NBER WORKING PAPER SERIES

ENDOGENOUS TRADE PROTECTION AND EXCHANGE RATE ADJUSTMENT

Stéphane Auray
Michael B. Devereux
Aurélien Eyquem

Working Paper 25517
http://www.nber.org/papers/w25517

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2019

We thank George Alessandria, Matteo Cacciattore, Larry Christiano, Martin Eichenbaum, Fabio Ghironi, Jean Imbs, Nir Jaimovich, Andrea Raffo, Morten Ravn, Sergio Rebelo, Pedro Teles, Shang-Jin Wei and seminar participants at the Frontiers in International Macroeconomics conference in Rennes, 2018 and at the Trade in Dynamic Stochastic General Equilibrium Models Conference in Santa Barbara, 2018 for comments. Devereux thanks SSHRC for research funding. Auray and Eyquem acknowledge the financial support of Projets Generique ANR 2015, Grant Number ANR-15-CE33-0001-01. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Endogenous Trade Protection and Exchange Rate Adjustment
Stéphane Auray, Michael B. Devereux, and Aurélien Eyquem
NBER Working Paper No. 25517
January 2019
JEL No. F13,F33,F41

ABSTRACT

This paper explores the relationship between exchange rate adjustment and trade policy in a simple New Keynesian open economy macro model. We show that movement in exchange rates have a direct implication for trade policy when governments choose tariffs endogenously. In particular, we show that the strategic incentive to impose trade restrictions is greater under flexible exchange rates than when exchange rates are fixed. This surprising result goes counter to conventional wisdom, which suggests that pressures to impose trade restrictions are greater when countries resist adjustments in exchange rates. But in fact, we show that the empirical evidence supports the model predictions. The paper goes on to characterize the path of equilibrium sustainable tariffs in the presence of sticky prices and flexible exchange rates. In our baseline model, tariff rates will rise in response to monetary policy shocks, but fall in response to productivity shocks. Estimating an SVAR model, we also find evidence in support of this prediction.

Stéphane Auray
CREST (UMR9194) - Ensai and
Université du Littoral Côte d'Opale.
ENSAI
Campus de Ker-Lann
Rue Blaise Pascal
BP 37203, 35172 BRUZ Cedex,
France
stephane.auray@ensai.fr

Michael B. Devereux
Department of Economics
University of British Columbia
997-1873 East Mall
Vancouver, BC V6T 1Z1
Canada
and NBER
mbdevereux@gmail.com

Aurélien Eyquem
Univ Lyon,
Université Lumière Lyon 2
GATE L-SE UMR 5824 and
Institut Universitaire de France
93 Chemin des Mouilles
BP167, 69131 Ecully Cedex
France
aurelien.eyquem@univ-lyon2.fr

1 Introduction

International macroeconomics typically treats the determination of trade policy separately from the determination of exchange rates. Exchange rate adjustment is a central feature of open economy business cycles and international macroeconomic transmission. But both exchange rate movements and trade policy, such as tariffs or quotas, affect relative prices, competitiveness, and trade flows. The factors which are important for the determination of exchange rate policy are likely to have implications for trade policy, and *vice versa*. Recent events suggest that protectionist trade policies are on the increase around the world. To the extent that trade restrictions attempt to affect macroeconomic outcomes such as competitiveness or the trade balance, their evaluation and impact may not be easily separated from the movements in exchange rates.

To take a concrete example, imagine a scenario where a country receiving a negative demand shock experiences an exchange rate depreciation which acts as a stabilizing mechanism, improving its trade balance and reducing the fall in domestic income. The exchange rate plays a role in cushioning the impact of the shock, increasing the trade balance, but at the same time, reducing the trade balance of partner countries. In this instance, one might expect that partner countries might have an incentive to respond to the appreciation of their currency by imposing trade restrictions. If trade restrictions follow exchange rate movements, then the stabilizing role of exchange rate adjustment in business cycle transmission may be diminished. It follows that one cannot analyze the mechanism of exchange rate adjustment without at the same time looking at the incentives behind trade policy.

A contemporary example of this perspective might be seen in the newly negotiated 'USMCA'; the Canada-US-Mexico trade pact which is set to replace NAFTA. Chapter 33 of this proposed agreement concerns the set-up of a 'Macroeconomic Committee' comprised of representatives from the three countries that will oversee macroeconomic policy in each country, designed to avoid 'competitive devaluation' and ensure 'free market' determination of exchange rates. Although the implications of this new body is unclear, it is not unrealistic to imagine that it could constrain a central bank from an easing of monetary policy following a negative shock, and thus prevent desired exchange rate adjustment, for fear of retaliatory trade restrictions.

This paper explores the relationship between exchange rate adjustment and trade policy in a simple open economy macro model. We show that movement in exchange rates have a direct implication for trade policy when governments choose tariffs endogenously. In particular, we show that the strategic incentive to impose trade restrictions is greater under flexible exchange rates than when exchange rates are fixed. This surprising result goes counter to conventional wisdom, which suggests that pressures to impose trade restrictions are greater when countries resist adjustments in exchange rates. But in fact, we show that the empirical evidence supports the model predictions.

The paper goes on to look at the business cycle determinants of trade policy and exchange rates in a dynamic game in which governments choose tariffs subject to time-varying incentive constraints. We identify the best sustainable tariff policies which satisfy this incentive constraint. The environment is one in which both demand and supply shocks may affect the dynamics of the best sustainable trade policy. In other words, we investigate whether the trade restrictions implied by the model are influenced by the business cycle.

We show that the cyclicality of trade policy is critically tied to the presence of sticky prices. In an economy with fully flexible prices, and persistent productivity shocks, tariffs will be essentially a-cyclical. In particular, a permanent shock to productivity in either country has no impact on the equilibrium tariff policy. This is because such a shock leads to an equal increase in the costs and benefits of deviating from a sustainable tariff policy, thus leaving the equilibrium sustainable tariff unchanged. By contrast, with sticky prices, in which trade policy is determined conditional on the preset prices, protectionism will vary at the business cycle frequency.

Are equilibrium tariffs pro-cyclical or countercyclical? The answer to this question depends both on the drivers of the business cycle; in our model these are both monetary shocks and productivity shocks, and the parameters of the underlying model – in particular the elasticity of inter-temporal substitution. In our baseline case, assuming a relatively high elasticity of intertemporal substitution, which we argue is appropriate for models of international trade, we show that demand shocks, in the form of innovations to monetary policy, will lead to an increase in trade restrictions, since these shocks will lead to a more binding incentive constraint on trade policy determination for each policy maker. On the other hand, persistent supply shocks (technology shocks) will relax the incentive constraint and reduce equilibrium trade restrictions. There is also empirical support for this implication of the model. Evidence from a simple structural VAR model for a subset of OECD countries with a flexible exchange rate regime provides support for this prediction about the link between business cycle frequency shocks and trade restrictions.

Our model has a further implication for the effects of cyclical macro shocks on protectionism. As noted above, with fully flexible prices, sustainable tariff rates are unresponsive to macro shocks, but in fact the average sustainable tariff level is higher, the greater the uncertainty in productivity. By contrast, with sticky prices, a rise in the volatility of productivity shocks leads to a large reduction in the mean sustainable tariff rate. We show that this is tied to the impact of uncertainty on the price level, when productivity shocks are unpredictable. An increase in the variance of productivity shocks, through its effect on price levels, causes a large increase in the costs of deviating from a sustainable tariff path, and as a result reduces the incentive for any country to deviate. Thus, our analysis implies that both first and second moments of equilibrium tariff rates are critically tied to movements in exchange rates and macro shocks in the basic New Keynesian open economy macro model.

Our paper is not the first to investigate the question of the interaction between exchange rates and trade restrictions or tariffs. Recently, Barattieri, Cacciatore, and Ghironi (2018) investigate empirically the impact of exogenous changes in tariffs, and show that they act as negative supply shocks, depressing GDP and raising inflation with little effects on the trade balance. They propose a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt a mirror perspective, considering tariffs as endogenous and ask how governments react to economic conditions to determine trade policies over the business cycle. Another recent paper by Erceg, Prestipino, and Raffo (2018) studies the effects of trade policies in the form of import tariffs and export subsidies. They show that the macroeconomic effects of these policies critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. Finally, a recent paper by Furceri, Hannan, Ostry, and Rose (2018) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary.

Focusing more closely on the endogenous determination of trade policies, Eaton and Grossman (1985) study optimal tariffs when international asset markets are incomplete and show that they can be used to partly compensate the lack of consumption insurance. Bergin and Corsetti (2015) also consider tariffs as policy instruments in addition to monetary policy but their focus is not specifically on tariffs, rather on the implications of monetary policy on the building of comparative advantages. Closer to our paper, Bagwell and Staiger (2003) propose a trade model featuring potential terms-of-trade manipulation by governments, and trade agreements as means to restrict this policy option. They find that the resulting trade restrictions are countercyclical, because the potential costs of opting out is pro-cyclical. Although our model has some differences, we adopt a similar logic by looking at the incentive behind sustainable tariffs, but our paper is complementary to theirs. In particular, it allows for different sources of business cycle fluctuations, and shows that the cyclical pattern of trade restrictions depends on the type of shocks. In addition, we also consider the importance of the exchange rate regime, as well as sticky prices in the determination of trade policies.

The paper is structured as follows. Section 2 develops a simple, tractable open economy model where tariffs play a role in affecting the exchange rate and the terms of trade. Section 3 looks at the welfare effects of tariffs in this model, shows that the incentive to set tariffs depends on the exchange rate regime and that incentives are higher under fixed exchange rate regimes. Section 4 then extends the analysis to allow for endogenous tariffs, where tariffs are determined as a sustainable equilibrium in a dynamic game between governments. This section shows that tariffs are acyclical under flexible prices, but with sticky prices (where prices are preset one period in advance), tariffs vary over the business cycle. In particular, tariffs respond negatively (respectively positively) to expansionary supply (resp. demand) shocks. Section 5 extends the analysis to a more elaborate dynamic model with gradual price adjustment and a Taylor-type monetary policy rule. We show that the results of the analytical model carry over to the more general specification. Section 6 is empirical. In Section 6.1, we use the Global Anti-Dumping database maintained by the WTO, and find evidence that protectionist tensions are conditionally more prevalent under flexible exchange rates. In Section 6.2, we present time series evidence from a sub-set of countries that maintain flexible exchange rates. Using SVAR identification assumptions, we show that supply shocks tend to reduce protectionism, while monetary policy shocks tend to increase protectionism, as predicted by the models. Section 7 presents some conclusions.

2 A tractable model with pre-set prices

To explore the ideas discussed in the introduction, we begin by developing a two country open economy framework with endogenous trade policy that may depend on macroeconomic conditions. Our first model is extremely simple. It can be solved with pen and paper, but despite this, it carries quite a rich set of implications for the relationship between exchange rate regimes, business cycle shocks, and protectionism. In a later section, we extend the analysis to a more elaborate dynamic model which is solved numerically.

2.1 Households

We start with a plain vanilla two-country model. The two countries are home and foreign. In each country, households earn wages and profits in each period, supply labor, and consume home and foreign goods. In this version of the model there is no capital mobility across countries. The home country utility is

$$U = \frac{C^{1-\sigma}}{1-\sigma} - \frac{H^{1+\psi}}{1+\psi}$$

where C is the consumption aggregator and H is labor supply. The consumption aggregator is Cobb Douglas, and depends on home goods and foreign goods in the following way:

$$C = C_x^{\omega} C_m^{1-\omega}$$

Households in the home country face the budget constraint:

$$M + P_x C_x + (1+\tau) S P_m C_m = WH + \Pi + T + M_0 \tag{1}$$

where τ is a tariff levied on the imported good, M represents money holdings, S is the nominal exchange rate, P_x is the home currency price of home goods and P_m is the foreign currency price of foreign goods. Variable W is the nominal wage rate, and T is a lump-sum rebate of the tariff revenue, so that:

$$T = \tau S P_m C_m \tag{2}$$

We assume that there is a binding cash in advance constraint for households, which acts so as to pin down nominal magnitudes:

$$M \geq P_x C_x + S P_m C_m$$

where M is home country money supply, set exogenously by the home monetary authority. In the dynamic model presented in Section 5, we expand the framework to allow monetary policy to be set as an interest rate rule. For simplicity, in this section we make the assumption that the cash in advance requirement exempts tariff payments. The consumer's first-order conditions are standard, and optimal consumption implies

$$P_x C_x = \frac{\omega}{1-\omega} S P_m (1+\tau) C_m \tag{3}$$

Optimal labor supply and the cash in advance constraint imply

$$W = H^{\psi} C^{\sigma-1} \frac{P_x C_x}{\omega} \tag{4}$$

$$M = P_x C_x + S P_m C_m \tag{5}$$

2.2 Firms

Firms choose prices to maximize profits. Sticky prices are a key factor in the analysis. To avoid intrinsic dynamics in the model, we assume that prices have to be set in advance of the within-period (monetary and productivity) shocks. But once the shocks are realized, prices can fully adjust before the next period. Firms operate with a linear technology:

$$Y_x = \theta H$$

and profits are given by:

$$\Pi = P_x Y_x - WH$$

The optimal price satisfies:[1]

$$P_x = \frac{E \frac{W}{\theta} Y_x \lambda}{E Y_x \lambda}$$

where E is the expectation operator, and λ is the household's Lagrange multiplier for nominal income:

$$\lambda = \frac{1}{C^\sigma P} \tag{6}$$

where

$$P = P_x^\omega ((1+\tau) S P_m)^{1-\omega} \tag{7}$$

The following further conditions define the equilibrium market clearing in goods and money:

$$\begin{aligned} Y_x &= C_x + C_x^* \\ Y_m &= C_m + C_m^* \\ M &= M_0 \\ M^* &= M_0 \end{aligned}$$

2.3 Solving the model

For given tariffs, using (1), (2), and (5) with $M = M_0$ we can derive:

$$C_x = \frac{\omega(1+\tau)}{\omega(1+\tau)+1-\omega} Y_x, \quad C_x^* = \frac{\omega^*}{(1-\omega^*)(1+\tau^*)+\omega^*} \frac{S P_m Y_m}{P_x}$$

$$C_m = \frac{(1-\omega)}{\omega(1+\tau)+1-\omega} \frac{P_x Y_x}{S P_m}, \quad C_m^* = \frac{(1-\omega^*)(1+\tau^*)}{(1-\omega^*)(1+\tau^*)+\omega^*} Y_m$$

We can also show that $PC = \frac{P_x C_x}{\omega}$, so that the wage W above is

$$W = H^\psi C^\sigma P$$

and therefore the expression for the pre-set price may be written as

$$P_x = \frac{E \frac{H^\psi Y_x}{\theta}}{E \frac{Y_x}{C^\sigma P}} \tag{8}$$

We write out the equilibrium in a simple form as

$$Y_x = \frac{\omega(1+\tau)}{\omega(1+\tau)+1-\omega} Y_x + \frac{\omega^*}{(1-\omega^*)(1+\tau^*)+\omega^*} Q Y_m \tag{9}$$

$$Y_m = \frac{1-\omega}{\omega(1+\tau)+1-\omega} \frac{Y_x}{Q} + \frac{(1-\omega^*)(1+\tau^*)}{(1-\omega^*)(1+\tau^*)+\omega^*} Y_m \tag{10}$$

where $Q = S P_m / P_x$ is the terms of trade, with

$$Y_x = \frac{M}{P_x} \tag{11}$$

$$Y_m = \frac{M^*}{P_m} \tag{12}$$

[1] Implicitly, we are assuming that the domestic good is differentiated into Dixit-Stiglitz substitutable products, so that firms have a well-defined maximization problem. In addition, we are assuming that price markups are offset with an optimal subsidy to eliminate monopoly distortions.

