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ENVIRONMENTAL POLICY ON THE BACK OF AN ENVELOPE: A COBB-DOUGLAS MODEL IS NOT JUST A TEACHING TOOL

Don Fullerton
Chi L. Ta

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ABSTRACT

To clarify and interpret the workings of a large computable general equilibrium (CGE) model of environmental policy in the U.S., we build an aggregated Cobb-Douglas (CD) model that can be solved easily and analytically. Its closed-form expressions show exactly how key parameters determine the sign and size of effects from a large new carbon tax on emissions, revenue, prices, output, and welfare. Data and parameters from the detailed, dynamic CGE model of Goulder and Hafstead (2018) are used in the CD model to calculate results that can be compared with theirs. Results from the CD model track those from the large CGE model quite closely, even though the CD model omits much detail such as the number of sectors, intermediate inputs, and international trade. A CGE model is quite useful to generate detailed numerical results and to reflect on particular aspects of environmental policy, but the simpler CD model provides a transparent view of exactly how the policy affects key outcomes.

Don Fullerton
Department of Finance
University of Illinois
515 East Gregory Drive, BIF Box#30 (MC520)
Champaign, IL 61820
and NBER
dfullert@illinois.edu

Chi L. Ta
Department of Agricultural and Consumer Economics
University of Illinois
Urbana, IL 61801
chita2@illinois.edu

Computable General Equilibrium (CGE) models are large numerical simulation models that apply detailed economic data to a model of producer and consumer behavior in a general framework to calculate supply, demand, and equilibrium prices across a great number of markets simultaneously. Such models have been widely used to study the effects of environmental regulation on the entire economy, including emissions, outputs, prices, tax revenue, and economics welfare.[1] The most recent and fully described CGE model for climate policy is in the book by Goulder and Hafstead (GH, 2018). It analyzes the myriad effects of a carbon tax, a cap-and-trade system, and some alternatives to emissions pricing. It includes not only careful modeling of the carbon-intensity of each different production sector, but also the intertemporal dynamics of all U.S. taxes on labor supply, investment, and the incomes of different household groups. The model is quite useful, but many readers who do not build their own CGE models may find the sheer size and complexity of such models impenetrable.

To understand such models, and to show exactly what features of the model drive key results, this paper builds a new but simplified Cobb-Douglas (CD) version of a CGE model that can be solved on "the back of an envelope" – using not a computer, but just paper, pencil, and a calculator. We use industry data from GH (2018) to calculate factor shares, elasticities, and tax rates. Then we perform the same kinds of simulations using the CD model to show how to solve for new post-reform general equilibrium prices and quantities. The CD model has only three sectors and four inputs, and it has no input-output matrix nor dynamics. Our results, therefore, must differ to some degree from those of Goulder and Hafstead, but we show that the simple model can generate results that are surprisingly close to those of the complicated CGE model. We also show derivations that are very transparent.

We believe this paper makes three contributions. First, it will be extremely useful for teaching general equilibrium or for teaching environmental economics – particularly pollution tax incidence and effects on economic efficiency. For the imposition of a pollution tax, this paper will show step-by-step derivations of all outcomes such as each quantity of output and its price, each input and its price, pollution, abatement costs, and money-metric utility measures of changes in household welfare (equivalent variations). Others have used Cobb-

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[1] Jorgenson and Wilcoxen 1990; Hazilla and Kopp 1990; Boyd *et al.* 1995; Bovenberg and Goulder 1996; Goulder *et al.* 1999; Parry and Williams 1999; Parry *et al.* 1999.

Douglas examples for teaching, but none of those is published, analytical, and available.[2]

Second, and more importantly, our paper provides an interpretation and commentary on Goulder and Hafstead (2018), the newest and most complete description of a large CGE model. This detailed model is fully capable of analyzing many facets of the U.S. economy, the tax system, and various proposed climate policies. But readers who do not build CGE models themselves may find it hard to penetrate its complexities. Our simpler exposition here is useful to understand the nature of this large, complicated CGE model. Moreover, its results are quite similar. While the complex model is necessary to get detailed numerical results on particular industries, and to solve for adjustment costs during transitions, we show that the detail is not necessary to understand the model nor to get broadly similar results.

In particular, we use their data and their calibration of parameters, adjusted to fit our more aggregated and simpler model. We then use the CD model to study the same questions as in Goulder and Hafstead: what are the effects of a pollution tax when its revenue is rebated to households in a lump-sum fashion or when its revenue is recycled to cut the labor tax? We compare results from our aggregated CD model to results from their dynamic disaggregated CGE model. For impacts of a carbon tax with either lump-sum rebates or with revenue used to cut labor taxes, our simpler model yields key results quite similar to theirs for carbon dioxide emissions, carbon tax revenue, and welfare costs of the carbon tax.

A third contribution is that we show how to use the simple CD model to analyze three other questions not studied by Goulder and Hafstead (2018). We calculate and show the entire marginal abatement cost curve for each type of carbon tax and revenue use. We calculate and show the second-best optimal rate of tax for each such policy. And we further explore the effect of a carbon tax in only one sector on emissions from the other sector (“leakage”). These results further demonstrate the usefulness of the simple model.

The first section of our paper describes the basic Cobb-Douglas model, and the second section shows how to solve it on the back of an envelope for expressions that show the impact of parameters on outcomes such as emissions, revenue, outputs, prices, and economic welfare. An appendix shows how we aggregate data and parameters from Goulder and Hafstead into

[2] A Cobb-Douglas example appears in the lecture notes of James Markusen (2018), but to obtain those lecture notes would require knowledge of their existence and sending an email to ask for them. Thomas Rutherford (1999) and Shoven and Whalley (1984, 1992) also provide simplified CGE models for pedagogical purposes, but solutions for these models require a computer. Kimbell and Harrison (1986) provide analytical solutions for more complicated models with constant elasticity of substitution production. In contrast, the goal here is to provide a complete and comprehensive Cobb-Douglas example in published form that can be solved analytically.

fewer sectors for our simple CD model. The third section uses the analytical solutions to calculate those outcomes and to interpret results. The fourth section compares those CD model results to the detailed results of Goulder and Hafstead (2018). The fifth section reports other results from our CD model. The sixth section discusses key findings and concludes.

1. Model Description

Here, we develop a simplified Cobb-Douglas version of a CGE model. We assume a closed economy with a large number n of identical households, perfect competition, perfect mobility, and no uncertainty. Each household is endowed with a fixed amount of capital ($\bar{K}$) and a fixed amount of time available for labor ($\bar{L}$). Each get utility from consumption of a composite good X, plus direct consumption by the household of electricity E_H, fossil fuels F_H, and leisure or home production L_H. In our model, household utility over consumption goods takes the Cobb-Douglas form:[3]

$$U = U(X, E_H, F_H, L_H) = X^a E_H^b F_H^c L_H^{1-a-b-c} \quad (1.1)$$

Consumers maximize that utility subject to their budget, but their welfare also negatively depends on total emissions of carbon dioxide (nC) from the use of fossil fuels. We assume a constant marginal effect on welfare from emissions, θ, so full welfare is $U - \theta nC$. With many identical households, each ignores the impact of their own emissions on total emissions nC.

In this simplified Cobb-Douglas model, goods are produced according to:[4]

$$X = X(K_X, L_X, E_X, F_X) = AK_X^\alpha L_X^\beta E_X^\gamma F_X^{1-\alpha-\beta-\gamma} \quad (1.2)$$

$$E = E_X + E_H = E(K_E, L_E, F_E) = BK_E^\delta L_E^\epsilon F_E^{1-\delta-\epsilon} \quad (1.3)$$

$$F = F_E + F_X + F_H = GL_F \quad (1.4)$$

$$C = \rho F \quad (1.5)$$

Production functions are all assumed to be Cobb-Douglas, but total-factor productivity (TFP) parameters differ (denoted A, B, and G). Production of X requires capital K_X, labor L_X,

[3] In contrast, the GH model uses constant elasticity of substitution (CES) utility between consumption and leisure, but a Cobb Douglas form to combine different goods into a composite consumption good. We comment later on which differences between the models are most relevant to driving differences in results.

[4] Constant returns to scale (CRTS) removes any importance of the scale of production, so we define all inputs and outputs as amounts per household. The GH model includes X as a potential input to X, E, and F.

electricity E_X, and direct use of fossil fuels F_X. Production of electricity itself requires capital K_E, labor L_E, and fossil fuels F_E. Fossil fuels are produced using labor, L_F, only.[5] In our model, emissions are proportional to fossil fuel use, where a unit of fossil fuels emits ρ tons of carbon dioxide, C.

In this static model, households have no saving decision; the capital stock is fixed.[6] Households allocate their fixed time endowment between labor supply (L) and leisure (L_H). The government levies a tax τ_L on market labor income and provides a lump-sum transfer of all tax revenue back to households as a per-household lump-sum rebate (R). This tax rate is defined as a fraction of the net wage or price of labor (P_L), so the gross wage is $P_L(1+\tau_L)$. We consider a policy reform that raises the pollution tax and uses all added revenue to cut the distorting labor tax (holding constant the initial level of government transfers).

This reform can include a tax τ_{CE} per metric ton of carbon dioxide (CO_2) emissions in the production of electricity and a tax τ_{CO} per metric ton of CO_2 emissions from all "other" uses of fossil fuels. Given a fixed ratio of emissions to fossil fuel use, ρ, each such rate is equivalent to a tax at rate $\rho\tau_{CE}$ per unit of fossil fuel used in electricity and $\rho\tau_{C0}$ per unit of fossil fuel used in other sectors (F_H and F_X). For simplicity, we ignore taxes on output and on capital, in order to focus only on the possible carbon tax in the presence of a pre-existing labor tax. The government budget constraint says that the sum of pollution tax revenues and labor tax revenues equals the lump-sum tax rebate:[7]

$$R = \rho\tau_{CE}F_E + \rho\tau_{CO}(F_H + F_X) + \tau_L P_L L \tag{1.6}$$

We can then show how this simple model produces the standard "double dividend" results when we calculate the effects of a carbon tax with revenue used to reduce the labor tax. We will also discuss two different kinds of carbon tax: a uniform tax on all carbon or a tax on carbon only in the electricity sector.

