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MAY THE FORCE BE WITH YOU:  
INVESTOR POWER AND COMPANY VALUATIONS

Thomas Hellmann  
Veikko Thiele

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**ABSTRACT**

This paper examines the effect of investor power in a model of staged equity financing. It shows how the usual effect where market power reduces valuations can be reversed in later rounds. Once they become insiders, powerful investors may use their market power to increase, not decrease valuations. Even though powerful investors initially lower valuations, companies prefer to bring them inside to leverage their power in later financing rounds. The paper also makes predictions about investor returns, and issues a warning that unrealized interim returns can be misleading predictors of final realized returns when powerful investors distort interim valuations.

Thomas Hellmann  
Saïd Business School  
University of Oxford  
Park End Street  
Oxford, OX1 1HP  
United Kingdom  
and NBER  
Thomas.Hellmann@sbs.ox.ac.uk

Veikko Thiele  
School of Business  
Queen's University  
Goodes Hall  
143 Union Street  
Kingston, Ontario  
Canada  
vthiele@business.queensu.ca

# 1 Introduction

Much of corporate finance assumes that investors are perfectly competitive, and the supply of capital is perfectly elastic. While this assumption may sometimes be suitable for public equity market, private equity markets typically face an inelastic supply of capital, and a limited number of investors who exercise market power. The standard implication of market power is that investors can take larger equity stakes, which drives down the valuation of the company. This result is based on a single round investment logic, where the investor is an outsider without a prior stake in the company. Many private equity markets, however, are characterized by staged financing where investors fund companies over several rounds. This is common with angel financing, venture capital, growth capital, or private placements. It also happens with publicly-listed companies that raise money in initial public offerings (IPOs), seasoned equity offerings (SEOs), or PIPEs (private investments in public equity). The fundamental difference with staged financing is that while an investor is an outsider the first time he invests, he becomes an insider thereafter. The question is whether an insider with market power would behave differently than an outsider?

This paper poses two main research questions. First, in a staged financing context, what is the effect of investor power on the valuation of a company? Second, when does a company want to bring a powerful investor inside?

To answer these questions, we build a parsimonious theory of staged financing. The company needs two financing rounds. Information is symmetric, and the company faces a supply of capital that is not perfectly elastic. Our focus is the effect of market power on the valuations of early and late rounds. As long as investors are competitive, the valuation of the late round is independent of the valuation that is obtained in the first round. However, with a powerful investor this is not true, because it matters what stake he obtained in the first round. If he is an outsider, the powerful investor always wants to push down the valuation of the second round. However, if he already has a stake in the company, there is a trade-off. On the one hand, the powerful investor invests new money and therefore wants a lower valuation just like an outsider. We call this the aggressive outsider logic. On the other hand, he already has a stake in the company, and prefers a higher valuation just like an insider. We call this the defensive insider logic. The net preference depends on the relative sizes of the existing stake versus the new investments. Our theory derives a simple condition that says that a powerful investor prefers a higher (lower) valuation whenever his second-round investment is below (above) the pro-rata threshold. In the next section we provide a simple numerical example to illustrate the logic of this result. The important novelty is that with staged financing, market power has an ambiguous effect on valu-

ations. In particular, the model identifies a large range of parameters where market power leads to higher, not lower valuations in the second-round.

Turning to the first round, we ask about the effect of market power on valuations, and whether the company wants to bring the powerful investor into the first round. We distinguish three scenarios. The benchmark scenario is perfect competition without any powerful investor. We then look at a scenario where the company obtains first-round funding from the powerful investor, and finally the scenario where the company delays bringing in the powerful investor until the second round. We find that first-round valuations are highest with competition, lower when postponing the investment of the powerful investor, and lowest when the powerful insider participates in the first round. At first glance this result seems to suggest that bringing in the powerful insider up-front is a bad idea, because it generates the lowest first-round valuation. However, as noted above, having an insider increases second-round valuations. The question becomes whether the higher second-round valuations can justify the lower first-round valuation? We formally show that the company always prefers to bring a powerful investor inside at the first round. The key intuition is that even though the company doesn't like investor power, if it exists, it is better to befriend it up-front. Once the powerful investor is inside, his market power gets used towards the benefit of the company: "May the force be with you."

The model also generates some predictions about the returns to different equity investors. Looking at the final returns of first-round investors, we note that powerful inside investors make the highest returns. However, powerful insiders have lower returns in the second round. Indeed, moderately powerful insiders who invest below pro-rata have even lower returns than the benchmark return of competitive investors. Powerful outsiders, by contrast, have the highest returns of all second-round investors. The analysis also shows that unrealized interim returns need to be interpreted with caution. For example, competitive investors experience low interim returns when the company is financed by a powerful outsider in the second round. However, these are not realized returns. In fact, these competitive investors achieve higher returns between the second round and the date where value is realized, fully recouping their benchmark returns. This last finding is a useful warning for empirical work that uses unrealized interim returns as a measure of performance. Our analysis suggests that market power invalidates the interpretation of unrealized returns as a proxy for expected realized returns.

Our paper builds on several prior literatures. Admati and Pfleiderer (1994) were the first to examine equity valuations in a staged financing context. Their model focuses on identifying robust financial contracts, and shows how an inside investor is neutral with respect to valuations when investing at pro-rata. Their model does not foresee inside investors investing below or above pro-rata, and market power is not directly accounted for. The issue of market power

does arise in models of hold-up, such as in the work of Grossman and Hart (1986), and Aghion and Tirole (1994). These models are based on incomplete contracting – no such assumption is used in our model. In this line of research, Khanna and Mathews (2015) is probably closest to our paper. They use a staged financing model where the inside investor reduces hold-up power by using high prices that also provide a signal to third parties. In our model inside investors sometimes bring about higher but other times lower prices, depending on how the new investment compares against the existing ownership stake.<sup>1</sup>

In addition to the literature on staged equity financing, there is a closely related literature about staged debt financing. The seminal work by Rajan (1992) identifies advantages and disadvantages of inside lenders who have better information on the company at the time of refinancing, but can also use that informational advantage to extract some information rents. Berglöf and Von Thadden (1994) and Dewatripont and Tirole (1994) further extend this type of analysis to allow for optimal securities and multiple investors. In our model we assume symmetric information, and therefore abstract away from informational advantages of insiders. Instead we look at sized-based market power. This approach is related to Burkhart, Gromb, and Panunzi (1997) who emphasize the importance of size for monitoring incentives. As an extension to our base model, we also consider the use of debt. We find a rationale for powerful insiders to use equity over debt. With equity they internalize valuation effects at the time of the second round, and therefore use their market power in a way that is more beneficial to the company. With debt, however, there is no need to protect prior stakes, and so market power is not used to defend the valuation.

The remainder of the paper is as follows. In the next section we provide a simple numerical example to illustrate how a powerful investor would like to change the company valuation. In Section 3 we provide our staged financing model with a powerful investor. Section 4 considers model extensions and empirical predictions. It is followed by a brief conclusion. All proofs are in the Appendix.

## 2 A Simple Numerical Example

To illustrate a central insight, we briefly consider a simple numerical example. Suppose a company needs to raise \$10M in a second-round financing. There is a powerful investor who already owns 20% of the company from the first round. Suppose that there are only two valuations under

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<sup>1</sup>Further theories about staged financing include the work of Neher (1999), Cornelli and Yosha (2003), Bergemann and Hege (2005), Hellmann and Thiele (2015), and Nanda and Rhodes-Kropf (2017).

consideration. Either the second-round investors receive 50% of the company for their \$10M investment, which implies a post-money valuation of \$20M ( $= \$10/50\%$ ). Or the second-round investors receive 40% of the company for their \$10M investment, which implies a valuation of \$25M ( $= \$10/40\%$ ). What deal would the powerful investor prefer?

Suppose first that the powerful investor plans to invest \$2M in the new round. In this case he provides 20% of the new money and receives 20% from the stake of the new investors. However, if he already owns 20% of the existing stake, and now receives 20% of the new stake, his overall stake simply remains at 20%. This is called investing at pro-rata, i.e., investing in the new round an amount that is proportional to the existing stake. It allows the existing investor to exactly maintain his ownership stake.

An insight going back to Admati and Pfleiderer (1994) is that if an investor invests at pro-rata, then he is indifferent between lower and higher valuations, precisely because he always maintains the same ownership stake. While Admati and Pfleiderer fix the investment of the insider at pro-rata, we consider what would happen if the insider invested above or below pro-rata. Suppose the powerful investor invested \$3M out of the \$10M in the second round. Under the low valuation he would get a stake of  $30\%*50\% = 15\%$ , and his existing stake would be worth  $20\%*50\%=10\%$ , so his overall stake would be 25%. Under the high valuation he would get a stake of  $30\%*40\% = 12\%$ , and his existing stake would be worth  $20\%*60\%=12\%$ , so his overall stake would be 24%. This shows that a powerful investor who invests above pro-rata prefers the low valuation. The outsider logic of investing at a low valuation dominates because the new outsider stake is relatively larger than the existing insider stake.

Consider next the case where the powerful investor invests below pro-rata. Suppose he only provides \$1M out of the \$10M in the second round. Under the low valuation he would get a stake of  $10\%*50\% = 5\%$ , and his existing stake would be worth  $20\%*50\%=10\%$ , so his overall stake would be 15%. Under the high valuation he would get a stake of  $10\%*40\% = 4\%$ , and his existing stake would be worth  $20\%*60\%=12\%$ , so his overall stake would be 16%. This shows that when the powerful investor invests below pro-rata, he prefers the high valuation. In this case the investment made in the second round is relatively small (below pro-rata) compared the existing stake. Consequently, the insider logic of preferring high valuations dominates.

This example illustrates the way that a powerful insider would like to influence the valuation of the company. Naturally, this simple example is built on several artificial assumptions. It also does not explain how the investor can influence the valuation, and what the equilibrium looks like. For all this we need a proper theory model. We turn to that in the next section.

## 3 Main Model

### 3.1 Base Assumptions

Consider a company, called  $A$ , which requires two rounds of financing for a project.<sup>2</sup> Specifically,  $A$  needs to raise an amount  $K_1$  in the first round, and  $K_2$  in the second round, with  $K_1 < K_2$ . The project generates an expected return  $x > 0$ . For parsimony we assume no discounting. There are three dates, date 1 for the first round, date 2 for the second round, and date 3 for the realization of returns.

There are  $n$  risk neutral competitive investors, which we denote by  $C$ . In Section 3.2.2 we will introduce a powerful investor.  $A$  uses a competitive auction in each round to raise the required capital. Competitive investors play a simultaneous move game where they all submit their supply schedules at the same time. The prices ( $\alpha, \beta$ , to be introduced shortly) clear the market in a standard Walrasian fashion.

