RANDOMIZATION WITH ASYMMETRIC INFORMATION

Richard Arnott

Joseph E. Stiglitz

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ABSTRACT

It is by now well-known that, in the presence of moral hazard or adverse selection, randomization of insurance premia and benefits may be Pareto efficient. This paper: i) provides a typology of the various forms that randomization may take; ii) derives necessary and/or sufficient conditions for the desirability of these various forms of randomization; iii) obtains some simple characterization theorems of the efficient random policies; iv) gives some intuition behind the results; and v) considers why randomization appears to occur less often in practice than the theory suggests it should.

Richard Arnott
Department of Economics
Queens University
Kingston, Ontario
Canada K7L 3N6

Joseph E. Stiglitz
Department of Economics
Dickinson Hall
Princeton University
Princeton, NJ 08544

During the past fifteen years, an extensive literature has developed analyzing the structure of adverse selection and moral hazard (principal - agent) problems. Early on it was recognized that these problems may not have the usual convexity-concavity properties so familiar to economists; and only a few years later, it was realized that this in turn implies that, in a variety of circumstances, randomization is efficient.[1]

The aims of this paper are: (i) to provide a typology of the various forms that randomization may take; (ii) to provide necessary and/or suffi- cient conditions for the desirability[2] of these various forms of randomization in the context of the standard models of an insurance market with moral hazard or adverse selection; (iii) to provide some simple characterization theorems of the efficient random policies; (iv) to provide some intuition behind the results; and (v) to consider why randomization appears to occur less often in practice than the theory suggests it should.

Many of the results in the paper are new. Some (Propositions 2, 4, and 6), however, are not. In particular, several recent papers have investi- gated the desirability of randomization of insurance contracts in the presence of moral hazard (Holmstrom [1979], Gjesdal [1982], and Fellingham, Kwon, and Newman [1984]). All treat a continuum of possible outcomes, which tends to obscure intuition. We rederive their results for the two-outcome case, em- ploying derivations which highlight the intuition.

Section 1 treats randomization with moral hazard, and section 2 ran- domization with adverse selection. The concluding section contains a discus- sion of why randomization is not as common as theory suggests it should be, which casts doubt on the appropriateness of several of the standard assump- tions employed in the incentives and contract literatures.

Throughout the paper, we cast the problem in terms of a monopoly in-

surer who maximizes profits subject to providing its clients with an exoge-
nously-specified level of expected utility and subject also to the relevant
incentive-compatibility constraints. This allows us to characterize Pareto
efficient contracts. Throughout the paper, competitive equilibrium (when it
exists) corresponds to the solution of the monopoly problem with expected
utility at the competitive equilibrium level(s).[3] Thus, when we say that
randomization is _desirable_, we mean that randomization is Pareto efficient, is
profitable for a monopoly insurer, and is a characteristic of competitive
equilibrium.

1. Moral Hazard

Consider the simplest moral hazard setting. There are a large number
of identical individuals, each of whom realizes one of two possible outcomes
or events. Either an accident does not occur in which case the individual
receives w prior to insurance, or an accident[4] does occur in which case the
individual receives w-d, where d is the accident damage. The probability that
an accident occurs depends on the individual's accident-prevention effort, e;
i.e. p=p(e). It is assumed that $p'<0$ and $p''>0$, and that the individual al-
ways expends some effort in the monopoly solution. Also, individuals' acci-
dent probabilities are statistically independent.

Moral hazard arises because an insurer is unable to observe effort.
As a result, insurance contracts cannot be written contingent on effort. We
assume that the insurer can, however, observe its clients' total insurance
purchases.[5] In these circumstances, the non-random insurance contract will
specify a (net) insurance payout or benefit α payable to an individual if an
accident occurs and an insurance premium β payable by the individual if an
accident does not occur. Thus, consumption (y) is w-d+α when an accident does

occur, and $w-\beta$ when it does not.

The individual's expected utility is

$$EU = (1-p(e))U_0(w-\beta,e) + p(e)U_1(w-d+\alpha,e), \qquad (1)$$

where U_0 is the no-accident utility function and U_1 the accident utility function. It is assumed that

$$\frac{\partial U_j}{\partial y} > 0, \quad \frac{\partial^2 U_j}{\partial y^2} < 0, \quad \frac{\partial U_j}{\partial e} < 0 \quad \text{for } j = 0,1.$$

Note that this specification permits utility functions to be outcome-contingent; the accident may have non-pecuniary effects or influence tastes. We shall have occasion to particularize (1):

(i) separable utility function

$$EU = (1-p(e))u_0(w-\beta)+p(e)u_1(w-d+\alpha)-e \qquad (1')$$

with $u_j' > 0$ and $u_j'' < 0$ for j=0,1, and effort is measured in terms of the disutility is causes.

(ii) separable, event-independent utility function

$$EU = (1-p(e))u(w-\beta)+p(e)u(w-d+\alpha)-e \qquad (1'')$$

The individual chooses effort to maximize expected utility, taking the parameters of the insurance contract as given. This yields a function or correspondence relating effort to the parameters of the insurance contract, $e=e(\alpha,\beta)$, which, when substituted into the expected utility function, EU, yields expected utility as a function of α and β, $v(\alpha,\beta)$. The insurer's problem is to choose (α,β) to maximize profits subject to providing its clients with a given level of expected utility.[6]

The question to be addressed is: Under what circumstances does randomization of the parameters of the insurance contract result in a welfare improvement?

The basic intuition behind why randomization might be desirable is

this: Moral hazard causes a tradeoff between risk-bearing and incentives, since if full insurance were provided the insured would take little or no accident-prevention effort. As a result, the deterministic monopoly insurance contract typically entails the insured receiving less than full insurance and expending less than the first-best (i.e., with effort observable) level of effort. A compensated randomization of insurance may cause the insured to increase effort by enough that profits are increased The techniques of Rothschild-Stiglitz [1970] can be used to ascertain conditions under which a mean-preserving randomization of, say, the payout will increase effort at accident avoidance; randomization will do so provided only that the first-order condition for effort is convex in the payout. Randomization also has a direct, negative effect on risk-averse individuals. Thus, whether randomization is desirable depends on a careful balancing of the welfare gains from the mitigation of the moral hazard problem with the welfare losses from increased risk. While the latter depend on the degree of risk aversion (which depends on first and second derivatives of the utility function), the former depend on the degree of concavity of the first-order condition for effort (which depends on first, second, and third derivatives of the utility function). There appears to be no a priori reason why the effort effect should not outweigh the risk effect. This intuition turns out to be correct.[7]

1.1 The forms of randomization

We will consider two types of randomization. If the realization of the random policy occurs before the insured individual makes his effort decision, we shall say that the randomization is ex ante; if it occurs after his effort decision, ex post. In all cases, we assume that the individual knows fully the nature of the randomization being undertaken. With ex post randomization, the insurer quotes a set of random policies $\{\alpha_i, \beta_i, \mathbb{Q}_i\}$, which we

term the insurance contract, where Q_i ($\sum_i Q_i = 1$) is the probability that policy
i will be assigned to the individual <u>after</u> he has made his effort decision.
The individual makes his effort decision and only after this finds out which
of the policies the insurance company has assigned him. With <u>ex ante</u> randomiz-
ization, the insurer again quotes a set of random policies $\{\alpha_i, \beta_i, Q_i\}$ but now
Q_i is the probability that policy i will be assigned to the individual <u>before</u>
he makes his effort decision. The individual is randomly assigned a policy
and <u>then</u> makes his effort decision. To simplify the analysis, we shall con-
consider each form of randomization in isolation. Thus, in the subsection on
<u>ex post</u> randomization, <u>ex ante</u> randomization is excluded by assumption, and in
the section on <u>ex ante</u> randomization, <u>ex post</u> randomization is excluded by
assumption.

