

NBER WORKING PAPER SERIES

THE GAINS FROM FISCAL COOPERATION
IN THE TWO COMMODITY
REAL TRADE MODEL

Stephen J. Turnovsky

Working Paper No. 2466

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 1987

The research reported here is part of the NBER's research program in International Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

The Gains from Fiscal Cooperation in the Two Commodity Real Trade Model

ABSTRACT

This paper analyzes the gains from fiscal cooperation within the context of the standard two commodity real trade model. It shows how the adjustment in terms of trade is the critical factor in determining the effects of moving from a noncooperative equilibrium. In general, a noncooperative equilibrium leads to an overexpansion of government expenditure on the export good and an underexpansion on the import good, relative to a cooperative equilibrium. The specific example of a logarithmic economy is also considered. The paper discusses further the welfare effects resulting from the formation of a coalition among two countries.

Stephen J. Turnovsky
Department of Economics
University of Washington
301 Savery Hall
Seattle WA 98195

(206) 543-5955

1. INTRODUCTION

The increase in international economic interdependence during the past several years has stimulated increased interest in questions pertaining to economic policy interdependence and coordination. One particular aspect of this is the recognition that policy making in such an environment necessarily involves strategic behavior. The formal analysis of such strategic behavior among international policy makers began with the pioneering work of Hamada (1976) and has been extended more recently in various directions by several authors.¹

This literature can be characterized by two features. First, it focuses on monetary policy and monetary interdependence. Second, the models employed are what one might characterize as Keynesian type macro models, in which the objective is the minimization of a quadratic loss function, typically involving inflation and output as tradeoffs. While the linear-quadratic stabilization framework is a well-established and convenient one, its limitations, in not focusing on any fundamental welfare objective, are well known.

Recent work in international macroeconomic policy has undergone somewhat of a change in emphasis. First, with the large U.S. government deficit, questions of *fiscal* interdependence have come more to the fore.² Secondly, this literature has tended to be cast in terms of utility-maximizing agents, with the effects of fiscal changes being evaluated in terms of some explicitly stated welfare function, typically the welfare of the representative individual; see, e.g., Frenkel and Razin (1985, 1986).

The present paper analyzes strategic fiscal policy making within the context of the standard two country-two good real trade model. We first derive the optimal degrees of government expenditures on both the export and the import good under the assumption that each policy maker behaves non-cooperatively, taking his rival's actions as given. Expressions are obtained for the optimal degrees of government expenditure, which are directly analogous to the expressions for the optimal tariff, originally due to Johnson (1954). We then determine the optimal degrees of government expenditures in a cooperative equilibrium, where joint utilities are maximized. Certain welfare conclusions are drawn

and comparisons between the two equilibria can be made.

As a special example, we consider the case of a logarithmic utility function, with output fixed and specialized in the two countries. Not only can this be solved explicitly very simply, but also some useful insights as to the numerical gains from cooperation can be obtained in this case. Indeed, following this line it would appear that the differences between noncooperative and cooperative equilibria may be significantly greater than the more traditional macro models suggest. The reason for this is that fiscal policies generate large international spillovers through relative price movements and these render an important role for strategic policy making.

Most of the literature focuses on a world consisting of two countries. In the latter part of the paper, the logarithmic utility function is used to address an important, but neglected, question. We consider a world consisting of three countries and assess the effects of the formation of a coalition between two of them, upon the welfare of the third.

Before proceeding with the formal analysis, we should relate the present paper to other recent studies. Devereux (1986) looks at the question of the gains from fiscal coordination using a model based on logarithmic preferences. A number of authors including Kehoe (1987), Cheri and Kehoe (1987), and Devereux (1986) examine the divergence between noncooperative and cooperative fiscal policies as the number of economies in the world increase. Also, Hamada (1986) considers optimal strategic fiscal policy in a two-period, single commodity framework. Although our analysis is conducted within the context of the two country-two good trade model, in which the transmission occurs through the terms of trade, with little more than notational change, one can interpret our results as applying to a two-period world, much closer to the framework of Hamada, or Frenkel and Razin. In this case the relative price is the intertemporal interest rate, rather than the intratemporal terms of trade.