We assume that the first round is small, so that only one investor provides  $K_1$ .<sup>3</sup> For each investor the cost of providing  $K_1$  is given by  $C_1(K_1) = \mu_1 K_1$ , with  $\mu_1 > 1$ .<sup>4</sup> In return the investor gets an equity share, denoted by  $\alpha$ . Similarly, the second-round investors invest  $K_2$ , and jointly receive a total equity share  $\beta$ . The post-money valuations are given by  $V_1 = K_1/\alpha$  and  $V_2 = K_2/\beta$ . We assume that  $K_2$  is large, requiring the company to raise funding from multiple investors; this is also known as a syndicated investment round. A competitive investor provides an amount  $k_2^j, j \in \{1, \dots, n\}$ , incurring a cost  $c_2^j = \mu_2 k_2^j + \frac{\gamma}{2} (k_2^j)^2$ . For  $\gamma > 0$  costs are convex, so that the supply of capital is not infinitely elastic. The increasing marginal costs of investing should be interpreted as increasing opportunity costs for investors who are allocating more capital into  $A$  (as opposed to all other investment opportunities). The more the investor allocates his portfolio to this deal, the more exposed (or less diversified) his portfolio becomes. Note that our model uses competitive but not atomistic investors. This means that the market has a finite number of investors, each facing increasing marginal investment costs. This approach

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<sup>2</sup>For simplicity we directly assume that financing comes in two stages. It is easy to extend the model to endogenously derive optimal staging. All that is required is to add some uncertainty, and the possibility that in case of failure the entrepreneur spends all remaining funds on private benefits.

<sup>3</sup>For simplicity we assume that the first round competitive investor does not participate in the second round. Relaxing this would increase the complexity of the exposition, but would not impact the results. This is because individual competitive investors are too small to affect market prices.

<sup>4</sup>To keep the model as simple as possible we assume that all investors have the same costs. However, allowing for heterogenous costs would not change the main insights from the model. Moreover, for simplicity we assume that costs in the first period are small and linear. We can think of this as the first term of a Taylor series expansion. Allowing for larger and non-linear costs would increase the complexity of the model.

will allow us to introduce market power in a simple and intuitive way, as discussed in Section 3.2.2.

## 3.2 second-round Investments and Valuations

We start by deriving the equilibrium investments and company valuation in the second round. To illustrate the effect of market power in our staged financing model, we first consider the competitive benchmark where no investor can influence the price ( $\beta$ ). We then show how market power affects investments and valuation.

### 3.2.1 Competitive Benchmark

Suppose there are  $n$  competitive investors in the second round. We assume that the second-round price, as represented by  $\beta$ , clears the market in a Walrasian fashion. We call this the  $NP$  case, which stands for “No Powerful” investor.

When investing  $k_2^j$ ,  $j = 1, \dots, n$ , in the second round, investor  $j$  gets the equity share  $\frac{k_2^j}{K_2}\beta$ . The objective function for each competitive investor is therefore given by

$$\max_{k_2^j} \frac{k_2^j}{K_2} \beta x - \mu_2 k_2^j - \frac{\gamma}{2} (k_2^j)^2.$$

Consequently, each competitive investor invests the amount  $k_2^j(\beta)$  so that the price per unit of capital equals the marginal cost:

$$\frac{1}{K_2} \beta x = \mu_2 + \gamma k_2^j. \quad (1)$$

Moreover, the equilibrium equity share for all second-round investors,  $\beta$ , is defined by the market clearing condition  $n k_2^j(\beta) = K_2$ .

Solving the system of two equations we find that the investment by a competitive investor  $j$  is  $k_2^j|^{NP} = K_2/n$ . Moreover, the equilibrium equity share  $\beta^{NP}$  is given by

$$\beta^{NP} = \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma}{n} K_2 \right]. \quad (2)$$

This implies the following company valuation:

$$V_2^{NP} = \frac{K_2}{\beta^{NP}} = \frac{x}{\mu_2 + \frac{\gamma}{n} K_2}. \quad (3)$$



For later comparison, we note that the first-round price  $\alpha$  does not affect the second-round valuation  $V_2^{NP}$ . This is because second-round investors take the first-round price  $\alpha$  as given. We will see that this is not true for the powerful investor.

It is easy to see some additional comparative statics results from the expression of  $V_2^{NP}$ . A higher cost of capital forces  $A$  to issue more equity, which implies a lower second-round valuation (i.e.,  $dV_2^{NP}/d\mu_2, dV_2^{NP}/d\gamma < 0$ ). By contrast, a higher expected return implies a lower price  $\beta$ , and therefore a higher valuation (i.e.,  $dV_2^{NP}/dx > 0$ ). Likewise, if  $A$  requires a larger amount in the second round ( $K_2$ ), then each investor needs to provide more capital (i.e.,  $dk_2^{j|NP}/dK_2 > 0$ ). Naturally the round can then only close if  $A$  issues more equity, which implies a lower valuation (i.e.,  $dV_2^{NP}/dK_2 < 0$ ). We find the opposite with respect to the number of investors  $n$ : The presence of more investors induces every single investor to invest less (so that  $dk_2^{j|NP}/dn < 0$ ). This implies a lower total cost of financing across all investors (due to the convexity of  $c_2^j$ ). The company can then raise the amount  $K_2$  with less equity, which in turn improves the company valuation ( $dV_2^{NP}/dn > 0$ ).

### 3.2.2 Powerful Investor

We now introduce a powerful investor, called  $P$ . For this it is convenient to assume that out of the  $n$  investors,  $m$  investors coordinate their investments, and therefore act as a single powerful investor ( $P$ ). Thus,  $m$  will be our measure of market power.<sup>5</sup>

Let us explain the rationale behind this way of modelling market power. In order to compare market constellations with more or less market power, we want to maintain an overall cost structure in the market. That is, we want to vary market power while holding constant the overall elasticity of capital supply. It would be tempting to simply add to the  $n$  competitive investors one powerful investor  $P$ , and give him his own convex cost function. However, this would increase the overall funds available, and thus alter the market cost structure. The approach chosen here allows us to preserve the overall cost structure. Specifically, we retain  $n$  “pockets” of financing, each with its convex cost  $c_2^j = \mu_2 k_2^j + \frac{\gamma}{2} (k_2^j)^2$ . Market power in our model is the ability to coordinate the action of a subset  $m$  of these pockets of capital. One way of thinking about this is that  $m$  investors in the market manage to collude and act as a single decision maker. This kind of coordination can be interpreted as a financial intermediary who invests as a single actor on behalf of the  $m$  “pockets” of capital. This would be a stylized description of venture capital, where the venture capital firm (aka the general partner) acts on

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<sup>5</sup>For simplicity we assume that  $n$  is sufficiently large so that the  $(n - m)$  competitive investors continue to have no market power, and thus continue to be price-takers.

behalf of the ultimate investors (aka limited partners). It can also be thought of as an investment syndicate (such as an angel group or a closely-knit group of venture capitalists), where one leader acts on behalf of the group to negotiate the deal.

We assume that the powerful investor makes investments, but cannot make any transfer payments. Due to our sparse set of assumptions, there is a theoretical possibility in this model that a powerful investor makes a transfer payment to the company, and then owns and runs the entire company. This is clearly unrealistic, and it is easy to augment the model to exclude this possibility.<sup>6</sup>

The model with a powerful investor distinguishes two cases. First, there is the case of a powerful inside investor, who participated in the first financing round, and therefore already holds a stake in the company. We denote this stake by  $\alpha^P$ . We call this the *PI* case, which stands for “Powerful Insider”. Second, there is the case of the powerful outside investor. He only arrives at the second round, and is assumed to be unavailable at the time of the first round. In this case a competitive outcome occurs at the first stage investment  $K_1$ . At the second stage we can simply set  $\alpha^P = 0$ . We call this the *PO* case, which stands for “Powerful Outsider”. We can think of two possible reasons why  $P$  is not around in the first round. One is that some investors do not like to invest in early stage deals, or find it too costly to do so (i.e.,  $\mu_1^P$  is very large). Another reason is that  $A$  does not yet want to approach  $P$ , i.e., the company wants to avoid facing the powerful investor. In the former case,  $P$ 's absence is exogenous, in the latter case it is endogenous. In Section 3.3.3 we analyze the latter case.

### 3.2.3 The Case of a Powerful Insider

Consider first the case of a powerful insider.  $P$  provides the amount  $K_2^P = mk_2^i$ ,  $i = 1, \dots, m$ , in the second round, while each of the competitive investors  $C$  invest  $k_2^j$ ,  $j = m + 1, \dots, n$ . It

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<sup>6</sup>The assumption of no transfer payments is standard in the literature (see Rajan (1992), or Hellmann (2002) for a discussion). One way to derive this constraint endogenously is to add a simple adverse selection problem. Suppose that in addition to the honest companies described so far, there are dishonest companies that can pretend to have a business that looks identical to the honest ones. The dishonest companies take the transfer payment and disappears, leaving investors with a total loss. If for every honest company there are enough dishonest ones, no investor would ever make a transfer payment, simply because the probability of investing in a dishonest company is too high. Other ways of justifying the absence of transfer payments are based on moral hazard models, where the owner/manager needs to retain as much equity as possible to continue providing effort for the success of the company.

is convenient to define each “pocket” of capital controlled by  $P$  as  $k_2^i = K_2^P/m$ ,  $i = 1, \dots, m$ . This way we derive the total investment cost for the powerful investor  $P$  as

$$C^P = m \left[ \mu_2 k_2^i + \frac{\gamma}{2} (k_2^i)^2 \right] = \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2. \quad (4)$$

When choosing his investment  $K_2^P$ ,  $P$  takes the effect on the equilibrium price  $\beta$  into account. This price is defined by the market clearing condition

$$K_2^P + (n - m) k_2^j(\beta) = K_2, \quad j = m + 1, \dots, n, \quad (5)$$

where  $k_2^j(\beta) = \frac{1}{\gamma} \left[ \frac{1}{K_2} \beta x - \mu_2 \right]$  is the amount provided by each competitive investor (which can be derived from (1) in Section 3.2.1).

$P$  chooses  $K_2^P$  to maximize his expected net return

$$\pi_2^{P|PI}(K_2^P) = (1 - \beta(K_2^P)) \alpha^P x + \frac{K_2^P}{K_2} \beta(K_2^P) x - \left[ \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2 \right], \quad (6)$$

where  $\beta(K_2^P)$  is the total equity issued to the second-round investors as a function of  $P$ 's investment (as defined by the market clearing condition (5)), and  $K_2^P/K_2$  is  $P$ 's share in the round. Intuitively  $P$ 's expected net payoff depends on three components. The second term represent his second-round equity stake ( $\frac{K_2^P}{K_2} \beta(K_2^P)$ ). The third term represents his cost of capital ( $C^P(K_2^P)$ ). So far this is similar to competitive investors. The most interesting component is the first term.  $P$  holds an equity stake  $\alpha^P$  from his first-round investment  $K_1$ . This gets diluted as  $A$  needs to issue equity to new investors (including to  $P$ ). This component therefore captures the fact that  $P$  is an inside investor who also cares about preserving his existing stake.

We informally note that the first component is decreasing in  $\beta$ . This means that as an insider  $P$  prefers a higher valuation (i.e., lower  $\beta$ ). We call this the “defensive” insider logic. The second component, however, is increasing in  $\beta$ . As an outsider  $P$  prefers a lower valuation (i.e., higher  $\beta$ ). We call this the “aggressive” outsider logic. The relative strength of these two logics will play a key role in determining how  $P$  exercises his market power. We discuss this in Section 3.3.

