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MISMATCH AND ASSIMILATION

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ABSTRACT

Income disparity across countries has been large and widening over time. We develop a tractable model where factor requirements in production technology do not necessarily match a country's factor input profile. Appropriate assimilation of frontier technologies balances such multi-dimensional factor input-technology mismatch, thus mitigating the efficiency loss. This yields a new measure for endogenous TFP, entailing a novel trade-off between a country's income level and income growth that depends critically on the assimilation ability and the factor input mismatch. Our baseline model accounts for 80%-92% of the global income variation over the past 50 years. The widening of mismatch and heterogeneity in the assimilation ability account for 41% and 20% of the global growth variation, whereas physical capital accounts for about one third with human capital largely inconsequential. In particular, about 30% of the output growth in miracle Asian economies comes from narrowing the gap arisen from mismatch, and 94% of the growth stagnation in trapped African economies due to the widening mismatch. A country may fall into a middle-income trap after a factor advantage reversal that changes the pattern of mismatch.

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"The sixteenth-century Dutch, on the verge of becoming economic leaders of the world, borrowed heavily from the techniques of the Italians, the outgoing leaders. By then, the English were already learning not only from the Low Countries but also from other parts of the continent. The Americans borrowed heavily from the English and from other European sources, particularly from the time they achieved independence up until the middle of the nineteenth century." (Baumol et al., 1991, p. 271-272)

1 Introduction

The world income distribution has been widening over the past 50 years. In 1960, the real GDP per worker of the top 10th percentile country was about 12 times higher than that of the bottom 10th percentile; in 2014 it has doubled to 24 times (the black line in the left panel of Figure 1). However, the same ratio for factor inputs, such as physical capital and human capital per worker, remained largely constant at 5 times (the red line). This evidence suggests a large and widening TFP gap between the rich and poor countries. Furthermore, the TFP gaps are widening in diverging directions. The dash line in the right panel of Figure 1 delineates the frontier of growth: the growth rate of real GDP per worker as the sum of the growth rate of factor inputs per worker and the growth rate of frontier productivity (represented by the TFP of the U.S.). If the TFP gaps were not widening, then countries should be well aligned on the frontier. If the TFP gap were widening but at a constant rate, then they should be parallelly shifted below the frontier. Instead, evidence suggests an amplification mechanism à la *anti-clockwise rotation* in the figure: countries having their factors accumulation faster than the U.S. tend to have their TFP gaps shrinking and their income growing *increasingly* faster than the U.S. - the growth miracles we have witnessed in the past half century. The rotation is also consistent with a poverty-trap puzzle where some countries (dots on the far left of the U.S.) experience negative income growth in spite of slower yet positive growth of factors in general. Indeed, in some of these trapped countries the positive factor accumulation is driven by strong growth in human capital comparable to the miracle economies (for example human capital in Zimbabwe is growing 1.16% faster than in the U.S., see Table 4 for details). Furthermore, their income level in 1970 were comparable with those of the Asian tigers and were ahead of some of their ASEAN counterparts. How can some countries successfully catch up with the rich countries? But why at the same time some other countries are worsening, even with similar initial condition and similar improvement in human capital? Explaining these

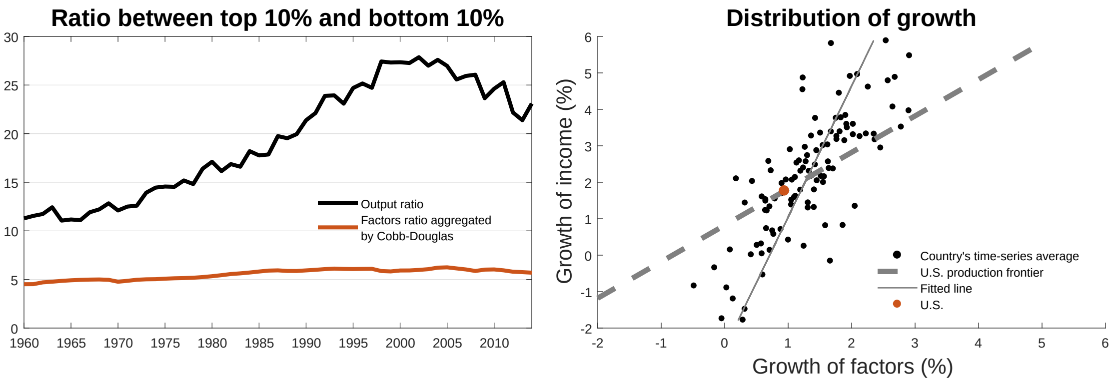

Figure 1: Left black: the ratio between the top and the bottom 10 percentiles of real GDP per worker. Left red: the top-bottom ratio of factors per worker. The factors are physical capital and human captial aggregated by a Cobb-Douglas function with a capital share of 1/3. Right: the average growth rate of per-worker real GDP against the average growth rate of per-worker factors, each dot represents a country in the sample. Outliners like oil rich countries in OPEC, and former members of USSR and Yugoslavia are excluded. Source: Penn World Table 9.0.

regularities of TFP gap is crucial to answering these questions. Unfortunately, it is challenging under the standard aggregate production function framework.

To address these issues, we revisit a microfoundation of the aggregate production function framework. Production technology often requires the composition of factor inputs very different from the country's available factor inputs, a concept entailing *factor input-technology mismatch*. This is particularly true when the technologies are adopted from developed countries: they are usually more productive but also require demanding factors such as sophisticated machines and skilled workers, not always available in less developed countries. In reality the mismatch is more severe in some factors and less in others. We refer the former the disadvantageous factors, and the latter the advantageous factors. To mitigate the impact of mismatch, it is natural to leverage the advantageous factors to compensate for the disadvantageous factors. We refer to this process as *technology assimilation*. In practice the ability to assimilate technologies is a key component of firms' intangible organizational capital – depending on entrepreneurs' understanding of foreign techniques, their learning from experimenting, the flexibility of organization and institution, and the infrastructure and policies relevant to adaptation.[1] It is therefore natural to expect that the cross-country variations in the

[1] The importance of assimilation is supported by a vast body of evidences from various case studies. For example Wan (2011), emphasizes the successful assimilation of technology as the common denominator in Asian economic development, whereas Rosenberg (1994) many developed countries today are successful cases for modified existing technology. Policy issues concerning the assimilation of foreign techniques are discussed in Dahlman et al. (1987) and Nelson and Pack (1999)

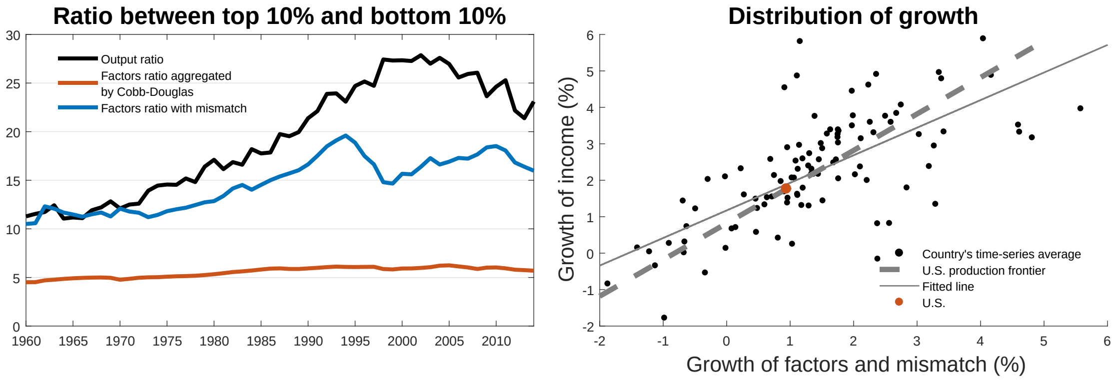

Figure 2: Left black and red are the same as those in Figure 1. Left blue: the top-bottom ratio of the same factors but aggregated by (3). Right: the average growth rate of per-worker real GDP against the average growth rate of per-worker factors in the assimilation model. Source: see Figure 1.

mismatch and in assimilation ability serve to explain part of the cross-country income disparities. Notwithstanding, mismatch and assimilation are missing in the standard aggregate production function.

The main contribution of this paper is to develop a tractable framework for analyzing factor input-technology mismatch under imperfect technology assimilation, subsequently accounting for cross-country income disparities. Our framework encompasses, for example, the technological frontier model, the stochastic techniques model, and the multi-tier CES production model. This mechanism can generate large income variations in both the *level* and the *growth rate* across countries. Our theoretical results show that, a lower ability of assimilation reduces production efficiency by deterring adaptation to the frontier technology and aggravating the mismatch, thus widens the variation of income level. However, a lower ability of assimilation increases factor complementarity, and the disadvantageous factors become more important in production. Thus, a modest variation in the growth of the disadvantageous factors translates to a much larger variation in the income growth. Moreover, disadvantageous factors may eventually become advantageous. If the *factor advantage reversal* takes place, mismatch would be widening again, slowing down the income growth.

Upon establishing the theory, we then verify its explanation power with the data. Identifying technology and assimilation ability in our framework faces the similar challenge from Diamond-McFadden impossibility theorem. We circumvent the impossibility theorem by using the solution to the firm's technology choice problem as an extra identification condition from the theory. The quantitative results show that our framework helps reducing the reliance on the unexplained TFP gap, illustrated in the left panel of Figure 2. In addition, the unexplained TFP gap is *growing much slower* in our framework, as illustrated by the flatter fitted line in the right panel of Figure 2, indicating that the mismatch story is promising in explaining the rotation pattern of TFP gaps. Indeed, our framework yields better fit in about 95% of the sample countries.

We then examine the quantitative importance of the mismatch mechanism by conducting development and growth accounting. In development accounting, accounting for the factor input-technology mismatch improves the success rate from 33% to 92%. In growth accounting, changes in the factor input-technology mismatch contribute to 41% of the income growth gap from the U.S., exceeding the contributions of physical capital accumulation (about one-third), human capital (almost negligible) and residual (about a quarter). Accumulation of factors matters a lot to growth when they are the disadvantageous factors. In development-trapped countries such as Congo, Zimbabwe, and many others, their disadvantageous factor, physical capital, has been decumulating and the advantageous factor, human capital, has been accumulating significantly over time. It widens the factor input-technology mismatch and lowers growth - our explanation to the poverty-trap puzzle. For these trapped countries, our mismatch channel explains on average 94% of the widening income gap from the U.S., whereas the standard approach would predict a converging gap otherwise. On the other hand, in miraculously fast growing countries such as Korea, China and Vietnam, the disadvantageous factor, physical capital, has out-grown the advantageous factor, human capital. As a result, factor input-technology mismatch has been improving over time, accounting for 30% of their prolonged growth. We further investigate the role of cross-country heterogeneity in assimilation ability, and find it to account for another 20% of the income growth gap from the U.S., on top of the 41% from the widening mismatch. We finally extend our analysis to alternative assimilation targets as well as early stop or late start in assimilation, finding a more important role played by mismatch.

The mechanism of factor advantage reversal can also serve as a plausible explanation for the *middle-income traps* that features a significant slowdown of once fast-growing economies in the midst of its development. We find that the timing of reversal in several countries, including Spain, Greece, Hong Kong and Taiwan, matches well the timing of middle-income trap [cf. Eichengreen et al. (2014)], as well as their structural breaks in relative incomes (identified by Chow test).

The main takeaway is that factor input-technology mismatch and assimilation ability are crucial for disparities in income levels and growth rates, accounting for a large portion of the otherwise unexplained residual TFP component. Successful assimilation narrowing mismatch has promoted economic development, with Western Europe and America taking off early, then Japan and followed first by the newly industrialized Asian Tigers in the 1960s and various emerging latecomers since the 1980s, as argued by Baumol et al. (1991) in our opening quote. This dynamic process of development generates an ever changing cross-section distribution as observed in data. Conversely, failure to assimilate technology has caused the backwardness seen in Sub-Saharan countries. What distinguishes miracles from traps is not just the speed of factor accumulation; it is the accumulation of the right factor.

Related Literature

To formalize the concepts of technology-factor input mismatch and assimilation, we refer to the now-classic pieces by Houthakker (1955) and recently by Kortum (1997), Jones (2005) - HKJ hereafter - and Lagos (2006). With different frictions and focuses, HKJ and Lagos (2006) obtain a Cobb-Douglas global production function as an aggregation of Leontief local productions in which firms' production techniques are drawn from the Pareto distribution. HKJ derive a measure of TFP that summarizes the production efficiency of the economy, and show how the TFP depends on frictions and fundamentals of the economy. Another theoretical lesson from HKJ and Lagos (2006) are that the macro elasticity of substitution (EOS) can be much higher than the micro EOS, because of factor reallocation across firms or techniques (in spite of various frictions and constraints). Our framework includes the results of HKJ and Lagos (2006) as special cases. Related but on the time dimension, León-Ledesma and Satchi (2017) show that with the presence of adjustment cost to techniques, a short-run EOS of about 0.2 can generate a long-run EOS of about 0.6 and explain the short- and medium-run behaviors of labor shares.[2]

Our framework emphasizes the fact that the frontier techniques available can be very restrictive. To this end, our paper is also closely related to Caselli and Coleman (2006). The Caselli-Coleman paper explains cross-country income disparities by assuming that the frontier technology is not available and countries choose from their own technology menus. Given that skilled and unskilled labor are gross substitutes under a CES production function, Caselli and Coleman show that rich (poor) countries who are skilled (unskilled)-labor abundant choose skilled (unskilled)-labor augmenting

[2] In this regard, our paper is also related to a growing literature that investigates the role of disaggregated (sectoral, firm or plant level) EOS on explaining the declining labor share, since the seminal work of Karabarbounis and Neiman (2014). Also see Oberfield and Raval (2014) and Lawrence (2015) for some recent studies on the EOS and labor share. See the Appendix for a detailed discussion about the methodology of estimating the EOS.

technologies. We show that the factor input-technology mismatch alone can explain much of income differences even when the frontier technology is available.

Some studies have focused on costly (or frictional) learning, including Caselli (1999). Atkinson and Stiglitz (1969) suggest a theory that country's production features localized learning by doing. Based on that idea, Basu and Weil (1998) construct a Solow-type one-factor growth model of technological progress that emphasizes that technological advances will only benefit the technologies with similar capital intensity. Parente and Prescott (1994) examine how barriers to technology adoption affect the process of development. With the exogenous growth of world knowledge, the amount of investment required for technological advances decreases, which enhances long-term growth. Acemoglu and Zilibotti (2001) study the economy where skilled labor available does not match the technology requirement. Therefore, even if all technologies are freely available and instantly transferred, a country may refrain from using a new but inappropriate technology.

In these studies, the mismatch or the inappropriateness is summarized by one factor (e.g. skill or capital). While it proves useful to explain the cross-sectional variation, it does not fully explain why over time the income gap is widening even though the inappropriateness in some aspects, for example the gap of human capital, is narrowing. And the importance of human capital has been emphasized to explain the Asian miracles [see the survey by Lucas (1993)], but it cannot explain why no miracle happens in those trapped countries with comparable growth in human capital. Insteads, our framework allows for multi-dimensional mismatch which is useful to generate various income dynamics.

In the equilibrium our production function falls into a class of normalized CES (NCES) function, popularized by Klump and de La Grandville (2000). The tractability of NCES allows us to estimate the model with cross-country time series, and to apply the estimated model to development accounting and growth accounting. León-Ledesma et al. (2010) demonstrate that EOS estimation without normalization can be seriously biased. A summary of the empirical literature using NCES and CES will be provided in the methodology section.

2 The Aggregate Production Function: A Prelude

We first preview the key innovation of this paper before the details of the theoretical and quantitative analyses. The existing literature of the neoclassical production adopts the following prototypical model for a country j:

$$y = z_j F(k_j, h_j). \tag{1}$$

According to (1), output y is produced with a constant-returns-to-scale technology, F, using physical capital k and human capital h with TFP z. Letting s denote the benchmark country with frontier technology, the literature [see the survey by Caselli (2005)] focuses on the Cobb-Douglas specification of F which is given by

$$y = \exp(\tau_j) z_s k_j^{\alpha} h_j^{1-\alpha}, \tag{2}$$

where $\alpha \in (0,1)$ is the output elasticity of physical capital and all countries share the TFP z_s, By normalizing $\tau_s = 0$, τ_j becomes the residual TFP gap from the leading country – or, in short, the *TFP residual.* Then the common conclusion is: only about 20%-40% of the variation in world income can be attributed to variation in factor inputs, with the TFP residual alone accounting for most of the variation. This result is viewed as unsatisfactory.

Departing from the neoclassical production framework, we generalize the concept of production techniques developed by HKJ. While we will lay out the micro-foundation to derive the general case in the next section, here we illustrate with a simple two-factor case. The resulting production function takes a *normalized CES* form:

$$y = \exp(\tau_j) y_s \left[\alpha \left(\frac{k_j}{k_s} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{h_j}{h_s} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\sigma \in [0,1]$ and the normalization is taken at the frontier techniques summarized by the triplet $\{y_s, k_s, h_s\}$. Figure 3 illustrates the isoquants of the normalized CES production function under assimilation. An isoquant consists of combination of factors (k,h) needed to produce a one unit of output. Point E_s represents the frontier technology on the isoquant F_s if the benmark country is endowed with country j's factors. Point E_j^1 represents the production of country j on the isoquant F_j with $\sigma < 1$ capturing the imperfect assimilation of the frontier technology. Isoquants with various σ are tangent at the production point E_j on the $(k/h)_s$ ray. Isoquants moving downward from E_j along the $(k/h)_s$ ray represents increasing TFP τ_j. The factor input-technology mismatch is represented by the intersection of these isoquants and the $(k/h)_j$ ray. When no factor input-technology mismatch can be assimilated, country j is producing on the kinked isoquant F_j with $\sigma = 0$ and the factors needed to produce one unit of output are captured by point E_j^0. Caselli (2005) is the special case of the isoquant F_j with $\sigma = 1$, associating with a lower point E_j^2. The ranking of factor

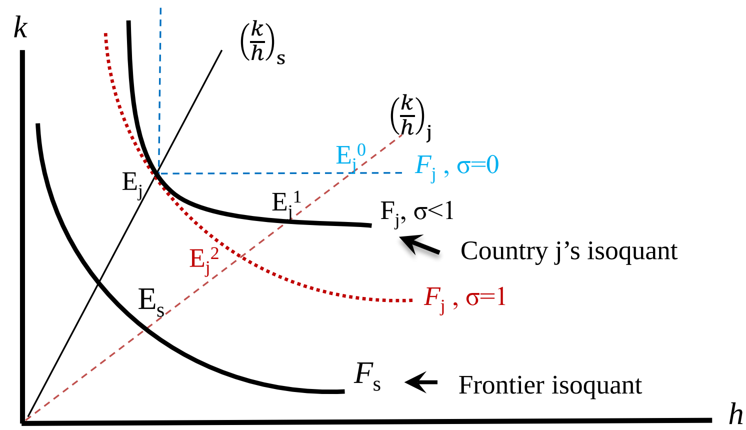

Figure 3: Assimilation and the isoquants.

requirements is $E_j^0 > E_j^1 > E_j^2$. The efficiency loss measured by the standard TFP residual from (2) is associated with the increased factor requirement given by $E_j^2 - E_s$. The efficiency loss due to the factor input-technology mismatch is the increased factor requirement given by $E_j^1 - E_j^2$. As the mismatch increases, the $(k/h)_j$ ray moves away from the $(k/h)_s$ ray and the factor requirement gap $E_j^1 - E_j^2$ increases. Similarly, as assimilation ability decreases, the isoquant moves closer to F_j with $\sigma = 0$, and the factor requirement gap $E_j^1 - E_j^2$ also increases. Thus, the assimilation model generates additional efficiency loss due to mismatch and imperfect assimilation.