1.2 Ex post randomization

The insurer chooses that insurance contract which maximizes his ex-
pected profits, subject to providing its clients with a given level of expect-
ed utility and taking into account the dependence of effort on the terms of
the contract. We can formalize the problem as a two-stage (indirect control)
problem. In the first stage, the individual chooses effort, taking the para-
meters of the insurance contract as given, to maximize his expected utility:

$$EU \equiv \sum_i \nu(\alpha_i, \beta_i, e) Q_i \qquad (2)$$

where

$$\nu(\alpha_i, \beta_i, e) \equiv U_0(w - \beta_i, e)(1 - p(e)) + U_1(w - d + \alpha_i, e)p(e).$$

Thus,

$$\tilde{e}(\{\alpha_i, \beta_i, Q_i\}) = \underset{e}{\text{argmax }} EU. \qquad (3)$$

In the second stage, the insurer maximizes its expected profit, taking the
dependence of the accident probability on effort, and the dependence of effort

on the parameters of the insurance contracts, into account. It turns out to be convenient, in analyzing the second problem, to embed in it the maximization condition for the first stage; we thus write the insurer's problem as

$$\max_{\{\alpha_i,\beta_i,Q_i\}} \; (\Sigma\beta_i Q_i)(1-p(\tilde{e})) - (\Sigma\alpha_i Q_i)p(\tilde{e})$$

s.t.   i) $\sum_i \nu(\alpha_i,\beta_i,\tilde{e})Q_i \geq U$ (expected utility constraint)

   ii) $\sum_i Q_i = 1,$

   iii) $Q_i \geq 0 \;\; \forall i,$ and   (4)

   iv) $\tilde{e} = \underset{e}{\text{argmax}} \; EU$

Before proceeding to analyse (4), we state:

Lemma 1:   In the general concave programming problem

$$\max_{\{x_i,Q_i\}} \; \sum_i F(x_i)Q_i \quad \text{s.t.} \quad \sum_i g^j(x_i)Q_i \leq c^j, \; \sum_i Q_i = 1$$

$$Q_i \geq 0 \; \forall i, \; x_i \geq 0 \; \forall i,$$

where x_i is a vector of control variables, applied with probability Q_i, j indexes constraints, $F(\cdot)$ is a weakly concave function and $g^j(\cdot)$ are strictly convex constraint functions, randomization of x is undesirable.

Lemma 1 is well-known and implies in this context that randomization is undesirable when the insurer's profit-maximization problem is concave.

Since (4) is not in general a concave programming problem, it appear that randomization may in some circumstances be desirable. Characterization of the solution to (4) is difficult. Two results may, however, be obtained:

Proposition 1:   With ex post randomization, the optimum can be achieved with at most three insurance policies.

Proposition 2:   With separable utility functions, ex post randomization is never desirable.[8]

The first Proposition is established as follows: The first-order condition of the individual's effort choice problem satisfies

$$\sum_i \frac{\partial \nu(a_i, \beta_i, \tilde{e})}{\partial e} \, \mathbb{Q}_i = 0. \tag{5}$$

(Because expected utility is not necessarily a concave function of effort, (5) is only a necessary condition; constraint iv) is the necessary and sufficient condition). One may imagine that the problem has been completely solved, and that one is told what the profit-maximizing random policies are and what the individual's level of effort is, but <u>not</u> the probabilities with which the optimal random policies are assigned. There are then three linear equations -- constraints (i) and (ii) in (4), and (5) -- that can be solved for the optimal assignment probabilities. Thus, the assignment probabilities are determinate with at most three random policies.

Proposition 2 is proved by writing the insurer's maximization problem in such a way that Lemma 1 may be applied. Suppose that the level of effort chosen by individuals in the solution to the monopoly insurer's problem, e^*, has been determined. And let $\bar{u}_0$ denote the probability-weighted average of u_0 (recall (1')) across the random contracts, i.e. $\bar{u}_0 \equiv \sum_i u_0(w - \beta_i)\mathbb{Q}_i$, and define $\bar{u}_1$ accordingly. Then the analog to constraint i) of (4), written in standard $\leq$ form, is

$$-\bar{u}_0(1 - p(e^*)) - \bar{u}_1 p(e^*) + e^* \leq -\bar{U} \tag{6a}$$

and the analog to constraint iv) of (4) is

$$(\bar{u}_1 - \bar{u}_0)p'(e^*) - 1 = 0 \tag{6b}$$

(since with separable utility, expected utility is a concave function of effort).

Substitute (6b) into (6a) to obtain

$$-\bar{u}_0 - \frac{p(e^*)}{p'(e^*)} + e^* \leq -\bar{U} \tag{7a}$$

Since constraints (6a) and (7a) together are equivalent to constraints (6a) and (6b) together, the insurer's problem may be rewritten as

$$\max_{\{\alpha_i,\beta_i,Q_i\}} \; (\Sigma_i \beta_i Q_i)(1 - p(e^*)) - (\Sigma_i \alpha_i Q_i)p(e^*)$$

$$\text{s.t.} \quad \text{i)} \quad \text{(6a)} \qquad \text{iii)} \quad Q_i \geq 0 \; \forall i \tag{8}$$

$$\text{ii)} \quad \Sigma_i Q_i = 1 \qquad \text{iv)} \quad \text{(7a)}$$

Since this programming problem has a linear objective function and constraint functions that are strictly convex in $\{\alpha_i, \beta_i\}$, Lemma 1 applies, and so randomization is undesirable.

The "trick" in the proof was to rewrite constraints (6a) and (6b), where (6b) is not a convex constraint, as (6a) and (7a), both of which are convex constraints.

Proposition 2 and its proof are useful because they provide the basis of a general theorem characterizing sufficient conditions for randomization to be undesirable. If the utility function is not separable, the equation analogous to (7a) is

$$\varphi(y_0, y_1, e^*) \equiv -\bar{U}_0 + \frac{p(e^*)}{p'(e^*)} \left[\frac{\partial \bar{U}_0}{\partial e} (1 - p(e^*)) + \frac{\partial \bar{U}_1}{\partial e} p(e^*) \right] \leq -\bar{U}, \tag{7b}$$

where $\bar{U}_0 \equiv \Sigma_i U_0(w - \beta_i, e^*)Q_i$, $\dfrac{\partial \bar{U}_0}{\partial e} \equiv \Sigma_i \dfrac{\partial U_0(w - \beta_i, e^*)}{\partial e} Q_i$ etc. Since (7b) is not necessarily a convex constraint, it appears that randomization may be desirable. Since the analog to (6a) for non-separable utility is a convex constraint, if constraint (7b) is also convex, then by Lemma 1 randomization is undesirable, i.e.:

Proposition 3: A sufficient condition for ex post randomization to be undesirable is that φ be convex in y_0 and y_1 jointly.

Proposition 3 has an immediate corollary:

Corollary 1: If $\dfrac{\partial^3 U_0}{\partial e \partial y_0^2} \leq 0$ and $\dfrac{\partial^3 U_1}{\partial e \partial y_1^2} \leq 0$, then ex post randomization is un-

desirable.

Note that Proposition 2 is a special case of Corollary 1.[9]

Deriving necessary and sufficient conditions for randomization to be desirable is difficult. A large randomization may be desirable, even when a small one is not, and the desirability of large randomizations depends on global rather than local properties of the functions $U_0(\cdot)$, $U_1(\cdot)$, and $p(\cdot)$, as well as the parameters w and d.[10] Determining necessary and sufficient conditions for a local randomization to be desirable is presumably possible, but since efficient randomization may entail three random policies (recall Proposition 1), doing so would be algebraically very tedious. As a result, we shall adopt the more modest goal of deriving necessary and sufficient conditions for a local randomization of the payout, holding the premium fixed, to be desirable.[11]

A necessary condition for a local randomization of the payout to be desirable, is that a small, mean-preserving spread in α, holding β fixed, increase effort. If this condition is not satisfied, then the randomization not only lowers expected utility, by exposing the individual to more risk, but also decreases profits by increasing the probability of accident. From first principles, or from Rothschild and Stiglitz [1970], it is straightforward to show that this randomization stimulates effort if and only if

$$p' \, \frac{\partial^2 U_1}{\partial y_1^2} + p \, \frac{\partial^3 U_1}{\partial e \partial y_1^2} > 0.$$

A similar necessary condition is that a small, expected-utility-preserving spread in α, holding β fixed, increase effort. If this condition is not

satisfied, the randomization unambiguously decreases profits; not only does the probability of accident rise, but also compensating the individual for the risk he faces is costly. From first principles or from Diamond-Stiglitz [1974], one obtains that this randomization stimulates effort if and only if

$$p \, \frac{\partial^3 U_1}{\partial e \partial y_1^2} - p \, \frac{\partial^2 U_1}{\partial e \partial y_1} \left[\frac{\partial^2 U_1/\partial y_1^2}{\partial U_1/\partial y_1} \right] > 0.$$