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[5] Production of fuels are capital intensive in GH, but adding capital to fuel production here would complicate our derivations and solutions. A different simplification could combine labor and capital into one composite input, but then we would have to use the labor tax rate from GH on the composite input, or else adjust the tax rate. Our simplified model is not "accurate", but still can obtain results close to those from the GH model.

[6] In contrast, GH have a dynamic model with quasi-fixed capital, giving rise to adjustment costs that imply windfall gains or losses with important distributional implications. GH also obtain short-run impacts that differ from long-run impacts. In these respects, we cannot compare our results to theirs, because (1) our static CD model has no such dynamics, and (2) we have only one type of household and thus no distributional results.

[7] In contrast, the government sector in the GH model provides fixed real transfers and services, where services are produced using labor, capital, fuel, electricity, and other inputs.

Households choose their consumption of goods and leisure to maximize utility subject to their budget constraint, which requires that their “full” income (I) not be exceeded by the sum of expenditures on the composite good, electricity, fossil fuel, and leisure. Their full income includes receipts from fixed factor endowments and the lump-sum tax rebate:

$$I = P_K\bar{K} + P_L\bar{L} + R = P_X X + P_E E_H + (P_F + \rho\tau_{CO})F_H + P_L L_H$$

where P_X and P_E are prices faced by consumers. Also, P_F denotes the net-of-tax price of fossil fuels, so $P_F + \rho\tau_{CO}$ is the price faced by consumers (gross of tax). The rental price for capital is denoted as P_K and is untaxed. Only relative prices matter in our general equilibrium model, so we anchor the overall price level by assuming that the nominal amount of full income is fixed ($I = \bar{I}$). Then the Cobb-Douglas utility function implies that expenditure on each consumption good is also fixed (as a fraction of that fixed income):

$$P_X X = aI \quad (1.7)$$

$$P_E E_H = bI \quad (1.8)$$

$$(P_F + \rho\tau_{CO})F_H = cI \quad (1.9)$$

$$P_L L_H = (1 - a - b - c)I \quad (1.10)$$

Competitive firms are price takers in input and output markets, and they choose input quantities to maximize profits subject to their production functions. The Cobb-Douglas forms yield very simple factor demands. For example, firms in X always use α of net sales revenue, $P_X X$, to buy units of capital K_X, each of which cost P_K. Similarly, for other inputs:

$$P_K K_X = \alpha P_X X \quad (1.11)$$

$$P_L(1 + \tau_L)L_X = \beta P_X X \quad (1.12)$$

$$P_E E_X = \gamma P_X X \quad (1.13)$$

$$(P_F + \rho\tau_{CO})F_X = (1 - \alpha - \beta - \gamma)P_X X \quad (1.14)$$

$$P_K K_E = \delta P_E E \quad (1.15)$$

$$P_L(1 + \tau_L)L_E = \epsilon P_E E \quad (1.16)$$

$$(P_F + \rho\tau_{CE})F_E = (1 - \delta - \epsilon)P_E E \tag{1.17}$$

$$P_L(1 + \tau_L)L_F = P_F F \tag{1.18}$$

In our Cobb-Douglas economy, the three conditions for general equilibrium are income balance, zero profit, and market clearance. Income balance conditions are the household and government budget constraints above. The zero profit conditions say that the value of output produced by each industry must equal the sum of the values of inputs employed in production [satisfied by equations (1.11) – (1.18)]. Market clearing implies that the total quantity of each commodity (e.g., F or E) supplied by firms must equal the total quantity demanded [in equations (1.3) and (1.4) above]. In addition, the quantity of each primary factor used in all sectors must sum to the household's endowment of that factor:

$$\overline{K} = K_X + K_E \tag{1.19}$$

$$\overline{L} = L + L_H = L_X + L_E + L_F + L_H \tag{1.20}$$

All equations above hold in the initial equilibrium (indicated by superscript "o") and in the new equilibrium (with primes). For a change in uniform carbon tax rate from τ_C^{o} to τ_C', we can then compare outcomes such as new emissions C' to old emissions C^o.

2. Deriving New Equilibrium Outcomes

This section is designed to demonstrate the simplicity and ease of successive substitution to solve for the new general equilibrium prices and quantities "on the back of an envelope". We study four of the GH model's scenarios: (1) a uniform carbon tax with lump-sum rebate, (2) a uniform carbon tax with a labor tax cut, (3) a carbon tax on the electricity sector only with lump-sum rebate, and (4) a carbon tax on the electricity sector only with a labor tax cut. In this section, we show the first ten equations that pertain to all of those scenarios, which are enough to demonstrate the method. Then Appendix A shows remaining additional steps, including details that differ for each of the four scenarios.

First, of course, Cobb-Douglas forms yield very simple factor demands. Equations (1.7) and (1.11) yield (2.1), while (1.13) and (1.15) yield (2.2):

$$K_X' = \frac{\alpha P_X' X'}{P_K'} = \frac{\alpha a I}{P_K'} \tag{2.1}$$

$$K_E' = \frac{\delta P_E' E'}{P_K'} = \frac{\delta(b + \gamma a)I}{P_K'} \tag{2.2}$$

Substitute (2.1) and (2.2) into the market-clearing condition (1.19) and manipulate to derive an expression for the new factor price of capital:

$$P_K' = \frac{\alpha a + \delta(b + \gamma a)}{\bar{K}} I \tag{2.3}$$

Then substitute (2.3) back into (2.1) and (2.2) to solve for new uses of capital K_X' and K_E'.

Since full income includes receipts from the lump-sum tax rebate and fixed factor endowments (labor and capital), that P_K' in (2.3) can be used to find the net-of-tax wage as:

$$P_L' = \frac{I - P_K'\bar{K} - R'}{\bar{L}} \tag{2.4}$$

Then use labor demands in (1.12), (1.16), and (1.18), plus the market-clearing equation (1.20) to obtain labor used in each sector. With $F' = GL_F'$, these equations yield:

$$L_X' = \frac{\beta a I}{P_L'(1 + \tau_L')} \tag{2.5}$$

$$L_E' = \frac{\epsilon(b + \gamma a)I}{P_L'(1 + \tau_L')} \tag{2.6}$$

$$L_H' = \frac{(1 - a - b - c)I}{P_L'} \tag{2.7}$$

$$F' = GL_F' = G(\bar{L} - L_H' - L_X' - L_E') \tag{2.8}$$

The production of fossil fuels (1.4) and zero profit condition (1.18) yield the first equation for the price of fuel below, and the expression for P_L' in equation (2.4) yields the second:

$$P_F' = \frac{P_L'(1 + \tau_L')}{G} = \frac{(I - P_K'\bar{K} - R')(1 + \tau_L')}{\bar{L}G} \tag{2.9}$$

In the case with lump-sum rebate of carbon revenue, we assume no change in labor tax rate ($\tau_L' = \tau_L^o$). Given the solution for P_K' in (2.3), the only unknown on the right side of (2.9) is R'. Our approach is to express all other unknown outcomes as functions of the equilibrium rebate R'. Then we solve for R' and substitute back to get all other outcomes. For example, for the uniform carbon tax case ($\tau_C' = \tau_{CE}' = \tau_{CO}'$), we use the government budget (1.6) to get:

$$R' = \rho\tau_C'(F_X' + F_E' + F_H') + \tau_L' P_L'(L_X' + L_E' + L_F'),$$

and manipulate to derive the new equilibrium total quantity of fossil fuels:

$$F' = \frac{R' - \tau_L' P_L'(\bar{L} - L_H')}{\rho \tau_C'} = \frac{R' - \tau_L'(I - P_K'\bar{K} - R') + \tau_L'(1-a-b-c)I}{\rho \tau_C'} \qquad (2.10)$$

We equate equations (2.8) and (2.10) and insert expressions for P_K' from equation (2.3) and P_L' from equation (2.4) to solve for R'.

In the case with labor tax recycling, an increase in carbon tax revenue is offset by a cut in labor tax rate, with no change in rebate ($R' = R^o$). Our approach in this case is to express all unknown variables as functions of the new labor tax rate τ_L'. Then we solve for τ_L' and substitute back that expression for τ_L' to obtain other new equilibrium outcomes. For example, for the uniform carbon tax case, we equate (2.8) and (2.10) to solve for τ_L'. Remaining steps are all listed explicitly in Appendix A. Here, we proceed to solve for welfare.

Demand functions for consumption goods and leisure are obtained from expenditure share equations (1.7)-(1.10). Here, we substitute these demands into the utility function (1.1) to obtain the indirect utility function:

$$V(prices, I) = \left(\frac{aI}{P_X}\right)^a \left(\frac{bI}{P_E}\right)^b \left(\frac{cI}{P_F(1+\rho\tau_{CO})}\right)^c \left(\frac{(1-a-b-c)I}{P_L}\right)^{1-a-b-c}$$

$$= \frac{I}{\left(\frac{P_X}{a}\right)^a \left(\frac{P_E}{b}\right)^b \left(\frac{P_F(1+\rho\tau_{CO})}{c}\right)^c \left(\frac{P_L}{1-a-b-c}\right)^{1-a-b-c}}$$

Then we define $\bar{P} \equiv \left(\frac{P_X}{a}\right)^a \left(\frac{P_E}{b}\right)^b \left(\frac{P_F(1+\rho\tau_{CO})}{c}\right)^c \left(\frac{P_L}{1-a-b-c}\right)^{1-a-b-c}$ as the "ideal" price index, a function of all prices faced by households.[8] The result is a simple expression for indirect utility, $U = I/\bar{P}$. We then invert that function to obtain the expenditure function $Exp(U, \bar{P})$:

$$I = Exp(U, \bar{P}) = U \times \bar{P}$$

In the case with fixed full income, Mas-Colell *et al*. (1995, p.82) show that the equivalent variation can be measured as:

$$EV = Exp(\bar{P}^o, U') - Exp(\bar{P}', U') = Exp(\bar{P}^o, U') - Exp(\bar{P}^o, U^o)$$

In the Cobb-Douglas case,

[8] Following prior literature, the bars over earlier variables indicate they are fixed ($\bar{K}$, $\bar{L}$, and $\bar{I}$). Following *other* literature, however, the bar over $\bar{P}$ is used to represent the "ideal" price index. Since all prices can change from the old equilibrium to new equilibrium, the value of this price index can change (from $\bar{P}^o$ to $\bar{P}'$).

$$EV = \bar{P}^o \times U' - \bar{P}^o \times U^o = \bar{P}^o(U' - U^o)$$

This expression has a ready interpretation, since households essentially purchase “utils” at the price $\bar{P}$ per util. Then the equivalent variation is the change in non-environmental utility $(U' - U^o)$ valued at old prices $(\bar{P}^o)$, while the compensating variation (CV) is the same change in U valued at new prices $(\bar{P}')$.

