NBER WORKING PAPER SERIES

GENERAL EQUILIBRIUM EFFECTS IN SPACE: THEORY AND MEASUREMENT

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Working Paper 25544
http://www.nber.org/papers/w25544

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2019, Revised June 2020

We thank David Atkin, David Autor, Marta Bengoa, Martin Beraja, Varadarajan V. Chari, Lorenzo Caliendo, Arnaud Costinot, Jonathan Dingel, Dave Donaldson, Farid Farrokhi, Gordon Hanson, Rich Hornbeck, Erik Hurst, Samuel Kortum, Andrew McCallum, Eduardo Morales, Elias Papaioannou, Amil Petrin, Steve Redding, Esteban Rossi-Hansberg, Jonathan Vogel, David Weinstein, as well as numerous participants at many seminars and conferences for helpful suggestions and comments. We also thank Ariel Boyarsky, Zijian He, Guangbin Hong, Jack Liang, and Josh Morris-Levenson for excellent research assistance. Rodrigo Adão thanks the NSF (grant 1559015) for financial support. All errors are our own. A previous version of this paper circulated under the title "Spatial Linkages, Global Shocks and Local Labor Markets: Theory and Evidence." The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 25544
February 2019, Revised June 2020
JEL No. F1,F14,F16,R1

ABSTRACT

How do international trade shocks affect spatially connected regional markets? We answer this question by extending shift-share empirical specifications to incorporate general equilibrium effects that arise in spatial models. In partial equilibrium, regional shock exposure has a shift-share structure: it is the average shock weighted by regional exposure shares in revenue and consumption. General equilibrium responses of employment and wages in each market are the sum, across all regions, of these shift-share measures times bilateral reduced-form elasticities determined by the economy's spatial links. We use this reduced-form representation of the model to efficiently estimate the bilateral elasticities exploiting exogenous variation in shock exposure across markets. Finally, we study the general equilibrium impact of the "China shock" on U.S. CZs using our model-consistent generalization of the specification in Autor et al. (2013). We find that indirect effects from the shock exposure of other markets reinforce the negative impact of the market's own shock exposure, leading to employment and wage losses that are significantly larger than those reported in the existing literature.

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A data appendix is available at http://www.nber.org/data-appendix/w25544

1 Introduction

What are the labor market consequences of international trade shocks, such as a trade policy change or a foreign productivity boom? Answering this question requires quasi-experimental variation in trade shocks affecting different labor markets. A growing literature obtains such a variation from regional measures of exposure to trade shocks that are constructed from the interaction of aggregate shocks and associated region-specific exposure shares, as in the shift-share designs in Autor et al. (2013) and Kovak (2013). These measures yield estimates of how labor market outcomes differentially respond in regions with higher shock exposure. However, these differential responses may not fully capture all the channels through which trade shocks affect regional labor markets in general equilibrium. Such is the case if spatial connections imply that the shock exposure of a region not only affects its own labor market, but also has spillover effects on other regions. Without estimates of this type of spatial spillover effects, any analysis of the general equilibrium impact of trade shocks on local labor markets is incomplete.[1]

In this paper, we analyze how trade shocks affect local labor markets by extending shift-share empirical specifications to incorporate the general equilibrium effects that arise from spatial links in a flexible model. We first show that a market's shock exposure in partial equilibrium can be written in terms of two shift-share variables based on market-specific exposure shares for revenue and consumption. We then establish that, in general equilibrium, responses of employment and wages in each market are the sum, across all regions, of these shift-share measures times bilateral reduced-form elasticities determined by the economy's spatial links. Thus, though the lens of our spatial model, these reduced-form elasticities are sufficient for computing the general equilibrium impact of observed measures of regional shock exposure on local labor market outcomes. We then show how to efficiently estimate these elasticities using the model's reduced-form representation for employment and wage responses to exogenous variation in the shock exposure of different markets. Finally, we study the impact of the "China shock" on U.S. Commuting Zones (CZs). Our theory yields a generalization of the shift-share specification in Autor et al. (2013) that accounts for both the direct effect of the CZ's own shock exposure in revenue and consumption, as well as the indirect effect of the shock exposures of other CZs. We find that indirect effects reinforce direct effects, leading to employment and wage losses that are significantly larger than those reported in the existing literature.

We consider a general equilibrium framework with three types of spatial links. Every market has multiple sectors, each featuring a gravity-type demand for the goods from different markets. Local labor supply is endogenous: it depends on wages and prices in all markets. Finally, we allow for local economies of scale and spatial productivity spillovers by making local labor productivity a

[1] This is related to the well-known problem that difference-in-difference empirical strategies do not recover the general equilibrium effect of the "treated" on "non-treated", as pointed out by Heckman et al. (1998) and, more recently, by Muendler (2017) in the context of regional regressions.

function of employment in all markets. Through the shape of the functions specifying spatial links in the economy, our model encompasses several of the mechanisms in existing trade and spatial frameworks – for example, the gravity trade models reviewed by Costinot and Rodríguez-Clare (2014) and the spatial models reviewed in Moretti (2011) and Redding and Rossi-Hansberg (2017).

We start by expressing equilibrium wages and employment in terms of each market's excess labor demand. This way studying the impact of trade shocks in our spatial framework becomes a traditional comparative statics exercise in general equilibrium – see e.g. Arrow and Hahn (1971). In partial equilibrium, for any given initial wage vector, trade shocks trigger shifts in the excess labor demand of each market. In general equilibrium, wages and employment in all markets respond to these partial equilibrium shifts to guarantee labor market clearing everywhere. Such responses depend on the Jacobian matrix of the excess demand system with respect to wages. This is the "spatial links" matrix that summarizes the combined strength of different types of spatial links.

We then separately analyze these two components of the impact of trade shocks on local labor markets. We first show that the partial equilibrium shifts in excess labor demand can be written in terms of two shift-share variables, given by the sum of the product of trade shocks and market-specific exposure shares for revenue and consumption. For sectoral foreign productivity shocks, a market's revenue exposure is the commonly used shift-share variable based on sectoral employment shares – e.g., Autor et al. (2013) and Kovak (2013). In addition, our theory yields a consumption exposure measure that is a shift-share variable where the "share" is instead the sectoral spending share.

In general equilibrium, responses of employment and wages in each market are the sum, across *all* markets, of their partial equilibrium excess demand shifts multiplied by bilateral reduced-form elasticities. These general equilibrium reduced-form elasticities control how much the shift in a market's excess labor demand affects directly its own market and indirectly other markets. Depending on their sign, indirect reduced-form elasticities may reinforce or attenuate the direct effect of the market's own excess demand shift. Thus, in our spatial model, these elasticities are ex-ante sufficient statistics that allow the general equilibrium aggregation of excess demand shifts across markets.

We open the black-box of spatial shock propagation in the economy by writing the reduced-form elasticities as a series expansion of the spatial links matrix. This implies that bilateral reduced-form elasticities are larger between markets with stronger spatial links, due to tighter bilateral or third-market connections (e.g., similar compositions of trade partners and sectoral employment, or tighter labor supply links).[2] Only when spatial links are identical for all markets, bilateral indirect effects are identical and yield a common endogenous variable that is absorbed by time fixed-effects.

[2]Intuitively, this arises from the several adjustment rounds triggered by the response of each market's excess labor demand to shock-induced wage changes elsewhere. This is similar to the channel creating percolation of sectoral shocks in production networks, as in Acemoglu et al. (2012) and Acemoglu et al. (2016).

We further show that, if trade demand links are strong enough, indirect and direct effects have the same sign and, therefore, reinforce each other.

Taken together, our theoretical results point to two critical components of any empirical analysis of how trade shocks affect local labor markets: (i) the partial equilibrium shock exposure in revenue and consumption, and (ii) the general equilibrium reduced-form elasticities to the shock exposure of different markets. Accordingly, we build on these two components to propose a new empirical methodology to study the labor market consequences of trade shocks.

To this end, we use the reduced-form representation of our general equilibrium model to specify how regional outcomes respond to an observed trade shock and other unobserved shocks. We combine the observed trade shock with trade data to compute the revenue and consumption exposures of each market. Our model yields two estimating equations linking observed changes in employment and wages in each market to linear combinations of the product of reduced-form elasticities and observed measures of shock exposure across markets, as well as additive unobserved residuals. We proceed in two steps to estimate the reduced-form elasticities using these equations and exogenous cross-regional variation in shock exposure.

First, we reduce the dimensionality of the reduced-form elasticity matrix by imposing that spatial links are known functions of observables and parameters. Thus, we only need to estimate the parameters regulating the strength of reduced-form effects associated with different observed variables (e.g., bilateral flows in migration and sectoral trade).[3] We obtain moment conditions to estimate these parameters by assuming that the observed shock exposure is mean-independent of unobserved residuals. This assumption is the same orthogonality condition underlying the shift-share strategies in Kovak (2013) and Autor et al. (2013). Notice however that our estimating equations generalize the specifications in this literature: they include direct and indirect reduced-form elasticities to the shock exposures in both revenue and consumption.

Second, we characterize the efficient estimator of the parameters that non-linearly regulate the general equilibrium reduced-form elasticities. We outline a class of consistent GMM estimators for the parameter vector that differ in how they aggregate the shock exposure of different markets. We follow Chamberlain (1987) to characterize the "optimal" variance-minimizing estimator within this class and its two-step feasible implementation. For each market, the model-implied optimal moment puts more weight on the observed exposure of markets whose bilateral reduced-form elasticities are more sensitive to changes in the parameter of interest.[4]

In the last part of the paper, we apply our methodology to study the impact of the "China

[3]Our approach is similar to the common practice in demand estimation of projecting cross-price elasticities on product characteristics – see e.g. Berry (1994), Berry et al. (1995) and, for a review, Nevo (2000).

[4]The approach of Chamberlain (1987) has been used in partial equilibrium models by Berry et al. (1995), Petrin (2002), and Reynaert and Verboven (2014). Our contribution is, for a flexible spatial model in general equilibrium, to formally establish a class of consistent estimators and among them the optimal estimator and its feasible implementation.

shock" on U.S. Commuting Zones (CZs), as in Autor et al. (2013) (henceforth ADH). We follow ADH by considering the exposure of each CZ to industry-level Chinese export growth to a group of developed countries (excluding the U.S.). In this case, the model-consistent revenue exposure is proportional to the shift-share instrument in ADH. However, our theory predicts that regional outcomes also respond to (i) the CZ's own consumption shift, and (ii) indirectly to the revenue and consumption shifts of other CZs. Therefore, we begin our empirical analysis by estimating a simple extension of the specification in ADH with the goal of qualitatively investigating the relevance of these two additional channels. We impose that each market's indirect effect is proportional to the average shock exposure of all other markets, weighted by their inverse bilateral distance. We find that, conditional on a CZ's own revenue exposure, growth of both employment and wages are significantly lower if nearby CZs suffer larger revenue shocks. In our theory, these reinforcing indirect effects are consistent with strong trade demand links across CZs (relative to labor supply links). In addition, we do not find evidence that regional outcomes significantly respond to shifts in consumption cost. This suggests weak labor supply responses to lower import costs.[5]

We then implement our methodology to estimate the general equilibrium reduced-form elasticities implied by our parametrization of the spatial links matrix. This allows general equilibrium reduced-form elasticities to depend on bilateral as well as third-market connections that arise in general equilibrium. In addition, it implies that, through the lens of our spatial model, neither time fixed-effects nor residuals include any component of the endogenous responses of regional outcomes to the observed sectoral average of the China shock. In line with the evidence discussed above, we find that the direct and indirect effects of revenue exposure have the same sign. These indirect reinforcing effects are mostly driven by spatial trade links. We estimate large reduced-form elasticities that are consistent with strong amplification through local economies of scale. In addition, a reduction in consumption cost has a positive direct and indirect impact on wages and employment. However, the positive responses to consumption cost shifts are much weaker than the negative responses to revenue shifts.

To gain confidence in our methodology, we propose a way of evaluating a spatial model's predictions based on their ability to match the impact of regional shock exposure on labor market outcomes across CZs – in line with the evaluation strategy suggested by Kehoe (2005). Specifically, we regress actual changes in employment and wages of U.S. CZs on the model-predicted responses to the China shock. We find that the estimated fit coefficients are close to one for our baseline predicted responses, indicating that their magnitude is consistent with the shock's differential

[5]We construct CZ-level sectoral spending shares using the CZ's sectoral employment shares interacted with national input-output matrices, so they capture also the CZ's input spending on different sectors. Thus, this result indicates that we do not find evidence of stronger employment growth in CZs intensively using inputs from industries with larger increases in Chinese exports. This is consistent with the evidence in Pierce and Schott (2016) and Acemoglu et al. (2016) who do not find a significant response of an industry's employment to stronger Chinese export growth in the industries supplying its inputs.

impact on employment and wages across CZs.[6]

Using the same methodology, we evaluate the fit implied by typical specifications of spatial links in the quantitative spatial literature. We find that they perform poorly: the fit coefficient is much higher than one and, depending on the specification, very imprecisely estimated. This finding indicates a troubling disconnect between the results of empirical analyses using cross-regional variation in shock exposure and those of quantitative analyses using general equilibrium spatial models. It implies that common specifications of spatial links generate regional responses to the China shock that are too small compared to –and often uncorrelated with– actual changes in employment and wages across CZs. We show that these results follow mainly from specifying weak agglomeration forces and strong sensitivity of employment to import prices.

Finally, we compute the impact of the China shock on U.S. CZs by aggregating their exposure to the China shock using our estimates of the general equilibrium reduced-form elasticities. On average, revenue losses dominate consumption gains, reducing employment and wage log-growth respectively by 2.8 and 4 log-points between 1990 and 2007. The consumption gains however lead to a small increase in the average real wage of 0.2 log-points. These responses vary substantially across CZs: the standard deviation of log-changes is 1.3 for wages, 3.3 for employment, and 1.7 for real wages. These employment and wage losses are significantly larger than those in the existing literature. Empirical specifications ignoring indirect reinforcing effects yield smaller losses. In addition, quantitative frameworks yield average responses that are close to zero because they rely on reduced-form elasticities that are too small compared to those necessary to match the observed cross-regional responses to the shock.

Building on the seminal works of Bartik (1991) and Blanchard and Katz (1992), a growing literature uses shift-share strategies to estimate how regional markets respond to economic shocks, in general, and trade shocks, in particular – see e.g. Topalova (2010), Autor et al. (2013), Kovak (2013), Dix-Carneiro and Kovak (2017), Autor et al. (2016), Pierce and Schott (2020) and Burstein et al. (2020). Moving beyond the differential responses documented in this literature, we propose a generalization of this empirical approach that captures the indirect general equilibrium impact of the shock exposures of different markets. Through the lens of our spatial model, such estimates can be used to aggregate the shock exposure of different regions to obtain the general equilibrium responses of local labor market outcomes.[7]

Our work provides a bridge between this empirical literature and the alternative popular approach used to study the labor market consequences of trade shocks: quantitative analyses using

[6]While we use this methodology to evaluate regional outcomes, the same logic can be applied to evaluate outcomes such as bilateral trade or commuting flows, see e.g. Dingel and Tiltenot (2020).

[7]Heterogeneity in spatial links also implies heterogeneity in the direct "treatment" effect of the own market's shock exposure – as shown by Monte et al. (2018). In a recent related paper, Hornbeck and Moretti (2018) empirically document indirect effects of regional productivity shocks through out-migration to other regions. Instead, we focus on the implications of spatial links for the measurement of shock exposure in partial equilibrium and the estimation of indirect reduced-form elasticities in general equilibrium.

general equilibrium spatial models. Several papers point out the challenge of extrapolating general equilibrium counterfactual predictions from estimated differential responses across regions – see e.g. Moretti (2011); Kline and Moretti (2014); Beraja et al. (2019); Muendler (2017); Kehoe et al. (2017); Redding and Rossi-Hansberg (2017). This motivated recent analyses based on quantitative spatial frameworks studying the effects of import growth from China on local labor markets in the U.S. – see e.g. Galle et al. (2017) and Caliendo et al. (2019). We depart in two ways from this quantitative approach. First, we characterize the reduced-form representation of our general equilibrium model that links actual changes in employment and wages to shift-share measures of regional shock exposure in revenue and consumption. Second, we use this representation to obtain estimating equations that can be used to identify parameters regulating the general equilibrium reduced-form elasticities of local outcomes to the shock exposure of different markets. Thus, our approach is an extension of existing shift-share empirical specifications that allows the aggregation of regional shock exposure in a way that is consistent with the predictions of general equilibrium spatial models.[8] As a result, compared to common specifications of spatial links in quantitative frameworks, our methodology yields a substantially better fit for the differential response of labor market outcomes to shock exposure across regions, increasing the credibility of the model's counterfactual predictions.

Finally, our paper is related to the so-called "market access" approach in Redding and Venables (2004) that has been recently used to study how regional labor markets respond to economic shocks in general equilibrium – e.g., Donaldson and Hornbeck (2016); Alder et al. (2015); Bartelme (2018). Our model-consistent exposure measures correspond to partial equilibrium versions of the changes in producer and consumer market access in this literature, if the latter were computed holding wages and employment constant. Our exposure measures can be immediately constructed using trade data and can be aggregated in a model consistent-way using our estimates of the general equilibrium reduced-form elasticities. Market access measures are instead endogenous variables obtained from solving the general equilibrium model under restrictive assumptions and whose aggregation requires additional general equilibrium shifters.

The rest of the paper is structured as follows. Section 2 describes our spatial model. Section 3 characterizes the partial equilibrium exposure measures and the general equilibrium reduced-form elasticities. Section 4 presents the linear expressions for changes in wages and employment in terms of the exposure measures. Section 5 describes our methodology to estimate the reduced-form elasticities using these expressions. In Section 6, we estimate the impact of the China shock on U.S. CZs. Section 7 concludes.

[8]Note that our methodology is different from that used in papers exploiting cross-regional variation in shock exposure to estimate parameters of particular structural equations of the model – e.g., Faber and Gaubert (2019); Fajgelbaum et al. (2018); Galle et al. (2017); Allen et al. (2020); Burstein et al. (2020). This is because they do not explicitly characterize nor estimate the model-implied reduced-form elasticities of regional outcomes to shock exposure.

2 Model

We consider a spatial general equilibrium model in which N "markets" are linked through trade flows, labor productivity, and labor supply. Each "market" i consists of a set of sectors, $s \in \mathcal{S}_i$, within a geographic unit r where producers face identical endogenous production costs.[9] In the rest of the paper, we use bold variables to denote stacked vectors of market outcomes, $\boldsymbol{x} \equiv \{x_i\}_i$, and bar bold variables to denote matrices with bilateral variables associated with origin market i and destination market j, $\bar{\boldsymbol{x}} \equiv [x_{ij}]_{i,j}$.

Labor Supply Labor is freely mobile within a market, so that w_i is market i's wage. Labor supply in i, L_i, is a function of the vectors of wages, $\boldsymbol{w} \equiv \{w_j\}$, and price indices, $\boldsymbol{P} \equiv \{P_j\}$:

$$L_i = \Phi_i(\boldsymbol{w}, \boldsymbol{P}), \tag{1}$$

where $\Phi_i(\cdot)$ is strictly positive, differentiable, bounded, and homogeneous of degree zero.

We use the matrices of labor supply elasticities to changes in wages and prices to summarize the economy's spatial links in labor supply,

$$\phi_{ij}^w \equiv \frac{\partial \ln \Phi_i(\boldsymbol{w}, \boldsymbol{P})}{\partial \ln w_j} \quad \text{and} \quad \phi_{ij}^p \equiv \frac{\partial \ln \Phi_i(\boldsymbol{w}, \boldsymbol{P})}{\partial \ln P_j}. \tag{2}$$

The own-market elasticities, ϕ_{ii}^w and ϕ_{ii}^p, control the response of employment to changes in the market's wages and prices. They allow employment to respond to the regional exposure to import competition – as documented in Autor et al. (2013) and Dix-Carneiro and Kovak (2017). Our specification allows, but does not require, employment responses to wages and prices to be different. As discussed below, this implies that our model can match different types of employment responses to shocks in import competition and import prices. This is important given the evidence in Acemoglu et al. (2016) and Pierce and Schott (2016) that these shocks have different implications for employment growth across industries. Finally, the cross-market elasticities, ϕ_{ij}^w and ϕ_{ij}^p, regulate employment responses in market i to wages and prices in other markets, capturing endogenous employment changes across markets – as in the literature reviewed by Moretti (2011).

Our general labor supply specification $\Phi_i(\boldsymbol{w}, \boldsymbol{P})$ encompasses the labor supply function implied by several micro-founded frameworks – for a formal discussion, see Online Appendix B. It can replicate the labor supply functions in spatial models with homogeneous individuals facing housing congestion forces, as in Helpman (1998), and heterogeneous individuals in market-specific amenity preferences, as in Allen and Arkolakis (2014), Redding (2016), and Bryan and Morten (2015).[10]

[9] Each market in our model can be seen as a group of industries within a geographic unit at a point in time. For example, a market may be the set of manufacturing industries in a U.S. Commuting Zone.

[10] If a market is a group of industries in a region, then $\Phi_j(\boldsymbol{w}, \boldsymbol{P})$ can also replicate the industry-level labor supply

Our specification also allows aggregate employment to respond to changes in wages and prices. The fact that labor supply is a separate function of wages and price indices may arise from the endogenous labor supply choice in the presence of unemployment benefits when preferences either imply income effects on leisure demand, as in Shimer (2009) and Keane (2011), or entail heterogeneity in the disutility to work across individuals, as in Rogerson (1988) and Chetty (2012). In fact, without migration, these two settings imply that labor supply is more sensitive to the market's wage than to the price index, $\phi_{ii}^{w} > -\phi_{ii}^{p} > 0$. We show that this feature is key to match employment responses to trade shock exposure in the empirical application in Section 6.

Production The region-sector pair (r, s) in market i has a competitive representative firm whose production function uses only labor,

$$Q_{r,s} = \Psi_i \left(\boldsymbol{L}\right) L_{r,s}, \tag{3}$$

where $\Psi_i(\cdot)$ is a strictly positive differentiable function that governs the endogenous component of labor productivity in all sectors and regions of market i. We use the matrix of labor productivity elasticity to changes in employment to summarize the economy's spatial links in productivity,

$$\psi_{ij} = \frac{\partial \ln \Psi_i(\boldsymbol{L})}{\partial \ln L_j}. \tag{4}$$

The own-market elasticity ψ_{ii} captures labor productivity gains caused by higher local employment – as documented, for example, by Greenstone et al. (2010), Kline and Moretti (2014) and Peters (2019). The cross-market elasticity ψ_{ij} regulates spatial spillovers in labor productivity.

Again, different micro-founded frameworks imply that labor productivity takes the general form in $\Psi_i(\boldsymbol{L})$. A setting with $\psi_{ii} = \psi$ and $\psi_{ij} = 0$ arises from firm entry and increasing returns to scale with homogeneous firms, as in Krugman (1980), or heterogeneous firms, as in Arkolakis et al. (2008) and Chaney (2008).[11] By properly defining markets, our specification allows for external economies of scale in a region-sector, as in Ethier (1982), and technology diffusion between regions, as in Fujita et al. (1999) and Lucas and Rossi-Hansberg (2003). When markets are industry groups in a region, our environment accommodates differences across sectors in market structure and economies of scale – e.g., Krugman and Venables (1995), Balistreri et al. (2010), Kucheryavyy et al. (2016). In addition, the cross-market elasticity of labor productivity, ψ_{ij}, may also incorporate congestion forces implied by the spatial re-allocation of other factors of production (see Online Appendix B). Finally, under combined parametric restrictions on $\Phi_i(\boldsymbol{w}, \boldsymbol{P})$ and $\Psi_i(\boldsymbol{L})$, our model is equivalent to existing quantitative gravity spatial models, such as those reviewed by Redding

implied by assignment models with heterogeneous individuals in terms of industry-specific productivity, as in Burstein et al. (2019); Galle et al. (2017); Adão (2015).

[11] Costinot and Rodríguez-Clare (2014) establish this in the context of gravity trade models.

and Rossi-Hansberg (2017).[12]

Under perfect competition, the linear production function in (3) and the labor supply equation in (1) imply that the endogenous production cost in market i, p_i, is

$$p_i = \frac{w_i}{\Psi_i\left(\boldsymbol{\Phi}(\boldsymbol{w}, \boldsymbol{P})\right)}. \tag{5}$$

Demand We consider a multi-sector nested CES trade demand. The spending share of j on goods of sector s is constant and given by $\xi_{j,s}$, with $\sum_s \xi_{j,s} = 1$. Within sector s, the demand for goods from different markets is a constant elasticity function. This implies that the spending share of market j on goods produced in i is

$$x_{ij}(\boldsymbol{p}|\boldsymbol{\tau}) \equiv \sum_{s\in\mathcal{S}_i} x_{ij,s}\xi_{j,s} = \sum_{s\in\mathcal{S}_i} \frac{\left(\tau_{ij,s}p_i\right)^{-\epsilon_s}}{\sum_{o:s\in\mathcal{S}_o}\left(\tau_{oj,s}p_o\right)^{-\epsilon_s}}\xi_{j,s}, \tag{6}$$

where $\epsilon_s > 0$ is the trade elasticity in sector s, $\boldsymbol{\tau} \equiv \{\tau_{ij,s}\}_{ijs}$ is the stacked vector of bilateral trade costs, and the summation in the denominator is over markets that produce in sector s, $\{o : s \in \mathcal{S}_o\}$. The sector-specific trade costs combine bilateral differences in tastes, iceberg trade costs, and productivity. In our analysis below, we consider the consequences of exogenous changes in these bilateral terms.