2.4 Price solutions

Expanding (8) using (6), (7), (11) and (12), we get the expression for the optimal price of home goods as

$$P_x^{\psi+\sigma} = \frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(M^{1-\sigma}((1+\tau)Q)^{(1-\omega)(\sigma-1)}\left(\frac{1+\tau}{\omega(1+\tau)+1-\omega}\right)^{-\sigma}\right)} \tag{13}$$

and for foreign goods:

$$P_m^{\psi+\sigma} = \frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(M^{*(1-\sigma)}\left(\frac{(1+\tau^*)}{Q}\right)^{\omega^*(\sigma-1)}\left(\frac{1+\tau^*}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{-\sigma}\right)} \tag{14}$$

where we have used $P = P_x((1+\tau)Q)^{1-\omega}$ and $P^* = P_m((1+\tau^*)/Q)^{\omega^*}$. Equations (13) and (14) also use the property that

$$C = \frac{P_x C_x}{\omega P}, \text{ and } C^* = \frac{P_m C_m^*}{(1-\omega^*)\,P^*}$$

The analysis of tariff setting in the model below depends critically on the way in which pre-set prices depend on money and productivity shocks. We can illustrate this linkage by explicitly solving (13) and (14). First, starting from (9), we can express the terms of trade as:

$$Q = \frac{Y_x}{Y_m}\left(\frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+1-\omega}\right) \tag{15}$$

Then, using (11) and (12) we get the following nominal exchange rate equation:

$$S = \frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+(1-\omega)}\frac{M}{M^*} \tag{16}$$

Finally, using (16) along with the price equations (13) and (14), we can express the home and foreign goods price as a function of the underlying shocks:

$$P_x = \left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+\omega^*(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{(1-\omega)(1-\sigma)}{\Delta}} \tag{17}$$

$$P_m = \left(\frac{E\left(\frac{M^*}{\theta^*}\right)^{1+\psi}}{E\left(\frac{M^{*(1-\omega^*)(1-\sigma)}}{M^{\omega^*(\sigma-1)}}\Lambda^{*-1}(\tau,\tau^*)\right)}\right)^{\frac{\psi+\sigma+(1-\omega)(1-\sigma)}{\Delta}}\left(\frac{E\left(\frac{M}{\theta}\right)^{1+\psi}}{E\left(\frac{M^{\omega(1-\sigma)}}{M^{*(1-\omega)(\sigma-1)}}\Lambda^{-1}(\tau,\tau^*)\right)}\right)^{\frac{\omega^*(1-\sigma)}{\Delta}} \tag{18}$$

where $\Delta = (\psi+\sigma)\delta$, $\delta = (1+\psi+(\omega^*-\omega)(1-\sigma))$, and the functions Λ and Λ^* are defined as:

$$\Lambda(\tau,\tau^*) = \left(\frac{1+\tau}{\omega(1+\tau)+1-\omega}\right)^{(1-\omega)(1-\sigma)}\left(\frac{1}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{-(1-\omega)(1-\sigma)}\left(\frac{1-\omega}{\omega^*}\right)^{(1-\omega)(1-\sigma)} \tag{19}$$

$$\Lambda^*(\tau,\tau^*) = \left(\frac{1+\tau^*}{(1-\omega^*)(1+\tau^*)+\omega^*}\right)^{1-(1-\omega^*)(1-\sigma)} \left(\frac{1}{\omega(1+\tau)+1-\omega}\right)^{-\omega^*(1-\sigma)} \left(\frac{1-\omega}{\omega^*}\right)^{-\omega^*(1-\sigma)} \tag{20}$$

Hence, the solutions (17) and (18) make clear that *ex-ante* pre-set prices depend on the distribution of money shocks, productivity shocks and home and foreign tariffs. We use (17) and (18) along with (11) and (12) to compute expected home and foreign output and expected utility below. First note that if prices were fully flexible, and could adjust to money, productivity, or tariff shocks, we would have the solutions:

$$\frac{P_x}{M} = \left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\psi+\sigma+\omega^*(1-\sigma))}{\Delta}} \left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{(1-\omega)(1-\sigma))}{\Delta}}, \quad \frac{P_m}{M^*} = \left[\frac{\Lambda^*}{\theta^{*(1+\psi)}}\right]^{\frac{\psi+\sigma+(1-\omega)(1-\sigma))}{\Delta}} \left[\frac{\Lambda}{\theta^{1+\psi}}\right]^{\frac{\omega^*(1-\sigma))}{\Delta}} \tag{21}$$

With fully flexible prices money is neutral, but normalized prices are negatively related to domestic productivity shocks, and positively related to own country tariffs.

2.5 Utility measures

In order to determine the path of tariffs, it is necessary to construct welfare measures for benevolent governments in each country. Given optimal price-setting, we can express expected utility for the home country as:

$$EU = E\left(\frac{C^{1-\sigma}}{1-\sigma} - \frac{H^{1+\psi}}{1+\psi}\right) = E\Gamma\frac{C^{1-\sigma}}{1-\sigma} \tag{22}$$

where

$$\Gamma(\tau) \equiv 1 - \frac{(1-\sigma)(\omega(1+\tau)+1-\omega)}{(1+\tau)(1+\psi)} \tag{23}$$

From the equilibrium terms of trade in the previous section, we can express the consumption aggregator as:

$$C = C_x^{\omega} C_m^{1-\omega} = Y_x^{\omega} Y_m^{1-\omega} \zeta(\tau,\tau^*)$$

where $\zeta = \frac{(1+\tau)^{\omega}}{\delta\omega(\tau)^{\omega}\delta_{\omega^*}(\tau^*)^{1-\omega}}(\frac{\omega^*}{1-\omega})^{1-\omega}$, $\delta_\omega = \omega(1+\tau)+1-\omega$ and $\delta_{\omega^*} = (1-\omega^*)(1+\tau^*)+\omega^*$.

So the equilibrium period expected utility expression can be written as:

$$EU = E\frac{\Gamma(\tau)}{1-\sigma}\left(\left(\frac{M}{P_x}\right)^{\omega}\left(\frac{M^*}{P_m}\right)^{1-\omega}\zeta(\tau,\tau^*)\right)^{1-\sigma} \tag{24}$$

where the prices are expressed as the above solutions (17) and (18), and depend on expected productivity and money shocks. Expression (24) indicates that expected utility depends on the tariff rates set by the home and foreign governments. In particular, it is easy see that for expected output levels, beginning at a zero home tariff, expected utility is increasing in the home tariff rate and (always) decreasing in the foreign tariff rate.

In the case of fully flexible prices, we may combine (21) with (24) to express expected utility solely as a function of productivity and tariff shocks:

$$U(\tau,\tau^*) = \frac{\Gamma(\tau_t)}{1-\sigma}\left(\mathcal{F}(\theta_t,\theta_t^*)\mathcal{H}(\tau_t,\tau_t^*)\right)^{1-\sigma} \tag{25}$$

where we define the following functions;

$$\mathcal{F}(\theta_t,\theta_t^*)=\theta_t^{\frac{(1+\psi)(\omega(\sigma+\psi)+(1-\sigma)\omega^*)}{(\sigma+\psi)\delta}}\,\theta_t^{*\frac{(1+\psi)^2(1-\omega)}{(\sigma+\psi)\delta}}$$

$$\mathcal{H}(\tau_t,\tau_t^*)=\Lambda(\tau_t,\tau_t^*)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}\Lambda^*(\tau_t,\tau_t^*)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}\zeta(\tau,\tau^*)$$

In section 4 below, we will use (25) to construct equilibrium value functions in the tariff game between countries.

3 Tariff setting

We first look at the motives for setting tariffs among non-cooperative, benevolent governments. The incentive to employ tariffs is critically dependent upon the timing of tariff setting, and the degree of price rigidity. Let us first assume that prices are fully flexible and tariffs are set by governments that internalize the price setting activities of firms.

3.1 Tariff setting with flexible prices

When prices are fully flexible, tariff setting must take account of both the direct effect on the country terms of trade and the indirect effect on home and foreign output through endogenous labor supply. From (15) and (24) above, we can easily show that holding home and foreign output constant, a home country tariff improves the home terms of trade, and increases home welfare. But the tariff will also affect domestic and foreign output. Note that from (21) above, home output with flexible prices may be written as:

$$Y_x=\left[\frac{\theta^{1+\psi}}{\Lambda(\tau,\tau^*)}\right]^{\frac{\psi+\sigma+\omega^*(1-\sigma))}{\Delta}}\left[\frac{\theta^{*(1+\psi)}}{\Lambda^*(\tau,\tau^*)}\right]^{\frac{(1-\omega)(1-\sigma))}{\Delta}}\tag{26}$$

The impact of a home tariff on home output depends on a mix of income and substitution effects. The tariff raises the domestic terms of trade, which increases the real wage and increases labor supply. But the rise in the terms of trade also increases consumption which reduces labor supply through an income effect. When $\sigma=1$, we see from (19) and (26) above that a home tariff reduces domestic output, but has no impact on foreign output. More generally, we can evaluate the impact of a tariff on Y_x, in the special case where home bias is symmetric across the two countries, so that $\omega^*=1-\omega$, and evaluated at zero initial tariffs. We obtain:

$$\frac{dY_x}{d\tau}|_{\{\tau=\tau^*=0\}}=-Y_x\frac{(1-\omega)(\omega\psi(\sigma-1)+\omega(\sigma^2-1)+\psi+1)}{(\sigma+\psi)((2\omega-1)\sigma+\psi+2(1-\omega)))}$$

This may be positive or negative. In the case where $\sigma<1$, it is possible that substitution effects are strong enough that the tariff increases home country output. The effect of a home tariff on foreign output may be expressed as

$$\frac{dY_m}{d\tau}|_{\{\tau=\tau^*=0\}}=Y_m\frac{(1-\omega)(\sigma-1)(1+\omega\psi+\omega(\sigma-1))}{(\sigma+\psi)((2\omega-1)\sigma+\psi+2(1-\omega)))}$$

When $\sigma<1$ this is negative, as the fall in the foreign terms of trade generates substitution effects in the opposite direction to those in the home country.

Whether the tariff increases or decreases home or foreign output, it is easy to show that the direct welfare benefit from terms of trade improvement always outweighs the indirect effects on output. Even under flexible prices and endogenous output, a country gains from imposing a small tariff, conditional on the zero tariff of the foreign country. Using (25) above, we can derive the impact of a tariff on home welfare, evaluated at $\tau = \tau^* = 0$, as:

$$\frac{dU(\tau,\tau^*)}{d\tau}|_{\{\tau=\tau^*=0\}} = \Lambda \frac{(1-\omega)(1+\omega\psi+\omega(\sigma-1))}{(2\omega-1)\sigma+\psi+2(1-\omega)}$$

where $\Lambda > 0$.

3.2 Tariff setting with pre-set prices

Now we ask what are the incentives to levy tariffs in the economy with sticky prices. For this we need to be careful about the timing of tariffs. We make the following assumption. Say that tariffs are levied at the end of a period, after prices have been set by firms. In addition, we assume that there is no commitment in tariff setting. Tariffs are set in the current period for this period alone, assuming that the future trade authority sets its own tariffs. So the trade authority this period faces a static problem, setting tariffs once the prices have been set.

Here we have to distinguish between fixed and flexible exchange rates. Under flexible exchange rates, we have M and M^* exogenous, and the exchange rate is, from (16)

$$S = \frac{1-\omega}{\omega^*}\frac{(1-\omega^*)(1+\tau^*)+\omega^*}{\omega(1+\tau)+(1-\omega)}\frac{M}{M^*}$$

From the point of view of the tariff authority, Y_x is taken as given, since P_x is fixed and M is outside of its control. But a tariff can tilt the terms of trade in its favour under flexible exchange rates. Utility, given output, is just captured by C, which from the authority's perspective, when exchange rates adjust to change the terms of trade, is

$$C = \left(\frac{M}{P_x}\frac{\omega(1+\tau)}{\delta_\omega}\right)^{\omega}\left(\frac{1-\omega}{\delta_\omega}\frac{\frac{M}{P_x}}{\frac{SP_m}{P_x}}\right)^{1-\omega} = \left(\frac{M}{P_x}\right)^{\omega}\left(\frac{M^*}{P_m}\right)^{1-\omega}\zeta(\tau,\tau^*) \tag{27}$$

This is increasing in τ, so the authority has an incentive to levy tariffs starting from a point of zero tariffs $\tau = 0$. In fact, it would want an infinite tariff, given the assumption of Cobb Douglas elasticity and no production of importable. To prevent this from happening in the sustainable tariff game, we will assume a maximum possible tariff rate of τ^H.

Now look at the same situation with fixed exchange rates. An important question arises as to which monetary authority fixes the exchange rate. If it is the foreign monetary authority, then from the point of view of the home tariff setter the movements in foreign money necessary to maintain an exchange rate peg have no consequences for home welfare, as can be seen from the middle equality in, (27), when S is fixed. But if it is the home monetary authority, this is not the case, because movements in M have direct effects on the domestic output level and the domestic endowment. To resolve this question, we use the following strategic rule. If the home country deviates from a sustainable tariff (as defined below), we assume that the foreign monetary authority adjusts policy to keep the exchange rate fixed, while if the foreign country deviates, then the home monetary authority adjusts policy to maintain a fixed exchange rate.

This rule is intuitive in the sense that attempts by the home country to manipulate its effective terms of trade by increasing tariffs are offset by the foreign monetary authority, and *vice versa.*

Given this assumption, let us look at the incentive of the home monetary authority to levy tariff under fixed exchange rates. When the exchange rate is fixed by the foreign country (where the foreign monetary authority does by adjusting M^* to keep S fixed), then from the definition of utility, we have:

$$C_x^{\omega}C_m^{1-\omega} = Y_x\left(\frac{(1+\tau)^{\omega}}{\omega(1+\tau)+1-\omega}\right)\left(\frac{P_x}{SP_m}\right)^{1-\omega}\omega^{\omega}(1-\omega)^{1-\omega}$$

This is decreasing in τ, starting at $\tau = 0$, given that S and Y_x are both fixed from the point of view of the tariff setter. Also, given fixed prices and domestic monetary policy set independently of the tariff M, home labor will be independent of τ. So there is no incentive to levy a tariff under fixed exchange rates. Intuitively, the trade authority cannot affect its terms of trade under a fixed exchange rate regime, so the tariff only reduces its own welfare.

We may summarize this section in the following way:

Result 1 Given the timing assumptions of tariff setting and policy response, the incentive to levy tariffs for domestic gain is greater in a flexible exchange rate environment than under fixed exchange rates.

4 Sustainable tariffs

We now extend this logic to the case of sustainable tariffs. The idea is that there is a utility that each authority achieves from sustainable tariffs $V(\tilde{\tau}, \tilde{\tau}^*)$. If the authority cheats, and sets the maximum tariff, it gets utility $V^{cheat}(\tau^H, \tilde{\tau}^*)$. But if it cheats, then the maximum possible punishment is imposed in the next period, and lasts forever. Since $\tau = \tau^* = \tau^H$ is a Nash equilibrium, this punishment is credible. As such, the maximum sustainable tariff is one which just offsets the benefits of cheating against the losses of reverting to the maximum Nash tariff equilibrium in the next period.

However, given the timing of decision making for the tariff authority, there is an extra complication. The tariff setter chooses τ once prices have been set. But she must take into account the future consequences of either cheating or sticking to the sustainable plan, *taking into account the nature of price setting for the future.* This means that the functional statements of the gains from cheating are stated in a different way than the future consequences of cheating. Simply speaking, the authority knows that prices will adjust, and takes this into account, whereas in the current time period, prices are taken as given.

4.1 Notation

Start with some notation. There are four exogenous state variables, the two states of monetary policy, and the two states of productivity. We define these as $z_t = \{M_t, M_t^*, \theta_t, \theta_t^*\}$. We define $z^t = \{z_0 ... z_t\}$ as a state history. In addition, from the perspective of the tariff authority at any time t, the state includes the preset prices P_{xt}, P_{mt}. Thus, sustainable tariffs will be conditioned on the state z_t and preset prices. We thus define the expanded state $\tilde{z}_t = \{z_t, P_{xt}, P_{mt}\}$. The task is to derive the sequences of *sustainable tariffs* $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$.

First, for the home country tariff setter, define the one-period payoff from cheating on the sustainable tariff sequence at time t. Let $C\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)$ and $H\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)$ respectively denote the consumption and the hours worked levels that realize if the local tariff setter cheats and applies τ^H (the highest level of tariffs) while the foreign tariff setter remains on the path of sustainable tariffs $\tilde{\tau}_t^*(\tilde{z}_t)$. Then the current-period value of cheating is:

$$v^{CH}\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right) = \frac{C\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1+\psi}}{\theta_t^{1+\psi}(1+\psi)}$$

and the value of cheating at time t in state $\tilde{z}_t$ for the home tariff-setter is

$$V^{CH}(\tilde{z}_t) = v^{CH}\left(\tilde{z}_t, \tau^H, \tilde{\tau}_t^*(\tilde{z}_t)\right) + \beta E_t V^N(\tilde{z}_{t+1}) \quad (28)$$

where β is the tariff setter's discount factor, and $V^N(\tilde{z}_{t+1})$ is the continuation value, given that cheating has happened in the past. We define $V^N(\tilde{z}_t)$ as follows. The one-period payoff in the worst Nash equilibrium is defined as

$$v^N\left(\tilde{z}_t, \tau^H, \tau^{*H}\right) = \frac{C\left(\tilde{z}_t, \tau^H, \tau^{*H}\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tau^H, \tau^{*H}\right)^{1+\psi}}{\theta_t^{1+\psi}(1+\psi)}$$

Given this, the recursive form of $V^N(\tilde{z}_t)$ is written as

$$V^N(\tilde{z}_t) = v^N\left(\tilde{z}_t, \tau^H, \tau^H\right) + \beta E_t V^N(\tilde{z}_{t+1}) \quad (29)$$

That is, $V^N(\tilde{z}_t)$ is the value to the tariff setter of being in the worst Nash equilibrium forever. The one-period value of being on the sustainable path of tariff is defined as

$$v^C\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right) = \frac{C\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1-\sigma}}{1-\sigma} - \frac{Y_x\left(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tilde{\tau}_t^*(\tilde{z}_t)\right)^{1+\psi}}{\theta_t^{1+\psi}(1+\psi)}$$

Using this, we define the continuation value of being on the sustainable tariff path as

$$V^C(\tilde{z}_t) = v^C(\tilde{z}_t, \tilde{\tau}_t(\tilde{z}_t), \tau_t^*(\tilde{z}_t)) + \beta E_t V^C(\tilde{z}_{t+1}) \quad (30)$$

Equivalent definitions apply to the foreign tariff setters decision. Given the above definitions, a pair of sustainable tariff sequences $\tilde{\tau}(\tilde{z}_t)$, $\tilde{\tau}^*(\tilde{z}_t)$ is constructed from the following conditions:

$$V^{CH}(\tilde{z}_t) = V^C(\tilde{z}_t) \quad (31)$$

$$V^{*CH}(\tilde{z}_t) = V^{*C}(\tilde{z}_t) \quad (32)$$

4.2 Sustainable tariffs with flexible prices

We first illustrate the result stated in the introduction; the cyclical nature of tariffs appears only when prices are sticky. To see this, we make the following assumptions regarding the money and productivity shocks. Specifically, we assume that

$$M_t = M_{t-1}(1+\mu_t), \quad M_t^* = M_{t-1}^*(1+\mu_t^*), \quad \{\mu_t, \ \mu_t^*\} \sim \text{i.i.d.}(0, \sigma_\mu^2) \quad (33)$$

$$\theta_t = \theta_{t-1}(1+\nu_t), \quad \theta_t^* = \theta_{t-1}^*(1+\nu_t^*), \quad \{\nu_t, \ \nu_t^*\} \sim \text{i.i.d.}(0, \sigma_\nu^2) \quad (34)$$

Hence, both productivity and money growth are i.i.d. processes.

