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CAN PUBLIC SPENDING CUTS BE INFLATIONARY?

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ABSTRACT

The paper uses a "demand for seigniorage revenue" and "supply of seigniorage revenue" approach to determine the consequences of cuts in public spending for the rate of inflation. Monetary financing is viewed as the residual financing mode, with tax rates and public debt-GDP ratios held constant. In a small open economy with an exogenous real interest rate, cuts in public consumption spending will lower the inflation rate in the revenue-efficient region of the seigniorage Laffer curve. When there are cuts in public sector capital formation, the inflation rate can rise even in the seigniorage-efficient region. This will be the case if the expenditure effect (which reduces the deficit one-for-one) is more than offset by direct and indirect revenue effects (which raise the deficit) and by an adverse money demand effect.

When the real interest rate is endogenous, the scope for inflation-increasing public spending cuts is enhanced.

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1. Introduction

This paper studies the consequences for inflation of public expenditure cuts. The setting is that of a small open economy, but most of the analysis is applicable to closed economic systems as well. While the emphasis is on the long run, transitional dynamics will be considered briefly.

While the purpose of this paper is to develop a general theoretical framework, the question it addresses was motivated by the recent experience of the Mexican economy (see Feltenstein and Morris (1987), Jarque (1987) and Jung (1987)).

With but minor abuse of language we can pose the question that is the title of this paper in terms of the demand for and supply of ***seigniorage revenue***. Monetary financing is viewed as the residual financing mode of the government. Specifically, real per capita public spending, current and capital, are treated as exogenous as are real per capita lump sum taxes and the proportional tax rate on domestic value added. Real per capita stocks of public debt, internal and external, are also taken as given. Cuts in public spending will therefore affect the demand for seigniorage revenue by changing the (inflation-and-real-growth-corrected) public sector deficit that must be financed by printing money.[1] The paper emphasises the potentially important distinction, as regards their effect on the public sector deficit, of cuts in public sector consumption and cuts in public sector capital formation.

Real seigniorage revenue (the real value of new issues of nominal high-powered or base money) can be written as the product of a "seigniorage tax rate" (the proportional rate of change of the high powered nominal money stock, μ) and the "seigniorage tax base" (the real stock of money balances or the real stock of money balances per capita, m).

Across steady states there is a one-for-one association between the seigniorage tax rate and the rate of inflation; in classical flex-price models, such a relationship may exist also in the short run, say for unanticipated, immediate and permanent changes in μ. The seigniorage tax is indeed often referred to as the inflation tax and we will follow this rather sloppy usage.

Given the seigniorage tax base, there will be a unique seigniorage tax rate that generates ("supplies") the required seigniorage revenue. However, because the velocity of circulation of high-powered money is not independent of the (expected) inflation rate, the seigniorage tax base is likely to vary (negatively) with the seigniorage tax rate, at any rate in the long run. This leads to the possibility of a "seigniorage Laffer curve": the elasticity of the seigniorage tax base with respect to the seigniorage tax rate can become greater than unity in absolute value,[2] so higher growth rates of the nominal money stock (and sooner or later higher actual and expected inflation) will appropriate a smaller amount of real resources for the government.

If there is perfect international capital mobility and the long-run real interest rate is independent of the government's spending behaviour, a cut in public consumption spending reduces the long-run rate of inflation as long as, if there are multiple rates of inflation yielding the same real seigniorage revenue, the authorities always choose the lower rate of inflation, i.e. as long as the equilibrium is on the (globally) revenue efficient segment of the seigniorage Laffer curve.

When cuts in public sector capital formation are considered, the ***expenditure effect***, which ***cet.par.*** reduces the budget deficit one-for-one, lowers inflation (and is the only effect in the case of public consumption cuts) is supplemented by three further effects. The ***direct revenue effect*** represents any cash returns the government may receive as a direct consequence of its ownership of the public sector capital stock (including infrastructure). The ***indirect revenue effect*** measures the implications of a lower public sector capital stock for domestic production and thus for production or income-related taxes. The ***money demand effect*** is the effect of a lower public sector capital stock on money demand (at a given real interest rate and inflation rate) through its effect on the scale variable (some measure of income or wealth) in the money demand function. While the direct revenue effect is ambiguous, the indirect revenue effect and the money demand affect will mitigate and may even reverse completely the inflation reducing expenditure effect of cuts in public sector capital formation.

When there is limited international capital mobility (the paper considers the zero capital mobility case) the real interest rate becomes endogenous. The response of the real interest rate to public spending cuts when money is the residual financing instrument is ambiguous, however. It is quite possible that the real interest rate would rise, even in the long-run.

Note that the paper studies the effect of spending cuts on the inflation rate, not on the *level* of the path of money prices. The possibility of a negative association between the volume of public spending and the general price level has been discussed by Hall (1987) and will only be reviewed here in brief. Let M be the nominal stock of high-powered or base money, V the income velocity of circulation of high-powered money, P the general price level and Y real income. The identity of exchange states that

$$MV \equiv PY$$

Let M be given at a point in time; through policy choice in a closed economy; in an open economy through a freely floating exchange rate or, if the exchange rate is managed, through capital or exchange controls. Velocity is generally taken to be a non-decreasing function of the opportunity cost of holding outside money, which is the nominal interest rate, i, when the monetary base does not earn any interest.[3] In many macromodels (but not necessarily in the one considered in this paper), a public spending cut has a non-positive effect on the real interest rate, r. Consider the case where r actually falls. Holding constant inflationary expectations (an assumption that is not in general consistent with rational expectation formation), i falls and velocity falls with it. Nominal income therefore declines. If real income stays constant or rises, the price level falls unambiguously. Only if real income falls by more (proportionally) than the decline in velocity, will the price level rise.

A ***necessary*** condition for such an association is a positive effect on real output of an increase in public spending. In a Keynesian model with demand-constrained output and employment, real output of course does decline in response to a cut in public spending,[4] but the price level is either taken to be predetermined or assumed to decline with output along a public spending-invariant upward-sloping aggregate supply schedule.

There are a whole range of possible direct and indirect, short-run and long-run effects of public spending on equilibrium output. These include ***direct*** substitutability or complementary between current public spending and current private spending (see Buiter (1977)) or between current public spending and the supply of labour. There also can be short-run real interest rate effects on the supply of labour (through the intertemporal substitution mechanism emphasised by Hall (1987)) and on the demand for labour (through working capital channels as in Blinder (1987)). This paper does not consider these effects but does allow for a long-run effect of public spending on the private capital stock.

The organisation of the paper is as follows. Section 2 develops the model. In Section (2a) we consider the public sector and its "demand for seigniorage". Section (2b) models production and factor demand with special emphasis on the role of public sector capital. Private consumption and portfolio behaviour (including money demand) are developed in Section (2c). Market equilibrium and the external accounts are given in Section (2d). Section 3 contains a brief discussion of seigniorage and the exchange rate regime. In Section 4 we consider the long-run effects of public spending cuts on the rate of inflation while Section 5 contains a brief discussion of dynamic issues. Section 6 concludes.

2. The Model

2a. The Public Sector

Equation (1) gives the budget identity of the consolidated public sector, i.e. general government, state enterprises and central bank

$$\frac{\dot{M}}{P} - \frac{E}{P}\dot{R}^{*} + \frac{\dot{B}}{P} + \frac{E}{P}\dot{B}^{*} \equiv C^{G} + I^{G} - T - J + \frac{iB}{P} + i^{*}\frac{E}{P}(B^{*}-R^{*}) \qquad (1)$$

M is the nominal stock of high-powered money, base money or reserve money. It consists of coin and currency held by the public and reserves held by the commercial banking system. It bears a zero nominal interest rate. B is the stock of domestic-currency-denominated interest-bearing public debt. It has a fixed nominal market value and a variable nominal interest rate i and is only held domestically. B^{*} is foreign-currency-denominated public debt. It bears a nominal interest rate i^{*} and is only held abroad. R^{*} is the stock of official foreign exchange reserves, which are assumed to bear the same interest rate, i^{*}, as foreign currency denominated public debt. C^{G} is public consumption spending, I^{G} gross public sector capital formation, T taxes net of transfers and J government cash income from the public sector capital stock K^{G}. C^{G}, I^{G}, T, J and K^{G} are all measured in terms of domestic output. E is the nominal spot exchange rate, P the domestic GDP deflator and P^{*} the foreign GDP deflator. Ignoring depreciation, gross (and net) public sector capital formation I^{G} and the public sector capital stock K^{G} are related by

$$\Delta K^{G} \equiv I^{G}.$$

The government's *direct* gross return from the stock of public sector capital (including social overhead capital), J, is the product of the capital stock K^{G} and its gross cash rate of return ρ^{G}, i.e.

$$J \equiv \rho^G K^G$$

$F^{*G} \equiv R^* - B^*$ is the government's net stock of foreign assets. We define the following ratios to the labour force (or to the population) L.

$$m \equiv \frac{M}{PL};\ k^G \equiv \frac{K^G}{L};\ b \equiv \frac{B}{L};\ f^{*G} \equiv \frac{EF^{*G}}{PL};\ \bar{R}^* \equiv \frac{ER^*}{PL};\ b^{*G} \equiv \frac{EB^{*G}}{PL};$$

$$c^G \equiv \frac{C^G}{L};\ i^G \equiv \frac{I^G}{L};\ \tau \equiv \frac{T}{L}$$

Let $\pi \equiv \dot{P}/P$ be the domestic rate of GDP inflation, $\pi^* \equiv \dot{P}^*/P^*$ the foreign rate of GDP inflation, $\varepsilon \equiv \dot{E}/E$ the proportional rate of depreciation of the nominal exchange rate and $n \equiv \dot{L}/L$ the proportional growth rate of the labour force and population. $\Gamma \equiv EP^*/P$ is the real exchange rate (defined here as the ratio of the foreign GDP deflator times the nominal spot exchange rate to the domestic GDP deflator) or the reciprocal of the terms of trade. γ is the proportional rate of depreciation of the real exchange rate, i.e.

$$\gamma \equiv \varepsilon + \pi^* - \pi$$

For the purposes of this paper the distinction between domestically produced goods and foreign goods and the endogeneity of the *real* exchange rate are not important. Using Occam's razor, we therefore restrict the analysis that follows Section 2a to a one good world, i.e. $\gamma \equiv 0$.