Our multi-sector gravity trade structure follows closely that of the recent quantitative gravity literature reviewed by Costinot and Rodríguez-Clare (2014). In fact, when labor supply is exogenous (i.e., $\Phi_i(\boldsymbol{w}, \boldsymbol{P}) = \bar{L}_i$), our framework becomes a standard multi-sector gravity trade model with perfect labor mobility across sectors within a market but no mobility across markets. With a single sector, our trade demand specification is equivalent to that of the Armington model in Anderson (1979), the Ricardian model in Eaton and Kortum (2002), and, more generally, the class of gravity trade models in Arkolakis et al. (2012).

The revenue of i is $Y_i(\boldsymbol{p}, \boldsymbol{E}|\boldsymbol{\tau}) = \sum_j x_{ij}(\boldsymbol{p}|\boldsymbol{\tau})E_j$ where E_j is j's total spending. To summarize the economy's spatial linkages in trade, we use the elasticity of revenue to production costs,

$$\chi_{ij} \equiv \frac{\partial \ln Y_i(\boldsymbol{p}, \boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln p_j} = \sum_{s\in\mathcal{S}_i}\sum_k \left(y_{ik}y_{ik,s}\right)\left(x_{jk,s} - \mathbb{I}_{[i=j]}\right)\epsilon_s, \tag{7}$$

where y_{ij} is the share of j in the revenue of i, and $y_{ij,s}$ is the share of sector s in the sales of i to j. Equation (7) has two properties that follow directly from our model's multi-sector gravity structure. First, all markets are substitutes in the trade demand: $\chi_{ij} \geq 0$ for all $i \neq j$. Second, markets i and j are closer substitutes whenever i gets more of its revenue from destinations and sectors in which j's market share is large: χ_{ij} is increasing in $\sum_{k,s}(y_{ik}y_{ik,s})(x_{jk,s}\epsilon_s)$.

[12] Online Appendix B shows that the formal equivalence with these settings requires specifying the regional allocation of payments to non-labor factors (e.g., rent payments to land and capital).

Lastly, the gravity trade demand in (6) implies that the price index in market j is

$$P_j(\boldsymbol{p}|\boldsymbol{\tau}) = \prod_s \left[\sum_{o:s\in\mathcal{S}_o} (\tau_{oj,s} p_o)^{-\epsilon_s} \right]^{\frac{\xi_{j,s}}{-\epsilon_s}}. \tag{8}$$

Equilibrium We assume that trade is balanced, so that market clearing requires $w_i L_i = \sum_j x_{ij} (w_j L_j)$. Using the labor supply in (1) and the trade demand in (6), this market clearing condition yields the excess labor demand in market i:

$$D_i(\boldsymbol{w}|\boldsymbol{\tau}) \equiv \sum_j \left[\sum_{s\in\mathcal{S}_i} \frac{\left(\frac{\tau_{ij,s} w_i}{\Psi_i(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}\right)^{-\epsilon_s}}{\sum_{o:s\in\mathcal{S}_o} \left(\frac{\tau_{oj,s} w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}\right)^{-\epsilon_s}} \xi_{j,s} \right] w_j \Phi_j(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})) - w_i \Phi_i(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})) \tag{9}$$

where $\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})$ is implicitly defined as the solution of (8) for all j with p_o given by (5).

We define the equilibrium as a wage vector, $\boldsymbol{w} \equiv \{w_i\}_i$, that satisfies

$$\boldsymbol{D}(\boldsymbol{w}|\boldsymbol{\tau}) = 0. \tag{10}$$

We now establish conditions for the existence and uniqueness of the equilibrium wage vector in terms of the normalized Jacobian matrix of the excess demand system with respect to the wage vector: $\gamma_{ij} = -Y_i^{-1} \nabla_{\ln \boldsymbol{w}} D_i(\boldsymbol{w}|\boldsymbol{\tau})$ where Y_i is the total revenue in i. In the rest of the paper, we refer to $\bar{\boldsymbol{\gamma}} \equiv [\gamma_{ij}]$ as the "spatial links" matrix. This guarantees that our counterfactual analysis yields unambiguous predictions for the impact of trade cost shocks on local labor markets.

Assumption 1. ***[Uniqueness]*** *There is a market m with $\lim_{w_m\to 0} \frac{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}{w_m} = \infty$. For any equilibrium wage vector $\boldsymbol{w}$, assume that (i) $\gamma_{ii} > 0$ for all i, and (ii) $h_i \gamma_{ii} > \sum_{j\neq i,m} |\gamma_{ij}| h_j$ for all $i \neq m$ and some vector $\{h_i\}_{i\neq m} \gg 0$.*

This assumption requires a weighted sum of the cross-market wage elasticities of excess labor demand to be lower than the own-market wage elasticity of excess labor demand.[13] Under this assumption, we establish the following result.

Proposition 1. ***[Uniqueness]*** *Suppose that Assumption 1 holds. There exists a unique wage vector, $\boldsymbol{w}$ with $w_m \equiv 1$, that satisfies (10).*

Proof. See Appendix 8.1.

Imposing diagonal dominance on $\bar{\boldsymbol{\gamma}}$ to achieve equilibrium uniqueness has a long tradition in general equilibrium theory. In Proposition 1, we apply the tools in Arrow and Hahn (1971) to

[13] This assumption is weaker than the gross substitution property (i.e., $\gamma_{ii} > 0$ and $\gamma_{ij} < 0$ for all $i \neq j$) that yields uniqueness of gravity trade models (e.g., Alvarez and Lucas (2007)). This is despite the fact that our environment entails endogenous labor supply and economies of scale.

establish uniqueness in our framework with spatial links in labor supply, labor productivity, and trade demand. Note that, whereas proving existence in the presence of such links is straightforward (see Mas-Colell et al. (1995) Chapter 17), characterizing uniqueness is typically harder and involves specific parametric restrictions (e.g., see Miyao et al. (1980), Allen et al. (2020), Allen et al. (2015)).

3 Theory of General Equilibrium Effects in Space

We now study how exogenous changes in bilateral trade costs $\tau_{ij,s}$ affect outcomes in different markets. We use the superscript 0 to denote variables in the initial equilibrium, z_j^0, and hats to denote log changes in variables between the initial and new equilibria, $\hat{z}_j \equiv \ln(z_j/z_j^0)$. Given the normalization $\hat{w}_m = 1$, relative wage changes follow directly from the equilibrium definition in terms of excess labor demand. By totally differentiating (10), we obtain

$$\underbrace{-(\bar{\boldsymbol{Y}}^0)^{-1}\left(\nabla_{\ln w}\boldsymbol{D}\left(\boldsymbol{w}^0|\boldsymbol{\tau}^0\right)\right)}_{\bar{\boldsymbol{\gamma}}}\hat{\boldsymbol{w}} = \underbrace{(\bar{\boldsymbol{Y}}^0)^{-1}\left(\nabla_{\ln \boldsymbol{\tau}}\boldsymbol{D}\left(\boldsymbol{w}^0|\boldsymbol{\tau}^0\right)\right)\hat{\boldsymbol{\tau}}}_{\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})}, \tag{11}$$

where $\bar{\boldsymbol{Y}}^0$ is the diagonal matrix of initial market revenue.

The system in (11) frames our analysis as a traditional comparative statics exercises in general equilibrium.[14] The right hand side measures the partial equilibrium shift in the excess labor demand of each market caused by the shock (holding wages constant). The left hand side is the general equilibrium change in excess demand, which depends on the "spatial links" matrix $\bar{\boldsymbol{\gamma}}$, triggered by relative wage responses that restore labor market clearing everywhere.

Remark 1. *Following shocks to bilateral trade shifters, relative wage changes combine: (i) the vector of partial equilibrium shifts in excess labor demand, $\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}})$, and (ii) the Jacobian matrix of the excess demand system, $\bar{\boldsymbol{\gamma}}$.*

In the rest of this section, we first establish that the excess demand shift can be written in terms of shift-share variables similar to those used to measure regional exposure to trade shocks in a recent empirical literature – e.g., Kovak (2013) and Autor et al. (2013). We then show that, because of the spatial links in $\bar{\boldsymbol{\gamma}}$, the general equilibrium elasticity of the wage in market i aggregates the direct effect of i's own demand shift and the indirect effect of the demand shift of other markets.

It is worth noting that our analysis focuses on the elasticity of labor market outcomes to trade cost shocks. As such, it is a good approximation for the response to small changes in trade costs.

[14] For example, see sections 10.2 in Arrow and Hahn (1971) and 17.G in Mas-Colell et al. (1995). Allen et al. (2020) use an analogous representation in a single-sector gravity economy with a logit function of effective labor supply – in our notation, $\Phi_i(\boldsymbol{w}, \boldsymbol{P}) = \nu_i(w_i/P_i)^\phi / \sum_j \nu_j(w_j/P_j)^\phi$ and $\Psi_i(\boldsymbol{L}) = L_i^\psi$.

For large shocks however, it is necessary to compute the integral of our formulas accounting for changes in trade outcomes along the adjustment path to the new equilibrium.

3.1 Partial Equilibrium Shifts in Excess Labor Demand

Our first result characterizes partial equilibrium shifts in the only two variables directly affected by the shock: revenue Y_i and consumption cost index P_i.

Proposition 2. *Partial equilibrium shifts in i's revenue,* $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) \equiv \sum_{j,o,s} \frac{\partial \ln Y_i(\boldsymbol{p}^0, \boldsymbol{E}^0 | \boldsymbol{\tau}^0)}{\partial \ln \tau_{oj,s}} \hat{\tau}_{oj,s}$*, and consumption cost,* $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) \equiv \sum_{o,s} \frac{\partial \ln P_i(\boldsymbol{p}^0 | \boldsymbol{\tau}^0)}{\partial \ln \tau_{oi,s}} \hat{\tau}_{oi,s}$*, are given by*

$$\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) = \sum_{j,o,s} \left(y_{ij}^0 y_{ij,s}^0 \right) \left(x_{oj,s}^0 - \mathbb{I}_{[i=o]} \right) \epsilon_s \hat{\tau}_{oj,s} \quad \textit{and} \quad \hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) = \sum_{o,s} \xi_{i,s} x_{oi,s}^0 \hat{\tau}_{oi,s}. \tag{12}$$

Proof. See Appendix 8.2.

Equation (12) shows that the partial equilibrium shifts, $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$ and $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$, have a shift-share structure: they interact the shock $\hat{\tau}_{oj,s}$ with i's initial exposure in terms of revenue and consumption. In particular, market i's revenue is more exposed to $\hat{\tau}_{oj,s}$ when i's revenue share in sector s and destination j is higher (i.e., $y_{ij}^0 y_{ij,s}^0$ is higher) and the spending share of j on market o is higher (i.e., $x_{oj,s}^0 \epsilon_s$ is higher). In addition, by Shepard's lemma, i's consumption cost is more exposed to $\hat{\tau}_{oi,s}$ when i spends more on sector s of market o (i.e., $\xi_{i,s} x_{oi,s}^0$ is higher). Thus, given any trade shock, the computation of $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$ and $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ only requires bilateral trade flows and trade elasticities.[15] We simplify the notation by denoting $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$ and $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ as $\hat{\eta}_i^R$ and $\hat{\eta}_i^C$.

To understand better $\hat{\eta}_i^R$ and $\hat{\eta}_i^C$, consider a shock in a single origin F, $\hat{\tau}_{ij,s} = 0$ for $i \neq F$ and $\hat{\tau}_{Fj,s} \neq 0$, under the assumption of $\epsilon_s = \epsilon$ for all s. For $i \neq F$, the exposure to the sectoral average of the demand-adjusted shock, $\hat{\zeta}_s \equiv N^{-1} \sum_j (\epsilon x_{Fj,s}^0 \hat{\tau}_{Fj,s})$, is[16]

$$\hat{\eta}_i^R = \sum_s y_{i,s}^0 \hat{\zeta}_s \quad \text{and} \quad \hat{\eta}_i^C = \epsilon^{-1} \sum_s \xi_{i,s} \hat{\zeta}_s \quad \text{for } i \neq F, \tag{13}$$

where $y_{i,s}^0 \equiv \sum_j y_{ij}^0 y_{ij,s}^0$ is the share of sector s in market i's revenue. Notice that $y_{i,s}$ is also the share of sector s in i's total employment since our model features a single wage rate in each market.

[15] The measures $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$ and $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ are the partial equilibrium (i.e. holding wages and employment constant in all markets) impact of trade shocks on the measures of firm and consumer market access introduced in Anderson and Van Wincoop (2003) and Redding and Venables (2004). However, while our measures can be readily recovered with data in the initial equilibrium, the definition of market access requires solving the full general equilibrium, which is typically done under the assumption of a single industry whose bilateral trade costs are symmetric and observed (e.g., Redding and Sturm (2008), Donaldson and Hornbeck (2016), Bartelme (2018)).

[16] Without loss of generality, we can decompose $\epsilon x_{Fj,s}^0 \hat{\tau}_{Fj,s}$ into a sector mean, $\hat{\zeta}_s \equiv N^{-1} \sum_j (\epsilon x_{Fj,s}^0 \hat{\tau}_{Fj,s})$, and a sector-market residual, $\hat{\varepsilon}_{j,s} \equiv \epsilon x_{Fj,s}^0 \hat{\tau}_{Fj,s} - \hat{\zeta}_s$.

In this case, $\hat{\eta}_i^R$ is a shift-share variable based on sectoral employment shares. It is thus the regional exposure to import competition used in a growing empirical literature. If F becomes more productive ($\hat{\zeta}_s < 0$ for all s), then every other market suffers a negative revenue shift, $\hat{\eta}_i^R < 0$. The effect is stronger in markets with a higher employment share in sectors (i.e., higher $y_{i,s}^0$) with a stronger foreign shock (i.e., lower $\hat{\zeta}_s$). Our theory also entails a consumption cost shift that is proportional to a shift-share variable based, instead, on sectoral spending shares. If $\hat{\zeta}_s < 0$, then consumption cost falls everywhere, $\hat{\eta}_i^C < 0$. The decline is stronger in markets with higher spending shares in sectors (i.e., higher $\xi_{i,s}$) with a stronger foreign shock (i.e., lower $\hat{\zeta}_s$).

The partial equilibrium excess demand shift can be now written in terms of the shifts in revenue and consumption cost.

Theorem 1. *The vector of excess labor demand shifts is*

$$\hat{\boldsymbol{\eta}}(\hat{\boldsymbol{\tau}}) = \underbrace{\hat{\boldsymbol{\eta}}^R}_{\textit{Demand shift}} - \underbrace{\bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\phi}}^p\hat{\boldsymbol{\eta}}^C}_{\textit{Supply shift}}, \tag{14}$$

where $\bar{\boldsymbol{\alpha}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\right)\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)^{-1}$, $\bar{\boldsymbol{y}}^0$ *is the revenue share matrix with entries* y_{ij}^0, *and* $\bar{\boldsymbol{x}}^{0\prime}$ *is the spending share matrix with entries* x_{ij}^0.

Proof. See Appendix 8.3.

The first component in (14), the revenue shift $\hat{\boldsymbol{\eta}}^R$, measures the shock's impact on the demand for goods from each market (holding wages and employment constant). The second component measures the labor supply shift caused by the import price shock. It arises only when labor supply is a function of consumption prices ($\bar{\boldsymbol{\phi}}^p \neq 0$). The total effect on the excess labor demand is amplified by the multiplier matrix $\bar{\boldsymbol{\alpha}}$ implied by the feedback effect of employment on productivity and market size (holding wages constant).

We convey the intuition for the supply shift in the special case without cross-market links in labor supply and productivity: $\bar{\boldsymbol{\psi}} = \psi\bar{\boldsymbol{I}}$ and $\bar{\boldsymbol{\phi}}^p = -\phi^p\bar{\boldsymbol{I}}$. If $0 < \phi^p\psi < 1$, then

$$-\bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\phi}}^p\hat{\boldsymbol{\eta}}^C = \underbrace{\left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0\right)\left(\phi^p\hat{\boldsymbol{\eta}}^C\right)}_{\text{Net labor supply}} + \underbrace{\bar{\boldsymbol{\chi}}^0\psi\left(\phi^p\hat{\boldsymbol{\eta}}^C\right)}_{\text{Productivity}} + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \psi\bar{\boldsymbol{\chi}}\right)\underbrace{\sum_{d=1}^{\infty}\left(\phi^p\psi\bar{\boldsymbol{x}}^{0\prime}\right)^d}_{\text{Amplification}}\left(\phi^p\hat{\boldsymbol{\eta}}^C\right). \tag{15}$$

Consider the supply shift implied by an increase in foreign productivity that lowers consumption cost everywhere, $\hat{\eta}_i^C < 0$. The shock causes a labor supply increase in i, pushing down excess labor demand by $\phi^p\hat{\eta}_i^C$. However, labor supply increases in every other market j, increasing i's excess demand proportionally to its revenue share from j, $-\phi^p\sum_j y_{ij}^0\hat{\eta}_j^C$. The net effect is the first term in (15). Second, through the productivity response, the higher labor supply in i lowers its good prices by $\psi\phi^p\hat{\eta}_i^C$. The second term in (15) is this cost reduction times the demand elasticity

$\bar{\boldsymbol{\chi}}^0$. The third term in (15) arises from the several feedback rounds of changes in labor supply and productivity. Because all entries of $\phi^p \psi \bar{\boldsymbol{x}}^{0\prime}$ are positive, every term in the series expansion is positive and, therefore, amplifies the effect of the net labor supply and productivity terms.

Remark 2. *The revenue shift $\hat{\boldsymbol{\eta}}^R$ has a direct impact on excess labor demand $\hat{\boldsymbol{\eta}}$. The impact of the consumption cost shift $\hat{\boldsymbol{\eta}}^C$ is proportional to the price elasticity of labor supply $\bar{\boldsymbol{\phi}}^p$.*

3.2 Direct and Indirect Effects in General Equilibrium

We now investigate the general equilibrium impact of partial equilibrium shifts in excess labor demand. We start by establishing that the spatial links matrix is a function of the spatial connections in labor supply, productivity, and trade demand.

Proposition 3. *The spatial links matrix is*

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{I}} - \underbrace{\left(\bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\right)}_{\text{Demand substitution}} + \underbrace{\bar{\boldsymbol{\alpha}}\left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime}\right)}_{\text{Supply substitution}}. \tag{16}$$

Proof. See Appendix 8.4.

The spatial links matrix combines two forces created by spatial connections. The first is the demand substitution effect implied by changes in trade demand, as summarized in $(\bar{\boldsymbol{y}}^0, \bar{\boldsymbol{\chi}}^0)$. It controls how much the wage change in one market affects demand for goods elsewhere (given initial employment). The second is the supply substitution effect that regulates how much excess labor demand changes due to employment responses. It is the sum of the wage's direct impact on labor supply, $\bar{\boldsymbol{\phi}}^w$, and its indirect impact on labor supply through changes in the price index, $\bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime}$.[17] Again, the labor supply response is amplified by the multiplier matrix $\bar{\boldsymbol{\alpha}}$ defined in Theorem 1.

We now characterize the general equilibrium response of relative wages.

Theorem 2. *Suppose that Assumption 1 holds. Consider shocks to bilateral shifters, $\hat{\boldsymbol{\tau}}$, with an associated vector of excess demand shifts given by $\hat{\boldsymbol{\eta}}$. Then,*

$$\hat{w}_i = \underbrace{\beta_{ii}\hat{\eta}_i}_{\text{Direct effect}} + \underbrace{\sum_{j \neq i} \beta_{ij}\hat{\eta}_j}_{\text{Indirect effect}}, \tag{17}$$

where

$$\beta_{ij} = \frac{1}{\gamma_{ii}}\left(\mathbb{I}_{[i=j]} - \frac{\gamma_{ij}}{\gamma_{jj}}\mathbb{I}_{[i \neq j]}\right) + \sum_{d=2}^{\infty} \frac{\tilde{\gamma}_{ij}^{(d)}}{\gamma_{jj}} \tag{18}$$

[17] This indirect response is an immediate implication of Shepard's lemma: a wage change of $\hat{w}_j$ in market j has an effect on i's price index that is proportional to i's spending share on j, $x_{ji}^0 \hat{w}_j$.

such that $\tilde{\gamma}_{ij}^{(d)}$ *is the i-j entry of* $(\bar{\tilde{\gamma}})^d$ *with* $\tilde{\gamma}_{ij} \equiv -\frac{\gamma_{ij}}{\gamma_{ii}} \mathbb{I}_{[i \neq j;\ i,j \neq m]}$.

Proof. See Appendix 8.5.

Theorem 2 characterizes the reduced-form impact of shifts in excess labor demand on relative wages (given the normalization $\hat{w}_m \equiv 1$). The term β_{ij} is the *reduced-form elasticity* of market i's relative wage to market j's partial equilibrium excess demand shift. Equation (17) shows that local shock exposure percolates to other markets: $\hat{\eta}_j$ has a direct effect of $\beta_{jj}\hat{\eta}_j$ on the own market j and an indirect effect of $\beta_{ij}\hat{\eta}_j$ on other markets i. Thus, i's relative wage responses *aggregates* the direct exposure to its own shock ($\hat{\eta}_i$) and the indirect exposure to the shock of other markets ($\hat{\eta}_j$ for $j \neq i$), where the aggregation weights are given by β_{ij}. Accordingly, conditional on knowing the excess demand shifts $\hat{\boldsymbol{\eta}}$, the matrix of reduced-form elasticities $\bar{\boldsymbol{\beta}}$ is sufficient to aggregate wage responses in general equilibrium. Thus, expression (17) can be used to construct an ex-ante analog of the type of sufficient statistic for welfare gains in Arkolakis et al. (2012) that only requires outcomes in the initial equilibrium.[18]

The theorem shows that β_{ij} is a series expansion of the spatial links matrix $\bar{\boldsymbol{\gamma}}$. A stronger bilateral link γ_{ij} yields a larger indirect effect β_{ij}: the first term of β_{ij} in (18) is proportional to γ_{ij}. The change in i's excess demand must also be corrected for changes in the wages of other markets, triggering higher-round responses captured by the power series in (18), whose magnitude depends on the combined strength of the connections across the network of markets.[19]

This result indicates that the spatial links matrix $\bar{\boldsymbol{\gamma}}$ determines the reduced-form elasticity matrix $\bar{\boldsymbol{\beta}}$. In fact, Appendix 8.6 shows that the connection between $\bar{\boldsymbol{\gamma}}$ and $\bar{\boldsymbol{\beta}}$ holds also in the opposite direction since $\bar{\boldsymbol{\beta}}$ is, up to a normalization, the inverse of $\bar{\boldsymbol{\gamma}}$.

Remark 3. *Given the vector of partial equilibrium shifts* $\hat{\boldsymbol{\eta}}$*, the general equilibrium wage responses aggregate the direct effect of the own market shock exposure,* $\beta_{ii}\hat{\eta}_i$*, and the indirect effects of other markets' exposure,* $\sum_{j \neq i} \beta_{ij}\hat{\eta}_j$*. The indirect effects* β_{ij} *are stronger among markets with stronger bilateral spatial links,* γ_{ij}*, or stronger third-market connections,* $\sum_{d=2}^{\infty} \tilde{\gamma}_{ij}^{(d)} / \gamma_{jj}$*.*

3.3 Understanding Indirect Effects in General Equilibrium

We now present two special cases to provide further intuition for the results above. They illustrate how indirect effects shape the general equilibrium impact of trade shocks on local labor markets. They also link the reduced-form representation in (17) to specifications used to investigate how regional markets respond to trade shocks in a recent empirical literature.

[18] Our characterization also complements the connection between outcomes and market access across regions introduced by Donaldson and Hornbeck (2016). While the aggregation of the reduced-form responses based on (17) immediately yields wage changes in general equilibrium, the aggregation of the impact of market access also involves an endogenous shifter implied by the full specification of the model.

[19] Similar power series of indirect effects arise in the percolation of shocks across the network of sectors e.g., Acemoglu et al. (2012) and Acemoglu et al. (2016).

Our first special case provides a sufficient condition for the direct and indirect effects to have the same sign. It imposes that $\bar{\boldsymbol{\gamma}}$ satisfies the gross substitution property, which effectively restricts demand substitution in $\bar{\boldsymbol{\gamma}}$ to dominate opposing supply substitution forces.