We calibrate this model to the U.S. economy in a way that allows us to study multiple questions as listed above. In each case, we compare results from our simple aggregated CD model to results from the large, dynamic, disaggregated CGE model in GH (2018). To see how results depend on model structure *per se,* we compare results of the two models using the same data and parameters. Therefore, we use the data and calibration from GH, adjusted to fit our more aggregated and simpler model. We also supplement their data with GDP by industry from the U.S. Bureau of Economic Analysis and primary energy use by source from the U.S. Energy Information Administration. Table 1 summarizes data and parameters for our CD model. All details of this calibration are shown in Appendix B. We undertake the same kinds of simulations as in GH, but using the CD model, and we show how to solve for new post-reform general equilibrium prices and quantities.

3. Derive and Interpret Numerical Results

We study the effects of a large change in the carbon tax on prices, inputs, outputs, and welfare.[9] In GH’s central case, the carbon tax starts at $6.67 per metric ton in 2017, rises to $20 in 2019, and then increases by 4% every year until it reaches $67 in 2050. This section illustrates how to obtain simple numerical results, so we first calculate effects of introducing an arbitrary round-number tax of $15 per metric ton on all sectors’ CO_2 emissions while using additional revenue to cut the labor tax rate. Table 2 summarizes key outcomes. The first two columns list variables and their definitions. The third column shows benchmark equilibrium values with no carbon tax (from GH’s data and our calibration). Column 4 presents the new equilibrium after implementing the carbon tax with labor tax recycling, which we solve using use all equations above. Column 5 of Table 2 presents percentage changes for key variables.

Impacts on Outputs, Inputs, and Prices

The carbon tax with revenue recycling implies no change in tax revenue, so a higher

[9] The model framework can be used to study other environmental policies, but we focus on carbon pricing.

emissions tax rate must be accompanied by a reduced labor tax rate. With fixed nominal income I, the net-of-tax wage and capital price are unchanged (in the bottom two rows of Table 2), but the cut in labor tax reduces the gross-of-tax wage cost (by 1.04%, not shown). Because of our simplifying assumption that the production of fossil fuels uses only labor in production, a tax on CO_2 is equivalently a tax on fossil fuels or on labor used in production of fossil fuels. Table 2 shows that both F and L_F fall by the same percentage (10.6%).

The first three rows of Table 2 show the impact of the carbon tax on outputs, where fossil fuels is the output most affected. Though the table shows that the net-of-tax price of F received by producers falls by -1.04%, the gross-of-tax cost of fossil fuels increases 12% (from $1/unit to $1.12/unit, not shown in Table 2). The production of electricity is much more fossil-fuel intensive than the production of X (the composite good).[10] Thus, due to the rise in the fuel input price, the consumer price of E rises by 4.5%, and output of E falls by 4.3%. In contrast, the consumer price of X falls by only 0.058% (and output of X rises by 0.058%).

The next nine rows of Table 2 show impacts of this reform on uses of primary factors in each sector. No change is reported in the use of capital, since the factor price of capital is unchanged (and our static model considers a fixed supply of factor endowments). The 10.6% reduction in labor used by sector F allows for labor use in larger sectors X and E to rise by 1.05%. Total labor supply does not change, so labor use at home (L_H) is also unaffected.[11]

Cobb-Douglas utility and production functions ensure that the *values* of fossil fuels and electricity used in production and at home are fixed proportions of fixed nominal income. So changes in fossil fuels and electricity are proportional to changes in their prices. Also, since the carbon tax is levied on all sectors, they all face the same gross-of-tax price of fossil fuels ($P_F + \rho\tau_C$). Thus, the percentage change in fossil fuels used in each sector is the same as the percentage change in total fossil fuels F. Similarly, the percentage change in electricity deployed in each sector is the same as the percentage change in total electricity E.

Impacts on Welfare

In our numerical results, the EV is -$7.996 billion from this tax-swap experiment – the

[10] Panel B of Table 1 shows that the cost share for fossil fuels in the production of the composite good (0.023) is much less than its cost share in the production of electricity (0.407).

[11] Cobb-Douglas utility means that households spend a fixed fraction of fixed nominal income on each good (e.g., $P_L L_H = (1 - a - b - c)I$). The unchanged net-of-tax wage P_L then implies unchanged L_H. Yet this result is consistent with numerical findings in GH, given their choice of elasticities (see GH's Table 5.7, p.106).

introduction of a \$15 tax per metric ton of CO_2 with revenue used to cut the labor tax rate from 60.2% to 58.6% of the net wage. Since the reform reduces welfare, this EV is negative, so we reverse its sign to represent the positive cost of the policy. The EV shows that the higher CO_2 tax has a negative effect on welfare that exceeds the positive effect of reducing the wage tax. Thus, in our numerical model, the revenue-neutral substitution of the pollution tax for an existing distortionary labor tax has a positive gross cost.[12]

How big is this welfare loss? We divide total welfare loss from the policy by total CO_2 abated to obtain the average abatement cost (AAC). In our example, the tax-swap experiment reduces 0.570 billion metric tons of carbon dioxide emissions compared to the benchmark, so the AAC is \$14.03 per ton (=7.996/0.570). This AAC means that on average, and before considering the environment-related benefit, the tax-swap reform costs \$14.03 to reduce each metric ton of CO_2 emissions. Below, we show marginal abatement cost (MAC) curves.

By including the amount of carbon dioxide emissions in the welfare function ($W = U - \theta n C$), we are also able to calculate environmental benefits and thus net benefits of the reform. The environmental benefit of the tax reform is total emission reduction (0.57 billion tons) times the social cost of carbon (SCC). GH use the U.S. government's estimate of SCC = \$43 per metric ton of CO_2.[13] Thus, after accounting for the environment-related benefit (0.570billion×43=\$B 24.52), the reform offers a net welfare gain of \$B 16.52.

4. Comparisons to Results from the Large CGE Model

As shown in previous sections, our simplified Cobb-Douglas GE model can be solved by readers using a paper and pencil on the back of an envelope. That is an advantage relative to large CGE models whose sheer size and complexity can make them impenetrable for many readers who do not build their own CGE models. Thus, the simple Cobb-Douglas GE model with its great accessibility allows readers to explore and understand exactly and explicitly what features of the model drive key results. And especially, it assists new researchers who seek to build their first own CGE models to decide what features to include in their models.

[12] Thus, these numerical results do not support the "strong double dividend" claim, defined by Goulder (1995) as the assertion that the environmental tax not only improves the environment but also increases non-environmental welfare through revenue recycling. GH study welfare costs of a carbon tax under various recycling methods such as cuts in payroll tax, individual income tax, or corporate tax. They find evidence of a strong double dividend only with low abatement amounts and with corporate income tax cuts.

[13] Page 318 of GH cites this study, which is available at https://www.epa.gov/sites/production/files/2016-12/documents/sc_co2_tsd_august_2016.pdf .

In this section, we show that the simple model generates results that are similar to those of a complicated CGE model but for which derivations are much more transparent. In particular, we compare results from our simple aggregated Cobb-Douglas GE model to results from the dynamic disaggregated CGE model of Goulder and Hafstead (2018). The book by GH includes not only careful modeling of carbon-intensity of each different production sector, but also of intertemporal dynamics and of all U.S. taxes on labor, on capital, and on the incomes of different household income groups. For our purpose and length restriction in this paper, we do not undertake every simulation in GH. Instead, we analyze effects of a carbon tax with lump-sum rebates and with revenue used to reduce employee payroll taxes, which enables direct comparison of our results to their results.[14]

We note several key differences between our model and their model. First, our Cobb-Douglas model implies unitary elasticities of substitution in both consumption and production, while GH's elasticities of substitutions are constant but can be different from one.[15] Second, the Cobb-Douglas model implies that the uncompensated leisure demand elasticity is -1. Thus, if we use GH's assumption that workers spend 66% of their time at work and 34% of their time as leisure, the uncompensated labor supply is 1×0.34/0.66=0.515, while the GH's uncompensated labor supply is 0.05. Third, for simplicity, our model assumes away all other taxes on capital and goods and considers only a preexisting tax on labor.[16] GH include treatments of various *ad-valorem* sales and exercise taxes, plus tax deductions and tax credits.[17] Fourth, as mentioned above, our model is a static GE model, while GH is a dynamic model with growth. Fifth, our aggregated model cannot consider distributional impacts across industries or household groups, as can the GH disaggregated model. GH also include import and export activities in their model, while the economy is closed in our model. Therefore, we certainly expect our results to differ to some degree from those of GH.

Goulder and Hafstead in their book study different carbon tax time profiles from 2016 to 2050. In their central case, the carbon tax starts at $6.67 in 2017, followed by $13.33 in

[14] GH consider four revenue recycling methods of a carbon tax revenue, such as: lump-sum rebates, cuts in employee payroll taxes, cuts in individual income taxes, and cuts in corporate income taxes.