3 The Framework

Let *production technique* specifies the organization of factor inputs for output. A typical technique is defined by the input-output parameters, $\mathbf{a} \equiv (a_1, ..., a_N)$, such that its output level given the factor inputs, $\mathbf{n} \equiv (n_1, ..., n_N)$, is

$$y = f(\mathbf{n}; \mathbf{a}) \equiv \left[\sum_{m=1,...,N} \alpha_m \left(\frac{n_m}{a_m} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad \sum_{m=1,...,N} \alpha_m = 1. \tag{4}$$

That is, the technique requires $\mathbf{n} = \mathbf{a}$ units of different factors to produce one unit of output. The tuple of ratios $\boldsymbol{\mu} \equiv (n_1/a_1, ..., n_N/a_N)$ represents the *factor input-technology mismatch* between factor inputs and the factor requirement of the technique. We interpret the EOS parameter $\sigma \in [0, 1]$ as the ability of assimilation.[3] In the absence

[3]This is a parsimonious setup to capture two sided matching between heterogeneous factor inputs with technical progress as in Chen et al. (2012). The idea of learning-by-using (LBU) or fixed adoption costs of dynamic technology adoption models [e.g., Arthur (1989) and David (1985)] can be captured

assimilation ($\sigma = 0$), the output level is determined entirely by the worst mismatched factor – one with the lowest n_m/a_m.

Given a combination of factor inputs, $\mathbf{n}$, firms to choose the best technique from a menu, $\mathbf{a} \in \mathcal{P}$. With its best selected techniques, a firm's output level is then given by the following *global production function*:

$$\overline{f}(\mathbf{n}) = \max_{\mathbf{a}\in\mathcal{P}} f(\mathbf{n};\mathbf{a}). \tag{5}$$

Assuming that $\mathcal{P}$ is a compact connected set in $\mathbb{R}^N_{++}$ with a differentiable envelope, Proposition 1 provides necessary and sufficient condition for the Cobb-Douglas specification to arise given a menu set.[4]

Proposition 1 *Given $\mathcal{P}$, there exists $z > 0$ such that the global production function (5) is given by*

$$\overline{f}(\mathbf{n}) = z \prod_{m=1,...,N} (n_m)^{\alpha_m},$$

if and only if $\mathbf{a}^\star(\mathbf{n}) \equiv \arg\max_{\mathbf{a}\in\mathcal{P}} f(\mathbf{n};\mathbf{a}) \propto \mathbf{n}$.[5]

Notably, $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$ if there exists a positive scalar k such that $a^\star_m = n_m/k$ for all $m = 1, ..., N$, i.e., the optimal technique is proportional to factor inputs. Under Proposition 1, if the optimal technique is proportional to factor inputs, then the global production function is Cobb-Douglas and the output effect of the optimal technique is completely summarized by the TFP parameter z. The "only if" part of Proposition 1 points out that having $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$ is necessary in any environments that "microfound" the Cobb-Douglas specification. We illustrate the following special cases to Proposition 1 of particular interest.

Example 1 *(Caselli and Coleman, 2006) Consider $N = 2$ and define $\mathcal{P} \equiv \{(a_1,a_2) \mid B \leq (a_1)^{\alpha_1}(a_2)^{\alpha_2}\}$, where $B > 0$ is a measure to the barrier to technology frontier. Then the optimal technique is $a^\star_1 = B(n_1/n_2)^{\alpha_2}$ and $a^\star_2 = B(n_2/n_1)^{\alpha_1}$ and the global production function is given by $\overline{f}(\mathbf{n}) = z(n_1)^{\alpha_1}(n_2)^{\alpha_2}$, where $z = B^{-1}$.*[6]

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by our flexibility parameter σ. We can translate a high cost of LBU or adoption to a low flexibility in production, i.e., a low σ.

[4] Define $\mathbf{a}_- \equiv \{a_1, ..., a_{N-1}\} \in \mathbb{R}^{N-1}_{++}$, and $\mathcal{A}(\mathbf{a}_-) \equiv \min a_{i,N}$ s.t. $\mathbf{a}_i \in \mathcal{P}$ and $a_m \leq a_{i,m}$ for all $m = 1, ..., N-1$. $\mathcal{P}$ has a differentiable envelope if $\mathcal{A}(\mathbf{a}_-)$ is differentiable for any interior $\mathbf{a}_-$.

[5] All proofs are relegated to Appendix A.

[6] Caselli and Coleman (2006) assume $Y = K^\alpha\left[(A_uL_u)^\psi + (A_sL_s)^\psi\right]^{(1-\alpha)/\psi}$, which is isomorphic to our framework by taking Y/K as our y, L_u/K as n_1, L_s/K as n_2, $A_u\alpha_1^{1/\psi}$ as a_1 and $A_s\alpha_2^{1/\psi}$ as a_2.

Our framework also encompasses the model of stochastic technique, for example Kortum (1997) and Jones (2005), where $\mathbf{a}_i$ is drawn from a stationary distribution that can lead to an extreme value distribution of the technology frontier. In this case $\mathcal{P}$ is the limited set as the number of technique goes to infinity.[7]

Example 2 *(Kortum, 1997; Jones, 2005) Consider $\sigma = 0$ and $N = 2$. Suppose $a_{i,m}$, $m = 1, 2$ and $i = 1, ..., I$, is drawn from independent inverse Pareto distributions with the shape parameter $\alpha_m \theta$ and the scale parameter $\gamma_m I^{-1/\theta}$, where $\theta > 1$ and $\gamma_m > 0$. When $I \to \infty$ the optimal technique converges to satisfy $n_1/a_1^\star = n_2/a_2^\star$ almost surely, and the global production function converges in distribution to $\overline{f}(\mathbf{n}) = z\,(n_1)^{\alpha_1}\,(n_2)^{\alpha_2}$, where $z = \varepsilon \gamma_1^{\alpha_1} \gamma_2^{\alpha_2}$ and ε follows the Fréchet distribution with parameter θ.*

Without much loss of generality, the local production production (4) consists of a single-tier structure. Proposition 1 can be extended to the local production function featuring multi-tier CES like in Krusell et al. (2000).

Example 3 *(Krusell et al., 2000) Consider $N = 3$ and the following local production function*

$$y = \max_{\mathbf{a} \in \mathcal{P}} \left[\alpha \left[\beta \left(\frac{n_1}{a_1} \right)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \beta) \left(\frac{n_2}{a_2} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1} \frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(\frac{n_3}{a_3} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \quad \alpha, \beta \in (0, 1) \,. \tag{6}$$

There exists $z > 0$ such that the global production function is given by $y = z n_1^{\alpha\beta} n_2^{\alpha(1-\beta)} n_3^{1-\alpha}$, if and only if the solution $\mathbf{a}^\star(\mathbf{n})$ to (6) satisfies that $n_1/a_1^\star = n_2/a_2^\star = n_3/a_3^\star$.

Krusell et al. (2000) use the local production function (6) to study capital-skill complementarity – think of n_1 as capital, n_2 skilled labor, n_3 unskilled. In their model there is a CES aggregation of capital and skilled labor with a complementary elasticity of substitution (EOS) $\varepsilon \leq 1$, whose outcome is then aggregated with unskilled labor with an EOS $\sigma \geq 1$. If $\varepsilon = \sigma$ then (6) is a special case to (4), analyzed in Proposition 1. For the generic values of ε and σ, rewrite (6) as

$$y = \max_{\mathbf{a} \in \mathcal{P}} \left[\alpha \,(\bar{n})^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha) \left(\frac{n_3}{a_3} \right)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \text{ s.t. } \bar{n} \equiv \left[\beta \left(\frac{n_1}{a_1} \right)^{\frac{\varepsilon - 1}{\varepsilon}} + (1 - \beta) \left(\frac{n_2}{a_2} \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}.$$

Applying Proposition 1 sequentially – first on $\bar{n}$ with inputs n_1 and n_2 given a_3, then on y with inputs n_3 and $\bar{n}$ – we reach the result in Example 3. While our results

[7] Wong and Yip (2014) extend the results of Jones (2005) to the general case with $N \geq 2$ and $\sigma \in [0, 1)$. A CDF of $a_{i,m}$ is given by $\Pr(a_{i,m} \leq x) = \left(x \gamma_m I^{-1/\theta} \right)^{\alpha_m \theta}$ for all $x \in \left[0, \gamma_m^{-1} I^{1/\theta}\right]$.

are readily extensible to more sophitiscated structure like (6), we will, for the sake of parsimony, focus primarily on the more commonly specified setting given by (4).

In Proposition 1 what matters to give rise to the Cobb-Douglas specification is that the technique chosen is proportional; the environment (like the shape of menu, cost or other frictions) do not matter much. To see it, consider that, alternatively, the technology choice is no longer restricted to any menu but the choice is costly. The firm's net output is given by the following global production function

$$\overline{f}(\mathbf{n}) \equiv \max_{\mathbf{a}} \left\{ f(\mathbf{n}; \mathbf{a}) - C(\mathbf{a}) \right\}, \tag{7}$$

where $C : \mathbb{R}_+^N \to \mathbb{R}_+$ is a twice-differentiable cost function. Analog to Proposition 1, Proposition 2 provides necessary and sufficient condition for the Cobb-Douglas specification to arise under the problem of costly technology choice.[8]

Proposition 2 *Given C, there exists $z > 0$ such that the global production function (7) is given by*

$$\overline{f}(\mathbf{n}) = z \prod_{m=1,\ldots,N} (n_m)^{\alpha_m},$$

if and only if $\mathbf{a}^\star(\mathbf{n}) \equiv \arg\max_{\mathbf{a}} \{ f(\mathbf{n}; \mathbf{a}) - C(\mathbf{a}) \} \propto \mathbf{n}$.

3.1 Assimilating the Frontier and Endogenous TFP

Consider a leader country s, say, the U.S. or a major advanced economy regarded as a regional leader (e.g., Japan in Asian; France, UK or Germany in Europe). The leader country is using the most productive "frontier" technique $\mathbf{a}_s$ (maybe because it has the largest menu $\mathcal{P}$ or lowest cost C) to produce y_s with its factor input $\mathbf{n}_s$. In this section we analyze the world development when $\mathbf{a}_s$ is avaliable to the rest of the world.

Depending on $\mathcal{P}$ or C, the chosen technique can be anything. In practice, we do not have sufficient information to separately identify the techniques and the parameter of assimilation ability [Diamond et al. (1978)]. To circumvent this problem, we assume that the resulted *global* production function, $\overline{f}(\mathbf{n}_s)$, in the leader country is Cobb-Douglas - probably the most common assumption in macroeconomics. This identification assumption is maintained only on the leader country, and the local production functions in the rest of world are not restricted. Applying Proposition 1 or

[8] Lagos (2006) derives a Cobb-Douglas global production function in the presence of labor search friction. One can formulate the firm's problem of Lagos (2006) in term of (7). Since the technology in Lagos (2006) is assumed to be Hick-neutral, with $n_1 = n_2$ in the equilibrium (or in term of his notations hours n as n_1 in our model and capital k as n_2, where the total labor is the total hours of employed workers) the techniques of the filled firms are always proportional to the factor inputs.

2, it is necessary (the part of "only if") that the frontier technique in equilibrium is proportional to the leader's factor inputs, i.e., $a_{s,m} = n_{s,m}/y_s$ for all $m = 1, ..., N$. Thus, the output level of a country with factor input $\mathbf{n}$ after assimilating the frontier technique is tractably given by

$$y = f(\mathbf{n}; \mathbf{a}_s) = y_s \left[\sum_{m=1,...,N} \alpha_m \left(\frac{n_m}{n_{s,m}} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = Z(\mathbf{n}; \mathbf{n}_s, \sigma) \prod_{m=1,...,N} (n_m)^{\alpha_m}, \quad (8)$$

where $Z(\mathbf{n}; \mathbf{n}_s, \sigma)$ is a TFP measure given by

$$Z(\mathbf{n}; \mathbf{n}_s, \sigma) = \underbrace{z_s}_{\text{source TFP}} \times \underbrace{\prod_{m=1,...,N} \left(\frac{n_{s,m}}{n_m} \right)^{\alpha_m} \left[\sum_{m=1,...,N} \alpha_m \left(\frac{n_m}{n_{s,m}} \right)^{1-1/\sigma} \right]^{1+1/(\sigma-1)}}_{\text{mismatch and assimilation}}. \quad (9)$$

Equation (8) is the key equation in this paper; we shall examine in the rest of this paper, theoretically and quantitatively, how assimilating the frontier technique is accountable for the world income disparity.

Since the frontier technique is chosen only based on the leader country's own interest, it does not necessarily match the factor inputs in other countries. More specifically, when a country assimilates the frontier technique, $\mathbf{a}_s$, its TFP measure, $Z(\mathbf{n}; \mathbf{n}_s, \sigma)$, becomes endogenous and contains two effects: (i) a source TFP effect captured by z_s; and (ii) an *assimilation effect*, jointly captured by the assimilation ability parameter, σ, and the mismatch from assimilating the frontier technique, $\boldsymbol{\mu}_s = (n_1/a_{s,1}, ..., n_N/a_{s,N})$. The following proposition relates the TFP with the assimilation ability:

Proposition 3 *The TFP, $Z(\mathbf{n}; \mathbf{n}_s, \sigma)$, is increasing in the assimilation ability, σ.*

Proposition 3 states that for a given degree of factor input-technology mismatch, production efficiency improves with more flexible assimilation that enables better adaptation to the frontier technique. If $\sigma \to 1$ then the country inherits the frontier productivity and output such that $Z(\mathbf{n}; \mathbf{n}_s, \sigma) = z_s$ and $\overline{f}(\mathbf{n}) = z_s \prod_{m=1,...,N} (n_m)^{\alpha_m}$; a special case studied in Caselli (2005).

The effects of the leader's factor input on the TFP are more involved. The leader's factor input, $\mathbf{n}_s$, now matters for the TFP in other countries, through assimilating the frontier technique, $\mathbf{a}_s$, which is chosen according to $\mathbf{n}_s$. Clearly from (9), assimilating the frontier technique does not always lead to a higher TFP. Define the most disadvantageous factor as $m_d \equiv \arg\min_{m=1,...,N} (n_m/n_{s,m})$ and the most advantageous factor as $m_a \equiv \arg\max_{m=1,...,N} (n_m/n_{s,m})$. The following proposition shows that the effects of $n_{s,m}$ depend crucially on the relative factor advantage:

Proposition 4 *There exists $\gamma_y \in [n_{m_d}/n_{s,m_d}, n_{m_a}/n_{s,m_a}]$ such that the TPF, $Z(\mathbf{n}; \mathbf{n}_s, \sigma)$, is decreasing in $n_{s,m}$ if and only if $n_m/n_{s,m} \leq \gamma_y$, i.e. when the factor is disadvantageous.*

On the one hand, an increase in the leader's factor input $n_{s,m}$ induces the use of more productive technique (higher $a_{s,m}$) and yields higher output via assimilation. This resembles the conventional positive productivity effect of technology adoption. On the other hand, an increase in $a_{s,m}$ widens the factor input-technology mismatch, thereby harming the assimilated production outcome. This is our novel mismatch effect as a result of discrepancies between factor inputs and factor requirements for assimilating the frontier technique. Proposition 4 concludes that the factor input-technology mismatch effect dominates when the m-th factor is an disadvantageous one ($n_m < \gamma_y n_{s,m}$). When the m-th factor is the advantageous one, the two effects align and always increase the assimilated output. In the exercise of development accounting, we will check whether the theoretical predictions drawn from our propositions are supported by the cross country data.

Finally, by taking log and totally differentiating with respect to time, the rate of growth assimilated output is derived as

$$\hat{y} = \underbrace{\hat{z}_s}_{\text{source TFP growth}} + \underbrace{\sum_{m=1,\ldots,N} \alpha_m \hat{n}_m}_{\text{factor growth}} + \underbrace{\sum_{m=1,\ldots,N} (\pi_m - \alpha_m)(\hat{n}_m - \hat{n}_{s,m})}_{\text{change in mismatch}}, \tag{10}$$

where $\hat{x} \equiv d \ln x/dt$ and $\pi_m \equiv \alpha_m \left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}} \Big/ \sum_{m=1,\ldots,N} \alpha_m \left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}$. Thus, output growth with assimilation can be decomposed into three components: (i) source TFP growth, (ii) the conventioanl factor growth component as in Caselli (2005) and others, and (iii) the new component highlighted by this paper measuring changes in factor input-technology mismatch over time. Focusing on the mismatch component, we stress that the difference in the relative growth of factor m between the two countries, $\hat{n}_m - \hat{n}_{s,m}$, must be properly weighted by the associated gap between the factor share (π_m) and the technology share (α_m). When this gap is larger, changes in the factor input difference becomes more important for driving output growth.

It is interesting to note that widening factor input-technology mismatch need not lower output growth. The following corollary to Propositon 4 provides the necessary and sufficient condition for the harmful effect of widening mismatch to arise.

Corollary 1 *Output growth, $\hat{y}$, is increasing in the relative factor growth, $\hat{n}_m - \hat{n}_{s,m}$, if and only if $n_m \leq \gamma_y n_{s,m}$.*

Corollary 1 states that the disadvantageous factors have a positive weight in the growth component of the mismatch, i.e., $\pi_m - \alpha_m > 0$ for m such that $n_m \leq \gamma_y n_{s,m}$. Given that the factor is a disadvantageous one, then the mismatch component diminishes with an increase in the factor input difference. So narrowing the mismatch of such disadvantageous factors, i.e., by having $\hat{n}_{m_d} - \hat{n}_{s,m_d} > 0$, will bring additional growth in the assimilated output.[9]

The next proposition further establishes the relationship between assimilation ability (σ) and relative factor advantage ($n_m/n_{s,m}$), as well as their interaction on growth.

