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NEGATIVE SWAP SPREADS AND LIMITED ARBITRAGE

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ABSTRACT

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Abstract

Since October 2008 fixed rates for interest rate swaps with a thirty year maturity have been mostly below treasury rates with the same maturity. Under standard assumptions this implies the existence of arbitrage opportunities. This paper presents a model for pricing interest rate swaps where frictions for holding bonds limit arbitrage. I show analytically that negative swap spreads should not be surprising. In the calibrated model, swap spreads can reasonably match empirical counterparts without the need for large demand imbalances in the swap market. Empirical evidence is consistent with the relation between term spreads and swap spreads in the model. Keywords: Swap spread, limited arbitrage, fixed income arbitrage (JEL: G12, G13).

1 Introduction

Interest rate swaps are the most popular derivative contracts. According to the Bank for International Settlements, for the first half of 2015, the notional amount of such contracts outstanding was 320 trn USD. In a typical interest rate swap in USD, a counterparty periodically pays a fixed amount in exchange for receiving a payment indexed to LIBOR. Since October 2008, the fixed rate on swaps with a thirty year maturity has typically been below treasuries with the same maturity, so that the spread for swaps relative to treasuries has been negative. What in 2008 may have looked like a temporary disruption related to the most virulent period of the financial crisis has persisted for years, see Figure 1.

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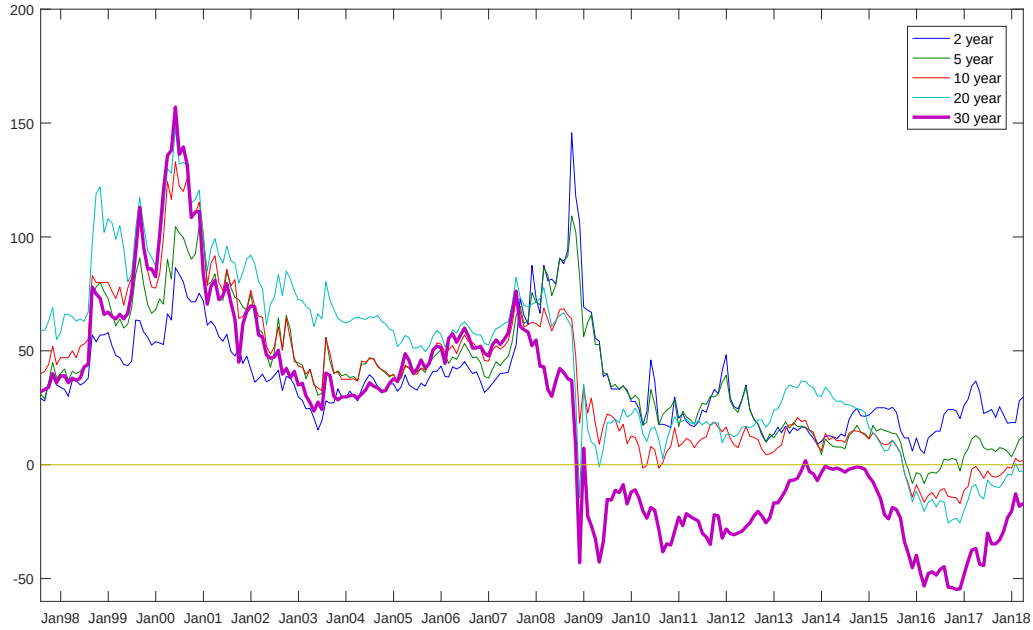

Figure 1: Swap Spreads. *Difference between fixed swap rate and treasury yield of same maturity. Units are in basis points.*

Negative swap spreads are challenging for typical asset pricing models as they seem to imply a risk-free arbitrage opportunity. By investing in a treasury bond and paying the lower fixed swap rate, an investor can generate a positive cash flow. With repo financing for the bond, the investor would also typically receive a positive cash flow from the difference between LIBOR and the repo rate. If the position is held to maturity, and if LIBOR remains above repo, this represents a risk-free arbitrage. In reality, a shorter horizon exposes the investor to the risk of an even more negative swap spread. Possible disruptions in the repo funding can also make such an investment risky, and capital requirements can add costs. While there seem to be good reasons for why arbitrage would be limited in this case, there are so far no equilibrium asset pricing models that are consistent with negative swap spreads.

This paper develops a model for pricing interest rates swaps that features limited arbitrage. In the model, dealers invest in fixed income securities. A dealer can buy and sell risk-less debt with different maturities, as well as interest rate swaps. Debt prices are exogenous, the model prices swaps endogenously. Without frictions, the price of a swap equals its no-arbitrage value, and the swap spread has to be positive. When frictions limit the size of the dealer's fixed income investments, swaps cannot be fully arbitraged, and swaps are priced with state prices that are not fully consistent with bond prices.

My main finding is that with limited arbitrage, negative swaps spreads are not surprising anymore, even without explicit demand effects. With frictions, dealers have smaller bond positions and are less exposed to long-term interest rate risk. They require less compensation for the exposure to the fixed swap rate and, therefore, the swap rate is lower. In the model, in the limit as frictions become more extreme, the unconditional expectations of the swap rate and LIBOR are equalized. With long-term treasury rates typically larger than LIBOR, the swap spread would then naturally be negative. Equivalently, because the TED spread is typically smaller than the term spread, the swap spread would be negative. Quantitatively, with moderate frictions for holding long-term bonds, the model can produce thirty-year swap spreads in the range observed since October 2008. Model extensions such as demand effects and swap holding costs can affect swap rates in meaningful ways, but they are unlikely to be the main drivers of recent negative swap spreads. Explicit leverage constraints or capital requirements are shown to affect swap spreads similarly to holding costs for long-term bonds. Another implication of the model is that, conditional on short-term rates, term spreads are negatively related to swap spreads. Empirical evidence consistent with this regularity is presented.

Practitioners have advanced a number of potential explanations for why swap spreads have turned negative, the so-called swap spread *inversion*. Consistently among the main reasons is the notion that stepped-up banking regulation in the wake of the global financial crisis has made it more costly for banks to hold government bonds. For instance, Bowman and Wilkie (2016) at Euromoney magazine write on this topic: "... there is little doubt about the impact of regulation – primarily the leverage ratio and supplementary leverage ratio – on bank balance-sheet capacity and market liquidity. ... The leverage ratio has made the provision of the repo needed to buy treasuries prohibitively expensive for banks." As it has become more costly for banks to hold treasuries, apparent arbitrage opportunities can persist. In my model, it is costly for dealers to hold treasuries and this reduces the size of their bond positions. This leads to the possibility that swaps are no longer priced in line with treasuries. A key insight provided by my model is that with arbitrage limited in this way, swap spreads should naturally be negative, even in the absence of explicit demand effects. Banks have been required to disclose their supplementary leverage ratios starting in 2015. Consistent with the model, as shown in Figure 1, empirical swap spreads declined sharply in 2015. More generally, the anticipation of new capital requirements, as well as other regulatory changes impacting dealer arbitrage trades between swaps and government bonds following the financial crisis, motivate the frictions in the model that lead to negative swap spreads.

A large literature has developed models with limited arbitrage where frictions faced by

specialized investors can affect prices. For instance, Shleifer and Vishny (1997) consider mispricing due to the limited capital of arbitrageurs, Dow and Gorton (1994) study the impact of holding costs when traders have limited horizons. Other examples include Garleanu, Pedersen and Poteshman (2009) on pricing options when risk-averse investors cannot hedge perfectly, Gabaix, Krishnamurthy and Vigneron (2007) on the market for mortgage-backed securities, and Vayanos and Vila (2009) who price long-term bonds with demand effects. Liu and Longstaff (2004) analyze portfolio choice for arbitrageurs with collateral constraints, and Tuckman and Vila (1992) with holding costs. For a survey of this literature, see Gromb and Vayanos (2010). As in most of these papers, in my model specialized investors determine the price of some security with other prices given exogenously. So far, this literature has not considered interest rate swaps.[1]

Empirical studies have documented the drivers of swap spreads with factor models, in particular Liu, Longstaff and Mandell (2006) and Feldhuetter and Lando (2008). More recently, Hanson (2014) documents the relation between MBS duration and swap spreads. Gupta and Subrahmanyam (2000) study swap prices relative to the prices of interest rate futures, and Eom, Subrahmanyam and Uno (2002) the links between USD and JPY interest rate swaps. Collin-Dufresne and Solnik (2001) focus on the impact of the LIBOR panel selection for swap pricing. Johannes and Sundaresan (2007) theoretically and empirically find an increase in swap rates due to collateralization. Studies that focus on the period with negative swap spreads include Smith (2015) who analyzes the principal components in swap spreads, and Klinger and Sundaresan (2016) who document a relation between pension funds duration hedging and negative swap spreads. Boyarchenko, Gupta, Steele and Yen (2018) present an example of how regulatory changes have affected the cost for supervised institutions to enter interest swap spread trades.[2]

My paper contributes to the literature by developing a model that determines swap spreads with limited arbitrage. It is shown analytically and quantitatively that the model can plausibly explain negative swap spreads. The model is also shown to be consistent with additional empirical evidence on the relation between swap spreads and term spreads. In my model long-term debt and swaps are modelled with geometric amortization, a feature used for tractability in models for corporate debt or sovereign debt, following Leland (1998). The

[1] Faulkender (2005) documents a relation between the term spread and interest swap usage for firms in the chemical industry, suggesting market timing motivation. Jermann and Yue (2018) present a model of nonfinancial firms' swap demand. Both of these abstract from the swap spread.

