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HETEROGENEITY IN EXPECTATIONS, RISK TOLERANCE, AND HOUSEHOLD STOCK SHARES: THE ATTENUATION PUZZLE

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ABSTRACT

This paper jointly estimates the relationship between stock share and expectations and risk preferences. The survey allows individual-level, quantitative estimates of risk tolerance and of the perceived mean and variance of stock returns. These estimates have economically and statistically significant association for the distribution of stock shares with relative magnitudes in proportion with the predictions of theories. Incorporating survey measurement error in the estimation model increases the estimated associations twofold, but they are still substantially attenuated being only about 5 percent of what benchmark finance theories predict. Because of the careful attention in the estimation to measurement error, the attenuation likely arises from economic behavior rather than errors in variables.

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The source of heterogeneity in portfolio choices is an important question for household finance (Campbell, 2006). Theories, such as consumption CAPM, predict that the share of risky assets should be positively related to their expected returns, negatively related to their risk, and positively related to investors' risk tolerance. These theories also have quantitative implications for the magnitudes of those relations. This paper assesses those implications by estimating how heterogeneity in preferences and beliefs explain heterogeneity in household portfolios.

In this paper we take a systematic attempt at quantitatively evaluating the implications of benchmark financial theories by using better data and more careful statistical modeling. We build a structural maximum likelihood model to estimate jointly quantitative measures of risk tolerance and the perceived mean and variance of stock returns from high-quality survey data while taking survey measurement error into account. We estimate their association with household stock shares at the intensive margin. Our approach is made possible by new data on portfolio composition for a large enough sample of stockholding households, combined with appropriate measures of preferences and beliefs. Our data set was created by the Vanguard Research Initiative (VRI) that combines administrative account data and survey responses for a large sample of Vanguard account holders. The VRI has multiple features that make it especially well-suited for estimation of the sources of heterogeneity in stock holdings.

Section I summarizes related literature and discusses how our approach improves upon previous analyses. Section II describes the VRI sample and the measurements of assets and stock share. Section III describes how we measure preferences and beliefs. To get individual-specific estimates of preference parameters, we use a modification of the Barsky, Juster, Kimball, and Shapiro (1997) approach of eliciting risk tolerance from hypothetical gambles over permanent income. To get individual-specific estimates of the moments of the perceived

distribution of returns, we use both the Manski (2004) approach of eliciting points in the CDF of perceived returns together with individuals' estimates of expected returns.

Survey measures of preferences have considerable external validity (i.e., that preference parameters explain a wide range of behaviors) and internal validity (i.e., test-retest validation and consistence across different measures). See Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2011), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2011), and Josef, Richter, Samanez-Larkin, Wagner, Hertwig, and Mata (2016) for evidence both of external and internal validity. Recent evidence suggests survey measures of risk preferences show more stability than measures based on small-stakes lottery experiments (Lönnqvist, Verkasalo, Walkowitz and Wichardt, 2015). Similarly, probabilistic measures of expectations have predictive validity (Hurd, 2009). See Manski (2017) for a summary of progresses made in eliciting subjective expectations on macroeconomic variables including equity returns. Carroll (2017) also stresses the role of expectations in explaining macroeconomic fluctuations and hence the importance of correctly measuring them and understanding their formation. This paper is the first attempt to measure both preferences and expectations and to use them jointly to explain portfolio choices.

Like many survey measures, preference and expectations are subject to response error. This paper uses a unified procedure accounting for response error to produce unbiased estimates of the subjective variables for both preferences and beliefs. Section IV combines these estimates to explain the cross-section of stock shares. We find that the stock share is positively related to the individuals' perceived expected stock returns, is negatively related to their perceived standard deviation of the returns, and is positively related to their risk tolerance. These

relationships are economically and statistically significant, they are robust across various specifications, and they are substantially larger in magnitude than corresponding estimates that do not take care of measurement error in the survey answers.

At the same time, the estimated associations are only about 5 percent of what benchmark theories predict. Some features of our estimates are in line with those implications: the signs and also the relative magnitudes of the estimated coefficients conform to the predictions of theories. They are substantially smaller in magnitude, though, a finding that we call the "attenuation puzzle." The empirical method advanced by this paper addresses measurement error in survey measures of preferences and beliefs, so it establishes that this attenuation reflects actual gap between benchmark portfolio choice theories and individuals' behavior.

I. Relationship to the Literature

Several papers estimated associations of household portfolio compositions with various measures of beliefs and preferences. Not all of them yield results that can be weighed against the quantitative predictions of finance theories. The results of those that do allow for such comparisons suggest that beliefs and preferences, as measured by the data, are related to household portfolios indeed, but those relations are substantially weaker than what benchmark finance theories would suggest. Most studies analyzed associations at the extensive margin, i.e., whether households hold any stocks, primarily due to constraints on sample size. Yet theories have the starkest quantitative predictions at the intensive margin, i.e., the share of stocks in the portfolio of stockholders. Most studies either examine the role of beliefs or preferences but not both.

Vissing-Jorgensen (2003), Glaser and Weber (2005), Hurd, van Rooij and Winter (2011), Hudomiet, Kezdi and Willis (2011), Amromin and Sharpe (2012), Hoffman, Post and Pennings (2013), and Guiso, Sapienza and Zingales (2018) focus on expectations and show that people with more optimistic expectations about future stock returns are more likely to hold stocks. Barsky, Juster, Kimball and Shapiro (1997), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2011) and Guiso, Sapienza and Zingales (2018) show that more risk tolerant individuals are more likely to hold stocks. Dominitz and Manski (2007) and Hurd, Rooij and Winter (2011) show that individuals with higher levels of stock market expectations and lower perceived risk are more likely to hold stocks. Kimball, Sahm and Shapiro (2008) model the intensive margin. Kezdi and Willis (2011) and Dimmock, Kouwenberg, Mitchell and Peijnenburg (2016) combine the extensive and intensive margins in Tobit-type models and establish associations with risk tolerance, expectations and ambiguity aversion, respectively. Weber, Weber and Nosic (2013) show that individual measures of risk tolerance and expectations predict the share of stocks respondents invest in a hypothetical financial portfolio but they do not consider beliefs. Hoffmann, Post and Pennings (2013) and Merkle and Weber (2011) analyze the role of expectations and risk tolerance in trading behavior of individual investors rather than the share of stocks in household portfolios. Brunnermeier and Nagel (2008) conclude that understanding the determinants of the share of stocks in the portfolio of stock market participants is very difficult.

Several related studies investigated the role of wealth and past experiences in household portfolios. See, for example, Vissing-Jorgensen (2003), Greenwood and Nagel (2009), Seru, Shumway and Stoffman (2010), Malmendier and Nagel (2011), Calvet and Sodini (2014). Another literature focuses on the role of preferences and beliefs in other household decisions.

Piazzesi and Schneider (2009) and Armona, Fuster and Zafar (2016) examine the role of expectations on the housing market, while Bruine de Bruin, Manski, Topa, and van der Klaauw (2011), Armantier, Bruine de Bruin, Potter, Topa, van der Klaauw, and Zafar (2013), Malmendier and Nagel (2016), and Botsch and Malmendier (2017) investigate inflation expectations.

Our approach improves on the previous literature in multiple ways. First, the VRI sample is a large sample of stock holders. Despite being drawn from the account holders of a single company, the characteristics of the sample are broadly representative of the targeted population of households with non-negligible financial assets. Unlike most studies that focus on the extensive margin for stock holdings, this sample allows for meaningful inferences about the intensive margin of portfolio choice.

Second, the VRI survey includes batteries of questions that we purposely designed to produce estimates of preference and belief parameters that should help to explain the cross-sectional distribution of portfolio choices. These survey questions yield quantitative estimates of individual-level moments of subjective returns distribution and of individual-level values of preference parameters. These estimates can then be related to portfolio decisions in ways that are quantitatively interpretable relative to benchmark economic models.

Third, the design of the VRI allows careful consideration of response errors along a variety of dimensions. These include errors in measuring stock shares in both survey and administrative account data and errors in eliciting preferences and expectations from survey responses. Few studies take survey measurement error into account in their estimation procedure. Yet there is strong evidence that survey measures of preferences and beliefs are subject to substantial response error leading to potentially severe attenuation bias (Kimball, Sahm and

Shapiro, 2008; Kezdi and Willis, 2011). These limitations may be in part responsible for why estimated associations in the literature are so much smaller than what finance theories would predict.