In the Appendix we show that in equilibrium  $P$  invests

$$K_2^{P|PI} = \frac{m}{n + m} [1 + \alpha^P] K_2. \quad (7)$$

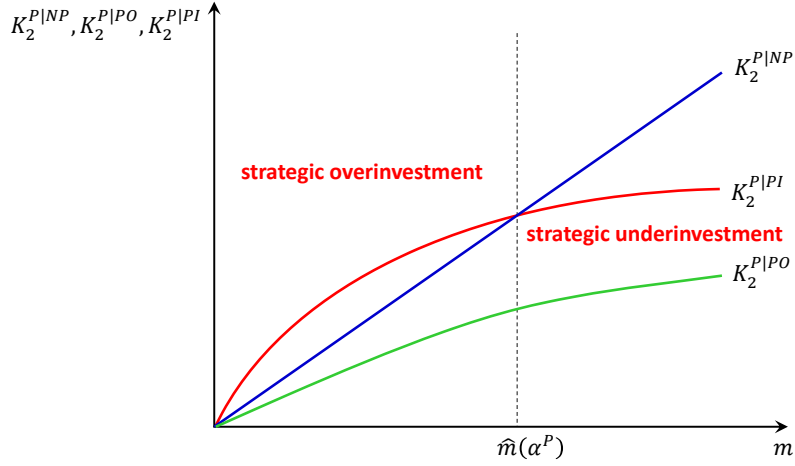


Figure 1: Second Round Investments

We can immediately see that  $P$ 's investment  $K_2^{P|PI}$  depends on his first-round equity stake  $\alpha^P$ . This also applies to the second-round valuation  $V_2^{PI}$ , given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2}. \quad (8)$$

Note that for  $m = 0$ , this simplifies back to the competitive valuation  $V_2^{NP}$  (see (3) in Section 3.2.1).

An important question is how  $P$ 's investment  $K_2^{P|PI}$  relates to the pro-rata investment. In other words, when does  $P$  invest below pro-rata and accept a dilution of his stake, and when does  $P$  invest above pro-rata to increase his stake? The next lemma provides a simple condition.

**Lemma 1**  $P$  invests above (below) pro-rata whenever  $m > (<) \hat{m} \equiv \alpha^P n$ .

Lemma 1 says that  $P$  invests above (below) pro-rata whenever his power ( $m$ ), is above (below) a critical value  $\hat{m}$ . This critical value varies with his prior equity stake ( $\alpha^P$ ). With this we can examine how  $P$ 's market power (as measured by  $m$ ), and his first-round equity stake  $\alpha^P$ , affect  $K_2^{P|PI}$  and  $V_2^{PI}$ .

**Proposition 1** Consider the PI case. The investment by  $P$ ,  $K_2^{P|PI}$ , is increasing in both  $m$  and  $\alpha^P$ . Moreover, there exists a threshold  $m'$ , with  $0 < m' < \hat{m}$ , such that the second-round valuation of  $A$ ,  $V_2^{PI}$ , is increasing in  $m$  for  $m < m'$ , and decreasing thereafter. The valuation  $V_2^{PI}$  is also increasing in  $\alpha^P$ .

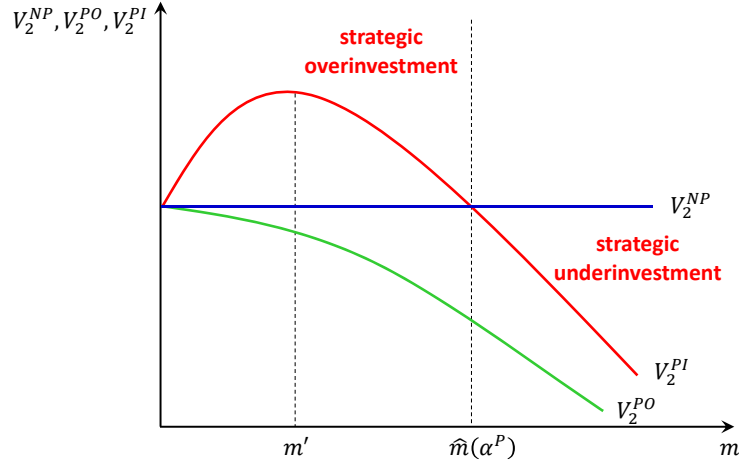


Figure 2: Second Round Valuations

Figures 1 and 2 illustrate the main insights from Proposition 1 (these two figures also compare the *PI* case with the other cases, which we discuss in more detail in Section 3.2.5). More market power ( $m$ ) encourages  $P$  to provide a larger share of the required second-round investment  $K_2$ . The effect on valuation is non-monotonic. To get an intuition, we first note that  $P$  behaves just like a competitive investor in two cases: when  $m$  is small, i.e. when there is not much concentration of power, and when  $m = \hat{m}$ , i.e., when  $P$  maintains exactly his pro-rata. Between these critical values, we find that the valuation first rises and then falls in  $m$ . The intuition for the valuation rising initially in  $m$  is that  $P$  is driven by the defensive insider logic that comes from the first component of equation (6). The intuition why it falls again is that as  $P$  invests larger amounts, he is increasingly driven by the aggressive outsider logic from the second component of (6). For  $m = \hat{m}$ , the insider and outsider logic exactly cancel out. For  $m > \hat{m}$ , the aggressive outsider logic always dominates. Note also that the intuition for why the valuation is always increasing in  $\alpha^P$  is that a higher  $\alpha^P$  increases the first component, and therefore strengthens the defensive insider logic.

### 3.2.4 The Case of a Powerful Outsider

So far we assumed that the powerful investor is an insider, with some ownership stake  $\alpha^P > 0$ . We now turn to the case of a powerful outside investor. Technically this is a special case of the model from the previous section with a powerful insider where we set  $\alpha^P = 0$  in (7) and (8).

**Proposition 2** *Consider the PO case. The investment by  $P$ ,  $K_2^{P|PO}$ , is increasing in  $m$ . The second-round valuation,  $V_2^{PO}$ , is always decreasing in  $m$ .*

In the  $PO$  case,  $P$  does not hold any equity when participating in the second financing round ( $\alpha^P = 0$ ). This means that  $P$  can only invest above pro-rata (since  $m > \alpha^P n = 0$ ).  $P$  is not concerned about equity dilution, and only uses his market power to drive the valuation down (see also Figures 1 and 2).

In the Appendix we derive some additional comparative statics results with respect to equilibrium investments and valuations for the  $PI$  and  $PO$  cases. The cost of capital ( $\mu_2, \gamma$ ), as well as the expected return ( $x$ ), do not affect  $P$ 's investment ( $K_2^{P|PI}, K_2^{P|PO}$ ). However, the cost of capital has a negative effect on valuation ( $V_2^{PI}, V_2^{PO}$ ), while the expected return has a positive effect on valuation. Furthermore, a higher capital requirement in the second round ( $K_2$ ) results in  $P$  making a larger investment ( $K_2^{P|PI}, K_2^{P|PO}$ ), leading to a lower company valuation ( $V_2^{PI}, V_2^{PO}$ ). The presence of more investors ( $n$ ) also implies a lower equilibrium investment by  $P$ , and a higher company valuation. Finally, we show in the Appendix that  $P$ 's participation constraint is always satisfied.<sup>7</sup>

### 3.2.5 Comparing Second-round Constellations

We now compare the second-round investments and company valuations when  $P$  is an insider versus outsider. We also compare both cases against the competitive benchmark ( $NP$  case). For this it is useful to define  $K_2^{P|NP} \equiv mk_2^i = \frac{m}{n}K_2$ . This is the total amount that  $P$  would provide in a (counterfactual) competitive benchmark.<sup>8</sup> All results derived in this section are illustrated in Figures 1 and 2.

We begin with a Proposition focussed on the powerful outsider ( $PO$ ) case.

**Proposition 3** *For all  $m$ , and all  $\alpha^P > 0$ , a powerful outsider makes the smallest second-round investment, i.e.,  $K_2^{P|PO} < \{K_2^{P|PI}, K_2^{P|NP}\}$ . This also results in the lowest second-round company valuation, i.e.,  $V_2^{PO} < \{V_2^{PI}, V_2^{NP}\}$ .*

Figures 1 and 2 illustrate how the investment and valuation in the  $PO$  case compare to the other scenarios. In the  $PO$  case,  $P$  does not have a stake in  $A$  ( $\alpha^P = 0$ ). It is then optimal for  $P$  to exploit his market power to drive down the valuation of the company. This can be achieved by investing a smaller amount compared to the competitive benchmark (i.e.,  $K_2^{P|PO} < K_2^{P|NP}$ , so that  $V_2^{PO} < V_2^{NP}$ ). This result is consistent with the standard economics result that market

<sup>7</sup>Note that this also implies that a constellation where  $P$  participates in the first round, but not the second round, can never arise in equilibrium.

<sup>8</sup>Note that the investment amount of an individual investment ‘‘pocket’’,  $k_2^j|NP$ , does not depend on  $m$ . However, the more individual pockets belong to  $P$ , the higher the total investment  $K_2^{P|NP}$ .

power leads to smaller quantities (here: smaller investment) and higher prices (here: lower valuation, which means higher cost of capital for the company).

To further understand why the powerful insider behaves differently from the powerful outsider, we note that having a stake in the company ( $PI$  case with  $\alpha^P > 0$ ) curbs  $P$ 's temptation to drive valuations down in the second round. That is, when choosing his second-round investment  $K_2^P$ ,  $P$  trades off the positive effect on his second-round equity stake ( $\frac{K_2^P}{K_2}\beta(K_2^P)$ ), and the negative effect on his first-round stake ( $\alpha^P$ ). A larger stake from the first financing round ( $\alpha^P$ ) makes it optimal for  $P$  to invest a larger amount in the second round, resulting in a higher valuation (see Proposition 1).

Proposition 3 shows that having a powerful outsider leads to the lowest possible second-round valuation. We also just discussed how the powerful insider increases second-round valuations, relative to the powerful outsider. It remains to be seen how the powerful insider compares to the competitive benchmark (the  $NP$  case).

**Proposition 4** *Comparing the powerful insider against the competitive benchmark, we distinguish two cases:*

- (i) **Moderately powerful insider:** *Suppose  $m \leq \hat{m}$ .  $P$  invests more in the second round compared to the competitive benchmark (i.e.,  $K_2^{P|PI} \geq K_2^{P|NP}$ ). The second-round valuation is higher compared to the competitive benchmark (i.e.,  $V_2^{PI} \geq V_2^{NP}$ ).*
- (ii) **Highly powerful insider:** *Suppose  $m > \hat{m}$ .  $P$  invests less in the second round compared to the competitive benchmark (i.e.,  $K_2^{P|PI} < K_2^{P|NP}$ ). The second-round valuation is lower compared to the competitive benchmark (i.e.,  $V_2^{PI} < V_2^{NP}$ ).*

The insights from Proposition 4 can be seen in Figures 1 and 2. At the critical level of market power ( $m = \hat{m}$ ), we know that  $P$  invests exactly at pro-rata, and therefore maintains his stake in the company. In this case the outcome is identical to the competitive benchmark, i.e.,  $P$  provides the same amount of capital in the second round ( $K_2^{P|PI} = K_2^{P|NP}$ ), leading to the same company valuation ( $V_2^{PI} = V_2^{NP}$ ).

If the inside investor is highly powerful ( $m > \hat{m}$ ), he invests more in absolute terms, but strategically reduces his investment compared to the competitive benchmark ( $K_2^{P|PI} < K_2^{P|NP}$ ), as shown in Figure 1. In doing so  $P$  accepts a dilution of his first-round equity stake, but obtains a lower company valuation in the second round ( $V_2^{PI} < V_2^{NP}$ ), as shown in Figure 2. This captures the problem of having an insider that is too powerful. At the same time we note that

it is better for  $A$  to have the powerful investor as an insider, than as an outsider. This follows from the fact that valuations are even lower with the powerful outsider.