Corollary 1. *If $\bar{\boldsymbol{\gamma}}$ satisfies gross-substitution ($\gamma_{ij} < 0$ for $i \neq j$), then $\beta_{ij} > 0$ for all (i, j).*

Proof. See Appendix 8.7.

To understand this result, consider again the productivity increase in foreign market F. If the negative revenue shift of i dominates the impact of its consumption cost shift ($\hat{\eta}_i < 0$), then market i's excess demand shift leads to reinforcing indirect effects everywhere, lowering not only i's wage relative to F, but also that of all other markets. Intuitively, a negative revenue shift in a market reduces the demand for goods produced in other markets, further decreasing revenues and wages in those markets. In this case, ignoring indirect effects underestimates the impact of the foreign productivity shock on local outcomes.

The second special case restricts spatial links to be identical.

Corollary 2. *If the entries of $\bar{\boldsymbol{\gamma}}$ satisfy $\gamma_{ij} = \gamma \mathbb{I}_{[i=j]} - \gamma_j$, then $\beta_{ij} = \beta \mathbb{I}_{[i=j]} + \beta_j \ \forall \ (i, j)$.*

Proof. See Appendix 8.8.

This case gives rise to an "endogenous" fixed-effect, which contains the sum of the indirect effects of all markets, $\sum_j \beta_j \hat{\eta}_j$. This common component can be ignored only if the shock affects a zero mass of markets. Notice that β_j is positive whenever $\bar{\boldsymbol{\gamma}}$ also satisfies gross substitution (i.e., $\gamma_j > 0$). In this case, a foreign productivity gain leading to $\hat{\eta}_i < 0$ for all $i \neq F$ creates a negative fixed-effect that reinforces the wage decline caused by the market's own shock exposure.

Remark 4. The off-diagonals of the spatial links matrix determine the sign and the heterogeneity of the indirect effects of excess labor demand shifts in other markets.

Connection to gravity trade models. The previous two corollaries illustrate the predictions of gravity trade models with exogenous labor supply ($\bar{\boldsymbol{\phi}}^w = \bar{\boldsymbol{\phi}}^p = 0$) – e.g., see Costinot and Rodríguez-Clare (2014). In fact, these models satisfy the gross-substitution property if the trade elasticity ϵ_s is positive (Alvarez and Lucas, 2007). By Corollary 1, in this case, excess labor demand shifts further propagate through the trade network, generating reinforcing indirect effects on other markets. Theorem 2 indicates that these indirect effects are stronger in nearby markets for which γ_{ij} is likely to be higher.[20]

[20] A gravity structure of bilateral trade is also present in quantitative spatial frameworks used in a recent literature (e.g. Allen and Arkolakis (2014); Redding and Rossi-Hansberg (2017)). However, in these models, the gross-substitution property may not hold due to the combination of endogenous responses in employment and productivity across markets. This creates a force for attenuating indirect effects as negative shocks elsewhere lead to local employment increases that have positive impacts on local market size and labor productivity.

This special case allows us to tightly connect our results to existing evidence on the response of regional wages to trade shocks – e.g., Kovak (2013); Autor et al. (2013). To see this, assume no trade costs (i.e., $y_{ij,s} = x_{ij,s} = x_i$), and consider again the shifts created by the sectoral component of productivity gains in the foreign country F (i.e., $\hat{\zeta}_s > 0$ for all $i \neq F$). By Corollary 2, equation (17) for any $i \neq F$ becomes

$$\hat{w}_i = \beta \hat{\eta}_i^R + \bar{\eta}^R \qquad \text{where} \qquad \bar{\eta}^R \equiv \sum_j \beta_j \hat{\eta}_j^R \quad \text{and} \quad \hat{\eta}_i^R \equiv \sum_s y_{i,s}^0 \hat{\zeta}_s. \tag{19}$$

This expression implies that the differential wage response of market i is proportional to that market's shift-share exposure (as measured by $\hat{\eta}_i^R$). The elasticity β measures how much more relative wages respond in markets experiencing stronger shifts in excess demand. The general equilibrium response also includes the common "endogenous" fixed-effect $\bar{\eta}^R$ that incorporates the indirect effects of labor demand shifts in other markets. This fixed-effect reinforces the impact of the market's own revenue shift caused by the foreign productivity gains ($\hat{\eta}_i^R < 0$ for all $i \neq F$). Its size is proportional to the size of the more severely exposed markets (see Appendix 8.8).

Our model thus generalizes shift-share specifications used in this recent empirical literature. In the general case considered in Theorem 2, the indirect effect of the shock exposure of other markets may reinforce or attenuate the direct effect of the local shock exposure. These indirect effects are stronger between markets with stronger spatial links. In Section 5, we use the heterogeneity in indirect effects to propose a strategy to estimate how local labor markets respond to the shock exposure of different markets. The strategy exploits the one-to-one mapping between $\bar{\boldsymbol{\beta}}$ and $\bar{\boldsymbol{\gamma}}$ to reduce the number of parameters in estimation. Before we do that, we derive our model's reduced-form expressions for changes in wages, employment, and real wages.

3.4 Extensions

In Online Appendix C, we derive versions of the results in this section for several extensions of our baseline model. We highlight potential differences in terms of sources of spatial links, which regulate the reduced-form general equilibrium elasticities, and the shock exposure measures, which take the form of shift-share variables.

Trade imbalances. We follow Dekle et al. (2007) to incorporate exogenous trade imbalances for each market (specified in terms of the world's average wage). All results above remain valid with a spatial links matrix that accounts for the effect of wage changes on transfers.

Bilateral migration. We follow Bryan and Morten (2015) to incorporate bilateral migration flows into our model. All results above remain the same. The only difference is that this extended model also yields predicted changes in bilateral migration flows following trade shocks.

Multiple worker groups. We introduce multiple worker groups, as in Cravino and Sotelo (2019).

In this case, the market definition includes worker groups and the spatial links matrix depends on the elasticity of substitution in production across groups. We show that revenue shifts for each group are based on the group's employment distribution across sectors.

General trade demand. We relax the nested CES demand structure by considering an one-factor economy with arbitrary within-sector heterogeneity in productivity across goods and markets, as in Adao et al. (2017). The main difference in this case is that spatial links in trade demand χ_{ij} take a more general form and they also enter the definition of the revenue shift.

Input-Output linkages. We finally extend our model to incorporate input-output linkages in production, as in Caliendo and Parro (2015). Bilateral trade demand for intermediate inputs are reflected in the spatial links matrix. This economy entails an additional exposure measure capturing shifts in the cost of imported inputs, which takes the form of a shift-share variable based on shares of intermediate input usage across sectors for each market (similar to the ones in Acemoglu et al. (2016)).

4 Reduced-Form Responses in General Equilibrium

In this section, we derive our theory's reduced-form representation for the impact of trade shocks on employment and wages across markets. In particular, we show how each labor market responds, directly and indirectly, to the shock-induced shifts in revenue and consumption cost of different markets. These reduced-form expressions are the cornerstone of our empirical strategy to measure the labor market consequences of trade shocks in general equilibrium. Appendix 8.9 contains the derivations for this section.

4.1 Wage and Employment Responses

The expressions for excess demand shifts in (14) and wage changes in (17) imply

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{\beta}}^R \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\beta}}^C \hat{\boldsymbol{\eta}}^C \quad \text{such that} \quad \bar{\boldsymbol{\beta}}^R \equiv \bar{\boldsymbol{\beta}} \quad \text{and} \quad \bar{\boldsymbol{\beta}}^C \equiv -\bar{\boldsymbol{\beta}} \bar{\boldsymbol{\alpha}} \bar{\boldsymbol{\phi}}^p. \tag{20}$$

Wage changes combine (i) the vector of shock-induced shifts in revenue and consumption costs, $\hat{\boldsymbol{\eta}}^R$ and $\hat{\boldsymbol{\eta}}^C$, and (ii) the reduced-form elasticity matrices of wage changes to these shifts, $\bar{\boldsymbol{\beta}}^R$ and $\bar{\boldsymbol{\beta}}^C$. The elasticity of wages to shifts in revenue and excess labor demand are identical, so it has all the properties discussed in Sections 3.2 and 3.3. The reduced-form elasticity of wages to consumption cost shifts also includes the supply component of the excess demand shift discussed in Section 3.1 and, as such, it is proportional to $\bar{\boldsymbol{\phi}}^p$.

Turning to employment, the labor supply function in (1) and the price index in (8) yield

$$\hat{\boldsymbol{L}} = \bar{\boldsymbol{\rho}} \left(\bar{\boldsymbol{\phi}} \hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C \right), \tag{21}$$

where $\bar{\boldsymbol{\rho}} \equiv (\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}})^{-1}$ and $\bar{\boldsymbol{\phi}} \equiv (\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime})$. This expression captures all the different channels through which trade shocks affect employment in our model. $\bar{\boldsymbol{\rho}}$ is the multiplier of employment-induced changes in productivity and prices that feedback into further employment changes. It is the matrix that introduces the amplifying series expansion in $\bar{\boldsymbol{\alpha}}$ – see equation (15). The terms inside brackets capture the labor supply responses to changes in wages and prices: $\bar{\boldsymbol{\phi}} \hat{\boldsymbol{w}}$ is the impact of wages through the elasticity structure of labor supply in $\bar{\boldsymbol{\phi}}$, and $\bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C$ is the impact of import prices through the price elasticity of labor supply in $\bar{\boldsymbol{\phi}}^p$.

We use this expression to write employment changes as only a function of shifts in revenue and consumption cost. The combination of (20) and (21) implies

$$\hat{\boldsymbol{L}} = \bar{\boldsymbol{\varphi}}^R \hat{\boldsymbol{\eta}}^R + \bar{\boldsymbol{\varphi}}^C \hat{\boldsymbol{\eta}}^C \quad \text{such that} \quad \bar{\boldsymbol{\varphi}}^R \equiv \bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}} \bar{\boldsymbol{\beta}}^R \quad \text{and} \quad \bar{\boldsymbol{\varphi}}^C \equiv \bar{\boldsymbol{\rho}} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\phi}} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{\alpha}} \right) \bar{\boldsymbol{\phi}}^p. \tag{22}$$

Employment responses depend again on $\hat{\boldsymbol{\eta}}^R$ and $\hat{\boldsymbol{\eta}}^C$, but they are now multiplied by different reduced-form elasticity matrices, $\bar{\boldsymbol{\varphi}}^R$ and $\bar{\boldsymbol{\varphi}}^C$. The revenue elasticity combines the revenue shift's impact on wages $\bar{\boldsymbol{\beta}}$ and the employment elasticity to wages $\bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}$. The consumption cost elasticity is the sum of employment responses to the consumption cost shift, $\bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}}^p$, and the wage change induced by the cost shift, $\bar{\boldsymbol{\rho}} \bar{\boldsymbol{\phi}} \bar{\boldsymbol{\beta}}^C$. Thus, $\bar{\boldsymbol{\varphi}}^C$ is proportional to $\bar{\boldsymbol{\phi}}^p$.

Through the lens of our model, (20) and (22) are the reduced-form responses of local labor market outcomes to trade shocks in general equilibrium. Accordingly, they allow us to aggregate any vector of partial equilibrium shifts $\hat{\boldsymbol{\eta}}^R$ and $\hat{\boldsymbol{\eta}}^C$ to obtain wage and employment responses in general equilibrium. This aggregation only requires the reduced-form elasticity matrices $(\bar{\boldsymbol{\beta}}^R, \bar{\boldsymbol{\beta}}^C, \bar{\boldsymbol{\varphi}}^R, \bar{\boldsymbol{\varphi}}^C)$.

Remark 5. *The reduced-form expressions in (20) and (22) connect responses of wage and employment to shock-induced vectors of shifts in revenue and consumption cost. The reduced-form elasticities are sufficient to aggregate any vector of shifts* $\hat{\boldsymbol{\eta}}^R$ *and* $\hat{\boldsymbol{\eta}}^C$ *to obtain wage and employment responses in general equilibrium.*

4.2 Real Wage Responses

We conclude our theoretical analysis by establishing how trade shocks affect real wages, defined as $\hat{W}_i = \hat{w}_i - \hat{P}_i$. We use the real wage change as measure of the shock's impact on welfare. In fact, Online Appendix B.3 shows that, under general preferences for consumption and leisure, the equivalent welfare variation in each market implied by the shock is increasing in the shock's impact on the market's real wage. The real wage changes can be decomposed into three terms:

$$\hat{\boldsymbol{W}} = \underbrace{(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^{0\prime})\hat{\boldsymbol{w}}}_{\text{Terms-of-trade gains}} + \underbrace{\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\hat{\boldsymbol{L}}}_{\text{Efficiency gains}} - \underbrace{\hat{\boldsymbol{\eta}}^{C}}_{\text{Consumption cost gains}}. \tag{23}$$

The terms in this expression represent the three main sources of welfare gains in the model: (i) changes in terms-of-trade due to the wage changes characterized in (20), (ii) changes in labor productivity due to the employment changes characterized in (22), and (iii) the partial equilibrium changes in consumption cost, $\hat{\boldsymbol{\eta}}^C$ in (12).

The main forces shaping real wage responses can be readily seen using again the example of the gravity trade model and a foreign productivity shock. In such case, $\bar{\boldsymbol{\phi}}^w = \bar{\boldsymbol{\phi}}^p = 0$ and, therefore,

$$\hat{\boldsymbol{W}} = \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^{0\prime}\right)\bar{\boldsymbol{\beta}}\hat{\boldsymbol{\eta}}^R - \hat{\boldsymbol{\eta}}^C$$

where $\bar{\boldsymbol{\beta}} > 0$. For a foreign productivity gain, $\eta_i^R < 0$ and $\eta_i^C < 0$ for all $i \neq C$. This creates two opposite forces on the real wage: it tends to fall because of adverse terms of trade movements, but it tends to rise due to lower import prices. The overall effect depends on the relative intensity of these forces as controlled by the reduced-form elasticity $\bar{\boldsymbol{\beta}}$.

5 Measurement of General Equilibrium Effects in Space

We now develop a methodology to estimate the general equilibrium reduced-form elasticities. We specify the data generating process using our theory's reduced-form representation for changes in wages and employment across markets. We then outline parametric restrictions on spatial links that make the reduced-form elasticity matrix a function of observable variables and unknown parameters. We finally characterize a GMM estimator of the vector of parameters based on the Optimal IV approach of Chamberlain (1987).

5.1 Data Generating Process

We observe changes in employment and wages between t and t_0, $\hat{\boldsymbol{L}}^t$ and $\hat{\boldsymbol{w}}^t$, as well as the sector-level trade matrix at t_0, $\{X_{ij,s}^{t_0}\}$. We decompose trade cost changes into observed and unobserved components such that $\hat{\boldsymbol{\tau}}^t = \hat{\boldsymbol{\tau}}_O^t + \hat{\boldsymbol{\tau}}_U^t$.[21] Since $\eta_i^R(\hat{\boldsymbol{\tau}}^t)$ and $\eta_i^C(\hat{\boldsymbol{\tau}}^t)$ in (12) are linear combinations of $\hat{\boldsymbol{\tau}}$, we can define the shifts implied by each component of the shock:

$$\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}^t) = \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}_O^t) + \hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}_U^t) \quad \text{and} \quad \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}^t) = \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}_O^t) + \hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}_U^t). \tag{24}$$

[21] The observed component may be sectoral productivity shocks in a foreign country or changes in import barriers in an importer country. The unobserved component includes all other sources of trade shocks.

Up to a first order approximation, expressions (20), (22) and (24) imply that

$$\begin{bmatrix} \hat{w}_i^t \\ \hat{L}_i^t \end{bmatrix} = \begin{bmatrix} \beta^t \\ \varphi^t \end{bmatrix} + \sum_j \begin{bmatrix} \beta_{ij}^R \\ \varphi_{ij}^R \end{bmatrix} \hat{\eta}_j^R(\hat{\boldsymbol{\tau}}_O^t) + \sum_j \begin{bmatrix} \beta_{ij}^C \\ \varphi_{ij}^C \end{bmatrix} \hat{\eta}_j^C(\hat{\boldsymbol{\tau}}_O^t) + \begin{bmatrix} \nu_i^{w,t} \\ \nu_i^{L,t} \end{bmatrix}. \tag{25}$$

Without loss of generality, we define the time fixed-effects, β^t and φ^t, to be the mean effect on wage and employment of the unobserved trade shocks, $\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}_U^t)$ and $\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}_U^t)$. The terms $\nu_i^{w,t}$ and $\nu_i^{L,t}$ are then the deviations from the mean for market i in wage and employment responses to $\hat{\boldsymbol{\eta}}^R(\hat{\boldsymbol{\tau}}_U^t)$ and $\hat{\boldsymbol{\eta}}^C(\hat{\boldsymbol{\tau}}_U^t)$. Notice that we can easily allow $\nu_i^{w,t}$ and $\nu_i^{L,t}$ to also include the effect of labor supply shocks – i.e., the reduced-form responses to the shifts in $L_i = \Phi_i(\{v_j^w w_j\}_j, \{v_j^p P_j\}_j)$.

Assumption 5a. ***[DGP]*** *Between periods t_0 and t, we observe a trade shock vector $\hat{\boldsymbol{\tau}}_O^t$. We also observe changes in wages and employment, $\hat{w}_i^t$ and $\hat{L}_i^t$, that are given by equation (25) with $\nu_i^{w,t}$ and $\nu_i^{L,t}$ denoting mean-zero unobserved residuals.*

5.2 Dimensionality Reduction: Parametrizing Spatial Links

The estimation of equation (25) entails one important challenge: while we only observe employment and wage changes for N markets, (25) has $4N^2$ unknown elasticities. To circumvent this problem, we parametrize spatial links to write reduced-form elasticities as non-linear functions of observable variables in the initial equilibrium and a small number of unknown parameters.[22] Our parametric assumptions yield a parsimonious specification while incorporating enough degrees of freedom to flexibly capture different response patterns in the data.

Definition of a Market. Each market i is an integrated regional labor market, in which workers are perfectly mobile across sectors. In the empirical analysis below, markets are U.S. Commuting Zones (CZs), as in Tolbert and Sizer (1996).

Labor Supply. The labor supply elasticities to changes in wages and prices are given by

$$\phi_{ij}^w = \phi^w \mathbb{I}_{[i=j]} - \phi^m m_{ij}^0 - (\phi^w + \phi^p) b_j^w, \quad \text{and} \quad \phi_{ij}^p = \phi^p \mathbb{I}_{[i=j]} + \phi^m m_{ij}^0 - (\phi^w + \phi^p) b_j^p, \tag{26}$$

where m_{ij}^0 is the observed share of i's population born in j at t_0, and b_j^w and b_j^p are observed j-specific attributes such that $\sum_j (b_j^w + b_j^p) = 1$.

The parameters ϕ^w and ϕ^p control, respectively, the sensitivity of regional employment to the local wage and price index. As discussed in Section 4, ϕ^w is proportional to the response of

[22] This procedure effectively projects the reduced-form elasticities in (25) onto observable variables regulating the strength of spatial links. It is similar to the common practice in demand estimation of specifying cross-price demand elasticities in terms of observable variables (Berry, 1994; Berry et al., 1995).

regional employment to the shock-induced change in the region's wage. It captures the evidence in Autor et al. (2013) and Dix-Carneiro and Kovak (2017) that both employment and wage growth are lower in regions more exposed to import competition shocks. In contrast, ϕ^p is proportional to i's employment response to its own import price shock $\hat{\eta}_i^C$. This allows the model to flexibly match the type of employment responses to lower import prices studied by Acemoglu et al. (2016) and Pierce and Schott (2016). The parameter ϕ^m controls the employment response in i to wage changes in the regions where a higher share of i's population was born (i.e., m_{ij}^0 is higher). It thus captures the gravity migration links documented by Bryan and Morten (2015). For simplicity, we impose identical absolute values for the cross-market labor supply elasticities associated with m_{ij} for changes in prices and wages.

Lastly, b_j^w and b_j^p in (26) guarantee the homogeneity of the labor supply function, so that employment changes are invariant to the numeraire choice. Our baseline specification uses $b_j^p = 0$ and $b_j^w = Y_j / \sum_o Y_o$, which can be interpreted as setting non-employment benefits in terms of the world's average wage (the same numeraire used for international transfers).[23]

Productivity. The labor productivity elasticity to changes in employment is

$$\psi_{ij} = \psi \mathbb{I}_{[i=j]}. \tag{27}$$

The parameter ψ is the elasticity of regional labor productivity to regional employment. As discussed in Section 3, when combined with the labor supply elasticity in (26), a higher value of ψ amplifies the reduced-form elasticities of wages and employment to the observed shifts in revenue and consumption. Intuitively, it regulates the strength of the feedback effect of employment on productivity and, therefore, excess labor demand. It captures the agglomeration forces documented by Greenstone et al. (2010), Kline and Moretti (2014) and Peters (2019). It is important to note that such channel is absent in recent quantitative spatial frameworks based on the Ricardian model of Eaton and Kortum (2002) – e.g. Caliendo et al. (2018), Caliendo et al. (2019) and Galle et al. (2017).

Trade Demand. The trade demand elasticity is

$$\epsilon_s = \epsilon \ \forall s, \quad \text{such that} \quad \chi_{ij} = -\epsilon \mathbb{I}_{[i=j]} + \epsilon \sum_{s,k} \left(y_{ik}^0 y_{ik,s}^0 x_{jk,s}^0 \right). \tag{28}$$

The parameter ϵ controls the sensitivity of good's demand to production costs. By increasing the demand substitution elasticity (i.e., higher χ_{ij}), a higher ϵ strengthens spatial links and,

[23] Online Appendix A.2.1 evaluates how our results change when we use alternative labor supply specifications with the normalization in terms of the average price index.

consequently, indirect effects among regions specialized in similar sectors or destinations (i.e., higher $\sum_{s,k} y^0_{ik} y^0_{ik,s} x^0_{jk,s}$). Thus, ϵ captures demand spillovers implied by the gravity trade links extensively documented in the literature reviewed by Head and Mayer (2014).

Estimating Equation. For $\boldsymbol{\theta} \equiv (\phi^w, \phi^p, \phi^m, \psi, \epsilon)$, equation (25) becomes, after imposing (26)–(28):

$$\begin{bmatrix} \hat{w}^t_i \\ \hat{L}^t_i \end{bmatrix} = \begin{bmatrix} \beta^t \\ \varphi^t \end{bmatrix} + \sum_j \begin{bmatrix} \epsilon \beta^R_{ij}(\boldsymbol{\theta}) \\ \epsilon \varphi^R_{ij}(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}^{R,t}_j + \sum_j \begin{bmatrix} \beta^C_{ij}(\boldsymbol{\theta}) \\ \varphi^C_{ij}(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}^{C,t}_j + \begin{bmatrix} \nu^{w,t}_i \\ \nu^{L,t}_i \end{bmatrix} \tag{29}$$

where, by equation (12),

$$\hat{\eta}^{R,t}_j \equiv \sum_{j,o,s} \left(y^{t_0}_{ij} y^{t_0}_{ij,s} \right) \left(x^{t_0}_{oj,s} - \mathbb{I}_{[i=o]} \right) \hat{\tau}^t_{oj,s} \quad \text{and} \quad \hat{\eta}^{C,t}_i \equiv \sum_{o,s} \xi^{t_0}_{i,s} x^{t_0}_{oi,s} \hat{\tau}^t_{oi,s}. \tag{30}$$

We summarize these parametric restrictions in the following assumption.

Assumption 5b. ***[Parametrization of spatial links]*** *Assume that (26)–(28) hold. Conditional on $\boldsymbol{\theta} \equiv (\phi^w, \phi^p, \phi^m, \psi, \epsilon)$, equation (29) relates observed changes in wage and employment to the observed shifts in revenue and consumption cost in (30) across markets.*

5.3 Model-Implied Optimal IV

We now derive an estimator of $\boldsymbol{\theta}$. We start by imposing the following orthogonality condition.

Assumption 5c. ***[Exogeneity]*** *Let $\nu^t_i \equiv \left[\nu^{w,t}_i, \nu^{L,t}_i \right]'$. In every period t, $E\left[\nu^t_i \middle| \left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t} \right) \right] = 0$.*

Assumption 5c imposes that the unobserved residuals in (29) are mean-independent from the observed shifts in revenue and consumption. This type of exogeneity condition is necessary for the causal interpretation of estimates of how regional labor markets respond to international trade shocks – e.g. Topalova (2010), Kovak (2013), Autor et al. (2013).[24]

Consider a function $H_i\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t} \right)$ with dimension $dim(\boldsymbol{\theta}) \times 2$. By the law of iterated expectations, Assumption 5c implies

$$E[H_i(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}) \nu^t_i] = 0, \tag{31}$$

which yields the following class of GMM estimators of $\boldsymbol{\theta}$.

