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ABSTRACT

We propose a model of sovereign debt in which countries vary in their level of financial development, defined as the extent to which they can issue debt denominated in domestic currency in international capital markets. We show that low levels of financial development generate the "debt intolerance" phenomenon that plagues emerging markets: it reduces overall debt capacity, increases credit spreads, and limits the country's ability to smooth consumption.

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1 Introduction

One intriguing fact about sovereign debt markets is that emerging economies pay high credit spreads on their sovereign debt, despite generally having much lower debt-output ratios than developed countries. Reinhart, Rogoff, and Savastano (2003) call this phenomenon "debt intolerance."

In this paper, we propose a model of sovereign debt in which countries vary in their level of financial development. By "financial development," we mean the extent to which a country can issue debt denominated in domestic currency in international capital markets.[1] We show that low levels of financial development generate debt intolerance.

As in the literature on rare disasters, we assume that there are sporadic downward jumps in output.[2] In our model, output follows the jump-diffusion process estimated by Barro and Jin (2011) in which the size distribution of jumps is governed by a power law. This process is consistent with the evidence presented in Aguiar and Gopinath (2007) that permanent shocks are the primary source of fluctuations in emerging markets.

Since domestic currencies generally depreciate in disaster periods (Farhi and Gabaix (2016)), the dollar value of domestic currency debt falls in these periods. This property makes domestic currency debt a natural partial hedge against rare disasters. Countries that can borrow more in domestic currency have a greater ability to manage disaster risk. As a result, they have higher debt capacity and pay lower spreads on foreign currency debt. In other words, they have less "debt intolerance."

We assume that countries face exogenous limits to their ability to issue domestic currency debt in international capital markets. This assumption is motivated by the key finding of the literature on the original-sin hypothesis: the ability to borrow in domestic currency is more closely related to the size of the economy than to the soundness of fiscal and monetary policy or other fundamentals (Hausmann and Panizza (2003) and Bordo, Meissner, and Redish (2004)).

Our key result is that the more limited is a country's ability to issue debt in domestic

[1] Another aspect of financial development is a country's access to commitment mechanisms such as posting collateral or depositing money in escrow accounts that can be seized by creditors. We do not consider these mechanisms because sovereign debt is generally unsecured in practice.

[2] This framework has proved useful in modeling many asset-pricing and macroeconomic phenomena. Examples include the equity premium (Rietz (1988), Barro (2006), Barro and Jin (2011), and Gabaix (2012)), business cycles (Gourio (2012)), the predictability of excess stock returns (Wachter (2013)), investment, interest rates, and equity returns (Pindyck and Wang (2013)), and the returns to the carry trade (Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and Farhi and Gabaix (2016)).

currency, the lower is its overall debt capacity and the more severe is its debt intolerance. In other words, domestic currency debt and foreign currency debt are complements. This implication is consistent with the key finding in Du, Pflueger, and Schreger (2020). These authors show that countries that can issue more domestic currency debt also issue more debt denominated in foreign currency.

An important question is: how much better would a country be if it could hedge rare-disaster risk with a full set of state-contingent hedging contracts? To answer this question, we compare two economies. The first has a high level of financial development and uses domestic currency debt to hedge its rare-disaster risk. The second is a "full-spanning" economy that uses a full set of state-contingent hedging contracts to hedge its rare-disaster risk, as in Kehoe and Levine (1993) and Kocherlakota (1996).

We find that the limited commitment, full-spanning economy has higher welfare than the economy that hedges rare-disaster risk by issuing debt denominated in domestic currency. But this difference is quantitatively small. In contrast, the welfare gain from increasing the ability of economies with low financial development to issue more domestic currency debt in international capital markets is much larger. These results suggest that expanding the ability of emerging markets to issue domestic currency debt might be an effective and expedient way to improve their welfare.

We write our sovereign debt model in continuous time. This approach has several significant advantages. First, our model can be solved in closed form for both the value function and the policy rules up to an ordinary differential equation (ODE) for certainty-equivalent wealth with intuitive boundary conditions. Second, the analytical expressions for optimal consumption, debt issuance, and default policies yield valuable insights into the key mechanisms at work in our model. Third, we obtain a sharp characterization of the properties of our model as the country approaches its debt capacity: the diffusion volatility of the debt-output ratio approaches zero, and the country's endogenous risk aversion approaches infinity.

Our method for characterizing global nonlinear dynamics is similar to that used in the dynamic optimal contracting and macro-finance diffusion-based literature (e.g., DeMarzo and Sannikov (2006), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Williams (2015), and Bolton, Wang, and Yang (2019)). As our model features jump shocks that cannot be fully hedged and equilibrium credit risk pricing, we generalize the numerical solution methodology used in these papers to accommodate these features.

The representative agent has the continuous-time version of the Epstein-Zin-Weil preferences proposed by Duffie and Epstein (1992). These preferences allow our model to generate empirically plausible average debt-to-output ratios without resorting to the very high discount rates used in the literature. Our calibration combines a conventional value of the discount rate (5.2 percent per year) with a low elasticity of intertemporal substitution (EIS = 0.025) and a conventional value for relative risk aversion ($\gamma = 2$). We interpret the low EIS as reflecting expenditure commitments that are difficult to change, as in Bocola and Dovis (2016). Recursive preferences are key to making this calibration work. With standard expected utility, a low EIS implies a high risk aversion that creates an incentive to avoid the debt region, generating a low average debt-to-output ratio.

Following Aguiar and Gopinath (2006) and Arellano (2008), we assume that, upon default, the country suffers a decline in output and loses access to international capital markets. It then regains access to these markets with constant probability per period. Outside of the default state, the country can issue debt denominated in both domestic and foreign currency that can be defaulted upon. The country can also invest at a risk-free rate and can hedge diffusion shocks.

As emphasized by Bulow and Rogoff (1989), autarky might be difficult to sustain because the rest of the world cannot commit ex ante to excluding the defaulting borrower from ex post risk-sharing arrangements. In our model, the permanent output loss that occurs upon default is sufficient to sustain the existence of sovereign debt. In this sense, our model is immune to the Bulow-Rogoff critique.

One virtue of our model is that it does not require the nonlinear default costs commonly used in the literature to generate plausible average debt-output ratios. Our linear specification of default costs is consistent with recent evidence by Hébert and Schreger (2016) and Trebesch and Zabel (2017).

In response to large jump shocks, it is optimal for countries to default on their debt. In response to moderate jump shocks, the country fully repays its domestic and foreign currency debt. However, the domestic currency depreciates, reducing the dollar value of domestic currency debt. The larger is the shock, the larger is the rate of depreciation and the lower is the ex post dollar value of domestic currency debt. Domestic currency debt serves as a natural partial hedge against rare disasters because the dollar value of output and domestic currency debt move in the same direction when disasters occur.

The equilibrium credit spread on foreign currency debt reflects its default risk. The

equilibrium credit spread on domestic currency debt reflects both default and currency depreciation risk.

We consider two variants of our model. In the first variant, lenders demand a credit risk premium calibrated using estimates in Longstaff, Pan, Pedersen, and Singleton (2011). In this setting, it is more costly to service debt, so debt capacity is lower. In the second variant, the output cost of defaulting is temporary instead of permanent. Since the overall cost of default is lower, the country is more tempted to default. As a result, debt capacity is lower than in our benchmark model.

The paper is organized as follows. Section 2 presents a limited commitment model in which the country can issue domestic currency debt up to a limit. Section 3 discusses this benchmark model's solution. Section 4 summarizes the properties of the first-best model solution. Section 5 calibrates our benchmark model and explores its quantitative properties. Section 6 compares our benchmark model with a model in which the country also has limited commitment but can hedge with a set of full-spanning hedging contracts. Section 7 generalizes our benchmark model to a setting with credit risk premia. Section 8 presents a version of our benchmark model with transitory default costs. Section 9 concludes.

2 Model Setup

We consider a continuous-time model in which the country's infinitely lived representative agent receives a perpetual, stochastic output stream and can issue both domestic and foreign currency debt.

2.1 Output Processes

Output Process in the Normal Regime. In the normal regime, the country can borrow and lend in international capital markets as well as hedge its diffusion shocks.

We model output in this regime, Y_t, as a jump-diffusion process. Diffusion shocks represent normal economic fluctuations. Large jump shocks represent rare disasters. The law of motion for output is given by

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1 - Z) d\mathcal{J}_t \,, \quad Y_0 > 0 \,, \tag{1}$$

where μ is the drift parameter, σ is the diffusion volatility parameter, $\mathcal{B}$ is a standard Brownian motion process, and $\mathcal{J}$ is a pure jump process with a constant arrival rate, λ. Let

$\tau^{\mathcal{J}}$ denote the jump arrival time. Since Brownian motion is continuous, if a jump does not occur at t ($d\mathcal{J}_t = 0$), we have $Y_t = Y_{t-}$, where $Y_{t-} \equiv \lim_{s \uparrow t} Y_s$ denotes the left limit of output. If a jump occurs at t ($d\mathcal{J}_t = 1$), output falls from Y_{t-} to $Y_t = ZY_{t-}$. We call $Z \in (0,1)$ the fraction of output recovered after a jump arrival. We assume that Z follows a well-behaved cumulative distribution function, $F(Z)$.

Since the expected percentage output loss upon the arrival of a jump is $1 - \mathbb{E}(Z)$, the expected growth rate of output in levels is given by

$$g = \mu - \lambda(1 - \mathbb{E}(Z)) \ . \tag{2}$$

Here, the term $\lambda(1 - \mathbb{E}(Z))$ represents the reduction in the expected growth rate associated with jumps.

We can write the dynamic equation for the logarithm of output, $\ln Y_t$, in discrete time as follows:

$$\ln Y_{t+\Delta} - \ln Y_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}\,\epsilon_{t+\Delta} - (1 - Z)\nu_{t+\Delta}\,, \tag{3}$$

where the time-t conditional distribution of $\epsilon_{t+\Delta}$ is a standard normal and $\nu_{t+\Delta} = 1$ with probability $\lambda\Delta$ and zero with probability $(1 - \lambda\Delta)$. Equation (3) implies that the expected change of $\ln Y$ over a time interval Δ is $(\mu - \sigma^2/2)\,\Delta - \lambda(1 - \mathbb{E}(Z))\Delta$. The term $\sigma^2/2$ is the Jensen inequality correction associated with the diffusion shock.

The Exchange Rate Process. Let S_t denote the spot exchange rate, defined as dollars per unit of domestic currency. We measure output, Y_t, and consumption, C_t, in U.S. dollars.

We assume that absolute purchasing power parity holds and that the price of output in U.S. dollars is constant and equal to one. Given these assumptions, there is no drift in the exchange rate process. So, the law of motion for S_t is given by the following martingale process:

$$\frac{dS_t}{S_{t-}} = \sigma_S d\mathcal{B}_t - \pi(Z)d\mathcal{J}_t + \lambda\mathbb{E}[\pi(Z)]dt\,, \tag{4}$$

where σ_S is a positive constant and $\pi(Z) \in (0,1)$ with $\pi'(Z) < 0$ and $\pi(1) = 0$. When a jump shock arrives at time t, the exchange rate changes from S_{t-} to $S_t = (1 - \pi(Z))S_{t-}$.

Specification (4) is consistent with empirical evidence that it is difficult to beat the random walk as a short-term forecast of the exchange rate (see Rossi (2013) for a recent survey of this evidence). This specification is also consistent with the joint fluctuations between rare disasters and exchange rates modeled by Farhi and Gabaix (2016) and the role of exchange rates in the financial adjustment considered by Gourinchas and Rey (2007).

Output Process If the Country Defaults on Its Debt. We assume that if the country decides to default, it defaults on both foreign currency and domestic currency debt.[3] After default, the country immediately enters autarky. Let $\mathbb{1}_t^{\mathcal{D}}$ be an indicator function that takes the value one if the country defaults on its debt at t and zero otherwise. Let $\tau^{\mathcal{D}}$ denote the endogenous time when the country chooses to default. We show below that, because markets are incomplete, jump shocks that cause sufficiently large output losses lead the country to optimally default on its debt.

Defaulting entails two costs. The first is that the country loses access to international capital markets and enters a state of autarky in which consumption equals output. The second is an output loss that proxies for the disruptions of economic activity associated with default. We assume that upon default, output drops permanently from $Y_{\tau^{\mathcal{D}}-} \equiv \lim_{s\uparrow\tau^{\mathcal{D}}} Y_s$, the output in the normal regime just prior to default, to $\alpha Z Y_{\tau^{\mathcal{D}}-}$, where $\alpha \in (0,1)$.

Let $\widehat{Y}_t$ denote the level of output in the autarky regime. We assume that $\widehat{Y}_t$ follows the same process as output in the normal regime:

$$\frac{d\widehat{Y}_t}{\widehat{Y}_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1-Z)d\mathcal{J}_t \ . \tag{5}$$

This process starts at time $\tau^{\mathcal{D}}$ with the value of $\widehat{Y}_{\tau^{\mathcal{D}}} = \alpha Z Y_{\tau^{\mathcal{D}}-}$.

While in autarky, the country regains access to international capital markets with probability ξ per unit of time. Let $\tau^{\mathcal{E}}$ denote the stochastic exogenous exit time from autarky. The stochastic duration of the autarky regime is $\tau^{\mathcal{E}} - \tau^{\mathcal{D}}$. Upon randomly exiting from autarky at time $\tau^{\mathcal{E}}$, the country starts afresh with no debt and regains access to international capital markets. Then, output follows the process given by equation (1) starting with $Y_{\tau^{\mathcal{E}}}$, which is equal to $\widehat{Y}_{\tau^{\mathcal{E}}-}$, the pre-exit output level under autarky: $Y_{\tau^{\mathcal{E}}} = \widehat{Y}_{\tau^{\mathcal{E}}-}$.

In this formulation, default results in the permanent loss of a fraction $1-\alpha$ of output. In Section 8, we discuss a version of the model in which the output loss associated with default is temporary.

2.2 Preferences

We assume that the lifetime utility of the representative agent, V_t, has the recursive form proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). We use

[3] See Reinhart and Rogoff (2011) for evidence of outright default on domestic currency debt.

the continuous-time version of these preferences developed by Duffie and Epstein (1992),

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_u, V_u) du \right], \tag{6}$$

where $f(C, V)$ is the normalized aggregator for consumption C and utility V. This aggregator is given by

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\chi}}{((1-\gamma)V)^{\chi-1}}. \tag{7}$$

Here, ρ is the subjective discount rate and

$$\chi = \frac{1 - \psi^{-1}}{1 - \gamma}. \tag{8}$$

This recursive, non-expected utility formulation allows us to separate the coefficient of relative risk aversion, γ, from the elasticity of intertemporal substitution (EIS), ψ. This separation plays an important role in our quantitative analysis. The time-additive separable CRRA utility is a special case of recursive utility where the coefficient of relative risk aversion, γ, equals the inverse of the EIS, $\gamma = \psi^{-1}$, implying $\chi = 1$. In this case, $f(C, V) = U(C) - \rho V$, which is additively separable in C and V, with $U(C) = \rho C^{1-\gamma}/(1-\gamma)$.

2.3 Foreign and Domestic Currency Debt

Both domestic and foreign currency debt markets are perfectly competitive. In the normal regime, the country chooses to: (1) issue domestic currency debt, B_t; (2) issue foreign currency debt (denominated in U.S. dollars), B_t^*; and (3) insure against diffusion shocks using hedging contracts.

The country's wealth measured in dollars, W_t, is

$$W_t = -\left(B_t S_t + B_t^*\right). \tag{9}$$

When $B_t S_t + B_t^* < 0$, the country is a net saver. We assume that the country can save in foreign currency but not in domestic currency (i.e. $B_t \geq 0$).

As in discrete-time settings, both domestic and foreign currency debt are borrower specific, non-contingent, unsecured, and short term.[4] Foreign currency debt is continuously

[4] Auclert and Rognlie (2016) show that sovereign debt models with short-term debt have a unique Markov-perfect equilibrium. Sovereign debt models with long-maturity debt include Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012).

repaid and reissued at the dollar interest rate $r + \delta_t^*$, where δ_t^* is the endogenous credit spread. The country either fully repays its debt or defaults.