The necessary and sufficient condition for a local randomization of the payout to be desirable is that a small, expected-utility-preserving spread in α, holding β fixed, increase effort by enough that profits are increased. To calculate this, we provide a payout of $\alpha+\Delta$ half the time and $\alpha-\Delta$ half the time, adjusting α so as to maintain expected utility constant, and ascertain whether doing so increases profits; that is, where $\hat{\Pi}(\alpha,\Delta;\bar{\beta}) \equiv$ $(1-p(e(\alpha,\Delta;\bar{\beta})))\bar{\beta}-p(e(\alpha,\Delta;\bar{\beta}))\alpha$, we calculate $\left.\frac{d\hat{\Pi}}{d\Delta}\right|_{\substack{\bar{U}\\\Delta=0}} = \frac{\partial\hat{\Pi}}{\partial\Delta} + \frac{\partial\hat{\Pi}}{\partial\alpha}\left[\left.\frac{d\alpha}{d\Delta}\right|_{\substack{\bar{U}\\\Delta=0}}\right]$. Since the randomization is small, $\left.\frac{d\hat{\Pi}}{d\Delta}\right|_{\substack{\bar{U}\\\Delta=0}} = \left.\frac{d\alpha}{d\Delta}\right|_{\substack{\bar{U}\\\Delta=0}} = 0$. Hence, it is necessary to examine second derivatives. Specifically, the local randomization is desirable if and only if $\left.\frac{d^2\hat{\Pi}}{d\Delta^2}\right|_{\substack{\bar{U}\\\Delta=0}} > 0$. Tedious manipulation gives

$$\left.\frac{d^2\hat{\Pi}}{d\Delta^2}\right|_{\substack{\bar{U}\\\Delta=0}} = \frac{p'(\alpha+\beta)}{EU_{ee}} \left[p \, \frac{\partial^3 U_1}{\partial e \partial y_1^2} - p \, \frac{\partial^2 U_1}{\partial e \partial y_1} \left[\frac{\partial^2 U_1/\partial y_1^2}{\partial U_1/\partial y_1} \right] \right] + p \, \frac{\partial^2 U_1/\partial y_1^2}{\partial U_1/\partial y_1} \ , \quad (9)$$

where

$$EU_{ee} = -p''(U_0-U_1) - 2p'\left[\frac{\partial U_0}{\partial e} - \frac{\partial U_1}{\partial e}\right] + \frac{\partial^2 U_0}{\partial e^2}(1-p) + \frac{\partial^2 U_1}{\partial e^2}\, p < 0$$

from the second-order conditions of the individual's effort choice problem.

Proposition 4:[12] A necessary and sufficient condition for a local randomization of the payout to be desirable, with the premium held fixed, is that

$\dfrac{d^2\hat{\Pi}}{d\Delta^2}\bigg|_{\bar{U} \atop \Delta=0} > 0$, the expression for $\dfrac{d^2\hat{\Pi}}{d\Delta^2}\bigg|_{\bar{U} \atop \Delta=0}$ being given in (9).

With separable utility, this condition cannot be satisfied, but theory imposes no natural restrictions on the values of $\dfrac{\partial^2 U}{\partial e \partial y}$ or $\dfrac{\partial^3 U}{\partial e \partial y^2}$; hence, it appears that randomization can be desirable.[13]

The result is depicted in Figure 1. The deterministic optimum is characterized by the contract (α^*, β^*), utility U^*, and profit Π^*. In the neighborhood of the deterministic optimum, $\dfrac{\partial \hat{\Pi}}{\partial \alpha} < 0$ (otherwise, both profits and expected utility could be increased with a rise in α). Since also both the indifference curve and iso-profit curve through the deterministic optimum have zero slope in Δ-α space at the deterministic optimum, randomization is desirable if the iso-profit curve has greater positive curvature than the indifference curve.

1.3 Ex ante randomization

The desirability of ex ante randomization depends on quite a different set of considerations.

Define EU(Π) to be the maximum expected utility as a function of the insurer's (expected) profit per client, Π, when only deterministic contracts are admitted. The desirability of ex ante randomization depends on the curvature properties of EU(Π). This is shown in Figure 2. Suppose, for the sake of argument, that the solution to the monopoly insurer's problem with deterministic contracts occurs at C, with expected profit per contract of Π_C. Now allow ex ante randomization. It is immediate that no more than one policy, with a given level of expected profits, need be offered in the profit-maximizing insurance contract. This implies that if the profit-maximizing contract contains more than one policy, each must have a different level of

profits associated with it. Suppose the insurer offers two <u>ex ante</u> random policies in his contract, policy A with profits $\Pi_A < \Pi_C$ and policy B with profits $\Pi_B > \Pi_C$, with the probability weights chosen so that the expected profit from the pair of contracts is Π_C. It is evident from the diagram that, as drawn, this <u>ex ante</u> randomization increases expected utility and is therefore desirable. It is also evident that: i) the desirability of <u>ex ante</u> randomization stems from the convexity of $EU(\Pi)$ near C; and ii) that no more than two random policies are needed in a profit-maximizing contract with <u>ex ante</u> randomization.

We formalize this intuition in a series of Propositions.

<u>Proposition 5:</u> If $EU(\Pi)$ is concave, <u>ex ante</u> randomization is never desirable.

<u>Proposition 6:</u> If $EU(\Pi)$ is convex at the level of profits at the deterministic monopoly insurer profit maximum, then <u>ex ante</u> randomization is desirable.[14]

<u>Proposition 7:</u> With <u>ex ante</u> randomization, no more than two random policies are required in the profit-maximizing insurance contract.

In the absence of any moral hazard problem, expected utility is a concave function of Π because of diminishing marginal utility. But with moral hazard, as Π changes, effort changes. In the normal case, as profit increases (decreases), the individual will expend more (less) effort at accident avoidance. If the absolute value of the individual's effort response is greater for increases in profits than to decreases (in a sense to be made more precise below), expected utility may be a convex function of Π and <u>ex ante</u> randomization may be desirable.

It should be clear from Figure 2 that convexity of $EU(\Pi)$ at C is not a necessary condition for <u>ex ante</u> randomization to be desirable; in particular, non-local randomization may be desirable even when $EU(\Pi)$ is concave at C.

The desirability of non-local randomization is difficult to characterize because it depends on global rather than local properties of the event-contingent utility functions and the probability of accident function. In what follows, we shall therefore consider only local randomizations.

To simplify, we hold β fixed at its level in the absence of randomization (since if expected utility can be increased via ex ante randomization with β fixed, it can a fortiori be increased with β variable) and treat the special case of a separable, event-independent utility function (1''). Recall that from the individual's maximization problem without randomization, we can express effort as a function of the contract parameters; i.e. $e=e(\alpha,\beta)$. Now $\Pi=\beta(1-p)-\alpha p$. Thus, we may write

$$e(\alpha,\beta) = e\left[\frac{\beta(1-p)-\Pi}{p} , \beta\right] = \hat{e}(\Pi,\beta). \qquad (10)$$

Substitute this into the expression for expected utility,

$$V(\Pi,\beta,\hat{e}(\Pi,\beta)) \equiv V(\Pi,\beta) \equiv (1-p(\hat{e}))u(w-\beta)+p(\hat{e})u\left[w-d + \frac{\beta(1-p(\hat{e}))-\Pi}{p(\hat{e})}\right] - \hat{e}. \quad (11)$$

Ex ante randomization is desirable if $\left.\frac{\partial^2 V}{\partial \Pi^2}\right|_{\substack{\beta \\ \Pi_C}} > 0$. The following expression for $\left.\frac{\partial^2 V}{\partial \Pi^2}\right|_{\substack{\beta \\ \Pi_C}}$ is derived in Appendix 1:

$$\left.\frac{\partial^2 V}{\partial \Pi^2}\right|_{\substack{\beta \\ \Pi_C}} = \left[\frac{1}{Z_e^3}\left\{Z_e(V_{\Pi\Pi}Z_e^2-2V_{e\Pi}Z_eZ_\Pi+V_{ee}Z_\Pi^2)-V_e(Z_{\Pi\Pi}Z_e^2-2Z_{e\Pi}Z_eZ_\Pi+Z_{ee}Z_\Pi^2)\right\}\right]_{\Pi=\Pi_C} , \quad (12)$$

where subscripts denote partial derivatives and

$$Z(\Pi,\beta,\hat{e}(\Pi,\beta)) \equiv -p'(\hat{e})\left[u(w-\beta)-u\left[w-d + \frac{\beta(1-p(\hat{e}))-\Pi)}{p(\hat{e})}\right]\right] - 1.$$

We have thus established

Proposition 8: A sufficient condition for ex ante randomization to be desir-

able is that the expression on the RHS of (12) be positive.