2. THE MODEL

Consider the standard two country-two good trade model, where both countries are competitive. The countries are indexed by A, B, the goods by x, y , and we assume without loss of generality that Country B imports x in return for y , which it exports to Country A. There are no impediments to trade, so that relative prices in the two countries are equal.

The basic model is described by the following set of equations:

$$U^i = U^i(C_x^i, C_y^i, G_x^i, G_y^i) \quad i = A, B \quad (1)$$

$$\sigma \frac{\partial U^i}{\partial C_x^i} = \frac{\partial U^i}{\partial C_y^i} \quad i = A, B \quad (2)$$

$$F^i(Q_x^i, Q_y^i) = 0 \quad i = A, B \quad (3)$$

$$\sigma \frac{\partial F^i}{\partial Q_x^i} = \frac{\partial F^i}{\partial Q_y^i} \quad i = A, B \quad (4)$$

$$Q_j^i + E_j^i = C_j^i + G_j^i \quad i = A, B; \quad j = x, y \quad (5)$$

$$\sigma E_y^A + E_x^A = 0 \quad (6a)$$

$$\sigma E_y^B + E_x^B = 0 \quad (6b)$$

$$E_x^A + E_x^B = 0 \quad (7a)$$

$$E_y^A + E_y^B = 0 \quad (7b)$$

where

C_j^i denotes the consumption of good j in Country i , $i = A, B$, $j = x, y$

Q_j^i denotes the production of good j in Country i ,

E_j^i denotes excess demand for good j in Country i ,

G_j^i denotes government expenditure on good j in Country i ,

σ denotes the relative price of good y to good x in the world economy.

Equations (1) define the utility functions of representative agents in Countries A, B. These are functions of: (i) private consumptions of the two goods; (ii) government expenditures on the two goods. The utility functions are assumed to be increasing functions of their respective arguments and to be continuous, twice differentiable, and quasi-concave. Equations (2) specify the optimality

conditions for consumers with respect to private consumption (C_j^i) in each country. The quantities of government expenditures are fixed, as far as private agents are concerned, but are determined by the two governments, in accordance with the strategic behavioral regimes to be discussed below.

Equations (3) and (4) apply analogously to production. Equation (3) defines the production possibility curve, which is assumed to be concave, while (4) specifies the usual marginal product conditions for optimality. Equations (5) define the excess demand for good j in Country i to be production less private consumption and domestic government expenditure. For the import good $E < 0$.³ We also assume that government expenditure in each country is financed by lump sum taxation. The government budget is therefore balanced and there are no distortionary effects of taxes on consumption.

Equation (6) describes the balance of trade equilibrium for the two countries, while (7) specifies market clearance in the world market for the two goods. It is clear from (6) and (7) that only one goods market is independent. Equilibrium in one implies equilibrium in the other, so that one equation, say (7b), can be dropped.

3. OPTIMAL FISCAL POLICIES: NONCOOPERATIVE EQUILIBRIUM

The welfares of the respective representative agents in the two economies are given by the utility functions U^A , U^B . We assume initially, that the two governments act noncooperatively to maximize the welfare of their respective citizens, taking the actions of the rival government as given. In each case, the government has two possible fiscal instruments: expenditure on the export good, and expenditure on the import good. Both instruments will be considered.

Consider first Country A. Using (5), (6), and (7b), its utility function may be expressed as:

$$U^A = U^A[Q_x^A - \sigma E_y^A - G_x^A, Q_y^A + E_y^A - G_y^A, G_x^A, G_y^A]$$

To determine the optimal government expenditure on the export good, we differentiate U^A with respect to G_x^A . Taking the differential of (3) and using the optimality conditions (2), (4), yields the optimality condition

$$\frac{\partial U^A / \partial G_x^A}{\partial U^A / \partial C_x^A} = 1 + E_y^A \frac{\partial \sigma}{\partial G_x^A} \quad (8)$$

This equation is obtained in precisely the same way as is the optimal tariff.⁴ It is therefore the direct analogue to the well-known optimal tariff formula; see, e.g., Takayama (1974), Bhagwati and Srinivasan (1983). Equation (8) can be given a simple interpretation. In a single commodity world, in which government expenditure is financed by lump sum taxation, the optimal level of government expenditure on the export good is to equate the marginal rate of substitution between government expenditure and private expenditure on that good to the relative price, namely unity. But in the present two-commodity world, the change in government expenditure on good x say, induces a change in the relative price of good x to good y . This causes further substitution between the two private goods, the result of which is that the marginal rate of substitution between good x and the public good is adjusted away from unity, in response to this relative price effect.