Similarly, domestic currency debt is continuously repaid and reissued at the nominal interest rate $r + \delta_t$, where δ_t is the endogenous credit spread in local currency. Since the exchange rate is a martingale, there is no expected currency depreciation and the two spreads coincide, $\delta_t = \delta_t^*$. Upon default, both foreign currency and domestic currency debt holders receive nothing.

As emphasized by Eaton and Gersovitz (1981), Zame (1993), and Dubey, Geanakoplos, and Shubik (2005), the possibility of default provides a partial hedge against risks that cannot be insured because of limited financial spanning.[5]

Financial Development. Following the original-sin literature, we assume that the ability to issue debt in local currency in international capital markets is limited. The dollar value of domestic currency debt, $B_t S_t$, has to satisfy the following constraint at all time t:

$$B_t S_t \leq \kappa Y_t \,. \tag{10}$$

Here, κ is a parameter that represents the level of financial development. The lower is κ, the less developed are financial markets.

Foreign and domestic currency debt are priced in competitive markets by well-diversified foreign investors. The maximal amount of foreign and domestic currency debt that the country can issue is stochastic and endogenously determined in equilibrium by the creditors' break-even conditions.

Diffusion Risk Hedging Contracts. We assume that diffusive shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. An investor who holds one unit of the hedging contract at time t receives no up-front payment, since there is no risk premium for bearing idiosyncratic risk, and receives a gain or loss equal to $\sigma d\mathcal{B}_t = \sigma \left(\mathcal{B}_{t+dt} - \mathcal{B}_t\right)$ at time $t + dt$. We normalize the volatility of this hedging contract so that it is equal to the diffusion volatility parameter, σ. This hedging contract is analogous to a futures contract in standard no-arbitrage models (see, e.g., Cox, Ingersoll, and Ross (1981)). We denote the country's holdings of diffusion risk contracts at time t by Θ_t.

[5]For simplicity, we consider only the possibility of complete default. Our model can easily be generalized to allow for partial default. See Yue (2010) and Asonuma, Niepelt, and Ranciere (2017) for models with partial default.

Optimality. The country chooses its consumption, C_t, diffusion risk hedging demand, Θ_t, domestic and foreign currency debt issuances, B_t and B_t^*, and default timing, τ, to maximize the agent's utility defined by equations (6)-(7), given the output processes specified in equations (1) and (5), the exchange rate process given in equation (4), and equilibrium pricing of domestic and foreign currency debt.

3 Model Solution

We solve our model using dynamic programming. Let $V(W_t, Y_t)$ and $\widehat{V}(\widehat{Y}_t)$ denote the representative agent's value function for the normal and autarky regime, respectively. The autarky value function depends only on contemporaneous output because financial wealth is always zero in autarky.

3.1 Normal Regime

In the normal regime, financial wealth expressed in dollars, W, evolves according to

$$dW_t = \left[Y_{t-} - C_{t-} - (r + \delta^*_{t-})B^*_{t-} - (r + \delta_{t-})B_{t-}S_{t-}\right] dt - B_{t-}dS_t + \sigma\Theta_{t-}d\mathcal{B}_t - \mathbb{1}_t^{\mathcal{D}} W_{t-}d\mathcal{J}_t \,. \quad (11)$$

The first term on the right side of equation (11) is the drift of financial wealth that occurs in the absence of jumps. This drift is equal to output minus the sum of consumption and interest payments for both domestic and foreign currency debt. The second term, $-B_{t-}dS_t$, is the realized gain or loss from exchange rate movements. When the domestic currency depreciates ($dS_t < 0$), the dollar value of domestic currency debt falls. Since the domestic currency depreciates in disaster periods, domestic currency debt is a natural hedge against rare disasters. The third term, $\sigma\Theta_{t-}d\mathcal{B}_t$, is the realized gain or loss from diffusion risk-hedging contracts. Given that diffusion shocks are idiosyncratic with zero mean, the country incurs no up-front payment. The last term states that upon default, the country enters autarky and starts with $W_t = 0$.

Since the diffusion shock influences both the output and exchange rate process, it is convenient to define and work with the following effective diffusion risk hedging demand:

$$\widetilde{\Theta}_{t-} = \Theta_{t-} + \sigma_S\sigma^{-1}\left(W_{t-} + B^*_{t-}\right) . \quad (12)$$

Given solutions for $\widetilde{\Theta}_{t-}$ and B^*_{t-}, we can back out the diffusion risk hedging demand Θ_{t-}.

Using the effective diffusion risk hedging demand, $\widetilde{\Theta}_{t-}$ defined in (12), we rewrite equation (11) as

$$\begin{aligned} dW_t &= \left[Y_{t-} - C_{t-} + (r + \delta^*_{t-})W_{t-} + (W_{t-} + B^*_{t-})\lambda\mathbb{E}[\pi(Z)]\right] dt + \sigma\widetilde{\Theta}_{t-}d\mathcal{B}_t \\ &\quad -(W_{t-} + B^*_{t-})\pi(Z)d\mathcal{J}_t - \mathbb{1}^{\mathcal{D}}_t W_{t-}d\mathcal{J}_t\,. \end{aligned} \tag{13}$$

Equation (13) shows that $\widetilde{\Theta}_{t-}$ controls the country's total exposure to the diffusion shock, which takes into account the partial natural hedge against the diffusion shock provided by domestic currency debt.

Next, we use dynamic programming to solve for optimal consumption, C_t, effective diffusion hedging demand, $\widetilde{\Theta}_t$, and domestic and foreign currency debt issuances, B_t and B^*_t.

Dynamic Programming. Let $\underline{W_t}$ denote the country's optimal default boundary. In the normal regime ($W \geq \underline{W}$), the country chooses C_{t-}, $\widetilde{\Theta}_{t-}$, B_{t-}, and B^*_{t-} to maximize the value function $V(W, Y)$ by solving the following Hamilton-Jacobi-Bellman (HJB) equation:[6]

$$\begin{aligned} 0 = \max_{C_{t-},\widetilde{\Theta}_{t-},B^*_{t-}} \; & f(C_{t-}, V_{t-}) + \left[Y_{t-} - C_{t-} + (r + \delta^*_{t-})W_{t-} + (W_{t-} + B^*_{t-})\lambda\mathbb{E}[\pi(Z)]\right] V_W(W_{t-}, Y_{t-}) \\ & + \frac{\sigma^2\widetilde{\Theta}^2_{t-}}{2}V_{WW}(W_{t-}, Y_{t-}) + \mu Y_{t-}V_Y(W_{t-}, Y_{t-}) + \frac{\sigma^2 Y^2_{t-}}{2}V_{YY}(W_{t-}, Y_{t-}) \\ & + \sigma^2\,\widetilde{\Theta}_{t-}Y_{t-}V_{WY}(W_{t-}, Y_{t-}) + \lambda\mathbb{E}_{t-}\left[V^{\mathcal{J}}(W_t, Y_t) - V(W_{t-}, Y_{t-})\right]\,, \end{aligned} \tag{14}$$

subject to the financial development constraint (10).

The HJB equation (14) states that at the optimum, the sum of the normalized aggregator, $f(C, V)$, and the expected change in the value function $V(W, Y)$ (the sum of all other terms on the right side of equation (14)) must equal zero.

The second and third terms of equation (14) describe the drift and diffusion volatility effects of wealth W on $V(W, Y)$. The fourth and fifth terms reflect the drift and volatility effects of output, Y, on $V(W, Y)$. The sixth term captures the effect of the intertemporal diffusion risk hedging demand on $V(W, Y)$. Diffusion shocks do not trigger default because it is always more efficient to hedge them with actuarially fair insurance contracts than default and incur default costs.

The last term, which appears in the third line of equation (14), represents the effect of jumps. We show that there are two possible outcomes upon a jump arrival at time t

[6]Duffie and Epstein (1992) generalize the standard HJB equation for the expected-utility case to allow for non-expected recursive utility such as the Epstein-Weil-Zin utility used here.

($d\mathcal{J}_t = 1$) depending on the fractional output loss ($1 - Z$), the debt-output ratio, and the level of financial development (κ). The first is no default. The second is default on both domestic and foreign currency debt. We characterize these state-contingent outcomes using an endogenous stochastic threshold, $\underline{Z_t}$.

If the output loss is sizable (i.e., $Z < \underline{Z_t}$), the country defaults on both domestic and foreign currency debt and enters autarky. Its output falls to $\widehat{Y}_t = \alpha Z Y_{t-}$ and its wealth drops to zero, so the value function at $t = \tau^{\mathcal{D}}$ is $V^{\mathcal{J}}(W_t, Y_t) = \widehat{V}(\widehat{Y}_t) = \widehat{V}(\alpha Z Y_{t-})$.

If $Z \geq \underline{Z_t}$, the country repays both its domestic and foreign currency debt. But the dollar value of domestic currency debt discretely falls so that the country's wealth changes from W_{t-} to $W_t = W_{t-} - (W_{t-} + B^*_{t-})\pi(Z)$ and the value function changes from the pre-jump value, $V(W_{t-}, Y_{t-})$, to $V^{\mathcal{J}}(W_t, Y_t) = V(W_{t-} - (W_{t-} + B^*_{t-})\pi(Z), ZY_{t-})$.

To sum up, the value function upon a jump arrival at time t is

$$V^{\mathcal{J}}(W_t, Y_t) = V(W_{t-} - (W_{t-} + B^*_{t-})\pi(Z), ZY_{t-})\mathbb{1}_{Z \geq \underline{Z_t}} + \widehat{V}(\alpha Z Y_{t-})\mathbb{1}_{Z < \underline{Z_t}}\,. \tag{15}$$

First-Order Conditions. As in Duffie and Epstein (1992), the first-order condition (FOC) for C is

$$f_C(C, V) = V_W(W, Y)\,. \tag{16}$$

This condition equates the marginal benefit of consumption, $f_C(C, V)$, to the marginal utility of savings, $V_W(W, Y)$. With expected utility, $f_C(C, V) = U'(C)$, we recover the standard FOC for consumption: $U'(C) = V_W(W, Y)$.

The FOC for the effective diffusion risk hedging demand is

$$\widetilde{\Theta} = -\frac{Y V_{WY}(W, Y)}{V_{WW}(W, Y)}\,. \tag{17}$$

Equation (17) is similar to the intertemporal hedging demand in Merton (1971) for expected utility and in Duffie and Epstein (1992) for recursive preferences.

The country chooses foreign currency debt issue, B^*_{t-}, to solve the following problem:

$$\max_{B^*_{t-}} \left[(r + \delta^*_{t-})W_{t-} + \lambda \mathbb{E}(\pi(Z))(W_{t-} + B^*_{t-})\right] V_W(W_{t-}, Y_{t-}) + \lambda\, \mathbb{E}_{t-}\left[V^{\mathcal{J}}(W_t, Y_t)\right]\,, \tag{18}$$

subject to the financial development constraint (10), which we write as $B^*_{t-} \geq -W_{t-} - \kappa Y_{t-}$. Both $V^{\mathcal{J}}(W_t, Y_t)$, given in (15), and the credit spread δ^*_{t-} depend on B^*_{t-}.

Domestic currency debt investors require a higher rate of return (in dollars) absent jumps in order to compensate for the depreciation of local currency that occurs when disasters hit.

As a result, it is more costly for the country to service domestic currency debt than foreign currency debt in normal times. The first term in equation (18) captures the cost of issuing domestic currency debt. The second term shows that domestic currency debt is a partial hedge against output losses caused by disasters. The country optimally chooses domestic currency debt issuance, $B_{t-}S_{t-} = -(W_{t-} + B^*_{t-})$, to maximize (18) subject to the financial development constraint (10).

After obtaining B^* from (18) and $\widetilde{\Theta}$ from (17), we obtain the diffusion risk hedging demand, Θ, by using

$$\Theta = -\frac{YV_{WY}(W,Y)}{V_{WW}(W,Y)} - \frac{\sigma_S}{\sigma}(W + B^*)\,. \tag{19}$$

Equation (19) has two terms. The first is the standard Merton intertemporal hedging demand. Since the country is endowed with a long position in domestic output, this hedging demand is negative. The second term is always positive because domestic currency debt issuance is weakly positive.

Value Function. We show that the value function in the normal regime, $V(W,Y)$, is given by

$$V(W,Y) = \frac{(aP(W,Y))^{1-\gamma}}{1-\gamma}\,, \tag{20}$$

where the coefficient a is given by

$$a = \rho\left[\frac{r + \psi(\rho - r)}{\rho}\right]^{\frac{1}{1-\psi}}\,. \tag{21}$$

To ensure that utility is finite, we impose the following regularity condition:

$$\rho > (1 - \psi^{-1})\, r\,. \tag{22}$$

We can interpret $P(W,Y)$ as the certainty-equivalent wealth, which is the total wealth that makes the agent indifferent between the status quo (with financial wealth W and output process Y) and having a wealth level $P(W,Y)$ and no output for all of the indefinite future:

$$V(W,Y) = V(P(W,Y),0)\,. \tag{23}$$

Next, we turn to the autarky regime.

3.2 Autarky Regime

In the autarky regime, wealth is zero and the country cannot borrow or lend, so consumption equals output and wealth is not an argument of the value function. This function, $\widehat{V}(\widehat{Y})$, satisfies the following differential equation:

$$0 = f(\widehat{Y}, \widehat{V}) + \mu \widehat{Y} \widehat{V}'(\widehat{Y}) + \frac{\sigma^2 \widehat{Y}^2}{2} \widehat{V}''(\widehat{Y}) + \lambda \mathbb{E}\left[\widehat{V}(Z\widehat{Y}) - \widehat{V}(\widehat{Y})\right] + \xi\left[V(0, \widehat{Y}) - \widehat{V}(\widehat{Y})\right]. \quad (24)$$

The first term on the right side of equation (24) is the net utility flow, often referred to as the normalized aggregator. The second and third terms represent the impact of the output drift and diffusion volatility, respectively. The fourth term describes the possibility of output jumping from $\widehat{Y}_t$ to $Z\widehat{Y}_{t-}$ while the country is in autarky. The last term reflects the possibility of exiting from autarky. This exit occurs at an exogenous rate, ξ. Upon exiting from autarky at time $\tau^{\mathcal{E}}$ and entering the normal regime, the country's value function is $V(0, Y_{\tau^{\mathcal{E}}})$, where $Y_{\tau^{\mathcal{E}}} = \widehat{Y}_{\tau^{\mathcal{E}}-}$.

We show that the value function in the autarky regime, $\widehat{V}(\widehat{Y})$, is

$$\widehat{V}(\widehat{Y}) = \frac{(a\,\widehat{p}\,\widehat{Y})^{1-\gamma}}{1-\gamma}, \quad (25)$$

where the coefficient a is given by equation (21) and $\widehat{p}$ is the endogenous (scaled) certainty-equivalent wealth in the autarky regime.

3.3 Characterizing Default Decisions

The value functions $V(W, Y)$ and $\widehat{V}(\widehat{Y})$ are connected by recurrent transitions between the normal and autarky regimes (see the two HJB equations, (14) and (24)).

Upon a jump arrival at t, output drops from Y_{t-} to $Y_t = ZY_{t-}$. If the country then defaults on domestic and foreign currency debt, output drops further from $Y_t = ZY_{t-}$ to $\alpha Y_t = \alpha Z Y_{t-}$ and the country enters autarky. Therefore, the value of wealth at t, W_t, that makes the country indifferent between repaying its debt and defaulting, which we denote by $\underline{W_t}$, satisfies the following value-matching condition:

$$V(\underline{W_t}, Y_t) = \widehat{V}(\alpha Y_t), \quad (26)$$

where $Y_t = ZY_{t-}$. Condition (26) defines the default boundary $\underline{W_t}$ as a function of Y_t:

$$\underline{W_t} = \underline{W}(Y_t). \quad (27)$$

We refer to $-\underline{W_t}$ as the country's debt capacity since it is the maximum amount of debt that the country can issue without triggering default in equilibrium. We need one more condition to determine $\underline{W_t}$, which is a free boundary. We present this condition in Section 3.4 after we simplify the model solution.