[2] Duffie (2016) documents increased financial intermediation costs for U.S. fixed income markets due to the tightening of leverage ratio requirements for banks. Du, Tepper and Verdelhan (2016) show that deviations from covered interest parity have persisted since 2008 and relate these to increased banking regulation. Andersen, Duffie and Song (2018) examine funding valuation adjustments in connection with violations of arbitrage-based pricing relations.

model remains challenging numerically because it includes a dynamic portfolio problem with potentially large short and long positions in multiple securities with incomplete markets that needs to be combined with the pricing of swap contracts with a long maturity. Only a global solution seems to be able to offer the required numerical precision.

In the rest of the paper the model is first presented, followed by analytical characterizations of the arbitrage-free case and the case with frictions. Section 4 contains the model's quantitative implications and additional empirical evidence on swap spreads. Section 5 concludes.

2 Model

A dealer with an infinite horizon invests in bonds and swaps. Bond prices are exogenous, the swap price is endogenous. The model is driven by the exogenous prices for the bonds and inflation. Long-term bonds and swaps have geometric amortization with a given maturity parameter.

2.1 Available assets

The dealer chooses among three securities: short-term risk-free debt (which we can think of as treasury or repo), long-term default-free debt (treasury bonds), and fixed-for-floating interest rate swaps. The risk averse dealer takes prices as given and maximizes the lifetime utility of profits. Prices for swaps are determined in equilibrium to clear the swap market. The demand for swaps is assumed to come from endusers such as corporations and insurance companies. Swap contracts are free of default risk, as they nowadays are mostly collateralized. The fixed swap rate can differ from the long-term bond with the same maturity because the floating leg pays LIBOR which typically exceeds the short-term treasury rate. A process for the LIBOR rate is assumed. Because holding bonds is costly, the dealer cannot arbitrage between securities in a frictionless way, and this creates an additional wedge between fixed swap rates and rates for long-term bonds.

Short-term riskless debt pays one unit of the numeraire (the dollar) next period and has a current price of

$$q_{ST}(z) = \exp\left(-y_{ST}(z)\right),$$

with y_{ST} the log of the short rate. The exogenous z follows a finite-state Markov process.

LIBOR debt pays one unit of the numeraire next period. The price of LIBOR debt is

$$q_{LIB}(z) = \exp\left(-y_{LIB}(z)\right)$$

with the log yield

$$y_{LIB}(z) = y_{ST}(z) + \theta(z).$$

We can think of θ as the so-called TED ("Treasury Euro-Dollar") spread, where LIBOR is referred to as the Euro-Dollar rate. Historically, 3-month TED spreads have never been negative; the model will satisfy this property. The TED spread can be thought of as compensating for some disadvantage of bank debt relative to the risk-free debt. This could be reduced liquidity or higher default risk. Explicitly modelling the sources of this spread would be conceptually straightforward, but would burden computations, without an obvious benefit for the current analysis.

Long-term default-free debt pays

$$c_{LT} + \lambda$$

per period, where c_{LT} is the coupon and λ the amortization rate, implying an average maturity of $1/\lambda$. In the next period the owner of the bond gets

$$c_{LT} + \lambda + (1-\lambda) q'_{LT}(z'),$$

where q'_{LT} is the market price of the long-term bond next period. The price of this bond is related to its yield to maturity, $\exp(y_{LT}) - 1$, which after solving for the infinite sum can be written as

$$q_{LT}(z) = \frac{c_{LT} + \lambda}{\exp(y_{LT}(z)) - 1 + \lambda}.$$

Clearly, with the bond at par, $q_{LT}(z) = 1$, we have $c_{LT} = \exp(y_{LT}(z)) - 1$. We model the exogenous yield process as

$$y_{LT}(z) = y_{ST}(z) + \tau(z),$$

with $\tau(z)$ the stochastic term spread. Note that this relation between q_{LT} and y_{LT} is without loss of generality; λ and c are constants.

Swaps pay a constant coupon in exchange for LIBOR. The value of a swap, or its price, is denoted by m. This price captures mark-to-market gains and losses for the swap. In particular, next period, the fixed rate receiver of the swap gets

$$c_{Sw} - (\frac{1}{q_{LIB}(z)} - 1) + (1-\lambda) m',$$

with c_{Sw} the fixed coupon rate. The maturity parameter for the swap, λ, is the same as for the long-term debt. This could easily be changed, but given our focus on the spreads of swaps and bonds with the same maturity, does not seem useful. My way of modelling

an interest rate swap with geometric amortization is inspired by Leland's (1998) model for long-term debt. As in this case, the advantage of this representation is that the swap does not age, and the model does not require swaps with multiple maturities.

A newly minted swap has its coupon rate set so that the swap has a market value of zero, $m = 0$. The coupon rate c_{Sw} for which the current price of the swap equals zero is called the swap rate, y_{Sw}. The swap spread is defined as

$$y_{Sw} - (\exp(y_{LT}) - 1).$$

Empirically, swaps have zero initial value, and new swap contracts are continuously offered with fixed coupon rates so that the contract value is zero. In the general model, where the net demand facing the dealer, $d(z)$, is not zero, we can think that the model has only one swap, whose coupon does not change but that is traded at its mark-to-market value. The dealer then only trades the swap with this fixed coupon, and not new at-market swaps. Having a new at-market swap every period would create an infinite dimensional state variable, and make the model intractable. For the special case with a net demand of swaps facing the dealer that is zero for all periods, $d(z) = 0$, the existence of swaps does not affect the equilibrium. Therefore, new swaps, with normalized coupons can be continuously introduced and priced, and a time-series of swap rates can be generated in the model. Given this obvious advantage, most of the numerical analysis is focused on this special case.

Long or short positions for bonds are potentially costly to hold for the dealer. Specifically, the cost for holding short-term debt is given by

$$h(\alpha'_{ST}) = \frac{\kappa_{ST}}{2}(\alpha'_{ST})^2. \tag{1}$$

The cost is incurred in the current period, with $\kappa_{ST} \geq 0$ the cost parameter and α'_{ST} the amount of the short-term bond bought this period and held into next period. Similarly, the cost for long-term debt is

$$j(\alpha'_{LT}) = \frac{\kappa_{LT}}{2}(\alpha'_{LT})^2. \tag{2}$$

These costs capture financing and regulatory costs and create the frictions that limit perfect arbitrage. The quantitative analysis below also considers capital requirements that can be seen as contributing to bond holding costs.

It is straightforward to introduce a cost function for holding swaps. To keep the theoretical analysis focused, this is postponed to the quantitative section of the paper, and the derivations are in the appendix. The costs and the consequences for equilibrium swap prices depend to a large extent on the exogenous demand the dealer is facing.

2.2 Maximization, equilibrium, and solution

The dealer maximizes lifetime utility of profits by selecting short and long bonds, swaps and payouts. Specifically, the dealer solves

$$V(\omega, z) = \max_{c, \alpha'_{ST}, \alpha'_{LT}, s'} u(c) + \beta(\omega, z) E(V(\omega', z'))$$

subject to

$$c = \omega - \alpha'_{ST} q_{ST}(z) - \alpha'_{LT} q_{LT}(z) - s' m - h(\alpha'_{ST}) - j(\alpha'_{LT})$$

and

$$\omega' = \frac{\alpha'_{ST}}{e^{\mu(z')}} + \frac{\alpha'_{LT}}{e^{\mu(z')}} [c_{LT} + \lambda + (1-\lambda) q_{LT}(z')] + \frac{s'}{e^{\mu(z')}} \left[c_{Sw} - \left(\frac{1}{q_{LIB}(z)} - 1 \right) + (1-\lambda) m' \right] + \pi(z').$$

$\mu(z')$ is the log inflation rate and s the amount of the swap. Other profits, $\pi(z)$, are included for generality of the analytical analysis; in the computations they are set to 0. Payouts or consumption c are valued with momentary utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and the discount factor is of the Uzawa-Epstein type, $\beta(\omega, z) = (1+\bar{c})^{-\nu}$, with $\nu > 0$, and $\bar{c}$ equilibrium consumption which the dealer does not internalize. With this specification the dealer discounts the future more when wealth and consumption/payouts are high. Relative to a constant discount factor, wealth accumulation is favored when wealth is low and limited when wealth is high. This helps make the model more tractable numerically, but does not directly produce pricing frictions. This specification is popular for inducing stationarity in small open models with incomplete markets, following Mendoza (1991) and Schmitt-Grohe and Uribe (2003).

