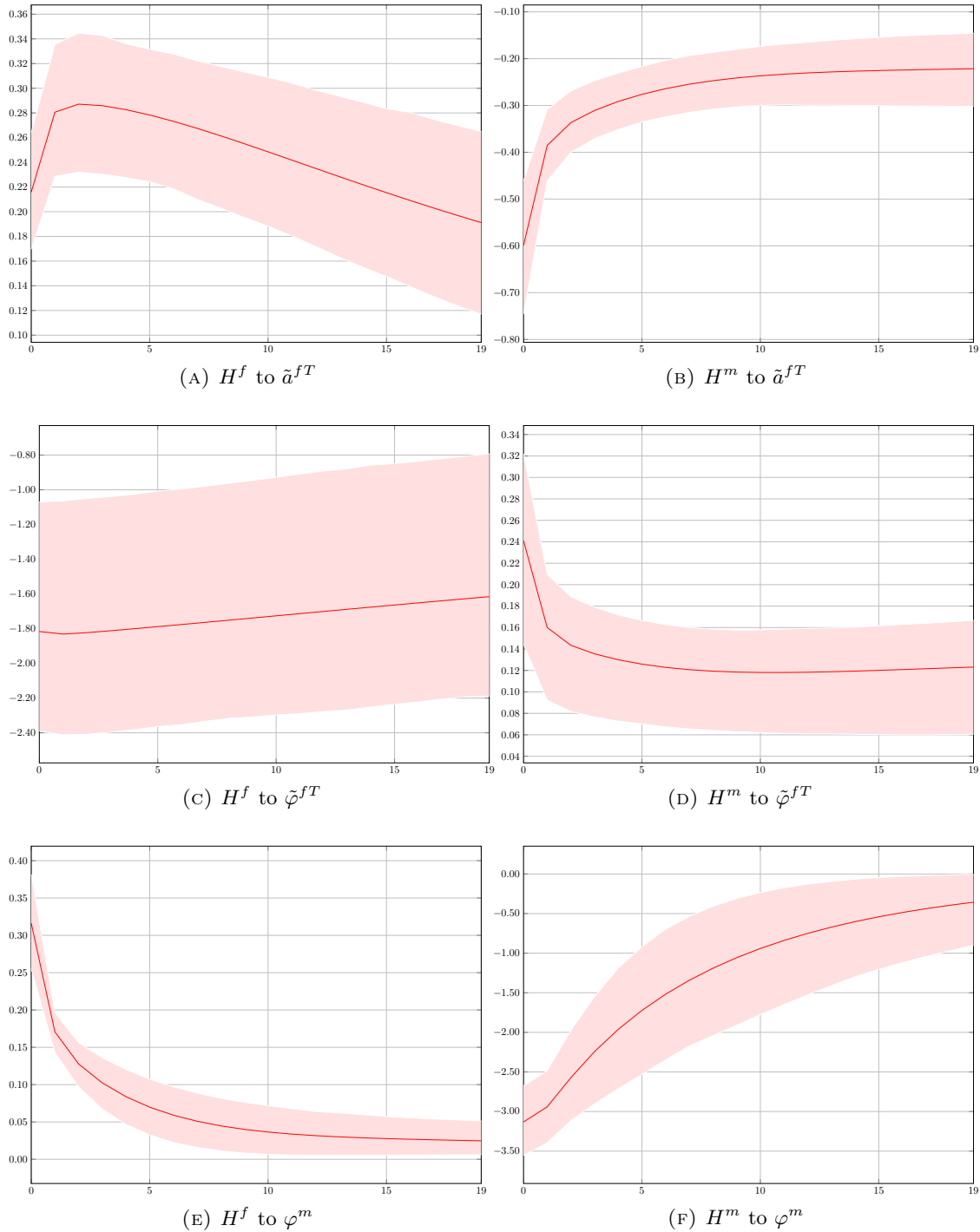


FIGURE 20: Impulse response functions with 5% confidence intervals. Full Model. Positive 1 percent shocks, percent log deviations from steady state. Sample period: 1969-2017.

shock contribute positively to the growth in output starting in the early 1980s. For female wages, the growth is primarily driven by the female productivity shock, while the female labor supply shock has a negative effect on wages. The female productivity shock also contributes to the growth in

male wages, and so does the female labor supply shock, as higher female hours even for given female productivity, increase the marginal product of male hours.

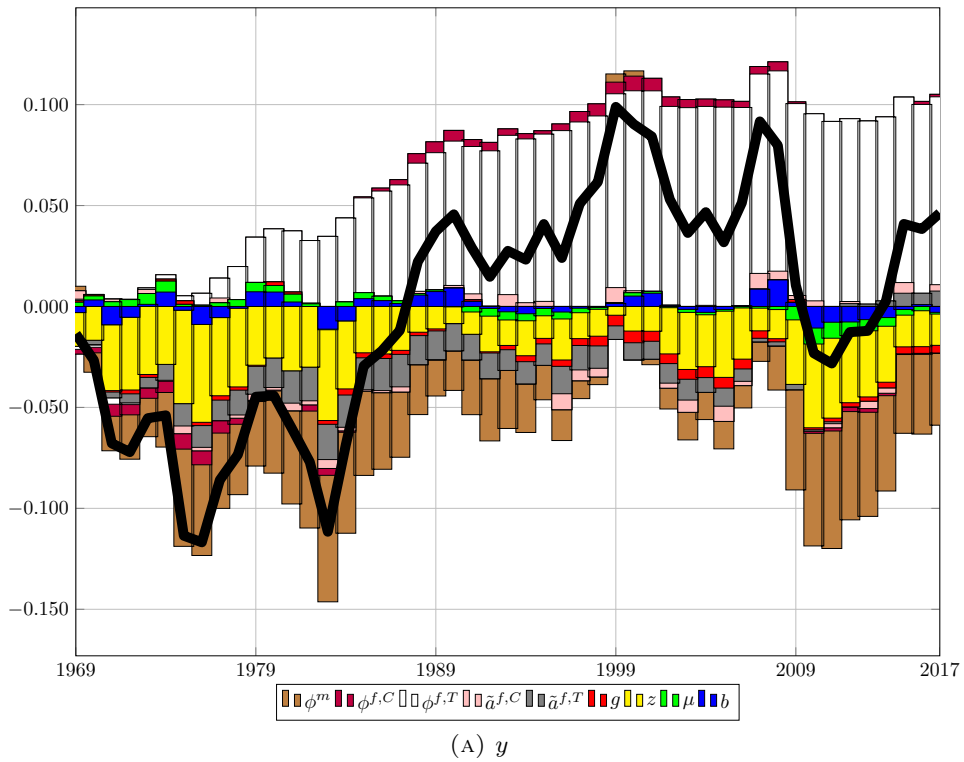


FIGURE 21: Shock decomposition, output. Full model, 1969-2017.

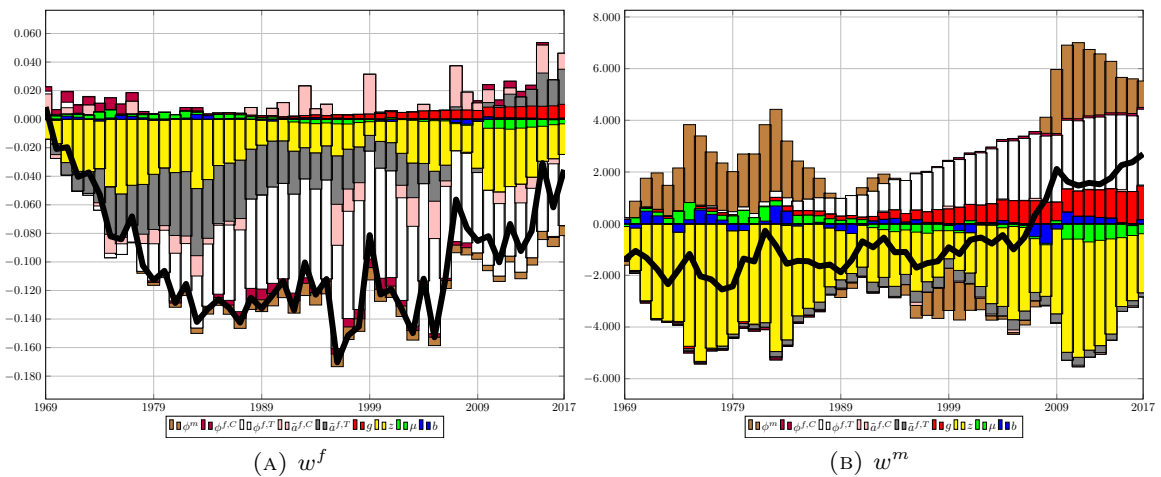


FIGURE 22: Shock decompositions, wages by gender. Full model, 1969-2017.

4.2 Comparison with Simple Model

We now compare the results for the estimated parameters, aggregate states and variance decompositions with the simple model. We consider two variants of this model. One with a labor supply shock, denoted with $\varphi_t = \varphi_t^T + \varphi_t^C$, with a trend and a cyclical component, similar to the female shock in the full model.²⁰ We also consider a version of the model in which this shock is reduced to a fixed parameter ϕ , that scales the disutility of hours and is estimated. The calibrated parameters that are common to the full and simple models are set to the same values. The curvature of the disutility of labor, which we denote with ν in the simple model is set to $1/0.75$, following the same calibration strategy adopted for the full model. All the aggregate shocks have the same specification as in the full model and the estimation starts from the same priors for all the common parameters.

We start by comparing estimated parameters, which are reported in Appendix D.3 for both versions of the simple model. For the version with the labor supply shock, the maximized log-likelihood about 2/3 of full model, and as in the full model. The trend component of the labor supply shock is very persistent, which reduces the estimated autocorrelation of the other aggregate shocks relative to estimates found in the literature, such as [32]. For the version with constant ϕ , the maximized log-likelihood is similar to the simple model with the labor supply shock, however, several important parameters display sizable differences in their estimated values. Specifically, the absence of a labor supply shock triples the estimated autocorrelation of the preference shock b relative to the simple model with variable φ and doubles it relatively to the full model. As we will see later, the preference shock will capture part of the trend in employment which in the full model is driven by the trends in the female labor supply and relative productivity shock. The standard deviation of the iid component of the investment shock is also about 1/3 higher in both versions of the simple model compared to the full model. There is very little difference in the estimated values of the other common parameters across models.

For the simple model with a labor supply shock, the estimated path of the trend and cyclical component of this shock are presented in figure 36 in Appendix D.3. There is a notable reduction in the volatility of both the trend and cyclical components of this shock starting respectively in the early 1990s and the early 1980s, and there is no decline in the trend component, despite the rise in aggregate hours. For the simple model without labor supply shock, the estimated value of the parameter for the disutility of labor is $\varphi = 0.091$.