Whenever the country's total debt exceeds its endogenous debt capacity (i.e., when $W_t < \underline{W_t}$), the country defaults and enters autarky. The value function in this region satisfies

$$V(W_t, Y_t) = \widehat{V}(\alpha Y_t)\,, \quad \text{when} \quad W_t < \underline{W_t}\,. \tag{28}$$

Next, we characterize the default threshold expressed in terms of the recovery fraction, Z, upon a jump arrival at t. The country defaults on its foreign and domestic currency debt provided that the following condition holds:

$$V(W_t, Y_t) \leq \widehat{V}(\alpha Y_t)\,, \tag{29}$$

where $Y_t = ZY_{t-}$. Let $\underline{Z_t}$ denote the highest fractional recovery Z at t that satisfies equation (29) (i.e., $\underline{Z_t}$ is the supremum for the set of Z satisfying equation (29)).

3.4 Simplifying the Model Solution

It is useful to define a scaled state variable, scaled financial wealth, w_t, as

$$w_t = \frac{W_t}{Y_t}. \tag{30}$$

Similarly, we define scaled versions of the control variables: consumption $c_t = C_t/Y_t$, diffusion hedging demand $\theta_t = \Theta_t/Y_t$, effective diffusion risk hedging demand $\widetilde{\theta}_t = \widetilde{\Theta}_t/Y_t$, foreign currency debt issuance $b^*_t = B^*_t/Y_t$, domestic currency debt $b_t = B_t S_t/Y_t$, and debt capacity $\underline{w_t} = \underline{W_t}/Y_t$. The scaled certainty-equivalent wealth, $p(w_t)$, is equal to $P(W_t, Y_t)/Y_t$. Euler's theorem implies that $P_W(W_t, Y_t) = p'(w_t)$. The value of $p'(w)$ plays a crucial role in our analysis.

The default boundary, $\underline{Z_t}$, is given by

$$\underline{Z_t} = \underline{Z}(w_{t-}) = \frac{w_{t-} - \pi(\underline{Z_t})(w_{t-} + b^*(w_{t-}))}{\underline{w}}\,. \tag{31}$$

Using the homogeneity property, we express δ^*_{t-} as functions of pre-jump scaled wealth w_{t-}, which we characterize below.

Equilibrium Credit Spreads. When the country issues foreign currency debt ($B^*_{t-} > 0$), the competitive market zero profit condition for diversified investors implies that the credit spread, δ^*_{t-}, satisfies

$$B^*_{t-}(1 + rdt) = B^*_{t-}(1 + (r + \delta^*_{t-})dt)\,[1 - \lambda\, F(\underline{Z}(w_{t-}))dt] + \lambda\, F(\underline{Z}(w_{t-}))dt \times 0\,. \tag{32}$$

The first term on the right side of equation (32) is the expected total payment to investors, which is the product of the probability of repayment, $[1 - \lambda\, F(\underline{Z}(w_{t-}))dt]$, and the cum-interest value of debt repayment, $B^*_{t-}(1 + (r + \delta^*_{t-})dt)$. The second term on the right side of equation (32) corresponds to the zero payment that occurs upon default. The left side of equation (32) is the investors' total expected payoff (including principal B^*_{t-}) at $t + dt$.

Equation (32) shows that jumps are necessary to generate default in our model. To see this result, suppose that there are no jumps. Then, equation (32) implies that the credit spread δ^*_t is zero. This result requires that diffusion shocks be hedgeable so that they do not trigger default.[7]

Simplifying equation (32), we obtain the following expression for $\delta^*_{t-} = \delta^*(w_{t-})$, where

$$\delta^*(w_{t-}) = \lambda\, F(\underline{Z}(w_{t-}))\,. \tag{33}$$

This equation ties the equilibrium credit spread to the country's default strategy. For a unit of debt per unit of time, the left side of equation (33) is the compensation for bearing credit risk and the right side is the expected loss given default. Both terms are of order dt. Because there is zero recovery upon default, investors are perfectly diversified, and default risk is idiosyncratic, the credit spread is equal to the probability of default.[8]

The competitive market zero profit condition for domestic currency debt ($B_{t-} > 0$) is given by

$$B_{t-}S_{t-}(1 + rdt) = \mathbb{E}_{t-}\left[B_{t-}S_t(1 + (r + \delta_{t-})dt)\right][1 - \lambda F(\underline{Z}(w_{t-}))dt] + \lambda\, F(\underline{Z}(w_{t-}))dt \times 0\,. \tag{34}$$

Since $\mathbb{E}_{t-}(S_t) = S_{t-}$, simplifying equation (34), we obtain $\delta_{t-} = \delta(w_{t-}) = \lambda\, F(\underline{Z}(w_{t-}))$.

[7] Short-term debt in pure diffusion models has to be risk free as creditors cannot break even for any *defaultable* short-term debt. The intuition is as follows. For a small time increment, dt, diffusion shocks can cause losses of order $\sqrt{dt}$ with strictly positive probability. These losses cannot be compensated with a finite credit spread, as this compensation is only of order dt, which is much lower than $\sqrt{dt}$. For this reason, other diffusion-based debt models work with term debt in order to generate default (see, e.g., Leland (1994), Nuño and Thomas (2015), and Tourre (2017)). DeMarzo, He, and Tourre (2020) show that it is optimal to have smooth debt issuance in their model. Bornstein (2020) generates default by assuming that output follows a Poisson process in a continuous-time version of Arellano (2008).

[8] When scaled wealth is positive, there is no debt outstanding, so the probability of default is zero.

Time-varying Endogenous Relative Risk Aversion $\widetilde{\gamma}_t$. To better understand our results, it is useful to introduce the following measure of endogenous relative risk aversion, denoted by $\widetilde{\gamma}_t$. We show that $\widetilde{\gamma}_t$ is a function of w_t, which we write as $\widetilde{\gamma}(w_t)$:

$$\widetilde{\gamma}_t \equiv -\frac{V_{WW}(W_t, Y_t)}{V_W(W_t, Y_t)} \times P(W_t, Y_t) = \gamma p'(w_t) - \frac{p(w_t)p''(w_t)}{p'(w_t)} = \widetilde{\gamma}(w_t)\,. \tag{35}$$

The first part of equation (35) defines $\widetilde{\gamma}_t$. The second part follows from the homogeneity property.

The economic interpretation of $\widetilde{\gamma}_t$ given in equation (35) is as follows. Because limited commitment and incomplete markets result in endogenous market incompleteness, the country's endogenous risk aversion is given by the curvature of the value function $V(W, Y)$ rather than by the risk aversion parameter, γ. We use the value function to characterize the coefficient of endogenous absolute risk aversion: $-V_{WW}(W, Y)/V_W(W, Y)$.

We can build a measure of relative risk aversion by multiplying $-V_{WW}(W, Y)/V_W(W, Y)$ with "total wealth." There is no well-defined market measure of the total wealth under either incomplete markets or limited commitment. However, the certainty-equivalent wealth $P(W, Y)$ is a natural measure, so we use it in our definition of $\widetilde{\gamma}_t$ in equation (35).

The marginal certainty-equivalent value of wealth exceeds one (i.e., $P_W(W, Y) = p'(w) \geq 1$). Also, in our model, $p''(w) < 0$, which implies that $\widetilde{\gamma}(w) > \gamma$ (see equation (35)). That is, the representative agent is endogenously more risk averse than indicated by the coefficient of relative risk aversion, γ. Moreover, as we show later, the endogenous risk aversion increases as the country becomes more indebted (i.e., as w_t becomes more negative). In contrast, in the first-best solution described below, the country fully hedges against diffusion and jump shocks and $\widetilde{\gamma}(w_t) = \gamma$ for all levels of w_t.

Using the homogeneity property, we obtain the following expression for the scaled effective diffusion risk hedging demand, $\widetilde{\theta}(w)$:

$$\widetilde{\theta}(w) = w - \frac{\gamma p(w)p'(w)}{\gamma(p'(w))^2 - p(w)p''(w)} = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)}\,, \tag{36}$$

where $\widetilde{\gamma}(w)$ is the endogenous relative risk aversion given by equation (35). The scaled diffusion risk hedging demand, $\theta(w)$, is then given by

$$\theta(w) = \widetilde{\theta}(w) - \frac{\sigma_S}{\sigma}(w + b^*(w)) = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)} - \frac{\sigma_S}{\sigma}(w + b^*(w))\,. \tag{37}$$

Equation (37) determines the hedging demand with respect to diffusion shocks. The

country hedges to avoid defaulting in response to diffusive shocks and to preserve the option to default in response to large downward jump shocks.

Dynamics for Scaled Financial Wealth, w_t. Using Ito's lemma, we obtain the following law of motion for w_t in the normal regime:

$$dw_t = \mu_w(w_{t-})\,dt + \sigma_w(w_{t-})\,d\mathcal{B}_t + \left(w_t^{\mathcal{J}} - w_{t-}\right)\,d\mathcal{J}_t\,. \tag{38}$$

The first term in equation (38) is

$$\mu_w(w_{t-}) = \left(r + \delta^*(w_{t-}) - \mu + \sigma^2\right) w_{t-} - \sigma^2\widetilde{\theta}(w_{t-}) + 1 + \lambda\mathbb{E}[\pi(Z)](w_{t-} + b^*(w_{t-})) - c(w_{t-})\,. \tag{39}$$

The second term in equation (38) is the volatility function, $\sigma_w(w_{t-})$, given by

$$\sigma_w(w_{t-}) = \left(\widetilde{\theta}(w_{t-}) - w_{t-}\right)\sigma = (\theta(w_{t-}) - w_{t-})\,\sigma + \sigma_S(w_{t-} + b^*(w_{t-}))\,. \tag{40}$$

The third term in equation (38) captures the effect of jumps on w, where the post-jump scaled financial wealth, $w_t^{\mathcal{J}}$, is given by

$$w_t^{\mathcal{J}} = \frac{w_{t-} - \pi(Z)(w_{t-} + b^*(w_{t-}))}{Z}\,. \tag{41}$$

Substituting equation (37) into equation (40), we obtain

$$\sigma_w(w) = \left(\widetilde{\theta}(w) - w\right)\sigma = -\sigma\frac{\gamma p(w)}{\widetilde{\gamma}(w)} < 0\,. \tag{42}$$

The absolute value of the volatility of w is proportional to the ratio between $p(w)$ and endogenous risk aversion, $\widetilde{\gamma}(w)$.

Scaled Debt Capacity $\underline{w}$. To maximize the country's debt capacity, the country hedges diffusion shocks so that they do not trigger default. However, even though the representative agent is risk averse, it is not optimal to fully hedge diffusion shocks, making certainty-equivalent wealth immune to diffusion shocks. The reason is that limited commitment and incomplete financial spanning make the first-best risk-sharing outcome infeasible. Technically speaking, the country optimally sets the volatility of w to zero at its endogenous debt capacity:[9]

$$\sigma_w(\underline{w}) = 0\,. \tag{43}$$

[9]Bolton, Wang, and Yang (2019) derive a similar boundary condition in a corporate finance continuous-time diffusion model in which the entrepreneur has inalienable human capital.

The intuition for this result is as follows. Suppose that the diffusion volatility $\sigma_w(w_t)$ evaluated at $w_t = \underline{w}$ is not zero. Then, over a small interval dt, the realized value of w_{t+dt} can cross the default boundary $\underline{w}$ with strictly positive probability in response to a small diffusive shock, triggering default. Such default is clearly inefficient since diffusive shocks can be hedged at an actuarially fair price. So, optimality requires $\sigma_w(\underline{w}) = 0$.

Substituting the zero-volatility condition (43) into equation (40), we obtain $\widetilde{\theta}(\underline{w}) = \underline{w}$, which is the effective diffusion risk hedging demand at $\underline{w}$. While this hedging strategy eliminates the volatility of w at $\underline{w}$, it does not in general eliminate the idiosyncratic volatility of *unscaled* consumption and *unscaled* certainty-equivalent wealth. In this sense, hedging is incomplete. We provide intuition for this incomplete-hedging result in Section 3.5 after describing the first-best and limited commitment solutions.

Finally, to ensure that w weakly moves toward zero and away from $\underline{w}$ in the absence of jumps, it is necessary to verify that $\mu_w(\underline{w}) \geq 0$. Substituting equation (43) into equation (39), we show that $\mu_w(\underline{w}) \geq 0$ is equivalent to the following constraint at $\underline{w} < 0$:

$$c(\underline{w}) \leq 1 + \mu \cdot (-\underline{w}) - [(r + \delta^*(\underline{w})) \cdot (-\underline{w}) + \lambda \mathbb{E}[\pi(Z)](-\underline{w} - b^*(\underline{w}))] \,. \tag{44}$$

Evaluating equation (42) at $\underline{w}$ and using $\sigma_w(\underline{w}) = 0$ and $p(\underline{w}) > 0$, we see that endogenous relative risk aversion, $\widetilde{\gamma}(w)$, approaches infinity, as $w \to \underline{w}$.

3.5 Summary

The following proposition summarizes the main properties of the solution.

Proposition 1 *The scaled certainty equivalent wealth in the normal ($p(w)$) and autarky ($\widehat{p}$) regimes satisfies the following two interconnected ODEs:*

$$\begin{aligned} 0 = &\left(\frac{\zeta (p'(w))^{1-\psi} - \psi\rho}{\psi - 1} + \mu - \frac{\gamma\sigma^2}{2} \right) p(w) + [\,(r + \delta^*(w) - \mu)\, w + 1 + \lambda \mathbb{E}[\pi(Z)](w + b^*(w))\,]\, p'(w) \\ &+ \frac{\gamma^2 \sigma^2 p(w) p'(w)}{2\,\widetilde{\gamma}(w)} + \frac{\lambda}{1-\gamma} \mathbb{E}\left[\left(\frac{Z p\left(w^{\mathcal{J}}\right)}{p(w)} \right)^{1-\gamma} - 1 \right] p(w)\,, \end{aligned} \tag{45}$$

$$0 = \frac{\rho \left[(a\,\widehat{p})^{-(1-\psi^{-1})} - 1 \right]}{1 - \psi^{-1}} + \mu + \frac{\lambda(\mathbb{E}(Z^{1-\gamma}) - 1)}{1-\gamma} - \frac{\gamma\sigma^2}{2} + \frac{\xi}{1-\gamma} \left[\left(\frac{p(0)}{\widehat{p}} \right)^{1-\gamma} - 1 \right], \tag{46}$$

where $w^{\mathcal{J}}$ is given by equation (41). When $w < \underline{w}$, the country defaults and hence

$$p(w) = \alpha\, \widehat{p}\,. \tag{47}$$

In addition, we have the following boundary conditions:

$$p(\underline{w}) = \alpha\, \widehat{p}\,, \tag{48}$$

$$p''(\underline{w}) = -\infty\,, \tag{49}$$

$$\lim_{w\to\infty} p(w) = w + h\,, \tag{50}$$

where h is given by

$$h = \frac{1}{r-g}\,. \tag{51}$$

The equilibrium credit spreads for foreign and domestic currency debt, $\delta(w_{t-}) = \delta^*(w_{t-})$, are given in (33). The country defaults when $\tau^{\mathcal{D}} = \inf\{t : w_t < \underline{w}\}$.

In the no-default region where $w \geq \underline{w}$, the following policy rules apply. The optimal consumption-output ratio, $c(w)$, is

$$c(w) = \zeta p(w)(p'(w))^{-\psi}\,, \tag{52}$$

where ζ is given by

$$\zeta = r + \psi\,(\rho - r)\ . \tag{53}$$

The scaled foreign currency debt issuance, b^*, maximizes

$$\max_{b^*\geq -w-\kappa} \quad [\delta^*(w)w + \lambda\mathbb{E}(\pi(Z))(w + b^*(w))]p'(w) + \frac{\lambda p(w)}{1-\gamma}\mathbb{E}\left(\frac{Zp\left(w^{\mathcal{J}}\right)}{p(w)}\right)^{1-\gamma}\ . \tag{54}$$

The scaled diffusion risk hedging demand, $\theta(w)$, is given by equation (37). The optimal default threshold, $\underline{Z}(w)$, is given by equation (31).