Proposition 5 *There exists $\gamma_g \in [n_{m_d}/n_{s,m_d}, n_{m_a}/n_{s,m_a}]$ such that the growth coefficient, $\pi_m - \alpha_m$, is decreasing in σ if and only if $n_m \leq \gamma_g n_{s,m}$. Moreover, $\gamma_g \leq \gamma_y$ where the equality holds if either $\sigma = 1$ or $n_m/n_{s,m} = \gamma_0 > 0$ for all m.*

Interestingly, although a higher assimilation ability always raises the *level* of output (Proposition 3), Proposition 5 points out that its effect on output *growth* is ambiguous and depends on the relative factor advantage. Specifically, if the factor is a disadvantageous one, the positive effect on output growth from its growth relative to the source country diminishes when assimilation ability is greater (higher σ). The key insight of Proposition 5 on how assimilation ability affects the relationship between the degree of mismatch and the relative factor advantage is elaborated intuitively as follows. Given the complementarity nature of factor inputs in the assimilation process, the more disadvantageous a factor is, the more essential it would be in production using assimilated techniques. With lower assimilation ability, factor complementarity rises and the disadvantageous factors become more important in production, so it is harder to assimilate the frontier techniques (see Propositions 3 and 4). As a result, *narrowing the mismatch of these disadvantageous factors has a much larger impact on output growth when assimilation ability is low.*

4 Data and Methodology

In the quantitative analysis, we estimate the assimilation model with the data from the Penn World Table 9.0 (PWT) for 107 countries over the period of 1950 to 2014.[10] We exploit several nice features of this new generation of the PWT. First, to compare

[9] Similarly, the m_a-th factor captures the most advantageous factor in assimilation. The explanation goes through in vice versa.

[10] See Feenstra et al. (2015) for details. We exclude outliners like the former USSR countries, former Yugoslav countries (long time series are not available anyway) and all OPEC countries (due to large dependence on energy production where the output price is heavily influenced by nonmarket factors). In the end, we have a panel of 106 countries and the U.S. While we always use maximal length of

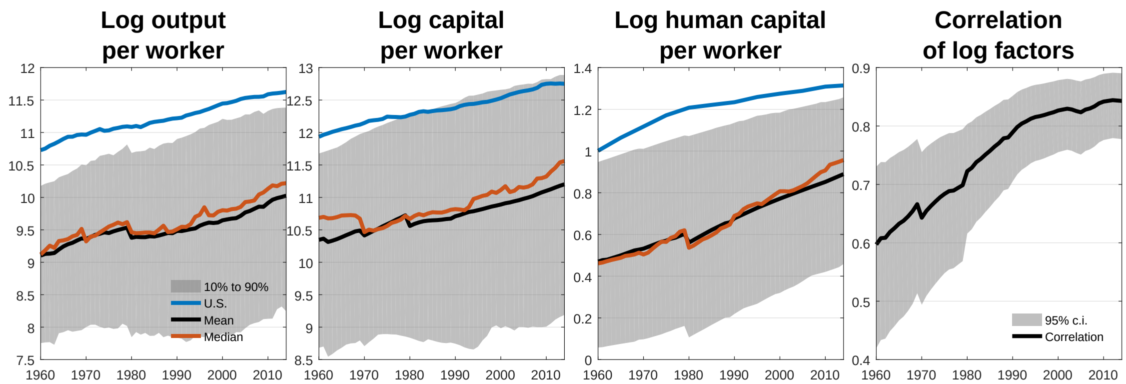

Figure 4: Global output, capital and human capital. Source: PWT 9.0

production capacity, an output-based measure of real GDP, rgdp^o, is used income measure, y_t. Second, consistent measures of physical capital, k_t, and (years-of-school based) human capital, h_t, are available to calibrate our assimilation model, where multi-dimensional factors in addition to per capita physical capital are crucial for the mismatch problem (thus, view $n_1 = k$ and $n_2 = h$, from now on). Here, we divide both output and physical capital measures by employment to obtain per-worker series.[11] Finally, this new generation of the PWT is based on multiple vintages of price data to construct the income time series. The PWT provides reliable information without the concerns about inconsistency raised by Johnson et al. (2013) for the earlier version.

Figure 4 illustrates the time-series of global incomes, physical capital and human capital. Over the last fifty years, while the gaps between the U.S. and the world average, in terms of output or factor input, are pretty much constant (the gap between the two series), the inequalities in physical capital and output among countries are widening (the grey area). Human capital inequality has been steadily improved, due to faster increases in years of schooling in poor countries compared to the U.S. Nevertheless, human and physical capitals are increasingly correlated, which also contribute to the overal inequality of the factor input.

Let the U.S. be the technology frontier. Following Propositions 1 or 2, the US technique in our model is $a_{1,t} = k_{US,t}/y_{US,t}$ and $a_{2,t} = h_{US,t}/y_{US,t}$. We estimate the

data in each country for our analysis, not many have data available prior to 1960, so in the rest of the paper we only report our post-1960 results. Results based on the earlier vintages of PWT are also available upon request.

[11] Note from Barro and Lee (1996) that human capital is based on the Mincerian returns from the year of schooling, so it is already a per capita measure.

following specification of the aggregate production function

$$y_{j,t} = \exp(\tau_{j,t})\, y_{US,t} \left[\alpha \left(\frac{k_{j,t}}{k_{US,t}} \right)^{1-1/\sigma} + (1-\alpha) \left(\frac{h_{j,t}}{h_{US,t}} \right)^{1-1/\sigma} \right]^{1-1/(1-\sigma)}, \tag{11}$$

where the TFP residual follows a standard specification:

$$\tau_{j,t+1} = \tau_j^0 + \rho_j \tau_{j,t} + \nu_{j,t+1}. \tag{12}$$

with i.i.d. zero-mean innovation, $\nu_{j,t}$, satisfying $\mathbb{E}(\nu_{i,t}, \nu_{j,t}) = \Omega$ and $\mathbb{E}(\nu_{j,t}\nu_{j,t'}) = 0$ for any $t \neq t'$.[12] In general, the assimilation mechanism and the TFP residual are the two competing forces in explaining the income variation. Here we restrict to a single σ to fit 107 countries for more than 60 years of time series, but allow that countries have its own process of TFP residual, where its innovation has country-specific variance and correlates with other countries' innovation contemporarily. This flexible specfication on $\tau_{j,t}$ gives the TFP residual the best explanation power to the data, implying that the explanation power of σ would be conservative.

Following Hall and Jones (1999), we set $\alpha = 1/3$ for the physical capital share such that our specification allows the standard Cobb-Douglas parameterizartion as a special case. The estimation of the EOS σ, altogether with $\{\tau_j^0, \rho_j\}$ and Ω, follows the non-linear feasible generalized least square estimator outlined in Antras (2004) and León-Ledesma et al. (2010). The non-linear feasible generalized least square estimator finds that $\sigma = 0.42$.

The empirical literature of estimating EOS is huge. Summarizing a large number of studies (mostly without normalization), Chirinko (2008) finding that "while the estimates range widely, the weight of the evidence suggests a value of EOS in the range of 0.40 and 0.60." León-Ledesma et al. (2010) demonstrate that the estimates of EOS can be biased, depending whether the technical changes are rightly modelled. Table 1 focuses on the literature that allows biased technical changes. Our estimate of $\sigma = 0.42$ is in line with this literature, which found σ between 0.2 and 0.9 [with a notable outlier $\sigma = 1.2$ by Karabarbounis and Neiman (2014)]. See Online Appendix B for a discusion about our estimate and the literature.[13]

[12] As commonly assumed in the literature, factor inputs are determined at the beginning of each period, so $\mathbb{E}\left(\nu_{j,t}\left[\frac{k_{j,t}}{k_{s,t}}\frac{h_{s,t}}{h_{j,t}}\right]\right) = 0$. We do, however, allow $\mathbb{E}(\nu_{j,t'}k_{j,t}) \neq 0$ and $\mathbb{E}(\nu_{j,t'}h_{j,t}) \neq 0$ for $t' \leq t$, so factor inputs can be contemporarily correlated with the TFP residual through the shocks in previous periods. These capture the standard environment in growth models that factor inputs take time to build and the aggregate shocks are usually persistent.

[13] Data on the labor shares is also available for some countries in PWT 9.0. We also estimate σ from the labor shares, after introducing more structure to pin down these shares as in the literature. Results are available upon request.

	EOS (σ)	Sample	Biased technical change
Brown and de Cani (1963)	0.08-0.35	1890-1958, U.S.	Constant growth
David and van de Klundert (1965)	0.32	1899-1960, U.S.	Constant growth, allowing incomplete capital adjustment
Wilkinson (1968)	0.5	1899-1953, U.S.	Constant growth
Sato (1970)	0.5-0.7	1909-1960, U.S.	Constant growth
Panik (1976)	0.76	1929-1966, U.S.	Constant growth, allowing learning-by-doing
Kalt (1978)	0.76	1929-1967, U.S.	Constant growth
Antràs (2004)	0.64-0.89	1948-1998, U.S.	Constant growth
Klump et al. (2007)	0.56	1953-1998, U.S.	Time-varying, Box-Cox model
Karabarbounis and Neiman (2014)	1.20	1975-2012, 129 countries	Capital-augmenting change proxied by TFP growth
Oberfield and Raval (2014)	0.71	1972-2007, U.S. manufacturing	Aggregating plant-level EOS
Lawrence (2015)	0.41-0.69	1947-2010, U.S. manufacturing	Constant growth
León-Ledesma and Satchi (2017)	0.2	1948-2013, U.S.	Adjustment cost to technical changes
This paper	0.42	1950-2014, 107 countries	Time-varying, assimilating the U.S.

Table 1: Empirical studies of the elasticity of substitution. References before 2007 are combined from Chirinko (2008) and León-Ledesma et al. (2010), see therein for details.

5 Development Accounting

Development accounting asks how much world income disparities can be explained by factor input differences, controlling for production efficiency (Caselli, 2005). This section conducts development accounting under our assimilation framework. A useful feature of our framework for accounting exercises is that it can allow the set of frontier techniques and its productivity changing over time (but each country still assimilates the same frontier technology). The relative outputs of country j to the frontier in the data and in the three models are given as follows.

- Data:

$$q_{j,t}^{\text{data}} = \frac{y_{j,t}}{y_{US,t}}.$$

- Our Assimilation Model:

$$q_{j,t} = \left[\alpha\left(\frac{k_{j,t}}{k_{US,t}}\right)^{1-1/\sigma} + (1-\alpha)\left(\frac{h_{j,t}}{h_{US,t}}\right)^{1-1/\sigma}\right]^{1-1/(1-\sigma)}. \tag{13}$$

- The Cobb-Douglas Model:

$$q_{j,t}^{\text{CD}} = \frac{z_{US,t}\,(k_{j,t})^{\alpha}\,(h_{j,t})^{1-\alpha}}{y_{US,t}} = \left(\frac{k_{j,t}}{k_{US,t}}\right)^{\alpha}\left(\frac{h_{j,t}}{h_{US,t}}\right)^{1-\alpha}. \tag{14}$$

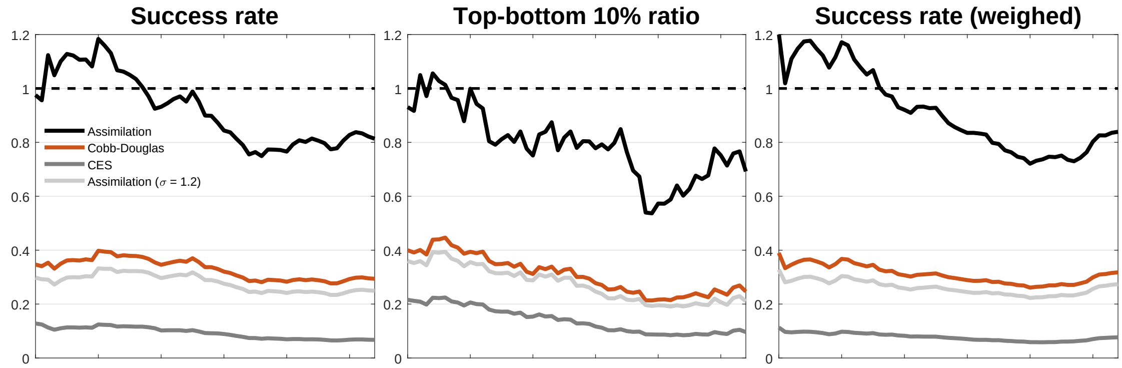

Figure 5: Comparison of success rates.

- The Hick-neutral CES Model:

$$q_{j,t}^{\text{CES}} = (k_{US,t})^{-\alpha} (h_{US,t})^{\alpha-1} \left[\alpha (k_{j,t})^{1-1/\epsilon} + (1-\alpha)(h_{j,t})^{1-1/\epsilon}\right]^{1-1/(1-\epsilon)}. \tag{15}$$

To facilitate comparison, we fix the EOS $\epsilon = \sigma$ for all countries. We also compare with the model with high assimilation ability $\sigma = 1.20$ [as in Karabarbounis and Neiman (2014)] to contrast the impact of imperfect assimilation under our estimate $\sigma = 0.42$.

In the standard development accounting exercise, we measure how much international income variation can be accounted by the model with the following success rate:

$$S_t \equiv \frac{VAR(\log q_{j,t})}{VAR\left(\log q_{j,t}^{\text{data}}\right)}. \tag{16}$$

Similarly, we construct S_t^{CD}, S_t^{CES} and S_t^{H} for the Cobb-Douglas model, the Hick-neutral CES model and the assimilation model with a high value $\sigma = 1.20$ respectively. In principle, the better the model is capable of explaining the income disparity, the closer the success rate is to 100%. Nonetheless, Caselli (2005) argues that the variations can be sensitive to outliers. A measure that is less sensitive to outliers, while at the cost of forgoing some information in the data, is a measure of the inter-percentile differential. As such, we also calculate the success rate based on the comparison between the top 10-th percentile, $q_{90th,t}$, and the bottom 10-th percentile, $q_{10th,t}$, as

$$S_{p,t} \equiv \frac{q_{90th,t}/q_{10th,t}}{q_{90th,t}^{\text{data}}/q_{10th,t}^{\text{data}}}. \tag{17}$$

with similar counterparts, $S_{p,t}^{\text{CD}}$, $S_{p,t}^{\text{CES}}$ and $S_{p,t}^{\text{H}}$. Figure 5 reports the results of the development accounting since 1960.

No matter which measure of success rates, our assimilation model consistently yields better success rates than the alternatives for almost the entire sample period. For instance, our assimilation model on average can explain 92% of the income disparities in terms of the conventional success rate, compared to a rate of about 33% of the Cobb-Douglas model, 9% of the CES model and 28% for the high-σ model; for the top-bottom 10-th percentile measures of the success rates, our assimilation model yields a rate of 80%, compared to 31%. 14% and 27% for the three alternative models.

Finally, one might also argue that the income variation is biased toward small countries; for example, China and India together are counted as a tenth of the twenty African countries in computing the income variation even though China and India are forty-times larger than in population. One way to address (at least partly) this concern is to weight the success rate with population:

$$S_{w,t} \equiv \frac{VAR_w\left(\log q_{j,t}\right)}{VAR_w\left(\log q_{j,t}^{\text{data}}\right)}. \tag{18}$$

with weighted success rates $S_{w,t}^{\text{CD}}$, $S_{w,t}^{\text{CES}}$ and $S_{w,t}^{\text{H}}$ for the the three alternatives. Again, as shown in Figure 5, the previous ranking on the success-rate remains valid. The success rate of our assimilation model is almost the same (92%), and the success rates of the three alternatives reduce slightly (31% for the Cobb-Douglas, 8% for the CES and 26% for the high-σ). Notice that the weighted success rate is calculated as if no inequality within the country, so the correction may remain biased in another direction.

5.1 Why do standard models underperform?

To understand why the standard models underperform in development accounting, Figure 6 compares the distributions generated by these models. Specifically, we classify the sample countries based on their (i) initial (relative) incomes in 1960, (ii) "current" (relative) incomes in 2010, and (iii) average growth rates (relative to the U.S.) during the sample period. The initial/current incomes indicate a country's initial/current development stage; the average growth indicates its catching up speed with the U.S.

Cobb-Douglas and high σ. Comparison with models of Cobb-Douglas and high σ highlighs the role of imperfect assimilation. Regardless of which income-level classifications we use, the Cobb-Douglas model always generates too much upper tail of the relative income distribution while missing the lower half of the distribution. Intuitively, Proposition 3 shows that, under our NCES setup, income level is increasing in σ. So a model with imperfect assimilation ($\sigma = 0.42$ in our estimation) can generate more lower tail in the level than the Cobb-Douglas model with $\sigma = 1$. Increasing assimilation ability to $\sigma = 1.2$ (the last column in Figure 6) only misses more lower tail than Cobb-Douglas since the better assimilation ability mitigates the output loss due to mismatch.

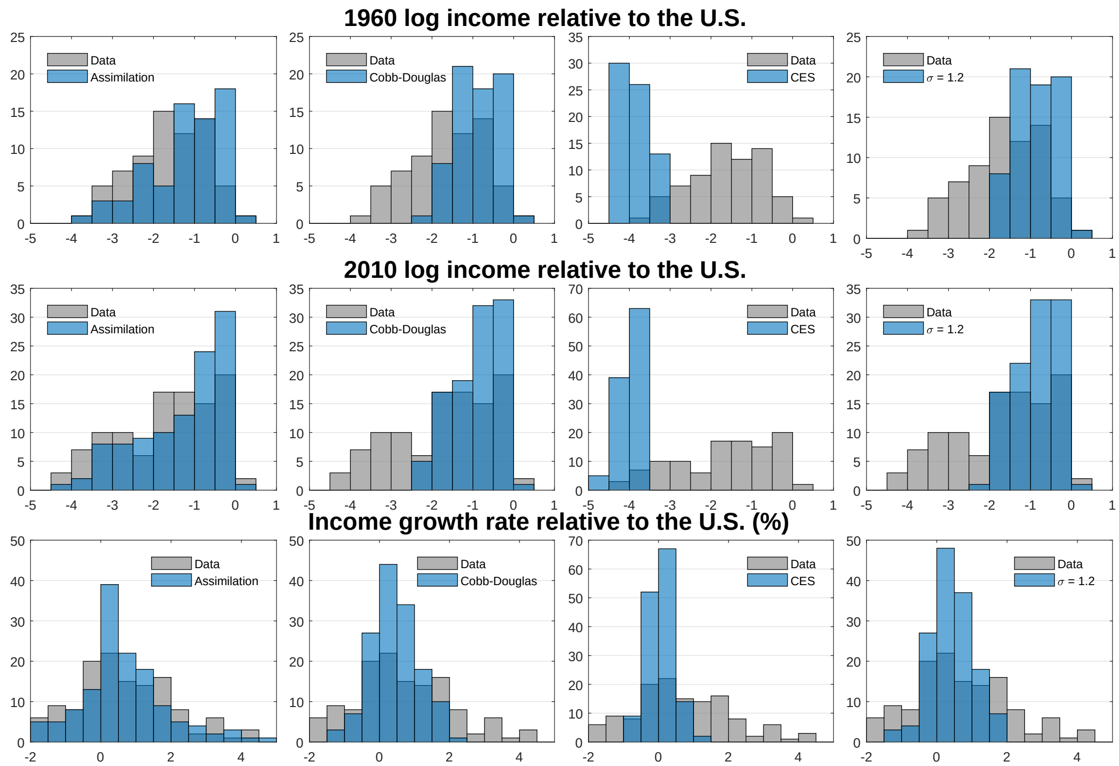

Figure 6: Distributions of $\log(q_{j,1960})$, of $\log(q_{j,2010})$ and of time-series average $\hat{q}_j$ according to different models.