The real interest rate on (domestically held) domestic-currency-denominated public debt r and the real interest rate on foreign-currency-denominated debt r^* are defined by

$$r \equiv i - \pi$$

$r^* \equiv i^*-\pi^*$

The government's budget identity (1) can now be rewritten as:

$$\dot{b}-\dot{f}^{*G}-\dot{k}^G \equiv c^G-\tau+(r^*-n)(b-f^{*G}-k^G)+(r-r^*)b-\gamma f^{*G}+[r^*-\rho^G]k^G-\sigma$$

The real value of high powered nominal money stock issues per worker or real seigniorage per capita, σ, can be written in three equivalent ways given in equations (2a,b,c). $\mu \equiv \dot{M}/M$ is the proportional rate of growth of the nominal high-powered money stock;

$$\sigma \equiv \frac{\dot{M}}{PL} \tag{2a}$$

$$\sigma \equiv \dot{m}+(n+\pi)m \tag{2b}$$

$$\sigma \equiv \mu m \tag{2c}$$

I shall refer to σ as "seigniorage" or the "inflation tax", although usage is not yet standardised here.[5]

Let $\tilde{d}$ denote the real per capita net non-monetary liabilities of the government.

$$\tilde{d} \equiv b-f^{*G}-k^G$$

The budget identity can be rewritten more completely as:

$$\dot{\tilde{d}} \equiv c^G-\tau+(r-n)\tilde{d}+\ell-\sigma \tag{3}$$

where

$$\ell \equiv (r-(r^*+\gamma))f^{*G}+(r-\rho^G)k^G \tag{4}$$

Equivalently, letting d denote the net real per capita *financial* non-monetary liabilities of the government, we have

$$d \equiv b-f^{*G}$$

and

$$\dot{d} \equiv c^G+i^G-\tau-\rho^G k^G+(r-n)d+(r-(r^*+\gamma))f^{*G}-\sigma \tag{5}$$

Total tax receipts are a linear function of GDP, i.e.

$$T = \theta Y+t_0, \qquad 0\leq\theta<1 \tag{6}$$

In what follows the proportional tax rate θ is interpreted as a value added tax.

The government's present value budget constraint, or solvency constraint is given in (7) or (7')

$$\int_t^\infty e^{-\int_t^s (r(u)-n)du}[\tau(s)-c^G(s)-\ell(s)]ds + \int_t^\infty e^{-\int_t^s (r(u)-n)du}\sigma(s)ds \geq \tilde{d}(t) \tag{7}$$

$$\int_t^\infty e^{-\int_t^s (r(u)-n)du}[\tau(s)+\rho^G(s)k^G(s)-(c^G(s)+i^G(s))-(r(s)-(r^*(s)+\gamma(s)))f^{*G}(s)]ds$$

$$+\int_t^\infty e^{-\int_t^s (r(u)-n)du}\sigma(s)ds \geq d(t) \tag{7'}$$

Equations (7) and (7') are derived from equations (3) and (5) respectively by imposing the "no Ponzi game" transversality conditions

$$\lim_{v\to\infty} e^{-\int_t^v (r(u)-n)du} \tilde{d}(v) \leq 0 \text{ and } \lim_{v\to\infty} e^{-\int_t^v (r(u)-n)du} d(v) \leq 0 \text{ respectively.}$$

2b. Production

Domestic output is produced by N identical competitive firms. The production function of the ith firm is given by

$$Y_i = F(K_i, \zeta(K^G,N), L_i) \tag{8}$$

Y_i, K_i and L_i are output, capital input and labour input of the ith firm respectively. K^G is the aggregate social overhead capital stock whose services are available free of charge to private firms. Y_i is increasing and linear homogeneous in K_i, ζ and L_i, strictly concave, twice continuously differentiable and satisfies the Inada conditions for K_i and L_i. ζ is increasing in K^G, non-increasing in N and twice continuously differentiable.

One simple special case of (8) is given in equation (9).

$$Y_i = F(K_i, K^G, L_i), \text{ i.e. } \zeta(K^G,N) \equiv K^G \tag{9}$$

In this version there is no "congestion effect" involved in the impact of the public sector stock on the output of firm i. The effect of K^G on the output of the ith firm is independent of N, the number of firms. Even with free entry, there will in general be positive pure profits $K^G F_{K^G} > 0$. In this case we can model aggregate output as if it were produced by a representative firm as in

$$Y = F(K, K^G, L). \tag{9'}$$

Letting $y \equiv Y/L$, $k \equiv K/L$ and $k^G \equiv K^G/L$ equation (9') implies, through constant returns, equation (10)

$$y = f(k,k^G) \tag{10}$$

Each individual firm takes K^G as given and optimally chooses K and L to maximise after tax profits given the real wage w and the real interest rate r. We therefore have the following first order conditions for competitive factor demands:

$$(1-\theta)f_1(k,k^G) = r \tag{11}$$

$$(1-\theta)[f(k,k^G)-kf_1(k,k^G)-k^G f_2(k,k^G)] = w \tag{12}$$

Equation (11) permits us to write the private capital-labour ratio as a function of the domestic real interest rate and the public sector capital-labour ratio:

$$k = k(\frac{r}{1-\theta}, k^G); \quad k_1 = \frac{1}{f_{11}}<0; \; k_2 = \frac{-f_{12}}{f_{11}}$$

All factors of production are assumed to be complements, i.e. $F_{LK}>0$, $F_{LK^G}>0$ and $F_{KK^G}>0$. The last of these implies $f_{12}>0$ and thus $k_2>0$.

The demand price for labour, w, can from (12) be written as:

$$w = (1-\theta)\omega(k,k^G); \; \omega_1 = -(kf_{11}+k^G f_{12})>0; \; \omega_2 = -(kf_{12}+k^G f_{22})>0 \qquad (14)^6$$

With the marginal product of public sector capital (f_2) positive and k^G positive, there will be positive pure profits ($k^G f_2$ per capita) in the economy if capital and labour are paid their marginal products. We assume that the ownership claims to this stream of pure profits are not traded internationally and that they are perfect substitutes in domestic private portfolios for domestic interest-bearing debt. The real per capita value of these equity claims, s, given in equation (15), is the present discounted value of future per capita pure profits (after tax), z.

$$s(t) = \int_t^{\infty} z(v)\, e^{-\int_t^v r(u)du}\, dv \tag{15}$$

After-tax pure profits per capita are given by

$$z = (1-\theta)y-rk-w \tag{16}$$

In the model under consideration this becomes:

$$z = (1-\theta)k^G f_2 \tag{16'}$$

Per capita taxes can be written as in equation (17) with $\tau_0 \equiv T_0/L$

$$\tau = \theta f\left[k(\tfrac{r}{1-\theta},k^G),k^G\right]+\tau_0 \tag{17}$$

The importance of the assumption that strong "congestion effects" do not affect the contribution of public capital to the output of an individual firm, becomes apparent when we consider the extreme alternative given in the following equation:

$$Y_i = F(K_i, \frac{K^G}{N} - \bar{k}^G, L_i), \quad \bar{k}^G>0; \text{ i.e. } \zeta(K^G,N) \equiv \frac{K^G}{N} - \bar{k}^G$$

This is a simple special case of a general formulation of congestion effects, according to which, for any K^G, there exists a unique finite value of N, $\bar{N}(K^G)$ say, such that $\zeta(K,\bar{N}(K^G))=0$. This critical value of N is assumed to increase with K^G. With free entry, pure profits will disappear ony if $N=\bar{N}$, i.e. in the special case under consideration if $N=K^G/\bar{k}^G$.

Applying the representative firm approach to this formulation we have, with free entry,

$$(1-\theta)\ f_1(k,0) = r$$

$$(1-\theta)(f(k,0)-kf_1(k,0)) = w$$

$$N = \frac{K^G}{\bar{k}^G}$$

Under this specification equilibrium output or output per worker is independent of the stock of public sector capital. A higher value of K^G simply induces more entry of new firms, competing away any pure profits. Each of the larger number of firms employs labour and private capital in the same proportions as before (at given w and r). Total output Y and the total private capital stock are independent of K^G. For reasons of space this alternative approach to social overhead capital in the aggregate production function will not be pursued here.[7]

2c. Private Consumption Behaviour

Aggregate private consumption behaviour is summarised in equations (18)-(23).

q denotes per capita "comprehensive" consumption, i.e. the consumption of the single commodity plus the imputed value of the "money services" consumed by domestic households. c is per capita consumption of the single commodity and m^d the per capita demand for real money balances. a denotes per capita financial wealth or non-human wealth, h the per capita stock of human capital. q, c, m^d, a and h are all measured in terms of domestic goods. f^{*P} is real per capita private ownership of net foreign assets, which are assumed to be denominated in terms of foreign currency.

$$q = \delta(a+h) \qquad \delta>0 \tag{18}$$

$$\dot{a} = (r-n)a+w-\tau_0-q \tag{19}$$

$$\dot{h} = rh+\tau_0-w \tag{20}$$

or

$$h(t) = \int_t^{\infty}[w(s)-\tau_0(s)]e^{-\int_t^s r(u)du} ds \tag{20'}$$

$$a \equiv m+b+k+s+f^*p \tag{21}$$

$$c = \eta(i)q \qquad 0<\eta<1;\ \eta' \gtrless 0 \tag{22}$$

$$m^d = \left[\frac{1-\eta(i)}{i}\right]q \qquad 1-\eta+i\eta'>0 \tag{23}$$

The model of consumer behaviour is based on Weil's (1985) reinterpretation of Blanchard's (1985) model. (See also Buiter (1987a)). Each individual consumer lives forever and has a time-additive utility functional with a constant pure rate of time preference, δ. Instantaneous utility is the logarithm of a homogeneous function of consumption of the one good and real money balances. Labour is supplied inelastically. This homogeneous function is increasing, strictly quasi-concave and twice continuously differentiable. It satisfies the Inada conditions. Internal capital markets are perfect. All domestic and internationally traded assets except money earn the same expected pecuniary rate of return. While each individual lives forever, there is a constant birth rate (and population growth rate) $n \geq 0$. Each individual earns the same wage and pays the same taxes. Human capital is therefore the same for all, regardless of age.

Per capita comprehensive consumption $q \equiv c+im$ is a constant multiple, δ, of the sum of non-human wealth, a, and human wealth, h, as shown in equation (18).[8] Equation (19) is the equation of motion for per capita non-human wealth and equation (20) that for human wealth. Given a standard transversality condition, (20) implies (20'): human capital of those alive today equals the present discounted value of the future after-tax wage income earned by those currently alive.

Note that aggregate consumption behaviour looks as though it represents the behaviour of a representative consumer who discounts his future human capital income at a higher rate, r, than the rate of return on his non-human capital, r-n. The excess of the human capital income discount rate over the rate of return on tangible assets is n, the birth rate. Absent operative intergenerational gift and bequest motives, private agents currently alive do not have as collateral (and do not act as if they had as collateral) the future after-tax labour income of those yet to be born, the "new entrants". A positive birth rate (n>0) therefore generates absence of ***debt neutrality*** : holding constant the path of current and future exhaustive public spending on goods and services, holding constant the path of current and future distortionary tax rates and holding constant the path of current and future nominal money issues, changes in the path of lump-sum taxes (which are consistent with the government satisfying its solvency constraint) will affect the real equilibrium of the eocnomy. With n=0, debt neutrality prevails.

Equation (21) defines private sector non-human wealth. Note that domestic money, domestic-currency-denominated-public debt, equity and the domestic capital stock are held entirely by the domestic private sector.

Aggregate consumption of the single good, c, depends positively on comprehensive consumption, q and ambiguously on the ***real*** price of holding money balances, the nominal interest rate, i (equation (22)). The demand for real money balances increases with q and decreases with i (equation (23)).

2d. Market equilibrium and the external accounts

When there is perfect international capital mobility and perfect substitutability between foreign currency denominated bonds and domestic-currency-denominated bonds, uncovered nominal interest parity prevails as in equation (24a)

$$i = i^{*}+\varepsilon \tag{24a}$$

Since this is a one good world and the law of one price holds, this means that uncovered real interest parity also holds, as in equation (24b)

$$r \equiv i-\pi = i^{*}-\pi^{*} \equiv r^{*} \tag{24b}$$

As the country is small it takes both i^{*} and π^{*} as given.