In the fully flexible price environment, Section 2 above showed that home utility may be written as

$$U(\tau_t, \tau_t^*) = \frac{\Gamma(\tau_t)}{1-\sigma} \left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau_t, \tau_t^*)\right)^{1-\sigma} \tag{35}$$

where $\Gamma(\tau_t)$, $\mathcal{F}(\theta_t, \theta_t^*)$, and $\mathcal{H}(\tau_t, \tau_t^*)$ are as defined above.

An equilibrium sustainable tariff sequence is defined as $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$, $t = 0..., \infty$. Using the value function definition (28) along with (35), the expected utility from cheating along a sustainable path may be written as

$$EV^{CH}(\tilde{z}_t) = \frac{\Gamma(\tau^H)}{1-\sigma} \left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}} \frac{\Gamma(\tau^H)}{1-\sigma} \left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tau^H, \tau^H)\right)^{1-\sigma} \tag{36}$$

where $\Xi_1 = (1+\nu_{t+1})^{\frac{(1+\psi)(\omega(\sigma+\psi)+(1-\sigma)\omega^*)}{(\sigma+\psi)\delta}} (1+\nu_{t+1}^*)^{\frac{(1+\psi)^2(1-\omega)}{(\sigma+\psi)\delta}}$ This expression uses the property that expected utility is homogeneous in productivity, and productivity shocks are i.i.d., so that $E_t(\Xi_{1t+1})^{1-\sigma}$ is constant.[2]

By contrast, the expected utility from remaining on the sustainable path is

$$EV^{S}(\tilde{\tau}_t, \tilde{\tau}_t^*) = \frac{\Gamma(\tilde{\tau}_t)}{1-\sigma} \left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}} \frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma} \left(\mathcal{F}(\theta_t, \theta_t^*)\mathcal{H}(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)\right)^{1-\sigma} \tag{37}$$

Equating (36) and (37) and canceling the term $\mathcal{F}(\theta_t, \theta_t^*)$ gives us

$$\frac{\Gamma(\tau^H)}{1-\sigma} \left(\mathcal{H}(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}} \frac{\Gamma(\tau^H)}{1-\sigma} \left(\mathcal{H}(\tau^H, \tau^H)\right)^{1-\sigma}$$

$$= \frac{\Gamma(\tilde{\tau}_t)}{1-\sigma} \left(\mathcal{H}(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} + \frac{\beta}{1-\beta E{\Xi_1}^{1-\sigma}} \frac{\Gamma(\tilde{\tau}_{t+1})}{1-\sigma} \left(\mathcal{H}(\tilde{\tau}_{t+1}, \tilde{\tau}_{t+1}^*)\right)^{1-\sigma} \tag{38}$$

Condition (38), and an equivalent condition for the foreign country, defines an equilibrium sequence of sustainable tariffs $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$, $t = 0..., \infty$. However, since (38) contains no time-varying coefficients, a stationary equilibrium for the sustainable tariff sequence for each country implies a constant tariff rate. We conclude from that the following result:

Result 2 With flexible prices, the stationary equilibrium sustainable tariffs are constant over time.

Hence, with flexible prices, protectionism is acyclical in our model. Even though countries are subject to random productivity shocks, the productivity shocks affect the costs and benefits of deviating from the sustainable path in the same way. Thus, the incentive towards increased protectionism is unaffected.

[2] We must also assume that expected utility converges, which requires $\beta E{\Xi_1}^{1-\sigma} < 1$.

4.3 Sustainable tariffs with pre-set prices

When prices are pre-set, the sustainable path of tariffs is characterized in a different manner, and critically, the equilibrium sustainable tariff sequence will be time-varying, depending on the outcome of productivity and money shocks. A key feature of the determination of tariffs is that the policy-maker in any time period takes the prices as pre-set, so that output is given. Hence, while monetary policy shocks will affect output and the gains to cheating by affecting demand, productivity shocks will not – although technology shocks affect the disutility of labor in the one-period utility of cheating, they affect the one-period utility of remaining on the sustainable path in exactly equivalent ways, so the two effects cancel out. However, productivity shocks are permanent, as described by (34) above, so these shocks will impact on future expected utility, and as such affect the costs of cheating on the sustainable tariff equilibrium.

In order to explore this trade-off, we compute the value functions faced by the tariff setters in each country when tariffs are set conditional on pre-set prices. To begin, we have the value of being in the worst Nash equilibrium as described by (29). This may be written more explicitly as

$$V^N(\tilde{z}_t) = \frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H,\tau^H)\right)^{1-\sigma} - \frac{1}{1+\psi}\left(\frac{M_t}{\theta_t P_{xt}}\right)^{1+\psi} + \beta E_{t-1}V^N(\tilde{z}_{t+1}) \tag{39}$$

where the term $\zeta(\tau^H,\tau^H)$ indicates that both countries set the highest possible tariff τ^H.

Taking expectations of (39) dated $t-1$, we can use the property (22) to show that

$$E_{t-1}V^N(\tilde{z}_t) = E_{t-1}\frac{\Gamma(\tau^H)}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H,\tau^H)\right)^{1-\sigma} + \beta E_{t-1}V^N(\tilde{z}_{t+1}) \tag{40}$$

Then, using the equilibrium home and foreign prices (13) and (14), we can express (40) as

$$E_{t-1}V_t^N(\tilde{z}_t) = \frac{\Gamma(\tau^H)}{1-\sigma}\left(\mathcal{F}(\theta_{t-1},\theta^*_{t-1})\mathcal{H}_1(\tau^H,\tau^H)\right)^{1-\sigma}E_{t-1}\left(\Xi_t\right)^{1-\sigma} + \beta E_{t-1}V^N(\tilde{z}_{t+1}) \tag{41}$$

where $\mathcal{F}$ is defined as before, and

$$\mathcal{H}_1(\tau_t,\tau_t^*) = \left(\Lambda(\tau^H,\tau^H)\Delta_1\right)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}\left(\Lambda^*(\tau^H,\tau^H)\Delta_2\right)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}\zeta(\tau^H,\tau^H)$$

In addition, the expressions Δ_1 and Δ_2 are defined as

$$\Delta_1 = \frac{E(\frac{1+\mu}{1+\nu})^{1+\psi}}{E\frac{(1+\mu)^{\omega(1-\sigma)}}{(1+\mu^*)^{-(1-\omega)(1-\sigma)}}}, \qquad \Delta_2 = \frac{E(\frac{1+\mu^*}{1+\nu^*})^{1+\psi}}{E\frac{(1+\mu^*)^{(1-\omega^*)(1-\sigma)}}{(1+\mu)^{-\omega^*(1-\sigma)}}} \tag{42}$$

Finally, Ξ_t represents a term in expected monetary shocks, defined as

$$\Xi_t = (1+\mu_t)^{\omega}(1+\mu_t^*)^{1-\omega} \tag{43}$$

Given the form of (40), we conjecture that $E_{t-1}V(\tilde{z}_t) = A_N\mathcal{F}(\theta_{t-1},\theta^*_{t-1})$, where A_N is a constant. We can show that

$$A_N = \frac{\frac{\Gamma(\tau^H)}{1-\sigma}\mathcal{H}_1(\tau^H,\tau^H)^{1-\sigma}E_{t-1}\left(\Xi_t\right)^{1-\sigma}}{1-\beta E_{t-1}\Xi_1^{1-\sigma}} \tag{44}$$

where

$$\Xi_1 = (1+\nu_t)^{\frac{(1+\psi)(\omega(\sigma+\psi)+(1-\sigma)\omega^*)}{(\sigma+\psi)\delta}}(1+\nu_t^*)^{\frac{(1+\psi)^2(1-\omega)}{(\sigma+\psi)\delta}} \tag{45}$$

Note that given the assumption of i.i.d. shocks to money and productivity growth, both $E_{t-1}(\Xi_t)^{1-\sigma}$ and $E_{t-1}(\Xi_{1t})^{1-\sigma}$ are constant, hence A_N is constant, as conjectured. So we have the full evaluation of utility from cheating at time t as:

$$V^{CH}(\tilde{z}_t) = \left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} - \frac{\left(\frac{M_t}{P_{xt}}\right)^{1+\psi}}{1+\psi} + \beta A_N \left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma} \tag{46}$$

where implicitly we are assuming that P_{xt} and P_{mt} are pre-set by firms on the assumption that the sustainable path for tariffs is in place.

Using the same logic, we may derive the full evaluation of remaining on the sustainable path as:

$$V^{S}(\tilde{z}_t) = \left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} - \frac{\left(\frac{M_t}{P_{xt}}\right)^{1+\psi}}{1+\psi} + \beta A_S \left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma} \tag{47}$$

where it can be shown that

$$A_S = \frac{E_{t-1}\left(\frac{\Gamma(\tilde{\tau}_t)}{1-\sigma}\mathcal{H}_1(\tilde{\tau}_t, \tilde{\tau}_t)^{1-\sigma}\Xi_t^{1-\sigma}\right)}{1-\beta E_{t-1}\Xi_1^{1-\sigma}} \tag{48}$$

The key difference between (44) and (48) is that the sequence of sustainable tariffs $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ is now stochastic. Thus, the value function conjecture is only verified if A_S is constant, which requires that sustainable tariffs are time-invariant functions of the shocks z_t. Since shocks are i.i.d. this would ensure that $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ are also i.i.d., verifying the conjecture. We will show below that the incentive constraint in fact ensures that $\{\tilde{\tau}_t, \tilde{\tau}_t^*\}$ is i.i.d.

Both (46) and (47) represent value functions pertaining to the home country, but equivalent functions taking analogous forms may be derived for the foreign country.

4.4 The dynamic incentive constraints

The conditions (31) and (32) may now be applied using (46) and (47) to characterize the equilibrium sequence of sustainable tariffs. In particular, the incentive constraint for the home country is

$$V(\tilde{z}_t)^N = V(\tilde{z}_t)^S$$

or

$$\frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tau^H, \tilde{\tau}_t^*)\right)^{1-\sigma} + \beta A_N \left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma}$$

$$= \frac{1}{1-\sigma}\left(\left(\frac{M_t}{P_{xt}}\right)^{\omega}\left(\frac{M_t^*}{P_{mt}}\right)^{1-\omega}\zeta(\tilde{\tau}_t, \tilde{\tau}_t^*)\right)^{1-\sigma} + \beta A_s \left(\mathcal{F}(\theta_t, \theta_t^*)\right)^{1-\sigma} \tag{49}$$

Note that we have dropped the disutility of labor terms in the period utility for both sides of the incentive constraint because they cancel out. This is because $H = \frac{M}{P_x}$ and the price is set on the assumption that the sustainable path is maintained, so employment and the disutility term for the period utility is that same for $V^N(\tilde{z}_t)$ and $V^S(\tilde{z}_t)$.

Now, we can use the homogeneity of the value function to cancel out the term $\mathcal{F}(\theta_{t-1}, \theta^*_{t-1})$ from both sides of (49), and again use the properties of the pricing equations (13) and (14) to restate (49) as:

$$\frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t, \tilde{\tau}^*_t))^{1-\sigma}\zeta(\tau^H, \tilde{\tau}^*_t)^{1-\sigma} + \beta A_N \Xi_{1t}^{1-\sigma}$$
$$= \frac{1}{1-\sigma}\Xi_t^{1-\sigma}(E_{t-1}\mathcal{J}_1(\tilde{\tau}_t, \tilde{\tau}^*_t))^{1-\sigma}\zeta(\tilde{\tau}_t, \tilde{\tau}^*_t)^{1-\sigma} + \beta A_S \Xi_{1t}^{1-\sigma} \tag{50}$$

where

$$\mathcal{J}_1(\tilde{\tau}_t, \tilde{\tau}^*_t) = (\Lambda(\tilde{\tau}_t, \tilde{\tau}^*_t)\Delta_1)^{-\frac{\omega(\sigma+\psi)+(1-\sigma)\omega^*}{(\sigma+\psi)\delta}}(\Lambda^*(\tilde{\tau}_t, \tilde{\tau}^*_t)\Delta_2)^{-\frac{(1+\psi)(1-\omega)}{(\sigma+\psi)\delta}}$$

An analogous condition holds for the foreign country, representing (32), and may be written as

$$\frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}^*_1(\tilde{\tau}_t, \tilde{\tau}^*_t))^{1-\sigma}\zeta^*(\tau^H, \tilde{\tau}^*_t)^{1-\sigma} + \beta A^*_N \Xi_{1t}^{*1-\sigma}$$
$$= \frac{1}{1-\sigma}\Xi_t^{*1-\sigma}(E_{t-1}\mathcal{J}^*_1(\tilde{\tau}_t, \tilde{\tau}^*_t))^{1-\sigma}\zeta^*(\tilde{\tau}_t, \tilde{\tau}^*_t)^{1-\sigma} + \beta A^*_S \Xi_{1t}^{*1-\sigma} \tag{51}$$

and as before, we define:

$$\mathcal{J}_1(\tilde{\tau}_t, \tilde{\tau}^*_t)^* = (\Lambda(\tilde{\tau}_t, \tilde{\tau}^*_t)\Delta_1)^{-\frac{\omega^*(1+\psi)}{(\sigma+\psi)\delta}}(\Lambda^*(\tilde{\tau}_t, \tilde{\tau}^*_t)\Delta_2)^{-\frac{(1-\omega^*)(\sigma+\psi)+(1-\sigma)(1-\omega)}{(\sigma+\psi)\delta}}$$

Here (51) differs from (50) due to home bias in preferences, since $\omega \geq \omega^*$. Conditions (50) and (51) indicate that the incentive constraints will depend on shocks to money growth Ξ_t and to productivity growth Ξ_{1t}. We can now confirm that the equilibrium sustainable tariff sequence is indeed i.i.d. Self-evidently, (50) and (51) depend only on current valued shocks, and the shocks themselves are i.i.d.

Although the incentive constraints are affected by both monetary growth and productivity growth, these shocks have very different effects on the incentive constraints. A money growth shock affects the immediate benefits from cheating, affecting current period utility but not expected future utility. By contrast, a productivity shock has no immediate effects on the current benefits from cheating, since it impacts only the disutility of labor supply, and it does so in an equal way for both the value of cheating and the value of remaining on the sustainable path. But a productivity growth shock affects the expected future path of utility, both for the Nash 'punishment' path, and the expected future utility along the sustainable path.

How will this difference in the time dimension of shocks affect the response of equilibrium sustainable tariffs? The critical feature of (50) and (51) is that there is a current benefit from cheating, but this brings future costs of cheating. As a result, we have:

$$\zeta(\tau^H, \tilde{\tau}_t^*) > \zeta(\tilde{\tau}_t, \tilde{\tau}_t^*)$$

and

$$A_S > A_N$$

This implies that a money growth shock will lead the response of equilibrium sustainable tariffs to move in a different direction than will a productivity growth shock. To make this concrete, assume that $\sigma < 1$. Then a money growth shock will raise the first expression on both the left and right hand side of (50), but will also raise the incentive to cheat, since it raises the left hand expression more than the right hand expression. As a result, given a higher incentive to cheat on the sustainable tariff sequence, equilibrium sustainable tariffs must rise to offset this incentive.

On the same logic, when $\sigma < 1$, a productivity shock will raise the future cost of cheating more than it raises the future benefits of remaining on the sustainable path. Hence, there is a reduced incentive to cheat, and equilibrium sustainable tariffs will fall to reflect this.