In equilibrium

$$s' = -d(z)$$

where $d(z)$ is the net market demand for the fixed receiver swap.

The state vector includes cash-at-hand (or the net asset position if $\pi = 0$) ω, and the exogenous state that determines bond prices, inflation and possibly swap demand ($y_{ST}(z)$, $\tau(z)$, $\theta(z)$, $\mu(z)$, $d(z)$, $\pi(z)$).

First-order conditions for bonds and swaps are given by

$$q_{ST}(z) = \beta E \frac{u_1(c')}{u_1(c) e^{\mu(z')}} - h_1(\alpha'_{ST}), \tag{3}$$

$$q_{LT}(z) = \beta E \left(\frac{u_1(c')}{u_1(c) e^{\mu(z')}} [c_{LT} + \lambda + (1-\lambda) q_{LT}(z')] \right) - j_1(\alpha'_{LT}), \tag{4}$$

$$m = \beta E\left(\frac{u_1\left(c'\right)}{u_1\left(c\right)e^{\mu(z')}}\left[c_{Sw} - \left(\frac{1}{q_{LIB}\left(z\right)} - 1\right) + \left(1-\lambda\right)m'\right]\right). \tag{5}$$

As is clear from the first-order conditions for short-term and long-term debt, holding costs introduce a wedge in the dealer's Euler equations. As a consequence, the price of the swap – given in equation 5 – is typically not equal to its no-arbitrage value.

3 Analytical characterization

Several properties of the swap spread can be derived analytically. Analytical expressions also help understand some of the quantitative findings. I focus on three cases. First, some properties of the no-arbitrage case are reviewed. Second, it is shown how frictions for holding short-term and long-term debt affect swap prices in general, and for the specific frictions considered in my quantitative model. For the third case, it is shown how with very strong frictions a negative swap spread should be expected.[3]

3.1 No-arbitrage case

In this subsection, the swap spread is characterized explicitly when arbitrage is ruled out, and it is shown why a limited arbitrage approach is needed to produce a negative swap spread. Specifically, ruling out arbitrage, this section establishes that if the TED spread (three-month LIBOR minus three-month treasury) is constant, the swap spread is equal to that constant value, and otherwise, if TED is nonnegative, the swap spread also needs to be nonnegative.

Rewriting the dealer's first-order condition for the swap for a more general state-price process, explicit sequential time-indexing, and with an analytically more convenient additive notation for the TED spread, the price of the swap is given as

$$m_t = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1\right) - \theta_t + \left(1-\lambda\right)m_{t+1}\right]\right).$$

Ruling out arbitrage implies that Λ_t prices not only swaps, but also short-term and long-term debt. Under this assumption, and after some algebra (detailed in the appendix), the value of the swap can be written as

$$m_t = \left(c^{Sw} + \lambda\right)\Omega_t\left(\{1\}\right) - 1 - \Omega_t\left(\{\theta_t\}\right), \tag{6}$$

[3] In this section, yields are compounded per period, while in the rest of the paper they are continuously compounded. This is for convenience. The notation does not explicitly aknowledge this difference.

with

$$\Omega_t\left(\{x_t\}\right) = \sum_{j=0}^{\infty} (1-\lambda)^j E_t \frac{\Lambda_{t+1+j}}{\Lambda_t} x_{t+j}.$$

Ω_t is the present value of a sequence of geometrically declining, potentially random, payoffs x, that are paid out with a one period lag. Intuitively, the term $c^{Sw}\Omega_t\left(\{1\}\right)$ captures the annuity value of receiving the fixed coupon, while $1-\lambda\Omega_t\left(\{1\}\right)$ represents the value of a floating rate note paying the risk-free short rate adjusted for the amortization payments. The last term in (6) represents the present value of the sequence of TED spreads.

Defining the (at-market) swap rate y_t^{Sw} as

$$0 = \left(y_t^{Sw} + \lambda\right)\Omega_t\left(1\right) - 1 - \Omega_t\left(\{\theta_t\}\right),$$

implies

$$y_t^{Sw} = \frac{1 + \Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)} - \lambda.$$

Consider a long term-bond with the same amortization rate as the swap and whose price can be written as

$$q_t^{LT} = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{LT} + \lambda + (1-\lambda)\, q_{t+1}^{LT}\right]\right) = \left(c^{LT} + \lambda\right)\Omega_t\left(1\right).$$

Combined with the implicit definition of the yield from above, $q_t^{LT} = \frac{c^{LT}+\lambda}{y_t^{LT}+\lambda}$,

$$y_t^{LT} = \frac{1}{\Omega_t\left(1\right)} - \lambda.$$

The swap spread then equals

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)}. \tag{7}$$

As is clear from equation (7), if $\theta_t = \theta$,

$$y_t^{Sw} - y_t^{LT} = \theta.$$

If $\theta_t \geq 0$,

$$y_t^{Sw} - y_t^{LT} \geq 0.$$

To summarize these results, ruling out arbitrage, the swap spread equals the present value of a TED annuity scaled by the present value of a constant annuity at 1. If TED is non-negative, the swap spread is non-negative. If TED is constant, the swap spread equals the constant TED spread. Clearly, without violation of arbitrage, the swap spread cannot be negative. In

my model, arbitrage is limited by the holding costs for bonds. Instead of the geometrically amortizing structures, the same argument can be made with standard swaps and bullet bonds. In this subsection, there is no advantage of using the geometrically amortizing bond; it is essential however for numerical tractability.

3.2 Bond holding frictions

I now assume that there are frictions for the one-period debt and for the long-term debt with the same maturity as the swap. Note that this is more general than the quantitative model presented in Section 2 because no assumptions are made about whether the dealer trades bonds for other maturities and because the frictions can be more general.

As above, assume that the value of the swap satisfies

$$m_t = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1\right) - \theta_t + (1-\lambda)\, m_{t+1}\right]\right).$$

Contrary to the no-arbitrage case, the dealer's marginal valuations, Λ, are now no longer necessarily consistent with the prices of risk-free debt.

Assume that frictions for short-term debt affect the relation between the market price and the dealer's marginal valuations such that

$$q_t^{ST} = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \mathfrak{h}_t,$$

where $\mathfrak{h}_t$ is a wedge coming from frictions of holding/selling short-term debt. For instance, this could be the derivative of a convex holding cost function, as in my quantitative model from the previous section, and $\mathfrak{h}_t$ could be positive or negative. Alternatively, $\mathfrak{h}_t$ could be capturing the Lagrange multiplier of a borrowing constraint, in which case it would be negative.

To achieve analytical tractability, I will now work with a first-order approximation of $1/q_t^{ST}$ around $\mathfrak{h} = 0$ and $1/E\left(\frac{\Lambda'}{\Lambda}\right) = 1$. The price of the swap can then be written as

$$m_t = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - \frac{1}{E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)} + \mathfrak{h}_t + 1 - \theta_t + (1-\lambda)\, m_{t+1}\right]\right) + o\left(\mathfrak{h}_t, E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)\right),$$

and, going forward, for notational ease, the approximation error will be omitted from the

equations. After some algebra, the swap rate satisfies

$$y_t^{Sw} = \frac{1 + \Omega_t\left(\{\theta_t + \mathfrak{h}_t\}\right)}{\Omega_t\left(1\right)} - \lambda.$$

Considering a pricing equation for the debt with the same maturity as the swap that is similarly distorted

$$q_t^{LT} = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{LT} + \lambda + (1-\lambda)\, q_{t+1}^{LT}\right]\right) - \mathfrak{j}_t,$$

where $\mathfrak{j}_t$ is the implied frictional marginal cost of holding one unit, which again can be positive or negative.

After some algebra, the long-term yield can be written as

$$y_t^{LT} = \frac{1}{\Omega_t\left(1\right) - \frac{1}{(c^{LT}+\lambda)}\Omega_t\left(\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\mathfrak{j}_t\right\}\right)} - \lambda.$$

The frictional cost $\mathfrak{j}_t$ is here multiplied by $\frac{\Lambda_t}{\Lambda_{t+1}}$ to account for the fact that, unlike for the other uses of $\Omega\left(.\right)$, $\mathfrak{j}_t$ is not lagged.

The swap spread now becomes

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)} + \frac{\Omega_t\left(\{\mathfrak{h}_t\} - \frac{1}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\mathfrak{j}_t\right\}\right)}{\Omega_t\left(1\right)}, \tag{8}$$

highlighting its dependence on current and future frictional marginal costs of short-term and long-term debt. Clearly, these frictional costs have the ability to produce negative swap spreads.