Let f^{*} denote the real per capita value of the nation's (private plus public sector's) stock of net foreign assets, i.e. $f^{*} \equiv f^{*P}+f^{*G}$. The current account of the balance of payments can be written as in equation (25) where i^{P} denotes real per capita private capital formation, i.e.

$$i^{P} \equiv \dot{k}+nk$$

$$\dot{f}^{*} \equiv y-c-c^{G}-i^{P}-i^{G}+(r^{*}-n)f^{*} \tag{25}$$

With zero international capital mobility, equations (24a,b) are inapplicable. Instead we have $\dot{f}^{*}\equiv 0$, i.e. absorption should equal national income. For simplicity we assume that in this case we also have a zero *stock* of net foreign assets, i.e. $f^{*}=0$. With zero international capital mobility therefore, we have

$$y = c+c^G+i^P+i^G \tag{26}$$

We shall apply the zero international capital mobility restriction not only to the nation as a whole, but to each of the two domestic sectors individually, i.e.

$$\dot{f}^{*G}=f^{*G}=\dot{f}^{*P}=f^{*P}=0$$

The real after-tax rates of return on capital and domestic government debt are equalised, so repeating equations (11) and (13) we have

$$(1-\theta)f_1(k,k^G) = r \tag{27a}$$

or

$$k = k\left[\frac{r}{1-\theta},k^G\right] \tag{27b}$$

Labour is supplied inelastically. The labour force grows at the exogenous constant proportional rate n and a flexible real wage ensures continuous full employment. Equations (12) and (14), reproduced below, can therefore be interpreted as labour market clearing conditions

$$(1-\theta)(f-kf_1-k^Gf_2) = w \tag{28a}$$

or

$$w = (1-\theta)\omega(k,k^G) \tag{28b}$$

The demand for real money balances equals the outstanding stock, i.e.

$$m^d = m \equiv \frac{M}{PL} \tag{29}$$

3. Seigniorage and the exchange rate regime

For reasons of space little attention is paid in this paper to the wide range of possible (and actually existing) exchange rate regimes. Only a freely floating exchange rate and a crawling peg with a constant proportional rate of depreciation or appreciation are considered explicitly. Since we are focussing on the long-run inflation consequences of public expenditure cuts, any exchange rate regime that is considered should be viable in the long run. Viability has several dimensions. We consider two : *long run seigniorage consistency* and international reserve sufficiency.

3a. Long run seigniorage consistency

In steady-state equilibrium all real per capita assets and liabilities of the government are constant (i.e. $\dot{b}=\dot{k}^G=\dot{f}^G=0$ and $i^G=nk^G$) and the real interest rate r is constant. c^G, τ_0, θ, ρ^G, n and $r*$ are parameters. In the one good world of this paper, the terms of trade Γ are of course identically equal to unity and $\gamma=0$. What follows holds, however, also for models with endogenous terms of trade or real exchange rate, since in steady state no real appreciation or depreciation occurs.

Real per capita steady state seigniorage is therefore given by[9]

$$\sigma \equiv (n+\pi)m \equiv c^G-\tau_0-\theta f(k,k^G)+(r-r*)f^{*G}+(n-\rho^G)k^G+(r-n)(b-f^{*G}) \qquad (30)$$

Through the money demand function, we can determine which rate(s) of inflation (there may be more than one) can generate the required steady state real seigniorage given in equation (30). The money demand function given in equation (23) together with equations (18), (19), (20), (27b), (28b) and (29) imply

$$m = \left[\frac{1-\eta(r+\pi)}{r+\pi}\right]\delta\frac{n}{(n+\delta-r)r}\left[(1-\theta)\omega(k(\frac{r}{1-\theta},k^G),k^G)-\tau_0\right] \qquad (31)$$

It should be noted that throughout we assume that $n>0$, $n+\delta>r$ and $w-\tau_0>0$.[10]

Holding constant the other arguments of (30) and (31) we can trace the *cet.par.* relationship between long-run real per capita seigniorage or the long-run real per capita stock of high-powered money and the (actual and expected) rate of inflation.

For a number of commonly used money demand functions including the linear one in (32a) and the log-linear one in (32b) the long-run relationship between the inflation rate (or the monetary growth rate) and real seigniorage has the "Laffer curve" shape shown in Figure 1.

$$m = \alpha-\beta\pi \qquad \alpha,\beta>0;\ \pi<\alpha\beta^{-1} \tag{32a}$$

$$\ln m = \alpha'-\beta'\pi \qquad \beta'>0 \tag{32b}$$

For the linear demand function shown in Figures 1A and 1B and for the log-linear demand function shown in Figures 1C and 1D (both drawn with $n=0$) there is a unique finite long-run seigniorage maximizing rate of inflation ($\hat{\pi}=(\alpha/(2\beta))-(n/2)$ in the linear case, $\hat{\pi}=(1/\beta)-n$ in the log-linear case). Such a unimodal long-run seigniorage Laffer curve would, of course, not exist if the money demand function were the rectangular hyperbola given in (32c)

$$m = \frac{\alpha''}{\beta''+\pi} \qquad \alpha''>0;\ \beta''>0;\ \pi>-\beta'' \tag{32c}$$

This money demand function, which would correspond to equation (31) in the Cobb-Douglas case ($\eta'=0$) and with $r>0$, has seigniorage increasing (decreasing) forever with π if $\beta''>n$ (if $\beta''<n$).

For the government's fiscal-financial-monetary strategy to be feasible if there is a seigniorage Laffer curve, the maximal amount of steady-state seigniorage $\hat{\sigma}$ should obviously be no less than the value required to satisfy the steady-state budget constraint (30). If there is a unimodal long-run seigniorage Laffer curve (and there are consequently two steady state rates of inflation that generate any given feasible amount of long-run real seigniorage), it seems reasonable to assume that the lower of these two inflation rates is actually *chosen*, by the policy makers (and that we are therefore always on the revenue-efficient side of the long-run seigniorage Laffer curve) if there is a managed exchange rate regime. With somewhat less confidence the lower inflation rate may be expected to emerge endogenously under a floating exchange rate regime. No rational policy making process would ever drive the steady state inflation rate beyond the seigniorage maximising value, as in addition to the costs of higher inflation, there would also be a reduction in inflation tax revenues. In what follows we assume that if there is a long-run seigniorage Laffer curve, it is unimodal.

The minimal long-run rate of inflation implied by the government's fiscal-financial-monetary strategy, $\bar{\pi}$, must be consistent with the long-run exchange rate rule. In our model, in which the law of one price holds, $\pi \equiv \varepsilon + \pi^*$ even outside the steady state. However, even if the real exchange rate is endogenous, the steady state rate of depreciation of the nominal exchange rate ε must equal the excess of the minimal long-run domestic rate of inflation required to generate the required steady state seigniorage, $\bar{\pi}$, over the exogenous world rate of inflation, π^* (assumed to be constant for simplicity), i.e.

$$\varepsilon = \bar{\pi} - \pi^* \qquad (33)$$

As our formal model is non-stochastic and strategic behaviour is not considered, it does not matter whether we view (33) as the steady state of an ***exchange rate management rule*** (i.e. a rule for E or its rate of change with the stock of international reserves and the money stock adjusting endogenously) or as the steady state of a ***reserve stock or money stock management rule*** (i.e. a rule for M or R* with the exchange rate adjusting endogenously).

In most of what follows it is convenient to interpret exchange rate behaviour as if it were generated by a crawling peg exchange rate management rule which generates the real seigniorage required to satisfy the government budget identity at each instant, given exogenously determined values of c^G, i^G, τ_0, θ, b and f^{*G} and without any "maxi" devaluations or revaluations, i.e. without any discontinuous changes in the ***level*** of the nominal exchange rate. The (managed) proportional rate of change of the nominal exchange rate, ϵ will in general vary over time in response to changes in c^G or i^G. Such an exchange rate management rule mimics a freely floating exchange rate except for the absence (under the exchange rate management rule) of discontinuous changes in E even at those instants when "news" about the fundamentals reaches the private sector. With a predetermined nominal exchange rate, any stock-shift changes in the demand for real money balances (say in response to a change in the nominal interest rate) will be reflected in changes in the stock of international reserves. As regards the behaviour of ***real*** money balances (per capita), m, this regime is identical to a freely floating exchange rate regime. Under the latter regime, discontinuous changes in the nominal exchange rate in response to news replace discontinuous changes in official international reserves. The exchange rate management interpretation may make the selection of seigniorage revenue efficient solutions more plausible.

3b. International reserve sufficiency

Current conventional wisdom on the collapse of managed exchange rate regimes (see eg Krugman (1979), Flood and Garber (1984) and Obstfeld (1986)) implies that a collapse can occur even if there is long-run seigniorage consistency as just defined. Even if the government achieves an inflation rate and nominal exchange rate depreciation rate that guarantee long-run seigniorage consistency, (and even if the government satisfies its intertemporal solvency constraint at each instant, in or out of steady state), international foreign exchange reserves could still be declining: nominal domestic credit expansion could exceed the change in the nominal money stock demanded in equilibrium. This can occur even when c^G, τ, k^G, ρ^G, b and f^{*G} are constant : the loss of official international reserves would be balanced in that case by a reduction in government borrowing abroad : $\dot{R}^* - \dot{B}^* = (n+\pi-\epsilon)(R^*-B^*)$ is consistent with any value of $\dot{R}^*$ (and thus, ultimately, of R*).[11]

In the model under consideration, international reserves and foreign borrowing carry the same interest rate. The government's solvency constraint is therefore unaffected by equal offsetting changes in R* and in B*. Note that if reserves have a lower interest rate than government debt (as would be the case under a pure gold standard with a positive nominal interest rate (see Buiter 1987)), then financing a given deficit by running down reserves rather than by borrowing abroad would gradually strengthen government solvency. Similarly, a stock-shift open market sale to replenish the stock of reserves would, because the authorities incur a high interest rate liability in exchange for a low interest rate asset, weaken government solvency immediately.

As shown in Buiter (1986), when the interest rates on reserves and on government debt are the same and when a government is solvent for any rate of domestic credit expansion (however low), then the government is solvent for any other rate of domestic credit expansion (however high). There can be no "international reserve problem" as long as there is no solvency problem.

The literature just referred to "creates" a reserve problem by postulating some ad-hoc lower bound on the stock of reserves (or in the context of our model on the real per capita stock of reserves) and by denying the government unlimited lines of credit even when solvency is guaranteed. If the actual stock of reserves falls below the critical threshold level, the current managed exchange rate regime collapses and something else takes its place. This "something else" can be a "free float" (possibly followed by an eventual return to another managed rate or rule), a maxi-devaluation, the imposition of capital or exchange controls or any other conceivable scheme.

I shall assume that in steady state, domestic credit expansion is such that any reserve threshold there may be is not breached.

