NBER WORKING PAPER SERIES

SOME EMPIRICAL EVIDENCE ON THE PRODUCTION LEVEL AND PRODUCTION COST SMOOTHING MODELS OF INVENTORY INVESTMENT

Martin Eichenbaum

Working Paper No. 2523

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 1988

Graduate School of Industrial Administration, Carnegie Mellon University and the Graduate School of Business, University of Chicago. This research was supported by a grant from the National Science Foundation. Helpful comments were made by Tony Braun and John Huizinga. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #2523
February 1988

Some Empirical Evidence on the Production Level and Production Cost Smoothing Models of Inventory Investment

ABSTRACT

The production smoothing model of inventories has long been the basic paradigm within which empirical research on inventories has been conducted. The basic hypothesis embedded in this model is that inventories of finished goods serve primarily to smooth production levels in the face of fluctuating demand and convex cost functions. However once we allow for shocks to technology and the costs of producing output firms will also use inventories to shift production from periods in which production costs are relatively high to periods in which production costs are relatively low. In this sense inventories can serve to smooth production costs rather production levels. In this paper we examine the empirical plausibility of the production level and production cost smoothing models of inventories. Our basic strategy is to derive and contrast a set of unconditional moment restrictions implied by these models in a way that minimizes the role of auxiliary assumptions regarding market structure and industry demand. We find overwhelming evidence against the production level smoothing model and very little evidence against the production cost smoothing model. We conclude that the variance of production exceeds the variance of sales in most manufacturing industries because the production cost smoothing role of inventories is quantitatively more important than the production level smoothing role of inventories.

Martin Eichenbaum
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, Illinois 60637
(312) 702–0459

1. Introduction

The production smoothing model of inventories has long been the basic paradigm within which empirical research on inventories has been conducted. The basic hypothesis embedded in this model is that inventories of finished goods serve primarily to smooth production levels in the face of fluctuating demand and convex cost functions. In fact, the variance of production exceeds the variance of shipments in virtually all manufacturing industries (see for example Blanchard (1983), Blinder (1986a) and West (1986)).

Broadly speaking the responses to this finding fall into one of two categories. First, a variety of authors have modified the traditional linear quadratic production smoothing model to allow for an "accelerator" target inventory level, which arises because it is costly for firms to allow inventories to deviate from some fraction of actual or expected sales (see for example Blanchard (1983), Eichenbaum (1984) and West (1986) for empirical formulations of this accelerator inventory target level). Kahn (1987a, 1987b) formally demonstrates that this accelerator effect can be justified by modeling the stockout avoidance motive for inventory accumulation. Once this effect is embedded in otherwise standard production smoothing models there is no a priori reason to expect the variance of sales to exceed the variance of production.

Second, a variety of authors have sought to modify the basic production smoothing models in ways which imply that firms hold inventories primarily to smooth production costs rather than production levels per se. For example Remy (1987), following a suggestion made in Blinder (1986a), argues that if imperfectly competitive firms operate in a region of declining marginal costs, then cost minimizing firms will choose to make production more variable than sales.

Other authors like Eichenbaum (1984), Maccini and Rossana (1984), Blinder (1986b), Christiano and Eichenbaum (1987), Miron and Zeldes (1987) and West (1987) retain the assumption of convex cost functions but convert the production level smoothing model into a production cost smoothing model by allowing for shocks to technology and the costs of producing output. Here firms use inventories to shift production to periods in which production costs are relatively low. In this sense inventories serve to smooth production costs rather than levels and there is no a priori reason to expect an unambiguous ordering between the variance of production and sales. The relative magnitudes of these two unconditional moments will depend on all of the structural parameters which describe agents' production possibilities, the preferences underlying demand for the goods in question, market structure and the laws of motion for the shocks to demand and costs.

It is difficult to judge the plausibility of these alternative responses based on the existing empirical evidence. On the one hand, the evidence presented by Blinder (1986b), Remy (1987), and West (1987) is based on models that do not lead to testable over–identifying restrictions. On the other hand, the evidence presented by Blanchard (1983), Eichenbaum (1984), Christiano and Eichenbaum (1987) and Miron and Zeldes (1987) is obtained from models which are formally over–identified. Unfortunately, when these over–identifying restrictions are tested, they are decisively rejected. In our view, these rejections ought to be interpreted with a great deal of caution, at least with respect to the issue of the importance of technology shocks. The models considered by Blanchard (1983), Eichenbaum (1984) and Christiano and Eichenbaum (1987) embed a host of strong auxiliary assumptions regarding the nature of demand and market structure. Under these circumstances it is simply very difficult to ascertain which aspects of the models are being rejected. And none of these authors focus their empirical analysis on the importance of costs shocks per se.

The purpose of this paper is to examine the empirical plausibility of the production level and production cost smoothing models of inventories. Our basic strategy is to derive and contrast a set of unconditional moment restrictions implied by these models in a way that minimizes the role of auxiliary assumption regarding market structure and industry demand. Consequently we focus our analysis on the necessary conditions for cost minimization when firms can meet sales, at least in part, out of inventories.