To derive the relative price effect, we first note that the optimality conditions (2), (4), together with (5), (6), determine the excess demand functions

$$E_j^i = E_j^i(\sigma, G_x^i, G_y^i) \quad i = A, B, \quad j = x, y$$

Substituting these expressions into the market clearing condition (7a) and differentiating, one can show that under reasonable conditions⁵

$$\frac{\partial \sigma}{\partial G_x^A} < 0 \quad (9)$$

Plausibly, an increase in government expenditure on good x raises its relative price, and we refer to this as being the *normal* case. But the opposite case where $\partial \sigma / \partial G_x^A > 0$ cannot be ruled out *a priori*.

The optimal government expenditure on the import good, obtained by differentiating U^A with respect to G_y^A , is derived similarly. Omitting details it is given by the expression

$$\frac{\partial U^A / \partial G_y^A}{\partial U^A / \partial C_y^A} = \left[1 + \frac{E_y^A}{\sigma} \frac{\partial \sigma}{\partial G_y^A} \right] \quad (10)$$

where we may show that under normal conditions $\partial \sigma / \partial G_y^A > 0$. Hence an increase in government

expenditure on the import good raises excess demand for that good, causing its relative price to rise. The marginal rate of substitution between private and government expenditure on the import good is equated to a quantity in excess of the relative private of unity.

Repeating these calculations for Country B yields analogous conditions, namely

$$\frac{\partial U^B / \partial G_x^B}{\partial U^B / \partial C_x^B} = 1 - E_y^A \left(\frac{\partial \sigma}{\partial G_x^B} \right) \quad (11)$$

$$\frac{\partial U^B / \partial G_y^B}{\partial U^B / \partial C_y^B} = 1 - \frac{E_y^A}{\sigma} \left(\frac{\partial \sigma}{\partial G_y^B} \right) \quad (12)$$

where under normal conditions, $\partial \sigma / \partial G_x^B < 0$, $\partial \sigma / \partial G_y^B > 0$, and the interpretation of the optimality conditions (11) and (12) are analogous to those for Country A.

The optimality conditions (8), (10), (11) and (12) are various reaction functions, some or all of which may apply, depending upon what fiscal instruments are being varied. For example, if the two countries optimize with respect to their respective export goods, then the reaction functions are (8) and (11) respectively, which jointly determine the Nash equilibrium noncooperative outcomes for G_x^A and G_y^B . It is also possible for both governments to optimize with respect to both commodities, in which case, these four equations will determine the equilibrium values of the four variables G_x^A , G_y^A , G_x^B , and G_y^B .

4. COOPERATIVE EQUILIBRIUM

We now assume that the two policy makers act cooperatively, to maximize their joint welfare function.

$$W = U^A[C_x^A, C_y^A, G_x^A, G_y^A] + U^B[C_x^B, C_y^B, G_x^B, G_y^B] \quad (13)$$

The optimality conditions with respect to the four government expenditure variables, obtained by differentiating (13), subject to the equations of the system (1)-(10), are given by

$$\frac{\partial W}{\partial G_x^A} = -\frac{\partial U^A}{\partial C_x^A} \left[1 + E_y^A \frac{\partial \sigma}{\partial G_x^A} \right] + \frac{\partial U^A}{\partial G_x^A} + \frac{\partial U^B}{\partial C_x^B} E_y^A \frac{\partial \sigma}{\partial G_x^A} = 0 \quad (14a)$$

$$\frac{\partial W}{\partial G_y^A} = -\frac{\partial U^A}{\partial C_y^A} \left[1 + \frac{E_y^A}{\sigma} \frac{\partial \sigma}{\partial G_y^A} \right] + \frac{\partial U^A}{\partial G_y^A} + \frac{\partial U^B}{\partial C_y^B} \frac{E_y^A}{\sigma} \frac{\partial \sigma}{\partial G_y^A} = 0 \quad (14b)$$