Equation (48) follows from the value-matching condition, (26). Equation (49) follows from the zero-volatility condition, (43) evaluated at $\underline{w}$, and $p(\underline{w}) > 0$. Equations (48) and (49) characterize the properties of the economy when $-w$ equals the country's debt capacity, $-\underline{w}$. Equation (50) states that, as $w \to \infty$, the effect of limited commitment wanes and $p(w)$ converges to $w + h$, the value of $p(w)$ in the first-best solution (see Section 4).

Equation (52) shows that consumption is a nonlinear function of w, which depends on both the certainty-equivalent wealth, $p(w)$, and its derivative, $p'(w)$. Later, we show that $p'(w) \geq 1$ and $p'(w)$ decreases with w. These properties imply that $c(w)$ is lower than the product of certainty-equivalent wealth $p(w)$ and the marginal propensity to consume in the first-best solution, ζ (i.e., $c(w) < \zeta p(w)$). Equation (54) determines the country's foreign currency debt issuance, $b^*(w)$.

4 First Best: Full Commitment and Full Spanning

Before discussing the quantitative properties of our model, we make a small digression to summarize the first-best (FB) solution to our model. This solution obtains when there is full commitment and full spanning. Full commitment means that the country always honors its contractual agreements, so it never defaults. Full spanning means that both diffusion and jump risks (for all values of Z) are hedgeable at actuarially fair prices. We use the superscript FB to denote the value of different variables in the FB solution.

As in Friedman (1957) and Hall (1978), we define non-financial wealth, H_t, for the case in which all risks are hedgeable as the present value of output, discounted at the constant risk-free rate, r,

$$H_t = \mathbb{E}_t \left(\int_t^\infty e^{-r(u-t)} Y_u du \right) . \tag{55}$$

Because Y is a geometric jump-diffusion process, we have $H_t = hY_t$. Scaled non-financial wealth, h, is given by equation (51). The expected growth rate of output, g, is given by equation (2). To ensure that non-financial wealth is finite, we require that $r > g$. This convergence condition is standard in asset-pricing and valuation models.

Let $P_t^{FB} \equiv P^{FB}(W_t, Y_t)$ denote the country's certainty-equivalent wealth, defined in equation (20), for the FB case. We show below that P_t^{FB} is equal to

$$P_t^{FB} \equiv P^{FB}(W_t, Y_t) = W_t + hY_t \, . \tag{56}$$

In other words, in the FB case, certainty-equivalent wealth coincides with total wealth, defined as the sum of financial wealth W_t and non-financial wealth H_t.

The following proposition summarizes the properties of the FB solution.

Proposition 2 *Scaled total wealth, $p^{FB}(w) = P^{FB}(W, Y)/Y = (W + H)/Y$, is*

$$p^{FB}(w) = w + h \, , \tag{57}$$

where h is given by equation (51) and $w_t \geq \underline{w}^{FB}$. The scaled endogenous debt capacity is $-\underline{w}^{FB} = h$. The optimal consumption-output ratio, $c_t = c^{FB}(w)$, is given by

$$c^{FB}(w) = \zeta\, p^{FB}(w) = \zeta(w + h)\,, \tag{58}$$

where ζ, the marginal propensity to consume (MPC) in the FB case, is given by equation (53). There is no default, meaning $\underline{Z} = 0$. The endogenous relative risk aversion defined in equation (35), $\widetilde{\gamma}_t$, is equal to γ for all t.

5 Quantitative Results

To explore the quantitative properties of our model, we calibrate it with the eleven parameter values reported in Table 1. We divide these parameters into two groups. The seven parameters in the first group are set to values that are standard in the literature. The four parameters in the second group are calibrated to match key features of data for Argentina.

5.1 Baseline Calibration

We first describe the parameters drawn from the literature.

Parameters from the Literature. Following Aguiar and Gopinath (2006), we set the coefficient of relative risk aversion (γ) to 2, the annual risk-free rate (r) to 4 percent, and the rate at which the country exits autarky (ξ) to 0.25 per annum. This choice of ξ implies that after default, the country stays in autarky for four years on average. This implication is consistent with the empirical estimates reported in Gelos, Sahay, and Sandleris (2011).

Following Barro (2009), we set the annual subjective discount rate (ρ) to 5.2 percent. Since $\rho > r$, the country wants to borrow to front-load consumption, holding everything else constant. This ability to front-load consumption is lost when the country defaults and enters autarky.

As in the rare-disasters literature, we assume that the cumulative distribution function of the recovery fraction, $F(Z)$, is governed by a power law:

$$F(Z) = Z^{\beta}\,, \quad 0 \leq Z \leq 1\,. \tag{59}$$

We choose $\beta = 6.3$, which is the point estimate obtained by Barro and Jin (2011). Since large disasters are rare, Barro and Jin (2011) obtain these estimates by pooling long time

Table 1: Parameter Values

Parameters	Symbol	Value
risk aversion	γ	2
subjective discount rate	ρ	5.2%
risk-free rate	r	4%
jump arrival rate	λ	0.073
power law parameter	β	6.3
autarky exit rate	ξ	0.25
financial development parameter	κ	10%
Parameters chosen to target observables		
output drift (in the absence of jumps)	μ	2.7%
output diffusion volatility	σ	4.5%
output recovery post default	α	97%
elasticity of intertemporal substitution	ψ	0.025
Targeted observables		
average output growth rate	g	1.7%
output growth volatility		6.85%
average debt-output ratio		15%
unconditional default probability		3%

All parameter values, whenever applicable, are continuously compounded and annualized.

series for different countries. Barro and Jin (2011) also estimate the probability of disasters to be 3.8 percent per annum. This estimate is based on their definition that a disaster is a macroeconomic contraction (measured in consumption or output) that exceeds 10 percent. Since the stochastic recovery fraction upon a jump arrival, Z, can take values from zero to one, our estimate of λ is given by $\lambda F(0.9) = \lambda F(0.9) = \lambda \times 0.9^{\beta} = 0.038$ with $\beta = 6.3$. We obtain an annual jump arrival rate, $\lambda = 0.073$. For simplicity, we assume that $\pi(Z) = 1 - Z$.

Calibrated Parameters from Argentinean Data. We set the financial development parameter κ to 10 percent. This choice results from the following calculations. Coppola, Maggiori, Neiman, and Schreger (2020) report that the share of gross Latin American domestic currency debt that foreigners hold is relatively low: 22 percent in Argentina, 23 percent in Chile, and 33 percent in Brazil and Mexico. During the period from 1992 to 2001, Argentina's gross domestic debt fluctuated between 25 and 48 percent of GDP. Combining a

domestic gross debt of 48 percent of GDP with a foreign ownership share of 22 percent yields a value of κ of about 10 percent.

Next, we choose the parameters that control the drift in the absence of jumps (μ), the diffusion volatility (σ), the default distress cost ($1-\alpha$), and the elasticity of intertemporal substitution (ψ) to target the following four moments estimated with Argentinean data: an average growth rate of output of 1.7 percent per annum, a standard deviation of the growth rate of output of 6.8 percent, an average debt-to-GDP ratio of 15 percent, and an unconditional default probability of 3 percent per annum. We obtain estimates of the average and standard deviation of the annual growth rate of real GDP for Argentina using Barro and Ursua's (2008) data for the period from 1876 to 2009.

Our model consolidates the expenditure and borrowing decisions of the private sector and the government. For this reason, we calibrate it to match the ratio of net debt to GDP. In Argentina, as in most countries, a significant fraction of government debt is owned by the domestic private sector, so debt held by foreigners is much smaller than the country's total debt. We compute our target for the debt-to-output ratio by calculating the difference between Argentina's debt liabilities and debt assets using the data compiled by Lane and Milesi-Ferretti (2007) for the period from 1970 to 2011. The average net debt-to-GDP ratio during this period is 15 percent. Since defaults are rare, it is helpful to use as much data as possible to estimate the probability of default. Argentina defaulted six times in roughly 200 years, so we target an annual default probability of 3 percent.[10]

Our calibration procedure yields the following parameter values: $\psi = 0.025$, $\mu = 2.7$ percent per annum, $\sigma = 4.5$ percent per annum, and $\alpha = 97$ percent. The calibrated value of α implies that the direct cost of defaulting is 3 percent of output. This cost of default is conservative relative to the estimates reported by Hébert and Schreger (2017) for Argentina.

In this calibration, the value of the EIS ($\psi = 0.025$) is low, so the representative agent has a strong preference for smooth consumption paths.[11]

We can interpret the low value of the EIS as resulting from rigidities in spending patterns and expenditure commitments that are difficult to change.

[10] Argentina defaulted in 1830, 1890, 1915, 1930, 1982, and 2001. See Sturzenegger and Zettlemeyer (2006) for a discussion.

[11] There is currently no consensus on what are empirically plausible values for the EIS (see Attanasio and Weber (2010) for a discussion). Our choice is consistent with Hall (1988), who argues that the elasticity of intertemporal substitution is close to zero. It is also consistent with the recent estimates by Best, Cloyne, Ilzetzki, and Kleven (2017), which are based on mortgage data.

5.2 Debt Intolerance

We now discuss how a country's average debt-output ratio, average default probability, and debt capacity vary with financial development, κ. We fix all other parameters at the values reported in Table 1.

Table 2: The effect of financial development, κ

κ	average debt-output ratio	default probability	debt capacity $\lvert\underline{w}\rvert$
10%	**14.5%**	**2.9%**	**18.4%**
20%	18.4%	0.6%	23.4%
50%	42.0%	0	42.4%

All parameter values other than κ are reported in Table 1.

Table 2 illustrates the key result in our paper: the higher is financial development, κ, the higher are debt capacity and the average debt-output ratio and the lower is the probability of default. As discussed above, the intuition for these results is that domestic currency debt is a natural hedge against disaster risk. A country that can issue more domestic currency debt has a greater ability to manage its disaster risk, takes on more debt, and defaults less.

5.3 Quantitative Results and Economic Mechanisms

In this subsection, we use the calibration described above to explore the quantitative properties of our model. Figures 1, 2, and 3 illustrate these properties for four levels of financial development: $\kappa = 10$, 20, 50, and 100 percent. In our benchmark calibration, we set $\kappa = 10$ percent, which means that the country can issue domestic currency debt only up to 10 percent of its output.

It is useful to first discuss the FB solution. In this solution, the country fully uses its debt capacity, which is the present discounted value of output, $h = 1/(r - g)$, and never defaults. Given our calibration, the country borrows 4,348 percent of its current output in the FB solution, an implication that is clearly unrealistic.

Certainty-equivalent wealth, Marginal Certainty-equivalent Value of Wealth, and Consumption. Improving financial development by increasing κ from 10 to 50 percent has

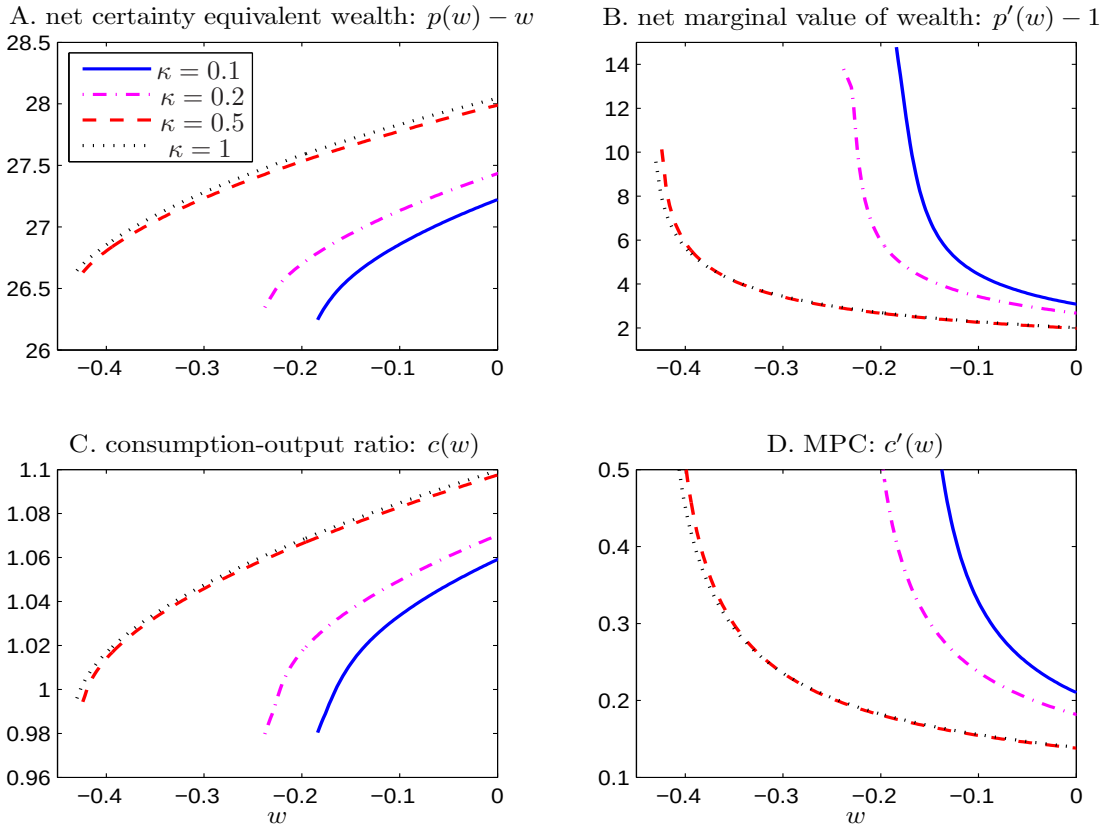

Figure 1: Scaled certainty-equivalent wealth, $p(w)$, marginal certainty-equivalent value of wealth, $p'(w)$, consumption-output ratio, $c(w)$, and $c'(w)$, for four levels of financial development: $\kappa = 10$, 20, 50, and 100 percent.

a large impact on debt capacity, average debt-output ratio, and default probability: debt capacity rises from 18.4 to 42.4 percent of output, the average debt-output ratio increases from 15 to 42 percent, and the probability of default drops from 3 percent to zero.

Further improving financial development by increasing κ from 50 to 100 percent has a negligible impact on debt capacity and average debt-output ratio. The reason for this small impact is that the ability to issue domestic currency debt up to 50 percent of output already provides the country with a sizable hedge against disaster shocks.

Panels A and B of Figure 1 display the scaled certainty-equivalent wealth, $p(w)$, and the marginal certainty-equivalent value of wealth, $P_W(W, Y) = p'(w)$, respectively. The function $p(w)$ is increasing and concave, which implies that $p'(w)$ is decreasing in w and $p'(w)$ is greater than one.[12] Panels C and D display the consumption-output ratio, $c(w)$, and the

[12]Wang, Wang, and Yang (2016) derive similar properties for certainty-equivalent wealth in a self-insurance

MPC out of wealth, $c'(w)$, respectively. The function $c(w)$ is increasing and concave, which implies that $c'(w)$ is decreasing in w. As w goes to infinity, $p(w)$ approaches $p^{FB}(w) = w + h$, $p'(w)$ approaches one, $c(w)$ approaches $c^{FB}(w) = \zeta(w + h)$, and $c'(w)$ approaches the MPC obtained in the FB, $\zeta = 0.0403$.

We see that the higher is financial development, κ, the higher is $p(w)$. The ability to issue more domestic currency debt, which is a natural hedge against disaster risk, increases the country's debt capacity, $|\underline{w}|$. As a result, the marginal certainty-equivalent value of wealth, $p'(w)$, is lower. Consumption is higher because both a higher $p(w)$ and a lower $p'(w)$ cause $c(w)$ to be higher (see equation (52)). Consider $w = -15$ percent, which is the average debt-to-output ratio in our baseline calibration. The net marginal certainty-equivalent value of wealth, $p'(-0.15) - 1$, is 7.92 in the economy with $\kappa = 10$ percent. This value is 130 percent higher than the value of $p'(-0.15) - 1$ in the economy with $\kappa = 50$ percent.

