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CONSUMPTION:
BEYOND CERTAINTY EQUIVALENCE

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Consumption:
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ABSTRACT

This paper discusses the recent research on the consumption function that has attempted to relax the assumption of certainty equivalence. While there remain many open questions, both theoretical and empirical, it is clear that the assumption of certainty equivalence can be misleading. Under more plausible specifications of preferences toward risk, uncertainty lowers the level of consumption, increases the expected rate of growth of consumption, and increases the response of consumption to news about income. Moreover, changes in the amount of uncertainty are a potentially important source of fluctuations in consumption.

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Twenty years ago, it was standard practice in describing macroeconomic behavior to build theoretical models assuming all current and future variables were known with certainty. When models were applied to the data, the only concession to the presence of uncertainty was more often than not the introduction of an unexplained error term in the regression.

Ten years ago, under injunctions to "take uncertainty seriously", macroeconomists started introducing uncertainty explicitly at the model building stage. Much theoretical and empirical progress has been made, most of it under assumptions yielding certainty equivalence, or the property that optimal behavior depended only on expectations of other variables, and not on their higher moments.

Certainty equivalence yields convenient characterizations of behavior. Assumptions yielding certainty equivalence, namely that constraints are linear and objective functions quadratic, are however quite stringent and, in most contexts, highly implausible. Recent research has attempted to go beyond certainty equivalence, and to characterize behavior under more appealing assumptions. It is a difficult endeavor, both theoretically and empirically. In most cases, closed form solutions are unavailable and one only gets glimpses into the nature of the solution. In most cases also, the decision rule depends on higher moments of the exogenous variables, about which little hard evidence is available, making empirical implementation perilous. Nevertheless, much has been learned; in this paper, we present recent developments on the consumption front.[1]

1. Consumption under certainty equivalence

Consider the decision problem of a consumer who maximizes at time t:

$$E\left[\sum_{i=0}^{T-t} U(C_{t+i}) \mid I_t\right] \tag{1}$$

subject to

$$A_{t+i+1} = A_{t+i} + Y_{t+i} - C_{t+i};\ A_t \text{ given};\ A_{T+1} = 0.$$

For simplicity, we assume that both the interest rate and the discount rate are equal to zero. A_t is wealth, Y_t labor income. The only source of uncertainty is labor income, which is random.

If utility is quadratic, the set of first order conditions is:

$$E(C_{t+i}|I_t) = C_t,\ \text{for } i = 1,\ldots,T-t \tag{2}$$

and the solution to the maximization problem has the familiar form:

$$C_t = (1/(T-t+1))(\,A_t + \sum_{i=0}^{T-t} E[Y_{t+i}|I_t]). \tag{3}$$

Consumption is a linear function of initial wealth and the present value of expected future income. Higher moments of income do not matter. The marginal propensity to consume out of total wealth is equal to the inverse of remaining number of years.

The assumption of quadratic utility is crucial to derive the "certainty equivalence" consumption function (3) in the presence of uncertain labor income. Yet, quadratic utility is an unappealing way of describing consumers' behavior towards risk. It implies increasing absolute risk aversion, a willingness to pay more to avoid a given bet as wealth increases. Introspection and casual evidence suggest that this is a poor description of behavior under uncertainty.

Simple utility functions with more plausible properties towards risk are available, of course. Two such functions are the exponential and the isoelastic utility functions. Yet, in the presence of risky labor income, neither yields certainty equivalence. Indeed, they imply systematic effects of uncertainty on consumption, to which we now turn.

2. The slope and variance of consumption

If we return to the set of first order conditions of the maximization problem above, this time without restrictions on utility beyond risk aversion, $U''<0$, we get

$$E[U'(C_{t+i})|I_t] = U'(C_t), \text{ for } i = 1 \text{ to } T-1. \tag{4}$$

Uncertainty affects the first order conditions, and thus optimal consumption, only if it affects expected marginal utility. If the third derivative of the utility function U''' is positive, as is true of most plausible utility functions, an increase in uncertainty raises expected marginal utility. Thus to maintain equality in (4), expected future consumption must increase compared to current consumption. Uncertainty leads consumers to defer consumption, to be more prudent. The role of the condition $U'''>0$ in generating more prudent behavior in the face of uncertainty was first derived by Leland (1968) and further analyzed by Sandmo (1970) and Dreze and Modigliani (1972).