The last row of Figure 6 reflects the prediction of Proposition 5. For the growth distribution, the Cobb-Douglas model misses the two tails because the mismatch contribution for the extreme (both positive and negative) growing economies are severely underpredicted. The assimilation model with $\sigma = 0.42$ gives rise to addition contribution (positive and negative) to growth so that the predicted number of extreme growing countries increases. Thus, the distribution spreads more evenly. These outcomes are consistent with the right panel of Figure 2**.** With a lower σ than the Cobb-Douglas technology, the fitted line in Figure 2 generated from the assimilation model tilts in the clockwise direction and reduces the bias of the two ends.

Hick-neutral CES. Comparison with the Hick-neutral CES models highlighs the role of mismatch. The Hick-neutral CES model shifts the distribution to the left but inevitably misses the upper half of the income distribution. Since the Hick-neutral CES model misses the factor biasness of technology, if we lower the EOS ϵ from Cobb-Douglas toward the our EOS estimate ($\sigma = 0.42$), then factor complementarity overcorrects the skewness of the Cobb-Douglas model with all the predicted outcomes concentrated at the left tail. This can be seen by rewriting (15) as

$$q_{j,t}^{\text{CES}} = \left[\alpha \left(\frac{k_{j,t}}{k_{US,t}^{\alpha} h_{US,t}^{1-\alpha}} \right)^{1-1/\epsilon} + (1-\alpha) \left(\frac{h_{j,t}}{k_{US,t}^{\alpha} h_{US,t}^{1-\alpha}} \right)^{1-1/\epsilon} \right]^{1-1/(1-\epsilon)} . \tag{19}$$

So the CES model can be understood as an NCES setup with the baseline normalization at the geometric average of the frontier factor inputs. It implies that one of the relative factor inputs in (15) must be smaller compared with those in (13). As a result, factor complementarity makes the predicted relative output smaller. This explains the difference in the first and last panels of the first two rows in Figure 6.

For the growth distribution, the Hick-neutral CES production function ignores the relative factor advantage so that there is no mismatch consideration. With the EOS $\epsilon = 0.42$, factor complementarity implies that the less-growing countries dominates the fast-growing ones.[14] This reduces the spread of the relative distribution so that the predicted growth outcomes are more concentrated around the zero benchmark than the Cobb-Douglas model.

5.2 Fitness

So far the development accounting distinguishes that how well our framework can explain the income inequality on the global level. It would also be useful to examine how well our framework can explain the income dynamics for each country. This diagnostics is pursued in this section with two fitness measures. Recall that the TFP residual in our model is given by $\tau_{j,t} = \log q_{j,t}^{\text{data}} - \log q_{j,t}$; similarly, $\tau_{j,t}^{\text{CD}}$ for the Cobb-Douglas model and $\tau_{j,t}^{\text{CES}}$ for the Hick-neutral CES model. Define relative TFP as

$$z_j \equiv \exp(\mathbb{E}\tau_{j,t}), \tag{20}$$

and similarly $z_j^{\text{CD}} \equiv \exp(\mathbb{E}\tau_{j,t}^{\text{CD}})$ and $z_j^{\text{CES}} \equiv \exp(\mathbb{E}\tau_{j,t}^{\text{CES}})$ for the Cobb-Douglas and the Hick-neutral CES models. A model that competently explains the income dynamics should feature $\tau_{j,t}$ close to zero on average and a value of z_j close to one; a lower-than-unity (higher-than-unity) relative TFP means the model implies a lower (higher) productivity and hence underpredicts (overpredicts) the output data. Thus, we can rank the model explanation power by comparing z_j, z_j^{CD} and z_j^{CES} to one.

A model that misses some significant movements in the output dynamics may still feature z_j close to one if these movements happen to cancel each other. In this regard

[14] This applies to countries with either positive or negative growth rates. See a discussion of this non-balanced growth result for the case of positive growth rates in Acemoglu and Guerrieri (2008).

	#	z_j	z_j^{CD}	z_j^{CES}	**MSE**	$\leq$ **MSE**^{CD}	$\leq$ **MSE**^{CES}
Fraction of U.S. income in 1960							
$\leq 25\%$	39	0.62	0.40	8.61	0.60	97%	95%
$(25\%, 50\%]$	16	0.74	0.69	18.93	0.20	94%	100%
$(50\%, 75\%]$	10	0.83	0.79	22.61	0.11	90%	100%
$> 75\%$	3	0.87	0.85	27.12	0.03	100%	100%
Fraction of U.S. income in 2010							
$\leq 25\%$	56	0.63	0.32	5.83	0.67	100%	92%
$(25\%, 50\%]$	21	0.75	0.62	14.60	0.27	86%	100%
$(50\%, 75\%]$	16	0.81	0.72	20.40	0.17	88%	100%
$> 75\%$	13	0.78	0.75	23.15	0.10	25%	100%
Fraction of U.S. growth rate							
$\leq 50\%$	23	0.66	0.32	5.52	0.56	96%	91%
$(50\%, 100\%]$	17	0.63	0.50	12.12	0.44	100%	100%
$(100\%, 200\%]$	47	0.70	0.56	14.88	0.34	96%	96%
$> 200\%$	19	0.81	0.52	11.95	0.57	89%	95%
Overall	106	0.70	0.49	11.94	0.44	95%	95%

Table 2: Comparison of explanation power.

we also complement our diagnostics with the following MSE measure:

$$MSE_j \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\tau_{j,t}\right)^2 . \tag{21}$$

Specifically, MSE_j captures the time-series average variation of the income gap of country j that cannot be generated by the assimilation model. We then construct comparable measures, MSE_j^{CD} and MSE_j^{CES}, for the two alternative models.

Table 2 summarizes our findings. For all the countries in the sample, the CES model yields the worst performance, in terms of both the relative TFP and the MSE metrics, just like when we judge the development accounting performances using success rates. As such, we shall focus on the comparison between the assimilation model and the Cobb-Douglas model.

The relative TFP of our assimilation model is 0.7 on average, about 40% higher than that of the Cobb-Douglas counterpart. In terms of MSEs, the overall average of our assimilation model is 0.44, lower than the Cobb-Douglas counterpart in 95% of the sample countries. We also group countries according to the income quartiles in 1960 and 2010, as well as according to their average income growth rates.[15] In

[15] The average income growth rate of the U.S. for the sample period is 1.79%.

terms of the relative TFPs, our assimilation model is always better than the Cobb-Douglas model in the low tail of the income distributions, and also for fast-growing countries. These results are robust in terms of the MSEs. Specifically, the MSEs of our model for the above three low-income or low-growth groups are smaller than the Cobb-Douglas model in 97%, 100% and 96% of the sample countries, respectively. For the fast-growing countries, the MSEs of the assimilation model is still lower than the Cobb-Douglas model in 89% of the sample countries.

Summarizing, our assimilation framework yields better fit in general, generating z_j closer to unity and lower MSEs. The highest explanatory power is obtained in the following three groups: (i) countries with sufficiently low income (no more than a quarter of the U.S. income); (ii) countries with very low growth (no more than half of the U.S. growth rate); (iii) countries with very high growth rates (more than doubling the U.S. rates). These conclusions are consistent with our findings given in Figure 6, showing that our assimilation model is better in capturing the tails observed in the data than the alternatives. In other words, technology assimilation can be regarded as crucial for understanding *why some developing countries advanced successfully and some lagged behind or even fell into the development trap.* As a result, the consideration of technology assimilation enables a sizable reduction in the unexplained income gap: lowering it from 51% under the standard Cobb-Douglas model to 30%.

6 Growth Accounting

To gain further insight into the role of technology assimilation played in driving a country's growth path, we check how much it is able to explain changes in income gap over time. By assimilating the U.S. technique, relative income growth of country j can be, utilizing (10), decomposed as

$$\hat{q}_{j,t}^{\text{data}} = \hat{q}_{j,t} + \Delta\tau_{j,t} = \alpha\left(\hat{k}_{j,t} - \hat{k}_{US,t}\right) + (1-\alpha)\left(\hat{h}_{j,t} - \hat{h}_{US,t}\right) + \hat{M}_{j,t} + \Delta\tau_{j,t}, \quad (22)$$

where

$$M_{j,t} \equiv \left[\alpha\left(\frac{k_{j,t}}{k_{US,t}}\frac{h_{US,t}}{h_{j,t}}\right)^{1-1/\sigma} + 1-\alpha\right]^{1-1/(1-\sigma)} \left(\frac{k_{j,t}}{k_{US,t}}\frac{h_{US,t}}{h_{j,t}}\right)^{-\alpha}. \quad (23)$$

The first two terms on the right side of (22) measure the conventional factor growth contribution, via the accumulation of physical capital and human capital, as in the Cobb-Douglas model. The difference of the factor growth contribution from the data is typically regarded as the Solow residual, $\Delta\tau_{j,t}^{\text{CD}} = \hat{M}_{j,t} + \Delta\tau_{j,t}$, which is usually found very large (see the literature cited as well as our replication below). In our

	#	Relative growth(pp)			Contribution to growth(%)			
		Income	Capital	Human capital	Capital	Human capital	Mismatch	Wedge
Fraction of U.S. income in 1960								
$\leq 25\%$	39	0.66	0.36	0.39	28.84	6.38	49.93	14.85
$(25\%, 50\%]$	16	0.97	1.13	0.23	25.46	5.52	10.21	58.81
$(50\%, 75\%]$	10	0.21	0.77	0.01	36.19	3.07	4.56	56.18
$> 75\%$	3	0.01	0.36	-0.11	-28.86	20.89	-1.95	109.93
Fraction of U.S. income in 2010								
$\leq 25\%$	56	-0.15	-0.31	0.47	32.55	-14.29	60.63	21.11
$(25\%, 50\%]$	21	1.16	1.65	0.49	35.49	26.00	23.79	14.73
$(50\%, 75\%]$	16	1.33	1.73	0.24	37.74	17.18	19.70	25.38
$> 75\%$	13	1.32	1.40	0.14	31.48	8.77	6.31	53.44
Fraction of U.S. growth rate								
$\leq 50\%$	23	-1.74	-1.45	0.51	21.58	-24.76	51.24	51.94
$(50\%, 100\%]$	17	-0.28	-0.20	0.17	39.81	-54.78	55.36	59.61
$(100\%, 200\%]$	47	0.96	1.13	0.34	38.89	28.91	36.13	-3.93
$> 200\%$	19	2.84	2.46	0.61	30.53	14.57	24.96	29.94
Overall	106	0.51	0.60	0.40	33.78	1.27	40.49	24.45

Table 3: Growth accounting in different groups of countries.

assimilation framework, we can further isolate the contribution of an endogenous TFP component due to changes in the factor input-technology mismatch, $\hat{M}_{j,t}$. We will show the importance of this new and interesting component and check its validity by referring back to the development accounting results obtained before.

Table 3 reports the growth accounting results in different groups of income levels and growth rates as classified before. Each time-series growth rate is obtained for each country; average within each subgroup and for the entire sample (overall) are subsequently computed. The third to fifth columns report the subgroup and overall average of the growth rates of income and factor inputs relative to the U.S. The last four columns report the contribution to the relative income growth rates due respectively to: (i) physical capital accumulation, (ii) human capital accumulation, (iii) changes in factor input-technology mismatch, and (iv) changes in the (unexplained) TFP residual. These contributions correspond to the four terms in (22).

The growth accounting based to the Cobb-Douglas model rely entirely on the contribution from factor accumulation, leading to a large TFP residual component and failing to capture the development proces facing poor and low-growth countries that locate in the left tail of the distribution. For example, for countries in the lowest growth subgroup, the Cobb-Douglas model predicts that there would have been positive income growth relative to the U.S., contradicting the data (-1.74). As a result, the Cobb-Douglas model contributes a negligible -3.18% (obtained from summing up the contributions from the accumulation of the two capitals), failing explain the lack of growth and hence leaving all the explanatory power to the Solow residual.

Turning now to the assimilation model, we are able to account for more than 60% of the Solow residual in the Cobb-Douglas alternative. Overall, changes in the factor input-technology mismatch contribute to 40% of relative income growth, even more important than the contribution of physical capital accumulation (about one-third) and TFP residual changes (about a quarter). Most dramatically, for the 47 moderately fast growing countries (with growth rates exceeding the U.S. but no more than double), TFP residual changes become inessential whereas the accumulation of each capital and the mismatch all have comparable contributions.

For initially low-income countries (no more than a quarter of the U.S. income in 1960), the consideration of technology assimilation lowers the contribution of the TFP residual (from the Cobb-Douglas counterpart of 64.78%) to 14.85%. Such large reductions in the unexplained component also arise in currently low-income countries (no more than a quarter of the U.S. income in 2010) and in the fastest growing economies (growing at a speed more than doubling the U.S.). In all these countries, the factor input-technology mismatch plays a key role, accounting for half (or more) of their economic growth. By focusing on initial development stage and development speed, human capital accumulation is important only in those subgroups with countries having initially high relative income (more than three quarters of the U.S.) or growing at least as fast as the U.S.

6.1 Traps and Miracles

Because our assimilation model is more powerful than the Cobb-Douglas in explaining the income stagnation observed in many development trapped countries and the miraculous development process of fast growing countries. To gain more insight, we conduct in-depth study of some representative countries from these two subgroups.

With regard to trapped countries, we select 18 with relatively longer time series and divergent movements. We depict the data and the predicted income dynamics by the Cobb-Douglas model and by our assimilation model in Figure 7. The average MSE of our assimilation model is 0.61 over this subsample, far lower than the counterpart in the Cobb-Douglas model (2.04).[16] Among them, our model only fails to track the

[16] The variation is huge with the maximum at 1.60 (Niger) and the minimum at 0.01 (Burundi)

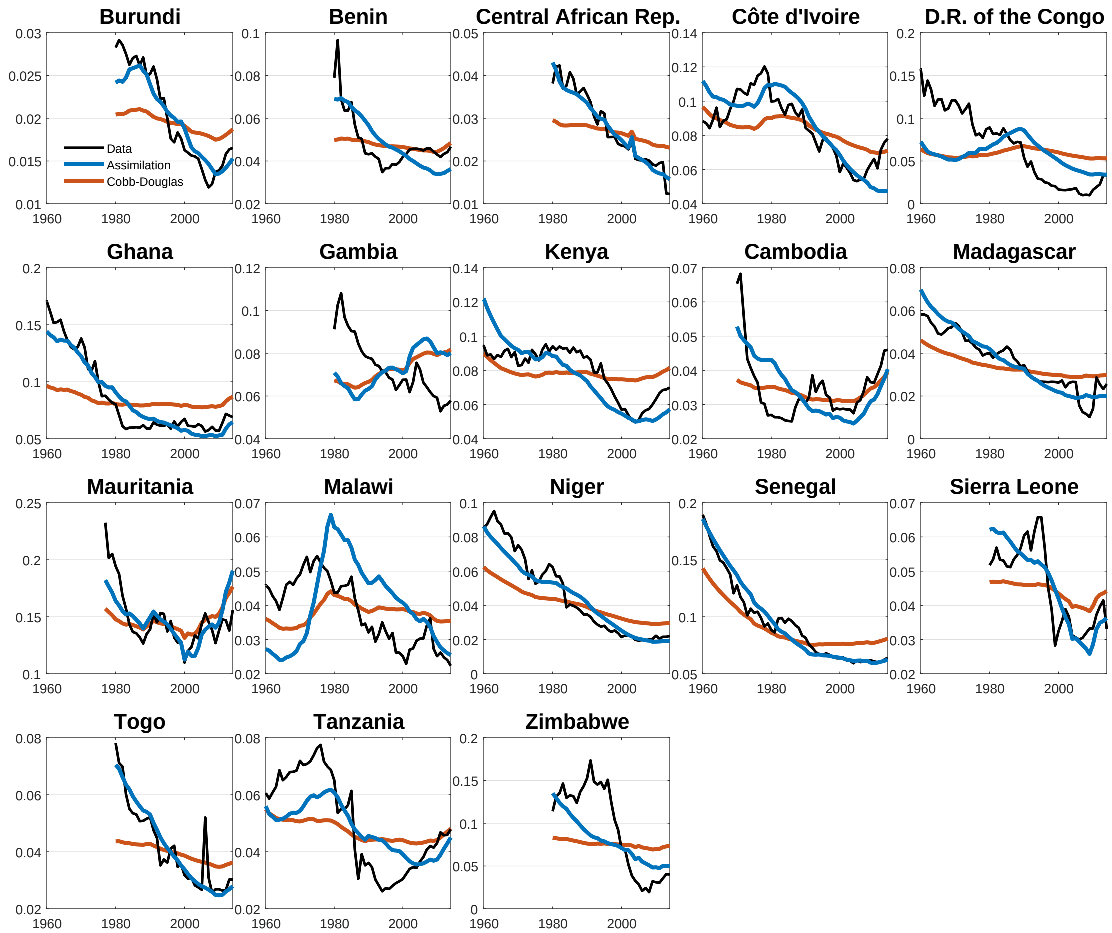

Figure 7: Income time-series for the development-trapped countries.

income dynamics of Gambia prior to 1990, but the Cobb-Douglas model misses many episodes of such movements in most of these countries.

For most of the time these trapped countries typically have less than 5% of the U.S. income and their income gaps are widening. While human capital is improving for most of the trapped countries (on average 0.41 percentage points faster than the U.S.), physical capital is common to see negative growth. The latter (former) is the (dis)advantageous factor, as reported in the third and fourth columns of Table 4. The fact that physical (human) capital is decumulating (accumulating) over time widens the factor input-technology mismatch.[17] Thus, from Proposition 5, this mismatch channel is translated to lower growth. Our quantitative results indicate that this new channel is significant in explaining the data observation: it explains on average 94%

in the assimilation model. For the Cobb-Douglas model, the variation is similarly large with the maximum at 3.71 (Central African) and the minimum at 0.40 (Mauritania).

[17] The only exception is Senegal, but it also has factor advantage reversal.