We use an empirical methodology that was suggested by Hansen (1982) and Hansen and Singleton (1982) who show how to exploit conditional moment restrictions which emerge from economic theory to estimate and test representative consumer models using generalized method of moments estimators. As Garber and King (1983) have emphasized, many applications of this methodology have assumed the absence of shocks to agents' criterion functions. So for example, in their analyses of the intertemporal capital asset pricing model, Hansen and Singleton (1982), Dunn and Singleton (1986), Eichenbaum and Hansen (1988) and Eichenbaum, Hansen and Singleton (1988) do not allow for shocks to the representative agent's preferences. Under these circumstances the economic theory being investigated generates relationships between the data and the parameter vector of interest which would be exact abstracting from the fact that the econometrician does not directly observe agents' conditional expectations. The only source of error terms in these econometric analyses is the difference between agents' conditional expected values of various functions of observable state variables and their ex post values. When we rule out shocks to firms' costs our analysis maps directly into this estimation and testing strategy. However, when we investigate the production cost smoothing model we must modify that strategy to allow for the fact that while agents' observe the shocks to their cost functions, the econometrician does not. Since the relationships which we investigate are linear in the state variables, we are able to utilize a strategy discussed in Cumby, Huizinga and Obstfeld (1983) and Hansen and Sargent (1982) to overcome the difficulties associated with unobserved shocks to agents' criterion functions.

The remainder of this paper is organized as follows. In section 2, we describe the technology facing firms and then, using this specification, derive relations among inventories, sales and technology shocks. In section 3, we describe a strategy for estimating and testing the two versions of our model. In section 4, we discuss our data and present our empirical results. Finally, concluding remarks are contained in section 5.

2. The Model

In this section we accomplish two tasks. First, we display a simple parameterization of the total cost function faced by firms which embodies both the production level and production cost smoothing motive for holding inventories. We then derive a first order condition for cost minimization which must be met in equilibrium.

Since we wish to accommodate two different types of inventory costs that have been considered in the literature, we assume that total inventory costs, denominated in units of some numeraire consumption good, are given by

$$(2.1) \quad C_{It} = (b/2)[S_t - cI_t]^2 + e_{1t}I_t + (e_2/2)I_t^2,$$

where b, c, and e_2 are nonnegative scalars, e_{1t} is a nonnegative deterministic scalar which may depend on time while S_t and I_t denote the representative firm's time t sales and stock of inventories at the beginning of time t. The last two terms in (2.1) correspond to the inventory holding cost function adopted by Blinder (1982, 1986a, 1986b), among others. This term reflects the costs of storing inventories of finished goods. The term $(b/2)[S_t - cI_t]^2$ represents the costs associated with allowing inventories to deviate from some fixed proportion of sales. This "accelerator" term has been used in a variety of empirical analyses (see for example Blanchard (1983), Eichenbaum (1984), Christiano and Eichenbaum (1986), West (1986,1987) and Remy (1987)).

We assume that the cost function, C_{Qt}, of producing output can be approximated by the quadratic function

$$(2.2) \quad C_{Qt} = \nu(t)Q_t + (a/2)Q_t^2.$$

Here Q_t denotes the time t output of the firm and $\nu(t)$ is a stochastic shock to the marginal cost of output. When the scalar a is positive, the marginal cost of producing output is an increasing function of output, so that (2.2) embodies the production level smoothing role of inventories emphasized by Blinder (1986a). When the scalar a is negative, the marginal cost of producing output is decreasing. This is the case emphasized by Remy (1987). Finally, when ν_t is stochastic, (2.2) embodies the production cost smoothing role of inventories considered by Blanchard (1983), Eichenbaum (1984), Christiano and Eichenbaum (1987), West (1987) and Miron and Zeldes (1987).

The firm's time t total costs are given by

$$(2.3)' \quad C_t = C_{Qt} + C_{It}.$$

The link between production, inventories of finished goods and sales is given by,

$$(2.4) \quad Q_t = S_t + \Delta I_t,$$

where Δ denotes the first difference operator. Substituting (2.1), (2.2) and (2.4) into (2.3)′ we can write C_t as

$$(2.3) \quad C_t = \nu_t[S_t + \Delta I_t] + (a/2)[S_t + \Delta I_t]^2 + (b/2)[S_t - cI_t]^2 + e_{1t}I_t + (e_2/2)I_t^2.$$

Consider the problem of a firm which seeks to maximize the expected discounted value of its profits:

$$(2.5) \quad E_0[\sum_{t=0}^{\infty} \beta^t \{p_tS_t - C_t\},$$

where p_t is the time t price of the good, measured in units of the numeraire good and β is a discount factor between zero and one. The operator $E_t(\cdot) = E[\cdot|\Omega_t]$ denotes the conditional expectations operator, where Ω_t denotes the firm's time t information set, $t \geq 0$. Throughout we assume that Ω_t includes the values of all variables which appear in the model dated t–j, $\forall\, j \geq 0$.