Definition 1. *Let $H_i\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t} \right)$ be a $dim(\boldsymbol{\theta}) \times 2$ function and $\nu^t_i(\boldsymbol{\theta})$ be the residual implied by (29) under Assumptions 5a and 5b. Define the GMM estimator:*

[24] Since $\hat{\boldsymbol{\tau}}^t_U$ generates the structural residuals $\nu^{w,t}_i$ and $\nu^{L,t}_i$, Assumption 5c is implied by the independence between $\hat{\boldsymbol{\tau}}^t_U$ and $\hat{\boldsymbol{\tau}}^t_O$ (given the initial trade matrix). This is similar to the requirements for identification and consistency in shift-share designs (see Borusyak et al. (2018) and Adão et al. (2019)).

$$\hat{\boldsymbol{\theta}}_H \equiv \arg min_{\boldsymbol{\theta}} \left[\sum_{i,t} H_i(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}) \nu_i^t(\boldsymbol{\theta}) \right]' \left[\sum_{i,t} H_i(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}) \nu_i^t(\boldsymbol{\theta}) \right]. \tag{32}$$

The estimator in (32) is consistent for $\boldsymbol{\theta}$ whenever $H_i(\cdot)$ satisfies the usual rank conditions establishing identification in GMM estimators.[25] Notice that a necessary condition for identification is that all elements of $\boldsymbol{\theta}$ are associated with heterogeneous responses to the shock exposure across markets (instead of an identical effect on all markets). In fact, the results in Section 3 show that bilateral reduced-form elasticities are increasing in bilateral spatial links, implying that identification relies on heterogeneity in the bilateral variables governing the spatial links associated with $\boldsymbol{\theta}$.

The implementation of the GMM estimator in (32) requires specifying $H_i(\cdot)$. Although any function yields a consistent estimator of $\boldsymbol{\theta}$, functions vary in terms of asymptotic variance – that is, the estimators differ in precision.[26] The following proposition uses the approach in Chamberlain (1987) to characterize the function $H_i^*(\cdot)$ that minimizes the asymptotic variance of $\hat{\boldsymbol{\theta}}_H$.

Proposition 4. *Under Assumptions 5a–5c, the function that minimizes the asymptotic variance of the class of estimators in Definition 1 is*

$$H_i^*\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right) \equiv E\left[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}) \,|\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right] \left(\Omega_i^t\right)^{-1} \tag{33}$$

where $\Omega_i^t \equiv E\left[\nu_i^t(\boldsymbol{\theta})\, \nu_i^t(\boldsymbol{\theta})' \,|\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right]$ *and*

$$E\left[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}) \,|\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right] = -\sum_j \begin{bmatrix} \nabla_{\boldsymbol{\theta}} \epsilon \beta_{ij}^R(\boldsymbol{\theta}) \\ \nabla_{\boldsymbol{\theta}} \epsilon \varphi_{ij}^R(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}_j^{R,t} - \sum_j \begin{bmatrix} \nabla_{\boldsymbol{\theta}} \beta_{ij}^C(\boldsymbol{\theta}) \\ \nabla_{\boldsymbol{\theta}} \varphi_{ij}^C(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}_j^{C,t}. \tag{34}$$

Proof. Appendix 8.10.

The optimal function $H_i^*\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right)$ has two components. The matrix Ω_i^t attributes larger weight to observations with a lower variance of unobserved residuals – under homoskedasticity, $\Omega_i^t = \Omega^t$ is the GMM optimal moment weight matrix. The second component is the predicted response of the endogenous variables associated with $\boldsymbol{\theta}$. Intuitively, through the Jacobian matrix in (34), the Optimal IV of each parameter in $\boldsymbol{\theta}$ puts more weight on the observed shifts of markets whose reduced-form effects on market i are more sensitive to changes in that parameter.

The Optimal IV in Proposition 4 is a function of the unknown vector $\boldsymbol{\theta}$. To simplify its implementation, we characterize an asymptotically equivalent two-step estimator: the Model-implied Optimal

[25] See Theorems 2.6 and 2.7 in Newey and McFadden (1994).

[26] Newey and McFadden (1994) provide regularity conditions for normality of GMM estimators of the form in (32) – see Theorem 3.4. This relies on the central limit theorem and, therefore, requires some sort of independence assumption. We assume that residuals are i.i.d across markets or clusters of markets. Notice that the number of moments is equal to the number of unknown parameters in $\boldsymbol{\theta}$. This does not imply any loss of generality, since it is always possible to define $H_i(\cdot)$ to include the optimal moment weighting matrix.

IV (MOIV). In the first-step, we use an arbitrary $\boldsymbol{\theta}_0$ to compute $H_i^0 = E[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}_0) | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}](\hat{\Omega}_i^t)^{-1}$ and obtain $\hat{\boldsymbol{\theta}}$ from (32). Since the instrument is a function of $(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t})$, the first-step estimator is consistent but not optimal. Thus, in the second-step, we use the consistent estimator $\hat{\boldsymbol{\theta}}$ to compute a consistent estimator of the Optimal IV, $\hat{H}_i^* = E[\nabla_{\boldsymbol{\theta}} \nu_i^t(\hat{\boldsymbol{\theta}}) || \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}](\hat{\Omega}_i^t)^{-1}$. We then use it to obtain $\hat{\boldsymbol{\theta}}^{MOIV}$ from (32).

Proposition 5. *The efficient estimator $H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t})$ in (33)–(34) is asymptotically equivalent to the Model-implied Optimal IV (MOIV) estimator obtained with the following two-step procedure.*

Step 1. *Using a guess $\boldsymbol{\theta}_0$, estimate $\hat{\boldsymbol{\theta}}$ from (32) using $H_i^0 = E[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}_0) | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}](\hat{\Omega}_i^t)^{-1}$.*

Step 2. *Using $\hat{\boldsymbol{\theta}}$, estimate $\hat{\boldsymbol{\theta}}^{MOIV}$ from (32) using $\hat{H}_i^* = E[\nabla_{\boldsymbol{\theta}} \nu_i^t(\hat{\boldsymbol{\theta}}) | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}](\hat{\Omega}_i^t)^{-1}$.*

Proof. Appendix 8.11.

6 Application: Measuring the Effects of The China Shock

In the last part of the paper, we use the results above to study how the rise in Chinese manufacturing exports affected U.S. CZs. We follow Autor et al. (2013) (henceforth ADH) to specify the observed sectoral intensity of the "China Shock." We first show that the methodology of Section 5 yields a generalization of the empirical specification in ADH. We then estimate our theory's reduced-form representation for employment and wage responses to the shock and use it to compute the shock's general equilibrium impact on U.S. CZs.

6.1 A General Equilibrium Extension of ADH

We start by connecting our theory's estimating equation (29) to the main specification in ADH. We focus on the labor market consequences of the sector-level average of the Chinese cost shock, $\hat{\zeta}_s^t \equiv N^{-1} \sum_j \left(\epsilon x_{China,j,s}^0 \hat{\tau}_{China,j,s} \right)$. We project $\hat{\zeta}_s^t$ onto ADH's sector-level shift: the per-worker Chinese exports growth to developed countries (excluding the U.S.), $\Delta M_s^{o,t}$.

Assumption 6a. ***[ADH Shock]*** *Assume that*

$$\hat{\zeta}_s^t = \kappa \Delta M_s^{o,t} + \hat{\varepsilon}_s^t \quad \text{such that} \quad \Delta M_s^{o,t} \perp \hat{\varepsilon}_k^t \ \forall s, k. \tag{35}$$

In equation (35), κ is the pass-through coefficient that connects the increase in per-worker Chinese exports $\Delta M_s^{o,t}$ to changes in Chinese production costs $\hat{\zeta}_s^t$. This parameter is negative because a sector with stronger cost reduction should have stronger export growth. Under Assumption 6a, equation (30) implies that $\hat{\eta}_i^{R,t}$ and $\hat{\eta}_i^{C,t}$ are proportional to shift-share variables based on employment and spending shares across sectors:

$$\hat{\eta}_i^{R,t} = \underbrace{\frac{\kappa}{\epsilon}\sum_s y_{i,s}^0 \Delta M_s^{o,t}}_{\equiv IPW_i^t} + \hat{\varepsilon}_i^{R,t} \quad \text{and} \quad \hat{\eta}_i^{C,t} = \underbrace{\frac{\kappa}{\epsilon}\sum_s \xi_{i,s}^0 \Delta M_s^{o,t}}_{\equiv IPC_i^t} + \hat{\varepsilon}_i^{C,t}, \tag{36}$$

where $\hat{\varepsilon}_i^{R,t}$ and $\hat{\varepsilon}_i^{C,t}$ are regional residuals implied by $\hat{\varepsilon}_s^t$.

The revenue shift is a linear function of the shift-share instrumental variable used in ADH – see equation (3) in ADH. In addition, the consumption cost shift yields a new shift-share measure, IPC_i^t, where the shift $\Delta M_s^{o,t}$ is interacted with the CZ's spending share on sector s. Under Assumption 6a, $\hat{\varepsilon}_i^{R,t}$ and $\hat{\varepsilon}_i^{C,t}$ are orthogonal to IPW_i^t and IPC_i^t. From (29) and (36),

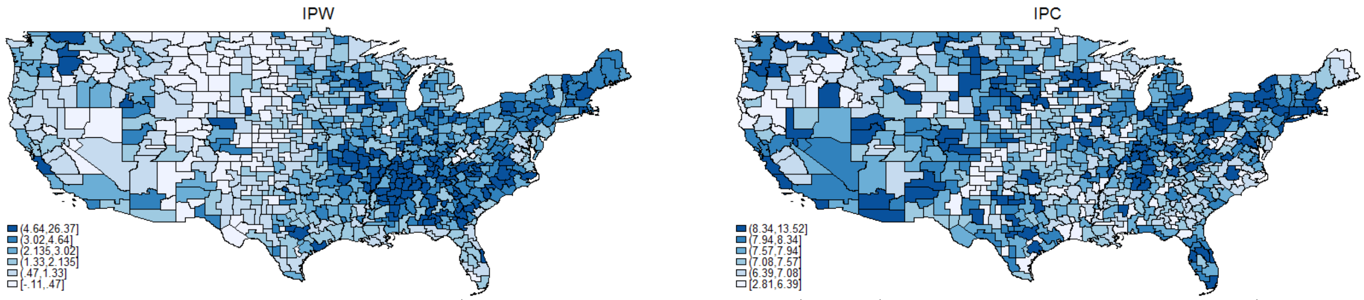
$$\begin{bmatrix} \hat{w}_i^t \\ \hat{L}_i^t \end{bmatrix} = \begin{bmatrix} \beta^t \\ \varphi^t \end{bmatrix} + \sum_j \begin{bmatrix} \beta_{ij}^R(\boldsymbol{\theta}) \\ \varphi_{ij}^R(\boldsymbol{\theta}) \end{bmatrix} \left(\kappa IPW_j^t\right) + \sum_j \begin{bmatrix} \epsilon^{-1}\beta_{ij}^C(\boldsymbol{\theta}) \\ \epsilon^{-1}\varphi_{ij}^C(\boldsymbol{\theta}) \end{bmatrix} \left(\kappa IPC_j^t\right) + \begin{bmatrix} \hat{v}_i^{w,t} \\ \hat{v}_i^{L,t} \end{bmatrix} \tag{37}$$

where $\hat{v}_i^{w,t}$ and $\hat{v}_i^{L,t}$ are structural residuals that contain the direct and indirect effects of all unobserved trade cost shocks, including $\hat{\varepsilon}_i^{R,t}$ and $\hat{\varepsilon}_i^{C,t}$.

Equation (37) is a strict generalization of ADH's main specification. In fact, ADH only allow for a common direct effect of the CZ's own revenue shift and the endogenous fixed-effect of revenue shifts: $\beta_{ij}^R(\boldsymbol{\theta}) = \beta\mathbb{I}_{[i=j]} + \beta_j$, $\varphi_{ij}^R(\boldsymbol{\theta}) = \varphi\mathbb{I}_{[i=j]} + \varphi_j$, and $\beta_{ij}^C(\boldsymbol{\theta}) = \varphi_{ij}^C(\boldsymbol{\theta}) = 0$. As discussed in Section 3.3, such a specification arises under two restrictions. First, consumption cost shifts can only be ignored when labor supply does not respond to prices (i.e., $\phi^p = \phi^m = 0$). Second, common indirect effects only arise when we restrict further the spatial links matrix to have identical off-diagonal elements in all rows – as in the case of equal spending shares ($x_{ij} = y_{ji} = x_i$ for all j). Notice that, even in this special case, the specification in ADH cannot separately identify the common indirect effect from other national shocks included in the time fixed-effect. Hence, whenever this common indirect effect is not zero, it delivers only part of the general equilibrium impact of the China shock on U.S. CZs.

In general, our theory implies that we must estimate how employment and wages in CZ i respond to both the IPW_j^t and IPC_j^t of the own CZ as well as those of other CZs. As discussed in Section 3.2, these reduced-form elasticities vary across markets because of the bilateral spatial links that we parametrize in terms of sector specialization, trade flows, and migration flows. Even in the presence of time fixed-effects, we identify each element of $\boldsymbol{\theta}$ from the response of outcomes in a CZ to the shock exposure of the CZs with higher values of the bilateral variables associated with each parameter. Thus, in the rest of this section, we rely on equation (37) to estimate the reduced-form elasticity matrices, $\{\beta_{ij}^R(\boldsymbol{\theta}), \varphi_{ij}^R(\boldsymbol{\theta}), \epsilon^{-1}\beta_{ij}^C(\boldsymbol{\theta}), \epsilon^{-1}\varphi_{ij}^C(\boldsymbol{\theta})\}_{ij}$, using the MOIV estimator described in Section 5.3. To adjust for the scale of the shock, we separately estimate the pass-through parameter κ using the linear expression in (35).

Figure 1: Exposure to Chinese export growth, 1990-2007



Notes: For each CZ, the left panel reports IPW_i^t and the right panel reports IPC_i^t. IPW_i^t has standard deviation of 2.52, IPC_i^t has standard deviation of 1.22. The spatial correlation between IPW_i^t and IPC_i^t is 0.34.

6.2 Data Construction

We now provide an overview of the steps to construct the data used in estimation. Appendix 9 presents a detailed discussion of the data construction methodology.

We follow ADH by considering the 722 CZs in mainland U.S. over 1990-2000 and 2000-2007. We use the procedure in the Online Appendix of ADH to construct the number of employed individuals and the average weekly log-wage in each CZ using the Census Integrated Public Use Micro Samples in 1990 and 2000 and the American Community Survey in 2006-2008. We also use the U.S. Census data in 2000 to measure m_{ij} as the share of working-age individuals in CZ i that report to be living in CZ j 5 years ago.

We construct the shift-share variables IPW_i^t and IPC_i^t as follows. We compute IPW_i^t by interacting ADH's sector-level shift $\Delta M_s^{o,t}$ and the CZ's ten-year-lagged employment share in that sector. For each CZ, we use the imputation methodology in ADH to obtain employment by 4-digit SIC industry from the County Business Patterns. Thus, our IPW^t is identical to the instrumental variable in ADH. To construct IPC_i^t, we interact ADH's sector-level shift $\Delta M_s^{o,t}$ and the CZ's sectoral spending share. We follow Gervais and Jensen (2019) to construct CZ-level spending shares by 4-digit SIC industries using the national input-output table. Specifically, for each industry, we combine the final consumption share from the national input-output table with the CZ's predicted input purchase share obtained by interacting national industry spending shares and regional industry employment.[27] Figure 1 reports the spatial variation in IPW_i^t and IPC_i^t. The two measures have a good degree of variation across space, and their spatial correlation is 0.34.

To implement our strategy, we also construct sectoral trade matrices between the 722 U.S. CZs and 52 foreign countries in 1990 and 2000. We assume a single labor market in each foreign country. First, we use trade data from UN Comtrade to construct a country-to-country matrix of trade flows

[27] In Appendix 9.3, we evaluate our procedure to construct CZ-level spending shares across 4-digit SIC industries. We run a regression of state-STCG shipment inflows in the Commodity Flow Survey (CFS) on the state-SCTG spending shares implied by the aggregation of our dataset, where SCTG is the commodity classification used in the CFS. We obtain a coefficient close to 1 and a R^2 of 0.95.

in 368 industries. We use the gravity structure of our model and data on domestic sales from Eora MRIO to impute domestic spending shares in each industry. Second, we distribute U.S. domestic and international trade flows across CZs using again the gravity structure of our model. We first split U.S. Census data on imports and exports for each industry-country across CZs using each CZ's share in that industry's national spending and production. We then impute bilateral trade shares across CZs using an industry-level gravity specification estimated with bilateral shipment data from the Commodity Flow Survey (CFS). Since our baseline model imposes trade balance, we adjust market sizes to balance trade flows given the bilateral trade shares.[28]

6.3 Simple Extension of ADH

We begin our analysis with a simpler extension of the specification in ADH. Our goal is to provide qualitative evidence for our main novel channels: how regional outcomes respond (i) directly to the CZ's own consumption shift, and (ii) indirectly to the revenue and consumption shifts of other CZs. To this end, we consider an intuitive approximation of the reduced-form elasticity matrices in equation (37):

$$\hat{Y}_i^t = \alpha^t + \alpha^R IPW_i^t + \alpha^C IPC_i^t + \alpha^{IR} \sum_{j \neq i} z_{ij} IPW_j^t + \alpha^{IC} \sum_{j \neq i} z_{ij} IPC_j^t + X_i^t \lambda + \nu_i^t \tag{38}$$

where, for CZ i in period t, $\hat{Y}_i^t$ is the change in log-employment or average weekly log-wage, α^t is a time fixed-effect, and X_i^t is a set of regional controls.[29] We equally weight all CZs when estimating (38) because our theory's unit of observation is a market.

As in ADH, α^R is the direct impact of higher exposure to Chinese import competition. We also include α^C to measure the direct effect of the CZ's own consumption cost shift, as well as α^{IR} and α^{IC} to measure the indirect effects of the revenue and consumption shifts of other CZs. We use z_{ij} to parametrize cross-regional variation in these indirect effects. Since general equilibrium reduced-form elasticities are stronger between CZs with stronger spatial links, we build on our model's gravity structure to set $z_{ij} \equiv \frac{L_j^0 D_{ij}^{-\delta}}{\sum_k L_k^0 D_{ik}^{-\delta}}$, where D_{ij} is the bilateral distance between i and j, so that CZ i responds more to shocks in nearby and larger CZs. We set the parameter δ controlling the relative importance of distance to five, following typical estimates of the trade elasticity.[30]

[28]Table 11 in Appendix 9.3 reports validation tests using the CFS data for 1997, 2002 and 2007. Regressions of actual on predicted trade flows across states and SCTGs yield coefficients close to 1 and R^2 around 0.5.

[29]We include the largest set of regional controls in ADH – i.e., column (6) of Table 3 in ADH. We also include controls for the CZ's exposure to the secular manufacturing decline in the period: the CZ's spending and employment shares in manufacturing ($y_{i,M}^0$ and $\xi_{i,M}^0$), as well as the weighted average of these shares across other CZs, $\sum_{j \neq i} z_{ij} y_{j,M}^0$ and $\sum_{j \neq i} z_{ij} \xi_{j,M}^0$. Finally, we follow Greenland et al. (2019) by including the CZ's lagged population growth to absorb the effect of persistent confounding shocks.

[30]Donaldson and Hornbeck (2016) rely on a similar specification to compute a proxy for their market access measure in partial equilibrium. Online Appendix A shows that we obtain similar qualitative results for different values for δ and functional forms for z_{ij}.

Table 1: Impact of the China Shock on Labor Market Outcomes across U.S. CZs

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Change in avg. log weekly wage						
IPW_i^t	-0.437***	-0.417**	-0.329**	-0.405**	-0.319**	-0.319**
	(0.156)	(0.170)	(0.138)	(0.155)	(0.152)	(0.152)
IPC_i^t		-0.092			-0.043	-0.043
		(0.214)			(0.208)	(0.197)
$\sum_{j \neq i} z_{ij} IPW_j^t$			-1.039***		-1.036***	-1.032***
			(0.309)		(0.309)	(0.333)
$\sum_{j \neq i} z_{ij} IPC_j^t$				-0.556		-0.010
				(0.452)		(0.473)
R^2	0.529	0.529	0.536	0.530	0.536	0.536
Panel B: Change in log of employment						
IPW_i^t	-0.561**	-0.593**	-0.423**	-0.555**	-0.468**	-0.467**
	(0.216)	(0.238)	(0.206)	(0.212)	(0.223)	(0.225)
IPC_i^t		0.149			0.212	0.154
		(0.416)			(0.419)	(0.423)
$\sum_{j \neq i} z_{ij} IPW_j^t$			-1.315***		-1.330***	-1.598***
			(0.340)		(0.345)	(0.425)
$\sum_{j \neq i} z_{ij} IPC_j^t$				-0.101		0.701
				(0.462)		(0.544)
R^2	0.472	0.472	0.476	0.472	0.476	0.477

Notes: Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Indirect effects computed with $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j, and L_j^0 is the population of CZ j in 1990. All specifications include the following three sets of controls. Regional controls in ADH: period dummies, college-educated population share in 1990, foreign-born population share in 1990, employment share of women in 1990, employment share in routine occupations in 1990, average offshorability in 1990, and Census division dummies. Initial manufacturing exposure: CZ's share of employment and spending in manufacturing ($\sum_s y_{i,s}^{t-20}$ and $\sum_s \xi_{i,s}^{t-20}$), CZ's indirect exposure to manufactruing employment and spending ($\sum_{j \neq i} z_{ij} \sum_s y_{j,s}^{t-20}$ and $\sum_{j \neq i} z_{ij} \sum_s \xi_{j,s}^{t-20}$). Lagged population growth from Greenland et al. (2019): growth of population with 15-34 years old and 35-64 years old in the previous 10-year period. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 1 reports the estimation of (38). Column (1) qualitatively replicates the main findings in ADH. Regions initially specialized in industries subject to stronger Chinese import competition exhibit slower growth in employment and wages. In contrast, column (2) shows that the shock-induced shift in consumption cost does not have a significant impact on both employment and wage growth across CZs. Notice that this is driven by lower point estimates with relatively tight confidence intervals – especially for the wage response in Panel A.[31]

[31] Notice that input-output connections can be incorporated in the reduced-form of our model as an additional shift-share variable, as discussed in Section 3.4. Since $\xi_{i,s}$ includes sectoral intermediate input spending in the CZ, our results indicate no systematic difference in the outcomes of CZs that intensively source inputs from industries with larger increases in Chinese export growth. This result is consistent with those in Pierce and Schott (2016) and

Columns (3) and (4) investigate the indirect effect of the shock exposure of other CZs. Given the CZ's own revenue exposure, column (3) indicates that employment and wage growth are weaker if nearby and larger CZs are more exposed to Chinese import competition. Compared to column (1), the point estimates for the direct effect are one-fourth lower in column (3), suggesting that the CZ's own revenue shift is partially correlated with the revenue shifts of closer and larger CZs. Column (4) shows however that the indirect effect of consumption exposure is not significant for both wages and employment – again, confidence intervals have the same magnitude as those of the indirect effect of revenue exposure. Finally, columns (5) and (6) indicates that results are qualitatively similar when we use the specification with all measures of shock exposure.

Our estimates show that revenue shifts percolate through the spatial link's network, amplifying the negative direct impact of the CZ's own exposure to Chinese import competition. The point estimates in column (5) indicate that an increase of \$1000 dollars in Chinese imports per U.S. worker in all nearby CZs is associated with a weaker growth of 1.3 log-points in employment and 1.0 log-points in wages. These negative indirect effects are three times larger than the differential direct impact of increasing the CZ's own import competition exposure by \$1000.

We can interpret these estimates using the theoretical insights presented in Section 3. First, the negative direct impact of exposure to the China shock on employment and wages suggests that revenue shifts have strong impacts on both employment and wages (i.e., ϕ^w is large). Second, the negative indirect effect of revenue shifts indicates that demand substitution dominates supply substitution in the spatial links matrix, as in Corollary 1. Lastly, the weak impact of consumption cost shifts on employment and wages suggests that labor supply does not respond much to import prices (i.e, ϕ^p is low relative to ϕ^w).

Robustness. Online Appendix A.1.1 investigates the robustness of the results in Table 1. We focus on our preferred specification in column (5). We first analyze the importance of the baseline controls. While the wage responses are not sensitive to the control set, the estimated employment responses are larger and more precise when we control for the lagged population growth, as in Greenland et al. (2019).[32] In addition, results are qualitatively similar when we compute z_{ij} (i) with other distance parameters $\delta \in (1, 8)$, (ii) without CZ size, and (iii) only with CZs in the same state. The indirect effects are negative and statistically significant when we weight CZs by their 1990 population (as in ADH), use the inference procedure in Adão et al. (2019), and use other measures of the CZ's spending shares $\xi_{i,s}$.