Maybe the most interesting insight pertains to the investments of a moderately powerful insider ( $m < \hat{m}$ ). Such an inside investor strategically invests above pro-rata ( $K_2^{P|PI} > K_2^{P|NP}$ ); see Figure 1. This increases the second-round valuation, and helps to preserve  $P$ 's stake from the first-round investment, as shown in Figure 2. The moderately powerful insider is therefore beneficial for the company, in the sense that his market power is now used to the benefit of the company: “May the force be with you!”

### 3.3 First-round Investments and Valuations

We now close our staged financing model by deriving the equilibrium investments and company valuation in the first financing round. Again, we distinguish three scenarios: (i) only competitive investors participate in the first and second round (the  $NP$  case), (ii) the powerful investor only invests in the second round (the  $PO$  case), and (iii) the powerful investor participates in both financing rounds (the  $PI$  case). We ask how the first-round valuations differ across these three scenarios. In addition, we derive whether in equilibrium the powerful investor enters in the first or second round.

#### 3.3.1 Investment by Competitive Investor

Suppose the required amount  $K_1$  in the first round is provided by a competitive investor. For parsimony we derive the outcome for the  $NP$  and  $PO$  case jointly (they are very similar, the only difference being  $\beta^{NP}$  versus  $\beta^{PO}$ ).

Suppose several identical investors submit their bids to finance the first round. The winning bid, which we denote by  $\alpha^i$  ( $i \in \{NP, PO\}$ ) is then defined by the zero-profit condition for a competitive investor:

$$(1 - \beta^i) \alpha^i x - \mu_1 K_1 = 0 \quad \Leftrightarrow \quad \alpha^i = \frac{\mu_1 K_1}{(1 - \beta^i) x}. \quad (9)$$

While the competitive investor gets a stake  $\alpha^i$ , he knows that this stake will get diluted in the second round when  $A$  issues  $\beta^i$  to new investors. The equilibrium first-round price  $\alpha^i$  is then such that the investor's diluted share of the expected payoff equals his cost of investing  $K_1$ .



Using the expressions for  $\beta^{NP}$  and  $\beta^{PO}$ , we get the following first-round equilibrium valuations:

$$V_1^{NP} = \frac{K_1}{\alpha^{NP}} = \frac{1}{\mu_1} \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma}{n} K_2 \right] \right) x$$

$$V_1^{PO} = \frac{K_1}{\alpha^{PO}} = \frac{1}{\mu_1} \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] \right) x.$$

The next proposition summarizes how  $P$ 's market power ( $m$ ) as an outsider affects the company valuation when the first-round investment is provided by a competitive investor.

**Proposition 5** *Suppose a competitive investor provides the amount  $K_1$  in the first round. If a powerful outsider provides capital in the second round, then the valuation  $V_1^{PO}$  is decreasing in  $m$ .*

This result is very intuitive, and can be seen from the expression of  $V_1^{PO}$ . As noted above, the difference between the company valuations is rooted in the amount of equity issued to the second-round investors ( $\beta^{NP}$  versus  $\beta^{PO}$ ). The presence of a more powerful outsider  $P$  implies that in equilibrium more equity is being issued to the second-round investors (i.e.,  $d\beta^{PO}/dm > 0$ ). This implies that the equity stake of the competitive first-round investor gets more diluted. Thus the company needs to provide a higher  $\alpha^{PO}$  to the competitive investors, to compensate for their subsequent dilution. This means that the first-round valuation  $V_1^{PO}$  is lower.

From the expressions of  $V_1^{NP}$  and  $V_1^{PO}$  it is easy to derive that the first and second-round costs of capital ( $\mu_1, \mu_2, \gamma$ ) have a negative effect on the first-round valuation, while the effect of the expected return ( $x$ ) is positive. The required amount in the second round ( $K_2$ ) has a negative effect on the first-round valuation. The presence of more second-round investors ( $n$ ) implies a higher first-round valuation. This is because it requires issuing less equity to second-round investors, and therefore curbs the dilution of the first-round investor's equity stake.<sup>9</sup>

### 3.3.2 Investment by Powerful Investor

Now suppose that  $P$  wants to finance the first round. We know from Section 3.2.2 that  $P$  always wants to participate in the second round. Consequently,  $P$  makes the first-round investment  $K_1$

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<sup>9</sup>Not surprising we find equivalent effects on company  $A$ 's expected profit, which we denote by  $\pi^{A|i}$ ,  $i \in \{NP, OP\}$ . The only exception concerns  $K_1$ , which has a negative effect on  $\pi^{A|i}$ . We provide more details in the Appendix (see Proof of Proposition 8).

(*PI* case) as long as it leads to a higher expected profit compared to waiting for the second round (*PO*). Formally,  $P$ 's participation constraint for the first round is given by  $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$ . In the Appendix we show that this can always be satisfied.

The next proposition characterizes  $P$ 's bid in the first round.

**Proposition 6**  $P$ 's bid in the first round is given by  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , where  $\hat{\alpha}$  satisfies

$$\hat{\alpha} \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n + (1 - \hat{\alpha}) m)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1. \quad (10)$$

The resulting first-round valuation  $V_1^{PI} = K_1/\alpha^P$  is decreasing in  $m$ .

When submitting a bid to  $A$  for the first financing round,  $P$  is competing with all the other competitive investors. The equity stake  $\hat{\alpha}$ , defined in Proposition 6, makes  $A$  just indifferent between accepting the bid  $\hat{\alpha}$  and taking the bid  $\alpha^P$  from a competitive investor. The optimal bidding strategy for  $P$  is to offer the investment  $K_1$  in exchange for a slightly lower equity stake, so that  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ .

Proposition 6 also shows that a more powerful investor asks for a higher equity stake  $\alpha^P$ , which leads to a lower company valuation in the first round (i.e.,  $dV_1^{PI}/dm < 0$ ). We know that the powerful outsider fully exploits his market power in the second period to maximize his economic rents. In addition, we know from Propositions 2 and 3 that more market power implies a lower second-round valuation (see also Figure 2). It follows that the powerful outsider is very unattractive for  $A$ . This situation improves for  $A$  when  $P$  becomes an insider. Knowing this,  $P$  can ask for a higher equity stake  $\alpha^P$  in the first round. This implies a lower valuation  $V_1^{PI}$ .

In the Appendix we derive additional comparative statics results. Specifically, we show that the first-round valuation  $V_1^{PI}$  is decreasing in the costs of capital ( $\mu_1, \mu_2, \gamma$ ), and increasing in the expected return ( $x$ ) and the number of second-round investors ( $n$ ). Moreover, we find that the required financing amount in the first round ( $K_1$ ) has a positive effect on  $V_1^{PI}$ , while the effect of the required amount in the second round ( $K_2$ ) is negative.<sup>10</sup>

### 3.3.3 Comparing first-round Constellations

We can now compare the first-round valuations for the different investor constellations.

<sup>10</sup>The effects on company  $A$ 's expected profit,  $\pi^{A|PI}$ , are equivalent; see Proof of Proposition 8 in the Appendix for more details.

**Proposition 7** *The first-round valuation is lowest with a powerful insider (PI case), higher with a powerful outsider (PO case), and highest when there is no powerful investor (NP case). Formally,  $V_1^{PI} < V_1^{PO} < V_1^{NP}$  for all  $m > 0$ .*

The company valuation in the first round is lowest in the *PI* case. To see why, we first note that *P*'s offer in the first period makes *A* marginally better off, compared to having a competitive investor in the first round and *P* only participating in the second round (*PO* case). According to Proposition 3, the second-round valuation is the lowest for the *PO* constellation. *P* then exploits the lower outside option for *A*, by asking for a higher equity stake, which implies a lower first-round valuation. This also explains why the equilibrium valuation  $V_1^{PI}$  in the first financing round is below the competitive benchmark.

A key insight so far is that the presence of a powerful investor can have opposite effects on company valuations across different financing rounds. For example, with a powerful insider (*PI* case) the first-round valuation is always below the competitive benchmark, while the second-round valuation may be above the benchmark level (see Proposition 4). It therefore remains to compare *A*'s expected profits under the different investor constellations.

**Proposition 8**  *$A$ 's expected profits for the different investor constellations are as follows:  $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$  for all  $m > 0$ . Furthermore,  $A$ 's expected profit in the *PO* case ( $\pi^{A|PO}$ ) and the *PI* case ( $\pi^{A|PI}$ ) is decreasing in  $m$ .*

It is intuitive that the presence of a powerful investor always leads to reduced profits for the company. The important insight pertains to the question whether a company should bring in a powerful investor early, thereby making him an insider in later financing rounds. Having an early stake encourages *P* to use his market power to defend the company valuation in later rounds. However, bringing in *P* in at the beginning comes at the price of a lower valuation. While *A* gets a higher valuation in the second round, it pays for it up-front. Still, Proposition 8 shows that “having the force with you” leads to a higher expected profit for the company.

## 4 Extensions and Discussions

### 4.1 Investor Returns

So far, our analysis looks at equilibrium valuations at date 1 and date 2. The model therefore provides some insights into the structure of investor returns. In particular, we distinguish three

types of returns. Define  $R_{13}$  ( $R_{23}$ ) as the return that a date 1 (date 2) investment generates at date 3 when the company realizes its value. Moreover, define  $R_{12}$  as the unrealized interim return (at date 2) for a date 1 investment. In the model these three returns are defined as follows:

$$R_{12} = \frac{V_2 - K_2}{V_1} \quad R_{23} = \frac{x}{V_2} \quad R_{13} = \frac{(1 - \beta)x}{V_1} = \frac{x}{V_1} \frac{V_2 - K_2}{V_2}.$$

These returns differ across the various model permutations. We distinguish four scenarios. First we look at the benchmark case where there is no powerful investor (*NP* case). Second, we look at the case with a powerful outsider (*PO* case). Third, we consider a moderately powerful insider where  $m < \hat{m}$ . We call this the *MPI* case. Finally, there is the highly powerful insider with  $m > \hat{m}$ , which we call the *HPI* case.

We immediately state the main result.

**Proposition 9** *The returns compare as follows:*

- $R_{13}$ :  $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{NP} = R_{13}^{PO} = \mu_1$
- $R_{23}$ :  $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$
- $R_{12}$ :  $R_{12}^{MPI} > R_{12}^{NP} > R_{12}^{PO}$  and  $R_{12}^{HPI} > R_{12}^{PO}$ .

Proposition 9 contains several intuitive results. The realized returns of date 1 investors ( $R_{13}$ ) can be ranked according to the amount of market power exercised. Returns are highest for highly powerful insiders, followed by moderately powerful insiders, followed by competitive investors in the *PO* and *NP* case. Note that  $R_{13}^{PO} = R_{13}^{NP} = \mu_1$  because these are the returns of the competitive first-round investors, not the powerful investor.

The realized returns of date 2 investors ( $R_{23}$ ) are also influenced by market power, but rank entirely differently. In particular we find that returns are highest in the *PO* case. This is because a powerful investor depresses the valuation to generate the highest possible returns to himself. The powerful insider moderates the use of his market power, because of his prior stake in the company, as discussed in Proposition 4. In fact, the realized returns of date 2 investors are lowest in the *MPI* case, which is when the moderately powerful investor boosts valuation above the competitive benchmark.