	Relative growth(pp)			Growth contribution(%)			Fitness			
	Income	**Capital**	**Human capital**	**Mismatch**	**Capital**	**Human capital**	z_j	$z_j^{\mathbf{CD}}$	$\mathbf{MSE}_j$	$\mathbf{MSE}_j^{\mathbf{CD}}$
Burundi	-1.58	-1.37	0.30	69.13	28.95	-12.49	1.00	0.19	0.01	2.77
Benin	-1.55	-2.35	1.04	116.55	50.59	-44.85	0.43	0.24	0.74	2.18
Central African	-3.33	-3.23	0.58	67.30	33.26	-11.62	0.32	0.15	1.31	3.71
Côte d'Ivoire	-0.23	-2.15	0.22	438.35	311.38	-64.39	0.41	0.32	0.84	1.32
Congo D.R.	-2.30	-1.30	0.14	40.67	18.82	-4.08	1.76	0.44	0.83	1.41
Ghana	-1.35	-2.50	0.74	91.53	61.60	-36.39	0.44	0.28	0.68	1.69
Gambia	-1.34	0.33	0.70	15.97	-8.11	-34.62	0.86	0.41	0.09	0.84
Kenya	-0.54	-1.78	0.48	212.50	108.59	-58.73	0.55	0.31	0.38	1.39
Cambodia	-0.80	-0.68	0.54	93.17	28.31	-44.80	0.64	0.21	0.25	2.54
Madagascar	-1.52	-2.42	0.01	98.41	53.17	-0.30	0.57	0.20	0.36	2.69
Mauritania	-1.08	-0.08	0.51	17.80	2.56	-31.42	0.71	0.54	0.13	0.40
Malawi	-1.14	-0.47	0.02	26.84	13.67	-1.02	0.48	0.19	0.69	2.75
Niger	-2.51	-3.58	-0.27	55.46	47.57	7.22	0.28	0.20	1.60	2.72
Senegal	-2.01	-3.36	0.11	48.09	55.71	-3.50	0.47	0.36	0.58	1.08
Sierra Leone	-1.32	-1.75	0.63	107.48	44.30	-31.68	0.64	0.26	0.22	1.83
Togo	-2.78	-3.04	0.70	78.23	36.38	-16.85	0.50	0.20	0.50	2.66
Tanzania	-0.43	-0.45	-0.15	36.42	34.23	22.52	0.33	0.20	1.25	2.62
Zimbabwe	-3.07	-3.40	1.16	83.57	36.92	-25.28	0.63	0.28	0.45	2.05
Average	-1.60	-1.87	0.41	94.25	53.22	-21.79	0.61	0.28	0.61	2.04

Table 4: Growth accounting for the development-trapped countries up to 2014.

of the widening income gap from the U.S., whereas the Cobb-Douglas model only accounts for 31%. Also, upon consideration technology assimilation, the unexplained income gap is reduced (on average) to 39% (from 72% in the Cobb-Douglas alternative). The detailed accounting results are reported in Table 4. Interestingly, in all but one country (Senegal), the mismatch component contributes more (mostly much more) than physical capital in their subpar development experiences.

We turn now to an in-depth study on how miracles happened. As shown in Figure 8, our assimilation model tracks the data quite well. Notably, although incomes in these countries were on average less than 15% of the U.S. income in the 1960s, they have maintained a prolong period of strong growth. By 2014 the relative income has been more than triple in Hong Kong, four times in Taiwan, five times in Singapore and six times in Korea. Overall, factor accumulation can explain 64% of the growth performance for these ten miracle countries. The disadvantageous factor – physical capital – has been growing much faster than the advantageous one – human capital: the average growth rate of the former is 2.5% faster than the U.S. whereas the latter is only 0.8%. The factor input-technology mismatch has therefore been improving over time and technology assimilation can account for another 30% of their prolonged growth. As a result, the TFP residual only accounts for 6%.

Our assimilation story is highly plausible in countries that are importing foreign technology based on their export-led growth policy, for instance, Korea and Taiwan.

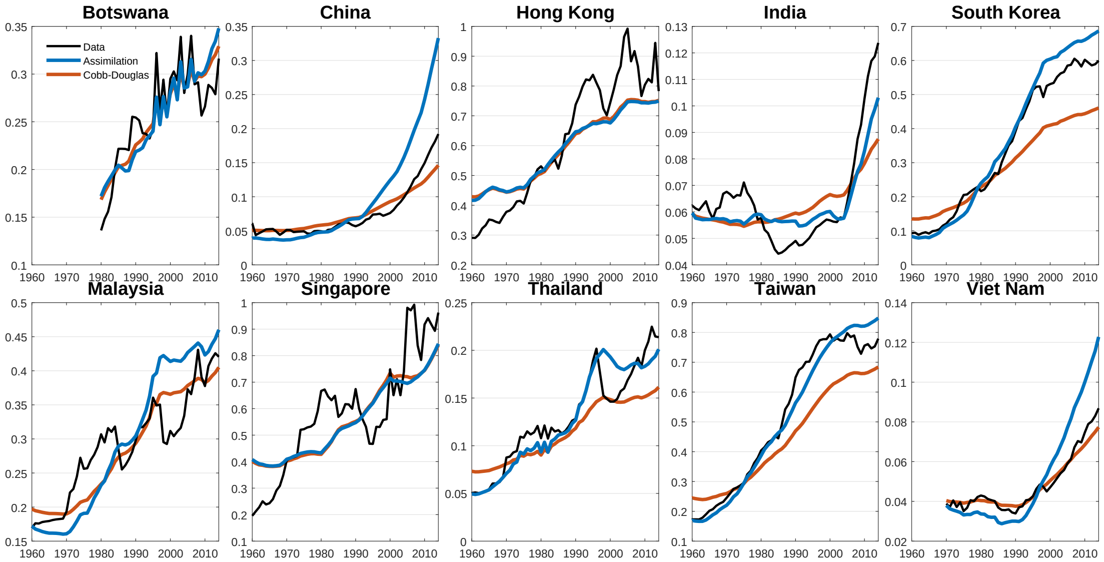

Figure 8: Income time-series for the miracle countries

Assimilation is important for Korea and Taiwan to catch up with the frontier, which contributes to 47% and 27%, respectively, to growth via narrowing the factor input-technology mismatch. It translates to extra 1.49% and 0.81% annual growth rates in Korea and Taiwan respectively. Similar conclusion can be apply to the ASEAN countries such as Malaysia and Thailand too, with the factor input-technology mismatch accounts for 18% and 31%. respectively.

Narrowing the mismatch is less important in some miracle countries. In Hong Kong there is a reversal in relative factor disadvantage such that phyiscal capital eventually becomes the advantageous factor: a phenomenon detailed in the next subsection.[18] In Botswana and India, although the factor input-technology mismatch channel does not contribute much to their growth, the assimilation model still performs much better than the Cobb-Douglas model (with the MSEs being reduced by at least 30%), and the assimilation model captures most of the important income dynamics, as illustrated in Figure 8. In Singapore, our assimilation model is largely indistinguishable from the Cobb-Douglas counterpart. The assimilation model becomes more distinguishable once we allow for alternative sources and early stop of assimilation when the local economy becomes highly advanced, shown in Section 8.

Finally, for China and Vietnam, factor accumulation alone has already accounted for 100% and 81% of their growth performance respectively, so our assimilation is

[18] As to be shown later, such a reversal also occurred in Taiwan but in a more moderate manner toward the end of the sample period, thereby not dampening much of the assimilation channel.

	Relative growth(pp)			Growth contribution(%)			Fitness			
	Income	Capital	Human capital	Mismatch	Capital	Human capital	z_j	z_j^{CD}	MSE_j	MSE_j^{CD}
Hong Kong	1.83	2.13	0.50	2.59	38.67	18.15	0.79	0.78	0.09	0.10
Korea	3.18	4.17	0.53	46.73	43.79	11.22	0.79	0.52	0.07	0.51
Singapore	2.95	2.01	1.05	-0.80	22.82	23.94	0.81	0.79	0.12	0.13
Taiwan	3.05	3.51	0.68	26.58	37.39	14.87	0.99	0.79	0.01	0.14
Botswana	2.48	2.27	1.82	4.04	30.56	48.99	0.59	0.53	0.29	0.42
China	1.79	4.08	0.71	101.41	76.10	26.63	0.83	0.31	0.18	1.43
India	0.97	0.37	0.36	0.19	12.62	24.97	0.44	0.26	0.68	1.84
Malaysia	1.64	1.71	0.80	17.78	34.67	32.63	0.79	0.64	0.09	0.22
Thailand	1.83	1.94	0.66	30.81	35.39	23.94	0.46	0.31	0.63	1.42
Vietnam	1.83	2.76	0.83	65.24	50.38	30.36	0.78	0.23	0.11	2.16
Average	2.15	2.50	0.80	29.46	38.33	25.55	0.73	0.52	0.23	0.84

Table 5: Growth accounting for the miracle countries up to 2014.

doomed to overpredict their growth, as illustrated in Figure 8. Nevertheless, for these economies, there are significant reduction in the unexplained income gap compared with the Cobb-Douglas model, from 69% to 17% for China, and from 77% to 22% for Vietnam. Similar to the case of Singapore, the selection of the assimilated frontier country matters as well, though rather than early stop they both face late start due to delayed implementation of market-oriented development policy.

In summary, the lack of assimilation is proven to prevent trapped countries from advancing; assimilation accompanied by accumulating the disadvantageous factor can help a country to produce miraculous development outcomes. Our findings echo the conclusion that the assimilation model is better than the Cobb-Douglas model in capturing the two tails of the growth distribution in the data, especially the left tail.

6.2 Factor Advantage Reversal and Middle-Income Trap

Proposition 5 states that rapid accumulation in the disadvantageous factors relative to the advantageous factors can drive a faster growth, nevertheless, it is possible that these disadvantageous factors may eventually become advantageous. In this circumstance, the factor input-technology mismatch will be reversed. Such reversal may arise naturally as an equilibrium outcome, or as a result of active development policy.

We have already mentioned such cases in the previous discussion, including the miracle ones such as Hong Kong and Taiwan. In the case of Hong Kong, factor advantage reversal may reflect the structural transformation of these countries from capital intensive manufacturing to service. The reversal may also happen in many middle-income countries such as Greece and Portugal, some high-income countries such as France and some low-income countries such as Senegal. For initially middle-income

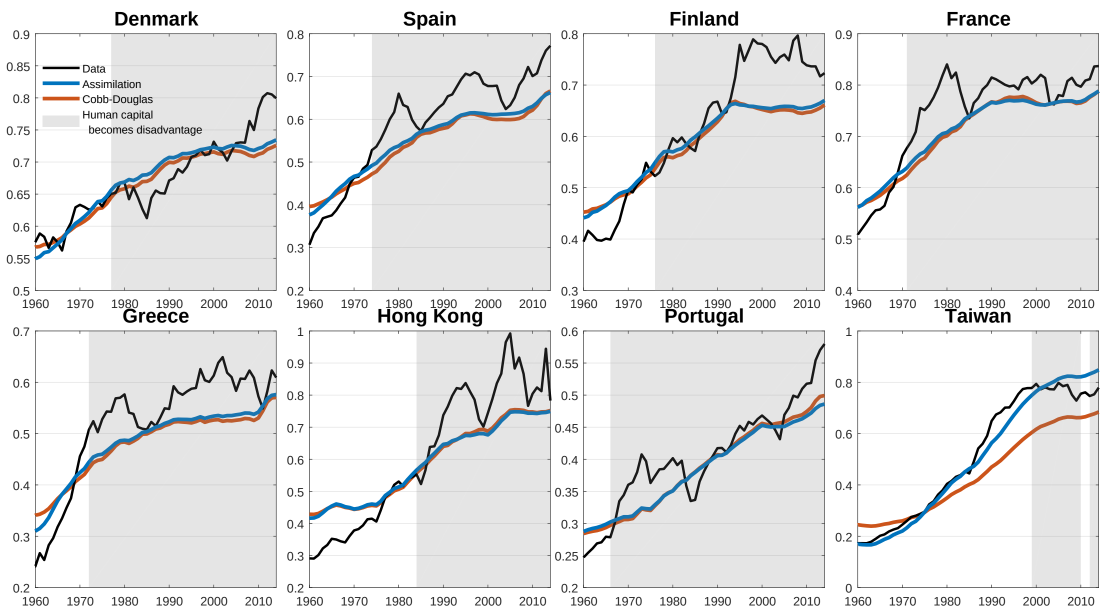

Figure 9: Income time-series for the countries with disadvantage reversal.

countries including development miracles, after the reversal has taken place, the factor input-technology mismatch will be working in the reverse direction. So the country's income will be bottlenecked at the middle level unless the once-advantageous factors keep up to the growing pace of the once-disadvantageous factors. Figure 9 highlights some of the countries that experience the relative factor advantage reversal.

In most of the countries experiencing reversal, the mismatch channel of the assimilation model does not contribute much in growth accounting. This is not difficult to understand because the contribution to growth accounting of the relative factor advantage is reversed after the reversal has taken place. For the eight countries depicted in Figure 9, the mismatch contributes to growth for only 9.4%. The detailed growth accounting results are reported in Table 6. After the reversal has taken place, almost all of the countries have their mismatch contribution turned negative. The only exception is Taiwan whose mismatch contribution falls but still remains positive (from 26.61% to 0.09%). This is likely due to the fact that the reversal has taken place three times in a relatively moderate manner only toward the last decade of the sample period, thus having less impact on the contribution of mismatch.

To better understand the mismatch contribution for the reversal countries, we compute the counterfactual increase in growth had the reversal been absent. Specifically, the growth differential is computed as the difference in the incremental growth caused by the mismatch component before and after the reversal. Such growth differentials,

	Relative growth(pp)			Growth contribution(%)			
	Income	Capital	Human capital	Mismatch	Capital	Human capital	Δgrowth (pp)
Denmark- overall	0.54	1.26	-0.24	26.28	77.26	-29.71	0.39
before reversal	0.50	2.06	-0.85	74.65	137.01	-113.16	
after reversal	0.56	0.60	0.18	-2.74	35.91	21.65	
Spain- overall	2.01	2.47	0.10	7.96	41.08	3.45	0.76
before reversal	3.63	3.50	-0.38	17.76	32.13	-6.90	
after reversal	0.95	1.80	0.39	-12.25	63.19	27.47	
Finland- overall	1.26	1.71	0.17	10.65	45.08	9.08	0.40
before reversal	2.00	2.67	-0.098	18.75	44.56	-3.26	
after reversal	0.85	0.95	0.35	-2.52	37.04	27.09	
France- overall	1.11	1.54	-0.0072	5.53	46.46	-0.43	0.35
before reversal	2.36	2.35	-0.56	12.86	33.11	-15.74	
after reversal	0.49	1.13	0.25	-10.25	76.28	33.34	
Greece- overall	1.53	1.43	0.51	-0.81	31.26	22.22	0.09
before reversal	3.56	1.70	0.72	1.26	15.88	13.53	
after reversal	0.44	1.13	0.42	-9.19	86.56	64.86	
Hong Kong- overall	1.83	2.13	0.50	2.59	38.67	18.15	0.26
before reversal	2.72	2.46	0.32	7.11	30.14	7.79	
after reversal	1.16	1.79	0.69	-5.46	51.52	34.64	
Portugal- overall	1.62	1.67	0.47	-3.63	34.42	19.58	0.14
before reversal	2.05	1.05	-0.23	2.17	17.03	-7.59	
after reversal	1.53	1.82	0.71	-6.03	39.63	31.17	
Taiwan- overall	3.05	3.51	0.68	26.58	37.39	14.87	1.09
before reversal	4.09	4.35	0.63	26.61	35.50	10.20	
after reversal	0.11	0.85	0.85	0.09	257.26	512.40	

Table 6: Growth accounting for the countries with disadvantage reversal

reported in the last column of Table 6 (Δgrowth), can be large if the differences in the mismatch contribution and in income growth caused by the reversal are sizable. For example, in the case of Denmark, the difference in the mismatch contribution is large but the difference in income growth is negligible, whereas both differences in Spain are large. This explains why the growth differential is larger in Spain than in Denmark (0.76 pp vs. 0.39 pp). Similar to Spain, differences in both the growth rates and in the mismatch contribution are large in Taiwan, leading to a large growth drop (by 1.09 pp). However, mismatch does not play much role after the reversal. Its sharp decline in growth is mainly associated with the widening of the TFP residual, despite decent factor accumulation.[19]

To close the section, we would like to point out that the mechanism of factor

[19] This can be explained by the fact that the relative factor advantage reversal of Taiwan is not significant in 2010. The relative k and relative h in 2010 are 0.84 and 0.83 respectively so that the relative ratio of k/h is 1.01 which is the lowest in the group.

Country	Chow's test	Eichengreen et. al. (2014)	This paper
Denmark	1969	1968-70, 1973	1977
Spain	**1975**	1966, 1969, **1972-7**	**1974**
Finland	**1974**	**1974-5**, 2002-3	**1976**
France	**1974**	**1973-4**	**1971**
Greece	**1972**	**1969-78**, 2003	**1972**
Hong Kong	1993	**1981-2**, 1990-4	**1984**
Portugal	1974,1990	1973-4, 1977, 1990-2	1966
Taiwan	**1995**	**1992-7**	**1999**, 2011, 2012

Table 7: Timing for the midde-income traps and for the factor advantage reversal.

advantage reversal can serve as a plausible explanation for the *middle-income traps* that features a significant slowdown of once fast-growing economies in the midst of its development. Specifically, Eichengreen et al. (2014) identify empirically the timing of middle-income trap with the following three criteria: (i) before the trap year the seven-year moving averages of the income growth rate are at least 3.5%; (ii) since the trap year the seven-year averages of the income growth rate decrease by at least 2 percentage points; and, (iii) at the trap year the per capita real GDP is at least USD10,000 at 2005 international PPP. Table 7 compares the middle-income traps identified by Eichengreen et al. (2014) to that based on that identified by factor advantage reversals. For robustness we also report the middle-income traps identified as the structural break by the Chow's test. For easy comparison we have boldfaced the matches with no more than two years apart from the finding of Eichengreen et al. (2014). For most countries in Table 7, it is found that the timing of the factor advantage reversal matches quite well the empirical timing of the middle-income trap. Thus, the mechanism of factor advantage reversal can be regarded as an empirically plausible cause of the slowdown of the middle-income traps experienced by these once fast-growing economies.

7 Country-Specific Assimilation and Counterfactual

So far we have assumed that all countries have the common ability to assimilate the frontier technology. In this section we allow for country-specific σ_j, which will allocate more explanation power to the assimilation mechanism by estimating σ_j that minimizes the MSE_j for each country j. In general, allowing country-specific assimilation significantly improves the fitness in all income groups (especially the lowest), by reducing the overall MSE from 0.44 to 0.15. The unexplained income gap falls from 30% to 14% (see Table 8 below for details).

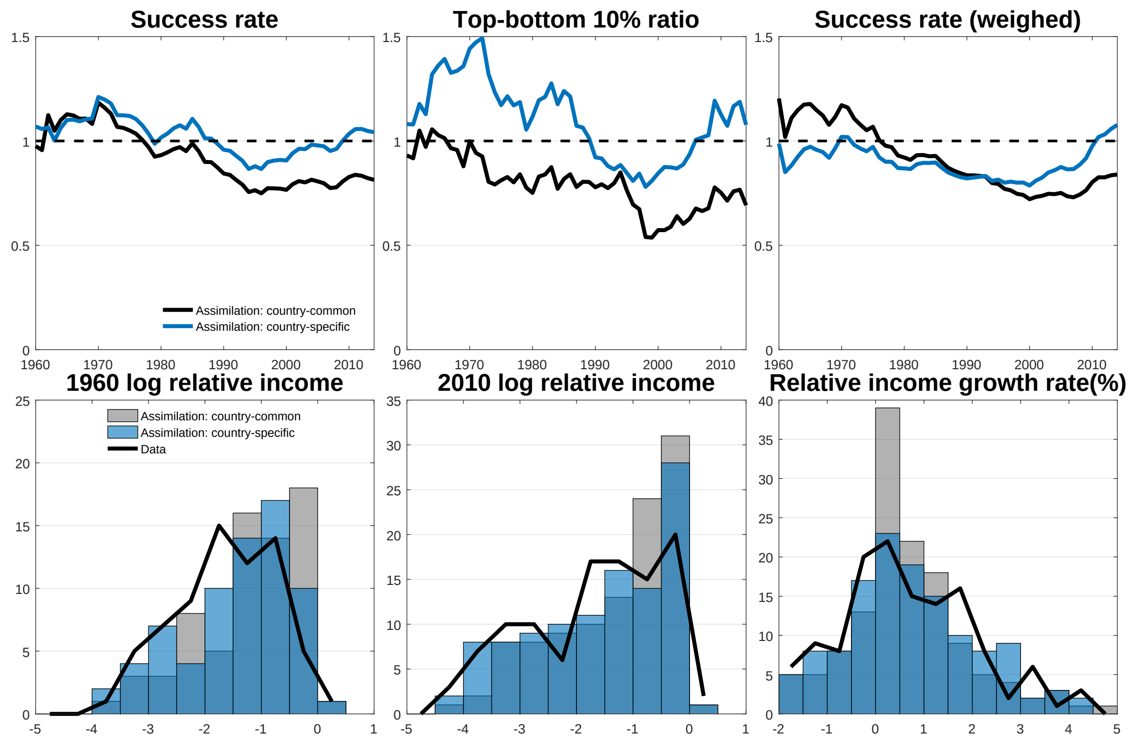

Figure 10: Development accounting with country-specific assimilation.