In order to provide a complete solution to this problem we must impose strong restrictions on the nature of market structure and industry demand (see for example Eichenbaum (1983)). Since we seek to minimize the role of auxiliary assumptions in our empirical analysis we work only with the necessary condition for cost minimization. For any given sales (and revenue) process, the first order necessary condition for cost minimization is given by the expectational difference equation:

$$(2.6)\quad E_t\{(1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1}\} = E_t\{\nu_t/a\beta - \nu_{t+1}/a - S_{t+1} + (1-bc/a)S_t/\beta + e_{1t}/a\beta\},$$

where λ and $\lambda^{-1}\beta^{-1}$ are the roots of $\beta^{-1}X^2 + \phi X + 1 = 0$, $|\lambda| < 1$ and

$$(2.7)\quad \phi = -\left[\frac{(1+\beta)}{\beta} + \frac{bc^2+e_2}{a\beta}\right]$$

Using methods in Hansen and Sargent (1980,1981) it is straightforward to show that the optimal plan for I_t must satisfy

$$(2.8)\quad I_t = \lambda I_{t-1} + a^{-1}(1-\lambda)E_t[\sum_{j=0}^{\infty}(\lambda\beta)^j\nu_{t+j}] - a^{-1}\nu_t - a^{-1}\lambda\, E_t[\sum_{j=0}^{\infty}(\lambda\beta)^j e_{1t+j}] + [1-\lambda(1-bc/a)]E_t[\sum_{j=0}^{\infty}(\lambda\beta)^j S_{t+j}] - S_t.$$

Suppose that $(1-bc/a) > 1$. Then $bc/a < 0$ which cannot be the case if b, c and a are all nonnegative. If we rule this case out for now then $(1-bc) < 1$. Since $0 < \lambda < 1$, it follows that $[1-\lambda(1-bc/a)] > 0$. Consequently relation (2.8) implies that I_t is an increasing function of current and expected future sales. This captures the notion that firms hold inventories in order to smooth production levels in the face of fluctuating demand. Also according to (2.8) I_t depends negatively on the current value of sales. Again this captures the notion that, in the presence of an increasing marginal cost function, firms would rather meet current sales out of current inventories than increase current output.

Relation (2.8) also implies that desired inventories depend negatively on ν_t, the time t shock to production costs. This is because firms wish to lower production levels in periods when marginal production costs are high. In addition I_t depends positively on current and expected future shocks to costs. This motive for inventory accumulation reflects the fact that firms wish to build up inventories (via production) in periods when costs are *relatively* low, and meet future sales out of these stocks of inventories. In this sense inventories can serve to smooth production costs rather than production levels. Finally relation (2.8) implies that I_t depends negatively on current and expected future values of the linear component in inventory holding costs, e_{1t}.

3. Testing the Production Level and Production Smoothing Versions of the Model

In this section we accomplish three tasks. First, we derive the set of testable implications which the model of section 2 imposes when there are no shocks to costs. Second, we derive the analogous restrictions which that model imposes when the marginal costs of production are stochastic. Finally, we show how to simultaneously estimate and test both versions of the model using the Generalized Method of Moments (GMM) procedure discussed in Hansen (1982) and Hansen and Singleton (1982).

Estimating The Production Level Smoothing Model

When there are no shocks to the marginal cost of production, relation (2.6) can be written as,

$$(3.1) \quad E_t\{(1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1} + S_{t+1} - (1-bc/a)S_t/\beta - e_{1t}/a\beta\} = 0.$$

It is convenient to define the vector valued function

$$(3.2) \quad X_{t+1} = [I_{t+1}, I_t, I_{t-1}, S_{t+1}, S_t],$$

and the parameter vector

$$(3.3) \quad \sigma_0 = \{\beta, \lambda, bc/a\}.$$

With this notation relation (3.1) can be written as

(3.4) $E_t H(X_{t+1}, \sigma_o) = 0, \forall\, t \geq 0,$

where

(3.5) $H(X_{t+1}, \sigma_o) = (1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1} + S_{t+1} - (1-bc/a)S_t - e_{1t}/a\beta.$

Relation (3.4) implies that

(3.6) $d_{t+1} = H(X_{t+1}, \sigma_o)$

satisfies the moment restrictions

(3.7) $E_t d_{t+1} = 0.$

According to (3.7) d_{t+1} is orthogonal to any random variables contained in Ω_t, including endogenous variables like I_{t-j} and S_{t-j}, $j \geq 0$. Consequently, the elements of Ω_t can be used as instruments in estimating σ_o. Note that (3.7) does not rule out the possibility that d_{t+1} is conditionally heteroscedastic.

In defining our estimation procedure it is to convenient to let Z_t denote an R–dimensional vector of elements in Ω_t, where R is greater than or equal to three, the dimensionality of the unknown parameter vector σ_o. Let $\otimes$ denote the Kronecker product operator. Then the R–dimensional function g_T

(3.8) $g_T(\sigma) = (1/T) \sum_{t=1}^{T} Z_t \otimes d_{t+1}(\sigma).$

can be calculated given a sample on $\{X_t$: t=1,2,...T+1$\}$.

Assuming that $\{I_t, S_t\}$ is a stationary and ergodic stochastic process, it follows from results in Hansen (1982) that σ_0 can be estimated by choosing that value of σ, say σ_T, that minimizes the quadratic criterion

$$(3.9) \quad J_T = g_T(\sigma)' W_T g_T(\sigma).$$

Here W_T is a positive definite matrix that can depend on sample information.