[15] Most elasticities of substitution range from 0.5 to 1.10, see Table 4.3 on Page 64 of GH.

[16] Our Cobb-Douglas model can easily accommodate other taxes on goods and capital, but we want to keep our model as simple as possible while still sufficient for the pedagogical purpose as well as for our main point that a simple Cobb-Douglas GE model can yield similar results to a complicated CGE model.

[17] See Tables 4.1 and 4.4 in GH for a list of tax features considered in GH.

2018, $20 in 2019, and increases 4% every year after 2020 until it is capped at $60 in 2048, 2049, and 2050. Our model is static, but we vary our tax rates from $5 to $60. We treat each tax rate as a separate new equilibrium. Then we calculate outputs, inputs, prices, emissions, and welfare at each tax rate. We plot those equilibrium outcomes against increasing levels of the carbon tax, and then we compare our plots with analogous plots from GH.

First, we compare the resulting impacts on economy-wide carbon emissions from our CD model and their CGE model. Since GH only shows the time profile for CO_2 in the case of a carbon tax with lump-sum rebates, we present a similar plot in Figure 1, which shows tax rates in ascending order from left to right on the horizontal axis and corresponding levels of emissions on the vertical axis. The dashed line shows results from our CD model, and the solid line represents GH's results.[18] The figure shows that a more stringent carbon tax policy results in a lower level of emissions. The solid line for the GH results and the dashed line for the CD results are very close to each other. Emissions fall by more than one-third from 5.3 billion metric tons in the initial equilibrium with no carbon tax to less than 3.5 billion metric tons by the year 2050, both in the GH model and in our Cobb-Douglas model when the carbon tax reaches $60 per metric ton (the 2050 tax rate in GH).

Second, also only for the case with lump-sum rebate, GH's Figure 5.4 on page 92 shows the time profile of carbon tax revenues (for years with rising tax rates). Similarly, we plot gross and net carbon tax revenues from our CD model against carbon tax rates in Figure 2. Gross carbon tax revenue is the product of the carbon tax rate and total CO_2 emissions. We also calculate the GH definition of net carbon tax revenue, which equals the gross carbon tax revenue minus the "tax-base effect". In GH, the tax-base effect refers to impacts of the carbon tax policy on the base of other taxes and thus on revenues generated by those taxes. In our CD model, the labor tax is the only other tax, so the tax-base effect is measured here by changes in labor tax revenue due to the imposition of the carbon tax. The solid lines show results from the GH model, and dashed lines show results from the CD model. Heavier lines present gross revenues, while lighter lines are net revenues. For example, the carbon tax rate in 2019 is $20 per metric ton, with a gross revenue of about $B100 in the GH model (about the same as for a rate of $20 in the CD model). Gross revenue reaches about $B230 by 2050 in GH with their

[18] In a similar figure, GH show emissions on the vertical axis and year on the horizontal axis, for both their reference case with no carbon tax and their central case with a carbon tax and lump-sum rebates (their Figure 5.5 on p. 90). In Figure 1 here, we replace their horizontal time axis with each year's corresponding tax rate.

tax rate of $60/ton, or at the tax rate of $60 in the CD model.

Figure 2 shows that gross tax revenues from our CD model and the GH model follow each other closely as carbon tax rates rise. Accounting for the tax base effect, however, impacts on net carbon tax revenue are much smaller and rather flat. Net carbon tax revenue is higher and steeper in our CD model than in the GH model, which can be explained by the fact that our model considers only the preexisting labor tax while GH have other preexisting taxes. Thus, our tax-base effect is less than their tax-base effect.[19]

Third, GH provide thorough welfare analysis of carbon tax policy under different recycling methods. They calculate various measures of the policy costs such as percent GDP reduction, welfare costs per ton of CO_2 reduced, or welfare costs per dollar of gross revenue. In particular, their Figure 5.6 graphs welfare cost per ton of emissions reduced against percent emission reductions, so our Figure 3 plots their results as well as our calculated welfare cost per ton of emissions reduced (average abatement cost, ignoring environmental benefits). Again, solid lines show GH results, while dashed lines are CD results. Heavier lines show the case of lump-sum rebates, and lighter lines show the labor tax recycling scenario.

Our welfare costs per ton of abatement follow the GH results closely. It is especially close for the labor tax recycling scenario, because the labor tax cut raises the real net wage by almost exactly the same amount that the carbon tax reduces it. And, for this reason, both our model and the GH model find no or little change in total labor supply. Thus, the difference in labor supply elasticities between our CD model and the GH model does not affect the welfare cost calculation. But, in the lump-sum rebate scenario, the real net wage falls, and so total labor supply falls – in a way that does depend on the elasticity of labor supply. More elastic labor supply exacerbates labor distortion via the tax interaction effect. Our CD model has a more elasticity labor supply than in the GH model, as discussed above, so our welfare cost per ton of emission reduction is larger than that of GH model. Also, Figure 3 shows that carbon pricing with lump-sum rebates is a costlier policy than a carbon tax with labor tax revenue recycling. Finally, welfare costs increase with emissions reductions in both models.

We can also compare the CD and GH model results on other outcomes such as prices, outputs, and labor use. Tables 5.5-5.7 in the GH book show detailed impacts of the uniform carbon tax with either lump-sum rebates or income tax cuts on these outcomes for each of

[19] The GH model also includes inflation-indexed transfers and government spending, so their tax base effect also accounts for increases in spending necessary to hold fixed the real the production of government services.

their disaggregated industries. For comparison, Table 3 here show our calculated policy impacts of both the uniform carbon tax and power-sector-only carbon tax, also with either lump-sum rebates or labor tax recycling (at our CD model's aggregated industry levels).

GH tables report changes in these outcomes from reference case values for years 2020 and 2035 (their carbon tax rate in 2020 is $20.8/ton and in 2035 is $37.5). Table 3 show our calculated policy impacts at carbon tax rates of $20 and $40. In general, our Cobb-Douglas results agree with GH's results that producer prices rise, while all outputs and labor demand falls. From Table 3 here and Tables 5.5-5.7 in GH, fossil-fuel output is the most affected sector, and it falls the most in percentage terms. Labor demand in the production of F also falls the most in both models. The percent reduction in total labor supply is a bit larger in the CD model, partially due to our assumption that labor is the only input to the production of fossil fuels. Therefore the reduction in L_F has more weight in determining the overall reduction of the equilibrium quantity of labor in our model than in theirs. Another influence is that our labor supply is more elastic than theirs. Prices received by producers fossil fuels fall in both models, while prices paid for fossil fuels rise and so other producer prices rise.

5. Other Results

The CD model is easily used to calculate other effects of a carbon tax that are not studied or fully reported in the book by Goulder and Hafstead (2018).

Marginal Abatement Costs

Another highly useful measure of the cost of a CO_2-emission policy is its marginal abatement cost (MAC), defined as the cost at each level of emission-reduction of eliminating one more metric ton of CO_2 emissions. Then the total cost of achieving a particular level of abatement is the integral under the MAC curve to that point. Policymakers can compare these costs across different policies to find the most cost-effective means of reducing emissions.

Therefore, we consider many small increases in the CO_2 tax, in increments of $1/ton. For example, consider the change from $15 to $16 per ton. As shown in previous sections, we obtain a set of outcomes at each CO_2 tax rate. The extra cost of abatement is the new welfare cost at $16/ton minus the prior welfare cost at $15/ton, equal to $0.773 billion. The extra CO_2 abated is 0.036 billion tons. Then the marginal cost of abatement (MAC) is the change in welfare cost per unit change in CO_2 abatement (0.773/0.036), equal to $22.70 per ton. The interpretation is that it costs an extra $22.70 for a marginal metric ton of CO_2 abatement.

Figure 4 graphs MAC curves for different carbon tax policies. It shows a carbon tax on all sectors with lump-sum rebates (heavy dashed line) and with labor tax recycling (light dashed line), a carbon tax in the power sector only with lump-sum rebates (heavy solid line) and with labor tax recycling (light solid line). These MAC curves all slope upward, indicating that costs increase with the quantity of emissions abated. For a given level of abatement, and for either type of carbon tax, the MAC with lump-sum rebates is higher than the MAC with labor tax recycling – because reducing the labor tax distortion reduces welfare cost.

Somewhat surprising in Figure 4 is that the MAC of the power-sector-only carbon tax is initially less than the MAC of the uniform carbon tax. Similar results are also found in GH for cap-and-trade policies (Figure 5.13 on page 138 of GH).[20] In the first best world with no preexisting tax, a uniform carbon tax can never be more costly than a specific-sector tax, because a uniform tax allows use of the lowest cost marginal abatement across sectors (as explained in GH on page 139). With preexisting taxes, however, the MAC includes firm-level abatement cost plus the tax interaction effect between the carbon tax policy and preexisting taxes. Our numerical simulation results in Table 3 show that the power-sector-only carbon tax reduces the real net wage by less than does the uniform carbon tax of the same tax rate. Thus, the uniform carbon tax policy exacerbates the labor tax distortion more than the other policy.

At a higher level of abatement, the tax interaction advantage of the power-sector-only carbon tax is weaker relative to its disadvantage of higher firm-level abatement cost. Thus in Figure 4, the MAC curves for a power-sector carbon tax start below those of a uniform tax, but rise steeply for more abatement, then cross and become higher than for a uniform tax.

Second-best optimal tax rates

In previous sections, we calculate the effects of the pollution-labor tax swap on welfare, pollution, the wage, and labor supply. This section answers a related but different question: given the pollution externality and labor supply distortion, what is the second best optimal (SBO) pollution tax, and how does it compare to the first best optimal (FBO) tax?