The upper panel of Figure 10 compares the success rates after applying the country-specific estimates. Allowing country-specific assimilation improves all three success rates starting from the 1990s. But before 1990, the comparison yields mixed outcomes: the weighted success rate is lower under country-specific than country-common assimilation, whereas the outcomes are reversed for the two other measures of success rates, which are always higher under country-specific assimilation. Notice that σ_j is estimated to maximize the average fitness of the entire sample period for each country, so the success rates can be lower under country-specific assimilation in some subsample period. On average, the three success rates yield close to 100% under country-specific assimilation, compared with 87% under country-common assimilation.

The lower panel of Figure 10 compares the predicted distribution, which are examined in different subgroups of countries in Table 8. In most countries, the country-specifc assimilation ability is much lower than the common value of $\sigma = 0.42$. Notice that the mismatch term under country-specific σ_j becomes

$$M_{j,t}\left(\sigma_j\right) \equiv \left[\alpha\left(\frac{k_{j,t}}{k_{US,t}}\frac{h_{US,t}}{h_{j,t}}\right)^{1-1/\sigma_j} + 1 - \alpha\right]^{1-1/(1-\sigma_j)} \left(\frac{k_{j,t}}{k_{US,t}}\frac{h_{US,t}}{h_{j,t}}\right)^{-\alpha}. \tag{24}$$

By allowing country-specific σ_j, the contribution to growth from the change in mis-

	#	Relative income growth (pp)	σ_j	Country-common			Country-specific		
				Mismatch contribution(%)	z_j	MSE	Mismatch contribution(%)	z_j	MSE
Fraction of U.S. income in 1960									
$\leq 25\%$	39	0.66	0.11	49.93	0.62	0.60	61.02	0.82	0.22
$(25\%, 50\%]$	16	0.97	0.09	10.21	0.74	0.20	22.09	0.86	0.11
$(50\%, 75\%]$	10	0.21	0.11	4.56	0.83	0.11	49.11	0.89	0.06
$> 75\%$	3	0.01	0.00	-1.95	0.87	0.03	-78.61	0.93	0.02
Fraction of U.S. income in 2010									
$\leq 25\%$	56	-0.15	0.13	60.63	0.63	0.67	89.08	0.85	0.22
$(25\%, 50\%]$	21	1.16	0.18	23.79	0.75	0.27	34.44	0.88	0.11
$(50\%, 75\%]$	16	1.33	0.12	19.70	0.81	0.17	30.61	0.88	0.06
$> 75\%$	13	1.32	0.03	6.31	0.78	0.10	19.01	0.87	0.06
Fraction of U.S. growth rate									
$\leq 50\%$	23	-1.74	0.18	51.24	0.66	0.56	69.89	0.81	0.14
$(50\%, 100\%]$	17	-0.28	0.07	55.36	0.63	0.44	133.59	0.86	0.15
$(100\%, 200\%]$	47	0.96	0.09	36.13	0.70	0.34	44.08	0.85	0.15
$> 200\%$	19	2.84	0.18	24.96	0.81	0.57	26.21	0.85	0.19
Overall	106	0.51	0.12	40.49	0.70	0.44	60.83	0.86	0.15

Table 8: Country-specific assimilation in different groups.

match increases from 40% to 61% on average (except the subgroup with three initially rich countries). As a result, Proposition 3 implies that the overshooting in the upper tail of the distribution under the country-common assimilation can now be corrected under the country-specific assimilation. Also, Proposition 5 signifies the factor input-technology mismatch so that the two tails of the country distribution in income growth are thickened. This increases the spread of the distribution as shown in Figure 10.

7.1 Revisiting Traps and Miracles

Table 9 summarizes the accounting results of the trapped countries based on the country-specific σ_j. The average σ_j for the trapped countries is 0.13, much lower than the country-common level of 0.42.[20] As a result, the lower ability of assimilation of this trapped group magnifies its significance in mitigating the factor input-technology mismatch, leading to a large contribution to growth from widening mismatch (more than 120%). This is consistent with Proposition 5. However, such a correction turns out to be overdone in some countries like Benin, Côte d'Ivoire, Ghana and Senegal, where the accounting outcome appears to overstate the contribution of mismatch. In these countries the estimated country-specific σ_j is at the zero lower bound - the complete inflexibility in adjusting the factor input ratio to the frontier technology may inflate the

[20] The only exception is Congo whose σ is 0.58 which is higher than the world average. As a result, growth contribution owing to the mismatch diminishes in the country-specific assimilation model. However, its fitness is the worst in the group with the highest MSE of 0.56.

	σ_j	Mismatch contribution(%)			MSE		z_j	
		Country-common	**Country-specific**	**Δgrowth (pp)**	**Country-common**	**Country-specific**	**Country-common**	**Country-specific**
Burundi	0.42	69.13	69.15	0.00	0.01	0.01	1.00	1.00
Benin	0.00	116.55	146.03	0.46	0.74	0.08	0.43	0.86
C. African	0.00	67.30	78.15	0.36	1.31	0.19	0.32	0.65
Côte d'Ivoire	0.00	438.35	687.14	0.57	0.84	0.23	0.41	0.66
Congo D.R.	0.58	40.67	32.78	-0.18	0.83	0.56	1.76	1.06
Ghana	0.00	91.53	159.60	0.92	0.68	0.10	0.44	0.79
Gambia	0.35	15.97	17.41	0.02	0.09	0.08	0.86	1.00
Kenya	0.06	212.50	275.92	0.35	0.38	0.06	0.55	1.00
Cambodia	0.22	93.17	101.37	0.07	0.25	0.06	0.64	1.00
Madagascar	0.14	98.41	106.65	0.13	0.36	0.04	0.57	1.00
Mauritania	0.17	17.80	35.51	0.19	0.13	0.01	0.71	1.00
Malawi	0.00	26.84	28.36	0.02	0.69	0.16	0.48	0.99
Niger	0.00	55.46	87.92	0.81	1.60	0.50	0.28	0.50
Senegal	0.00	48.09	108.37	1.21	0.58	0.14	0.47	0.76
Sierra Leone	0.21	107.48	120.21	0.17	0.22	0.03	0.64	1.00
Togo	0.03	78.23	89.61	0.32	0.50	0.02	0.50	1.00
Tanzania	0.00	36.42	45.94	0.04	1.25	0.24	0.33	0.65
Zimbabwe	0.18	83.57	99.02	0.47	0.45	0.21	0.63	1.00
Average	0.13	94.25	127.17	0.33	0.61	0.15	0.61	0.88

Table 9: Growth accounting for the development-trapped countries, with country-specific assimilation.

role of mismatch in growth accounting.[21] Nevertheless, country-specific assimilation significantly improves the fitness, largely reducing the MSE (from 0.61 to 0.15) and the unexplained income gap (from 39% to 12%).

We also perform the following counterfactual exercise: how much would growth have changed if we remove all the country-specific heterogeneity in assimilation ability? The results are reported in the fifth column of Table 9. Cross-country variations in assimilation ability lead to, on average, 0.33 percentage points difference in growth. This indicates that inability to assimilate can further explain growth stagnation by one-third a percentage point, which is a nonnegligible figure to the trapped countries and greatly raises the contribution of mismatch. Of course the counterfactual exercise is preliminary in the sense that it abstracts from the endogenous effect on factor accumulation after the change in assimilation ability. Nevertheless, these results suggest that the widening of mismatch is the key to understand the development trap.

Table 10 summarize the results obtained from the miracle countries. Without repeating, allowing for country-specific assimilation ability generates better fit (lowers MSEs from 0.23 to 0.06), reduces unexplained income gap (from 27% to 8%), raises

[21] It should be noted, however, that having a lower bound estimate of the assimilation parameter does not always lead to over-correction (for example, in Central Africa, Malawi and Tanzania).

	σ_j	Mismatch contribution(%) Country-common	Country-specific	Δgrowth (pp)	MSE Country-common	Country-specific	z_j Country-common	Country-specific
Hong Kong	0.00	2.59	22.80	-0.37	0.09	0.04	0.79	0.88
Korea	0.32	46.73	57.67	-0.35	0.07	0.05	0.79	0.88
Singapore	0.00	-0.80	-0.02	-0.02	0.12	0.07	0.81	0.91
Taiwan	0.39	26.58	28.95	-0.07	0.01	0.01	0.99	1.01
Botswana	0.00	4.04	12.14	-0.20	0.29	0.03	0.59	0.90
China	0.36	101.41	111.33	-0.18	0.18	0.16	0.83	0.95
India	0.00	0.19	0.27	-0.00	0.68	0.04	0.44	0.87
Malaysia	0.24	17.78	31.28	-0.22	0.09	0.06	0.79	0.95
Thailand	0.00	30.81	46.84	-0.29	0.63	0.06	0.46	0.83
Vietnam	0.32	65.24	69.55	-0.08	0.11	0.05	0.78	1.00
Average	0.16	29.46	38.08	-0.18	0.23	0.06	0.73	0.92

Table 10: Growth accounting for the miracle countries, with country-specific assimilation.

the contribution of the mismatch component in growth accounting (from 29% to 38%), and explains an additional 0.18 percentage points of income growth in these countries. Of particular interest, mismatch countribution rises significantly in the case of Hong Kong (from a negligible impact to more than one-fifth); nonetheless, it does not help much for the cases of Singapore, Taiwan, India and Vietnam. While the case of Taiwan is easier to understand as the country-specific σ_j of 0.39 is close to the common value of 0.42, the other three cases worth further investigation in the Appendix.

7.2 Taking Stock

In the baseline exercisees before Section 7, we first restrict to the country-common, time-invariant assimilation but allow for flexible, country-specific processes of TFP residual. While these exercises are designed to be conservative, the results have illustrated the significant role of mismatch on the growth performance. By introducing country-specific assimilation in Section 7, we further explore the effects of countries' assimilation abilities. Imperfect assimilation of mismatch explains for 61% of the growth gap in the data. In sum, technology assimilation influences growth in two aspects. On the one hand, imperfect assimilation ability (low σ) yields a lower income level via its direct negative effect from limiting production flexibility. On the other hand, it interacts with the country's mismatch to yield an indirect effect on income growth via amplifying the growth effect from the disadvantageous factors. The outcomes of the success rates shown in Figure 10 illustrate these two opposing forces are at work.

Back to our theme question, which are the main forces widening the growth gap? In our assimilation model, it can be due to differences in relative factor advantage or

	#	Relative income growth (pp)	σ_j	Overall (%)	Mismatch contribution from country-specific assimilation: Country-specific assimilation ability(%)	Country-specific relative factor advantage(%)	Country-common assimilation ability(%)
Fraction of U.S. income in 1960							
$\leq 25\%$	39	0.66	0.11	61.02	11.08	49.43	0.50
$(25\%, 50\%]$	16	0.97	0.09	22.09	11.87	11.54	-1.33
$(50\%, 75\%]$	10	0.21	0.11	49.11	44.55	2.29	2.26
$> 75\%$	3	0.01	0.00	-78.61	-76.65	18.84	-20.79
Fraction of U.S. income in 2010							
$\leq 25\%$	56	-0.15	0.13	89.08	28.45	63.29	-2.66
$(25\%, 50\%]$	21	1.16	0.18	34.44	10.65	21.64	2.14
$(50\%, 75\%]$	16	1.33	0.12	30.61	10.92	18.53	1.17
$> 75\%$	13	1.32	0.03	19.01	12.70	6.13	0.18
Fraction of U.S. growth rate							
$\leq 50\%$	23	-1.74	0.18	69.89	18.65	54.29	-3.04
$(50\%, 100\%]$	17	-0.28	0.07	133.59	78.24	78.22	-22.86
$(100\%, 200\%]$	47	0.96	0.09	44.08	7.95	28.78	7.35
$> 200\%$	19	2.84	0.18	26.21	1.25	23.38	1.58
Overall	106	0.51	0.12	60.83	20.34	41.27	-0.78

Table 11: Mismatch decomposition

in assimilation ability. To gain further insight, Table 11 reports the decomposition of the log difference of factor input-technology mismatch, $\hat{M}_{j,t}(\sigma_j)$ of equation (24), into three components:

$$\underbrace{\hat{M}_{j,t}(\sigma_j)}_{\text{country-specific mismatch}} = \underbrace{\hat{M}_{j,t}(\sigma_j) - \hat{M}_{j,t}}_{\text{country-specific assimilation ability}} + \underbrace{\hat{M}_{j,t} - \hat{M}_t}_{\text{country-specific relative factor advantage}} + \underbrace{\hat{M}_t}_{\text{country-common assimilation ability}} \tag{25}$$

where both $\hat{M}_{jt}$ and $\hat{M}_t$ are both measured under the same $\sigma = 0.42$: while the former is the log difference of the mismatch terms associated with country-specific $k_{j,t}$ and $h_{j,t}$ the latter is associated with the cross-country averages of $k_{j,t}$ and $h_{j,t}$. Thus, the last term on the right side of (25) captures the global effect of assimilation ability; the middle term measures the effect of a country's relative factor advantage over the world trend but controlling the heterogeneity in assimilation ability; the first term measures the effect of a country's assimilation ability controlling the heterogeneity in relative factor advantage. Notice that the sum of the middle and last terms gives us the mismatch contribution to growth in Section 6 captured by $\hat{M}_{jt}$.

Overall, cross-country heterogeneity in assimilation ability accounts for one-third of the mismatch countribution (41% out of 61%), whereas cross-country heterogeneity in relative factor advantage trend accounts for the remaining two-third of the mismatch countribution (20% out 61%). For initially and currently poor countries and countries growing slower than the U.S., the contribution of country-specific relative factor advantage is even greater, becoming the most important driver for relative income growth. In the absence of cross-country heterogeneities in relative factor advantage and assimilation ability, mismatch becomes inconsequential, implying that there is not much global trend in the factor input-technology mismatch.

8 Extensions

So far we have assumed that countries assimilate the frontier technology from the U.S. throughout the entire sample period. In reality, countries at different stages of development may look for alternative assimilable targets, and they may either start assimilating late or stop it early to suit their specific economic backgrounds. We then compare these alternative "tailor-made" assimilation schemes to selected countries with those of our country-specific assimilation model to see whether alternative assimilation schemes may make a difference.

For brevity, we only summarize new insights below while relegating the detailed discussion to Appendix C: (i) alternative targets (e.g., Thailand assimilating Hong Kong; Malaysia assimilating Singapore; China and Vietnam assimilating Taiwan), (ii) early stop (e.g., Singapore, stopping assimilation in 1990), (iii) late start (e.g., India, starting assimilation from 1985) and (iv) middle-income laggards (e.g., Colombia, Fiji, and Nicaragua, where mismatch contributes to more than half of relative income growth with human capital contributing negatively due to over-accumulation of the advantageous factor).

9 Concluding Remarks

Adopting frontier technology does not improve income without successfully assimilating the associated mismatch between technology and factor inputs. The goal of this paper is to develop a useful accounting framework based on this idea to explain world income disparities, subsequently setting forth a first but crucial step before modeling the assimilation process in a fully fleged dynamic general equilibrium model. Our technology assimilation framework highlights the factor input-technology mismatch generated from the interaction between production flexibility associated with assimilation ability and the relative factor advantage owing to factor input ratio gap from the assimilated leader. By performing development and growth accounting exercises, we have shown that our assimilation model outperforms the standard models, particularly in trapped countries having consistently low income ratios throughout the sample period and in miracle countries experiencing fast growth. That is, we have identified the successful assimilation of the frontier technology as the key to understand the widening gap between fast-growing miracles and poverty-trapped economies. We have further established the possibility of relative factor advantage reversal and explained how such reversal may lead to middle-income traps of countries having fast growth in their disadvantageous factor.

The main implication is clear: to pull a poor country out of the trap or to prevent a developing country from falling into the middle-income trap, it requires an adequate provision of correct incentives and institutional settings. These factors are crucial for domestic firms to assimilate relevant frontier technologies in a way that is suitable for the country's own development stages. For instance, in the earlier stages of development where the assimilation ability of a labor-intensive country is low, policies should be focused on providing incentives for adequate accumulation of the disadvantageous factor. In the later stages of development when the disadvantageous capital accumulation becomes satiated so that the levels of factor inputs relative to the frontier are stable, then one has to avoid overaccumulation (factor input reversal) and the correct incentives should be to enhance the assimilation ability while maintaining the right balance of relative factor input ratios. As a country becomes more advanced, assimilation may stop and innovation should take the center of the stage. Notably, the establishment of science parks can be rewarding, because with the clustering of high-tech firms, local learning about specific needs can greatly enhance the effectiveness of technology assimilation. Moreover, better platforms to encourage the acquisition of know-how and tacit knowledge are essential as well for technology assimilation. Both science parks and knowledge exchange platforms can further ensure technological innovation and sustain economic growth. Unfortunately, we have also observed various barriers to capital accumulation, learning and knowledge flows in developing (and even developed) countries. Such barriers may result from incorrect incentives/institutions, protectionism, blocks by incumbents, and any other forms of frictions in financial, labor and international trade markets. These frictions would lead to factor-input misallocation and cause a country failing to mitigate the factor input-technology mismatch, thereby slowing down its development process.

Along these lines are several interesting avenues for future studies, but for brevity, we shall discuss only two. The first is to evaluate various human capital, industrial, and trade policies for their effectiveness in promoting growth through the technology assimilation channel. Specifically, our framework of technology assimilation may be incorporated with that of Jovanovic (2009) to explain the interdependence of vintage technology and human capital distribution. As a consequence, directional human capital policies may possibly affect cross-industry human capital distributions as well as encourage the development of techniques, industry-specific assimilation, and vintage technology. Moreover, the promotion of key industries in some developing countries may possibly be harmful to growth because of the lack of proper and efficient assimilation. Furthermore, the reluctance to further reduce tariffs associated with imported technology may have differential effects on the development processes depending on the effectiveness in the assimilation of world technologies. The second avenue for future study is to apply our framework to a country-by-country analysis across sub-industries. As firms in different industries can be expected to have different frontier technologies and different assimilation processes, this allows us to have a better understanding of the growth contribution of the aggregate factor input-technology mismatch from the micro-level firm assimilation process.