Shocks We now compare the estimated path of the aggregate states in the three versions of the model. We begin with the technology shock z , for which the estimated path for the three models is displayed in the top panel of figure 23. In both versions of the simple model, there is a marked reduction in the volatility of the technology shock starting in the mid 1980s, whereas the full model displays an increase in volatility in the same period. This difference is due to the fact that in the full model the procyclical behavior of the labour supply shock and the fact that it intensifies in the

²⁰This approach is similar to [11], however, they assume that the labor supply shock is non-stationary, whereas here we impose it is stationary.

second part of the sample contributes to a decline in the volatility of aggregate hours and output, even if the volatility of the technology shock does not change or even increases.

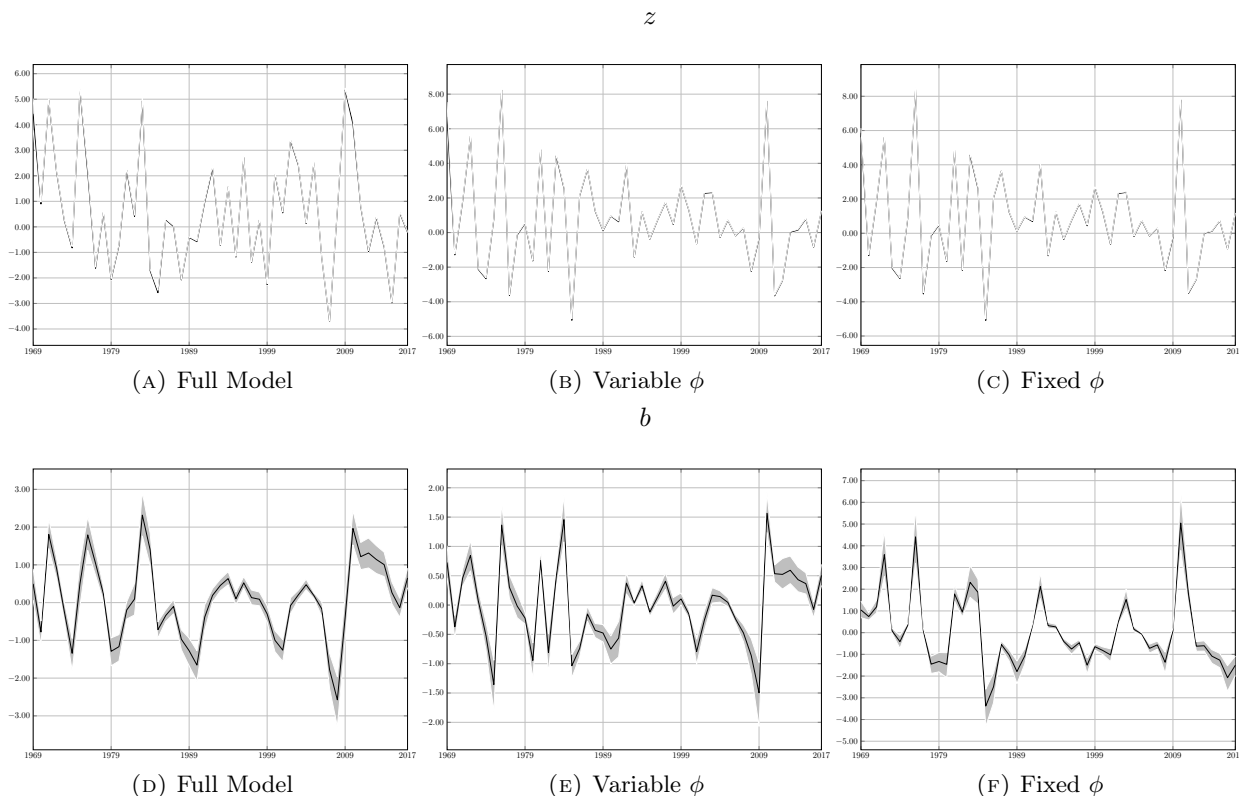


FIGURE 23: Model comparison, aggregate shocks: technology shock z and preference shock b . Sample period: 1969-2017.

The bottom panel of figure 23 presents the estimated path of the preference shock b in the three versions of the model. There only minor differences between the full model and the simple model with a labor supply shock. Instead, for the simple model, the estimated path of b shows a marked trend decline starting in the early 1990s. A lower value of b tends to increase labor supply in the model as it reduces the disutility of labor in the current period relative to other periods. This suggests that in the absence of a trend decline in the labor supply shock, the rise in aggregate per capita hours can only be matched with a decline in the b shock, so that a *demand shock* absorbs the pattern of the missing labor supply shock.

The estimated paths for the investment shock μ for the three models are displayed in figure 37 in Appendix D.3. For this shock, there are no major differences in the estimated paths across models. The estimated path for the g shock is also presented in figure 37, and for this shock the variations across models are very similar to those found for b . For all aggregate shocks other than the technology shock, we find a reduction in volatility starting in the mid-1980s, consistent with previous work on the great moderation discussion in [23].

Impulse Responses To gain further insight on the role of the gender specific shocks for aggregate variables, we examine the impulse response functions to the aggregate shocks in the full model and the simple model. Figure 24 reports the response of aggregate hours per capita to a productivity shock and the government consumption shock. The response to g are about 1/3 higher on impact in the two versions of the simple model, and the magnitude of the response to z is also about 1/3 higher in the simple models. To understand this result, one needs to consider that the response to z is driven by the combination of a negative wealth effect on impact and a positive substitution effect, while the response to g derives from a positive wealth effect. Both these effects, given log preferences over consumption, are driven by the curvature of the disutility of labor in the utility function. While the values for this parameters are set to be the same in the aggregate for both versions of the model, in the full model men comprise the majority of hours and they have lower Frisch elasticity and lower wealth effects. Additionally, both the technology shock and the government consumption shock are more persistent in the simple models, so the response of hours is more long lived. A similar but muted pattern holds for the other aggregate variables.

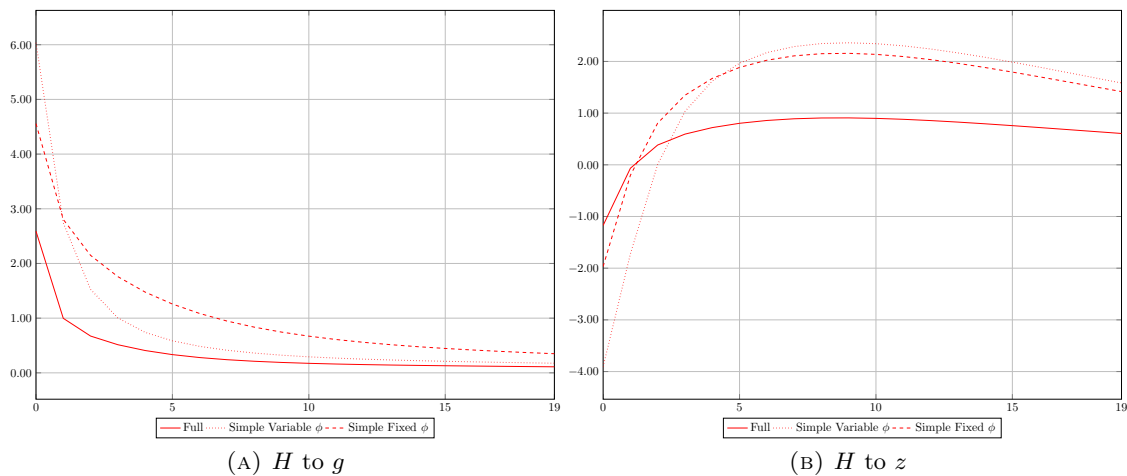


FIGURE 24: Impulse response functions, full model and simple model. Sample period: 1969-2017.

4.3 Comparison of Time Periods

Tables 8 and 9 in Appendix D.4 present the parameter estimates for the 1969-1992 and 1993-2017 periods. There are a number of differences in the estimates across the two sub periods. The autocorrelation coefficient for the female relative productivity and labor supply shocks are similar across time periods. However, the autocorrelation of the cyclical component of both the relative female productivity and the female labor supply shock is lower in the second period. The male labor supply shock, which does not have a persistent component, also displays lower autocorrelation during this period. The standard deviation of the error term for the female relative productivity shock and the female labor supply shock is stable across periods for the trend component. However, for the cyclical component, the standard deviation of the error term for both female and male labor supply

shock rises in the second period. Turning to the aggregate shocks, the estimated autocorrelation of the technology shock is lower after 1993.

The variance decompositions for female and male hours for the two periods is displayed in figure 25. The figure clearly shows that for 1993-2017, there is a smaller role of the trend female labor supply shock though a larger role of the cyclical component, and a larger role for the trend component of the female relative productivity shock. At short horizons, the variance explained by the preference shock b rises from close to 0 to approximately 10%, and there is also larger role of the technology shock z though at long horizons. This pattern also holds for output and aggregate hours.

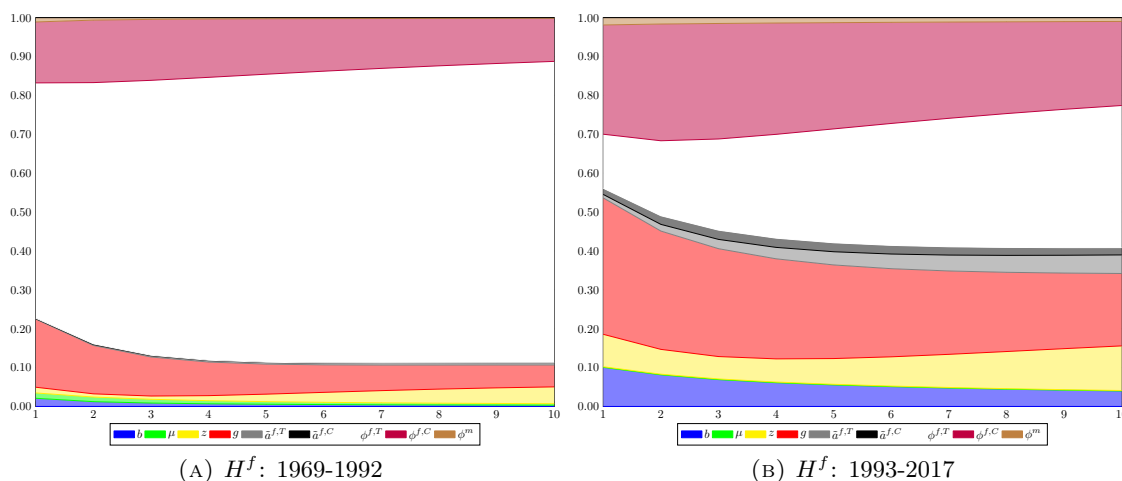


FIGURE 25: Variance decomposition, time comparison. Female hours per capita.