The unrealized returns of date 1 investors at date 2 ( $R_{12}^{PO}$ ) follow yet another pattern. They rank lowest in the *PO* case. This is not because date 1 investors made bad investments, but because the powerful outside investor depresses the valuation. The final realized returns of

these date 1 investors ( $R_{13}^{PO}$ ) are in fact the same as the returns in the benchmark  $NP$  case (i.e.,  $R_{13}^{PO} = \mu_1$ ). This generates an important insight about the dangers of looking at unrealized interim returns: In the presence of market power, unrealized interim returns can be a misleading indicator of expected realized returns.<sup>11</sup>

## 4.2 The Role of Debt

So far our model considers equity investors. In this section we ask whether our results change in the presence of debt. The main question is whether it would be more efficient for the powerful investor to hold debt instead of equity? To examine this, suppose  $A$  wants to raise capital by issuing debt claims to investors. The (endogenous) interest rates are denoted by  $r_1$  and  $r_2$ . The expected return from the project is uncertain, and given by  $x = \rho x_H + (1 - \rho)x_L$ , with  $x_H > x_L$ . For brevity's sake we focus on the case where debt is safe, i.e.,  $x_L \geq (1 + r_1)K_1 + (1 + r_2)K_2$ .<sup>12</sup>

Consider the second financing round, and assume for a moment that  $A$  issued equity to  $P$  in the first round – this allows us to focus exclusively on the effect of debt financing in the second round. In the Appendix we formally derive the equilibrium investments in the second round under debt financing, and derive the equilibrium cost of capital  $r_2^*$ . We find that the costs of debt and equity financing in the second round are identical, i.e.,  $(1 + r_2^*)K_2 = \beta^{PI}x$ . This is essentially the Modigliani-Miller theorem. In the second round it is irrelevant whether  $A$  raises the required amount  $K_2$  through debt or equity, because the financial structure doesn't affect any behaviors.

Now consider the first financing round, and suppose that  $A$  issues a debt claim to raise the amount  $K_1$ . In the Appendix we establish the following two results. First, when  $K_1$  is raised from a competitive investor ( $PO$  case), then debt and equity financing are again equivalent. Second, when the powerful investor provides  $K_1$  ( $PI$  case), then equity is optimal. The intuition is that issuing equity in the first round makes the powerful investor defend the valuation in the second round, to limit the dilution of his first-round equity stake. This strategic effect is not present when  $P$  holds debt, because in this case the value of his existing stake is not affected by the valuation in the second round. Our model therefore identifies a new reason for the use of equity, which is similar but not identical to the traditional incentive argument. The

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<sup>11</sup>For completeness, note also that date 1 investors in the  $MPI$  case achieve both higher unrealized interim returns ( $R_{12}^{MPI}$ ) and higher realized final returns ( $R_{13}^{MPI}$ ), compared to the benchmark returns ( $R_{12}^{NP}$  and  $R_{13}^{NP}$ ), and also compared to the  $PO$  case ( $R_{12}^{PO}$  and  $R_{13}^{PO}$ ). Finally, comparisons for the  $HPI$  case cannot always be signed unambiguously.

<sup>12</sup>In this simple model with only two states, any risky debt is equivalent to equity. Hence the combination of safe debt and equity covers all relevant combinations.

traditional argument is that investors need equity to have incentives for providing more value-adding services. This is typically modelled as a two-sided moral hazard problem (see, e.g., Da Rin et al. 2012, and Serfes, Hong, and Thiele (2018)). The reason for using equity in our model is also related to incentives, but not to moral hazard. Instead the argument is that by giving equity to the powerful investor, he takes the value of insiders into consideration when exercising his market power. This argument is reminiscent of Da Rin and Hellmann (2002) who look at how market power allows a bank to finance “big push” investments.

### 4.3 Empirical Predictions

Our theory generates several testable empirical predictions. The most important empirical predictions concern the relationship between valuations and investor power. When investing in a company for the first time, powerful investors drive valuations down. However, in follow-up investments, they only drive valuations down if they invest above pro-rata. If they invest below pro-rata, they drive valuations up. This effect is non-monotonic. It is smaller when insider investments are either sufficiently small, or else sufficiently close to the pro-rata benchmark.

The model also generates empirical predictions about investor returns. Powerful investors achieve higher investment returns, compared to the competitive benchmark. The interesting exception concerns follow-on investments of moderately powerful insiders who invest below pro-rata. The theory also provides a warning against the use of unrealized interim returns. Empirical studies of investor returns sometimes makes use of unrealized interim valuations to estimate investor portfolio returns (see Harris, Jenkinson, and Kaplan (2014)). However, our model shows that this can be misleading. For example, the presence of a powerful outsider reduces unrealized interim returns, yet going forward investor returns are higher in this case. More generally, the theory suggests that if unrealized interim returns are used to estimate expected realized returns, they would need to be adjusted to take market power into consideration.

Let us briefly consider how these empirical predictions might be tested, and what challenges occur. The theory generates predictions about two main dependent variables: company valuations ( $V_1$  and  $V_2$ ) and investor returns ( $R_{13}$ ,  $R_{23}$ , and  $R_{12}$ ).<sup>13</sup> In public markets this information is readily available, but in private markets this data is more difficult to obtain. Even if valuations can be found, it is often difficult to find data on realized returns. The most important dependent variables from the theory are market power ( $m$ ) and prior equity stakes ( $\alpha$ ). Measuring prior equity stakes is straightforward in principle, provided the data is disclosed. Measuring market

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<sup>13</sup>The model also makes predictions about the share of investments provided by powerful inside investors in later rounds ( $K_2^P/K_2$ ). The data required for this is a breakdown of round investments by individual investors.

power, however, is considerably more challenging. Our theory suggests looking at investor fund sizes. One complicating factor is that different investors specialize in different deals, and face different competitors at different stages. For example, a venture capital fund might be a large powerful investor in an early stage deal, when all others are smaller angel investors. However, that same venture capital fund might be a small player in later stage deals where there are much larger investors, such as private equity funds, mutual funds, hedge funds, or institutional investors. Alternative measures of market power might relate to an investor's reputation and track record. Hsu (2004), for example, establishes that venture capitalists with higher reputations offer lower valuations. In addition to these data measurement problem, there is the empirical challenge of finding exogenous variation in the independent variables. This requires finding exogenous shocks to investor power and market structures. The work of Nanda and Rhodes-Kropf (2013) contains some useful ideas for that.

## **5 Conclusion**

In this paper we develop a model of staged equity financing with investor market power. Standard economic reasoning suggests that a powerful investor obtains lower valuations, and thereby achieves higher returns. We show that while this result holds the first time a powerful investor invests, it may not hold in subsequent financing rounds. As an insider, a powerful investor faces dual motives. One is that higher valuations preserve his existing stake, the other is that lower valuations are more attractive for his new investments. We show that the former motive dominates when the investor is moderately powerful and invests below pro-rata in later financing rounds. In this case the effect of market power is reversed, i.e., the investor uses his market power to increase, not decrease valuations. We explain how this is an equilibrium behavior, and describe the circumstances under which this result obtains.

Our model also asks whether the company prefers to have the powerful investor up-front, or delay him to a later round. Even though the powerful investor can extract a lower valuation in the first round, the company prefers to bring him in up-front. This is because once he becomes an insider, the company can leverage his power to defend its valuation. Hence the title of the paper: "May the force be with you."

## Appendix

### *PI* Case: Equilibrium Investment and Valuation.

Using  $k_2^j(\beta) = \frac{1}{\gamma} \left[ \frac{1}{K_2} \beta x - \mu_2 \right]$  we can write the market clearing condition (5) as

$$K_2^P + (n - m) \frac{1}{\gamma} \left[ \frac{1}{K_2} \beta x - \mu_2 \right] = K_2.$$

Solving for  $\beta$  we get

$$\beta(K_2^P) = \frac{K_2}{x} \left[ \frac{\gamma}{n - m} (K_2 - K_2^P) + \mu_2 \right]. \quad (11)$$

Using the expression for  $\beta(K_2^P)$  we can write the objective function of  $P$  as follows:

$$\begin{aligned} \max_{K_2^P} \pi_2^{P|PI}(K_2^P) &= \alpha^P x - \alpha^P K_2 \left[ \frac{\gamma}{n - m} (K_2 - K_2^P) + \mu_2 \right] + K_2^P \left[ \frac{\gamma}{n - m} (K_2 - K_2^P) + \mu_2 \right] \\ &\quad - \left[ \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2 \right]. \end{aligned}$$

The optimal investment,  $K_2^{P|PI}$ , is then defined by the first-order condition:

$$\alpha^P K_2 \frac{\gamma}{n - m} + \frac{\gamma}{n - m} (K_2 - K_2^P) + \mu_2 = K_2^P \frac{\gamma}{n - m} + \mu_2 + \frac{\gamma}{m} K_2^P.$$

Solving for  $K_2^P$  we get  $K_2^{P|PI} = \frac{m}{n+m} [1 + \alpha^P] K_2$ . Substituting  $K_2^{P|PI}$  in (11) then yields the equilibrium equity share for all investors:

$$\beta^{PI} = \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} K_2 \right]. \quad (12)$$

Thus, the equilibrium valuation,  $V_2^{PI}$ , is given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x(n^2 - m^2)}{\gamma (n - \alpha^P m) K_2 + \mu_2 (n^2 - m^2)}.$$



**Proof of Lemma 1.**

Note that  $P$  invests above (below) pro-rata when

$$(1 - \beta^{P-P})\alpha^P + \frac{K_2^{P|P-P}}{K_2}\beta^{P-P} > (<)\alpha^P.$$

Using  $K_2^{P|PI} = \frac{m}{n+m} [1 + \alpha^P] K_2$ , we can write this condition as

$$\frac{m}{n+m} [1 + \alpha^P] \beta^{P-P} > (<)\alpha^P \beta^{P-P},$$

which can be simplified to  $\frac{m}{n} > (<)\alpha^P$ . □

**Proof of Proposition 1.**

We can immediately see that  $dK_2^{P|PI}/d\alpha^P > 0$  and  $dV_2^{PI}/d\alpha^P > 0$ . Moreover,

$$\frac{dK_2^{P|PI}}{dm} = \frac{n}{[n+m]^2} [1 + \alpha^P] K_2 > 0,$$

$$\frac{dV_2^{PI}}{dm} = \frac{-2mx [\gamma (n - \alpha^P m) K_2 + \mu_2(n^2 - m^2)] + x(n^2 - m^2) [\gamma \alpha^P K_2 + 2m\mu_2]}{[\gamma (n - \alpha^P m) K_2 + \mu_2(n^2 - m^2)]^2}.$$

We have  $dV_2^{PI}/dm > 0$  when

$$\begin{aligned} (n^2 - m^2) [\gamma \alpha^P K_2 + 2m\mu_2] &> 2m [\gamma (n - \alpha^P m) K_2 + \mu_2(n^2 - m^2)] \\ \Leftrightarrow \gamma \alpha^P n^2 K_2 + 2\mu_2 mn^2 - \gamma \alpha^P m^2 K_2 &> 2\gamma mn K_2 - 2\gamma \alpha^P m^2 K_2 + 2\mu_2 mn^2 \\ \Leftrightarrow \underbrace{\alpha^P (n^2 + m^2) - 2mn}_{\equiv Z} &> 0 \end{aligned}$$