Hansen (1982) also shows that the estimator which results in the minimum asymptotic covariance matrix of σ_T is obtained by choosing W_T^{-1} to be a consistent estimator of

$$(3.10) \quad Y_0 = \sum_{k=-\infty}^{\infty} E(Z_{t+k} \otimes d_{t+k+1})(Z_{t+k} \otimes d_{t+k+1})'.$$

Relation (3.7) and the fact that d_{t+1} is contained in Ω_t implies that $Z_t \otimes d_{t+1}$ is serially uncorrelated. Thus according to our theory,

$$(3.11) \quad E(Z_t \otimes d_{t+1})(Z_{t+k} \otimes d_{t+k+1})' = 0 \quad \text{for all } k \neq 0.$$

so that

$$(3.12) \quad Y_0 = E(Z_t \otimes d_{t+1})(Z_t \otimes d_{t+1})'.$$

Proceeding as in Hansen (1982) and Hansen and Singleton (1982) we estimate Y_0 by replacing the population moments in (3.12) by their sample counterparts evaluated at σ_T.

The previous discussion assumed that I_t and S_t are stationary and ergodic processes. In fact both of these random variables exhibit marked trends. The practice in the existing empirical inventory literature has been to model these trends as polynomial functions of time and apply the models to detrended data. It is possible to justify this practice in our context given the the linearity of (3.1). To see this suppose that inventories and sales do in fact have trends which are deterministic polynomial functions of time. Let the superscript I_t^* and S_t^* denote the time t demeaned and detrended value of I_t and S_t, respectively. Then (3.1) can be written as

$$(3.1)' \quad E_t\{(1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1}^* + S_{t+1}^* - (1-bc/a)S_t^*/\beta + g(t) - e_{1t}/a\beta\} = 0,$$

where g(t) is a deterministic function of time. Suppose we assume $g(t) \equiv e_{1t}/a\beta$. Alternatively we could allow ν_t to be a nonstochastic function of time. Then we could impose the assumption that g(t) equals $e_{1t}/a\beta$ plus a linear function of ν_t. Either of these restrictions amount to assuming that the observable state variables in the system inherit the trend properties of the unobserved exogenous shocks to the system. This type of assumption has been extensively used in maximum likelihood analyses of linear rational expectations models (see for example Sargent (1978)). Under this (untestable) assumption, relation (3.1)′ implies that the estimation methodology discussed above can be applied to detrended (and demeaned) inventory and sales data.

Estimating The Production Cost Smoothing Model

Consider now the situation in which we do not impose the a priori restriction that the cost function is deterministic. To see the nature of the problem that emerges here suppose for the moment that the shock to marginal costs, ν_t, is serially uncorrelated over time. Then (2.6) can be written as:

$$(2.6)' \quad E_t\{(1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1} + S_{t+1} - (1-bc/a)S_t/\beta - e_{1t}/a\beta\} = \nu_t/a\beta,$$

or

(3.13) $H(X_{t+1},\sigma_o) = \nu_t + \varphi_{t+1}$

where

(3.14) $\varphi_{t+1} = H(X_{t+1},\sigma_o) - E_t H(X_{t+1},\sigma_o).$

Then the random variable

(3.15) $d_{t+1} = \nu_t + \varphi_{t+1} = H(X_{t+1},\sigma_0).$

does not satisfy the condition $E_t d_{t+1} = 0$ since $\nu_t \in I_t$. We conclude that the restrictions implied by the production level smoothing model summarized by (3.7) do not hold for any version of the production cost smoothing model. The previous argument also shows that the presence of *any* stochastic measurement error in I_t or S_t will overturn condition (3.7).

Given assumptions regarding the time series representation of ν_t, it is still possible to derive moment restrictions analogous to those given by (3.7) for the production cost smoothing model. In our empirical analysis we proceed as in Eichenbaum (1984), Blinder (1986b) and Christiano and Eichenbaum (1987), among others, and assume that ν_t has the AR(1) representation

(3.16) $\nu_t = \rho\nu_{t-1} + \epsilon_t,$

where $|\rho| < 1$, ϵ_t is fundamental for the ν_t process, with finite unconditional second moment, and $E_{t-1}\epsilon_t = 0 \ \forall \ t \geq 0$.[3.1] Since $E_t\nu_{t+1} = \rho\nu_t \ \forall \ t \geq 0$, relation (2.6) can be expressed as

$$(3.17)\quad E_t\{(1-\lambda L)(1-\lambda^{-1}\beta^{-1}L)I_{t+1} + S_{t+1} - (1-bc/a)S_t/\beta - e_{1t}/a\beta\} = a^{-1}(\beta^{-1}-\rho)\nu_t.$$

It is convenient to write (3.17) as

$$(3.18)\quad d_{t+1} = (1-\rho/a)\nu_t + \varphi_{t+1} = H(X_{t+1},\sigma_0).$$

where φ_t is defined in (3.14). Applying the operator $(1-\rho L)$ to both sides of (3.18) we obtain

$$(3.19)\quad (1-\rho L)H(X_{t+1},\sigma_0) = k_{t+1}$$

where

$$(3.20)\quad k_{t+1} = (1/\beta - \rho/a)\epsilon_t + \varphi_{t+1} - \rho\varphi_t.$$

Since $E_t\varphi_{t+1} = 0$ and $E_{t-1}\epsilon_t = 0 \ \forall\ t \geq 0$, it follows that

$$(3.21)\quad E_{t-1}k_{t+1} = 0 \qquad \forall\ t \geq 0.$$

According to (3.21) k_{t+1} is orthogonal to any random variable in Ω_{t-1}. Consequently any element of Ω_{t-1} can be used as an instrument in estimating σ_0.