Outside the steady state, the initial stock of reserves is assumed to be sufficient to keep reserves above the exogenously imposed reserve threshold (if there is one), despite any stock-shift or gradual reserve losses incurred during the adjustment process. The focus is therefore firmly on solvency rather than on (badly understood) international liquidity problems that may occur despite solvency.[12]

4. Long run effects on inflation of public spending cuts

Putting together equations (30) and (31) we obtain equation (34).

$$(n+\pi)\left[\frac{1-\eta(r+\pi)}{r+\pi}\right]\frac{\delta n}{(n+\delta-r)r}\left[(1-\theta)\omega(k(\frac{r}{1-\theta},k^G),k^G)-\tau_0\right]$$

$$= c^G-\tau_0-\theta f(k(\frac{r}{1-\theta},k^G),k^G)+(r-r*)f^{*G}+(n-\rho^G)k^G+(r-n)(b-f^{*G}) \qquad (34)$$

Remember that $n,\delta,\theta,\tau_0,c^G,r*$ and ρ^G are taken as exogenous. The government financing rule (or an (ad hoc) internal debt ceiling) keeps b constant in and out of steady state. The government financing rule (or an (ad hoc) external debt ceiling) keeps f^{*G} constant in and out of steady state. Since $\dot{k}^G \equiv i^G-nk^G$, the steady state public sector capital stock is simply given by : $k^G=i^G/n$.

The cases of perfect capital mobility and zero capital mobility are considered in turn :

Perfect international capital mobility

With perfect international capital mobility ($r=r*$), the long-run effects on inflation of changes in public consumption spending, public sector capital formation and lump-sum taxes are given by equations (35a,b,c). Note that because the real interest rate is constant the nominal interest rate varies one-for-one with the rate of inflation.

$$\frac{d\pi}{dc^G} = a_{11}^{-1} \qquad (35a)$$

$$\frac{d\pi}{di^G} = a_{11}^{-1}b_{12} \qquad (35b)$$

$$\frac{d\pi}{d\tau_0} = a_{11}^{-1}b_{13} \qquad (35c)$$

where

$$a_{11} = m\left[1-\frac{(n+\pi)}{i(1-\eta)}(1-\eta+i\eta')\right] \tag{36a}$$

and

$$b_{12} = 1 - \frac{\rho^G}{n} - \frac{\theta}{n}\left[-f_1\frac{f_{12}}{f_{11}}+f_2\right] + \frac{(n+\pi)(1-\eta)\delta(1-\theta)}{i(n+\delta-r)r}k^G\left[\frac{f_{11}f_{22}-f_{12}^2}{f_{11}}\right] \tag{36b}$$

$$b_{13} = \frac{(n+\pi)(1-\eta)\delta n}{i(n+\delta-r)r} - 1 = \frac{\sigma}{w-\tau_0} -1 \tag{36c}$$

The coefficient a_{11} is positive if we are on the revenue-efficient side of the seigniorage Laffer curve, where the elasticity of money demand with respect to the inflation rate is less than 1 in absolute value.[13] A cut in public consumption spending reduces the amount of real seigniorage that needs to be extracted. In the revenue-efficient region of the Laffer curve, lower real seigniorage means lower inflation (see equations (35a) and (36a)). Note that since money is the "residual" mode of financing, lower public consumption spending does not imply lower explicit taxes.

The effect of higher lump-sum taxes on long-run inflation is actually ambiguous in this model. As shown in equations (35c) and (36c), while higher lump sum taxes reduce the amount of long-run seigniorage required to satisfy the government budget constraint, it also reduces the demand for money (at given nominal and real interest rates), which is an increasing function ***disposable*** labour income. If in the initial steady state seigniorage is less than disposable labour income (which is likely empirically), higher lump-sum taxes will lead to lower long-run inflation (in the revenue-efficient region of the long-run seigniorage Laffer curve).

Lower public sector capital formation may (but need not) lead to higher long-run inflation (see equations (35b) and (35c)). *Cet.par.* it reduces the deficit one-for-one, thus reducing long-run required seigniorage and (in the seigniorage-revenue-efficient region) long-run inflation. This we shall call the ***expenditure effect***, given by the first term on the RHS of (36b). However, there may be direct cash returns to the government, associated with its ownership of the public sector capital stock. If the (net) rate of return ρ^G exceeds the real growth rate n (or equivalently if $\rho^G k^G$ exceeds i^G) the direct effect of lower public sector capital formation on long-run seigniorage and (in the seigniorage-revenue-efficient region) on inflation will be positive. Empirically, it is of course quite possible for ρ^G to be negative. The permanent subsidisation (for good or bad reasons) of secular public sector loss makers would be a case in point. This ***direct revenue effect*** is given by the second term $-\rho^G/n$ on the RHS of equation (36b).

There is a further possible effect of a lower long-run stock of public sector capital on the deficit, seigniorage and inflation. Lower k^G means lower national income, both given k and by inducing a reduction in k (assuming $f_{12}>0$). Lower national income means lower income-related tax receipts if θ is positive. This ***indirect revenue effect*** is captured in the third term on the RHS of equation (36b).

Finally, there is an effect on the demand for money, captured by the last term on the RHS of equation (36b). A lower value of k^G means a lower real wage (since $f_{11}f_{22}-f_{12}^2>0$ by concavity). *Cet.par.* a lower real wage means a lower demand for real money balances; in the revenue-efficient segment of the seigniorage Laffer curve this implies higher inflation. This will be referred to as the ***money demand effect***.

In the seigniorage-revenue-efficient region, the expenditure effect of a cut in public sector capital formation reduces inflation. The direct revenue effect is ambiguous even as regards its sign. The indirect revenue effect will worsen inflation as long as the marginal product of public sector capital is positive ($f_2>0$) or public and private capital are complements ($f_{12}>0$). θ is likely to be quite small in many developing countries, however. The money demand effect also raises inflation. Whatever the scale measure, income or wealth term in the money demand function, it is likely to be adversely affected, *cet.par.* by a lower social overhead capital stock as long as the marginal product of public sector capital is positive.

Zero international capital mobility

When there is international capital mobility, domestic income equals private plus public absorption, i.e. equation (26) holds.

Substituting for output using the production function, for private consumption using equations (18), (19), (20) and (22) and for private investment using the steady-state condition $i^P=nk$, we obtain :

$$f\left[k\left[\frac{r}{1-\theta},\frac{i^G}{n}\right],\frac{i^G}{n}\right] = \eta(r+\pi)\delta\frac{n}{(n+\delta-r)r}\left[(1-\theta)\omega\left[k\left[\frac{r}{1-\theta},\frac{i^G}{n}\right],\frac{i^G}{n}\right]-\tau_0\right]$$

$$+c^G+nk\left[\frac{r}{1-\theta},\frac{i^G}{n}\right]+i^G \tag{37}$$

Equations (34) (with $k^G=i^Gn^{-1}$) and (37) can be used to solve for the long-run equilibrium values of the rate of inflation and the now endogenous real interest rate.

The long-run comparative statics of changes in c^G, i^G and τ_0 can be obtained from equations (38a-k).

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} d\pi \\ dr \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \begin{bmatrix} dc^G \\ di^G \\ d\tau_0 \end{bmatrix} \tag{38a}$$

$$a_{11} = m\left\{1-\frac{(n+\pi)}{i(1-\eta)}(1-\eta+i\eta')\right\} \tag{38b}$$

$$a_{12} = \frac{(n+\pi)m}{i(1-\eta)}(1-\eta+i\eta')+\frac{(n+\pi)m(n+\delta-2r)}{(n+\delta-r)r}+\frac{(n+\pi)}{i}\ \frac{(1-\eta)\delta n}{(n+\delta-r)r}\left[k+k^G\frac{f_{12}}{f_{11}}\right]$$

$$-\left[\frac{\theta}{1-\theta}\right]\frac{f_1}{f_{11}}+b \tag{38c}$$

$$a_{21} = \frac{\eta' c}{\eta} \tag{38d}$$

$$a_{22} = \frac{\eta' c}{\eta} - \frac{c(n+\delta-2r)}{(n+\delta-r)r} - \frac{\eta\delta n}{(n+\delta-r)r}\left[k+k^G\frac{f_{12}}{f_{11}}\right]+\frac{n-f_1}{f_{11}(1-\theta)} \tag{38e}$$

$$b_{11} = 1 \tag{38f}$$

$$b_{12} = 1-\frac{\rho^G}{n} - \frac{\theta}{n}\left[\frac{f_2 f_{11}-f_1 f_{12}}{f_{11}}\right]+\frac{(n+\pi)(1-\eta)\delta(1-\theta)k^G}{i(n+\delta-r)r}\ \frac{(f_{11}f_{22}-f_{12}^2)}{f_{11}} \tag{38g}$$

$$b_{13} = \frac{\sigma}{w-\tau_0} - 1 \tag{38h}$$

$$b_{21} = -1 \tag{38i}$$

$$b_{22} = \frac{1}{n}\left[f_2-\frac{f_1 f_{12}}{f_{11}}\right]-\frac{(1-\theta)\eta\delta k^G}{(n-\delta-r)r}\left[\frac{f_{12}^2-f_{11}f_{22}}{f_{11}}\right]+\frac{f_{12}}{f_{11}} - 1 \tag{38j}$$

$$b_{23} = \frac{\eta\delta n}{(n+\delta-r)r} \tag{38k}$$

Equation (34) defines the long-run equilibrium seigniorage schedule AA_0 or AA_1 in Figure 2. The long-run income-absorption equilibrium schedule corresponding to equation (37) is drawn as BB in Figure 2. The slope of the AA schedule in π-r space is given by

$$\frac{d\pi}{dr}\bigg|_{AA} = \frac{-a_{12}}{a_{11}}.$$

The coefficient a_{12} given in (38c) measures the effect of a higher real interest rate (*cet.par.*) on the "demand for seigniorage", i.e. on the inflation-and-growth-corrected government deficit on the RHS of equation (34), relative to its effect on the "supply of seigniorage", $(n+\pi)m$, on the LHS of equation (34). A higher value of r raises the deficit both by raising the interest to be paid on the debt and by generating a lower private capital stock, output and income-related tax receipts. These two effects are captured by the terms b and $-(\theta/(1-\theta))\ f_1/f_{11}$ respectively in equation (38c). *Cet.par.* a higher real interest rate also represents a higher nominal interest rate. This nominal interest rate effect will tend to reduce m, the seigniorage tax base, since $1-\eta+i\eta'>0$. A higher real interest rate will, however, also raise the long-run ratio of non-human wealth to disposable wage income. It will also, provided $r>(n+\delta)/2$, raise the long-run ratio of comprehensive consumption q to disposable wage income and thus raise m.[14]

Finally, a higher real interest rate will tend to be associated (through a lower value of k) with a lower real wage. This effect, measured by the third term on the RHS of (38c) will tend to reduce m. We will only consider the a priori more plausible case in which a_{12} is positive, i.e. a higher value of r raises the demand for seigniorage relative to its supply. (Note that we also assume throughout that the nominal interest rate is positive). The coefficient a_{11}, already discussed, will be positive if the inflation elasticity of money demand is less than unity in absolute value, negative otherwise. Without a seigniorage Laffer curve, the AA_0 curve is downward-sloping as shown in Figure 2. The AA_1 schedule represents the unimodal long-run seigniorage Laffer curve generated by an inflation elasticity of money demand which is below unity in absolute value for low values of π and rises monotonically with π, crossing unity for some finite value of π.