When $\sigma > 1$, this logic is reversed, and monetary growth shocks will lead to a fall in the sustainable equilibrium tariff rates, while productivity growth shocks will lead to a rise in sustainable tariffs. Note however that the evidence in Section 6 above suggests that protectionism in increasing (falling) in expansionary (contractionary) monetary shocks, while decreasing in productivity shocks, suggesting that the case $\sigma < 1$ is more pertinent in the context of our static model.

Given this discussion, we may conclude:

Result 3. (i) When $\sigma < 1$ (resp. > 1), a positive shock to M or M^* raises (reduces) the benefits of cheating on the sustainable tariff policy, leaving the costs of cheating unchanged, leading to an increase (decrease) in the equilibrium sustainable tariff. (ii) When $\sigma < 1$ (resp. > 1), a rise in home or foreign productivity θ, θ^* raises (reduces) the costs of cheating on the sustainable tariff equilibrium, leaving the benefits of cheating unchanged. As a result, the equilibrium sustainable tariff will fall (rise).

Thus, we find that the cyclical pattern of tariffs depend on the source of shocks and the value of σ. We take the case $\sigma < 1$ as a baseline. While macro and asset pricing models typically assume $\sigma > 1$, it is typical in trade models which abstract from inter-temporal asset trade to assume $\sigma = 0$. In any case, we present the $\sigma > 1$ case below, following our main discussion. Thus, in the case $\sigma < 1$, protectionism is procyclical when the business cycle is driven by monetary (or demand) shocks, but countercyclical when productivity shocks are the main sources of business cycle variation.

4.5 Calibration and simulations

We calibrate the baseline model with the following parameters; $\sigma = .5$, and $\psi = 2$. We assume a moderate degree of home bias in preferences, so that $\omega = 1 - \omega^* = 0.7$. We set the maximum feasible tariff rate at sixty-two percent, so that $\tau^H = .62$. This is the average tariff rate estimated by Ossa (2014) that would apply in a full scale world "tariff war", and hence represents the appropriate limit for the static Nash equilibrium tariff rate within our model – the implications of varying τ^H are also explored below.

Given this, we choose a discount factor $\beta = \beta^*$ so that the mean tariff rate in the sustainable equilibrium in the baseline case is 10 percent, which is approximately the average degree of trade restriction (including both tariff and non-tariff barriers), reported by UNCTAD (2013) for advanced economies. This leads to a value of $\beta = \beta^* = 0.6$. We then choose independent money and productivity shocks in the home and foreign country, assuming a standard deviation of 2 percent for each shock.[3]

Figure 1 illustrates the relationship between tariffs and *ex-post* productivity and monetary policy shocks, under various scenarios. Figure 1a illustrates the variation in home and foreign tariffs as a function of home country productivity. The blue line shows the base case, and the red line shows the impact of an unanticipated home country money shock. Absent the money shock tariff rates are effectively equal in the two countries, so that tariffs respond in the same way to productivity shocks in either country. After a home country money shock, the tariff schedule shifts up in both countries, but to a greater degree in the home country, since the money shock gives a greater incentive for the home country to deviate from the sustainable tariff equilibrium.

Figure 1b shows the effect of differential discounting among the two countries. For this Figure we set $\beta = 0.5$ and $\beta^* = 0.6$. Thus, the home country is more impatient than the foreign country. As expected, it will therefore set a higher tariff rate than the foreign country, for any pattern of monetary and productivity shocks, which reflects the higher relative valuation of the current benefits from cheating, compared to the patient country. However, the response to productivity and money shocks is qualitatively the same as in Figure 1a.

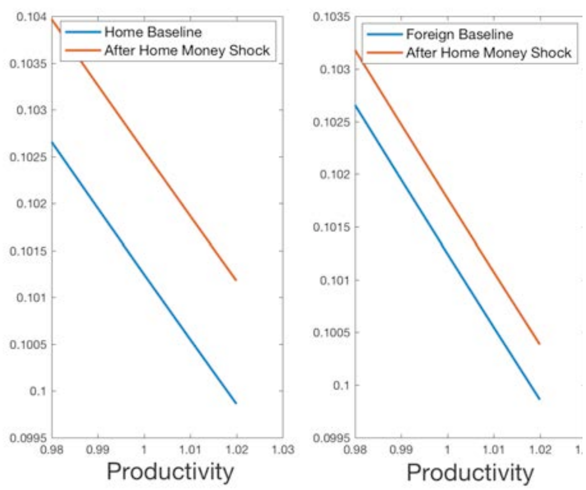
Figure 1c illustrates an opposite parameterization for the elasticity of intertemporal substitution, where $\sigma = 1.5$. As explained above, in this case, the cyclical pattern of tariffs is the reverse of that described in the previous Figures. Tariffs are *increasing* in productivity shocks, and *decreasing* in money shocks. Since our empirical results described below are more consistent with the previous parameterization, we choose instead to concentrate on the case $\sigma < 1$. As noted above, this is more consistent with most models of international trade, where intertemporal consumption smoothing is not explicitly modeled. In particular, our model is quasi-static, and consumption smoothing motives are not key to the dynamics of the model, which makes the assumption more innocuous.

Further, Table 1 reports the mean and coefficient of variation of tariff rates under the baseline case and various alternative scenarios. The most important implication of the Table is the large impact of productivity shocks on the average tariff levels. In the absence of variation in productivity growth, average tariff rates would be 56 percent, close to the maximum Nash tariff levels. By contrast, variance in monetary shocks has almost no effect on the mean tariff levels.

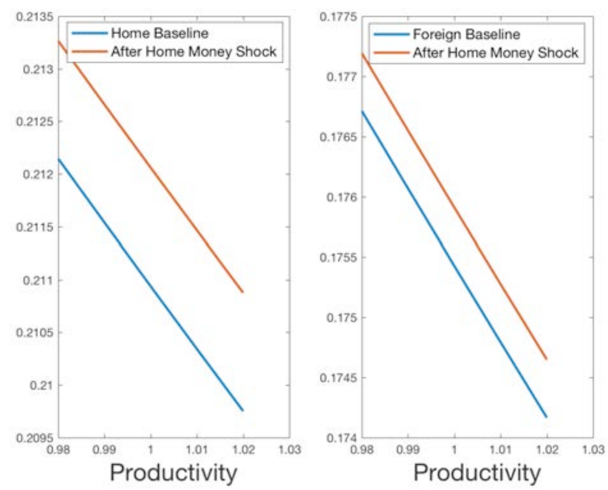
[3] The model is solved assuming each shock takes on a five point distribution with equal probabilities, with a standard deviation of 2 percent.

Tariff schedules

(a) Figure 1a



(b) Figure 1b



(c) Figure 1c

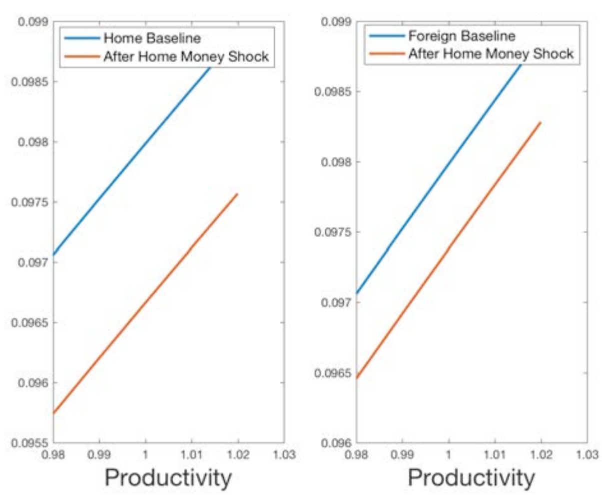


Table 1: Tariffs in the simulated model

		Mean	Coefficient of variation	
Baseline		10.2%	3%	
Flexible prices		62%	0	
Money shocks		56%	0.6%	
Productivity shocks		10.2%	2.3%	
No shocks		56%	0	
High punishment $\tau^H = 1$		6.2%	7.1%	
$\sigma = 1.5$		9.7%	2.8%	
	Home mean	Foreign mean	Home CV	Foreign CV
Large economy	6.7%	3.9%	4.8%	5.7%
Home impatient	21.2%	17.6%	1.2%	1.6%%

Calibration is as follows. $\omega = 1 - \omega^* = 0.7$, $\tau^H = 0.62$, $\beta = \beta^* = 0.6$, $\sigma = .5$, $\psi = 2$. In large economy case $\omega = \omega^* = 0.525$. In home impatient case, $\beta = .5$.

What explains the large impact of productivity variance on tariff levels? The key intuition is due to the impact of productivity uncertainty on sustainable tariffs, and through this channel, the effect on equilibrium nominal prices. The intuition can be gleaned from Equations (17) and (18) above, which give the pre-set prices in the presence of money and productivity shocks. With independent money and productivity shocks across countries, an increase in the volatility of either shock will lead to a lower price set by firms in each country. This is particularly more important for productivity shocks. A fall in the level of pre-set prices will increase the continuation value of the game for each country, whether on the sustainable path or in the Nash punishment equilibrium. However, there is a critical difference between the sustainable path and the Nash punishment equilibrium in that, under the sustainable path, tariff rates in the future are uncertain, since they will respond to realized productivity shocks. This implies that the effect of uncertain productivity on price levels is much stronger for the continuation values under the sustainable path than the effect under the Nash punishment path. As a result, a rise in the variance of productivity shocks makes the continuation value in the sustainable path more attractive, allowing for a lower mean level of sustainable tariffs required to offset the incentive to cheat in any period. Hence, a higher variance of productivity shocks reduces the mean level of tariffs in the sustainable equilibrium.

However, the above result is critically dependent on the price setting assumption. With fully flexible prices, as captured by (37), tariff rates are constant, but the level of tariffs will depend on the distribution of productivity shocks due to effect of this distribution on the discount factor. In fact in this case, uncertainty in productivity has the *opposite* effect on the level of tariffs in a sustainable equilibrium: a higher variance of productivity shocks will reduce the term $E\Xi_1{}^{1-\sigma}$ in the effective discount factor. This reduces the expected benefit from the continuation game, and reduces the cost of cheating. As result, with flexible prices, productivity uncertainty *raises* the mean tariff rate in a sustainable equilibrium.[4]

Table 1 also illustrates the effect of differences in the discount factor between countries. When the home country discount factor falls from 0.6 to 0.5, the mean sustainable tariff rate

[4]Note that, in our current calibration, sustainable tariffs under fully flexible prices are equal to the maximum Nash tariff rates. With a higher discount factor, sustainable tariff rates would be lower. In that case, it is easy to see from (37) that a rise in the variance of productivity shocks will raise the level of sustainable tariffs.

rises substantially for the home country. The mean tariff rate rises as well for the foreign country, although not as much as that of the home country, even though the foreign countries discount factor is unchanged.

Finally, country size plays an interesting role. Country size may be captured by variations in ω and ω^*. In particular, allowing for a rise in both ω and ω^*, implies that the home country produces a larger share of the world goods than the foreign country, and thus is the larger country.[5] Here we set $\omega = 0.725$ and $\omega^* = 0.325$. We find in this case that the mean tariff rates falls for both countries, although the home country (the larger country) tariff rates remains higher than that of the foreign country.

5 A dynamic model with sticky prices

5.1 Model set-up

We extend the previous model to relax some of the restrictive assumptions. In the dynamic model, prices are set subject to Rotemberg adjustment costs, financial markets are incomplete and the trade elasticity is allowed to differ from unity. In addition, we consider an alternative way of conducting monetary policy assuming that Central Banks commit to Taylor-type rules, subject to unexpected monetary policy shocks. Notations are deliberately kept as close as possible to the tractable model. The economy is made of two countries, home and foreign. The home economy produces good x and the foreign economy produces good m. The representative household of the domestic economy maximizes a welfare index

$$E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\sigma}}{1-\sigma} - \chi \frac{H_s^{1+\psi}}{1+\psi} \right) \right\}, 0 < \beta < 1, \sigma > 0, \chi > 0, \psi > 0 \tag{52}$$

subject to the following budget constraint

$$S_t B_t^* + B_t + P_{xt} C_{xt} + (1+\tau_t) S_t P_{mt} C_{mt} + P_t AC_t = S_t R_{t-1}^* B_{t-1}^* + R_{t-1} B_{t-1} + W_t H_t + P_{xt} \Pi_t - T_t \tag{53}$$

In Equation (52), β is the subjective discount factor, C_t is the level of consumption, made of domestic goods in quantity C_{xt} and foreign goods in quantity C_{mt}, and H_t is the level of hours worked. In Equation (53), B_t^* is the holdings of one-period international bonds denominated in the foreign currency, that pay a gross return R_t^* between period $t-1$ and period t, and B_t the holdings of domestic bonds. Variables P_{xt} and P_{mt} respectively denote the prices of the domestic and foreign goods, P_t (defined below) is the domestic CPI and S_t is the nominal exchange rate. Imports are subject to tariffs at the rate τ_t where the proceeds are be rebated to households through lump-sum transfers. The nominal wage is W_t, $\Pi_t = \int_0^1 \Pi_t(i) di$ is the profit paid by the monopolistic firms to the domestic households, and T_t is a lump-sum tax. Finally, $AC_t = \frac{\phi}{2} \left(\frac{S_t B_t^*}{P_t} - \frac{SB^*}{P} \right)^2$ is a portfolio adjustment cost paid on international bonds.

The domestic consumption basket has the following composition

$$C_t = \left(\omega^{\frac{1}{\mu}} C_{xt}^{\frac{\mu-1}{\mu}} + (1-\omega)^{\frac{1}{\mu}} C_{mt}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \tag{54}$$

[5] Obstfeld and Rogoff (1995) develop a model in which country size (population) has the same measure of a country's share of differentiated traded goods. We follow that interpretation here.

Domestic and foreign goods are imperfectly substitutable with elasticity of substitution $\mu > 0$. In addition, households' preferences are biased towards local goods, as $\omega \in [1/2, 1]$ denotes the share of domestic goods in the consumption bundle of the domestic households. Optimal good demands are derived maximizing C_t for a given total expenditure on goods $P_{xt}C_{xt} + (1+\tau_t) S_t P_{mt} C_{mt}$:

$$C_{xt} = \omega \left(\frac{P_{xt}}{P_t}\right)^{-\mu} C_t \tag{55}$$

$$C_{mt} = (1-\omega)\left(\frac{(1+\tau_t) S_t P_{mt}}{P_t}\right)^{-\mu} C_t \tag{56}$$

where

$$P_t = \left(\omega P_{xt}^{1-\mu} + (1-\omega)\left((1+\tau_t) S_t P_{mt}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{57}$$

$$= P_{xt}\left(\omega + (1-\omega)\left((1+\tau_t) Q_t\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{58}$$

and $Q_t = S_t P_{mt}/P_{xt}$ denotes the terms of trade. These conditions imply that

$$P_{xt}C_{xt} + (1+\tau_t) S_t P_{mt} C_{mt} = P_t C_t. \tag{59}$$

The remaining first-order conditions imply

$$\beta E_t \left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{R_t}{\pi_{t+1}}\right\} = 1 \tag{60}$$

$$\beta E_t \left\{\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{R_t^*}{\pi_{t+1}} \frac{S_{t+1}}{S_t \Omega_t}\right\} = 1 \tag{61}$$

$$\chi H_t^{\psi} C_t^{\sigma} = \frac{W_t}{P_{xt}} \frac{P_{xt}}{P_t} \tag{62}$$

where $\pi_t = P_t/P_{t-1}$ and $\Omega_t = 1 + \phi\left(\frac{S_t B_t^*}{P_t} - \frac{S B^*}{P}\right)$. The foreign consumption basket is

$$C_t^* = \left(\omega^{*\frac{1}{\mu}} C_{xt}^{*\frac{\mu-1}{\mu}} + (1-\omega^*)^{\frac{1}{\mu}} C_{mt}^{*\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}} \tag{63}$$

where $1-\omega^* \in [1/2, 1]$ denotes the share of foreign goods in the foreign consumption bundle. Optimization yields

$$C_{mt}^* = (1-\omega^*)\left(\frac{P_{mt}}{P_t^*}\right)^{-\mu} C_t^* \tag{64}$$

$$C_{xt}^* = \omega^* \left(\frac{(1+\tau_t^*) P_{xt}}{S_t P_t^*}\right)^{-\mu} C_t^* \tag{65}$$

where τ_t^* is the tariff rate on imports imposed by the foreign tariff setter. The corresponding foreign CPI writes

$$P_t^* = \left((1-\omega^*) P_{mt}^{1-\mu} + \omega^* \left(\frac{(1+\tau_t^*) P_{xt}}{S_t}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{66}$$

$$= P_{mt}\left(1-\omega^* + \omega^*\left((1+\tau_t^*) Q_t^{-1}\right)^{1-\mu}\right)^{\frac{1}{1-\mu}} \tag{67}$$

