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ABSTRACT

We use Belgian data with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. The data allow us to accurately construct the domestic production network of the Belgian economy, revealing several new empirical facts about firms' indirect exposure to foreign trade through their domestic suppliers and buyers. We use this data to develop and estimate models of domestic production networks and international trade. We first consider a model of trade with an exogenous network structure, which gives analytical solutions for the effects of a change in the price of foreign goods on firms' production costs and real wages. To examine how gains-from-trade calculations change if buyer-supplier links are allowed to form or break in response to changes in the price of foreign goods, we next develop a model of trade with endogenous network formation. We take both models to the data and compare the empirical results to those we obtain using existing approaches. This comparison highlights the relevance of data on and modeling of domestic production networks in studies of international trade.

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1 Introduction

Over the past few decades, the focus of research on international trade has shifted from countries and industries towards firms. This shift is in no small part due to the increased availability of firm-level transaction data on trade. One important insight from these data is that few firms directly import or export goods (Bernard, Jensen, Redding, and Schott, 2007). However, the concentration of imports and exports does not necessarily imply that few firms benefit from foreign trade. Even if firms themselves do not import or export, they may still buy from or sell to domestic firms that trade internationally. Capturing this channel, however, is challenging since domestic firm-to-firm transactions are rarely observed. In the absence of such data, quantification of the effects of foreign trade on all firms requires strong assumptions, such as a common intermediate good (Eaton and Kortum, 2002, and Blaum, Lelarge, and Peters, 2016) or the same import shares across firms within broad industries (Caliendo and Parro, 2015, and Costinot and Rodríguez-Clare, 2014).

The goal of this paper is to combine data on domestic firm-to-firm sales with information on foreign trade transactions to study how international trade affects real wages and efficiency of all firms, including those that do not directly export or import. Our analysis employs a detailed dataset of Belgian firms, which is based on several data sources that we have linked through identifiers. Annual accounts provide data on input factors and output, custom records and intra-EU declarations give information on exports and imports, and a value-added tax (VAT) registry provides information on domestic firm-to-firm transactions. Using these data, we empirically examine several new dimensions of firms in international trade before developing and estimating a model of trade and domestic production networks.

In Section 2, we describe the data, construct the domestic production (buyer-supplier) network of the Belgian economy, and provide new empirical findings on firms' indirect exposure to foreign trade. Our data reveal that most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct imports observed in firm-level transaction data on trade. For example, in a majority of firms, at least 39% of input costs are spent on goods that are imported directly or indirectly. By comparison, exports are concentrated in a relatively small number of firms, even if one includes indirect exports through sales to domestic buyers which subsequently trade internationally.

In Sections 3 and 4, we develop and estimate a model of domestic production networks and international trade. In our model, firms combine imports, inputs produced by domestic firms, and labor to produce differentiated products that can be sold either to households, other firms, or abroad. In Section 3, we assume a fixed network structure (i.e., the buyer-supplier links are exogenous). Firms endogenously decide their input purchases from their suppliers as well as how much labor to hire. We show that the effects of a change in the price of foreign goods on firms' production costs and real wages depends on two pieces of information that require data on domestic firm-to-firm sales. First, the effect on firms' production costs depends on the share of input costs that is spent on goods that are imported directly or indirectly. Second, the effect on real wages depends both on the change in a firm's production costs and on how much it sells to households versus other domestic firms.

We next assess whether, and in what situations, these two pieces of information may alter the usual gains-from-trade computations. To this end, we compare estimates from our model with a fixed network structure to those we obtain using existing approaches that either assume a common or industry-specific intermediate good. For sizable changes in foreign price, we find that data on and modeling of the domestic production network is quantitatively important for accurately measuring the impacts on production costs and real wages. By comparison, for small changes in foreign prices the effect on real wages depends little on whether we use our model with a fixed network structure or the alternative approaches.

While assuming a fixed network structure is convenient to take the model to the data, it does not allow us to capture how buyer-supplier relationships may change in response to trade shocks. In Section 4, we therefore develop a model of trade with endogenous network formation. In particular, we let firms optimally choose their set of suppliers (i.e. the firm's sourcing strategy) subject to a buyer-supplier-specific fixed cost for adding a supplier. Allowing for endogenous network formation is challenging for two reasons. First, firms face a large discrete choice problem of which suppliers to include in their sourcing strategy. Second, firms' sourcing strategies are interdependent, creating a large fixed point problem: Firms take into account the expected sourcing strategies of others in order to determine their own optimal sourcing strategy, all while knowing that other firms are thinking in the same way.

Building on Jia (2008) and Antras, Fort, and Tintelnot (2017), we overcome the first challenge by using lattice theory to solve firms' large combinatorial discrete choice problems. To address the second challenge, we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.[1] While restrictive, this assumption allows us also to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. After estimating the model, we use it to quantify how international trade affects firms' production costs and consumer prices in a small open economy with and without endogenous network formation. We find that endogenous formation of buyer-supplier relationships tends to attenuate the costs of a sizable negative trade shock (i.e., a large increase in the foreign price) and amplify the gains of a sizable positive trade shock (i.e., a large decrease in the foreign price).

Our paper contributes to a growing literature on the economy-wide effects of *foreign*

[1]See Spiegler (2016) for a recent contribution in economics studying belief formation in a directed acyclic network.

sourcing.[2] Many studies use aggregate data only, relying on the assumption that firms' import intensities are equalized — which is at odds with the data. Using firm-level data on trade transactions, Blaum et al. (2016) show that accounting for heterogeneity in import exposure significantly affects the measurement of the gains from international trade. Their model assumes that firms can import directly and purchase a common intermediate good. Taking advantage of data on domestic firm-to-firm transactions, we relax the assumption of a common intermediate good and derive a parsimonious sufficient statistics formula for a model with a fixed production network. We also go beyond the fixed network structure, solving a model of endogenous network formation with a finite number of firms and fixed costs for adding suppliers. This contribution builds on the global sourcing model of Antras et al. (2017). While they distinguish between final good and intermediate good sectors, we consider a more general input-output structure between firms. In addition, our model captures not only the firms' decisions with respect to foreign sourcing but also their choices of domestic sourcing strategies.[3]

Our paper also relates to a literature on the *formation of domestic production networks.*[4] Bernard, Moxnes, and Saito (2016b) adapt the model of Antras et al. (2017) to search for domestic suppliers in different locations, where each location has a continuum of intermediate-good-producing firms. They find significant improvements in firm performance from a reduction in internal search costs in Japan. Furusawa, Inui, Ito, and Tang (2017) develop a variant of the global sourcing model of Antras et al. (2017) and use Japanese buyer-supplier link data to test the model's predictions. Oberfield (2017) develops a theory in which the network structure of production forms endogenously among firms that each purchase a single input. Lim (2015) develops a dynamic model of network formation in which each firm has a continuum of domestic suppliers. However, with a continuum of suppliers and buyers, the sales from one firm to another are negligibly small, and a link between two particular firms has no effect on aggregate outcomes.[5] In contrast to these papers, we develop a model of endogenous network formation with a finite set of suppliers and incorporate both firm exporting and importing decisions. With a discrete set of firms, we are able to match granular

[2]See for example Antràs and Helpman (2004), Antràs and Helpman (2008), Rodríguez-Clare (2010), Garetto (2013), Halpern, Koren, and Szeidl (2015), Gopinath and Neiman (2014), Amiti and Konings (2007), Goldberg, Khandelwal, Pavcnik, and Topalova (2010), De Loecker, Goldberg, Khandelwal, and Pavcnik (2016), and Amiti, Dai, Feenstra, and Romalis (2017).

[3]Our work is also related to the analysis in Caliendo and Parro (2015) and Ossa (2015). They find the gains from trade to be larger when taking sectoral input-output linkages into account.

[4]A growing body of work studies how firms meet international trading partners. See for example Chaney (2014), Chaney (2016), Morales, Sheu, and Zahler (2015), and Eaton, Kinkins, Tybout, and Xu (2016). Our work is also related to the literature on the measurement and formation of global value chains such as Johnson and Noguera (2012) and Antràs and de Gortari (2017), recently surveyed by Johnson (2018).

[5]Taschereau-Dumouchel (2018) also considers a finite number of firms in his theory of cascades and fluctuations in an economy with a fixed set of connections between — potentially inactive — firms and endogenous firm entry.

firm-level outcomes such as the number of domestic suppliers and customers.

Finally, our paper relates to a literature that analyzes the *propagation of idiosyncratic firm-level shocks*. Barrot and Sauvagnat (2016), Boehm, Flaaen, and Pandalai-Nayar (2015), and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016) use natural disasters to study the propagation of shocks in production networks. Carvalho and Voigtländer (2015) analyze the adoption of inputs by innovators and the evolution of the domestic production network.[6] Building on this empirical evidence, we extend the analysis of shock propagation to foreign trade shocks and demonstrate that firms that are not directly exposed to international trade can still be significantly affected by changes in the price of foreign goods.[7] We additionally allow buyer-supplier relationships to change in response to these shocks and characterize how these extensive margins affect firm-level and aggregate outcomes.[8]

[6]This paper is also related to the literature on how micro-shocks can have aggregate outcomes. Hulten (1978) provides conditions under which the underlying network structure is irrelevant for quantifying the propagation of shocks — up to a first-order approximation — as long as firms' initial size and the magnitudes of the idiosyncratic shocks are observed. Gabaix (2011) provides conditions under which granular shocks can affect aggregate fluctuations. Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) study the transmission of shocks along sectoral input-output networks. Magerman, De Bruyne, Dhyne, and Van Hove (2016) test both channels with the Belgium domestic firm-to-firm data. In recent work, Baqaee and Farhi (2017) illustrate that the second-order effects of shock propagation arising from networks can be large.

[7]While our theory assumes simple solutions for the pricing game between firms, Kikkawa, Magerman, and Dhyne (2017) explore the segmentation of markets for different buyers, with supplier firms having heterogeneous bargaining power in the supplier-buyer relationships.

[8]Other recent contributions to determining the effects of networks include Baqaee (2014), Carvalho and Grassi (2017), as well as in the context of financial frictions, Bigio and La'o (2016) and Liu (2016). In terms of studying the structure of the network itself, Atalay, Hortacsu, Roberts, and Syverson (2011) characterize the buyer-supplier network of the US economy, Bernard, Dhyne, Magerman, Manova, and Moxnes (2018) study how much of the Belgian firm size heterogeneity is coming from firms' relationships with their suppliers and customers, and Eaton, Kortum, and Kramarz (2018) study firm-product-level heterogeneity in supplier-buyer networks in French customs data.

2 Trade and production networks: Data and evidence

This section describes the data and documents firms' direct and indirect exposure to foreign trade.

2.1 Data sources and sample selection

Our analysis draws on three administrative data sources from Belgium, accessible only at the National Bank of Belgium, for the years 2002-2014. These data sources can be linked through unique identifiers, assigned and recorded by the government for the purpose of collecting value-added taxes (VAT). Below we briefly describe our data and sample selection, while additional details are given in Appendix C.

The first data source is the Business-to-Business (B2B) transactions database. (See also Dhyne, Magerman, and Rubinova, 2015.) By law, all Belgian firms are required to file the annual sales to each buyer (provided the annual sales to a given buyer exceeds 250 Euro). Thus, the B2B dataset allows us to measure accurately the identity of the firms' suppliers and buyers. The second data source is the annual accounts filed by Belgian firms merged with firms' VAT tax declarations. These data contain detailed information from the firm's balance sheets on output (such as revenues) and inputs (such as capital, labor, intermediates) as well as 4-digit (NACE) industry codes and geographical identifiers at the zip code level. In addition, the annual accounts include information about ownership shares in other enterprises. The third datasource is the Belgian customs records and the intra-EU trade declarations. These data contain information about international trade transactions in each year and for every firm. Both imports and exports are disaggregated by product and origin or destination.

One challenge with using the Belgian data is that the information is recorded at the level of the VAT-identifier. The problem is that a given firm may have several VAT-identifiers (for accounting or tax reasons).[9] While organizational choices and transactions across units within a firm are of interest, our paper is centered on trade between firms. Thus, if a firm has multiple VAT-identifiers, we aggregate all data up to the firm level using information from the balance sheets about ownership structure. Details of the aggregation are outlined in Appendix C.1. In 2012, for example, the aggregation converts 896,000 unique VAT-identifiers into 860,000 unique firms. Of these firms, 842,000 had a single VAT-identifier. However, the 18,000 firms with multiple VAT-identifiers are important, accounting for around 60% of the total output in the dataset.

[9] Existing papers tend to ignore this issue, analyzing the data as if each firm has a single unique VAT-identifier. See e.g. Amiti, Itskhoki, and Konings (2014), Magerman et al. (2016), and Bernard, Blanchard, Van Beveren, and Vandenbussche (2016a)).

After constructing a firm-level dataset, we impose a few sample restrictions. We restrict our analysis to firms in the private and non-financial sector with positive labor costs, at least one full-time-equivalent employee, and positive output. Following De Loecker, Fuss, and Van Biesebroeck (2014), we also restrict our analysis to firms with tangible assets of more than 100 Euro and positive total assets in at least one year during our sample period. Applying these criteria reduces the number of firms significantly. In 2012, for example, only 98,745 firms satisfy the above criteria. The large reduction in sample size is mostly driven by the exclusion of local firms without employees (self-employment) from the sample (750,100 firms in 2012). These criteria also remove foreign firms with no local production activity in Belgium from the sample. These account for a sizable fraction of imports and exports but have no domestic production activity in Belgium.

Table 1 illustrates that our selected estimation sample of firms provides relatively good coverage of aggregate value added, gross output, exports, and imports. However, total sales in our sample is larger than what are reported in the national statistics. The reason is that the output of trade intermediaries in the national statistics is measured by their value added instead of their total sales. We refer to Appendix C.2 for the same statistics for all Belgian firms and to Appendix C.4 for their sectoral composition.[10]

Table 1: Coverage of selected sample

Year	GDP (Excl.	Output Gov. & Fin.)	Imports	Exports	Selected sample				
					Count	V.A.	Sales	Imports	Exports
2002	182	458	178	193	88,301	231	604	175	185
2007	230	593	254	267	95,941	299	782	277	265
2012	248	671	317	319	98,745	356	874	292	292

Notes: All numbers except for Count are denominated in billion Euro in current prices. Belgian GDP and output are for all sectors excluding public and financial sector. See Appendix C.2 for the same statistics for the total economy. Data for Belgian GDP, output, imports, and exports are from Eurostat. Firms' value added is computed as their sales minus imports and purchases from other Belgian firms in the sample.

2.2 Direct and indirect exposure to foreign trade

The Belgian data allow us to construct the buyer-supplier relationships of the Belgian economy and therefore document firms' direct and indirect exposure to foreign trade. While the importance of intermediaries in international trade has been well documented (Bernard, Jensen, Redding, and Schott 2010; Ahn, Khandelwal, and Wei 2011; Bernard et al. 2016a; Ganapati 2018), the linkages of domestic firms to these intermediaries are commonly unob-

[10]In Appendix C.3 we outline the definition of each variable that we use in the analyses.

served.[11]

We define firm j's *total foreign input share* as the sum of firm j's *direct foreign input share*, s_{Fj}, and the direct foreign input share of firm j's suppliers, suppliers' suppliers, and so forth, each weighted by the firm-pair-specific input shares ($s_{ij}, s_{ki}, ...$):

$$s_{Fj}^{Total} = s_{Fj} + \sum_{i \in Z_j^D} s_{ij} \underbrace{\left[s_{Fi} + \sum_{k \in Z_i^D} s_{ki} \left(s_{Fk} + \cdots \right) \right]}_{s_{Fi}^{Total}}, \tag{1}$$

where Z_j^D denotes the set of domestic suppliers of firm j, and the denominator of the input shares is the sum of labor costs, purchases from other firms, and imports. Note that the definition of the total foreign input share is recursive: A firm's total foreign input share is the sum of its direct foreign input share and the share of its inputs from other firms multiplied by those firms' total foreign input shares. While many firm-level datasets contain information about the direct foreign input share s_{Fj}, our data also offer information about firm-pair-specific input shares, s_{ij}.[12] As a result, we are able to calculate the total foreign input share for every firm. We note that there is one inherent assumption in our definition of the total foreign input share: When a firm sells its output to multiple firms or final consumers, the foreign input share in the costs of producing these goods is assumed to be the same for all buyers (i.e., independent of the identity of the buyer). This assumption is consistent with the model we develop in Sections 3 and 4, where each firm produces a single product.[13]

In Figure 1a, we display a histogram of the total and direct foreign input shares of the Belgian firms. While only 19% of firms import directly, nearly all firms obtain some foreign inputs either directly or indirectly through domestic suppliers which use foreign inputs in their production process.[14] Indeed, most firms are heavily dependent on foreign inputs, but only a small number of firms show that dependence through the direct foreign input shares observed in firm-level transaction data on trade. In the median firm, for example, the total

[11] A notable exception is Fujii, Ono, and Saito (2017) who use data on firm-to-firm linkages as recorded by a Japanese credit reporting company. The data suggest that 19% of manufacturing firms have a shipment to an exporting wholesaler. Another exception is Dhyne and Duprez (2017). Using the Belgian data on domestic firm-to-firm sales, they show that nearly all firms in Belgium are at most three links away from an importing firm, and about 65% of the firms are at most three links away from an exporting firm.

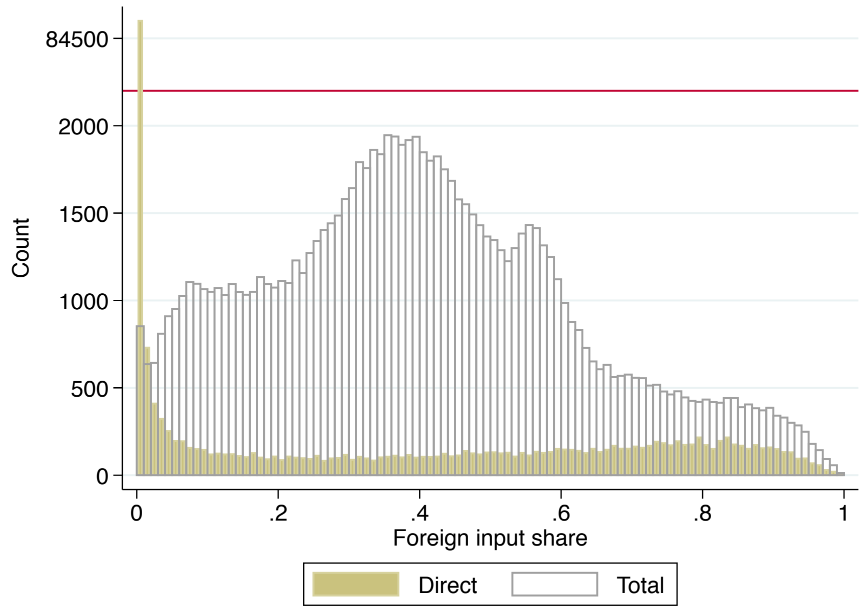
[12] Consistent with the our model of production network, we defined the denominator of s_{ij} as the sum of all input costs of firm j. Alternatively, one could have considered to use the sales of firm j in the denominator.

[13] Ideally, one would measure the foreign input share by product of each firm and the product-to-product input shares. We are not aware of the existence of such data for an entire economy to conduct the calculations at this more granular level. Naturally, the aggregation to the firm-level will lead us to overstate the total foreign input shares of some firms, and understate the total foreign input shares of other firms.

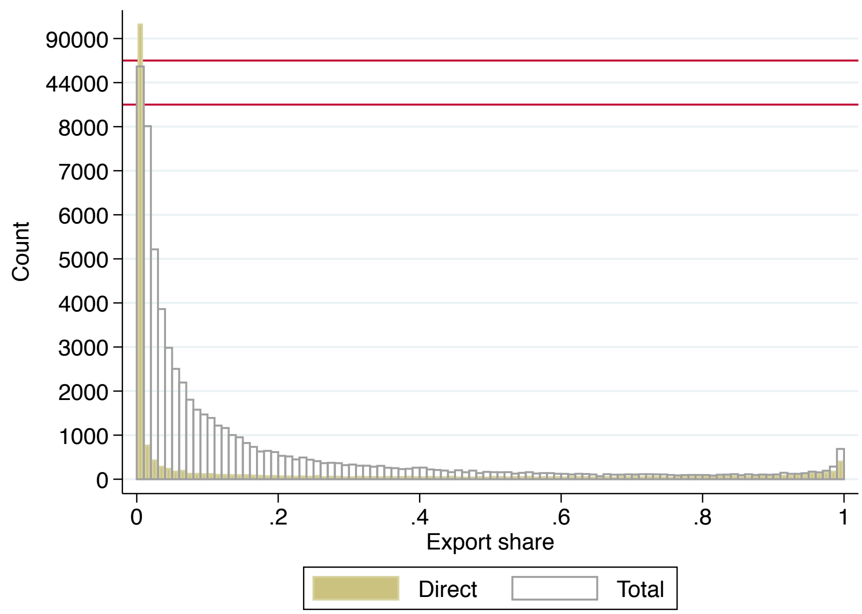
[14] Note that virtually all firms are likely to use certain types of materials that are not produced in Belgium, such as gasoline. While we may not observe all these purchases due to the 250 Euro reporting threshold for domestic firm-to-firm transactions, the use of such inputs could help explain why nearly all firms (over 99%) in our sample are obtaining some foreign input directly or indirectly.

Figure 1: Histograms of direct and indirect linkages to foreign trade

(a) Direct and total foreign input share



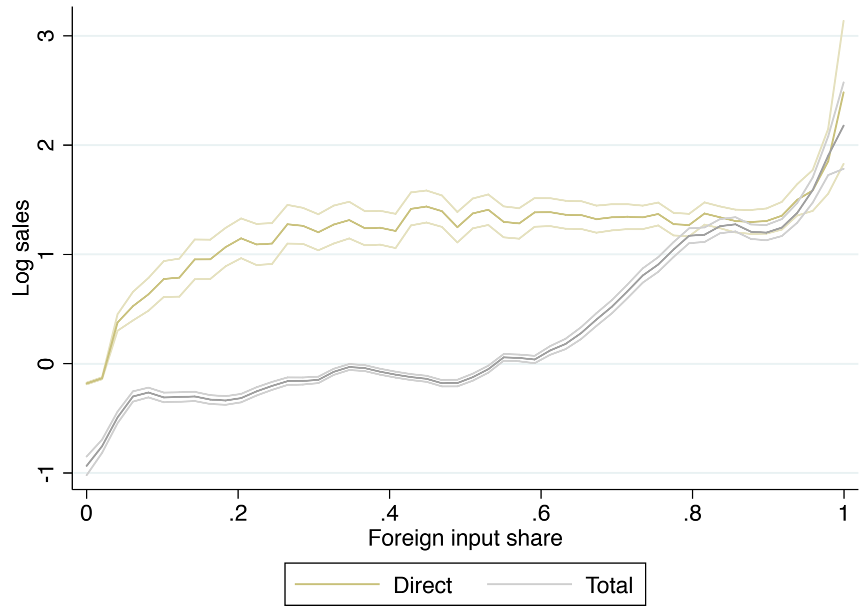
(b) Direct and total export share



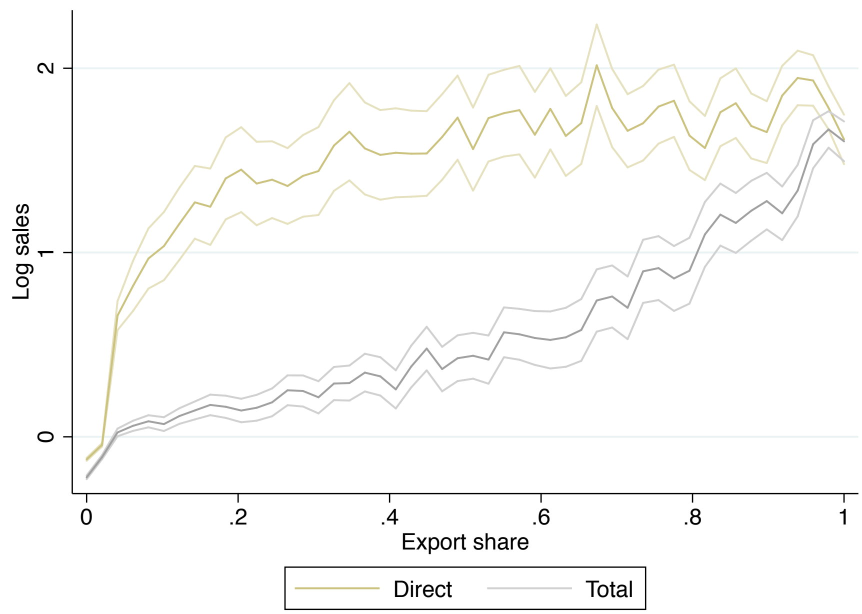
Notes: Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share and s_{ji} is j's share among i's inputs. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue and r_{ij} is share of i's revenue that arises from sales to firm j. The figures are based on the analysis of 98,745 private sector firms in Belgium in 2012. The horizontal lines represent scale breaks on the vertical axis.

Figure 2: Size premium of direct and indirect linkages to foreign trade

(a) Foreign inputs



(b) Exports



Notes: The two figures display the smoothed values with 95% confidence intervals of kernel-weighted local polynomial regression estimates of the relationship between firms' sales and their levels of participation in foreign trade. We use the Epanechnikov kernel function with kernel bandwidth of 0.01, pilot bandwidth of 0.02, degree of polynomial smooth at 0, and smooth obtained at 50 points. Log sales are demeaned with 4-digit industry fixed effects. The figures are based on the analysis of 98,745 private sector firms in Belgium in 2012.

foreign input share is 39%. While nearly all firms obtain some foreign inputs either directly or indirectly, the exposure to foreign trade varies substantially across firms. For example, the total foreign input shares are 20% and 58% at at the 20th and 80th percentile, respectively.[15]

In Appendix D.2, we break down the direct and total foreign input shares by sector. Interestingly, foreign inputs are important in all sectors once one accounts for the indirect import. Even in the service sector, in which firms have little if any direct import, the median firm's total foreign input share is as large as 24%. Breaking down the data by sector also reveals that indirect import is not entirely driven by firms obtaining foreign inputs through the wholesale and retail sector. As shown in Appendix D.3, even when excluding direct imports by the wholesale and retail sector (29% of aggregate imports) from the calculation of s_{Fi}^{Total}, most firms continue to indirectly import a significant amount of foreign inputs through purchases from their domestic suppliers.[16] For example, the median firm's total foreign input share is still as large as 18%.

Figure 1b performs a similar exercise but now looks at *total export shares* and *direct export shares*. We calculate the total export share of firm j, r_{jF}^{Total}, as the sum of the share of revenue of firm j coming from directly exported goods, r_{jF}, and the share of revenue coming from goods sold to other domestic firms, multiplied by those firms' total export shares:

$$r_{jF}^{Total} = r_{jF} + \sum_{i \in W_j} r_{ji} r_{iF}^{Total}, \tag{2}$$

where W_j denotes the set of domestic buyers of firm j and the denominator of the export shares is the total revenue of the firm. Direct export is even more concentrated than direct import, both on the intensive and extensive margin. While only 12% of firms export directly, 88% of firms export either directly or indirectly by selling to domestic buyers which subsequently trade internationally. In terms of trade volume, however, export remains relatively concentrated even after taking the indirect export into account. The total export share is only 2% in the median firm, whereas it is 14% at the 80th percentile. In Appendix D.3, we compute the total export shares when excluding any exports by wholesalers and retailers (which account for 18% of aggregate exports). Among firms in sectors other than wholesale or retail, the total export share is then 13% at the 80th percentile and 1% at the median.