The slope of the BB schedule in π-r space is given by

$$\left.\frac{d\pi}{dr}\right|_{BB} = \frac{-a_{22}}{a_{21}}$$

Homotheticity is consistent with any sign for η', as long as $1-\eta+i\eta'>0$. In what follows it's assumed that $\eta'\geq 0$, i.e. given total comprehensive consumption of goods and money services, an increase in the real price of money services does not reduce the demand for goods. This implies $a_{21}\geq 0$.

The coefficient a_{22} will be positive if *cet.par.* a higher real interest rate raises absorption relative to output. Assuming that the marginal product of private capital exceeds the real growth rate ($f_1>n$), the last term on the RHS of (38e) will be positive. $\eta'\geq 0$ makes the first-term on the RHS of (38e) non-negative. A higher value of r also tends to raise the ratio of comprehensive consumption q to disposable wage income (if $n+\delta<2r$), so the second term on the RHS of (38e) is positive. Human wealth is lower, however, when r is higher and this will make the third term on the RHS of (38e) negative. Figure 2 shows both the case where a_{22} is positive (the downward-sloping curve B_0B_0) and the case where a_{22} is negative (the upward-sloping curve B_1B_1). For reasons of space we only consider the case where in the low-inflation (seigniorage-revenue-efficient) equilibrium at E_0, the B_0B_0 curve cuts the AA_0 (or AA_1) curve from above.[15]

A cut in public consumption spending shifts the long-run seigniorage equilibrium schedule A_0A_0 to the left in Figure 3a and 3b. (We consider the case of the unimodal long-run seigniorage Laffer curve). If a higher real interest rate raises long-run absorption relative to long-run output ($a_{22}>0$), the long-run output-absorption equilibrium schedule is negatively sloped. It shifts to the right when c^G is cut as in Figure 3a. In the seigniorage-efficient region, the equilibrium moves from E_0 to E_1 : inflation falls and the real interest rate rises. Note that an increase in the real interest rate is required in the long run because, with $a_{22}>0$, a higher real interest rate is required to boost absorption and reduce output, thus restoring output-absorption balance following the cut in public consumption spending. In the seigniorage-inefficient region the stationary equilibrium in Figure 3a moves from $\tilde{E}_0$ to $\tilde{E}_1$. The inflation rate rises while the effect on the real interest rate is ambiguous. If given q, a rise in the nominal interest rate has only a small effect on the consumption of goods, i.e. if $\eta'c/\eta$ is small, the output-absorption equilibrium schedule will be near vertical and r will increase.

Figure 3b shows the case where $a_{22}<0$ and a higher value of r reduces absorption relative to output. In the seigniorage-revenue-efficient region the equilibrium moves from E_0 to E_1 or E_1'. The real interest rate falls and the inflation rate either falls (at E_1) or rises (at E_1'). In the seigniorage-revenue-inefficient region the stationary equilibrium moves from $\tilde{E}_0$ to $\tilde{E}_1$ or $\tilde{E}_1'$.

An increase in lump-sum taxes will shift the long-run seigniorage equilibrium schedule to the left if $b_{13}=[\sigma/(w-\tau_0)]-1<0$ as seems plausible empirically. This is shown both in Figure 4a and in Figure 4b. If $a_{22}>0$ (a higher real interest rate raises long-run absorption relative to long-run real output) the absorption output equilibrium schedule shifts to the right as shown in Figure 4a. If $a_{22}<0$, the absorption-output equilibrium schedule shifts to the left as in Figure 4b. In Figure 4a the seigniorage-revenue-efficient equilibrium shows a higher real interest rate and a lower rate of inflation.

A lower rate of public sector capital formation, unlike a cut in public sector consumption spending, need not shift the long-run equilibrium seigniorage schedule to the left. Figures 5a and 5b show the case where the expenditure effect is dominated by the direct and indirect revenue effects and the money demand effect i.e. $b_{12}<0$ and the long-run equilibrium seigniorage schedule shifts to the right.

The direction of the shift of the long-run output-absorption equilibrium schedule as i^G decreases is ambiguous. There is the direct expenditure effect on absorption (the term -1 in (38j)) which is reinforced by the reduction in long-run private consumption due to the lower real wage associated with the lower stock of public sector capital (the term

$$\frac{-(1-\theta)\eta\delta k^G}{(n+\delta-r)r}\left[\frac{f_{12}^2-f_{11}f_{22}}{f_{11}}\right]$$

in (38j)). Finally, there is the depressing effect on absorption due to the lower long-run rate of private capital formation, reflecting the lower value of k that accompanies a lower value of k^G (the term f_{12}/f_{11} in (38j)). Absorption therefore declines unambiguously. Output, however, also falls, both through the direct effect of a lower value of k^G and through the depressing effect on k of a lower value of k^G (the term $1/n(f_2-f_1f_{12}/f_{11})$ in (38j)).

In Figure 5a we consider the case where a_{22} is positive (a higher real interest rate raises long-run absorption relative to long-run output) and b_{22} is positive, (i.e. a lower value of i^G reduces output by more than absorption). The output-absorption equilibrium schedule shifts to the left in this case, moving the seigniorage-revenue-efficient equilibrium from E_0 to E_1. The long-run responses of both the inflation rate and the real interest rate in the seigniorage-revenue-efficient region are unambiguous: a cut in public sector capital formation leads to a higher rate of inflation and a lower real interest rate. The higher rate of inflation is needed to extract the larger real seigniorage that is required and the lower real interest rate raises output relative to absorption.

In Figure 5b we continue to assume that a_{22} is positive, but b_{22} is now assumed to be negative: a cut in public sector capital formation lowers absorption more than output. The output-absorption equilibrium schedule moves to the right in this case. In the seigniorage-revenue-efficient region the response of the inflation rate and the real interest rate are ambiguous. It is possible for the inflation rate to rise and the real interest rate to fall (the case of a very small shift to the right of the output-absorption equilibrium schedule, not shown in Figure 5b). The inflation rate can rise while the real interest rate rises (from E_0 to E_1 in Figure 5b) or the inflation rate can fall while the real interest rate rises (from E_0 to E_2 in Figure 5b).

With $a_{22}<0$ we obtain in the seigniorage-revenue-efficient region an unambiguously positive effect of a cut in public sector capital formation on the rate of inflation when $b_{22}<0$, i.e. when a cut in public sector capital formation reduces absorption by more than output. The real interest rate could rise if η' is large and the output-absorption equilibrium schedule shifts to the left only slightly; a lower real interest rate is more likely, however. It is easily checked that with $a_{22}<0$ and $b_{22}>0$, a cut in public sector capital formation lowers the real interest rate and has an ambiguous effect on the rate of inflation in the seigniorage-revenue-efficient region.[16]

The two cases where a cut in public sector capital formation unambiguously raises the long-run rate of inflation are depicted in Figures 5a and 5c.. In each case the seigniorage-revenue-efficient equilibrium is considered, i.e. $a_{11}>0$. In addition a cut in public sector capital formation is assumed to raise (at given values of r and π) the "demand for seigniorage" relative to the "supply of seigniorage", i.e. $b_{12}<0$. Finally, we have one of the two following output-absorption equilibrium configurations: either the case of Figure 5a in which a higher real interest rate raises absorption relative to output ($a_{22}>0$) and a cut in public sector capital formation reduces absorption by less than output ($b_{22}>0$), or the case of Figure 5c in which a higher real interest rate raises output relative to absorption ($a_{22}<0$) and a cut in public sector capital formation reduces absorption by more than output ($b_{22}<0$). Note that these are sufficient conditions only. As Figure 5b shows, they are not necessary.

5. Stabllity and dynamic adjustment

Perfect capital mobility

Under perfect capital mobility, the dynamic adjustment of the economy is given by equations (39)-(44).

$$\dot{q} = (r^*-\delta)q-\delta na \tag{39}$$

$$\dot{a} = (r^*-n)a+(1-\theta)\omega(k(\frac{r^*}{1-\theta},k^G),k^G)-\tau_0-q \tag{40}$$

$$\dot{k}^G = i^G-nk^G \tag{41}$$

$$\dot{m} = c^G+i^G-\tau_0-\theta f(k(\frac{r^*}{1-\theta},k^G),k^G)-\rho^G k^G+(r^*-n)d-(n+\pi)m \tag{42}$$

$$m = \left[\frac{1-\eta(r^*+\pi)}{r^*+\pi}\right]q \tag{43}$$

$$\pi = \varepsilon+\pi^* \tag{44}$$

Exogenous are r^*, ρ^G, τ_0, θ, i^G, c^G, n and δ.

Note that with the real interest rate fixed exogenously, q, a and k^G are determined independently of m, π and ε.

The subsystem represented by equations (39), (40) and (41) governing q, a and k^G is (locally) a saddlepoint with one unstable and two stable roots (governing the non-predetermined state variable q and the predetermined state variables a and k^G [17] provided (45) holds

$$r^*(r^*-(n+\delta))<0 \tag{45}$$

Since we assume that the real interest rate is positive, this is the viability condition $r^*<n+\delta$ required for positive steady-state consumption.[18]

From equations (42), (43) and (39) we can, solving out m and $\dot{m}$, obtain $\dot{\pi}$ as a function of π, q, a, k^G and the exogenous variables. The coefficient of $\dot{\pi}$ on π in the linear approximation to this $\dot{\pi}$ equation is also the fourth characteristic root of the linearised $\{q, a, k^G, \pi\}$ system (the root "governing" π). This coefficient, Ω_1 say, is given by

$$\Omega_1 = \frac{(1-\eta)i}{1-\eta+i\eta'} - (n+\pi) \tag{46}$$

It is easily checked that, provided $n+\pi>0$, Ω_1 is positive if and only if a_{11} is positive. The coefficient a_{11} given in (38b) is positive at a seigniorage-revenue-efficient stationary equilibrium, negative otherwise. Around a seigniorage-revenue-efficient stationary equilibrium the dynamic system governing q, a, k^G and π will have two stable and two unstable characteristic roots. There is a unique continuously convergent solution trajectory and, after any shock, a unique set of initial values of q and π that are consistent with convergence to the new seigniorage-revenue-efficient long-run equilibrium. Around a seigniorage-revenue-inefficient long-run equilibrium, Ω_1 is negative and there is a local continuum of initial values for π (but not for q) that are consistent with convergence to the seigniorage-revenue-inefficient long-run equilibrium. Note that this analysis is applicable both to the freely floating exchange rate case and to the managed exchange rate case outlined in Section 3a.