If there are no frictions for these two types of debts, ST and LT, then the swap spread is priced by the dealer's valuations Λ implicit in the present value operator $\Omega_t\left(\{.\}\right)$. Without complete markets, the dealer's valuations do not necessarily equal those of other market participants. Through this channel, demand effects can also affect the price of the swap. However, such market incompleteness or segmentation cannot produce a negative swap spread as long as θ_t has a zero probability of becoming negative. To produce negative swap spreads, the dealer needs to be subject to frictions either for the short rate or the long rate corresponding to the maturity of the swap. The intuitive reason why this has to be the case is that the payout of a swap can be replicated synthetically (except for the TED spread) with a combination of these two types of debt, one-period debt and debt with the same maturity as the swap.

For additional insights, I am now specializing the frictional cost terms to replicate my quantitative model. In this case,

$$\mathfrak{h}_t = \kappa_{ST}\alpha_{t+1}^{ST}, \text{ and } \mathfrak{j}_t = \kappa_{LT}\alpha_{t+1}^{LT}.$$

Marginal costs are linearly increasing in the size of the positions, and

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t\left(\{\theta_t\}\right)}{\Omega_t\left(1\right)} + \frac{\Omega_t\left(\kappa_{ST}\left\{\alpha_{t+1}^{ST}\right\} - \frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\alpha_{t+1}^{LT}\right\}\right)}{\Omega_t\left(1\right)}. \tag{9}$$

This equation shows that if the dealer has a long position in long-term bonds, an increase in the frictional cost – every else equal – lowers the swap spread. Intuitively, the long-term bond holding cost lowers the net-of-cost return of long-term bonds and the required return on the fixed swap leg also declines. This connection is even more transparent for the special case presented in the next subsection.

Equation (9) also illustrates the impact of an increase in the term spread, everything else equal. In response, the dealer will increase the position on long-term debt and typically reduce the position in short-term debt. Together, this will lead to a lower swap spread. Terms other than α_{t+1}^{LT} and α_{t+1}^{ST} in the equation can change too, but they are not likely to overturn this relation. In particular, as q_t^{LT} declines, it also contributes to a lower swap spread. Movements in the dealer's valuations cannot change the sign of the effect, they only reweigh the different periods' effects. Possibly, the movement in $\frac{\Lambda_t}{\Lambda_{t+1}}$ multiplying the position can go in the opposite direction. However, a higher term spread implies higher future expected short rates, which on average lead to increases in the $\frac{\Lambda_t}{\Lambda_{t+1}}$ terms. The quantitative model confirms this negative relation between term spreads and swap spreads.

Equation (9) also shows that if the dealer's positions are smaller in absolute value, everything else equal, the swap spread will be less distorted by holding costs. This will be the case when the dealer's equity is low.

3.3 Swap pricing with very strong frictions

To illustrate the main mechanism that allows the model to produce negative swap spreads, I am now presenting an analytic characterization of the quantitative model for the case where bond holdings costs are very large.

In the limit, as holding costs increase, the dealer will not hold any bonds. That is, as κ_{LT} and κ_{ST} get larger, with $d = 0$ and constant endowment of other profits, $\pi\left(z\right) = \pi$, consumption/payouts tend to equal endowment and be constant. In this limiting case, the

price of the swap is given by

$$m_t = \beta E_t \left(\frac{1}{\mu_{t,1}} \left[c^{Sw} - y_t^{LIB} + (1-\lambda)\, m_{t+1} \right] \right).$$

The at-market swap rate for which $m_t = 0$ satisfies

$$y_t^{Sw} = \frac{\sum_{j=1}^{\infty} \beta^j (1-\lambda)^{j-1} E_t \left(\frac{1}{\mu_{t,j}} y_{t+j-1}^{LIB} \right)}{\sum_{j=1}^{\infty} \beta^j (1-\lambda)^{j-1} E_t \frac{1}{\mu_{t,j}}}. \tag{10}$$

In the quantitative model, inflation uncertainty does not play a big role. To get a sharper characterization, consider the case with no inflation uncertainty, $\mu_{t,j} = \mu^j$. Taking unconditional expectations,

$$E\left(y_t^{Sw}\right) = \sum_{j=1}^{\infty} w^j E\left(y_{t+j-1}^{LIB}\right),$$

and

$$E\left(y_t^{Sw}\right) = E\left(y_t^{LIB}\right) = E\left(y_t^{ST}\right) + E\left(\theta_t\right)$$

because the weights, w^j, implicitly defined by Equation 10, are constant with $\mu_{t,j} = \mu^j$ and add up to one. Therefore, in this case, the swap rate equals the unconditional expected value of LIBOR, or equivalently, the unconditional expected short rate plus the TED spread.

For comparison, the unconditional mean of the long-term treasury yield can be written as

$$E\left(y_t^{LT}\right) = E\left(y_t^{ST}\right) + E\left(\tau_t\right)$$

where $E\left(\tau_t\right)$ is the unconditional mean term spread. Combining the two, the unconditional expectation of the swap spread equals

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\left(\theta_t\right) - E\left(\tau_t\right). \tag{11}$$

Historically, based on time-series averages, $E\left(\theta_t\right) = 0.6\%$ in annualized terms, and $E\left(\tau_t\right) = 1.7\%$, so that

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\left(\theta_t\right) - E\left(\tau_t\right) = -1.1\%.$$

Therefore, in the limiting case for which very strong frictions prevent arbitrage, the expected swap spread should be roughly -1%. Intuitively, the high holding costs drive down the dealer's bond positions and reduce the exposure to long-term interest rate risk. The swap is perceived to be less risky, and the required fixed rate declines. Of course, this is an extreme and unrealistic benchmark case. Nevertheless, it shows that in a world with limited arbitrage

Symbol	Parameter	Value
$\bar{y}_{ST}$	Short rate level	0.01156
$\bar{\tau}$	Term spread level	0.00429
$\bar{\mu}$	Inflation level	0.00938
$\bar{\theta}$	TED spread level	0.00158
γ	Risk aversion	2
ν	Discount elasticity	1
$1/\lambda$	Maturity of long-term debt and swap	120

Table 1: Model Parameters.

possibilities one should not be surprised by low or negative swap spreads, even without strong demand effects.

4 Quantitative analysis

In this section the model is calibrated and solved numerically. Quantitative model implications for swap prices are presented. I show that as bond holding costs are increased, the swap spread declines away from its arbitrage-free benchmark. The calibrated model has no difficulty generating negative swap spreads even without explicit demand pressure. The section concludes with additional empirical evidence in support of the model.

4.1 Parameterization

Processes for bond prices and inflation are specified so that the model matches key empirical facts. A period in the model is a quarter. The joint process for the short rate, the term spread, inflation and the TED spread,

$$\left[y_{ST}\left(z\right), \tau\left(z\right), \mu\left(z\right), \theta\left(z\right)\right],$$

is based on an estimated first-order vector autoregression. The data is for U.S. treasuries with 3-month and 30-year maturities and CPI inflation covering 1960-Q1 to 2015-Q3. TED spreads are available from 1986-Q1 onwards. TED spreads do not significantly enter the other three variables' equations. Innovations in the TED spread have very low correlations with the short rate and the term spread; I set these correlations to zero. The elements in the transition matrix that are not statistically significant are set to zero and equations are re-estimated with the zero restrictions.

As shown below, all series are quite persistent, and the only significant off-diagonal

interaction terms go from lagged inflation to the short rate and from lagged inflation to the TED spread. The transition matrix is

$$\begin{bmatrix} .91 & 0 & .07 & 0 \\ 0 & .87 & 0 & 0 \\ 0 & 0 & .76 & 0 \\ 0 & 0 & .06 & .72 \end{bmatrix}$$

and the covariance matrix for the innovations

$$10^{-6} \times \begin{bmatrix} 5.1 & -3.4 & 4.3 & 0 \\ & 3.2 & -2.6 & 0 \\ & & 24.8 & -1.1 \\ & & & 0.5 \end{bmatrix}.$$

The VAR is approximated by a finite-state Markov chain following Gospodinov and Lkhagvasuren (2014) with a total of $3^4 = 81$ possible realizations. Two sets of adjustments are made to the Markov chain obtained in this way. First, it is made sure that there are no arbitrage opportunities between the short-term and long-term bonds. This requires a slight reduction in the term spread for the highest realization of the long-term yield. Second, realizations for the TED spread are limited by a lower bound of 0.0003, which corresponds to the lowest historical end-of-quarter value. This is to make sure that negative swap spreads cannot come from negative TED spreads, which a linear VAR does not rule out. This requires increasing negative TED realizations to 0.0003 and adjusting small positive realizations so as to keep the unconditional expectation of the TED spread at the targeted (per quarter) level of $\bar{\theta} = 0.00158$. Additional profits are set to $\pi(z) = 0$.