Why does a majority of firms have significant indirect import but little indirect export? It is not reflecting a trade deficit in Belgium, as aggregate imports and exports are roughly equal. Instead, it reflects that most firms have a large number of domestic suppliers (i.e.,

[15] Note that we in these calculations exclude the user cost of capital in the calculation of the input shares. The reason is that the firm-to-firm may include purchases of capital goods and, therefore, adding the user cost of capital from as another input cost could lead to double counting of capital goods. As a robustness check, we nevertheless included the user cost of capital in the calculation of the input shares, finding that in the median firm the total foreign input share is 35% (see Appendix D.1 for details).

[16] Specifically, s_{Fi}^{Total} are computed by setting $s_{Fj} = 0$ for all firm j in the wholesale and retail sector.

indegrees) as compared to domestic buyers (i.e., outdegrees). While the first, second, and third quartiles of the indegree distribution are 19, 33, and 55, the quartiles of the outdegree distribution equal 2, 9, and 34. Hence the distribution of the number of buyers is much more skewed than the distribution of the number of suppliers, leading the majority of firms to have high indirect imports but low indirect exports. In Appendix D.4, we investigate the direct and total foreign export shares by sector. One sector that stands out is the service sector. While many firms in this sector rely on foreign inputs, relatively few export directly or sell to domestic firms that are exporting directly or indirectly.

Across a wide range of countries and industries, firms that directly export or import tend to be larger than other firms. A natural question is whether the positive association between firm size and international trade also carries over to indirect export or import. Figure 2 investigates this, calculating the average size of firms by direct and total foreign input shares as well as by direct and total export shares. We demean the log of firm sales using the firm's four-digit industry average, so that a firm with log sales of zero is the size of an average firm in its industry. Figure 2 illustrates that average firm size is increasing in both direct and total foreign input shares. However, firms that import directly tend to be much larger than firms that buy foreign inputs through domestic firms. Indeed, firms with less than 60% in total foreign input shares are, on average, of similar size as the average firm in their industry. A similar pattern is evident for size and export. Firms with very high total export shares tend to be large. However, over most of the total export share distribution, there is only a weak relationship between firm size and total export share. Firms with a total export share of 60% are only about 50% larger than the average firm in their industry. Taken together, the results in Figure 2 suggest that firms do not have to be large to rely heavily on foreign inputs or to have most of their sales going ultimately to a foreign country.

3 Model with fixed production networks

We now develop a model of trade with fixed production networks and use it to quantify how international trade affects firms' production costs and consumer prices. By fixed networks, we mean that the buyer-supplier links are exogenous. Given these links, firms decide how much to buy from and sell to the existing suppliers and buyers. While the assumption of exogenous buyer-supplier links is admittedly strong, it allows us to derive several analytical results and it makes it convenient to take the model to the data.

3.1 Model

To match our data from Belgium, we consider a small open economy. Before describing the model, we briefly discuss the notation. Since there exist many bilateral directed flows in our model, we will often have two subscripts. In such cases, the first subscript denotes the origin of the good, and the second subscript denotes the destination of the good.

3.1.1 Preferences and Demand

Each consumer supplies one unit of labor inelastically. Consumers are assumed to have identical, homothetic CES preferences over consumption goods:

$$U = \left(\sum_{k \in \Omega} \left(\beta_{kH} q_{kH} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where Ω denotes the set of available products in the small open economy, k denotes a product, and H denotes domestic final demand from households. Since all consumers have the same homothethic CES preferences for consumption, we can write the aggregate final consumer demand (in quantities) for product k, given price p_{kH}, as:

$$q_{kH} = \beta_{kH}^{\sigma-1} \frac{p_{kH}^{-\sigma}}{P^{1-\sigma}} E, \tag{4}$$

where E denotes the aggregate expenditure in Belgium, and P denotes the domestic consumer price index:

$$P = \left(\sum_{j \in \Omega} \beta_{jH}^{\sigma-1} p_{jH}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{5}$$

We assume that final goods are substitutes, and therefore $\sigma > 1$.

Demand from abroad for product k takes a similar functional form:

$$q_{kF} = \beta_{kF}^{\sigma-1} \frac{p_{kF}^{-\sigma}}{P_F^{1-\sigma}} E_F, \tag{6}$$

where β_{kF} is a product-k-specific foreign demand shifter, p_{kF} is the price of product k abroad, and P_F and E_F denote the foreign price index and expenditure, respectively .

3.1.2 Market structure and production

Firms produce single products. We will use i, j, k to index firms or products. The products are differentiated across firms. Firms sell the same product to final consumers and to other firms as an intermediate input, though not all firms sell to other firms, and not each pair of firms has a buyer-supplier relationship. Note that we allow Belgian firms to sell directly to foreign consumers, while all foreign products reach Belgian consumers indirectly through the importing of inputs by Belgian firms.[17]

When selling to households, we treat every firm as infinitesimal and assume monopolistic competition. Firms then charge a constant mark-up over marginal costs, $\mu = \frac{\sigma}{\sigma-1}$. When selling to other firms, the assumption of infinitesimal size is no longer reasonable, since most firms just have a few selected suppliers. We assume that in the Nash bargaining between buyer and supplier, the buyer has the full bargaining power. Given the assumptions on technology described below, this will imply that the supplier sells at marginal cost to the buyer firm. Note, however, that our theoretical results for the model with a fixed network structure do not change if firms charge positive and possibly heterogeneous mark-ups to buyer firms as long as these are fixed.[18] The assumption of the bargaining power in firm-to-firm transactions being on the buyer's side is convenient for modeling the network formation in a tractable manner in Section 4.[19]

In this section we assume a fixed network structure, meaning that we take as given the set of firms, Z_j, from which each firm j is eligible to purchase inputs. For importing firms, Z_j contains also foreign, F, as an eligible supplier. Sometimes we will refer to the set of domestic suppliers of firm j, which we denote by Z_j^D. In order to sell abroad, firms incur iceberg transport costs, τ. In this section, we take export participation, I_{jF}, as given ($I_{jF} = 1$ for all exporting firms and $I_{jF} = 0$ otherwise) and endogenize it in Section 4.

[17]The assumption that foreign goods reach Belgian consumers only through Belgian firms is reasonable because in the data nearly all imports are carried out by firms. We make the assumption that Belgian firms can reach foreign consumers directly to avoid modeling foreign firms in detail.

[18]Positive and fixed mark-ups by firms would only affect the system of equations in the Appendix to calculate the change in the domestic wage. See Kikkawa et al. (2017) for a model of trade with production networks and variable firm-to-firm mark-ups.

[19]Also in the industrial organization literature, corner solutions in the bargaining game are sometimes assumed to obtain tractable solutions for network formation problems. For example, when studying the determinants of the hospital networks offered by health plans, Ho (2009) assumes that hospitals make take-it-or-leave-it offers to all health plans in the market.

Firms use a CES input bundle of workers and domestic and foreign inputs with elasticity of substitution $\rho > 1$ in the production function. We assume that $\sigma > \rho$, implying that consumers are more price-elastic than firms in their purchase of goods. Given the CES production function, we can write the unit cost function of firm j as:

$$c_j(Z_j) = \frac{1}{\phi_j}\left(\sum_{k\in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho}\right)^{1/(1-\rho)}. \tag{7}$$

The first term in the cost function, ϕ_j, denotes the exogenous total factor productivity of firm j. Inside the parentheses, the term α_{kj} reflects how salient the good produced by supplier k is as an input for firm j, and p_{kj} denotes the price that supplier k charges for its input to firm j. Furthermore, the term α_{Lj} captures the importance of labor input for firm j, and the price of labor is denoted by w.

3.1.3 Firm behavior and dependence on foreign inputs

The share of variable costs by firm j that is spent on intermediate inputs produced by firm $k \in Z_j$ is:

$$s_{kj} = \frac{p_{kj} q_{kj}}{c_j q_j} = \frac{\alpha_{kj}^{\rho-1} p_{kj}^{1-\rho}}{\Theta_j(Z_j)}, \tag{8}$$

where $\Theta_j(Z_j) = \sum_{k\in Z_j} \alpha_{kj}^{\rho-1} p_{kj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho}$ denotes the *sourcing capability* of firm j, and Z_j denotes the *sourcing strategy* of firm j, following the terminology of Antras et al. (2017).

Firm j spends a larger fraction of variable costs on inputs produced by firm k if the saliency term α_{kj} is large or the price p_{kj} is low, relative to its sourcing capability. Analogously, the share of variable costs by firm j that is spent on labor is:

$$s_{Lj} = \frac{wL_j}{c_j q_j} = \frac{\alpha_{Lj}^{\rho-1} w^{1-\rho}}{\Theta_j(Z_j)}, \tag{9}$$

while the *direct* foreign input share of firm j (assuming $F \in Z_j$) is:

$$s_{Fj} = \frac{p_{Fj} q_{Fj}}{c_j q_j} = \frac{\alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho}}{\Theta_j(Z_j)}. \tag{10}$$

Based on the firm-pair-specific input share, s_{kj}, the direct foreign input share, s_{Fj}, and the set of domestic suppliers, Z_j^D, we are able to calculate the *total* foreign input share, s_{Fj}^{Total}, defined in equation (1). The total foreign input share summarizes the exposure of firms to foreign inputs and is intuitive in a model with single product firms in which each firm uses the same fraction of foreign inputs in the good sold to every buyer. Proposition 1 shows that the total foreign input share is useful for calculating the firm-level cost change in response to a change in the foreign input price. Let z' denote the value for any variable z after an exogenous shock and $\hat{z} = \frac{z'}{z}$ denote the change in variable z.

Proposition 1 (Cost changes in response to foreign price changes)
Given fixed linkages between firms, the change in firm j's unit cost, $\hat{c}_j \mid^{\hat{p}_{F\cdot}}$, given an uniform change in foreign price, $\hat{p}_{F\cdot}$, is

$$\hat{c}_j \mid^{\hat{p}_{F\cdot}} = \left(\left(1 - s_{Fj}^{Total}\right)\hat{w}^{1-\rho} + s_{Fj}^{Total}\hat{p}_{F\cdot}^{1-\rho}\right)^{1/(1-\rho)}. \tag{11}$$

For a small percentage point change in the foreign price, $\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$, the first-order approximation to firm j's unit cost is

$$\frac{\mathrm{d}c_j}{c_j} = \left(1 - s_{Fj}^{Total}\right)\frac{\mathrm{d}w}{w} + s_{Fj}^{Total}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}. \tag{12}$$

For both large and small changes to the foreign price, the changes in the firm-level cost are a weighted aggregate of the change in the foreign price and the change in the domestic wage. The change in the domestic wage is determined by the equilibrium, as described below. The weights are firm-specific and summarized by the the firm's total foreign input share.[20] As can be seen from equation (1), a firm's cost will not only change according to its own direct foreign input share, but also according to its suppliers' foreign input shares, suppliers' suppliers' foreign input share, and so forth. Note that we leveraged the assumption that there is perfect pass-through of any cost changes along the production chain.[21] For large changes to the foreign price, the elasticity of substitution in the production function, ρ, indicates how easy it is to switch to alternative inputs, including labor. Given the observed total foreign input share, a lower value of ρ leads to a larger cost increase from an increase in the foreign price. In the extreme case when the foreign price change is infinite and the economy goes to autarky, the change in firm-level cost becomes $\hat{c}_j \mid^{p_{F\cdot}\to\infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)}\hat{w}^{aut}$, where $\hat{w}^{aut}$ denotes the change in the domestic wage when going to autarky. For small changes to the foreign price, the first-order approximation to the firm's cost change does not depend on the elasticity of substitution. Furthermore, we show in the proof that the result in equation (12) holds more generally for any constant returns to scale production function. While Proposition 1 considers a uniform change in the foreign price, we derive expressions

[20] Equivalently, we could express the cost change using the firm's total labor input share: $s_{Lj}^{Total} = 1 - s_{Fj}^{Total} = s_{Lj} + \sum_{i\in Z_j^D} s_{ij}s_{Li}^{Total}$.

[21] Note that we would derive the same equation under positive, but constant, firm-to-firm mark-ups. However, the equilibrium change in $\hat{w}$ would be different in such a model. See Kikkawa et al. (2017) for results on cost changes under pair-wise variable mark-ups.

for firms' cost changes when the import price changes are heterogeneous across firms, $\{\hat{p}_{Fj}\}$ in Appendix A.2.[22]

3.1.4 Firm-level sales and profits

Firms' total sales consist of the sum of domestic sales to final consumers, foreign sales to final consumers, and domestic sales to other firms. Let firm j's total sales be denoted by:

$$
\begin{aligned}
x_j = & \underbrace{\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}}}_{\text{Domestic sales to final consumers}} + \underbrace{I_{jF}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}}_{\text{Exports}} \\
& + \underbrace{\sum_k I(j \in Z_k)\phi_j^{\rho-1}\alpha_{jk}^{\rho-1}\Theta(j)\frac{x_k/\mu_k}{\Theta_k(Z_k)}}_{\text{Domestic sales to firms}},
\end{aligned}
\tag{13}
$$

where μ_k denotes the average mark-up of firm k. Recall that the firm charges a constant mark-up to final consumers and a zero mark-up to other firms. Hence, μ_k depends on the distribution of firm k's sales.

Given that firms make their profits only on sales to final consumers, we can write the *variable* profits of firm j given a sourcing strategy, Z_j, and export participation, I_{jF}, as

$$
\begin{aligned}
\pi_j^{var}(Z_j, I_{jF}) = & \frac{1}{\sigma}\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}} \\
& + I_{jF}\frac{1}{\sigma}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}}.
\end{aligned}
\tag{14}
$$

3.1.5 Aggregation and equilibrium

We now describe the aggregation of our model, discuss how firm profits are redistributed to consumers, define the equilibrium, and discuss the determination of the domestic wage change in response to any foreign price change. In the model with a fixed production network, we abstract from fixed costs of linkage formation, and hence $\pi_j = \pi_j^{var}$.[23]

We assume that the set of Belgian firms is fixed and that firm profits are distributed to workers in Belgium. We consider Belgium as a small-open economy — i.e., foreign ex-

[22] Consider arbitrary changes in the import price at the firm-level, $\{\hat{p}_{Fj}\}$. Firm-level cost changes given these shocks can be expressed as $\hat{c}_j^{1-\rho} = \left(1 - s_{Fj}^{Total}\right)\hat{w}^{1-\rho} + t_{Fj}$, where t_{Fj} is computed from $t_{Fj} = s_{Fj}\hat{p}_{Fj}^{1-\rho} + \sum_{i \in Z_j^D} s_{ij}t_{Fi}$, and $\hat{w}$ is computed from the system of equations presented in Appendix A.2. In this system of equations, we also allow for arbitrary changes in export demand at the firm-level, $\left\{\hat{\beta}_{jF}\right\}$.

[23] Note that the counterfactual results are identical even in the presence of fixed costs, as long as the network structure is fixed.

penditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, are exogenous. We further assume that there are no foreign asset holdings and that trade is balanced. Hence aggregate household expenditure in Belgium is given by

$$E = wL + \sum_k \pi_k. \tag{15}$$

Balanced trade implies that aggregate exports are equal to aggregate imports:

$$\sum_j I_{jF} \beta_{jF}^{\sigma-1} \mu^{1-\sigma} \phi_j^{\sigma-1} \Theta_j(Z_j)^{(\sigma-1)/(\rho-1)} \tau^{1-\sigma} \frac{E_F}{P_F^{1-\sigma}} = \sum_j \frac{1}{\mu_j} s_{Fj} x_j. \tag{16}$$

Labor market clearing implies that labor income is equal to firms' labor costs:

$$wL = \sum_j \frac{1}{\mu_j} s_{Lj} x_j. \tag{17}$$

The equilibrium for the small open economy is defined as follows.

Definition 1 (Equilibrium given a fixed network structure)
Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, an equilibrium for the model with a fixed network structure and fixed export participation is a wage level, w, price index for the consumer, P, and aggregate expenditure, E, such that equations (5)*,* (7)*,* (8)*,* (9)*,* (13)*,* (14)*,* (15)*,* (16)*, and* (17) *hold.*

Using the system of equations in the above definition and the result in Proposition 1, we describe the system of equations to determine the change in the nominal wage for a foreign price change in Appendix A.7, while Appendix A.8 contains the analogous system of equations under first-order approximation.

In order to describe the change in the real wage, it will be useful to define the share of household consumption that is produced by firm i, s_{iH}:

$$s_{iH} = \frac{p_{iH} q_{iH}}{E} = \frac{\beta_{iH}^{\sigma-1} p_{iH}^{1-\sigma}}{P^{1-\sigma}}. \tag{18}$$

Given this definition, the change in the real wage for a uniform foreign price change can be summarized as follows:

Proposition 2 (Real wage change in response to foreign price changes)
Given fixed linkages between firms, the change in the real wage, $\frac{\hat{w}}{\hat{P}}$, due to an uniform

change in foreign price, $\hat{p}_{F\cdot}$, is:

$$\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot}} = \hat{w} \left(\sum_i s_{iH} \left(\hat{c}_i \mid^{\hat{p}_{F\cdot}} \right)^{1-\sigma} \right)^{\frac{1}{\sigma-1}}$$

$$= \left(\sum_i s_{iH} \left(\left(1 - s_{Fj}^{Total}\right) + s_{Fj}^{Total} \frac{\hat{p}_{F\cdot}^{1-\rho}}{\hat{w}^{1-\rho}} \right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}. \tag{19}$$

For a small percentage point change in the foreign price, $\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$, the first-order approximation to the change in the real wage is:

$$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \frac{\mathrm{d}w}{w} - \sum_j s_{jH} \frac{\mathrm{d}c_j}{c_j}$$

$$= \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \sum_j s_{jH} s_{Fj}^{Total}. \tag{20}$$

The change in the real wage is given by the change in the domestic nominal wage and a weighted aggregate of firms' cost increases, with the weights equal to the firms' share in domestic household demand, s_{iH}. In the extreme case where the foreign price change is infinite and the economy goes to autarky, the change in the real wage becomes $\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot} \to \infty} = \left(\sum_i s_{iH} \left(1 - s_{Fj}^{Total}\right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma-1}}$. When going to autarky, the change in the nominal wage doesn't have to be calculated since it cancels out. Moreover, we provide equations for calculating the change in real income in Appendix A.9.[24] The first-order-approximation to the change in the real wage in equation (20) holds for a wider class of production and utility functions, provided that they are homogeneous of degree one.

The above proposition highlights the importance of the change in the nominal wage, the firms' shares in domestic household demand, and the firms' total foreign input shares in calculating the real wage changes in response to a foreign price shock. In general, computing these statistics require knowledge (or strong assumptions) about the network structure. This contrasts with the result from Hulten (1978) for a closed economy, which states that for an efficient economy, and under some technical conditions, the knowledge of the network structure becomes redundant in making aggregate counterfactual predictions up to a first-order approximation.[25] Applying Hulten's insights to a small open economy, one may consider exports as inputs and imports as outputs of a "foreign sector." To calculate the

[24]Furthermore, we show in Appendix A.4 that analogous results to Propositions 1 and 2 can be obtained under the assumption of a continuum of firms.

[25]Hulten's theorem states that the change in aggregate real consumption, C, can be expressed as $\frac{\mathrm{d}C}{C} = \sum_s \frac{\text{Output}_s}{\text{VA}} \frac{\mathrm{d}A_s}{A_s}$, where $\frac{\mathrm{d}A_s}{A_s}$ denotes the change in TFP of sector s.

percentage change in the real wage of a small open economy, one would compute the product of the foreign sector's Domar (1961) weight, aggregate imports over aggregate value added, and the percentage change in the foreign sector's productivity. The change in the foreign sector's productivity can be captured by the change in the terms-of-trade (i.e., the change in the price of exports minus the change in the price of imports). Therefore, under perfect competition, the change in the real wage expressed in equation (20) becomes equivalent to the expression:

$$
\begin{aligned}
\underbrace{\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P}}_{\substack{\text{Real wage change} \\ \text{under perfect competition}}} &= \underbrace{\frac{\text{Imports}}{\text{VA}}}_{\text{Domar weight}} \left(\underbrace{\sum_j s_{jF} \left(\left(1 - s_{Fj}^{Total}\right) \frac{\mathrm{d}w}{w} + s_{Fj}^{Total} \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right)}_{\substack{\text{Change in} \\ \text{export price}}} - \underbrace{\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}}_{\substack{\text{Change in} \\ \text{import price}}} \right) \\
&= \frac{\text{Imports}}{\text{VA}} \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \sum_j s_{jF} \left(1 - s_{Fj}^{Total}\right), \qquad (21)
\end{aligned}
$$

where s_{jF} is firm j's export share in total exports.[26]

To compute the terms $\frac{dw}{w}$ and $\sum_j s_{jF} \left(1 - s_{Fj}^{Total}\right)$ in equation (21), knowledge or strong assumptions about the network structure is required. The reason the network structure matters even in the first-order approximation is because not all foreign shocks are absorbed domestically in a small open economy.

When going beyond first-order approximations, the substitution of consumers across firms in response to firms cost changes becomes important. Figure 1a shows that the majority of firms are meaningfully affected by a foreign price change, and therefore the consumer has less scope for substitution across firms. For a closed economy, Baqaee and Farhi (2017) also point out the importance of the network structure to capture the second-order effects of economic shocks.

3.2 Alternative approaches and data requirements

As we have shown above, the structure of the domestic production network is relevant for measuring changes of firm-level cost, real wages, and real income in response to a foreign price shock. Our results imply that one can find different aggregate effects from foreign price changes across two economies with the same elasticities of substitutions in production and in the utility functions and the same levels of aggregate imports and exports, GDP, *and* gross production. In fact, even all firm-level outcomes on domestic sales, imports, exports, value added, and intermediate good purchases can be identical, and the aggregate effects can still

[26] See Appendix A.5 for the derivation.

be different. We illustrate this in a simple numerical example in Appendix B and discuss this further in the quantitative application with the Belgian data in Section 3.3.[27]

Below, we describe several alternative approaches that have lower data requirements.

3.2.1 Back of the envelope calculation for an efficient economy

One may want to use the analogue of Hulten's theorem for a small open economy in equation (21) to calculate a first-order approximation to the change in real wages, assuming that the economy is efficient. As discussed above, the network structure is still needed to compute several of the terms in this equation. Here we discuss additional simplifying assumptions to approximate the term $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$. If one assumes no import content in the production of exports, then $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$ is equal to 1. Alternatively, if one assumes perfect competition and the import content being the same in both exports and domestic final demand, then $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$ becomes $\frac{\text{VA}}{\text{VA+Exports}}$.[28] This approach nevertheless only will yield to a back-of-the-envelope calculation as it ignores the change in the domestic wage, $\frac{dw}{w}$, which adjusts differently depending on the network structure.

3.2.2 Roundabout production economies

In the absence of firm-level data on the structure of the domestic production network, researchers can approximate linkages between firms, for example, by assuming a simple roundabout production economy or by using sectoral input-output tables. Importantly, these assumptions will lead to different predictions for s_{iH} and s_{Fi}^{Total}, which are not observed in standard data sets.[29] While we lay out the key difference in the assumptions here, we describe the changes in firm-level cost, real wage, and real income analogous to Propositions 1 and 2, in addition to the full set of equilibrium equations, for these alternative models in Appendices A.7, A.8, and A.9.

Simple roundabout production economy

The seminal work by Eaton and Kortum (2002) assumes that that goods are produced with

[27]This example is related to the discussion in Melitz and Redding (2014) that the gains from trade can be arbitrarily large in a model with sequential production as the number of stages of production increases. However, note that the ratio of gross production to GDP also rises in their example as the number of stages gets larger. Here we hold the level of gross production and GDP fixed and illustrate that the gains from trade can still differ.

[28]See Appendix A.6 for the derivation. Consistent with De Gotari (2017), we find in Appendix D.5 that the import content in exports tend to be different from that in domestic final demand for all sectors. In particular, we find that in the manufacturing sector the import content in exports is much larger than that in domestic final demand.

[29]Researchers with access to firm-level data usually observe domestic sales and exports, but not the split of the domestic sales between sales to households and sales to other firms. The sales to households are important for calculating s_{iH} and the sales between firms are important for calculating s_{Fi}^{Total}.

labor input and a composite good that is used as an intermediate (and also equal to the final good consumption bundle). While their model assumes that the production function is Cobb-Douglas in labor and intermediates, we stay close to the functional form in equation (7) and capture the roundabout approach by assuming a production function that leads to the following cost function for firm j:

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} P^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{22}$$

where α_{Dj} captures the saliency of intermediate goods for firm j and P is the standard CES price index defined above. We note that this production function differs in two dimensions from the baseline model. First, all firms are using every other firms' inputs through the composite good. Second, the composite good is aggregated with the elasticity of substitution in final demand, σ, even when these goods are used in production.

Sectoral roundabout production economy

The important works by Caliendo and Parro (2015) and Blaum et al. (2016) extend the roundabout approach to incorporate sectoral linkages. We follow their approach by assuming that the composite intermediate good is sector-specific and a Cobb-Douglas aggregate of sectoral composite input bundles. To be specific, we consider the following production function for firm j in sector u:

$$c_j = \phi_j^{-1} \left(\alpha_{Dj}^{\rho-1} \left(\prod_v P_v^{\gamma_{vu(j)}}\right)^{1-\rho} + \alpha_{Fj}^{\rho-1} p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1} w^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{23}$$

where $P_v = \left(\sum_{i\in v} \beta_{iD}^{\sigma-1} p_{iD}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ denotes the sector-v-specific CES price index for the composite sector v good, and γ_{vu} is the Cobb-Douglas share of sector v inputs in the production of sector u goods. For the sectoral model, we assume a sectoral Cobb-Douglas layer in the utility function which implies a price index for consumption $P = \prod_v P_v^{\gamma_{vH}}$.

3.3 Empirical results

In this section, we provide a quantitative analysis of how international trade affects firms' production costs and the consumer price index. We consider a counterfactual change in the import price of foreign goods. The analysis utilizes data from year 2012 on firm-to-firm transactions, firm-level output, international trade flows, and labor input, in combination with estimates or assumptions for the elasticity of substitution in the production function, ρ, and the utility function, σ. Throughout the paper, the baseline specification assumes σ

is equal to 4 and ρ is equal to 2.[30] For small changes and first-order approximations, these elasticities will matter only in so far as they affect the calculation of the domestic wage change to ensure trade balance.

As we solve in the counterfactual exercise for the change in endogenous wages, we require the model equations and the data to be consistent with each other. Under the assumptions made so far, the model is too stylized to rationalize all observed ratios of sales to input costs (mark-ups). Therefore, throughout this section we assume that the firm charges a fixed mark-up, μ_i on all its sales that is calculated from the data as the ratio of sales over input costs.[31] This assumption does not affect the results in Propositions 1 and 2, except for the calculation of the endogenous wage change.

