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A GLOBAL SAFE ASSET FOR AND FROM EMERGING MARKET ECONOMIES

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ABSTRACT

This paper examines how a newly designed global safe asset can mitigate international capital flows induced by flight-to-safety. In the model domestic investors have to co-invest in a safe asset along with their physical capital. At times of crisis, investors replace the initially safe domestic government bonds with safe US Treasuries and re-sell part of their capital. The reduction in physical capital lowers GDP and tax revenue, leading to increased default risk justifying the loss of the government bond's safe-asset status. We compare two ways to mitigate this self-fulfilling scenario. In the "buffer approach" international reserve holding reduces the severity of a crisis. In the "rechannelling approach" flight-to-safety capital flows are rechannelled from international cross-border flows to flows across two EME asset classes. The two asset classes are the senior and junior bond of tranched portfolio of EME sovereign bonds.

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1 Introduction

International capital flows are fickle. Short-term debt funding is especially subject to sudden stops. Sudden flight into safe-haven currencies can cause large disruptions and sharp currency movements, ultimately leading to a crisis. When markets shift from a risk-on to a risk-off mood, cross-country capital flows are triggered if the safe asset is not supplied symmetrically across counties. Advanced economies, which supply safe assets, experience capital inflows, while most emerging economies suffer sudden outflows. Hence, the design of global safe assets is paramount in creating a stable global financial architecture.

The focus of the international monetary system has, so far, been on leaning against these flight-to-safety capital flows. The International Monetary Fund offers various lending facilities that allow governments to borrow in order to counterbalance these capital outflows. Similarly, international swap line arrangements among various central banks allow central banks to offset sudden capital outflows. Absent these facilities, countries' primary precautionary strategy is to acquire large reserve holdings in good times that they can deploy in crisis times in order to lean against sudden outflows. The South East Asia crisis in 1997 was a wake-up call for most emerging economies. IMF funding was attached with conditionality and hence was not very popular in Asia. Many emerging countries subsequently opted for a self-reliant precautionary buffer approach by building-up large reserve holdings. This resulted in global imbalances, which possibly distorted interest and exchange rates. Holding reserves also incurs carry cost for the emerging economy, as the interest on safe foreign reserve assets is typically significantly lower than on domestic assets. This drains resources, lowers a country's fiscal space, and hence paradoxically can make a crisis more likely. However, when a crisis occurs, reserve holdings soften the severity of a crisis as they can be used to lean against the sudden capital outflows.

An alternative, more direct approach is to address the root of the problem, namely, that safe assets are asymmetrically supplied, since only a few advanced economies supply them. We analyze the alternative institutional arrangement which involves introducing sovereign bond-backed securities (SBBS) in order to rechannel the destabilizing flight-to-safety capital flows. Instead of facing cross-border flows from emerging economies to some advanced economies, one could redirect these capital flows to move across different asset classes.

Even a single country on its own could create SBBS by setting up a special-purpose vehicle (SPV) that buys some of the country's sovereign bonds and tranches them into a senior and a junior bond. The junior bond absorbs the losses and protects the senior bond. As long as the junior bond tranche is sufficiently thick and covers the maximum haircut of the sovereign debt, the senior bond is free of default risk, and can acquire safe-asset status. With SBBS, investors can at times of crises flee into the senior bond instead of, say, the US dollar.

Tranching a diversified pool of emerging-market government bonds, instead of those of a single country, exploits diversification benefits if the pool contains bonds from sufficiently heterogeneous countries. This allows for a "thinner" junior bond tranche without sacrificing the safety of the senior bond. The senior bond serves as an additional global safe asset.

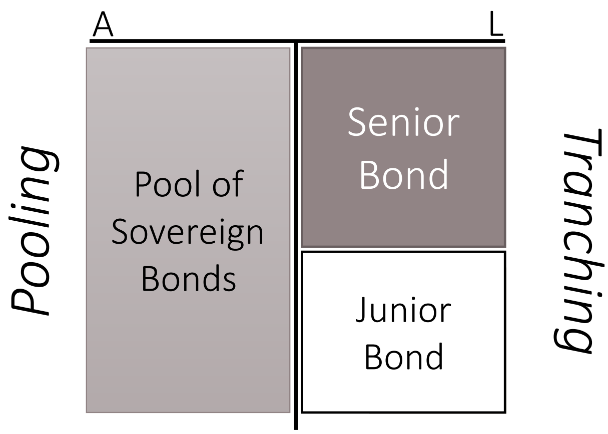

Figure 1: Structure of GloSBies

Such a global safe asset follows the same idea as the SBBS or the European Safe Bonds (ESBies) proposal for the Euro area, proposed by Brunnermeier et al. (2011). The Euro area suffered similar flight-to-safety capital flows from its peripheral countries to a few core countries. While within the Euro area there is no exchange rate risk, for the global SBBS the junior bond also has to absorb currency risk if the underlying national bonds are denominated in local currency. SBBS have a second advantage besides rechanneling flight-to-safety capital flows: as shown in Brunnermeier et al. (2016), SBBS can eliminate the doom (diabolic) loop between sovereign and banking risk that arises when banks hold domestic sovereign bonds that are subject to default risk. As default risk rises and the sovereign bond price tanks, banks suffer losses, thus increasing the likelihood that the government will have to bail them out, which in turn lowers the sovereign bond price. Brunnermeier et al. (2017) studies diversification and contagion interactions, carries out numerical simulations, and analyzes various implementation details of SBBS for Europe.[1]

In Asia, the Executives' Meeting of East Asia-Pacific Central Banks (EMEAP)[2], is involved in the so-called Asian Bond Fund. This fund pools bonds from 11 countries, but does not tranche the pooled cash flows into a senior bond that could serve as a regional safe asset[3].

Metaphorically, tranching is like building a second, stronger line of defense within a fort. With only a single defense line, some knights might be tempted to flee for safety, thereby weakening the overall defense of the fort. Having a "safe haven" within the same fort, e.g., the keep of a castle, to withdraw to lowers the knights' temptation to flee and thereby reduces the fort's overall vulnerability.

In this paper, we formally examine the flight-to-safety mechanism. Firms and banks hold safe assets in addition to physical capital for precautionary reasons. The domestic bond is considered safe if its default probability is very small (say, below 1%). Since the domestic bond's yield is significantly higher than that of the US Treasury, firms prefer the former as their safe asset in normal times. After an adverse shock, the probability of domestic sovereign bond default rises and the domestic bond loses its safe-asset status. Consequently, firms try to swap all their domestic bond holdings for US Treasuries. By doing so, they suffer losses on their bond position, which also forces them to shed some of their physical capital at fire-sale prices. As they scale back their production capacity, the domestic government's tax revenues also decline. This, in turn, leads to a partial default of the sovereign bond, which justifies the initial loss of the domestic bond's safe-asset status.

Going beyond the baseline setting, we analyze the implications of foreign-reserve holdings, the "buffer approach," whose objective is to insulate the economy from sudden stops. If the government initially issues more sovereign bonds in order to hold US Treasuries as reserves, it has to pay the interest rate differential but enjoys capital gains after an adverse shock. The "buffer approach" lowers the severity of a crisis, but the interest rate differential makes it an expensive proposition. In contrast, the "rechanneling approach" involves tranching the domestic sovereign bond into a junior and a senior bond. Since the latter does not lose its safe-asset status, this is a strictly superior solution. After an adverse shock, firms hold on to their senior bond and fire-sales are avoided. Production capacity (and with it tax revenue) remains high and consequently a default is also averted.

[1]The European Union Commission refined the SBBS proposal and proposed in May 2018 the necessary regulatory changes.

[2]See http://www.emeap.org.

[3]In 2009 the introduction of a similarly structured Latin America Bond Fund was studied.

As the safe-asset status plays a crucial role in our analysis, it begs the question of what defines a safe asset. In our setting, an asset is considered safe if its Value-at-Risk entails no losses, i.e., losses occur only with a probability smaller than, say, 1%. Brunnermeier and Haddad (2012) argue that safe assets possess the following two characteristics: the "good friend analogy" and the "safe asset tautology". Similar to a good friend who is around when needed, a safe asset is valuable and liquid exactly when needed. Like gold, a safe asset holds its value or even appreciates in times of crisis. While a risk-free asset is risk-free at a particular horizon, e.g., overnight or over 10 years, a safe asset is valuable at an ex-ante random horizon, when one needs it. They are, therefore, held as a precautionary buffer in addition to risky assets. Indeed, holding a safe asset allows one to scale up risky investment. The second property of safe assets is the safe-asset tautology. A safe asset is safe because it is perceived to be safe. Paradoxically, a safe asset might appreciate even though its fundamental value declines. For example, in August 2011 the US Congress seemed likely to refuse to lift the US debt ceiling; US Treasuries were about to default and the S&P rating agency downgraded them; nevertheless, the same Treasuries appreciated in value. Similarly, the German Bund gained in value during the Euro Crisis even though Credit Default Swap (CDS) spreads indicated that the German bund default risk was rising. In sum, safe assets share some features of bubbles or multiple equilibria. That is, the link to the assets' fundamentals is weak.

Dang et al. (2010) emphasize the feature that safe assets are informationally insensitive to shifts in fundamentals. Hence, asymmetric information frictions like Akerlof's lemons problem are limited. Gorton et al. (2012) argue that the share of safe assets as a fraction of total assets is roughly stable over time. In Caballero et al. (2017), safe assets are held by very risk-averse individuals who do not want to hold any risky investments, and a shortage of safe assets arises when monetary policy is constrained by the zero lower bound. He et al. (2017) model the safe-asset tautology in a global games framework. Our paper is also related to the literature on international debt crisis featuring multiple equilibria, e.g., Calvo (1988) and Cole and Kehoe (2000). While this strand of literature emphasizes the strategic default of the government due to limited commitment friction, our work focuses on the safe-asset demand of domestic entrepreneurs and the default in our model is a mechanical outcome of tax revenue (output) losses.

2 Baseline Model

In our baseline model, domestic entrepreneurs demand safe assets to complement their risky capital investment. Initially, the domestic sovereign bond is more attractive, since it offers a higher yield than the US Dollar Treasury. After an adverse shock, one of two possible equilibria can emerge. In the flight-to-safety equilibrium, the public suddenly expects that the domestic bond might default. Hence, it loses its safe-asset status and entrepreneurs flee to dollars to meet their demand for safe assets. The price of the domestic bond drops as more patient domestic investors dump domestic bonds to less patient foreign investors. If the decline in the domestic bond price is severe, proceeds from selling the domestic bond are not sufficient to buy enough US Treasuries as safe assets. Domestic entrepreneurs are thus forced to fire-sell capital to foreigners as well. The economy's output (and with it the government's tax revenue) declines, justifying the possible default of the domestic bond. This vicious cycle makes the flight-to-safety equilibrium self-fulfilling. In the second equilibrium, the fundamental equilibrium, no fire-sales occur, production and tax revenue remain high, and the absence of any default ensures that the domestic bond does not lose its flight-to-safety status.

In this section we study the baseline model before examining the implications of reserve holdings ("the buffer approach") in Section 3, national tranching in Section 4, and pooling and tranching in Section 5. We evaluate and compare these settings according to two criteria: (i) vulnerability/likelihood of a flight-to-safety crisis and (ii) severity of the crisis.

2.1 Model Setup

Consider a small open economy with three dates $t \in \{0, 1, 2\}$ and three types of agents: domestic entrepreneurs, domestic households, and foreign investors.

Physical capital produces $A_t K_1$ units of a single output good at date $t = 2$, where K_t is the physical capital employed in period t and A_t is the random productivity of that capital. Productivity can take one of the following three values:

$$\underline{A} < \overline{A} < \overline{\overline{A}}. \tag{1}$$

Uncertainty unfolds over time as depicted in Figure 2.

At $t = 1$, either the "worry-free" productivity state $\overline{\overline{A}}$ realizes or an adverse shock occurs with probability π_1. In that case, uncertainty remains and is only resolved at

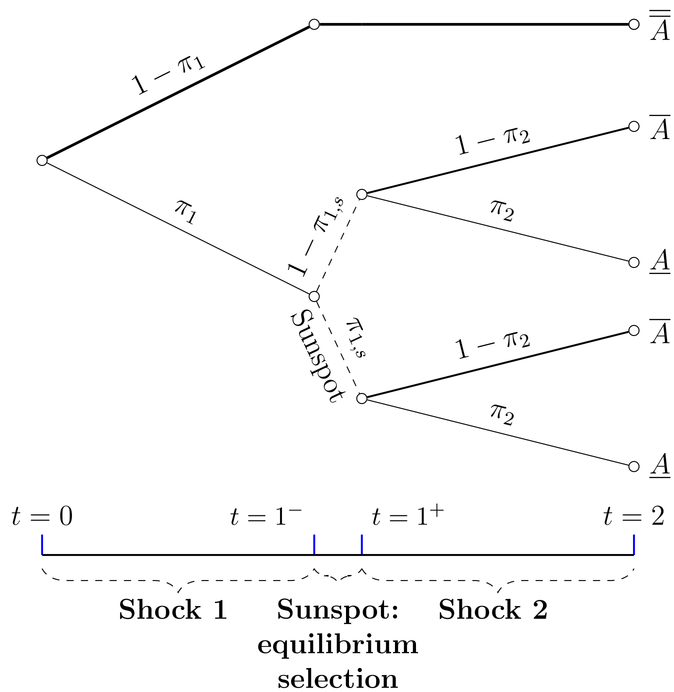

Figure 2: Timeline

the final date $t = 2$: Productivity will either be $\overline{A}$ or $\underline{A}$ with probability π_2. It turns out that, in the case at $t = 1$ in which uncertainty remains, two subgame equilibria can arise: a fundamental equilibrium and a flight-to-safety equilibrium. We assume that a sunspot arriving with probability $\pi_{1,s}$ selects the flight-to-safety (subgame) equilibrium. Both fundamental shocks and the sunspot shock are assumed to be independent.[4]

2.1.1 Assets

Agents can trade three assets in the economy: physical capital, domestic bonds, and a foreign safe asset, the US dollar. All assets pay off only at $t = 2$ and cannot be sold short.

[4]Note that we introduce the adverse shock at time $t = 1$ only to ensure the adverse scenario is sufficiently unlikely such that the domestic bond enjoys safe-asset status at $t = 0$.

Real investment. Domestic entrepreneurs have investment opportunities to build physical capital at time $t = 0$. The investment is a constant return to scale, and one unit of capital requires a physical investment of the consumption good at time $t = 0$. Both domestic entrepreneurs and foreign investors can trade physical capital at $t = 0$ and $t = 1$. At $t = 2$, output is produced. Domestic entrepreneurs have projects that pay off $\tilde{A}$ consumption goods per unit of capital, where $\tilde{A}$ is the state-dependent productivity at time $t = 2$ specified above. In contrast, foreign investors face lower productivity levels; they produce only a fraction $\eta < 1$ of output $\tilde{A}$ per unit of capital.

Domestic bonds. At time $t = 0$, the government issues zero-coupon domestic bonds with a total face value of B_0, which mature at time $t = 2$. At time $t = 0$, the price is p_0. At time $t = 1$, in the "worry-free" (uneventful) state, i.e., when no adverse shock hits at $t = 1$, the debt price is $p_{1,u}$. After an adverse shock, the fundamental price at $t = 1$ is $p_{1,f}$ and, if a sunspot occurs, the flight-to-safety price at $t = 1$ is denoted by $p_{1,s}$. For convenience, we also use subscripts $\{1, u\}, \{1, f\}, \{1, s\}$ to distinguish various variables across the scenarios in $t = 1$.

Our analysis will show that the domestic bond fully pays off its face value B at $t = 2$, except in the flight-to-safety (subgame) equilibrium. In the flight-to-safety equilibrium, domestic bonds may partially default and only repay a fraction $1 - h$ proportion of their face value. That is, a haircut h is subtracted since government tax revenue is not sufficient to fully pay off the debt.