It is argued in Appendix 1 that $\frac{\partial^2 V}{\partial \Pi^2}$ can be positive if p′′′ is sufficiently

negative.[15] The reason for the complexity of the result is that what is rele-

vant is the extent to which successive increases in Π lead to increases in

effort (and how these increases in effort lead to increases in the likelihood

of a loss). Since the level of effort depends on first derivatives (say of

p(e)), changes in levels depend on second derivatives, and differences in

changes in levels (which are critical for ascertaining the convexity or con-

cavity of profit as a function of Π) depend on third derivatives.

To simplify, we have considered ex post and ex ante randomization in

isolation. It should be evident that there are circumstances in which it is

desirable to employ both forms of randomization simultaneously.

2. Adverse Selection

It seems plausible that randomization can be desirable when adverse

selection is present, by altering the position of the self-selection locus.

To investigate this, we treat the case where there are two groups in the popu-

lation, one high-risk (H) and one low-risk (L).

Suppose that, in the absence of randomization, the monopoly insurer

provides group H with contract $(\hat{\alpha}^H, \hat{\beta}^H)$ and group L contract $(\hat{\alpha}^L, \hat{\beta}^L)$, as de-

picted in Figure 3. Group H utility is $\hat{U}^H$, and group L utility $\hat{U}^L$. The basic

adverse selection mechanism should be familiar. If the insurer were able to

distinguish between individuals in the two groups, the contracts would be

$(\hat{\alpha}^H, \hat{\beta}^H)$ and $(\tilde{\alpha}^L, \tilde{\beta}^L)$, providing the insurer with maximum profits consistent

with the exogenous utility levels. But with adverse selection, the insurer is
unable to distinguish between high- and low-risk individuals. If it were to
offer $(\hat{\alpha}^H, \hat{\beta}^H)$ and $(\tilde{\alpha}^L, \tilde{\beta}^L)$, both high- and low-risk individuals would choose
$(\tilde{\alpha}^L, \tilde{\beta}^L)$. If the proportion of the population in the high-risk group is above
a certain critical level, which we assume is the case, the insurer maximizes
its profits by offering the pair of contracts $(\hat{\alpha}^H, \hat{\beta}^H)$ and $(\hat{\alpha}^L, \hat{\beta}^L)$, where
$(\hat{\alpha}^L, \hat{\beta}^L)$ is the most profitable contract that can be offered to the low-risk
group consistent with their obtaining $\hat{U}_L$ and with the high-risk group prefer-
ring to "self-select", i.e. to choose $(\hat{\alpha}^H, \hat{\beta}^H)$, which is more expensive but
provides more insurance, rather than the one designed for low-risk individ-
uals. $(\hat{\alpha}^L, \hat{\beta}^L)$ lies "just below" the intersection point of $\hat{U}^H$ and
$\beta^L(1-p^L) - \alpha^L p^L = \hat{\Pi}^L$. $\hat{U}^H$ is called the self-selection locus. Throughout this
section, we assume that profit-maximization entails the insurer offering a
different contract to each group.

We now consider the use of randomization as a self-selection device.
Again, randomization can take on two forms. (Both are depicted in Table 1).
In the first, the insurer announces two contracts, denoted by A and B. If the
individual chooses contract A, he will be assigned policy (α_i^A, β_i^A) with proba-
bility Q_i^A; similarly if he chooses contract B, he will be assigned (α_i^B, β_i^B)
with probability Q_i^B. The individual reveals his type by his choice of con-
tract. We say that offering such a pair of contracts entails ex post random-
ization, since the uncertainty is resolved after the individual chooses be-
tween the two contracts, i.e. after he has revealed his type. Traditional
adverse selection models can be viewed as treating the case where A and B both
consist of a single policy; we ask here, when is this desirable?

The second form that randomization may take is also depicted in Table
1. The insurer gives the individual a lottery of policy pairs,

$L_i = [(\alpha_i^A, \beta_i^A), (\alpha_i^B, \beta_i^B), Q_i], \sum_{i=1}^{I} Q_i = 1$. The firm then randomly assigns him a

policy pair, e.g. $L_{i'}$ with probability $Q_{i'}$. Having been assigned a policy

pair, say $L_{i'}$, the individual can then choose between $A_{i'} = (\alpha_{i'}^A, \beta_{i'}^A)$ and

$B_{i'} = (\alpha_{i'}^B, \beta_{i'}^B)$. The individual reveals his type through his choice of $A_{i'}$ or

$B_{i'}$. We say that such a contract entails _ex ante_ randomization, since the

uncertainty is resolved before the individual makes his choice, and therefore

before he reveals his type. In the standard adverse selection model, the in-

dividual is offered a single pair of policies, rather than a lottery of pairs

of policies.

We shall employ the following additional notation: n^k, the popula-

tion of group k, k=H,L; p^k, the accident probability of group k; u_0^k, the

utility function for group k in the event of no accident; and u_1^k, the corre-

sponding utility function when an accident occurs. Superscript k(=H,L) on a

contract parameter indicates that the corresponding contract is _designed_ for

group k.

We first prove:

Proposition 9: The insurer can always do at least as well with _ex post_ ran-

    domization, as with _ex ante_ randomization.

Proof: Suppose the exogenous utility levels are $\bar{U}^H$ and $\bar{U}^L$. Then the profit-

    maximizing _ex ante_ random contract solves

$$\max_{\{Q_i, (\alpha_i^H, \beta_i^H), (\alpha_i^L, \beta_i^L)\}} n^H((1-p^H)\sum_i Q_i \beta_i^H - p^H \sum_i Q_i \alpha_i^H) + n^L((1-p^L)\sum_i Q_i \beta_i^L - p^L \sum_i Q_i \alpha_i^L)$$

s.t. i) $(1-p^H)\sum_i u_0^H(w-\beta_i^H)Q_i + p^H \sum_i u_1^H(w-d+\alpha_i^H)Q_i \geq \bar{U}^H$ \quad (expected \hfill (13)
\quad utility
\quad constraints)

   ii) $(1-p^L)\sum_i u_0^L(w-\beta_i^L)Q_i + p^L \sum_i u_1^L(w-d+\alpha_i^L)Q_i \geq \bar{U}^L$

  iii) $(1-p^H)(u_0^H(w-\beta_i^H) - u_0^H(w-\beta_i^L)) + p^H(u_1^H(w-d+\alpha_i^H) - u_1^H(w-d+\alpha_i^L)) \geq 0$ \quad $\forall i$
\quad (self-selection constraints)

iv) $\sum_{i} Q_i = 1$, $Q_i \geq 0$ $\forall i$.

Consider the profit maximization problem identical to (13) in all respects except that the set of constraints iii) is replaced by

$$(1-p^H)\sum_{i}(u_0^H(w-\beta_i^H)-u_0^H(w-\beta_i^L))Q_i+p^H\sum_{i}(u_1^H(w-d+\alpha_i^H)-u_1^H(w-d+\alpha_i^L))Q_i \geq 0.$$

We term this the modified maximization problem. Since this constraint is weaker than the set of constraints iii) of (13), profits are at least as high for the modified maximization problem as for (13). But the modified maximization problem is that of a monopoly insurer who undertakes _ex post_ randomization. ∎

We have established that _ex ante_ randomization is unnecessary by showing that _ex post_ randomization is always at least as profitable. If there are restrictions on _ex post_ randomization, _ex ante_ randomization may still be desirable. However, we shall not attempt to determine conditions under which _ex ante_ randomization is, in fact, desirable.

Proposition 9 is useful because it enables us to focus on _ex post_ randomization. _Ex post_ randomization of the high-risk group's policy is unambiguously harmful since it reduces the profitability of the high-risk group's contract, while leaving the low-risk group's contract unchanged. _Ex post_ randomization of the low-risk group's policy, meanwhile, has two possibly offsetting effects. On one hand, the low-risk individuals must be compensated for the increased risk they then face. On the other hand, such randomization may weaken the self-selection constraint. If high-risk individuals are very risk-averse, they will shy away from the randomized policy, allowing the low-risk individuals to obtain more insurance on average than they otherwise would. The Propositions that follow capture this intuition.