These features—a large, broadly representative sample of stockholders together with quantitative measurements of the potential sources of heterogeneity in stockholding—make the VRI a unique platform for understanding why different households make different portfolio choices.

II. VRI Data and Stock Share Measurement

A. VRI sample and wealth measurement

The Vanguard Research Initiative (VRI) consists of linked survey and administrative data of account holders who have non-negligible financial assets at Vanguard, are at least 55 years old, and use the internet to access their Vanguard accounts. This last requirement is necessary because the VRI is an internet survey. The VRI is an individual level survey, but it includes questions about household-level wealth and income as well as questions about spouses' or partners' demographics and labor supply. The survey oversampled older account holders and singles. The VRI draws respondents from two lines of business—individual account holders and employer-sponsored account holders. The employer-sponsored are enrolled at Vanguard through 401(k) or similar defined-contribution accounts. While both individual and employer-sponsored account holders are selected via ownership of a Vanguard account, the selection into individual and employer-sponsored accounts is presumably quite different. We will present separate estimates to get a sense of whether selection matters for our results. See Appendix A for more details on the VRI surveys and sample.

There are features of the VRI that make it well-suited for this analysis. First, it has a new approach to wealth and portfolio measurement. Second, it provides a larger sample of respondents with relevant levels of assets and stock holding compared to leading surveys such as the Health and Retirement Study (HRS) and the Survey of Consumer Finances (SCF). Third, demographics of the VRI are nonetheless comparable to those with similar asset levels in the HRS and SCF.

The VRI survey measure of wealth is based on a comprehensive account-by-account approach. The survey first asked about types of accounts respondents have (e.g. IRA, checking, money market funds) and the number each type of account held by the respondent or her spouse. For each account they indicated owning, the respondents were asked to provide the balance as well as the share of stock-market assets. When finished with all accounts, respondents were presented a summary table consolidating their responses and were invited to make corrections, if any. Measuring wealth and stock shares account by account matches the way respondents keep track of their own wealth, and it does not require them to sum balances across accounts to provide total figures for asset categories that are familiar to economists but less so to survey respondents. In contrast, the HRS and SCF—other leading surveys with state of the art wealth measurement—use account-by-account approaches but only for selected sets of account types. Item non-response in the wealth section of the VRI affects less than 1 percent of the observations.

Table 1 compares the VRI sample to the HRS and SCF. The HRS and SCF are nationally representative samples (of those above age 50 in the case of the HRS). Table 1 compares the VRI sample to the subsample of the HRS and SCF after imposing restrictions similar to VRI eligibility: being at least 55 years old, having access to internet at home, and having at least

$10,000 financial wealth. The number of respondents in Survey 1 is substantially larger than the VRI-eligible subsample of the HRS and the SCF. The difference in the number of respondents in stock-holding households is even larger: the comparable samples have slightly over 1,000 stock-holding households in the SCF and slightly over 2,000 in the HRS; the entire VRI sample has more than 8,000 stock holders and the sample used in our analysis (those who completed all the first three VRI surveys, see below) has more than 4,000.

Table 1. Sample Means: VRI, HRS, and SCF

	VRI		HRS	SCF
	Entire sample	Analysis sample	VRI-eligible subsample	
Household-level variables				
Number of households	8,950	4,414	3,684	1,275
Number of stockholding households	8,636	4,323	2,356	1,216
Average financial wealth ($'000)	1,207	1,148	578	970
Average total wealth ($'000)	1,589	1,551	804	1,764
Average stock share among stockholders	0.56	0.56	0.55	0.46
Respondent-level variables				
Married	0.67	0.68	0.70	0.71
Male	0.64	0.65	0.56	0.79
Age	67.8	67.8	64.9	64.1
Less than college degree	0.30	0.26	0.51	0.45
College degree but not more	0.32	0.33	0.23	0.27
Post-college degree	0.38	0.41	0.26	0.28
Retired	0.56	0.60	0.53	0.34

Notes: For the HRS and SCF, the VRI-eligible subsamples are those who are not younger than 55, have access to the internet at home, and have at least $10,000 in non-transactional accounts. Respondent-level variables for the HRS refer to the financial respondents; for the SCF they refer to the household heads. Variables in the VRI measured in 2013; HRS and SCF are from 2012 and 2013, respectively. Respondent-level variables are {0,1} binary variables except for age. Summary statistics of the wealth measures are shown in Table A1 in the Appendix. Table A2 in the Appendix shows the summary statistics of the variables we use as controls in our analysis, together with the definition of those variables. For more detailed comparisons with the HRS and SCF sample as well as for the effectiveness of the account-by-account approach in producing unbiased estimates of assets with low response error, see Ameriks, Caplin, Lee, Shapiro and Tonetti (2014).

The demographic composition of the VRI sample is broadly similar to the parallel subsamples of the HRS and the SCF. Average total wealth and average financial wealth in the

VRI are close to corresponding estimates from the SCF; the HRS averages are lower. The average stock share in financial wealth among stock holders is very similar in the VRI and the HRS; the SCF estimates are somewhat smaller. VRI respondents are slightly less likely to be married, and they are somewhat older, more educated and more likely to be retired. The differences in marital status, age and retirement are largely due to the fact that the VRI oversampled older individuals and singles. 65 percent of the VRI sample is male, compared to 79 percent in the SCF and 56 percent in the HRS. Within households, men are overrepresented as respondents: account holders in the VRI, financial respondents in the HRS, and household heads in the SCF.

B. Measuring stock shares

Our analysis focuses on the share of stock-market-based assets in total financial wealth. Specifying stock share in financial wealth is standard in the literature. Alternative measures may include housing wealth and human capital wealth in the denominator. We include such wealth items as control variables in the analysis and show that their inclusion leads to very similar results for the parameters of interest. We also show that our main findings are robust to including housing wealth as either risky or safe assets in the risky asset share calculation.

The VRI asks individual the share of stock held in each account. The stock share in financial wealth is the weighted average of the stock shares of the accounts. Respondents who did not answer all of the account-by-account stock share questions were asked the overall stock share of their financial portfolio. Ninety-five percent of respondents answered all the account-by-account stock share questions; the distribution of stock share is very similar across the two groups.

The VRI account data also allow us to calculate stock share using the administrative records, but of course only for assets held at Vanguard. Appendix A compares the survey and administrative measures of the stock share. Appendix B also presents the empirical results using the administrative stock share as the dependent variable. Individuals might hold stocks disproportionately at one provider or another, so there is no reason to expect portfolio theories to obtain for holding at each provider. Yet, despite the fact that individuals at sample tend to have a higher stock share at Vanguard, the results using the administrative share are quite similar to those using the survey share.

III. Measuring Preferences and Expectations

A. Measuring risk tolerance

Survey 2 of the VRI included Strategic Survey Questions (SSQs) that ask respondents to make choices between hypothetical financial products under hypothetical situations. In this paper, we use the VRI's risk tolerance questions that pose gambles over consumption. These are based on the questions used in Barsky, Juster, Kimball, and Shapiro (1997) and Kimball, Sahm, and Shapiro (2008) that are implemented in the HRS. The VRI risk tolerance questions are refined relative to those in the HRS to be more specific about the economic setting and to ask about consumption rather than income gambles. The HRS uses lifetime income rather than consumption because when the HRS questions were crafted, there was a concern that consumption was too abstract a concept to implement in the survey. The VRI approach frames the question in terms of consumption, the economically more-relevant flow. Earlier successes with the SSQ approach suggest that it is possible to elicit the more precisely model-relevant

measure using a survey instrument that has both more detailed scenarios and comprehension tests.

The VRI SSQs ask about preference between the following two options:

- Having a certain level of consumption;
- Having double that level of consumption or having it fall by *x*% with a 50-50 chance.

The question then alters the downside risk *x* and repeats the question in order to partition respondents into risk tolerance groups. There are some other differences between the VRI and HRS questions. In the VRI, the same question is asked with two different levels of guaranteed consumption for the safe option. Having two consumption treatments in this survey provides a test-retest measurement that is instrumental for separating true preference heterogeneity from survey response error. In contrast, Kimball, Sahm, and Shapiro (2008) relies on variation across multiple survey waves, which assumes time-invariant preferences, an assumption not needed in this paper. Additionally, using two different levels of guaranteed consumption allows identification of non-homothetic preferences. The VRI questions are more specific about the hypothetical situations to better assure that structural preference parameter estimates are independent from respondents' economic, health, and family conditions. Table A3 in Appendix A gives the exact wording of the risk tolerance question in the VRI.