Counterfactual To assess the contribution of the change in trends in female labor supply on the cyclical behavior of hours after 1993, we run the following counterfactual simulation. The female labor supply trend shock, $\tilde{\varphi}^{fT}$, is adjusted so that female hours grow at the same average rate that they did in 1969-1992 and aggregate shocks (z, b, μ, g) follow their historical path for 1993-2017. The cyclical components of the female labor supply and relative productivity shocks are simulated based on 1969-1992 estimated parameters, starting with the actual value estimated for 1993. The trend component of the female relative productivity follows the process estimated for 1969-1992 starting from its 1993 actual value, and the male labor supply shock is maintained at its 1993-2017 estimated historical values. The simulated path of the endogenous variables other than H^f in the counterfactuals corresponds to mean values of over 5000 realizations of the exogenous shocks, in log variation from 1993 levels.

Figure 26 reports the behavior of the endogenous variables. Female wages do not display a sizable response in the counterfactual. Counterfactual male hours are only marginal lower than the actual, stemming from negative wealth effect from higher female earnings. However, male wages are substantially higher than the actual, due to the higher marginal product of male labor resulting

from higher female hours. Aggregate hours grow at a very fast rate in the counterfactual, reaching a peak of 13 long point in 2008 and of 17 log points by 2017. They drop by about 5 log points during the Great Recession, while actual hours drop by over 10 log points starting from a much lower value. Counterfactual output is 5-10 log points higher dropping by only 4 log points as a result of the Great Recession, while actual output falls by 7 log points. Consumption largely reflects the behavior of output, while investment is mostly higher than the actual with the counterfactual path of female hours.

These results suggest that continued growth in female hours at the average pace registered for 1969-1992 would have resulted in shallower recessions in 2001 and 2007-2009, in terms of aggregate hours, output and consumption, and in a stronger rate of expansion in the mid to late 1990s and the early 2000s. Additionally, aggregate hours would have exhibited faster recoveries after both the 2001 and 2007-2009 recessions. Finally, male wages would have been substantially higher, due to the higher marginal product of male labor with higher female hours. The conclusion is that the flattening of female hours starting in the early 1990s had a substantial effect on business cycles and male wages and on overall economic performance in the United States.

5 Conclusion

This paper builds a real DSGE model with gender differences in labor supply and productivity. The model is used to assess the impact of changing trends in female labor supply on productivity and TFP growth and aggregate business cycles. We find that the growth in women's labor supply and relative productivity contributed substantially to TFP growth starting from the early 1980s, even if it depressed average labor productivity growth. We also show that the lower cyclicalities of female hours and their growing share in aggregate hours is able to account for a large fraction of the decline in the cyclicalities of aggregate hours during the great moderation, as well as the decline in the correlation between average labor productivity and hours. Finally, we show that the discontinued growth in female labor supply after the 1990s plays a substantial role in the jobless recoveries following the 2001 and 2007-2009 recession. Moreover, it also depresses aggregate hours and output growth during the late 1990s and mid 2000s expansions and it reduces male wages. These results suggest that continued growth in female hours since the early 1990s would have significantly improved economic performance in the United States.

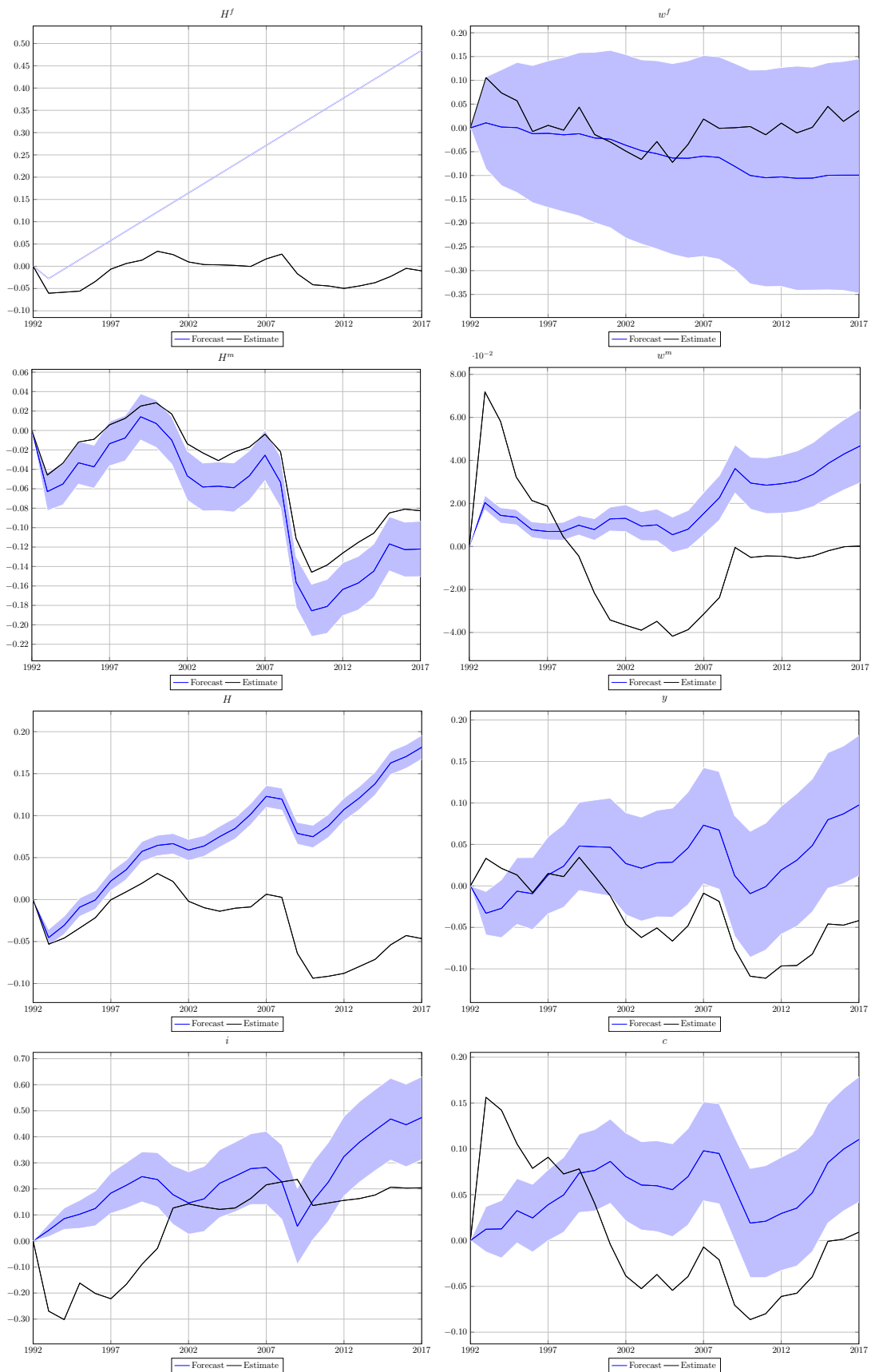


FIGURE 26: Counterfactual: Endogenous variables.

References

- [1] Acemoglu, Autor and Lyle. 2004. Women, War and Wages. *Journal of Political Economy*.
- [2] Autor, D.H., Katz, L.F. and Kearney, M.S., 2008. Trends in US wage inequality: Revising the revisionists. *The Review of economics and statistics*, 90(2), pp.300-323.
- [3] Albanesi, S. and C. Olivetti, "Gender Roles and Medical Progress," *Journal of Political Economy*, (2016). 1.1
- [4] Albanesi, S. and M. Prados, "Inequality and Household Labor Supply," manuscript (2014). 2
- [5] Albanesi, S. and A. Sahin. 2018. The Gender Unemployment Gap. *The Review of Economic Dynamics*. 2.2, 8, 14
- [6] An, Sungbae, and Frank Schorfheide. 2007. Bayesian analysis of DSGE models." *Econometric Reviews* 26.2-4: 113-172. 4
- [7] Baily, Martin Neil, Robert J. Gordon. 1989. The Productivity Slowdown, Measurement Issues, and the Explosion of Computer Power. NBER Reprint No. 1199. 4
- [8] Black, S. and C. Juhn. 2000. The Rise of Female Professionals: Women's Response to Rising Skill Demand," *American Economic Review Papers and Proceedings*, 90(2): 450-455. 1.1
- [9] Black, Sandra and A. Spitz-Oener. 2007. Explaining Women's Success: Technological Change and the Skill Content of Women's Work. NBER Working Paper 13116. 1.1
- [10] Blundell and Macurdy. 1999. *Handbook of Labor Economics* Vol. 3A 4
- [11] Chang, Y., Doh, T. and Schorfheide, F., 2007. Non-stationary Hours in a DSGE Model. *Journal of Money, credit and Banking*, 39(6), pp.1357-1373. 20
- [12] Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. *American Economic Review* 101(2). 4
- [13] Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy* 113(1): 1-45. 3
- [14] Christiano L.J., M. Eichenbaum , R. Vigfusson. 2003. What Happens After a Technology Shock?" NBER Working Papers 9819, National Bureau of Economic Research. 3
- [15] Christiano L.J., M. Eichenbaum , R. Vigfusson. 2004. The Response of Hours to a Technology Shock: Evidence Based on Direct Measures of Technology," *Journal of the European Economic Association*, MIT Press, vol. 2(2-3): 381-395. 3

- [16] Davis, Steven J. and James A. Kahn. 2008. Interpreting the Great Moderation: Changes in the Volatility of Economic Activity at the Macro and Micro Levels. *Journal of Economic Perspectives*, American Economic Association, vol. 22(4), pages 155-80, Fall.
- [17] Fernald, John G. 2015. Productivity and Potential Output before, during, and after the Great Recession. *NBER Macroeconomics Annual*, 29: 1-51. [3.3.2](#)
- [18] Fernandez, Raquel. Cultural Change as Learning: The Evolution of Female Labor Force Participation over a Century. *American Economic Review*. [2](#)
- [19] Fogli, Alessandra and Laura Veldkamp. 2007. Nature or Nurture? Learning and Female Labor Force Participation. Federal Reserve Bank of Minneapolis, Research Department Staff Report 386. [2](#)
- [20] Foote, Christopher and R. Ryan. 2012. Labor Market Polarization Over the Business Cycle. Federal Reserve Bank of Boston Public Policy Discussion Paper No.12-8. [2.3](#)
- [21] Francis, Neville and Valerie Ramey. 2009. Measures of Per Capita Hours and Their Implications for the Technology-Hours Debate. *Journal of Money, Credit, and Banking*. [3](#)
- [22] Fukui, Masao, Emi Nakamura, Jón Steinsson. 2018. Women, Wealth Effects, and Slow Recoveries. NBER Working Paper No. 25311. [1.1](#)
- [23] Galí, Jordi, and Luca Gambetti. 2009. "On the Sources of the Great Moderation." *American Economic Journal: Macroeconomics*, 1(1): 26-57. [1](#), [2.2](#), [2.2](#), [3.3.2](#), [4.2](#)
- [24] Galor, Oded and David N. Weil. 1996. The Gender Gap, Fertility, and Growth. *American Economic Review*, 86 (3): 374-387. [1.1](#)
- [25] Goldin, Claudia and Lawrence Katz. 2002. The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions. *Journal of Political Economy* 100, 730-770. [1.1](#)
- [26] Greenwood, Jeremy and Boyan Jovanovic. 1998. Accounting for Growth. NBER Working Paper No. 6647. [4](#)
- [27] Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorugoklu. 2005. Engines of Liberation. *The Review of Economic Studies* 72: 109-133. [1.1](#)
- [28] Jaimovich N. and H. and Siu. 2012. The Trend is the Cycle: Job Polarization and Jobless Recoveries. Manuscript. [2.3](#)
- [29] Juhn, Chinhui, and Simon Potter. 2007. Is there still an added worker effect? National Bureau of Economic Research. [7](#)
- [30] Jones, Larry, Manuelli Rodolfo E. and Ellen R. McGrattan. 2003. Why are Married Women Working So Much? Research Department Staff Report 317, Federal Reserve Bank of Minneapolis.