The average maturity of the swap and long-term debt, $1/\lambda$, corresponds to 120 quarterly periods, that is 30 years. For the benchmark case, risk aversion is set to 2. With this value, the dealer has long/short positions about 80% of the time; for higher values, positive positions for both bonds are more common. The discount elasticity parameter equals $\nu = 1$, and the cost parameter for short term debt is set to $\kappa_{ST} = 0$. I consider several values for the cost parameter for long-term debt κ_{LT}. Focusing on holding costs for long-term bonds as the main friction is consistent with the example of a typical swap spread trade presented in Boyarchenko et al (2018). The model is solved globally with an algorithm that shares features with Judd (1992), Judd, Kubler and Schmedders (2002), and Stepanchuk and Tsyrennikov (2015).

	E(30Y SS)	Std(30Y SS)	κ_{LT}E(α_{LT})	E(TED)
Data				
$7/1997 - 9/2008$	57	27		58
$10/2008 - 10/2015$	-18	12		35
Model				
$\kappa_{LT} = 0.0001$	62	8	21	63
$\boldsymbol{\kappa_{LT}}$**= 0.0032**	**−18**	**61**	**102**	**63**
$\kappa_{LT} = 0.01$	-54	89	119	63
Post 10/2008 TED level and $\kappa_{LT} = 0.0014$	-18	41	80	35
Higher risk aversion, $\gamma = 4$	-36	66	119	63
recalibrated, $\kappa_{LT} = 0.00195$	-18	54	106	63
Lower discount elast., $\nu = 0.8$	-26	67	106	63
recalibrated, $\kappa_{LT} = 0.00255$	-18	61	102	63
Short term debt cost, $\kappa_{ST} = 0.0032$	-15	82	110	63
Constant TED	-24	68	102	63
Constant Inflation	-27	64	115	63

Table 2: Swap Spreads in the Data and the Model. Units are annualized basis points (that is multiplied by 40000). Unless indicated otherwise, the cost parameter for long-term bonds $\kappa_{LT} = 0.0032$. $\kappa_{LT}E(\alpha_{LT})$ is the mean marginal cost for holding long-term bonds.

4.2 Model properties

Swap spreads with a thirty-year maturity from the model are compared to their empirical counterparts in Table 2. Unconditional expectations and standard deviations are presented for a set of values for κ_{LT}, the holding cost parameter for the long-term debt, with short-term debt costs at $\kappa_{ST} = 0$.

As κ_{LT} increases, arbitrage becomes more costly and the unconditional mean of the swap spread goes from positive 62 basis points with a low cost of $\kappa_{LT} = .0001$ to negative -54 basis points for the highest cost presented, $\kappa_{LT} = .01$. Clearly, the model has no difficulty producing realistic negative values for swap spreads. The pattern shown in Table 2 is consistent with the analytical characterization for the high friction case: as arbitrage becomes more costly, swap rates decline and spreads become negative. The mean level post-2008 is matched with a cost parameter at $\kappa_{LT} = .0032$. The low cost case with $\kappa_{LT} = .0001$ produces a mean swap spread corresponding roughly to the pre-2008 average. Therefore, with two different levels of the long-term bond cost parameter κ_{LT}, everything else equal, the model can roughly match the swap spread levels of the periods before and after 2008, respectively.

To get a sense of the magnitudes of the frictional costs implied by the long-term bond cost parameters κ_{LT}, we can consider the marginal cost implied in equilibrium. Given

the quadratic cost function, the unconditional expectation of the marginal cost equals $\kappa_{LT} E\left(\alpha_{t+1}^{LT}\right)$, with α_{t+1}^{LT} the (endogenous) long-term bond position. Table 2 reports these in units of annualized basis points, which are the same units as the swap spreads. For the benchmark case with $\kappa_{LT} = .0032$, the marginal cost amounts to 102 basis points in annualized terms and the swap spread has a mean of -18 basis points. Over the entire sample period available for the TED spread, its mean is 63 basis points; for the post 10/2008 period, the mean is lower, at 35 basis points. To get a tighter benchmark for the post 10/2008 period, I target the post 10/2008 level for the TED spread without changing its dynamics, and search over the cost parameter κ_{LT} that can produce the post 10/2008 average swap spread of -18 basis points. As shown in the table, a cost parameter of .0014 is required. This corresponds to a mean marginal cost of 80 basis points.

As one of the holding costs for long-term bonds, large U.S. banks are subject to the Supplementary Leverage Ratio (Bowman and Wilkie (2016), Davis Polk (2014), Boyarchenko et al (2018)) that requires them to hold 5% or 6% of tier 1 capital against total assets at the holding or the bank level, respectively. Assuming a required return on equity of 10%, the implied cost would be $5\% \times 10\% = 50$ or 60 basis points.[4] Therefore, this requirement, if binding, would have a cost not far from the magnitude needed for the model to match the expected swap spread. Even if this capital requirement is not binding, banks would likely want to consider a precautionary buffer. It is interesting to note that the steep decline in the longer maturity swap spreads starting January 2015, see Figure 1, coincides with the time banks were required to start publicly disclosing their SLR. There are several other potential costs that have increased since the financial crises, including risk-weighted capital requirements and FDIC insurance premiums, in addition to the restrictions due to the Volcker rule.[5] Overall, it appears that the extent of the friction in the model is empirically plausible. As we show below, demand effects have the potential to further affect the swap spread in a meaningful way.

Rearranging equation (9) for the case without short-term debt costs, $\kappa_{ST} = 0$, yields

$$E\left(y_t^{Sw} - y_t^{LT}\right) = E\theta_t - \kappa_{LT} \cdot E\alpha_{t+1}^{LT} \cdot \chi + E\frac{\Omega_t\left(\left\{\theta_t - E\theta_t\right\} - \frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\left(\alpha_{t+1}^{LT} - E\alpha_{t+1}^{LT}\right)\right\}\right)}{\Omega_t\left(1\right)},$$

with $\chi = \left[E\frac{\Omega_t\left(\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\right\}\right)}{q_t^{LT}\Omega_t(1)}\right] \simeq 1$. This highlights the key roles of the level of the TED spread,

[4] Whether it is reasonable to assume that equity is expensive is subject to debate. For it to matter in this context it is sufficient that dealer banks consider it to be expensive.

[5] The Volcker rule has as its main objective to prohibit proprietary trading while allowing market making. Practically, the distinction between the two activities is not always clearcut. The complexity of the rules implementing the Volcker rule can itself be a source of friction.

$E\theta_t$, and the expected marginal cost, $\kappa_{LT} \cdot E\alpha_{t+1}^{LT}$, in determining the swap spread. The last term captures corrections for risk in θ_t and α_{t+1}^{LT}. For the cases reported in Table 2, changes in the implied marginal cost are the main drivers of the swap spread. For instance, when considering the high risk aversion case reported in the table, with $\gamma = 4$, the swap spread declines by $-18 = -(36 - 18)$ basis points relative to the benchmark case, while the negative of the expected marginal cost, $-\kappa_{LT} \cdot E\alpha_{t+1}^{LT}$, changes by almost the same amount, $-17 = -(119 - 102)$. When the bond holding cost parameter κ_{LT} is recalibrated to produce the same mean swap spread as for the benchmark case, the implied marginal cost for the high risk aversion case is very close to the benchmark case, differing by only $4 = 106 - 102$ basis points. Therefore, the main conclusion about the frictional costs required to produce realistic post 10/2008 swap spreads is robust to changing risk aversion within a reasonable range. With respect to the discount rate elasticity, the link between the swap spread and the marginal cost is even more robust. Indeed, the case with a lower discount elasticity at $\nu = 0.8$ – once recalibrated to produce the same mean swap spread as for the benchmark calibration – implies the same mean marginal cost of 102 basis points as the benchmark case.

While the model can easily match empirical mean levels of swap spreads, it cannot also simultaneously match the standard deviation of swap spreads. In particular, when the bond holding cost parameter is raised to match post 10/2008 negative levels of swap spreads, the volatility of swap spreads exceeds its empirical counterpart. The tightly parameterized model offers limited degrees of freedom to more closely match this property of the data. As shown in Table 2, increasing risk aversion can lower the standard deviation relative to the mean. However, this also reduces the fraction of the time a dealer has long/short positions, so that the investment behavior resembles less to that of a typical levered dealer the model aims to represent. Dealers in the model have relatively modest amounts of leverage, as is more explicitly discussed below when studying leverage constraints. To bring the model closer to the data, a richer representation of the dealer's objective function would be required. Table 2 includes two additional cases illustrating that risk in inflation and risk in the TED spread have relatively moderate effects on the swap spread.

4.2.1 Demand effects and swap costs

While demand effects and swap costs are not necessary to produce realistic swap spreads, I am considering here the potential impact, qualitatively and quantitatively, of extending the model to include these features. Overall, these frictions have the potential to make meaningful contributions to the determination of the swap rate, but they are not likely to be able to explain on their own the decline in swap spreads since 2008.