Acemoglu et al. (2016) that find no evidence of differential employment growth in industries using more intensively inputs from sectors in which the China shock was stronger.

[32]This is consistent with the results in ADH who find a significant negative effect of Chinese import exposure only on manufacturing employment. In their baseline specification, the response of total employment is negative but non significant at usual levels (see also Bloom et al. (2019)).

Additional results. Online Appendix A.1.2 complements our baseline estimates. We find that the indirect effect is negative and statistically significant for employment responses in both manufacturing and non-manufacturing. In contrast, similar to ADH, wage responses are mainly driven by the non-manufacturing sector. Finally, we document that results are qualitatively similar when the sector-level shifter is the NTR gap used by Pierce and Schott (2016).

6.4 Reduced-Form Elasticities to Revenue and Consumption Shifts

We now turn to the implementation of the empirical strategy described in Section 5. In comparison to the simple linear specification presented above, this strategy imposes that general equilibrium reduced-form elasticities are implied by a parametrization of the spatial links matrix. This has two main advantages. It guarantees that the reduced-form elasticities capture both bilateral and third-market connections among CZs that arise in general equilibrium (as parametrized by the observable variables in our specification). In addition, it implies that, through the lens of our spatial model, neither time fixed-effects nor residuals include any component of the endogenous responses of regional outcomes to the observed sectoral average of the China shock. For these reasons, the predicted reduced-form responses implied by our elasticity estimates can be used to properly aggregate regional shock exposures when computing the general equilibrium impact of the China shock on U.S. CZs.

We present our results in three steps. We first estimate the pass-through coefficient κ in equation (35). We then estimate equation (37) and present the implied reduced-form elasticities. Lastly, we evaluate the fit of our model by comparing its predicted responses to the China shock to actual changes in employment and wages across CZs.

6.4.1 Pass-through of Chinese Export Growth to Chinese Cost Shock

We start by estimating the pass-through coefficient κ in equation (35). Since the reduced-form elasticities in (37) are a non-linear function of $\boldsymbol{\theta}$, κ is necessary to adjust the scale of the sectoral import changes in ADH to be consistent with the sectoral cost shocks in our theory. Thus, the estimation of (35) can be seen as the first-stage in the estimation of $\boldsymbol{\theta}$.

We measure $\hat{\zeta}_s^t$ in two steps. We first estimate destination-sector-period fixed-effects, $\rho_{j,s}^t$, in a gravity equation of changes in bilateral trade shares across countries. Using the gravity trade demand in (6), we approximate $\hat{\zeta}_s^t$ using $\hat{\zeta}_s^t \approx N^{-1} \sum_j \left(x_{China,j,s}^{t_0} \rho_{j,s}^t - \Delta x_{China,j,s}^t \right)$.[33] We consider the same periods and countries used in the construction of $\Delta M_s^{o,t}$.

[33] The gravity trade demand in (6) implies that $\Delta x_{China,j,s}^t \approx -\epsilon_s x_{China,j,s}^{t_0} (\hat{w}_{China}^t + \hat{\tau}_{China,j,s}^t - \hat{P}_{j,s}^t)$ if $\Psi_{China}\left(\boldsymbol{L}^t\right) \equiv \Psi^t$. By setting the Chinese wage to be the numeraire ($w_{China}^t \equiv 1$), the definition of $\hat{\zeta}_s^t$ implies that $\hat{\zeta}_s^t = N^{-1} \sum_j \left(x_{China,j,s}^{t_0} \epsilon_s \hat{P}_{j,s}^t - \Delta x_{China,j,s}^t \right)$. We obtain the expression above by noting that $\rho_{j,s}^t \equiv \epsilon_s \hat{P}_{j,s}^t$ is the destination-sector-period fixed-effect in the sector-level gravity equation for changes in bilateral trade shares.

Table 2 presents the estimation of (35). It indicates that industries with stronger growth of per-worker Chinese exports experienced stronger decline in Chinese production costs (as measured by $\hat{\zeta}_s^t$). Specifically, a sector with $1000 higher per-worker Chinese export growth had a decline in Chinese production cost of 0.38 percentage points in 1990-2000 and 0.10 percentage points in 2000-2007. We then set κ to the average estimate over the two periods: $\kappa = -0.0024$.

Table 2: Estimation of the Pass-through Parameter κ

Dependent variable:	Chinese production cost shock, $\hat{\zeta}_s^t$	
	(1)	(2)
$\Delta M_s^{o,t}$	-0.0038***	-0.0010***
	(0.0012)	(0.0003)
Period:	$1990-2000$	$2000-2007$

Notes: Sample of 368 4-digit SIC manufacturing industries. $\hat{\zeta}_s$ is the Chinese cost shock described in the main text, and $\Delta M_s^{o,t}$ is the growth in Chinese exports to non-U.S. developed countries normalized by the initial U.S. sector-level employment (as in ADH). All specifications also include a constant. Standard errors in parenthesis clustered by 3-digit industry. *** $p < 0.01$

6.4.2 Estimation of the Reduced-form Elasticity Matrices

We now turn to the estimation of the reduced-form elasticities in estimation of (37). Table 3 reports our baseline estimates of $\boldsymbol{\theta}$ obtained from the estimation of (37) with the two-step procedure in Proposition 5. We consider the same set of baseline controls in Table 1.

In Panel A, we present the estimation results without migration links in labor supply (i.e., $\phi^m = 0$). We estimate an elasticity of labor productivity to local employment of 0.56. This is a direct consequence of the large employment and wage responses to revenue shifts that we document in Table 1. In order to rationalize such large reduced-form responses, the model requires strong agglomeration forces that are higher than typical calibrations in the quantitative spatial literature (we return to this point below). It is roughly twice the agglomeration elasticity implied by firm entry in Krugman (1980) (as specified in Monte et al. (2018)) and much higher than the elasticity of zero in Ricardian models such as Eaton and Kortum (2002) (as specified in Galle et al. (2017), Caliendo et al. (2018) and Caliendo et al. (2019)). Instead, our estimate is closer to the elasticity of manufacturing productivity to population density estimated from regional demand shocks in the U.S. – for instance, Kline and Moretti (2014) estimate this elasticity to be around 0.4.

Second, we estimate a large elasticity of labor supply to wages, but a lower labor supply elasticity to consumption prices. In fact, our estimate is closer to estimates based on the aggregate employment responses to the business cycle, being three times higher than the median micro-estimate reviewed by Chetty et al. (2013). This high value is necessary to rationalize the strong responses of employment to revenue shifts, both directly and indirectly. In contrast, we estimate

Table 3: Estimates of the Structural Parameters

ψ	ϕ^w	ϕ^p	ϵ	ϕ^m
Panel A:				
0.56	2.11	-1.36	3.94	-
(0.07)	(0.25)	(0.24)	(0.41)	-
Panel B:				
0.55	2.11	-1.36	3.94	-0.06
(0.22)	(1.26)	(0.72)	(1.03)	(0.05)

Notes: Estimation of $\boldsymbol{\theta}$ using the reduced-form expressions in (37) for the pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Estimation uses the two-step procedure in Proposition 5. All specifications include the set of baseline controls in Table 1. Standard errors in parentheses are clustered by state.

a lower labor supply elasticity to prices, $\hat{\phi}^p = -1.36$. This parameter controls the response of employment and wages to consumption cost shifts across CZs. Accordingly, the low $\hat{\phi}^p$ relative to $\hat{\phi}^w$ follows from the weak responses of wages and employment to IPC_j^t in Table 1. Notice that, in the full structural estimation, we obtain a small confidence interval for ϕ^p because estimation uses all the channels through which this parameter affects reduced-form elasticities in the model.[34]

Third, we find that gravity trade links across regions are important, implying a trade elasticity of roughly four. This lies within the range of estimates in the literature – e.g., see Simonovska and Waugh (2014) and Costinot and Rodríguez-Clare (2014). This parameter captures the negative indirect effects of revenue shifts on both employment and wages that we document in Table 1.

Panel B presents results when we also estimate the parameter controlling migration links in labor supply across markets. Our estimate of ϕ^m is not statistically different from zero at usual significance levels. This implies that employment in a CZ, conditional on its own wage change, does not respond much to revenue shifts in CZs with stronger migration links. This is consistent with the evidence in ADH of weak responses in the CZ's working-age population to its own revenue shift.[35] Notice that the point estimates of all other parameters are almost the same in Panels A and B. However, relative to Panel A, standard errors are two to five times higher in Panel B. This

[34] In Online Appendix B.1.2, we show that two rudimentary formulations of the intensive and extensive margins of labor supply imply that employment is more sensitive to wages than to prices (i.e., $\phi^w > \phi^p$). In a setting with a representative household deciding the number of hours worked, this arises from the fact that a higher wage affects labor supply only through changes in the opportunity cost of leisure (i.e., the real wage), whereas a lower price index has ambiguous effects on labor supply because it increases both the real wage and the real value of lump-sum transfers. In a setting with heterogeneous individuals in terms of disutility to work, the labor supply function emerges from the comparison between the nominal wage and the home sector's payoff in each region and, therefore, it is not a direct function of the region's price index.

[35] Greenland et al. (2019) find that, in response to the China shock, population responses are weak for all working-age individuals, but are much stronger among young individuals aged below 30 years old. Our results are also consistent with the evidence in Cadena and Kovak (2016) who find weak migration responses of U.S. native workers to regional labor demand shocks, especially for non-college graduates.

follows from the high correlation between bilateral migration and trade shares across CZs, which makes it hard to separately estimate the different parameters in the model.[36] Given these results, our preferred specification uses the estimates in Panel A. In Appendix A.2.1, we investigate the sensitivity of reduced-form responses to different values of ϕ^m.

We now turn to the estimates of the reduced-form elasticities implied by the parameters in Table 3. Table 4 reports percentiles of the empirical elasticity distribution for 2000. The top panel reports the direct effects and the bottom panel reports the indirect effects.

The direct and indirect effects of revenue shifts are positive. Thus, a negative demand shock in a CZ triggers reductions in wages and employment in that CZ as well as in other CZs. Since the labor supply elasticity to wages is around two, revenue shifts affect more employment than wages. In addition, indirect effects are typically smaller than direct effects: while the median direct wage elasticity is 0.67, the median indirect wage elasticity is 0.002 (for employment, the median elasticities are 1.46 and 0.003, respectively). This reflects the fact that there are 721 CZs indirectly affecting each CZ.

Moreover, our estimates indicate that both wages and employment respond less to consumption cost shifts than to revenue shifts. The median direct elasticity to $\hat{\eta}_i^C$ is -0.35 for wages and -1.21 for employment. For indirect effects, the difference is even starker: the median elasticity to consumption shifts is close to 0 for both wages and employment.

Importantly, the difference between the 90^{th} and 10^{th} percentiles of the estimated elasticities suggests a large dispersion in direct and indirect effects across U.S. CZs. As discussed in Section 3, this arises from the observed heterogeneity in the variables controlling bilateral spatial linkages. Online Appendix A.2.2 shows that the indirect effects are increasing in the intuitive measure of gravity links z_{ij} used in Table 1, but z_{ij} explains only a small fraction of the variation in indirect effects. Instead, the elements of the spatial links matrix, y_{ij} and χ_{ij}, explain roughly 50% of the variation in indirect effects across pairs of CZs.

Robustness. Online Appendix A.2.1 investigates the robustness of the estimated reduced-form elasticities to the assumptions in the baseline parametrization of spatial links in Section 5.2. In particular, we re-estimate the specification in Panel A of Table 3 when we (i) allow for trade imbalances, (ii) use a calibration of migration links from the literature, and (iii) impose labor supply homogeneity in terms of the national price index (rather than the world's average wage). For all alternative specifications, despite the estimated parameters being different, the estimated reduced-form elasticities are highly correlated with our baseline estimates.

[36] It may be possible to improve on the estimation of spatial links in labor supply by extending our empirical strategy to include one additional estimating equation for reduced-form responses in bilateral migration flows – as in the extension presented in Online Appendix C.2.

Table 4: Percentiles of the Empirical Distribution of Reduced-form Elasticities

	Revenue			Consumption Cost		
	10^{th}	50^{th}	90^{th}	10^{th}	50^{th}	90^{th}
Panel A: Direct elasticities						
Wages	0.436	0.665	1.666	-0.899	-0.349	-0.190
Employment	0.924	1.461	3.965	-2.776	-1.206	-0.757
Panel B: Indirect elasticities						
Wages	0.000	0.002	0.021	-0.002	0.000	0.001
Employment	0.000	0.003	0.039	-0.008	-0.001	0.000

Notes: Percentiles of the 2000 empirical distribution of reduced-form elasticities implied by the estimates in Panel A of Table 3.

6.4.3 Model Fit

Finally, we investigate how observed changes in employment and wages across CZs relate to our baseline predicted responses to the China shock. As a benchmark, we compare this relationship to that obtained with alternative specifications of spatial links motivated by the existing literature. Specifically, we consider the following linear regression:

$$\hat{Y}_i^t = \alpha^t + \rho \hat{Y}_i^t \left(\boldsymbol{IPW}, \boldsymbol{IPC} | \boldsymbol{\theta} \right) + X_i^t \lambda + \nu_i^t \tag{39}$$

where, in CZ i in period t, $\hat{Y}_i^t \left(\boldsymbol{IPW}, \boldsymbol{IPC} | \boldsymbol{\theta} \right)$ is the response to the China shock implied by our model with parameter vector $\boldsymbol{\theta}$, and X_i^t is the same control set used in Table 1.

The coefficient ρ summarizes the relationship between actual and predicted changes in labor market outcomes across CZs. A coefficient of one means that predicted reduced-form responses have the correct magnitude to match cross-regional variation in wage and employment growth. In contrast, an estimated coefficient much larger than one implies that reduced-form responses in the model need to be multiplied by a large re-scaling coefficient ρ to match the observed variation in wage and employment growth between CZs. In this case, the predicted responses in the model are too small compared to the differential effect of higher shock exposure observed in the data. Finally, a non-significant coefficient indicates that the model's predicted responses are not correlated with the observed changes in labor market outcomes.

Table 5 reports the estimates of (39) under alternative parameterizations of our model. The coefficients close to one in column (1) indicate that our baseline estimates yield reduced-form responses to the China shock that are aligned with observed changes in wages and employment for U.S. CZs. This is a consequence of the fact that we estimate $\boldsymbol{\theta}$ precisely from the effect of shock exposure on employment and wages, as specified in equation (37). Column (2) reports similar fit

Table 5: Predicted Impact of China Shock and Actual Labor Market Outcomes across U.S. CZs

	Structural Estimates		Alternative Calibrations			
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Change in avg. weekly log wage						
Predicted response	0.67**	0.66**	3.56**	3.97**	3.70**	3.72**
	(0.27)	(0.26)	(1.50)	(1.76)	(1.57)	(1.57)
Panel B: Change in log of employment						
Predicted response	0.90***	0.84***	6.60***	8.95***	10.42	9.60
	(0.14)	(0.16)	(1.74)	(2.45)	(6.55)	(6.29)
Parameters:						
ψ	0.56	0.55	0.20	0.00	0.00	0.00
ϕ^w	2.11	2.11	2.11	2.11	0.70	0.70
ϕ^p	-1.36	-1.36	-1.36	-1.36	-0.70	-0.70
ϵ	3.94	3.94	3.94	3.94	3.94	3.94
ϕ^m	0.00	-0.06	0.00	0.00	0.00	0.25

Notes: Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. All specifications include the set of baseline controls in Table 1. Robust standard errors in parentheses are clustered by state. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

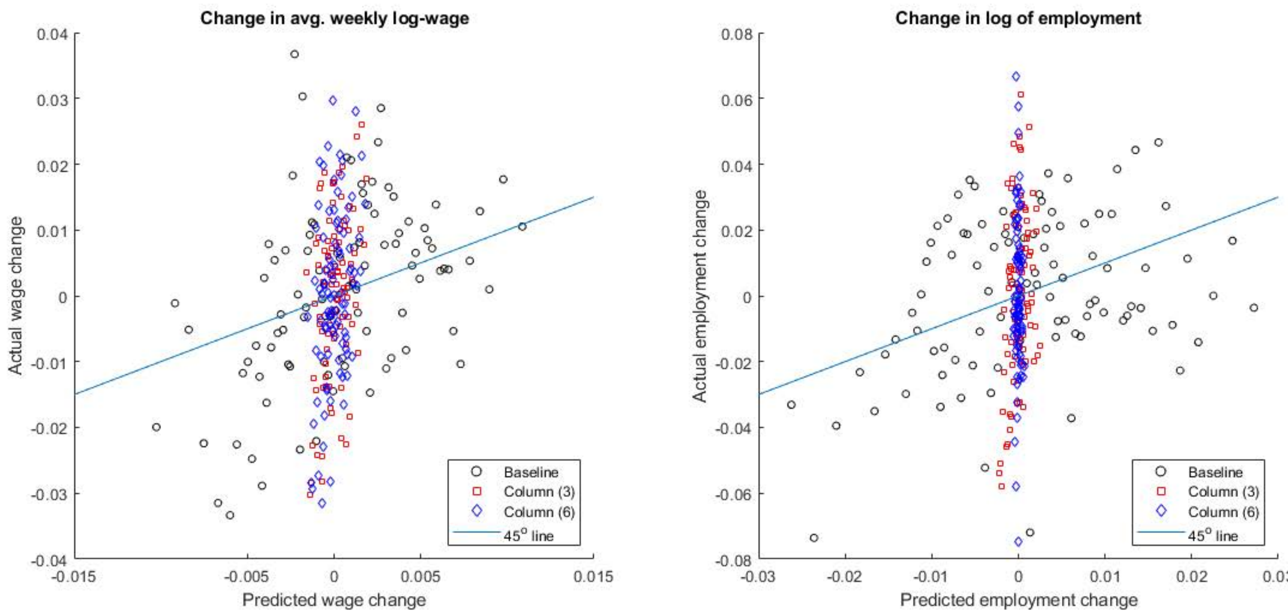
coefficients when we consider the alternative specification with migration links.[37]

Columns (3) and (4) show how the fit coefficients change when we calibrate agglomeration forces to be weaker. In column (3), we set $\psi = 0.2$ as in the increasing returns framework of Krugman (1980) and, more recently, Monte et al. (2018) and, in column (4), we set $\psi = 0$ as in the Ricardian framework of Eaton and Kortum (2002) and, more recently, in Galle et al. (2017), Caliendo et al. (2018) and Caliendo et al. (2019). The fit coefficient is substantially higher when ψ is lower. As discussed in Section 3, higher values of ψ amplify the reduced-form responses in the model. Thus, as we reduce ψ, a higher ρ is necessary to match the large differential responses to shock exposure that we observed across U.S. CZs.

Columns (5) and (6) investigate how the specification of labor supply links affects the model fit. In column (5), we set $\phi^w = -\phi^p = 0.7$, so that the CZ's labor supply responds to changes in the local real wage with an elasticity given by the median estimate reported by Chetty et al. (2013). This parametrization is consistent with that in Caliendo et al. (2019) who specify the non-employment benefit in terms of the local price index – see equation (60) in their Appendix C5. In this case, the fit coefficient for employment growth is very imprecise, indicating that this specification greatly reduces the model's ability to match cross-regional variation in employment responses to shock exposure. As discussed in Section 3, by setting $\phi^w = -\phi^p$, we increase the relative magnitude of the positive employment response to cheaper Chinese imports (relative to

[37] As a robustness check, Table A.10 in Online Appendix A.2.1 shows that our estimates of the fit coefficients are similar when we consider the alternative specifications of spatial links described in Section 6.4.2.

Figure 2: Predicted Impact of China Shock and Actual Labor Market Outcomes across U.S. CZs



Notes: Bin scatter plot of predicted and actual changes in avg. weekly log wages (left panel) and log of employment (right panel) after partialling out the baseline controls in Table 1. Pooled sample of 1,444 Commuting Zones in 1990-2000 and 2000-2007. Plots report average predicted and actual changes in percentile bins based on predicted changes. Baseline predicted changes are computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. Alternative calibration computed with the reduced-form responses in equation (37) using parameters in the corresponding columns of Table 5.

the negative response to the decline in revenue). However, as Table 1 shows, the impact of higher consumption exposure on regional outcomes is not statistically significant at usual level. Lastly, column (6) sets the elasticity of migration links in labor supply to 0.25 – an elasticity similar to the (annual) estimate of the migration elasticity in Caliendo et al. (2019). We can see that allowing for a larger migration elasticity does not affect much the fit of the model.

Figure 2 graphically illustrates the results reported in Table 5 using bin scatter plots of actual changes in labor market outcomes and predicted responses to the China shock across U.S. CZs. In line with the estimates in column (1) of Table 5, the black circles show that our baseline predicted responses are around the 45^o degree line, indicating that they are not only correlated with actual changes in employment and wages across CZs, but also their magnitude is consistent with the differential regional responses in labor market outcomes. We also report the relationship for the alternative specifications in columns (3) and (6) of Table 5. In both cases, the points are concentrated around zero, indicating that the magnitude of these predicted responses is too small compared to differential changes in labor market outcomes across U.S. CZs.[38]

[38] Online Appendix A.2.2 investigates whether different specifications of spatial links can match the findings in ADH: namely, the magnitude of the differential employment response to higher regional exposure to Chinese import competition. Our baseline estimates yield differential responses that are similar to those in column (1) of Table 1. In contrast, the alternative specifications in columns (3) and (6) of Table 5 yield much smaller differential responses that are less than one-fourth of those reported in column (1) of Table 1.

6.5 The Impact of the China Shock in General Equilibrium

We conclude our analysis by quantifying the impact of the China shock on U.S. CZs. Based on the theoretical results in Sections 3 and 4, we directly aggregate our estimates of equation (37) to obtain responses of employment and wages in general equilibrium. We then use expression (23) to compute the real wage response in each CZ. Table 6 reports the average and standard deviation of shock-induced changes in wages (relative to the Chinese wage), employment and real wages.[39] We consider the predicted responses over the entire period between 1990 and 2007.[40]

The first column of Table 6 shows that, on average, the wage of U.S. CZs fell by 4 log-points relative to the Chinese wage. Most of the wage decline was driven by the indirect effect of negative revenue shifts in other CZs. This is a consequence of our finding that indirect effects reinforce direct effects. The impact of lower consumption costs is positive, but it is not sufficient to offset the impact of the revenue decline. This follows from the fact that consumption elasticities are smaller than revenue elasticities. Interestingly, our results indicate large differential effects across CZs: all component of wage responses exhibit large cross-regional dispersion.

We then turn to the effect of the China shock on employment. Again, revenue shifts lead to average employment losses, but consumption cost shifts create partially offsetting employment gains. The sum of all components yields an average employment loss of 2.8 log-points between 1990 and 2007. The impact of the shock on employment varied greatly across U.S. CZs as can be seen by the large standard deviation of employment responses.

The last two columns report real wage responses across CZs. On average, the China shock generated a small real gain of 0.16 log-points. This gain was mostly driven by the reduction in import prices that is captured by the large positive effect of the CZ's own consumption cost shift. This more than compensates for the loss in terms-of-trade that followed from the wage reduction induced by the revenue decline. Notice that, again, the large standard deviation indicates that these consumption gains varied substantially across CZs. In fact, 39% of U.S. CZs experienced declines in real wages due to the China shock.

Robustness. Online Appendix A.3 investigates the robustness of our counterfactual predictions to the alternative specifications of spatial links discussed in Section 6.3. All alternative specifications yield predicted responses that have a high correlation with the baseline responses. Similar to the baseline, the average wage decline is close to 4 log-points in all cases. However, the average employment decline may be stronger or weaker depending on the specification. When we allow for trade imbalances and migration links, the average employment losses are respectively -4.1 and -5.4

[39] Recall that, by Walras' law, only *relative* wages are determined in general equilibrium (see discussion in Sections 2 and 3). Without loss of generality, we specify the Chinese wage as the numeraire. This implies that our predicted wage changes must be interpreted as relative to the Chinese wage.

[40] Online Appendix A.3 reports the same statistics for 1990-2000 and 2000-2007. We find that most of the impact of the China shock happened in the second period, after China's accession to WTO in 2001.

Table 6: Effect of the China Shock on U.S. CZs, 1990-2007

	Response in the log of					
	Wage		Employment		Real wage	
	Avg.	St. Dev.	Avg.	St. Dev.	Avg.	St. Dev.
Total effect	-3.98	1.30	-2.78	3.31	0.16	1.75
Direct effect of η^R	-0.81	1.79	-1.94	4.75	-0.98	2.53
Direct effect of η^C	0.98	1.36	3.18	3.92	3.14	2.23
Indirect effect of η^R	-4.24	1.71	-4.95	4.59	-2.88	2.44
Indirect effect of η^C	0.09	1.18	0.93	3.38	0.88	1.84

Notes: Predicted changes in employment and wages computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. Prediced real wage change computed with expression (23).

log-points. When we specify the labor supply normalization in terms of the U.S. price index, the employment decline is only 0.5 log-points because this specification entails stronger employment gains to lower import prices.