How strong is the effect of uncertainty on the slope of the consumption path likely to be? Kimball (1987) has shown that, in the same way as the Arrow-Pratt coefficients of risk aversion help study the effects of uncertainty on expected utility, coefficients of absolute and relative prudence help study the effects of uncertainty on expected marginal utility and thus on consumption. In parallel to the Arrow-Pratt coefficients, the coefficient of absolute prudence is defined as $-U'''/U''$, and the coefficient of relative prudence as $-U'''C/U''$. Constant absolute prudence implies that the increase in consumption required to keep the same level of expected marginal utility in the face of a small increase in risk is independent of the initial level of consumption, and a parallel interpretation applies to the coefficient of relative prudence.

In general, there need not be any tight relation between the coefficients of risk aversion and the coefficients of prudence. Conveniently -- and perhaps misleadingly-- however, the exponential utility function, $U(C) = -(1/\gamma)\exp(-\gamma C)$, exhibits both constant absolute risk aversion, γ, and constant absolute prudence, also equal to γ.

Similarly, the isoelastic utility function $U(C) = (1/(1-\xi))C^{1-\xi}$ exhibits both constant relative risk aversion, ξ and constant relative prudence, $(\xi+1)$. Thus, under those two specifications, specifying the degree of risk aversion also pins down the degree of prudence.

Equipped with those definitions, we can take a second order approximation of (4) around $U'(C_t)$.[2] Rearranging gives:

$$E[(C_{t+i}-C_t)|I_t] = 1/2\ a\ E[(C_{t+i}-C_t)^2|I_t] \tag{5}$$

or, dividing both sides by C_t:

$$E[((C_{t+i}-C_t)/C_t)|I_t] = 1/2\ r\ E[((C_{t+i}-C_t)/C_t)^2|I_t] \tag{6}$$

where a and r are the coefficients of absolute and relative prudence. Equation (5) gives a relation between the slope of the consumption path and the variance of the change in consumption (around zero). Equation (6) gives a relation between the expected growth rate and its variance.

While they still only give a relation between two endogenous variables, those two equations show the basic effects of uncertainty on consumption. Uncertainty, by increasing the variance of consumption, leads to a more steeply sloped consumption path. The effect is stronger the larger the coefficient of absolute or relative prudence. And, as increases in uncertainty do not affect the budget constraint, any increase the slope of the consumption path implies a decrease in the initial level of consumption.

3. Uncertainty and the consumption function

To go beyond equations (5) and (6) requires solving for consumption as a function of the income process. This is in general difficult.

The case of exponential utility, of constant absolute prudence, has proven analytically tractable (Kimball and Mankiw 1987, Caballero 1987). Yet what makes it tractable however also makes it somewhat unattractive. To see why, we consider a simple example, which follows Caballero. Suppose that utility is exponential with exponent $-\gamma$, and that labor income follows a random walk with normally distributed innovations with standard deviation σ. It is easy to verify that optimal consumption satisfies:

$$E[C_{t+1}|I_t] = C_t + \gamma\sigma^2/2. \tag{7}$$

Using the budget constraint, one can show that the level of C_t is given by:

$$C_t = (1/(T-t+1))A_t + Y_t - (\gamma(T-t)/4)\sigma^2. \tag{8}$$

The slope of the expected consumption path, rather than being equal to zero as under certainty equivalence, is positive and constant; it depends both on the degree of absolute prudence, γ, and the variance of income changes. This in turn implies that the consumption function is the same as under certainty equivalence, except for a negative term which depends on the degree of uncertainty, the degree of prudence, and the horizon.

We can use (7) and (8) to get a feel for magnitudes. If we evaluate the expected rate of growth of consumption at a point where consumption and labor income are roughly equal, equation (7) implies:

$$(E[C_{t+1}|I_t]-C_t)/C_t = (\gamma C_t)(\sigma/Y_t)^2/2. \tag{9}$$

Using panel data, Hall and Mishkin (1982) found that the standard deviation of the change in permanent income was about $1200; as median household income was about $12,000 during the period (1972), this finding suggests a value of σ/Y of about .1. The term γC_t is equal to the coefficient of relative risk aversion. If we assume this coefficient to be equal to 4, then equation (9) implies an expected growth rate of consumption of 2 percent. This number is roughly the same as the growth in aggregate consumption per capita. Since the cross-sectional age-consumption profile is upward sloping, the growth in individual consumption must be at least 2 percent.[3]

Cumulated over many years, such a tilt in the consumption path implies substantially lower consumption at the beginning of life, and thus much higher average wealth. Indeed, and this reveals the unattractive aspect of the assumption of constant absolute prudence, equation (8) can easily generate negative initial consumption as a result of uncertainty. Negative consumption is not ruled out by the exponential utility.