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Appendix

(Not Intended for Publication)

The Appendix contains mathematical details, detailed discussion on global EOS and detailed results of extensions.

A Appendix A: Mathematical Details

A.1 Proof of Proposition 1

Proposition 1 is proved by the following steps. We focus on $N \geq 2$ as the case of $N = 1$ is obvious. First we show that, instead of choosing $\mathbf{a} \in \mathcal{P}$, the global production function can be transformed to choosing any $\mathbf{a}_{-} \in \mathbb{R}^{N-1}_{++}$ such that

$$\overline{f}(\mathbf{n}) = \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-})),$$

where $\mathbf{a}_{-} = \{a_1, ..., a_{N-1}\}$, $\mathcal{A}(\mathbf{a}_{-})$ is on the differentiable envelope of $\mathcal{P}$ and

$$\mathcal{B}(\mathbf{a}_{-}) \equiv \{a_1, a_2, ...a_{N-1}, \mathcal{A}(\mathbf{a}_{-})\}.$$

Second, we show that the first-order condition implies that $\mathcal{A}(\mathbf{a})$ is in power form. It implies the global production function is Cobb-Douglas. Finally, we show that the first-order approach is valid if $\sigma \in [0, 1]$.

Step 1. We want to show $\overline{f}(\mathbf{n}) \geq \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-}))$. First we show $\overline{f}(\mathbf{n}) \leq \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-}))$. Since $\mathcal{P}$ is compact, the solution to (5) exists, denoted as $\mathbf{a}^{\star}$. Thus, we have

$$\begin{aligned} \overline{f}(\mathbf{n}) &= f(\mathbf{n}; \mathbf{a}^{\star}), \\ &\leq f(\mathbf{n}; \mathcal{B}((\mathbf{a}^{\star})_{-})), \\ &\leq \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-})). \end{aligned}$$

On the other hand, we want to show $\overline{f}(\mathbf{n}) \geq \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-}))$. Suppose $\mathbf{a}_{-}^{\star} = \arg\max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-}))$. Thus there exist some $\mathbf{a}_i$ such that $\mathbf{a}_{i-} \leq \mathbf{a}_{-}^{\star}$ and $\mathbf{a}_{i,N} = \mathcal{A}(\mathbf{a}_{-}^{\star})$. Then we have

$$\begin{aligned} \max_{\mathbf{a}_{-}} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_{-})) &= \left[\sum_{m=1,...,N-1} \alpha_m \left(\frac{n_m}{a_m^{\star}} \right)^{\frac{\sigma-1}{\sigma}} + \alpha_N \left(\frac{n_N}{\mathcal{A}(\mathbf{a}_{-}^{\star})} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \\ &\leq \left[\sum_{m=1,...,N} \alpha_m \left(\frac{n_m}{a_m} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \\ &\leq \overline{f}(\mathbf{n}). \end{aligned}$$

Step 2. From the envelope theorem, we have

$$\overline{f}_m(\mathbf{n}) = \frac{1}{a_m} f_m(\mathbf{n}; \mathcal{B}(\mathbf{a}_-)). \tag{A3}$$

Suppose the first-order approach is valid. The first-order condition of $\max_{\mathbf{a}_-} f(\mathbf{n}; \mathcal{B}(\mathbf{a}_-))$ with respect to a_m, $m = 1, ..., N-1$, is given by

$$-\frac{n_N}{\mathcal{A}(\mathbf{a}_-)^2} f_N(\mathbf{n}; \mathcal{B}(\mathbf{a}_-))\mathcal{A}_m(\mathbf{a}_-) = \frac{n_m}{(a_m)^2} f_m(\mathbf{n}; \mathcal{B}(\mathbf{a}_-)), \tag{A4}$$

where $\mathcal{A}_m(\mathbf{a}_-) \equiv \partial \mathcal{A}(\mathbf{a}_-)/\partial a_m$, and $f_m(\mathbf{n}; \mathbf{a}) = \partial \left[\sum_m \alpha_m \kappa_m^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} / \partial \kappa_m$. Using (A3) from the envelope theorem, we have

$$-\frac{a_m \mathcal{A}_m(\mathbf{a}_-)}{\mathcal{A}(\mathbf{a}_-)} \frac{n_N \overline{f}_N(\mathbf{n})}{\overline{f}(\mathbf{n})} = \frac{n_m \overline{f}_m(\mathbf{n})}{\overline{f}(\mathbf{n})}.$$

Since $\overline{f}(\mathbf{n})$ is Cobb-Douglas if and only if $n_m \overline{f}_m(\mathbf{n})/\overline{f}(\mathbf{n} = \alpha_m$ for all $\mathbf{n}$, the first-order condition implies that $\overline{f}(\mathbf{n})$ is Cobb-Douglas if and only if $\mathcal{A}$ solves the following partial differential equation (PDE):

$$\mathcal{A}_m(\mathbf{a}_-) = \frac{-\alpha_m}{\alpha_N} \frac{\mathcal{A}(\mathbf{a}_-)}{a_m}, \text{ for } m = 1, ..., N-1. \tag{A5}$$

Clearly, the solution to the above PDE has the following closed-form solution

$$\mathcal{A}(\mathbf{a}_-) = z^{-1/\alpha_N} \prod_{m=1,...,N-1} (a_m)^{-\frac{\alpha_m}{\alpha_N}},$$

where z is a positive constant. Substituting the above solution of $\mathcal{A}$ into (A4) we have $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$.

On the other hand, given that the first-order approach is valid, the optimal technique satisfies $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$ if and only if that $\mathbf{a}^\star(\mathbf{n}) = k(\mathbf{n})\mathbf{n}$, where $k : \mathbb{R}_+^N \to \mathbb{R}_+$, is a solution to (A4). By using the funtional form (4), it implies that $\mathcal{A}$ solves the PDE (A5). Thus, if the first-order approach is valid, then the global production function is Cobb-Douglas if and only if $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$.

Step 3. Suppose that $\sigma \in [0, 1]$. The second-order condition evaluated at $\mathbf{a} = \mathbf{n}/k$ is

$$\begin{aligned}
&\frac{-f_{Nm}(\mathbf{n}; \mathbf{n}/k)}{\mathcal{A}(\mathbf{a}_-)} \frac{\alpha_m n_N n_m}{\alpha_N a_m^3} - \frac{f_{NN}(\mathbf{n}; \mathbf{n}/k)}{\mathcal{A}(\mathbf{a}_-)} \frac{\alpha_m n_N^2 \mathcal{A}_m(\mathbf{a}_-)}{\alpha_N a_m \mathcal{A}(\mathbf{a}_-)^2} - \frac{f_N(\mathbf{n}; \mathbf{n}/k)}{\mathcal{A}^2(\mathbf{a}_-)} \frac{\alpha_m n_N}{\alpha_N a_m} \mathcal{A}_m(\mathbf{a}_-) \\
&- \frac{f_N(\mathbf{n}; \mathbf{n}/k)}{\mathcal{A}(\mathbf{a}_-)} \frac{\alpha_m n_N}{\alpha_N a_m^2} + \frac{2n_m}{(a_m)^3} f_m(\mathbf{n}; \mathbf{n}/k) + \frac{n_m^2}{(a_m)^4} f_{mm}(\mathbf{n}; \mathbf{n}/k) + \frac{n_m n_N}{(a_m)^2} f_{Nm}(\mathbf{n}; \mathbf{n}/k) \frac{\mathcal{A}_m(\mathbf{a}_-)}{\mathcal{A}(\mathbf{a}_-)^2}, \\
= \; & -f_{Nm}(\mathbf{n}; \mathbf{n}/k) \frac{\alpha_m k^2}{\alpha_N a_m^2} + f_{NN}(\mathbf{n}; \mathbf{n}/k) \left(\frac{\alpha_m k}{\alpha_N a_m}\right)^2 + f_N(\mathbf{n}; \mathbf{n}/k) \left(\frac{\alpha_m}{\alpha_N a_m}\right)^2 k \\
&- f_N(\mathbf{n}; \mathbf{n}/k) \frac{\alpha_m k}{\alpha_N a_m^2} + f_m(\mathbf{n}; \mathbf{n}/k) \frac{2k}{(a_m)^2} + \frac{k^2}{(a_m)^2} f_{mm}(\mathbf{n}; \mathbf{n}/k) - \frac{k^2}{a_m^2} \frac{\alpha_m}{\alpha_N} f_{Nm}(\mathbf{n}; \mathbf{n}/k),
\end{aligned}$$

$$
\begin{aligned}
&= -\alpha_m \alpha_N (\sigma k)^{-1} \frac{\alpha_m k^2}{\alpha_N a_m^2} - \alpha_N (1-\alpha_N)(\sigma k)^{-1} \left(\frac{\alpha_m k}{\alpha_N a_m}\right)^2 + \alpha_N \left(\frac{\alpha_m}{\alpha_N a_m}\right)^2 k \\
&\quad -\alpha_N \frac{\alpha_m k}{\alpha_N a_m^2} + \alpha_m \frac{2k}{(a_m)^2} - \frac{k^2}{(a_m)^2} \alpha_m (1-\alpha_m)(\sigma k)^{-1} - \frac{k^2}{a_m^2} \frac{\alpha_m}{\alpha_N} \alpha_m \alpha_N (\sigma k)^{-1}, \\
&= \left[-\alpha_N \left(\frac{\alpha_m}{\alpha_N a_m}\right)^2 - \frac{\alpha_m}{(a_m)^2} \right] \frac{k}{\sigma} + \frac{\alpha_m k}{(a_m)^2} + \left(\frac{\alpha_m}{\alpha_N a_m}\right)^2 \alpha_N k, \\
&= \frac{k \alpha_m}{(a_m)^2} \left(\frac{\alpha_m}{\alpha_N} + 1\right) \left(1 - \frac{1}{\sigma}\right), \\
&\leq 0,
\end{aligned}
$$

where $f_{mm}(\mathbf{n}; \mathbf{n}/k) = -\alpha_m (1-\alpha_m)(\sigma k)^{-1}$ and $f_{Nm}(\mathbf{n}; \mathbf{n}/k) = \alpha_m \alpha_N (\sigma k)^{-1}$.

A.2 Proof of Proposition 2

The proof is similar to the proof of Proposition 1 so we only highlight the key steps. From the envelope theorem, we have the condition similar to (A3). The first-order condition of 7 with respect to a_m, $m = 1, ..., N$, is given by

$$
C_m(\mathbf{a}) = \frac{n_m}{(a_m)^2} f_m(\mathbf{n}; \mathbf{a}). \tag{A6}
$$

Thus, similar to the proof of Proposition 1, $\overline{f}(\mathbf{n})$ is Cobb-Douglas if and only if $n_m \overline{f}_m(\mathbf{n}) / \overline{f}(\mathbf{n}) = \alpha_m$ for all $\mathbf{n}$, then the first-order condition implies that $\overline{f}(\mathbf{n})$ is Cobb-Douglas if and only if C has the power form, which is equivalent to $\mathbf{a}^\star(\mathbf{n}) \propto \mathbf{n}$.

A.3 Proof of Proposition 3

To prove $\partial Z(\mathbf{n};\mathbf{n}_s,\sigma)/\partial\sigma \geq 0$, notice that

$$
\begin{aligned}
\frac{\partial}{\partial\sigma}\log Z(\mathbf{n};\mathbf{n}_s,\sigma) &= \frac{-1}{(\sigma-1)^2}\log\left[\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\right] \\
&\quad + \frac{1}{\sigma(\sigma-1)}\frac{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\log\left(\frac{n_m}{n_{s,m}}\right)}{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}}, \\
&= \frac{1}{(\sigma-1)^2}\left[\begin{array}{c} -\log\left[\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\right] \\ +\frac{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\log\left[\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\right]}{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}} \\ -\frac{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\log\alpha_m}{\sum_{m=1,\ldots,N}\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}} \end{array}\right], \\
&= \frac{1}{(\sigma-1)^2}\left[\sum_{m=1,\ldots,N}\pi_m\log\left(\frac{\pi_m}{\alpha_m}\right)\right], \\
&\geq \frac{-1}{(\sigma-1)^2}\left[\sum_{m=1,\ldots,N}\pi_m\left(\frac{\alpha_m}{\pi_m}-1\right)\right], \\
&= 0,
\end{aligned}
$$

where the last inequality follows the fact that $\log(x) \geq -(1/x-1)$ and the last eqaulity follows the fact that $\sum_{m=1,\ldots,N}\pi_m = 1 = \sum_{m=1,\ldots,N}\alpha_m$ and

$$
\pi_m \equiv \frac{\alpha_m\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{m'=1,\ldots,N}\alpha_m\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}}.
$$

To prove the rest of Proposition 3, notice that

$$
\frac{\partial}{\partial n_{s,m}}\log Z(\mathbf{n};\mathbf{n}_s,\sigma) = \frac{\alpha_m}{n_{s,m}}\left[1-\frac{\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{m'=1,\ldots,N}\alpha_m\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}}\right].
$$

Also notice that since $n_{m_d}/n_{s,m_d} \leq n_{m_a}/n_{s,m_a}$ and $\sigma \in [0,1]$, we have

$$
\frac{\left(\frac{n_{m_d}}{n_{s,m_d}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{m'=1,\ldots,N}\alpha_m\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}} \geq 1 \geq \frac{\left(\frac{n_{m_a}}{n_{s,m_a}}\right)^{\frac{\sigma-1}{\sigma}}}{\sum_{m'=1,\ldots,N}\alpha_m\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}}.
$$

Hence we have

$$\frac{\partial}{\partial n_{s,m_d}} \log Z(\mathbf{n};\mathbf{n}_s,\sigma) \leq 0 \leq \frac{\partial}{\partial n_{s,m_a}} \log Z(\mathbf{n};\mathbf{n}_s,\sigma).$$

Finally, since $\partial \log Z(\mathbf{n};\mathbf{n}_s,\sigma)/\partial n_{s,m}$ is strictly increasing in $n_m/n_{s,m}$, using the intermediate value theorem there exists $\gamma_y \in [n_{m_d}/n_{s,m_d}, n_{m_a}/n_{s,m_a}]$ such that $\partial \log Z(\mathbf{n};\mathbf{n}_s,\sigma)/\partial n_{s,m} \leq 0$ if and only if $n_m/n_{s,m} \leq \gamma_y$.

A.4 Proof of Propositon 4

Notice that

$$\frac{\partial \pi_m}{\partial \sigma} = \frac{\alpha_m}{\sigma^2}\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}\left[\frac{\left[\sum_{m'=1,\ldots,N}\alpha_{m'}\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}\left[\log\left(\frac{n_m}{n_{s,m}}\right)-\log\left(\frac{n_{m'}}{n_{s,m'}}\right)\right]\right]}{\left[\sum_{m'=1,\ldots,N}\alpha_{m'}\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}\right]^2}\right]$$

Since $n_{m_d}/n_{s,m_d} \leq n_{m_a}/n_{s,m_a}$, we have

$$\frac{\partial \pi_{m_d}}{\partial \sigma} \leq 0 \leq \frac{\partial \pi_{m_a}}{\partial \sigma}.$$

Finally, since $\partial \pi_m/\partial \sigma$ is strictly increasing in $n_m/n_{s,m}$, using the intermediate value theorem there exists $\gamma_g \in [n_{m_d}/n_{s,m_d}, n_{m_a}/n_{s,m_a}]$ such that $\partial \pi_m/\partial \sigma \leq 0$ if and only if $n_m/n_{s,m} \leq \gamma_g$.

To show $\gamma_g \leq \gamma_y$, recall from the proof of Proposition 3 that $\frac{\partial}{\partial n_{s,m}} \log Z(\mathbf{n};\mathbf{n}_s,\sigma) = \frac{\alpha_m}{n_{s,m}}\left(1-\frac{\pi_m}{\alpha_m}\right)$. Consider any m such that $\pi_m \leq \alpha_m$ and hence $\frac{\partial}{\partial n_{s,m}} \log Z(\mathbf{n};\mathbf{n}_s,\sigma) \geq 0$. Then Proposition 3 implies that $n_m/n_{s,m} \geq \gamma_y$. Notice that $\partial \pi_m/\partial \sigma$ can be rewritten as

$$\begin{aligned}
\frac{\partial \pi_m}{\partial \sigma} &= \frac{\pi_m}{\sigma(\sigma-1)}\left[\sum_{m'=1,\ldots,N}\pi_{m'}\left[\begin{array}{c}\log\left(\frac{n_m}{n_{s,m}}\right)^{\frac{\sigma-1}{\sigma}}-\log\left(\sum_{m'=1,\ldots,N}\alpha_{m'}\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}\right)\\ -\log\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}+\log\left(\sum_{m'=1,\ldots,N}\alpha_{m'}\left(\frac{n_{m'}}{n_{s,m'}}\right)^{\frac{\sigma-1}{\sigma}}\right)\end{array}\right]\right], \\
&= \frac{\pi_m}{\sigma(1-\sigma)}\left[\log\left(\frac{\alpha_m}{\pi_m}\right)+\sum_{m'=1,\ldots,N}\pi_{m'}\log\left(\frac{\pi_{m'}}{\alpha_{m'}}\right)\right], \\
&\geq 0,
\end{aligned}$$

where the last inequality follows the fact that $\sum_{m'=1,\ldots,N}\pi_{m'}\log\left(\frac{\pi_{m'}}{\alpha_{m'}}\right) \geq 0$ (shown in the proof of Proposition 1) and the premise that $\pi_m \leq \alpha_m$. Thus, the proof of the first part of Proposition 4 we have $n_m/n_{s,m} \geq \gamma_g$ and hence $\gamma_g \leq \gamma_y$. Actually the weak inequality becomes equality if and only if $\sum_{m'=1,\ldots,N}\pi_{m'}\log\left(\frac{\pi_{m'}}{\alpha_{m'}}\right) = 0$, which happens when $\pi = \alpha_m$ for all m. But we have $\pi = \alpha_m$ if and only if either $\sigma = 1$ or $\mathbf{n} \propto \mathbf{n}_s$.

B Appendix B: Discussion on the Global EOS Estimation

The following specification of the production function is usually maintained in the empirical literature:

$$Y_t = \left[\alpha \left(A_{K,t}K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(A_{L,t}L_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

After some manipulation, the firm's FOC with respect to capital service can be expressed as the following specifications

$$\log\left(\frac{Y_t}{K_t}\right) = \beta_1 + \sigma \log R_t + (1-\sigma)\log A_{K,t} + \varepsilon_{1,t},$$

$$\log\left(\frac{R_t K_t}{Y_t}\right) = \beta_2 + (1-\sigma)\log R_t - (1-\sigma)\log A_{K,t} + \varepsilon_{2,t}, \tag{A7}$$

where R_t is the rental rate of the capital service and R_tK_t/Y_t is the capital income share. We can obtain similar specifications from the FOC with respect to labor service. The empirical literature basically estimates different combinations of these specifications, with different treatment on factor compensations and different estimation methods. In spite of that, $\sigma < 1$ is found in general.