6.5.1 Comparison to other specifications of spatial links

In Table 7, we compare our baseline estimates of the effect of the China shock on U.S. CZs to those obtained from two types of alternative specifications of spatial links in the literature. Panel A presents changes in employment and wages implied by the simple specifications of spatial links embedded in the linear regressions presented in Table 1. That is, we aggregate the predicted responses implied by the linear specification in ADH and its extension with intuitive parametrizations of indirect effects in equation (38).[41] Panel B reports changes in employment and wages that we compute using the alternative calibrations of spatial links used in Table 5.

The first row of Panel A shows that the simple aggregation of ADH's specification (column (1) of Table 1) implies average reductions of 1.2 log-points in wages and 1.5 log-points in employment. These average changes are less than half of those implied by our baseline specification. The second row of Panel A shows that, when we extend ADH's specification by including intuitive measures of indirect effects (column (3) of Table 1), the average predicted response is much closer to our baseline average response. This is a consequence of the fact that both our baseline estimates and the extension of ADH's specification yield indirect effects that reinforce the negative impact of the local exposure to import competition. Notice however that the correlations with our baseline predicted responses are below 0.5, indicating that these simple extensions still miss an important fraction of the heterogeneity in predicted responses of wages and employment across CZs.

[41] We only consider specifications that ignore consumption cost shifts since the effect of these variables is not statistically significant (see Table 1). Thus, researchers using the reduced-form results in Table 1 would conclude that consumption cost shifts are not relevant drivers of labor market outcomes across CZs.

In Panel B, we compare our baseline results to those implied by two alternative calibrations of spatial links used in Table 5. In the first row, we eliminate agglomeration forces, but keep other parameters unchanged. In this case, predicted responses remain highly correlated with our baseline responses. However, they become substantially smaller: the standard deviation of predicted responses falls by more than 80%. In fact, the average employment loss is close to zero. This results from the removal of the amplification channel of agglomeration forces (see Section 3). This is also consistent with the model fit coefficients in column (4) of Table 5, which suggest that employment responses were too small, such that they had to be multiplied by a factor of nine to match the cross-regional variation in the data.

In the last row of Panel B, we further change the labor supply specification imposing that $-\phi^p = \phi^w = 0.7$, as in column (5) of Table 5. In this case, the positive impact of lower import prices on employment dominates in the aggregate, implying a small increase in average employment. Across CZs, the correlation with our baseline predicted responses falls substantially – it was 0.65 in the first row, but it is only 0.35 in the second row. This is because this parametrization induces responses to import prices that are too strong compared to those in the data.

Table 7: Effect of the China Shock on U.S. CZs, 1990-2007 – Alternative Approaches

	Response in the log of					
	Wage			Employment		
	Avg.	St. Dev.	Corr.	Avg.	St. Dev.	Corr.
Baseline	-3.98	1.30	1	-2.78	3.31	1
Panel A: Aggregation of ADH extensions						
Table 1 column (1)	-1.17	1.10	0.47	-1.50	1.42	0.42
Table 1 column (3)	-3.55	2.12	0.47	-4.51	2.70	0.42
Panel B: Alternative calibrations						
Table 5 column (4)	-3.17	0.21	0.78	-0.40	0.24	0.65
Table 5 column (5)	-3.28	0.23	0.77	0.23	0.08	0.35

Notes: Baseline predicted changes are computed with the reduced-form responses in equation (37) using estimates in Panel A of Table 3. In panel A, predicted changes are computed with the linear specification in (38) using estimates in the corresponding columns of Table 1. In panel B, predicted changes are computed with the reduced-form responses in equation (37) using parameters in the corresponding columns of Table 5. For each row, column "Corr." indicates the correlation between predicted responses implied by the specification in the row and our baseline predicted changes.

7 Conclusions

For a general class of spatial models, we show that changes in labor market outcomes, as a result of foreign shocks, can be written as the product of two components. First, the partial equilibrium impact of trade shocks on markets, which takes the form of shift-share variables. Second, the general equilibrium reduced-form elasticities that summarize how local outcomes respond directly to the local shock exposure and indirectly to the shock exposure of other markets through spatial links. We use these results to propose a novel estimation strategy that extends existing empirical shift-share specifications to measure the general equilibrium impact of trade shocks on local labor markets. Our empirical strategy recovers the general equilibrium reduced-form elasticities using the response of labor market outcomes to the exposure of different regions to observed trade shocks, such as the China shock in ADH. These elasticities can then be used to aggregate the shock exposure of different markets to compute the general equilibrium impact of trade shocks on employment, wages, and real wages.

While our methodology differs from existing quantitative approaches, our empirical findings are ultimately ones that quantitative analyses need to reckon with. For that purpose, we devise a new model validation procedure that makes additional use of cross-regional variation in shock exposure. The procedure uncovers the ability of different variants of the model to come to grips with empirical estimates of the differential response of labor market outcomes in regions with higher exposure to observed trade shocks. We feel, to some extent, that this procedure achieves the standards set by Kehoe (2005): *"Ex-post performance evaluations of applied GE models are essential if policymakers are to have confidence in the results produced by these models. Such evaluations also help make applied GE analysis a scientific discipline in which there are well-defined puzzles with clear successes and failures for competing theories"*. We hope, therefore, that our approach will lead to a better understanding of the role of spatial links in shaping the impact of trade shocks on regional labor markets.

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8 Proofs

8.1 Proof of Proposition 1

We now establish the existence and uniqueness of the equilibrium wage vector of our economy. Consider the solution $\boldsymbol{w}^*$ of the system $D_i(\boldsymbol{w}^*|\boldsymbol{\tau}) = 0$ for all i, and the following two lemmas.

Lemma 1. *[Arrow and Hahn (1971) T.1.3 (p. 33)] Suppose that $F_i(\cdot)$ is a function defined for every $\boldsymbol{w} \in \mathbb{R}^N_{++}$ such that $F_i(\cdot)$ is (i) differentiable, (ii) homogeneous of degree 0, (iii) satisfies Walras' law, $\sum_{i=1}^N w_i F_i(\boldsymbol{w}) = 0$ for all $\boldsymbol{w}$, (iv) there exists a scalar s such that $F_i(\boldsymbol{w}) < s$ for every $\boldsymbol{w}$, and (v) if $\boldsymbol{w}^n \to \boldsymbol{w}$ with $w_m = 0$, then $\sum_i F_i(\boldsymbol{w}^n) \to -\infty$. Then, there exists $\boldsymbol{w}^* \in \mathbb{R}^N_{++}$ such that $F_i(\boldsymbol{w}^*) = 0$ for all i.*

Lemma 2. *[Arrow and Hahn (1971) T.9.12 (p. 234)] Suppose that $F_i(\cdot)$ satisfies the conditions in Lemma 1. Assume that, for any $\boldsymbol{w}^* \in \mathbb{R}^N_+$ with $F_i(\boldsymbol{w}^*) = 0$ for all i, $f_{ij}(\boldsymbol{w}^*) \equiv \frac{\partial F_i(\boldsymbol{w}^*)}{\partial w_j}$ satisfies (i) $f_{ii}(\boldsymbol{w}^*) > 0$ and (ii) $\exists\{h_i(\boldsymbol{w}^*)\}_{i,m} \gg 0$ such that $h_i(\boldsymbol{w}^*) f_{ii}(\boldsymbol{w}^*) > \sum_{j \neq i,m} |f_{ij}(\boldsymbol{w}^*)| h_j(\boldsymbol{w}^*)$ for all $i \neq m$. Then, there is a unique $\boldsymbol{w}^* \in \mathbb{R}^N_{++}$ such that $F_i(\boldsymbol{w}^*) = 0$ for all i.*

We consider the function $F_i(\boldsymbol{w}) \equiv -\frac{1}{w_i} D_i(\boldsymbol{w}|\boldsymbol{\tau})$ with $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ defined by (9) and $\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})$ implicitly defined as the solution of (8) with p_o given by (5). In this proof, we simplify notation by denoting $\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})$ as $\boldsymbol{P}(\boldsymbol{w})$. Notice that the definition of $\boldsymbol{P}(\boldsymbol{w})$ implies that $\boldsymbol{P}(\kappa\boldsymbol{w}) = \kappa\boldsymbol{P}(\boldsymbol{w})$. We now establish the existence and uniqueness of the equilibrium wage vector by verifying that Assumption 1 implies all conditions in Lemmas 1 and 2.

1. The function $F_i(\boldsymbol{w})$ is differentiable because it only combines differentiable functions.

2. We now verify that the system is homogeneous of degree 0. Since $F_i(\kappa\boldsymbol{w}) = -\frac{1}{\kappa w_i} D_i(\kappa\boldsymbol{w}|\boldsymbol{\tau})$,

$$F_i(\kappa\boldsymbol{w}) = \Phi_i(\kappa\boldsymbol{w}, \boldsymbol{P}(\kappa\boldsymbol{w})) - \sum_j \sum_{s \in \mathcal{S}_i} \frac{\left(\tau_{ij,s} \frac{\kappa w_i}{\Psi_i(\boldsymbol{\Phi}(\kappa\boldsymbol{w}, \boldsymbol{P}(\kappa\boldsymbol{w})))}\right)^{-\epsilon_s} \xi_{js}}{\sum_o \left(\tau_{oj,s} \frac{\kappa w_o}{\Psi_o(\boldsymbol{\Phi}(\kappa\boldsymbol{w}, \boldsymbol{P}(\kappa\boldsymbol{w})))}\right)^{-\epsilon_s}} \frac{\kappa w_j \Phi_j(\kappa\boldsymbol{w}, \boldsymbol{P}(\kappa\boldsymbol{w}))}{\kappa w_i}.$$

As discussed above, $\boldsymbol{P}(\kappa\boldsymbol{w}|\boldsymbol{\tau}) = \kappa\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})$. Thus,

$$F_i(\kappa\boldsymbol{w}) = \Phi_i(\kappa\boldsymbol{w}, \kappa\boldsymbol{P}(\boldsymbol{w})) - \sum_j \sum_{s \in \mathcal{S}_i} \frac{\left(\tau_{ij,s} \frac{w_i}{\Psi_i(\boldsymbol{\Phi}(\kappa\boldsymbol{w}, \kappa\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}}{\sum_o \left(\tau_{oj,s} \frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\kappa\boldsymbol{w}, \kappa\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}} \xi_{js} \frac{w_j \Phi_j(\kappa\boldsymbol{w}, \kappa\boldsymbol{P}(\boldsymbol{w}))}{w_i}.$$

This implies that $F_i(\kappa\boldsymbol{w}) = F_i(\boldsymbol{w})$ because $\Phi_i(\kappa\boldsymbol{w}, \kappa\boldsymbol{P}) = \Phi_i(\boldsymbol{w}, \boldsymbol{P})$ for all i.

3. We now verify that Walras' law holds:

$$\begin{aligned}
\textstyle\sum_i w_i F_i(\boldsymbol{w}) &= \textstyle\sum_i \left[w_i \Phi_i(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) - \sum_j x_{ij}\left(\left\{ \frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})))} \right\}_o |\boldsymbol{\tau} \right) w_j \Phi_j(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) \right] \\
&= \textstyle\sum_i w_i \Phi_i(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) - \sum_j \left[\sum_i x_{ij}\left(\left\{ \frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})))} \right\}_o |\boldsymbol{\tau} \right) \right] w_j \Phi_j(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) \\
&= \textstyle\sum_i w_i \Phi_i(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) - \sum_j w_j \Phi_j(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w})) \\
&= 0
\end{aligned}$$

4. By assumption, $\Phi_i(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}))$ is bounded from above, so $F_i(\boldsymbol{w}) \le \Phi_i(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})) < \bar{\Phi}$ for all i.

5. Let $\bar{\boldsymbol{w}}$ be a real vector with $w_m = 0$. Since $\Phi_i(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}))$ is bounded from above by assumption, $\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \Phi_i(\boldsymbol{w}^n,\boldsymbol{P}(\boldsymbol{w}^n)) \in [0,\bar{\Phi}]$ for all i. Notice also that the assumption of $\lim_{w_m \to 0} \frac{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}{w_m} = \infty$ implies that

$$\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \frac{\left(\tau_{mj,s}\frac{w_m}{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}}{\sum_o \left(\tau_{oj,s}\frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}} = \lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \frac{\left(\frac{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}{\tau_{mj,s}w_m}\right)^{\epsilon_s}}{\sum_o \left(\frac{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}{\tau_{oj,s}w_o}\right)^{\epsilon_s}} = 1.$$

So,

$$\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \sum_j \sum_{s\in\mathcal{S}_i} \frac{\left(\tau_{mj,s}\frac{w_m}{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}}{\sum_o \left(\tau_{oj,s}\frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}} \xi_{js} w_j \Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})) = \lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \sum_j w_j \Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})) > 0$$

and, therefore,

$$\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \frac{1}{w_m} \sum_j \sum_{s\in\mathcal{S}_i} \frac{\left(\tau_{mj,s}\frac{w_m}{\Psi_m(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}}{\sum_{o:s\in\mathcal{S}_o} \left(\tau_{oj,s}\frac{w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w})))}\right)^{-\epsilon_s}} \xi_{js} \left(w_j \Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}))\right) = \infty.$$

Because $F_i(\boldsymbol{w})$ is bounded from above for all i (see condition 4 above), this implies that $\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} F_m(\boldsymbol{w}) = -\infty$ and, therefore, $\lim_{\boldsymbol{w}^n \to \bar{\boldsymbol{w}}} \sum_i F_i(\boldsymbol{w}) = -\infty$.

6. For any equilibrium wage vector $\boldsymbol{w}^*$,

$$f_{ij}(\boldsymbol{w}^*) \equiv \frac{\partial F_i(\boldsymbol{w}^*)}{\partial w_j} = \mathbb{I}_{[i=j]} \frac{1}{(w_i^*)^2} D_i(\boldsymbol{w}^*|\boldsymbol{\tau}) - \frac{1}{w_i^*}\frac{\partial D_i(\boldsymbol{w}^*|\boldsymbol{\tau})}{\partial w_j} = \frac{Y_i^*}{w_i^* w_j^*}\gamma_{ij}$$

where the last equality follows from $D_i(\boldsymbol{w}^*|\boldsymbol{\tau}) = 0$ and the definition $\bar{\boldsymbol{\gamma}} \equiv -[Y_i^{-1}\nabla_{\ln \boldsymbol{w}} D_i(\boldsymbol{w}|\boldsymbol{\tau})]$ in Assumption 1. Since Assumption 1 imposes that $\gamma_{ii} > 0$, we have that $f_{ii}(\boldsymbol{w}^*) > 0$. In addition, let $\{h_i\}_{i\neq m}$ be the vector such that $h_i\gamma_{ii} > \sum_{j\neq i,m} |\gamma_{ij}| h_j$ for all $i \neq m$ (by Assumption 1). If $\tilde{h}_i(\boldsymbol{w}^*) \equiv h_i w_i^*$, then

$$\tilde{h}_i(\boldsymbol{w}^*) f_{ii}(\boldsymbol{w}^*) = \frac{Y_i}{w_i^*} h_i \gamma_{ii} > \frac{Y_i}{w_i^*} \sum_{j\neq i,m} |\gamma_{ij}| h_j = \sum_{j\neq i,m} |f_{ij}(\boldsymbol{w}^*)| \tilde{h}_j(\boldsymbol{w}^*).$$

8.2 Proof of Proposition 2

Consumption price shift. From the price index definition in (8), $\frac{\partial \ln P_i(\boldsymbol{p}|\boldsymbol{\tau})}{\partial \ln \tau_{oj,s}} = 0$ for all $i \neq j$ and

$$\frac{\partial \ln P_i(\boldsymbol{p}|\boldsymbol{\tau})}{\partial \ln \tau_{oi,s}} = -\frac{\xi_{i,s}}{\epsilon_s}\frac{\partial}{\partial \ln \tau_{oi,s}}\left[\sum_{o':s\in\mathcal{S}_{o'}} \left(\tau_{o'i,s}p_{o'}\right)^{-\epsilon_s}\right] = \xi_{i,s}\frac{\left(\tau_{oi,s}p_o\right)^{-\epsilon_s}}{\sum_{o':s\in\mathcal{S}_{o'}} \left(\tau_{o'i,s}p_{o'}\right)^{-\epsilon_s}}$$

By the definition of $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$,

$$\hat{\eta}_i^C(\hat{\boldsymbol{\tau}}) \equiv \sum_{o,s} \frac{\partial \ln P_i(\boldsymbol{p}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{oi,s}} \hat{\tau}_{oi,s} = \sum_{o,s} \xi_{i,s} x_{oi,s}^0 \hat{\tau}_{oi,s},$$

which is equivalent to $\hat{\eta}_i^C(\hat{\boldsymbol{\tau}})$ in (12).

Revenue shift. The definitions $Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau}) \equiv \sum_k x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})E_k$ and $x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})$ in (6) imply that $\frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln \tau_{oj,s}} = 0$ for $s \notin \mathcal{S}_i$. The same definitions also imply that, for all $s \in \mathcal{S}_i$,

$$\begin{aligned}\frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln \tau_{oj,s}} &= \sum_k \frac{\partial \ln x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})}{\partial \ln \tau_{oj,s}} \frac{x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})E_k}{\sum_{k'} x_{ik'}(\boldsymbol{p}|\boldsymbol{\tau})E_{k'}} \\ &= (-\epsilon_s)\left(\mathbb{I}_{[i=o]} - x_{oj,s}(\boldsymbol{p}|\boldsymbol{\tau})\right) \frac{x_{ij,s}(\boldsymbol{p}|\boldsymbol{\tau})\xi_{j,s}}{x_{ij}(\boldsymbol{p}|\boldsymbol{\tau})} \frac{x_{ij}(\boldsymbol{p}|\boldsymbol{\tau})E_j}{\sum_k x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})E_k}\end{aligned}$$

where the second equality uses the fact that $\frac{\partial \ln x_{ik}(\boldsymbol{p}|\boldsymbol{\tau})}{\partial \ln \tau_{oj,s}} = 0$ for all $k \neq j$.

From the definition of $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$,

$$\hat{\eta}_i^R(\hat{\boldsymbol{\tau}}) = \sum_{j,o,s} \frac{\partial \ln Y_i(\boldsymbol{p}^0,\boldsymbol{E}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{oj,s}} \hat{\tau}_{oj,s} = \sum_{j,o,s} \frac{x_{ij,s}^0 \xi_{j,s}}{x_{ij}^0} \frac{x_{ij}^0 E_j^0}{Y_i^0} \left(x_{oj,s}^0 - \mathbb{I}_{[i=o]}\right) \epsilon_s \hat{\tau}_{oj,s}.$$

We obtain $\hat{\eta}_i^R(\hat{\boldsymbol{\tau}})$ in (12) by noting that the share of j in the revenues of i is $y_{ij}^0 \equiv x_{ij}^0 E_j^0 / Y_i^0$ and the share of sector s in the sales of i to j is $y_{ij,s}^0 \equiv x_{ij,s}^0 \xi_{j,s} E_j^0 / x_{ij}^0 E_j^0 = x_{ij,s}^0 \xi_{j,s} / x_{ij}^0$.

8.3 Proof of Theorem 1

In order to characterize the shift in excess labor demand, we define the following two functions. First, we define the price index of i as a function of $\boldsymbol{w}$ and $\boldsymbol{\tau}$:

$$P_j = \Pi_s \left[\sum_{o:s\in\mathcal{S}_o} \left(\tau_{oj,s} \frac{w_o}{\Psi_o\left(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P})\right)}\right)^{-\epsilon_s}\right]^{\frac{\xi_{j,s}}{-\epsilon_s}} \quad \text{for all } j. \tag{40}$$

Second, we define the revenue of i as a function of $\boldsymbol{w}$ and $\boldsymbol{\tau}$:

$$\begin{aligned}\tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau}) &\equiv Y_i\left(\left\{\frac{w_j}{\Psi_j(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}\right\}_j, \left\{w_j \Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau}))\right\}_j |\boldsymbol{\tau}\right) \\ &= \sum_j \left[\sum_{s\in\mathcal{S}_i} \frac{\left(\frac{\tau_{ij,s} w_i}{\Psi_i(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}\right)^{-\epsilon_s}}{\sum_{o:s\in\mathcal{S}_o}\left(\frac{\tau_{oj,s} w_o}{\Psi_o(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})))}\right)^{-\epsilon_s}} \xi_{j,s}\right] w_j \Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})).\end{aligned} \tag{41}$$

Using these definitions, the excess demand function in market i can be written as

$$D_i(\boldsymbol{w}|\boldsymbol{\tau}) = \tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau}) - w_i \Phi_i(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau})). \tag{42}$$

We now characterize $\frac{\partial \ln P_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}}$ using the implicit function theorem and the system in (40). By defining $p_j(\boldsymbol{w}|\boldsymbol{\tau}) \equiv w_j/\Psi_j(\boldsymbol{\Phi}(\boldsymbol{w},\boldsymbol{P}))$,

$$\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} = \mathbb{I}_{[d=j]} \xi_{j,s} x_{oj,s}^0 + \sum_{o'} \left(\sum_k \xi_{j,k} x_{o'j,k}^0\right) \frac{\partial \ln p_{o'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}$$

where

$$\frac{\partial \ln p_{o'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} = -\sum_{j'} \frac{\partial \ln \Psi_{o'}(\boldsymbol{L}^0)}{\partial \ln L_{j'}} \sum_{k'} \frac{\partial \ln \Phi_{j'}(\boldsymbol{w}^0,\boldsymbol{P}^0)}{\partial \ln P_{k'}} \frac{\partial \ln P_{k'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}. \tag{43}$$

Thus, the elasticity matrices defined in (2) and (4) imply that

$$\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} = \mathbb{I}_{[d=j]}\xi_{j,s}x^0_{oj,s} - \sum_{o'}\sum_{j'} x^0_{o'j}\psi_{o'j'}\sum_{k'}\phi^p_{j'k'}\frac{\partial \ln P_{k'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}},$$

where we have used the fact that $x^0_{o'j} = \sum_k \xi_{j,k}x^0_{o'j,k}$.

Define the vector $\boldsymbol{x}^{od,s}$ with d-th entry equal to $\xi_{d,s}x^0_{od,s}$ and all other entries equal to zero. In matrix notation,

$$\nabla_{\ln \tau_{od,s}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0) = \boldsymbol{x}^{od,s} - \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p \nabla_{\ln \tau_{od,s}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)$$

and, therefore,

$$\nabla_{\ln \tau_{od,s}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0) = \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1}\boldsymbol{x}^{od,s}.$$

The combination of this expression and the definition of $\hat{\eta}^C_i$ in (12) yields the total change in the price index vector:

$$\sum_{o,d,s}\left(\nabla_{\ln \tau_{od,s}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)\right)\hat{\tau}_{od,s} = \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1}[\sum_{o,s}\xi_{j,s}x^0_{oj,s}\hat{\tau}_{oj,s}]_j = \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1}\hat{\boldsymbol{\eta}}^C. \tag{44}$$

We now use the total change in the price index vector to derive the excess labor demand shift. The expression of $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (42) implies that

$$\frac{\partial D_i(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}}\frac{1}{\tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})} = \frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} - \frac{w_i\Phi_i(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau}))}{Y_i(\boldsymbol{w}|\boldsymbol{\tau})}\sum_j \frac{\partial \ln \Phi_i(\boldsymbol{w},\boldsymbol{P})}{\partial \ln P_j}\frac{\partial \ln P_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} \tag{45}$$

We then combine this expression with the definition of $\tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (41) to obtain

$$\frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} = \frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} + \sum_j \frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln p_j}\frac{\partial \ln p_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} + \sum_j \frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln E_j}\frac{\partial \ln E_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}}, \tag{46}$$

where $E_j(\boldsymbol{w}|\boldsymbol{\tau}) \equiv w_j\Phi_j(\boldsymbol{w},\boldsymbol{P}(\boldsymbol{w}|\boldsymbol{\tau}))$ and

$$\frac{\partial \ln E_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} = \sum_{j'}\frac{\partial \ln \Phi_j(\boldsymbol{w},\boldsymbol{P})}{\partial \ln P_{j'}}\frac{\partial \ln P_{j'}(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}}.$$

Consider the elasticity matrices defined in (2), (4) and (7). By combining these definitions with expressions (43) and (46), we obtain that

$$\begin{aligned}\frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \quad &= \quad \frac{\partial \ln Y_i(\boldsymbol{p}^0,\boldsymbol{E}^0|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} - \sum_j \chi_{ij}\sum_{j'}\psi_{jj'}\sum_k \phi^p_{j'k}\frac{\partial \ln P_k(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \\ &+ \quad \sum_j y^0_{ij}\sum_{j'}\phi^p_{jj'}\frac{\partial \ln P_{j'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}.\end{aligned} \tag{47}$$

Since $\tilde{Y}_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0) = Y^0_i = w^0_iL^0_i = w^0_i\Phi_i(\boldsymbol{w}^0,\boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0))$, (45) implies that

$$\sum_{o,d,s}\frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}\frac{\hat{\tau}_{od,s}}{Y^0_i} = \sum_{o,d,s}\frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s} - \sum_{o,d,s}\sum_j \phi^p_{ij}\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}}\hat{\tau}_{od,s}$$

and, by (47),

$$\sum_{o,d,s} \frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \frac{\hat{\tau}_{od,s}}{Y_i^0} = \sum_{o,d,s} \frac{\partial \ln Y_i(\boldsymbol{p},\boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} - \sum_j \chi_{ij} \sum_{j'} \psi_{jj'} \sum_k \phi^p_{j'k} \left(\sum_{o,d,s} \frac{\partial \ln P_k(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} \right)$$
$$+ \sum_j y^0_{ij} \sum_{j'} \phi^p_{jj'} \left(\sum_{o,d,s} \frac{\partial \ln P_{j'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} \right) - \sum_j \phi^p_{ij} \left(\sum_{o,d,s} \frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln \tau_{od,s}} \hat{\tau}_{od,s} \right).$$

We use the definition of $\hat{\eta}^R_i$ in (12) to write the stacked vector of excess demand shifts:

$$\hat{\boldsymbol{\eta}} = \hat{\boldsymbol{\eta}}^R - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \bar{\boldsymbol{\phi}}^p \sum_{o,d,s} \left(\nabla_{\ln \tau_{od,s}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0) \right) \hat{\tau}_{od,s},$$

which, by the expression in (44), is equivalent to

$$\hat{\boldsymbol{\eta}} = \hat{\boldsymbol{\eta}}^R - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \bar{\boldsymbol{\phi}}^p \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \right)^{-1} \hat{\boldsymbol{\eta}}^C.$$

Finally, note that

$$\begin{aligned} \bar{\boldsymbol{\phi}}^p \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \right)^{-1} &= \left(\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \right) \left(\bar{\boldsymbol{\phi}}^p \right)^{-1} \right)^{-1} \\ &= \left(\left(\bar{\boldsymbol{\phi}}^p \right)^{-1} \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \right) \right)^{-1} \\ &= \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \right)^{-1} \bar{\boldsymbol{\phi}}^p. \end{aligned}$$

Thus,

$$\hat{\boldsymbol{\eta}} = \hat{\boldsymbol{\eta}}^R - \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \right)^{-1} \bar{\boldsymbol{\phi}}^p \hat{\boldsymbol{\eta}}^C,$$

which corresponds to equation (14) given the definition of $\bar{\boldsymbol{\alpha}} \equiv \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}} \bar{\boldsymbol{\psi}} \right) \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \right)^{-1}$.