The question is asked for two different levels of riskless consumption, $100K and $50K per year, and downside risks of 1/10, 1/5, 1/3, 1/2, and 3/4. Table 2 shows the distribution of the answers to the two questions. Most respondents have low tolerance for risk. About half of the respondents chose the first two categories, indicating that they would not accept a risk of more than 20% drop in their consumption to take a chance to double their consumption. Only a small fraction chose the last two categories with a risk of more than a 50% drop. Overall, the

distribution is similar to the distribution of the answers to a similar question in the HRS except that the fraction of respondents in the two extreme categories (0-10% and 75-100%) is slightly lower in the VRI (see Kimball, Sahm, and Shapiro, 2008 for the HRS). The table also shows that more respondents fall into the lower risk categories when riskless consumption is $50,000 instead of $100,000. We handle this increase in relative risk tolerance by positing a utility function with a subsistence level of consumption.

Following Barsky, Juster, Kimball, and Shapiro (1997) and Kimball, Sahm and Shapiro (2008), we use the multiple responses to identify the heterogeneity of the preference parameter and survey response errors. Estimation of a cardinal risk tolerance parameter requires specifying a utility function. We assume that the flow utility function is a generalization of CRRA with a subsistence level of consumption

$$u_i(c) = \frac{(c+\kappa)^{1-1/\theta_i}}{1-1/\theta_i}, \tag{1}$$

where subscript i denotes heterogeneity across individuals, c is consumption, the negative of κ is the subsistence level of consumption, assumed to be the same for all individuals, and θ is the risk tolerance parameter. To allow for heterogeneity in both θ and κ and to allow for survey response errors, we would need at least three responses for each respondent; the VRI asked only two. Therefore, we allow for heterogeneity only in θ. We do allow κ to be a function of observed covariates in specifications using those covariates (see Section IIIC). Appendix Tables B3, B4 and B10 show that the main results are almost the same when we use a CRRA utility function (i.e., setting $\kappa=0$) as in Kimball, Sahm and Shapiro (2008), except that the estimated risk tolerance parameter is lower.

Table 2. Risk Tolerance: Distribution of Responses to SSQ

Response	Downside risk		Percent of answers	
category	accepted	rejected	riskless consumption $100K	riskless consumption $50K
1	none	1/10	23	28
2	1/10	1/5	26	34
3	1/5	1/3	26	26
4	1/3	1/2	13	9
5	1/2	3/4	10	3
6	3/4	none	2	1
Total			100	100

Notes: Choice between two plans. Plan A guarantees $c consumption next year. Plan B: doubles $c with 50% chance and cuts it by a fraction x with 50% chance. $c=100K or 50K, shown in the two columns; the x values are shown in second and third columns. 4414 observations.

For this utility function, relative risk tolerance (RRT_i) is

$$RRT_i = \theta_i \frac{c+\kappa}{c} < \theta_i,$$

where the risk tolerance parameter θ_i is relative risk tolerance in the $\kappa = 0$ case. Empirically, the coefficient of risk tolerance is very close to what is implied by θ_i, as the level of average wealth (Table 1) and annual income before retirement ($90,000) are substantially larger than our estimate of $-\kappa$. At levels of consumption implied by the average before-retirement income, the difference is less than 20%, and its variation between individuals is small. See Appendix Figure B1 for the relationship of relative risk tolerance and θ as a function of consumption.

To parameterize the heterogeneity of the risk tolerance parameter, we assume that the parameter is distributed lognormally in the population according to

$$\log(\theta_i) = \bar{\theta} + u_{\theta i},\ u_{\theta i} \sim N(0, \omega_{u\theta}^2). \quad (2)$$

We model the measurement error as a log additive term to the parameter, such that

$$\log(\tilde{\theta}_{ij}) = \log(\theta_i) + \varepsilon_{\theta ij} \qquad \text{for } j = 1, 2$$
$$\varepsilon_{\theta ij} \sim N(0, \omega^2_{\varepsilon\theta j}) \tag{3}$$

where θ_i is the true risk tolerance parameter for individual *i*, $\varepsilon_{\theta ij}$ is measurement error, and $\tilde{\theta}_{ij}$ is the error-ridden risk tolerance parameter that provides the basis for individual *i*'s response to the j^{th} question (c=$100,000 for j=1 and c=$50,000 for j=2). Thus, in answering question *j* given the level of resource *c* and risk *x* that are associated with the risky gamble, the respondent compares

$$\frac{(c+\kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} \quad \text{vs. } 0.5\frac{(2c+\kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} + 0.5\frac{((1-x)c+\kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} \tag{4}$$

to determine whether to accept the risky gamble or not. Equation (4) translates each response category in Table 2 into an interval of $\tilde{\theta}_{ij}$. This approach generalizes that of Kimball, Sahm, and Shapiro (2008) by allowing for non-homothetic preferences. (Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2018) also exploits multiple responses within survey to identify individual level preference parameters for relating to decisions about long-term care and bequest. Ameriks, Briggs, Caplin, Shapiro, and Tonetti (2017) estimates the same parameters for a representative agent using a method-of-moments approach.) We carried out the estimation procedure jointly for risk tolerance and stock market expectations, so will defer discussion of estimation until Section IIIC below.

B. Measuring beliefs about stock returns

Survey 3 of the VRI asked about beliefs about the one-year return of the U.S. stock market, represented by a stock market index such as the Dow Jones Industrial Average (DJIA). Respondents had to answer three questions: the expected return on the stock market in the 12 months following the interview (*m*); the percent chance that the stock market will be higher in 12

months following the interview (*p0*) and the percent chance that it will be at least 20% higher (*p20*). The exact wording of the questions is in Table A4 in the Appendix. (Bruine de Bruin, Manski, Topa, and van der Klaauw (2011) and Armantier, Bruine de Bruin, Potter, Topa, van der Klaauw, and Zafar (2013) examine the reliability of the percent chance questions for inflation as well as how they relate to questions about point expectations of inflation.)

Answers to the expected value questions were constrained to be integers. Answers to the percent chance questions were constrained to be 5 point increments between 0 and 15 and between 85 and 100, and they were constrained to be 10 point increments between 15 and 85 (the set $\{0,5,10,15,25,35,45,55,65,75,85,90,95,100\}$). Answers to percent chance questions tend to be rounded to the nearest ten when they are not constrained, with an especially large fraction answering 50 percent (Hurd, 2009). The VRI survey instrument requires people to round to other values; in particular, they cannot give 50 percent probabilities. It also allows for finer rounding at the tails, in line with the findings of Manski and Molinari (2010). The survey also requires that $p20 \leq p0$. Respondents whose initial answer to *p20* violated this constraint are reminded of the constraint by the survey software and asked for a new reply to either $p0$ or $p20$ (or both). The survey imposes no constraints on *m* versus *p0* and *p20*. (A randomly selected half of the respondents received the *m* question first, followed by $p0$ and $p20$, while the other half received $p0$ and $p20$ first, followed by *m*. The distribution of the responses is slightly different across the two sequences. Nevertheless, we find similar relationships between the belief measures and portfolio choice from the two sequence groups.)

Table 3 shows the summary statistics of the answers to the questions about the distribution of stock market returns. The survey responses for expected returns (*m*) are distributed around the historical average of 4 to 7 percent depending on sample period, and their

dispersion is moderate. In contrast, most answers to the probability questions are lower than the historical probabilities, and they have substantial heterogeneity. (Individuals may use different sample windows for inferring expected returns, see Malmendier and Nagel, 2011. The table shows some different windows for realized returns. Average returns are quite variable owing to the well-known problem of estimating the expected return on the market.) A non-negligible fraction of the respondents gave a positive number to the expected return question (*m*) and a less than 50 percent chance answer to the probability of a positive return (*p0*). Taken together these answer patterns are consistent with many individuals implicitly applying a positive threshold when they answer the *p0* question (by thinking that the stock market goes up only if it goes up by at least some positive amount). Glaser, Langer, Reynders and Weber (2007) document a similar pattern when they compare stock market expectations elicited in terms of returns versus prices. They label the phenomenon as "framing effect," and our explanation can be viewed as a source of such a framing effect. Note that, although skewed returns could explain the phenomenon we observe, it is an unlikely explanation. The combination $m_i>0$ and $p0_i<0.5$ would correspond to long positive tails, implying mean above the median and infrequent large gains. This skewedness is the opposite of what one would expect from a "black swan" theory of infrequent stock market crashes.