$$\frac{\partial W}{\partial G_x^B} = -\frac{\partial U^B}{\partial C_x^B} \left[1 - E_y^A \frac{\partial \sigma}{\partial G_x^B} \right] + \frac{\partial U^B}{\partial G_x^B} - \frac{\partial U^A}{\partial C_x^A} E_y^A \frac{\partial \sigma}{\partial G_x^B} = 0 \quad (14c)$$

$$\frac{\partial W}{\partial G_y^B} = -\frac{\partial U^B}{\partial C_y^B} \left[1 - \frac{E_y^A}{\sigma} \frac{\partial \sigma}{\partial G_y^B} \right] + \frac{\partial U^B}{\partial G_y^B} - \frac{\partial U^A}{\partial C_y^A} \frac{E_y^A}{\sigma} \frac{\partial \sigma}{\partial G_y^B} = 0 \quad (14d)$$

Again, only some subset of these equations may apply, if only some subset of the fiscal variables are being optimized.

Consider, for example, equation (14a). It is evident by comparison with (8), that when government A acts noncooperatively, it fails to take account of the fact that its actions, by influencing the world relative price σ , generates an externality which affects the welfare of agents in Country B. In particular, the noncooperative level of G_x^A is generally too large, relative to the cooperative equilibrium level. To see this, we evaluate $\partial W / \partial G_x^A$ at the noncooperative equilibrium level. At this equilibrium (denoted by subscript N), equation (8) applies, so that

$$\left. \frac{\partial W}{\partial G_x^A} \right|_N = \frac{\partial U_B}{\partial C_x^B} E_y^A \frac{\partial \sigma}{\partial G_x^A} \quad (15)$$

Thus, in general, in the normal case, when $\partial \sigma / \partial G_x^A < 0$ (15) implies $\left. \partial W / \partial G_x^A \right|_N < 0$; i.e., aggregate welfare is increased if Country A reduces its expenditure on the export good from its Nash equilibrium level. In the less likely case, when $\partial \sigma / \partial G_x^A > 0$, its expenditure on that good should be increased.⁶ Similar comments apply to the other forms of government expenditure, enabling us to summarize with the following:

Proposition 1: Provided goods are normal in the sense defined, then a noncooperative equilibrium leads to an *overexpansion* of government expenditure on export goods and an *underexpansion* of government expenditure on import goods, relative to the

cooperative equilibrium where joint utilities are being maximized.

The welfare function in (13) assigns equal (arbitrary) weights to the two economies. This is the criterion used throughout both the monetary and fiscal literature analyzing the gains from policy coordination. As seen in (14), the cooperative equilibrium involves the redistribution of income between the economies through terms of trade effects. But this may not represent a world social optimum. Chari and Kehoe (1987) suggest an alternative benchmark in which the cooperative equilibrium and the social optimum coincide. As they note, this may be attained if in addition to setting expenditure levels, the government sets lump sum taxes to achieve the appropriate distribution.

The social optimum equilibrium can be formally derived as the solution to a planning problem in which private consumptions, outputs, and government expenditures are chosen to maximize a weighted average of utilities

$$W^* = \lambda^A U^A + \lambda^B U^B$$

subject to the market clearing conditions (5), (7a), (7b). Performing the maximization, the optimality conditions characterizing the social optimum can be shown to include

$$\frac{\partial W^*}{\partial G_j^i} = \lambda^i \left[\frac{\partial U^i}{\partial G_j^i} - \frac{\partial U^i}{\partial C_j^i} \right] = 0 \quad i = A, B \quad j = x, y$$

For example, evaluating $\partial W^* / \partial G_x^A$ at the noncooperative equilibrium, yields

$$\left. \frac{\partial W^*}{\partial G_x^A} \right|_N = \lambda^A \frac{\partial U^A}{\partial C_x^A} E_y^A \frac{\partial \sigma}{\partial G_x^A} \quad (16)$$

which analogously to (15) implies an overexpansion on G_x^A in general. Indeed, an analogous proposition to Proposition 1 holds with respect to the social optimum, although the degree of underexpansion is clearly different, reflecting the distortion of the cooperative equilibrium from the social optimum.