We start by considering the case where high- and low-risk individuals

have the same tastes (event-contingent utility functions). Let b be the certainty-equivalent insurance premium corresponding to the <u>ex post</u> random contract $\{\alpha_i, \beta_i, \mathbb{Q}_i\}$, defined implicitly by

$$u_0(w-b) \equiv \sum_i u_0(w-\beta_i)\mathbb{Q}_i; \tag{14a}$$

and a be the certainty-equivalent insurance payout, defined implicitly by

$$u_1(w-d+a) \equiv \sum_i u_1(w-d+\alpha_i)\mathbb{Q}_i. \tag{14b}$$

The expected utility for an individual in group k is, using (14),

$$\begin{aligned} EU^k &= (1-p^k)(\sum_i u_0(w-\beta_i)\mathbb{Q}_i) + p^k(\sum_i u_1(w-d+\alpha_i)\mathbb{Q}_i) \\ &= (1-p^k)u_0(w-b) + p^k u_1(w-d+a); \end{aligned} \tag{15}$$

thus, expected utility can be expressed in terms of the certainty-equivalent premium and payout corresponding to any <u>ex post</u> random contract. The self-selection constraint, meanwhile, can be written as

$$\begin{aligned} (1-p^H) \ &(u_0(w-b^H) - u_0(w-b^L)) \\ &+ p^H(u_1(w-d+a^H) - u_1(w-d+a^L)) \geq 0, \end{aligned} \tag{16}$$

where (a^k, b^k) is the certainty-equivalent of the random contract $\{\alpha_i^k, \beta_i^k, \mathbb{Q}_i^k\}$ designed for group k. Next, define $\bar{\beta}^k \equiv \sum_i \mathbb{Q}_i^k \beta_i^k$ to be the average premium in the random policy designed for group k, and $\bar{a}^k \equiv \sum_i \mathbb{Q}_i^k \alpha_i^k$ to be the corresponding average payout. Risk aversion implies that $a^k \leq \bar{a}^k$ and $b^k \geq \bar{\beta}^k$, with at least one of the inequalities being strict if the corresponding contract is randomized.

Recall that we have assumed that profit-maximization entails different contracts for the high- and low-risk groups. Thus, where $\hat{U}^k$ is the exogenous level of expected utility for group k, the group k expected utility constraint, using (15), may be written as

$$(1-p^k)u_0(w-b^k) + p^k u_1(w-d+a^k) \geq \hat{U}^k, \quad k=H,L. \tag{17}$$

The insurer's profit-maximization problem, with <u>ex post</u> randomization, may be
written as

$$\begin{array}{c} \max \\ \{a^k, \bar{a}^k, b^k, \bar{\beta}^k\} \\ k=H,L \end{array} \qquad \begin{array}{c} \Sigma \\ k=H,L \\ s.t. \end{array} \; n^k((1-p^k)\bar{\beta}^k - p^k\bar{a}^k) \qquad\qquad (18)$$

   i)   (16) (the self-selection constraint)

   ii)   (17) (the expected utility constraints)

   iii) $a^k \leq \bar{a}^k,\; b^k \geq \bar{\beta}^k$

From this formulation, it is evident that the profit-maximizing contract pair
has the properties $\bar{a}^k = a^k$ and $\bar{\beta}^k = b^k$ for k=H,L, and therefore that <u>ex post</u>

randomization is undesirable.

    This result is stated in[16]

<u>Proposition 10:</u> With adverse selection and two groups who differ in risk but

      not in tastes, <u>ex post</u> randomization is never desirable.

    The intuition underly...g this result can be seen from Figure 4. Sup-
pose that in the absence of randomization, the profit-maximizing deterministic
contracts are $(\hat{a}^H, \hat{\beta}^H)$ and $(\hat{a}^L, \hat{\beta}^L)$, corresponding to which are profit levels,
$\hat{\Pi}^H$ and $\hat{\Pi}^L$. Because the contracts are deterministic, the premiums and payouts
equal their corresponding certainty-equivalents, which we denote by $(\hat{a}^H, \hat{b}^H)$
and $(\hat{a}^L, \hat{b}^L)$, respectively. Since expected utility can be expressed in terms
of the certainty-equivalent premium and payout, we may portray the determinis-
tic profit-maximizing pair of contracts in a-b space: $(\hat{a}^H, \hat{b}^H)$ lies at the
point of tangency of $\hat{U}^H$ and the profit line $b^H(1-p^H) - a^H p^H = \hat{\Pi}^H$, while
$(\hat{a}^L, \hat{b}^L)$ lies at the point of intersection of $\hat{U}^H$ and $\hat{U}^L$, E_0 in the Figure.

    Now consider randomizing the contract for the low-risk group, hold-
ing $\hat{U}^H$ and $\hat{U}^L$ fixed. Since the randomization does not alter the position of
the self-selection constraint in a-b space, the low-risk group's certainty-

equivalent policy remains $(\hat{a}^L, \hat{b}^L)$. Since, with randomization, $\bar{a}^L \geq a^L$ and $\bar{\beta}^L \leq b^L$, with at least one of the inequalities holding strictly, then $\bar{\beta}^L(1-p^L) - \bar{a}^L p^L < b^L(1-p^L) - a^L p^L = \hat{\Pi}^L$. Thus, the randomization reduces profits on the low-risk group's policy, while leaving profits on the high-risk group's policy unaffected.

We now examine the situation where individuals differ in terms of both risk class and tastes for risk. We derive a condition under which a small amount of randomization is desirable.

We randomize the contract for low-risk individuals in such a way that:[17] i) there are two policies, both with probability .5; ii) low-risk individuals' utility is the same with either policy and equals their certainty utility; and iii) the profit level on the low-risk contract is maintained. Randomization will then be desirable if (and only if) it lowers the utility of the high-risk individuals when they hold the low-risk contract, since if this happens, the self-selection constraint is relaxed, and the low-risk contract can then be adjusted to increase its profitability. To simplify the algebra, it is assumed that both groups' utility functions are event-independent.

The randomization satisfies the above conditions:

$$(1-p^L)u^L(w-\beta_1^L) + p^L u^L(w-d+\alpha_1^L) = \bar{U}^L \tag{19a}$$

$$(1-p^L)u^L(w-\beta_2^L) + p^L u^L(w-d+\alpha_2^L) = \bar{U}^L \tag{19b}$$

$$(1-p^L)(\beta_1^L+\beta_2^L) - p^L(\alpha_1^L+\alpha_2^L) = 2\hat{\Pi}^L \tag{20}$$

The expected utility of a high-risk individual when he purchases a low-risk contract is

$$EU^H = \tfrac{1}{2}\{(1-p^H)(u^H(w-\beta_1^L)+u^H(w-\beta_2^L)) + p^H(u^H(w-d+\alpha_1^L)+u^H(w-d+\alpha_2^L))\}. \tag{21}$$

It is shown in Appendix 2 that the first-order effect of an infinitesmal randomization is zero; i.e. $\dfrac{dEU^H}{d\alpha_1^L}\bigg|_{\alpha_1^L=\alpha_2^L} = 0$. To ascertain the desirability of a

small randomization, we investigate the sign of $\dfrac{d^2 EU^H}{d(a_1^L)^2}\bigg|_{a_1^L = a_2^L}$. Define

$$\theta = \left[\frac{p^L}{1-p^L}\right] \frac{(u_1^L)'}{(u_0^L)'} \ , \qquad \begin{array}{l}\text{the slope of the low-risk}\\ \text{indifference curve with}\\ \text{the contract } (a_1^L, \beta_1^L)\end{array} \qquad (22a)$$

$$r^k = \frac{(u_0^k)'}{(u_1^k)'} \ , \qquad \begin{array}{l}\text{the ratio of the marginal utility}\\ \text{of consumption in the no-accident}\\ \text{to accident events for group k with}\\ \text{the contract } (a_1^L, \beta_1^L)\end{array} \qquad (22b)$$

$$A_j^k \equiv -\frac{(u_j^k)''}{(u_j^k)'} \ , \qquad \begin{array}{l}\text{the (local) coefficient of absolute}\\ \text{risk aversion for group k and event}\\ \text{j with the contract } (a_1^L, \beta_1^L)\end{array} \qquad (22c)$$

$$\eta \equiv \frac{(1-p^H)p^L}{(1-p^L)p^H} \qquad (22d)$$

Note that: i) $\eta < 1$; ii) with risk aversion and incomplete insurance, $r^k < 1$ for k=H,L; and iii) the greater the degree of risk aversion, the smaller is r^k (holding (a_1^L, β_1^L) constant). Then it can be shown (the procedure of the derivation is explained in Appendix 2) that for $A_j^k \gg 0$ (finitely greater than zero) for j=0,1; k=H,L:

$$\text{sgn}\left[\frac{d^2 EU^H}{d(a_1^L)^2}\right] = \text{sgn}\left[-A_0^H(\eta r^H \theta(1-r^L)) - A_1^H r^L(1-r^L)\right.$$

$$\left. + A_0^L r^L \theta(1-\eta r^H) + A_1^L r^L(1-\eta r^H)\right]. \qquad (23)$$