- [31] Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2010. Investment Shocks and Business Cycles. *Journal of Monetary Economics* 57(2): 132-145. 9, C.7
- [32] Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. 2012. Is There a Trade-Off Between Inflation and Output Stabilization? *American Economic Journal: Macroeconomics*. 4, 4, 4.2
- [33] Lundberg, S., 1985. The added worker effect. *Journal of Labor Economics*, 3(1, Part 1), pp.11-37. 2.2
- [34] Nordhaus, William. 2004 Retrospective on the 1970s Productivity Slowdown. NBER Working Paper No. 10950. 4
- [35] Olivetti, Claudia, 2006. Changes in women's hours of market work: The role of returns to experience. *Review of Economic Dynamics*, 9(4): 557-587. 3.3.1
- [36] Rendall, Michelle. 2010. Brain versus Brawn: The Realization of Women's Comparative Advantage. Manuscript, University of Zurich. 1.1
- [37] Stephens, Jr, M., 2002. Worker displacement and the added worker effect. *Journal of Labor Economics*, 20(3), pp.504-537. 7
- [38] Stock. J. and M. Watson. 2003. Has the Business Cycle Changed and Why? " NBER Macroeconomics Annual, Mark Gertler and Ken Rogoff (eds), MIT Press. 5
- [39] Stock. J. and M. Watson. 2003. Has the Business Cycle Changed? Evidence and Explanations," FRB Kansas City Symposium, Jackson Hole, Wyoming.
- [40] Stock. J. and M. Watson. 2012. Disentangling the Channels of the 2007-2009 Recession. *Brookings Papers on Economic Activity*.

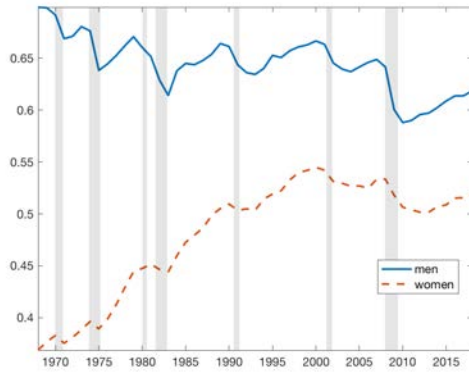
A Data

Our dataset spans 1969-2011 and is a collection of aggregate data extracted from the Haver Analytics database, the Federal Reserve Bank of St. Louis FRED database, supplemented with series constructed from the CPS March Supplement micro data. We construct real GDP per capita by dividing the nominal GDP series by population and the GDP deflator. Real per-capita consumption is defined to be the personal consumption of non-durables and services divided by population and the GDP deflator. Similarly, real per-capita investment is defined to be the sum of personal consumption of durables and gross private domestic investment divided by population and the GDP deflator.

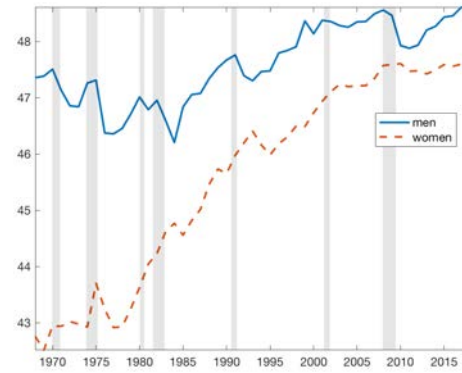
Because total monthly hours worked are not available in the aggregate for men and women, we use the CPS March Supplement to construct a series of monthly hours and average weekly wages by gender. For each gender, monthly hours are calculated as the product of the number of employed workers, the average hours worked each week, and the average number of weeks worked in a year. Wages are reported in 1984 dollars. Throughout the micro data analysis, we restrict the sample to people 15 and older and employed in the non-farm business sector. Figure 27 plots the series constructed from micro data used in the calculation of aggregate hours per capita by gender and hourly wages by gender.

A.1 Comparability to Aggregate Series

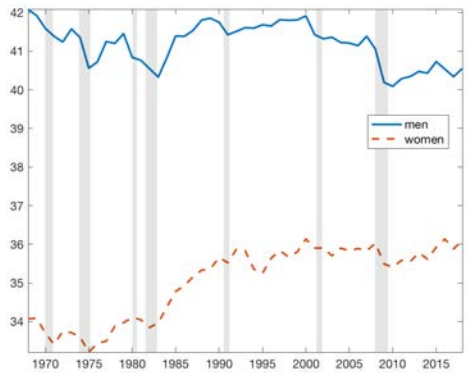
We compare our constructed employment, hours and wage series by gender with other series by gender directly available from the Bureau of Labor Statistics starting in later periods. The results are displayed in figure 28. Panel (A) reports the employment to population ratio by gender we constructed with the micro data with the value of this variable directly available from the CPS. We also include the corresponding value from the CES, obtained by dividing the number non-farm employees by the population. To derive the number of men employees, we take the difference between the total number of employees and the number of women employees. The three series match quite closely for both genders. Panel (B) reports hour estimate of hours worked per week by gender to the values reported by the CPS, which start in 1976. Again, for the periods in which we have both measures, they track each other very closely. Panel (C) reports our estimate for hourly wages by gender and the corresponding series provided by the CPS, which starts in 1990. Here, the time pattern is also very similar for the overlapping period. For our analysis, the most important series is the female/male wage ratio. We report three different values of it on panel (D). The solid line corresponds to the value we estimate from micro data, the dashed line to the value obtained from the CPS hourly wage series by gender also described in panel (C), while the dotted line corresponds to the ratio of usual weekly earnings by gender, which are available directly from the CPS starting in 1979. The three series have a very similar pattern for the overlapping periods. The ratios coming from CPS hourly wages and weekly earnings are somewhat higher, which may reflect smaller gender wage gaps for wage and salary workers than in the overall sample.



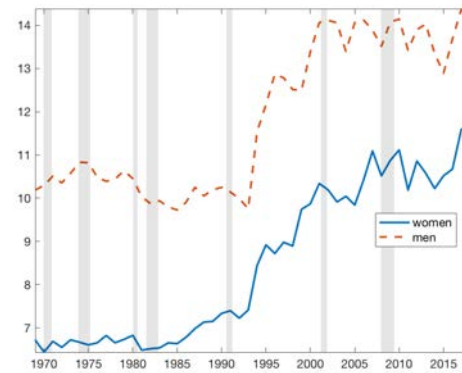
(A) E/P



(B) Average weeks worked per year



(C) Average number of hours worked per week



(D) Hourly wages

FIGURE 27: E/P , average weeks worked per year, average number of hours worked per week and hourly wages. Source: Author's calculation based on CPS March Supplement.

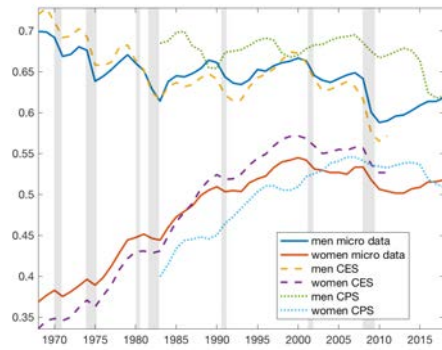
We also compare the aggregate values of hours and wages implied by our micro based estimates with commonly used aggregate series in the literature. Specifically, we consider the following series for aggregate hours:

- Hours of Wage and Salary Workers on Nonfarm Payrolls: Total (TOTLA). Source: U.S. Bureau of Labor Statistics.
- Nonfarm Business Sector: Hours of All Persons (HOANBS). Source: U.S. Bureau of Labor Statistics. Release: Productivity and Costs. Frequency: Quarterly, Seasonally Adjusted

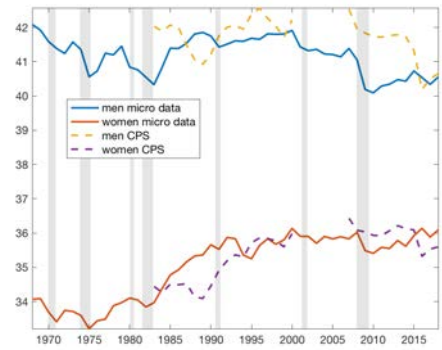
The series are obtained from the FRED Database <https://fred.stlouisfed.org>. For more information, see <http://www.bls.gov/lpc/hoursdatainfo.htm>.

For wages, we consider:

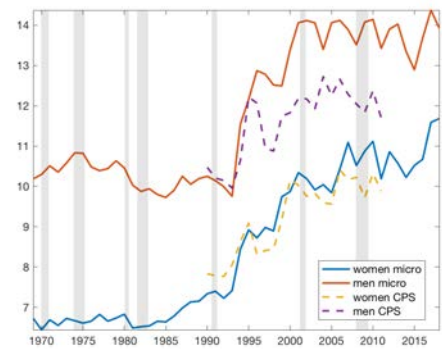
- Hourly wages for wage and salaried workers estimated from weekly earnings and usual weekly hours. Frequency: Monthly. Source: Current Population Survey.



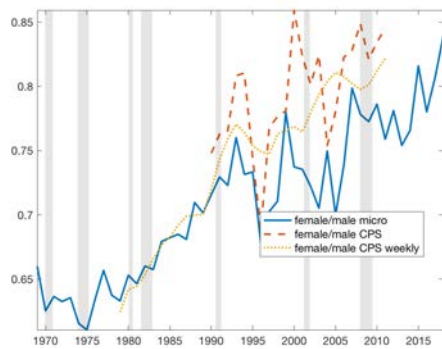
(A) E/P



(B) Hours worked per week



(C) Hourly wages



(D) Female/male wages

FIGURE 28: E/P , average number of hours worked per week, hourly wages and female/male wage ratios, comparison between micro data and aggregate series. Source: Author's calculation based on CPS March Supplement and CES.