The government can levy a lump-sum tax up to τ fraction of potential output at $t = 2$. Specifically, total "fiscal space" is

$$T_2 = \tau \tilde{A} K_1^E, \tag{2}$$

where K_1^E is the capital held by domestic entrepreneurs at the end of $t = 1$ and $\tilde{A}$ is the realized productivity at $t = 2$. If the collected tax revenue falls short of the bond's face value, the domestic government bond defaults and pays off only partially. For simplicity, we assume that capital that was "fire-sold" to foreign investors is shipped abroad and therefore does not contribute to domestic tax revenue.[5]

US Dollar Treasury. There is an outside storage technology in the form of US Treasuries offering return $R^\$$ in every period regardless of the state. That is, dollar

[5] This assumption is innocuous. If the government can also tax output produced with foreign-held capital, one obtains a qualitatively similar outcome.

Treasuries are always perfectly safe.

2.1.2 Agents

There are three groups of investors: domestic entrepreneurs, domestic households, and foreign investors. They trade at times $t = 0$ and $t = 1$.

Domestic entrepreneurs. The continuums of domestic entrepreneurs are risk-neutral and have a time-preference discount factor β:

$$\max \mathbb{E}_0[C_0 + \beta C_1 + \beta^2 C_2]. \tag{3}$$

Entrepreneurs have an initial wealth W_0^E at $t = 0$ at their disposal and can invest in all three assets.

Importantly, they have to complement physical investment with some safe-asset holdings. Specifically, they have to hold a quantity of safe assets in their portfolio that exceeds a risk measure α times their capital holdings, i.e.,

$$S_t^E \geq (\beta^{2-t})\alpha K_t^E, \tag{4}$$

where S_t^E is the market value of holdings of safe assets. This "safe-asset requirement" can be justified simply by bank regulation or as a shield to fend off bank runs.

Our analysis focuses on parameter values for which both the domestic bond and US Treasuries are safe at $t = 0$. Strictly speaking, domestic bonds still have default risk as long as $\pi_{1,s} > 0$, but an asset is considered safe as long as its default risk is negligible. For example, an asset is considered safe as long as its Value-at-Risk is sufficiently low, where the Value-at-Risk neglects tail risk that occurs with a probability of less than, say, 1 %.[6]

Domestic households. Households are similar to entrepreneurs. They have the same preferences, but they cannot produce with or hold physical capital. Also, their initial wealth W_0^H at time $t = 0$ is large enough such that they are able to buy all residual domestic bonds net of demand from entrepreneurs. This allows us to vary the total indebtedness of the country without affecting the initial domestic bond price.

[6] Formally, we can define the safe asset as an asset with default probability lower than ϵ threshold. With sufficiently small probability of $\pi_1\pi_{1,s}$, this condition always holds.

Foreign investors. Foreign investors can buy all three assets. They are also risk-neutral, but less patient than domestic agents. They solve

$$\max \mathbb{E}_0[C_0 + \beta^* C_1 + (\beta^*)^2 C_2]. \tag{5}$$

Foreign investors that are potentially invested in the emerging country are less patient than domestic investors. They also find the low US Treasury yield $R^\$$ unattractive, that is,

$$\frac{1}{R^\$} > \beta > \beta^*. \tag{6}$$

Patient home investors value assets more than less patient foreign/international investors. When domestic investors dump assets to foreign investors a fire-sale discount arises. The dollar is a perfectly safe but unattractive outside option. Its yield is very low since "other investors" that are never active in our emerging economy enjoy some convenience yield from holding the US Treasury.

Let B_0 be the face value of the domestic bond and B_0^E and B_0^H the part of the face value of the bond held by entrepreneurs and households. The key state variable in our model is the country's debt-to-GDP ratio, which is proportional to the country's indebtedness relative to physical capital. We denote the debt-to-capital ratio with d and the ratio held by entrepreneurs and households by b^E and b^H, respectively. That is,

$$d = \frac{B_0}{K_0}, b^E = \frac{B_0^E}{K_0}, b^H = \frac{B_0^H}{K_0}. \tag{7}$$

Note that

$$d = b^E + b^H, \tag{8}$$

where we refer to the ratio d simply as the total bond level outstanding and b^E and b^H as bond positions held by entrepreneurs and households, respectively.

Assumptions: We make the following parametric assumptions:

1. $\alpha < d < \overline{d} =: \tau \underline{A}$, $\underline{A} < \overline{A} \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^*\alpha}{(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 \frac{\alpha}{\tau\underline{A}}}$,
2. $\tau\underline{A} < \frac{\beta}{\beta^*}\alpha$,
3. $\beta > \beta^*(1+\alpha)$ and $\beta^2\{(1-\pi_1)\overline{\overline{A}} + \pi_1 \mathbb{E}_1[A]\} > 1$,
4. $W_0^H > \beta^2(B_0 - \alpha K_0)$,
5. $\frac{1}{\eta\beta^*} > \frac{\mathbb{E}_1[A] + \alpha\beta R^\$}{\beta^*\eta \mathbb{E}_1[A] + \alpha\beta} > \frac{1}{\beta^*}$,

6. $\pi_{1,s} = 0$ (unanticipated crisis),

where $\mathbb{E}_1[A] = \pi_2\underline{A} + (1-\pi_2)\overline{A}$. Assumption 1 guarantees that fire-sales of physical capital are necessary for the domestic bond to partially default. Assumption 2 ensures that there exists $\overline{d}$ such that there are multiple equilibria whenever $d \in [\alpha, \overline{d}]$. Assumption 3 ensures entrepreneurs choose to hold capital with safe assets at $t = 0$ instead of selling capital to foreigners, buying US Treasuries, or consuming. Assumption 4 ensures households have enough initial wealth to buy all residual domestic bonds at $t = 1$. Assumption 5 concerns the behavior of entrepreneurs in the debt crisis. It posits that entrepreneurs prefer to hold capital with a binding safe-asset constraint to holding price-depressed domestic bonds. Assumption 6 states that the flight-to-safety crisis due to a sunspot occurs with zero probability. This assumption significantly simplifies the analysis but can be relaxed. In Appendix A.3 we show that our main results continue to hold for a sufficiently small but strictly positive likelihood of a crisis.

2.2 Equilibrium

This section characterizes the equilibrium allocation and prices for our baseline setting, in which there are no reserve holdings, tranching, or pooling.

Equilibrium at $t = 0$**.** Assumptions 3 and 4 imply that domestic entrepreneurs invest their initial wealth W_0^E in physical capital and hold along with it $\beta^2\alpha K_0^E$ of the domestic bond at $t = 0$ as an accompanying safe-asset investment. Since entrepreneurs perceive little risk in the future, they reduce their low-yielding safe-asset holdings to the minimum given by the safe-asset constraint (4). Formally, entrepreneurs' bond holdings are

$$b^E = \alpha. \tag{9}$$

Meanwhile, the domestic bond, which is not expected to default, carries a price of

$$p_0 = \beta^2. \tag{10}$$

Consequently, the initial physical capital holding is

$$K_0 = K_0^E = \frac{W_0^E}{1 + \alpha\beta^2}. \tag{11}$$

Since initial capital investment is a deterministic function of initial wealth, we will use K_0 instead of initial wealth W_0 as the key exogenous parameter hereafter. Alternatively, capital K_0 could be viewed as an initial endowment.

Domestic households buy the remaining supply of the domestic bond and plan to hold it until maturity. They consume the rest of their wealth, since US Treasuries are unattractive as a saving vehicle. To ensure that domestic households are indifferent between consuming at $t = 0$ and buying a domestic bond and consuming in $t = 2$, the equilibrium return of the domestic bond over two periods is $\frac{1}{\beta^2}$. Hence,

$$p_0 B_0^E = \alpha \beta^2 K_0^E. \tag{12}$$

The following proposition summaries our results for time $t = 0$.

Proposition 2.1. *The time $t = 0$ equilibrium allocation is*

$$\begin{aligned}
K_0^E &= K_0, & K_0^H &= 0, & K_0^* &= 0, \\
B_0^E &= b^E K_0, & B_0^H &= B_0 - b^E K_0, & B_0^* &= 0, \\
\$_0^E &= 0, & \$_0^H &= 0, & \$_0^* &= 0.
\end{aligned} \tag{13}$$

Debt ratios are

$$b^E = \alpha, \quad b^H = d - \alpha. \tag{14}$$

The equilibrium domestic bond price is $p_0 = \beta^2$.

Next, we analyze three subgame equilibria: First, the subgame at $t = 1$ when no initial adverse shock, i.e., $A = \overline{\overline{A}}$ realizes. After an adverse shock the expected total factor productivity (TFP) is $\mathbb{E}[A]$, and either a fundamental equilibrium or a sunspot equilibrium with flight to safety can arise.

$\overline{\overline{A}}$-Subgame Equilibrium at $t = 1$. If at $t = 1$ no adverse shock occurred, the economy's fundamentals are sufficiently positive to rule out any crisis. In this subgame, capital and domestic bonds have the same return $\frac{1}{\beta}$. Domestic agents will be indifferent between holding the asset and consuming. Foreign investors strictly prefer not to buy any assets. Proposition 2.2 summarizes the result.[7]

[7] Note that there are also other (subgame) equilibria with the same allocation but different equilibrium prices. For example, any capital price $\eta\beta^* A < q_{1,u} < \beta A$ would be a valid equilibrium price. In these equilibria, domestic entrepreneurs prefer to invest in projects but are wealth-constrained. This equilibrium price indeterminacy is innocuous to our result.

Proposition 2.2. *($\overline{\overline{A}}$-Equilibrium at $t = 1$) Absent an adverse shock, the allocation remains unchanged compared to $t = 0$. The price of capital changes to*

$$q_{1,u} = \beta\overline{\overline{A}}. \tag{15}$$

The price of domestic bonds changes to

$$p_{1,u} = \beta \tag{16}$$

due to time discounting.

After an initial adverse shock, two possible subgame equilibria can emerge: a fundamental equilibrium and a self-fulfilling flight-to-safety equilibrium with (partial) default. If no sunspot occurs, the subgame ends up in the "fundamental equilibrium" at $t = 1$.

Fundamental $\mathbb{E}_1[A]$-Equilibrium at $t = 1$. The fundamental equilibrium resembles the $\overline{\overline{A}}$-equilibrium and results in the same allocation. Also, the domestic bond and dollar bond are default-free. Only the economic fundamentals are worse, since expected productivity is $\mathbb{E}_1[A]$ instead of $\overline{\overline{A}}$. Proposition 2.3 characterizes the fundamental (subgame) equilibrium.[8]

Proposition 2.3 (Fundamental equilibrium at $t = 1$). *After an adverse $t = 1$ shock, a (default-free) fundamental equilibrium exists for debt levels $d \in [\alpha, \tau\underline{A}]$. The equilibrium allocation remains unchanged compared to $t = 0$ while equilibrium prices adjust to*

$$q_{1,f} = \beta\,\mathbb{E}_1[A], \tag{17}$$

and

$$p_{1,f} = \beta. \tag{18}$$

Flight-to-Safety Equilibrium at $t = 1$. For a high enough debt level, there also exists a flight-to-safety equilibrium after a negative shock at $t = 1$. The domestic bond partially defaults and hence loses its safe-asset status. As a consequence, only

[8] Similar to the $\overline{\overline{A}}$-Subgame Equilibrium, there is an indeterminacy in equilibrium prices, which is irrelevant to our results.

US Treasuries remain as safe assets. Domestic entrepreneurs fire-sell their domestic bonds and scale back their physical capital holdings as well. This lowers total output and tax revenue, which in turn is the cause of the partial default. As foreigners become the marginal investors in physical capital and domestic bonds, their prices drop to

$$q_{1,s} = \beta^* \eta \, \mathbb{E}_1[A], \tag{19}$$

$$p_{1,s} = \beta^*(1 - \pi_2 h). \tag{20}$$

Recall that foreigners are less patient ($\beta^* < \beta$) and less productive at operating physical capital by a factor η.

Since holding the dollar bond yields a low return, domestic entrepreneurs hold just enough dollars to satisfy the safe-asset constraint $S_1 \geq \alpha\beta K_1^E$. That is, for each unit of capital, the entrepreneur must spend $q_{1,s}$ for capital plus $\alpha\beta$ on US Treasuries. With a net worth of $q_{1,s}K_0 + p_{1,s}B_0$ in crisis times, the entrepreneur can only hold capital

$$K_{1,s}^E = \frac{q_{1,s}K_0 + p_{1,s}B_0}{q_{1,s} + \alpha\beta} = \frac{\beta^*\eta \, \mathbb{E}_1[A] + \beta^*(1 - \pi_2 h)b^E}{\beta^*\eta \, \mathbb{E}_1[A] + \alpha\beta} K_0. \tag{21}$$

Due to the flight to safety, the capital holdings of entrepreneurs are linearly decreasing in entrepreneurs' expectations of the haircut h. Recall that the government only collects tax revenue proportional to entrepreneurs' capital holdings. The tax revenue in the lowest productivity state ($\tilde{A} = \underline{A}$) thus is also decreasing in h:

$$T(h) = \tau \underline{A} K_{1,s}^E(h)/K_0. \tag{22}$$

Figure 3 illustrates how the domestic bond haircut h is determined in equilibrium. The green dot is the fundamental $\mathbb{E}_1[A]$ equilibrium. Since the minimal tax revenue $\tau\underline{A}$ is larger than the required debt repayment d, the domestic bond remains safe, i.e., $h = 0$.

There is another possibility, namely, the flight-to-safety equilibrium denoted by the red dot. The black line plots the government's debt repayment after a partial default, $d(1 - h)$. The dashed red line plots tax revenue $T(h)$ against the haircut. The equilibrium haircut level h^* can be seen as a result of a vicious loop between tax revenue and the debt haircut. This loop occurs in four steps:

1. With the possibility of any haircut $h > 0$, domestic bonds become unsafe. Entrepreneurs then no longer have a reason to hold them and sell them off to impatient domestic investors, who value them less (at price $\frac{1}{\beta^*}(1 - h)$).

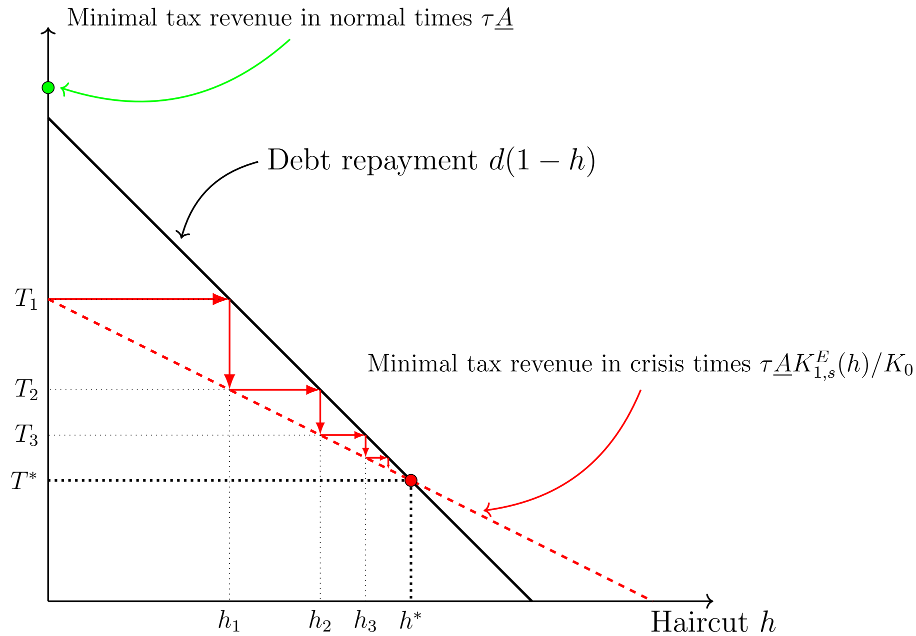

Figure 3: Determination of domestic bond haircut h

2. Entrepreneurs take losses on their domestic bond positions and are forced to sell capital.

3. Tax revenue declines and the government faces a shortfall on its debt repayment.

4. The expected haircut on government debt increases, after which the loop restarts from step (2).

In Figure 3, the revenue shortfall after the initial fire-sale of capital (when the debt is considered unsafe but the perceived haircut is near zero) is the distance between the black line and the point $(0, T_1)$. Once investors realize the government will not be able to repay its debt, the perceived haircut is updated to h_1. Then entrepreneurs take further losses and sell more capital, which decreases revenues to T_2, and so forth. This continues until the haircut reaches h^* such that

$$d(1-h^*) = T(h^*). \tag{23}$$

Proposition 2.4 (Flight-to-safety equilibrium at $t = 1$)**.** *The flight-to-safety equilibrium at $t = 1$ exists only if $d \in [\max\{\underline{d}, \alpha\}, \tau\underline{A}]$ ($\underline{d}$ defined below). In this equilibrium* *the domestic bond loses its safe-asset status, and entrepreneurs fire-sell the domestic bond and physical capital to foreign investors, causing a loss of output and with it a decline in tax revenues, which, in turn, causes the partial default of the domestic bond. The equilibrium allocation is*

$$
\begin{aligned}
K_{1,s}^E &= \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E}{\eta\beta^* \mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^*\pi_2 \frac{b^E}{b^E+b^H}} K_0, \qquad & K_{1,s}^H &= 0, \qquad & K_{1,s}^* &= K_0 - K_{1,s}^E,\\
B_{1,s}^E &= 0, & B_{1,s}^H &= B_0 - \alpha K_0, & B_{1,s}^* &= \alpha K_0,\\
\$_{1,s}^E &= \beta\alpha \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E}{\eta\beta^* \mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^*\pi_2 \frac{b^E}{b^E+b^H}} K_0, & \$_{1,s}^H &= 0, & \$_{1,s}^* &= 0
\end{aligned}
\tag{24}
$$

with $b^E = \alpha, \quad b^H = d - \alpha$. *Foreign investors are the marginal holders of both capital and domestic bonds and hence the asset prices are*

$$q_{1,s} = \beta^* \eta\, \mathbb{E}_1[A], \tag{25}$$

$$p_{1,s} = \beta^*(1 - \pi_2 h), \tag{26}$$

with a haircut of domestic bonds

$$h(b^E, b^H) = 1 - \frac{\tau\underline{A}}{b^E + b^H}\frac{K_{1,s}^E}{K_0} = 1 - \tau\underline{A}\frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E}{(b^E+b^H)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E}. \tag{27}$$

The minimal debt level for the flight-to-safety crisis $\underline{d}$ *is*

$$\underline{d} = \tau\underline{A}\frac{\beta^*\eta\, \mathbb{E}_1[A] + \beta^*\alpha}{\beta^*\eta\, \mathbb{E}_1[A] + \beta\alpha}. \tag{28}$$

Equation (27) reveals that the haircut is decreasing with $K_{1,s}^E/K_0$, the fraction of physical capital that entrepreneurs can retain after their fire-sales.