Karabarbounis and Neiman (2014) study a monopolistic competition model in final goods, which implies that $R_tK_t/Y_t = 1 - \mu s_t$ and $\hat{R}_t = \hat{\xi}_t$ around the steady state, where $\mu \in [0,1]$ is the markup, $s_t \in [0,1]$ is the labor income share, $\hat{R}_t$ is the growth rate of the rental rate and $\hat{\xi}_t$ is the growth rate of the relative price of the investment goods. Log-linearizing (A7) and averaging the log differences across time for each country, the cross-section observations of the labor shares are given by

$$\frac{\bar{s}_j}{1-\mu\bar{s}_j}\hat{s}_j = \beta_3 - (1-\sigma)\left(\hat{\xi}_j - \hat{A}_{K,j}\right) + \varepsilon_{3,j}, \tag{A8}$$

The data since 1980 documents global declines in the labor income share and in the relative price of the investment goods, i.e. $\hat{s}_j < 0$ and $\hat{\xi}_j < 0$ in most of the countries. The decline in the labor income share tends to be stronger in countries with a stronger decline in the relative price of the investment goods. Ignoring the effect from A_K, which is maintained in most of the paper, a regression on (A8) finds that $\tilde{\sigma} = 1.25$. When A_K is not negligible, the bias is given by $\tilde{\sigma} - \sigma = (1-\sigma)\, cov\left(\hat{\xi}_j, \hat{A}_{K,j}\right)/var\left(\hat{\xi}_j\right)$. Avoiding direct identification of $\hat{A}_{K,j}$, Karabarbounis and Neiman (2014) use TFP estimates from Conference Board as a proxy to $\hat{\lambda}_{K,j}$, and find that $cov\left(\hat{\xi}_j, \hat{A}_{K,j}\right)/var\left(\hat{\xi}_j\right) = -0.25$ and $\sigma = 1.20$.

To reconcile why our estimate of σ is different from Karabarbounis and Neiman (2014), first notice that given the biased estimate $\tilde{\sigma} > 1$ in their model, a unbiased estimate $\sigma < 1$ is possible if a direct identification of $\hat{A}_{K,j}$ finds that $cov\left(\hat{\xi}_j, \hat{A}_{K,j}\right) > var\left(\hat{\xi}_j\right)$. For example, an increase in $A_{K,j}$ induces the demand for investment and pushes up ξ_j. This mechanism can give a strong correlation between $\hat{\xi}_j$ and $\hat{A}_{K,j}$ such that $cov\left(\hat{\xi}_j, \hat{A}_{K,j}\right) > var\left(\hat{\xi}_j\right)$. Second, we are estimating different equations with different data. We estimate σ directly from the NCES production function with the time series of output, capital and human capital. Instead, Karabarbounis and Neiman (2014) estimate σ from the firm's FOC, (A8), with the time series of the labor income share and the relative price of the investment goods. Our choice of data is in line with the goal to explain the international income difference with the factor inputs and technology. Our specification is closest to Klump et al (2007), which found a slightly higher σ, despite that we are using a panel of time series from 110 countries instead of only the U.S.

Klump et al (2007) and León-Ledesma et al (2010) are recent examples of directly estimating the NCES production function. Compared with an indirect estimation of σ through the firm's FOCs, it has the following advantages. First, it does not require the data on factor prices, which avoid some classification problems like how to treat the compensation of the self-employed pointed out by Gollin (2003). The compensation data also tends to be noisy especially in the less developed countries. Second, to make use of the information on the factor prices and shares, we need to assume no distortion and perfect competition in the factor markets, which seems at odd with recent studies in development accounting like Hsieh and Klenow (2004) that emphasize the importance of the factor wedges. By estimating the production function directly, we do not need to take a stance on how factors are compensated. Finally, since factor prices tends to be endogenous to the innovation terms, using factor prices or shares as regressors likely results in biasedness. On the other hand, the upside of estimating the firm's FOC is the resulting linear equations relating factor demands to factor prices, which are straightforward to estimate. Of course, the computation cost of non-linear estimation is less relevant nowadays.

C Appendix C: Details of Extensions

To illustrative purposes, we begin by focusing on the miracle and trapped countries. We then investigate what alternative assimilation can do for better understanding the development process of the two Asian giants: China and India. In another extension, we will examine a group of countries which we call "middle-income laggards," characterized by two criteria: (i) the initial relative income is reasonable high, and (ii) the average growth rate of relative income is very low if not negative. We like to examine the applicability of our assimilation model these medium-income but low-growth countries that have lagged behind over time.

Best-Alternative Assimilation

In the 1960s and the 1970s, large flows of Japanese FDI entered Taiwan and then to South Korea and Hong Kong; in late 1970s, Japanese firms have also expanded production facilities to Singapore. Learning from the success of their neighbor, the Asian Tigers followed the Japanese footsteps and realized that export expansion was the main momentum to growth. Weiss (2005) notes a wave of Asian countries after Japan that illustrates the successful application of export-led growth: (i) first tier of Asian Miracle countries, namely, the four Asian tigers, whose takeoff began in the 1960s, (ii) second tier, namely, all the remaining miracles, whose takeoff began in the mid 1980s. These Asian growth experiences exhibit the *Flying Geese Pattern* (FGP) of economic development, as documented by Akamatsu (1962).

To capture this FGP development process, we consider an alternative assimilation scheme, with Japan assimilating the U.S. frontier and the early birds Asian Tigers subsequently assimilating their regional leader, Japan. So, $a_{k,t} = k_{JAP,t}/y_{JAP,t}$ and $a_{h,t} = h_{JAP,t}/y_{JAP,t}$, and the relative income growth can be written as:

$$\hat{q}_{j,t}^{\text{data}} = \hat{q}_{j,t}^{\text{alt}} + \Delta\tau_{j,t}^{\text{alt}} = \alpha\left(\hat{k}_{j,t} - \hat{k}_{US,t}\right) + (1-\alpha)\left(\hat{h}_{j,t} - \hat{h}_{US,t}\right) + \hat{M}_{j,t}^{\text{alt}} + \Delta\tau_{j,t}^{\text{alt}}, \quad (26)$$

where

$$q_{j,t}^{\text{alt}} = \frac{y_{JAP,t}}{y_{US,t}}\left[\alpha\left(\frac{k_{j,t}}{k_{JAP,t}}\right)^{1-1/\sigma_j^{\text{alt}}} + (1-\alpha)\left(\frac{h_{j,t}}{h_{JAP,t}}\right)^{1-1/\sigma_j^{\text{alt}}}\right]^{1-1/\left(1-\sigma_j^{\text{alt}}\right)}, \quad (27)$$

$$M_{j,t}^{\text{alt}} \equiv \underbrace{\left[\frac{y_{JAP,t}}{(k_{JAP,t})^{\alpha}(h_{JAP,t})^{1-\alpha}}\right]\Bigg/\left[\frac{y_{US,t}}{(k_{US,t})^{\alpha}(h_{US,t})^{1-\alpha}}\right]}_{\text{ratio of bridging TFP to frontier TFP}} \quad (28)$$
$$\underbrace{\times\left(\frac{k_{j,t}}{k_{JAP,t}}\frac{h_{JAP,t}}{h_{j,t}}\right)^{-\alpha}\left[\alpha\left(\frac{k_{j,t}}{k_{JAP,t}}\frac{h_{JAP,t}}{h_{j,t}}\right)^{1-1/\sigma_j^{\text{alt}}} + 1 - \alpha\right]^{1-1/\left(1-\sigma_j^{\text{alt}}\right)}}_{\text{mismatch from using the bridging technology}}.$$

The alternatives serve like a bridge with the frontier technology. Now the factor input-technology mismatch is reduced because the "bridging" technology is less demanding than the frontier technology as a result of a narrower factor input gap (i.e., $a_{JAP,k} = k_{JAP}/y_{JAP} < a_{US,k} = k_{US}/y_{US}$ and $a_{JAP,h} = h_{JAP}/y_{JAP} < a_{US,h} = h_{US}/y_{US}$). The trade-off is that this bridging technology of Japan is also less productive, as $y_{JAP}/y_{US} < 1$. Because of the rapid advancement of the tigers, we set the assimilation process to have an *early stop* in 1990. Further, we allow other latecomers in Asian to also target their neigboring Tigers based on stronger FDI ties. All such subsequent assimilation is assumed to take place with a *late start* from mid-1980s to present (2014). We have also considered switching targets from one (say, Japan) to another (say, the U.S.). But the improvement in fitness is negligible.

The results for the alternative assimilation scheme with the best-performing targets are reported in Table 12. Overall, the above results are consistent with the FGP. Aamong the Asian tigers, having Japan as the assimilation target greatly improves the fitness for Hong Kong and Singapore, where the unexplained income gaps reduce to almost zero. The corresponding increases in the growth differential gain, from the country-specific assimilation to assimilating Japan, are also large, at 1.42 and 2.30 percentage points, respectively. Taiwan also improve by having almost zero unexplained income gap with a nonnegligible growth differential gain of 0.93 percentage point. In these tigers the mismatch contribution to growth greatly increases to more than 50%. Korea is the only exception where the MSE and the unexplained income gap worsens, thus indicating the FGP may not be applicable. For the latecomers, switching the assimilation target from the U.S. to the Asian tigers performs better in reducing in the unexplained income gaps to alomost zero. Of particular interest, having Vietnam assimilating Taiwan instead of the U.S.not only improves fitness significantly (by reducing the MSE essentially to zero), but also corrects the overprediction of assimiation when targeting the U.S. (see the country-common assimilation case illustrated in Figure 8 and note from Table 10 that overprediction in mismatch contribution could not be fixed even with country-specific assimilation ability).

We next move to re-account the economic performance of the African trapped countries. The alternative targets are set among the choices of (i) the most technologically advanced large economy close to Africa (Germany), (ii) their continent leader (South Africa), and (iii) their colonial origins (e.g., France, the U.K.). Again we report the results from the best performing alternative targets in Table 13. It turns out that in 14 of these 17 countries, the alternative targets are more in line with the FGP rather than the colonial origins. Overall, in all but two countries, the fitness improves (at

	Best alternative assimiltion	σ_j^{alt}	σ_j	Mismatch contribution(%) Country-specific	Alter-native	Δgrowth (pp)	MSE Country-specific	Alter-native	z_j Country-specific	Alter-native
Hong Kong	Japan,1960-1990	0.59	0.00	22.80	76.08	1.42	0.04	0.01	0.88	0.97
Korea	Japan,1960-1990	0.00	0.32	57.67	41.23	-0.77	0.05	0.29	0.88	0.61
Singapore	Japan,1960-1990	0.68	0.00	-0.02	52.38	2.30	0.07	0.03	0.91	1.06
Taiwan	Japan,1960-1990	0.33	0.39	28.95	50.77	0.93	0.01	0.01	1.01	1.01
China	Taiwan,1985-2014	0.06	0.36	111.33	66.92	-0.90	0.16	0.01	0.95	1.00
India	Singapore,1985-2014	0.26	0.00	0.27	35.18	0.44	0.04	0.03	0.87	1.00
Malaysia	Singapore,1985-2014	0.41	0.24	31.28	51.27	0.23	0.06	0.02	0.95	1.00
Thailand	HongKong,1985-2014	0.04	0.00	46.84	29.45	-0.34	0.06	0.02	0.83	1.00
Vietnam	Taiwan,1985-2014	0.24	0.32	69.55	42.67	-1.41	0.05	0.00	1.00	1.00

Table 12: Growth accounting for the miracle countries, with alternative periods and assimilation targets.

	Best alternative assimiltion	σ_j^{alt}	σ_j	Mismatch contribution(%) Country-specific	Alter-native	Δgrowth (pp)	MSE Country-specific	Alter-native	z_j Country-specific	Alter-native
Burundi	Belgium	0.56	0.42	69.15	73.24	-0.06	0.01	0.01	1.00	1.00
Benin	S. Africa	0.00	0.00	146.03	148.98	-0.04	0.08	0.03	0.86	0.95
Central African	France	0.00	0.00	78.15	94.62	-0.55	0.19	0.01	0.65	0.97
Côte d'Ivoire	Germany	0.00	0.00	687.14	753.42	-0.15	0.23	0.01	0.66	0.97
Congo D.R.	S. Africa	0.61	0.58	32.78	39.21	-0.15	0.56	0.52	1.06	1.11
Ghana	Germany	0.11	0.00	159.60	78.59	1.10	0.10	0.02	0.79	0.97
Gambia	S. Africa	0.45	0.35	17.41	45.58	-0.38	0.08	0.08	1.00	0.94
Kenya	Germany	0.33	0.06	275.92	132.06	0.79	0.06	0.05	1.00	0.91
Madagascar	Germany	0.31	0.14	106.65	117.25	-0.16	0.04	0.06	1.00	0.99
Mauritania	S. Africa	0.27	0.17	35.51	53.46	-0.19	0.01	0.01	1.00	0.99
Malawi	S. Africa	0.02	0.00	28.36	56.29	-0.32	0.16	0.09	0.99	1.00
Niger	Germany	0.00	0.00	87.92	77.19	0.27	0.50	0.17	0.50	0.68
Senegal	Germany	0.14	0.00	108.37	39.02	1.39	0.14	0.04	0.76	0.95
Sierra Leone	Germany	0.37	0.21	120.21	83.69	0.48	0.03	0.02	1.00	1.00
Togo	S. Africa	0.11	0.03	89.61	91.25	-0.05	0.02	0.01	1.00	1.00
Tanzania	UK	0.00	0.00	45.94	36.36	0.04	0.24	0.05	0.65	0.95
Zimbabwe	Germany	0.34	0.18	99.02	79.25	0.61	0.21	0.25	1.00	1.03

Table 13: Growth accounting for the development-trapped countries with alternative assimilation targets.

least weakly). By switching the heterogeneous assimilation from the U.S. to the alternative, the unexplained income gap reduces from 12% to 4%, whereas the MSE is lowered from 0.15 to 0.09. The growth differential from the counterfactual yields a bonus of 0.15 percentage point, indicating that inability to assimilate can further explain growth stagnation by one-sixth a percentage point. Of particular interest, among the four trapped countries where the accounting outcome overstate the contribution of mismatch (Benin, Côte d'Ivoire, Ghana and Senegal, as shown in Table 9), two, Ghana and Senegal, have the problem corrected under the alternative target of Germany, with mismatch contribution dropped from over 100% to 79% and 39%, respectively.

The Asian Giants: China and India

In studying the economic performance of the two Asian giants, China and India, we first describe what the Cobb-Douglas model tell us from the accounting exercises. In

China, both factor inputs are growing rapidly at an average rate of 4.08% for relative physical capital and 0.71% for relative human capital. As a result, relative physical capital increases by eight times and relative human capital increases by 1.4 times over the sample period. Given the rapid growth in capitals, factor accumulation of the Cobb-Douglas model accounts for more than 100% of the income growth with a residual of only -2.74%. But owing to the fact that physical capital is the disadvantageous factor in the assimilation model, the factor input-technology mismatch alone also account for 101.41% of the growth performance. Nevertheless, the country-specific assimilation model seems informative to understand the China growth because it has a much lower MSE than the Cobb-Douglas model (0.16 vs 1.43). Another support of the country-specific assimilation model is that it has an unexplained income gap of only 5% which is far lower than the Cobb-Douglas counterpart (69%). By switching the assimilation target from the U.S. to Taiwan, as proposed by the FGP, the MSE further reduces from 0.16 to less than 0.01 and the unexplained income gap from 5% to 0.6%. More interestingly, the overprediction of mismatch contribution to growth is corrected: falling from 111% to 67%. Thus, having China assimilating Taiwan seems to best capture its rapidly development process after 1980s.

For the case India, the situation is different. Relative factor growth of the two capitals are around 0.36%, so that relative physical capital increases by 1.4 times and relative human capital increases by 1.3 times over the sample period. As a result, factor accumulation of the Cobb-Douglas model accounts only for 37% of the income growth. But balanced growth in the two relative factors also implies little change in the factor input-technology mismatch, so the country-specific assimilation channel only accounts for 0.27% of the growth performance. It is no longer the case when India assimilate alternative technologies. By switching the assimilation target from the U.S. to the best tied Asian tiger, Singapore, the growth contribution of factor input-technology mismatch raises from essentially a negligible 0.27% to 35%, which successfully corrects the underprediction with U.S. as the target (see Figure 8 and 10).

Middle-Income Laggards

To the end, we shift our focus to a group of middle-income laggards. We have already discussed the connection between relative factor advantage reversal and middle-income trap. In this subsection, we study the development process of countries with decent initial relative income but very low average growth rate. We have selected six countries for this laggard group, where the average initial income is 0.32 and the

	Relative growth(pp)			Growth contribution(%)					Fitness		
	Income	**Capital**	**Human capital**	**Country-common /specific**	**Capital**	**Human capital**	σ_j	z_j	$z_j^{\mathbf{CD}}$	$\mathbf{MSE}_j$	$\mathbf{MSE}_j^{\mathbf{CD}}$
Colombia	-0.30	-0.61	0.28	84.67 / 196.36	67.59	-61.18	0.00	0.64	0.57	0.22	0.36
Costa Rica	-0.25	-0.04	0.19	35.27 / 43.82	4.91	-49.59	0.35	0.94	0.74	0.03	0.13
Fiji	-1.45	-1.12	0.61	49.94 / 78.80	25.64	-28.15	0.17	0.69	0.49	0.15	0.54
Honduras	-0.83	-0.99	0.50	80.65 / 119.19	39.71	-39.78	0.00	0.50	0.37	0.49	1.03
Mexico	-0.38	-0.27	0.30	24.61 / 96.68	23.35	-52.32	0.08	0.87	0.84	0.07	0.10
Nicaragua	-3.37	-2.58	0.71	37.64 / 65.00	25.51	-13.98	0.00	0.54	0.41	0.44	0.91
Average	-1.10	-0.94	0.43	52.13 / 99.96	31.11	-40.83	0.10	0.70	0.57	0.23	0.51

Table 14: Growth accounting for the laggard countries up to 2014.

average relative income growth rate is -1.10%. Their accounting results are given in Table 14.

Our assimilation model again outperforms the Cobb-Douglas model, reducing the unexplained income gap from 43% to 30% and lowering the MSE by more than half (from 0.51 to 0.23). In these countries, despite that physical capital is the disadvantage factor, their accumulation of physical capital fell behind the U.S. whereas human capital outgrew the U.S. Widening factor input-technology mismatch thereby results in growth stagnation, which accounts for about half of the relative income decline under common assimilation. With country-specific assimilation, the accounting results improve further, though in two of the six cases overprediction occurs. From Proposition 3, we learn that the predicted income levels are low because the average country-specific σ is far below the country-common level (0.01 vs. 0.42). Proposition 5 points out that the low country-specific σ signifies the factor input-technology mismatch. As a result, we have larger contributions of the country-specific assimilation for the laggards than the trapped countries. Nonetheless, in Colombia and Honduras, the country-specific parameters are at its lower bound (zero); as a result, mismatch turns out to overpredict their negative growth outcomes.