Introducing demand effects, so that $d(z) \neq 0$, makes the equilibrium computation more

	Swap rate	Demand sensitivity	LT bond position	Swap cost impact	Marginal cost for swap
Benchmark ($\kappa_{LT} = 0.0032$)					
$d = 0$	6.08		1.05		
$d = .2$	5.92	$-.99$	1.14		
$d = .2, \kappa_{Sw} = .005$	5.52			$-.4056$	.4000
Low friction ($\kappa_{LT} = 0.0001$)					
$d = 0$	7.00		3.52		
$d = .2$	6.99	$-.14$	3.72		
$d = .2, \kappa_{Sw} = .005$	6.58			$-.4056$	.4000
High friction ($\kappa_{LT} = 0.01$)					
$d = 0$	5.67		.46		
$d = .2$	5.40	-1.28	.51		
$d = .2, \kappa_{Sw} = .005$	5.00			$-.4028$	.4000

Table 3: Demand Effects and Swap Costs. Units for rates and marginal costs are annualized percentage points (that is multiplied by 400). The demand sensitivity measures the change in basis points for a one percent change in the demand relative to the dealer's net asset position (which is set to the mean for the case with $d = 0$). The marginal cost for the swap is computed as $d \times \kappa_{Sw}$.

complex. With $d(z) = 0$, the equilibrium swap price does not affect the dealer's bond investments, and the equilibrium computation can proceed recursively by first solving the bond investment problem and then the swap pricing function. With $d(z) \neq 0$, the swap price matters for the investment problem, and the investment problem and the swap pricing function have to be solved jointly. To preserve numerical tractability, I am limiting the model to one swap whose coupon is determined for an initial state of the model (ω_0, z_0).

I am considering models with different levels of a fixed demand $d(z) = d$ and compare the resulting swap rates for the same initial state. Specifically, am solving for the swap rates $y^{Sw}(\omega_0, z_0, d = 0)$ and $y^{Sw}(\omega_0, z_0, d = \delta)$ and compute the demand sensitivity measure

$$\varepsilon^d(\omega_0, z_0) \equiv \lim_{\delta \to 0} \frac{y^{Sw}(\omega_0, z_0, d = \delta) - y^{Sw}(\omega_0, z_0, d = 0)}{\delta/\omega_0}.$$

This measures the change in percentage points of the swap spread in response to a demand change as a percent of the net asset position ω_0. I am setting ω_0 to its mean for the case $d(z) = 0$ and z_0 to the neutral shock that has all exogenous variables at their mean log levels. For small values of δ/ω_0, say 20% or less, the demand sensitivity measure does not depend in a significant way on δ. Table 3 reports demand sensitivity measures for $d(z) = 0.2$

which corresponds to $10-20\%$ of ω_0 depending on the case. For the benchmark case with $\kappa_{LT}=.0032$ the computed demand elasticity is $-.99$. This means that for a 1% increase in demand as a share of the dealer's net asset value ω_0, the swap rate (equivalently the swap spread) declines by .99 of a basis point. As the table shows, this demand sensitivity depends crucially on the extent of the friction for long-term bonds. In the model, even without bond costs, the dealer cannot perfectly hedge the swap, but that effect is small. For the case with a very low long-term bond cost of $\kappa_{LT}=.0001$, the demand sensitivity reported declines to $-.14$.

A non-zero swap demand creates a risk exposure for the dealer that can be partially offset with long-term bonds. With a positive d, the dealer becomes a net fixed rate payer on the swap, and this exposure can be partially hedged by adding long exposure to long-term bonds. As is shown in Table 3, the position in long-term bonds increases in each case when d is increased. This implies an increase in the marginal bond holding cost, leading to a lower swap rate. Very intuitively, an increase in the demand for receiving fixed rate payments lowers the fixed payments to be received. There is no obvious empirical counterpart available for the size of such demand effects. The results in the table suggest that while this type of demand effect has the potential to be meaningful, for demand effects to play a major role in driving swap spreads, the size of the demand imbalances would have to be very large relative to dealers' asset holdings, and the frictional costs should be nonnegligible. Remember also that, as shown in Section 3.2, to get swap spreads to be negative at all some frictions are needed.

Dealer holding costs for swaps represent another friction that has the potential to affect swap rates. Assuming a quadratic cost function for swaps that parallels the bond cost functions implies a marginal cost of $\kappa_{Sw}s'$, and in equilibrium $s'=-d$. Extending the decomposition from the previous section we have

$$E\left(y_t^{Sw}-y_t^{LT}\right)=E\theta_t-\kappa_{LT}\cdot E\alpha_{t+1}^{LT}\cdot\chi_1-\kappa_{Sw}d\cdot\chi_2+E\frac{\Omega_t\left(\left\{\theta_t-E\theta_t\right\}-\frac{\kappa_{LT}}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\left(\alpha_{t+1}^{LT}-E\alpha_{t+1}^{LT}\right)\right\}\right)}{\Omega_t\left(1\right)}, \tag{12}$$

with $\chi_1=E\left[\frac{1+(1-\lambda)\Omega_t(1)}{q_t^{LT}\Omega_t(1)}\right]\simeq 1$, $\chi_2=E\frac{1+(1-\lambda)\Omega_t(1)}{\Omega_t(1)}\simeq 1$. The main effect of the swap cost is straightforward. The dealer has to be compensated for the marginal holding cost, $\kappa_{Sw}d$, and this affects the swap rate essentially one for one. That is, one basis point in marginal cost corresponds essentially to a one basis point change in the swap rate. The swap rate increases or decreases depending on whether the exogenous demand is negative or positive, respectively. For the examples in the table, the marginal cost in all cases is exactly $\kappa_{Sw}d=0.005\times 0.2=.001$, in annualized terms (as in the table) this is 0.4%. The

decline in the swap rate due to the introduction of the swap cost is reported to be very close to this value as suggested by Equation (12) in all cases; for instance, 0.4052% for the case with $\kappa_{LT} = 0.0032$. Recent regulation has likely also increased dealer's swap holding costs. However, these costs are unlikely to be in the same order of magnitude as for long-term bonds, and it is unlikely that dealers face direct marginal costs for holding swaps that would be large enough to be solely responsible for the decline in the swap rates since 2008. See for instance the example in Boyarchenko et al (2018).

4.2.2 Leverage constraint

The quadratic bond holding cost functions used so far offer a tractable and parsimonious way to quantify the costs induced by the multitude of regulatory changes since 2008. In this subsection, the model is specialized to specifically study the impact of a leverage constraint or capital requirement. Recent regulatory changes have tightened existing capital requirements and introduced new ones (see for instance, Greenwood et al (2017) for a survey of these changes), and these have likely contributed to increased costs for typical swap dealers.

The following constraint is added to the dealer's problem

$$\max\left(\alpha'_{ST} q_{ST}\left(z\right), 0\right) + \max\left(\alpha'_{LT} q_{LT}\left(z\right), 0\right) \leq \xi \cdot \omega,$$

with the parameter $\xi > 1$, the leverage multiplier. With this constraint, a long bond position needs to be supported by a minimal amount of dealer book equity, the net asset position, ω. The Supplementary Leverage Ratio is an example of such a constraint. The constraint could also account for the derivative exposure of the swap. Given the regulatory complexity in that regard, and our analysis in the previous section suggesting that this is unlikely to be quantitatively significant, this is abstracted from here.

With this constraint, the first-order conditions become

$$q_{ST}\left(z\right)\left[1 + \frac{\eta}{u_1\left(c\right)} I^+\left(\alpha'_{ST}\right)\right] + h_1\left(\alpha'_{ST}\right) = \beta E\left\{\frac{u_1\left(c'\right) + \eta'\xi}{u_1\left(c\right)} \frac{1}{e^{\mu(z')}}\right\}, \tag{13}$$

$$q_{LT}\left(z\right)\left[1 + \frac{\eta}{u_1\left(c\right)} I^+\left(\alpha'_{LT}\right)\right] + j_1\left(\alpha'_{LT}\right) = \beta E\left\{\frac{u_1\left(c'\right) + \eta'\xi}{u_1\left(c\right)} \frac{1}{e^{\mu(z')}}\left[c_{LT} + \lambda + \left(1-\lambda\right) q_{LT}\left(z'\right)\right]\right\}, \tag{14}$$

$$m = \beta E\left\{\frac{u_1\left(c'\right) + \eta'\xi}{u_1\left(c\right)} \frac{1}{e^{\mu(z')}}\left[c_{Sw} - \left(\frac{1}{q_{LIB}\left(z\right)} - 1\right) + \left(1-\lambda\right) m'\right]\right\} \tag{15}$$

with η the multiplier on the constraint and $I^+\left(x\right)$ an indicator function that equals 1 for

Leverage multiplier	E(30Y SS)	Std(30Y SS)	freq(SS<0)	freq($\eta > 0$)	freq($\alpha_{LT} > 0 \vert \eta > 0$)
$\xi = \infty$	62	8	0	0	
$\xi = 20$	48	9	.001	.05	1
$\xi = 10$	19	24	.20	.14	1
$\xi = 5$	-25	53	.66	.29	.90

Table 4: Model with Leverage Constraint. This is the benchmark model version with very low long-term bond cost $\kappa_{LT} = 0.0001$, augmented to include the leverage constraint. α_{LT} is the long-term bond position, η is the multiplier on the constraint.