In general, the optimal pollution tax induces the level of pollution where the marginal social cost of pollution reduction equals the marginal social benefit of pollution reduction (the marginal environmental damage from pollution). In the first-best case without distortionary

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[20] Results are similar despite multiple differences between the two models. Unlike the CD model, the GH model incorporates adjustment costs, and it has disaggregated electricity generators differentiated by carbon intensity. Some of the many differences between the models likely affect results in opposite directions.

taxes, the optimal pollution tax rate equals the marginal social damage of pollution (the Pigouvian tax level, in Baumol and Oates, 1988, chapter 4). In the second-best case with distortionary taxes, the optimal pollution tax can lie either below or above the marginal environmental damage, depending on the use of additional pollution tax revenue, the marginal cost of public fund, the magnitude of the revenue-recycling effect, the magnitude of the tax-interaction effect, and the normalization of other tax rates.[21] Our simple numerical Cobb-Douglas model provides an example of those prior findings.

Using the simple Cobb-Douglas model, Figure 5 plots household utility against the CO_2 tax rate for different carbon tax policies. We obtain hump-shaped curves, where utility rises with the pollution tax rate up to some utility-maximizing tax rate and then falls with further increases in the tax rate. It falls because the adverse distorting effect of increasing the emission tax rate outweighs the benefit of its effect on emissions reduction. The heavy dashed curve represents a uniform carbon tax scenario with lump-sum rebates. The light dashed curve represents the case of a uniform carbon tax with labor tax recycling. The solid curves show the case of a carbon tax on the power sector only.

The calculated utility-maximizing SBO tax rate in the case of a uniform carbon tax with lump-sum tax rebates is \$22/metric ton of CO_2 emissions – and with labor tax recycling it is \$37/ton. These SBO tax rates for a uniform tax are below the FBO Pigouvian tax rate of \$43/ton of CO_2 emissions (the SCC). In contrast, the SBO tax rates of a carbon tax on power sector only with lump-sum rebates and labor tax recycling are \$46 and \$63, respectively, which are above the FBO tax rates.

To explain these results, note that the amount of abatement is not shown in Figure 5. The SBO uniform carbon tax rates are lower than the power-sector carbon tax rates, because each dollar per ton can achieve more abatement. Also, for each dollar per ton, the uniform carbon tax has a bigger tax interaction effect than the power-sector-only carbon tax. Our numerical results in Table 3 show that each dollar per ton of uniform tax reduces the real net wage much more than does the carbon tax just on the power sector. But, with labor tax recycling, the revenue recycling effect offsets the tax interaction effect, and thus the SBO tax rates in the case with labor tax recycling are always higher than with lump-sum rebates.

[21] See Bovenberg and de Mooij (1994), Bovenberg and van der Ploeg (1994), Goulder (1995), Bovenberg and Goulder (1996), and Fullerton (1997).

Leakage

A concern with any pollution restriction on one sector is that it might cause "leakage", an increase in emissions in other sectors that offset part of the reduction in pollution-restricted sectors. Goulder and Hafstead do not report leakage results when they discuss the carbon tax (on p.136). They do discuss leakage results for a cap-and-trade program in the power sector only (on p.140). They find leakage of 1.2% to 1.7%, but that page does not report the permit price. In our numerical example, we consider a carbon tax on the power sector only. With lump-sum rebates, we calculate a leakage rate of 3.9%, meaning that additional emissions in other sectors offset 3.9% of emission reductions in the power sector. With labor tax recycling, the leakage rate is 6%, which is one and a half times the leakage with lump-sum rebates.[22] Accuracy of calculated leakage in the CD model is probably limited because of its high level of aggregation and the absence of a foreign sector. The point of the CD model is not accuracy, however, but understanding the nature of results from large CGE models.

6. Discussion and Conclusions

For any proposed policy, political leaders are most often concerned about actual effects on jobs, outputs of each industry, changes in trade deficits, and overall GDP. They have little inclination to discuss conceptual issues in environmental policy analysis, or even to sort out the cause-and-effect relationships. They therefore need the best possible numerical results from the most recent and most detailed computable general equilibrium model available. As of 2018, one of the best and most recent CGE models for climate policy is described in the book by Goulder and Hafstead. It is indeed a monumental achievement. And such models can only be produced by the best-trained academics who do understand the conceptual issues and all the cause-and-effect relationships.

For political leaders, our paper does not produce the best possible numbers for the effects of a carbon tax on outputs, GDP, or even emissions. For academics, however, our paper does indeed provide some interesting results. It shows very directly, and in simple terms, exactly what are the key ingredients of a CGE model that does generate detailed

[22] We are unable to decompose total leakage into various channels, as in Baylis *et al*. (2014). Their paper finds that a pollution limitation on one sector can lead to "negative leakage", emissions *reductions* in the other sector under certain conditions. They show that negative leakage is more likely if the elasticity of substitution in the production of electricity is greater than the elasticity of substitution in utility. In our CD model, all elasticities of substitution are unity, which explains why we are unlikely to see a net negative leakage in our CD model.

numerical results. In particular, with just three industries and without the input-output matrix of the GH model, we obtain very similar results on aggregated outcomes such as economy-wide emissions, the welfare cost of carbon taxes, and total labor supply. These results are not much affected by the sheer number of industries (35 in GH) or the many interactions between them.[23] The main intuitions about climate policy also need not depend on having a dynamic model such as in GH, with capital investments, taxes on capital, or even changes in growth. Our model has none of those, nor international trade.

A key force is the assumed elasticities in production and consumption. More elastic behavior in production or in consumption means that a carbon tax has a larger impact on carbon emissions, output, and welfare. Our Cobb-Douglas model makes solutions for outcomes very simple, but the elasticities all equal to 1.0 are not that different from the elasticities in GH (between 0.5 and 1.0). Also relevant, of course, is carbon-intensity of production, factor shares, and the shares of total expenditure on each carbon-intensive product. We borrow these data directly from GH. A third key feature is whether the carbon tax applies to all carbon in the economy or just to carbon use in one sector. A fourth influence on the effect of carbon policy is the pre-existence of distorting taxes and the use of carbon revenue. When our CD model is built to replicate the large GH model in all these key elements, we show that results are very close to those of the large complicated CGE model.

Our numerical CD model also provides a clean illustration of many concepts and analytical findings in the prior literature on optimal carbon policy and on the impacts of a carbon tax. Our simple CD model has labor tax as the only pre-existing distortionary tax, for example, so the CD model allows us to study and compare magnitudes of the tax interaction effect between different carbon tax policies and the pre-existing labor tax in a very transparent fashion. As in the analytical chapter and the later simulation chapters of the GH book, we show that the tax interaction effect raises the cost of the carbon tax policy, and its magnitude increases with the carbon tax rate. Revenue recycling effects work in the opposite direction and reduce the cost of the carbon tax policy. Magnitudes of these tax interaction effects and revenue recycling effects help determine second-best optimal carbon tax rates.

In conclusion, our simple Cobb-Douglas model shows readers how to solve a CGE

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[23] With no short-run rigidity in the CD model, our results might approximate the medium-run results in the GH model. Their short-run results differ because of adjustment costs, and their long-run results differ because of growth. Also, as discussed, our results with identical households cannot match their distributional results.

model using a paper and pencil on the "back of an envelope." Thus, it is a useful tool for teaching general equilibrium to graduate students or for those who have no experience building or solving a CGE model. Also, by comparing numerical results from a simple static general equilibrium CD model to those from a dynamic CGE model, we show how a well-constructed simple CD model can yield several results that match with those from a large-scale CGE model. An advantage of a Cobb-Douglas model is that results are easier to understand using exact analytical solutions for each price, output, welfare loss, and the utility-maximizing pollution tax rate. Therefore, our paper is not just a teaching tool for a graduate economics, class, but it can also be used by new researchers as an entrée to building their own CGE model and by other seasoned researchers to understand CGE results.

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Table 1: Data and Parameters for the Cobb-Douglas Model

Panel A: Data for Inputs and Outputs (*in $B, also quantities since initial prices are 1.0*)

	Production Sectors			Household	
Inputs	*X*	*E*	*F*	Sector	Row Sum
K	K_X^o=7334.49	K_E^o=243.213	---	---	$\bar{K}$=7578.50
L	L_X^o=3862.92	L_E^o=50.593	L_F^o=387.658	L_H^o=2215.29	$\bar{L}$=6515.56
E	E_X^o=377.600	---	---	E_H^o=169.100	E^o=546.700
F	F_X^o=320.972	F_E^o=222.416	---	F_H^o= 77.8000	F^o=621.188
X	---	---	---	X^o=14221.6	---
τ_L	$\tau_L^o L_X^o$=2326.22	$\tau_L^o L_E^o$=30.4779	$\tau_L^o L_F^o$=233.530	---	R^o=2590.53
Column Sum	X^o=14,221.6	E^o=546.700	F^o=621.188	*I*=16683.8	---

Panel B: Share Parameters (*the corresponding entry in Panel A, divided by the column sum*)

	Production Sectors			Household
Inputs	*X*	*E*	*F*	Sector
K	α=0.516	δ=0.445	---	---
L	β=0.435	ϵ=0.148	1.00	(1-a-b-c)=0.133
E	γ=0.026	---	---	b=0.010
F	(1-α-β-γ)=0.023	(1-δ-ϵ)=0.407	---	c=0.005
X	---	---	---	a=0.852
Column Sum	1.00	1.00	1.00	1.00
TFP	A=2.978	B=2.942	G=1.602	---

Panel C: Other Central Case Parameters, as described in Appendix B

Parameter	Definition	Value
θ	Marginal effect on welfare from emissions	26.720
ρ	Emission conversion rate (ton CO_2 per dollar of fossil fuels)	0.00863

The numbers in Panel A are all provided with at least six digits (regardless of how many digits are left or right of the decimal point). Also, in Panel A, the value of national income is not the sum of the last column, because some of *F* and *E* are inputs to *X*. Instead, I=16683.8=$\bar{L}P_L + \bar{K}P_K + R$. Panel B shows parameters specific to production and utility. Panel C shows other parameters described in the text (and calibrated in Appendix B).