Table 3. Stock Market Returns: Survey Responses versus Historical Statistics

	Survey answers				Historical statistics			
	Mean	25th pctile	Median	75th pctile	1959-2014	1995-2014	1995-2009	2010-2014
m	0.06	0.04	0.06	0.10	0.04	0.07	0.05	0.13
*p*0	0.51	0.25	0.55	0.75	0.58	0.65	0.53	1.00
*p*20	0.15	0.05	0.15	0.25	0.18	0.25	0.27	0.20

Notes: *m* is expected one-year ahead returns of the stock market index DJIA; *p0* is the probability that the DJIA would be higher a year from the date of the interview; *p20* is the probability that it would be higher by at least 20%. Historical statistics computed from yearly relative returns of the Dow Jones Industrial Average (year on year changes divided by base year value, first days of July in each year), deflated using the PCE chain price index (available beginning in 1959). Historical average values shown for *m*; the fraction of years when positive or greater than 0.2 are shown for *p0* and *p20*. 4414 observations.

In order to use our data more efficiently and in a way that is more informative from a theoretical point of view we map the three survey responses, *m*, *p0*, and *p20* into a perceived returns distribution. The procedure closely parallels that for the risk tolerance questions: the survey responses are based on individual beliefs drawn from a distribution plus survey response error. We assume that individual *i* believes that yearly returns follow a lognormal distribution with individual-specific mean and standard deviation of log stock returns of μ_i and σ_i. Similar to how we handle the cross-sectional distribution of the risk tolerance parameter, these parameters are drawn across individuals as

$$\begin{aligned} \mu_i &= \bar{\mu} + u_{\mu i} \\ \sigma_i &= \bar{\sigma} + u_{\sigma i} \end{aligned}, \quad \begin{pmatrix} u_{\mu i} \\ u_{\sigma i} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_{u\mu}^2 & \rho_{\mu\sigma}\omega_{u\mu}\omega_{u\sigma} \\ . & \omega_{u\sigma}^2 \end{pmatrix} \right). \tag{5}$$

Individuals answer the survey questions *m*, *p*0 and *p*20 based on their beliefs, but their answers contain survey noise, that is, measurement error specific to the survey situation. Using the structure of the survey questions on expected returns and the two points of the probability distribution, applying the assumption of lognormal returns, and adding survey response error yields

$$\tilde{m}_i = \mu_i + \varepsilon_{mi}, \ \varepsilon_{mi} \sim N(0, \omega_{\varepsilon m}^2) \tag{6}$$

$$\tilde{p}_{0i} = \Phi(\frac{\mu_i}{\sigma_i} + \varepsilon_{0i}),\ \varepsilon_{0i} \sim N(\psi, \omega_{\varepsilon p}^2) \tag{7}$$

$$\tilde{p}_{20i} = \Phi(\frac{\mu_i - 0.2}{\sigma_i} + \varepsilon_{20i}),\ \varepsilon_{20i} \sim N(0, \omega_{\varepsilon p}^2) \tag{8}$$

where $\tilde{m}_i$, $\tilde{p}_{0i}$, and $\tilde{p}_{20i}$ are the error-ridden latent variables that determine survey responses. Survey error is assumed to be independent across the three answers, with mean zero except for *p*0 where its mean is ψ , which allows for the documented gaps between *m* and *p*0. An interpretation of ψ is that, on average, respondents answer the question about positive returns (*p*0) as if they had some positive threshold in mind instead of zero ($\Phi\left(-\psi / \sigma_i\right),\ \psi < 0$). The survey responses are transformed versions of latent variables $\tilde{m}_i$, $\tilde{p}_{0i}$, and $\tilde{p}_{20i}$ because of rounding. Recall that the VRI probability scale is for rounded responses. Similarly, as discussed above, the risk tolerance questions yield discrete responses. In the following subsection we discuss how our estimation procedure handles this issue.

C. Joint estimation of heterogeneity in stock market expectations and risk tolerance

Given the models of heterogeneity in preferences and beliefs (equations (2) and (5)) and the structural interpretation of the survey questions together with the additive survey response errors ((3), (4), (6), (7) and (8)), we can now move to estimation of the model. The parameters to be estimated are $\Xi \equiv \{\bar{\theta}, \bar{\mu},\ \bar{\sigma},\ \omega_{u\theta}^2,\ \omega_{u\mu}^2,\ \omega_{u\sigma}^2,\ \rho_{\mu\sigma},\ \psi,\ \omega_{\varepsilon\theta 1}^2,\ \omega_{\varepsilon\theta 2}^2,\ \omega_{\varepsilon m}^2,\ \omega_{\varepsilon p}^2\}$. We allow for $\bar{\theta}$, $\bar{\mu}$, $\bar{\sigma}$, and ψ to vary with covariates. Additionally, we allow the beliefs about returns to be correlated with risk preference, so the covariates of $\bar{\mu}$ and $\bar{\sigma}$ include the latent θ_i .

Note that the variables $\tilde{m}_i$, $\tilde{p}_{0i}$, and $\tilde{p}_{20i}$ in (6), (7), and (8) are before rounding. Actual survey response m_i is a rounded version of $\tilde{m}_i$ as m_i is restricted to take an integer value.

Survey responses p_{0i} and p_{20i} are to take a value from the set $\{0,5,10,15,25,35,\ldots,75,85,90,95,100\}$, we assume that $\tilde{p}_{0i}$ and $\tilde{p}_{20i}$ are rounded to the closest values allowed for each response. Also note that the survey does not allow for p_{20i} to be larger than p_{0i}. Hence when we observe $p_{20i} = p_{0i}$, we consider the possibility that the survey response error actually generated $\tilde{p}_{20i} > \tilde{p}_{0i}$ but after imposing the constraint we observe the equality in the actual responses. Together with interval responses, these formulae tell the range of survey response error terms that generate the responses of individual *i* that we observe, given μ_i, σ_i, and θ_i.

Let Z_i be the set of responses from respondent *i* to the five questions used for the estimation and $[\varepsilon_{ki}^{low}(\mu_i,\sigma_i,\theta_i \mid Z_i), \varepsilon_{ki}^{high}(\mu_i,\sigma_i,\theta_i \mid Z_i)]$ (for $k = \theta_1, \theta_2, m, p_0, \text{and } p_{20}$) be the range of survey responses error term that is consistent with the observed response for each question given the latent preference and belief parameters, calculated from (3), (4), and (6)-(8). Then the individual likelihood function for respondent *i* is calculated as:

$$
\begin{aligned}
L_i(\Xi \mid Z_i) = \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \prod_{k=\theta 1,\theta 2,m,20} & [\Phi(\frac{\varepsilon_{ki}^{high}(\mu_i,\sigma_i,\theta_i \mid Z_i)}{\omega_{\varepsilon k}}) - \Phi(\frac{\varepsilon_{ki}^{low}(\mu_i,\sigma_i,\theta_i \mid Z_i)}{\omega_{\varepsilon k}})]\times \\
& [\Phi(\frac{\varepsilon_{0i}^{high}(\mu_i,\sigma_i,\theta_i \mid Z_i)-\psi}{\omega_{\varepsilon 0}}) - \Phi(\frac{\varepsilon_{0i}^{low}(\mu_i,\sigma_i,\theta_i \mid Z_i)-\psi}{\omega_{\varepsilon 0}})]\times \\
& \phi(\frac{\log\theta_i - \bar{\theta}}{\omega_{u\theta}})\, f(\mu_i,\sigma_i) d\theta_i d\sigma_i d\mu_i,
\end{aligned}
\tag{9}
$$

where $\phi(\bullet)$ and $\Phi(\bullet)$ are the PDF and CDF of the standard univariate normal distribution and $f(\bullet,\bullet)$ is the PDF of a bivariate normal distribution with the mean vector $\begin{pmatrix} \bar{\mu} + \beta_{\mu\theta}\theta_i \\ \bar{\sigma} + \beta_{\sigma\theta}\theta_i \end{pmatrix}$ and the

covariance matrix $\begin{pmatrix} \omega_{u\mu}^2 & \rho_{\mu\sigma}\omega_{u\mu}\omega_{u\sigma} \\ . & \omega_{u\sigma}^2 \end{pmatrix}$. (The specification $\begin{pmatrix} \bar{\mu}+\beta_{\mu\theta}\theta_i \\ \bar{\sigma}+\beta_{\sigma\theta}\theta_i \end{pmatrix}$ means that estimates of $\bar{\mu}$ and $\bar{\sigma}$ are for θ_i =0. However, the estimated parameters $\beta_{\mu\theta}$ and $\beta_{\sigma\theta}$ are small in magnitude so that this affects only the third digits of the $\bar{\mu}$ and $\bar{\sigma}$ estimates throughout the estimated distribution of θ_i . Also note that, for technical purposes, we left-truncate the distribution of σ_i at zero. Under the estimated parameters the chance of σ_i being less than zero in the non-truncated distribution is essentially zero, so this is an innocuous assumption. $\beta_{\mu\theta}$ and $\beta_{\sigma\theta}$ capture potential dependence of the belief parameters on the preference parameter.)