The results in this section are focused on the change in firm-level costs and the change in the real wage due to a 10% foreign price increase. We also analyze the change in real wage and real income when going to autarky. We emphasize that when we compare the predictions from the baseline model with those from the alternative models outlined in Section 3.2, we keep unchanged the firm-level variables on domestic sales (aggregated over households and to other firms), imports and exports, domestic inputs, and labor costs. Therefore, we also hold the aggregates of these variables constant when comparing across models.[32]

We start by analyzing the effects of a 10% increase in the foreign input prices on firm-level costs. As foreign inputs are heavily used in Belgium exports and, by assumption, the export demand is relatively elastic compared to import demand, the domestic nominal wage falls by 2% to ensure trade balance. Figure 3a displays the effects on firms' production costs, as measured by equation (11) of Proposition 1. The horizontal axis represents the change in firm-level costs, which are bounded between .98 (the change in the nominal wage) and 1.1 (the change in the foreign price). The histograms of cost changes show that most firms would be significantly affected by the change in the foreign input prices. This is as expected since most firms are heavily dependent on foreign inputs (see Figure 1a). Even though the price of domestic labor falls by approximately 2%, the majority of firms would have 2.7% or larger nominal cost increases. Figures 3b and 3c displays the histograms of the change in firm-level costs as measured by the roundabout models. Both in the simple and the sectoral roundabout economy (with 52 different sectors), the change in the firm-level cost

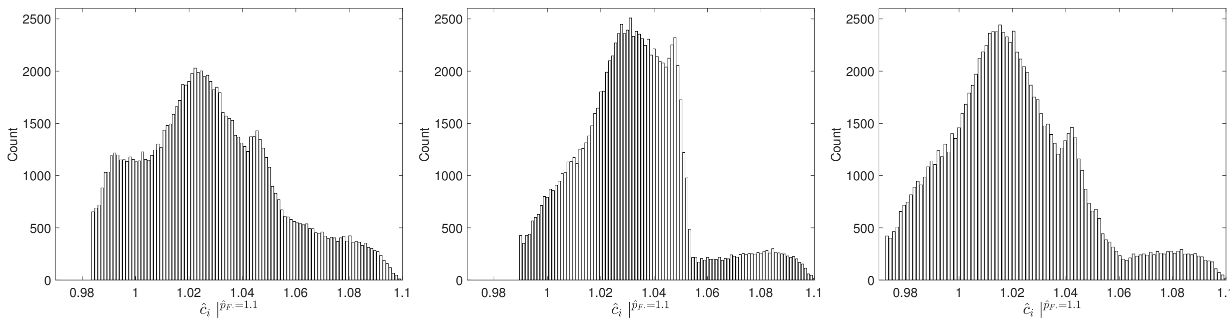
[30] Using data for the US, Antras et al. (2017) estimate $\sigma = 3.85$ and Oberfield and Raval (2014) estimate a level of σ between 3 and 5 among various manufacturing industries. In Appendix D.12, we perform sensitivity analysis under different sets of values for both ρ and σ.

[31] Note that for some firms this ratio is less than one.

[32] We aggregate the firm-to-firm transactions to 52 sectors to calculate the sectoral input-output coefficients. We do this aggregation based on the firm-to-firm transaction data to keep the aggregates unchanged across the various models that we are comparing. This would not be possible when using published sectoral input-output tables, which are not consistent with aggregated firm-to-firm transaction data; in particular, these published tables lead to much smaller coefficients for the wholesale and retail sector.

is considerably more compressed than when using the baseline model. This is because these models fail to capture the large amount of heterogeneity in firms' exposure to foreign inputs that is observed in the data. At the 90th percentile, for example, the change in the firm-level cost is 25 percent larger in the baseline model as compared to the roundabout models.

Figure 3: Histograms of firm-level cost changes



(a) Baseline model (b) Simple Roundabout (c) Sectoral Roundabout

Notes: Note that all three histograms are bounded from above with 1.1, which is the change in the foreign price. The changes in the nominal wages are different across the three models, which generates different lower bounds. See Appendix D.6 for analogous figures plotting firm-level cost changes relative to the change in nominal wage, $\frac{\hat{c}_i}{\hat{w}}$, in which all three histograms are bounded from below with 1 but have different upper bounds.

To investigate the effect of the increase in the foreign input prices, we use equation (19) in Proposition 2. Applied to our data, this equation suggests that a 10% foreign price increase leads to a 6% decline in the real wage. This sizable fall in real wages reflects the combination of a large import content in domestic final demand ($\sum_j s_{jH} s_{Fj}^{Total}$ equals 0.584) and a 2% decline in the nominal wage. The fall in real income is even larger as firms and their owners lose profits from exports (see Appendix D.11).

In Table 2, we decompose the change in the real wage into the effect from the direct cost increase and the indirect increase through intermediate goods. We consider both the case of a 10% foreign price increase and the extreme case of autarky. In the price index calculation, all firms are weighted by their sales to households. We find that about half of the increase in the price index is due to the direct effect, where the other half is coming from the indirect effect. In Appendix D.10, we furthermore calculate how final good prices (weighted by household expenditure on these goods) change across broad sectors in the economy. We find that the increase in the price index is coming primarily from firms that have their main activity in manufacturing, wholesale or retail.

How do these results compare to the back-of-the-envelope calculations motivated by Hulten (1978)? Using equation (21) and assuming perfect competition, no import content in the production of exports, and no change in the nominal wage, one can approximate the change in the real wage, $\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P}$, with $-\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \frac{\text{Imports}}{\text{VA}} = -\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \times 0.817$. Alternatively, under

Table 2: Changes in real wage, decomposition

	Total	Direct	Indirect
10% increase in $p_{F\cdot}$	0.940	0.968	0.973
Autarky	0.558	0.818	0.795

Notes: The number for *Direct* is computed by replacing s_{Fj}^{Total} in equation (19) with the direct foreign input share, s_{Fj}. The number for *Indirect* is computed by replacing s_{Fj}^{Total} in equation (19) with the indirect foreign input share, $s_{Fj}^{Total} - s_{Fj}$. The nominal wage change $\hat{w}$ is the same across the three columns and computed according to Appendix A.7. Due to non-linearities, the direct effects and the indirect effects do not necessarily add up. In Appendix D.8 we report an analogous decomposition under a first-order approximation where the decomposition is additive, leading to quantitatively similar results.

the assumption of perfect competition, import content being the same in both exports and domestic final demand, and no change in the nominal wage, one can approximate the change in the real wage with $-\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \frac{\text{Imports}}{\text{VA+Exports}} = -\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \times 0.449$. These back-of-the-envelope calculations, which are summarized in Appendix D.9, are simple to perform but unfortunately lead to unreliable estimates of the decline in the real wage as compared to those we obtain from the baseline model.

In contrast to the back-of-the-envelope approaches, the simple and the sectoral roundabout models capture much better the change in the real wage, as we show in Table 3a. To understand why, we decompose the change in the real wage for each of the models into two terms. The first is the log-change in the domestic nominal wage relative to the foreign price and the second captures the import content in domestic final demand. We find that both the simple and sectoral roundabout economies have a similar import content in domestic final demand (0.61) as the baseline economy (0.58). The changes in the nominal wage are similar as well. These results are shown in Table 17 of Appendix D.8.

Further, Table 3b shows that the differences between the aggregate predictions of the two alternative models and that of the baseline model are noticeably larger when one considers a large foreign price change ($\hat{p}_{F\cdot} \to \infty$) so that the economy is going to autarky. The real wage falls by 44% in the baseline model and by 40% in the simple roundabout model. The differences between models for large changes in foreign prices is driven by two opposing forces: On the one hand, firms are much more homogeneously affected by trade under the roundabout economy assumption, so it is more difficult for the consumer to substitute away from the firms with the largest price increases. All else equal, this effect leads to a larger fall in the real wage in the roundabout economy. On the other hand, in the roundabout economy the intermediate good is aggregated using the same elasticity as in final demand, σ, hence allowing firms to substitute more than in the baseline model, which in turn leads to a smaller fall in real wages.[33] The real wage would fall the most under the sectoral roundabout

[33] This force disappears when one assumes $\sigma = \rho$. Table 22c in Appendix D.12 shows that the real wage decline is larger in the simple roundabout economy than in the baseline model when we assume $\sigma = \rho = 2$.

economy assumption as the utility function was changed to Cobb-Douglas across sectors. In Appendix D.11 we discuss the analogous results for the changes in real income. The ordering in the real income changes is the same as the ordering in the real wage changes when going to autarky across the three models.

Table 3: Changes in real wage

(a) Changes in real wage upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\ \vert^{\hat{p}_{F\cdot}=1.1}$	0.940	0.941	0.931

(b) Changes in real wage upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{w}}{\hat{P}}\ \vert^{\hat{p}_{F\cdot}\to\infty}$	0.558	0.596	0.441

Our analysis so far has taken the network structure of firms as fixed. This has allowed us to derive analytic solutions for the firm-level cost and aggregate price index changes in the absence of international trade that could then be calculated easily with the firm-to-firm transaction and international trade data. In the following section, we analyze how the network forms endogenously and the implications of endogenous networks for the response of the economy to a change in the foreign price.

4 Model with endogenous production networks

This section develops a model of trade with endogenous network formation, allowing buyer-supplier relationships to change in response to trade shocks. The model builds on the theoretical framework presented in Section 3.1. We assume the same preferences, demand functions, production technology, and market structure as in the model with fixed production networks. What is new is that firms now optimally choose their set of suppliers (i.e. the firm's sourcing strategy) and decide whether to import and export. We first describe the model with endogenous network formation and discuss how it can be solved. Then we estimate the model and use it to quantify how international trade affects firms' production costs and consumer prices with and without endogenous network formation.

4.1 Model

4.1.1 Sourcing strategies and extensive margins of trade

We assume that only buyers initiate linkages with other domestic firms. Forming linkages to suppliers is costly, and firm j incurs a random, firm-pair-specific fixed cost $f_{kj}w$ to add supplier k. The realization of fixed costs is known to the firm at the time it selects suppliers. Firms in our model make profits due to positive mark-ups in sales to domestic and foreign final consumers. Since the buyer is assumed to have all the bargaining power, firms do not make profits from sales to other firms. Hence, variable profits are proportional to firm-level sales to final consumers.

Given a sourcing strategy, Z_j, and export participation choice, I_{jF}, the profits of firm j are equal to variable profits less the fixed costs of domestic and foreign sourcing, $\sum_{k \in Z_j} f_{kj} w$, and the fixed costs of exporting, $I_{jF} f_{jF} w$:

$$\pi_j(Z_j, I_{jF}) = \pi_j^{var}(Z_j, I_{jF}) - \sum_{k \in Z_j} f_{kj} w - I_{jF} f_{jF} w. \tag{24}$$

We assume that firm j exogenously meets a set of eligible suppliers, $\mathbf{Z}_j$. Firm j then endogenously decides on the set of suppliers and whether or not to export:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j,\ I_{jF} \in \{0, 1\}. \tag{25}$$

When deciding on whether to add a supplier to its sourcing strategy, the firm is trading off a reduction in its variable cost (leading to higher variable profits) for an increase in its fixed cost. Solving this problem is difficult for two reasons. First, each firm faces a large discrete choice problem of selecting Z_j and I_{jF}. Second, the profit function, $\pi_j^{var}(Z_j, I_{jF})$, depends on the cost of other firms in the economy, which in turn are equilibrium objects. We now describe how we tackle these two challenges.

We overcome the first challenge by combining theoretical insights on complementarities in the firms' decision making with an iterative algorithm developed by Jia (2008).[34] Given our assumption that final demand is more elastic than the substitution between inputs in the production function, $\sigma > \rho$, the marginal benefit of adding a supplier is increasing in the set of existing suppliers, and it is higher if the firm exports. These complementarities become useful when applying the iterative algorithm to solve the problem described in (25) for a given firm given its knowledge about the costs of the eligible suppliers in $\mathbf{Z}_j$. The algorithm works as follows. Holding the export participation choice fixed, one obtains a lower bound for the optimal sourcing strategy by evaluating the profitability of adding each supplier given a starting guess of no suppliers. The suppliers for which the marginal benefit of inclusion in the sourcing strategy is positive are then included in the updated guess of the sourcing strategy. Then again one evaluates the marginal benefit of each supplier and adds those with positive marginal benefit to the updated guess. Due to complementarities, the marginal benefit of a

[34] Antras et al. (2017) adopted this approach to solve a multi-country model of global sourcing. In recent work, Arkolakis and Eckert (2017) have extended this approach to the case where suppliers are substitutes in the firm's profit function.

supplier is more likely to be positive as the set of suppliers in the current guess increases. Once this iterative procedure stalls, one has obtained a lower bound for the optimal sourcing choice given the exporting choice. Similarly, one obtains an upper bound by including all eligible suppliers in the initial guess and dropping those suppliers with negative marginal benefit from the updated guess of suppliers. The optimal sourcing set, for a given export participation choice, then must be in between the two bounds. Eventually, one also repeats the procedure for the alternative export participation choice, and finally selects the sourcing strategy and exporting choice that yields the maximum profits. In Appendix F, we describe the algorithm more formally and in detail.[35]

To overcome the second challenge, it is necessary to deal with the fixed point problem that arises because firms' sourcing strategies are interdependent. Not only would it be extremely challenging computationally to find a fixed point in the set of costs for all firms so that these costs are consistent with everyone's optimal sourcing decision, the uniqueness of such a fixed point is also unlikely. On the one hand, if firms guess that suppliers have very high unit costs, this could result in the formation of very few linkages and lead to high unit costs overall. On the other hand, if firms guess that suppliers have very low unit costs, this could result in the formation of many linkages and lead to low unit costs overall. To get around these problems, we consider the formation of an acyclic network, postulating an ordering of firms and restricting the eligible set of suppliers to firms that appear prior to the buyer.

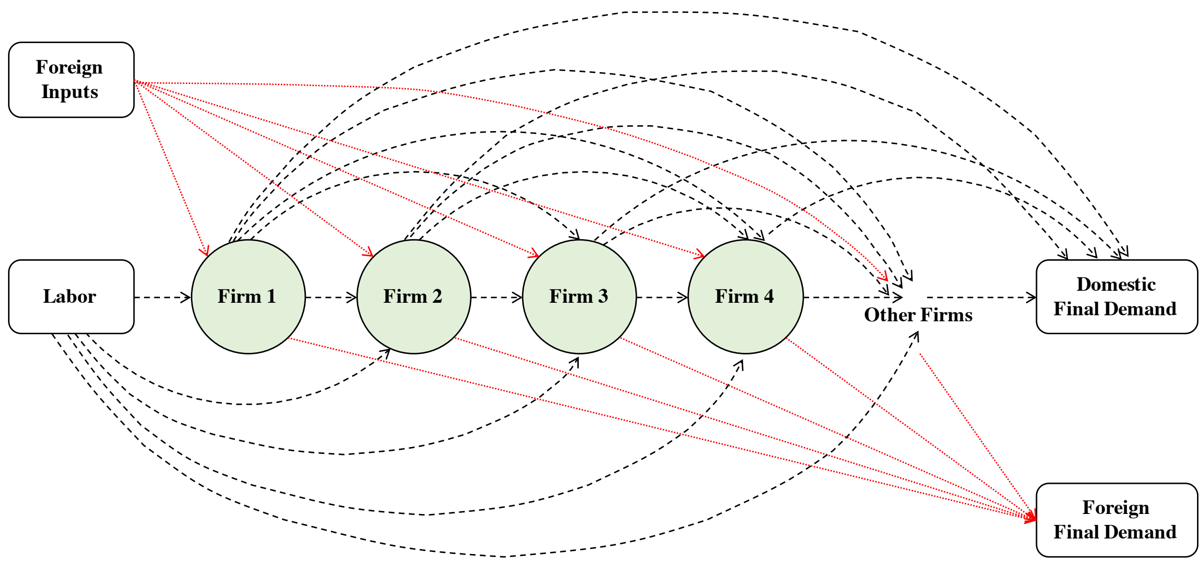
To be concrete, all firms can choose to import foreign inputs and to export their output abroad. However, the set of eligible domestic suppliers varies across firms. Specifically, we order firms in a sequence $S = \{1, 2, 3, ..., N\}$ that restricts the set of eligible suppliers, $\mathbf{Z}_j$, as illustrated in Figure 4. Because we assumed buyers have the full bargaining power in any firm-to-firm transactions, firms only need to know the choices of the firms prior in the sequence. Taken together, these assumptions make the network formation tractable, as we describe below.

Firm 1 is first in the sequence and can only hire labor inputs. To make its decision of how much labor to hire, firm 1 only needs to know the wage level, w, and domestic market demand, $\frac{E}{P^{1-\sigma}}$. Firm 2 is second in the sequence and can hire both labor inputs as well as purchase the input produced by firm 1. To make these decisions, firm 2 needs to know the wage and market demand level as well as the cost of its eligible supplier (firm 1). Firm 3 can hire labor and purchase the inputs from firm 1, firm 2, or both. And so on. Given a guess for equilibrium wages, w, and domestic market demand, $\frac{E}{P^{1-\sigma}}$, one can solve the problems of

[35] The procedure is very similar to the one described in Antras et al. (2017). Here, we also develop a greedy algorithm in case that the differences in the lower and upper bounds for the optimal solution are too wide to evaluate the profits of all feasible combinations in between. As in Antras et al. (2017), we find that in about 99% of the cases the lower and upper bounds are perfectly overlapping (see Table 24 in the Appendix).

the firms sequentially.

Figure 4: Endogenous network formation — eligible connections



Although the above ordering of firms simplifies the problem, we are still limited in the number of possible sourcing strategies we can feasibly evaluate. We therefore restrict the set of eligible suppliers for firm j, $\mathbf{Z}_j$, to be a random subset from the set of firms prior to firm j in the sequence. The suppliers for firm j are then optimally chosen as the solution to the problem in (25). In practice, we choose the maximum cardinality of $\mathbf{Z}_j$ to be 300. We have experimented with the threshold and found that our model fit and counterfactual predictions are very similar when, alternatively, imposing a maximum of 200 or 250 eligible suppliers. The firm's order in the sequence of supplier choices and its set of eligible suppliers are becoming attributes of the firm and therefore primitives of the model.

Imposing a tie-breaking rule that in the case of indifference a supplier is included ensures a unique solution to the problem in (25). As a consequence, the network formation will also be unique given a set of wages and a guess for the price index. We can then alter wages, price index, and expenditure to achieve labor market clearing, trade balance, and a fixed point for the price index and expenditure. Importantly, we are searching here only for a fixed point in wages and price index (only 2 scalars,) as opposed to searching for a large fixed point vector in every firm's costs and searching strategies. In other words, the ordering approach implies that even with a rich micro structure and firm-level heterogeneity, knowing only two equilibrium variables is sufficient to solve sequentially the firms' problems. We discuss the aggregation and the equilibrium with endogenous network structure more formally below.

4.1.2 Aggregation and equilibrium

The model aggregation and equilibrium are broadly similar to the case with fixed networks. However, there are a few notable differences. The first is that firms incur fixed costs — paid in units of labor — to add a domestic buyer, import, or export. Therefore, the labor market clearing condition becomes:

$$wL = \sum_j \frac{\mu_j - 1}{\mu_j} s_{Lj} x_j + w \sum_j \left(\sum_{k \in Z_j} f_{kj} + I_{jF} f_{jF} \right). \tag{26}$$

Additionally, a firm's profit function now subtracts the incurred fixed costs, and a firm's sourcing strategy and export participation are now endogenous choices. However, the trade balance condition remains unchanged. The following definition formally describes equilibrium with an endogenous network structure.

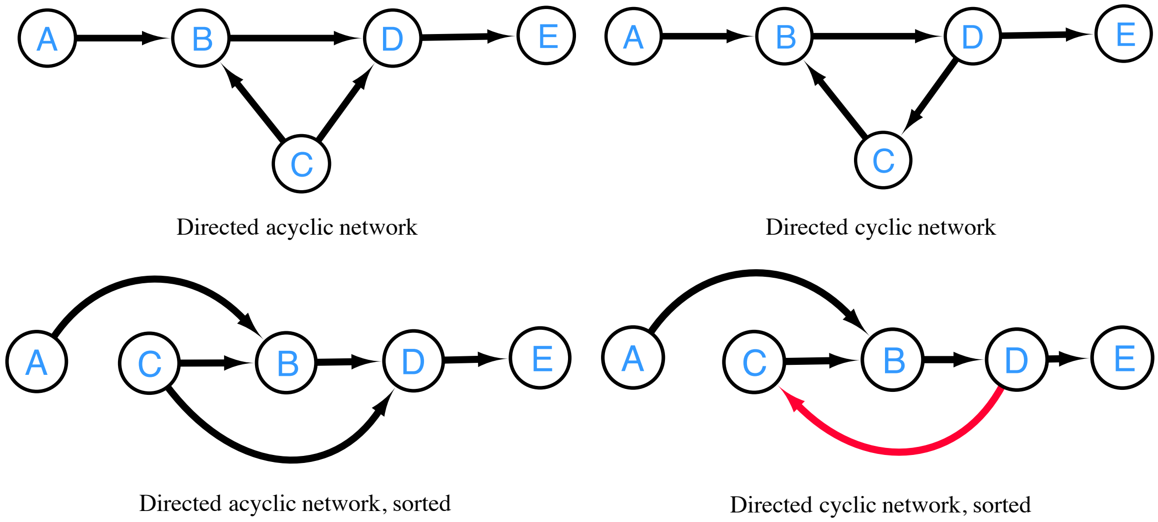
Definition 2 (Equilibrium with endogenous network structure)
Given foreign expenditure, E_F, foreign price index, P_F, and a set of prices by foreign suppliers, $\{p_{Fj}\}_j$, as well as set of eligible suppliers, $\mathbf{Z}_j$, that satisfies acyclicity of the network, an equilibrium for the model with endogenous network structure and endogenous export participation is a wage level, w, price index for the consumer, P, and aggregate expenditure, E, as well as a set of sourcing strategies and export participation choices, such that the firm's optimization problem in (25)*, and equations* (5), (7), (8), (9), (13), (14), (15), (16)*, and* (26) *hold.*

We note that the equilibrium of the economy may not be necessarily efficient. A source for a potential inefficiency is that the firm, when deciding on its sourcing strategy, is not internalizing the full benefits its lowered variable cost have for its buyers. We will return to discussing the efficiency of the economy later in the counterfactual analysis.

4.2 Assessing the assumptions about the shape of the network

Given the assumptions we invoked to solve the endogenous network formation, the resulting network will be acyclic. As shown in Figure 5, in an acyclic firm network, there exists at least one way to sort firms so that all directed edges face one direction. In contrast, in a cyclic network at least one edge will face the opposite direction. This feature of our network formation mechanism is admittedly restrictive. We now perform two checks to assess how well the Belgian data can be approximated by an acyclic network.

Figure 5: Examples of acyclic and cyclic networks



4.2.1 How cyclic is the production network?

Let $\nu(i)$ be an ordering of firms that maps firms $\{i, j, k, \cdots\} \in \Theta$ into numbers from $\{1, \cdots N\}$. To describe how cyclic the Belgian production network is, we want to find the optimal $\nu(k)$ that minimizes the following objective function:

$$\min_{\{\nu(k)\}} \sum_{i,j} \mathbf{1}\{i \in Z_j\} \mathbf{1}\{\nu(i) > \nu(j)\},$$

where Z_j is the supplier set of firm j. Solving this problem corresponds to minimizing the number of directed edges that are facing the direction opposite to that of the sorting order. In other words, we try to find an ordering that minimizes the number of arrows facing to the left in the cyclic network in Figure 5.

To solve this problem, which is also known as the feedback arc set problem, we adopt an algorithm proposed by Eades, Lin, and Smyth (1993). The details of the computational algorithm and implementation are presented in Appendix E. Intuitively, the algorithm places firms with a high net outdegree (number of outgoing links minus number of incoming links) at the beginning of the ordering and those firms with a low net outdegree towards the end of the ordering. The algorithm offers a local minimum, showing that at most 18% of edges in the whole firm-to-firm network in 2012 violate acyclicity.[36] We also search for an ordering that minimizes the value of firm-to-firm sales in violation of acyclicity.[37] We find that no more

[36]While there is no perfect reference point for this figure, we can compare it to the structure of the directed social network Twitter. Simpson, Srinivasan, and Thomo (2016) calculate that 23% of edges are in violation of acyclicity in the Twitter network in the year 2010.

[37]Specifically, we solve the following problem: $\min_{\{\nu(k)\}} \sum_{i,j} x_{ij} \mathbf{1}\{\nu(i) > \nu(j)\}$, where x_{ij} is the value

than 23% of firm-to-firm sales are in violation of acyclicity. We will refer to the former as the unweighted ordering algorithm and the latter as the weighted ordering algorithm. Note that under both approaches, this heuristic algorithm does not guarantee a global minimum solution. Likely there exists another ordering where the approximation of an acyclic network is even better than the one we find.[38]

A natural question that arises is how different the structure of an economy with an acyclic network is in comparison to the economy observed in the data. One way to make this comparison is to calculate input-output tables with and without the firms in buyer-supplier relationships that violate acyclicity. We find that when calculating input-output tables with 72 sectors, the correlations between the input-output table coefficients from the full data and the data without links in violation of the ordering are high.[39] When we use the unweighted ordering algorithm output, the correlation is 0.90, and it is even higher, 0.97, when using the weighted ordering algorithm output.[40]

4.2.2 Gains from trade under fixed networks: cyclic versus acyclic production networks

Another way to assess the assumption of an acyclic network is to examine how the results based on the exogenous network model change if we exclude transactions that violate acyclicity. It is reassuring to find that estimated effect of international trade on consumer prices and firms' costs of production are very similar if we only use firm-to-firm sales that are consistent with the acyclic network obtained by the ordering algorithm described in Section

of the sales from firm i to firm j.

[38]Coleman and Wirth (2009) and Simpson et al. (2016) find that compared to other algorithms proposed in the computer science literature, the algorithm of Eades et al. (1993) does well in terms of speed and finding a low objective.

[39]To construct the input-output table coefficients, we aggregate firm-to-firm transactions within the supplying and buying sector. We note that this procedure differs from the national account definition of an input-output table. First, the rows and columns of our aggregated tables are referring to the main sectors of the buyers and suppliers, but these firms can also have a significant share of their production in other sectors. Second, in national account tables, the contribution of the wholesale and retail sectors to the production of the other goods only refer to the trade margin of retailers and wholesalers. In our data, the wholesale and retail sectors are accounted based on their total sales and total input consumption and not on their trade margin.

[40]These correlations remain high when we drop the diagonal coefficients. With the unweighted ordering algorithm output, the correlation is 0.89, and with the weighted ordering algorithm output, the correlation stays at 0.97. We also find that these high correlations are robust when doing the same exercise at the 4-digit level with 524 industries. The correlation is 0.89 when using the unweighted ordering algorithm output and 0.93 when using the weighted ordering algorithm output. Finally, this heuristic algorithm produces orderings that are highly correlated with upstreamness measures in the existing literature, such as that from Antràs, Chor, Fally, and Hillberry (2012). The rank correlation between the upstreamness measure from Antràs et al. (2012) and the ordering from the unweighted algorithm's is -0.53, and the rank correlation between the ordering from the weighted algorithm is -0.76. However, this upstream measure produces much larger objectives, with 36% of links and 31% of firm-to-firm sales in violation of acyclicity.