Let $\underline{A}^k$ denote the minimum of the coefficient of absolute risk aversion for $u^k(y)$ with $y \in [w-d, w]$, and $\bar{A}^k$ the corresponding maximum, k=H,L. Propositions 11 and 12 then follow from the derivation of (23).

Proposition 11: For any $(w, d, u^L(\cdot), p^H, p^L)$ such that $d \gg 0$, $p^H \gg p^L$, and $\bar{A}^L$ finite, there is a finite number $\hat{A}^H$ such that with $\underline{A}^H > \hat{A}^H$, ex post ran-

domization is desirable.

The intuition for this result is straightforward. When a high-risk individual is infinitely risk-averse, he will care only about the worst possible outcome. Since the randomization worsens the worst possible outcome when the low-risk contract is bought, it reduces the utility of the high-risk individual when he purchases the low-risk contract, which relaxes the self-selection constraint.

<u>Proposition 12:</u> For any $(w, d, u^H(\cdot), p^H, p^L)$ such that $d \gg 0$, and $p^H \gg p^L$, there is a finite number $\hat{\lambda}^L$ such that with $\bar{\lambda}^L < \hat{\lambda}^L$, <u>ex post</u> randomization is desirable.

Consider the extreme case where $\hat{\lambda}^L$ is close to zero. Then the randomization undertaken is almost a randomization along the low-risk budget line. Because the high-risk group is risk-averse, its expected utility increases less from the increase in insurance in one policy than its expected utility decreases from the almost same-sized decrease in insurance in the other policy. Hence, the randomization loosens the self-selection constraint.

Note that none of the above results depend on constancy of absolute risk aversion and that finite randomizations may be Pareto-improving even when infinitesmal randomizations are not.

We have examined the circumstances under which randomization is desirable with <u>either</u> moral hazard <u>or</u> adverse selection, and explained the results. Future research should extend these results to the general situation where both moral hazard and adverse selection are present.

3. <u>Discussion</u>

In the previous two sections we employed standard assumptions to derive necessary and/or sufficient conditions for the randomization of insurance

contracts to be desirable. We investigated the desirability of randomization by analyzing whether randomization could increase the profits of a monopoly insurer faced with expected utility constraints. It bears repeating that if each insurer can observe his clients' total purchases of insurance, competitive equilibrium[18] coincides with the solution to the monopoly insurer's problem when the expected utility levels are those that would obtain in competitive equilibrium. Hence, all the results of the paper apply to competitive equilibrium.

In the absence of relevant empirical work, it is not possible to say how stringent the conditions we derived for randomization to be desirable in fact are. However, none of our arguments implies that randomization is 'normally' undesirable. Thus, it is remarkable that randomization of insurance contracts is not in fact observed. This statement needs to be qualified somewhat since the seeming capriciousness of the tax collector and insurance adjustor could be veiled forms of randomization.[19] Nevertheless, we know of no explicit random contracts in competitive insurance markets. How can we explain this apparent discrepancy between theory and fact?

Along with the possibility that the conditions for randomization are never met, we have come up with six reasons why randomization might not occur as frequently as the theory suggests. Each of these reasons leads one to question the appropriateness of some of the assumptions underlying our analysis and more generally underlying much of the literature on incentives and contracts, particularly that focussing on non-linear contracts.

1. Firms may not have discovered the advantages of randomization or else customers may be reluctant to purchase random insurance policies since they do not understand them. To the extent that this explanation is valid, one must question the appropriateness of assuming that the contracting parties exhibit

unbounded rationality.

2. A related argument is that it is costly to write down and enforce complex contracts. This argument is consistent with the observation that non-stochastic contracts are almost invariably considerably less complex than contract theory predicts. If this argument carries force, then contract theory should treat the costs of complexity.

Simple contracts may be observed not only because the costs of complexity are high, but also because the benefits are low.[20,21]

3. Even in those circumstances where randomization of insurance contracts is desirable, insured individuals would obtain insurance against the randomization if they could. They would perceive the benefits of such randomization insurance, but because they act non-cooperatively would neglect that, in response to everyone's purchasing the randomization insurance, accident insurance companies would be forced to offer less attractive contracts in order to break even. Full randomization insurance would completely neutralize the effects of randomization of the accident insurance contract, and so accident insurance firms would have no incentive to randomize. The analysis assumed away such randomization insurance.[22] But with limited observability, which underlies both moral hazard and adverse selection, the assumption may be unwarranted.[23] A more satisfactory analysis would describe the economy in such a way that which markets are present and what contracts are enforceable, are derived rather than assumed.

4. In some contexts (e.g., the draft in the U.S.) lotteries are commonly regarded as fair, in other contexts as unfair (horizontal equity). While the persuasive modelling of fairness and horizontal equity has proved elusive, it may be that consumers would find random insurance policies unattractive because they view them as unfair.

5. For other, familiar reasons, one may question the appropriateness of employing von Neumann-Morgenstern expected utility theory (Machina [1987]).

6. A sixth reason why such contracts may not be employed is that individuals may not trust their insurers to randomize in the manner specified in their contracts. To know whether a firm is complying with the contract, an individual must know not only which policy he has been assigned, but also which policies have been assigned to all its other clients. Even if he were able to verify that the firm assigned the policies with the contractually-specified probabilities, the firm could still cheat by assigning the policies in a systematic way, e.g. to friends or to those offering kickbacks. Whether reputation would be an effective enforcement mechanism in this context is moot.

A final point is that precluding randomization by assumption is not a satisfactory way of treating the empirical fact that random contracts are not observed. First, by prohibiting randomization from entering through the front door, one might find it entering in disguised form through the back door. For example, suppose that the moral hazard model of the paper were enriched to allow for several symmetric types of accidents. If random insurance contracts were excluded by assumption, then in some circumstances in which ex ante randomization is desirable, competitive equilibrium would entail extended exclusivity (each individual would purchase all his insurance from a single firm) and firms randomly cross-subsidizing between types of insurance, e.g. one firm might run a loss on cancer insurance financed by its profits from automobile accident insurance. Doing so would allow firms to indirectly randomize. Second, one cannot be sure that whatever considerations (discussed above) result in random insurance contracts not being observed do not also have implications for the structure of nonrandom contracts. For example, if complexity is important, the competitive equilibrium contract may be simple, linear, and non-

random. But if one were simply to exclude random contracts by assumption, without explicitly treating complexity, the solution might indicate that the equilibrium contract would be highly non-linear and complex. For both these reasons, disallowing random contracts by assumption is likely to generate spurious results.

Footnotes

1. The earliest studies were in the context of the problems of optimal taxation: Weiss [1976], Stiglitz [1982a, 1982b], and Balcer and Sadka [1982].

2. When we say that randomization is "desirable", we mean that there exists a Pareto-improving randomization. Also, if randomization is desirable, it will be employed by insurance firms in competitive equilibrium (as long as they can observe their clients' total purchases of insurance) and by a monopoly insurer.

3. This equivalence can be established on the basis of arguments presented in Arnott and Stiglitz [1987b].

4. "Accident" may be reinterpreted as "large damage conditional on an accident occurring", and "no accident" as "small damage conditional on an accident occurring".

5. The assumption of a monopoly insurer implies this assumption. This assumption is, however, consequential in the competitive equilibrium interpretation - see Arnott and Stiglitz [1987b].