More generally, if ν_t followed an AR(q) process, $\rho(L)\nu_t = \epsilon_t$, then the random variable $k_{t+1} = \rho(L)H(X_{t+1},\sigma_0)$ would be orthogonal to I_{t-q}. Thus the presence of shocks to technology systematically change the nature of the set of variables which can be used as legitimate instruments in the estimation procedure.

Absent any restrictions on the time series process for ν_t it is not in general possible to use S_{t-j} and I_{t-j} as instruments for any finite value of j. Under these circumstances, the analyst must use as instruments variables which are plausibly argued to be uncorrelated with all current and lagged values of the cost shocks. (See for example Remy (1987) who uses current and lagged values of aggregate military expenditures in her empirical analysis.)

Proceeding as before we define the R dimensional function

$$(3.22) \quad g_T(\sigma) = (1/T) \sum_{t=1}^{T} [Z_{t-1} \otimes k_{t+1}(\sigma)],$$

where Z_{t-1} is an R dimensional vector of elements of I_{t-1}. The parameter vector $\sigma_o = \{\beta,\lambda,\rho,bc/a\}$ can estimated by choice of σ_T which minimizes $J_T = g_T(\sigma)' W_T g_T(\sigma)$ where W_T^{-1} is a consistent estimator of

$$(3.23) \quad Y_0 = \sum_{k=-\infty}^{\infty} E(Z_{t+k-1} \otimes k_{t+k+1})(Z_{t+k-1} \otimes k_{t+k+1})'$$

Unlike the disturbance term in the estimation equation for the production level smoothing model the random variable k_{t+1} is not serially uncorrelated. However, it follows from (3.20) and the restrictions $E_t \varphi_{t+1} = 0$ and $E_{t-1} \epsilon_t = 0 \; \forall \; t \geq 0$, that

$$(3.24) \quad E(Z_{t-1} \otimes k_{t+1})(Z_{t+k-1} \otimes k_{t+k+1})' = 0 \quad \text{for all } |k| > 1.$$

Consequently,

$$(3.25) \quad Y_0 = \sum_{k=-1}^{1} E(Z_{t+k-1} \otimes k_{t+k+1})(Z_{t+k-1} \otimes k_{t+k+1})'.$$

Again we can estimate Y_0 by replacing the the population moments in (3.25) by their sample analog moments evaluated at σ_T. Using the same arguments as we made for the production cost smoothing model we can rationalize applying our estimation strategy to demeaned and detrended inventory and sales data.

The estimation strategy discussed above also gives rise to a straightforward test of the production level and production cost smoothing models. Hansen (1982) shows that the minimized value of the GMM criterion function, J_T, is asymptotically distributed as a chi–square random variable with degrees of freedom equal to the difference between the total number of unconditional moment restrictions and the number of coordinates in σ. This fact can be exploited to test the over–identifying restrictions imposed by the two models.

4. Empirical Results

The models discussed in sections 2 and 3 were estimated and tested using monthly sales and inventory data from aggregate nondurables manufacturing and the six (two digit SIC) industries identified by Belsely (1969) as being of the production to stock type: Tobacco, Rubber, Food, Petroleum, Chemicals and Apparel. The data on inventories and sales were obtained from the Department of Commerce and cover the period 1959:5 – 1984:12. The inventory data are end of month inventories of finished goods, adjusted by the Bureau of Economic Analysis from the book value reported by firms in constant dollars. The data were also adjusted using a procedure suggested by West (1983) which ensures that inventories and shipments are measured in comparable units. Unfortunately, seasonally unadjusted data on inventories and sales are not available. Miron and Zeldes (1987) suggest an approximate procedure for reinserting the seasonal factors back into the data. They find, however, that their results are quite insensitive to this correction. Consequently, we proceeded as much of the empirical literature does and used seasonally adjusted data which were demeaned and detrended using a second order polynomial function of time..

In implementing the estimation procedure discussed in section 3 to the production level smoothing model we specified the instrument vector Z_t to be:

$$(4.1) \quad Z_t = [1, S_{t-j}, I_{t-j}; j = 1,2]$$

In addition we set β a priori equal to .995. With five unconditional moment restrictions and a two dimensional parameter vector, the J_T statistic is asymptotically distributed as a chi–square with three degrees of freedom. We report our results in the right hand column of Tables 1 through 4 labelled "Production Level Smoothing Model". A number of results are worth mentioning. First, and perhaps most importantly, there is overwhelming evidence against the over–identifying restrictions implied by the production level smoothing model. For every industry these can be rejected at even the .001 significance level. In light of these results, we did not re–estimate the model for specifications of Z_t which included values of S_{t-j} and I_{t-j}, $j > 2$. Second, the estimated values of $\alpha = 1-bc/a$ are quite large, although in no instance do they exceed the maximum admissible value of 1.0. Third, with the exception of the Tobacco industry, the estimated values of λ are estimated to be quite large.