4.2.1. Specifically, we keep the direct import share of each firm the same as in the data, set all transactions in violation of the ordering to zero, and adjust all other domestic firm-to-firm input shares such that share of each firm j's input purchases, $\sum_{i\in Z_j} s_{ij}$, is unchanged.[41] The results presented in Table 23 in the Appendix show that the gains from trade under an exogenous network are virtually identical if we only use the subset of transactions for which the domestic production network is acyclic.

4.3 Determining the model parameters

When allowing for endogenous network formation, we are not able to analytically solve the model. Instead, we use the data to determine the model parameters and then provide numerical results for several counterfactual analyses. The goal of these numerical analyses is to draw inference about how endogenous network formation may affect gains-from-trade calculation in an economy that matches our data in important ways.

In the estimation of our model, we simulate 100,000 Belgian firms, approximately the same size as our sample in 2012. A firm is characterized by a set of eligible suppliers that satisfies the ordering, a core productivity level, a vector of firm-pair-specific cost shifters, a foreign input cost shifter, a foreign demand shifter, a vector of fixed cost draws for all eligible suppliers, and fixed costs of importing and exporting. We normalize firms' labor productivity shifters, $\alpha_{Lj} = 1$, and firms' domestic final demand shifters, $\beta_{jH} = 1$.[42] Firms are positioned in an order from 1 to 100,000. The sets of up to 300 eligible suppliers are drawn randomly such that the eligible suppliers are positioned earlier than the buyer in the order of firms.

As a first step of the estimation, we recover the productivity distribution of firms (scaled by some general equilibrium objects) from the identity

$$\frac{x_{iH}^{1/(\sigma-1)}}{s_{Li}^{1/(1-\rho)}} = \phi_i \frac{PE^{1/(\sigma-1)}}{\mu w}. \tag{27}$$

Observing all the terms on the left hand side enables us to estimate the distribution $\phi_i \frac{PE^{1/(\sigma-1)}}{\mu w}$. After visually inspecting the distribution, we assume it is log-normal and estimate the scale parameter to be -1.52 and the dispersion parameter to be 0.85.

We next turn to the parameterization for the distributions of cost and demand shifters. We choose a parameterization for the firm-pair-specific shifter in the production function,

[41] The only exception is when there is no other domestic supplier of a firm, in which case in the data with only acyclic transactions the domestic firm-to-firm input share is set to zero.

[42] Note that α_{Lj} as well as α_{kj} ($\forall k \in Z_j$) enter multiplicatively with the inverse of firm productivity, $1/\phi_j$ in the cost function in equation (7). We therefore normalize $\alpha_{Lj} = 1$. As we have only data on revenues and costs (not separated by price and quantity), we normalize the domestic final demand shifter, $\beta_{jH} = 1$.

α_{kj}, such that it can generate a more skewed distribution of firms' outdegrees than of firms' indegrees. Specifically, we assume that the α_{kj} draw is a product of a supplier-k-specific random variables and an i.i.d. firm-pair-specific random variable. Outside the steps of the estimation described below, we choose the supplier-k-specific random variable to be drawn from a Beta distribution with shape parameters 0.1 and 0.9. We assume that firm-pair-specific random variables contained in α_{kj}, as well as α_{Fj}, and β_{jF} are independent draws from three log-normal distributions which share a common dispersion parameter, $\Phi^{\alpha,\beta}_{disp}$, and have different scale parameters, $\Phi^{\alpha_{\mathrm{dom}}}_{scale}$, $\Phi^{\alpha_F}_{scale}$, $\Phi^{\beta_F}_{scale}$, respectively.

Similarly, the fixed cost draws for domestic purchases from other firms, imports, and exports, are drawn independently from three log-normal distributions with scale parameters $\Phi^{f_{\mathrm{dom}}}_{scale}$, $\Phi^{f_{\mathrm{imp}}}_{scale}$, and $\Phi^{f_{\mathrm{exp}}}_{scale}$ and a common dispersion parameter, Φ^{f}_{disp}. Overall, there are 8 parameters to be estimated.

We use a method of simulated moments to estimate our parameters. We target four sets of moments to match. The first set of moments describes the domestic sourcing patterns of firms. For example, we target the model to match the quartile distribution of number of suppliers. Following the procedure used by Eaton, Kortum, and Kramarz (2011), we include in the first vector of moments generated by the model, $\hat{m}_1(\Phi)$, the proportion of firms that has a number of suppliers equal to the first, second, third, and fourth quartile in the data. We also target the distribution of customers, share of labor costs in firms' costs, and the firm-to-firm input shares (conditional on observing trade between firms). Using the same procedure as above, we include the fraction of firms in the four quartile bins of the distribution (using as thresholds the quartiles observed in the data). This generates 16 elements in the vector $\hat{m}_1(\Phi)$. These moments are helpful in estimating the parameters that affect domestic sourcing.

The second set of moments characterizes firm-level patterns of imports and exports. We include in $\hat{m}_2(\Phi)$ the share of firms that import and export, respectively. We also include the fraction of firms falling into the bins of the first, second, third, and fourth quartile of firms' direct share of foreign inputs (conditional on importing). Furthermore, we target the quartiles of the total share of foreign inputs shown in Figure 1a. Similarly, on the export side, we target the quartiles of the direct share of exports (conditional on exporting) as well the total share of exports (as shown in Figure 1b). There are 18 elements in the vector $\hat{m}_2(\Phi)$. These moments are helpful in estimating the parameters that affect foreign sourcing and exporting.

As a third set of moments, we include important aggregate targets such as the ratio of aggregate exports to aggregate final demand and the import content of domestic final demand. There are 2 elements in the vector $\hat{m}_3(\Phi)$.

We describe the difference between the moments in the data and in the simulated model by $\hat{y}(\Phi)$:

$$\hat{y}(\Phi) = m - \hat{m}(\Phi) = \begin{bmatrix} m_1 - \hat{m}_1(\Phi) \\ m_2 - \hat{m}_2(\Phi) \\ m_3 - \hat{m}_3(\Phi) \end{bmatrix},$$

and the following moment condition is assumed to hold at the true parameter value Φ_0:

$$E\left[\hat{y}(\Phi_0)\right] = 0. \tag{28}$$

The method of simulated moments selects the model parameters that minimize the following objective function:

$$\hat{\Phi} = \arg\min_{\Phi} \left[\hat{y}(\Phi)\right]^{\top} \mathbf{W} \left[\hat{y}(\Phi)\right], \tag{29}$$

where $\mathbf{W}$ is a weighting matrix.[43]

4.3.1 Estimation results

Table 4 shows the values of the estimated parameters.

Table 4: Estimated parameters

Preference and production				Fixed costs			
$\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$	$\hat{\Phi}^{\alpha_F}_{scale}$	$\hat{\Phi}^{\beta_F}_{scale}$	$\hat{\Phi}^{\alpha,\beta}_{disp}$	$\hat{\Phi}^{f_{\text{dom}}}_{scale}$	$\hat{\Phi}^{f_{\text{imp}}}_{scale}$	$\hat{\Phi}^{f_{\text{exp}}}_{scale}$	$\hat{\Phi}^{f}_{disp}$
-4.42	-2.22	-2.01	2.34	-3.21	2.64	6.75	6.98

We note that the scale of the estimated parameters is affected by the choice of normalizations for the foreign market size and the price of the foreign input. We therefore focus on the model fit given these parameter estimates. Table 5 shows the targeted moments generated by the estimated parameters, compared with the moments from the data. Note that instead of showing the moments directly (i.e., the fraction of firms falling into each quartile bin), we show the values of the 25th, 50th, and 75th percentiles in both the data and model. The estimated model fits well the targeted statistics of firm-to-firm transactions. In particular, the model succeeds in generating a more skewed distribution of the number of domestic buyers than of the domestic suppliers. This enables the model to match also the differences in the distribution of total foreign input shares and total export shares. Analogous to Figure 1, we plot the distributions of direct and total foreign input shares and export shares from the estimated model and show them in Appendix H.1. While we succeed in generating different distributions of firms' exposure to imports and exports, both directly and indirectly, the model also matches well the magnitude of aggregate imports relative to domestic final

[43] We weight the moments equally, hence the weighting matrix is the identity matrix.

demand. Finally, we also match quite well the import content in domestic final demand, $\sum_i s_{iH} s_{Fi}^{Total}$, which is a key statistic that determines the magnitude of the change in real wage in response to a change in the foreign price (see equation (20)).

Table 5: Model fit: targeted moments

	Data	Model
Number of dom. suppliers 25th percentile	19	17
Number of dom. suppliers 50th percentile	33	30
Number of dom. suppliers 75th percentile	55	49
Number of dom. buyers 25th percentile	2	2
Number of dom. buyers 50th percentile	9	13
Number of dom. buyers 75th percentile	34	46
Share of labor costs 25th percentile	0.17	0.12
Share of labor costs 50th percentile	0.34	0.25
Share of labor costs 75th percentile	0.54	0.49
Firm-to-firm input share 25th percentile	0.0002	0.0000
Firm-to-firm input share 50th percentile	0.0012	0.0003
Firm-to-firm input share 75th percentile	0.0053	0.0034
Share of firms that import	0.19	0.19
Direct foreign input share (among importers) 25th percentile	0.01	0.01
Direct foreign input share (among importers) 50th percentile	0.28	0.05
Direct foreign input share (among importers) 75th percentile	0.67	0.25
Total foreign input share 25th percentile	0.24	0.18
Total foreign input share 50th percentile	0.39	0.31
Total foreign input share 75th percentile	0.55	0.44
Share of firms that export	0.12	0.10
Direct export share (among exporters) 25th percentile	0.01	0.02
Direct export share (among exporters) 50th percentile	0.10	0.14
Direct export share (among exporters) 75th percentile	0.62	0.50
Total export share 25th percentile	0.0008	0.0000
Total export share 50th percentile	0.0156	0.0328
Total export share 75th percentile	0.1005	0.2982
Ratio of aggregate exports to aggregate sales to domestic final demand	0.82	0.90
Import content of domestic final demand	0.58	0.65

Notes: Percentiles are calculated based on all firms in the sample. Share of labor costs refers to the fraction of labor costs in costs (labor costs + domestic purchases + imports) and the percentiles are calculated based on all firms the sample. Firm-to-firm share refers to the fraction of costs a firm spends on one particular supplier and the percentiles are calculated for all firm-to-firm transactions. The percentiles for share of exports in total firm sales are calculated for all firms with positive export sales. The percentiles for share of imports in firm inputs are calculated for all firms with positive import purchases.

We also examine how well the model fits moments that were not directly targeted in the estimation. Specifically, we have not targeted directly the association of size between buyers and suppliers that trade with each other. Consistent with the data, the model predicts a weak negative correlation between the number of suppliers of the buying firms (indegree buyer) and the number of buyer firms of suppliers (outdegree supplier). Similarly, the correlation between sales of the buying and selling firm is close to zero both in the data and in the model. Analogous to Figure 2, the model generates the pattern that the size premium increases sharply for low values of direct import and export shares and only gradually increase for higher direct shares. In comparison, the sales premium for total share measures increases more gradually (Figure 15 in Appendix H.2).

Table 6: Model fit: non-targeted moments

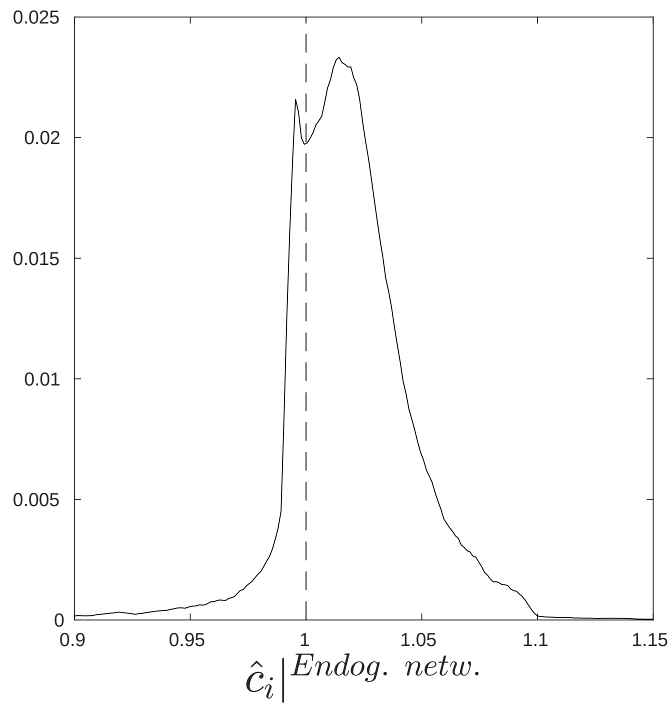
	Data	Model
Corr (Indegree buyer, Outdegree supplier)	-0.05	-0.13
Corr (Sales buyer, Sales supplier)	-0.02	0.01

4.4 Effects of changes in the price of foreign goods with endogenous network formation

Equipped with the parameter estimates of our model, we next turn to the same counterfactual experiment as we studied under fixed linkages, i.e., a 10% increase in the price of the imported goods. As before, we solve for the change in the domestic wage such that trade is balanced and the labor market clears. Here, however, we allow firms to also change their set of suppliers (i.e. the firm's sourcing strategy) or alter the decision to import and export in response to the trade shock.

Figure 6 presents the density of the change in firms costs in the model with endogenous network formation. Similar to the what we found in the model with a fixed network structure, a large majority of firms experience increases in their production costs. To directly compare the two models, we also compute the analogous cost changes under fixed networks. To compute the cost changes under fixed networks, we start from the economy generated by our endogenous network model and consider the same change in the foreign price while holding fixed the choice of suppliers as well as the decisions to import and export. Under fixed networks, we showed analytically that the total share of foreign inputs are the key firm-level predictor for firms' cost increases in response to a change in the import price. Figure 7 depicts the univariate correlations between firms' cost changes and observable firm-level variables prior to the change. Even under endogenous networks, the total share of foreign inputs is still a strong indicator of how much firms' costs will change in response to a change in the price of the imported good. The cost change is also negatively correlated with firms' share of labor costs in variable costs, and positively correlated with firms' size variables such as log total sales and log sales to domestic final demand.

Figure 6: Cost changes under endogenous networks

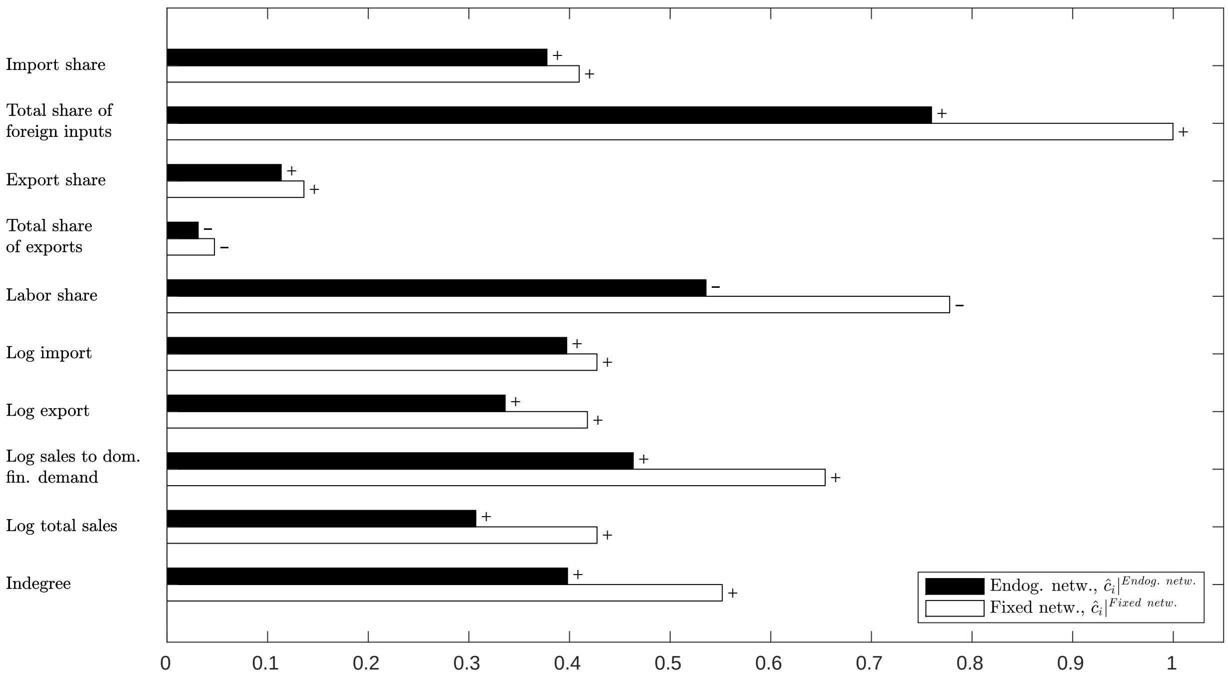


Notes: The figure shows the density of cost changes from a 10% increase in the price of foreign goods under endogenous networks.

We also find that firms that have a high total foreign input share, high export share, or a low labor share are more likely to reduce their number of suppliers in response to a 10% increase in the price of imported goods (see Table 25 in the Appendix for supporting regression results). Due to complementarity in the exporting, importing, and domestic sourcing decisions, it is intuitive that the firms that lose the most from the price increase in the foreign goods may also be the ones that reduce the most the number of domestic suppliers. Other firms, however, benefit from the rise in the domestic price index, which increases their overall scale and enables them to add more suppliers. Overall, we find that under endogenous networks the number of domestic linkages would increase by 1% in response to a 10% increase in the price of foreign goods. Underlying this net increase, however, is churn in the domestic linkages between firms in the economy.

We next turn to the changes in the real wage that are summarized in Table 7. It is easy to show that for an efficient economy, the flexibility of the network structure must dampen the aggregate effect of a negative shock (i.e., increase of the foreign price) and amplify the benefits of a positive shock (i.e., a decrease in the foreign price). However, as discussed above, the endogenous network economy is not necessarily efficient. In particular, firms do

Figure 7: Correlations with firm-level variables



Notes: The black bars depict the univariate correlations between firms' cost changes under endogenous networks and observable firm-level variables prior to the counterfactual change. The white bars depict the univariate correlations between firms' cost changes under exogenous networks and observable firm-level variables.

not fully internalize the benefits their cost reductions bring to their buyers, their buyers' buyers, etc.

Therefore, whether endogenous networks amplify/ dampen the effects of economic shocks is ultimately an empirical question. We find that for a 10% increase in the foreign price (a negative shock), the real wage would decrease slightly less under an endogenous network structure than under a fixed network structure (hence the dampening occurs). For large changes in the price of the foreign goods, it becomes more even more costly for the economy to be tied to the fixed network structure and the dampening effect gets stronger. The real wage would fall by 7 percentage points less under endogenous than under fixed network structure when going to autarky.

For a 10% decrease in the price of foreign goods, we find that allowing for endogenous network formation generates a smaller increase in the real wage (hence we find no amplification in this case). By comparison, when considering a 50% decrease in the price of foreign goods, we find that the endogeneity of the network amplifies the benefits of this positive shock, and yields a 25 percentage points real wage gain over and above the fixed network benchmark.

We have tried various alternative parameterizations of our endogenous networks model and robustly obtained the result that the endogenous network structure dampens the effect of a large negative shock (i.e., autarky) and amplifies the benefits of a large positive shock (i.e., 50% reduction in the foreign price).

Table 7: Changes in real wage upon change in foreign price

	Endogenous Network	Fixed Network
10% increase ($\hat{p}_{F\cdot} = 1.1$)	0.9388	0.9321
Autarky ($\hat{p}_{F\cdot} \to \infty$)	0.4844	0.4157
10% decrease ($\hat{p}_{F\cdot} = 0.9$)	1.0681	1.0881
50% decrease ($\hat{p}_{F\cdot} = 0.5$)	2.2779	2.0170

5 Conclusion

In this paper, we used administrative data from Belgium with information on domestic firm-to-firm sales and foreign trade transactions to study how international trade affects firm efficiency and real wages. Our paper offered three sets of results. First, we documented that most firms that do not directly import or export still have large indirect exposure to foreign trade and in particular, to foreign inputs.

Second, we derived new sufficient statistics results for how international trade affects firms' production costs. Assuming a fixed network structure, the cost reduction for an individual firm due to international trade depends only on the share of input costs that is spent on goods that are imported directly or indirectly, the change in the domestic wage, and the elasticity of substitution in the production function. We applied this sufficient statistics formula to our data and compared the results to those we obtain using existing approaches. This comparison highlights the importance of data on and modeling of domestic production networks in studies of international trade.

Lastly, we developed a novel framework for analyzing the endogenous formation of the production network. We make the model tractable by focusing on the formation of an acyclic rather than a cyclic production network. While restrictive, this allowed us to solve a model of firm trade with endogenous formation of domestic buyer-supplier relationships. Reassuringly, we found that the vast majority of buyer-supplier relationships in Belgium can be described by an acyclic production network. Moreover, both sectoral input-output tables and the gains from trade under a fixed network structure do not change materially if we restrict attention to transactions for which the domestic production network is acyclic. Our approach to endogenous network formation may prove useful in contexts other than trade where researchers are increasingly interested in the formation and consequences of domestic production networks.

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A Theoretical Results

A.1 Proof of Proposition 1

Proof.

We have

$$\begin{aligned}
s_{Fj}^{Total} &= s_{Fj} + \sum_i s_{ij} s_{Fi}^{Total} \\
&= s_{Fj} + \sum_i s_{ij} \left[s_{Fi} + \sum_k s_{ki} \left(s_{Fk} + \cdots \right) \right]
\end{aligned}$$

and

$$c_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{Lj}^{\rho-1} \phi_j^{\rho-1} w^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} p_{Fj}^{1-\rho},$$

where p_{Fj} is common across j.

Consider a uniform foreign price change, where firm j's import price changes to $\tilde{p}_{F\cdot}$. Then firm j's new cost function is

$$\tilde{c}_j^{1-\rho} = \sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{Lj}^{\rho-1} \phi_j^{\rho-1} \tilde{w}^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} \tilde{p}_{F\cdot}^{1-\rho}.$$

Computing the change in the costs yields:

$$\begin{aligned}
\hat{c}_j^{1-\rho} \mid^{\hat{p}_{F\cdot}} &= \frac{\sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} \tilde{c}_k^{1-\rho} + \alpha_{Lj}^{\rho-1} \phi_j^{\rho-1} \tilde{w}^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} \tilde{p}_{F\cdot}^{1-\rho}}{\sum_k \alpha_{kj}^{\rho-1} \phi_j^{\rho-1} c_k^{1-\rho} + \alpha_{Lj}^{\rho-1} \phi_j^{\rho-1} w^{1-\rho} + \alpha_{Fj}^{\rho-1} \phi_j^{\rho-1} p_{Fj}^{1-\rho}} \\
&= \sum_k s_{kj} \hat{c}_k^{1-\rho} + s_{Lj} \hat{w}^{1-\rho} + s_{Fj} \hat{p}_{F\cdot}^{1-\rho} \\
&= \sum_k s_{kj} \left(\sum_l s_{lk} \hat{c}_l^{1-\rho} + s_{Lk} \hat{w}^{1-\rho} + s_{Fk} \hat{p}_{F\cdot}^{1-\rho} \right) + s_{Lj} \hat{w}^{1-\rho} + s_{Fj} \hat{p}_{F\cdot}^{1-\rho} \\
&= s_{Fj} \hat{p}_{F\cdot}^{1-\rho} + \sum_k s_{kj} s_{Fk} \hat{p}_{F\cdot}^{1-\rho} + \cdots + s_{Lj} \hat{w}^{1-\rho} + \sum_k s_{kj} s_{Lk} \hat{w}^{1-\rho} + \cdots \\
&= \left(1 - s_{Fj}^{Total} \right) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_{F\cdot}^{1-\rho}
\end{aligned}$$

For small changes, log-linearize equation (11) around $\hat{p}_{F\cdot} = \hat{c}_j = \hat{w} = 1$ and obtain

$$\frac{\mathrm{d}c_j}{c_j} = \left(1 - s_{Fj}^{Total} \right) \frac{\mathrm{d}w}{w} + s_{Fj}^{Total} \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}.$$

This result can be obtained in a more general setting. Consider a constant returns to scale production function. Denote firm j's cost function to produce y units of output with $c(\{p_{kj}\}_{k\in Z_j}, w, y)$. Taking the total derivatives of the cost function yields:

$$\mathrm{d}c(\{p_{kj}\}_{k\in Z_j}, w, y) = \sum_{k\in Z_j} \frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial p_{kj}} \mathrm{d}p_{kj} + \frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial w} \mathrm{d}w + \frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial y} \mathrm{d}y$$

Dividing both sides with $c(\{p_{kj}\}_{k\in Z_j}, w, y)$, we get:

$$\frac{\mathrm{d}c(\{p_{kj}\}_{k\in Z_j}, w, y)}{c(\{p_{kj}\}_{k\in Z_j}, w, y)} - \frac{y\frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial y}}{c(\{p_{kj}\}_{k\in Z_j}, w, y)} \frac{\mathrm{d}y}{y} = \sum_{k\in Z_j} \frac{p_{kj}\frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial p_{kj}}}{c(\{p_{kj}\}_{k\in Z_j}, w, y)} \frac{\mathrm{d}p_{kj}}{p_{kj}} + \frac{w\frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial w}}{c(\{p_{kj}\}_{k\in Z_j}, w, y)} \frac{\mathrm{d}w}{w}.$$

By Shephard's lemma, we also have $\frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial p_{kj}} = x_{kj}(\{p_{kj}\}_{k\in Z_j}, w, y)$ and $\frac{\partial c(\{p_{kj}\}_{k\in Z_j}, w, y)}{\partial w} = \ell_j(\{p_{kj}\}_{k\in Z_j}, w, y)$. In addition, from the constant returns to scale assumption, $c(\{p_{kj}\}_{k\in Z_j}, w, y) = yc(\{p_{kj}\}_{k\in Z_j}, w, 1)$. Rearrange and obtain:

$$\frac{\mathrm{d}c(\{p_{kj}\}_{k\in Z_j}, w, 1)}{c(\{p_{kj}\}_{k\in Z_j}, w, 1)} = \sum_{k\in Z_j} s_{kj} \frac{\mathrm{d}p_{kj}}{p_{kj}} + s_{Lj} \frac{\mathrm{d}w}{w}, \forall y.$$

That is, if the production function is CRS, the percentage change in the unit cost equals a weighted average of percentage change in factor prices, with a factor's weight equal to the expenditure share on the factor. We can then iterate forward to arrive at equation (12).

■

A.2 Firm-level cost changes under general foreign shocks

Here we derive expressions analogous to Proposition 1, but considering more general foreign shocks. We consider import price changes that are heterogeneous across firms, $\{\hat{p}_{Fj}\}$, and in addition consider firm-level changes in export demand, $\left\{\hat{\beta}_{jF}\right\}$. To compute the changes in equilibrium variables given these shocks, we solve the system of equations by following the steps below.