Finally, since the foreign economy does not have access to home bonds but only to foreign bonds paying symmetric portfolio costs $AC_t^* = \frac{\phi}{2}\left(\frac{B_t^{**}}{P_t^*} - \frac{B^{**}}{P^*}\right)^2$, where B_t^{**} are the holding of foreign bonds by foreign households. The households first-order conditions imply

$$\beta E_t \left\{ \left(\frac{C_t^*}{C_{t+1}^*} \right)^{\sigma} \frac{R_t^*}{\pi_{t+1}^* \Omega_t^*} \right\} = 1 \tag{68}$$

$$\chi H_t^{*\psi} C_t^{*\sigma} = \frac{W_t^*}{P_{mt}} \frac{P_{mt}}{P_t^*} \tag{69}$$

where $\Omega_t^* = \left(1 + \phi\left(\frac{B_t^{**}}{P_t^*} - \frac{B^{**}}{P^*}\right)\right)$. In each country, a unit continuum of firms indexed in i produce varieties of each type of good according to

$$Y_{xt}(i) = \theta_t H_t(i) \text{ and } Y_{mt}(i) = \theta_t^* H_t^*(i) \tag{70}$$

where θ_t and θ_t^* are exogenous measures of productivity, following AR1 processes

$$\log \theta_t = \rho_\theta \log \theta_{t-1} + \xi_{\theta t} \tag{71}$$

$$\log \theta_t^* = \rho_{\theta^*} \log \theta_{t-1}^* + \xi_{\theta t}^* \tag{72}$$

Prices are set optimally subject to Rotemberg price adjustment costs, and firms maximize the discounted sum of profits

$$\max_{P_{xt}(i)} \sum_{s=t}^{\infty} \beta^{s-t} \lambda_s \Pi_s \text{ and } \max_{P_{mt}(i)} \sum_{s=t}^{\infty} \beta^{s-t} \lambda_s^* \Pi_s^* \tag{73}$$

where λ_t and λ_t^* are the Lagrange multipliers associated with the households budget constraints, and

$$\Pi_t = \left(\frac{(1-\tau_y) P_{xt}(i)}{P_{xt}} - \frac{W_t}{\theta_t P_{xt}} - \frac{\varphi}{2}\left(\frac{P_{xt}(i)}{P_{xt-1}(i)} - 1 \right)^2 \right) Y_{xt}^d(i) \tag{74}$$

$$\Pi_t^* = \left(\frac{(1-\tau_{y^*}) P_{mt}(i)}{P_{mt}} - \frac{W_t^*}{\theta_t^* P_{mt}} - \frac{\varphi}{2}\left(\frac{P_{mt}(i)}{P_{mt-1}(i)} - 1 \right)^2 \right) Y_{mt}^d(i) \tag{75}$$

The demands for varieties depend on total demand, on the relative price of variety i and on the elasticities of substitution between varieties $\eta > 1$ and $\eta^* > 1$:[6]

$$Y_{xt}^d(i) = \left(P_{xt}(i)/P_{xt}\right)^{-\eta} \left(C_{xt} + C_{xt}^* + AC_{xt} + AC_{xt}^*\right) \tag{76}$$

$$Y_{mt}^d(i) = \left(P_{mt}(i)/P_{mt}\right)^{-\eta} \left(C_{mt}^* + C_{mt} + AC_{mt} + AC_{mt}^*\right) \tag{77}$$

Finally, τ_y and τ_{y^*} are subsidies introduced by the government to offset the steady-state distortions implied by monopolistic competition. Optimal pricing conditions are symmetric, *i.e.* $P_{xt}(i) = P_{xt}$ and $P_{mt}(i) = P_{mt}$ and imply

$$(1-\tau_y)(\eta - 1) = \eta MC_t - \varphi\left((\pi_{xt} - 1)\pi_{xt} - \beta E_t \left\{ \frac{(\pi_{xt+1} - 1)\pi_{xt+1} Y_{xt+1} C_{t+1}^{-\sigma}}{Y_{xt} C_t^{-\sigma}} \right\} \right) \tag{78}$$

$$(1-\tau_{y^*})(\eta^* - 1) = \eta^* MC_t^* - \varphi\left((\pi_{mt} - 1)\pi_{mt} - \beta E_t \left\{ \frac{(\pi_{mt+1} - 1)\pi_{mt+1} Y_{mt+1} C_{t+1}^{*-\sigma}}{Y_{mt} C_t^{*-\sigma}} \right\} \right) \tag{79}$$

[6] Portfolio costs are paid in units of final goods and therefore give rise to similar demands for domestic and foreign goods by households.

The optimal steady-state levels of subsidies are negative, $\tau_y = \frac{1}{1-\eta}$ and $\tau_{y^*} = \frac{1}{1-\eta^*}$, and financed using the proceeds from import taxes and a lump-sum tax on households. Government budget constraints are thus

$$\begin{aligned} T_t + \tau_t S_t P_{mt} C_{mt} + \tau_y P_{xt} Y_{xt} &= 0 \quad (80) \\ T_t^* + \tau_t^* S_t^{-1} P_{xt}^* C_{xt}^* + \tau_{y^*} P_{mt} Y_{mt} &= 0 \quad (81) \end{aligned}$$

The aggregation of the model is straightforward as goods market clearing conditions are:[7]

$$\begin{aligned} Y_{xt}\left(1 - \frac{\varphi}{2}(\pi_{xt}-1)^2\right) &= \omega\left(\frac{P_{xt}}{P_t}\right)^{-\mu}(C_t + AC_t) + \omega^*\left(\frac{(1+\tau_t^*)P_{xt}}{S_t P_t^*}\right)^{-\mu}(C_t^* + AC_t^*) \quad (86) \\ Y_{mt}\left(1 - \frac{\varphi}{2}(\pi_{mt}-1)^2\right) &= (1-\omega^*)\left(\frac{P_{mt}}{P_t^*}\right)^{-\mu}(C_t^* + AC_t^*) + (1-\omega)\left(\frac{(1+\tau_t)S_t P_{mt}}{P_t}\right)^{-\mu}(C_t + AC_t) \quad (87) \end{aligned}$$

We assume that domestic bonds are in zero net supply $B_t = 0$ and equilibrium on the foreign bonds market yields

$$B_t^* + B_t^{**} = 0 \quad (88)$$

Aggregating all budget constraints in the home economy gives the dynamics of net foreign assets[8]

$$NFA_t = \left(\Delta S_t R_{t-1}^*/\pi_t\right) NFA_{t-1} + \frac{P_{xt}}{P_t} Y_{xt}\left(\frac{\varphi}{2}(\pi_{xt}-1)^2\right) + \tau_t \frac{S_t P_{mt}}{P_t} C_{mt} - C_t - AC_t \quad (89)$$

where $NFA_t = S_t B_t^*/P_t$. Defining $NFA_t^* = B_t^{**}/P_t^*$ and using the bonds market clearing condition also gives

$$Q_t^r NFA_t^* = -NFA_t \quad (90)$$

where $Q_t^r = S_t P_t^*/P_t$ is the real exchange rate.[9] Finally, under flexible exchange rates, we consider the following simple monetary policy rules for each Central Bank

$$\begin{aligned} \frac{R_t}{R} &= \left(\frac{R_{t-1}}{R}\right)^{\rho_r}\left(\frac{\pi_t}{\pi}\right)^{(1-\rho_r)d_\pi} e^{\xi_{rt}} \quad (91) \\ \frac{R_t^*}{R^*} &= \left(\frac{R_{t-1}^*}{R^*}\right)^{\rho_r}\left(\frac{\pi_t^*}{\pi^*}\right)^{(1-\rho_r)d_\pi} e^{\xi_{rt}^*} \quad (92) \end{aligned}$$

---

[7] Notice that

$$\begin{aligned} P_{xt}/P_t &= \left(\omega + (1-\omega)\left((1+\tau_t)Q_t\right)^{1-\mu}\right)^{\frac{1}{\mu-1}} \quad (82) \\ (1+\tau_t) S_t P_{mt}/P_t &= \left(\omega\left((1+\tau_t)Q_t\right)^{\mu-1} + 1 - \omega\right)^{\frac{1}{\mu-1}} \quad (83) \\ P_{mt}/P_t^* &= \left(1 - \omega^* + \omega^*\left((1+\tau_t^*)Q_t^{-1}\right)^{1-\mu}\right)^{\frac{1}{\mu-1}} \quad (84) \\ \frac{(1+\tau_t^*)P_{xt}}{S_t P_t^*} &= \left((1-\omega^*)\left((1+\tau_t^*)Q_t^{-1}\right)^{\mu-1} + \omega^*\right)^{\frac{1}{\mu-1}} \quad (85) \end{aligned}$$

[8] Notice also that $S_t P_{mt}/P_t = \left(\omega Q_t^{\mu-1} + (1-\omega)(1+\tau_t)^{1-\mu}\right)^{\frac{1}{\mu-1}}$.

[9] The real exchange rate can be expressed as a function of terms of trade by

$$Q_t^r = \left(\frac{1 - \omega^* + \omega^*\left(Q_t/(1+\tau_t^*)\right)^{\mu-1}}{\omega Q_t^{\mu-1} + (1-\omega)(1+\tau_t)^{1-\mu}}\right)^{\frac{1}{1-\mu}}$$

where

$$\pi_t = \pi_{xt} \left(\frac{\omega + (1-\omega)\left((1+\tau_t) Q_t\right)^{1-\mu}}{\omega + (1-\omega)\left((1+\tau_{t-1}) Q_{t-1}\right)^{1-\mu}} \right)^{\frac{1}{1-\mu}} \tag{93}$$

$$\pi_t^* = \pi_{mt} \left(\frac{1-\omega^* + \omega^*\left((1+\tau_t^*) Q_t^{-1}\right)^{1-\mu}}{1-\omega^* + \omega^*\left(\left(1+\tau_{t-1}^*\right) Q_{t-1}^{-1}\right)^{1-\mu}} \right)^{\frac{1}{1-\mu}} \tag{94}$$

and where ξ_{rt} and ξ_{rt}^* are monetary policy shocks following AR1 processes.

Under fixed exchange rates, we assume that the home economy pegs its interest rate to the foreign nominal rate. As shown by Benigno, Benigno, and Ghironi (2007), this requires assuming:

$$\frac{R_t}{R_t^*} = \left(\frac{S_t}{S_{t-1}}\right)^{\Phi} \tag{95}$$

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\rho_r} \left(\frac{\pi_t^*}{\pi^*}\right)^{(1-\rho_r)d_\pi} e^{\xi_{rt}^*} \tag{96}$$

where $\Phi > 0$ and large, and guarantees equilibrium determinacy along with $\S_t = S_t{-}1 = S$.

The value functions are defined in the following way. Let us denote variables under the Nash equilibrium where $\tau_t = \tau_t^* = \tau^H$ by a 'N' superscript, the variables under the equilibrium where the home (foreign) tariff setter cheats by a 'HC' ('FC') superscript, and the equilibrium under sustainable tariffs with a 'S' superscript, in which case $\tau_t = \tilde{\tau}_t$ and $\tau_t^* = \tilde{\tau}_t^*$. The value functions for home and foreign when tariffs are on the sustainable path are:

$$V_t^S = \frac{C\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)^{1+\psi}}{1+\psi} + \beta^g E_t V_{t+1}^S \tag{97}$$

$$V_t^{S*} = \frac{C^*\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H^*\left(\tilde{\tau}_t, \tilde{\tau}_t^*\right)^{1+\psi}}{1+\psi} + \beta^{g*} E_t V_{t+1}^{S*} \tag{98}$$

where β^g and β^{g*} respectively denote the home and foreign discount factors of tariff setters. The value of home cheating (for the home country) and the value of foreign cheating (for the foreign country) are respectively

$$V_t^{HC} = \frac{C\left(\tau^H, \tilde{\tau}_t^*\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H\left(\tau^H, \tilde{\tau}_t^*\right)^{1+\psi}}{1+\psi} + \beta^g E_t V_{t+1}^N \tag{99}$$

$$V_t^{FC*} = \frac{C^*\left(\tilde{\tau}_t, \tau^H\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H^*\left(\tilde{\tau}_t, \tau^H\right)^{1+\psi}}{1+\psi} + \beta^{g*} E_t V_{t+1}^{N*} \tag{100}$$

while the home and foreign value functions associated with the Nash equilibrium write

$$V_t^N = \frac{C\left(\tau^H, \tau^H\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H\left(\tau^H, \tau^H\right)^{1+\psi}}{1+\psi} + \beta^g E_t V_{t+1}^N \tag{101}$$

$$V_t^{N*} = \frac{C^*\left(\tau^H, \tau^H\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H^*\left(\tau^H, \tau^H\right)^{1+\psi}}{1+\psi} + \beta^{g*} E_t V_{t+1}^{N*} \tag{102}$$

The condition that determines sustainable tariffs is then

$$E_t\left(V_{t+1}^S\right) = E_t\left(V_{t+1}^{HC}\right) \tag{103}$$

$$E_t\left(V_{t+1}^{S*}\right) = E_t\left(V_{t+1}^{FC*}\right) \tag{104}$$

5.2 Model solution and calibration

The above dynamic model can be solved for a single equilibrium when tariffs are exogenous. Alternatively, when tariffs are endogenous and on the sustainable path, the form of the value functions shows that we need to solve the model under four equilibria: one for the (worst) Nash equilibrium, one for the case in which the home tariff setter cheats, one for the case in which the foreign tariff setter cheats and one for sustainable tariffs. In the first equilibrium, tariffs are simply $\tau_t = \tau_t^* = \tau^H$, in the second one, we have $\tau_t = \tilde{\tau}_t$ and $\tau_t^* = \tau^H$, in the third one, we have $\tau_t = \tau^H$ and $\tau_t^* = \widetilde{\tau_t^*}$, and in the last one we have $\tau_t = \tilde{\tau}_t$ and $\tau_t^* = \widetilde{\tau_t^*}$, where $\tilde{\tau}_t$ and $\widetilde{\tau_t^*}$ are respectively determined by (103) and (104).

The model is solved under rational expectations up to a first-order approximation around the steady state using Dynare.[10] The calibration is as follows. The discount factor of households is $\beta = 0.99$ while the discount factor of tariff setters is $\beta^g = \beta^{g*} = 0.6$ and the highest tariff rate is $\tau^H = 0.62$. The consumption risk-aversion parameter is $\sigma = 0.5$, the inverse of the Frisch elasticity of labor supply is $\psi = 2$, the home bias parameters are $\omega = 1 - \omega^* = 0.5$, the trade elasticity is $\mu = 1$ and the portfolio adjustment cost parameter is $\phi = 0.001$. On the production side, we assume $\eta = \eta^* = 6$, impose a subsidy that removes steady-state distortions induced by monopolistic competition $\tau_y = 1/(1-\eta)$ and $\tau_{y^*} = 1/(1-\eta^*)$, and assume that the Rotemberg parameter is $\varphi = 80$. Parameters of the Taylor rules are $\rho_r = 0.7$ and $d_\pi = 1.5$. The previous Section looks at growth (permanent) shocks. To remain as close as possible to this situation, we look at quasi-permanent shocks and impose $\rho_\theta = \rho_{\theta^*} = 0.9999$. Finally, the standard deviations of productivity and monetary policy shocks are $\sigma(\xi_\theta) = \sigma(\xi_\theta^*) = 0.02$ and $\sigma(\xi_r) = \sigma(\xi_r^*) = 0.002$.

5.3 Exogenous tariff shock

Consider the case where the economy is initially at zero tariffs in both countries. Consider that the home tariff setter imposes a very persistent tariff on imports. Figure 2 reports the resulting Impulse Response Functions (IRFs hereafter) under a flexible exchange rate regime.[11]

Figure 2 shows that a home tariff shock appreciates the real exchange rate, which eventually boosts domestic consumption through the wealth effect, that overturns the expenditure switching effect. Home and foreign output and hours worked fall, but more so in the Home economy, since home products are affected by the negative expenditure switching effect implied by the real appreciation. Home consumption rises and hours worked fall together, which raises the utility of home consumers, while the fall in foreign consumption – induced by the negative wealth effect – and the relatively smaller fall in hours worked – since home and foreign consumers buy more of the foreign good – trigger a drop in the utility of foreign consumers. Starting from a zero tariff situation, the dynamic model fully preserves the incentive to impose tariffs under flexible exchange rate regime, and the latter very clearly depend on the relative size of wealth and substitution effects, as in the static model.