To provide a behavioral interpretation for λ it is convenient to show how the standard stock adjustment model can be mapped into our framework. According to that model inventories evolve according to

$$(4.2) \quad \Delta I_t = (1-\lambda)[I_{t-1} - I^*_{t-1}]$$

where I^*_{t-1} denotes the actual or "target" stock of inventories at the end of time t–1. The parameter λ governs the speed of adjustment of actual to target inventories.

Let the target level of inventories I^*_{t-1} be the level of inventories such that if $I_{t-1} = I^*_{t-1}$, then actual inventory investment, ΔI_t, equals zero. Relation (2.8) implies that

$$(4.3) \quad I^*_{t-1} = a^{-1}E[\sum_{j=0}^{\infty}(\lambda\beta)^j\nu_{t+j}|I_t] - a^{-1}\nu_t/(1-\lambda)$$
$$[1-\lambda(1-bc/a)]E[\sum_{j=0}^{\infty}(\lambda\beta)^j S_{t+j}|I_t]/(1-\lambda) - S_t/(1-\lambda).$$

Substituting (4.3) into (2.8) and subtracting I_{t-1} from both sides of the resulting equation we recover the stock equation specification for ΔI_t given by (4.2). According to relation (4.3) the number of days to close 95% of a gap between actual and target inventories is

$$(4.4) \quad T^S = -30\log(.05)/\log(\lambda),$$

where 30 is the approximate number of days in a month. T^s turns out to be a useful summary statistic of the speed with which actual inventories adjust to their target levels. The estimated values of λ reported in Tables 1 to 4 imply that for all industries, excluding, Tobacco, it takes more than 300 days to close 95% of the gap between actual and target inventories. These results are implausible to say the least. Overall we conclude, both on the basis of the formal statistical tests and the behavioral interpretations of the estimated parameter values, that there is overwhelming evidence against the production level smoothing model.

In implementing the estimation procedure discussed in section 2 to the production cost smoothing model we specified the instrument vector Z_{t-1} to be:

(4.5) $$Z_t = [1, S_{t-1-j}, I_{t-1-j}; j = 1,2,\ldots,\text{Klag}]$$

where Klag equaled 2, 3 or 4. As before we set β a priori equal to .995. With (2xKlag +1) unconditional moment restrictions and a three dimensional parameter vector, the J_T statistic is asymptotically distributed as a chi–square with (2xKlag–2) degrees of freedom.

We report our results in the left hand column of Tables 1 through 4 labelled "Production Cost Smoothing Model". First, notice that there is very little evidence against this version of the model. While the probability values of the J_T statistics varied across the values of Klag and industries, we cannot reject, at the five percent significance level, the over–identifying restrictions implied by the model in any case except for the Chemicals industry when Klag equals 3. In the large majority of cases we cannot reject the model at even the 10% significance level.

Second, while imprecisely estimated, in the large majority of cases, the parameter $\alpha = (1-bc/a)$ is estimated to be less than than zero. In no case was α estimated to be larger than one. Assuming that $c > 0$, these estimates are consistent with decreasing marginal costs only if we believe that b is negative, so that firms are actually rewarded for allowing inventories to deviate from $(1/c)S_t$ each period. In our view a more plausible interpretation of the estimated values of α is that both b and and a are positive, i.e. marginal costs are increasing and the accelerator effect is operative.

Third, the parameter ρ is estimated quite accurately and indicates substantial serial correlation in the stochastic component of marginal costs. Fourth, the parameter λ is estimated quite accurately and for most industries is consistent with the notion that firms close any gap between actual and target inventories quite quickly. For example when Klag equals 3, the estimated values of λ imply the values of T^S summarized below:

	T^S
Nondurables[4.2]	56 (37,77)
Tobacco[4.3]	41 (0,77)
Rubber	59 (42,77)
Food	79 (71,88)
Petroleum	73 (65,81)
Chemicals	133 (101,181)
Apparel	133 (81, 195)

In all cases the point estimates indicate that firms close 95% of a gap between actual and target inventories within approximately 4 months. Excluding Chemicals and Apparel this adjustment occurs well within three months. Overall we conclude, both on the basis of our formal statistical tests and the behavioral interpretations of the estimated structural parameters, that there is very little evidence against the production cost smoothing model of inventories.

We conclude this section by discussing an important caveat concerning our results, namely the power of the statistical tests used to assess the empirical plausibility of the production cost smoothing model. As we indicated in section 3 the presence of cost shocks (or for that matter measurement error) imples that random variables which are contained in agents' time t–1 information set but which are not contained in agents' time t information set cannot be used as instruments in testing and estimating the model.

One possible interpretation of our test results is that while the production cost smoothing is false, our test is simply not sufficiently power to detect this fact. Needless to say this interpretation of our results cannot be ruled out a priori. Given a well specified alternative model of inventory investment it would be possible to investigate the power of our tests using a variety of Monte Carlo methods. Unfortunately there does not seem to be any well developed alternative model of inventory investment which would violate the optimality conditions investigated in this paper (at least if we abstract from functional form considerations). Absent such an alternative, we take our results to be suggestive rather than definitive.