2.3 Crisis Vulnerability and Severity

We evaluate various domestic bond market settings based on two criteria: (i) the vulnerability of the economy to a crisis and (ii) the severity of the crisis.

Definition 2.1. For an emerging economy parameterized by x,
(i) the **crisis vulnerability region** is the set of debt-to-capital ratios d defined as

$$\mathcal{V}(x) = [\alpha, \tau\underline{A}] \cap \{d \mid \text{ A flight to safety equilibrium exists}\}, \tag{29}$$

(ii) the **crisis severity** $\mathcal{S}(d,x)$ is defined as the fraction of physical capital that has to be fire-sold, i.e.,

$$\mathcal{S}(d,x) \equiv 1 - \frac{K^E_{1,s}(d,x)}{K_0}. \tag{30}$$

Note that in our model, alternative measures of crisis severity such as total debt losses $(b^E + b^H)h = dh$ or output losses $\underline{A}(1 - \frac{K^E_{1,s}}{K_0})$ all map one-to-one to our measure $\mathcal{S}$, which is based on the fraction of fire-sold capital.

As a benchmark, Proposition 2.5 derives the crisis vulnerability region and severity denoted with a superscript B for baseline setting.

Proposition 2.5.
(i) The crisis vulnerability region is $\mathcal{V}^B = [\max\{\underline{d}, \alpha\}, \tau\underline{A}]$, *and*
(ii) the crisis severity in the baseline model is

$$\mathcal{S}^B(d) = \max\{0, \frac{\eta\beta^*\,\mathbb{E}_1[A] + (1-\pi_2)\beta^*\alpha}{\eta\beta^*\,\mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^*\pi_2\frac{\alpha}{d}}\}. \tag{31}$$

The following sections show that central bank reserve holdings, tranching, and pooling and tranching alter the crisis vulnerability region as well as the severity of the crisis.

3 Reserve Holdings: The "Buffer Approach"

Financial crises associated with fight to safety capital flows have historically led to large economic dislocations and social hardship. The Southeast Asia crisis of 1997 and the Euro crisis beginning in 2008 are two prominent examples of crises in which flight to safety played a significant role. Especially after the Southeast Asia crisis, many emerging economies in Asia decided to accumulate large holdings of foreign reserves as a precautionary measure. By 2018, these holdings amounted to 6.45 trillion dollars, of which 3.42 trillion are held by China.[9] Emerging economies try to fend off crises, but also to mitigate the consequences of cross-border flight-to-safety capital flows. This section analyzes the implications of holding safe assets in the form of foreign reserves as a precautionary measure. Specifically, we examine, within our

[9] The source is IMF data template on International Reserves and Foreign Currency Liquidity (IRFCL). See `http://www.imf.org/external/np/sta/ir/IRProcessWeb/index.aspx`.

model, how US Treasury holdings funded by the issuance of extra domestic bonds affect equilibrium outcomes. Interestingly, we find that foreign reserve holdings do not necessarily reduce the likelihood of a crisis, but they do make the crisis less severe when it occurs.

3.1 Model Setup with Official US Treasury Holdings

We generalize our baseline model by allowing the government to raise some additional funds. It can now issue additional domestic bonds at $t = 0$ and promise to repay an additional $b^R K_0$ at $t = 2$. Since households have sufficient wealth at $t = 0$, they cut back their $t = 0$ consumption as long as the bond yields a (gross) interest rate of $1/\beta$. The government invests the proceeds of $(1/\beta)^2 b^R K_0$ into US Treasuries yielding $R^\$$ per period. That is, reserve holdings come with a cost of carry of $(1 - (\beta R^\$)^2)$. Total debt is now $d = b^E + b^H + b^R$, where b^E is held by entrepreneurs and $b^H + b^R$ by domestic households.

3.2 Equilibria

Fundamental Equilibrium. Absent any flight to safety, the equilibrium allocation and prices are essentially the same as in the baseline model, but with an important difference: the cost of carry of US Treasuries funded by issuing extra domestic bonds reduces the government's "fiscal space," as part of the tax revenue has to be used to finance the extra carry costs. This additional fiscal burden lowers the maximal sustainable debt level. Moreover, domestic households consume less in $t = 0$ and hold a larger amount of the domestic bond.

Proposition 3.1. *The (non-flight-to-safety) fundamental equilibrium with a reserve policy b^R exists if and only if $d \in [\alpha, \tau\underline{A} - (1-(\beta R^\$)^2) b^R]$, i.e, the maximal sustainable debt level is lower than the one in the baseline model. Households' domestic bond holdings increase to $(b^R + b_0 - \alpha)K_0$, and*
(i) at $t = 0$ the allocation and prices are as in Proposition 2.1,
(ii) at $t = 1$ after a positive shock, the $\overline{\overline{A}}$-equilibrium is as in Proposition 2.2,
(iii) at $t = 1$ after a negative shock, the fundamental $\mathbb{E}_1[A]$-equilibrium is as in Proposition 2.3.

Flight-to-Safety (Subgame) Equilibrium at $t = 1$. Reserve holdings help to mitigate the flight-to-safety crisis. As before, in a flight-to-safety equilibrium domestic entrepreneurs sell off domestic bond holdings and reduce their physical capital to

$$K^E_{1,s} = \frac{q_{1,s}K_0 + p^s_{1,s}B^E_0}{q_{1,s} + \alpha\beta} = \frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1-\pi_2 h^R)b^E}{\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta}K_0, \tag{32}$$

where h^R denotes the haircut for the case with government reserve holdings. The government budget constraint in the low fundamental state (when $\underline{A}$ realizes in $t = 2$) now generalizes to

$$(b^E + b^H + b^R)(1 - h^R) = \tau\underline{A}\frac{K^E_{1,s}}{K_0} + b^R(\beta R^\$)^2, \tag{33}$$

$$\Leftrightarrow (b^E + b^H)(1 - h^R) - b^R h^R = \tau\underline{A}\frac{K^E_{1,s}}{K_0} - b^R[1 - (\beta R^\$)^2],$$

where the second equation simply rearranges terms such that the left-hand side reflects the government's repayment after debt restructuring in the baseline model minus the debt reduction that arises from partial default on the extra debt raised for reserve holdings, and the right-hand side reflects tax revenue minus the cost of carry. Next, we modify Figure 3 to be the new Figure 4. The debt repayment after restructuring is reduced by $b^R h^R$, i.e., the slope of the block solid line becomes more negative compared to the baseline model. The dashed red line reflects the minimum tax revenue in crisis time, which now has to be further reduced by the cost of carry $b^R[1 - (\beta R^\$)^2]$, hence the parallel shift in the dotted red line.

The fact that the fundamental equilibrium (green dot in Figure 4) is now closer to the black debt-repayment line reflects the fact that the cost of carry reduces the sustainable debt level.

Formally, Equations (32) and (33) lead to an endogenous haircut with reserve holdings of

$$h^R(b^E, b^H, b^R) = 1 - \tau\underline{A}\frac{\eta\beta^*\,\mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E + b^R\frac{(\beta R^\$)^2}{\tau\underline{A}}(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)}{(b^E + b^H + b^R)(\eta\beta^*\,\mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E}. \tag{34}$$

Note that the new haircut h^R is only lower than the one in the baseline model h if the latter exceeds the cost of carry. In this case, the benefit from haircut reduction outweighs the extra cost of carry.

Lemma 3.1. *If equilibrium haircuts absent reserve holdings are sufficiently large, then reserve holdings reduce the haircut in case of a flight-to-safety crisis. Formally,*

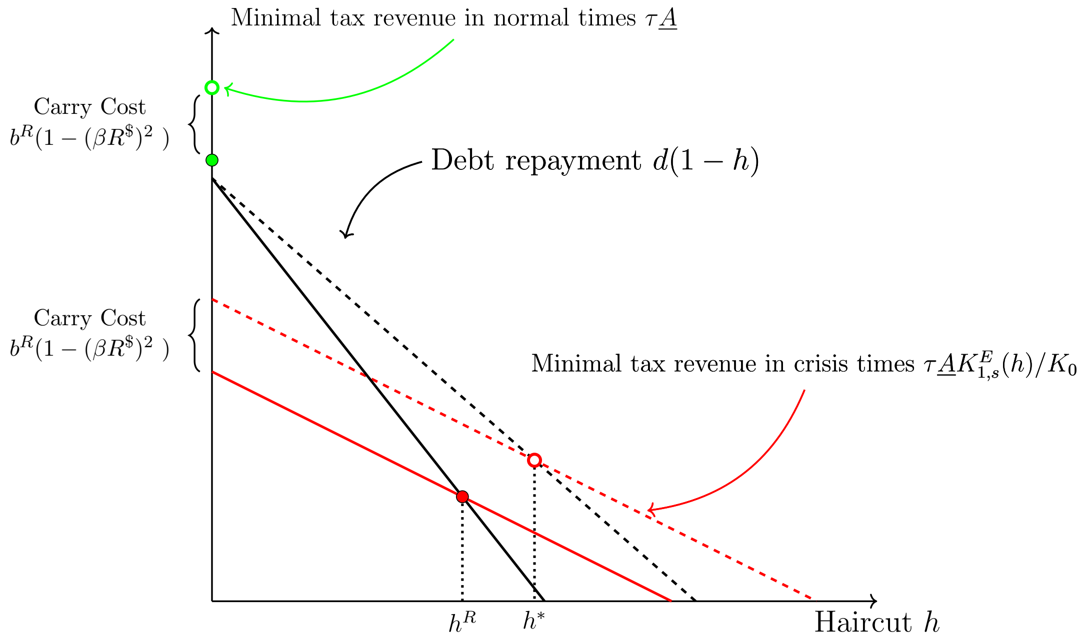

Figure 4: Determination of domestic bond haircut h

(i) $h^R(b^E, b^H, b^R) < h(b^E, b^H) \quad \Leftrightarrow h(b^E, b^H) > 1-(\beta R^\$)^2.$

(ii) $h^R(b^E, b^H, b^R)$ *is decreasing in* $b^R \Leftrightarrow h(b^E, b^H) > 1-(\beta R^\$)^2.$

We defer to the appendix the full characterization of the flight-to-safety (subgame) equilibrium at $t = 1$, as it does not add much economic insight beyond that discussed above.

3.3 Crisis Vulnerability and Severity with Reserves

Interestingly, the cost of carry of foreign reserve holdings makes the economy *more* vulnerable to a flight-to-safety crisis. Importantly, however, the severity of the crisis is lower if the haircut exceeds the cost of carry. Proposition 3.2, which follows directly from Lemma 3.1, states these results formally.

Proposition 3.2. *Reserve holdings* b^R *lead to*
(i) a vulnerability region that is at least as large as in the baseline model due to

reserves' cost of carry,

$$\mathcal{V}^R(b^R) \supset \mathcal{V}^B = [\max\{\underline{d}, \alpha\}, \tau \underline{A}]. \tag{35}$$

(ii) a reduced crisis severity compared to the baseline model if and only if the haircut in the baseline model is greater than the cost of carry $1 - (\beta R^{\$})^2$*,*

$$\mathcal{S}^R(d, b^R) \leq \mathcal{S}^B(d) \Leftrightarrow h \geq 1 - (\beta R^{\$})^2 \tag{36}$$

The fact that the reserve holdings b^R reduce the severity of the flight-to-safety crisis raises the question of why individual households do not hold US Treasuries on their own. Why does it require a government intervention to hold reserves? Recall that the US Treasury's yield is very low compared to the expected yield of the (safe but ultimately tail risk afflicted) domestic bond. This makes individual investors reluctant to hold US Treasuries despite their awareness that the total holding of US Treasuries reduces the severity of a possible flight-to-safety crisis. Each household prefers to free-ride on other households' US Treasury holdings. Individually, they do not internalize the positive externality that reserve holdings would have on the whole economy.[10]

4 Tranching

Instead of building up reserves, a country could split its debt into a senior and a junior bond. While it might be legally difficult for a country to commit to a specific seniority structure, it is always possible for an international private-sector bank to set up special purpose vehicles (SPV) that purchases some of a country's government bond and issues a senior and a junior bond. The issued securities are referred to as Sovereign Bond-Backed Securities (SBBS). Any losses due to partial default are then first absorbed by the junior bond. Only after the junior bond is fully wiped out does the senior bond begin to take losses. It is easy to see that the senior bond (with a yield higher than that of the US Treasury) is much less likely to lose its safe-asset status. Hence, domestic entrepreneurs, who hold the senior bond as a safe asset, do not have to fire-sell any bond or any physical capital. They can keep operating

[10] In a more general model, households might even want to undo government reserve holdings by taking on a carry trade that shorts the low-yielding US Treasury and investing in the higher-yielding domestic government bond.

at full capacity and consequently tax revenues will be high enough to fully pay off not only the senior bond but even the junior bond as well. Our main result in this section is that the government's debt capacity with tranching is the same as if the country had only senior bonds outstanding.

4.1 Model Setup with Tranching of Domestic Bonds

In a setting with tranching, we maintain the assumptions of the baseline model of Section 2. For simplicity, we switch off the reserve holdings, i.e., $b^R = 0$. We denote the (total) face value of the senior bond by $B_0^S = sK_0$ in total and hence the junior bond's face value is $B_0^J = B_0 - sK_0 = (d - s)K_0$. We assume there is a sufficient amount of the senior bond outstanding such that entrepreneurs can fully satisfy their safe-asset requirement, i.e., $s \geq \alpha$, and focus on the case in which the entrepreneurs only hold senior debt at time $t = 0$. For convenience, we use the capital letters S and J as superscripts for variables related to the senior and junior bonds, respectively. For example, the debt holdings (relative to K_0) of entrepreneurs and households are $b^{S,E}, b^{S,H}, b^{J,E}, b^{J,H}$.

4.2 Equilibria Outcomes with Tranching

Tranching makes the senior bond a much more stable asset. Since it is protected by the junior bond, it is much less likely to default and, if it does so, the haircut h^S is smaller.