The MPC out of wealth, $c'(-0.15)$, is equal to 63.4 percent in the economy with $\kappa = 10$ percent. This value is 382 percent higher than the value of $c'(-0.15)$ in the economy with $\kappa = 50$ percent.

Domestic and Foreign Currency Debt, Default Threshold, and Credit Spreads. Panels A and B of Figure 2 plot, as a function of w, the dollar value of domestic and foreign currency debt, $b(w)$ and $b^*(w)$, respectively. Consider the case in which $\kappa = 0.10$. In this case, the country's debt capacity is $-\underline{w} = 18.4$ percent. The country issues domestic currency debt, $b(w) = 0.1$, and saves $(0.1 + w) > 0$ in foreign currency assets in the region where $w \in (-0.1, 0)$. In the region where $w \in (-\underline{w}, -0.1) = (-0.184, -0.1)$, the country issues foreign currency debt, $b^*(w) = -(w + 0.1) > 0$, and exhausts its domestic currency debt issuance capacity ($\kappa = 0.1$).

With $\kappa = 0.5$, the country only issues domestic currency debt, $b(w) = 0.5$, for all levels of w up to its debt capacity, $-\underline{w} = 0.424$, and saves the amount $-(w + 0.424) > 0$ in foreign currency assets.

With $\kappa = 1$, the financial development constraint (10) binds in the region where $w \in (-0.4, 0)$ but does not bind in the region where $w \in (-0.43, -0.4)$. The reason is that the "implicit" jump insurance premium from using domestic currency debt, which is the domestic currency appreciation that occurs absent jumps, becomes much more costly when the country is near its debt capacity (very large $p'(w)$).

model in which labor income shocks are uninsurable and the agent can only save via a risk-free asset.

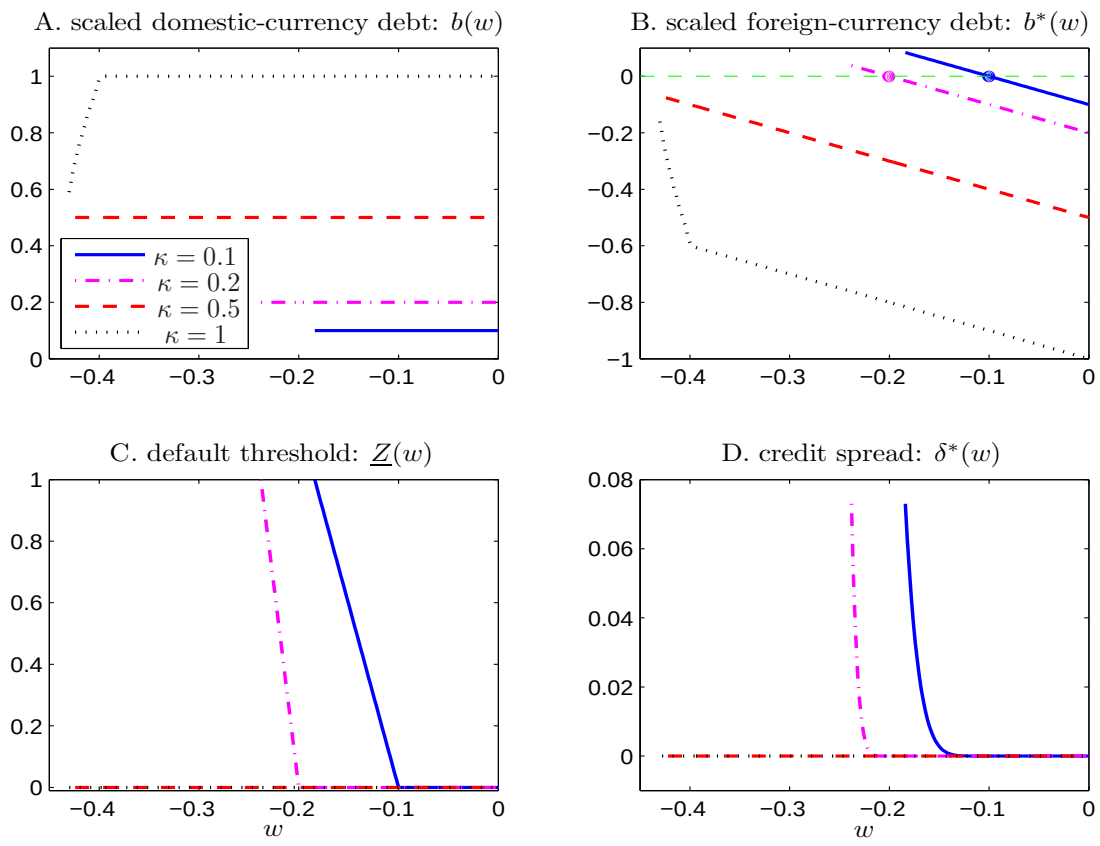

Figure 2: Scaled foreign currency debt, $b^*(w)$, scaled currency domestic debt, $b(w) = -(w + b^*(w))$, default boundary, $\underline{Z}(w)$, and credit spread, $\delta^*(w)$, for four levels of financial development: $\kappa = 10$, 20, 50, and 100 percent.

Panels C and D plot the optimal default threshold, $\underline{Z}(w)$, and the credit spread $\delta^*(w)$, respectively. Low values of κ are associated with debt intolerance. Debt capacity is lower, default is more likely, and the credit spread, $\delta^*(w)$, is higher for a given level of w.

The country never defaults when $\kappa \geq |\underline{w}|$ because all its debt is denominated in domestic currency, which is a natural hedge against disaster risk. For example, when $\kappa = 50$ percent, the country's debt capacity is 42.4 percent. The country never defaults, so its credit spread is zero.

Diffusion Hedging Demand, Drift and Volatility of w, and Endogenous Risk Aversion. Panel A of Figure 3 shows that the scaled effective hedging demand, $\widetilde{\theta}(w)$, is negative and that its absolute value increases with w. That is, a less indebted country hedges more diffusive risk. Hedging and financial wealth are complements. The higher is the level of

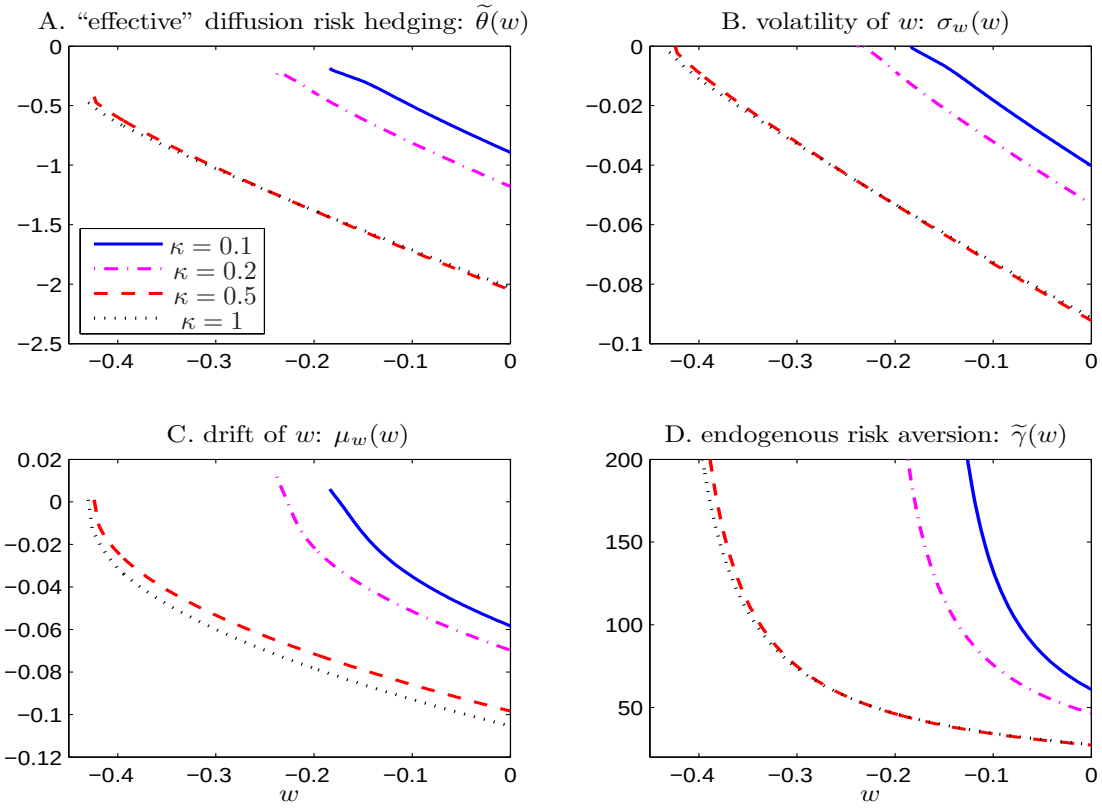

Figure 3: The effective diffusion risk hedging, $\widetilde{\theta}(w)$, volatility of w, $\sigma_w(w)$, drift of w, $\mu_w(w)$, and endogenous risk aversion, $\widetilde{\gamma}(w)$, for four levels of financial development: $\kappa = 10$, 20, 50, and 100 percent.

financial development, the more the country hedges for a given value of w. So hedging and financial development are also complements. Even though the country incurs no up-front cost to hedge diffusion shocks, it is not optimal to fully hedge the diffusion risk.

Panel B plots the volatility function, $\sigma_w(w)$. Because a less indebted country has a higher $p(w)$ and a lower endogenous relative risk aversion, $\widetilde{\gamma}(w)$, the absolute value of $\sigma_w(w)$ increases with w (see equation (42)). In the limit, as $w \to \underline{w}$, the absolute value of σ_w reaches zero, $\sigma_w(\underline{w}) = 0$. The intuition for this property, which is visible in panel B, is that it is inefficient for the country to use default to manage diffusive shocks. Since these shocks do not trigger default, $\sigma_w(\underline{w}) = 0$.

Panel C shows the drift function for w, $\mu_w(w)$, which is negative for most values of w. This result follows from two observations. First, the country's consumption is often larger than output (see Figure 1). Second, interest payments for domestic and foreign currency

debt drain the country's financial wealth. Both forces increase the country's average debt level. However, as debt approaches debt capacity, $-\underline{w}$, the country voluntarily adjusts its consumption, diffusion hedging demand, and domestic and foreign currency debt issuances so that $\mu_w(\underline{w}) \geq 0$. The property $\mu_w(w) \geq 0$ together with $\sigma_w(\underline{w}) = 0$ are necessary to ensure that the country does not default in response to diffusive shocks.

Panel D shows the behavior of endogenous risk aversion, $\widetilde{\gamma}(w)$. We see that the less indebted is the country, the lower is its endogenous risk aversion. In addition, the higher is the level of financial development, the lower is its endogenous risk aversion. As w approaches the lower bound, $\underline{w}$, $\widetilde{\gamma}(w)$ approaches infinity. This result follows from equation (42), $p(\underline{w}) > 0$, and the zero-volatility condition at the boundary, $\sigma_w(\underline{w}) = 0$.

6 Domestic Currency Debt versus Hedging Contracts

So far, we have discussed the role of domestic currency debt as a natural hedge against jump shocks. An alternative approach to managing rare-disaster risk, emphasized by Cantú and Chui (2020), is to use state-contingent hedging contracts.

In this section, we compare two economies. The first economy hedges its disaster risk using domestic currency debt and has a sufficiently high level of financial development, κ, so that the financial development constraint, (10), does not bind.

The second economy hedges its disaster risk with a set of full-spanning, state-contingent hedging contracts that are actuarially fair. This economy has limited commitment and full spanning, so it corresponds to the case considered by Kehoe and Levine (1993) and Kocherlakota (1996). In this economy, it is more cost effective to manage risk by hedging than by using default. So, the country never defaults and its credit spread is zero. The country's debt capacity is reduced to a level such that, in equilibrium, the country weakly prefers repaying its outstanding debt over defaulting on it.[13]

We find that the full-spanning economy has higher certainty-equivalent wealth than the domestic currency economy, but this difference is not large. The domestic currency economy hedges less diffusion and jump risk than the full-spanning economy. The full-spanning economy hedges the risk of large disasters much more than the domestic currency economy.

[13]Other work that emphasizes the importance of limited commitment includes Alvarez and Jermann (2000, 2001), Kehoe and Perri (2002), Albuquerque and Hopenhayn (2004), Cooley, Marimon, and Quadrini (2004), Krueger and Perri (2006), Krueger and Uhlig (2006), Chien and Lustig (2010), and Lustig, Syverson, and Van Nieuwerburgh (2011).

Jump Insurance Contracts and Insurance Premium Payments. We assume that jump shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. Bulow and Rogoff (1989) define a "cash-in-advance" contract as a "conventional insurance contract under which a country makes a payment up front in return for a state contingent, nonnegative future payment." Following Bulow and Rogoff (1989), we consider an insurance contract initiated at time t that covers the following jump event: the *first* stochastic arrival of a downward jump in output with a recovery fraction in the interval $(Z, Z + dZ)$ at jump time $\tau^{\mathcal{J}} > t$.

The buyer of a unit of this insurance contract makes continuous insurance premium payments. Once the jump event occurs at time $\tau^{\mathcal{J}}$, the buyer stops making payments and receives a one-time *unit* lump-sum payoff. The insurance premium payment is equal to $\lambda dF(Z)$, the product of the jump intensity, λ, and the probability $dF(Z)$ that the recovery fraction falls inside the interval $(Z, Z + dZ)$. Conceptually, this insurance contract is analogous to one-step-ahead Arrow securities in discrete-time models. In practice, this insurance contract is similar to a credit default swap.[14]

We denote the country's holdings of jump-risk insurance contracts at time t contingent on a recovery fraction Z by $X_t(Z)$. The country pays an insurance premium to hedge jump risk at a rate $X_t(Z)\lambda\, dF(Z)$ before the *first* jump with recovery fraction Z arrives at time $\tau^{\mathcal{J}}$. At this time, the country receives a lump-sum payment $X_t(Z)$ if the recovery fraction is in the interval $(Z, Z + dZ)$. The total jump insurance premium payment per unit of time is given by

$$\Phi_t = \lambda \int_0^1 X_t(Z) dF(Z) \equiv \lambda\, \mathbb{E}\left[X_t(Z)\right], \tag{60}$$

where the expectation, $\mathbb{E}[\,\cdot\,]$, is calculated with respect to the cumulative distribution function, $F(Z)$.

In the normal regime, financial wealth, W_t, evolves according to

$$dW_t = [(r + \delta^*_{t-})W_{t-} + Y_{t-} - C_{t-} - \Phi_{t-}]dt + \sigma\Theta_{t-}d\mathcal{B}_t + X_{t-}(Z)\, d\mathcal{J}_t - \mathbb{1}^{\mathcal{D}}_t W_{t-} d\mathcal{J}_t\,. \tag{61}$$

The jump insurance premium payment, Φ_{t-}, is deducted in the first term in equation (61). The last two terms reflect the effect of a jump arrival. The first term is the jump-insurance payment to the country, $X_{t-}(Z)$, for the insurance purchased at $t-$. The second term captures the effect of default.

[14] Pindyck and Wang (2013) discuss a similar insurance contract in a general equilibrium setting with economic catastrophes.

The value function $V(W,Y)$ in the normal regime satisfies the following HJB equation:

$$\begin{aligned}0 = \max_{C,\Theta}\ & f(C,V(W,Y)) + \left[(r+\delta^*)W + Y - C - \Phi\right] V_W(W,Y) \\ & + \frac{\Theta^2\sigma^2}{2} V_{WW}(W,Y) + \mu Y V_Y(W,Y) + \frac{\sigma^2 Y^2}{2} V_{YY}(W,Y) + \Theta\sigma^2 Y V_{WY}(W,Y) \\ & + \lambda\mathbb{E}\left[V(W+X,ZY)\mathbb{1}_{Z\geq\underline{Z}} + \widehat{V}(\alpha ZY)\mathbb{1}_{Z<\underline{Z}} - V(W,Y)\right]. \end{aligned} \tag{62}$$

If $Z < \underline{Z}$, the country defaults and enters autarky. Its output falls to $\widehat{Y}_t = \alpha Y_t$, where $Y_t = ZY_{t-}$, so the value function at $t = \tau^{\mathcal{D}}$ is $\widehat{V}(\widehat{Y}_t) = \widehat{V}(\alpha ZY_{t-})$.