5. Conclusion

In this paper we investigated the empirical plausibility of the production level and production cost smoothing models of inventory investment. We find overwhelming evidence against the former model and very little evidence against the latter model. West (1987), working from an exactly identified model, also finds evidence that cost shocks play at least as important a role as demand shocks in determining the time series properties of inventory investment. In this sense our study is complementary to his. Based on this evidence we conclude that the variance of production exceeds the variance of sales because one of the primary functions of inventories is to allow firms to shift production from periods in which production costs are relatively high to periods in which production costs are relatively low.

As we noted in the introduction, a variety of authors have incorporated cost shocks into their empirical analyses. Yet when those models are tested they are decisively rejected. The results in this paper suggest that what is being rejected are subsets of the auxiliary assumptions maintained by those authors, not the basic production cost smoothing model of inventories. Results in Eichenbaum (1983) and Aiyagari, Eckstein and Eichenbaum (1988) suggest that the empirical implications of a large class of inventory models are not very sensitive to specification of industry structure. In our view a more promising avenue for improving the empirical performance of fully specified empirical models of inventory investment lies in a more careful analysis of industry demand and the impact of measurement errors.

TABLE 1

Nondurables

Parameters*	Production Cost Smoothing Model Klag** 2	3	4	Production Level Smoothing Model Klag 2
λ	.19 (.12)	.20 (.11)	.25 (.11)	.60 (.04)
ρ	.95 (.02)	.94 (.02)	.95 (.02)	–
α	-1.44 (.60)	-1.42 (.60)	-0.77 (.36)	.88 (.04)
J_T***	.33 (.15)	3.29 (.49)	11.08 (.91)	52.38 (1.00)

* Standard errors in parentheses
** Klag refers to the number of lags of sales and inventories used in the estimation procedure.
*** Probability levels in parenetheses.

TABLE 2

Tobacco

	Production Cost Smoothing Model			Production Level Smoothing Model
Parameters*			Klag**	Klag
	2	3	4	2
λ	-.02 (.02)	.11 (.20)	.18 (.13)	.63 (.06)
ρ	.85 (.01)	.80 (.11)	.70 (.11)	–
α	-2.02 (1.11)	-0.32 (.77)	.09 (.34)	.79 (.06)
J_T***	4.36 (.89)	8.56 (.93)	9.86 (.87)	30.23 (1.00)

Rubber

λ	.13 (.13)	.22 (.10)	.22 (.09)	.61 (.04)
ρ	.86 (.03)	.88 (.03)	.88 (.02)	–
α	-4.18 (6.22)	-1.63 (1.61)	-1.76 (1.51)	.88 (.04)
J_T***	.35 (.16)	3.69 (.55)	4.47 (.39)	32.60 (1.00)

* Standard errors in parentheses.
** Klag refers to the number of lags of sales and inventories used in the estimation procedure.
*** Probability levels in parentheses.

TABLE 3

Food

Parameters*	Production Cost Smoothing Model Klag** 2	3	4	Production Level Smoothing Model Klag 2
λ	.28 (.05)	.32 (.04)	.33 (.05)	.62 (.05)
ρ	.83 (.05)	.85 (.05)	.85 (.05)	–
α	-0.32 (0.44)	-0.13 (.36)	-0.26 (.32)	.05 (.05)
J_T***	1.00 (.39)	3.03 (.45)	6.54 (.63)	27.75 (1.00)

Petroleum

Parameters*	Production Cost Smoothing Model Klag** 2	3	4	Production Level Smoothing Model Klag 2
λ	.21 (.10)	.29 (.04)	.30 (.13)	.58 (.16)
ρ	.88 (.11)	.92 (.17)	.85 (.10)	–
α	-0.18 (.98)	.24 (.84)	.46 (.49)	.93 (.08)
J_T***	..49 (.22)	4.18 (.62)	6.30 (.61)	30.33 (1.00)

* Standard errors in parentheses.
** Klag refers to the number of lags in sales and inventories used in the estimation procedure.
*** Probability values in parentheses

TABLE 4

Chemicals

Parameters*	Production Cost Smoothing Model Klag** 2	3	4	Production Level Smoothing Model Klag 2
λ	.36 (.13)	.51 (.10)	.54 (.06)	.73 (.08)
ρ	.92 (.04)	.90 (.04)	.90 (.04)	–
α	–0.39 (0.89)	0.43 (.28)	0.26 (.22)	.95 (.03)
J_T***	5.38 (.93)	10.91 (.97)	11.21 (.92)	30.30 (1.00)

Apparel

Parameters*	Production Cost Smoothing Model Klag** 2	3	4	Production Level Smoothing Model Klag 2
λ	.42 (.18)	.48 (.15)	.45 (.11)	.63 (.06)
ρ	.82 (.04)	.81 (.04)	.85 (.04)	–
α	–1.50 (1.18)	–0.58 (.58)	0.01 (.50)	.79 (.06)
J_T***	1.12 (.43)	3.90 (.58)	9.15 (.83)	30.49 (1.00)

* Standard errors in parentheses.
** Klag refers to the number of lags of sales and inventories used in the estimation procedure.
*** Probability levels in parentheses.