Allocation and fundamental equilibrium. At time $t = 0$, the fundamental equilibrium allocation is the same as in the baseline model. We only have to adjust households' and entrepreneurs' bond holdings. Note that with unanticipated sunspots ($\pi_{1,s} = 0$), investors consider the senior and junior bonds as perfect substitutes. We assume that entrepreneurs have a slight preference for the senior bond at time $t = 0$.[11] The remaining senior bonds and all junior bonds are purchased by households. Formally, the fundamental equilibrium is summarized by the following proposition.

[11] In the more general case with positive sunspot probability, entrepreneurs strictly prefer senior bonds. As one lets the sunspot probability $\pi_{1,s}$ go to zero, entrepreneurs maintain this preference. In short, our assumption would be the natural outcome of a refinement argument.

Proposition 4.1. *The (non-flight-to-safety) equilibrium with tranching features a debt capacity as if only senior bonds were outstanding and*
(i) at $t = 0$ the allocation and prices are as in Proposition 2.1,
(ii) at $t = 1$ after a positive shock, the $\overline{\overline{A}}$-equilibrium allocation is as in Proposition 2.2,
(iii) at $t = 1$ after a negative shock, the fundamental $\mathbb{E}_1[A]$-equilibrium is as in Proposition 2.3,
while debt holdings and prices with tranching are

$$\begin{aligned} B_{1,f}^{S,E} &= b^{S,E} B_0, & B_{1,f}^{S,H} &= sK_0 - b^{S,E} K_0, & B_{1,f}^{S,*} &= 0, \\ B_{1,f}^{J,E} &= 0, & B_{1,f}^{J,H} &= B_0 - sK_0, & B_{1,f}^{J,*} &= 0. \end{aligned} \tag{37}$$

with $b^{S,E} = \alpha$. Bond prices are

$$p_t^S = \beta^{2-t}, \quad p_t^J = \beta^{2-t}, \quad t \in \{0, 1\}. \tag{38}$$

Flight-to-safety (subgame) equilibrium at $t = 1$. Despite the fact that the senior bond, the safe asset supplied by the emerging market economy in this section, is protected by the junior bond, it might still be subject to default and suffer a haircut of h^S. In this (more extreme) case, entrepreneurs fire-sell physical capital and senior bonds to foreign investors, who have a lower discount factor β^*. The senior bond price is then given by

$$p_{1,s}^S = \beta^*(1 - \pi_2 h^S). \tag{39}$$

Since this will only happen if junior bonds are completely wiped out, the government only repays senior bonds partially. The government budget constraint in the lowest productivity state, $\underline{A}$, is then

$$sK_0(1 - h^S) = \tau \underline{A} K_{1,s}^E. \tag{40}$$

Note that compared to Equation (23) in the baseline model, we now have sK_0 instead of dK_0.

The share of physical capital retained by the domestic entrepreneurs is

$$K_{1,s}^E = \frac{q_{1,s} K_0 + p_{1,s}^S B_0^{S,E}}{q_{1,s} + \alpha\beta} = \frac{\beta^* \eta\, \mathbb{E}_1[A] + \beta^*(1 - \pi_2 h^S) b^{S,E}}{\beta^* \eta\, \mathbb{E}_1[A] + \alpha\beta} K_0, \tag{41}$$

which differs from Equation (21) in the baseline model: now we have a smaller haricut h^S on the senior bond and b^E is replaced by $b^{S,E}$. In fact, the debt haircut function h^S is

$$h^S(b^{S,E}, b^{S,H}, b^{J,H}) = h^S(b^{S,E}, b^{S,H}) = h(b^{S,E}, b^{S,H}), \tag{42}$$

that is, the senior bond's haircut depends only on senior bond holdings. This observation leads to the following proposition:

Proposition 4.2. *For $\alpha < s < d < \tau \underline{A}$, after an adverse shock at $t = 1$ a flight-to-safety equilibrium can exist. The equilibrium allocation and senior bond price are as in the baseline flight-to-safety equilibrium of Proposition 2.4 after replacing the total debt d with only the senior debt s. The junior bonds are held by households and foreigners with a flight-to-safety price*

$$p^J_{1,s} = \beta^*(1 - \pi_2). \tag{43}$$

Proposition 4.2 states that the flight-to-safety equilibrium with tranching is almost as if junior bonds do not exist. To understand the intuition, recall that in times of crisis the entrepreneurs sell capital and domestic bonds to gain enough liquidity to buy safe assets. Domestic bonds have two roles. First, domestic bonds are quasi-safe assets backed by the government's fiscal capacity. At $t = 1$ they might lose their safe-asset status. Second, the domestic bond also serves as a liquid asset at $t = 1$ that can be counted on even when the only remaining safe asset is the US Treasury. In other words, even when the price of the senior bond is somewhat depressed it can still be sold and transformed into US Treasury holdings. The junior bond plays neither role in the flight-to-safety equilibrium.

Note also that entrepreneurs only hold senior bonds, the amount of junior bonds is irrelevant for the liquidity entrepreneurs receive when they fire-sale bonds. In sum, junior bonds neither act as a fiscal burden ex-post at $t = 2$ nor provide liquidity in the interim period $t = 1$. As a result, they play no role in the fire-sale of capital and consequently in most aspects of the equilibrium. We relegate to the appendix a full characterization of equilibrium, including the more general case with a strictly positive sunspot probability.

4.3 Crisis Vulnerability and Severity with Tranching

The next proposition follows directly from Proposition 4.2.

Proposition 4.3.

1. *With tranching into a senior and junior bond,*

   (i) if $\alpha \leq \underline{d}$*, the optimal tranching policy is* $s \in [\alpha, \underline{d}]$*. The crisis region is empty following optimal tranching policy.*

   (ii) if $\alpha > \underline{d}$*, the optimal tranching policy is* $s = b = \alpha$*, for which the crisis vulnerability region is*

$$\mathcal{V}^T(s) = [\alpha, \tau \underline{A}]. \tag{44}$$

2. *The crisis severity* $\mathcal{S}^T(d, s)$ *is as if the senior* s *is the only debt in the baseline model,*

$$\mathcal{S}^T(d, s) = \mathcal{S}^B(s) \leq \mathcal{S}^B(d). \tag{45}$$

Tranching can either completely eliminate crises or mitigate the magnitude of the flight to safety. Interestingly, higher total outstanding debt does not make the economy more crisis-prone as long as the additional debt is financed with the junior bond. This is the case, since the junior bond can be wiped out without adverse consequences. The junior bond provides a cushion and ensures that the senior bond maintains its safe-asset status. As a result, tranching shrinks the crisis vulnerability region. Moreover, even if a flight to safety occurs nevertheless, the haircut of the senior bond is significantly smaller, as the junior bond is fully wiped out first. This feature reduces the fire-sale of physical capital and stabilizes the overall economy.

Finally, note that $s = b^{S,E} = \alpha$ is the best tranching policy among all the possible ones. Setting $s = \alpha$ as the tranching point (subordination level) maximizes the size of the loss-absorbing cushion provided by the junior bond, while ensuring that entrepreneurs' total demand, α, for safe assets is met by the senior bond supply.

5 Pooling and Tranching

So far, we have focused on a single country. Next, we turn to an international setting with many countries to show that pooling several countries' government bonds and subsequently tranching the pool exploits in additional some diversification benefits. The pooling and tranching can be done by an international bank setting up an SPV acquiring government bonds from several countries (weighted according to relative GDP) and issuing a senior and a junior bond.

5.1 Model Setup with Pooling and Tranching

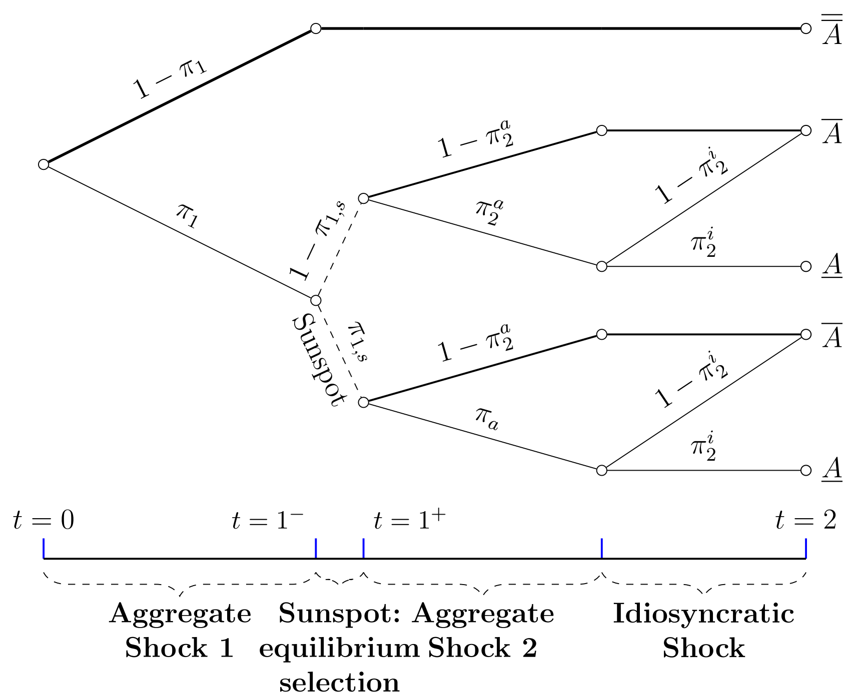

Figure 5: Timeline for pooling case

To study pooling and tranching, we generalize our baseline framework to a setting with a continuum of ex-ante identical countries indexed by m. The environment within each country is the same as in the baseline framework. However, we modify the structure of shocks: (i) sunspot shocks are perfectly correlated across countries, while (ii) productivity A-shocks are imperfectly correlated. The fundamental productivity shock at $t = 1$ is assumed to be perfectly correlated and occurs with probability π_1. After an adverse shock, the productivity shock at $t = 2$ follows in two waves. The first wave at $t = 2$ is an aggregate shock and hits all countries the same way with probability π_2^a. Meanwhile, the second wave is purely idiosyncratic across countries occurring with probability π_2^i. If the aggregate shock is not realized, no further idiosyncratic shock happens and all countries enjoy a productivity level of

$\overline{A}$. The details of the shock structure are depicted in Figure 5. We assume emerging economies do not trade with each other. Within each country, three groups of agents trade with each other at the beginning of time $t = 0$ and time $t = 1$.

5.2 Crisis Vulnerability and Severity with Tranching and Pooling

Combining pooling and tranching does make a difference. The economic intuition is that combining both policy tools exploits the fact that ex-post there is only a π_2^i fraction of countries that truly default. In this case, we need a much smaller cushion to ensure the safety of senior bonds. We retain earlier superscripts S and J for the two classes of bonds. Let *GloSBies* be the superscript for this global safe-asset policy. Recall $\underline{d}$ defined in Equation (28) is the threshold of the crisis vulnerability region in the baseline model.

Proposition 5.1. *Combining pooling and tranching yields the following:*

1. *With tranching into a senior and junior bond,*[12]

   (i) If $\alpha \leq (1-\pi_2^i)d+\pi_2^i\underline{d}$, the optimal tranching policy is $s \in [\alpha, (1-\pi_2^i)d+\pi_2^i\underline{d}]$ and the crisis vulnerability is eliminated.

   (ii) If $\alpha > (1-\pi_2^i)d + \pi_2^i\underline{d}$, the optimal tranching policy is $s = b^{S,E} = \alpha$ and the economy is still vulnerable to a less severe crisis.

2. *Whenever a crisis exists after adopting any tranching policy s, the crisis severity $\mathcal{S}^{GloSBies}(d, s)$ is*

$$\mathcal{S}^{GloSBies}(d, s) = \mathcal{S}^B(s) - \frac{\beta^*\pi_2^a\alpha d(1-\pi_2^i)}{(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)s - \beta^*\pi_2^a\pi_2^i\tau\underline{A}\alpha} \leq \mathcal{S}^B(s) \leq \mathcal{S}^B(d), \tag{46}$$

   where $\mathcal{S}^B(\cdot)$ is defined in Equation (31) with $\pi_2 = \pi_2^a\pi_2^i$.

[12] In the appendix, we also show the vulnerability region is

$$\mathcal{V}^{GloSBies}(s) = [\max\{\alpha, \underline{d}\}, \min\{\frac{s-\pi_2^i\underline{d}}{1-\pi_2^i}\,\tau\underline{A}\}], \quad d \geq s \geq \alpha.$$

We find such a measure is less intuitive for comparison purposes, since the crisis existence condition $\alpha \leq (1-\pi_2^i)d+\pi_2^i\underline{d}$ is better described in the two-dimensional (s, d) space. In contrast, crisis region is defined as the set of possible d for a fixed policy s.

Notice that with pooling, the tranching-only threshold for the vulnerability region $\underline{d}$ is replaced by $(1-\pi_2^i)d + \pi_2^i\underline{d}$. Recall that without pooling, the relevant condition to eliminate the crisis vulnerability was $\alpha \leq \underline{d}$ (Proposition 4.3), and for $d \leq \underline{d}$ there was no crisis vulnerability even in the benchmark economy (see Section 2.3). In the nontrivial case $d \geq \underline{d}$, pooling yields a relaxed condition compared to the tranching-only case. The intuition for this is straightforward: to avoid a flight to safety, we need to guarantee a sufficient supply of safe assets α even during a crisis. A fraction $(1-\pi_2^i)$ of countries that do not default repay their full debt of d, while a fraction π_2^i of countries defaulting in a crisis repay only $\underline{d}$, the amount of tax revenue the governments in these countries can collect. Combining both improves their capacity to back safe assets. Given a total demand α of safe assets, the required supply of safe assets is

$$(1-\pi_2^i) \underbrace{d}_{\substack{\text{repayment of} \\ \text{default-free countries}}} + \pi_2^i \underbrace{\underline{d}}_{\substack{\text{repayment of} \\ \text{defaulted countries}}} \geq \underbrace{\alpha}_{\text{safe asset demand}} . \tag{47}$$

Notice that the global safe-asset policy precisely exploits the fact that ex-post some country will be safe and thus a good supplier of safe assets. The same intuition extends to the case when a crisis does occur. The sovereign bonds that do not default provide a good source of liquidity. Entrepreneurs can sell these sovereign bonds in exchange for dollars at a favorable price. Consequently, entrepreneurs need to sell less capital and the severity of the crisis is mitigated.

6 Conclusion

Flight to safety is a major contributor to financial crises. This paper sets up a simple three-period model in which entrepreneurs hold a safe asset in addition to physical capital. When the domestic government bond loses its safe-asset status, domestic entrepreneurs shed it and replace it with US Treasuries. The resulting losses force entrepreneurs to also reduce their productive capital holdings. The associated loss in aggregate output and tax revenue makes a default in government bonds likely, which justifies the initial loss of the safe-asset status.

The current global financial architecture relies on a "buffer approach" to avoid cross-border flight-to-safety capital flows. The most prominent such method is self-insurance via a buildup of precautionary foreign reserves, to a large extent in the form of US Treasury holdings. This is, however, costly as they yield a lower interest rate compared to the domestic government bond. These extra costs do not make crises less likely, but they do significantly reduce their severity.

This paper examines in detail the less costly and self-stabilizing "rechanneling approach." This approach requires a global safe asset that is symmetrically supplied, including by emerging economies. Sovereign Bond-Backed Securities could be such a global safe asset. While the sovereign bond of an emerging economy might lose its safe-asset status after an adverse shock, a senior bond that is backed by several sovereign bonds does not. Hence, flight-to-safety capital flows do not have to leave the country. By pooling many sovereign bonds and tranching the pool, governments can exploit diversification benefits and increase the size of the senior tranche, thereby increasing the total quantity of safe assets (GloSBSies) supplied by and for emerging economies.

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A Appendix

A.1 Detail on the flight-to-safety equilibrium under different policies

Proposition A.1 characterizes the flight-to-safety equilibrium with reserves policy.