<u>Zero capital mobility</u>

With zero international capital mobility, the behaviour of q and a is only independent of the behaviour of the inflation rate if aggregate consumption of goods, $c=\eta(i)q$ is independent of the nominal interest rate ($\eta'=0$). Unless this is the case, goods market equilibrium and thereby capital formation will depend on π. With k dependent on π, a and q also can only by determined simultaneously with π. The relevant dynamic system is given in equations (47)-(52).

$$\dot{q} = ((1-\theta)f_1(k,k^G)-\delta)q-\delta na \tag{47}$$

$$\dot{a} = ((1-\theta)f_1(k,k^G)-n)a+(1-\theta)\omega(k,k^G)-\tau_0-q \tag{48}$$

$$\dot{k}^G = i^G-nk^G \tag{49}$$

$$\dot{k} = f(k,k^G)-nk-\eta((1-\theta)f_1(k,k^G)+\pi)q-c^G-i^G \tag{50}$$

$$\dot{m} = c^G+i^G-\tau_0-\theta f(k,k^G)-\rho^G k^G+((1-\theta)f_1(k,k^G)-n)b-(n+\pi)m \tag{51}$$

$$m = \left[\frac{1-\eta((1-\theta)f_1(k,k^G)+\pi)}{(1-\theta)f_1(k,k^G)+\pi}\right]q \tag{52}$$

As the analytical characterisation of the unrestricted five-dimensional dynamic system given in equations (47)-(52) is too messy and complicated, I restrict the analysis to the case where $\eta'=0$ and the classical dichotomy holds.

Note that when $\eta'=0$, real per capita seigniorage σ increases with the inflation rate if $r>n$ (decreases if $r<n$). We only consider dynamically efficient equilibria with $r>n$. This means that the long-run equilibrium seigniorage schedule is always downward-sloping in π-r space: there is no seigniorage-revenue-inefficient segment. The output-absorption equilibrium schedule is vertical when $\eta'=0$. Changes in c^G, i^G and τ_0 will however shift this vertical schedule as before. In the linearised version of the dynamic model, the characteristic root governing π (or m) is r-n.[19] With r>n this root is therefore unstable, as desired for a locally unique solution.

The linear approximation to the subsystem governing q, a, k and k^G is given in equation (53).

$$\begin{bmatrix} \dot{q} \\ \dot{a} \\ \dot{k} \\ \dot{k}^G \end{bmatrix} = \begin{bmatrix} r-\delta & -\delta n & q(1-\theta)f_{11} & q(1-\theta)f_{12} \\ -1 & r-n & -(1-\theta)((k-a)f_{11}+k^G f_{12}) & -(1-\theta)((k-a)f_{12}+k^G f_{22}) \\ -\eta & 0 & f_1-n & f_2 \\ 0 & 0 & 0 & -n \end{bmatrix} \begin{bmatrix} q \\ a \\ k \\ k^G \end{bmatrix} \quad (53)$$

The root governing k^G is again $-n$. For there to be a saddlepoint configuration that matches the structure of one non-predetermined state variable (q) and three predetermined ones (a, k and k^G), two of the other three roots should be stable and one unstable. The determinant of the three-by-three submatrix obtained by deleting the last row and column of the coefficient matrix in equation (53) should therefore be positive, i.e.

$$\frac{(n-f_1)}{f_{11}(1-\theta)}(n+\delta-r)r-\eta\delta n\left[k+k^G\frac{f_{12}}{f_{11}}\right]-c(n+\delta-2r) < 0 \quad (54)$$

Provided $n+\delta>r$ (which is required for positive steady state consumption), equation (54) is equivalent to the condition $a_{22}<0$ (see (38e) with $\eta'=0$). When the output-absorption equilibrium schedule is vertical therefore, the situation depicted in Figure 5a cannot arise when the dynamic system has, locally, the appropriate saddlepoint configuration. This leaves the configuration shown in Figure 5c as the only one for which inflation unambiguously rises as public sector capital formation is reduced. The intuition is clear. Private capital deepening ($\dot{k}>0$) means a lower real interest rate. Unless a lower real interest rate raises absorption relative to output ($a_{22}<0$), the rate of private capital formation increases further, setting off an unstable process. (When $\eta'>0$, the condition that a_{22} be negative is not necessary for saddlepoint stability).

The eigenvalues of a matrix are continuous functions of the coefficients of that matrix. Small perturbations of η' from zero will therefore not cause a qualitative change in the local saddlepoint configuration just discussed. For η' far from zero (including any model with a seigniorage-revenue-inefficient region) numerical simulation methods are required to determine local (and of course global) stability characteristics.

6. Conclusions

The answer to the question : "can public spending cuts be inflationary?" is a clear "yes". The paper specifies the conditions under which a cut in public spending can permanently raise the rate of inflation. Monetary financing is taken to be the "residual" financing mode. Real public spending per capita (current and capital) is taken to be exogenous. So are the share of income-related taxes in GDP, GDP-independent real per capita taxes and the real per capita public debt, both internal and external. This set-up helps us pose the issue clearly (it is one of the two fiscal-financial regimes considered in Sargent and Wallace's "Unpleasant Monetarist Arithmetic Paper" (1981)) and it also may be a quite reasonable approximation to the real-world situation of a number of highly indebted semi-industrial countries such as Mexico, Argentina and Brazil.

These countries face the need to reduce the public sector deficit. They tend to be credit-rational in the private international financial markets and are unable or reluctant to add to the internal public debt burden. Raising revenues (which tend to be a low proportion of national income relative to those found in most industrial countries) often appears to be politically or administratively infeasible. Barring internal or external debt repudiation (or economically equivalent capital levies on public debt holders), this leaves public expenditure cuts as the only instrument of fiscal retrenchment. The paper emphasises and elaborates the important distinction between cuts in public consumption expenditure which will tend to reduce the deficit[20] and cuts in public sector capital formation which may have the perverse effect of increasing the deficit. This will happen if the expenditure effect is swamped by the direct and indirect effects of a reduced public sector capital stock on government revenues.

If they increase the (inflation-and-real-growth-corrected) public sector deficit, the cuts in public sector capital formation will raise the "demand for seigniorage revenue". Such cuts are also likely to lower the demand for money (at a given inflation rate and real interest rate) by reducing the scale factor (real income) in the money demand function. This reduces the "supply of seigniorage revenue". In the seigniorage-revenue-efficient portion of the "seigniorage Laffer curve", the price of seigniorage, i.e. the rate of inflation, will increase.

When the domestic real interest rate is governed by the world real interest rate, the empirical information required to determine whether an economy finds itself in the situation just outlined, is rather limited. Needed are (1) an estimate of the high-powered money demand function, including an estimate of the effect of the public sector capital stock on the "scale factor" in the money demand function, (2) an estimate of the production function which includes the contribution of the public sector capital stock [21] and (3) an estimate of the government revenue function. When the domestic real interest rate is endogenous, things are considerably more complicated as private consumption and equilibrium in the domestic credit market now have to be modelled.

A major caveat in interpreting these results is that they are concerned with long-run (steady-state) effects. The adjustment dynamics which supported the long-run comparative statics in addition did not allow for any kind of non-Walrasian equilibrium in the labour market, the output market or the credit market. Demand-deficient, Keynesian temporary equilibria in response to public expenditure cuts were not considered, but may well be an important part of the story in the larger semi-industrial debtor countries.

The assumption made in the paper that real government revenue is independent of the rate of inflation is unrealistic. With low rates of inflation, "bracket creep" in income tax systems that are progressive in nominal terms makes for a positive relationship between the price level and the real tax burden. This is unlikely to be important in most of the semi-industrial debtor countries, as they tend not to have (de facto) progressive tax structures. More important is the erosion of real tax receipts at high rates of inflation due to tax collection lags and a failure to charge and/or collect the appropriate interest rate on overdue taxes (see Tanzi (1978)).

Finally, when public sector capital formation is interpreted in its broadest possible sense (as it should be for the purposes of this paper) as any public outlays that yield a stream of future returns, it is likely that some of the expenditure on communications, transportation, education etc which are classified as current will in fact enhance the future revenue raising capacity of the government. Indeed some current spending (on public administration, law enforcement) may have an immediate effect on current public revenues.

While it is not difficult to come up with horror stories about instances of wasteful or socially undesirable public expenditure, it may not be as simple to ensure that the broad-brush cuts so often proposed (and occassionally implemented) do in fact achieve their declared purpose of achieving a lasting reduction in the public sector deficit and a lower rate of inflation.

FOOTNOTES

** This paper was written while the author was a consultant for the World Bank (Country Economics Department, Public Economics Division (CECEM)). The findings, interpretations, and conclusions are the results of research supported by the World Bank; they do not necessarily represent the official policy of the Bank. The Bank does not accept responsibility for the views expressed herein which are those of the author and should not be attributed to the World Bank or to its affiliated organisations. The designations employed, the presentation of material, and any maps used in this document are solely for the convenience of the reader and do not imply the expression of any opinion whatsoever on the part of the World Bank or its affiliates concerning the legal status of any country, territory, city, area, or of its authorities, or concerning the delimitation of its boundaries, or national affiliations.

1 Running down foreign exchange reserves is included in net external borrowing by the public sector.

2 In the cases of the linear or log-linear money demand functions the absolute value of the elasticity of money demand with respect to the nominal interest rate increases with the nominal interest rate. The long-run seigniorage Laffer curve (holding constant the real interest rate and the real scale variable in the money demand function) is unimodal in this case.

3 In Hall (1987), the real rate of return on high-powered money is kept constant through policy by varying the nominal interest rate on base money with the (expected) rate of inflation.

4 Exceptions in the form of Keynesian models with more than 100 per cent "financial crowding out" are reviewed e.g. in Buiter (1985).

5 Some authors refer to πm or to $(n+\pi)m$ as the inflation tax.

6 $\omega_1>0$ and $\omega_2>0$ follow from $F_{LK}>0$ and $F_{LK}G>0$ respectively.

7 The socially efficient solution would of course be to have a single firm (N=1) and to choose K^G, K and L such that $f_1=f_2=r$ (ignoring the distortionary tax rate θ).

8 If the instantaneous utility function is of the constant elasticity of marginal utility or constant relative risk aversion variety with constant of relative risk aversion $1-\gamma$, $\gamma<1$, comprehensive consumption would be given by

$$q(t) = \lambda(t)(a(t)+h(t))$$

where

$$\lambda(t) = \left[\int_t^\infty e^{-\left[\frac{\gamma}{\gamma-1} \int_t^v r(u)du + (v-t)\frac{1}{1-\gamma}\delta \right]^{-1}} dv \right]^{-1}$$

in a steady state with constant r,

$$\lambda = \frac{\gamma}{\gamma-1}r+\frac{\delta}{1-\gamma}$$

provided this expression is positive. The logarithmic utility function considered in most of the paper corresponds to the special case $\gamma=0$.

9 Since $\sigma\equiv\dot{m}+(n+\pi)m$ and in steady state $\dot{m}=0$.

10 When n=0, steady state equilibrium with positive and bounded per capita consumption only exists if $r=\delta$. Steady state non-human wealth a is "hysteric" in this case, i.e. it cannot be determined from the steady state conditions alone but depends on the initial conditions and the

values of the exogenous variables during the adjustment process to the steady state. For reasons of space we do not consider this case here. $n+\delta>r$ is a viability condition for this economy, ensuring positive long-run consumption provided after-tax disposable wage income is positive.

11 Alternatively, the government could run down reserves with B* constant but B declining. The private sector would in that case reduce its foreign indebtedness.

12 The distinction between ***ability*** to service external debt and ***willingness*** to service this debt isn't relevant here. Voluntary default or repudiation risk creates external credit rationing, but no special role for a subcategory of external assets labelled "reserves".