When we turn to more attractive utility function which do rule out negative consumption, such as isoelastic utility, obtaining closed form solutions becomes generally impossible. But, from some analytical results (Kimball 1987), and from simulations (Zeldes 1984, Barsky, Mankiw, and Zeldes 1986), we know the consumption function has the following property. Under decreasing absolute prudence, the convenient dichotomy between the effects of expected income and the effects of uncertainty which is exhibited in (8) disappears.

On the one hand, the impact of uncertainty on consumption depends on the level of wealth. At higher levels of wealth, a larger portion of lifetime income is certain -- under our assumption of a constant interest rate -- and the variance of the percentage change in consumption decreases. This in turn implies a flatter consumption path. Zeldes shows for example that under isoelastic utility, the consumption path is initially very steep and flattens as wealth accumulates. This effect is very much in accordance with empirical evidence. Kotlikoff and Summers (1981, Figure 1) for example show that the annual rate of change of consumption for the cohort born in 1910 was over 3 percent from age 18 to age 50, but was 1 percent thereafter.

On the other hand, the marginal propensity to consume depends on the amount of uncertainty. An increase in income decreases the need for precautionary savings, leading to a larger response in consumption than would be the case under certainty equivalence. As a result, consumption can show what appears as excess sensitivity to income movements (Zeldes 1984).

4. Changes in uncertainty and movements in consumption

If uncertainty is an important determinant of the level of consumption, changes in uncertainty can potentially be an important source of fluctuations in consumption. Measuring changes in individual income uncertainty is difficult given the typically short time series on individuals in panel data.

A useful starting point is to look at changes in aggregate income uncertainty. To do so, we computed the standard deviation of n-period ahead forecasts of GNP by DRI, probably a good proxy for the relevant measure of subjective uncertainty. Each month, in addition to its main forecast, DRI issues a set of two or three alternative forecasts for the next three years. Each forecast is given a probability by DRI. When we computed DRI's subjective uncertainty, three results stood out. (1) At each date, the subjective standard deviation increases roughly as the square root of the forecast horizon, indicating that the uncertainty about the future level of output increases with the horizon. (2) The subjective standard deviation, three years ahead, fluctuates substantially: it varies between 1.14 percent in 1978, and 2.70 percent in 1981. (3) The level of aggregate uncertainty is small relative to the standard deviation of income uncertainty facing individuals, roughly 17 percent over three years (based on Hall and Mishkin).

This last fact suggests that if all consumers share fluctuations equally, movements in aggregate uncertainty are unlikely to have a large impact on aggregate consumption. But if fluctuations fall more heavily on some individuals, the aggregate effect can be much larger. If we assume for example that all consumers follow equation (8), and that only a fraction α of the consumers is subject to the aggregate shocks, it is easy to show that the effect on aggregate consumption is proportional to $1/\alpha$. The more concentrated the effect of aggregate fluctuations, the stronger the impact of uncertainty on aggregate consumption.

This impact of changing uncertainty underlies the papers by Barsky, Mankiw, and

Zeldes and by Kimball and Mankiw. Both emphasize the deviations from Ricardian equivalence caused by the interaction between precautionary saving and idiosynchratic income risk. If taxes vary with income, increases in taxes have, in addition to their direct effect on expected after-tax income, an insurance effect which works in the opposite direction. Barsky, Mankiw, and Zeldes use simulations to show, assuming isoelastic utility, that debt finance--a decrease in taxes today financed by higher proportional taxes later--can have a significant impact on consumption. They conclude that, for plausible parameter values, the marginal propensity to consume out of a tax cut is approximately half the marginal propensity to consume out of wealth. Kimball and Mankiw derive analytic results for the case of exponential utility. They show that, if individual income is serially correlated, the initial effect of deficit finance on consumption is stronger, the larger the anticipated length of time to the eventual tax increase. The reason is a simple one: the longer the deferral, the more uncertain individual income and the higher the insurance effect of future taxes.

5. Conclusion

While macroeconomists have long understood the behavior of consumers under certainty equivalence, the behavior of consumers with plausible utility functions facing uncertain future income has remained largely a mystery. Recent research has begun to reveal some the properties of optimal consumer behavior under uncertainty. Perhaps most important, this research has taught us that, in many ways, the assumption of certainty equivalence can be highly misleading.

Notes

1. Developments on other fronts would also warrent a report. See in particular Bertola (1987) for an analysis of the interaction of uncertainty and irreversibility in determining investment.

2. As is usual, these formula can be derived exactly under appropriate assumptions if the consumer's problem is set in continuous time. See Breeden (1986).

3. Kuehlwein (1987) studies the relation between the growth and variance of consumption in panel data.

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