Note that this condition is satisfied for  $m = 0$ . Thus,  $dV_2^{PI}/dm > 0$  for  $m \rightarrow 0$ . Moreover, for  $m = n$  this condition simplifies to  $\alpha^P - 1 > 0$ , which is clearly violated. Therefore,  $dV_2^{PI}/dm < 0$  for  $m \rightarrow n$ . Next, note that  $dZ/dm = 2\alpha^P m - 2n < 0$ . Consequently, there exists a unique  $m' > 0$  so that  $dV_2^{PI}/dm > 0$  for  $m < m'$ , and  $dV_2^{PI}/dm \leq 0$  for  $m \geq m'$ . Finally, evaluating  $Z$  at  $m = \hat{m} = \alpha^P n$  we get  $\alpha^P n^2 (\alpha^{P^2} - 1) < 0$ . Thus,  $\left. \frac{dV_2^{PI}}{dm} \right|_{m=\hat{m}} < 0$ , which implies that  $m' < \hat{m}$ . □

**Proof of Proposition 2.**

Setting  $\alpha^P = 0$  in (7) and (8) we get

$$K_2^{P|PO} = \frac{m}{n+m} K_2 \quad (13)$$

$$V_2^{PO} = \frac{x(n^2 - m^2)}{\gamma n K_2 + \mu_2(n^2 - m^2)}. \quad (14)$$

Thus,

$$\begin{aligned} \frac{dK_2^{P|PO}}{dm} &= \frac{n}{[n+m]^2} K_2 > 0 \\ \frac{dV_2^{PO}}{dm} &= \frac{-2mx[\gamma n K_2 + \mu_2(n^2 - m^2)] + x(n^2 - m^2)2\mu_2 m}{[\gamma n K_2 + \mu_2(n^2 - m^2)]^2} \\ &= -\frac{2\gamma m n x K_2}{[\gamma n K_2 + \mu_2(n^2 - m^2)]^2} < 0. \end{aligned}$$

□

**Second Round: Additional Comparative Statics.**

Consider first  $K_2^{P|PI}$ . We can immediately see that  $dK_2^{P|PI}/d\mu_2 = dK_2^{P|PI}/d\gamma = dK_2^{P|PI}/dx = 0$ ,  $dK_2^{P|PI}/dK_2 > 0$ , and  $dK_2^{P|PI}/dn < 0$ . Likewise, for  $V_2^{PI}$  we can see that  $dV_2^{PI}/d\mu_2$ ,  $dV_2^{PI}/dK_2$ ,  $dV_2^{PI}/dK_2 < 0$ , and  $dV_2^{PI}/dx > 0$ . Moreover,

$$\begin{aligned} \frac{dV_2^{PI}}{dn} &= \frac{2xn[\gamma(n - \alpha^P m)K_2 + \mu_2(n^2 - m^2)] - x(n^2 - m^2)[\gamma K_2 + 2n\mu_2]}{[\gamma(n - \alpha^P m)K_2 + \mu_2(n^2 - m^2)]^2} \\ &= \frac{\overbrace{[n^2 - 2\alpha^P mn + m^2]}^{\equiv Z(\alpha^P)} x\gamma K_2}{[\gamma(n - \alpha^P m)K_2 + \mu_2(n^2 - m^2)]^2}. \end{aligned}$$

Note that  $Z(\alpha^P)$  is decreasing in  $\alpha^P$ . Moreover, evaluating  $Z(\alpha^P)$  at  $\alpha^P = 1$  we get  $Z(1) = (n - m)^2 > 0$ . This implies that  $Z(\alpha^P) > 0$  for all  $\alpha^P \in [0, 1]$ . Thus,  $dV_2^{PI}/dn > 0$ .

Next consider  $K_2^{P|PO}$  (see (13) in Proof of Proposition 2). Note that  $dK_2^{P|PO}/d\mu_2$ ,  $dK_2^{P|PO}/d\gamma$ ,  $dK_2^{P|PO}/dx = 0$ ,  $dK_2^{P|PO}/dK_2 > 0$ , and  $dK_2^{P|PO}/dn < 0$ . In addition, for  $V_2^{PO}$  (see (14) in

Proof of Proposition 2) we can immediately see that  $dV_2^{PO}/d\mu_2 < 0$ ,  $dV_2^{PO}/d\mu_2 < 0$ ,  $dV_2^{PO}/dK_2 < 0$ , and  $dV_2^{PO}/dx > 0$ . Furthermore,

$$\begin{aligned}\frac{dV_2^{PO}}{dn} &= \frac{2xn[\gamma nK_2 + \mu_2(n^2 - m^2)] - x(n^2 - m^2)[\gamma K_2 + 2\mu_2n]}{[\gamma nK_2 + \mu_2(n^2 - m^2)]^2} \\ &= \frac{[n^2 + m^2]\gamma xK_2}{[\gamma nK_2 + \mu_2(n^2 - m^2)]^2} > 0.\end{aligned}$$

### Second Round Participation of Powerful Investor.

Note that  $K_2^{P|P-P} > 0$  for all  $\alpha^P \in [0, 1]$ ; see (7). Given the structure of  $P$ 's profit function (see (6)), this implies that  $P$  always chooses to invest in the second round for all  $\alpha^P \in [0, 1]$ . Consequently, his participation constraint is always satisfied in the second round.

### Proof of Proposition 3.

We can immediately see that  $K_2^{P|PO} < K_2^{P|PI}$  for all  $\alpha^P > 0$ , and  $K_2^{P|PO} < K_2^{P|NP}$  for all  $m > 0$ . Moreover, note that  $V_2^{PO} = V_2^{PI}$  at  $\alpha^P = 0$ , and recall from Proposition 1 that  $dV_2^{PI}/d\alpha^P > 0$ . Thus,  $V_2^{PO} < V_2^{PI}$  for all  $\alpha^P > 0$ . Finally,  $V_2^{PO} < V_2^{NP}$  because

$$\begin{aligned}\frac{x(n^2 - m^2)}{\gamma nK_2 + \mu_2(n^2 - m^2)} &< \frac{x}{\mu_2 + \frac{\gamma}{n}K_2} \\ \Leftrightarrow (n^2 - m^2) \left( \mu_2 + \frac{\gamma}{n}K_2 \right) &< \gamma nK_2 + \mu_2(n^2 - m^2) \\ \Leftrightarrow n^2 - m^2 &< n^2.\end{aligned}$$

□

### Proof of Proposition 4.

We have  $K_2^{P|PI} \geq K_2^{P|NP}$  when

$$\frac{m}{n+m} [1 + \alpha^P] K_2 \geq \frac{m}{n} K_2 \quad \Leftrightarrow \quad \alpha^P \geq \hat{\alpha}^P(m) = \frac{m}{n},$$

which is equivalent to  $m \leq \hat{m} = \alpha^P n$ . Moreover, note that  $V_2^{PI} \geq V_2^{NP}$  when

$$\begin{aligned} \frac{x(n^2 - m^2)}{\gamma(n - \alpha^P m) K_2 + \mu_2(n^2 - m^2)} &\geq \frac{x}{\mu_2 + \frac{\gamma}{n} K_2} \\ \Leftrightarrow (n^2 - m^2) \frac{\gamma}{n} K_2 &\geq \gamma(n - \alpha^P m) K_2 \\ \Leftrightarrow \alpha^P &\geq \hat{\alpha}^P(m) = \frac{m}{n}, \end{aligned}$$

which, again, is equivalent to  $m \leq \hat{m} = \alpha^P n$ . □

### Participation Constraint of Powerful Investor (First Round).

Using the expressions for  $K_2^{P|PI}$  and  $\beta^{PI}$  (see (7) and (12)) we get

$$\begin{aligned} \pi^{P|PI}(\alpha^P) &= (1 - \beta^{PI})\alpha^P x + \frac{K_2^{P|PI}}{K_2} \beta^{PI} x - \mu_1 K_1 - \left[ \mu_2 K_2^{P|PI} + \frac{\gamma}{2m} \left( K_2^{P|PI} \right)^2 \right] \\ &= \left( x - K_2 \left[ \mu_2 + \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2 \right] \right) \alpha^P + \frac{m}{n + m} [1 + \alpha^P] \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2^2 \\ &\quad - \mu_1 K_1 - \frac{\gamma m}{2} \frac{1}{[n + m]^2} [1 + \alpha^P]^2 K_2^2. \end{aligned} \tag{15}$$

Likewise, by setting  $\alpha^P = 0$  and  $\mu_1 K_1 = 0$ , we get

$$\pi^{P|PO} = \frac{m}{n + m} \frac{\gamma n}{n^2 - m^2} K_2^2 - \frac{\gamma m}{2} \frac{1}{[n + m]^2} K_2^2$$

We then find that  $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$  is equivalent to  $Z \geq \mu_1 K_1$ , where

$$\begin{aligned} Z &= \left( x - K_2 \left[ \mu_2 + \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2 \right] \right) \alpha^P - \frac{m}{n + m} \frac{\gamma \alpha^P m}{n^2 - m^2} K_2^2 + \frac{m}{n + m} \alpha^P \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2^2 \\ &\quad - \frac{\gamma m}{2} \frac{1}{[n + m]^2} [2\alpha^P + [\alpha^P]^2] K_2^2 \\ &= \left( x - K_2 \left[ \mu_2 + \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2 \right] \right) \alpha^P - \frac{1}{2} \alpha^P \frac{\gamma m}{(n + m)^2} \frac{\alpha^P}{(n^2 - m^2)} K_2^2 [m^2 + 2mn + n^2] \\ &= \left( x - K_2 \left[ \mu_2 + \frac{\gamma(n - \frac{1}{2}\alpha^P m)}{n^2 - m^2} K_2 \right] \right) \alpha^P. \end{aligned}$$

Thus,  $P$ 's participation constraint can be written as

$$\alpha^P \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n - \frac{1}{2} \alpha^P m)}{n^2 - m^2} K_2 \right] \right) \geq \mu_1 K_1. \quad (16)$$

**Proof of Proposition 6.**

Let  $\hat{\alpha}$  denote the first round equity share which makes  $A$  indifferent between accepting  $\hat{\alpha}$  from  $P$ , and  $\alpha^{PO}$  from a competitive investor. Formally,  $\hat{\alpha}$  is defined by  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$ . Using (12) with  $\alpha^P = \hat{\alpha}$  we get

$$\pi^{A|PI}(\hat{\alpha}) = (1 - \hat{\alpha}) (1 - \beta^{PI}(\hat{\alpha})) x = (1 - \hat{\alpha}) \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma (n - \hat{\alpha} m)}{n^2 - m^2} K_2 \right] \right) x.$$

Moreover, recall from the comparative statics analysis for the  $PO$  case (see (22)) that

$$\pi^{A|PO} = \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] \right) x - \mu_1 K_1.$$

Using these two expressions we then find that  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$  is equivalent to

$$\hat{\alpha} \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n + (1 - \hat{\alpha}) m)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1, \quad (17)$$

which defines  $\hat{\alpha}$ .