- Hourly wages of production and supervisory employees. Frequency: Monthly. Source: Current Establishment Survey.
- Hourly wages of all employees. Frequency: Monthly. Initial availability: 2006 January. Source: Current Establishment Survey.
- Nonfarm Business Sector: Compensation Per Hour (COMPNFB). Source: U.S. Bureau of Labor Statistics. Release: Productivity and Costs. Frequency: Quarterly, Seasonally Adjusted. Obtained from the FRED Database.

All wage series are expressed in 1982-1984 dollars. The Compensation Per Hour is reported as an index and we rescale it to be the same as Nonfarm Payrolls Hourly Compensation for All Employees in 2011 (this series is only available since 2006).

The results are displayed in figure 29. Hour aggregate hours per capita series closely follows the ones available from the Current Employment Survey, though it growth at a slightly faster rate starting in the mid-1980s. This may reflect that we also include the self-employed and we consider total hours, which may include hours on a second job, which would not be considered by the CES. Our aggregate wage series is considerably higher than aggregate wages for wage and salaried workers from the CPS and wages for production and supervisory employees from the CES. However, for the available years it is lower than the CES measure of hourly wage for all workers. The growth in our aggregate hourly wage series reflects quite closely the growth rate of compensation per hour from the Productivity and Costs release.

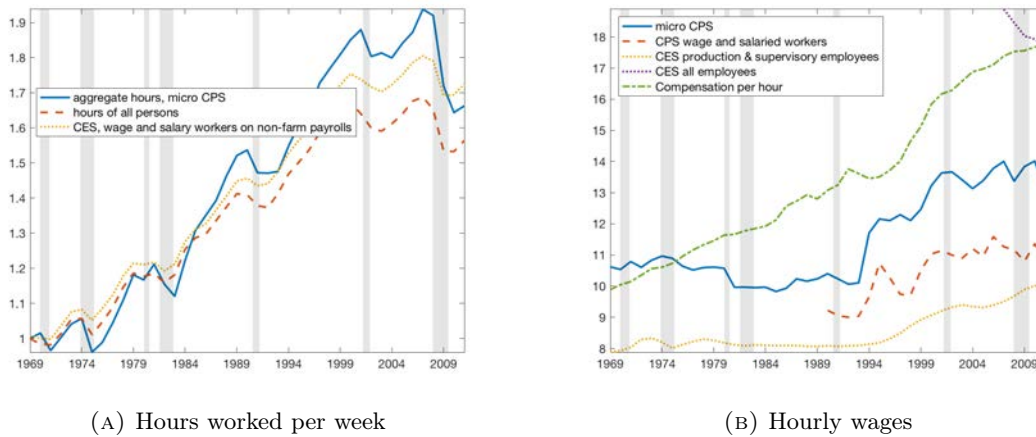
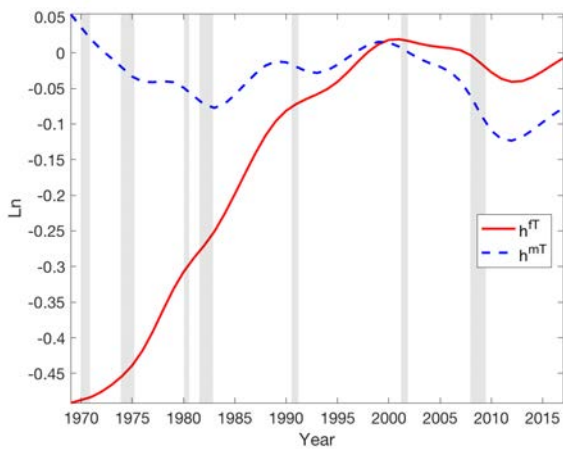
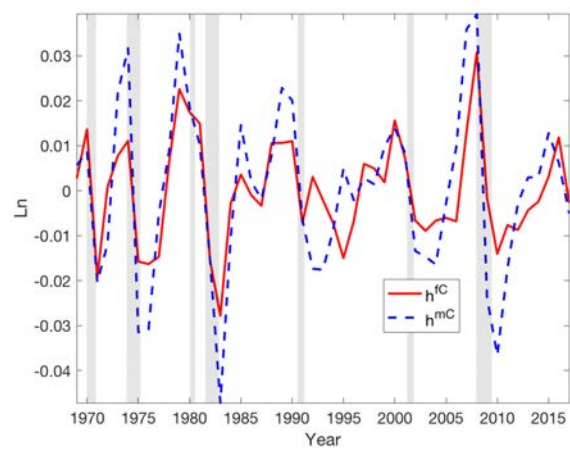


FIGURE 29: Aggregate hours per capita, index 1969=1, and hourly wages in 1982-1984 dollars, comparison between micro based estimates for economy wide averages and aggregate series. Source: Author's calculation based on CPS March Supplement and CES.

B Additional Figures: Empirical Analysis



(A) Trend component of hours by gender



(B) Cyclical component of hours by gender

FIGURE 30: Female and male hours per capita, difference from 1995-2004 average. Trend component (A) and cyclical component (B) of female and male hours per capita, obtained with Hodrick-Prescott filter with $\lambda = 6.5$. Source: Author's calculations based on CPS March Supplement.

C Model: Additional Results and Derivations

C.1 Gender Specific Shocks

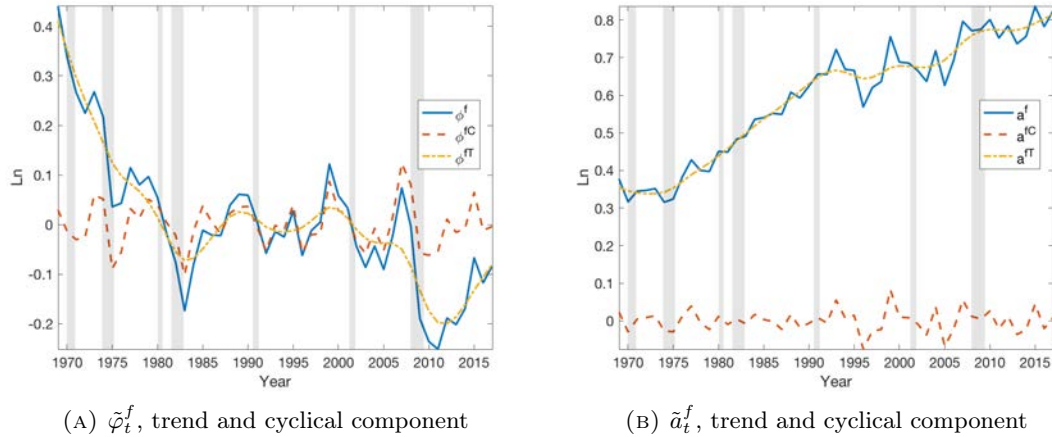


FIGURE 31: Female labor supply shock and relative productivity shock, trend and cyclical components. Calibrated model. HP-filtered with parameter $\lambda = 6.5$. Source: Author's calculations based on Current Population Survey.

C.2 Average Labor Productivity and Solow Residual

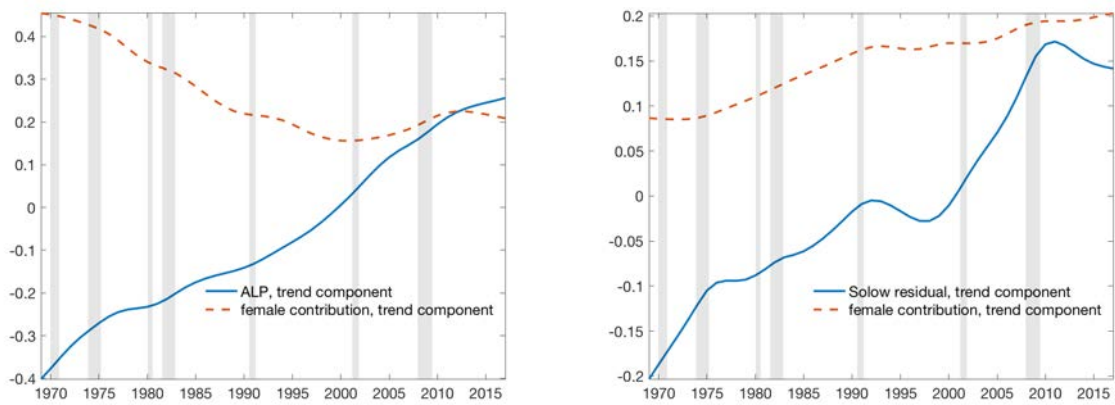


FIGURE 32: Aggregate labor productivity (ALP, left panel) and Solow residual (right panel) and corresponding female contributions. Trend components

C.3 Household and Firm Optimization

The household's first order necessary conditions for optimality are as follows:

- Consumption:

$$\Lambda_t = \frac{b_t}{C_t - \eta C_{t-1}} - \beta \eta E_t \frac{b_{t+1}}{C_{t+1} - \eta C_t}$$

where Λ_t is the multiplier of the budget constraint;

- Physical capital (\bar{K}_t):

$$\Phi_t = \beta E_t \left[\Lambda_{t+1} \left(r_{t+1}^k u_{t+1} - a(u_{t+1}) \right) \right] + (1 - \delta) \beta E_t \Phi_{t+1}$$

- Investment:

$$\Lambda_t = \Phi_t \mu_t \left[1 - S \left(\frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S' \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta E_t \left[\Phi_{t+1} \mu_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S' \left(\frac{I_{t+1}}{I_t} \right) \right]$$

- Utilization:

$$r_t^k = a'(u_t)$$

- Labor supply (H_t^j for $j = f, m$):

$$W_t^j = \frac{\varphi_t^j (H_t^j)^{\nu_j}}{\Lambda_t}. \quad (19)$$

The firms' first order necessary conditions are given by the following system of equations.

- Effective capital (K_t):

$$r_t^k = \alpha A_t^{1-\alpha} \left(\frac{L_t}{K_t} \right)^{1-\alpha};$$

- Demand for female labor (H_t^f)

$$W_t^f = (1 - \alpha) \frac{Y_t}{H_t^f} \omega^f \left(\frac{\tilde{a}_t^f (H_t^f / H^f)}{L_t} \right)^\rho$$

- Demand for male labor (H_t^m):

$$W_t^m = (1 - \alpha) \frac{Y_t}{H_t^m} \omega^m \left(\frac{(H_t^m / H^m)}{L_t} \right)^\rho$$

C.4 Characterizing the Solution for the Normalized Model

We adopt the following normalization to make the model stationary:

$$\begin{aligned}
 y &= Y/A \\
 k &= K/A \\
 \bar{k} &= \bar{K}/A \\
 c &= C/A \\
 i &= I/A \\
 w^j &= W^j/A \\
 \lambda &= \Lambda A \\
 \phi &= \Phi A.
 \end{aligned}$$

We now rewrite the equilibrium conditions in terms of the normalized variables.