$x \geq 0$, and 0 otherwise.[6]

As equation (14) shows, the current period multiplier, η, can increase the dealer's implicit marginal cost for long-term bonds, just as the convex cost function $j_1(\alpha'_{LT})$ does. This leads to lower returns for long-term bonds and to a reduced swap rate in equilibrium, as discussed for the benchmark model above. In addition, the leverage constraint can increase the implicit cost for short-term bonds. The leverage constraint also affects the implied valuations directly through η' in each of the first-order conditions. But as discussed above, this type of distortion cannot alone make swap spreads negative. To examine the overall effect and the quantitative impact of this constraint, the computational algorithm is augmented to include features from the approach presented in Judd, Kubler and Schmedders (2002).

Table 4 presents model properties for the benchmark calibration with very low bond holding costs $\kappa_{LT} = 0.0001$ (with no costs for short term bonds and swaps) and compares model versions with different constraint parameters ξ.

The case without the leverage constraint, equivalently $\xi = \infty$, is the low friction case seen in Table 2 and Table 3. In this case, the swap spread has an unconditional expectation of 62 basis points and the swap spread never gets negative. As the table shows, tightening the constraint lowers the mean swap spread and increases the frequency of swap spreads being negative. The tightest constraint presented uses $\xi = 5$ and produces negative swap spreads 66% of the time and a mean swap spread of -25 basis points. As these results show, the leverage constraint acts on the swap spread like the cost for long-term bonds (as suggested by the first-order condition (14)).

From a quantitative perspective, to produce a negative mean swap spread would require a leverage parameter smaller than $\xi = 10$, equivalently a leverage ratio $1/\xi$ of more than 10%. Therefore the SLR requirement of 5% or 6% would not alone be sufficient to produce

[6] The constraint is not differentiable at $\alpha'_{ST} = 0$ and $\alpha'_{LT} = 0$. To have well-defined first-order conditions, one can use smooth approximations of the max functions around the kink. Practically, with $\xi > 1$, the constraint does not bind with either α'_{ST} or α'_{LT} close to 0, because to have the left hand side of the leverage constraint exceed ω, the budget constraint, with $\omega > 0$ and $\pi(z) = 0$, would require that either α'_{ST} or α'_{LT} be negative.

negative swap spreads on average. This may not be too surprising as the model does not include any of the features typically being thought of as rationalizing dealer banks' views that equity is expensive. If anything, it can seem surprising that the capital requirements have such a significant impact on the swap price in this setting. As shown in the table, the capital requirement mostly limits the size of the long position in long-term bonds. For instance, with $\xi = 10$, the constraint binds 14% of the time, and in essentially 100% of these cases, it is the long-term debt that is constrained. In the model, the dealer values the ability to invest in long-term bonds when the term premium is high. The leverage constraint limits this strategy. Overall, for a single leverage constraint alone to produce negative mean swap spreads would require a constraint that is tighter than the SLR. However, a leverage constraint can have a significant impact on swap prices even without explicitly including model features that make equity expensive from a bank's perspective.

4.3 Additional empirical evidence

A key model mechanism is the relation between the term spread and the swap rate. As shown in Subsection 3.3, in the limiting case where the dealer holds no long-term bonds, the swap spread declines one for one with the term spread, because the risk premium in long-term bonds does not get incorporated into the price of the swap. In the time series dimension, in the model, this relation is not very strong unconditionally. However, conditional on a given level of the short rate, the term spread is robustly negatively related to the swap spread. Figure 2 shows how term spreads and swap spreads are related conditional on given levels of the short rate. In particular, each set of lines of a given color corresponds to a fixed level of the short rate. The dealer's equity, ω, is also a state variable, but its impact on the swap rate is relatively less important. The figure shows swap spreads for the mean level of equity for a given combination of the term spread, the short rate, inflation and TED. Inflation and the TED spread do not significantly affect this relationship. Indeed, for a given color, multiple lines represent different inflation rates and TED spreads; these lines are close to each other. The rest of this section presents empirical evidence that is consistent with this model property.

Table 5 displays the coefficients from regressing swap spreads with different maturities on a number of factors one would expect to be relevant. As suggested by my model, this includes the term spread between thirty-year and three-month treasuries, three-month treasuries rates, and the TED spread. In line with Feldhuetter and Lando (2008) and Hanson (2014), the duration of mortgage backed securities (MBSD) is included as it appears to capture hedging demands for investors in MBS. To control for other factors that can affect swap

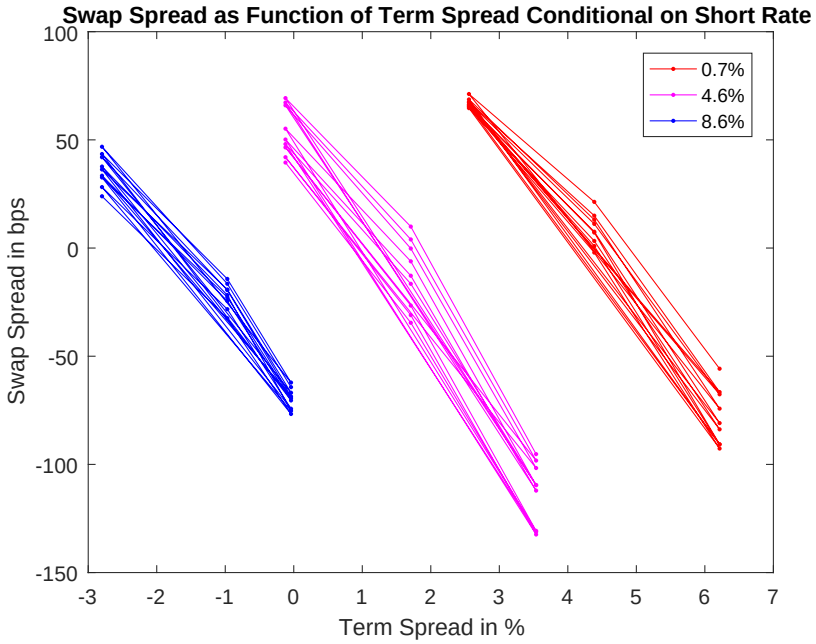

Figure 2: Swap Spread as a Function of Term Spread Conditional on Short Rate. *The holding cost parameters equal* $\kappa_{LT} = 0.0032$ and $\kappa_{ST} = 0$.

spreads, the market volatility index (VIX) is also included.

Table 5 covers the period 7/1997 to 5/2018. Regressions are based on monthly growth rates for all the variables. Consistent with the model, the term spread, TERM, is negatively related to the thirty-year swap spread and significant at the 1% level. Running the regression without TERM produces an adjusted R2 of 0.08; including TERM boosts that value to 0.18. The relationship is weaker for the shortest maturities, particularly 2 and 5 years.

As expected, the TED spread factors in positively. It is also not surprising that this effect is weaker for longer maturities. As the swap spread is driven by the sequence of future TED spreads over the period to maturity of the swap, the current TED spread should be relatively less important for longer maturities. Consistent with the prior research cited above, MBSD is positively related to the swap spread. VIX matters only for the shorter maturities.

Table 6 presents the same regression for the post-crisis sample. For the thirty-year maturity there is again a negative effect from the term spread. Somewhat unexpectedly TED now is significantly negatively related to the longer maturity swap spreads.

Large U.S. banks have been required to publicly disclose their Supplementary Leverage Ratio starting 2015. As shown in Figure 1, this marks the beginning of a steep decline in the swap spread that went on throughout 2015. Focusing on the period from 1/2015 until 5/2018, Table 7 shows a strong negative connection between the term spread and the thirty

Regressor \ Swap Maturity	2	5	10	20	30
TERM	-0.036	-0.044	-0.157^{***}	-0.128^{***}	-0.205^{***}
TED	0.220^{***}	0.089^{***}	0.025	0.012	0.007
MBSD	0.030^{**}	0.043^{***}	0.088^{***}	0.058^{***}	0.104^{***}
3MTB	0.012	0.000	0.083^{*}	-0.096	-0.100
VIX	0.004^{***}	0.006^{***}	0.002	-0.000	-0.000
adj. R2	0.50	0.28	0.22	0.04	0.18
adj. R2 without TERM	0.49	0.27	0.12	0.00	0.08

Table 5: Swap Spread Regressions 7/1997 - 5/2018. All variables are in monthly growth rates. TERM stands for the difference between the 30-year treasury rate minus the 3-month rate, TED is the TED spread, and 3MTB the 3-month treasury rate, MBSD is the duration of mortgage backed securities, VIX the market volatility index. Significance levels: *** 1 %, ** 5 %, * 10 %. The regressions are estimated by OLS with a constant, with Newey-West standard errors.