1. Guess $\hat{w}$. Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j = \left(\left(1 - s_{Fj}^{Total}\right) \hat{w}^{1-\rho} + t_{Fj}\right)^{\frac{1}{1-\rho}},$$

where t_{Fj} is the obtained by solving the following system:

$$t_{Fj} = s_{Fj}\hat{p}_{Fj}^{1-\rho} + \sum_{i \in Z_j^D} s_{ij} t_{Fi}.$$

This is in contrast with the definition of s_{Fj}^{Total}, where the analogous system is $s_{Fj}^{Total} = s_{Fj}^{Total} + \sum_{i \in Z_j^D} s_{ij} s_{Fi}^{Total}$.

2. Compute the following hat variables.

$$\begin{aligned} \hat{x}_{iF} &= \hat{\beta}_{iF}^{1-\sigma}\hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\ \hat{P}^{1-\sigma} &= \sum_i s_{iH}\hat{c}_i^{1-\sigma} \\ \hat{s}_{Li} &= \hat{w}^{1-\rho}\hat{c}_i^{\rho-1} \\ \hat{s}_{ij} &= \hat{c}_i^{1-\rho}\hat{c}_j^{\rho-1} \quad (\text{if } i \in Z_j), \end{aligned}$$

where $\hat{x}_{iF}$ is defined for exporting firms.

3. Solve for $\{\hat{x}_i\}$ from

$$\hat{x}_i = \frac{x_{iF}}{x_i}\hat{x}_{iF} + \frac{x_{iH}}{x_i}\hat{c}_i^{1-\sigma}\hat{P}^{\sigma-1}\left(\frac{wL}{E}\hat{w} + \sum_k \frac{\mu_k - 1}{\mu_k}\frac{x_k}{E}\hat{x}_k - \frac{TB}{E}\right) + \sum_{j \in W_i} \frac{\hat{s}_{ij} x_{ij}}{x_i}\hat{x}_j.$$

4. Update the guess of $\hat{w}$ with

$$\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L}\hat{s}_{Li}\hat{x}_i,$$

and iterate from Step 1 until $\hat{w}$ converges.

A.3 Proof of Proposition 2

Proof.

From equation (5), we have the expression for the price index after the shock,

$$\tilde{P} = \left(\sum_i \beta_i^{\sigma-1}\mu^{1-\sigma}\tilde{c}_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Combining this expression with the pre-shock price index P, we have

$$\begin{aligned}
\hat{P} \mid^{\hat{p}_{F\cdot}} &= \frac{\tilde{P}}{P} \\
&= \left(\frac{\sum_i \beta_i^{\sigma-1}\mu^{1-\sigma}\tilde{c}_i^{1-\sigma}}{P^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\
&= \left(\sum_i s_{iH}\left(\hat{c}_i \mid^{\hat{p}_{F\cdot}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}
\end{aligned}$$

where s_{iH} denotes firm i's share in final consumption. Combine with equation (11) and obtain

$$\begin{aligned}
\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot}} &= \hat{w}\left(\sum_i s_{iH}\left(\hat{c}_i \mid^{\hat{p}_{F\cdot}}\right)^{1-\sigma}\right)^{\frac{1}{\sigma-1}} \\
&= \left(\sum_i s_{iH}\left(\left(1-s_{Fj}^{Total}\right)+s_{Fj}^{Total}\frac{\hat{p}_{F\cdot}^{1-\rho}}{\hat{w}^{1-\rho}}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

For small changes, first obtain the log-linearized change in the aggregate price index,

$$\frac{\mathrm{d}P}{P} = \sum_j s_{jH}\frac{\mathrm{d}c_j}{c_j}.$$

Combine with equation (12) and obtain

$$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right)\sum_j s_{jH}s_{Fj}^{Total}.$$

Again, as in Proposition 1, the result for small changes holds more generally. Now consider a constant returns to scale utility function and describe it as $e(\{p_{jH}\},U) = Ue(\{p_{jH}\},1)$. That is, the minimized expenditure to achieve utility level U equals U times the minimized expenditure to obtain an unit utility. Furthermore, with homothetic preferences, the ideal price index P is the minimized cost of buying one unit of utility, i.e., $P = e(\{p_{jH}\},1)$.

Take the total derivative of $e(\{p_{jH}\},U)$ and obtain:

$$\frac{\mathrm{d}e(\{p_{jH}\},U)}{e(\{p_{jH}\},U)} = \sum_j \frac{p_{jH}\frac{\partial e(\{p_{jH}\},U)}{\partial p_{jH}}}{e(\{p_{jH}\},U)}\frac{\mathrm{d}p_{jH}}{p_{jH}} + \underbrace{\frac{U\frac{\partial e(\{p_{jH}\},U)}{\partial U}}{e(\{p_{jH}\},U)}}_{=1}\frac{\mathrm{d}U}{U}.$$

Then total differentiate $Ue(\{p_{jH}\},1)$:

$$\frac{\mathrm{d}e(\{p_{jH}\},U)}{e(\{p_{jH}\},U)} = \frac{\mathrm{d}P}{P} + \frac{\mathrm{d}U}{U}.$$

By Shephard's Lemma, $\frac{\partial e(\{p_{jH}\},U)}{\partial p_{jH}} = x_{jH}(\{p_{jH}\},U)$. Equating the previous two equations and dropping $\frac{\mathrm{d}U}{U}$, the percentage change in the ideal price index for any level U is:

$$\frac{\mathrm{d}P}{P} = \sum_j s_{jH}\frac{\mathrm{d}p_{jH}}{p_{jH}}.$$

The percentage change in the real wage is thus:

$$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \frac{\mathrm{d}w}{w} - \sum_j s_{jH}\frac{\mathrm{d}p_{jH}}{p_{jH}}.$$

As $\frac{\mathrm{d}p_{jH}}{p_{jH}} = \frac{\mathrm{d}c_j}{c_j}$, the rest of Proposition 2 follows.
■

A.4 Proposition 1 and 2 under continuum of firms

Here we show that analogous results to Propositions 1 and 2 can be obtained under the assumption of a continuum of firms. To represent a network of a continuum of firms, we use a notation similar to Lim (2015). Denote $m(\chi',\chi)$ the probability a type$-\chi$ firm sourcing from a type$-\chi'$ firm. The key difference between the continuous network representation from the model described in the main text is that the continuous network is characterized by a probability measure that any two types of firms are matched, while the discrete network features a finite number of possible linkages between firms taking values of either 0 (not connected) or 1 (connected). χ is a collection of firm characteristics which are all continuous and have bounded support: $\{\beta_H(\chi), \beta_F(\chi), \alpha_L(\chi), \alpha_F(\chi)\}$. $g(\chi',\chi)$ measures the efficiency of a match between a firm with type-χ' and a firm with type-χ. In addition, $v(\chi)$ is the probability density function of type-χ firms. We assume that the functions $m(\cdot,\cdot)$, $g(\cdot,\cdot)$, and $v(\chi)$ are all continuous and have bounded supports.

We first list the key variables analogous to ones derived in the main text. The consumer preference is:

$$U = \left(\int (\beta_H(\chi)\, q(\chi,H))^{\frac{\sigma-1}{\sigma}}\, \mathrm{d}V(\chi)\right)^{\frac{\sigma}{\sigma-1}}.$$

The aggregate final consumer demand for a type$-\chi$ firm is:

$$q\left(\chi,H\right)=\beta_{H}\left(\chi\right)^{\sigma-1}\frac{p\left(\chi,H\right)^{-\sigma}}{P^{1-\sigma}}E.$$

The price index is represented by:

$$P^{1-\sigma}=\int\beta_{H}\left(\chi\right)^{\sigma-1}p\left(\chi,H\right)^{1-\sigma}\mathrm{d}V\left(\chi\right).$$

Foreign sales takes the following form:

$$q\left(\chi,F\right)=\beta_{F}\left(\chi\right)^{\sigma-1}\frac{p\left(\chi,F\right)^{-\sigma}}{P_{F}^{1-\sigma}}E_{F}.$$

The unit cost function for a type$-\chi$ firm:

$$c\left(\chi\right)=\frac{1}{\phi}\left(\alpha_{L}\left(\chi\right)^{\rho-1}w^{1-\rho}+m\left(F,\chi\right)\alpha_{F}\left(\chi\right)^{\rho-1}p_{F.}^{1-\rho}+\int m\left(\chi',\chi\right)g\left(\chi',\chi\right)^{\rho-1}p\left(\chi',\chi\right)^{1-\rho}\mathrm{d}V\left(\chi'\right)\right)^{\frac{1}{1-\rho}}.$$

The sourcing capability of firm with type χ:

$$\Theta\left(\chi\right)=\alpha_{L}\left(\chi\right)^{\rho-1}w^{1-\rho}+m\left(F,\chi\right)\alpha_{F}\left(\chi\right)^{\rho-1}p_{F.}^{1-\rho}+\int m\left(\chi',\chi\right)g\left(\chi',\chi\right)^{\rho-1}p\left(\chi',\chi\right)^{1-\rho}\mathrm{d}V\left(\chi'\right).$$

The share of variable cost of type$-\chi$ firm spent on type$-\chi'$ firm:

$$s\left(\chi',\chi\right)=\frac{m\left(\chi',\chi\right)g\left(\chi',\chi\right)^{\rho-1}p\left(\chi',\chi\right)^{1-\rho}}{\Theta\left(\chi\right)}.$$

Cost share spent on labor is:

$$s\left(L,\chi\right)=\frac{\alpha_{L}\left(\chi\right)^{\rho-1}w^{1-\rho}}{\Theta\left(\chi\right)}.$$

Cost share spent on foreign inputs is:

$$s\left(F,\chi\right)=\frac{m\left(F,\chi\right)\alpha_{F}\left(\chi\right)^{\rho-1}p_{F.}^{1-\rho}}{\Theta\left(\chi\right)}.$$

The total foreign share becomes:

$$s^{Total}\left(F,\chi\right)=s\left(F,\chi\right)+\int s^{Total}\left(F,\chi'\right)s\left(\chi',\chi\right)\mathrm{d}V\left(\chi'\right).$$

Now, Proposition 1 holds for a continuum of firms: Given fixed linkages between firms,

$m(\chi', \chi)$, the change in firm j's unit cost, $\hat{c}_j|^{\hat{p}_{F\cdot}}$ given a uniform change in foreign prices, $\hat{p}_{F\cdot}$, is:

$$\hat{c}(\chi)\,|^{\hat{p}_{F\cdot}} = \left(\left(1 - s^{Total}(F, \chi)\right)\hat{w}^{1-\rho} + s^{Total}(F, \chi)\,\hat{p}_{F\cdot}^{1-\rho}\right)^{1/(1-\rho)}.$$

For a small percentage point change in the foreign price, $\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$, the first-order approximation to firm j's unit cost is

$$\frac{\mathrm{d}c(\chi)}{c(\chi)} = \left(1 - s^{Total}(F, \chi)\right)\frac{\mathrm{d}w}{w} + s^{Total}(F, \chi)\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}.$$

The proof largely follows the one for Proposition 1.

$$\begin{aligned}
\left(\hat{c}(\chi)\,|^{\hat{p}_{F\cdot}}\right)^{1-\rho} =& s(L, \chi)\,\hat{w}^{1-\rho} + s(F, \chi)\,\hat{p}_{F\cdot}^{1-\rho} + \int s(\chi', \chi)\,\hat{p}(\chi', \chi)^{1-\rho}\mathrm{d}V(\chi') \\
=& \Bigg(s(L, \chi) + \int s(\chi', \chi)\,s(L, \chi')\,\mathrm{d}V(\chi') + \\
& \int s(\chi', \chi)\int s(\chi'', \chi')\,s(L, \chi'')\,\mathrm{d}V(\chi')\,\mathrm{d}V(\chi'') + ...\Bigg)\hat{w}^{1-\rho} \\
& + \Bigg(s(F, \chi) + \int s(\chi', \chi)\,s(F, \chi')\,\mathrm{d}V(\chi') + \\
& \int s(\chi', \chi)\int s(\chi'', \chi')\,s(F, \chi'')\,\mathrm{d}V(\chi')\,\mathrm{d}V(\chi'') + ...\Bigg)\hat{p}_{F\cdot}^{1-\rho} \\
=& \left(1 - s^{Total}(F, \chi)\right)\hat{w}^{1-\rho} + s^{Total}(F, \chi)\,\hat{p}_{F\cdot}^{1-\rho}.
\end{aligned}$$

Proposition 2 also holds for a continuum of firms. Given fixed linkages between firms, the change in the real wage, $\frac{\hat{w}}{\hat{P}}$, due to an uniform change in foreign price, $\hat{p}_{F\cdot}$, is:

$$\begin{aligned}
\frac{\hat{w}}{\hat{P}}\,|^{\hat{p}_{F\cdot}} &= \hat{w}\left(\int s(\chi, H)\left(\hat{c}(\chi)\,|^{\hat{p}_{F\cdot}}\right)^{1-\sigma}\mathrm{d}V(\chi)\right)^{\frac{1}{\sigma-1}} \\
&= \left(\int s(\chi, H)\left(\left(1 - s^{Total}(F, \chi)\right) + s^{Total}(F, \chi)\frac{\hat{p}_{F\cdot}^{1-\rho}}{\hat{w}^{1-\rho}}\right)^{\frac{1-\sigma}{1-\rho}}\mathrm{d}V(\chi)\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

For a small change in the foreign price, $\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$, the first-order approximation to the change in the real wage is:

$$\begin{aligned}
\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} &= \frac{\mathrm{d}w}{w} - \int s(\chi, H)\frac{\mathrm{d}c(\chi)}{c(\chi)} \\
&= \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right)\int s(\chi, H)s^{Total}(F, \chi)\,\mathrm{d}V(\chi).
\end{aligned}$$

A.5 Derivation of equation (21)

Here we show that under perfect competition, the change in the real wage in equation (20) can be expressed as the product of $\frac{\text{Imports}}{\text{VA}}$ and the change in terms of trade.

The change in the terms of trade can be written as the change in the average export price minus the change in the import price, $\sum_j s_{jF} \frac{\mathrm{d}c_j}{c_j} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$, where s_{jF} is firm j's share in aggregate exports. Combining with equation (12), we have

$$\begin{aligned}\frac{\text{Imports}}{\text{VA}} \left(\sum_j s_{jF} \frac{\mathrm{d}c_j}{c_j} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) &= \frac{\text{Imports}}{\text{VA}} \left(\sum_j s_{jF} \left(\left(1 - s_{Fj}^{Total}\right) \frac{\mathrm{d}w}{w} + s_{Fj}^{Total} \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \\ &= \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \frac{\text{Imports}}{\text{VA}} \sum_j s_{jF} \left(1 - s_{Fj}^{Total}\right).\end{aligned}$$

Now consider the following identity under perfect competition:

$$\text{Imports} = \sum_j \left(x_{jH} + x_{jF}\right) s_{Fj}^{Total}, \tag{30}$$

where it says the total imports equal the sum of firms' total input used for final output, that is, sales to domestic consumers and foreign, multiplied by the total foreign input share of the firm. Rearranging this equation yields the following:

$$\frac{\text{Imports}}{\text{VA}} = \sum_j s_{jH} s_{Fj}^{Total} + \frac{\text{Exports}}{\text{VA}} \sum_j s_{jF} s_{Fj}^{Total}.$$

Using the trade balance condition where Imports = Exports, we obtain

$$\frac{\text{Imports}}{\text{VA}} \sum_j s_{jF} \left(1 - s_{Fj}^{Total}\right) = \sum_j s_{jH} s_{Fj}^{Total}.$$

Therefore, under perfect competition we have

$$\begin{aligned}\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} &= \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \sum_j s_{jH} s_{Fj}^{Total} \\ &= \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} \right) \frac{\text{Imports}}{\text{VA}} \sum_j s_{jF} \left(1 - s_{Fj}^{Total}\right).\end{aligned}$$

A.6 Conditions under which the knowledge of the network structure becomes irrelevant

Here we outline conditions under which the knowledge of the network structure is unnecessary to compute the term $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$, in equation (21).

If we assume that there is *no import content in the production of exports*, then the total foreign input share of firm j that export, s_{Fj}^{Total}, should be 0. Therefore $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$ would simply be 1.

Alternatively, if we assume *perfect competition* and that there is *import content being the same in both exports and domestic final demand*, then $\sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right)$ can be expressed as $\frac{\text{VA}}{\text{VA+Exports}}$. To see this, combine equation (30) with the trade balance condition and obtain

$$
\begin{aligned}
\frac{\text{Exports}}{\text{VA} + \text{Exports}} &= \sum_j \frac{x_{jH} + x_{jF}}{\text{VA} + \text{Exports}} s_{Fj}^{Total} \\
&= \sum_j \frac{\text{VA}}{\text{VA} + \text{Exports}} s_{jH} s_{Fj}^{Total} + \sum_j \frac{\text{Exports}}{\text{VA} + \text{Exports}} s_{jF} s_{Fj}^{Total}.
\end{aligned}
$$

From the assumption of import content being the same in both exports and domestic final demand, $s_{jH} = s_{jF}$, we then have

$$
\frac{\text{Exports}}{\text{VA} + \text{Exports}} = \sum_j s_{jF} s_{Fj}^{Total},
$$

which then yields

$$
\frac{\text{VA}}{\text{VA} + \text{Exports}} = \sum_j s_{jF}\left(1 - s_{Fj}^{Total}\right).
$$

A.7 System of hat equations allowing for changes in wages

Here we describe the system of equations to solve for the changes in equilibrium variables in general equilibrium given $\hat{p}_{F}$.. We also outline the firm-level cost changes and change in real wage when foreign price goes to infinity and the economy goes into autarky. We do this for three different models: the baseline model that utilizes the observed firm-to-firm linkages, the roundabout production economy, and the sectoral roundabout production economy.

Throughout the three models, the labor market clearing condition and the household's budget constraint are the same. The labor market clearing condition is

$$
wL = \sum_i \frac{1}{\mu_i} s_{Li} x_i,
$$

where μ_i is the average firm-level markups that are constant, $\mu_i = \frac{x_i}{c_i q_i}$. Household's budget constraint is

$$\begin{aligned} E &= wL + \sum_i \pi_i - TB \\ &= wL + \sum_i \frac{\mu_i - 1}{\mu_i} x_i - TB \end{aligned}$$

where TB is the exogenous trade balance, $TB = \sum_i x_{iF} - \sum_i x_{Fi}$.

A.7.1 Baseline

Given a shock of $\hat{p}_{F\cdot}$, we solve the system of equations by following the steps below. In the special case of autarky, $\hat{p}_{F\cdot} \to \infty$, the firm-level cost changes are given by $\hat{c}_j \mid^{p_{F\cdot} \to \infty} = \left(1 - s_{Fj}^{Total}\right)^{1/(1-\rho)}$, and the change in real wage is $\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot} \to \infty} = \left(\sum_i s_{iH} \left(1 - s_{Fi}^{Total}\right)^{\frac{1-\sigma}{1-\rho}}\right)^{\frac{1}{\sigma-1}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting $\hat{w}$ with 1.

1. Guess $\hat{w}$. Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = \left(1 - s_{Fj}^{Total}\right) \hat{w}^{1-\rho} + s_{Fj}^{Total} \hat{p}_{F\cdot}^{1-\rho}.$$

2. Compute the following hat variables.

$$\begin{aligned} \hat{x}_{iF} &= \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\ \hat{P}^{1-\sigma} &= \sum_i s_{iH} \hat{c}_i^{1-\sigma} \\ \hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\ \hat{s}_{ij} &= \hat{c}_i^{1-\rho} \hat{c}_j^{\rho-1} \quad (\text{if } i \in Z_j), \end{aligned}$$

where $\hat{x}_{iF}$ is defined for exporting firms.

3. Solve for $\{\hat{x}_i\}$ from

$$\hat{x}_i = \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iH}}{x_i} \hat{c}_i^{1-\sigma} \hat{P}^{\sigma-1} \left(\frac{wL}{E} \hat{w} + \sum_k \frac{\mu_k - 1}{\mu_k} \frac{x_k}{E} \hat{x}_k - \frac{TB}{E} \right) + \sum_{j \in W_i} \frac{\hat{s}_{ij} x_{ij}}{x_i} \hat{x}_j.$$

4. Update the guess of $\hat{w}$ with

$$\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,$$

and iterate from Step 1 until $\hat{w}$ converges.

A.7.2 Roundabout

For general shocks of $\hat{p}_{F\cdot}$, we solve the system of equations by following the steps below. In the special case of autarky, the firm-level cost changes are given by solving the system

$$\left(\hat{c}_j \mid^{p_{F\cdot}\to\infty}\right)^{1-\rho} = s_{Dj} \left(\sum_i s_{iD} \left(\hat{c}_i \mid^{p_{F\cdot}\to\infty}\right)^{1-\sigma} \right)^{\frac{1-\rho}{1-\sigma}} + s_{Lj},$$

and the change in real wage is $\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot}\to\infty} = \left(\sum_i s_{iD} \left(\hat{c}_i \mid^{p_{F\cdot}\to\infty}\right)^{1-\sigma}\right)^{\frac{-1}{1-\sigma}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting $\hat{w}$ with 1.

1. Guess $\hat{w}$. Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = s_{Dj} \hat{P}^{1-\rho} + s_{Lj} \hat{w}^{1-\rho} + s_{Fj} \hat{p}_{F\cdot}^{1-\rho}$$

where $\hat{P}^{1\text{-}\sigma} = \sum_i s_{iD} \hat{c}_i^{1-\sigma}$.

2. Compute the following hat variables.

$$\begin{aligned}
\hat{x}_{iF} &= \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\
\hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\
\hat{s}_{Di} &= \hat{P}^{1-\rho} \hat{c}_i^{\rho-1} \\
\hat{s}_{iD} &= \hat{c}_i^{1-\sigma} \hat{P}^{\sigma-1}.
\end{aligned}$$

3. Solve for $\{\hat{x}_i\}$ from

$$\begin{aligned}
\hat{x}_i &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \hat{x}_{iD} \\
&= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \frac{s_{iD} E}{x_{iD}} \hat{s}_{iD} \hat{E} + \frac{x_{iD}}{x_i} \frac{s_{iD}}{x_{iD}} \hat{s}_{iD} \left(\sum_j s_{Dj} \frac{1}{\mu_j} x_j \hat{s}_{Dj} \hat{x}_j \right) \\
\hat{E} &= \frac{wL}{E} \hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i} \frac{x_i}{E} \hat{x}_i - \frac{TB}{E}.
\end{aligned}$$

Combined, the above two equation can be expressed as

$$\hat{x}_i = \frac{x_{iF}}{x_i}\hat{x}_{iF} + \frac{s_{iD}}{x_i}\hat{s}_{iD}\left(wL\hat{w} - TB\right) + \frac{s_{iD}}{x_i}\hat{s}_{iD}\left(\sum_j \frac{\mu_j - 1}{\mu_j} x_j \hat{x}_j + \sum_j s_{Dj}\frac{1}{\mu_j} x_j \hat{s}_{Dj}\hat{x}_j\right).$$

4. Update the guess of $\hat{w}$ with

$$\hat{w} = \sum_i \frac{s_{Li}x_i}{\mu_i wL}\hat{s}_{Li}\hat{x}_i,$$

and iterate from Step 1 until $\hat{w}$ converges.

A.7.3 Sectoral roundabout

Consider firm i in sector v, firm j in sector u. Let the price index of sector v be P_v. We calculate the share of sector u goods in household consumption as $\gamma_{vH} = \frac{\sum_{i\in v} x_{iH}}{\sum_k x_{kH}}$ and the share of sector v goods in the domestic intermediate input bundle for sector u as $\gamma_{vu} = \frac{\sum_{i\in v}\sum_{j\in u} x_{ij}}{\sum_k \sum_{j\in u} x_{kj}}$.

For general shocks of $\hat{p}_{F\cdot}$, we solve the system of equations by following the steps below. In the special case of autarky, the firm-level cost changes are given by solving the system

$$\left(\hat{c}_j \mid^{p_{F\cdot}\to\infty}\right)^{1-\rho} = s_{Dj}\left(\prod_v \left(\hat{P}_v \mid^{p_{F\cdot}\to\infty}\right)^{\gamma_{vu(j)}}\right)^{1-\rho} + s_{Lj}$$

$$\left(\hat{P}_{v(i)} \mid^{p_{F\cdot}\to\infty}\right)^{1-\sigma} = \sum_{i\in v} s_{iv(i)}\left(\hat{c}_i \mid^{p_{F\cdot}\to\infty}\right)^{1-\sigma},$$

and the change in real wage is $\frac{\hat{w}}{\hat{P}} \mid^{\hat{p}_{F\cdot}\to\infty} = \prod_v \left(\hat{P}_v \mid^{p_{F\cdot}\to\infty}\right)^{-\gamma_{vH}}$. Using these firm-level cost changes one can proceed with the Steps 2 and 3 below by substituting $\hat{w}$ with 1.

1. Guess $\hat{w}$. Compute $\{\hat{c}_i\}$ from

$$\hat{c}_j^{1-\rho} = s_{Dj}\left(\prod_v \hat{P}_v^{\gamma_{vu(j)}}\right)^{1-\rho} + s_{Lj}\hat{w}^{1-\rho} + s_{Fj}\hat{p}_{F\cdot}^{1-\rho},$$

where $\hat{P}_{v(i)}^{1-\sigma} = \sum_{i\in v} s_{iv(i)}\hat{c}_i^{1-\sigma}$. $s_{iv(i)}$ is computed as firm i's share of domestic sales among other firms in the same sector.

2. Compute the following hat variables.

$$
\begin{aligned}
\hat{P} &= \prod_v \hat{P}_v^{\gamma_{vH}} \\
\hat{s}_{Li} &= \hat{w}^{1-\rho} \hat{c}_i^{\rho-1} \\
\hat{s}_{iv(i)} &= \hat{c}_i^{1-\sigma} \hat{P}_{v(i)}^{\sigma-1} \\
\hat{x}_{iF} &= \hat{c}_i^{1-\sigma} \quad (\text{if } I_{iF} = 1) \\
\hat{s}_{Dj} &= \left(\prod_v \hat{P}_v^{\gamma_{vu(j)}} \right)^{1-\rho} \hat{c}_j^{\rho-1}.
\end{aligned}
$$

3. Solve for $\{\hat{x}_i\}$ from

$$
\begin{aligned}
\hat{x}_i &= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{x_{iD}}{x_i} \hat{x}_{iD} \\
&= \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{s_{iv(i)} \gamma_{v(i)H} E}{x_i} \hat{s}_{iv(i)} \hat{E} + \frac{1}{x_i} s_{iv(i)} \hat{s}_{iv(i)} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} \hat{s}_{Dj} \hat{x}_j \right) \\
\hat{E} &= \frac{wL}{E} \hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i} \frac{x_i}{E} \hat{x}_i - \frac{TB}{E},
\end{aligned}
$$

which can be summarized as

$$
\begin{aligned}
\hat{x}_i =& \frac{x_{iF}}{x_i} \hat{x}_{iF} + \frac{s_{iv(i)} \hat{s}_{iv(i)} \gamma_{v(i)H}}{x_i} \left(wL\hat{w} - TB + \sum_j \frac{\mu_j - 1}{\mu_j} x_j \hat{x}_j \right) \\
&+ \frac{s_{iv(i)} \hat{s}_{iv(i)}}{x_i} \left(\sum_u \sum_{j \in u} s_{Dj} \gamma_{v(i)u(j)} \frac{x_j}{\mu_j} \hat{s}_{Dj} \hat{x}_j \right).
\end{aligned}
$$

4. Update the guess of $\hat{w}$ with

$$
\hat{w} = \sum_i \frac{s_{Li} x_i}{\mu_i w L} \hat{s}_{Li} \hat{x}_i,
$$

and iterate from Step 1 until $\hat{w}$ converges.