Then the overall likelihood function is obtained as:

$$L(\Xi \mid Z) = \Pi_i L_i(\Xi \mid Z_i).$$

We calculate $L_i(\Xi \mid Z_i)$ using the Gaussian quadrature approximation. See Appendix C for the detailed algorithm.

Table 4. Distribution of Preferences and Beliefs

		Mean	Standard deviation	25th pctile	Median	75th pctile
Preferences						
Risk tolerance parameter	θ_i	0.41	0.33	0.20	0.32	0.50
Subsistence consumption	$-\kappa$	17,000				
Beliefs						
Mean of return	μ_i	0.06	0.06	0.02	0.06	0.10
Standard deviation of return	σ_i	0.12	0.03	0.10	0.12	0.14

Notes: Statistics are calculated from the estimated parameters in Table B1; see the notes to Table B1 for more detail. The summary statistics in this table are from estimates without covariates. The estimation model constrains κ to be constant across individuals. Appendix Table B2 reports the estimates of the statistical model with covariates.

Table 4 shows key estimated statistics of the distribution of preferences and beliefs based on the estimated statistical model of preferences, beliefs, and response error. Table B1 in the appendix shows the estimates of the underlying parameters of the model. The subsistence level

of consumption ($-\kappa$) is estimated to be \$17,000. The negative value of κ generates decreasing relative risk aversion as in the basic/luxury good model of Wachter and Yogo (2010). The design of the SSQ does not allow heterogeneity in κ to be readily identified, although it tightly identifies its mean. The estimated mean of the risk tolerance parameter (θ) implies low risk tolerance on average. A respondent with the mean level of θ and κ has relative risk tolerance 0.34 (relative risk aversion 2.9) when the consumption level is \$100,000. In terms of the SSQ question, she would be indifferent between a fixed consumption of \$100,000 and the 50-50 gamble of doubling that consumption and losing 20 percent. There is a considerable heterogeneity in risk tolerance. At the 25th percentile of risk tolerance parameter, the point of indifference is the downside risk of losing 13 percent; at the 75th percentile the point of indifference is the downside risk of losing 29 percent. These numbers indicate higher levels of risk tolerance than in a representative sample of Americans older than 50 years of age. Kimball, Sahm and Shapiro (2008) estimate the corresponding risk tolerance percentiles (25th, 50th and 75th) to imply indifference to 7, 12 and 20 percent of downside risk, respectively.

Beliefs about mean stock returns are in line with historical mean returns, on average. Beliefs about standard deviation are slightly lower than the historical value of 0.16. Heterogeneity in perceived mean returns (μ) is substantial, with the lowest 25 percent believing expected returns to be 2 percent or less and the top 25 percent believing 10 percent or more. At the same time, estimated heterogeneity in the perceived standard deviation of stock returns (σ) is small, perhaps because it is easier for people to estimate the second moment of the returns distribution than the first moment, as pointed out by Merton (1980).

According to our estimates heterogeneity in preferences and beliefs are weakly related. More risk tolerant respondents believe that stock returns are slightly higher, but we don't find

association of risk tolerance and beliefs about the standard deviation of returns. Beliefs about the mean and the standard deviation of returns are weakly positively correlated. Preferences and beliefs are significantly related to observable right hand side variables in our sample (Table B2 in the Appendix). However, when interpreting these associations, one has to keep in mind that the VRI sample is selected on wealth and stock ownership. For example, sample selection may explain the negative correlation of wealth and stock market expectations. Almost all households in the VRI sample have nonzero stockholding. With fixed costs of stock market participation wealth should matter at the extensive margin on top of expectations. As a result, we expect wealthier stockholders to have lower expected returns than less wealthy stockholders.

Based on the estimated distribution summarized in Table 4, 17 percent of the population expects negative stock returns. As we will see, this part of the population holds less stock than on average, but still has substantial stock market exposure. Symmetrically, 17 percent expect returns to be larger than 12 percent, rates of return that should make people hold the vast majority of their wealth in stocks given the distribution of risk and risk preferences. Though this part of the population holds more stock than on average, very high stock shares are uncommon. Taken together, these facts suggest that expectations are correlated with stock shares in an attenuated fashion, a finding that our analysis will verify in the next section.

The Table 4 results take into account substantial estimated survey noise. The parameters of the survey noise distributions are presented in Appendix Table B1. To understand the magnitude of noise, consider the differences in terms of the survey responses of individuals with the estimated averages of latent preferences and beliefs, one without measurement error and one with a positive standard deviation unit shock of measurement error. A one standard deviation unit measurement error in the first risk tolerance SSQ would make the survey response imply a

point of indifference of a 38% drop of consumption instead of the 20% implied by an error-free answer. A one standard deviation unit measurement error in the second risk tolerance SSQ would make the response imply an indifference point of 27% instead of 17%. One standard deviation unit measurement error in the response to the expected stock returns question would result in a response of 14% instead of 6%; one standard deviation unit measurement error in the stock market probability answers would change *p0* responses to 67% from 48% and *p20* responses to 25% from 12%. The estimated bias of the measurement error in the *p0* response (ψ) suggests that, on average, people think of positive gains only when they exceed 4 percent when answering the *p*0 question. Allowing for covariates (Appendix Table B2), ψ is estimated to be substantially less negative among more educated and wealthier people, indicating that their threshold value is closer to the nominal threshold zero.

D. Estimating individual-specific cardinal proxies of risk tolerance and beliefs

In the previous sections, we show how to separately identify the true heterogeneity in preferences and beliefs and the survey response errors in the survey measures of them. In this subsection, we explain how we construct the individual-specific belief and preference parameters based on those estimates that are immune from the standard effects of using generated regressors.

1. *Constructing individual-specific preference and belief parameters*

Using the estimation results we calculate individual-specific proxy variables $\hat{\mu}_i$, $\hat{\sigma}_i$ and $\hat{\theta}_i$. These proxies are the expected values of the corresponding latent variables: the individual-specific expected value and standard deviation of the distribution of stock market returns perceived by the individual (μ_i, σ_i), and the individual-specific latent risk tolerance parameter (θ_i). They are expected values conditional on the individual's responses to the survey questions

on stock market returns (m_i, p_{0i}, p_{20i}) and to the SSQ's with the two hypothetical gambles. To get these expected value of the latent individual-specific parameters conditional on the survey response and the statistical model, there are two steps. First, the distribution of the latent variables conditional on the observed responses can be obtained from the likelihood function using Bayes' theorem. Second, integrating out this function yields the individual-specific proxy variables ($\hat{\Omega}_i \equiv \{\hat{\mu}_i, \hat{\sigma}_i, \hat{\theta}_i\}$) as the conditional expectations of the latent variables given the observed survey responses. To be more specific, they are calculated as:

$$\begin{aligned}\hat{\Omega}_i \equiv E[\Omega_i \mid \hat{\Xi}, Z_i] = \frac{1}{L_i(\hat{\Xi} \mid Z_i)} \times \\ \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \Omega_i \prod_{k=\theta 1,\theta 2,m,20} [\Phi(\frac{\varepsilon_{ki}^{high}(\mu_i,\sigma_i,\theta_i \mid Z_i)}{\hat{\omega}_{\varepsilon k}}) - \Phi(\frac{\varepsilon_{ki}^{low}(\mu_i,\sigma_i,\theta_i \mid Z_i)}{\hat{\omega}_{\varepsilon k}})] \times \\ [\Phi(\frac{\varepsilon_{0i}^{high}(\mu_i,\sigma_i,\theta_i \mid Z_i) - \hat{\psi}}{\hat{\omega}_{\varepsilon 0}}) - \Phi(\frac{\varepsilon_{0i}^{low}(\mu_i,\sigma_i,\theta_i \mid Z_i) - \hat{\psi}}{\hat{\omega}_{\varepsilon 0}})] \times \\ \phi(\frac{\log\theta_i - \hat{\bar{\theta}}}{\hat{\omega}_{u\theta}}) f(\mu_i,\sigma_i) d\theta_i d\sigma_i d\mu_i,\end{aligned} \tag{10}$$

where $L_i(\hat{\Xi} \mid Z_i)$ is the individual likelihood function (calculated from equation (9)) evaluated under the estimated parameters. We use the same numerical approximation used in the estimation for this calculation. See Appendix C for details.