8.4 Proof of Proposition 3

We start by characterizing $\frac{\partial \ln P_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o}$ using the implicit function theorem and the system in (40):

$$\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} = \sum_{o'} \left(\sum_k \xi_{j,k} x^0_{o'j,k} \right) \frac{\partial \ln p_{o'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} = \sum_{o'} x^0_{o'j} \frac{\partial \ln p_{o'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}$$

where

$$\frac{\partial \ln p_{o'}(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o} = \mathbb{I}_{[o'=o]} - \sum_{j'} \frac{\partial \ln \Psi_{o'}(\boldsymbol{L})}{\partial \ln L_{j'}} \left(\frac{\partial \ln \Phi_{j'}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln w_o} + \sum_{k'} \frac{\partial \ln \Phi_{j'}(\boldsymbol{w},\boldsymbol{P})}{\partial \ln P_{k'}} \frac{\partial \ln P_{k'}(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o} \right). \tag{48}$$

Thus, the elasticity matrices defined in (2) and (4) imply that

$$\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} = x^0_{oj} - \sum_{o'} \sum_{j'} x^0_{o'j} \psi_{o'j'} \left(\phi^w_{j'o} + \sum_{k'} \phi^p_{j'k'} \frac{\partial \ln P_{k'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} \right).$$

Define the vectors $\boldsymbol{x}^o \equiv [x^0_{oj}]_j$ and $\boldsymbol{\phi}^o \equiv [\phi^w_{jo}]_j$. In matrix notation,

$$\nabla_{\ln w_o} \ln \boldsymbol{P} = \boldsymbol{x}^o - \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \boldsymbol{\phi}^o - \bar{\boldsymbol{x}}^{0\prime} \bar{\boldsymbol{\psi}} \bar{\boldsymbol{\phi}}^p \nabla_{\ln w_o} \ln \boldsymbol{P}.$$

Thus, the Jacobian of the price index with respect to $\ln \boldsymbol{w}$ is

$$\nabla_{\ln \boldsymbol{w}} \ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0) = \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1} \bar{\boldsymbol{x}}^{0\prime} \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^w\right). \tag{49}$$

We now derive the Jacobian of the excess labor demand with respect to $\ln \boldsymbol{w}$. The expression of $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (42) implies that

$$\frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\frac{1}{Y_i^0} = \frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} - \left(\mathbb{I}_{[i=o]} + \frac{\partial \ln \Phi_i(\boldsymbol{w}^0, \boldsymbol{P}^0)}{\partial \ln w_o} + \sum_j \frac{\partial \ln \Phi_i(\boldsymbol{w}^0, \boldsymbol{P}^0)}{\partial \ln P_j}\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\right). \tag{50}$$

The definition of $\tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})$ in (41) yields

$$\frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o} = \sum_j \frac{\partial \ln Y_i(\boldsymbol{p}, \boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln p_j}\frac{\partial \ln p_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o} + \sum_j \frac{\partial \ln Y_i(\boldsymbol{p}, \boldsymbol{E}|\boldsymbol{\tau})}{\partial \ln E_j}\frac{\partial \ln E_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o}, \tag{51}$$

where

$$\frac{\partial \ln E_j(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o} = \mathbb{I}_{[j=o]} + \frac{\partial \ln \Phi_j(\boldsymbol{w}, \boldsymbol{P})}{\partial \ln w_o} + \sum_{j'} \frac{\partial \ln \Phi_j(\boldsymbol{w}, \boldsymbol{P})}{\partial \ln P_{j'}}\frac{\partial \ln P_{j'}(\boldsymbol{w}|\boldsymbol{\tau})}{\partial \ln w_o}.$$

Consider the elasticity matrices defined in (2), (4) and (7). By combining these definitions with expression (51), we obtain that

$$\begin{aligned}\frac{\partial \ln \tilde{Y}_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} &= \chi_{io} - \textstyle\sum_j \chi_{ij} \sum_{j'} \psi_{jj'}\left(\phi^w_{j'o} + \sum_k \phi^p_{j'k}\frac{\partial \ln P_k(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\right) \\ &+ \quad y^0_{io} + \textstyle\sum_j y^0_{ij}\phi^w_{jo} + \sum_j y^0_{ij}\sum_{j'}\phi^p_{jj'}\frac{\partial \ln P_{j'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}.\end{aligned} \tag{52}$$

The combination of (50) and (52) implies that

$$\begin{aligned}\frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\frac{1}{Y_i^0} &= \chi_{io} - \textstyle\sum_j \chi_{ij}\sum_{j'}\psi_{jj'}\left(\phi^w_{j'o} + \sum_k \phi^p_{j'k}\frac{\partial \ln P_k(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\right) + y^0_{io} + \sum_j y^0_{ij}\phi^w_{jo} \\ &+ \textstyle\sum_j y^0_{ij}\sum_{j'}\phi^p_{jj'}\frac{\partial \ln P_{j'}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o} - \left(\mathbb{I}_{[i=o]} + \phi^w_{io} + \sum_j \phi^p_{ij}\frac{\partial \ln P_j(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\right).\end{aligned}$$

Since $\bar{\boldsymbol{\gamma}} \equiv [-\frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial \ln w_o}\frac{1}{Y_i^0}]_{i,o}$, we have that

$$\begin{aligned}\bar{\boldsymbol{\gamma}} &= -\bar{\boldsymbol{\chi}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p\nabla_{\ln \boldsymbol{w}}\ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)\right) - \bar{\boldsymbol{y}}^0 - \bar{\boldsymbol{y}}^0\bar{\boldsymbol{\phi}}^w \\ &-\bar{\boldsymbol{y}}^0\bar{\boldsymbol{\phi}}^p\nabla_{\ln \boldsymbol{w}}\ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0) + \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p\nabla_{\ln \boldsymbol{w}}\ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)\right),\end{aligned}$$

which can be written as

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{I}} - \left(\bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\right) + \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\right)\left(\bar{\boldsymbol{\phi}}^w + \bar{\boldsymbol{\phi}}^p\nabla_{\ln \boldsymbol{w}}\ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)\right).$$

We now substitute for $\nabla_{\ln \boldsymbol{w}}\ln \boldsymbol{P}(\boldsymbol{w}^0|\boldsymbol{\tau}^0)$ using (49) to obtain

$$\begin{aligned}\bar{\boldsymbol{\gamma}} &= \bar{\boldsymbol{I}} - \left(\bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\right) \\ &+ \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}^p\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1}\bar{\boldsymbol{x}}^{0\prime} \\ &+ \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\right)\left(\bar{\boldsymbol{\phi}}^w - \bar{\boldsymbol{\phi}}^p\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1}\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^w\right).\end{aligned}$$

As shown in Appendix 8.3, $\bar{\boldsymbol{\phi}}^p\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^p\right)^{-1} = \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)^{-1}\bar{\boldsymbol{\phi}}^p$. This relationship and the definition of $\bar{\boldsymbol{\alpha}}$ imply that

$$\begin{aligned}\bar{\boldsymbol{\gamma}} \quad = \quad & \bar{\boldsymbol{I}} - \left(\bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\right) + \bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime} \\ & \left(\bar{\boldsymbol{I}} - \bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\bar{\boldsymbol{\psi}}\right)\left(\bar{\boldsymbol{\phi}}^w - \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)^{-1}\bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\bar{\boldsymbol{\phi}}^w\right).\end{aligned}$$

By re-arranging the term in the second row, we obtain

$$\begin{aligned}\bar{\boldsymbol{\gamma}} \quad = \quad & \bar{\boldsymbol{I}} - \left(\bar{\boldsymbol{y}}^0 + \bar{\boldsymbol{\chi}}^0\right) + \bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime} \\ & \bar{\boldsymbol{\alpha}}\left(\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right) - \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)\bar{\boldsymbol{\phi}}^w,\end{aligned}$$

which is equivalent to the expression for $\bar{\boldsymbol{\gamma}}$ in (16).

8.5 Proof of Theorem 2

We re-define the system in (11) to set the change in the wage of market m to zero. Consider the matrix $\bar{\boldsymbol{M}}$ obtained by deleting the m-th row from the identity matrix with dimension equal to the number of markets. If $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is nonsingular, then we can write

$$\bar{\boldsymbol{M}}\hat{\boldsymbol{w}} = \left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}},$$

which yields the representation in (17) when we define $\bar{\boldsymbol{\beta}} \equiv \bar{\boldsymbol{M}}'(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}')^{-1}\bar{\boldsymbol{M}}$.

In the rest of the proof, we first show that $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'$ is nonsingular and then establish that $\bar{\boldsymbol{\beta}}$ admits the series representation in (18). To simplify exposition, we abuse notation by defining

$$\bar{\boldsymbol{\gamma}} \equiv \bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}', \quad \hat{\boldsymbol{w}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{w}} \qquad \text{and} \qquad \hat{\boldsymbol{\eta}} \equiv \bar{\boldsymbol{M}}\hat{\boldsymbol{\eta}}.$$

This modified system does not include the row associated with the market clearing condition of market m and imposes that $\hat{w}_m = 0$. To obtain a characterization for the solution of this system, let $\bar{\boldsymbol{\kappa}}$ be the diagonal matrix with the diagonal elements of $\bar{\boldsymbol{\gamma}}$: $\bar{\boldsymbol{\kappa}} = [\kappa_{ij}]$ s.t. $\kappa_{ii} = \gamma_{ii}$ and $\kappa_{ij} = 0$ for $i \neq j$. Thus, we can write the system as

$$\bar{\boldsymbol{\gamma}} = \bar{\boldsymbol{\kappa}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\boldsymbol{\gamma}}}\right) \quad st \quad \bar{\tilde{\boldsymbol{\gamma}}} \equiv \bar{\boldsymbol{I}} - \bar{\boldsymbol{\kappa}}^{-1}\bar{\boldsymbol{\gamma}},$$

which implies that $\tilde{\gamma}_{ii} = 0$ and $\tilde{\gamma}_{ij} = -\gamma_{ij}/\gamma_{ii}$.

Consider the vector $\{h_i\}_{i\neq m} \gg 0$ that guarantees the diagonal dominance of $\bar{\boldsymbol{\gamma}}$ in the initial equilibrium. Let $\bar{\boldsymbol{h}}$ be the diagonal matrix such that h_i is the diagonal entry in row i. Thus, the system in (11) is equivalent to

$$\bar{\boldsymbol{\kappa}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\boldsymbol{\gamma}}}\right)\left(\bar{\boldsymbol{h}}\bar{\boldsymbol{h}}^{-1}\right)\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}$$

$$\bar{\boldsymbol{\kappa}}\left(\bar{\boldsymbol{h}} - \bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}$$

$$\left(\bar{\boldsymbol{\kappa}}\bar{\boldsymbol{h}}\right)\left(\bar{\boldsymbol{I}} - \left(\bar{\boldsymbol{h}}^{-1}\bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}\right)\right)\bar{\boldsymbol{h}}^{-1}\hat{\boldsymbol{w}} = \hat{\boldsymbol{\eta}}$$

which implies that

$$\hat{\boldsymbol{w}} = \bar{\boldsymbol{h}}\left(\bar{\boldsymbol{I}} - \bar{\tilde{\tilde{\boldsymbol{\gamma}}}}\right)^{-1}\left(\bar{\boldsymbol{\kappa}}\bar{\boldsymbol{h}}\right)^{-1}\hat{\boldsymbol{\eta}}, \quad \bar{\tilde{\tilde{\boldsymbol{\gamma}}}} \equiv \bar{\boldsymbol{h}}^{-1}\bar{\tilde{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}. \tag{53}$$

Notice that, for all i, $\tilde{\tilde{\gamma}}_{ii} = 0$ and $\tilde{\tilde{\gamma}}_{ij} = -\gamma_{ij}h_j/\gamma_{ii}h_i$.

First, we show that $(\bar{\boldsymbol{I}} - \bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$ is non-singular, so that we can write the expression in (53). We proceed by contradiction. Suppose that $(\bar{\boldsymbol{I}} - \bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$ is singular, so $\lambda = 0$ is an eigenvalue of $(\bar{\boldsymbol{I}} - \bar{\tilde{\tilde{\boldsymbol{\gamma}}}})$. Take the eigenvector $\boldsymbol{x}$ associated with the zero eigenvalue and normalize it such that $x_i = 1$ and $|x_j| \leq 1$. Notice that $(\bar{\boldsymbol{I}}-\bar{\bar{\boldsymbol{\gamma}}})\boldsymbol{x}=0$, so that the i-row of this system is

$$1+\sum_{j\neq i,m}-\bar{\bar{\gamma}}_{ij}x_j=0 \qquad\Longrightarrow\qquad 1+\sum_{j\neq i,m}\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j=0$$

Thus,

$$\gamma_{ii}h_i=-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq|\sum_{j\neq i,m}\gamma_{ij}h_jx_j|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$$

where the last inequality holds because $|x_j|\leq 1$ and $h_j>0$. Thus, $\gamma_{ii}h_i\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts Assumption 1.

Second, we show that $(\bar{\boldsymbol{I}}-\bar{\bar{\boldsymbol{\gamma}}})^{-1}$ admits the series representation in (18). This is true whenever the largest eigenvalue of $\bar{\bar{\boldsymbol{\gamma}}}$ is below one. To show this, we proceed by contradiction. Assume that the largest eigenvalue λ is weakly greater than one. Take the eigenvector $\boldsymbol{x}$ associated with the largest eigenvalue and normalize it such that $x_i=1$ and $|x_j|\leq 1$. Notice that $\lambda\boldsymbol{x}=\bar{\bar{\boldsymbol{\gamma}}}\boldsymbol{x}$ so that the i-row of this system is

$$1\leq\lambda=\sum_{j\neq i,m}-\frac{\gamma_{ij}}{\gamma_{ii}}\frac{h_j}{h_i}x_j$$

Since γ_{ii} and h_i are positive,

$$\gamma_{ii}h_i\leq-\sum_{j\neq i,m}\gamma_{ij}h_jx_j\leq|\sum_{j\neq i,m}\gamma_{ij}h_jx_j|\leq\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|$$

Since $|x_j|\leq 1$ and $h_j>0$, $\sum_{j\neq i,m}|\gamma_{ij}||h_j||x_j|\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$. Thus, $\gamma_{ii}h_i\leq\sum_{j\neq i,m}|\gamma_{ij}|h_j$, which contradicts Assumption 1. Thus, the largest eigenvalue of $\bar{\bar{\boldsymbol{\gamma}}}$ is below one, allowing us to write $(\bar{\boldsymbol{I}}-\bar{\bar{\gamma}})^{-1}=\sum_{d=0}^{\infty}(\bar{\bar{\gamma}})^d$. Substituting this series expansion into (53) yields

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\boldsymbol{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\bar{\boldsymbol{\kappa}}^{-1}\hat{\boldsymbol{\eta}}.$$

Finally, to establish the result, we now show that $\bar{\boldsymbol{h}}(\bar{\bar{\boldsymbol{\gamma}}})^d\bar{\boldsymbol{h}}^{-1}=(\bar{\boldsymbol{\gamma}})^d$. We proceed by induction. For $d=1$, it is trivial to see that $\bar{\boldsymbol{h}}(\bar{\bar{\boldsymbol{\gamma}}})^d\bar{\boldsymbol{h}}^{-1}=\bar{\boldsymbol{\gamma}}$. Then,

$$\bar{\boldsymbol{h}}\left(\bar{\bar{\boldsymbol{\gamma}}}\right)^{d+1}\bar{\boldsymbol{h}}^{-1}=\left(\bar{\boldsymbol{h}}\left(\bar{\bar{\boldsymbol{\gamma}}}\right)^d\bar{\boldsymbol{h}}^{-1}\right)\left(\bar{\boldsymbol{h}}\bar{\bar{\boldsymbol{\gamma}}}\bar{\boldsymbol{h}}^{-1}\right)=(\bar{\boldsymbol{\gamma}})^d\left(\bar{\boldsymbol{h}}\left(\bar{\boldsymbol{h}}^{-1}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{h}}\right)\bar{\boldsymbol{h}}^{-1}\right)=(\bar{\boldsymbol{\gamma}})^{d+1}.$$

Thus,

$$\hat{\boldsymbol{w}}=\sum_{d=0}^{\infty}(\bar{\boldsymbol{\gamma}})^d\,\bar{\boldsymbol{\kappa}}^{-1}\hat{\boldsymbol{\eta}},$$

which immediately implies the result.

8.6 One-to-one mapping between $\bar{\boldsymbol{\beta}}$ and $\bar{\boldsymbol{\gamma}}$

From Appendix 8.5, we know that $\bar{\boldsymbol{\beta}}=\bar{\boldsymbol{M}}'\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}\bar{\boldsymbol{M}}$. Since $\bar{\boldsymbol{M}}\bar{\boldsymbol{M}}'=\bar{\boldsymbol{I}}_{N-1}$, $\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{M}}'\right)=\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'\right)^{-1}$ and, therefore, $\bar{\boldsymbol{M}}\bar{\boldsymbol{\gamma}}\bar{\boldsymbol{M}}'=\left(\bar{\boldsymbol{M}}\bar{\boldsymbol{\beta}}\bar{\boldsymbol{M}}'\right)^{-1}$. This implies that knowledge of $\bar{\boldsymbol{\beta}}$ yields knowledge of γ_{ij} for all $i,j\neq m$.

To recover γ_{im}, recall that, from Appendix 8.1, $D_i(\boldsymbol{w}|\boldsymbol{\tau})$ is homogeneous of degree one on $\boldsymbol{w}$, implying that $\sum_j \gamma_{ij} = -\frac{1}{Y_i^0}\sum_j \frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial w_j} w_j = -\frac{D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{Y_i^0}$. Since $\boldsymbol{w}^0$ is an equilibrium wage vector, $D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0) = 0$ and $\sum_j \gamma_{ij} = 0$ for all i. So, $\gamma_{im} = -\sum_{j\neq m}\gamma_{ij}$.

To recover γ_{mj}, recall that $\sum_i D_i(\boldsymbol{w}|\boldsymbol{\tau}) = 0$ for all $\boldsymbol{w}$ due to trade balance in the world economy. Thus, for every j, $\sum_i Y_i^0\gamma_{ij} = -\sum_i \frac{\partial D_i(\boldsymbol{w}^0|\boldsymbol{\tau}^0)}{\partial w_j} w_j = 0$, implying that $\gamma_{mj} = -\sum_{i\neq m}\frac{Y_i^0}{Y_m^0}\gamma_{ij}$.

8.7 Proof of Corollary 1

We start by showing that the gross substitution property implies the diagonal dominance in Assumption 1. As shown in Appendix 8.6, $\sum_j \gamma_{ij} = 0$ for all i, which implies that $\gamma_{ii} = -\sum_{j\neq i}\gamma_{ij}$ for all i. Thus, $\gamma_{ij} < 0$ for all $i \neq j$ implies that $\gamma_{ii} = \sum_{j\neq i}|\gamma_{ij}| > \sum_{j\neq i,m}|\gamma_{ij}|$ for any numeraire m. Thus, Assumption 1 holds and Theorem 2 immediately implies the expression in (18). Since $\gamma_{ij} < 0$ for all $i \neq j$, $\tilde{\gamma}_{ij} \geq 0$ for all i and j and, therefore, $\tilde{\gamma}_{ij}^{(d)} \geq 0$ for all i, j and d.

8.8 Proof of Corollary 2

We abuse notation by defining the matrix $\bar{\boldsymbol{\gamma}}$ after dropping the numeraire market: $\bar{\boldsymbol{\gamma}} \equiv \gamma\left(\bar{\boldsymbol{I}} - \mathbb{I}\tilde{\boldsymbol{\gamma}}'\right)$, where $\mathbb{I}$ is a column vector of ones and $\tilde{\boldsymbol{\gamma}} \equiv \{\gamma_j\}_{j\neq m}$ is column vector. We verify that $\bar{\boldsymbol{\gamma}}^{-1} = \gamma^{-1}\left(\bar{\boldsymbol{I}} + (1+\gamma_m)^{-1}\mathbb{I}\tilde{\boldsymbol{\gamma}}'\right)$.

$$
\begin{aligned}
\bar{\boldsymbol{\gamma}}^{-1}\bar{\boldsymbol{\gamma}} &= \bar{\boldsymbol{I}} + (1+\gamma_m)^{-1}\mathbb{I}\tilde{\boldsymbol{\gamma}}' - \mathbb{I}\tilde{\boldsymbol{\gamma}}' - (1+\gamma_m)^{-1}\mathbb{I}\tilde{\boldsymbol{\gamma}}'\mathbb{I}\tilde{\boldsymbol{\gamma}}' \\
&= \bar{\boldsymbol{I}} + (1+\gamma_m)^{-1}\mathbb{I}\tilde{\boldsymbol{\gamma}}' - \mathbb{I}\tilde{\boldsymbol{\gamma}}' + (1+\gamma_m)^{-1}\gamma_m\mathbb{I}\tilde{\boldsymbol{\gamma}}' \\
&= \bar{\boldsymbol{I}}
\end{aligned}
$$

where the second equality follows from $\tilde{\boldsymbol{\gamma}}'\mathbb{I} = \sum_{j\neq m}\gamma_j = -\gamma_m$ (because $\sum_j \gamma_{ij} = 0$ for all j).

Overall, we can write $\boldsymbol{\beta} = \gamma^{-1}\bar{\boldsymbol{I}} + \gamma^{-1}(1+\gamma_m)^{-1}\mathbb{I}\tilde{\boldsymbol{\gamma}}'$. Notice that the second term in case of a gravity model with symmetric trade costs can be written as $\gamma^{-1}x_m^{-1}x_j$ where $x_j = y_j$ is the import share of market j, or, equivalently, the size of the market. In that case, it can be shown that $\beta_j \equiv \gamma^{-1}x_m^{-1}x_j$ for $j \neq m$ and $\beta_m = 0$, so that $\sum_j \beta_j\hat{\eta}_j = \sum_{j\neq m}\gamma^{-1}x_m^{-1}x_j\hat{\eta}_j$. Therefore, conditional on the size of the numeraire country, if the countries that are large have larger shocks, then the fixed effect term will be larger.