Table 2: Summary of Key Values and their Changes

(1) Variable	(2) Definition	(3) Value at τ_c=0	(4) Value at τ_c=15	(5) Change (%)
Panel A	*Quantities*			
X	Output of *X*	14221.600	14229.829	0.058
E	Output of *E*	546.700	523.068	-4.323
F	Output of *F*	621.188	555.127	-10.635
K_X	Capital in Production of *X*	7334.487	7334.487	0.000
K_E	Capital in Production of *E*	243.213	243.213	0.000
K	Fixed Total Capital Supply	7577.700	7577.700	0.000
L_X	Labor in Production of *X*	3862.017	3902.711	1.054
L_E	Labor in Production of *E*	50.953	51.126	1.054
L_F	Labor in Production of *F*	387.658	346.432	-10.635
L	Market Labor Supply	4300.269	4300.269	0.000
L_H	Labor at Home	2215.290	2215.290	0.000
$\bar{L}$	Fixed Total Labor Endowment	6515.559	6515.559	0.000
E_X	Electricity in Production of *X*	377.600	361.278	-4.323
E_H	Electricity Consumption at Home	169.100	161.790	-4.323
F_X	Fossil Fuel in Production of *X*	320.972	286.838	-10.635
F_E	Fossil Fuel in Production of *E*	222.416	198.763	-10.635
F_H	Fossil Fuel Consumption at Home	77.800	69.526	-10.635
R	Tax Revenue	2590.531	2590.531	0.000
Panel B	*Prices*			
τ_L	Labor Tax Rate	0.602	0.586	-2.774
P_X	Consumer Price of *X* (no tax)	1.000	0.999	-0.058
P_E	Consumer Price of *E* (no tax)	1.000	1.045	4.518
P_F	Net-of-Tax Price of *F*	1.000	0.990	-1.043
P_K	Factor Price of Capital (no tax)	1.000	1.000	0.000
P_L	Net-of-Tax Wage	1.000	1.000	0.000

Table 3: Impacts on Output, Producer Prices, and Labor Demand
(Percentage Changes from Initial Equilibrium Values)

Variable	Definition	Lump-sum Rebate		Labor Tax Cut	
		(1) Value at τ_c=20	(2) Value at τ_c=40	(3) Value at τ_c=20	(4) Value at τ_c=40
Panel A: Overall Economy Carbon Tax					
X	Output of X	-0.153	-0.335	0.065	0.048
E	Output of E	-5.859	-10.740	-5.643	-10.409
F	Output of F	-14.071	-24.795	-13.730	-24.334
P_X	Consumer Price of X (no tax)	0.153	0.336	-0.065	-0.048
P_E	Consumer Price of E (no tax)	6.224	12.033	5.980	11.618
P_F	Net-of-Tax Price of F	-0.882	-1.544	-1.342	-2.354
P_L	Net-of-Tax Wage	-0.882	-1.544	0.000	0.000
L_X	Labor in Production of X	0.890	1.569	1.360	2.411
L_E	Labor in Production of E	0.890	1.569	1.360	2.411
L_F	Labor in Production of F	-14.071	-24.795	-13.730	-24.334
L	Market Labor Supply	-0.459	-0.808	0.000	0.000
Panel B: Power-Sector-Only Carbon Tax					
X	Output of X	-0.003	-0.013	-0.286	-0.244
E	Output of E	-6.125	-11.143	-6.049	-11.029
F	Output of F	-4.958	-8.724	-4.837	-8.482
P_X	Consumer Price of X (no tax)	0.003	0.013	0.287	0.244
P_E	Consumer Price of E (no tax)	6.525	12.541	6.439	12.396
P_F	Net-of-Tax Price of F	-0.314	-0.549	-0.477	-0.833
P_L	Net-of-Tax Wage	-0.314	-0.549	0.000	0.000
L_X	Labor in Production of X	0.315	0.552	0.479	0.840
L_E	Labor in Production of E	0.315	0.552	0.479	0.840
L_F	Labor in Production of F	-4.895	-8.724	-4.837	-8.482
L	Market Labor Supply	-0.162	-0.284	0.000	0.000

Figure 1: Impacts on Carbon Dioxide Emissions (Uniform Carbon Tax with Lump-sum Rebates)

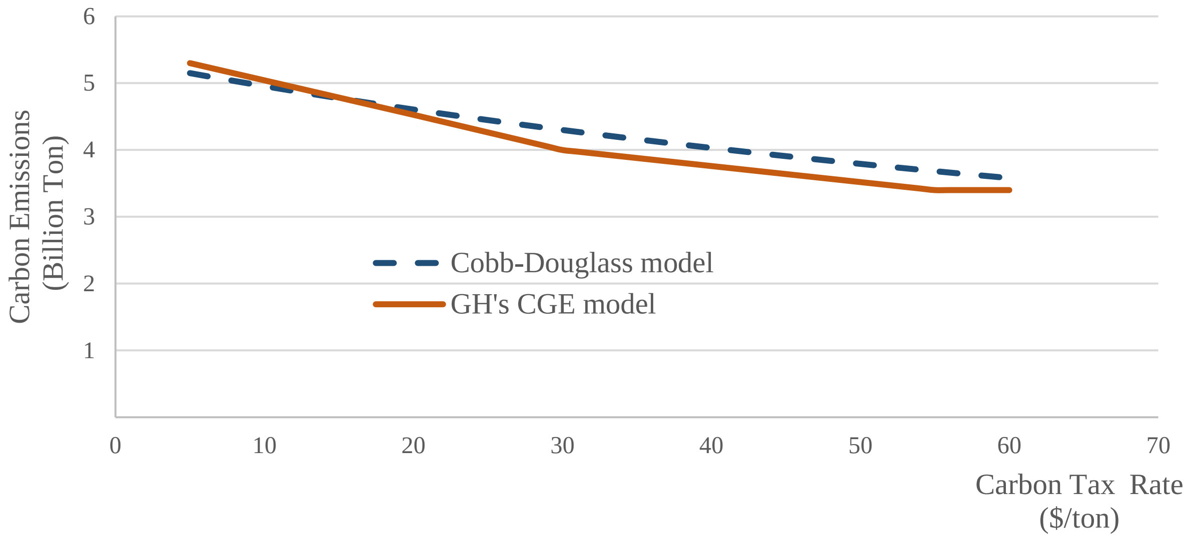


Figure 2: Revenue from a Uniform Carbon Tax (with Lump-sum Rebates)

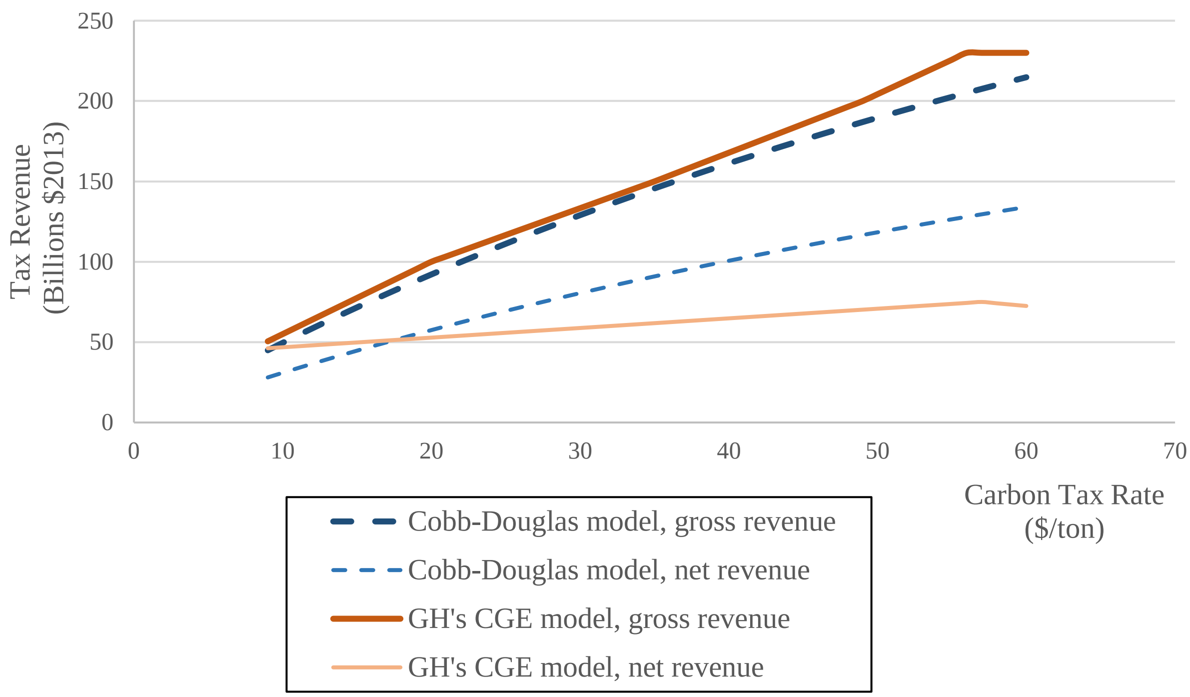


Figure 3: Welfare Costs of a Uniform Carbon Tax under Alternative Recycling Methods