Footnotes

3.1 In our model the shock ν_t is perhaps best viewed as an exogenous technology or productivity shock. Of course there are a variety of interpretations to the stochastic component of firms' marginal costs. For example, these costs will be stochastic as long as the prices of factors of production, such as labor, are stochastic. In order to be consistent with our formulation, the representative firm must view these prices parametrically. However it is possible to construct stochastic processes for these prices in which the univariate innovation, ϵ_t is orthogonal to lagged values of the prices, but for which it is not the case that $E_{t-1}\epsilon_t = 0$.

4.1 We experimented with a variety of values of β between .95 and .999. We found that our results were almost completely insensitive to the specification of β.

4.2 Standard errors for T^S were calculated by evaluating (4.4) at values of λ plus one and minus one estimated standard errors of the corresponding point estimates of λ.

4.3 For the Tobacco industry our point estimate λ minus one standard error results in a negative number so that T^S is undefined. Consequently we report a value of T^S equal to zero, which is appropriate for values of λ which are arbitrarily close to zero but positive.

References

Aiyagari, S.R., Eckstein, Z. and M. Eichenbaum, "Inventories and Price Fluctuations Under Perfect Competition and Monopoly," forthcoming in T. Kolintzas (ed.) ***The Rational Expectations Equilibrium Inventory Model: Theory and Applications,*** Springer Verlag, 1988.

Belsley, David A. (1969), **Industry Production Behavior: The Order – Stock Distinction**, Amsterdam: North Holland.

Blanchard, Olivier J. (1983), "The Production and Inventory Behavior of the American Automobile Industry," **Journal of Political Economy** 91, 265–400.

Blinder Alan S. (1982), "Inventories and Sticky Prices," **American Economic Review** 72, 334–348.

Blinder, Alan S, (1986a), "Can the Production Smoothing Model of Inventories be Saved?," **Quarterly Journal of Economics** CI, 431–454.

Blinder, Alan S. (1986b), "More on the Speed of Adjustment in Inventory Models," **Journal of Money, Credit and Banking**, 335–365.

Christiano, Lawrence J. and Martin Eichenbaum (1987) "Temporal Aggregation and Structural Inference in Macroeconomics," **Carnegie Rochester Conference** on Public Policy: **Bubble and Other Essays**, Karl Brunner and Allan H. Meltzer (ed.s), 26: 63–130.

Cumby Robert E., Huizinga, John and Maurice Obstfeld (1983) "Two – Step Two – Stage Least Square Estimation in Models With Rational Expectations," **Journal of Econometrics** 21, 333–354.

Dunn, Kenneth and Kenneth J. Singleton, (1986) "Modeling the Term Structure of Interest Rates Under Nonseparable Utility and Durability of Goods," **Journal of Financial Economics**. 17, 27–56.

Eichenbaum, Martin (1983), "A Rational Expectations Equilibrium Model of Inventories of Finished Goods and Employment," **Journal of Monetary Economics** 12, 259–278.

Eichenbaum, Martin (1984), "Rational Expectations and the Smoothing Properties of Inventories of Finished Goods," **Journal of Monetary Economics** 14, 71–96.

Eichenbaum, Martin and Lars P. Hansen (1988), "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data," forthcoming **Journal of Business and Economic Statistics**.

Eichenbaum, Martin, Hansen, Lars P. and Kenneth J. Singleton, (1988) "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Under Uncertainty," forthcoming, **Quarterly Journal of Economics.**

Garber, Peter M. and Robert G. King (1983) "Deep Structural Excavation? A Critique of Euler Equation Methods," NBER Working Paper No. 31.

Hansen, Lars P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators," **Econometerica** 50, 1029–1054.

Hansen, Lars P. and Kenneth J. Singleton (1982), "Generalized Instrumental Variables Estimation of Nonlinear Rational Models," **Econometrica** 50, 1269–1286.

Hansen Lars P. and Thomas J. Sargent (1982) "Instrumental Variables Procedures for Estimating Linear Rational Expectations Models," **Journal of Monetary Economics** 9, 263–296.

Kahn, James A. (1987a), "Inventories and the Volatility of Production," **American Economic Review** 77, 667–679.

Kahn James A. (1987b), "Some Theory and Evidence on Inventories Fluctuations," manuscript, Rochester University.

Maccini, L. and R. Rossana (1984), "Joint Production, Quasi – Fixed Factors of Production and Investment in Finished Good Inventories," **Journal of Money, Credit and Banking** 16, 218–236.

Miron, Jeffrey A. and Stephen P. Zeldes (1987), "Seasonality, Cost Shocks and the Production Smoothing Model of Inventories, manuscript, University of Pennsylvania.

Remy, Valerie A. (1987), Non–Convex Costs and the Behavior of Inventories", manuscript, University of California, San Diego.

Sargent, T.J (1978), "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations," ***Journal of Political Economy*** 86, 1009–1044.

West, Kenneth D. (1983), "A Note on the Economteric Use of Constant Dollar Inventory Series," **Economic Letters** 13, 337–341.

West, Kenneth D. (1986), "A Variance Bounds Test of the Linear Quadratic Inventory Model," **Journal of Political Economy** 91, 374–401.

West, Kenneth D. (1987), "The Sources of Fluctuations in Aggregate GNP and Inventories", manuscript, Princeton University.