First-Order Conditions (FOCs). The FOCs for consumption and diffusion risk hedging are the same as in the benchmark model. The optimal jump-risk hedging demand for each value of Z, $X_{t-}(Z) \equiv X(Z; W_{t-}, Y_{t-})$, solves the following problem

$$\max_{X_{t-}(Z)} \quad V(W_{t-} + X_{t-}, ZY_{t-}) - X_{t-}V_W(W_{t-}, Y_{t-}). \tag{63}$$

Equation (63) applies for all values of Z, and this flexibility in choosing Z-contingent hedging demand $X_{t-}(Z)$ enhances the country's ability to manage risk, creating value.

The FOC for $X(Z;W,Y)$ is

$$V_W(W + X(Z;W,Y), ZY) = V_W(W,Y). \tag{64}$$

Without jump insurance, output falls upon a jump arrival, $V_W(W,Y) < V_W(W,ZY)$. The country chooses $X(Z;W,Y) > 0$ to equate the pre- and post-jump marginal utility of wealth.

The homogeneity property allows us to solve $p(w)$ using the following equation:

$$\begin{aligned}0 = & \left(\frac{\zeta(p'(w))^{1-\psi} - \psi\rho}{\psi - 1} + \mu - \frac{\gamma\sigma^2}{2}\right) p(w) + \left[\,(r + \delta^*(w) - \mu)\, w + 1 - \lambda\mathbb{E}[x(Z,w)]\,\right] p'(w) \\ & + \frac{\gamma^2\sigma^2 p(w)p'(w)}{2\,\widetilde{\gamma}(w)} + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp\left(w^{\mathcal{J}}\right)}{p(w)}\right)^{1-\gamma} - 1\right] p(w), \end{aligned} \tag{65}$$

where $x(Z,w) = X(Z;W,Y)/Y$ is the scaled jump-risk hedging demand,

$$w_t^{\mathcal{J}} = \frac{w_{t-} + x(Z,w)}{Z}, \tag{66}$$

and the FOC for $x(Z,w)$ is

$$\frac{p'(w)}{p'(w^{\mathcal{J}})} = \left(\frac{Zp(w^{\mathcal{J}})}{p(w)}\right)^{-\gamma}. \tag{67}$$

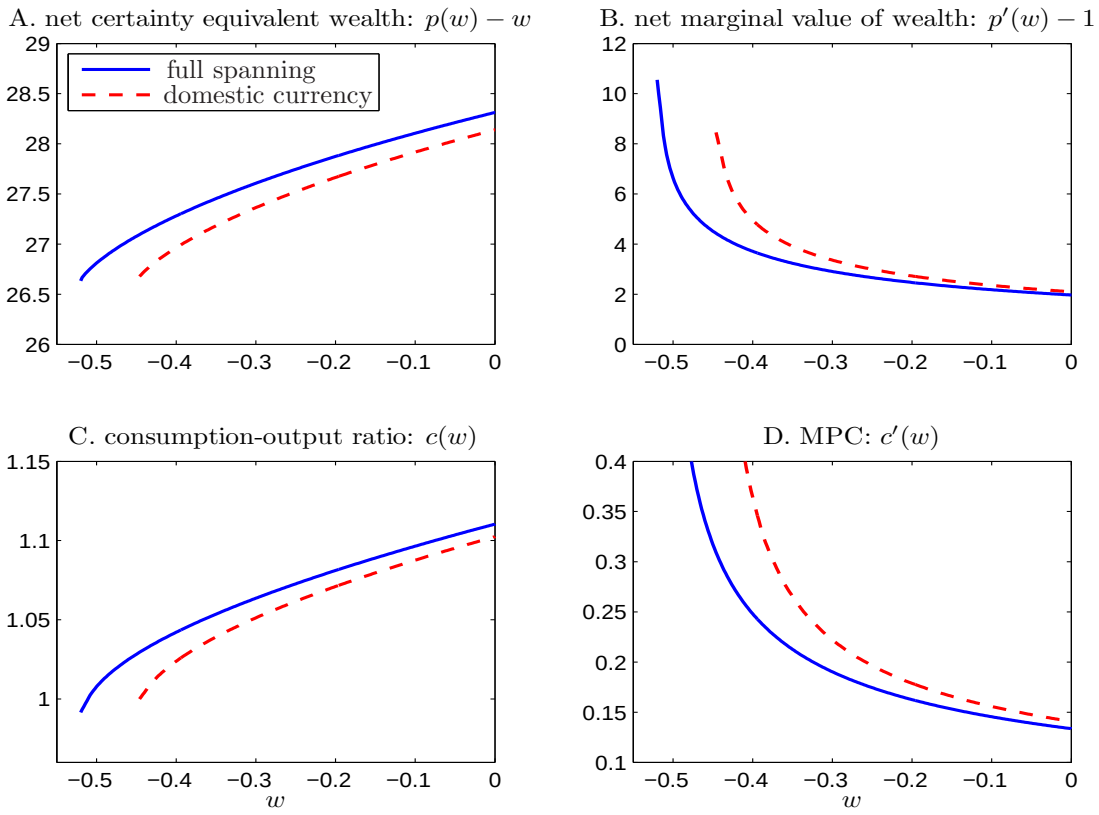

Figure 4: Scaled certainty-equivalent wealth, marginal certainty-equivalent value of wealth, $p(w)$, $p'(w)$, consumption-output ratio, $c(w)$, and MPC, $c'(w)$, for the domestic currency and full-spanning economies. The debt capacity is 44.6 percent for the domestic currency economy and 52 percent for the full-spanning economy. All other parameters are reported in Table 1.

To compare the two economies, stating the following result is useful. The domestic currency economy is observationally equivalent to a full-spanning economy that is constrained to using a hedging policy where $x(Z, w)$ is proportional to the percentage depreciation of domestic currency upon a jump arrival, $\pi(Z)$, where the constant of proportionality is the level of domestic currency debt:

$$x(Z, w_{t-}) = b(w_{t-})\pi(Z)\,. \tag{68}$$

The corresponding scaled jump insurance premium is $\phi_{t-} = \lambda b(w_{t-})\mathbb{E}[\pi(Z)]$. This premium reflects the domestic currency appreciation that occurs in non-disaster states. This appreciation makes domestic currency debt more costly to service in U.S. dollars in these states.

Panel A of Figure 4 shows that the full-spanning economy has higher net certainty-

equivalent wealth (e.g., for $w = 0$, $p(0) = 28.31$) than the domestic currency economy (e.g., for $w = 0$, $p(0) = 28.14$). As we can see, this difference is modest (less than 0.6 percent for $p(0)$). Recall that $p(0) = 27.22$ for the domestic currency economy with $\kappa = 0.1$. Improving financial development from $\kappa = 0.1$ up to the point where the domestic currency debt issuance constraint, (10), is not binding increases welfare by about 3.3 percent.

Figure 4 also shows that, for a given w, the full-spanning economy has higher debt capacity, lower marginal value of wealth, higher consumption-output ratio, and lower MPC than the domestic currency economy. These results are consistent with our intuition that improving risk management opportunities makes the country better off.

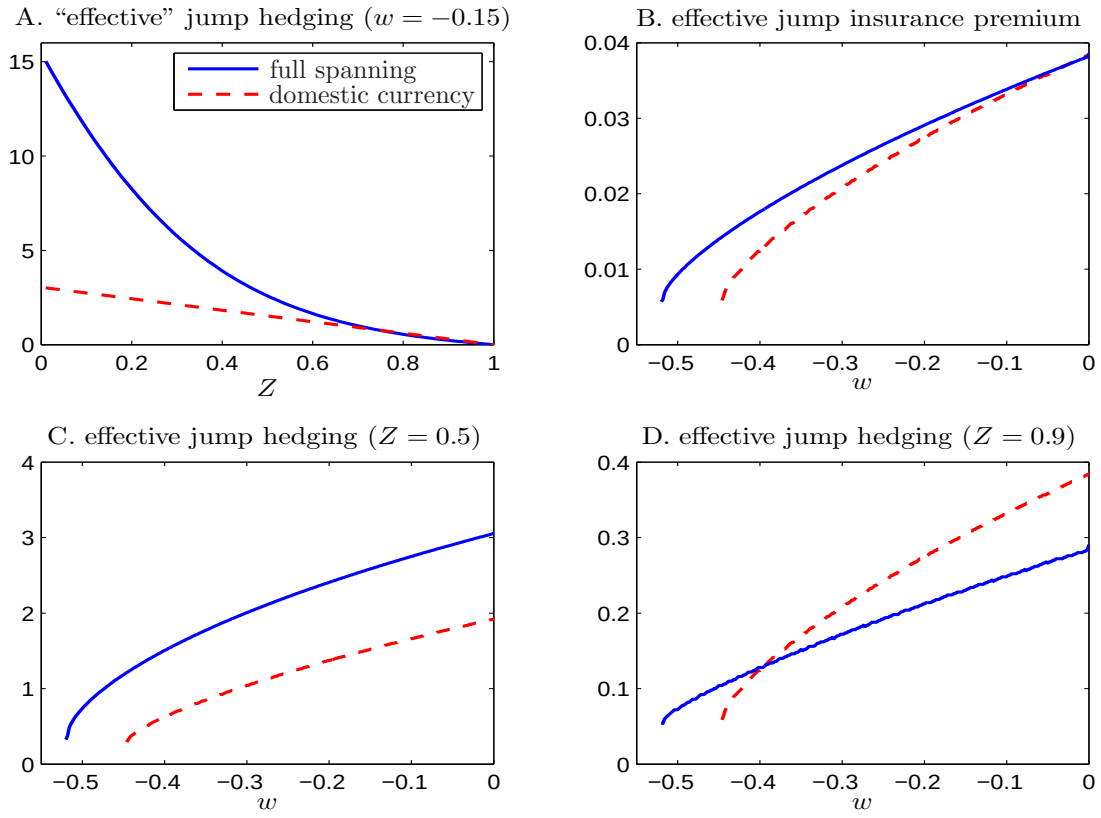

Figure 5: The effective jump-risk hedging demand and insurance premium for the domestic currency and full-spanning economies. The debt capacity is 44.6 percent for the domestic currency economy and 52 percent for the full-spanning economy. All other parameters are reported in Table 1.

Panel A of Figure 5 compares the state-contingent jump hedging policy in the full-spanning economy to the effective hedging demand delivered by domestic currency debt

issuance (equation (68)). We see that the two policies are similar for small jump shocks. However, the optimal hedging policy in the full-spanning economy involves much higher hedging of large disasters (low-Z states).

Panel B of Figure 5 shows the effective jump hedging costs ($\phi_{t-} = \lambda b(w_{t-})\mathbb{E}[\pi(Z)]$) for the domestic currency economy. This cost is the extra interest payment in U.S. dollars on domestic currency debt to compensate for the stochastic depreciation of domestic currency upon a jump arrival. The jump insurance premium payments in the full-spanning economy are higher than the effective jump hedging costs in the domestic currency economy.

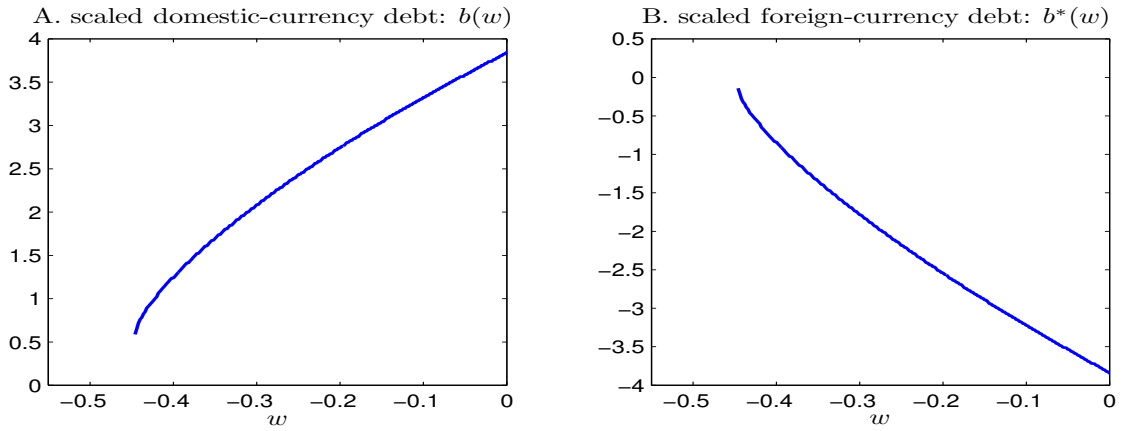

Figure 6: Scaled foreign currency debt, $b^*(w)$ and scaled domestic currency debt, $b(w) = -(w + b^*(w))$. All other parameters are reported in Table 1.

Panel C of Figure 5 compares the hedging demand for a $Z = 0.5$ disaster in the domestic currency and full-spanning economies for different levels of w. This panel shows that for large disasters, the domestic currency economy hedges less than the full-spanning economy. Also, for both economies, hedging decreases as the country gets closer to its debt capacity.

Panel D of Figure 5 contains the analogous information for a relatively small disaster ($Z = 0.9$). We see that the domestic currency economy hedges more than the full-spanning economy when w is close to the origin but hedges less when w is close to the debt capacity. This property reflects the constraint that the effective jump hedging demand implied by the issuance of domestic currency debt has to be proportional to $\pi(Z) = 1 - Z$. So, it is not possible to increase hedging against the risk of large disasters without also increasing hedging against the risk of small disasters. Since the country has only one instrument, $b(w_{t-})$, it cannot choose how much to hedge each jump, Z. So, it has to balance overhedging small-

disaster states with underhedging large-disaster states. The net effect is that the domestic currency economy spends less to hedge. As a result, this economy is more exposed to disaster shocks.

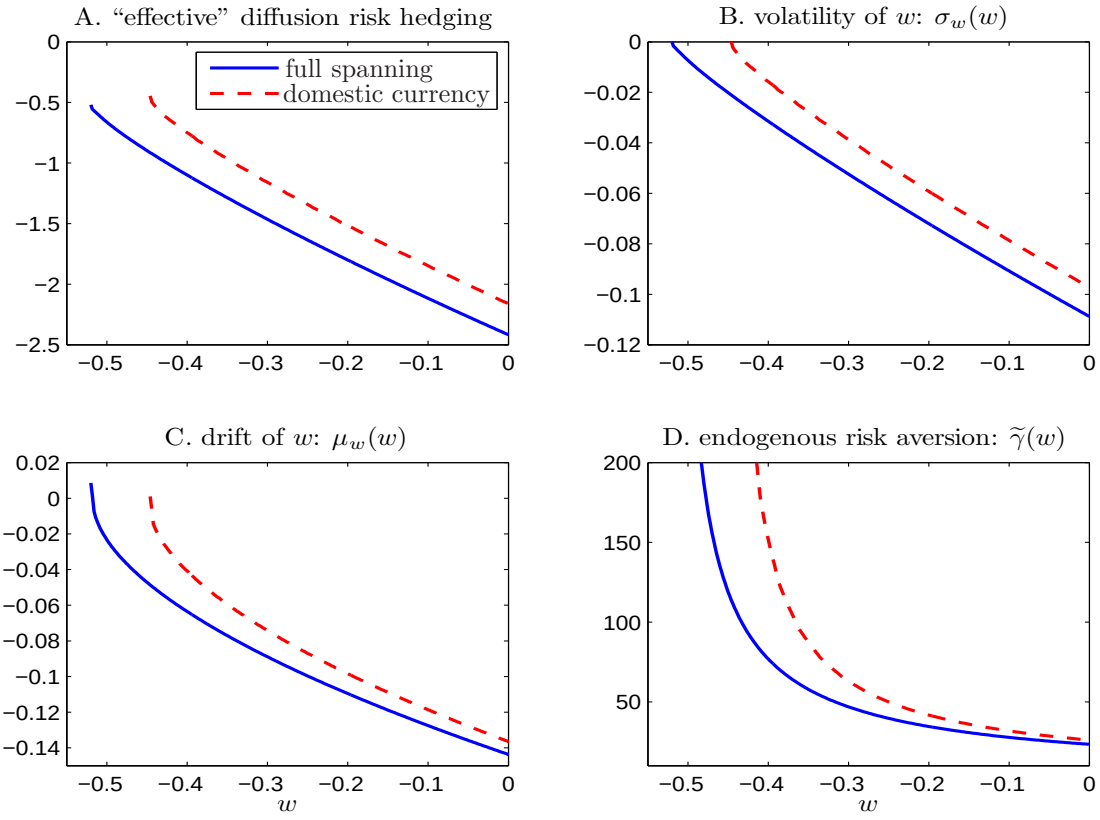

Figure 7: Effective diffusion risk hedging, $\theta(w)$, volatility and drift of w, and endogenous risk aversion, $\widetilde{\gamma}(w)$, for the domestic currency and full-spanning economies. The debt capacity is 44.6 percent for the domestic currency economy and 52 percent for the full-spanning economy. All other parameters are reported in Table 1.