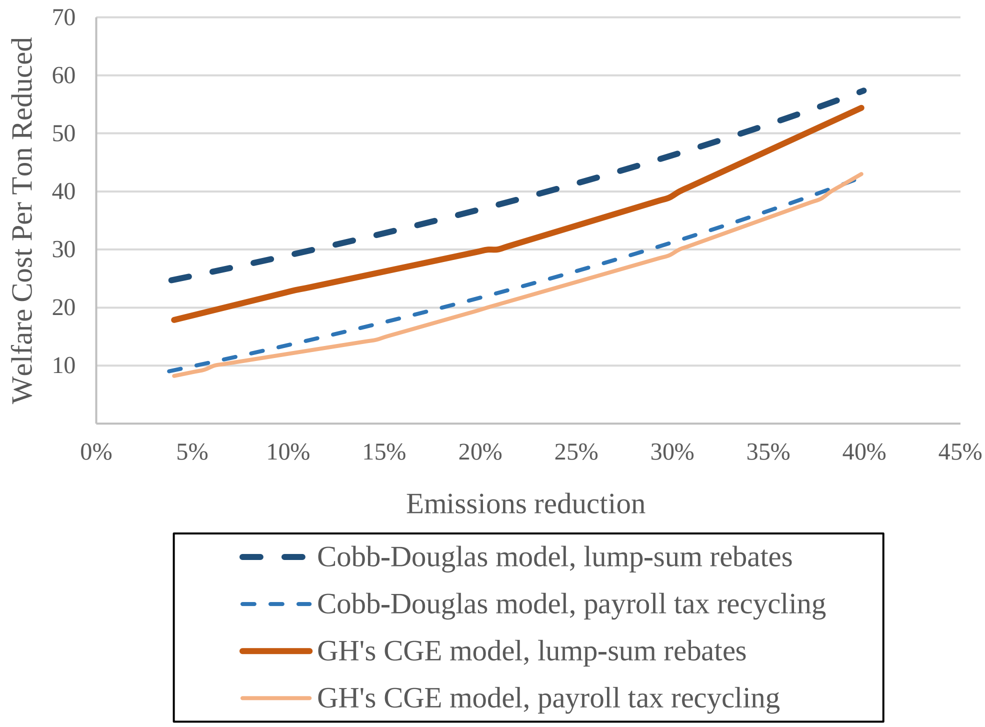


Figure 4: Marginal Abatement Cost Curve (Cobb-Douglas Model Only)

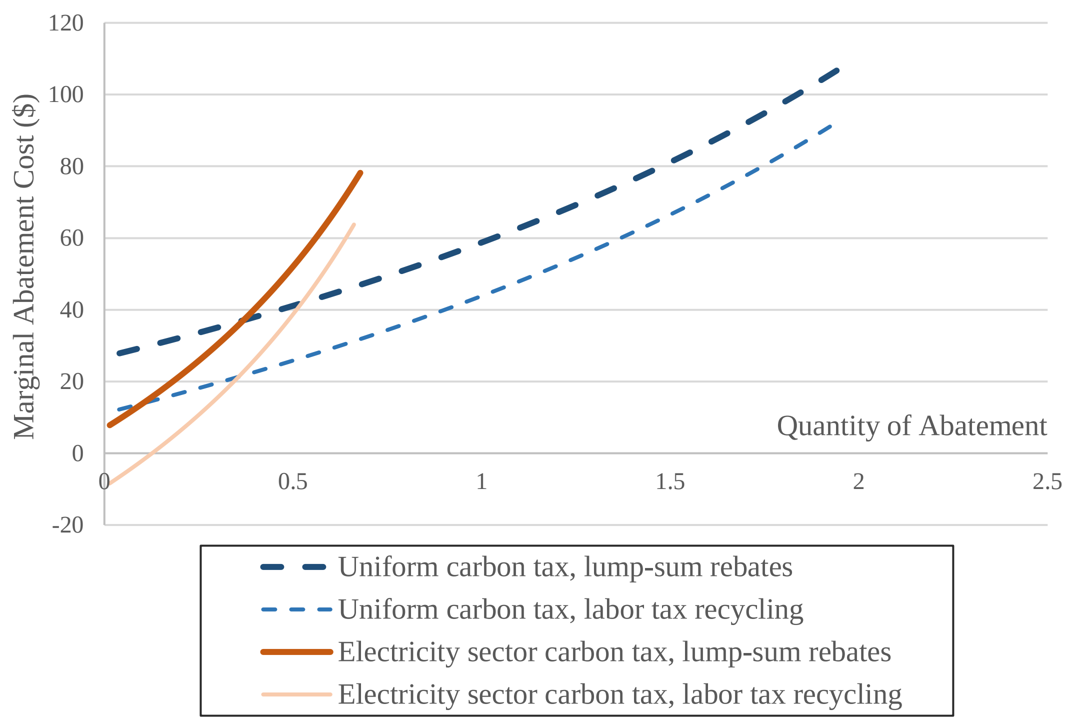
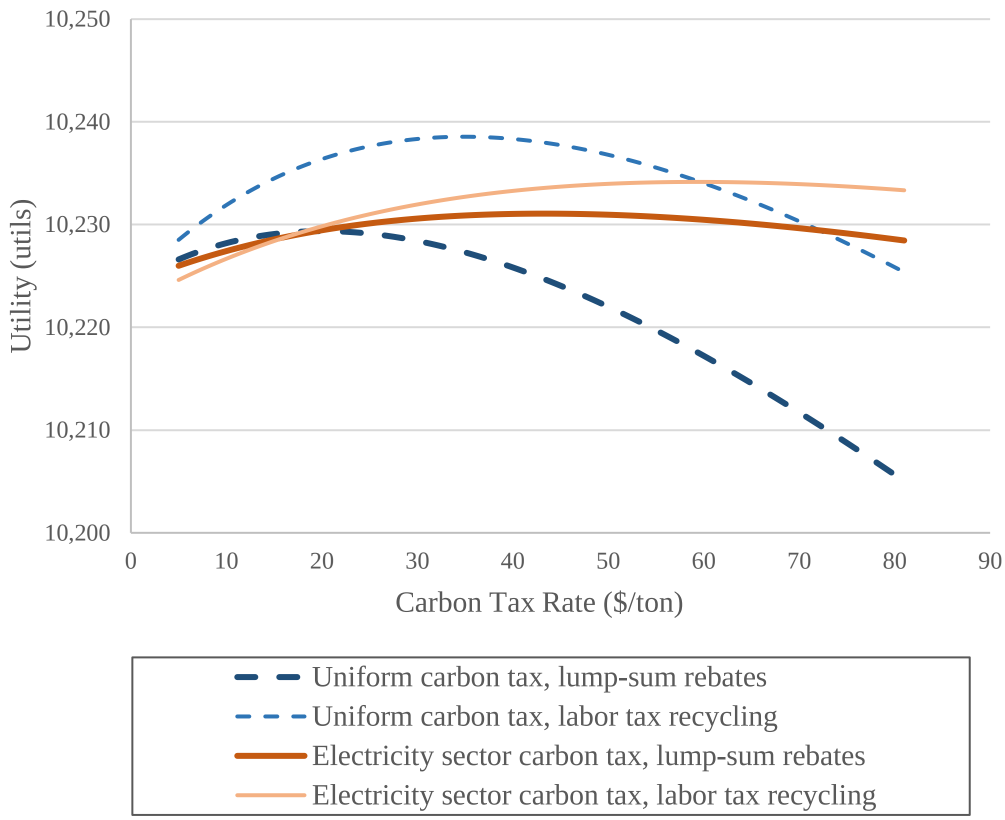


Figure 5: Second-Best Optimal Carbon Tax Rates



Appendix A: Remaining Steps to Solve for New Equilibrium Outcomes

Uniform carbon tax with lump-sum rebate

The initial equilibrium has no carbon tax ($\tau_{CE}^{o} = \tau_{CO}^{o} = \tau_{C}^{o} = 0$), but imposition of a uniform carbon tax implies a known $\tau_{CE}' = \tau_{CO}' = \tau_{C}' > 0$. We assume no change in labor tax ($\tau_L' = \tau_L^o$). We use (2.8) and (2.10) to get an equation with only one unknown variable, R':

$$G\bar{L}\left(1 - \frac{(1+\tau_L')(1-a-b-c)+\beta a+\epsilon(b+\gamma a)}{(I-P_K'\bar{K}-R')(1+\tau_L')}I\right) \tag{A.1}$$
$$= \frac{R' - \tau_L'(I-P_K'\bar{K}-R') + \tau_L'(1-a-b-c)I}{\rho\tau_C'}$$

To solve for R', first shorten that expression using $d \equiv (I - P_K'\bar{K} - R')(1+\tau_L')$, and obtain:

$$-d^2 + d[I - P_K'\bar{K} + \tau_L'(1-a-b-c)I - \rho\tau_C' G\bar{L}]$$
$$+\rho\tau_C' G\bar{L}I\big((1+\tau_L')(1-a-b-c)+\beta a+\epsilon(b+\gamma a)\big) = 0$$

We solve this quadratic equation to get d and then R'.

Then substitute R' into (2.4) to get P_L', into (2.9) to get P_F', and into (2.5), (2.6), and (2.8) to get labor used for each sector. Use equation (1.14), (1.17), and (1.18) to get fossil fuels and thus carbon emissions in each sector. We use the production function for E to get output E and price P_E, and then find E_X, E_H, X, and P_X.

Uniform carbon tax with labor-tax recycling

In the case with labor tax recycling and no change in rebate ($R' = R^o$), we use (2.8) and (2.10) to get an equation with τ_L' as the only unknown variable:

$$G\left(\bar{L} - L_H' - \frac{\beta a I}{P_L'(1+\tau_L')} - \frac{\epsilon(b+\gamma a)I}{P_L'(1+\tau_L')}\right) = \frac{R' - \tau_L' P_L'(\bar{L} - L_H')}{\rho\tau_C'} \tag{A.2}$$

Through algebraic derivations, we solve this quadratic equation for τ_L' using (A.2) to get:

$$(\bar{L} - L_H')(\tau_L' + 1)^2 + \left(\left(\bar{L} - L_H' - \frac{R'}{G\rho\tau_C{}'}\right)G\rho\tau_C' - (\bar{L} - L_H')\right)(1+\tau_L')$$
$$-\big(\beta a + \epsilon(b+\gamma a)\big)IG\rho\tau_C' = 0$$

Then substitute τ_L' into (2.9) to get P_F', and into (2.5), (2.6), and (2.8) to get labor used in each sector. Use equation (1.14), (1.17), and (1.18) to get fossil fuels and carbon emissions in each sector. We use the production function for E to get output E and price P_E, and then find E_X, E_H, X, and P_X.