A.8 System of equations under small changes of $p_{F\cdot}$

A.8.1 Baseline

Under the baseline case where we take the observed Belgian firm-to-firm trade data, the change in real wage given $\frac{\mathrm{d}P}{P}$ is expressed as follows (Lemma 2):

$$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right) \sum_j s_{jH} s_{Fj}^{Total}.$$

The term $\sum_j s_{jH} s_{Fj}^{Total}$ corresponds to the import content in domestic final demand (ICD), and analogously, import content in exports (ICE) is $\sum_j s_{jF} s_{Fj}^{Total}$ where $s_{jF} = \frac{x_{jF}}{\sum_j s_{jF}}$. We obtain the system of equations that determines $\frac{\mathrm{d}w}{w}$ by log-linearizing the system described in Section A.7.1.

1. Guess $\frac{\mathrm{d}w}{w}$. Compute $\left\{\frac{\mathrm{d}c_j}{c_j}\right\}$ from

$$\frac{\mathrm{d}c_j}{c_j} = \left(1 - s_{Fj}^{Total}\right) \frac{\mathrm{d}w}{w} + s_{Fj}^{Total} \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}.$$

2. Compute the following variables.

$$\begin{aligned}
\frac{\mathrm{d}x_{iF}}{x_{iF}} &= (1-\sigma)\frac{\mathrm{d}c_i}{c_i} \\
\frac{\mathrm{d}P}{P} &= \sum_j s_{jH} \frac{\mathrm{d}c_j}{c_j} \\
\frac{\mathrm{d}s_{Li}}{s_{Li}} &= (1-\rho)\frac{\mathrm{d}w}{w} + (\rho - 1)\frac{\mathrm{d}c_i}{c_i} \\
\frac{\mathrm{d}s_{ij}}{s_{ij}} &= (1-\rho)\frac{\mathrm{d}c_i}{c_i} + (\rho - 1)\frac{\mathrm{d}c_j}{c_j}.
\end{aligned}$$

3. Solve for $\left\{\frac{\mathrm{d}x_i}{x_i}\right\}$ from

$$\begin{aligned}
\frac{\mathrm{d}x_i}{x_i} =& \frac{x_{iF}}{x_i}\frac{\mathrm{d}x_{iF}}{x_{iF}} + \frac{x_{iH}}{x_i}(1-\sigma)\left(\frac{\mathrm{d}c_i}{c_i} - \frac{\mathrm{d}P}{P}\right) + \frac{x_{iH}}{x_i}\frac{wL}{E}\frac{\mathrm{d}w}{w} + \frac{x_{iH}}{x_i}\sum_k \frac{\mu_k - 1}{\mu_j}\frac{x_k}{E}\frac{\mathrm{d}x_k}{x_k} \\
&+ \sum_{j \in W_i} \frac{x_{ij}}{x_i}\left(\frac{\mathrm{d}s_{ij}}{s_{ij}} + \frac{\mathrm{d}x_j}{x_j}\right).
\end{aligned}$$

4. Update the guess of $\frac{\mathrm{d}w}{w}$ with

$$\frac{\mathrm{d}w}{w} = \sum_i \frac{s_{Li}x_i}{\mu_i w L}\left(\frac{\mathrm{d}s_{Li}}{s_{Li}} + \frac{\mathrm{d}x_i}{x_i}\right),$$

and iterate from Step 1 until $\frac{\mathrm{d}w}{w}$ converges.

A.8.2 Roundabout

Under the roundabout production model, the production functions are

$$\begin{aligned} c_j^{1-\rho} &= \phi_j^{\rho-1}\left(\alpha_{Dj}^{\rho-1}P^{1-\rho} + \alpha_{Fj}^{\rho-1}p_{Fj}^{1-\rho} + \alpha_{Lj}^{\rho-1}w^{1-\rho}\right) \\ P^{1-\sigma} &= \sum_j \beta_{jH}^{\sigma-1}\mu^{1-\sigma}c_j^{1-\sigma}. \end{aligned}$$

By log-linearizing the system by considering small changes, one obtains:

$$\begin{aligned} \frac{\mathrm{d}c_j}{c_j} &= s_{Dj}\frac{\mathrm{d}P}{P} + s_{Fj}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} + s_{Lj}\frac{\mathrm{d}w}{w} \\ \frac{\mathrm{d}P}{P} &= \sum_j s_{jD}\frac{\mathrm{d}c_j}{c_j}. \end{aligned}$$

Rearranging yields the change in real wage given $\frac{\mathrm{d}w}{w}$:

$$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right)\frac{\sum_j s_{jD}s_{Fj}}{1-\sum_j s_{jD}s_{Dj}}.$$

The import content in domestic final demand (ICD) in this model is $\frac{\sum_j s_{jD}s_{Fj}}{1-\sum_j s_{jD}s_{Dj}}$, and the import content in exports (ICE) is $\sum_j s_{jF}\left(s_{Fj} + s_{Dj}\mathrm{ICD}\right)$. We obtain the system of equations that determines $\frac{\mathrm{d}w}{w}$ by log-linearizing the system described in Section A.7.2.

1. Guess $\frac{\mathrm{d}w}{w}$. Compute $\left\{\frac{\mathrm{d}c_j}{c_j}\right\}$ from

$$\frac{\mathrm{d}c_j}{c_j} = s_{Dj}\frac{\mathrm{d}P}{P} + s_{Lj}\frac{\mathrm{d}w}{w} + s_{Fj}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}.$$

   where $\frac{\mathrm{d}P}{P} = \sum_j s_{jD}\frac{\mathrm{d}c_j}{c_j}$.

2. Compute the following variables.

$$
\begin{aligned}
\frac{\mathrm{d}x_{iF}}{x_{iF}} &= (1-\sigma)\frac{\mathrm{d}c_i}{c_i} \text{ (if } I_{iF}=1) \\
\frac{\mathrm{d}s_{Li}}{s_{Li}} &= (1-\rho)\frac{\mathrm{d}w}{w} + (\rho-1)\frac{\mathrm{d}c_i}{c_i} \\
\frac{\mathrm{d}s_{Di}}{s_{Di}} &= (1-\rho)\frac{\mathrm{d}P}{P} + (\rho-1)\frac{\mathrm{d}c_i}{c_i} \\
\frac{\mathrm{d}s_{iD}}{s_{iD}} &= (\sigma-1)\frac{\mathrm{d}P}{P} + (1-\sigma)\frac{\mathrm{d}c_i}{c_i}.
\end{aligned}
$$

3. Solve for $\left\{\frac{\mathrm{d}x_i}{x_i}\right\}$ from

$$
\begin{aligned}
\frac{\mathrm{d}x_i}{x_i} =& \frac{x_{iF}}{x_i}\frac{\mathrm{d}x_{iF}}{x_{iF}} + \frac{s_{iD}}{x_i}wL\frac{\mathrm{d}w}{w} + \frac{s_{iD}}{x_i}\left(E + \sum_j s_{Dj}\frac{x_j}{\mu_j}\right)\frac{\mathrm{d}s_{iD}}{s_{iD}} \\
&+ \frac{s_{iD}}{x_i}\left(\sum_j \frac{\mu_j - 1}{\mu_j}x_j\frac{\mathrm{d}x_j}{x_j} + \sum_j s_{Dj}\frac{x_j}{\mu_j}\frac{\mathrm{d}x_j}{x_j}\right) \\
&+ \frac{s_{iD}}{x_i}\left(\sum_j s_{Dj}\frac{x_j}{\mu_j}\frac{\mathrm{d}s_{Dj}}{s_{Dj}}\right).
\end{aligned}
$$

4. Update the guess of $\frac{\mathrm{d}w}{w}$ with

$$
\frac{\mathrm{d}w}{w} = \sum_i \frac{s_{Li}x_i}{\mu_i wL}\left(\frac{\mathrm{d}s_{Li}}{s_{Li}} + \frac{\mathrm{d}x_i}{x_i}\right),
$$

and iterate from Step 1 until $\frac{\mathrm{d}w}{w}$ converges

A.8.3 Sectoral roundabout

Log-linearizing the system of equations for firm-level cost changes yields:

$$
\begin{aligned}
\frac{\mathrm{d}c_j}{c_j} &= s_{Dj}\sum_v \gamma_{vu(j)}\frac{\mathrm{d}P_v}{P_v} + s_{Fj}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} + s_{Lj}\frac{\mathrm{d}w}{w} \\
\frac{\mathrm{d}P_v}{P_v} &= \sum_{i\in v} s_{iv(i)}\frac{\mathrm{d}c_i}{c_i} \\
\frac{\mathrm{d}P}{P} &= \sum_v \gamma_{vH}\frac{\mathrm{d}P_v}{P_v}.
\end{aligned}
$$

Rearranging and solving the system of equations below yields the change in sectoral goods' prices, $\frac{\mathrm{d}P_v}{P_v}$, and thus the change in real wage given $\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$:

$$\begin{aligned}\frac{\mathrm{d}P_v}{P_v} &= \sum_{i\in v} s_{iv(i)}\left(s_{Di}\sum_u \gamma_{uv(i)}\frac{\mathrm{d}P_u}{P_u} + s_{Fi}\frac{\mathrm{d}P_{F\cdot}}{P_{F\cdot}} + s_{Li}\frac{\mathrm{d}w}{w}\right)\\ \frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} &= \frac{\mathrm{d}w}{w} - \sum_v \gamma_{vH}\frac{\mathrm{d}P_v}{P_v}.\end{aligned}$$

The import content in domestic goods at the sector level, IC_v, and the import content in domestic final demand, ICD, can be obtained by solving the following:

$$\begin{aligned}\mathrm{IC}_v &= \sum_{i\in v} s_{Fi}s_{iv(i)} + \sum_u \gamma_{uv}\sum_{i\in v} s_{Di}s_{iv(i)}\mathrm{IC}_u\\ \mathrm{ICD} &= \sum_v \gamma_{vH}\mathrm{IC}_v.\end{aligned}$$

The import content in exports is computed by:

$$\mathrm{ICE} = \sum_i s_{iF}\left(s_{Fi} + s_{Di}\sum_u \gamma_{uv(i)}\mathrm{IC}_u\right).$$

We obtain the system of equations that determines $\frac{\mathrm{d}w}{w}$ by log-linearizing the system described in Section A.7.3.

1. Guess $\frac{\mathrm{d}w}{w}$. Compute $\left\{\frac{\mathrm{d}c_j}{c_j}\right\}$ from

$$\begin{aligned}\frac{\mathrm{d}c_j}{c_j} &= s_{Dj}\sum_v \gamma_{vu(j)}\frac{\mathrm{d}P_v}{P_v} + s_{Fj}\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} + s_{Lj}\frac{\mathrm{d}w}{w}\\ \frac{\mathrm{d}P_v}{P_v} &= \sum_{i\in v} s_{iv(i)}\frac{\mathrm{d}c_i}{c_i}.\end{aligned}$$

2. Compute the following variables.

$$\begin{aligned}
\frac{\mathrm{d}P}{P} &= \sum_v \gamma_{vH} \frac{\mathrm{d}P_v}{P_v} \\
\frac{\mathrm{d}s_{Li}}{s_{Li}} &= (1-\rho)\left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}c_i}{c_i}\right) \\
\frac{\mathrm{d}s_{iv(i)}}{s_{iv(i)}} &= (1-\sigma)\left(\frac{\mathrm{d}c_i}{c_i} - \frac{\mathrm{d}P_{v(i)}}{P_{v(i)}}\right) \\
\frac{\mathrm{d}x_{iF}}{x_{iF}} &= (1-\sigma)\frac{\mathrm{d}c_i}{c_i} \quad (\text{if } I_{iF}=1) \\
\frac{\mathrm{d}s_{Dj}}{s_{Dj}} &= (1-\rho)\left(\sum_v \gamma_{vu(j)} \frac{\mathrm{d}P_v}{P_v} - \frac{\mathrm{d}c_j}{c_j}\right).
\end{aligned}$$

3. Solve for $\left\{\frac{\mathrm{d}x_i}{x_i}\right\}$ from

$$\begin{aligned}
\frac{\mathrm{d}x_i}{x_i} =& \frac{x_{iF}}{x_i}\frac{\mathrm{d}x_{iF}}{x_{iF}} + \frac{s_{iv(i)}}{x_i}\left(\sum_u \sum_{j\in u} s_{Dj}\gamma_{v(i)u(j)}\frac{x_j}{\mu_j} + \gamma_{v(i)H}E\right)\frac{\mathrm{d}s_{iv(i)}}{s_{iv(i)}} \\
&+ \frac{s_{iv(i)}\gamma_{v(i)H}}{x_i}\left(wL\frac{\mathrm{d}w}{w} + \sum_j \frac{\mu_j - 1}{\mu_j} x_j \frac{\mathrm{d}x_j}{x_j}\right) \\
&+ \frac{s_{iv(i)}}{x_i}\left(\sum_u \sum_{j\in u} s_{Dj}\gamma_{v(i)u(j)}\frac{x_j}{\mu_j}\left(\frac{\mathrm{d}s_{Dj}}{s_{Dj}} + \frac{\mathrm{d}x_j}{x_j}\right)\right).
\end{aligned}$$

4. Update the guess of $\frac{\mathrm{d}w}{w}$ with

$$\frac{\mathrm{d}w}{w} = \sum_i \frac{s_{Li}x_i}{\mu_i wL}\left(\frac{\mathrm{d}s_{Li}}{s_{Li}} + \frac{\mathrm{d}x_i}{x_i}\right),$$

and iterate from Step 1 until $\frac{\mathrm{d}w}{w}$ converges.

A.9 Real income changes

Here we present the expression for the change in real income. Across the three models, the change in real income can be expressed in terms of the change variables that are computed from the system of equations in Appendix A.7:

$$\frac{\hat{E}}{\hat{P}} = \left(\frac{wL}{E}\hat{w} + \sum_i \frac{\mu_i - 1}{\mu_i}\frac{x_i}{E}\hat{x}_i - \frac{TB}{E}\right)\hat{P}^{-1}. \tag{31}$$

We then present the expression for the first-order approximated change in real income. Across the three models, the log-change in real income can be expressed in terms of the log-change variables that are computed from the system of equations in Appendix A.8:

$$\frac{\mathrm{d}E}{E} - \frac{\mathrm{d}P}{P} = \frac{wL}{E}\frac{\mathrm{d}w}{w} + \sum_k \frac{\mu_k - 1}{\mu_k}\frac{x_k}{E}\frac{\mathrm{d}x_k}{x_k} - \frac{\mathrm{d}P}{P}. \tag{32}$$

B Numerical example

Here we demonstrate that economies with identical sets of aggregate exports, aggregate imports, aggregate gross production and GDP, but with different firm-to-firm network structures, can potentially generate different import content in domestic final demand. We consider two economies with both consisting of three firms. Table 8 lays out the details of the two economies. In the two economies, firm-level imports, exports, gross production, domestic sales, labor costs, and value added are the same. The only difference between the two economies are how firms allocate their domestic sales to sales to households or to sales to other firms. In Table 8, the first seven rows are identical across the two economies, but the entries for Firm-to-firm sales and sales to households differ.

Table 8: Two economies

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
Imports	100	0	0	100	0	0
Exports	0	0	100	0	0	100
Gross production	200	200	200	200	200	200
Domestic Sales	200	200	100	200	200	100
Labor cost	50	100	50	50	100	50
Domestic purchases	0	50	100	0	50	100
Profits	50	50	50	50	50	50
Firm-to-firm sales	$x_{12} = 50$	$x_{23} = 50$			$x_{23} = 100$	$x_{32} = 50$
	$x_{13} = 50$					
Sales to households	100	150	100	200	100	50

Now let us compute the direct and total shares of foreign inputs, as well as the firms' shares in household consumption. Table 9 summarizes the firms' shares. Firms' direct shares of foreign inputs are the same across the two economies, as firm-level imports and total inputs are the same. But because the firm-to-firm sales structure is not the same in the two economies, the total shares of foreign inputs and firms' shares in household consumption are different.

Table 9: Direct and total shares of foreign inputs, and shares in household consumption

	Economy 1			Economy 2		
	Firm 1	Firm 2	Firm 3	Firm 1	Firm 2	Firm 3
s_{Fi}	2/3	0	0	2/3	0	0
s_{Fi}^{Total}	2/3	2/9	2/27	2/3	0	0
s_{iH}	2/7	2/7	3/7	4/7	1/7	2/7

Next, we compute the import content in domestic final demand in the two economies, $\sum_i s_{iH} s_{Fi}^{Total}$. In economy 1, the import content in domestic final demand is 6/21, while in economy 2 it is 8/21. Because there is no leakage of imports in economy 2 to Foreign, the import content in domestic final demand is about 10 percentage points higher in economy 2.

We can also easily calculate what that implies for the real wage change of going to autarky. Under $\rho = 2$ and $\sigma = 4$,

$$\frac{\hat{w}}{\hat{P}} |^{\hat{p}_{F\cdot} \to \infty} = \left(\sum_i s_{iH} \left(1 - s_{Fj}^{Total}\right)^{\frac{1-\sigma}{1-\rho}} \right)^{\frac{1}{\sigma - 1}} = \begin{cases} 0.79 & \text{if economy 1} \\ 0.77 & \text{if economy 2} \end{cases} .$$

Hence, as expected, the real wage changes are larger in economy 2.

C Data Appendix

C.1 Grouping VAT-identifiers into firms

As mentioned in the main text, all our datasets are recorded at the VAT-identifier level. We utilize ownership filings in the annual accounts and information from the Balance of Payments survey in order to aggregate multiple VAT-identifiers into firms. In the ownership filings, each enterprise reports a list of all other enterprises of which it has an ownership share of at least 10% and the value of the share. In the Balance of Payments survey, Belgian enterprises with international financial linkages have to report their stock and flows of financial links. They have to report both the international participation they own and the foreign owners of financial participation in their capital if the participation represents at least 10% of the capital. The survey is designed to cover the population of Belgian enterprises involved in international financial transactions.

We group all VAT-identifiers into firms if they are linked with more than or equal to 50% of ownership. In addition, we group all VAT-identifiers into firms if they share the same foreign parent firm that holds more than or equal to 50% of their shares. We use a "fuzzy string matching" method to determine whether they share the same foreign parent firm, by obtaining similarity measures of all possible pairs of foreign firms' names. Lastly, in order to correct for misreportings, we also add links to the VAT-identifier pairs if the two were linked one year before and one year after. We define a firm as the group of VAT-identifiers that are directly and indirectly linked.

Given these groupings of VAT-identifiers, we then choose the "most representative" VAT-identifier for each firm. We use this "head VAT-identifier" as the identifier of the firm.[44] Then, in order to make the identifiers consistent over time, we make the following adjustment: We take firms whose head VAT-identifier was not an identifier of any firm in the previous year. For such firms, if there exists a VAT-identifier within the firm which was a head VAT-identifier in the previous year, then we switch the firm identifier to that former head VAT-identifier.[45]

Having determined the head VAT-identifier for each firm with multiple VAT-identifiers, we aggregate all the variables up to the firm level. For variables such as total sales and inputs, we adjust the aggregated variables with the amount of B2B trade that occurred

[44]The criteria for determining the head VAT-identifier is as follows: (i) If there is only one VAT-identifier in the firm that filed all the full annual accounts, the VAT declarations, and the B2B filings, then this VAT-identifier is chosen as the head. (ii) If there are no such VAT-identifiers or multiple of them, then we choose the VAT-identifier that has the largest total assets reported. (iii) If there are no VAT-identifier that filed the annual accounts, then we choose the VAT-identifier that has the largest amount of total inputs, which is the sum of labor costs, B2B inputs, and imports.

[45]If there are multiple such VAT-identifier, then we choose the "most representative" VAT-identifier, using the same criteria as above.

within the firm, correcting for double counting. For other non-numeric variables such as firms' primary sector, we take the value of its head VAT-identifier.

The number of VAT-identifiers for firms with multiple VAT-identifiers are shown in Table 10.

Table 10: Number of VAT-identifier in firms with multiple VAT-identifiers

	Mean	10%	25%	50%	75%	90%	max
Num. VAT-identifier	3	2	2	2	3	5	372

C.2 Firm selection

Table 11 displays the same numbers for Table 1, with statistics for all Belgian firms added.

Table 11: Coverage of all Belgian firms and selected sample

Year	All Belgian Firms					Selected sample				
	Count	V.A.	Sales	Imports	Exports	Count	V.A.	Sales	Imports	Exports
2002	714,469	226	812	204	217	88,301	231	604	175	185
2007	782,006	315	1080	294	282	95,941	299	782	277	265
2012	860,373	322	1244	320	317	98,745	356	874	292	292

Notes: All numbers except for Count are denominated in billion Euro in current prices. Firms' value added in the selected sample is computed as their sales minus imports and their purchases from other Belgian firms that are in the selected sample. Firms' value added for all Belgian firms is computed as their sales minus imports and their purchases from all other Belgian firms.

C.3 Definition of variables

We describe how we compute each variable that we use in the analyses in the paper. Firms' variable inputs consist of their labor costs reported in the annual accounts, their imports reported in the international trade dataset, and the goods purchased from other Belgian firms that are reported in the B2B dataset. Note that we do not include goods purchased from firms that do not meet the sample selection criteria. Firms' sales consist of their sales to other Belgian firms that meet the sample selection criteria, their exports reported in the international trade dataset, and their sales to domestic final demand. A firm's sales to domestic final demand is computed as the residual of the firm's total turnover reported in the annual accounts, after subtracting B2B sales and exports. This procedure counts firms' sales to other firms that do not meet the sample selection criteria as part of sales to domestic final demand.

C.4 Sectoral composition

Table 12 shows the sectoral composition of our selected sample. Values for value added and output are in billion Euro.

Table 12: Sectoral composition in 2012

Sector	Count	V.A.	Output	Imports	Exports
Agriculture and Mining	2,805	28.5	49.4	16.9	10.9
Manufacturing	16,577	138	272	146	193
Utility and Construction	20,421	23.3	77.0	27.8	17.5
Wholesale and Retail	31,117	87.8	241	84.1	53.4
Service	27,825	79.1	127	17.6	16.9
Total	98,745	356	874	292	292

Notes: Agriculture and Mining corresponds to NACE 2-digit codes 01 to 09, Manufacturing corresponds to NACE 2-digit codes 10 to 33, Utility and Construction corresponds to NACE 2-digit codes 35 to 43, Wholesale and Retail corresponds to NACE 2-digit codes 45 to 47, and Service corresponds to NACE 2-digit codes 49 to 63, 68 to 82, and 94 to 96.

C.5 Reporting thresholds of the international trade dataset

There are different reporting thresholds for the international trade dataset, depending on if the trade occurred with an extra-EU country or within the EU. The dataset covers all extra-EU exports and imports by firms with values higher than 1,000 Euro or with weights bigger than 1,000kg. Nevertheless, we also observe values less than 1,000 Euro as more firms use electronic reporting procedures. For intra-EU trade prior to 2006, the dataset covers all exports and imports by firms whose combined imports from intra-EU countries that are more than 250,000 Euro a year. For intra-EU trade from 2006 onward, the thresholds for exports and imports changed to 1,000,000 Euro and 400,000 Euro, respectively. Import reporting thresholds became 700,000 Euro per year in 2010. While these reporting threshold for intra-EU trade imply we miss some trade transaction, they are set to capture at least 93% of aggregate Belgian trade in the micro-data, hence our data still contains the overwhelming majority of the value of Belgian trade.

C.6 Mapping CN codes into NACE codes

Our international trade dataset records products in Combined Nomenclature (CN) codes, up to 8 digits. On the other hand, all other datasets that we use record the enterprise's primary sector in NACE Rev.2 code. To concord the two classifications, we convert the CN

8 digit codes into NACE Rev.2 codes. As the first 6 digits of CN codes are identical to the contemporary Harmonized System (HS) codes, we first convert those HS 6-digit codes to Classification of Products by Activity (CPA) codes. We then convert CPA codes to NACE codes, using the fact that CPA 2008 codes are identical to NACE Rev.2 codes up to 4 digits. This conversion allows us to convert more than 98% of all international trade recorded in our dataset, in terms of values (in 2012).

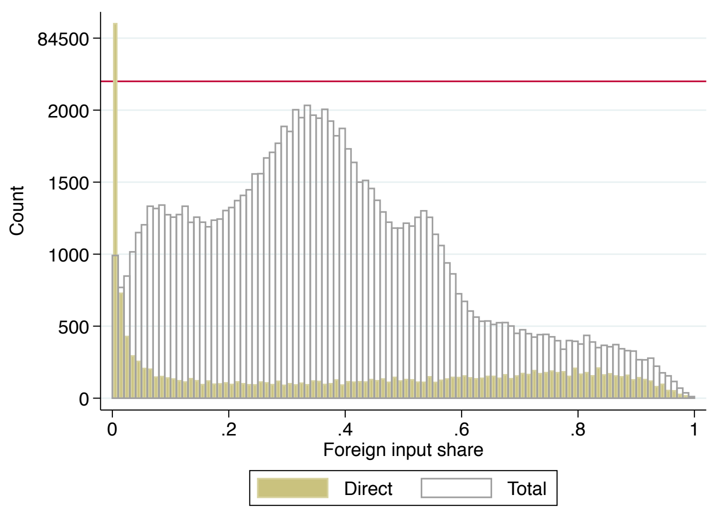
D Descriptive statistics

D.1 Taking into account capital usage

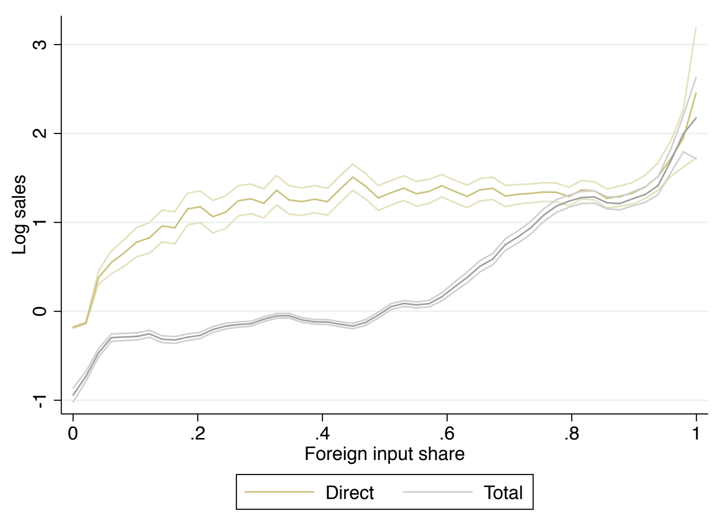
In Figure 8 we show figures analogous to Figures 1a and 2a, but taking into account firms' capital usage. Following Dhyne, Petrin, Smeets, and Warzynski (2017), we set the yearly depreciation rate as 8% and set the interest rate as the long-term interest rate in Belgium. We compute the capital rental costs using fixed tangible assets reported in the annual accounts.