2. *Using individual-specific preference and belief parameters in regressions*

Our aim is to use the survey-based estimates of individual-specific parameters to explain heterogeneity in portfolio choice. In contrast with classical measurement error that is uncorrelated with true values but correlated with measured values, the error in the proxy variables is the error of optimal prediction, which is uncorrelated with measured values. Thus, when entered on the right-hand-side of linear regressions, this type of non-classical measurement

error does not induce attenuation bias in the regression coefficients of these proxy variables (see Kimball, Sahm and Shapiro, 2008).

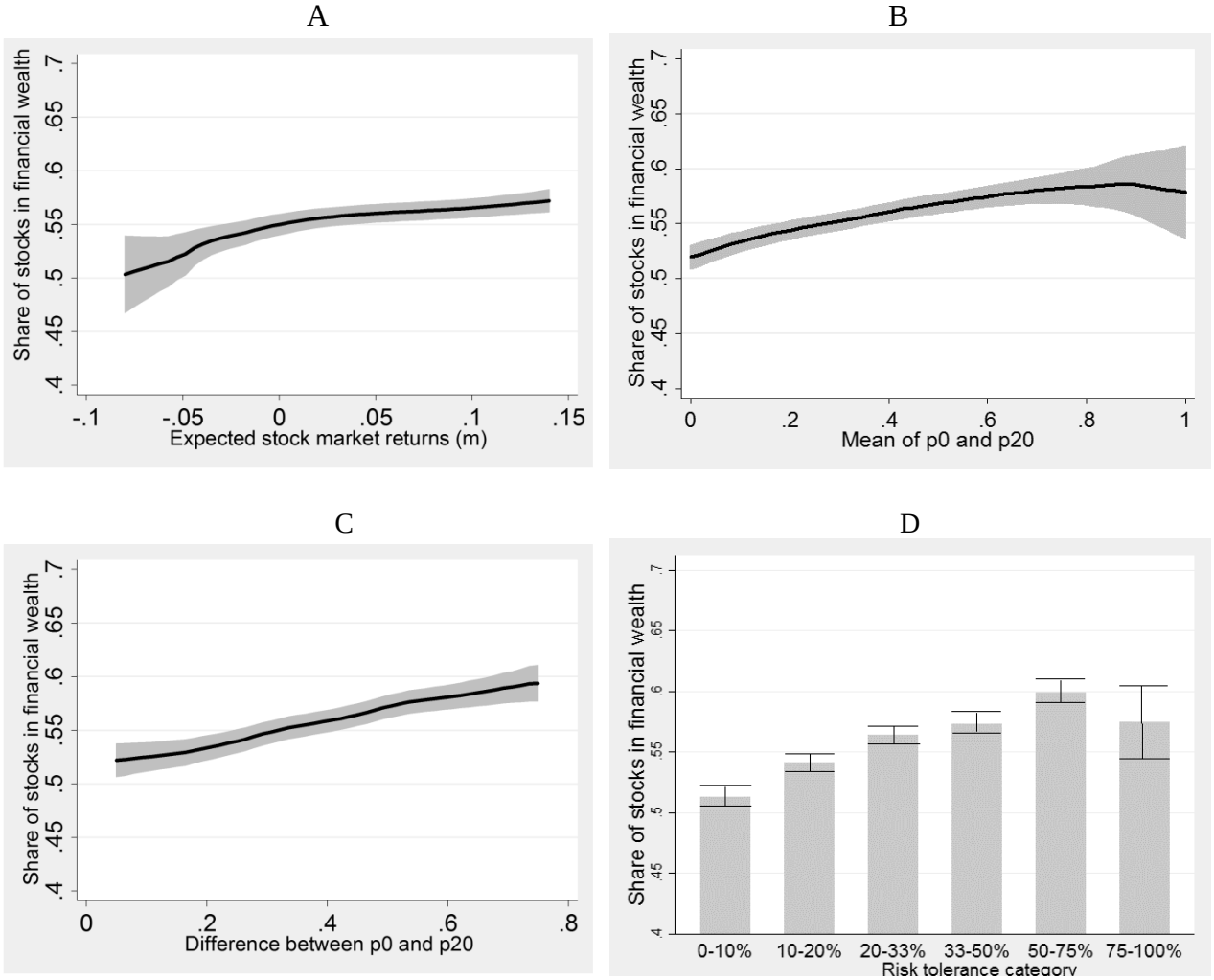
When the regressions include other covariates as well the OLS estimates are unbiased if the proxies are estimated conditional on those covariates, too. We therefore estimate two sets of proxies. The first set is conditional on the survey answers to the risk tolerance and the stock market belief questions only. The second set is conditional on other covariates as well. We use the second set of proxy estimates as right-hand-side variables in regressions that also include those covariates. In the next section, we present such regressions to explain portfolio behavior based on our estimates of preferences and beliefs.

IV. Explaining Heterogeneity in Portfolio Choice

A. Stock share and answers to survey questions

Before turning to the regressions based on our structural estimates of the latent preferences and beliefs, we investigate the relationship between the stock share of household portfolios and the raw survey responses. Figure 1 shows non-parametric regressions of the stock share in total financial assets on the survey answers to expected stock market returns (m_i), the average between the probability that the stock market would go up and that of an increase of 20 percent or more $((p0_i + p20_i)/2)$, the difference between those two ($p0_i - p20_i$), and the answer to the risk tolerance question with income level $100,000. (Figure B2 in Appendix B shows the analogous non-parametric regression results on $p0_i$ and $p20_i$ separately.)

Figure 1. Stock Share versus Raw Survey Responses



The results indicate a positive relationship between the stock share of household portfolios and expected stock market returns (m_i) and the mean of the two probability responses $((p0_i + p20_i)/2)$. The stock share is also positively related to the difference between the responses to the probability questions ($p0_i - p20_i$), suggesting a negative relationship with perceived risk of stock returns. Finally, the stock share is monotonically positively related to the answers to the risk tolerance question except for the last categories that has relatively few responses, suggesting a monotonic positive relationship with risk tolerance. Hence, the

relationship between the raw survey responses and the stock share has the direction benchmark theories of portfolio choice would suggest.

We also estimate linear regressions with the survey measure and the administrative measure of stock share as alternative left-hand-side variables and the same right-hand-side variables entered with and without the control variables that include detailed measures of demographics, education, employment, income, wealth, as well as background risks of long-term care and longevity. The results are included in Tables B5 and B6 in the Appendix. The results imply similar relationships of stock share with the survey answers with or without the control variables. The magnitudes of the associations are difficult to interpret because not all measures have a cardinal interpretation and because of the presence of survey noise. These problems are addressed in the next section.

B. Stock share and cardinal proxies of expectations and risk tolerance

Our more structural analysis has two goals. First, it relates the stock share of household portfolios to cross-sectional heterogeneity in preferences and expectations in a way that is related to portfolio choice theory thus making magnitudes easier to interpret. Second, it aims at incorporating survey noise in the estimation thus reducing its effect on the estimated magnitudes. This is a structural analysis in the sense that it makes use of additional assumptions in order to relate stock shares to heterogeneity in latent preferences and expectations. The analysis is still reduced form in the sense that it aims at uncovering associations without claims for causality. Nonetheless, since the explanatory variables are proxies that have cardinal interpretations relevant for economic theories, they potentially convey much more information than the relationship of raw survey responses to economic outcomes.