Proposition A.1. *(Assume unexpected crisis) The flight-to-safety equilibrium at* $t = 1$ *exists if and only if* $d \in [\max\{\underline{d}^R(b^R), \alpha\}, \tau\underline{A}]$. *The minimal threshold for total debt level* $\underline{d}^R(b^R)$ *is defined as*

$$h^R(\alpha, \underline{d}^R - \alpha, b^R) = 0. \tag{A.1}$$

The equilibrium allocation is

$$\begin{aligned}
K^E_{1,s} &= \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E + b^R \frac{(\beta R^{\$})^2)}{\tau\underline{A}}(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)}{\eta\beta^* \mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^*\pi_2 \frac{b^E}{b^E + b^H + b^R}} K_0, \\
K^H_{1,s} &= 0, \qquad K^*_{1,s} = K_0 - K^E_{1,s}, \\
B^E_{1,s} &= 0, \qquad B^H_{1,s} = b^R K_0 + B_0 - \alpha K_0, \qquad B^*_{1,s} = \alpha K_0, \\
\$^E_{1,s} &= \beta\alpha \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E + b^R \frac{(\beta R^{\$})^2)}{\tau\underline{A}}(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)}{\eta\beta^* \mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^*\pi_2 \frac{b^E}{b^E + b^H + b^R}} K_0, \\
\$^H_{1,s} &= 0, \qquad \$^*_{1,s} = 0.
\end{aligned} \tag{A.2}$$

with $b^E = \alpha$ *and* $b^H = d - \alpha$. *Foreign investors are the marginal holders for both domestic bonds and capital and hence the asset prices are*

$$q_{1,s} = \beta^*\eta\,\mathbb{E}_1[A], \tag{A.3}$$

$$p_{1,s} = \beta^*(1 - \pi_2 h^R(\alpha, d - \alpha, b^R)), \tag{A.4}$$

with a haircut of domestic bonds $h^R(b^E, b^H, b^R)$ *defined in Equation* (34).

Proof. See section A.2. □

Proposition A.2 characterizes the flight-to-safety equilibrium with tranching policy.

Proposition A.2. *(Assume unexpected crisis) For $\alpha < s < d < \tau\overline{A}$, a flight-to-safety equilibrium exists if and only $s \in [\max\{\underline{d}, \alpha\}, \tau\overline{A}]$. The equilibrium allocation is*

$$
\begin{aligned}
K_{1,s}^{E} &= \frac{\eta\beta^{*}\,\mathbb{E}_1[A] + (1-\pi_2)\beta^{*}b^{S,E}}{\eta\beta^{*}\,\mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^{*}\pi_2\frac{b^{S,E}}{s}}K_0, \qquad && K_{1,s}^{H} = 0, \qquad && K_{1,s}^{*} = K_0 - K_{1,s}^{E},\\
B_{1,s}^{S,E} &= 0, && B_{1,s}^{S,H} = sK_0 - b^{S,E}K_0, && B_{1,s}^{S,*} = b^{S,E}B_0,\\
B_{1,s}^{J,E} &= 0, && B_{1,s}^{J,H} = B_0 - sK_0, && B_{1,s}^{J,*} = 0,\\
\$_{1,s}^{E} &= \beta\alpha\frac{\eta\beta^{*}\,\mathbb{E}_1[A] + (1-\pi_2)\beta^{*}b^{S,E}}{\eta\beta^{*}\,\mathbb{E}_1[A] + \alpha\beta - \tau\underline{A}\beta^{*}\pi_2\frac{b^{S,E}}{s}}K_0, && \$_{1,s}^{H} = 0, && \$_{1,s}^{*} = 0,
\end{aligned}
\tag{A.5}
$$

with $b^{S,E} = \alpha$. Foreign investors are the marginal holders for both domestic bonds and capital and hence the asset prices are

$$q_{1,s} = \beta^{*}\eta\,\mathbb{E}_1[A], \tag{A.6}$$

$$p_{1,s}^{S} = \beta^{*}(1 - \pi_2 h(\alpha, s-\alpha)), \tag{A.7}$$

$$p_{1,s}^{J} = \beta^{*}(1-\pi_2). \tag{A.8}$$

Proof. See section A.2. □

A.2 Proofs of Results in Main Text

To ease exposition, we introduce two lemmas first.

Lemma A.1. *Households always hold domestic bonds they bought at $t = 0$ to maturity ($t = 2$). Entrepreneurs hold their asset position unchanged at $t = 1$ as long as there is no fire-sale.*

Proof. We argue households will not sell domestic bonds in time 1. There are two cases. First, if foreigners hold some domestic bonds at $t = 1$, the bonds must have expected return $\frac{1}{\beta^{*}}$ since foreigners are risk neutral and wealth unconstrained. Since $\beta > \beta^{*} > \frac{1}{R^{\$}}$, households prefer holding domestic bonds over holding dollars and consumption. Second, if foreigners hold no bonds at $t = 1$, there are no fire-sales in both capital and bonds. By assumption, entrepreneurs weakly prefer holding capital over holding bonds. They do not sell capital unless the bonds are unsafe in the first place. As a result, there is no resource exchange between the domestic economy and the foreigners. Since there is no production in $t = 1$, the total consumption of domestic agents at $t = 1$ is zero. Since entrepreneurs do not sell their capital or consume, they do not have extra wealth to buy additional bonds. We therefore have entrepreneurs and foreigners buy no additional bonds. Entrepreneurs' asset positions are unchanged at $t = 1$. To clear the bonds market, households must keep their bond holdings unchanged. □

Lemma A.2. *In time 0, the domestic bonds price is*

$$p_0 = \beta^2, \tag{A.9}$$

and bond positions held by entrepreneurs are

$$b^E = \alpha. \tag{A.10}$$

Proof of Lemma A.2. The proof is already outlined in the main text. By assumptions, entrepreneurs are not wealthy enough to buy all domestic bonds at $t = 0$. The residual bonds must be purchased either by households or the foreigners. With short-sales constraint, the households have a higher discount rate and therefore buy residual domestic bonds. Because domestic bonds have finite supply, households consume the rest of their wealth.

The above discussion together with Lemma A.1 implies households consume only at $t = 0$ and $t = 2$. They must be indifferent about consuming at $t = 0$ and at $t = 2$. Explicitly, they face a reduced optimization problem between date 0 and date 2.

$$\max C_0 + \beta^2 C_2,$$
$$\text{subject to } W_0 = C_0 + p_0 B_0^H,$$
$$B_0^H = C_2.$$

The Euler equation between $t = 0$ and $t = 2$ is

$$\beta^2 \frac{1}{p_0} = 1 \Rightarrow p_0 = \beta^2.$$

This proves the first part. For the second part, notice that entrepreneurs have binding safe-asset constraints at $t = 0$,

$$p_0 B_0^E = \alpha\beta^2 K_0 \Rightarrow \beta^2 b_0^E = \alpha\beta^2 \Rightarrow b_0^E = \alpha.$$

□

Proof of Proposition 2.1. Following Lemma A.2 , $b^E = \alpha$ and $b^H = d - \alpha$. This gives the allocation of domestic bonds. For capital allocation, we verify that entrepreneurs strictly prefer to hold capital instead of selling it to foreigners at $t = 0$. The return of a portfolio of capital and safe assets from $t = 0$ to $t = 1$ is

$$\frac{\mathbb{E}(q_1) + \alpha\beta^2 \frac{\mathbb{E}(p_1)}{p_0}}{1 + \alpha\beta^2}.$$

At $t = 0$, for each unit of capital, entrepreneurs invest 1 into capital and $\alpha\beta^2$ in domestic bonds. $E(p_1)/p_0 = \frac{1}{\beta}$ is the expected return of domestic bonds since default is unexpected at $t = 0$. $\mathbb{E}(q_1)$ is the expected price of capital. It is also the expected return of capital because of the unit marginal cost of investment at $t = 0$. If capital is sold to foreigners, the selling price of capital at $t = 0$ would be $\beta^* \mathbb{E}(q_1)$ due to their impatience. Entrepreneurs prefer holding capital if[13]

$$\beta \frac{\mathbb{E}(q_1) + \alpha\beta}{1 + \alpha\beta^2} > \beta^* \mathbb{E}(q_1),$$

which holds under Assumption 3, $\beta > \beta^*(1 + \alpha)$. We therefore know entrepreneurs hold all capital at $t = 0$. At last, no agents would prefer to buy dollars due to their low yield (Equation (6)). □

Proof of Proposition 2.2. By Assumption 1, there are no fire-sales with sufficiently good fundamentals. By Lemma A.1, entrepreneurs and households keep their asset positions unchanged at $t = 1$. The equilibrium allocation is the same as in Proposition 2.1. Also notice that, given the asset prices in Proposition 2.2, households and entrepreneurs will be indifferent between consumption and holding assets. Any finite demand is possible. Consequently, the asset market clears. □

Proof of Proposition 2.3. An argument similar to that for Proposition 2.2 gives the equilibrium allocation. The capital price is different now since the expected dividend of unit capital changed even though the expected return from $t = 1$ to $t = 2$ is still $\frac{1}{\beta}$. The bond price is β as there is no default. For the existence result, a negative fundamental equilibrium can be constructed as specified in the proposition. It is straightforward to verify the optimization problems and market-clearing conditions. □

[13]Implicitly here we assume the marginal utility of the wealth of entrepreneurs is 1, which is the case when entrepreneurs are not wealth constrained in any future states. In a flight-to-safety equilibrium at $t = 1$, entrepreneurs are indeed wealth constrained when there are price-depressed assets available to purchase. However, since a crisis is unexpected, this case has a probability weight of 0 ex-ante.

Proof of Proposition 2.4. Substituting Equations (21) and (22) into Equation (23), we have

$$(b^E + b^H)(1-h) = \tau \underline{A} \frac{\beta^* \eta\, \mathbb{E}_1[A] + \beta^*(1-\pi_2 h(b^E, b^H))b^E}{\beta^* \eta\, \mathbb{E}_1[A] + \alpha\beta},$$

from which we solve the haircut $h(b^E, b^H)$,

$$h(b^E, b^H) = 1 - \tau \underline{A} \frac{\eta \beta^* \, \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E}{(b^E + b^H)(\eta\beta^* \, \mathbb{E}_1[A] + \alpha\beta) - \tau \underline{A} \beta^* \pi_2 b^E}. \tag{A.11}$$

The haircut can be rewritten as

$$h(b^E, b^H) = \frac{\tau \underline{A} - (b^E + b^H)}{b^E + b^H} \frac{1}{\frac{-\frac{\partial T(h)}{\partial h}}{b^E + b^H} - 1},$$

with $-\frac{\partial T(h)}{\partial h}$ being the sensitivity of tax revenue to the bond haircut h (in absolute value), which highlights the haircut spiral. Given the haircut $h(b^E, b^H)$, we can solve $K^E_{1,s}$ from Equation (21). The entrepreneurs' dollar holdings follow from the binding safe-asset constraint $\$^E_{1,s} = \beta \alpha K^E_{1,s}$. Households have their domestic bond positions unchanged by Lemma A.1. Because foreigners demand return $1/\beta^*$ for both capital and domestic bonds, the asset prices follow from discounting the expected dividend at $t = 2$ ($1 - \pi_2 h$ for domestic bonds and $\mathbb{E}_1[A]$ for capital).

It remains to be verified that the government (partially) defaults and domestic investors fire-sale part of their physcial capital, i.e., $h(b^E, b^H) > 0$ and $K^E_{1,s} < K_0$. Since at $t = 0$, $b^E = \alpha, b^H = d - \alpha$. For a fixed α, the haircut function $h(\alpha, d-\alpha)$ is increasing in d. That is, a higher ex-ante total debt level leads to a larger ex-post default. To see that, define $h_2(d)$ as

$$h_2(d) = h(\alpha, d - \alpha) = 1 - \tau \underline{A} \frac{\eta\beta^* \, \mathbb{E}_1[A] + (1-\pi_2)\beta^* \alpha}{d(\eta\beta^* \, \mathbb{E}_1[A] + \alpha\beta) - \tau \underline{A} \beta^* \pi_2 \alpha}.$$

By Assumption 1 and 2, the denominator $d(\eta\beta^* \, \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E$ is positive and to verify $\frac{\partial h_2(d)}{\partial d} > 0$ is straightforward. Since $h(\alpha, d-\alpha)$ is increasing in d, there is a unique debt level $\underline{d}$ such that $h(\alpha, \underline{d} - \alpha) = 0$. Solving $\underline{d}$ yields Equation (28). Assumption 1 and 2 ensure $\underline{d} \leq \tau \underline{A}$. By definition,

$$h(\alpha, d - \alpha) > 0 \Leftrightarrow d > \underline{d}.$$

From Equation (21),

$$h > 0 \Rightarrow K^E_{1,s}/K_0 < 1.$$

Together the flight-to-safety equilibrium exists if and only if $d \in [\max\{\underline{d}, \alpha\}, \tau \underline{A}]$. □

Proof of Proposition 3.1. We first show the characterization of equilibrium allocation and prices assuming equilibrium existence and then proceed to verify the condition for equilibrium existence.

At $t = 0$, the only difference between the setup with reserves and the baseline model is the extra domestic debt issuance. Similar to the baseline model, households buy all the residual old debt and all new debt at $t = 0$, i.e.,

$$B_0^H = \underbrace{B_0 - \alpha K_0}_{\text{Residual old debt}} + \underbrace{b^R K_0}_{\text{All new debt}} .$$

This proves the first claim.

At $t = 1$, note that Lemma A.1 still holds. Conditional on no fire-sales, domestic agents keep their positions unchanged. The equilibrium allocation follows from the $t = 0$ allocation. It is straightforward to verify that equilibrium prices are the same as those in the baseline model. This proves the second and third claims.

For the existence result, we need to check that domestic bonds do not default in the state with the lowest productivity at $t = 2$, i.e., the lowest tax revenue plus reserves is enough to cover maturing debt.

$$\underbrace{\tau \underline{A} K_0}_{\text{tax revenue}} + \underbrace{\beta^2 b^R K_0 (R^\$)^2}_{\text{Reserves}} \geq \underbrace{d K_0}_{\text{the old debt}} + \underbrace{b^R K_0}_{\text{the new debt}} .$$

Rearrange the equation to get

$$d \leq \tau \underline{A} - (1 - \beta^2 (R^\$)^2) b^R, \tag{A.12}$$

which is the upper bound for the existence region. The lower bound follows from the model's assumption. □

Proof of Lemma 3.1. To prove the first claim, notice that the difference between the two haircut functions is

$$\begin{aligned}
&h^R(b^E, b^H, b^R) - h(b^E, b^H) \\
&= \frac{b^R(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta)\{\tau\underline{A}(\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* b^E)}{[(b^E + b^H)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E][(b^E + b^H + b^R)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E]} \\
&\quad + \frac{-[(b^E + b^H)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E](\beta R^\$)^2\}}{[(b^E + b^H)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E][(b^E + b^H + b^R)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E]} \\
&= \frac{b^R(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta)(1 - h(b^E, b^H) - (\beta R^\$)^2)}{(b^E + b^H + b^R)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau\underline{A}\beta^*\pi_2 b^E}.
\end{aligned}$$

It follows

$$h^R(b^E, b^H, b^R) - h(b^E, b^H) < 0 \Leftrightarrow 1 - (\beta R^\$)^2 < h(b^E, b^H).$$

For the second claim, the partial derivative $\frac{\partial h^R(b^E,b^H,b^R)}{b^R}$ is

$$\frac{\partial h^R(b^E, b^H, b^R)}{b^R} = \frac{(1 - h^R(b^E, b^H, b^R) - (\beta R^\$)^2)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta)}{(b^E + b^H + b^R)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau \underline{A}\beta^*\pi_2 b^E},$$

which implies

$$\frac{\partial h^R(b^E, b^H, b^R)}{b^R} < 0 \Leftrightarrow 1 - (\beta R^\$)^2 < h^R(b^E, b^H, b^R).$$

Also notice

$$\begin{aligned} &h^R(b^E, b^H, b^R) - (1 - (\beta R^\$)^2) \\ &= \frac{(\beta R^\$)^2 - 1 + h(b^E, b^H)}{[(b^E + b^H)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau \underline{A}\beta^*\pi_2 b^E][(b^E + b^H + b^R)(\eta\beta^* \mathbb{E}_1[A] + \alpha\beta) - \tau \underline{A}\beta^*\pi_2 b^E]}, \end{aligned}$$

which implies

$$\frac{\partial h^R(b^E, b^H, b^R)}{b^R} < 0 \Leftrightarrow 1 - (\beta R^\$)^2 < h^R(b^E, b^H, b^R) \Leftrightarrow 1 - (\beta R^\$)^2 < h(b^E, b^H).$$

Together, the second result is proved. □

Proof of Proposition A.1. We first show the equilibrium allocation and prices assuming equilibrium existence. Then we proceed to verify the condition of equilibrium existence.