6. This problem is treated exhaustively in Arnott and Stiglitz [1987a].

7. This intuition applies to what we shall, in the next subsection, identify as ex post randomization. A rather different set of arguments are relevant for ex ante randomization; see below, section 1.3.

8. This Proposition has been proven previously for the continuum of outcomes case by Holmstrom [1979]. Our proof, however, is new.

9. Since they are expressed in terms of the characteristics of the constraint set, Propositions 2 and 3, as well as Corollary 1, apply to any (α,β) (and the associated levels of $\bar{U}$ and e) and not just to

profit-maximizing α and β associated with a given level of expected utility. The same remark applies to Proposition 4.

10. Furthermore, with non-separable utility, effort need not be a continuous function of the contract parameters. We have chosen not to treat this complication.

11. We have considered other two-dimensional perturbations: i) randomizing the premium, holding the payout fixed; and ii) randomizing the payout, while varying the premium to maintain profits constant, and investigating the change in expected utility.

12. This Proposition has been proven previously by Gjesdal [Proposition 3, 1982] for the continuum of outcomes case. Our proof, however, is new.

13. Gjesdal [1982, pps. 382-3] provides an example in which ex post randomization is desirable.

14. This Proposition is similar to Theorem 3 of Fellingham, Kwon, and Newman [1984].

15. Fellingham, Kwon and Newman [1984, pps. 296-297] provide an example in which ex ante randomization is desirable.

16. It can also be proven, rather obviously, that "with adverse selection and two groups, randomization is never desirable if the high-risk group is less risk-averse than the low-risk group".

17. We would like to thank Stephen Slutsky for suggesting this randomization procedure to us in another context.

18. By competitive equilibrium, we mean a Nash equilibrium in contracts with free entry and exit. In the case of adverse selection, this statement applies only when competitive equilibrium exists - see Rothschild and Stiglitz [1971] for a discussion of non-existence of

competitive equilibrium in this context.

19. We consider this unlikely, however. A more plausible explanation for the randomness of tax audits is that decreasing the probability of audit, while holding the expected fine constant, reduces administrative costs.

20. In moral hazard theory, it is assumed that insurance companies know the accident technology and consumers' tastes perfectly, and in adverse selection theory that they know consumers' tastes and the composition of the population perfectly. Intuition suggests that the more imperfect is insurers' knowledge, the smoother the optimal contract and the smaller the gains from complexity.

21. In this context it should be noted that the literature on the randomization of insurance contracts, including this paper, has not attempted to measure the benefits from randomization, though doing so would not be excessively difficult.

22. The problem of "side-contracts" is a pervasive, though neglected, one in contract theory. For instance, whenever payments schedules are non-linear, there is an incentive for side-payments and the creation of secondary markets. If the payment function is concave, individuals have an incentive to smooth measured output (selling output on the secondary market when output is high, and buying when it is low) and to make measured output more uneven when the payment schedule is convex (e.g. by introducing randomization).The presence of such secondary markets provides one explanation for the linearity of payment schedules.

23. By treating a monopoly insurer, the analysis assumed that individuals' total purchases of accident insurance are observable. In these

circumstances, competitive equilibrium is characterized by exclusiv-
ity - each insurance company will require that its clients purchase
insurance only from itself. In this case, it is not inconsistent to
assume away randomization insurance, since accident insurance com-
panies may be able to observe their clients' purchases of randomiza-
tion insurance as well. Exclusive accident contracts are also en-
forceable if the accident insurance companies cannot observe indi-
viduals' total purchases of accident insurance, but can enforce re-
fusing to make a payment in the event of accident if the insured re-
ceives payment from any other company for the accident. This re-
quires that accident insurance companies monitor only victims. With
this more limited form of observability, it is not inconsistent to
assume that accident insurance firms can enforce exclusive accident
insurance contracts but cannot monitor their clients' purchases of
randomization insurance.

Bibliography

Arnott, R.J. and J.E. Stiglitz, "The Basic Analytics of Moral Hazard", 1987a, mimeo.

Arnott, R.J. and J.E. Stiglitz, "Equilibrium in Competitive Insurance Markets with Moral Hazard", 1987b, mimeo.

Balcer, Y., and E. Sadka, "Horizontal Equity, Income Taxation and Self-Selection with an Application to Income Tax Credits", Journal of Public Economics, 1982, 19, 291-310.

Diamond, P. and J. Stiglitz, "Increases in Risk and Risk Aversion", Journal of Economic Theory, 1974, 8, 337-360.

Fellingham, J.C., Y.K. Kwon, and D.P. Newman, "Ex Ante Randomization in Agency Models", Rand Journal of Economics, 1984, 15, 290-301.

Gjesdal, F., "Information and Incentives: The Agency Information Problem", Review of Economic Studies, 1982, 49, 373-390.

Holmstrom, B., "Moral Hazard and Observability", Bell Journal of Economics, 1979, 10, 74-91.

Laffont, J.-J., and E. Maskin, "The Theory of Incentives: An Overview" in Advances in Economic Theory, W. Hildenbrand, ed. (Cambridge, U.K.: Cambridge University Press, 1982).

Machina, M., "The Choice under Uncertainty: Problems Solved and Unsolved", Journal of Economic Perspectives, 1987, 1, 121-154.

Rothschild, M., and J.E. Stiglitz, "Increasing Risk: A Definition", Journal of Economic Theory, 1970, 2, 225-243.

Rothschild, M., and J.E. Stiglitz, "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information", Quarterly Journal of Economics, 1976, 629-649.

Stiglitz, J.E., "Self-selection and Pareto Efficient Taxation", Journal of Public Economics, 1982a, 17, 213-240.

Stiglitz, J.E., "Utilitarianism and Horizontal Equity: The Case for Random Taxation", Journal of Public Economics, 1982b, 18, 1-33.

Weiss, L., "On the Desirability of Cheating, Incentives and Randomness in the Optimal Income Tax", Journal of Political Economy, 1976, 84, 1343-1352.

Appendix 1

Derivation of $\dfrac{\partial^2 V}{\partial \Pi^2}\Big|_\beta$

Define

$$V(\Pi,\beta,\hat{e}(\Pi,\beta))=(1-p(\hat{e}))u(w-\beta)+p(\hat{e})u\left[w-d+\frac{\beta(1-p(\hat{e}))-\Pi}{p(\hat{e})}\right] - \hat{e} \qquad (A1.1)$$

and

$$Z(\Pi,\beta,\hat{e}(\Pi,\beta) = -p'(\hat{e})\left[u(w-\beta)-u\left[w-d+\frac{\beta(1-p(\hat{e}))-\Pi}{p(\hat{e})}\right]\right]. \qquad (A1.2)$$

Then

$$\frac{\partial V}{\partial \Pi}\Big|_\beta = \frac{dV}{d\Pi}\Big|_\beta = V_\Pi + V_e\frac{\partial \hat{e}}{\partial \Pi} , \qquad (A1.3)$$

where subscripts denote partial derivatives. From the individual's effort choice problem

$$\frac{\partial \hat{e}}{\partial \Pi} = -\frac{Z_\Pi}{Z_e} . \qquad (A1.4)$$

Combining (A1.3) and (A1.4) gives

$$\frac{\partial V}{\partial \Pi}\Big|_\beta = \frac{1}{Z_e} (V_\Pi Z_e - V_e Z_\Pi). \qquad (A1.5)$$

Then

$$\frac{\partial^2 V}{\partial \Pi^2}\Big|_\beta = \frac{d^2 V}{d\Pi^2}\Big|_\beta = \frac{\partial\left[\frac{dV}{d\Pi}\Big|_\beta\right]}{\partial \Pi} + \frac{\partial\left[\frac{dV}{d\Pi}\Big|_\beta\right]}{\partial \hat{e}}\frac{\partial \hat{e}}{\partial \Pi} \qquad (A1.6)$$

$$= \left[\frac{1}{Z_e^3}\{Z_e(V_{\Pi\Pi}Z_e^2 - 2V_{e\Pi}Z_e Z_\Pi + V_{ee}Z_\Pi^2) - V_e(Z_{\Pi\Pi}Z_e^2 - 2Z_{e\Pi}Z_e Z_\Pi + Z_{ee}Z_\Pi^2)\}\right] . \qquad (A1.7)$$

After substitution of the expressions for the various partial derivatives, the resulting expression is extremely messy. It is not, however, difficult to establish that the sign of $\dfrac{\partial^2 V}{\partial \Pi^2}\Big|_\beta$ can be positive. The only place where p''' enters the expression is through Z_{ee}. The full term containing p''' is

$$- \frac{V_e Z_\Pi^2}{Z_e^3} \left(-p'''(u_0 - u_1) \right).$$ Since $V_e > 0$, $Z_e < 0$, and $u_0 - u_1 > 0$, then if p''' is nega-

tive and sufficiently large in absolute value, $\left. \dfrac{\partial^2 V}{\partial \Pi^2} \right|_\beta$ will be positive.