Power-sector-only carbon tax with lump-sum rebate

The power-sector-only carbon tax implies a known $\tau'_{CE} > 0, \tau_{CO} = 0$, and unknown R'. From equations (1.9), (1.14), and (2.9), we derive fossil fuels used in sectors X and H:

$$F'_X = \frac{G(1-\alpha-\beta-\gamma)aI}{P'_L(1+\tau'_L)} = \frac{G(1-\alpha-\beta-\gamma)aI\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \tag{A.3}$$

$$F'_H = \frac{GcI}{P'_L(1+\tau'_L)} = \frac{GcI\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \tag{A.4}$$

Sum equations (A.3), (A.4), and (1.10) to get total fossil fuels, and then set that equal to the new total fossil fuels in equation (2.8) to get:

$$\begin{aligned} G\bar{L}\left(1-\frac{(1+\tau'_L)(1-a-b-c)+\beta a+\epsilon(b+\gamma a)}{(I-P'_K\bar{K}-R')(1+\tau'_L)}I\right) \\ = \frac{R-\tau'_L(I-P'_K\bar{K}-R')+\tau'_L(1-a-b-c)I}{\rho\tau'_{CE}} \\ + \frac{GcI\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} + \frac{G(1-\alpha-\beta-\gamma)aI\bar{L}}{(I-P'_K\bar{K}-R')(1+\tau'_L)} \end{aligned} \tag{A.5}$$

Manipulate the above equation (A.5):

$$-d^2 + (I - P'_K\bar{K} + \tau'_L(1-a-b-c)I - \rho\tau'_{CE}G\bar{L})d$$

$$+\rho\tau'_{CE}G\bar{L}(\tau'_L(1-a-b-c) + (\epsilon-1)(b+\gamma a) - \alpha a + 1)I = 0$$

where $d \equiv (I - P'_K\bar{K} - R')(1+\tau'_L)$. Solve for R', and then substitute R' successively into other equations, as in the case above for the uniform carbon tax with lump-sum rebate.

Power-sector-only carbon tax with labor-tax recycling

This case has a known $\tau'_{CE} > 0, \tau_{CO} = 0$, and $R' = R^o$, but τ'_L is unknown. From equations (1.9), (1.14), and (2.9), we derive fossil fuels used in sectors X and H as equations (A.3) and (A.4). Then we use the government budget constraint equation (1.6) and manipulate to derive the new equilibrium quantity of total fossil fuels used in the power sector:

$$R' = \rho\tau'_{CE}F'_E \ + \tau'_L P'_L(L'_X + L'_E + L'_C)$$

$$F_E' = \frac{R' - \tau_L' P_L'(\bar{L} - L_H')}{\rho \tau_{CE}'} \tag{A.6}$$

Sum equations (A.3), (A.4), and (A.6) to get total fossil fuels, and then equate that total fossil fuels with equation (2.8) to get:

$$\frac{G(1-\alpha-\beta-\gamma)aI}{P_L'(1+\tau_L')} + \frac{GcI}{P_L'(1+\tau_L')} + \frac{R' - \tau_L' P_L'(\bar{L} - L_H')}{\rho {\tau_{CE}}'} = G\left(\bar{L} - L_H' - \frac{\beta a I}{P_L'(1+\tau_L')} - \frac{\epsilon(b+\gamma a)I}{P_L'(1+\tau_L')}\right) \tag{A.7}$$

Manipulate the above equation (A.7) to get:

$$0 = (1+\tau_L')^2 \frac{(\bar{L} - L_H')}{G\rho\tau_{CE}'} + \left(\bar{L} - L_H' - \frac{R' + (\bar{L} - L_H')}{G\rho\tau_{CE}'}\right)(1+\tau_L')$$

$$-(\epsilon(b+\gamma a) + (1-\alpha-\gamma)a + c)I$$

and then solve for τ_L'. To get solutions for all new outcomes, substitute that τ_L' successively into other equations, as in the case of a uniform carbon tax with labor-tax recycling.

Appendix B. Calibration

We define our electricity sector E to include four GH industries: electric transmission and distribution, coal-fired electricity generation, other-fossil electricity generation, and non-fossil electricity generation. We aggregate seven industries of GH into our fossil fuel production sector F, including coal mining, oil and gas extraction, mining support activities, natural gas distribution, petroleum refining, and pipeline transportation. All other private sector industries are aggregated into X, our composite good sector.

Panel A of Table 1 shows data for inputs and outputs in the production and household sectors (both dollar values and quantities). We employ the "unit convention," which defines a unit of each good or primary factor as the amount such that its net-of-tax cost is one dollar in the initial equilibrium: $P_X^o = P_E^o = P_F^o = P_K^o = P_L^o = 1$. No tax is imposed on capital K or on goods $X, E,$ and F. Thus, we infer the initial quantity of each such good from its production value, and the quantity of capital K from the sum of its values in production of all outputs. Because of the unit convention ($P_L^o = 1$), labor supply equals total labor compensation divided by (1+τ_L). Then, using data in Panel A of Table 1, we calculate and show share parameters in production and household sectors in Panel B of the same table.

We consider the choice between labor and leisure, using the GH assumption that workers spend 66% of their time at work (e.g., 40 hour week of 60 hours maximum).[1] We aggregate their various labor income taxes into a single rate. Their calculated labor income tax rate is 26.70%, employer payroll tax rate is 6.23%, and employee payroll tax rate is 6.95%. Thus, our single labor tax rate is 60.24% of the *net* wage.[2] Total labor compensation is \$6,890.8 billion, so L=6,890.8 billion. Then we calculate the dollar value of non-market time (leisure at home) as $P_L^o L_H^o$= (34/66)×6,890.80/(1.6024)=B\$2,215.29, so the quantity of non-market labor is L_H^o=2,215.29. Fixed nominal "full" income I is the sum of this value of leisure plus gross domestic product in the private sector from the U.S. Bureau of Economic Analysis (BEA).[3] We also take household spending on electricity and fossil fuels directly from GH, and then our spending on the composite good X is the income left after paying for electricity, fossil fuels, and non-market time. By the unit convention, these dollar value expenditures are also quantities (see Household Sector column of Panel A of Table 1).

We also calculate quantities and dollar values of electricity and fossil fuel used in each sector. First, based on our aggregation and the GH data, we calculate the total dollar value of the electricity sector, and E^o = B546.70. Electricity is used either at home or in sector X in our model, so the dollar value of electricity input in the production of X is just the dollar value of total electricity, E, minus the dollar value of electricity used at home, E_H. These numbers all appear in Panel A of Table 1. Next, we need dollar values of fossil fuels used in sectors E and X, which are not readily available from GH, so we use household expenditures on fossil fuels and reported sectoral shares of fossil-fuel use from the U.S. Energy Information Administration (EIA).[4] By our calculation for 2013, the residential sector uses about 12.52% of total fossil fuels, the electricity sector uses 35.80%, and all other sectors use 51.67%. Thus, F_E is about three times the fossil fuels used at home, F_H (and F_X is about four times F_H). Total fossil fuel used in the economy is the sum of fossil fuel use in all sectors.

[1] The relevance and impact of this assumption is demonstrated in Ballard (1999).

[2] Suppose w is the wage actually paid, so the gross-of-all-tax wage paid for each unit of labor is w×(1+0.0623). The net-of-all-tax wage is P_L = w×(1–0.0695–0.267). Thus, our single labor tax rate as a fraction of the net wage is: τ_L^o = (1+0.0623)/(1–0.0695–0.267) –1 = 0.6024. This labor tax rate is 60.24% of the net wage, equivalent to a tax rate of 0.6024/(1.6024) = 37.59% of the gross wage.

[3] We choose the BEA's 2013 data to be consistent with input-output data from GH.

[4] https://www.eia.gov/energyexplained/

Next, we find labor and capital used in each sector. Capital input data are not reported in GH, and our production functions do not include all GH inputs (capital, labor, energy, and materials). Therefore, we supplement GH data with Bureau of Economic Analysis (BEA) data. The production of electricity in our model uses only three inputs: capital K_E, labor L_E, and fossil fuels F_E. Thus, this sector's total payment to labor and capital must equal the value of electricity minus the cost of fossil fuels calculated above. BEA data are used only to calculate the ratio of labor to capital payments for our aggregated electricity sector (which is about one-third).[5] This ratio is then used to separate payments to labor and capital.

As labor is the only input to production of F, the payment to labor in F is the value of F calculated above. The payment to labor used in X is total labor compensation minus labor compensation paid in E and F. Total payment to capital in X is the dollar value of X minus the above-calculated costs of electricity, fossil fuel, and labor inputs. Using our unit convention and the initial labor tax rate, we get quantities of labor and capital used in each sector, as shown in the first two columns of panel A in Table 1.

Next, we use output value for each sector to calculate the household expenditure share for each consumption good, and we use input values in each sector to calculate production cost shares. In the Cobb-Douglas model, those shares are also parameters in utility and production functions. Also, we calculate the TFP parameters in Cobb-Douglas production functions using benchmark equilibrium quantities of outputs and inputs.[6]

To analyze welfare impacts of the pollution tax, we follow GH by using $43 per metric ton of CO_2 as the social cost of carbon (SCC). We take this SCC (in dollars per ton) times the marginal utility of income (in utils per dollar) to get the marginal utility cost of emissions (in utils per ton). This marginal disutility is assumed to be a constant parameter θ in household welfare ($W = U - \theta nC$). In the Cobb-Douglas case, with homothetic utility, the marginal utility of income equals the ratio of total utility from consumption of goods and leisure to the full income (U/I). This calculation yields θ=26.72 (utils per ton).

Finally, to calculate the emission conversion parameter ρ, divide total CO_2 emissions (5.360 billion metric tons as in GH) by total fossil fuels F (621.188 billion). Thus, ρ=0.00863.

[5] The BEA data on composition of output by industry only allow us to calculate the labor-capital compensation ratio for "Utilities," which include our aggregated electricity industry plus natural gas distribution and water utilities. However, output from the electricity industry is the major part of all utilities.

[6] Given $X = AK_X^{\alpha} L_X^{\beta} E_X^{1-\alpha-\beta}$, for example, we solve backwards for the parameter $A = X/(K_X^{\alpha} L_X^{\beta} E_X^{1-\alpha-\beta})$.