Households

- Consumption

$$\lambda_t = \frac{b_t e^{z_t}}{e^{z_t} c_t - \eta c_{t-1}} - \beta \eta E_t \frac{b_{t+1}}{e^{z_{t+1}} c_{t+1} - \eta c_t} \quad (20)$$

- Physical capital (\bar{K}_t)

$$\phi_t = \beta E_t \left\{ e^{-z_{t+1}} \lambda_{t+1} \left[r_{t+1}^k u_{t+1} - a(u_{t+1}) \right] \right\} + (1 - \delta) \beta E_t (\phi_{t+1} e^{-z_{t+1}}) \quad (21)$$

- Investment

$$\begin{aligned}
 \lambda_t = \phi_t \mu_t & \left[1 - S \left(\frac{i_t}{i_{t-1}} e^{z_t} \right) - \frac{i_t}{i_{t-1}} e^{z_t} S' \left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \right] \\
 & + \beta E_t \left[\phi_{t+1} e^{-z_{t+1}} \mu_{t+1} \left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right)^2 S' \left(\frac{i_{t+1}}{i_t} e^{z_{t+1}} \right) \right] \quad (22)
 \end{aligned}$$

- Utilization

$$r_t^k = a'(u_t) \quad (23)$$

- Definition of effective capital

$$k_t = u_t \bar{k}_{t-1} e^{-z_t} \quad (24)$$

- Physical capital accumulation

$$\bar{k}_t = (1 - \delta) e^{-z_t} \bar{k}_{t-1} + \mu_t \left[1 - S \left(\frac{i_t}{i_{t-1}} e^{z_t} \right) \right] i_t \quad (25)$$

- Labor supply H_t^j for $j = f, m$

$$w_t^j = \frac{\varphi_t^j (H_t^j)^{\nu^j}}{\lambda_t} \quad (26)$$

Firms

- Production function

$$y_t = k_t^\alpha (L_t)^{1-\alpha} \quad (27)$$

- Labor input

$$L_t = \left[\omega^f \left(\tilde{a}_t^f \frac{H_t^f}{H^f} \right)^\rho + \omega^m \left(\frac{H_t^m}{H^m} \right)^\rho \right]^{1/\rho} \quad (28)$$

- Effective capital (K_t)

$$r_t^k = \alpha \left(\frac{L_t}{k_t} \right)^{1-\alpha} \quad (29)$$

- Female labor demand (H_t^f)

$$w_t^f = (1 - \alpha) \frac{y_t}{H_t^f} \omega^f \left(\frac{\tilde{a}_t^f (H_t^f / H^f)}{L_t} \right)^\rho \quad (30)$$

- Male labor demand (H_t^m)

$$w_t^m = (1 - \alpha) \frac{y_t}{H_t^m} \omega^m \left(\frac{(H_t^m / H^m)}{L_t} \right)^\rho \quad (31)$$

Resource Constraint and Summary

- Resource constraint

$$c_t + i_t + \frac{a(u_t)}{u_t} k_t = \frac{y_t}{g_t} \quad (32)$$

Equations (20)-(32) comprise a system of 13 dynamic equations in the 13 unknowns:

$\{\lambda, c, \phi, r^k, u, i, k, \bar{k}, w^f, w^m, H^f, H^m, y, L\}$.

C.5 Steady State

The aggregate variables in the steady state for the rescaled version of the model are characterized by the following system of equations, using the normalization $L = 1$:

$$1 = \beta e^\gamma r^k + (1 - \delta) \beta e^\gamma \quad (33)$$

$$r^k = \frac{e^{-\gamma}}{\beta} - (1 - \delta) \quad (34)$$

$$k = \left(\frac{\alpha}{r^k} \right)^{\frac{1}{1-\alpha}}$$

$$y = k^\alpha$$

$$\bar{k} = k e^\gamma$$

$$i = [1 - (1 - \delta)e^{-\gamma}] \bar{k}$$

$$c = \frac{y}{g} - i$$

$$\lambda = \frac{1}{c} \frac{e^\gamma - \beta\eta}{e^\gamma - \eta}.$$

Equations 7, 8 and 9 provide four conditions to pin down wages and hours, with two extra degrees of freedom represented by the steady state values of the disutility of labor φ^j :

$$w^f = (1 - \alpha) \frac{y}{H^f} \omega^f,$$

$$w^m = (1 - \alpha) \frac{y}{H^m} \omega^m,$$

$$w^j = \frac{\varphi^j (H^j)^{\nu^j}}{\lambda},$$

for $j = f, m$. The value of φ 's is not necessary to solve this model. However, it might be of interest to compute a value for the relative disutility of work that is compatible with the observed relative hours and wages. Given an observed value for average relative hours $h^f = \frac{H^f}{H^m}$, and normalizing $H^m = 1$, we have:

$$\frac{w^f}{w^m} = \frac{\omega^f}{\omega^m} \left(\tilde{H}^f \right)^{-1},$$

from which we can finally pin down the relative disutility:

$$\tilde{\varphi}^f = \frac{w^f}{w^m} \left(\tilde{H}^f \right)^{-\nu^f}.$$

The value of φ^m compatible with the normalization $H^m = 1$ is:

$$\varphi^m = (1 - \alpha) y \omega^m \lambda.$$

C.6 Log Linear Approximation

We can now derive the model's log-linear approximation. Log-linear deviations from steady state are defined as follows, for a generic variable x_t with s.s. value x :

$$\hat{x}_t \equiv \log x_t - \log x,$$

except for $\hat{z}_t \equiv z_t - \gamma$. The set of state equations that will be used in the estimation comprise (35)-(47) derived below.

Households

- Consumption

$$(e^\gamma - \eta\beta)(e^\gamma - \eta)\hat{\lambda}_t = \eta\beta e^\gamma E_t \hat{c}_{t+1} - (e^{2\gamma} + \eta^2\beta)\hat{c}_t + \eta e^\gamma \hat{c}_{t-1} \\ + \eta e^\gamma (\beta\rho_z - 1)\hat{z}_t + (e^\gamma - \eta\beta\rho_b)(e^\gamma - \eta)\hat{b}_t \quad (35)$$

- Physical capital (\bar{K}_t)

$$\hat{\phi}_t = (1 - \delta)\beta e^{-\gamma} E_t (\hat{\phi}_{t+1} - \hat{z}_{t+1}) + (1 - (1 - \delta)\beta e^{-\gamma}) E_t [\hat{\lambda}_{t+1} - \hat{z}_{t+1} + \hat{r}_{t+1}^k] \quad (36)$$

- Investment

$$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - e^{2\gamma}\zeta(\hat{u}_t - \hat{u}_{t-1} + \hat{z}_t) + \beta e^{2\gamma}\zeta E_t [\hat{u}_{t+1} - \hat{u}_t + \hat{z}_{t+1}] \quad (37)$$

- Utilization

$$\hat{r}_t^k = \chi \hat{u}_t \quad (38)$$

- Definition of effective capital

$$\hat{k}_t = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t \quad (39)$$

- Physical capital accumulation

$$\hat{\bar{k}}_t = (1 - \delta)e^{-\gamma} (\hat{\bar{k}}_{t-1} - \hat{z}_t) + (1 - (1 - \delta)e^{-\gamma}) (\hat{\mu}_t + \hat{u}_t) \quad (40)$$

- Labor supply (H_t^j for $j = f, m$)

$$\hat{w}_t^j = \hat{\varphi}_t^j + \nu^j \hat{H}_t^j - \hat{\lambda}_t \quad (41)$$

Firms

- Production function

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{L}_t \quad (42)$$

- Labor input

$$\hat{L}_t = \omega^f (\hat{a}_t^f + \hat{H}_t^f) + \omega^m \hat{H}_t^m \quad (43)$$

- Return to capital

$$\hat{r}_t^k = (1 - \alpha) (\hat{L}_t - \hat{k}_t) \quad (44)$$

- Female labor demand (H_t^f)

$$\hat{w}_t^f = \hat{y}_t + (\rho - 1) \hat{H}_t^f + \rho \hat{a}_t^f - \rho \hat{L}_t \quad (45)$$

- Male labor demand (H_t^m)

$$\hat{w}_t^m = \hat{y}_t + (\rho - 1) \hat{H}_t^m - \rho \hat{L}_t \quad (46)$$

Resource Constraint

- Resource constraint

$$\frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + \frac{r^k k}{y} \hat{u}_t = \frac{1}{g} \hat{y}_t - \frac{1}{g} \hat{g}_t \quad (47)$$

C.7 Shocks

Following [31], we normalize the intertemporal preference shock as:

$$\hat{b}_t^* = \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1 - \rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma} \hat{b}_t$$

so as to make the coefficient on consumption in the Euler equation equal to one. The Euler equation pricing a real one-period bond with interest rate r_t , which we did not consider explicitly, reads:

$$\hat{\lambda}_t = E_t \left[\hat{\lambda}_{t+1} - \hat{z}_{t+1} \right] + \hat{r}_t$$

and substituting equation (35), we obtain:

$$\begin{aligned} \frac{e^{2\gamma} + \eta^2\beta + \eta e^\gamma}{(e^\gamma - \eta\beta)(e^\gamma - \eta)} \hat{c}_t + (\dots) &= (\dots) + \frac{(e^\gamma - \eta\beta\rho_b)(1 - \rho_b)}{e^\gamma - \eta\beta} \hat{b}_t \\ \hat{c}_t + (\dots) &= (\dots) + \frac{(e^\gamma - \eta)(e^\gamma - \eta\beta\rho_b)(1 - \rho_b)}{e^{2\gamma} + \eta^2\beta + \eta e^\gamma} \hat{b}_t. \end{aligned}$$

Using this normalization, the resulting set of exogenous shocks in the model is summarized in 4.

C.8 Observation Equations

We adopt the following observation equations, where all series are measured in deviations from their mean over the period of the estimation.