Figure 6 plots the optimal debt policy for the domestic currency economy. Recall that financial development is such that constraint (10) does not bind. The economy issues domestic currency debt and saves in the form of foreign currency bonds. As discussed before, a country with high net debt (more negative w) issues less domestic currency debt. That is, it effectively hedges its disaster risk less because, as the country approaches its debt capacity, the marginal value of wealth is very high (Figure 6), so hedging is more costly.

Figure 7 shows that the diffusion risk hedging demand is larger in the full-spanning economy than in the domestic currency economy. The intuition for this result is that the

full-spanning economy can better manage its risk by choosing how much to hedge each value of Z. As a result, its debt capacity is larger, and the endogenous risk aversion for each given w is lower. The drift of w is also lower. The country can manage risk better, and hence it can take on more debt to smooth consumption. In the full-spanning economy, there is no equilibrium default, as in Kehoe and Levine (1993), because using default is more costly than hedging risk with contingent contracts. In this domestic currency economy in which the financial development constraint does not bind, there is no default either.

7 Credit Risk Premium

In this section, we relax the assumption that all shocks are idiosyncratic and generalize our model to incorporate a credit risk premium. We model credit risk along the lines of Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), and Borri and Verdelhan (2015). To simplify, we assume that realizations of the recovery fraction, Z, and diffusion shocks are idiosyncratic. As a result, credit risk results from jump-arrival timing risk.

Model Setup and Solution. As in the credit risk literature (see, e.g., Duffie and Singleton, 2012), we model the jump-arrival timing risk by working under the risk-neutral measure, $\mathbb{Q}$. We assume that the risk-neutral jump-arrival intensity, $\lambda^{\mathbb{Q}}$, is larger than the physical jump-arrival intensity, λ. This assumption captures the idea that well-diversified, risk-averse investors demand a premium for downward jump arrivals. As a result, they perceive these jump arrivals as more likely under $\mathbb{Q}$ than under the physical measure, $\lambda^{\mathbb{Q}}/\lambda > 1$.

The higher the ratio $\lambda^{\mathbb{Q}}/\lambda$, the higher the default risk premium. Given that the realizations of the recovery fraction, Z, and diffusion shocks are idiosyncratic, their distributions are the same under the risk-neutral and physical measures.

The Exchange Rate Process. Since foreign investors price the domestic currency, we assume that the exchange rate process is a martingale under the risk-neutral measure, $\mathbb{Q}$,

$$\frac{dS_t}{S_{t-}} = \sigma_S d\mathcal{B}_t - \pi(Z) d\mathcal{J}_t^{\mathbb{Q}} + \lambda^{\mathbb{Q}} \mathbb{E}[\pi(Z)] dt \,. \tag{69}$$

Under the physical measure,

$$\frac{dS_t}{S_{t-}} = \sigma_S d\mathcal{B}_t - \pi(Z) d\mathcal{J}_t + \lambda^{\mathbb{Q}} \mathbb{E}[\pi(Z)] dt \,, \tag{70}$$

where $\mathcal{J}_t$ is a pure jump process with arrival rate λ. When a jump shock arrives at time t, the exchange rate changes from S_{t-} to $S_t = (1-\pi(Z))S_{t-}$. The expected currency appreciation is $\mathbb{E}_{t-}(dS_t/S_{t-}) = \left(\lambda^{\mathbb{Q}} - \lambda\right)\mathbb{E}[\pi(Z)]dt$, which is positive given that $\lambda^{\mathbb{Q}} > \lambda$.

Under these assumptions, the competitive market zero-profit condition for diversified risk-averse investors implies that the credit spread for foreign currency debt, $\widetilde{\delta}^*$, satisfies

$$B^*_{t-}(1+rdt) = B^*_{t-}(1+(r+\widetilde{\delta}^*_{t-})dt)\left[1-\lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))dt\right] + \lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))dt \times 0\,. \quad (71)$$

Simplifying this equation, we obtain the following credit spread equation:

$$\widetilde{\delta}^*(w_{t-}) = \lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))\,. \quad (72)$$

The key difference between this equation and the one that applies when shocks are idiosyncratic (equation (32)) is that $\lambda^{\mathbb{Q}}$ instead of λ appears in equation (72).

When the country issues domestic currency debt ($B_{t-} > 0$), the competitive market zero-profit condition for diversified investors implies that the credit spread for domestic currency debt, $\widetilde{\delta}$, satisfies

$$B_{t-}S_{t-}(1+rdt) = \mathbb{E}^{\mathbb{Q}}_{t-}\left[B_{t-}S_t(1+(r+\widetilde{\delta}_{t-})dt)\right]\left[1-\lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))dt\right] + \lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))dt \times 0\,. \quad (73)$$

Simplifying (73), we obtain $\widetilde{\delta}_{t-} = \widetilde{\delta}(w_{t-}) = \lambda^{\mathbb{Q}}F(\underline{Z}(w_{t-}))$.

The law of motion for W_t under the physical measure is given by

$$\begin{aligned} dW_t &= \left[Y_{t-} - C_{t-} + (r+\widetilde{\delta}^*_{t-})W_{t-} + (W_{t-}+B^*_{t-})\lambda^{\mathbb{Q}}\mathbb{E}[\pi(Z)]\right]dt + \sigma\widetilde{\Theta}_{t-}d\mathcal{B}_t \\ &\quad -(W_{t-}+B^*_{t-})\pi(Z)d\mathcal{J}_t\,. \end{aligned} \quad (74)$$

The corresponding ODE for $p(w)$ is given by

$$\begin{aligned} 0 = &\left(\frac{\zeta(p'(w))^{1-\psi} - \psi\rho}{\psi-1} + \mu - \frac{\gamma\sigma^2}{2}\right)p(w) + \left[\left(r+\widetilde{\delta}^*(w)-\mu\right)w + 1 + \lambda^{\mathbb{Q}}\mathbb{E}[\pi(Z)](w+b^*(w))\right]p'(w) \\ &+ \frac{\gamma^2\sigma^2 p(w)p'(w)}{2\widetilde{\gamma}(w)} + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp\left(w^{\mathcal{J}}\right)}{p(w)}\right)^{1-\gamma} - 1\right]p(w)\,. \end{aligned} \quad (75)$$

The optimal level of foreign currency debt issuance is the solution to the following maximization problem

$$\max_{b^*\geq -w-\kappa} \quad \left[\lambda^{\mathbb{Q}}F(\underline{Z}(w))w + \lambda^{\mathbb{Q}}\mathbb{E}(\pi(Z))(w+b^*(w))\right]p'(w) + \frac{\lambda p(w)}{1-\gamma}\mathbb{E}\left(\frac{Zp\left(w^{\mathcal{J}}\right)}{p(w)}\right)^{1-\gamma}. \quad (76)$$

All other first-order and boundary conditions are the same as those stated in Proposition 1.

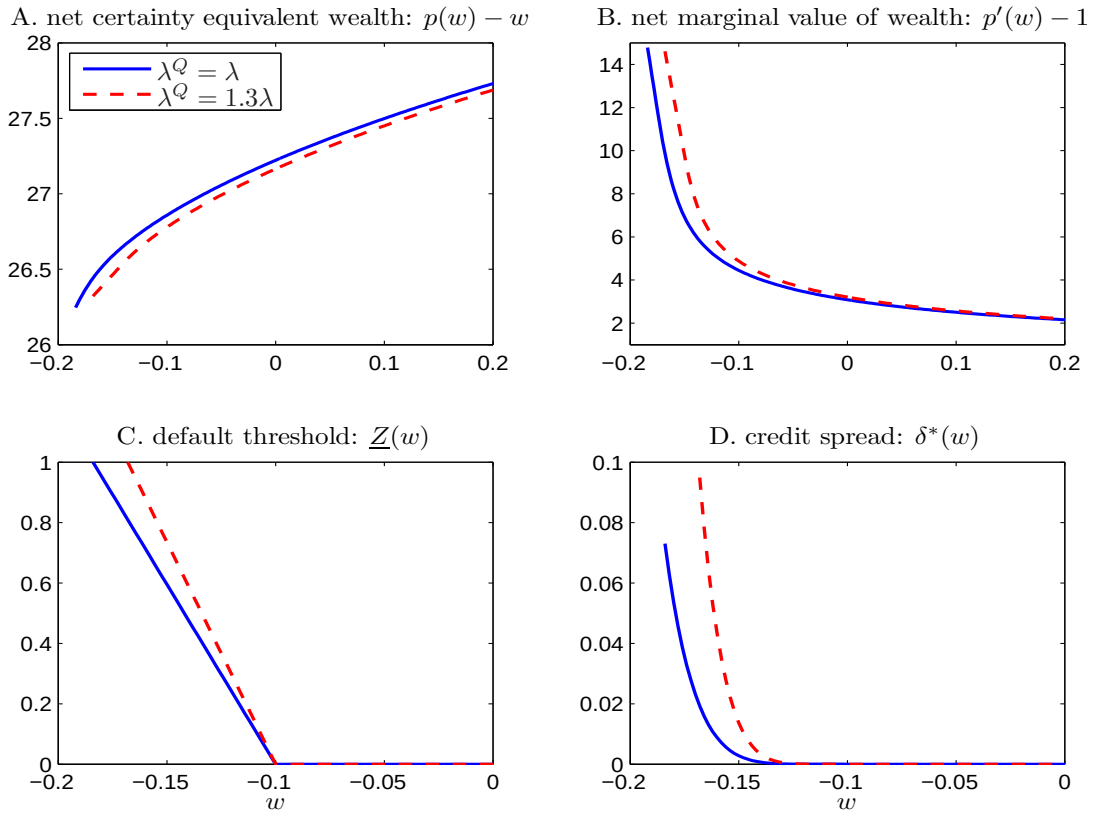

Figure 8: Scaled certainty-equivalent wealth, $p(w)$, and marginal certainty-equivalent value of wealth, $p'(w)$, for our baseline case ($\lambda^{\mathbb{Q}}/\lambda = 1$) and the case with a credit risk premium ($\lambda^{\mathbb{Q}}/\lambda = 1.3$). Debt capacity is equal to $-\underline{w} = 18.4$ percent and $-\underline{w} = 16.8$ percent for the $\lambda^{\mathbb{Q}}/\lambda = 1$ and $\lambda^{\mathbb{Q}}/\lambda = 1.3$ cases, respectively. All other parameters are reported in Table 1.

Calibration and Quantitative Results. To calibrate the default risk premium, we use the estimates in Longstaff, Pan, Pedersen, and Singleton (2011) for Brazil and Colombia since they do not have estimates for Argentina. In that paper, the authors assume that the logarithmic default intensity under both the physical and the risk-neutral measures follow an Ornstein-Uhlenbeck process (analogous to an AR(1) process in discrete time) with different mean reversion and long-run mean parameter values. We use their estimates to infer the long-run mean and variance of the logarithmic default intensity under both the physical and risk-neutral measures.

Since the long-run default intensity under both measures is log-normal, we can calculate the long-run mean of the default intensity under both measures. The ratio between the average default intensity under the risk-neutral and the physical measure is 1.33 and 1.31 for

Brazil and Colombia, respectively. Since we assume that the distribution of Z is idiosyncratic, we set the ratio between the jump-arrival rate under the risk-neutral measure ($\lambda^{\mathbb{Q}}$) and the physical measure (λ) to $\lambda^{\mathbb{Q}}/\lambda = 1.3$.

Figure 8 compares the $\lambda^{\mathbb{Q}}/\lambda = 1.3$ model with the one without a credit risk premium ($\lambda^{\mathbb{Q}}/\lambda = 1$). In the economy with a credit risk premium, risk-averse foreign investors price disaster risk as if they were risk neutral, and the arrival rate is 30 percent higher than under the physical measure. As a result, for a given value of w, credit spreads are larger in the economy with credit risk premia. In equilibrium, these higher credit risk premia translate into a lower debt capacity, which in turn is associated with a lower $p(w)$ and a higher $p'(w)$, (see panels A and B of Figure 8).

Panels C and D show that a country that is close to its debt capacity is more likely to default in the economy with a credit risk premia. For a given level of w, the default threshold, $\underline{Z}(w)$, is higher in the economy with a credit risk premium.

In sum, credit risk premia exacerbate debt intolerance by reducing debt capacity and increasing conditional credit spreads. However, these effects are not strong in our calibration.

8 Transitory Default Costs

In this section, we consider an extension of our model in which output losses associated with default are temporary and disasters are followed by recoveries.[15] To conserve on the number of state variables, we model recoveries as coinciding with the country's resumption of access to international capital markets.

When the country enters the autarky regime at $t = \tau^{\mathcal{D}}$, output falls to $\widehat{Y}_t = \alpha Y_t$. Output follows the stochastic process (5) as long as the country is in the autarky regime. We denote by $\tau^{\mathcal{E}}$ the time at which the country exits autarky and enters the normal regime. At this time, output discretely jumps upward from $\widehat{Y}_{\tau^{\mathcal{E}}-}$ to $Y_{\tau^{\mathcal{E}}} = \widehat{Y}_{\tau^{\mathcal{E}}-}/\alpha = \alpha Y_{\tau^{\mathcal{E}}-}/\alpha = Y_{\tau^{\mathcal{E}}-}$, so the output loss associated with default is temporary. After exiting autarky, output follows the process (1).

We rewrite the HJB equation (24) for $\widehat{V}(\widehat{Y})$ as follows:

$$0 = f(\widehat{Y}, \widehat{V}) + \mu \widehat{Y} \widehat{V}'(\widehat{Y}) + \frac{\sigma^2 \widehat{Y}^2}{2} \widehat{V}''(\widehat{Y}) + \lambda \mathbb{E}\left[\widehat{V}(Z\widehat{Y}) - \widehat{V}(\widehat{Y})\right] + \xi\left[V\left(0, \widehat{Y}/\alpha\right) - \widehat{V}(\widehat{Y})\right]. \quad (77)$$

[15] See Nakamura, Steinsson, Barro, and Ursua (2013) for a rare-disaster model in which disasters are followed by recoveries.