13 The elasticity of money demand with respect to the nominal interest rate is $-(1/(1-\eta))(1-\eta+i\eta')$.

14 It is easily checked that in the long run,

$$a = \frac{(r-\lambda)}{(n+\lambda-r)r}(w-\tau_0)$$

where λ, the ratio of comprehensive consumption to total (human plus non-human) wealth is (from Footnote 8) given by

$$\lambda = \frac{\gamma}{\gamma-1}r+\frac{\delta}{1-\gamma} \ ; \quad \gamma<1$$

Therefore,

$$a = \frac{(r-\delta)}{[(1-\gamma)n+\delta-r]r}(w-\tau_0)$$

The change in the ratio $a/(w-\tau_0)$ as r changes is

$$\frac{d\left[\frac{a}{w-\tau_0}\right]}{dr} = \frac{(r-\delta)^2+\delta(1-\gamma)n}{\{[(1-\gamma)n+\delta-r]r\}^2} > 0$$

Also,

$$q = \frac{\lambda n}{r(n+\lambda-r)}(w-\tau_0)$$

or

$$q = \frac{(\delta-\gamma r)n}{[(1-\gamma)n+\delta-r]r}(w-\tau_0)$$

so

$$\frac{d\left[\frac{q}{w-\tau_0}\right]}{dr} = \frac{-n[((1-\gamma)n+\delta-2r)\delta+\gamma r^2]}{\{[(1-\gamma)n+\delta-r]r\}^2}$$

When $\gamma=0$ (i.e. in the logarithmic case considered in the body of the paper) $d(q/(w-\tau_0))/dr$ will be positive if $n+\delta<2r$. This will hold if the economy is dynamically efficient ($r>n$) and if $r>\delta$, which is necessary for $a>0$ (if $q>0$).

15 If a_{12} is positive, this will certainly be the case if $a_{21}\geq 0$ and $a_{22}<0$. In Section 5 it is shown that if $\eta'=0$ (i.e. $a_{21}=0$) $a_{22}<0$ is necessary for the appropriate kind of saddlepoint stability.

16 It is even possible for the long-run inflation rate to rise (in the seigniorage-revenue-efficient region) in response to a cut in public sector capital formation when b_{12} is positive, i.e. when the long-run seigniorage equilibrium schedule shifts to the left. This will occur for sufficiently large shifts to the left of the output-absorption

equilibrium schedule. We do not consider this case for reasons of space.

17 Note that $a \equiv m+b+k+s+f^{*P}$ will jump discontinuously at a point in time if $m \equiv M/PL$, $b \equiv B/PL$, $f^{*P} \equiv EF^{*P}/PL$ or s jump discontinuously. (Since the law of one price holds, $E/P=1/P^*$, so even with a freely floating exchange rate, f^{*P} will not move discontinuously when the exchange rate jumps.) Any jump in a can be reduced to the underlying jumps in E and s using the wealth identity. See Buiter (1984) for some linear examples.

18 Note that public sector capital formation is, with i^G exogenous, governed by the single stable root $-n$. More generally we could represent i^G by a first-order partial adjustment process such as $i^G=\alpha(\bar{k}^G-k^G)+nk$; $\alpha>0$; $\bar{k}^G>0$.

19 This is the special case of Ω_1 in equation (46) with $\eta'=0$.

20 When $a_{22}<0$, as shown in Figure 3b the real interest rate may fall so much that inflation actually increases.

21 In our model the production function provides the relevant information on the scale factor in the money demand function.

FIGURE 1

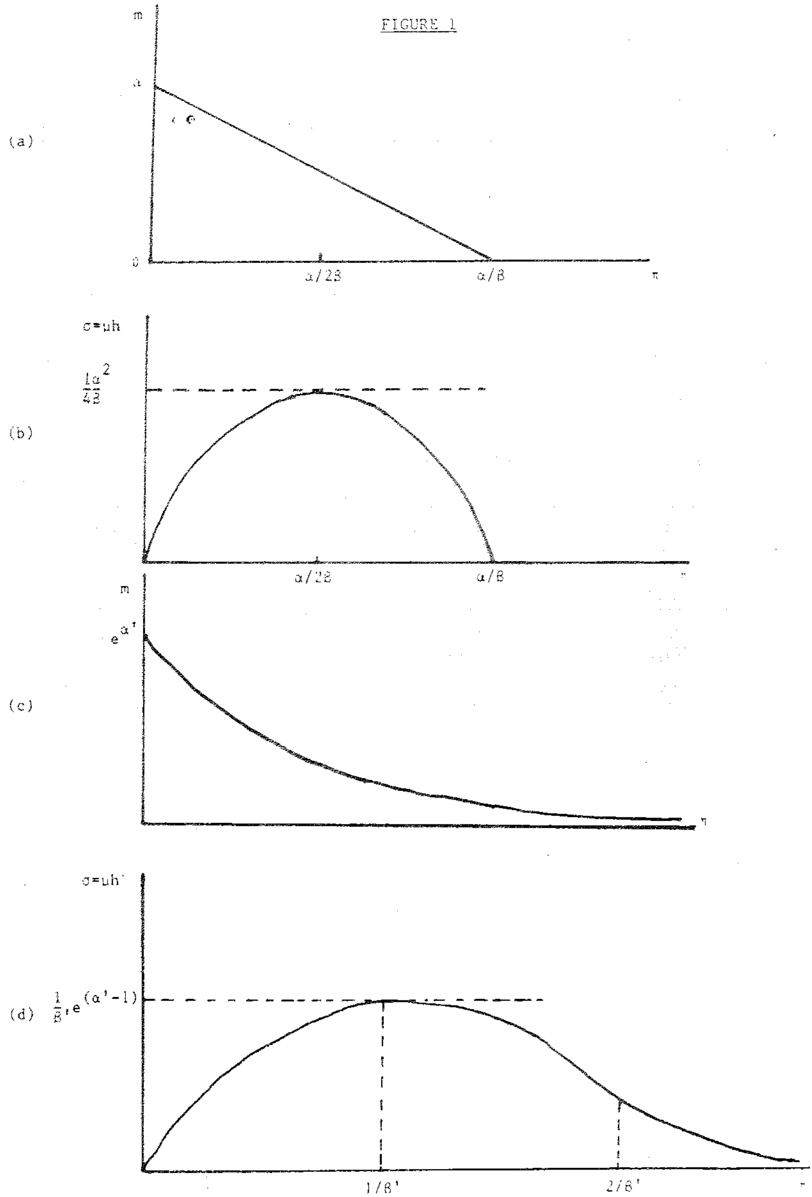

(a)
m
α
0
α/2β
α/β
π
(b)
σ=μh
$\frac{1}{4}\frac{\alpha^2}{\beta}$
α/2β
α/β
(c)
m
$e^{\alpha'}$
π
(d)
σ=μh'
$\frac{1}{\beta'}e^{(\alpha'-1)}$
1/β'
2/β'