The proof here is similar to the one for Proposition 2.4. Substituting Equation (32) into Equation(33), we solve $K^E_{1,s}$ and h^R (Equation (34)). Entrepreneurs' dollar holdings can be solved from the binding safe-asset constraint. Households have their domestic bond positions from $t = 0$ unchanged by Lemma A.1. Because foreigners demand return $1/\beta^*$ for both capital and domestic bonds, the asset prices follows from discounting the expected dividend at $t = 2$ ($1 - \pi_2 h$ for domestic bonds and $\mathbb{E}_1[A]$ for capital).

For the existence result, we need to verify that domestic bonds have default risk, i.e., $h^S > 0$. From equation (34), the equilibrium haircut $h^R(\alpha, d-\alpha, b^R)$ is increasing in d. Define $\underline{d}^R(b^R)$ as the unique solution of

$$h^R(\alpha, \underline{d}^R - \alpha, b^R) = 0. \tag{A.13}$$

The flight-to-safety equilibrium exists if and only if $d \in [\max\{\alpha, \underline{d}^R\}, \tau\underline{A}]$. □

Proof of Proposition 3.2. For the first claim, we can compute the crisis region directly. Following Proposition A.1, the crisis region with tranching policy b^R is $\mathcal{V}^R(b^R) = [\max\{\alpha, \underline{d}^R(b^R)\}, \tau\underline{A}]$ whereas the one in the baseline is $\mathcal{V}^B = [\max\{\alpha, \underline{d}\}, \tau\underline{A}]$. It therefore suffices to show

$$\underline{d}^R(b^R) \leq \underline{d}. \tag{A.14}$$

It can be verified that $h^R(\alpha, d-\alpha, 0) = h(\alpha, d-\alpha)$. It follows that $\underline{d}^R(0) = \underline{d}$. We show that $\underline{d}^R(b^R)$ is decreasing in b^R and the conclusion follows. By implicit function theorem,

$$\frac{\partial \underline{d}(\alpha, b^R)}{\partial b^R} = -\frac{\frac{\partial h^R(b^E, \underline{d}(b^E, b^R) - b^E, b^R)}{\partial b^R}}{\frac{\partial h^R(b^E, \underline{d}(b^E, b^R) - b^E, b^R)}{\partial b^H}\big|_{b^H = \underline{d}(b^E, b^R) - b^E}}$$

By the second result in Lemma 3.1, the numerator is positive. It is straightforward to check that $\frac{\partial h^R(b^E, b^H, b^R)}{\partial b^H} > 0$. Consequently, the denominator in the equation above is also positive. We have

$$\frac{\partial \underline{d}(\alpha, b^R)}{\partial b^R} < 0,$$

which proves the first claim.

For the second claim, we note that the share of fire-sold capital $\mathcal{S}^R(d, b^R)$ is linked to haircut $h^R(\alpha, d-\alpha, b^R)$ through entrepreneurs' wealth. Specifically, Equation (32) implies

$$\mathcal{S}^R(d, b^R) = 1 - \frac{K^E_{1,s}}{K_0} = 1 - \frac{\beta^* \eta\, \mathbb{E}_1[A] + \beta^*(1 - \pi_2 h^R(\alpha, d-\alpha, b^R))\alpha}{\beta^* \eta\, \mathbb{E}_1[A] + \alpha\beta}. \tag{A.15}$$

The counterpart for the baseline is

$$\mathcal{S}^B(d) = 1 - \frac{\beta^* \eta\, \mathbb{E}_1[A] + \beta^*(1 - \pi_2 h(\alpha, d-\alpha))\alpha}{\beta^* \eta\, \mathbb{E}_1[A] + \alpha\beta}. \tag{A.16}$$

Therefore

$$\begin{aligned} &\mathcal{S}^R(d, b^R) < \mathcal{S}^B(d) \\ \Leftrightarrow\ & 1 - \pi_2 h^R(\alpha, d-\alpha, b^R) > 1 - \pi_2 h(\alpha, d-\alpha) \\ \Leftrightarrow\ & h^R(\alpha, d-\alpha, b^R) < h(\alpha, d-\alpha). \end{aligned} \tag{A.17}$$

By Lemma 3.1,

$$h(\alpha, d-\alpha) - h^R(\alpha, d-\alpha, b^R) < 0 \Leftrightarrow h(\alpha, d-\alpha) < 1 - (\beta R^{\$})^2.$$

We therefore conclude that

$$\mathcal{S}^R(d, b^R) < \mathcal{S}^B(d) \Leftrightarrow h^R(\alpha, d - \alpha, b^R) > 1 - (\beta R^{\$})^2,$$

which proves the second claim. □

Proof of Proposition 4.1. Since the crisis is unexpected at $t = 0$, junior bonds and senior bonds are perfect substitutes at $t = 0$. All results except for bond holdings in Proposition 2.1 hold, with the additional restriction on entrepreneurs' preference for senior bonds. The bond holdings are naturally pinned down. A similar argument holds for all non-flight-to-safety equilibria at $t = 1$. □

Proof of Proposition 4.2. Similar to the baseline, the haircut of senior bonds h^S can be solved from equations (40) and (41), which is

$$h^S(b^{S,E}, b^{S,H}) = h^S(b^{S,E}, s - b^{S,E}) = h(b^{S,E}, s - b^{S,E}). \tag{A.18}$$

The equilibrium allocation and prices other than the part for junior bonds follows the same argument for Proposition 2.4 with the replacement of domestic bonds as senior bonds. For the allocation and price of junior bonds, one feasible equilibrium is that households hold all junior bonds at $t = 1$ and the junior bonds have a return equal to $1/\beta^*$. In this case, the households are indifferent between junior bonds and senior bonds, but they prefer holding bonds over consumption and holding dollars. The market clears since households simply hold their initial bond positions unchanged.

In general, we argue that junior bonds and senior bonds are perfect substitutes in the flight-to-safety equilibrium. By perfect substitutes, we mean that they both are risky assets and have the same return $1/\beta^*$. They are equivalent as far as portfolio choice is concerned. The idea is as follows. Since entrepreneurs prefer to hold no domestic bonds, bonds are held by households and foreigners. To trigger a flight to safety, there have to be some bonds sold to foreigners, which have an expected return $1/\beta^*$. Therefore households can obtain returns no lower than $1/\beta^*$ by investing in particular bonds. As a result, households strictly prefer holding bonds over consumption and holding dollars. Since households' wealth is stored in bonds at the beginning of $t = 1$, they have to buy the same market value of bonds from the market. This implies that they must hold both junior bonds and senior bonds as their initial wealth is larger than the total market value of junior bonds. This happens only when the expected return is equalized between the two bonds. The pricing of junior bonds immediately follows from their expected return $\frac{1}{\beta^*}$ and expected dividend $1 - \pi_2$.

The above discussion shows that there is equilibria indeterminacy up to the bond holdings that divides between households and foreigners. In all these equilibria, both senior bonds and junior bonds have the same expected return $\frac{1}{\beta^*}$ and asset price. The indeterminacy is innocuous to our main insight. □

Proof of Proposition 5.1. The characterization of non-crisis states are standard following the discussion in the baseline model. We focus on the flight-to-safety equilibrium here. When the adverse $t = 2$ aggregate shock hits, the global junior bonds are wiped out and the global safe assets default partially. This leads to a flight to safety at $t = 1$. The price of the global senior bond at $t = 1$ is

$$p^S_{1,s} = \beta^*(1 - \pi^a_2 h^S)$$

As in the case of a single economy with tranching, entrepreneurs sell all of their senior bonds in exchange for dollars. The capital holdings at the end of $t = 1$ are

$$K^E_{1,s} = \frac{q_{1,s}K_0 + p^S_{1,s}B^{S,E}_0}{q_{1,s} + \alpha\beta} = \frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1 - \pi^a_2 h^S)b^{S,E}}{\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta}K_0. \tag{A.19}$$

Because the crisis is unanticipated, we know $b^{S,E} = \alpha$ following the argument in the case of a single- economy model with tranching. After the adverse $t = 2$ aggregate shock, the idiosyncratic shock follows. $1 - \pi^i_2$ fraction of countries have final productivity $\overline{A}$. They do not default and repay their debt in full value dK_0. In contrast, the remaining π^i_2 fraction of countries have final productivity $\underline{A}$. They default and repay recovery value $\tau\underline{A}K^E_{1,s}$. We have the balance sheet identity for SUV sector (scaled) as

$$(1 - \pi^i_2)d + \pi^i_2\tau\underline{A}\frac{K^E_{1,s}}{K_0} = (1 - h^S)s. \tag{A.20}$$

We can solve h^S from Equations (A.19) and (A.20), which is

$$h^S(d, s) = 1 - \frac{\tau\underline{A}(\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1 - \pi^a_2)\alpha) + \frac{d(1-\pi^i_2)}{\pi^i_2}(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)}{(\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta)\frac{s}{\pi^i_2} - \beta^*\pi^a_2\tau\underline{A}\alpha}.$$

We need to verify $h^S(d, s) \geq 0$ and $\tau\underline{A}K^E_{1,s}/K_0 < d$. The second condition states that countries experience adverse idiosyncratic shock default. This condition coincides with the first haircut condition in the case of a single country. The first condition can be simplified as

$$s > (1 - \pi^i_2)d + \pi^i_2\tau\underline{A}\frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*\alpha}{\beta^*\eta\,\mathbb{E}_1[A] + \beta\alpha} = (1 - \pi^i_2)d + \pi^i_2\underline{d}.$$

Notice $s \leq d$. The first condition implies $\underline{d} < d$, which further implies $\tau \underline{A} K^E_{1,s}/K_0 < d$. Therefore,

$$s > (1 - \pi^i_2)d + \pi^i_2 \underline{d} \tag{A.21}$$

is the necessary sufficient condition for flight-to-safety equilibrium existence. The existence condition can be written in terms of vulnerability region $\mathcal{V}^{GloSBies}(s)$:

$$\mathcal{V}^{GloSBies}(s) = [\max\{\alpha, \underline{d}\}, \min\{\frac{s - \pi^i_2 \underline{d}}{1 - \pi^i_2}, \tau \underline{A}\}], \quad d \geq s \geq \alpha. \tag{A.22}$$

The first claim follows naturally from the above more general results. As for the second claim, the share of fire sold capital $\mathcal{S}^{GloSBies}(d, s)$ is computed from Equation (A.19) once we know $h^S(d, s)$:

$$\begin{aligned}
\mathcal{S}^{GloSBies}(d, s) &= 1 - \frac{\beta^* \eta \, \mathbb{E}_1[A] + (1 - \pi_2)\beta^* \alpha}{\beta^* \eta \, \mathbb{E}_1[A] + \alpha\beta - \beta^* \pi_2 \tau \underline{A} \frac{\alpha}{s}} - \frac{\beta^* \pi^a_2 \alpha d (1 - \pi^i_2)}{(\beta^* \eta \, \mathbb{E}_1[A] + \alpha\beta)s - \beta^* \pi^a_2 \pi^i_2 \tau \underline{A} \alpha} \\
&= \mathcal{S}^B(s) - \frac{\beta^* \pi^a_2 \alpha d (1 - \pi^i_2)}{(\beta^* \eta \, \mathbb{E}_1[A] + \alpha\beta)s - \beta^* \pi^a_2 \pi^i_2 \tau \underline{A} \alpha}
\end{aligned} \tag{A.23}$$

The last equality follows from $\mathcal{S}^B(d, s) = 1 - \frac{\eta\beta^* \mathbb{E}_1[A] + (1-\pi_2)\beta^* \alpha}{\eta\beta^* \mathbb{E}_1[A] + \alpha\beta - \beta^* \pi_2 \tau \underline{A} \frac{\alpha}{s}}$ with $\pi_2 = \pi^a_2 \pi^i_2$. □

A.3 Extension to Anticipated Flight to Safety

In this appendix, we (partially) relax the assumption that flight to safety is unanticipated at $t = 0$. ur major results hold as long as the ex-ante probability of flight to safety is sufficiently small. This can be due to either the fundamental being strong (π_1 is small) or a sunspot unlikely ($\pi_{1,s}$ is small). The interpretation is that no shock outcome captures normal times, and the $t = 1$ productivity shock is ex-ante unlikely to be initial bad news, from which things might grow worse.

We maintain all earlier assumptions except for the assumption of unanticipated crisis. In addition, we restrict ourselves to equilibria that two more properties hold. First, entrepreneurs optimally choose to hold capital at $t = 0$. Second, households optimally choose to hold only domestic bonds at $t = 0$. The first property requires that the investment opportunity of capital have sufficiently high yield compared to other means. The second property requires that the perceived likelihood of crisis in

$t = 0$ is low such that dollar is not too attractive even in $t = 0$. [14]. We verify that the two properties hold for sufficiently small $\pi_1 \pi_{1,s}$.

A.3.1 Baseline Model

To fix ideas, recall that $\pi_{1,s}$ is the sunspot probability. For $\pi_{1,s} > 0$, we have a new lemma generalizing the results in Lemma A.2.

Lemma A.3. *Suppose equilibria exist. At $t = 0$, the domestic bonds price is* [15]

$$p_0 = \beta^2(1 - \pi_1 \pi_{1,s} \pi_2 \max\{0, h(b^E, d - b^E)\}), \tag{A.24}$$

and entrepreneurs' bond positions are

$$b^E = \frac{\alpha}{1 - \pi_1 \pi_{1,s} \pi_2 \max\{0, h(b^E, d - b^E)\}}. \tag{A.25}$$

Proof. Notice Lemma A.1 still holds since the argument is only about equilibrium at $t = 1$. Following the same argument for Lemma A.2, households must be indifferent between consuming at $t = 0$ and $t = 2$. The only difference here is that households expect bonds to default with probability $\pi_1 \pi_{1,s} \pi_2$. Therefore, a bond with unit face value is sold at price p_0 at $t = 0$ and gives full face value 1 when debt is safe and recovery value $1 - h$ when there is default at $t = 2$. The expected payoff at $t = 2$ is

$$1(1 - \pi_1 \pi_{1,s} \pi_2) + (1 - h)\pi_1 \pi_{1,s} \pi_2 = 1 - \pi_1 \pi_{1,s} \pi_2 h.$$

The Euler equation for households between $t = 0$ and $t = 2$ is

$$\beta^2 \frac{1 - \pi_1 \pi_{1,s} \pi_2 h}{p_0} = 1 \Rightarrow p_0 = \beta^2(1 - \pi_1 \pi_{1,s} \pi_2 h),$$

which gives the price of domestic bonds at $t = 0$. Since entrepreneurs have a binding safe-asset constraint,

$$p_0 B_0^E = \alpha \beta^2 K_0 \Rightarrow \beta^2(1 - \pi_1 \pi_{1,s} \pi_2 h) b_0^E = \alpha \beta^2 \Rightarrow b_0^E = \frac{\alpha}{1 - \pi_1 \pi_{1,s} \pi_2 h}.$$

□

[14] The two properties listed make sure that $t = 0$ is a tranquil period for the economy. If any of the properties are not true, then flight to safety already happens at $t = 0$, which is ill-suited for our purpose of characterizing a possible flight to safety at $t = 1$.

[15] The max operator is to incorporate the case where no flight-to-safety equilibrium exists. In such a case, the equation has solution $b^E = \alpha$ and $h(\alpha, d - \alpha) < 0$.

At $t = 0$, the likelihood of flight to safety is $\pi_1\pi_{1,s}$. As the likelihood of flight to safety increases, the ex- ante price of domestic bonds decreases to reflect the crisis and entrepreneurs buy more bonds as safe assets.[16] Formally, define entrepreneurs' endogenous bond positions at $t = 0$ as functions of probability of flight to safety and total debt (ratio): $b^E(\pi_1\pi_{1,s}, d)$. We have following lemma.