Regressor \ Swap Maturity	2	5	10	20	30
TERM	-0.045^{**}	-0.008	-0.116^{***}	-0.115^{*}	-0.187^{**}
TED	0.337^{***}	0.179^{***}	-0.102^{*}	-0.285^{***}	-0.253^{**}
MBSD	0.026^{***}	0.015	0.067^{***}	0.065^{**}	0.100^{***}
3MTB	0.151^{*}	0.154	-0.011	0.022	0.036
VIX	0.003	0.003^{**}	0.002^{**}	0.000	-0.001
adj. R2	0.56	0.38	0.28	0.32	0.31
adj. R2 without TERM	0.55	0.39	0.15	0.27	0.20

Table 6: Swap Spread Regressions 10/2008 - 5/2018. All variables are in monthly growth rates. TERM stands for the difference between the 30-year treasury rate minus the 3-month rate, TED is the TED spread, and 3MTB the 3-month treasury rate, MBSD is the duration of mortgage backed securities, VIX the market volatility index. Significance levels: *** 1 %, ** 5 %, * 10 %. The regressions are estimated by OLS with a constant, with Newey-West standard errors.

swap spread. However, given the short sample period, these results should be interpreted with some caution.

5 Conclusion

Negative swap spreads are inconsistent with an arbitrage-free environment. In reality, arbitrage is not costless. I have presented a model where specialized dealers trade swaps and bonds of different maturities. Costs for holding bonds can put a price wedge between bonds and swaps. I show a limiting case with very high bond holding costs, expected swap spreads should be negative. In this case, no term premium is required to price swaps, and this results in a significantly lower fixed swap rate. As a function of the level of bond holding costs,

Regressor \ Swap Maturity	2	5	10	20	30
TERM	0.030	−0.052	−0.169***	−0.238***	−0.455***
TED	0.310***	0.146***	−0.076*	0.050	0.099*
MBSD	−0.003	0.013	0.075***	0.111***	0.201***
3MTB	0.151	−0.020	−0.214***	−0.326***	−0.487**
VIX	−0.001	0.001	0.001	0.001	0.000
adj. R2	0.36	0.08	0.09	0.10	0.26
adj. R2 without TERM	0.38	0.10	0.01	0.00	0.00

Table 7: Swap Spread Regressions 1/2015 - 5/2018. All variables are in monthly growth rates. TERM stands for the difference between the 30-year treasury rate minus the 3-month rate, TED is the TED spread, and 3MTB the 3-month treasury rate, MBSD is the duration of mortgage backed securities, VIX the market volatility index. Significance levels: *** 1 %, ** 5 %, * 10 %. The regressions are estimated by OLS with a constant, with Newey-West standard errors.

the model can move between this benchmark and the arbitrage-free case. The quantitative analysis of the model shows that under plausible holding costs, expected swap spreads are consistent with the values observed since 2008.

Demand effects operate in the model, but are not necessary for these results. As frictions for bonds increase, the impact of demand effects becomes stronger. Quantitatively, demand effects are shown to have the potential to affect swap spreads in a meaningful way. For demand effects to play a major role in driving swap spreads, the size of demand imbalances would have to be very large relative to dealers' asset holdings.

My model can capture relatively rich interest rate and inflation dynamics. Conditional on the short rate, the model predicts a negative link between the term spread and the swap spread. The paper has presented some empirical evidence consistent with this property.

The objective of this paper has been to present a parsimonious derivative pricing model for interest rate swaps that goes beyond the typical no-arbitrage requirement to be consistent with the possibility of negative swap spreads. For most of the analysis, frictions have been modelled with cost functions for bonds and swaps that can be interpreted as summarizing the overall increase in costs of financial intermediation following various regulatory changes. The model could be further extended to, for instance, include additional specific regulatory measures, or to explicitly represent repo transactions. Another possibly fruitful extension could be to embed the dealer's optimization into a diversified financial institution with additional frictions.

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Appendix A: Swap value

Iterating on the different parts in the square brackets of

$$m_t = E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - \left(\frac{1}{q_t^{ST}} - 1\right) - \theta_t + (1-\lambda)\, m_{t+1}\right]\right),$$

gives for the first term

$$\begin{aligned} & c^{Sw} E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t} + (1-\lambda)\frac{\Lambda_{t+2}}{\Lambda_t} + (1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}..\right\} \\ = \;& c^{Sw}\sum_{j=1}(1-\lambda)^{j-1} E_t\frac{\Lambda_{t+j}}{\Lambda_t} \\ \equiv \;& c^{Sw}\Omega_t(1), \end{aligned}$$

where Ω_t is the present value of a geometrically declining annuity. For the second part with the $-\left(\frac{1}{q_t^{ST}} - 1\right)$ terms, assuming that Λ_t prices short-term debt,

$$E_t\left\{\begin{array}{c} -1 + \left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - (1-\lambda)\frac{\Lambda_{t+1}}{\Lambda_t} \\ +(1-\lambda)\frac{\Lambda_{t+2}}{\Lambda_t} - (1-\lambda)^2\frac{\Lambda_{t+2}}{\Lambda_t} \\ +(1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}... = \end{array}\right\}$$

$$E_t\left\{\begin{array}{c} -1 - (1-\lambda)\frac{\Lambda_{t+1}}{\Lambda_t} - (1-\lambda)^2\frac{\Lambda_{t+2}}{\Lambda_t}... \\ +\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) + (1-\lambda)\frac{\Lambda_{t+2}}{\Lambda_t} + (1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}... = \end{array}\right\}$$

$$E_t\left\{\begin{array}{c} -1 - (1-\lambda)\left\{\frac{\Lambda_{t+1}}{\Lambda_t} + (1-\lambda)^1\frac{\Lambda_{t+2}}{\Lambda_t} + (1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}...\right\} \\ +\left\{\frac{\Lambda_{t+1}}{\Lambda_t} + (1-\lambda)^1\frac{\Lambda_{t+2}}{\Lambda_t} + (1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}...\right\} = \end{array}\right\}$$

$$= -1 + \lambda\Omega_t(1).$$

And the third part

$$E_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\theta_t + (1-\lambda)\frac{\Lambda_{t+2}}{\Lambda_t}\theta_{t+1} + (1-\lambda)^2\frac{\Lambda_{t+3}}{\Lambda_t}\theta_{t+2}..\right\} =$$

$$\sum_{j=1}(1-\lambda)^{j-1} E_t\frac{\Lambda_{t+j}}{\Lambda_t}\theta_{t+j-1} \equiv \Omega_t(\{\theta_t\}).$$

Combining terms yields

$$\begin{aligned} m_t &= c^{Sw}\Omega_t(1) - 1 + \lambda\Omega_t(1) - \Omega_t(\{\theta_t\}) \\ &= \left(c^{Sw} + \lambda\right)\Omega_t(1) - 1 - \Omega_t(\{\theta_t\}). \end{aligned}$$

Appendix B: Swap holding costs

With explicit holding costs for swaps, the pricing equation can be written as

$$m_t = E\left(\frac{\Lambda_{t+1}}{\Lambda_t}\left[c^{Sw} - y_t^{LIB} + (1-\lambda)\, m_{t+1}\right]\right) - \mathfrak{i}_t$$

where $\mathfrak{i}_t$ is the marginal cost from a cost function with a timing in line with the bond cost functions used. This implies that

$$m_t = c^{Sw}\Omega_t(1) - 1 + \lambda\Omega_t(1) - \Omega_t\left(\left\{\theta_t + h_t + \frac{\Lambda_t}{\Lambda_{t+1}}\mathfrak{i}_t\right\}\right),$$

and the swap spread

$$y_t^{Sw} - y_t^{LT} = \frac{\Omega_t(\{\theta_t\})}{\Omega_t(1)} + \frac{\Omega_t\left(\{h_t\} - \frac{1}{q_t^{LT}}\left\{\frac{\Lambda_t}{\Lambda_{t+1}} j_t\right\}\right)}{\Omega_t(1)} + \frac{\Omega_t\left(\left\{\frac{\Lambda_t}{\Lambda_{t+1}}\mathfrak{i}_t\right\}\right)}{\Omega_t(1)}.$$

The last term differentiates this from equation (8) in the main text. With a quadratic cost, $\frac{\kappa_{Sw}}{2}\left(s_{t+1}\right)^2$, the marginal cost is $\mathfrak{i}_t = \kappa_{Sw} s_{t+1}$, and the additional price wedge depends on the process for current and future swap holdings.