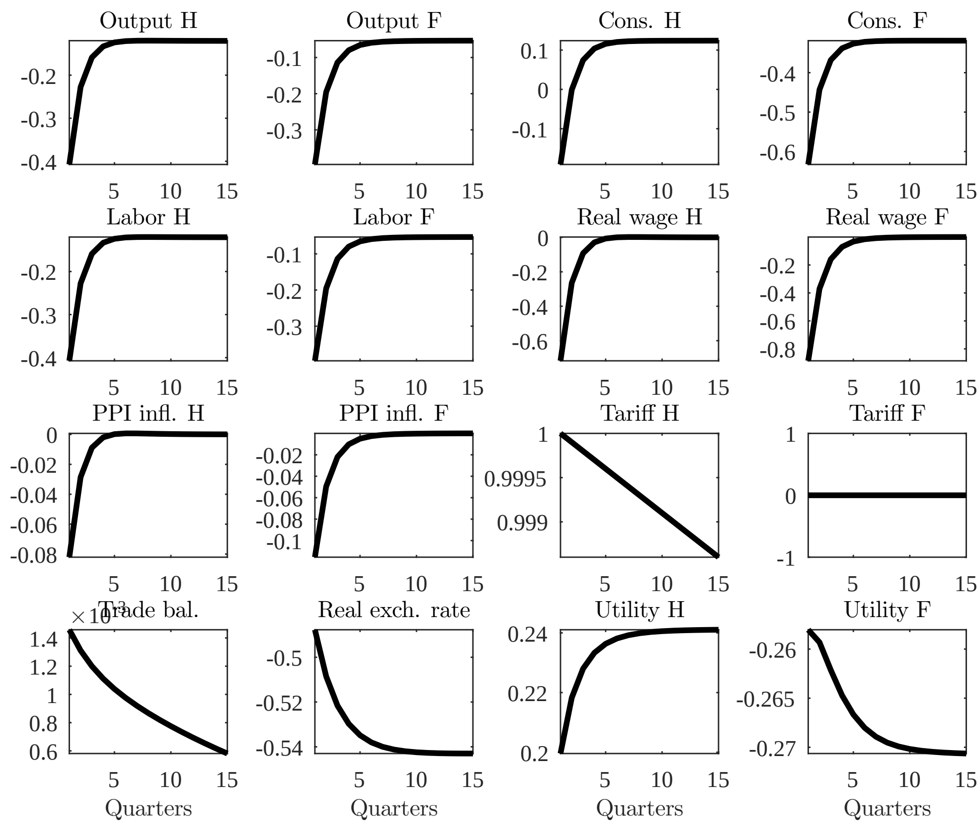
5.4 Sustainable tariffs dynamics

Let us now look at the dynamics of sustainable tariffs when those are endogenously determined by the incentive constraints (103) and (104). Figure 3 below reports the dynamics of sustainable

[10] See Adjemian, Bastani, Juillard, Karamé, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot (2011).

[11] IRFs to a home productivity shock and to a home restrictive monetary policy shock are also reported in Appendix A.

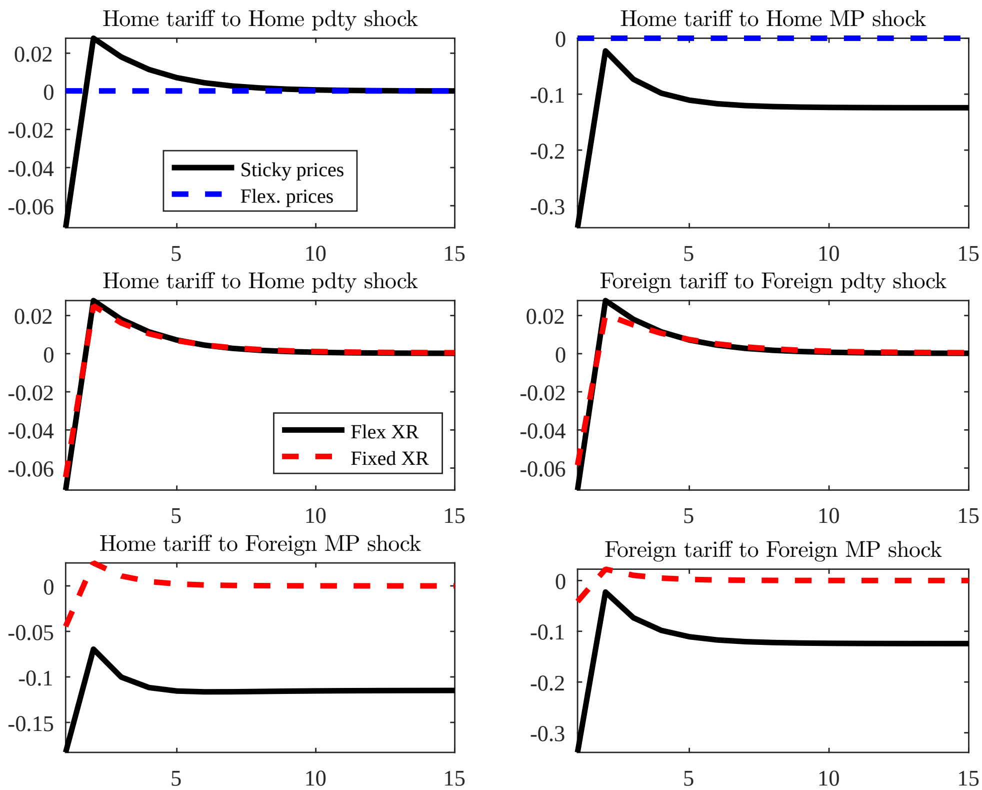
Figure 2: IRFs to a quasi-permanent home tariff shock



Note: All variables are reported in percent deviation from their steady-state values, except for the trade balance, that is reported in absolute deviation.

tariffs to a quasi-permanent productivity shock and to a restrictive monetary policy shock under various configurations.

Figure 3: Response of tariffs to various shocks under alternative configurations



Note: tariffs are reported in percent deviation from their steady state values. Monetary policy shocks consist in a rise in the nominal interest rate.

First, we want to know whether the intuition from the tractable model about sticky vs. flexible prices applies to the dynamics set-up. In the tractable model, price flexibility, beyond making the difference between exchange rate regime irrelevant, predicted that tariffs should be constant under flexible prices (Result 2). Under sticky prices, the tractable model found (under some conditions) that positive local productivity shocks led local tariffs to fall and that restrictive local monetary policy shocks led local tariffs to rise (Result 3).

Figure 3 (top row) shows that these results carry over to the dynamic model. Under flexible prices, the impact response of the home tariff is and remains zero after restrictive local monetary policy shock. It is and remains zero as well after a local positive productivity shock. Result 2 thus holds in our dynamic model for the calibration considered. In addition, under sticky prices, the sustainable tariff set by the home tariff setter decreases on impact after a positive local productivity shock, and falls (resp. rises) after a restrictive (resp. expansionary) local monetary policy shock, as in the static model.

In addition, we also want to know about the incentive to change tariffs under flexible vs. fixed exchange rate regimes, as the static model had a sharp prediction about it (see Result 1). Our dynamic model first implies that the mean sustainable tariff with a flexible exchange rate is 0.4393 while the mean sustainable tariff with a fixed exchange rate is lower, at 0.4391. In addition, Figure 3 (bottom rows) suggests that changes in sustainable tariffs with a fixed exchange rate after productivity shocks are either similar, or smaller than with a flexible exchange rate, and that these changes are *much* smaller after monetary policy shocks. In terms of volatility, the standard deviation of sustainable tariffs is 0.1198 under flexible exchange rates while it is only 0.0016 under fixed exchange rates. Sustainable tariffs are thus lower on average and less volatile under fixed exchange rates according to the dynamic model, which comforts our Result 1, obtained in the static model.

6 Empirical evidence

In this Section, we take a look at the data to see whether our theoretical predictions can find some empirical support. We first investigate the effects of the exchange rate regime on the incentive to initiate trade disputes, which often result in applying retaliatory tariffs. We then focus on a small sub-set of countries with flexible exchange rates and look at the empirical responses of trade disputes/tariffs to positive supply shocks and restrictive monetary policy shocks. In both case, we find empirical evidence in favor of the theoretical results described in the previous Sections.

6.1 Exchange rate regimes

We start our empirical investigation by looking at the impact of various variables on the incentive to apply tariffs, with a special focus on the exchange rate regime. Intuitively, the exchange rate regime should matter because exchange rate manipulation or currency wars, that are more likely under flexible exchange rate regimes, can lead to retaliation through tariffs. Flexible exchange rate regimes should therefore lead to stronger incentives to manipulate tariffs. Following Barattieri, Cacciatore, and Ghironi (2018), our empirical analysis of this potential effect relies on Bown (2016)'s Global Anti-Dumping database that collects anti-dumping or trade disputes initiatives at the Dispute Settlement Body of the WTO, that are usually followed by the imposition of tariffs.

For each country covered by the dataset, we count the number of trade disputes initiated each year and relate this variable to a set of explanatory variables. Those include the level of GDP, the GDP growth rate, net exports to GDP, a measure of trade openness, a measure of exchange rate flexibility, and a trade-weighted measure of real exchange rate.[12] We have no prior about the effect of size (measured by the log of GDP) and openness (measured by the trade openness ratio) on the occurrence of trade disputes. We expect GDP growth, net exports and competitiveness (measured by the level of the trade-weighted real exchange rate) to reduce the probability of trade disputes while exchange rate flexibility should raise the probability of trade disputes, since the former makes currency manipulations easier both by the country and by partners, and thus enhances the probability that trade restrictions are used as a tool in a potential currency war. Table 2 below summarizes the characteristics of the different variables contained in our dataset.

The dependent variable – the number of trade disputes initiated – is a count variable that is non-negative and takes integer values. As such, it requires that a Poisson or a negative binomial

[12]The GAD database covers different periods for each country and the longest period covered ranges from 1977 to 2015. It is completed by various indicators taken from the PennWorld database, the Ilzetzki, Reinhart, and Rogoff (2017) classification of exchange rate regimes and a trade-weighted measure of the real exchange rate taken from the EQCHANGE CEPII Database. Appendix B details the characteristics of our dataset.

Table 2: Summary statistics of the dataset

	Mean	Min.	Max.	Stdv.	P_{20}	P_{40}	P_{60}	P_{80}
Trade disputes initiated annually	10.9	0.0	94.0	15.0	1.0	3.0	8.0	17.1
log(GDP)	13.3	2.7	16.7	2.3	12.5	13.2	13.8	14.5
GDP growth, in %	4.3	−19.4	28.6	5.6	0.5	2.8	4.9	8.4
Net exports to GDP, in %	0.4	−24.5	31.9	6.4	−3.6	−1.2	1.4	4.0
Trade openness, in %	44.0	6.7	123.4	26.5	20.1	30.3	44.6	67.0
Exchange rate flexibility	9.9	2.0	15.0	3.2	8.0	10.0	12.0	13.0
Trade weighted real exchange rate	1.1	0.4	2.7	0.3	0.9	1.0	1.1	1.2

Note: Based on 612 observations.

regression is used. Under the Poisson model, the dependent variable is distributed as a Poisson describing the probability that a number of events realizes within a given time interval. One limitation of the Poisson model however is that it does not account for potential over-dispersion given that the Poisson distribution imposes a variance of the dependent variable that is equal to the mean, a condition that is likely not to be met in our sample. The negative binomial model is a generalization of the Poisson model that loosens this restrictive assumption by specifying a Poisson-gamma mixture distribution, according to which overdispersion can be estimated. We include a constant, abstract from any country fixed-effect, pool our panel data and report the results of the Poisson and negative binomial regression in Table 3 below.

Table 3: The impact of key macro variables on trade disputes.

	Dependent : Trade disputes	
	Poisson	*Neg. Bin.*
Cst.	−1.3713**	−0.4761
	(-2.2299)	(-1.2349)
log(GDP)	0.3172***	0.2643***
	(8.8202)	(13.5361)
GDP growth	−0.0145	−0.0256***
	(-1.4586)	(-2.9315)
Net exports	−0.0200**	−0.0053
	(-2.0729)	(-0.7383)
Openness	−0.0076***	−0.0099***
	(-3.4974)	(-5.6510)
Exchange rate flexibility	0.0216	0.0418***
	(1.2763)	(2.7417)
Trade-weighted RER	−0.4597**	−0.6763***
	(-2.2171)	(-3.9684)
Observations	612	612
Dispersion parameter	1.0000	1.3569
Loglikelihood	−4532.61	−1974.60

Note: *p<0.1; **p<0.05; ***p<0.01.

Table 3 shows that positive GDP growth and net exports lower the probability of trade disputes. Depending on the specification, net exports or GDP growth are statistically non-significant. In both cases the signs make sense: countries with a growing economy and positive trade balance are less likely to initiate trade disputes. The log of GDP affects positively the probability of trade disputes under both specifications: larger countries initiate more trade disputes all else equal, potentially making strategic use of their home market size. Openness significantly lowers the probability of trade disputes, which can also be rationalized by the fact that more open economies have more to lose to deter international trade flows. Exchange rate flexibility is statistically non-significant in the Poisson model but has the same positive sign than in the negative binomial model, where it is statistically significant: all else equal, countries with a more flexible exchange rate are more likely to initiate trade disputes. Finally, competitiveness affects trade disputes in the way it is expected to: countries with a relatively depreciated trade-weighted real exchange rate are less likely to initiate trade disputes. Overall, we conclude that most macroeconomic variables inspected in this empirical work affect trade disputes in a significant and sensible way, and that our main variable of interest, exchange rate flexibility raises the probability of trade disputes.

6.2 Business cycle frequency evidence

We now investigate the effects of macroeconomic shocks on trade disputes/tariffs. In particular, we would like to uncover the response of tariffs, approximated by our trade dispute variable, to standard supply and monetary policy shocks. To do so, we estimate country-specific VAR models using quarterly data, and identify structural shocks using sign restrictions (see Uhlig (2005)).

Our previous analysis suggests that countries with flexible exchange rates are more likely to initiate trade disputes, and hence to apply tariffs. We thus focus on a subset of countries with flexible exchange rates for which we have a data sample that is long enough: Australia, Canada, and the USA. Each VAR is estimated with the following specification:

$$X_t = \alpha_1 + \alpha_2 \times t + \alpha_3 (L) X_t + \alpha_4 \xi_t \tag{105}$$

where $X_t = [log(1 + TD_t)\ log(Y_t)\ 100 \times \pi_t\ 100 \times i_t]'$, TD_t being the number of trade disputes, $log(Y_t)$ the log of GDP, $100 \times \pi_t$ and $100 \times i_t$ respectively being the core inflation rate and the nominal interest rate, in percents. The datasets are quarterly. In the above specification, α_1 is a vector of constants, α_2 is a vector of coefficients attached to the linear trend, and α_3 a matrix of lagged coefficients where L is the lag operator. Lag selection is achieved using the Akaike information criterion and yields 3 lags for Australia, 2 lags for Canada and 3 lags for the USA. Finally, α_4 is a variance-covariance matrix and ξ_t is vector of normally distributed innovations.

As usual, ξ_t contains the reduced-form innovations and those are not structural. The structural shocks are identified using the sign restriction method proposed by Uhlig (2005). Let u_t denote the set of structural shocks and define $\xi_t = B^{-1} u_t$ as the relation between reduced-form residuals and structural shocks. The identification method imposes restrictions on matrix B using commonly used assumptions about the sign of the implied responses of variables for some periods after the shock. Technically, this is done by drawing random orthonormal candidate B matrices, compute the associated IRFs and keep candidates that satisfy the imposed scheme of sign restrictions on the response of variables. If the restrictions are satisfied, IRFs and the associated B matrix are kept as a valid draw for the computation of median IRFs and confidence bands. We use the set of restrictions summarized in Table 4 below.

We impose that a trade dispute shock raises the number of trade disputes, and leave other the response of other variables unrestricted. A positive supply shock is assumed to raise GDP,

Table 4: Identifying restrictions for the SVAR analysis.

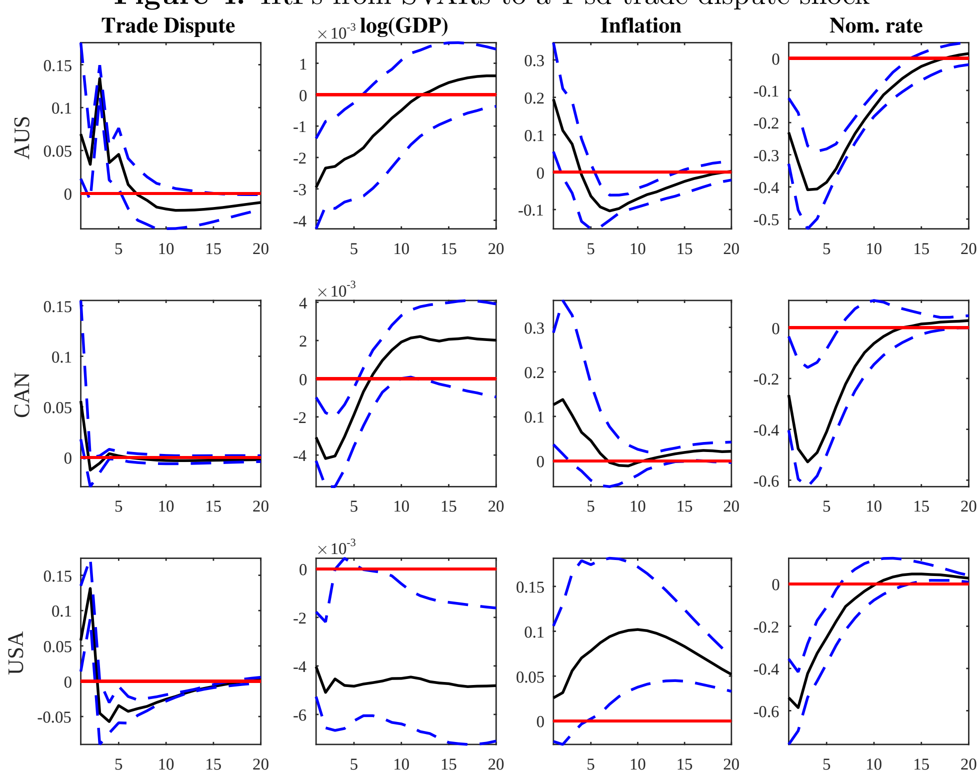
	Trade disputes	GDP growth	Inflation	Nominal interest rate
Trade dispute shock	+(1)	×	×	×
Supply shock	×	+(3)	−(3)	−(3)
Demand shock	×	+(1)	+(1)	+(1)
Monetary policy shock	×	−(2)	−(2)	+(2)

Note: A + (−) indicates that the response of the variable is positive (negative). A × indicates that the response of the variable is left unrestricted. The numbers in parentheses indicates the number of quarters after the shock for which the restriction is imposed.

lower inflation and the nominal interest rate. The response of trade disputes to a supply shock is left unrestricted. A demand shock raises GDP, inflation and the nominal interest rate, while the response of trade disputes is, once again, left unrestricted. Finally, we assume that a (restrictive) monetary policy shock raises the nominal interest rate, lowers the inflation rate as well as the level of GDP, and we leave the response of trade disputes unrestricted. Our main interest is the response of trade disputes to other structural shocks, with a special focus on monetary policy and supply shocks.