Lemma A.4. *For each $\alpha \leq d \leq \tau A$, there exists a threshold for the probability of flight to safety $\pi^*(d)$. For $\pi_1\pi_{1,s} \in [0, \pi^*(d)]$, there exists a solution $b^E(\pi_1\pi_{1,s}, d) \in [\alpha, d]$ to Equation* (A.25).

Proof. Rewrite Equation (A.25) as

$$b^E(1 - \pi_1\pi_{1,s}\pi_2 \max\{0, h(b^E, d - b^E)\}) = \alpha. \tag{A.26}$$

Since $h(b^E, d - b^E)$ is decreasing in b^E (see Equation (27)), the left-hand side of the above equation is strictly increasing in b^E. The equation therefore has at most one solution. To ensure $b^E \in [\alpha, d]$, we show this holds if and only if $\pi_1\pi_{1,s} \in [0, \pi^*(d)]$ for some threshold level $\pi^*(d)$. For $\pi > \pi^*(d)$, the domestic bonds are so cheap that entrepreneurs buy all domestic bonds and might even buy additional dollars to meet the safe-asset constraint.

Define function $F(b, \Pi, d)$ to be

$$F(b, \Pi, d) = b(1 - \Pi\pi_2 \max\{0, h(b, d - b)\}) - \alpha. \tag{A.27}$$

$F(b, \Pi, d)$ is nonincreasing in Π and strictly increasing in b. For given Π and d, function $F(b, \Pi, d) = 0$ has at most one solution for $b^*(\Pi, d)$. Notice $F(\alpha, \Pi, d) \leq 0$ and $F(d, 0, d) = d - \alpha > 0$. There are two cases. First, if $F(d, 1, d) > 0$, we know

$$0 < F(d, 1, d) \leq F(d, \Pi, d).$$

Intermediate value theorem applies and there exists unique solution $b^*(\Pi, d) \in [\alpha, d]$ for $\Pi \in [0, 1]$. Second, if $F(d, 1, d) < 0$, we can define $\pi^*(d)$ such that

$$F(d, \pi^*(d), d) = 0 \tag{A.28}$$

since $F(d, 0, d) > 0$ and the function is continuous in Π. In this case, for $\Pi \in [0, \pi^*(d)]$, we know that

$$F(\alpha, \Pi, d) \leq 0 \quad \text{and} \quad F(d, \Pi, d) \geq F(d, \pi^*(d), d) = 0.$$

[16] Entrepreneurs still buy the same market value of bonds, but the total face value of bonds they bought increases.

Apply the intermediate value theorem to augment b of function $F(b, \Pi, d)$. We have unique solution $b^*(\Pi, d) \in [\alpha, d]$. If we defined the thresholds, $\pi^*(d) = 1$ for the first case. We have shown the claim. □

The following propositions characterize results parallel to those in Section 2 when $\pi_1\pi_{1,s}$ is strictly positive but sufficiently small.

Proposition A.3. *We obtain following results regarding the baseline model with the anticipated flight to safety.*

1. *For sufficiently small $\pi_1\pi_{1,s}$, the flight-to-safety equilibrium exists (crisis vulnerability region) if and only if $h(b^E(\pi_1\pi_{1,s}, d), d - b^E(\pi_1\pi_{1,s}, d)) > 0$. The characterization of equilibria in Propositions 2.1, 2.2, 2.3, and 2.4 hold as long as b^E and b^H are replaced with a unique pair of solutions from equations*

$$b^E = \frac{\alpha}{1 - \pi_1\pi_{1,s}\pi_2 \max\{0, h(b^E, d - b^E)\}}, \tag{A.29}$$

$$b^H = d - b^E \tag{A.30}$$

   and the bond price at $t = 0$ is replaced with the one in Lemma A.3.

2. *The result is continuous at $\pi_1\pi_{1,s} = 0$ provided $\pi_1 > 0$.*

Proof*.* It is straightforward that the second claim follows from the first claim.

For the first claim, we have following observation. If we assume entrepreneurs optimally choose to hold capital and bonds at $t = 0$ and households optimally choose to hold bonds at $t = 0$, the allocation and prices at $t = 0$ are by construction pinned down as long as we know b^E and p_0, which are provided in Lemma A.3. Besides, the arguments for Proposition 2.2, 2.3, and 2.4 go through as long as we replace b^E with the one defined in Equation A.25. Lemma A.4 shows that b^E exists for sufficiently small $\pi_1\pi_{1,s} < \pi^*(d)$.

To finish the proof, we must verify the optimality of entrepreneurs' and households' choices at $t = 0$. In the non-flight-to-safety equilibria at $t = 1$, both types of agents have marginal utility of wealth (MUW) of 1. However, when flight to safety happens, entrepreneurs invest all their wealth in capital and dollars for the high return in times of asset fire-sales. Households invest all their wealth in domestic bonds similarly. The optimality of such actions is ensured by Assumption 5. Consequently, the marginal utility of wealth in crisis times is higher than 1 for both agents. For households, their MUW is

$$\xi^H_{1,s} = \frac{\beta}{\beta^*} > 1,$$

since domestic bonds have an expected return $\frac{1}{\beta^*}$ in a flight-to-safety episode. For entrepreneurs, their MUW is

$$\xi^E_{1,s} = \beta \frac{\mathbb{E}_1[A] + R^\$ \alpha\beta}{\beta^* \eta\, \mathbb{E}_1[A] + \alpha\beta} > \xi^H_{1,s} > 1,$$

where letter ξ stands for marginal utility of wealth. $\xi^E_{1,u} = \xi^E_{1,f} = \xi^H_{1,u} = \xi^H_{1,f} = 1$.

At $t = 0$, households optimally choose to consume and buy domestic bonds instead of buying dollars, which requires

$$1 = \beta\, \mathbb{E}(\xi^H_1 \frac{p_1}{p_0}) > \beta\, \mathbb{E}(\xi^H_1 R^\$).$$

The first equality states that households are indifferent between consumption and holding domestic bonds.

For entrepreneurs, their available options are 1) holding capital and using domestic bonds as safe assets, 2) building capital and selling to foreigners, 3) holding domestic bonds only, 4) holding dollars only, and 5) consuming only. We need to ensure they optimally choose to hold capital and use domestic bonds as safe assets, i.e., the marginal utility of wealth from holding capital and using domestic bonds as safe assets is the highest among the five possible choices. We have four inequalities:

$$\beta\, \mathbb{E}(\xi^E_1 \frac{q_1 + \alpha\beta^2 \frac{p_1}{p_0}}{1 + \alpha\beta^2}) > \beta\, \mathbb{E}(\xi^E_1 \frac{q_1 + \alpha\beta^2 R^\$}{1 + \alpha\beta^2}) \tag{A.31}$$

$$\beta\, \mathbb{E}(\xi^E_1 \frac{q_1 + \alpha\beta^2 \frac{p_1}{p_0}}{1 + \alpha\beta^2}) > \beta^*\, \mathbb{E}(q_1) \tag{A.32}$$

$$\beta\, \mathbb{E}(\xi^E_1 \frac{q_1 + \alpha\beta^2 \frac{p_1}{p_0}}{1 + \alpha\beta^2}) > 1 \tag{A.33}$$

$$\beta\, \mathbb{E}(\xi^E_1 \frac{q_1 + \alpha\beta^2 \frac{p_1}{p_0}}{1 + \alpha\beta^2}) > \beta\, \mathbb{E}(\xi^E_1 R^\$) \tag{A.34}$$

Notice all inequalities hold strictly if $\pi_{1,s} = 0$ by Assumptions 1-5. In that case, the inequalities reduce to

$$\beta\frac{\beta\,\mathbb{E}(A)+\alpha\beta}{1+\alpha\beta^2} > \beta\frac{\beta\,\mathbb{E}(A)+\alpha\beta^2 R^{\$}}{1+\alpha\beta^2}$$
$$\beta\frac{\beta\,\mathbb{E}(A)+\alpha\beta}{1+\alpha\beta^2} > \beta^*\,\mathbb{E}(A)$$
$$\beta\frac{\beta\,\mathbb{E}(A)+\alpha\beta}{1+\alpha\beta^2} > 1 > \beta R^{\$}.$$

By continuity, there exists a threshold $\pi^{**}(d) \leq \pi^*(d)$ such that Equations (A.31)-(A.34) hold.[17] □

Proposition A.3 shows that our main results hold in the neighborhood of an unanticipated flight to safety for a sufficiently small ex-ante likelihood of flight to safety.

For future reference, we characterize the crisis vulnerability and crisis intensity for the baseline model in the following proposition.

Proposition A.4. *We obtain the following results regarding crisis vulnerability and intensity.*

1. *The crisis vulnerability region $\mathcal{V}^B$ is*

$$\mathcal{V}^B = [\max\{\alpha, \underline{d}^*\}, \tau\underline{A}], \tag{A.35}$$

where $\underline{d}^*$ is the unique solution of $h(b^E(\pi_1\pi_{1,s}, \underline{d}^*), \underline{d}^* - b^E(\pi_1\pi_{1,s}, \underline{d}^*)) = 0$.

2. *The crisis intensity $\mathcal{S}^B$ is*

$$\mathcal{S}^B = 1 - \frac{K_{1,s}^E}{K_0} = 1 - \frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1-\pi_2 h(b^E, d-b^E))b^E}{\beta^*\eta\,\mathbb{E}_1[A]+\alpha\beta}. \tag{A.36}$$

Proof. We only need to verify

$$h(b^E(\pi_1\pi_{1,s}, d), d - b^E(\pi_1\pi_{1,s}, d)) > 0 \Leftrightarrow d > \underline{d}^*.$$

The rest follows from earlier analysis. It suffices to show $h(b^E(\pi_1\pi_{1,s}, d), d - b^E(\pi_1\pi_{1,s}, d))$ is increasing in d. We only sketch the key steps here. By using the implicit function theorem in (A.25), we can show $b^E(\pi_1\pi_{1,s}, d)$ is increasing in d. The result then follows as h must decrease if b^E increases to ensure Equation (A.25) holds . □

[17] $\pi^{**}(d) \leq \pi^*(d)$ is required to ensure that the valid solution b^E exists for Equation (A.25).

A.3.2 Reserves

The analysis for the reserves case is similar. Lemma A.5 characterizes bond holdings and pricing at $t = 0$.

Lemma A.5. *Suppose equilibria exist. At $t = 0$, the domestic bond price is given by*

$$p_0 = \beta^2(1 - \pi_1\pi_{1,s}\pi_2 \max\{0, h^R(b^E, d - b^E, b^R)\}), \tag{A.37}$$

and bond positions held by entrepreneurs are

$$b^E = \frac{\alpha}{1 - \pi_1\pi_{1,s}\pi_2 \max\{0, h^R(b^E, d - b^E, b^R)\}}. \tag{A.38}$$

where haircut $h^R(b^E, b^H, b^R)$ is defined in Equation (34).

Proof. The same proof of Lemma A.3 applies here with haircut $h(b^E, d - b^E)$ replaced by haircut function $h^R(b^E, d - b^E, b^R)$ under reserve policies. □

Lemma A.6 shows Equation (A.38) has unique solution for a sufficiently small probability of crisis $\pi_1\pi_{1,s}$.

Lemma A.6. *For each $\alpha \leq d \leq \tau A$ and given reserve policy b^R, there exists a threshold for the probability of flight to safety $\pi^{R,*}(d, b^R)$. For $\pi_1\pi_{1,s} \in [0, \pi^{R,*}(d, b^R)]$, there exists a solution $b^E(\pi_1\pi_{1,s}, d, b^R) \in [\alpha, d]$ to Equation (A.38).*

Proof. Notice in the proof of Lemma A.4 that we only need the comparative static that $h(b, d - b)$ is decreasing in b. The same comparative static is true for function $h^R(b^E, d - b^E, b^R)$. As a result, we can apply the proof of Lemma A.4 with function $h(b^E, d - b^E)$ replaced by $h^R(b^E, d - b^E, b^R)$. □

Proposition A.5 characterizes the flight-to-safety equilibrium, which generalizes Proposition A.1.

Proposition A.5. *We obtain the following results regarding the case of an anticipated flight to safety with reserve policies.*

1. *For sufficiently small $\pi_1\pi_{1,s}$, the (non-flight-to-safety) fundamental equilibrium exists if and only if $d \in [\alpha, \tau\underline{A} - (1 - \beta^2(R^\$)^2)b^R]$, whereas the flight-to-safety equilibrium exists if and only if $h^R(b^E, d - b^E, b^R) > 0$. The characterizations*

of equilibria in Propositions 3.1 and A.1 hold as long as b^E *and* b^H *are replaced with a unique pair of solutions from equations*

$$b^E = \frac{\alpha}{1 - \pi_1 \pi_{1,s} \pi_2 \max\{0, h^R(b^E, d - b^E, b^R)\}}, \tag{A.39}$$

$$b^H = d - b^E. \tag{A.40}$$

2. *Result 1 is continuous at* $\pi_1 \pi_{1,s} = 0$ *provided* $\pi_1 > 0$.

Proof. The proof is almost the same as the one for Proposition A.3. The result of equilibria at $t = 1$ in Proposition A.1 is still valid as long as we pin down endogenous domestic bond positions b^E at $t = 0$. And we already show there is a solution to the b^E given in Lemma A.5 if $\pi_1 \pi_{1,s} \leq \pi^{R,*}(d, b^R)$ (Lemma A.6). At last, we can check that the optimality of entrepreneurs' and households' actions at $t = 0$ holds given sufficiently small $\pi_1 \pi_{1,s}$. □

Notice Lemma 3.1 still holds, as only function h^R is concerned. The following proposition gives the policy implication of the reserves policy, which generalizes Proposition 3.2.

Proposition A.6. *Given a reserve policy* b^R, *in equilibria characterized in Proposition A.5,*

1. *the crisis vulnerability region is not smaller than the crisis vulnerability region in the baseline model,*

$$\mathcal{V}^R(b^R) \supset \mathcal{V}^B = [\max\{\underline{d}^*, \alpha\}, \tau \underline{A}], \tag{A.41}$$

2. *the crisis intensity is less than that in the baseline model if and only if the haircut before implementing the policy is greater than* $1 - (\beta R^\$)^2$,

$$\mathcal{S}^R(\pi_1 \pi_{1,s}, d, b^R) \leq \mathcal{S}^B(\pi_1 \pi_{1,s}, d) \Leftrightarrow h(b^E, d - b^E) \geq 1 - (\beta R^\$)^2, \tag{A.42}$$

where b^E *is implicitly defined in Equation* (A.38).

Proof. To prove the first claim, define function $F(d, b^R)$ as

$$F(d, b^R) = h^R(b^E(\pi_1 \pi_{1,s}, d, b^R), d - b^E(\pi_1 \pi_{1,s}, d, b^R), b^R). \tag{A.43}$$

Similar to $\underline{d}^R$ in Proposition 3.2, define $\underline{d}^{R,*}(b^R)$ as $F(\underline{d}^{R,*}(b^R), b^R) = 0$. The crisis region now becomes $[\max\{\alpha, \underline{d}^{R,*}(b^R)\}, \tau\underline{A}]$. It can be verified that $\underline{d}^{R,*}(0) = \underline{d}$. It suffices to show that $\underline{d}^{R,*}(b^R)$ is decreasing in b^R. By implicit function theorem,

$$\frac{d\underline{d}^{R,2}(b^R)}{db^R} = \frac{-\frac{\partial F(d,b^R)}{\partial b^R}}{\frac{\partial F(d,b^R)}{\partial d}} \tag{A.44}$$

Taking the partial derivative of both sides of Equation (A.38) and rearranging the terms[18], we have

$$\frac{\partial F(d,b^R)}{\partial b^R} = \frac{1-\pi_1\pi_{1,s}\pi_2 h^R}{\pi_1\pi_{1,s}\pi_2 b^E}\frac{\partial b^E}{\partial b^R} = \frac{1}{\pi_1\pi_{1,s}\pi_2 b^E}\frac{\partial b^E}{\partial b^R},$$

$$\frac{\partial F(d,b^R)}{\partial d} = \frac{1}{\pi_1\pi_{1,s}\pi_2 b^E}\frac{\partial b^E}{\partial d}.$$

Notice that

$$\frac{\partial F(d,b^R)}{\partial b^R} = \frac{\partial h^R(b^E, d-b^E, b^R)}{\partial b^E}\frac{\partial b^E}{\partial b^R} + \frac{\partial h^R(b^E, d-b^E, b^R)}{\partial b^R}.$$

We can solve $\frac{\partial b^E}{\partial b^R}$ as

$$\frac{\partial b^E}{\partial b^R} = -\frac{\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial b^R}}{\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial b^E} - \frac{1}{\pi_1\pi_{1,s}\pi_2 b^E}}.$$

From Lemma 3.1 and $h^R = 0$, we know $\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial b^R} > 0$ and $\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial b^E} < 0$. As a result,

$$\frac{\partial b^E}{\partial b^R} > 0 \Rightarrow \frac{\partial F(d,b^R)}{\partial b^R} > 0.$$

Similarly,

$$\frac{\partial F(d,b^R)}{\partial d} = \frac{\partial h^R(b^E, d-b^E, b^R)}{\partial b^E}\frac{\partial b^E}{\partial d} + \frac{\partial h^R(b^E, d-b^E, b^R)}{\partial d}.$$

And it follows that

$$\frac{\partial b^E}{\partial d} = -\frac{\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial d}}{\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial b^E} - \frac{1}{\pi_1\pi_{1,s}\pi_2 b^E}}.$$

[18]Since $h^R = 0$, we take a one-sided derivative due to the max operator.