Next we need to show that  $\hat{\alpha}$  satisfies  $P$ 's participation constraint (16). Using (17) we can write (16) as

$$\begin{aligned} \hat{\alpha} \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n - \frac{1}{2} \hat{\alpha} m)}{n^2 - m^2} K_2 \right] \right) &\geq \hat{\alpha} \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n + (1 - \hat{\alpha}) m)}{n^2 - m^2} K_2 \right] \right) \\ \Leftrightarrow 0 &\leq 1 - \frac{1}{2} \hat{\alpha}, \end{aligned}$$

which is clearly satisfied for all  $\hat{\alpha} \in [0, 1]$ . Thus, the optimal bid for  $P$ , which is strictly preferred by  $A$ , is then given by  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ .

Finally, using (17), we can implicitly differentiate  $\alpha^P = \hat{\alpha} - \epsilon$ ,  $\epsilon \rightarrow 0$ , w.r.t.  $m$ :

$$\frac{d\alpha^P}{dm} = \frac{\alpha^P \gamma K_2^2 \left( \frac{(1-\alpha^P)(n^2-m^2)+2m(n+(1-\alpha^P)m)}{[n^2-m^2]^2} \right)}{\underbrace{x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right]}_{>0} + \alpha^P \frac{\gamma m}{n^2-m^2} K_2^2} > 0.$$

Consequently,

$$\frac{dV_1^{PI}}{dm} = \frac{d}{dm} \left[ \frac{K_1}{\alpha^P} \right] < 0.$$

### Powerful Investor in First Round – Comparative Statics.

Recall that  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , is defined by

$$Z \equiv \alpha^P \left( x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right] \right) - \mu_1 K_1 = 0.$$

Moreover, note that

$$\frac{\partial Z}{\partial \alpha^P} = \underbrace{x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right]}_{>0} + \alpha^P \frac{\gamma m}{n^2-m^2} K_2^2 > 0. \quad (18)$$

Using  $Z$  we can then implicitly differentiate  $\alpha^P$  and get

$$\begin{aligned} \frac{d\alpha^P}{d\mu_1} &= \frac{K_1}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_1} = \frac{d}{d\mu_1} \left[ \frac{K_1}{\alpha^P} \right] < 0 \\ \frac{d\alpha^P}{d\mu_2} &= \frac{\alpha^P K_2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_2} = \frac{d}{d\mu_2} \left[ \frac{K_1}{\alpha^P} \right] < 0 \\ \frac{d\alpha^P}{d\gamma} &= \frac{\alpha^P \frac{n+(1-\alpha^P)m}{n^2-m^2} K_2^2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\gamma} = \frac{d}{d\gamma} \left[ \frac{K_1}{\alpha^P} \right] < 0 \\ \frac{d\alpha^P}{dx} &= -\frac{\alpha^P}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dx} = \frac{d}{dx} \left[ \frac{K_1}{\alpha^P} \right] > 0 \\ \frac{d\alpha^P}{dn} &= -\frac{\alpha^P K_2^2 \frac{\gamma(n^2+m^2)+2\gamma(1-\alpha^P)mn}{[n^2-m^2]^2}}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dn} = \frac{d}{dn} \left[ \frac{K_1}{\alpha^P} \right] > 0. \end{aligned}$$

Likewise,

$$\frac{d\alpha^P}{dK_1} = \frac{\mu_1}{\frac{\partial Z}{\partial \alpha^P}}, \quad (19)$$

so that

$$\frac{dV_1^{PI}}{dK_1} = \frac{d}{dK_1} \left[ \frac{K_1}{\alpha^P} \right] = \frac{\overbrace{\alpha^P - K_1 \frac{d\alpha^P}{dK_1}}^{\equiv T}}{[\alpha^P]^2}.$$

Using (19) with (18), we can write  $T$  as

$$\begin{aligned} T &= \frac{\overbrace{\alpha^P \left[ x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right] \right]}^{=Z=0} - \mu_1 K_1 + (\alpha^P)^2 \frac{\gamma m}{n^2-m^2} K_2^2}{x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right] + \alpha^P \frac{\gamma m}{n^2-m^2} K_2^2} \\ &= \frac{(\alpha^P)^2 \frac{\gamma m}{n^2-m^2} K_2^2}{\underbrace{x - K_2 \left[ \mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2} K_2 \right]}_{>0} + \alpha^P \frac{\gamma m}{n^2-m^2} K_2^2} > 0. \end{aligned}$$

Thus,  $dV_1^{PI}/dK_1 > 0$ .

Finally note that  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , where  $\hat{\alpha}$  is defined by  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$ . Consequently, we get the same comparative statics results for  $\pi^{A|PI}$  as for  $\pi^{A|PO}$ .

**Proof of Proposition 7.**

Recall that  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , where  $\hat{\alpha}$  is defined by (10) (see Proposition 6). For  $m = 0$  we can write condition (10) as

$$\hat{\alpha} = \frac{\mu_1 K_1}{\left(1 - \frac{K_2}{x} \left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x}.$$

Clearly,  $\hat{\alpha} = \alpha^{NP}$  for  $m = 0$ . Thus,  $\alpha^P = \alpha^{NP} - \epsilon$ , with  $\epsilon \rightarrow 0$ , for  $m = 0$ , which implies that  $V_1^{PO} \rightarrow V_1^{NP}$  for  $m = 0$ . Moreover, recall from Proposition 6 that  $dV_1^{PI}/dm < 0$ . Consequently,  $V_1^{PI} < V_1^{NP}$  for all  $m > 0$ .

Next, note that the condition  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$ , which defines  $\hat{\alpha}$ , can be written as

$$(1 - \hat{\alpha})(1 - \beta^{PI}) - (1 - \alpha^{PO})(1 - \beta^{PO}) = 0. \quad (20)$$

Moreover, recall that  $\beta^{PI}$  and  $\beta^{PO}$  are given by

$$\beta^{PI} = \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2 \right] \quad \beta^{PO} = \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right].$$

We can immediately see that  $\beta^{PI} = \beta^{PO}$  for  $m = 0$ . This implies that  $\hat{\alpha} = \alpha^{PO}$  for  $m = 0$ , and therefore  $\alpha^P = \alpha^{PO} - \epsilon$ , with  $\epsilon \rightarrow 0$ , for  $m = 0$ . Hence,  $V_1^{PI} \rightarrow V_1^{PO}$  for  $m = 0$ . Moreover, it is easy to see that  $\beta^{PI} < \beta^{PO}$  for all  $m > 0$ . And using (20) we get

$$\frac{\partial \hat{\alpha}}{\partial \beta^{PI}} = -\frac{1 - \hat{\alpha}}{1 - \beta^{PI}} < 0.$$

This implies that  $\alpha^P = \hat{\alpha} - \epsilon > \alpha^{PO}$ , with  $\epsilon \rightarrow 0$ , for all  $m > 0$ . Consequently,  $V_1^{PI} < V_1^{PO}$  for all  $m > 0$ .  $\square$



**Proof of Proposition 8.**

The expected profit for  $A$  is given by

$$\pi^{A|i} = (1 - \alpha^i) (1 - \beta^i) x = (1 - \beta^i) x - \mu_1 K_1,$$

with  $i \in \{NP, PO, PI\}$ . Using the expressions for  $\beta^{NP}$  (see (2)) and  $\beta^{PO}$  (see (12) with  $\alpha^P = 0$ ) we get

$$\pi^{A|NP} = \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma}{n} K_2 \right] \right) x - \mu_1 K_1 \quad (21)$$

$$\pi^{A|PO} = \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] \right) x - \mu_1 K_1. \quad (22)$$

It is easy to see that  $\pi^{A|NP} = \pi^{A|PO}$  when  $m = 0$ . Moreover, note that  $d\pi^{A|PO}/dm < 0$ . Thus,  $\pi^{A|NP} > \pi^{A|PO}$  for all  $m > 0$ .

Likewise, we can immediately see that  $d\pi^{A|i}/d\mu_1, d\pi^{A|i}/d\mu_2, d\pi^{A|i}/d\gamma, d\pi^{A|i}/dK_1, d\pi^{A|i}/dK_2 < 0$ , and  $d\pi^{A|i}/dx > 0, i \in \{NP, PO\}$ . Moreover,  $d\pi^{A|NP}/dn > 0$ , and

$$\frac{d\pi^{A|PO}}{dn} = -\frac{\gamma [n^2 - m^2] - 2\gamma n^2}{[n^2 - m^2]^2} K_2^2 = \frac{\gamma [n^2 + m^2]}{[n^2 - m^2]^2} K_2^2 > 0.$$

Next, recall from Proof of Proposition 7 that  $\alpha^P = \alpha^{PO} - \epsilon$ , with  $\epsilon \rightarrow 0$ , when  $m = 0$ . For  $m > 0$  we have  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , where  $\hat{\alpha}$  is defined by  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$ . This implies that  $\pi^{A|PI}(\alpha^P) = \pi^{A|PO} + \epsilon$ , with  $\epsilon \rightarrow 0$ . Consequently,  $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$  for all  $m > 0$ .

We can immediately see from (21) that  $d\pi^{A|NP}/dm = 0$ . Moreover, it is easy to see from (22) that  $d\pi^{A|PO}/dm < 0$ . And because  $\pi^{A|PI}(\alpha^P) = \pi^{A|PO} + \epsilon$ , with  $\epsilon \rightarrow 0$ , this also implies that  $d\pi^{A|PI}/dm < 0$ .

Finally recall that  $\alpha^P = \hat{\alpha} - \epsilon$ , with  $\epsilon \rightarrow 0$ , where  $\hat{\alpha}$  is defined by  $\pi^{A|PI}(\hat{\alpha}) = \pi^{A|PO}$ . Consequently, we get the same comparative statics results for  $\pi^{A|PI}$  as for  $\pi^{A|PO}$ .  $\square$

**Proof of Proposition 9.**

We first derive and compare  $R_{23}$  for the different investor constellations. Using the expressions of  $V_2^{NP}$ ,  $V_2^{PO}$ , and  $V_2^{PI}$ , we get

$$R_{23}^{NP} = \mu_2 + \frac{\gamma}{n}K_2 \quad R_{23}^{PO} = \mu_2 + \gamma \frac{n}{n^2 - m^2}K_2 \quad R_{23}^{PI} = \mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2}K_2.$$

Recall from Propositions 3 and 4 that  $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$ . Thus,  $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$ .