The IRFs resulting from our estimations and identification are reported in Figure 4 to 6.[13] Figure 4 reports the IRFs to a trade dispute shock for the different countries considered.

Figure 4: IRFs from SVARs to a 1-sd trade dispute shock

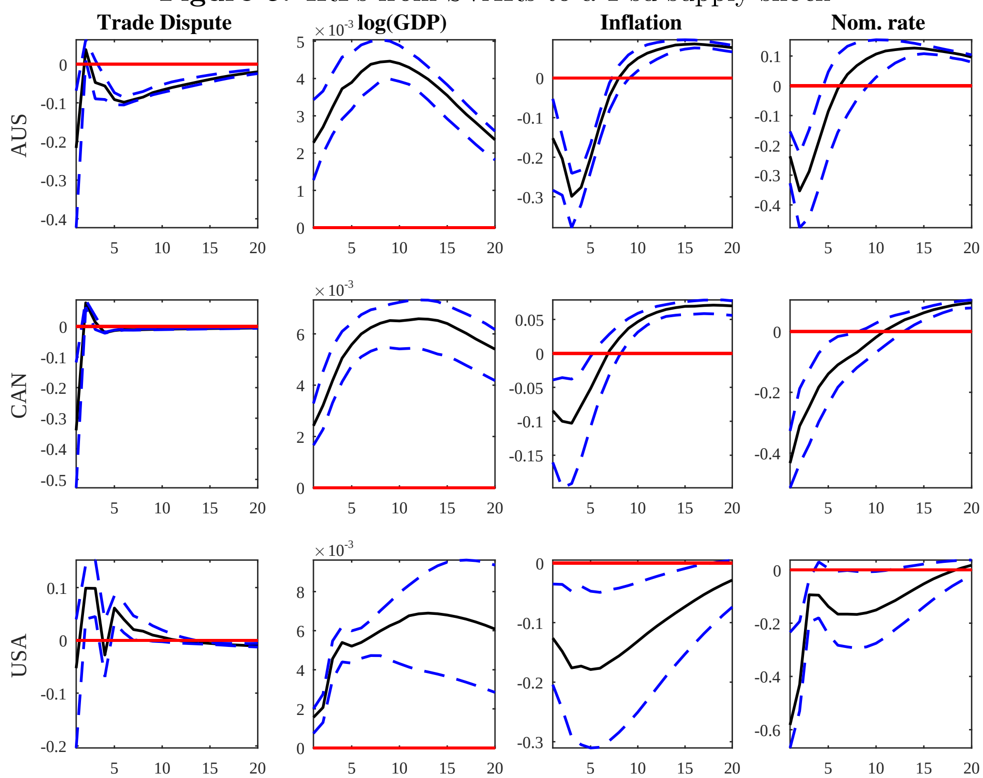


Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

[13] We draw enough candidate matrices until 100 matrices satisfy the imposed restrictions, which implies drawing thousands or tens of thousands of candidate matrices.

First, Figure 4 shows that a trade dispute shock is self-correcting in several countries, *i.e.* that that a rise in the number of trade disputes is followed by a subsequent statistically significant fall in trade disputes. Second, although we do not restrict the response of GDP to a trade dispute shock, we find that a rise in trade disputes lowers GDP quite substantially for at least 4 to 5 quarters. For the U.S., the effects on the level of GDP even more persistent. For all three countries, we also find that a trade dispute shock is inflationary, as core inflation increases significantly. This is consistent with the views that trade disputes or trade restrictions act as negative supply shocks, in line with the findings of Barattieri, Cacciatore, and Ghironi (2018). While interesting, these results are not our main focus though. Our chief interest is indeed in the response of trade disputes/tariffs to supply and monetary policy shocks. As such, Figure 5 reports the IRFs to a supply shock for the three countries.

Figure 5: IRFs from SVARs to a 1-sd supply shock

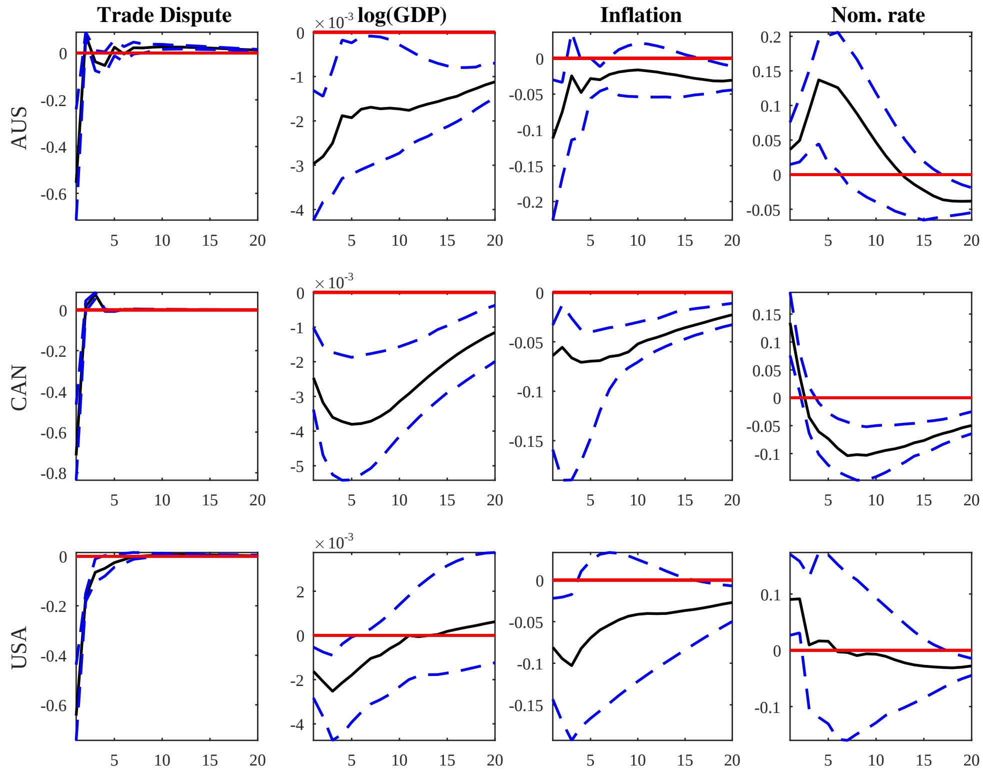


Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

Under our identifying assumptions, Figure 5 shows that a positive supply shock generates a fall in trade disputes on impact. The latter is statistically significant for Australia and Canada but not for the U.S. This pattern is consistent the empirical exercise of the previous section: a positive supply shock raises GDP growth and competitiveness, which makes the use of trade disputes/tariffs less appealing from a policy perspective, perhaps by raising the output costs of a tariff/trade war. Finally, Figure 6 reports the IRFs to a (restrictive) monetary policy shock for the different countries.

Figure 6 shows that the response of trade disputes to a tightening of monetary policy is significantly negative. This result is consistent with our theoretical findings, showing that a

Figure 6: IRFs from SVARs to a 1-sd restrictive monetary policy shock



Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

restrictive monetary policy shock lowers the current gains from raising tariffs while leaving the future costs of deviating from the sustainable path of tariffs basically unaffected.

Are those results robust to alternative identification schemes? To answer this question, Appendix C performs three different exercises.

In the first one, for the same three countries, bi-variate VARs are estimated using quarterly data on the growth rate of labor productivity and the total number of hours worked. Following Debortoli, Galì, and Gambetti (2018), supply shocks are identified using long-run restrictions. Supply shocks are assumed to be the only shocks that affect the level of labor productivity permanently. The identified shocks are then used to estimate a bi-variate VAR with trade disputes, assuming that supply shocks do not affect trade disputes permanently. Appendix C shows that supply shocks tend to lower trade disputes for all countries on impact, although the response of trade disputes is significant only for the U.S.

In the second one, monetary policy shocks are identified using a Cholesky decomposition where the nominal interest rate is ranked last, *i.e.* assuming that monetary policy shocks do not have contemporary effects on other variables. The VAR specification includes the trade dispute variable, the log of GDP, an index of oil prices, the core inflation rate and the nominal interest rate. Appendix C shows that restrictive monetary policy shocks identified using a Cholesky decomposition also produce a significant drop in trade disputes, as in the baseline case where monetary policy shocks are identified using sign restrictions.

In the third one, we perform SVAR estimations for the same three countries using sign restrictions but using tariffs instead of trade disputes, and including the real exchange rate as an additional variable. Appendix C shows that for these countries, tariffs tend to respond negatively to positive supply shocks (except for Canada), and negatively to restrictive monetary policy shocks (except for Australia). Overall, we conclude that our results are relatively robust to alternative identification schemes/specifications.

7 Conclusion

We discussed the joint determination of trade policies and exchange rates in a stylized open-economy model with sticky prices, productivity and money growth shocks. Despite its relative simplicity, the model is able to deliver a surprisingly large number of results that are consistent with empirical evidence. We found that policy-makers faced higher incentive to set tariffs in flexible exchange rate economies than in fixed exchange rate economies. We also found that tariffs resulting from a dynamic game were constant with flexible prices, and time-varying with sticky prices. Whether they fell or rose was shown to depend on whether the shock was to money growth or productivity, and on whether the intertemporal elasticity of substitution was above or below unity, reflecting the relative importance of income versus substitution effects of exchange rates on aggregate consumption.

Future research on the subject might involve considering the effects of alternative financial market structures on the incentive to apply tariffs. One could also be interested in looking at different pricing schemes for exports, considering local currency or dominant currency pricing for instance.

References

Adjemian, Stéphane, Houtan Bastani, Michel Juillard, Frédéric Karamé, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto, and Sébastien Villemot. 2011. "Dynare: Reference Manual, Version 4." Dynare Working Papers 1, CEPREMAP.

Bagwell, Kyle and Robert W. Staiger. 2003. "Protection and the Business Cycle." *The B.E. Journal of Economic Analysis & Policy* 3 (1):1–45.

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi. 2018. "Protectionism and the Business Cycle." NBER Working Paper 24353.

Benigno, Gianluca, Pierpaolo Benigno, and Fabio Ghironi. 2007. "Interest Rate Rules for Fixed Exchange Rate Regimes." *Journal of Economic Dynamics and Control* 31 (7):2196–2211.

Bergin, Paul R. and Giancarlo Corsetti. 2015. "Beyond Competitive Devaluations: The Monetary Dimensions of Comparative Advantage." CEPR Discussion Paper 10718.

Bown, Chad P. 2016. "Global Antidumping Database." The World Bank (available at http://econ.worldbank.org/ttbd/gad/).

Debortoli, Davide, Jordi Galì, and Luca Gambetti. 2018. "On the Empirical (Ir)Relevance of the Zero Lower Bound Constraint." Barcelona GSE Working Paper 1013.

Eaton, Jonathan and Gene M. Grossman. 1985. "Tariffs as Insurance: Optimal Commercial Policy When Domestic Markets Are Incomplete." *Canadian Journal of Economics* 18 (2):258–272.

Erceg, Christopher, Andrea Prestipino, and Andrea Raffo. 2018. "The Macroeconomic Effect of Trade Policy." 2018 Meeting Papers 221, Society for Economic Dynamics.

Furceri, Davide, Swarnali A. Hannan, Jonathan D. Ostry, and Andrew K. Rose. 2018. "Macroeconomic Consequences of Tariffs." Annual research meeting, IMF.

Ilzetzki, Ethan, Carmen M. Reinhart, and Kenneth S. Rogoff. 2017. "Exchange Arrangements Entering the 21st Century: Which Anchor Will Hold?" NBER Working Paper 23134.

Obstfeld, Maurice and Kenneth Rogoff. 1995. "Exchange Rate Dynamics Redux." *Journal of Political Economy* 103 (3):624–660.

Ossa, Ralph. 2014. "Trade Wars and Trade Talks with Data." *American Economic Review* 104 (12):4104–46.

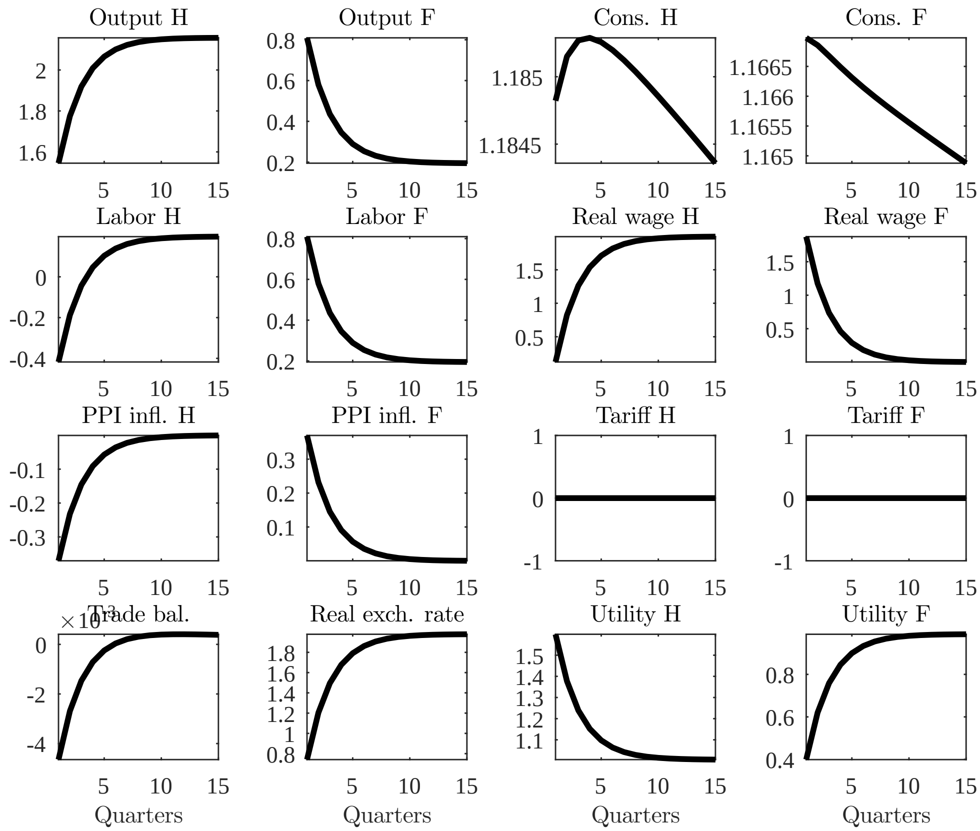
Uhlig, Harald. 2005. "What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure." *Journal of Monetary Economics* 52 (2):381–419.

UNCTAD. 2013. "NON-TARIFF MEASURES TO TRADE: Economic and Policy Issues for Developing Countries." *United Nations* .

A Impulse responses under sticky prices with flexible exchange rate and exogenous tariffs

Figure 7 to 2 report the impulse response functions respectively to a quasi-permanent Home productivity shock, a temporary monetary policy shock and a quasi-permanent tariff shock.

Figure 7: IRFs to a quasi-permanent Home productivity shock



Note: All variables are reported in percent deviation from their steady state values, except for the trade balance, that is reported in absolute deviation.