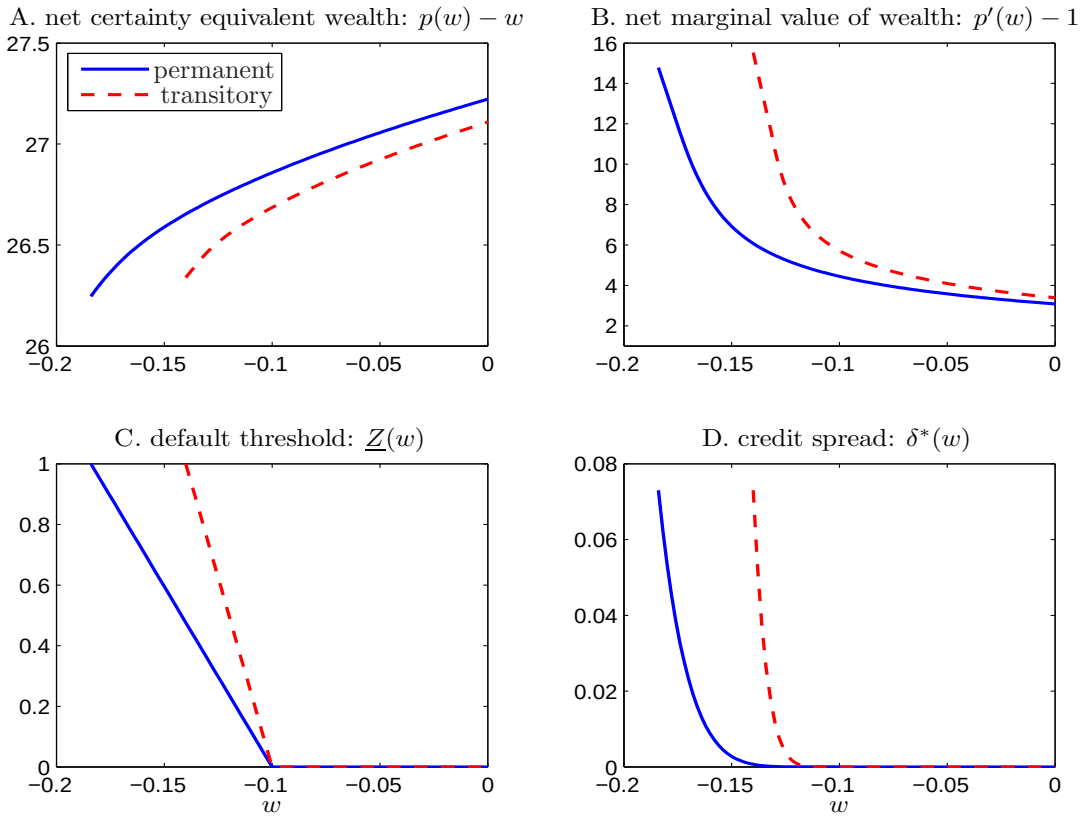

Figure 9: Scaled certainty-equivalent wealth, $p(w)$, and marginal certainty-equivalent value of wealth, $p'(w)$, for both the permanent and transitory default cost cases. Debt capacity is equal to $-\underline{w} = 18.4$ percent and $-\underline{w} = 14$ percent for the permanent and transitory default cost cases, respectively. All other parameters are reported in Table 1.

The last term in equation (77) captures the effect of the upward jump in output from $\widehat{Y}$ to $\widehat{Y}/\alpha$ that occurs when the country exits autarky. The corresponding implicit equation for scaled certainty-equivalent wealth, $\widehat{p}$, is given by

$$0 = \frac{\rho\left[\left(a\,\widehat{p}\right)^{-(1-\psi^{-1})} - 1\right]}{1-\psi^{-1}} + \mu + \frac{\lambda(\mathbb{E}(Z^{1-\gamma}) - 1)}{1-\gamma} - \frac{\gamma\sigma^2}{2} + \frac{\xi}{1-\gamma}\left[\left(\frac{p(0)}{\alpha\widehat{p}}\right)^{1-\gamma} - 1\right]. \quad (78)$$

The remaining first-order and boundary conditions are the same as in Proposition 1.

Figure 9 compares the model with permanent and temporary default costs using our benchmark calibration. Given that the output losses associated with default are temporary, the cost of default is lower than in a model with permanent default costs. In equilibrium, this lower default cost translates into a lower debt capacity, which in turn is associated with a lower welfare ($p(w)$) and a higher $p'(w)$.

Panels C and D show that a country that is close to its debt capacity is more likely to default when the output loss is temporary rather than permanent.

In sum, making default costs transitory exacerbates debt intolerance by reducing debt capacity and increasing credit spreads for a given level of debt.

9 Conclusions

We present a tractable model of sovereign debt that features a jump-diffusion process for output used in the rare-disasters literature and recursive preferences that separate the role of intertemporal substitution and risk aversion.

We define financial development as the ability to issue domestic currency debt in international capital markets. Since domestic currency depreciates when disasters occur, domestic currency debt is a natural hedge against disaster risk.

We show that countries with low levels of financial development suffer from debt intolerance: they have low debt capacity and pay high conditional credit spreads even when their debt level is modest.

To focus on the impact of financial development on sovereign debt, we abstracted from two forces that could influence demand and supply of sovereign debt. The first is the moral hazard problem that is associated with insurance. The second is the impact of sudden stops (Calvo (1998) and Mendoza (2010)) and debt roll-over risk. We plan to address these issues in future research.

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A Appendices

A.1 Derivation of Proposition 1

We show that the value function in the normal regime, $V(W,Y)$, is given by equation (20) and the value function in the autarky regime, $\widehat{V}(\widehat{Y})$, is given by equation (25).

Substituting equation (20) and the first and second derivatives of $V(W,Y)$ into the HJB equation (14) and using the homogeneity property of the value function, we obtain

$$\begin{aligned} 0 = \max_{c,\,\widetilde{\theta},\,b^*\geq -w-\kappa} & \frac{\left(\frac{c(w)}{bp(w)}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho p(w) + \widetilde{\theta}(w)\sigma^2\left(-wp''(w) - \frac{\gamma p'(w)(p(w)-wp'(w))}{p(w)}\right) \\ & + \left[\,(r+\delta^*(w)-\mu)\,w + 1 - c(w) + \lambda\mathbb{E}(\pi(Z))(w+b^*(w))\,\right]p'(w) \\ & + \frac{(\widetilde{\theta}(w)\sigma)^2}{2}\left(p''(w) - \frac{\gamma(p'(w))^2}{p(w)}\right) + \frac{\sigma^2}{2}\left(w^2p''(w) - \frac{\gamma(p(w)-wp'(w))^2}{p(w)}\right) \\ & + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(w^{\mathcal{J}})}{p(w)}\right)^{1-\gamma}-1\right]p(w)\,, \end{aligned} \tag{A.1}$$

where $w^{\mathcal{J}}$ is given by equation (41).

We simplify the FOCs for consumption (equation (16)) and diffusion risk hedging demand (equation (17)) to obtain equations (52) and (36)-(37). Simplifying (18), we obtain the condition for the optimal scaled foreign currency debt issue, $b^*(w)$, which is given by equation (54). Substituting equations (52) and (37) into equation (A.1), we obtain the ODE (45) for $p(w)$.

Substituting the value functions (20) and (25) into the HJB equation (24), we obtain equation (46) for $\widehat{p}$. The value-matching condition that equates the cost of repaying debt and defaulting, given by equation (26), implies the boundary condition (48). Substituting equation (37) into (43), we obtain the boundary condition (49).

A.2 Limited Commitment, Full-Spanning Economy in Section 6

The value function in the normal regime is also homothetic (in W and Y) and given in equation (20). Substituting equation (20) and the first and second derivatives of $V(W,Y)$

into the HJB equation (62) and simplifying, we obtain

$$\begin{aligned}
0 \;=\; \max_{c,\theta,x} \; & \frac{\left(\frac{c(w)}{bp(w)}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho p(w) + \frac{(\theta(w)\sigma)^2}{2}\left(p''(w) - \frac{\gamma(p'(w))^2}{p(w)}\right) \\
& + \left[\,(r+\delta^*(w)-\mu)\,w + 1 - c(w) - \lambda\mathbb{E}[x(Z,w)]\,\right]p'(w) + \frac{\sigma^2}{2}w^2p''(w) \\
& + \frac{\sigma^2}{2}\left(-\frac{\gamma(p(w)-wp'(w))^2}{p(w)}\right) + \theta(w)\sigma^2\left(-wp''(w) - \frac{\gamma p'(w)(p(w)-wp'(w))}{p(w)}\right) \\
& + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(w^{\mathcal{J}})}{p(w)}\right)^{1-\gamma}-1\right]p(w)\,, \qquad \text{(A.2)}
\end{aligned}$$

where $w^{\mathcal{J}}$ is given by equation (66). Simplifying the FOC for the jump-risk hedging demand ($X(Z;W,Y)$) given by equation (64), we obtain the following condition for the optimal scaled hedging demand for jump risk, $x(Z,w) = X(Z;W,Y)/Y$:

$$p'(w) = \left(\frac{Zp((w+x(Z,w))/Z)}{p(w)}\right)^{-\gamma} p'((w+x(Z,w))/Z)\,, \qquad \text{(A.3)}$$

which implies equation (67). Using the FOCs for consumption and diffusion risk hedging demand, we obtain $c(w) = \zeta p(w)(p'(w))^{-\psi}$ and $\theta(w) = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)}$. Substituting these expressions into equation (A.2), we obtain the ODE (65) for $p(w)$.

Finally, $\widehat{p}$ and the boundary conditions for $p(w)$ are the same as those in our benchmark domestic currency economy stated in equations (46)-(51), Proposition 1.

First Best. We conjecture and verify that the scaled certainty-equivalent wealth is given by $p(w) = w + h$. Substituting this expression into $c(w) = \zeta p(w)(p'(w))^{-\psi}$ and the scaled diffusion hedging demand, $\theta(w) = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)}$, we obtain the following closed-form expressions:

$$\begin{aligned}
c(w) &= \zeta(w+h)\,, & \text{(A.4)}\\
\theta(w) &= -h\,. & \text{(A.5)}
\end{aligned}$$

Substituting $p(w) = w + h$ into equation (A.3), we obtain the following expression for the optimal scaled jump-risk hedging demand: $x = (1-Z)h$.

Substituting $p(w) = w + h$ into the ODE (A.2), we obtain

$$\begin{aligned}
0 &= \left(\frac{\zeta-\psi\rho}{\psi-1}+\mu\right)(w+h) + [(r-\mu)w+1] + \lambda(\mathbb{E}(Z)-1)h & \text{(A.6)}\\
&= \left(\frac{\zeta-\psi\rho}{\psi-1}+r\right)w + \left(\frac{\zeta-\psi\rho}{\psi-1}+\mu-\lambda(1-\mathbb{E}(Z))\right)h + 1\,. & \text{(A.7)}
\end{aligned}$$

As equation (A.7) must hold for all levels of w, $\frac{\zeta-\psi\rho}{\psi-1}+r=0$ has to hold. This condition implies that $\zeta=r+\psi(\rho-r)$ (see equation (53)). Using $\zeta=\rho^{\psi}a^{1-\psi}$, we obtain equation (21) for the coefficient a. Finally, substituting $\zeta=r+\psi(\rho-r)$ into equation (A.7), we obtain

$$h=\frac{1}{r-[\mu-\lambda(1-\mathbb{E}(Z))]}=\frac{1}{r-g}\,. \tag{A.8}$$

A.3 Credit Risk Premium in Section 7

Using the dynamics of W_t under the physical measure given by (74) and the standard principle of optimality, we obtain the following HJB equation for the value function $V(W,Y)$:

$$\begin{aligned}0=\max_{C_{t-},\widetilde{\Theta}_{t-},B^*_{t-}}\; & f(C_{t-},V_{t-})+\left[Y_{t-}-C_{t-}+(r+\widetilde{\delta}^*_{t-})W_{t-}+(W_{t-}+B^*_{t-})\lambda^{\mathbb{Q}}\mathbb{E}[\pi(Z)]\right]V_W(W_{t-},Y_{t-})\\ &+\frac{\sigma^2\widetilde{\Theta}^2_{t-}}{2}V_{WW}(W_{t-},Y_{t-})+\mu Y_{t-}V_Y(W_{t-},Y_{t-})+\frac{\sigma^2Y^2_{t-}}{2}V_{YY}(W_{t-},Y_{t-})\\ &+\sigma^2\,\widetilde{\Theta}_{t-}Y_{t-}V_{WY}(W_{t-},Y_{t-})+\lambda\mathbb{E}_{t-}\left[V^{\mathcal{J}}(W_t,Y_t)-V(W_{t-},Y_{t-})\right]\,,\end{aligned} \tag{A.9}$$

where $\widetilde{\delta}^*_{t-}=\lambda^{\mathbb{Q}}\,F(\underline{Z}(w_{t-}))$. Using the value function given by (20), we obtain the following simplified equation for $p(w)$:

$$\begin{aligned}0=\max_{c,\widetilde{\theta},b^*\geq-w-\kappa}\; & \frac{\left(\frac{c(w)}{bp(w)}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho p(w)+\frac{(\widetilde{\theta}(w)\sigma)^2}{2}\left(p''(w)-\frac{\gamma(p'(w))^2}{p(w)}\right)+\frac{\sigma^2}{2}w^2p''(w)\\ &+\Big[\,(r+\widetilde{\delta}^*(w)-\mu)\,w+1-c(w)+\lambda^{\mathbb{Q}}\mathbb{E}[\pi(Z)](w+b^*(w))\,\Big]\,p'(w)\\ &-\frac{\sigma^2}{2}\frac{\gamma(p(w)-wp'(w))^2}{p(w)}+\widetilde{\theta}(w)\sigma^2\left(-wp''(w)-\frac{\gamma p'(w)(p(w)-wp'(w))}{p(w)}\right)\\ &+\frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(w^{\mathcal{J}})}{p(w)}\right)^{1-\gamma}-1\right]p(w)\,,\end{aligned} \tag{A.10}$$

where $w^{\mathcal{J}}$ is given by equation (41). We obtain equation (76) for the optimal level of foreign currency debt issuance. The implied FOCs for diffusion risk hedging demand and consumption are given by equations (36) and (52). Substituting the optimal consumption and diffusion risk hedging demand into (A.10), we obtain the ODE (75) for $p(w)$.

Finally, $\widehat{p}$ and the boundary conditions for $p(w)$ are the same as those in our benchmark domestic currency economy, stated in equations (46)-(51) in Proposition 1.

B Solution Algorithm

We numerically solve the ODE in Proposition 1 using the following algorithm.

1. Start with a sufficiently large region $(\underline{w}, \overline{w})$ by setting $\underline{w} = -h$ and a sufficiently large $\overline{w}$ (e.g., $\overline{w} = 10^4$). We use the superscript (i) to denote the i-th iteration value for $p(w)$, $b^*(w)$, $w^{\mathcal{J}}$, and $\widehat{p}$. That is, $p^{(i)}(w)$, $b^{*(i)}(w)$, $w^{\mathcal{J}(i)}$, and $\widehat{p}^{(i)}$.

2. Assign an initial value for the scaled certainty-equivalent wealth $p(w)$, which we denote by $p^{(1)}(w)$. For example, we start with the following initial linear function for $p(w)$: $p^{(1)}(\underline{w}) = \alpha h$, $p^{(1)}(\overline{w}) = \overline{w} + h$, and $p^{(1)}(w) = p^{(1)}(\underline{w}) + \frac{w-\underline{w}}{\overline{w}-\underline{w}}(p^{(1)}(\overline{w}) - p^{(1)}(\underline{w}))$ for $\underline{w} < w < \overline{w}$.

3. For a given $p^{(i)}(w)$, where $i = 1, 2, \cdots$, compute $b^{*(i)}(w)$, $w^{\mathcal{J}(i)}$ and $\widehat{p}^{(i)}$ by using equation (54), equation (41), and equation (46), respectively.

4. Substitute the policy rules, $b^{*(i)}(w)$, $w^{\mathcal{J}(i)}$, and $\widehat{p}^{(i)}$ obtained in step 3 into ODE (45). Use the Matlab function **ode45** (or another variant of the finite-difference method) to solve for $p^{(i+1)}(w)$ given by ODE (45).

5. Repeat steps 3 and 4 until $|p^{(i+1)}(w) - p^{(i)}(w)|$ is sufficiently low (e.g., $|p^{(i+1)}(w) - p^{(i)}(w)| < 10^{-10}$).

6. Compute $p''(\underline{w})$ until $p''(\underline{w})$ becomes sufficiently low (e.g., $p''(\underline{w}) < -10^{10}$), that is, until the program converges. Otherwise, go back to step 1 and increase $\underline{w}$ with a new guess and iterate until the program converges.