Derivation of $\left.\dfrac{d^2 EU^H}{d(a_1^L)^2}\right|_0$

Let

$$I^k = \frac{d}{dy}\ (u^k(w\text{-}d\text{+}a_2^L)) \qquad\qquad I^k = \frac{d}{dy}\ (u^k(w\text{-}\beta_2^L)) \qquad\qquad (A2.1a,b)$$

$$X^k = \frac{d}{dy}\ (u^k(w\text{-}d\text{+}a_1^L)) \qquad\qquad Y^k = \frac{d}{dy}\ (u^k(w\text{-}\beta_1^L)) \qquad\qquad (A2.1c,d)$$

Total differentiation of (19) and (20) gives

$$\frac{d\beta_1^L}{da_1^L} = \left[\frac{p^L}{1\text{-}p^L}\right]\left[\frac{X^L}{Y^L}\right] \qquad\qquad \frac{da_2^L}{da_1^L} = -\left[\frac{X^L\text{-}Y^L}{I^L\text{-}I^L}\right]\left[\frac{I^L}{Y^L}\right] \qquad\qquad (A2.2a,b)$$

$$\frac{d\beta_2^L}{da_2^L} = \left[\frac{p^L}{1\text{-}p^L}\right]\left[\frac{I^L}{I^L}\right] \qquad\qquad \frac{d\beta_2^L}{da_1^L} = \left[\frac{d\beta_2^L}{da_2^L}\right]\left[\frac{da_2^L}{da_1^L}\right] \qquad\qquad (A2.2c,d)$$

Now, from (21),

$$\frac{dEU^H}{da_1^L} = (1\text{-}p^H)\ \left[-Y^H\frac{d\beta_1^L}{da_1^L} - I^H\frac{d\beta_2^L}{da_1^L}\right] + p^H\left[X^H\text{+}I^H\frac{da_2^L}{da_1^L}\right]\ . \qquad (A2.3)$$

When the amount of randomization is infinitesmal, $a_1^L\text{=}a_2^L$, $\beta_1^L\text{=}\beta_2^L$, $Y^H\text{=}I^H$,

$X^H\text{=}I^H$, $\dfrac{d\beta_1^L}{da_1^L} = -\dfrac{d\beta_2^L}{da_1^L}$, and $\dfrac{da_2^L}{da_1^L} = -1$. Hence,

$$\left.\frac{dEU^H}{da_1^L}\right|_0 = 0. \qquad\qquad\qquad (A2.4)$$

To determine the effect of an infinitesmal randomization, we investigate $\left.\dfrac{d^2 EU^H}{d(a_1^L)^2}\right|_0$. Substituting (A2.2) into (A2.3), totally differentiating the resulting expression, and using the above relations when the amount of randomization is infinitesmal gives (23).

Notational Glossary

Symbols in order of introduction in text

w	pre-insurance, no accident income
d	accident damage
e	accident-prevention effort
$p(e)$	probability-of-accident function
α	net insurance payout with an accident
β	premium payable if there is no accident
y	consumption
EU	expected utility
j	subscript denoting event ($j=0$, no accident; $j=1$, accident)
$U_j(y,e)$	event-dependent utility function
$u_j(y)$	utility-from-consumption function with separable utility function
$v(\alpha,\beta)$	indirect utility function
i	subscript indexing randomized policy
Q_i	probability of random policy i
$\nu(\alpha_i,\beta_i,e)$	expected utility function with policy i
x	control variable
$g^j(\cdot)$	constraint function j
$\bar{U}$	level of expected utility
e^*	level of effort at the solution to the monopoly insurer's problem
$\bar{u}_j$	average value of u_j
Δ	size of randomization of α
Π	expected profit on a contract

$\hat{e}(\Pi,\beta)$ $= e(\alpha,\beta)$

$V(\Pi,\beta)$ $= v(\alpha,\beta) = V(\Pi,\beta,\hat{e})$

Z net marginal benefit of effort

k superscript denoting risk groups (H=high, L=low)

n^k population of group k

p^k accident probability of group k

u_j^k utility function for group k with event j

a certainty-equivalent net payout

b certainty-equivalent premium

η $\equiv [(1-p^H)p^L]/[(1-p^L)p^H]$

θ slope of low-risk indifference curve

r^k ratio of no-accident marginal utility to accident marginal utility for group k

A_j^k (local) coefficient of absolute risk aversion of group k in event j

Ex post randomization	Ex ante randomization

Ex post randomization

- Insurance firm offers two contracts

A	B
$(\alpha_1^A, \beta_1^A), Q_1^A$	$(\alpha_1^B, \beta_1^B), Q_1^B$
$(\alpha_2^A, \beta_2^A), Q_2^A$	$(\alpha_2^B, \beta_2^B), Q_2^B$
$\vdots$	$\vdots$
$(\alpha_N^A, \beta_N^A), Q_N^A$	$(\alpha_M^B, \beta_M^B), Q_M^B$
$\sum\limits_1^N Q_i^A = 1$	$\sum\limits_1^M Q_i^B = 1$

- Individual chooses A or B

- Individual randomly assigned insurance policy (α_i^A, β_i^A) with probability Q_i^A if chose A, etc.

- Uncertainty resolved <u>after</u> individual's choice (of lotteries).

Ex ante randomization

- Insurance firm offers a lottery of policy pairs

$$L_i = [(\alpha_i^A, \beta_i^A), (\alpha_i^B, \beta_i^B), Q_i]$$

$$\sum\limits_1^I Q_i = 1$$

- Individual randomly assigned L_i with probability Q_i

- Having been assigned L_i, individual chooses between

$$A_i = (\alpha_i^A, \beta_i^A) \text{ and}$$

$$B_i = (\alpha_i^B, \beta_i^B).$$

- Uncertainty resolved <u>before</u> individual's choice of policies.

Table 1: The timing of <u>ex ante</u> and <u>ex post</u> randomization with adverse selection
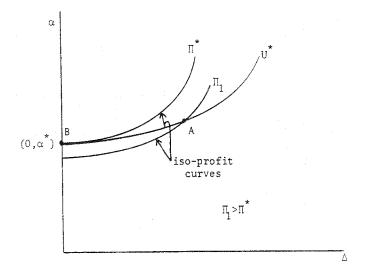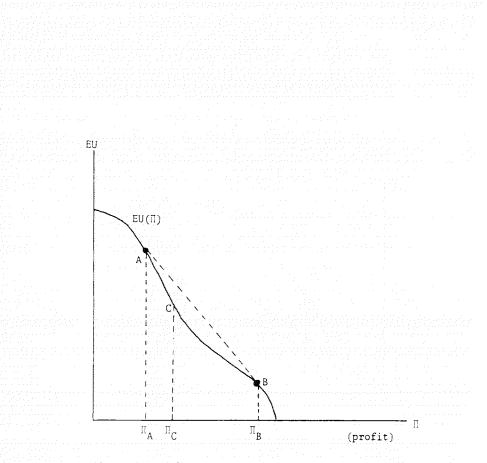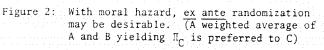
Figure 1: With moral hazard, ex post randomization may be desirable (A is preferred to B)

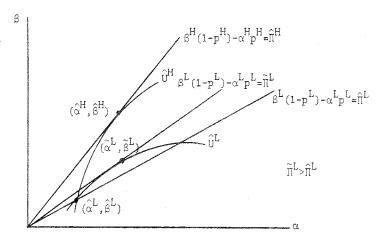Figure 2: With moral hazard, ex ante randomization may be desirable. (A weighted average of A and B yielding Π_C is preferred to C)

Figure 3: Adverse selection.

Figure 4: When both groups have the same tastes, randomization is undesirable.