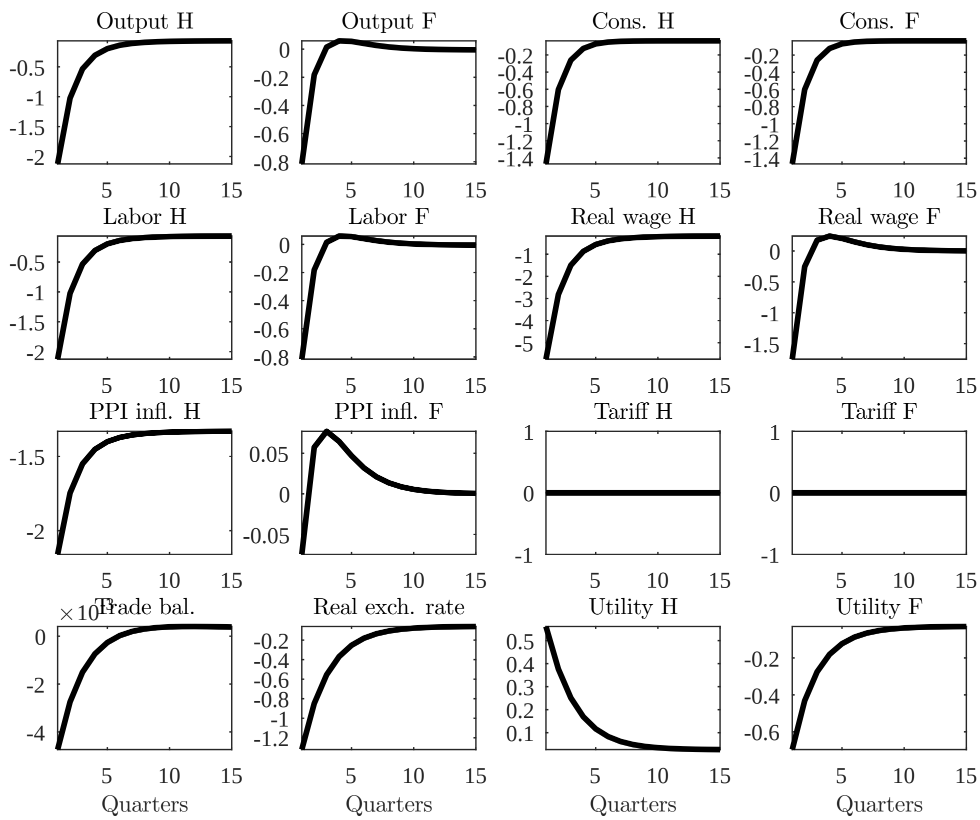
B Data description

B.1 Exchange rate regimes

We test the effects of various variables on trade disputes. The explained variable is the total number of trade disputes observed for country i at time t. The dataset is annual and covers the following countries: Argentina, Australia, Brazil, Canada, Chile, China, Columbia, Costa Rica, the European Union, India, Indonesia, Israel, Japan, South Korea, Mexico, Malaysia, New Zealand, Pakistan, Peru, Philippines, Russia, Thailand, Turkey, Taiwan, the USA, Venezuela and South Africa. The longest time range is 1977-2015 and most countries have more limited time ranges. The potential explanatory variables are:

- The log of GDP (PennWorld Tables 9.0)

Figure 8: IRFs to a restrictive Home monetary policy shock



Note: All variables are reported in percent deviation from their steady state values, except for the trade balance, that is reported in absolute deviation.

- The annual rate of GDP growth (PennWorld Tables 9.0)
- The depreciation rate of nominal exchange rate (vs the US dollar). Positive = depreciation, negative = appreciation. (PennWorld Tables 9.0)
- Net exports to GDP (PennWorld Tables 9.0)
- The openness ratio: exports plus imports divided by GDP (PennWorld Tables 9.0)
- The exchange rate regime according to the Ilzetzki, Reinhart, and Rogoff (2017) classification (http://www.carmenreinhart.com/data/browse-by-topic/topics/11/)
- The trade-weighted measure of the real exchange rate take from the EQCHANGE CEPII Database (http://www.cepii.fr/CEPII/fr/bdd_modele/presentation.asp?id=34). Weights are time-varying (averaged over the last 5 years). This variable is an index (100 in 2010) and expressed with opposite signs as usual in the litt. We thus take $log(100/x)$ where x is the variable expressed in the database to get a measure of depreciation in the usual sense (positive = depreciation, negative = appreciation).

B.2 Trade disputes at the business cycle frequency

In this subsection, we focus on a narrower set of countries: Australia, Canada, and the U.S. Each VAR contains the log of one plus the number of trade disputes, the log of GDP, the annualized rate of core inflation and the nominal interest rate. The time series for these last three variables are taken from the OECD Economic Outlook dataset for a time range that corresponds to the time range of available data for trade disputes:

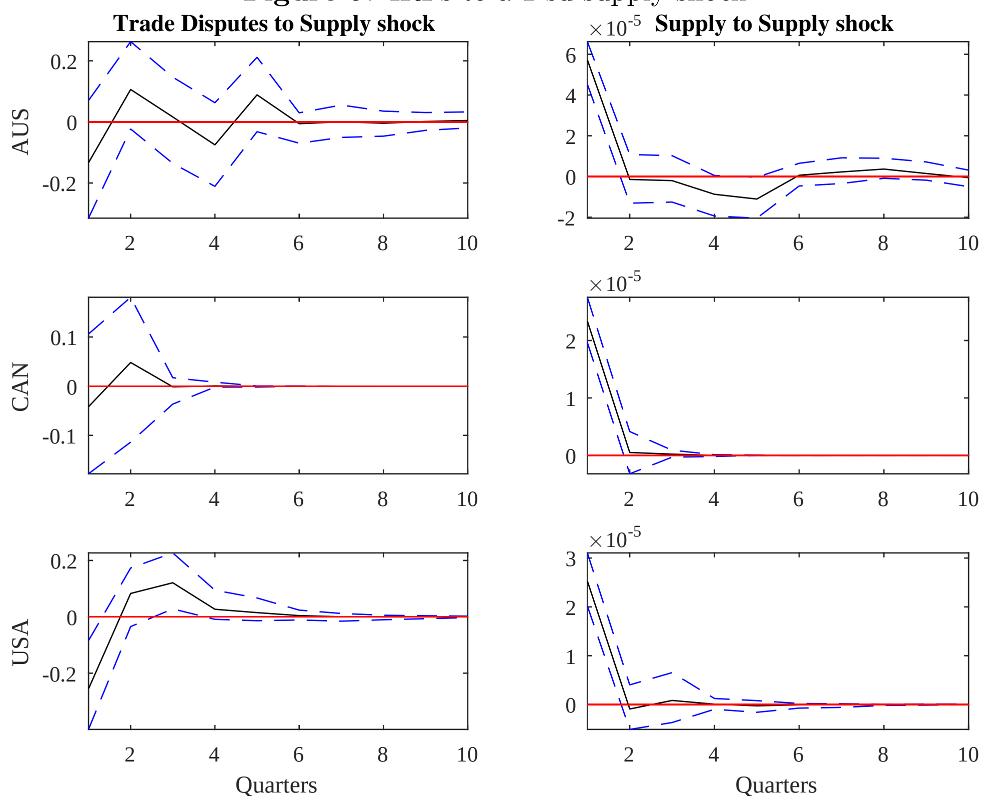
- Australia: 1989-2015
- Canada: 1985-2015
- USA: 1979-2015

C Robustness checks for the VAR analysis

C.1 Supply shocks

An alternative way of identifying supply shocks is the one proposed by Debortoli, Galì, and Gambetti (2018). According to this approach, a bi-variate VAR is estimated using the log-difference (growth rate) of labor productivity and the total number of hours worked. For the three countries, we use OECD data for the log level of GDP (y_t), for the average number of hours worked annually (h_t) and total employment (e_t) to build a measure of the total number of hours worked ($n_t = log(h_t \times e_t)$) and the associated growth rate of labor productivity ($\Delta(y_t - n_t)$). The time range remains unchanged for the three countries. The vector of variables included in the VAR is thus $X_t = [\Delta(y_t - n_t)\ n_t]'$, the VAR includes a constant and a trend, and lag selection proceeds through Akaike information criterion. The structural shocks are identified using long-run restrictions *à la* Blanchard and Quah: demand shocks are assumed to be those that do have permanent effects on the *level* of labor productivity. By contrast, the other shock is a supply shock. Once the supply shocks have been identified for the three countries, we estimate a bi-variate VAR with $X_t = [TD_t\ supply_t]'$, and report the effects of an innovation to the supply shock in Figure 9 below.

Figure 9: IRFs to a 1-sd supply shock

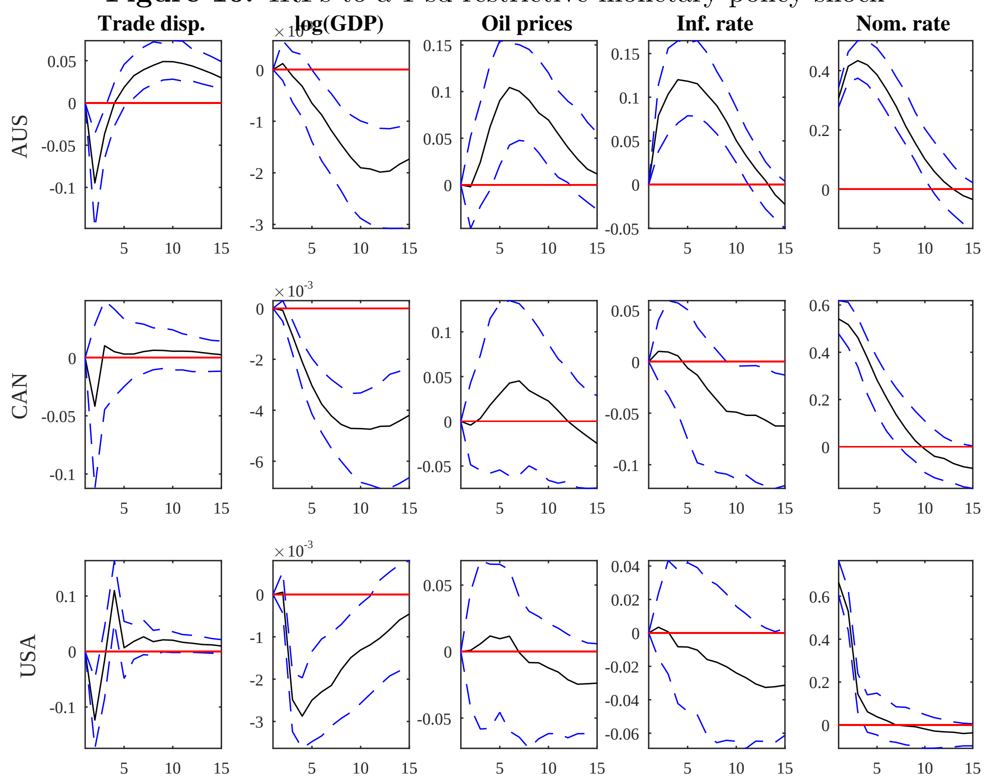


Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

C.2 Monetary policy shocks

We also investigate the robustness of our results to an alternative identification scheme for monetary policy shocks. The usual alternative to sign restriction is the Cholesky decomposition of the B matrix where the nominal interest rate is ranked last. The assumption then amounts to impose that monetary policy shocks do not have any contemporaneous effects on all the variables included in the VAR. We consider the following variables and ranking: the log of one plus trade disputes, the log of GDP, an oil price index, core inflation and the nominal interest rate. GDP, core inflation and the nominal interest rate are taken from the OECD Economic Outlook database. The oil price index is computed from the Spot Crude Oil Price (WTI), taken from the FRED database. The specification includes a constant and a trend, and lag selection proceeds through Akaike information criterion. Figure 10 below reports the effects of a monetary policy shock identified through short-run restrictions in the three countries.

Figure 10: IRFs to a 1-sd restrictive monetary policy shock

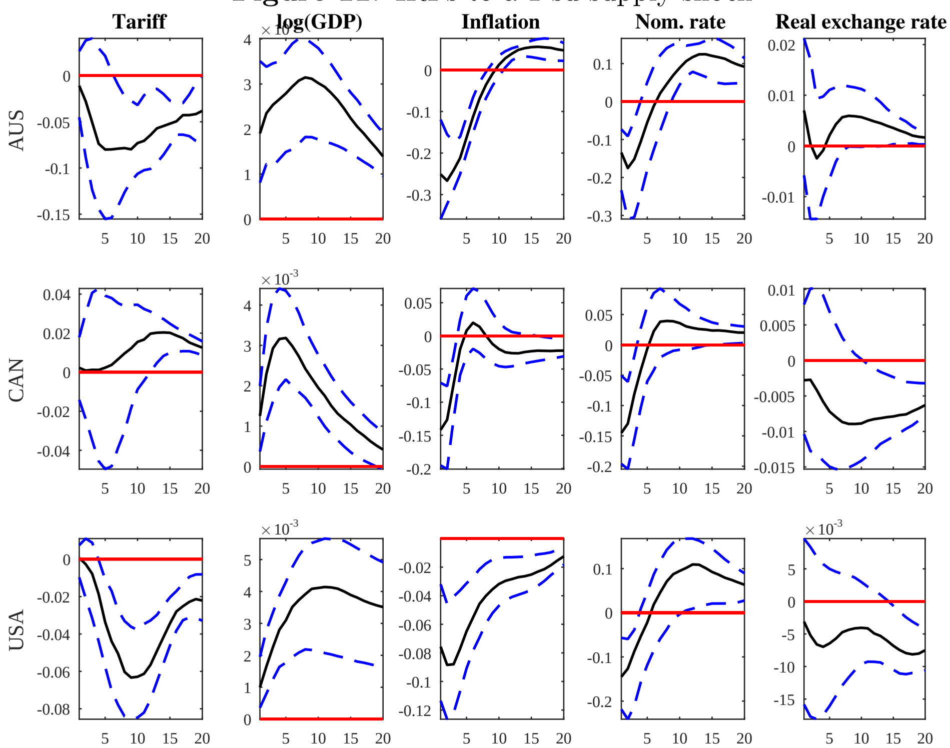


Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

C.3 Alternative specification

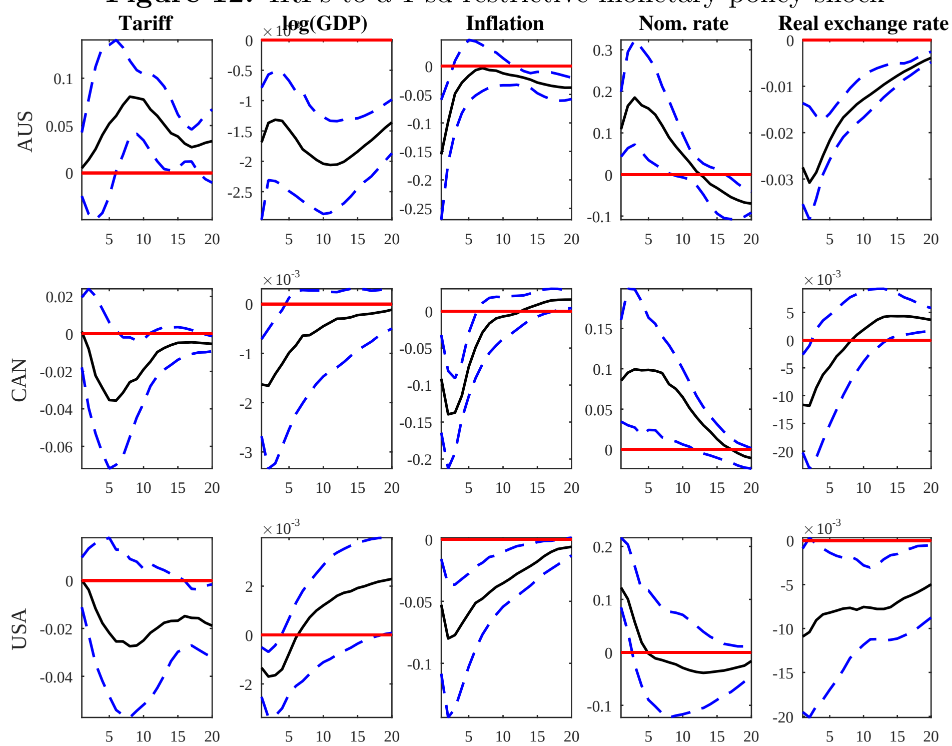
One might wonder what are the results of considering tariffs instead of trade disputes in our VAR analysis. In addition, one might also wonder what happens if the real exchange rate is introduced. We perform both exercises in this subsection. Instead of trade disputes, we consider the average tariff rate. We use data from the World Bank Development Indicators, that reports the weighted-average tariff rate applied by several countries from 1988 to 2015. One important potential caveat is that the dataset for average tariffs is annual, which explains why we do not use this specification as our benchmark. We convert annual time series into quarterly times series using a spline interpolation method, and include this variable along with the log of quarterly GDP, the quarterly core inflation rate and the quarterly series for the nominal interest rate. We also include the real exchange rate at a quarterly frequency using the competitiveness indicator reported by the OECD Economic Outlook database. The latter is computed based on trade-weighted relative CPIs, and has to be inverted to be made consistent with the model, *i.e.* so that it goes up in case of a real depreciation. We consider two-lags, include a constant and a linear trend, and the identification scheme remains broadly the same. In particular, the response of the real exchange rate to supply and monetary policy shocks is left unrestricted. We get the results reported in Figure 11 and 12 below.

Figure 11: IRFs to a 1-sd supply shock



Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.

Figure 12: IRFs to a 1-sd restrictive monetary policy shock



Note: Confidence bands respectively represent the 16th and 84th percentiles of IRFs.