TABLE 4: Exogenous Shocks

\hat{b}_t^*	$= \rho_b \hat{b}_{t-1}^* + \varepsilon_{b,t}$
$\hat{\mu}_t$	$= \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t}$
\hat{z}_t	$= \rho_z \hat{z}_{t-1} + \varepsilon_{z,t}$
\hat{g}_t	$= \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}$
\hat{a}_t^f	$= \hat{a}_t^{fT} + \hat{a}_t^{fC}$
\hat{a}_t^{fT}	$= \rho_{\hat{a}^{fT}} \hat{a}_{t-1}^{fT} + \varepsilon_{\hat{a}^{fT},t}$
\hat{a}_t^{fC}	$= \rho_{\hat{a}^{fC}} \hat{a}_{t-1}^{fC} + \varepsilon_{\hat{a}^{fC},t}$
$\hat{\varphi}_t^f$	$= \hat{\varphi}_t^{fT} + \hat{\varphi}_t^{fC}$
$\hat{\varphi}_t^{fT}$	$= \rho_{\hat{\varphi}^{fT}}^T \hat{\varphi}_{t-1}^{fT} + \varepsilon_{\hat{\varphi}^{fT},t}^T$
$\hat{\varphi}_t^{fC}$	$= \rho_{\hat{\varphi}^{fC}}^T \hat{\varphi}_{t-1}^{fC} + \varepsilon_{\hat{\varphi}^{fC},t}^T$
$\hat{\varphi}_t^m$	$= \rho_{\varphi^m} \hat{\varphi}_{t-1}^m + \varepsilon_{\varphi^m,t}$

TABLE 5: Observation Equations

log GDP Growth	$= \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$
log Consumption Growth	$= \hat{c}_t - \hat{c}_{t-1} + \hat{z}_t$
log Investment Growth	$= \hat{i}_t - \hat{i}_{t-1} + \hat{z}_t$
log Hours, Men	$= \hat{H}_t^m$
log Hours, Relative	$= \hat{H}_t^f - \hat{H}_t^m$
log Labor Share, Men	$= \hat{w}_t^m + \hat{H}_t^m - \hat{y}_t$

D Estimation: Additional Results

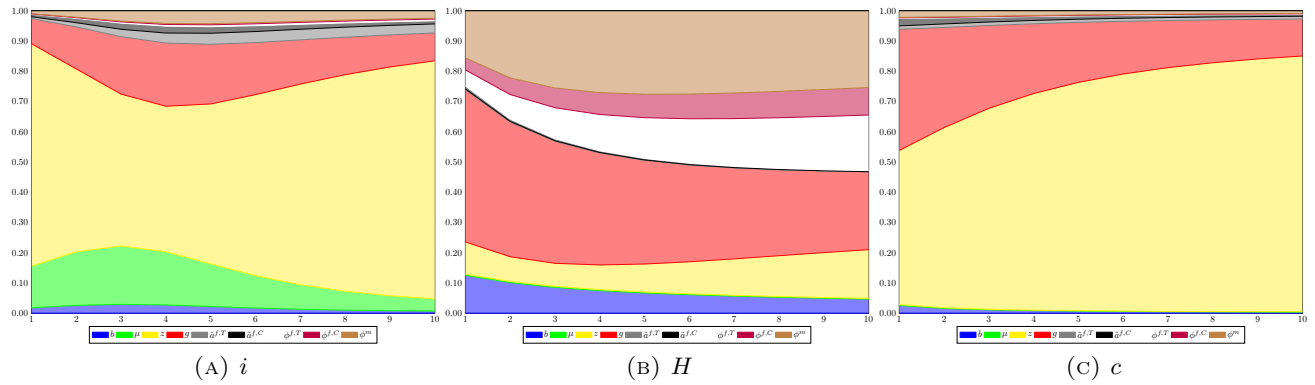


FIGURE 33: Variance decomposition, aggregate variables. Full model, 1969-2017.

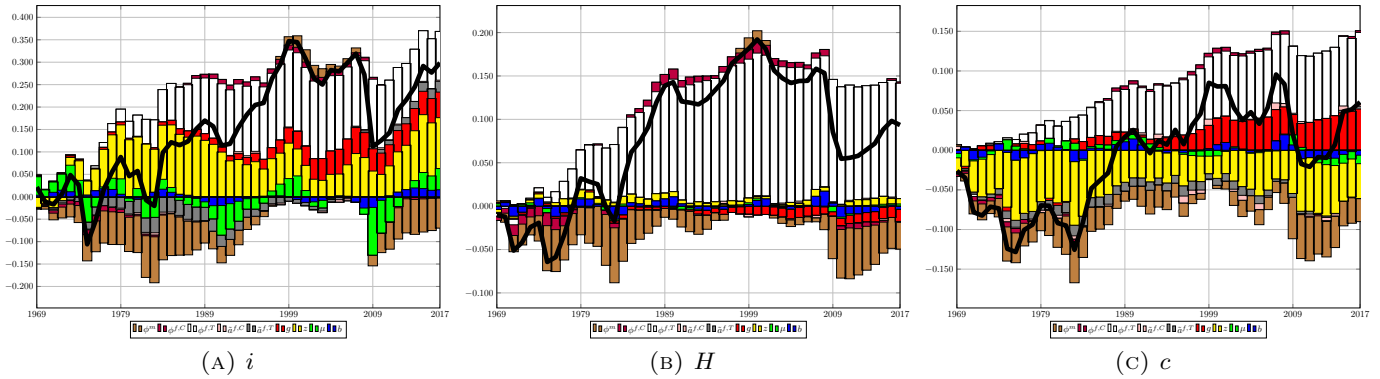


FIGURE 34: Shock decompositions, aggregate variables. Full model, 1969-2017.

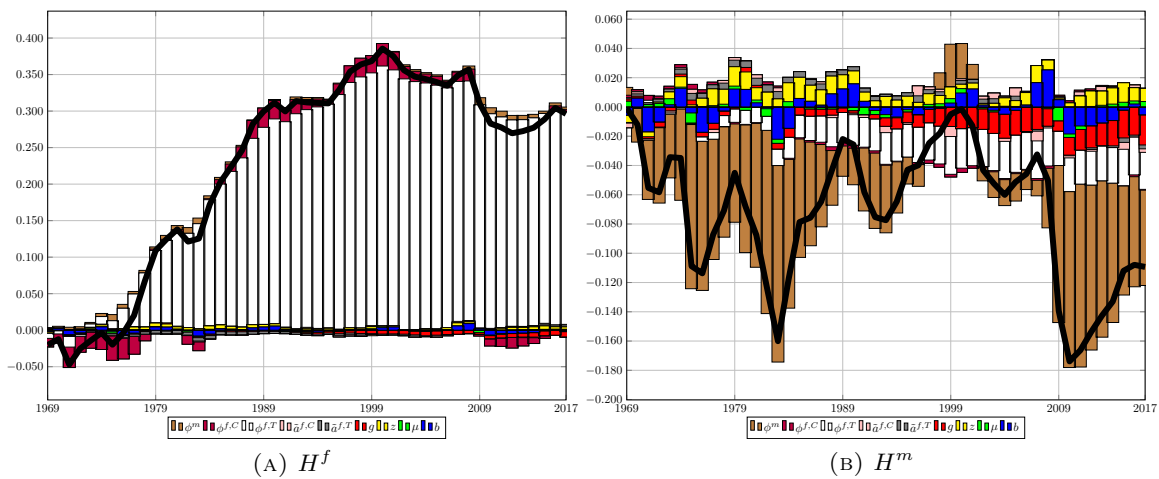


FIGURE 35: Shock decompositions, aggregate hours per capita by gender. Full model, 1969-2017.

D.1 Simple Model with Variable φ

TABLE 6: Estimated Parameters: Simple Model with Stochastic φ

	Dist	Prior			Posterior						
		10%	Median	90%	Mode	Mean	SE	10%	Median	90%	
γ	N	3.0000	0.4616	0.5000	0.5384	0.5005	0.5005	0.0296	0.4647	0.5008	0.5392
χ	G	2.0000	3.7689	4.9335	6.3167	5.4097	5.4097	1.0045	4.2974	5.4779	6.9277
η	B	1.0000	0.3701	0.5000	0.6299	0.5454	0.5454	0.0758	0.4255	0.5296	0.6204
ζ	G	2.0000	1.5236	2.5776	4.0345	4.3303	4.3303	0.8382	3.8007	4.7161	5.9664
ρ_b	B	1.0000	0.3205	0.6143	0.8574	0.0607	0.0607	0.0465	0.0342	0.0815	0.1515
ρ_μ	B	1.0000	0.3205	0.6143	0.8574	0.2569	0.2569	0.1075	0.1363	0.2644	0.4129
ρ_z	B	1.0000	0.1426	0.3857	0.6795	0.1935	0.1935	0.1623	0.0978	0.2687	0.5302
ρ_g	B	1.0000	0.3205	0.6143	0.8574	0.8004	0.8004	0.1643	0.4606	0.7284	0.8913
ρ_{ϕ^T}	B	1.0000	0.9599	0.9836	0.9954	0.9822	0.9822	0.0212	0.9389	0.9752	0.9911
ρ_{ϕ^C}	B	1.0000	0.2306	0.5000	0.7694	0.3351	0.3351	0.1757	0.1739	0.3836	0.6376
σ_b	IG1	4.0000	0.0260	0.0600	0.1880	0.0152	0.0152	0.0024	0.0134	0.0159	0.0193
σ_μ	IG1	4.0000	0.1470	0.3250	0.9390	0.1281	0.1281	0.0253	0.1094	0.1363	0.1732
σ_z	IG1	4.0000	0.3760	0.7480	0.9990	0.1564	0.1564	0.0110	0.1580	0.1659	0.1844
σ_g	IG1	4.0000	0.1470	0.3250	0.9390	0.0645	0.0645	0.0042	0.0651	0.0683	0.0755
σ_{ϕ^T}	IG1	4.0000	0.0530	0.1220	0.3760	0.0469	0.0469	0.0092	0.0406	0.0504	0.0635
σ_{ϕ^C}	IG1	4.0000	0.1130	0.2530	0.7520	0.0868	0.0868	0.0155	0.0712	0.0875	0.1112

Posterior log density at mode: 309.8249 Estimation period: 1969-2017.

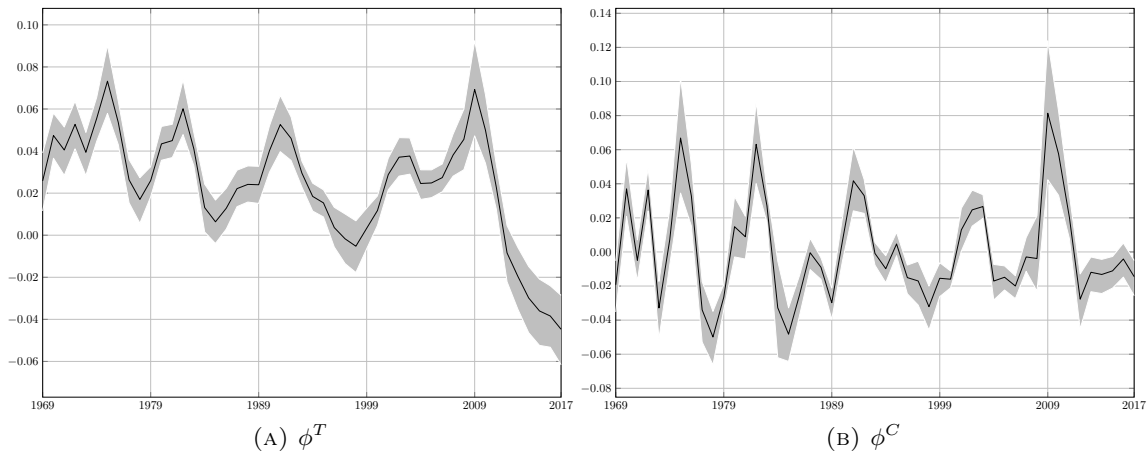


FIGURE 36: Simple model with labor supply shock. Estimated path of ϕ^T and ϕ^C . Sample period: 1969-2017.