Start from a general function of the solution of optimal stock share

$$s_i^* = s^*\left(\mu_i, \sigma_i, \theta_i, \kappa; x_i, u_i\right) \tag{11}$$

where μ_i and σ_i are the beliefs of person *i* about the mean and the standard deviation of one-year-ahead stock returns, θ_i is the risk tolerance parameter, x_i is a vector of wealth, demographic variables and other risk factors that are measured in our data, and u_i combines all unobservables. We assume that unobservables are independent of observables.

The relative deviation of s^* around its mean value is related to relative deviations of the other variables around their mean values, holding values of x_i constant by

$$\frac{s_i^* - \overline{s^*}}{\overline{s^*}} \approx \beta_0 + \beta_1 \frac{\mu_i - \bar{\mu}}{\bar{\mu}} + \beta_2 \frac{\sigma_i - \bar{\sigma}}{\bar{\sigma}} + \beta_3 \frac{\theta_i - \bar{\theta}}{\bar{\theta}} + \beta_4' x_i + u_i. \tag{12}$$

The coefficients approximate the first derivatives of the function around the mean values, with $\beta_1 = \partial \tilde{s}^* / \partial \tilde{\mu}$, $\beta_2 = \partial \tilde{s}^* / \partial \tilde{\sigma}$ and $\beta_3 = \partial \tilde{s}^* / \partial \tilde{\theta}$, where the tilde denote relative differences from mean values. This approximation is a way of log-linearizing the function that allows observations with nonpositive values of some of the variables, which is relevant for μ_i in our case. We linearize about the risk tolerance parameter rather than relative risk tolerance to avoid the ambiguity that relative risk tolerance depends on the level of consumption.

We estimate (12) using the observed stock share s_i to approximate the target stock share s_i^* and the individual proxies $\hat{\mu}_i$, $\hat{\sigma}_i$, and $\hat{\theta}_i$ approximating the latent variables μ_i , σ_i , and θ_i as described earlier. We estimate the equation by OLS both with and without covariates. (We do not use a Tobit-type procedure to account for the truncation at 0 and 1 because there are very few observations (less than 2 percent of the sample) at these boundaries.) When we control for covariates in the stock share equation, we enter the structural parameters that were estimated conditional on the same covariates. Kimball, Sahm, and Shapiro (2008) show that it is necessary

to construct the proxies conditional on the same covariates as included in the main regression to deliver unbiased coefficient estimates. As the proxies are generated regressors, we estimated the standard errors by bootstrapping the entire estimation procedure including the structural estimation of the model underlying the proxies. We estimated two versions of each regression: one with the survey measure of the share of stocks in total financial wealth on the left hand side and one with the administrative measure of stock share in wealth held at Vanguard. The main results using the survey measure of stock shares are in Table 5.

The estimates show that the share of stocks is positively related to the perceived mean of stock market returns, negatively related to the perceived standard deviation of stock market returns, and positively related to the risk tolerance parameter. The estimated coefficients are precisely estimated for the expected return ($\hat{\mu}_i$) and risk tolerance ($\hat{\theta}_i$) variables and somewhat less precisely estimated for the standard deviation of returns ($\hat{\sigma}_i$). The coefficients are very similar whether we enter them with or without the covariates. The results are similar using the administrative measure of stock shares as the dependent variable except for smaller and less precise estimates for risk tolerance (see Appendix Table B7).

Table 5. Stock Shares versus Cardinal Proxies for Preferences and Beliefs

	Optimal Proxies		Error-Ridden Proxies	
Expected return	0.058	0.055	0.017	0.020
	(0.010)	(0.009)	(0.004)	(0.004)
Perceived standard deviation	-0.093	-0.083	-0.029	-0.019
	(0.046)	(0.051)	(0.006)	(0.007)
Risk tolerance parameter	0.034	0.033	0.021	0.020
	(0.009)	(0.010)	(0.005)	(0.005)
constant	-0.001	1.136	-0.001	1.120
	(0.008)	(0.649)	(0.007)	(0.565)
covariates	N	Y	N	Y
R^2	0.019	0.045	0.013	0.039
N	4414	4414	4414	4414

Notes: Stock share in total financial wealth (survey measure) are regressed on proxies for the expected stock returns, perceived standard deviation of stock returns, and the risk tolerance parameter. For the first two columns, right-hand-side variables are the optimal cardinal proxies ($\hat{\mu}_i$, $\hat{\sigma}_i$, and $\hat{\theta}_i$) calculated from (10). For the next two columns with the error-ridden proxies, the right-hand-side variables are the raw survey answers to the stock market expectation question (m_i), a crude transformation of the probability answers to approximate perceived risk defined as $\breve{\sigma}_i = 0.2 \Big/ \left(\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i}) \right)$, and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions (κ set to zero). (The parameter $\breve{\sigma}_i$ is not defined if the denominator is zero so we imputed 0.2 for the denominator for such observations to obtain $\breve{\sigma}_i = 1$, which is larger than the maximum of the non-imputed values. This imputation affects less than 10% of the observations. Alternative imputations that replace the denominator with other values yield very similar estimates.) All variables are expressed as relative differences normalized to their mean values (as specified in equation (12)). Control variables: married, male, age, whether respondent comes from the employer-sponsored subsample, education (below college; college; MBA; PhD, other higher degree); log financial wealth, log wage, dummy for owning a house, log annuity income (Social Security and DB pensions) for retired, log expected annuity income for non-retired; dummy for retired, log home stock; subjective probability of needing long-term care, and longevity expectations. Bootstrap standard errors in parentheses. See Appendix Tables B8 and B9 for full results including coefficients of covariates.

According to the point estimates, a one percent higher perceived mean is associated with one twentieth of a percent higher stock share; a one percent higher perceived standard deviation is associated with around one tenth of a percent lower stock share; and a one percent higher risk tolerance parameter is associated with one thirtieth of a percent higher stock share. Converting the relative magnitudes to absolute ones, our estimates imply that for the stock share to be higher by 1 percentage point expected returns need to be higher by 2.1 percentage points, the perceived

standard deviation needs to be lower by 2.4 percentage points, or the risk tolerance parameter needs to be higher by 0.24.

Comparing our estimates to the literature is not straightforward as most papers do not have cardinal proxies for the expectations and risk tolerance variables, and those that do estimate functional forms that are different from ours. Wherever we can make the comparison we find magnitudes that are very similar to our estimates. The closest to our specification are the estimates of Amromin and Sharpe (2012). On a sample of stockholders with positive expected returns they regress the log of the stock share on the log of their proxies of μ and σ. Their point estimates are +0.04 and -0.11, respectively. These magnitudes are very close to ours. The results of the Tobit model of Vissing-Jorgensen (2003), estimated on a sample of investors, imply that one percentage point higher returns expectations are associated with about 0.5 percentage point higher equity share. Kezdi and Willis (2011) estimate a coefficient of 0.3 in a truncated regression model estimated on a representative sample with stock shares on the left hand-side. Our log-linearized estimates imply that, around its mean, a one point difference in μ is associated with a 0.45 percentage point difference in stock shares. In a Tobit model of stock shares that combines the extensive and intensive margins Kimball, Sahm and Shapiro (2008) find a small magnitude for the association with the cardinal proxy of risk tolerance.

The columns labeled "error-ridden proxies" in Table 5 show results of analogous estimations that do not account for measurement error in the survey answers. Instead of the cardinal proxies $\hat{\mu}_i, \hat{\sigma}_i, \hat{\theta}_i$, these regressions include the raw survey answers to the stock market expectation question (m_i), a crude transformation of the probability answers to approximate perceived risk, and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions. (See note to Table 5 for the precise

definitions of these error-ridden proxies.) The coefficient estimates are qualitatively similar to the baseline results reported in column (1) and (2) of Table 5, but the magnitudes are considerably attenuated. The absolute values of the point estimates are one third to one half of the baseline estimates. These results are consistent with substantial measurement error in the raw survey answers. They show the importance of taking into account measurement error in the construction of the proxies and in using them in econometric models.

C. Alternative samples and specifications

The findings are similar for different subsamples and specifications. Appendix Tables B11 – B18 present complete results. This section discusses them briefly.