We know $\frac{\partial h^R(b^E,d-b^E,b^R)}{\partial d} > 0$. Therefore

$$\frac{\partial b^E}{\partial d} > 0 \Rightarrow \frac{\partial F(d, b^R)}{\partial d} > 0.$$

Combine the above two comparative statics and Equation (A.44). The first claim is proved.

Now we move on to the second claim. For convenience, we omit the argument for endogenous function $b^E(\pi_1\pi_{1,s}, d, b^R)$ whenever there is no ambiguity. From Equation (32), $\mathcal{S}^R$ is

$$\mathcal{S}^R(d, b^R) = 1 - \frac{K_{1,s}^E}{K_0} = 1 - \frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1 - \pi_2 h^R(b^E, d - b^E, b^R))b^E}{\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta}.$$

And $\mathcal{S}^R(\pi_1\pi_{1,s}, d, 0) = \mathcal{S}^B(\pi_1\pi_{1,s}, d)$. Define function $G(d, b^R) = (1 - \pi_2 h^R(b^E, d - b^E, b^R))b^E$. It suffices to show that

$$G(d, b^R) \leq G(d, 0) \Leftrightarrow h(b^E(\pi_1\pi_{1,s}, d, 0), d - b^E(\pi_1\pi_{1,s}, d, 0)) \geq 1 - (\beta R^\$)^2.$$

We prove the sufficient condition

$$\frac{\partial G(d, b^R)}{\partial b^R} > 0 \Leftrightarrow h(b^E(\pi_1\pi_{1,s}, d, 0), d - b^E(\pi_1\pi_{1,s}, d, 0)) \geq 1 - (\beta R^\$)^2.$$

Taking the partial derivative of Equation (A.38) again, we have

$$\frac{\partial G(d, b^R)}{\partial b^R} = -\frac{1 - \pi_1\pi_{1,s}}{\pi_1\pi_{1,s}}\frac{\partial b^E}{\partial b^R},$$

which implies

$$\frac{\partial G(d, b^R)}{\partial b^R} > 0 \Leftrightarrow \frac{\partial b^E}{\partial b^R} > 0.$$

Note earlier that we proved the following result in proving the first claim:

$$\frac{\partial F(d, b^R)}{\partial b^R} > 0 \Leftrightarrow \frac{\partial b^E}{\partial b^R} > 0 \Leftrightarrow \frac{\partial h^R(b^E, d - b^E, b^R)}{\partial b^R} > 0.$$

Also from Lemma 3.1,

$$\frac{\partial h^R(b^E, d - b^E, b^R)}{\partial b^R} > 0 \Leftrightarrow F(d, b^R) = h^R(b^E, d - b^E, b^R) < 1 - (\beta R^\$)^2.$$

From the above two inequalities, we have $\frac{\partial F(d,b^R)}{\partial b^R} > 0 \Leftrightarrow F(d,b^R) < 1-(\beta R^\$)^2$. We guess this equivalence implies

$$\frac{\partial F(d,b^R)}{\partial b^R} > 0 \Leftrightarrow F(d,0) = h(b^E(\pi_1\pi_{1,s},d,0), d-b^E(\pi_1\pi_{1,s},d,0)) < 1-(\beta R^\$)^2.$$

If the guess is correct, it immediately follows that

$$G(d,b^R) \leq G(d,0) \Leftrightarrow h(b^E(\pi_1\pi_{1,s},d,0), d-b^E(\pi_1\pi_{1,s},d,0)) < 1-(\beta R^\$)^2, \quad \text{(A.45)}$$

which is the second claim.

It remains to show that our guess is correct. We prove by contradiction. Suppose $\frac{\partial F(d,b^R)}{\partial b^R} > 0$ and $F(d,0) \geq 1-(\beta R^\$)^2$. By mean value theorem, there exists $b_0 \in [0, b^R]$ such that

$$\frac{\partial F(d,b^R)}{\partial b^R}(d,b_0) = \frac{F(d,b^R)-F(d,0)}{b^R} < 0 \Leftarrow F(d,b_0) > 1-(\beta R^\$)^2.$$

We can construct a sequence $\{b_n\}_{n=1}^{\infty}$ by repeating mean value theorem between point b_n and b^R. Since $b_n \in [b_{n-1}, b^R]$, the sequence is non-decreasing. It has limit b^*. By continuity of $F(d,b^R)$, $F(d,b^*) \geq 1-(\beta R^\$)^2$. If $b^* \neq b^R$, we can construct $b' < b^*$ by applying mean value theorem once more, which contradicts b^* being the lower bound of the sequence. If $b^* = b^R$, it contradicts $F(d,b^R) < 1-(\beta R^\$)^2$. Together, the only possibility is that the assumption is not true. □

A.3.3 Tranching

Similar to Lemmas A.3 and A.5, Lemma A.7 characterizes the bond holdings and pricing in $t=0$ in the case with tranching.

Lemma A.7. *At $t=0$, the domestic senior bond price p_0^S and junior bond price p_0^J are*

$$p_0^S = \beta^2(1-\pi_1\pi_{1,s}\pi_2\max\{0, h^S(b^{S,E}, s-b^{S,E})\}), \quad \text{(A.46)}$$

$$p_0^J = \beta^2(1-\pi_1\pi_{1,s}\pi_2). \quad \text{(A.47)}$$

Entrepreneurs have senior bond positions

$$b^{S,E} = \frac{\alpha}{1-\pi_1\pi_{1,s}\pi_2\max\{0, h(b^{S,E}, s-b^{S,E})\}}, \quad \text{(A.48)}$$

and junior bond positions

$$b^{J,E} = 0, \tag{A.49}$$

where $h^S = h(b^{S,E}, b^{S,H})$ is the haircut for senior bonds in the flight-to-safety equilibrium.

Proof. For bond prices, the same argument in the proof of Lemma A.3 can be applied to senior bonds and junior bonds separately. For their bond positions at $t = 0$, entrepreneurs strictly prefer to hold senior bonds over junior bonds to fulfill their safe-asset constraints. The reason is that compared to households, entrepreneurs have higher marginal utility of wealth in a crisis state due to their ability to manage capital. While households are indifferent between the two types of bonds on the margin, entrepreneurs would strictly prefer the one offering the higher return in a crisis state. Formally, households' indifference condition is

$$1 = \beta\, \mathbb{E}(\xi_1^H \frac{p_1^S}{p_0^S}) = \beta\, \mathbb{E}(\xi_1^H \frac{p_1^J}{p_0^J}).$$

Comparing the marginal utility of wealth between entrepreneurs and households, we have $\xi^E \geq \xi^H$ with strict inequality in a crisis state (A.3.1). Also notice that junior bonds get wiped out in a crisis, $p_{1,s}^S > p_{1,s}^J = 0$. Combining both inequalities, we have entrepreneurs' valuation of both types of bonds at equilibrium price,

$$\beta\, \mathbb{E}(\xi_1^E \frac{p_1^S}{p_0^S}) = 1 + \pi_1 \pi_{1,s}(\xi_{1,s}^E - \xi_{1,s}^H)\frac{p_{1,s}^S}{p_0^S} > 1 + \pi_1 \pi_{1,s}(\xi_{1,s}^E - \xi_{1,s}^H)\frac{p_{1,s}^J}{p_0^J} = \beta\, \mathbb{E}(\xi_1^E \frac{p_1^J}{p_0^J}).$$

Consequently, entrepreneurs hold no junior bonds as safe assets at $t = 0$, $b^{J,E} = 0$. The position in senior bonds can be derived similarly as in the proof of Lemma A.3. □

Equation (A.48) implicitly defines entrepreneurs' endogenous bond positions at $t = 0$ as a function of the likelihood of crisis and the tranching policy: $b^{S,E}(\pi_1 \pi_{1,s}, d, s)$. Note that the total debt-to-capital ratio d is irrelevant here, just like in the model with an unanticipated crisis (Equation (42)). Similar to Lemma A.4, the following lemma establishes the existence of solution $b^{S,E}$ in Equation (A.48).

Lemma A.8. *For each $\alpha \leq s \leq d \leq \tau A$, there exists a threshold for the probability of flight to safety $\pi^*(s)$. For $\pi_1 \pi_{1,s} \in [0, \pi^*(s)]$, there exists a solution $b^{S,E}(\pi_1 \pi_{1,s}, s) \in [\alpha, s]$ to Equation* (A.48)*.*

Proof. Apply Lemma A.4 with $s = d$. □

Lemma A.8 is the same as Lemma A.4 except the total debt ratio d is replaced by total senior debt ratio s. This is consistent with the results in section 4 of the main text. It is the amount of senior debt s instead of the amount of total domestic bonds d that matters. Moreover, we have a proposition corresponding to Proposition A.3.

Proposition A.7. *We obtain the following results regarding the model with anticipated flight to safety and tranching policy.*

1. *For sufficiently small $\pi_1\pi_{1,s}$, the flight-to-safety equilibrium exists (crisis vulnerability region) if and only if $h(b^{S,E}, s - b^{S,E}) > 0$. The characterization of equilibria in Propositions 4.1, 4.2, and A.2 hold as long as $b^{S,E}$ and $b^{S,H}$ are replaced with a unique pair of solutions from equations*

$$b^{S,E} = \frac{\alpha}{1 - \pi_1\pi_{1,s}\pi_2 \max\{0, h(b^{S,E}, d - b^{S,E})\}}, \tag{A.50}$$

$$b^{S,H} = s - b^{S,E}, \tag{A.51}$$

   and the bond price at $t = 0$ is replaced with the one in Lemma A.7.

2. *The result is continuous at $\pi_1\pi_{1,s} = 0$ provided $\pi_1 > 0$.*

Proof. We point out that Proposition 4.2 holds in the anticipated crisis case, since the proposition is about $t = 1$. The rest of the proof is the same as the proof for Proposition A.3 but replaces d with s. Again, we need to verify all optimality of asset positions held by entrepreneurs and households. These conditions hold as long as $\pi_1\pi_{1,s}$ is sufficiently small. □

Proposition A.7 shows that our main results in tranching still hold in the neighborhood of unanticipated flight to safety as long as the ex-ante likelihood of flight to safety is sufficiently small. Proposition A.8 characterizes the policy implication of tranching. It shows that the results in Proposition 4.3 are robust.

Proposition A.8. *Given a feasible tranching policy s, consider the equilibria characterized in Proposition A.7. One of the following cases holds:*

1. *The flight-to-safety equilibrium does not exist.*

2. *The flight-to-safety equilibrium exists. In the flight-to-safety equilibrium, the share of fire-sold capital $\mathcal{S}^T(d, s, \pi_1\pi_{1,s})$ will be the same as that in the baseline model with total debt level s,*

$$\mathcal{S}^T(d, s, \pi_1\pi_{1,s}) = \mathcal{S}^B(s, \pi_1\pi_{1,s}) \leq \mathcal{S}^B(d, \pi_1\pi_{1,s}). \tag{A.52}$$

Proof. Suppose the first case does not hold. From Proposition A.7, we know $h(b^{S,E}, s - b^{S,E}) > 0$. Similar to the argument for Proposition 4.3, we link the share of fire-sold capital $\mathcal{S}^T$ to the haircut of senior bonds $h^S = h(b^{S,E}, s - b^{S,E})$. Specifically, Equation (40) gives

$$(1 - h^S)s = \tau \underline{A} \frac{K_{1,s}^E}{K_0} = \tau \underline{A}(1 - \mathcal{S}^T),$$

which still holds in our case. We obtain

$$\mathcal{S}^T(d, s, \pi_1\pi_{1,s}) = \mathcal{S}^T(s, \pi_1\pi_{1,s}) = 1 - \frac{(1 - h^S)s}{\tau \underline{A}} = 1 - \frac{(1 - h(b^{S,E}, s - b^{S,E}))s}{\tau \underline{A}},$$

where the first equality follows from the irrelevance of d. A similar equation holds for the baseline case,

$$\mathcal{S}^B(d, \pi_1\pi_{1,s}) = 1 - \frac{(1 - h(b^E, d - b^E))d}{\tau \underline{A}},$$

where b^E is implicitly defined by Equation (A.25). Comparing the expression of the share of fire-sold capital in both cases, we know that

$$\mathcal{S}^T(s, \pi_1\pi_{1,s}) = \mathcal{S}^B(s, \pi_1\pi_{1,s}).$$

It remains to show that $\mathcal{S}^B(d, \pi_1\pi_{1,s})$ is increasing in argument d. Thereafter the inequality in the claim would follow. We note that the result is nontrivial as b^E endogenously depends on d. For convenience, in the following discussion we omit the argument $(\pi_1\pi_{1,s}, d)$ in function $b^E(\pi_1\pi_{1,s}, d)$ whenever there is no ambiguity. From Equation (21),

$$\mathcal{S}^B(d, \pi_1\pi_{1,s}) = 1 - \frac{K_{1,s}^E}{K_0} = 1 - \frac{\beta^*\eta\,\mathbb{E}_1[A] + \beta^*(1 - \pi_2 h(b^E, d - b^E))b^E}{\beta^*\eta\,\mathbb{E}_1[A] + \alpha\beta}.$$

It thus is equivalent to show $(1 - \pi_2 h(b^E, d - b^E))b^E$ is decreasing in d. To do that, we note $b^E(\pi_1\pi_{1,s}, d)$ is strictly increasing in d when $h(b^E, d - b^E) > 0$, which we mentioned in the proof of Proposition A.4. From Equation (A.25), we have

$$(1 - \pi_1\pi_{1,s})b^E + \pi_1\pi_{1,s}(1 - \pi_2 h(b^E, d - b^E))b^E = \alpha,$$

provided $h(b^E, d - b^E) > 0$. Now suppose $h(b^E, d - b^E) > 0$. Notice the first term in the left-hand side is increasing in d and the right-hand side is a constant. It must be that the second term $\pi_1\pi_{1,s}(1-\pi_2 h(b^E, d-b^E))b^E$ is decreasing in d. Or $(1-\pi_2 h(b^E, d-b^E))b^E$ is decreasing in d. It also follows that $h(b^E, d-b^E)$ is increasing in d.

Notice that for the second case we have $h(b^{S,E}, s-b^{S,E}) = h(b^E(\pi_1\pi_{1,s}, s), s-b^E(\pi_1\pi_{1,s}, s)) > 0$, since $t \in [s,d]$, $h(b^E(\pi_1\pi_{1,s}, t), t-b^E(\pi_1\pi_{1,s}, t)) \geq h(b^E(\pi_1\pi_{1,s}, s), s-b^E(\pi_1\pi_{1,s}, s)) > 0$. Our assumption therefore holds for all $t \in [s,d]$. Consequently, $(1-\pi_2 h(b^E(\pi_1\pi_{1,s}, t), t-b^E(\pi_1\pi_{1,s}, t)))b^E(\pi_1\pi_{1,s}, t)$ is increasing in t for $t \in [s,d]$. The second claim follows.

□

Proposition A.8 shows tranching is still effective as long as the ex-ante likelihood of flight to safety is sufficiently small.