D.2 Simple Model with Fixed ϕ

TABLE 7: Estimated Parameters: Simple Model with Stochastic ϕ

	Dist	Prior			Mode	Mean	Posterior				
		10%	Median	90%			SE	10%	Median	90%	
γ	N	3.0000	0.4616	0.5000	0.5384	0.4894	0.4894	0.0236	0.4478	0.4741	0.5053
χ	G	2.0000	3.7689	4.9335	6.3167	5.8217	5.8217	1.1742	3.5706	5.0215	6.9035
η	B	1.0000	0.3701	0.5000	0.6299	0.3352	0.3352	0.0751	0.2237	0.3278	0.4306
ζ	G	2.0000	1.5236	2.5776	4.0345	4.2510	4.2510	0.6648	3.6192	4.4208	5.3990
ρ_b	B	1.0000	0.3205	0.6143	0.8574	0.1557	0.1557	0.0673	0.0539	0.1386	0.2358
ρ_μ	B	1.0000	0.3205	0.6143	0.8574	0.3211	0.3211	0.0833	0.1480	0.2652	0.3744
ρ_z	B	1.0000	0.1426	0.3857	0.6795	0.1829	0.1829	0.1135	0.0996	0.2377	0.3801
ρ_g	B	1.0000	0.3205	0.6143	0.8574	0.8002	0.8002	0.0816	0.7378	0.8594	0.9294
σ_b	IG1	4.0000	0.0260	0.0600	0.1880	0.0182	0.0182	0.0034	0.0158	0.0197	0.0246
σ_μ	IG1	4.0000	0.1470	0.3250	0.9390	0.1374	0.1374	0.0265	0.1224	0.1506	0.1901
σ_z	IG1	4.0000	0.3760	0.7480	0.9990	0.1564	0.1564	0.0104	0.1580	0.1661	0.1827
σ_g	IG1	4.0000	0.1470	0.3250	0.9390	0.0645	0.0645	0.0040	0.0651	0.0683	0.0750
ϕ	N	3.0000	-0.0563	0.2000	0.4563	0.0362	0.0362	0.0799	-0.0596	0.0333	0.1370

Posterior log density at mode: 327.9962. Estimation period: 1969-2017.

D.3 Model Comparison

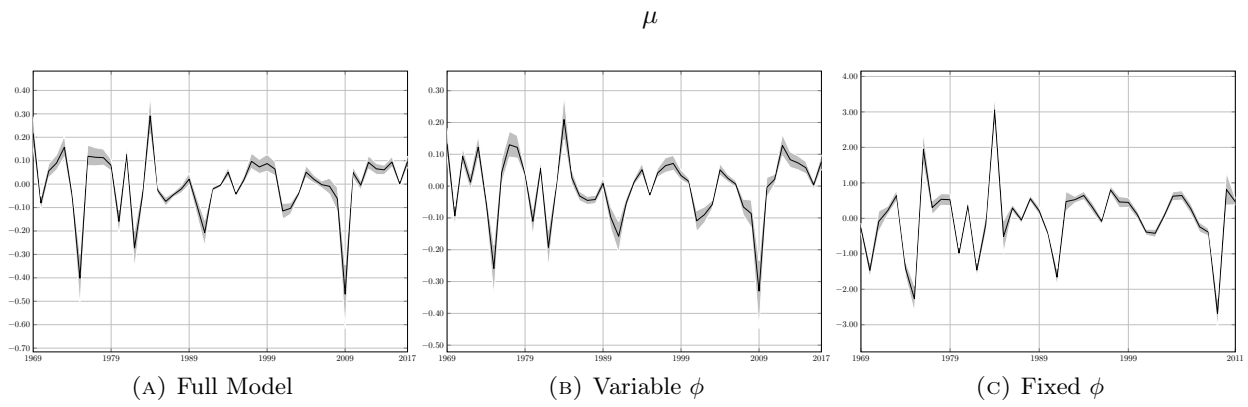


FIGURE 37: Model comparison, aggregate shocks: investment technology shock μ . Sample period: 1969-2017.

D.4 Time Comparison

TABLE 8: Estimated Parameters: Full Model, 1969-1992

	Dist	Prior			Mode	Mean	Posterior				
		10%	Median	90%			SE	10%	Median	90%	
γ	N	3.0000	0.4616	0.5000	0.5384	0.5000	0.5000	0.0283	0.4635	0.5003	0.5371
χ	G	2.0000	3.7689	4.9335	6.3167	5.0738	5.0738	1.0006	3.9887	5.2093	6.5405
η	B	1.0000	0.3701	0.5000	0.6299	0.3661	0.3661	0.0837	0.2487	0.3548	0.4618
ζ	G	2.0000	1.5236	2.5776	4.0345	2.2766	2.2766	0.7495	1.8423	2.6479	3.7696
ρ_b	B	1.0000	0.3205	0.6143	0.8574	0.1958	0.1958	0.1092	0.0947	0.2178	0.3781
ρ_μ	B	1.0000	0.3205	0.6143	0.8574	0.3275	0.3275	0.1636	0.1823	0.3737	0.6130
ρ_z	B	1.0000	0.1426	0.3857	0.6795	0.2249	0.2249	0.1679	0.0979	0.2819	0.5458
ρ_g	B	1.0000	0.3205	0.6143	0.8574	0.8096	0.8096	0.1543	0.4965	0.7448	0.8954
$\rho_{\bar{a},T}$	B	1.0000	0.9599	0.9836	0.9954	0.9866	0.9866	0.0134	0.9592	0.9803	0.9928
$\rho_{\bar{a},C}$	B	1.0000	0.2306	0.5000	0.7694	0.7077	0.7077	0.1990	0.3058	0.6149	0.8432
$\rho_{\phi,T}$	B	1.0000	0.9599	0.9836	0.9954	0.9952	0.9952	0.0055	0.9837	0.9927	0.9975
$\rho_{\phi,C}$	B	1.0000	0.2306	0.5000	0.7694	0.6712	0.6712	0.2221	0.3220	0.6357	0.9237
ρ_{ϕ^m}	B	1.0000	0.3205	0.6143	0.8574	0.7884	0.7884	0.1459	0.4989	0.7290	0.8808
σ_b	IG1	4.0000	0.0260	0.0600	0.1880	0.0235	0.0235	0.0065	0.0197	0.0251	0.0350
σ_μ	IG1	4.0000	0.1470	0.3250	0.9390	0.2169	0.2169	0.0695	0.1744	0.2431	0.3499
σ_z	IG1	4.0000	0.3760	0.7480	0.9990	0.2023	0.2023	0.0278	0.1815	0.2096	0.2495
σ_g	IG1	4.0000	0.1470	0.3250	0.9390	0.0862	0.0862	0.0127	0.0753	0.0890	0.1077
$\sigma_{\bar{a},T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.0520	0.0520	0.0120	0.0440	0.0564	0.0731
$\sigma_{\bar{a},C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.0797	0.0797	0.0122	0.0701	0.0837	0.1007
$\sigma_{\phi,T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.2010	0.2010	0.0428	0.1590	0.2102	0.2619
$\sigma_{\phi,C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.1270	0.1270	0.0335	0.1053	0.1402	0.1915
σ_{ϕ^m}	IG1	4.0000	0.1470	0.3250	0.9390	0.1250	0.1250	0.0255	0.1073	0.1330	0.1679

TABLE 9: Estimated Parameters: Full Model, 1993-2017

	Dist	Prior			Posterior						
		10%	Median	90%	Mode	Mean	SE	10%	Median	90%	
γ	N	3.0000	0.4616	0.5000	0.5384	0.5000	0.5000	0.0296	0.4614	0.4993	0.5404
χ	G	2.0000	3.7689	4.9335	6.3167	5.0669	5.0669	0.9682	3.9992	5.1292	6.4362
η	B	1.0000	0.3701	0.5000	0.6299	0.3445	0.3445	0.0791	0.2476	0.3462	0.4537
ζ	G	2.0000	1.5236	2.5776	4.0345	4.7082	4.7082	1.0479	4.0194	5.2667	6.7027
ρ_b	B	1.0000	0.3205	0.6143	0.8574	0.2139	0.2139	0.1043	0.1055	0.2176	0.3757
ρ_μ	B	1.0000	0.3205	0.6143	0.8574	0.2922	0.2922	0.1519	0.1486	0.3301	0.5473
ρ_z	B	1.0000	0.1426	0.3857	0.6795	0.1552	0.1552	0.1382	0.0673	0.2020	0.4158
ρ_g	B	1.0000	0.3205	0.6143	0.8574	0.8343	0.8343	0.1718	0.4857	0.7648	0.9157
$\rho_{\bar{a}.T}$	B	1.0000	0.9599	0.9836	0.9954	0.9847	0.9847	0.0171	0.9508	0.9775	0.9927
$\rho_{\bar{a}.C}$	B	1.0000	0.2306	0.5000	0.7694	0.6756	0.6756	0.1903	0.3246	0.6119	0.8306
$\rho_{\phi.T}$	B	1.0000	0.9599	0.9836	0.9954	0.9861	0.9861	0.0182	0.9481	0.9782	0.9929
$\rho_{\phi.C}$	B	1.0000	0.2306	0.5000	0.7694	0.7821	0.7821	0.1612	0.4470	0.6951	0.8596
ρ_{ϕ^m}	B	1.0000	0.3205	0.6143	0.8574	0.8281	0.8281	0.1321	0.5910	0.7909	0.9135
σ_b	IG1	4.0000	0.0260	0.0600	0.1880	0.0230	0.0230	0.0061	0.0190	0.0242	0.0340
σ_μ	IG1	4.0000	0.1470	0.3250	0.9390	0.1472	0.1472	0.0339	0.1239	0.1630	0.2117
σ_z	IG1	4.0000	0.3760	0.7480	0.9990	0.1979	0.1979	0.0270	0.1733	0.1995	0.2402
σ_g	IG1	4.0000	0.1470	0.3250	0.9390	0.0834	0.0834	0.0118	0.0729	0.0861	0.1035
$\sigma_{\bar{a}.T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.0512	0.0512	0.0111	0.0426	0.0540	0.0715
$\sigma_{\bar{a}.C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.0801	0.0801	0.0129	0.0705	0.0839	0.1021
$\sigma_{\phi.T}$	IG1	4.0000	0.0530	0.1220	0.3760	0.0703	0.0703	0.0245	0.0587	0.0810	0.1153
$\sigma_{\phi.C}$	IG1	4.0000	0.1130	0.2530	0.7520	0.1180	0.1180	0.0227	0.0968	0.1209	0.1548
σ_{ϕ^m}	IG1	4.0000	0.1470	0.3250	0.9390	0.1156	0.1156	0.0226	0.1016	0.1224	0.1507