Our sample is drawn from two groups of Vanguard clients: those with employer-sponsored plans and those with individual accounts. We can gain insight into whether the selection of being a Vanguard customer affects the results. For employer-sponsored plans, the employer selected Vanguard. For individual accounts, the individual selected Vanguard. Self-selection on unobserved attributes is arguably substantially less severe for the employer-sponsored sample. Appendix Tables B11 and B12 show estimates of the same regression as the first two columns of Table 5. Estimated coefficients are quite similar across sample suggesting that individual-level selection of Vanguard is not driving the results.

Appendix Tables B13 and B14 show results for individuals with at least 50 or 70 percent of their financial wealth at Vanguard. These individuals are more selected toward Vanguard, but also may have better measurements owing to higher engagement with Vanguard or having accounts spread over fewer providers. The results are again similar.

Appendix Table B15 shows results for individuals who have some directly-held stock. This subsample result is again similar. Appendix Table B16 shows results by education levels.

In the survey data, MBAs show less attenuation with respect to expected returns. There are no other differences by education.

Appendix Table B17 shows that the results are similar even if we include housing wealth either as a safe asset or as a risky asset in the calculation of the share of risky assets.

Finally, the columns labeled "Administrative stock share" in Appendix B show the results for using the fraction of stocks in assets held at Vanguard using the administrative account measure. The results in the text (and those in Appendix B labeled "Survey stock share") use the stock share of all assets from the survey response. In brief, there is little difference in the results across the measures of stock shares. Appendix Table B18 confirms that the difference between the two stock share measures is not correlated with the estimated beliefs and only marginally correlated with the estimated preferences.

D. Interpreting the magnitudes: The Attenuation Puzzle

How might one evaluate the estimates relative to an economic model? The simplest model of Merton (1969) with CRRA utility would imply that the coefficient on log μ should be 1, the coefficient on log σ should be -2, and the coefficient on log θ should be 1 again. The same implications hold if we modify the utility function in the Merton model to incorporate the subsistence level of consumption as in equation (1) above, or if we include deterministic labor income (Merton, 1971). The Merton model is based on strong assumptions: it requires continuous rebalancing, no background risk, and it allows for unlimited leverage and short sales. We therefore investigate whether adding these realistic features would move the predictions of the benchmark model more in line with what we observe in the data. Appendix D has an analysis of a more realistic lifecycle model. It shows, for the purposes of the cross-section regression explaining portfolio choice, the Merton model is a very good approximation.

The *relative* magnitudes of the estimated coefficients reported in Table 5 are remarkably close to these theoretical implications of the Merton benchmark. In the regressions on the survey measure of stock share, the coefficient on the (approximately log-linearized) expected value and risk tolerance proxies are close to each other, and the coefficient on the standard deviation proxy is close to be negative two times their magnitudes. At the same time, the magnitudes are much smaller than in the benchmark model: each estimate is about one twentieth of what a simple theory implies.

In principle, the attenuation bias may arise from classical errors in variables on the right hand-side or appropriate non-classical errors in the left hand-side variable. Recall that our measures of beliefs and preferences already take care of substantial survey noise that arise from noisy responses conditional on the latent variables. While it is of course possible for those latent variables to exhibit additional noise, due to, for example, mood effects, that noise would have to be extremely large for the observed attenuation. The magnitude of the attenuation and its similar strength across the coefficients call for an explanation beyond these measurement issues.

We can represent the substantially attenuated association of stock holding with beliefs and preferences by expressing observed stock shares as a linear combination of the individual optimum s_i^* and the average stock share $\bar{s}$ plus additional heterogeneity

$$s_i = \lambda s_i^*\left(\mu_i, \sigma_i, \gamma_i\right) + \left(1-\lambda\right)\bar{s} + v_i \tag{13}$$

where λ is the weight on the individual optimum given beliefs and preferences, (1- λ) is the weight on the average stock share, and v_i is heterogeneity in stock shares due to other factors. This model can be viewed as a simple statistical representation of the attenuation. It can also be interpreted as a behavioral model, in which investors consider the possibility that everyone else may choose the average stock share even if their own beliefs and preferences imply a different

choice, and their decision combines the two. Such behavior could also account for the finding we discussed earlier that those who report negative expected returns in the survey continue to hold stock and those who are very optimistic do not have extreme exposure to the stock market.

Expressing equation (13) in deviations from averages, denoting the coefficients of the log-linearized optimal stock share by β^0 and decomposing heterogeneity due to other factors into observed and unobserved parts yields

$$\frac{s_i - \bar{s}}{\bar{s}} \approx \beta_0 + \lambda\beta_1^0 \frac{\mu_i - \bar{\mu}}{\bar{\mu}} + \lambda\beta_2^0 \frac{\sigma_i - \bar{\sigma}}{\bar{\sigma}} + \lambda\beta_3^0 \frac{\theta_i - \bar{\theta}}{\bar{\theta}} + \beta_4' x_i + u_i. \qquad (14)$$

This is a constrained version of equation (12), with the Merton solution implying $\beta_1^0 = 1$, $\beta_2^0 = -2$, and $\beta_3^0 = 1$.

Table 6. Observed Stock Shares versus Theoretically-Warranted Index of Expectations and Preferences

λ	0.046	0.045
	(0.006)	(0.006)
covariates	N	Y
R^2	0.017	0.044
N	4414	4414
p-value of Wald test on restriction	0.240	0.258

Notes: Regression results from equation (12) imposing $\beta_1^0 = 1$, $\beta_2^0 = -2$, and $\beta_3^0 = 1$. Bootstrap standard errors in parentheses.

We estimate the constrained versions of each unconstrained regression presented in Table 5. Table 6 shows the results. The estimated λ is around 0.05. The proportionality restriction holds reasonably well in the data as one would expect from inspection of the results in Table 5. (The Wald test does not reject the null of proportionality.) These results imply that the theoretically-warranted index expressed in equation (14) that gives a weight of one for expected return

$\frac{\mu_i - \bar{\mu}}{\bar{\mu}}$, negative two for expected standard deviation $\frac{\sigma_i - \bar{\sigma}}{\bar{\sigma}}$, and one for risk tolerance $\frac{\theta_i - \bar{\theta}}{\bar{\theta}}$ does explain heterogeneity in stock returns, but with strong attenuation.

V. Conclusion

This paper uses a distinctive measurement and analytic strategy that combines high-quality measurement of portfolio shares, preferences about risk, and beliefs about returns in an attempt to explain heterogeneity in the composition of household portfolios. The approach uses purposely-constructed measures to elicit measures of preferences and beliefs that have quantitative interpretations. This paper does find that risk preference and moments of the subjective returns distribution—both mean and variance—do have a role in understanding why portfolio choices differ. Relative to each other, the magnitudes of the coefficients on expected returns, perceived risk, and risk tolerance are in proportion with the predictions of benchmark theories. That the survey measures of preference and belief do align with portfolio choices provides external validation of our approach to measuring them.

The size of the estimated associations of the risk and belief parameters is, nonetheless, substantially smaller in magnitude than benchmark theories would suggest. We call this finding the "attenuation puzzle." Our methods produce risk and belief parameters that measure the precise, quantitative variables that should explain portfolio choice. Moreover, the statistical procedure deals with measurement error in these parameters, which is one most obvious source of such attenuation. Hence, the attenuation cannot be dismissed because the measures of preference and belief are only loosely related to what people have in mind when choosing their portfolio.

Instead, the attenuation puzzle exists because people behave in ways that deviate substantially from what standard finance theories prescribe. Those theories say that people should react strongly to their preferences and beliefs. For example, changes in their beliefs should lead to a substantial reallocation of their portfolios, possibly with frequent transactions. Moreover, many people with mild risk aversion should hold levered portfolios (see, for example, Ayres and Nalebuff, 2010). All of these prescriptions run counter to standard household finance advice, most of which suggest buy and hold strategies of portfolios with stocks between zero and 100 percent. Typical life cycle funds offer those kinds of portfolios, too, and none of the more extreme ones standard finance theories imply for many investors. Perhaps the way to make sense of the attenuation puzzle is to recognize that people do react to their own preferences and beliefs when choosing their investment portfolios, but that reaction is greatly damped because of the advice they receive (either from their advisors or from their peers; see Arrondel, Calvo-Pardo, Giannitsarou and Haliassos, 2017, for the effect of peers on portfolio choice) and the products offered, combined with limited attention to their own portfolios, strong inertia in portfolio, and sensitive beliefs.

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