Figure 8: Figures 1a and 2a with capital usage

(a) Direct and total foreign input share



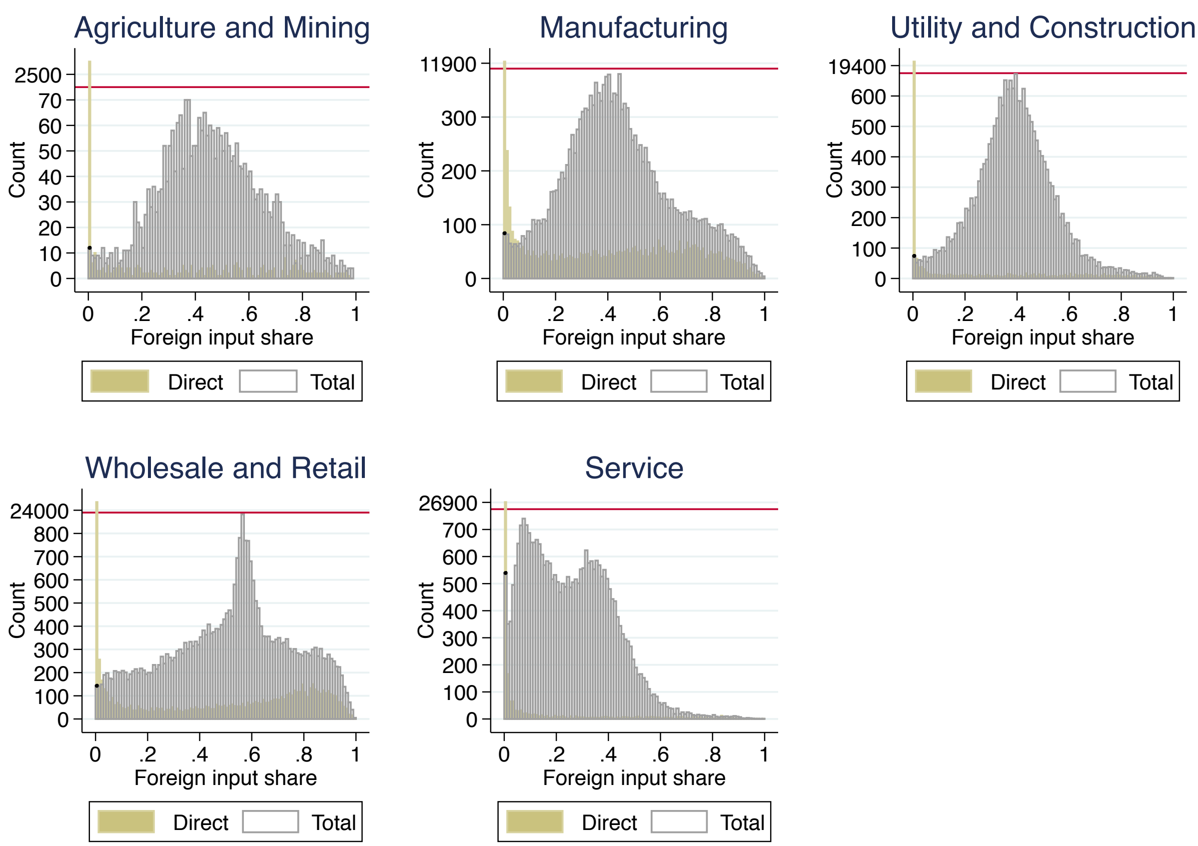
(b) Sales and foreign input shares



D.2 Direct and Total foreign input shares

In Figure 9 we present the histograms of both the direct and total foreign input shares, for each major sector. We also summarize statistics on these distributions in Table 13.

Figure 9: Histograms of direct and total foreign input share by firms' sector



Notes: The black dot indicates the ending of the bar for the total foreign input share. Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. The horizontal lines represent scale breaks on the vertical axis.

Table 13: Distribution of direct and total foreign input share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.70	0	0.46	0.85	0.45
Manufacturing	0.12	0.59	0	0.44	0.75	0.42
Utility and Construction	0.02	0.31	0	0.39	0.59	0.39
Wholesale and Retail	0.12	0.43	0	0.52	0.75	0.55
Service	0.01	0.19	0	0.25	0.41	0.24
Total	0.07	0.45	0	0.40	0.68	0.39

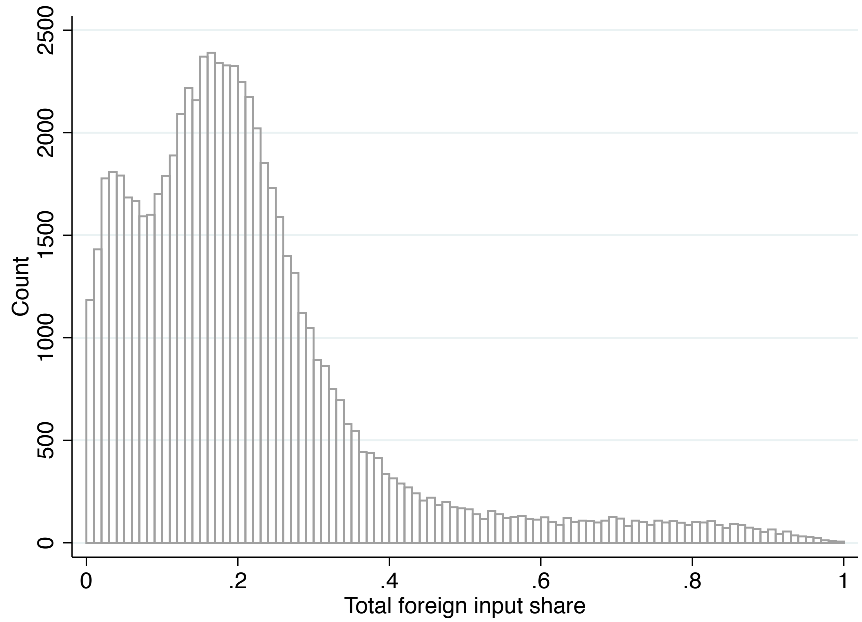
Notes: The numbers for the weighted mean are calculated using total input purchases of firms as the weights.

D.3 Total exposures to foreign trade when excluding wholesale and retail sectors

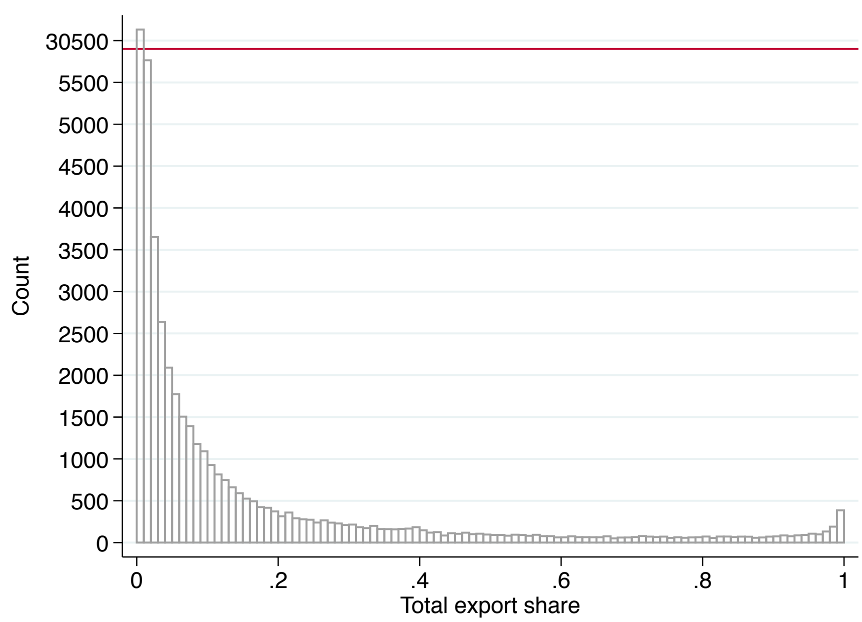
Figure 10a plots the distribution of total foreign input shares for firms outside the wholesale and retail sectors, where the total foreign input shares are computed by setting direct foreign input shares for firms in wholesale and retail sectors as 0. 15% of firms outside the wholesale and retail sectors were importers. Analogously, Figure 10b plots the distribution of total export shares for firms outside the wholesale and retail sectors, where the total export shares are computed by setting direct export shares for firms in wholesale and retail sectors as 0. 10% of firms outside the wholesale and retail sectors were exporters.

Figure 10: Histograms of total exposures to foreign trade, when excluding direct imports and exports by wholesale and retail sectors

(a) Total foreign input share



(b) Total export share

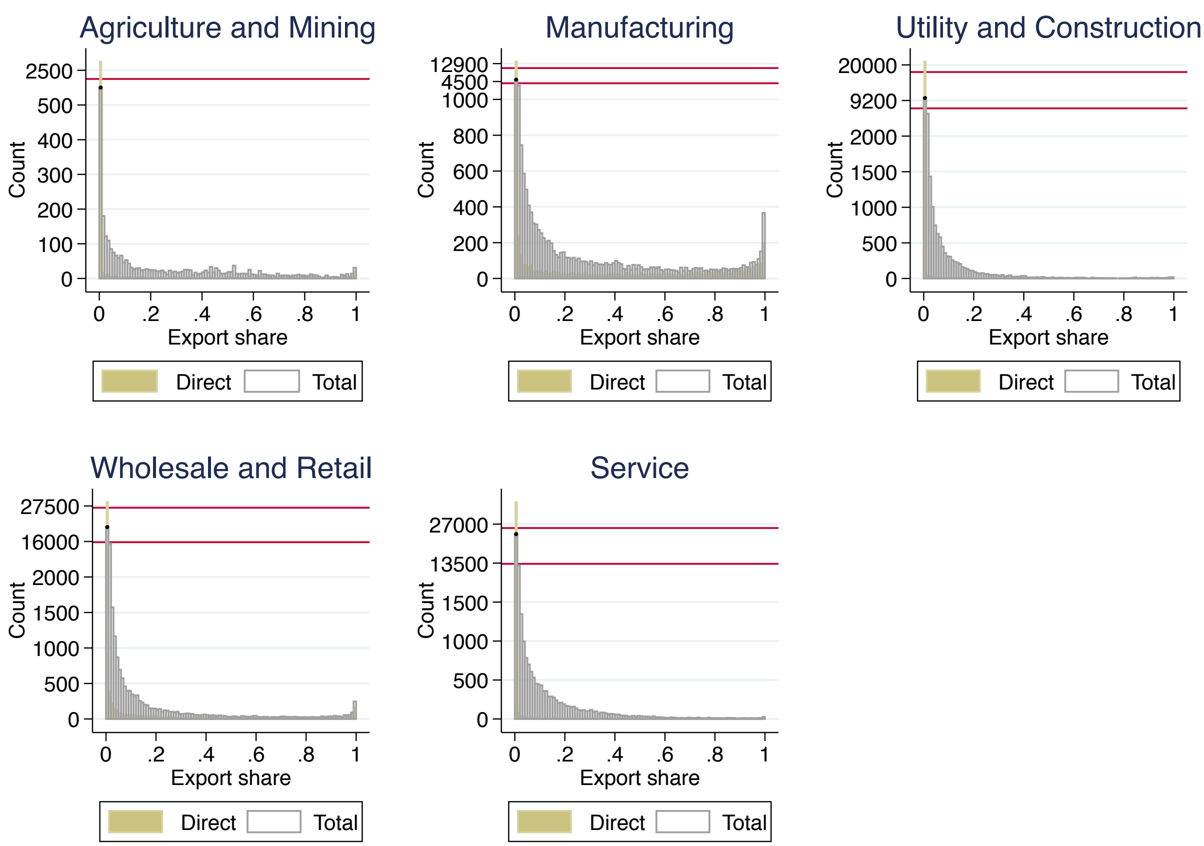


Notes: The histograms plot the total foreign input shares and total export shares for firms not in the wholesale or retail sectors. Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. s_{Fi} for firms in wholesale and retail sectors are set to be 0. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue, and r_{ij} is share of i's revenue that arises from sales to firm j. r_{iF} for firms in wholesale and retail sectors are set to be 0.

D.4 Direct and Total export share

In Figure 11 we present the histograms of both the direct and total foreign input shares, for each major sector. We also summarize statistics on these distributions in Table 14.

Figure 11: Histograms of direct and total export share by firms' sector



Notes: The black dot indicates the ending of the bar for the total export share. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue, and r_{ij} is share of i's revenue that arises from sales to firm j. W_i is the set of customers of i. The horizontal lines represent scale breaks on the vertical axis.

Table 14: Distribution of direct and total export share by firms' sector

Sector	Direct			Total		
	Mean	Weighted Mean	Median	Mean	Weighted Mean	Median
Agriculture and Mining	0.05	0.22	0	0.24	0.30	0.11
Manufacturing	0.11	0.56	0	0.23	0.62	0.07
Utility and Construction	0.01	0.19	0	0.06	0.25	0.01
Wholesale and Retail	0.04	0.21	0	0.09	0.26	0.01
Service	0.01	0.13	0	0.07	0.23	0.01
Total	0.04	0.33	0	0.11	0.40	0.02

Notes: The numbers for the weighted mean are calculated using total sales of firms as the weights.

D.5 Import content in domestic final demand and import content in exports

As explained in Appendix A.8.1, we define the import content in domestic final demand (ICD) as $\sum_j s_{jH} s_{Fj}^{Total}$ as , and analogously, import content in exports (ICE) as $\sum_j s_{jF} s_{Fj}^{Total}$ where $s_{jF} = \frac{x_{jF}}{\sum_j s_{jF}}$. Table 15 reports the two numbers across broad sectors.

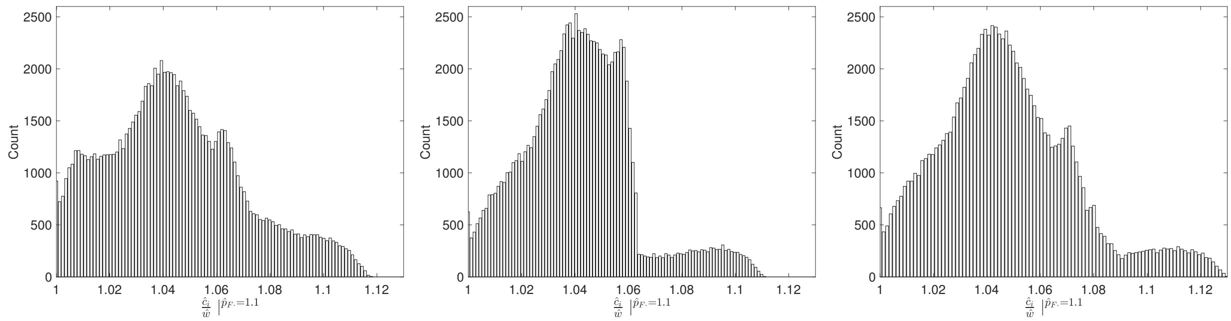
Table 15: ICD and ICE across sectors

Sector	ICD	ICE
Agriculture and Mining	0.068	0.033
Manufacturing	0.158	0.522
Utility and Construction	0.059	0.052
Wholesale and Retail	0.237	0.155
Service	0.062	0.038
Total	0.584	0.800

D.6 Histogram of $\frac{\hat{c}_i}{\hat{w}} |^{\hat{p}_F = 1.1}$

Figure 12 plots histograms of firm-level cost changes relative to the change in nominal wage, $\frac{\hat{c}_i}{\hat{w}}$, under 10% increase in the foreign price.

Figure 12: Histograms of firm-level cost changes relative to change in nominal wage

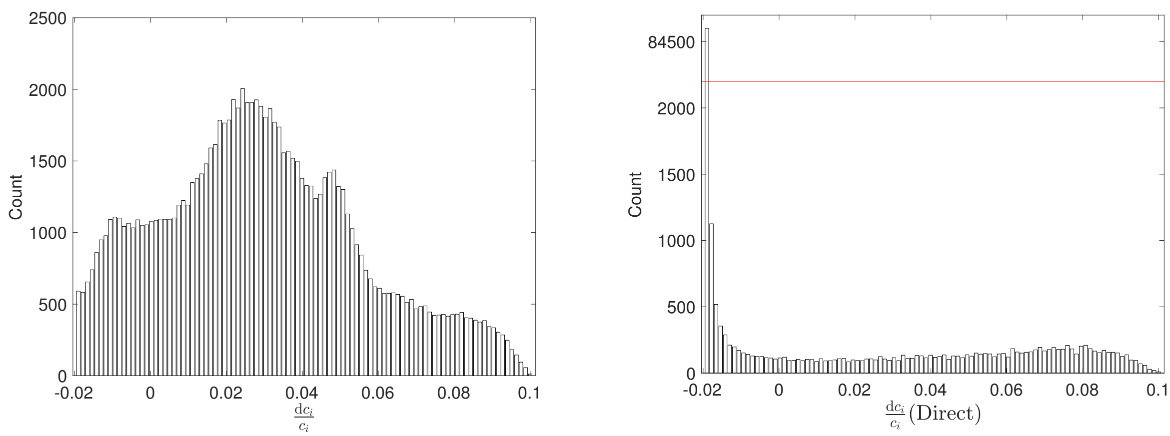


(a) Baseline model (b) Simple Roundabout (c) Sectoral Roundabout

D.7 Histogram of $\frac{\mathrm{d}c_i}{c_i}$

Figure 13 plots the first-order approximated cost increases at the firm-level, upon a 10% increase in the foreign input price. Figure 13a shows the firm-level cost changes in equation (12) from Proposition 1, and Figure 13b plots $\frac{\mathrm{d}c_i}{c_i}$ in which total foreign input share, s_{Fj}^{Total}, is substituted to direct foreign input share, s_{Fj}.

Figure 13: Histograms of firm-level cost changes (First-order approximation)



(a) Total (b) Direct

D.8 Decomposition of real wage change

Table 16 shows the numbers analogous to those of Table 2, but in first-order approximations. Under a first-order approximation (see equation (20)), we can also decompose the log-change in the real wage upon a 10% increase in the foreign price into two multiplicative terms: the log-change in the domestic nominal wage and the foreign price as well as the level of import content in domestic final demand, $\sum_j s_{jH} s_{Fj}^{Total}$. Table 17 shows that the import content in domestic final demand is about 2 percentage points higher in the two alternative models. In the data, firms with higher export intensities tend to have higher input shares on foreign inputs. Our baseline case is able to capture this heterogeneity, while the two alternative models cannot. This difference results in the alternative models predicting larger import content in domestic final demand.

Table 16: Log-changes in real wage upon 10% increase in foreign price, direct and indirect effects

	Total	Direct	Indirect
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P}$	-0.069	-0.038	-0.031

Notes: The number for *Direct* is computed by replacing s_{Fj}^{Total} in equation (20) with the direct foreign input share, s_{Fj}. The number for *Indirect* is computed by replacing s_{Fj}^{Total} in equation (20) with the indirect foreign input share, $s_{Fj}^{Total} - s_{Fj}$. The nominal wage change $\frac{\mathrm{d}w}{w}$ is the same across the three columns and computed according to Appendix A.8.

Table 17: Log-changes in real wage upon 10% increase in foreign price, decomposition using equation (20)

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}$	-0.118	-0.112	-0.130
$\sum_j s_{jH} s_{Fj}^{Total}$	0.584	0.609	0.607
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P} = \left(\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}}\right) \sum_j s_{jH} s_{Fj}^{Total}$	-0.069	-0.068	-0.079

D.9 Back-of-the-envelope calculations of the changes in real wage

In Table 18 we present the first-order approximated changes in real wage that are computed from the back-of-the-envelope calculations motivated by Hulten (1978). Notice that all calculations lead to noisy estimates compared to the baseline result in Table 17.

Table 18: Back-of-the-envelope calculations of the changes in real wage, upon 10% increase in foreign price

	No import content in exports	Same import content in exports and domestic final demand
(1)	$-\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} = -0.1$	$-\frac{\mathrm{d}p_{F\cdot}}{p_{F\cdot}} = -0.1$
(2)	$\frac{\text{Imports}}{\text{VA}} = 0.817$	$\frac{\text{Imports}}{\text{VA+Exports}} = 0.449$
$\frac{\mathrm{d}w}{w} - \frac{\mathrm{d}P}{P}$	-0.082	-0.045

Notes: The third row is computed as (1) × (2). The first column computes the real wage change under assumptions of perfect competition and no import content being in the production of exports. The second column computes the real wage change under assumptions of perfect competition and same import contents in both exports and domestic final demand. Both calculations in the two columns also assume no change in the nominal wage, $\frac{\mathrm{d}w}{w} = 0$.

D.10 Real wage changes across sectors

Here we investigate how final good prices (weighted by household expenditure on these goods) change across broad sectors in the economy. Table 19 reports the increase in the price index, $\frac{\mathrm{d}P}{P}$, upon a 10% increase in the foreign price. We take the additive decomposition coming from the first-order approximation of the price index changes in equation (20).

Table 19: Log-changes in price index across sectors

Sector	$\frac{\mathrm{d}P}{P}$
Agriculture and Mining	0.007
Manufacturing	0.014
Utility and Construction	0.005
Wholesale and Retail	0.022
Service	0.004
Total	0.051

D.11 Changes in real income

Analogous to Table 3, in Table 20 we report the change in real income across the three models both upon 10% increase in the foreign price and upon autarky. In addition, analogous to Table 17, in Table 21, we decompose the change in real income. Rearranging equation (32), the log-change in real income can be decomposed into four additive terms: (1) the term that arises from the change in nominal wage, (2) the term arising from the changes in firms' profits from their domestic sales, (3) the term arising from the changes in firms' profits from their exports, and (4) the term coming from the change in the price index.

$$\frac{\mathrm{d}E}{E} - \frac{\mathrm{d}P}{P} = \underbrace{\frac{wL}{E}}_{=0.28}\underbrace{\frac{\mathrm{d}w}{w}}_{(1)} + \underbrace{\sum_k \frac{\mu_k - 1}{\mu_k}\frac{x_{kD}}{E}\frac{\mathrm{d}x_{kD}}{x_{kD}}}_{(2)} + \underbrace{\sum_k \frac{\mu_k - 1}{\mu_k}\frac{x_{kF}}{E}\frac{\mathrm{d}x_{kF}}{x_{kF}}}_{(3)} - \underbrace{\frac{\mathrm{d}P}{P}}_{(4)}. \quad (33)$$

Table 20: Changes in real income

(a) Changes in real income upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}}\|^{\hat{p}_{F\cdot}=1.1}$	0.883	0.878	0.887

(b) Changes in real income upon autarky

	Baseline	Simple Roundabout	Sectoral Roundabout
$\frac{\hat{E}}{\hat{P}}\|^{\hat{p}_{F\cdot}\to\infty}$	0.374	0.390	0.291

Table 21: Log-changes in real income upon 10% increase in foreign price

	Baseline	Simple Roundabout	Sectoral Roundabout
(1)	-0.018	-0.012	-0.030
(2)	-0.040	-0.042	-0.036
(3)	-0.043	-0.044	-0.040
(4)	0.051	0.056	0.049
$\frac{\mathrm{d}E}{E} - \frac{\mathrm{d}P}{P}$	-0.139	-0.145	-0.133

Notes: The fourth row is computed as $0.28 \times (1) + (2) + (3) - (4)$ in equation (33), where 0.28 is the laborcost to income ratio in the data. Note that labor is the only primary input our model, and all other primary inputs such as capital enter this calculation as part of profits.

D.12 Sensitivity results under exogenous network

Table 22 reports the sensitivity results on $\hat{c}_i$ and $\frac{\hat{w}}{\hat{P}}$ under different values of σ and ρ. Throughout we consider a 10% increase in the price of foreign inputs.

Table 23 reports how the change in real wage is affected when one consideres an acyclic network structure. In the fourth and sixth columns, we make use of the acyclic network from the algorithm explained in Appendix E, for the weighted case where we minimize the values of violating transactions.

Table 22: Results on $\hat{c}_i$ and $\frac{\hat{w}}{\hat{P}}$ under different values of σ and ρ

(a) Median $\hat{c}_i$

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.035	1.038	1.030
1.5	4	1.017	1.025	1.008
1.5	6	1.009	1.019	0.998
2	2	1.046	1.047	1.041
2	4	1.027	1.032	1.018
2	6	1.017	1.024	1.006

(b) 90th percentile $\hat{c}_i$

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	1.069	1.058	1.059
1.5	4	1.061	1.047	1.045
1.5	6	1.057	1.042	1.038
2	2	1.075	1.064	1.065
2	4	1.065	1.052	1.051
2	6	1.060	1.046	1.043

(c) $\frac{\hat{w}}{\hat{P}}$

ρ	σ	Baseline	Simple Roundabout	Sectoral Roundabout
1.5	2	0.945	0.944	0.939
1.5	4	0.931	0.934	0.922
1.5	6	0.927	0.932	0.915
2	2	0.955	0.953	0.949
2	4	0.940	0.941	0.931
2	6	0.934	0.937	0.923

Table 23: Change in real wage $\frac{\hat{w}}{\hat{P}}$ under acyclic network

ρ	σ	10% increase in p_F.		Autarky	
		$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic	$\frac{\hat{w}}{\hat{P}}$, Baseline	$\frac{\hat{w}}{\hat{P}}$, acyclic
1.5	2	0.945	0.942	0.251	0.223
1.5	4	0.931	0.927	0.437	0.408
1.5	6	0.927	0.922	0.541	0.519
2	2	0.955	0.952	0.391	0.388
2	4	0.940	0.936	0.558	0.530
2	6	0.934	0.929	0.634	0.609

E Ordering algorithm

In this section we describe the implementation of the ordering algorithm to solve the feedback arc set problem. We begin by defining some terms and notation.

E.1 Terms and notation

- *graph / network*, $G = (V, E)$ - A collection of a set of edges E and set of vertices V. Edges describe the relationship between vertices. Two basic classifications of graphs are based on whether the edges are *directed* or *undirected* and whether they are *weighted* or *unweighted*

- $n = |V|$, $m = |E|$

- *cycle* - A path within a graph where a vertex is reachable from itself

- $d^+(u)$ - For a vertex $u \in V$ in a directed graph, number of outgoing edges

- $d^-(u)$ - For a vertex $u \in V$ in a directed graph, number of incoming edges

- $w^+(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of outgoing edges

- $w^-(u)$ - For a vertex $u \in V$ in a directed graph, cumulative sum of weights of incoming edges

- *sink* - A vertex $u \in V$ in a directed graph with $d^+(u) = 0$

- *source* - A vertex $u \in V$ in a directed graph with $d^-(u) = 0$

- *feedback arc set* - A set of edges from a directed cyclic graph that when removed make the graph acyclic

- $s = s_{left}s_{right}$ - Given 2 finite sequences s_{left} and s_{right} with the indicated notation we symbolize the *concatenation* operation. For example, if $s_{left} = (A, B, C)$ and $s_{right} = (X, Y, Z)$, then $s = s_{left}s_{right} = (A, B, C, X, Y, Z)$

- $\lfloor x \rfloor$ is the greatest integer less than or equal to x

E.2 Overview

The Belgian B2B data describes a weighted directed graph $G = (V, E)$. Vertices are firms and edges are sales between firms. The goal of the ordering algorithm is to order firms in a way such that a given firm only sells to firms further along in the ordering and only buys from firms that precede it. The condition desired by this ordering is known in graph theory as a *topological ordering* (Black, 1999). A topological ordering exists if and only if a graph is directed and acyclic. The B2B data is cyclic. For the *unweighted* case our motivation is to find a feedback arc set of minimal cardinality, that is, what is the minimum number of transactions that we need to drop (i.e., the "violators") from our network to satisfy our ordering condition? For the *weighted case*, we seek to find a feedback arc set such that the cumulative weight of the violating transactions is minimized. Finding a minimum feedback arc set is computationally difficult but approximation algorithms exist.