8.9 Derivation of Expressions (21) and (23)

Expression (21). The combination of the labor supply equation in (1) and the price index in (8) implies that

$$
\hat{L}_i = \sum_j \phi_{ij}^w\hat{w}_j + \sum_j \phi_{ij}^p\hat{P}_j = \sum_j \phi_{ij}^w\hat{w}_j + \sum_j \phi_{ij}^p\left(\hat{\eta}_j^C + \sum_o x_{oj}^0\hat{p}_o\right)
$$

Using the unit cost expression in (5),

$$
\hat{L}_i = \sum_j \phi_{ij}^w\hat{w}_j + \sum_j \phi_{ij}^p\left(\hat{\eta}_j^C + \sum_o x_{oj}^0\left(\hat{w}_o - \sum_{o'}\psi_{oo'}\hat{L}_{o'}\right)\right)
$$

In matrix form,

$$
\hat{\boldsymbol{L}} = \bar{\boldsymbol{\phi}}^w\hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p\left(\hat{\boldsymbol{\eta}}^C + \bar{\boldsymbol{x}}^{0\prime}\left(\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}\hat{\boldsymbol{L}}\right)\right)
$$

$$
\left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^p\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)\hat{\boldsymbol{L}} = \bar{\boldsymbol{\phi}}^w\hat{\boldsymbol{w}} + \bar{\boldsymbol{\phi}}^p\left(\hat{\boldsymbol{\eta}}^C + \bar{\boldsymbol{x}}^{0\prime}\hat{\boldsymbol{w}}\right),
$$

which immediately yields the expression in (21) with the definition $\bar{\boldsymbol{\rho}} \equiv \left(\bar{\boldsymbol{I}} + \bar{\boldsymbol{\phi}}^{p}\bar{\boldsymbol{x}}^{0\prime}\bar{\boldsymbol{\psi}}\right)^{-1}$ and $\bar{\boldsymbol{\phi}} \equiv \left(\bar{\boldsymbol{\phi}}^{w} + \bar{\boldsymbol{\phi}}^{p}\bar{\boldsymbol{x}}^{0\prime}\right)$.

Expression (23). By definition, $\hat{W}_i = \hat{w}_i - \hat{P}_i = \hat{w}_i - \left(\hat{\eta}_i^C + \sum_o x_{oi}^0 \hat{p}_o\right)$. Using the unit cost expression in (5),

$$\hat{W}_i = \hat{w}_i - \left(\hat{\eta}_i^C + \sum_o x_{oi}^0 \left(\hat{w}_o - \sum_{o'} \psi_{oo'} \hat{L}_{o'}\right)\right).$$

In matrix notation,

$$\hat{\boldsymbol{W}} = \hat{\boldsymbol{w}} - \hat{\boldsymbol{\eta}}^C - \bar{\boldsymbol{x}}^{0\prime}\left(\hat{\boldsymbol{w}} - \bar{\boldsymbol{\psi}}\hat{\boldsymbol{L}}\right),$$

which immediately yields Expression (23).

8.10 Proof of Proposition 4

The asymptotic variance of the GMM estimator for any function $H_i(\cdot)$ is

$$V(H) = \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[H_i^t \nu_i^t \nu_i^{t\prime} H_i^{t\prime}\right]\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime} \tag{54}$$

where $G_i^t \equiv E\left[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}) | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right]$ and $H_i^t \equiv H_i\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right)$.

From (29), the gradient of $\nu_i^t(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ is

$$G_i^t \equiv E\left[\nabla_{\boldsymbol{\theta}} \nu_i^t(\boldsymbol{\theta}) | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right] = -\sum_j \begin{bmatrix} \nabla_{\boldsymbol{\theta}} \epsilon \beta_{ij}^R(\boldsymbol{\theta}) \\ \nabla_{\boldsymbol{\theta}} \upsilon \varphi_{ij}^R(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}_j^{R,t} - \sum_j \begin{bmatrix} \nabla_{\boldsymbol{\theta}} \beta_{ij}^C(\boldsymbol{\theta}) \\ \nabla_{\boldsymbol{\theta}} \varphi_{ij}^C(\boldsymbol{\theta}) \end{bmatrix} \hat{\eta}_j^{C,t}. \tag{55}$$

We now show that function that minimizes the asymptotic variance is

$$H_i^*\left(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right) \equiv G_i^t \left(\Omega_i^t\right)^{-1} \tag{56}$$

where $\Omega_i^t \equiv E\left[\nu_i^t(\boldsymbol{\theta}) \nu_i^t(\boldsymbol{\theta})' | \hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}\right]$. For this function, the asymptotic variance is

$$V(H^*) = \left(E\left[G_i^{t\prime}\left(\Omega_i^t\right)^{-1} G_i^t\right]\right)^{-1}. \tag{57}$$

To establish the result, we show that $V(H) - V(H^*)$ is positive semi-definite for any $H_i(\cdot)$:

$$V(H) - V(H^*) = \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[\left(H_i^t \nu_i^t\right)\left(H_i^t \nu_i^t\right)'\right]\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime} - \left(E\left[G_i^{t\prime}\left(\Omega_i^t\right)^{-1} G_i^t\right]\right)^{-1}$$
$$= \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[\left(H_i^t \nu_i^t\right)\left(H_i^t \nu_i^t\right)'\right] - E\left[H_i^t G_i^t\right] \left(E\left[G_i^{t\prime}\left(\Omega_i^t\right)^{-1} G_i^t\right]\right)^{-1} E\left[H_i^t G_i^t\right]'\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime}.$$

Let us define

$$U_i^t \equiv H_i^t \nu_i^t - E\left[\left(H_i^t \nu_i^t\right)\left(G_i^{t\prime}\left(\Omega_i^t\right)^{-1} v_i^t\right)'\right] \left(E\left[G_i^{t\prime}\left(\Omega_i^t\right)^{-1} G_i^t\right]\right)^{-1} G_i^{t\prime}\left(\Omega_i^t\right)^{-1} v_i^t,$$

which implies that

$$E\left[U_i^t U_i^{t\prime}\right] = E\left[\left(H_i^t \nu_i^t\right)\left(H_i^t \nu_i^t\right)'\right] - E\left[H_i^t G_i^t\right] \left(E\left[G_i^{t\prime}\left(\Omega_i^t\right)^{-1} G_i^t\right]\right)^{-1} E\left[H_i^t G_i^t\right]'.$$

Therefore,

$$V(H) - V(H^*) = \left(E\left[H_i^t G_i^t\right]\right)^{-1} \left(E\left[U_i^t U_i^{t\prime}\right]\right) \left(E\left[H_i^t G_i^t\right]\right)^{-1\prime}.$$

Since $E\left[U_i^t U_i^{t\prime}\right]$ is positive semi-definite, $V(H) - V(H^*)$ is also positive semi-definite. Therefore, the asymptotic variance is minimized at H^*.

8.11 Proof of Proposition 5

We use the strategy in Section 6.1 of Newey and McFadden (1994) to establish asymptotic properties of two-step estimators. To this end, we define the joint moment equation for the two estimating steps:

$$\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) \equiv \arg\min_{\theta_2, \theta_1} \left(\sum_{i,t} e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)\right)' \left(\sum_{i,t} e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)\right) \tag{58}$$

where

$$e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1) \equiv \left[\begin{array}{cc} H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}_1) v_i^t(\boldsymbol{\theta}_2) & H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}_0) v_i^t(\boldsymbol{\theta}_1) \end{array}\right]$$

We have that $\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) \xrightarrow{p} (\boldsymbol{\theta}, \boldsymbol{\theta})$, with an asymptotic variance given by

$$Var\left(\hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_1\right) = \left(\tilde{G}'\tilde{\Omega}^{-1}\tilde{G}\right)^{-1}$$

where $\tilde{G} \equiv \left[\nabla_{(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)} e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)\right]$ and $\tilde{\Omega} \equiv E\left[\left(e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)\right)\left(e_i^t(\boldsymbol{\theta}_2, \boldsymbol{\theta}_1)\right)'\right]$.

Define $h_i^t \equiv H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}) e_i^t(\boldsymbol{\theta})$ and $\bar{h}_i^t \equiv H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}_0) e_i^t(\boldsymbol{\theta})$. Thus, $\tilde{G}$ and $\tilde{\Omega}$ are given by

$$\tilde{\Omega} = E\left[\begin{array}{cc} h_i^t h_i^{t\prime} & h_i \bar{h}_i^{t\prime} \\ \bar{h}_i^t h_i^{t\prime} & \bar{h}_i^t \bar{h}_i^{t\prime} \end{array}\right] \quad \text{and} \quad \tilde{G} = \left[\begin{array}{cc} G & G_1 \\ 0 & G_2 \end{array}\right]$$

where

$$G \equiv E\left[H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}) \nabla_{\boldsymbol{\theta}} v_i^t(\boldsymbol{\theta})\right]$$

$$G_1 \equiv E\left[\nabla_{\boldsymbol{\theta}} H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}) v_i^t(\boldsymbol{\theta})\right]$$

$$G_2 \equiv E\left[H_i^*(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t}|\boldsymbol{\theta}_0) \nabla_{\boldsymbol{\theta}} v_i^t(\boldsymbol{\theta})\right].$$

By Assumption 4c, any function of $(\hat{\boldsymbol{\eta}}^{R,t}, \hat{\boldsymbol{\eta}}^{C,t})$ is orthogonal to $v_i^t(\boldsymbol{\theta})$, which implies that $G_1 = 0$. Thus, $(\tilde{G}'\tilde{\Omega}^{-1}\tilde{G})^{-1}$ is block diagonal and the marginal distribution of $\hat{\boldsymbol{\theta}}_2$ is asymptotically normal with variance

$$Var\left(\hat{\boldsymbol{\theta}}_2\right) = \left(G'\Omega^{-1}G\right)^{-1},$$

which is equivalent to the asymptotic distribution of the Optimal IV in (57).

9 Data Construction

This appendix describes the procedure to construct bilateral trade flows among 722 US CZs and 52 countries in 1990 and 2000 for 367 4-digit SIC manufacturing industries and one non-manufacturing sector.

9.1 Summary Statistics

We start by reporting in Table 8 summary statistics for the main variables in our empirical application. The dispersion of employment changes is higher than that of wage changes in both periods. Our main shock exposure measures IPW_i^t and IPC_i^t have larger average and dispersion in 2000-2007 than in 1990-2000. This arises because the increase in imports from China was stronger after China's accession to WTO in 2001. Finally, the measures of the CZ's indirect shock exposure have lower dispersion than the measure of the CZ's own exposure.

Table 8: Summary Statistics

	1990-2000		2000-2007		1990-2007	
	Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
	(1)	(2)	(3)	(4)	(5)	(6)
100 x Change in log w_i	12.39	4.65	3.84	5.51	16.23	6.47
100 x Change in log L_i	11.73	11.64	8.85	10.21	20.58	18.70
IPW_i^t	0.91	0.96	1.76	1.78	2.67	2.52
IPC_i^t	2.27	0.52	5.19	0.92	7.45	1.22
$\sum_{j\neq i} z_{ij} IPW_j^t$	0.84	0.59	1.73	1.00	2.57	1.51
$\sum_{j\neq i} z_{ij} IPC_j^t$	2.33	0.32	5.35	0.63	7.68	0.85

Notes: Sample of 722 Commuting Zones. Indirect effects computed with $z_{ij} \equiv L_j^0 D_{ij}^{-\delta} / \sum_k L_k^0 D_{ik}^{-\delta}$ where $\delta = 5$, D_{ij} is the distance between CZs i and j, and L_j^0 is the population of CZ j in 1990.

9.2 Methodology

Country-to-country bilateral trade matrix. We start by creating a country-to-country matrix of trade flows at the 4-digit SIC classification. We consider the countries listed in Table 9.

We obtain international trade flows at the product-country level from the BACI dataset, assembled by CEPII, which we aggregate at the 4-digit SIC level. Since the starting year of the BACI dataset is 1995, we use the trade flows for 1995 and 2000.[42] To obtain domestic spending shares for each country, we note first that our gravity model implies that $X_{ij,s}^t = \tau_{ij,s}^t (p_i^t)^{-\epsilon_s} (P_{j,s}^t)^{\epsilon_s} E_{j,s}^t$. We define two aggregate sectors: manufacturing and non-manufacturing. For any sector s within an aggregate sector S, assume that, for $i \neq j$, $\tau_{ij,s}^t = \zeta_{i,s}^t \tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{j,S}^{D,t} e^{\tilde{\tau}_{ij,s}^t}$ with $E_i[\tilde{\tau}_{ij,s}^t] = 0$ and $E_j[\tilde{\tau}_{ij,s}^t] = 0$. For $i = j$, $\tau_{ii,s}^t = \zeta_{i,s}^t$, implying that only productivity affects domestic sales. So,

$$\ln X_{ij,s}^t = \tilde{\tau}_{ij,s}^t + \alpha_{i,s}^t + \varphi_{j,s}^t, \tag{59}$$

where $\alpha_{i,s}^t \equiv \ln \tilde{\tau}_{i,S}^{O,t} \zeta_{i,s}^t (p_i^t)^{-\epsilon_s}$ and $\varphi_{j,s}^t \equiv \ln \tilde{\tau}_{j,S}^{D,t} (P_{j,s}^t)^{\epsilon_s} E_{j,s}^t$.

[42] Although there is trade data available for 1990 from UN Comtrade, the data is quite sparse across countries and industries.

Table 9: Sample of Countries

Argentina	Malaysia
Australia	Mexico
Austria	Netherlands
Baltic Republics	New Zealand
Belarus	Norway
Benelux	Pakistan
Brazil	Philippines
Bulgaria	Poland
Canada	Portugal
Chile	Rest of World
China	Romania
Colombia	Russia
Croatia	Saudi Arabia
Czech Republic	Singapore
Denmark	Slovakia
Finland	South Africa
France	South Korea
Germany	Spain
Greece	Sweden
Hungary	Switzerland
India	Taiwan
Indonesia	Thailand
Ireland	Ukraine
Italy	United Kingdom
Japan	Uruguay
Kazakhstan	Venezuela

Notes: Baltic Republics includes Estonia, Lithuania and Latvia.

To get the domestic trade flows, notice that $X_{ii,s}^t = \zeta_{i,s}^t (p_i^t)^{-\epsilon_s} (P_{i,s}^t)^{\epsilon_s} E_{i,s}^t = \left(e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t}\right) / \left(\tilde{\tau}_{i,S}^{O,t} \tilde{\tau}_{i,S}^{D,t}\right)$. Since $X_{ii,S}^t = \sum_{k \in S} X_{ii,k}^t$,

$$X_{ii,s}^t = X_{ii,S}^t \frac{e^{\alpha_{i,s}^t} e^{\varphi_{i,s}^t}}{\sum_{k \in S} e^{\alpha_{i,k}^t} e^{\varphi_{i,k}^t}} \tag{60}$$

We use (60) to compute $X_{ii,s}^t$. In each year t, we obtain $\alpha_{i,s}^t$ and $\varphi_{j,s}^t$ from the estimation of (59) with bilateral trade flows by sector, and $X_{ii,S}^t$ from the domestic sales in manufacturing and non-manufacturing in the Eora MRIO dataset.

CZ employment share. We use the same imputation procedure of ADH to compute employment in each 4-digit SIC manufacturing industry for 1980, 1990 and 2000 using the County Business Pattern (CBP). In year t, we use $L_{i,s}^t$ to denote employment in CZ i and 4-digit SIC industry s and $y_{i,s}^t = L_{i,s}^t / L_i^t$ to denote the associated employment share.

CZ spending shares. We construct spending by sector and CZ, $\xi_{i,s}^t$, using

$$\xi_{i,s}^t \equiv \frac{E_{i,s}^t}{E_i^t} = \frac{\gamma_s^t + \sum_k \theta_{sk}^t b_k^t y_{i,k}^t}{1 + \sum_k b_k^t y_{i,k}^t}. \tag{61}$$

where, in year t, θ^t_{sk} is the share of spending on intermediates of sector s by sector k (common to all CZs), b^t_k is the ratio of intermediate cost to labor cost of sector k (common to all CZs), and γ^t_s is consumers' spending share on final goods of sector s (common to all CZs). We compute $\theta^t_{sk} \equiv \frac{M^t_{sk}}{\sum_{s'} M^t_{s'k}}$ where M^t_{sk} is the spending of industry k on industry s in 1992 in the BEA 1992 U.S. Input-Output table used in Acemoglu et al. (2016). For manufacturing SIC-4 industries, we compute b^t_k using total material costs divided by payroll in the NBER manufacturing database for year t. For non-manufacturing industries, we compute b^t_k as average the material to payroll ratio across all U.S. non-manufacturing industries in the WIOD database. Finally, we obtain γ^t_s from the BEA 1992 U.S. Input-Output table.[43]

CZ exports and imports. We follow three steps to create exports and imports for each CZ and industry. First, we compute the CZ spending on sector s as $E^t_{i,s} = \xi^t_{i,s} L^t_i$ where $\xi^t_{i,s}$ is the sectoral spending share described above and L^t_i is the total employment in the CZ. Second, for each sector s, we compute the share of CZ i in national spending, $\tilde{\xi}^t_{i,s} = E^t_{i,s}/\sum_j E^t_{j,s}$, and in national employment, $\tilde{y}^t_{i,s} = L^t_{i,s}/\sum_j L^t_{j,s}$. Third, we use the US Census data at the state-sector level for 1997 to compute the share of exports/imports of each state for each foreign country in a SCTG category, which is the 40-sector classification used by the US Census.[44] This yields $\beta_{state,i,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign importer, and $\beta_{i,state,s} = \frac{X_{state,i,s}}{X_{US,i,s}}$, where i is any of 52 foreign exporters. We use the same share $\beta_{state,i,s}$ and $\beta_{state,i,s}$ for all SIC-4 industries within the same SCTG category. Finally, in each year t, we take US imports $X^t_{i,US,s}$ and US exports $X^t_{US,i,s}$ in each sector s and foreign country i, and split them across CZs using the following expression:

$$X^t_{ij,s} = \frac{\tilde{\xi}^t_{j,s}}{\sum_{j \in state} \tilde{\xi}^t_{j,s}} \beta_{i,state,s} X^t_{i,US,s} \quad \text{and} \quad X^t_{ji,s} = \frac{\tilde{y}^t_{j,s}}{\sum_{j \in state} \tilde{y}^t_{j,s}} \beta_{state,i,s} X^t_{US,i,s}.$$

CZ-to-CZ bilateral trade matrix. We follow three steps to impute trade flows across CZs using the gravity trade structure of our model. First, for each SCTG category, we use state-to-state shipment data from the Commodity Flow Survey in 1997 to estimate

$$\ln X_{ij,s} = \delta_s + \beta_1 \ln D_{ij} + \beta_2 \ln E_{j,s} + \beta_3 \ln Y_{i,s} + \beta_4 d_{i=j} + \varepsilon_{ij,s} \tag{62}$$

where i is the origin state, j is the destination state, s is the SCTG sector, D_{ij} is the distance between i and j, $E_{j,s}$ are expenditures, $Y_{i,s}$ are production, $d_{i=j}$ is a dummy that equals 1 when $i = j$.

Second, we use the estimated coefficients to impute trade flows across CZs with the following gravity specification:

$$\ln X^t_{ij,s} \equiv \hat{\beta}_1 \ln D_{ij} + \hat{\beta}_2 \ln \tilde{\xi}^t_{j,s} + \hat{\beta}_3 \ln \tilde{y}^t_{i,s} + \hat{\beta}_4 d_{state(i)=state(j)} \tag{63}$$

where D_{ij} is the distance between CZ i and j, and $d_{state(i)=state(j)}$ is a dummy equal 1 if i and j belong to the same state.

Lastly, we re-scale the imputed CZ-to-CZ trade flows to sum to the total US domestic sales in each SIC sector (as in the country-to-country trade matrix).

[43] The final consumption shares vary across sectors but not across CZs. We take this approach because we are not aware of any comprehensive data on final consumption shares by CZ. The Consumer Expenditure Survey produced by the BLS only covers 26 selected MSA and does not vary by manufacturing sectors. We verify that, across these MSAs, consumption shares in the CEX display little variation – for instance, the average share of consumption on food and apparel is 19%, with a standard deviation of only 2% across MSAs.

[44] We construct state-sector exports and imports as follows. First, we use the US Merchandise Trade Data for 1997 released by the US Census to create a mapping from each of the 44 US districts to the 50 US states, in terms of share of imports and exports to each foreign country. Note that this is done at the aggregate level as this information is not available at the industry-level. We then use US Census data to create district-level exports and imports at the HS-6 level for 1997. Finally, we use the mapping previously constructed to obtain state-HS6, and then state-SIC 4 digit, trade flows with our sample of foreign countries.

Trade balance. Finally, we impose that trade is balanced at the regional level, as in the baseline model. We use the trade flows obtained above to compute matrix $\bar{\boldsymbol{x}}^t$ whose entries correspond to the share of spending of each region j on another region i. Under trade balance, the vector of total revenue in the world economy, $\boldsymbol{Y}^t$, must satisfy $\bar{\boldsymbol{x}}^t\boldsymbol{Y}^t = \boldsymbol{Y}^t$ and, therefore, $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)\boldsymbol{Y}^t = 0$. Notice that it is always possible to find a vector $\boldsymbol{Y}^t$ that satisfies this system since $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ is singular ($\sum_i x_{ij}^t = 1$ for every j). Thus, we find the vector $\boldsymbol{Y}^t$ as the eigenvector of $(\bar{\boldsymbol{I}} - \bar{\boldsymbol{x}}^t)$ associated with the eigenvalue of zero. We then normalize it such that world GDP is one, $\sum_i Y_i^t = 1$.

9.3 Validation Tests

We first evaluate the correlation between the expenditure shares $\xi_{i,s}^t$ constructed in equation (61) and the spending shares implied by the shipment data for US states. To this end, for each of the 40 SCTG categories, we compute state-level total shipment inflow in the Commodity Flow Survey (CFS) for 1997. We then aggregate our expenditure shares at the SCTG level using a crosswalk between SIC-4 and SCTG categories, and compute total spending by state-SCTG using the size of the CZs in the state. Table 10 reports the result of a regression of the expenditure shares in the CFS on our constructed spending shares in 1990 and 2000. We can see that they are positively and significantly correlated, with an OLS coefficient close to 1 and a R2 of 0.95.

We then proceed to assess whether our constructed CZ-level trade matrix reproduces the patterns of observed trade flows for US states. We use the CFS to measure bilateral shipments between US states in each SCTG category for 1997, 2002 and 2007. To obtain comparable data, we use the methodology in Appendix 9.2 to construct the trade matrix for CZs and SIC industries for 1997, 2002, and 2007. We then aggregate this data at the state-SCTG level in each year.

Table 11 reports the results of regressing actual shipment data on the corresponding trade flow obtained from our trade matrix. Column (1) considers domestic flows between US states, column (2) considers export flows from US states to foreign countries, and column (3) considers import flows from foreign countries to US states. All specifications include sector fixed-effects. We can see that the predicted trade flows are significantly and positively related to the actual flows, with coefficients close to 1. Notice also that our imputed data captures a large share of the variation in bilateral trade flows. The R2 is above 0.8 for exports and imports of US states, and around 0.5 for domestic flows between US states.

Table 10: Validation Test – Predicted Expenditure Shares

Dependent variable:	Observed expenditure shares, 1997	
	(1)	(2)
Constructed expenditure shares, 1990	1.275***	
	(0.01)	
Constructed expenditure shares, 2000		1.265***
		(0.01)
Constant	-0.009***	-0.009***
	(0.00)	(0.00)
Observations	1,392	1,392
R^2	0.95	0.95

Notes: Sample of 1,392 state-SCTG pairs, where SCTG is the industry classification used in the CFS. Dependent variable is the observed expenditure share in 1997 computed from the CFS. The regressors are the expenditure shares computed in equation (61), aggregated at the state-SCTG level. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Table 11: Validation Test – Bilateral Trade Flows

	(1)	(2)	(3)
***Panel A:* Log of Actual Flows in 1997**			
Log of Predicted Flows in 1997	1.068***	0.973***	0.993***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.512	0.950	0.950
***Panel B:* Log of Actual Flows in 2002**			
Log of Predicted Flows in 2002	1.024***	0.847***	0.884***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.509	0.816	0.837
***Panel C:* Log of Actual Flows in 2007**			
Log of Predicted Flows in 2007	1.047***	0.797***	0.861***
	(0.01)	(0.00)	(0.00)
Observations	64,512	68,544	68,544
R^2	0.477	0.806	0.827
Flow type:			
U.S. state to U.S. state	Yes	No	No
U.S. state to Country	No	Yes	No
Country to U.S. state	No	No	Yes

Notes: The dependent variable in column (1) is the actual shipment flow reported in the CFS for state-state-SCTG triples. The dependent variables in columns (2) and (3) are trade flows constructed from the US Census trade data for state-country-SCTG triples. The regressors are the trade flows constructed using our methodology for the years 1997, 2002 and 2007, aggregate at the state-state-SCTG or state-country-SCTG level. All regressions include sector fixed effects. Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$