Next consider  $R_{12}$ . Using the expressions of  $V_1^{PO}$  and  $V_2^{PO}$  we get

$$R_{12}^{PO} = \frac{V_2^{PO} - K_2}{V_1^{PO}} = \frac{\frac{x}{n} - K_2}{\frac{1}{\mu_1} \left(1 - \frac{K_2}{x} \left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2\right]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2}.$$

Likewise, using the expressions of  $V_1^{NP}$  and  $V_2^{NP}$  we find

$$R_{12}^{NP} = \frac{V_2^{NP} - K_2}{V_1^{NP}} = \frac{\frac{(1 - \frac{K_2}{x} [\mu_2 + \frac{\gamma}{n} K_2]) x}{\mu_2 + \frac{\gamma}{n} K_2}}{\frac{1}{\mu_1} \left(1 - \frac{K_2}{x} [\mu_2 + \frac{\gamma}{n} K_2]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n} K_2}.$$

Finally, using  $V_1^{PI}$  and  $V_2^{PI}$  we get

$$R_{12}^{PI} = \frac{V_2^{PI} - K_2}{V_1^{PI}} = \frac{x - K_2 \left[\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2\right]}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2} \frac{\alpha^P}{K_1},$$

where  $\alpha^P$  satisfies

$$\alpha^P \left( x - K_2 \left[ \mu_2 + \frac{\gamma (n + (1 - \alpha^P) m)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1. \quad (23)$$

Note that we can rewrite  $R_{12}^{PI}$  as

$$R_{12}^{PI} = \frac{\alpha^P \left( x - K_2 \left[ \mu_2 + \gamma \frac{n + (1 - \alpha^P) m}{n^2 - m^2} K_2 \right] \right) \frac{1}{K_1} + \left[ \gamma \frac{m}{n^2 - m^2} K_2^2 \right] \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2}.$$

Using (23) we then get

$$R_{12}^{PI} = \frac{\mu_1 + \gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2}.$$

We know from Propositions 3 and 4 that  $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$ . And according to Proposition 7,  $V_1^{PI} < V_1^{PO} < V_1^{NP}$ . Moreover, we know from Proposition 6 that  $dV_1^{PI}/dm < 0$ . Consequently,  $V_1^{HPI} < V_1^{MPI} < V_1^{PO} < V_1^{NP}$ . This implies that (i)  $R_{12}^{HPI} > R_{12}^{PO}$  because  $V_2^{PO} < V_2^{HPI}$  and  $V_1^{HPI} < V_1^{PO}$ , (ii)  $R_{12}^{PO} < R_{12}^{MPI}$  because  $V_2^{PO} < V_2^{MPI}$  and  $V_1^{MPI} < V_1^{PO}$ , and (iii)  $R_{12}^{NP} < R_{12}^{MPI}$  because  $V_2^{NP} < V_2^{MPI}$  and  $V_1^{MPI} < V_1^{NP} < V_1^{NP}$ . Moreover, it is straightforward to show that  $R_{12}^{PO} < R_{12}^{NP}$ . Consequently,  $R_{12}^{PO} < R_{12}^{NP} < R_{12}^{MPI}$  and  $R_{12}^{PO} < R_{12}^{HPI}$ .

Finally, using the above expressions for  $R_{12}$  and  $R_{23}$  we get

$$R_{13}^{PO} = R_{12}^{PO} R_{23}^{PO} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2} \left[ \mu_2 + \gamma \frac{n}{n^2 - m^2} K_2 \right] = \mu_1$$

$$R_{13}^{NP} = R_{12}^{NP} R_{23}^{NP} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n} K_2} \left[ \mu_2 + \frac{\gamma}{n} K_2 \right] = \mu_1.$$

Likewise,

$$\begin{aligned} R_{13}^{PI} &= R_{12}^{PI} R_{23}^{PI} = \frac{\mu_1 + \gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2} \left[ \mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2 \right] \\ &= \underbrace{\mu_1 + \gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}_{\equiv Z}. \end{aligned}$$

We can immediately see that  $R_{13}^{PO} = R_{13}^{NP} = \mu_1$ . And because  $Z > 0$ , we have  $R_{13}^{PI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$ . Moreover, recall from Proof of Proposition 6 that  $d\alpha^P/dm > 0$  – this implies that

$dZ/dm > 0$ . Consequently,  $R_{13}^{HPI} > R_{13}^{MPI}$ . All this implies that  $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$ .  $\square$

### The Role of Debt – Derivations.

Consider the second financing round. Given  $r_2$  each competitive investor will invest  $k_2^j$ ,  $j = m+1, \dots, n$ , so that the price per unit of capital equals the marginal cost:  $1 + r_2 = \mu_2 + \gamma k_2^j$ . Consequently,  $k_2^{jD}(r_2) = [1 + r_2 - \mu_2]/\gamma$ . Using  $k_2^{jD}(r_2)$  in the market clearing condition,  $K_2^P + (n - m) k_2^{jD}(r_2) = K_2$ , and solving for  $1 + r_2$ , we get

$$1 + r_2 = \frac{\gamma}{n - m} [K_2 - K_2^P] + \mu_2. \quad (24)$$

$P$  then chooses  $K_2^P$  to maximize his expected net return

$$\pi_2^{P|D}(K_2^P) = \alpha^P (x - (1 + r_2) K_2) + (1 + r_2) K_2^P - \left[ \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2 \right]. \quad (25)$$

Using (24) we can write (25) as

$$\pi_2^{P|D}(K_2^P) = \alpha^P x + \left( \frac{\gamma}{n - m} [K_2 - K_2^P] + \mu_2 \right) [K_2^P - \alpha^P K_2] - \left[ \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2 \right].$$

The optimal investment,  $K_2^{P|D}$ , is then defined by the first-order condition:

$$-\frac{\gamma}{n - m} [K_2^P - \alpha^P K_2] + \frac{\gamma}{n - m} [K_2 - K_2^P] + \mu_2 = \mu_2 + \frac{\gamma}{m} K_2^P.$$

Solving for  $K_2^P$  we get

$$K_2^{P|D} = \frac{m}{n + m} [1 + \alpha^P] K_2. \quad (26)$$

Next, using (26) we can rewrite (24) as

$$1 + r_2^* = \frac{\gamma}{n - m} \left[ K_2 - \frac{m}{n + m} [1 + \alpha^P] K_2 \right] + \mu_2 = \mu_2 + \left[ \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} \right] K_2. \quad (27)$$

Thus, the cost of debt financing in the second round is given by

$$(1 + r_2^*) K_2 = K_2 \left[ \mu_2 + \left[ \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} \right] K_2 \right].$$

Moreover, note that the cost of equity financing is given by

$$\beta^{PI} x = K_2 \left[ \mu_2 + \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} K_2 \right].$$

Consequently,  $(1 + r_2^*) K_2 = \beta^{PI} x$ .

Next we derive the outcome for the first financing round. Consider first the *PO* case (where  $\alpha^P = 0$ ), and let

$$T_2 \equiv (1 + r_2^*) K_2 = \beta^{PO} x = K_2 \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right]$$

denote the cost of financing in the second round (which is the same for debt and equity financing). Under debt financing the competitive investor's zero profit condition is given by  $(1 + r_1) K_1 - \mu_1 K_1 = 0$ . Consequently, the equilibrium return is  $r_1^C = \mu_1 - 1$ . The expected profit for *A*, denoted by  $\pi^{A|D(PO)}$ , is then given by

$$\pi^{A|D(PO)} = x - (1 + r_1^C) K_1 - T_2 = \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] \right) x - \mu_1 K_1.$$

We can immediately see that  $\pi^{A|D(PO)} = \pi^{A|PO}$  (see (22), i.e., for the *PO* case debt and equity financing are equivalent for *A*).

Now consider the *PI* case. The expected profit for *A* is then given by

$$\pi^{A|D(PI)} = x - (1 + r_1) K_1 - T_2 = x - (1 + r_1) K_1 - K_2 \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right].$$

Let  $\hat{r}_2$  denote the return which makes *A* indifferent between accepting  $\hat{r}_2$  from *P*, and  $r_1^C$  from a competitive investor. Formally,  $\hat{r}_2$  is defined by  $\pi^{A|D(PI)}(\hat{r}_2) = \pi^{A|D(PO)}$ , which immediately implies that  $\hat{r}_2 = r_1^C = \mu_1 - 1$ . Consequently, *P* offers  $r_1^P = \hat{r}_2 + \epsilon = \mu_1 - 1 + \epsilon$ , with  $\epsilon \rightarrow 0$ . The expected profit for *A* is then given by  $\pi^{A|D(PI)} = \pi^{A|D(PO)} + \epsilon$ , with  $\epsilon \rightarrow 0$ . And because  $\pi^{A|D(PO)} = \pi^{A|PO}$ , we find again that debt and equity financing lead to the same expected profit for *A*.

It remains to show that  $P$  is strictly better off under equity financing. For this it is sufficient to compare the joint surplus of  $A$  and  $P$  under equity and debt financing, since  $\pi^{A|D(PO)} = \pi^{A|PO}$ . Note that  $P$ 's expected profit under debt financing can be written as

$$\pi^{P|D(PI)} = (1 + r_1^P) K_1 + (1 + r_2^*) K_2^P - \mu_1 K_1 - \left[ \mu_2 K_2^P + \frac{\gamma}{2m} (K_2^P)^2 \right].$$

Using  $r_1^P = \mu_1 - 1$ , (26) with  $\alpha^P = 0$ , and (27), we can write  $\pi^{P|D(PI)}$  as

$$\pi^{P|D(PI)} = K_2^P \left[ (1 + r_2^*) - \left[ \mu_2 + \frac{\gamma}{2m} K_2^P \right] \right] = \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2.$$

Thus, the joint surplus for  $A$  and  $P$  under debt financing,  $\Pi^D \equiv \pi^{A|D(PI)} + \pi^{P|D(PI)}$ , is given by

$$\begin{aligned} \Pi^D &= x - K_2 \left[ \mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] - \mu_1 K_1 + \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2 \\ &= x - \mu_1 K_1 - \mu_2 K_2 - \frac{\gamma (n - \frac{1}{2}m)}{n^2 - m^2} K_2^2. \end{aligned}$$

Next we derive the joint surplus for equity financing (with  $\alpha^P > 0$ ). We first note that  $A$ 's expected profit is given by

$$\pi^{A|PI} = (1 - \alpha^{PI}) (1 - \beta^{PI}) x = (1 - \alpha^{PI}) \left( 1 - \frac{K_2}{x} \left[ \mu_2 + \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} K_2 \right] \right) x.$$

Moreover, the expected profit for  $P$  is given by (15). Thus, the joint surplus under equity financing,  $\Pi^E \equiv \pi^{A|PI} + \pi^{P|PI}$ , can be written as

$$\begin{aligned} \Pi^E &= x - K_2 \left[ \mu_2 + \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} K_2 \right] - \mu_1 K_1 + \frac{\gamma (1 + \alpha^P) m}{(n + m)^2} K_2^2 \left[ \frac{(n - \alpha^P m)}{(n - m)} - \frac{1}{2} [1 + \alpha^P] \right] \\ &= x - K_2 \left[ \mu_2 + \frac{\gamma (n - \alpha^P m)}{n^2 - m^2} K_2 \right] - \mu_1 K_1 + \frac{1}{2} \frac{\gamma (1 - (\alpha^P)^2) m}{n^2 - m^2} K_2^2. \end{aligned}$$

Finally, we have  $\Pi^E > \Pi^D$  if

$$\begin{aligned}
-\frac{\gamma(n - \alpha^P m)}{n^2 - m^2} K_2^2 + \frac{1}{2} \frac{\gamma(1 - (\alpha^P)^2) m}{n^2 - m^2} K_2^2 &> -\frac{\gamma(n - \frac{1}{2}m)}{n^2 - m^2} K_2^2 \\
\Leftrightarrow \alpha^P m + \frac{1}{2} (1 - (\alpha^P)^2) m &> \frac{1}{2} m \\
\Leftrightarrow 2 &> \alpha^P,
\end{aligned}$$

which is clearly satisfied. Thus,  $\Pi^E > \Pi^D$ . And because  $\pi^{A|D(PO)} = \pi^{A|PO}$ , we can infer that  $\pi^{P|D(PO)} < \pi^{P|PO}$ .

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