E.3 Unweighted case

The algorithm we use for the paper was first presented by Eades et al. (1993). This algorithm was chosen because it has a linear run time complexity, $O(m + n)$, and because of its relative implementation simplicity. The algorithm uses a greedy heuristic through which it builds the proposed ordering $s = s_{left}s_{right}$.[46] Vertices are initialized into several buckets: sinks, sources, and δ buckets, where for a vertex $u \in V$, $\delta(u) = d^{-}(u) - d^{+}(u)$.[47] At each iteration, the algorithm removes all sinks from the network and prepends them to a sequence s_{right}, removes all sources and appends them to a sequence s_{left}, and then removes the vertex with the lowest δ score (the most "source"-like vertex) and appends it to s_{left}.[48] Each removal requires updating the buckets to reflect the modified graph. The algorithm stops when the graph is empty. There will be $2n - 1$ buckets, which can be formalized as follows:[49]

[46] According to Black (2005), a greedy algorithm is "an algorithm that always takes the best immediate, or local, solution while finding an answer. Greedy algorithms find the overall, or globally, optimal solution for some optimization problems, but may find less-than-optimal solutions for some instances of other problems."

[47] We have flipped the sign here compared to Eades et al. (1993) to be consistent with the diagrams elsewhere in our paper.

[48] Eades et al. (1993) take the vertex with the maximum δ score.

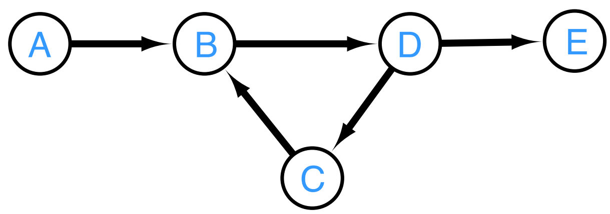
[49] Eades et al. (1993) assume that the graph G is simple (no bidirectional edges), and hence their original algorithm only requires $2n - 3$ buckets.

$$V_{-n+1} = V_{sources} = \{u \in V \mid d^-(u) = 0;\ d^+(u) > 0\}$$
$$V_{n-1} = V_{sinks} = \{u \in V \mid d^-(u) > 0;\ d^+(u) = 0\}$$
$$V_d = \{u \in V \mid d = \delta(u);\ d^-(u) > 0;\ d^+(u) > 0\}$$

The bucket V_{-n+1} contains all the vertices that are only the sources of edges. The bucket V_{n-1} contains all the vertices that are only the sinks of edges (in other words, vertices that are only receiving edges). Each V_d bucket contains vertices with d net incoming edges (conditional on these vertices having both outgoing and incoming edges).

E.4 Example execution on unweighted network

Consider the following network:



Let's trace the execution of the algorithm described by Eades et al.

E.4.1 Initialization

Buckets:

A			D	C	B			E
sources	-3	-2	-1	0	1	2	3	*sinks*

$Ordering: \ s = s_{left} = s_{right} = ()$

E.4.2 First iteration:

Remove sinks

Updated buckets:

A				C, D	B			
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = ()$, $s_{right} = (E)$, $s = s_{left}s_{right} = (E)$

Remove sources

Updated buckets:

				C, D, B				
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, E)$

Remove vertex with lowest delta score

Updated buckets:

B								D
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (E)$, $s = s_{left}s_{right} = (A, C, E)$

E.4.3 Second iteration

Remove sinks

Updated buckets:

B								
sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, D, E)$

Remove sources

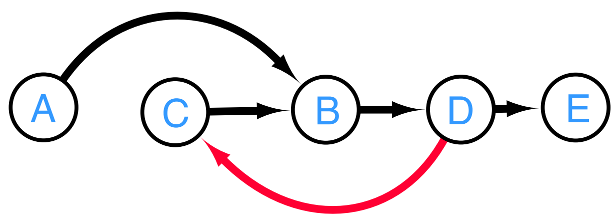
Updated buckets:

sources	-3	-2	-1	0	1	2	3	*sinks*

Updated ordering : $s_{left} = (A, C, B)$, $s_{right} = (D, E)$, $s = s_{left}s_{right} = (A, C, B, D, E)$

E.4.4 Final output

Ordering : $s = s_{left}s_{right} = (A, C, B, D, E)$, *Violator edge set*: $\{(D, C)\}$



E.5 Weighted case

Simpson et al. (2016) have proposed a modification to adapt the Eades et al. (1993) algorithm to solve the weighted problem. The required changes are:

1. In the initialization step, all edge weights need to be normalized to be between 0 and 1.

2. $\delta(u)$ is redefined as $\delta(u) = \lfloor w^{-}(u) - w^{+}(u) \rfloor$.

The key motivation behind these steps is to reformat the network so that the unweighted version of the algorithm could be used in an identical fashion as before, specifically without increasing the number of buckets. Without the floor in step 2, for any given network the number of buckets could become large.

F Algorithm to solve for the firm's sourcing strategy and export participation

A firm is solving the problem described in (25), where profits are defined in equation (24) and variable profits are defined in equation (14). For convenience, we re-state the problem of firm j here:

$$\max_{Z_j, I_{jF}} \pi_j(Z_j, I_{jF}) \quad \text{s.t.} \quad Z_j \subseteq \mathbf{Z}_j \ , \ I_{jF} \in \{0,1\},$$

where

$$\begin{aligned}
\pi_j(Z_j, I_{jF}) =& \frac{1}{\sigma}\beta_{jH}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\frac{E}{P^{1-\sigma}} \\
&+ I_{jF}\frac{1}{\sigma}\beta_{jF}^{\sigma-1}\mu^{1-\sigma}\phi_j^{\sigma-1}\Theta_j(Z_j)^{(\sigma-1)/(\rho-1)}\tau^{1-\sigma}\frac{E_F}{P_F^{1-\sigma}} \\
&- \sum_{k\in Z_j} f_{kj}w - I_{jF}f_{jF}w. \\
=& \pi_j^{var}(Z_j, I_{jF}) - \sum_{k\in Z_j} f_{kj}w - I_{jF}f_{jF}w
\end{aligned}$$

In words, the firm is choosing its sourcing strategy, Z_j, and export participation, I_{jF}. We solve the firm's problem of choosing its sourcing strategy separately for $I_{jF} = 0$ and $I_{jF} = 1$. We then calculate the profits for these two cases and determine the firm is an exporter if and only if the profits under exporting are higher than under non-exporting.

Below we describe how we solve for the firm's optimal sourcing strategy for a given export participation choice.

F.1 Lower and upper bounds for the optimal sourcing strategy

We determine the lower and upper bound for the firm's sourcing strategy following the procedure in Jia (2008) and Antras et al. (2017).

F.1.1 Lower bound

We start from a guess of no sourcing from any other domestic supplier and no importing, $S_l^{(0)}$. We then check supplier by supplier whether the benefit of adding a supplier (given the current guess of not purchasing from any supplier) is positive. At iteration t, starting from $S_l^{(t)}$, we calculate the marginal benefit of adding supplier $k \notin S_l^{(t)}$, $k \in \mathbf{Z}_j$:

$$\pi_j^{var}(S_l^{(t)} \cup k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) - f_{kj}w.$$

If the marginal benefit to include supplier k is positive, in the next iteration we include supplier k in the guess for the sourcing strategy of firm j. Note that given $\sigma > \rho$, one is the least likely to determine the benefit of a supplier is positive when the current guess is no supplier. Hence if it is possible to include a supplier in this iteration, in all the next iterations the marginal benefit from this supplier will be positive as well.

Starting from $S_l^{(t)}$, we consider firm-by-firm if trading with a firm not contained in $S_l^{(t)}$ brings positive marginal benefit (i.e., the additional variable profits under this sourcing strategy exceed the additional fixed cost) or not. Then, we add all those firms which bring positive benefit to form $S_l^{(t+1)}$.

The process ends when $S_l^{(t)} = S_l^{(t+1)}$ or all eligible suppliers are in $S_l^{(t)}$ already. When the process ends (i.e., $S_l^{(t)} = S_l^{(t+1)}$), we denote the lower bound of the sourcing strategy for firm j as $S_l^* = S_l^{(t)} = S_l^{(t+1)}$.

F.1.2 Upper bound

To determine the upper bound we start from a guess of purchasing from every supplier (incl. foreign), $S_u^{(0)}$. We then check supplier-by-supplier whether the marginal benefit from dropping the supplier is positive. At iteration t, starting from $S_u^{(t)}$, we calculate the marginal benefit of dropping supplier $k \in S_u^{(t)}$ as:

$$\pi_j^{var}(S_l^{(t)} \setminus k, I_{jF}) - \pi_j^{var}(S_l^{(t)}, I_{jF}) + f_{kj}w.$$

The remainder of the procedure is very similar to the iteration for the lower bound but starting from the opposite direction (i.e., we drop from the next iteration $S_u^{(t+1)}$ all those suppliers for which the marginal benefit of dropping is positive). The ending criteria is the same. We denote the upper bound for the sourcing strategy as S_u^*.

F.2 From lower and upper bounds to optimal sourcing strategy

Once we obtain S_u^* and S_l^*, we consider 3 alternative cases. Let $D = \{x \in S_u^* \mid x \notin S_l^*\}$ denote the set with the elements that are in the upper bound but not in the lower bound for the sourcing strategy.

F.2.1 $S_u^* = S_l^*$

If the upper and lower bounds for the sourcing strategy are the same, then we have obtained the optimal sourcing strategy for the firm (for a given exporting choice).

F.2.2 S_u^* is close to S_l^*

When the cardinality of set D is less than or equal to 15, we consider S_u^* to be close to S_l^*.

In that case we evaluate the profits at all possible combinations of sourcing strategies in between S_u^* and S_l^*, including S_u^* and S_l^* themselves. We choose the combination that yields the highest total profit as the optimal sourcing strategy for the firm.

F.2.3 S_u^* is far from S_l^*

When the cardinality of set D is larger than 15, then evaluating the profits at all combinations of feasible sourcing strategies in between the two bounds would be too computationally intensive. For that case, we have developed the following greedy algorithm to determine the firm's sourcing strategy:

Starting from S_l^*, we calculate the marginal benefit from adding separately each supplier in D to the sourcing strategy S_l^*. Note that by construction the marginal benefit from adding each of these suppliers individually to S_l^* is negative (otherwise the algorithm in Section F.1 would have already added those suppliers to the lower bound). We order the suppliers in D by their marginal benefit of being added to S_l^*. If the cardinality of D is K, we consider $K-1$ alternative sourcing strategies. We first add the top 2 suppliers in D (those with the highest marginal benefit of being added evaluated at S_l^*) to S_l^*, then add the top3 suppliers to S_l^*, and so forth. Hence, we calculate the profits for $K-1$ alternative sourcing strategies.

In addition, we also follow a similar process starting from S_u^* and calculate the marginal benefit from dropping separately each supplier in D from the sourcing strategy S_u^*. Again, by construction, the benefit from dropping each of the suppliers individually is negative. We order the suppliers in D by their marginal benefit of being dropped from S_u^*. We then consider $K-1$ alternative sourcing strategies, in which $2, 3, ..., K$ suppliers are removed from S_u^* (following the ranking of their marginal benefit of dropping individually at S_u^*).

Then, out of these $2K-2$ sourcing strategies we choose the one with the highest total profit for the firm.

Note that, using the approach in Section F.2.2, the number of sourcing strategies we would need to calculate profits for would be 2^K (growing exponentially in K). The greedy algorithm developed here, requires evaluations of alternative sourcing strategies that grow linearly in K, making it feasible even in the rare case that the difference between S_u^* and S_l^* is large.

We present statistics on the cardinality of the differences in the bounds in Table 24.

F.3 Statistics on the algorithm

Table 24: Cardinality of differences in the upper and lower bounds

Number of firm draws $\times$ parameter iterations	Percent of cases in which		
	Bounds are perfectly overlapping	Differences in bounds ≤ 15	Differences in bounds > 15
2,215,600,000	99.26	0.61	0.13

Notes: During the estimation we have to solve for each firm and parameter guess the firm's optimal sourcing strategy and exporting choice. This table presents aggregate statistics on the cardinality of the differences in the upper and lower bounds for the sourcing strategy summing over the outcomes for each firm, parameter guess, and exporting choice.

G Algorithm for network formation

Below we describe the algorithm to solve for the network formation in three contexts: In Section G.1, we describe the algorithm to solve for the network formation and equilibrium for a given set of parameters. In Section G.2, we describe the algorithm to estimate the parameters of the model. In Section G.3, we describe the algorithm to solve for network formation and equilibrium in a closed economy.

G.1 Network formation given parameters

Given a set of parameters, size of the labor force, price of foreign goods, and foreign demand, we follow the steps below to simulate the network formation.

1. Firms with productivities ϕ_i are randomly sorted, and indexed with $i = 1, 2, 3, \cdots$. A firm's index determines the firm's set of eligible suppliers, $\mathbf{Z}_i$. The set of eligible suppliers is such that all feasible networks will be acyclic. Firms' draws of firm-pair-specific fixed cost of sourcing, fixed cost of importing and exporting, export demand, and benefits of importing, and firm-pair-specific cost shifters are also known at this point.

2a. All firms make a common guess of the wage level w.

2b. All firms make a common guess of aggregate demand term: $A = EP^{\sigma-1}$.

3. We assume that firms decide on their sourcing strategies in sequence of i. Firm 1 decides its sourcing strategy and determines c_1, then firm 2 decides its sourcing strategy and determines c_2, and so on. When firms make their sourcing decisions, we assume that all firms are able to use labor and foreign inputs, but firm i is only able to choose its suppliers from its eligible supplier set $\mathbf{Z}_i$. We determine which suppliers among $\mathbf{Z}_i$ firm i sources from, using the algorithm described in Section F, and compute c_i. After the final firm $i = N$ decides its sourcing strategy, the whole vector $\boldsymbol{c}$ and the supplier sets of all firms are determined. At this point we have also solved for the firm's optimal export participation choice and export sales.

4. Given the network, the guesses for A and w, we are able to compute the equilibrium variables.

   (a) Sales to domestic final demand of firm i is computed as $X_{iH} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} A$ and to foreign final demand is computed as $X_{iF} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} c_i^{1-\sigma} \beta_{kF}^{\sigma-1} \frac{E_F}{P_F^{1-\sigma}}$.

   (b) The cost of inputs used for firm i's sales to domestic final demand is thus $C_{iH} = \frac{\sigma-1}{\sigma} X_{iH}$ and to foreign final demand is $C_{iF} = \frac{\sigma-1}{\sigma} X_{iF}$.

(c) The total input costs of firms, C_i are calculated by solving the system of linear equations below:

$$C_i = C_{iH} + C_{iF} + \sum_j s_{ij} C_j,$$
$$\rightarrow \mathbf{C} = (I - S)^{-1} (\mathbf{C}_H + \mathbf{C}_F)$$

where $\mathbf{C}$, $\mathbf{C}_H$, and $\mathbf{C}_F$ are vectors of C_i, C_{iH}, and C_{iF}, respectively, and the i, j element of matrix S is s_{ij}.

(d) The total sales of firm i is then $X_i = X_{iH} + X_{iF} + C_i - C_{iH}$.

(e) Firm profits and total expenditure on fixed costs.

5. We solve for equilibrium variables of A and w in the following way: In the outer loop, we solve for wages such that the labor market clearing condition (26) is solved. In the inner loop, we iterate over steps 2b-4, such that a fixed point for the market demand level, A, is found.

G.2 Parameter estimation and network formation

One possible approach to estimating the parameters of the model is to simulate the model for each parameter guess according to the algorithm outlined in Section G.1, calculate the objective function in equation (29), and vary the parameter guesses to maximize the objective function. However, this requires for each parameter guess finding a fixed point in both the market demand, A, and a wage level, w. Below, we describe a more computationally attractive algorithm to estimate the model.

Throughout the estimation, we set the domestic wage, $w = 1$, as well as the domestic market demand, $A = 1$. We ensure labor market clearing condition and the fixed point in market demand in the following way:

1. Of the 8 parameters to estimate, the mean foreign demand parameter is implicitly pinned down to take the value that satisfies the trade balance condition.

2. Instead of treating the size of the labor force as data (note that the units are arbitrary), we choose its level by setting: $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w}$.

Note that $A = w = 1$. Under this level of the size of the labor force, $L = \frac{AP^{1-\sigma} - \sum_i \pi_i}{w}$, the fixed point for the market demand, A is automatically satisfied. Also, since the trade balance holds, the domestic labor market clears as well.

Given a parameter guess for $\hat{\Phi}^{\alpha_{\text{dom}}}_{scale}$, $\hat{\Phi}^{\alpha_F}_{scale}$, $\hat{\Phi}^{\alpha,\beta}_{disp}$, $\hat{\Phi}^{f_{\text{dom}}}_{scale}$, $\hat{\Phi}^{f_{\text{imp}}}_{scale}$, $\hat{\Phi}^{f_{\text{exp}}}_{scale}$, and $\hat{\Phi}^{f}_{disp}$, we vary $\hat{\Phi}^{\beta_F}_{scale}$ and go through steps 3 and 4 in Section G.1 to calculate the aggregate trade balance. Given the other parameters, the level of $\hat{\Phi}_{scale}^{\beta_F}$ is pinned down implicitly such that aggregate trade balance is equal to zero. Hence, instead of search for a fixed point in both A and w, we hold those fixed throughout the estimation and only need to find one fixed point in $\hat{\Phi}_{scale}^{\beta_F}$, that satisfies trade balance, for each guess of the other seven parameters.

G.3 Network formation in the closed economy

In the closed economy, we can normalize the domestic wage $w = 1$. We therefore only have to follow steps 2b-4 in Section G.1 to pin down the level of domestic market demand, A, in the closed economy.

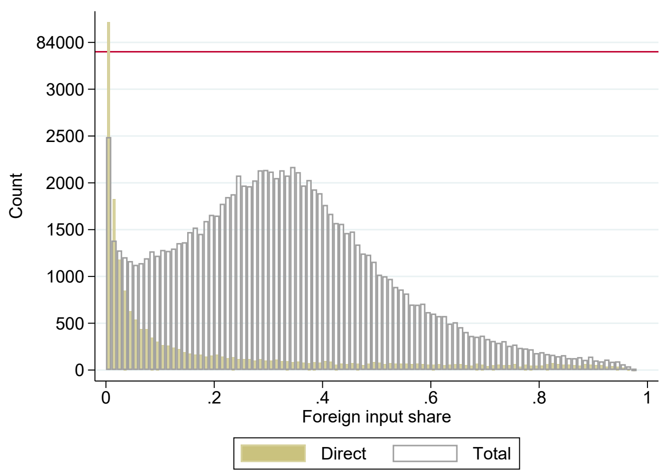
H Results from the estimated model

H.1 Direct and indirect linkages to foreign trade

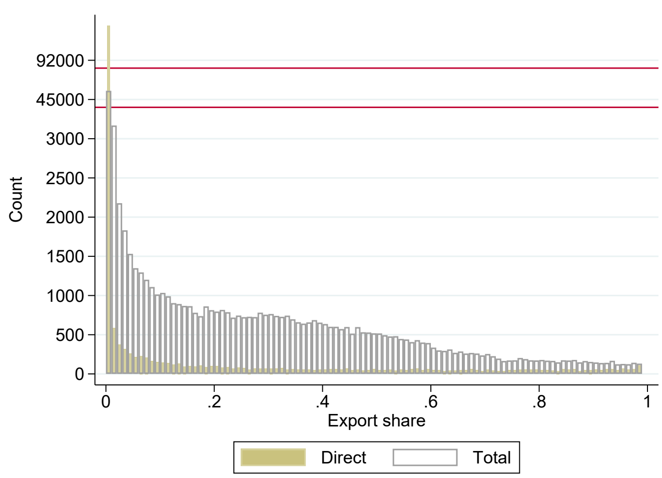
Figure 14 shows the histograms analogous to those in Figure 1, generated from the estimated model.

Figure 14: Histograms of direct and indirect linkages to foreign trade, from the estimated model

(a) Direct and total foreign input share



(b) Direct and total export share



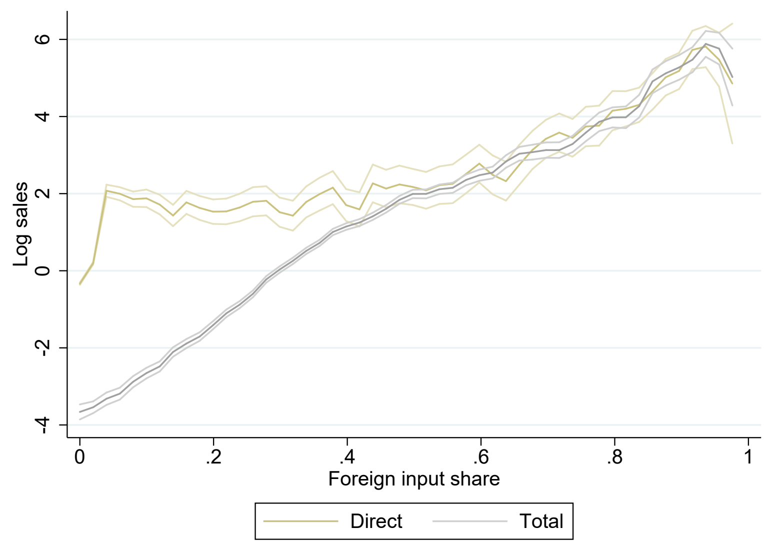
Notes: Total foreign input share of firm i, s_{Fi}^{Total} is calculated by solving $s_{Fi}^{Total} = s_{Fi} + \sum_{j \in Z_i} s_{ji} s_{Fj}^{Total}$ where s_{Fi} is i's direct foreign input share, and s_{ji} is j's share among i's inputs. Total export share firm i, r_{iF}^{Total} is calculated by solving $r_{iF}^{Total} = r_{iF} + \sum_{j \in W_i} r_{ij} r_{jF}^{Total}$ where r_{iF} is i's share of exports in its revenue, and r_{ij} is share of i's revenue that arises from sales to firm j. The figures are based on the estimated model. The horizontal lines represent scale breaks on the vertical axis.

H.2 Size premium of direct and indirect linkages to foreign trade

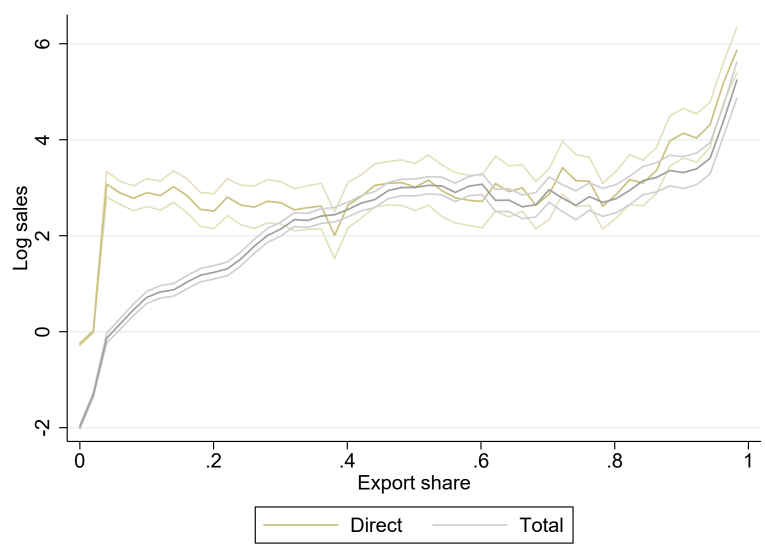
Figure 15 shows the plots analogous to those in Figure 2, generated from the estimated model.

Figure 15: Size premium of direct and indirect linkages to foreign trade, from the estimated model

(a) Foreign inputs



(b) Exports



Notes: The two figures display the smoothed values with 95% confidence intervals of kernel-weighted local polynomial regression estimates of the relationship between firms' sales and their levels of participation in foreign trade. We use the Epanechnikov kernel function with kernel bandwidth of 0.01, pilot bandwidth of 0.02, degree of polynomial smooth at 0, and smooth obtained at 50 points.

H.3 Statistics from the counterfactual exercise

Table 25 shows the results from linear and probit regressions where in its first four columns we regress the probability of having a lower unit cost upon a 10% increase in the foreign price, on firm-level variables. 21% of firms experienced a reduction in their costs. In its last four columns, we also show the results from regressing the probability of having a smaller number of domestic suppliers upon 10% increase in the foreign price, on firm-level variables. 4% of firms experienced a reduction in the number of domestic suppliers.

Table 25: Probabilities when cost increases by 10%

	Probability of having a cost reduction				Probability of having a reduction in the number of domestic suppliers			
	Linear		Probit		Linear		Probit	
	Est.	R^2	Est.	Pseudo R^2	Est.	R^2	Est.	Pseudo R^2
Import share	-0.40	0.02	-1.71	0.03	0.33	0.05	0.14	0.07
Total share of foreign inputs	-1.12	0.29	-1.32	0.40	0.28	0.08	0.22	0.20
Export share	-0.25	0.01	-0.40	0.01	0.45	0.09	0.16	0.11
Total share of exports	0.10	0.00	0.10	0.00	0.10	0.01	0.07	0.03
Labor share	0.86	0.31	0.63	0.29	-0.12	0.03	-0.23	0.12
Import status	-0.18	0.03	-0.24	0.04	0.07	0.02	0.05	0.05
Export status	-0.15	0.01	-0.19	0.01	0.16	0.06	0.08	0.11
Log import	-0.01	0.01	-0.01	0.03	0.02	0.05	0.02	0.08
Log export	-0.01	0.02	-0.01	0.04	0.03	0.09	0.03	0.10
Log sales to dom. fin. demand	-0.04	0.17	-0.04	0.18	0.01	0.04	0.01	0.12
Log total sales	-0.02	0.06	-0.02	0.05	0.01	0.02	0.01	0.07
Indegree	-0.01	0.11	-0.01	0.14	0.00	0.05	0.00	0.12

Notes: In the first four columns, the dependent variable equals to 1 if the firm's cost becomes lower after 10% increase in the foreign price, under endogenous networks. In the last four columns, the dependent variable equals to 1 if the number of domestic suppliers of the firm drops after 10% increase in the foreign price. All regressions are univariate. Estimates of Probit specification are scaled to match the interpretations of linear probability model.