FIGURE 2

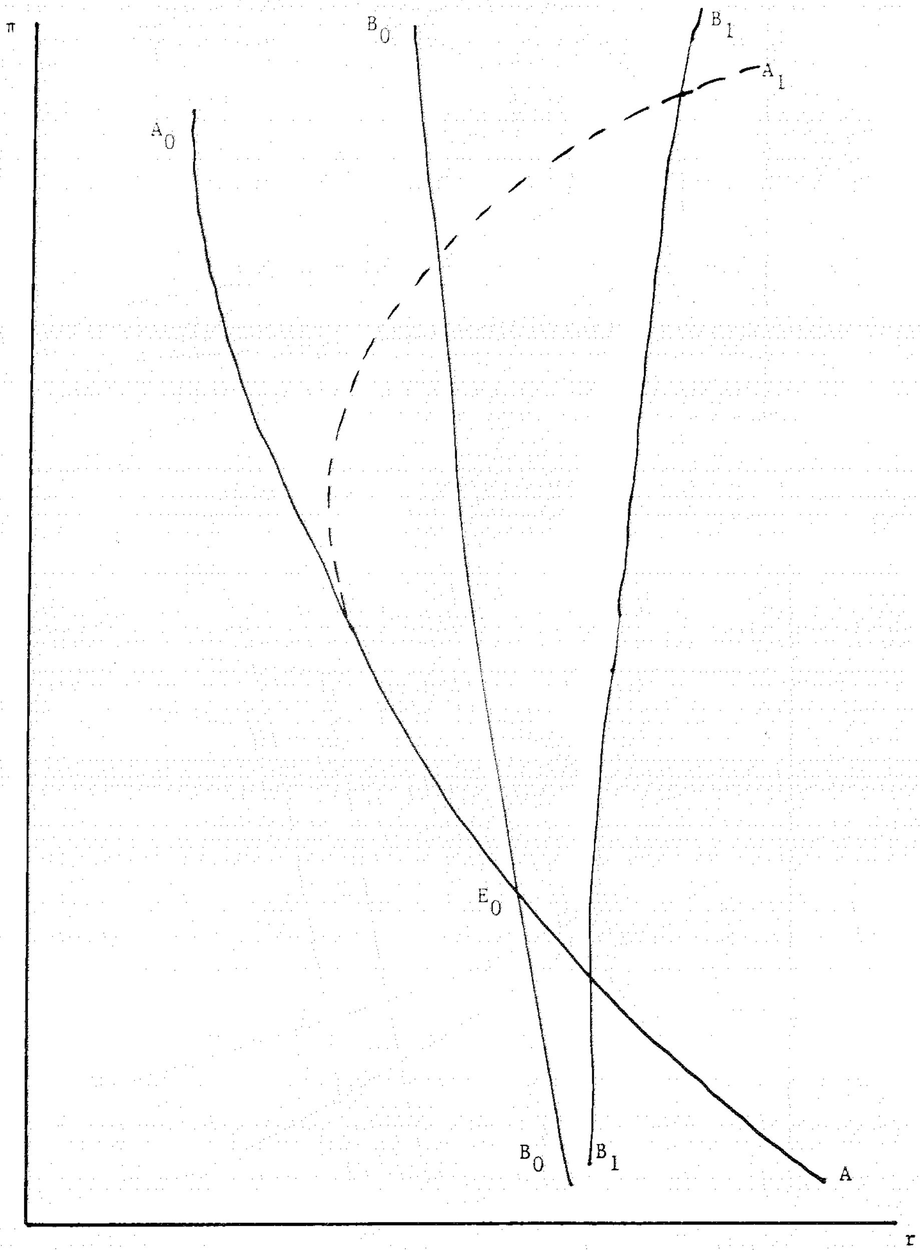


FIGURE 3A

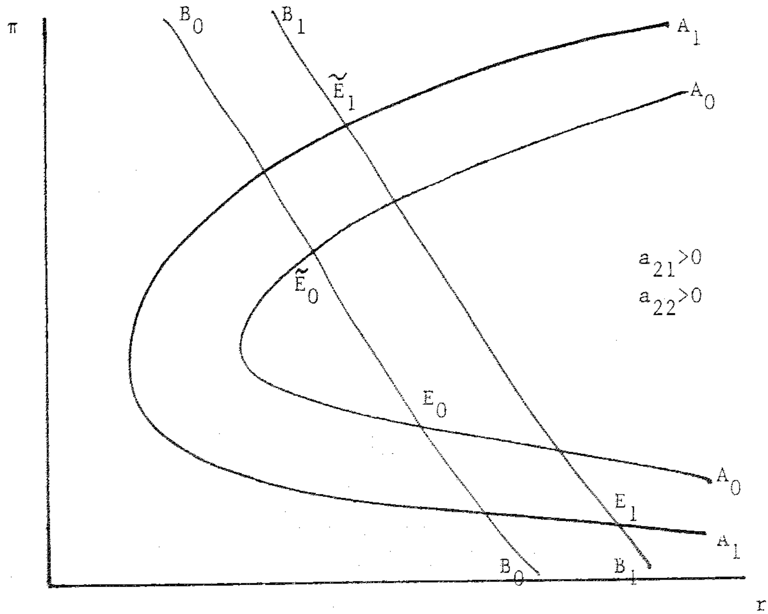


FIGURE 3B

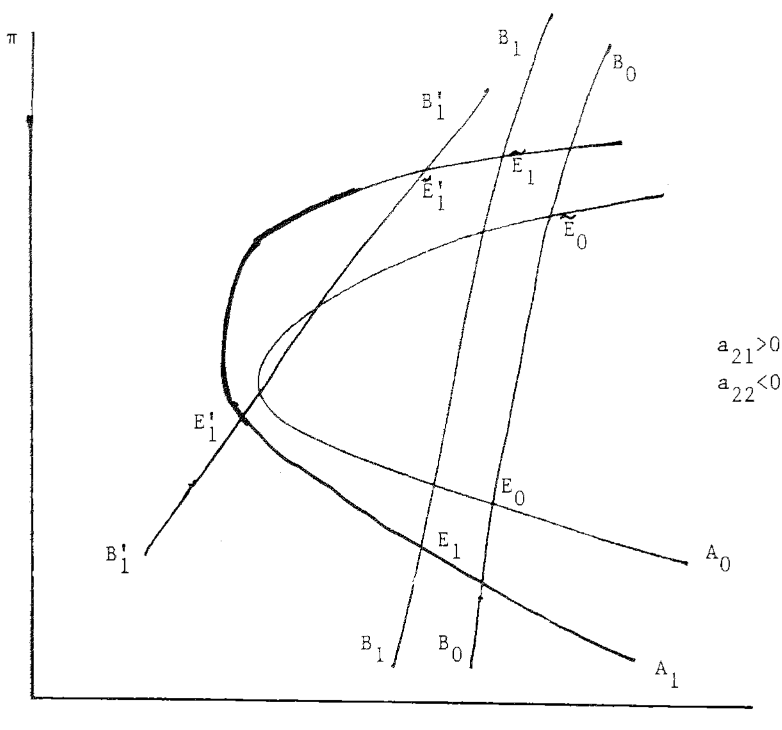


FIGURE 4A

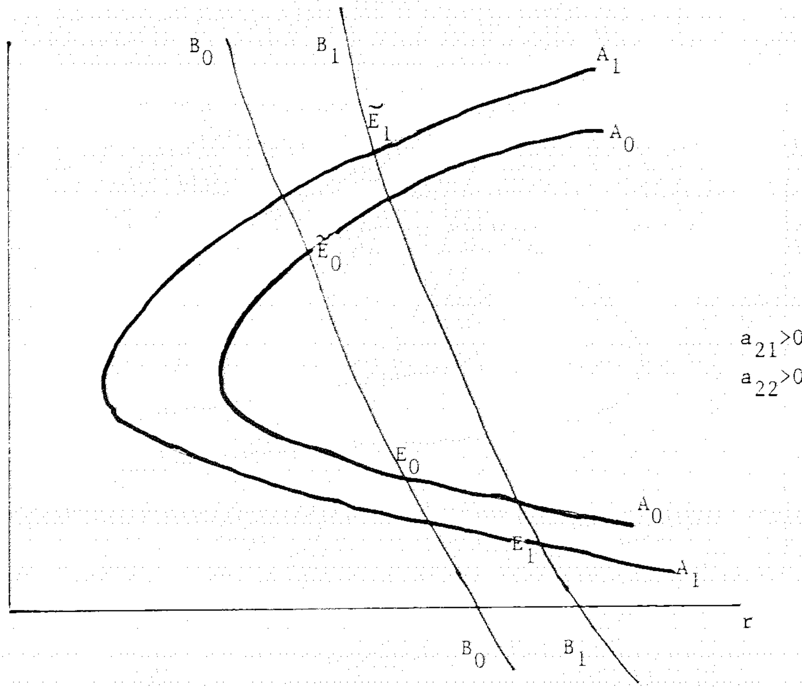


FIGURE 4B

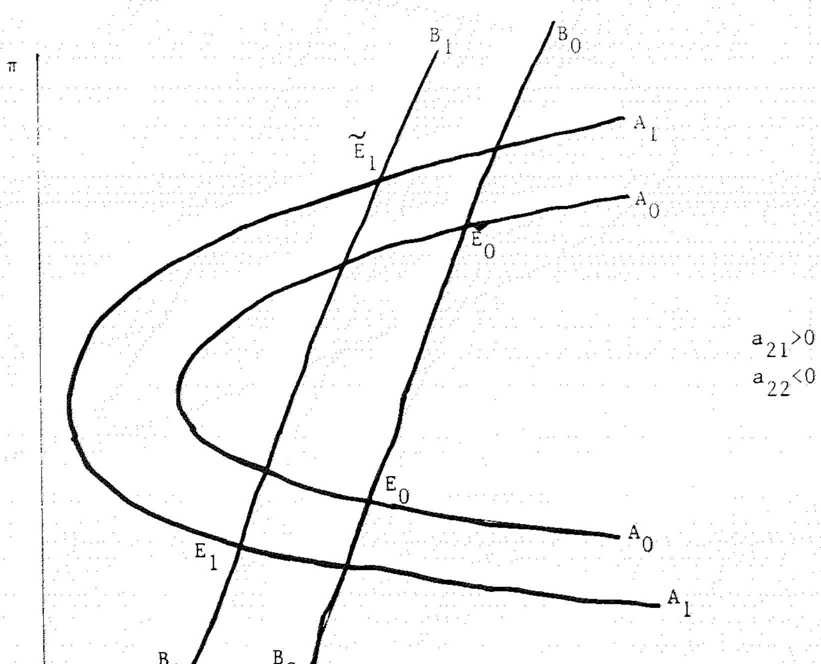


FIGURE 5A

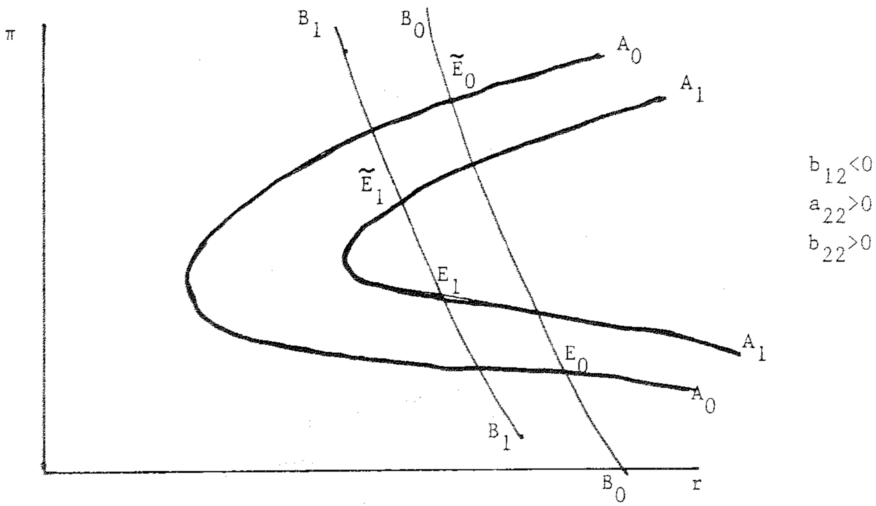


FIGURE 5B

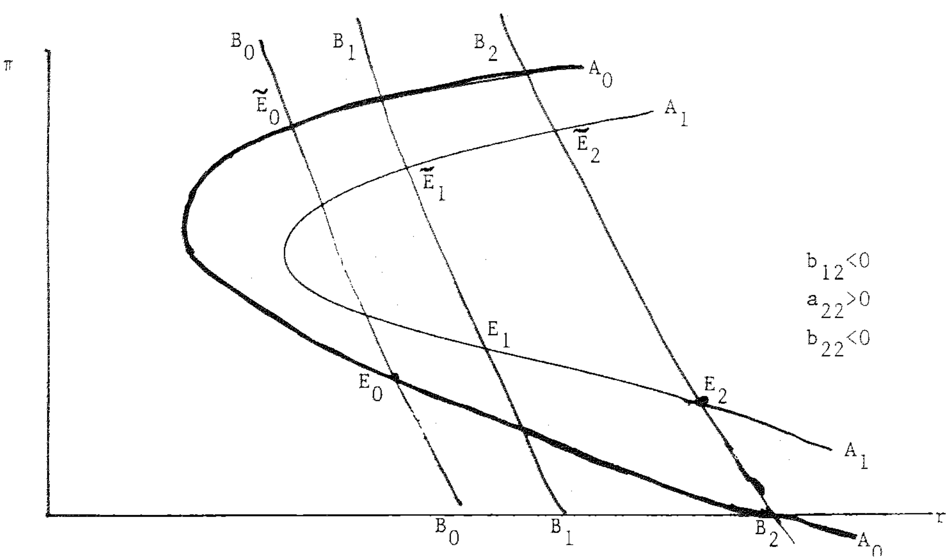
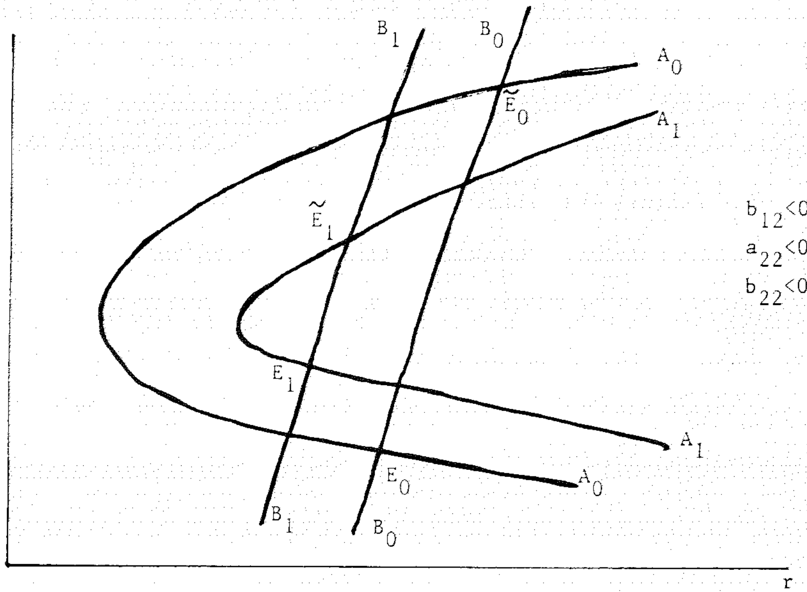


FIGURE 5C



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