5. A LOGARITHMIC EXAMPLE

We now consider the special case where: (i) the two countries are each specialized in the production of a single good; (ii) the government spends only on the domestic good; (iii) the respective utility functions are logarithmic:

$$U^i = \alpha_x^i \ln C_x^i + \alpha_y^i \ln C_y^i + \beta^i \ln G^i \quad i = A, B \quad (17)$$

where $\alpha_x^i + \alpha_y^i = 1$, and good x is produced by Country A and y by Country B. The budget constraints for the respective economies are:

$$C_x^A + \sigma C_y^A = \bar{Q}^A - G^A \quad (18a)$$

$$\frac{1}{\sigma} C_x^B + C_y^B = \bar{Q}^B - G^B \quad (18b)$$

where \bar{Q}^A, \bar{Q}^B , denote the fixed levels of output. The demand functions for the two goods in the two economies, obtained by maximizing the utility functions subject to the respective budget constraints, are:

$$C_x^A = \alpha_x^A (\bar{Q}^A - G^A); \quad C_y^A = \alpha_y^A \left(\frac{\bar{Q}^A - G^A}{\sigma} \right) \quad (19a)$$

$$C_x^B = \alpha_x^B (\bar{Q}^B - G^B) \sigma; \quad C_y^B = \alpha_y^B (\bar{Q}^B - G^B) \quad (19b)$$

The relative price, σ , is obtained from the balance of trade equilibrium condition

$$C_x^B = \sigma C_y^A \quad (20)$$

$$\sigma = \left(\frac{\alpha_y^A}{\alpha_x^B} \right) \left(\frac{\bar{Q}^A - G^A}{\bar{Q}^B - G^B} \right)$$

The relative price is therefore inversely proportional to the disposable incomes in the two economies.

Furthermore, substituting (20) into (19), leads to

$$C_x^A = \alpha_x^A (\bar{Q}^A - G^A); \quad C_y^A = \alpha_y^B (\bar{Q}^B - G^B) \quad (21a)$$

$$C_x^B = \alpha_x^A (\bar{Q}^A - G^A); \quad C_y^B = \alpha_y^B (\bar{Q}^B - G^B) \quad (21b)$$

The interesting feature is that through the adjustment in the relative price, the equilibrium quantities

of the imported good in each case depend upon the other country's disposable income. This fact gives rise to a large spillover in the effects of fiscal policy in one country on the other.

The next step is to substitute (21a), (21b) into the utility functions (17a), (17b) and this yields

$$U^i(G^A, G^B) = K^i + \alpha_x^i \ln(\bar{Q}^A - G^A) + \alpha_y^i \ln(\bar{Q}^B - G^B) + \beta^i \ln(G^i) \quad i = A, B \quad (22)$$

where the constants K^i are functions of the parameters α_j^i . The noncooperative Nash equilibrium is obtained by calculating

$$\frac{\partial U^A}{\partial G^A} = -\frac{\alpha_x^A}{\bar{Q}^A - G^A} + \frac{\beta^A}{G^A} = 0 \quad (23a)$$

$$\frac{\partial U^B}{\partial G^B} = -\frac{\alpha_y^B}{\bar{Q}^B - G^B} + \frac{\beta^B}{G^B} = 0 \quad (23b)$$

Equations (23a), (23b) are of course the respective reaction functions in the two economies. Observe that each is independent of the other's decision. This means that the Nash equilibrium is dominant and the assumption of a zero conjectured response is correct.⁷ The reaction functions are plotted in Figure 1 as horizontal and vertical lines respectively, which intersect at the point A, where

$$G^A = \left(\frac{\beta^A}{\alpha_x^A + \beta^A} \right) \bar{Q}^A; \quad G^B = \left(\frac{\beta^B}{\alpha_y^B + \beta^B} \right) \bar{Q}^B \quad (24)$$

The implied levels of utility in the two countries are obtained by substituting these two solutions in (19a), (19b).

Suppose now that the two policy makers act cooperatively, and maximize their joint utility functions

$$U^A + U^B = K^A + K^B + (\alpha_x^A + \alpha_x^B) \ln(\bar{Q}^A - G^A) + (\alpha_y^A + \alpha_y^B) \ln(\bar{Q}^B - G^B) + \beta^A \ln G^A + \beta^B \ln G^B$$

The optimality conditions are

$$-\frac{(\alpha_x^A + \alpha_x^B)}{\bar{Q}^A - G^A} + \frac{\beta^A}{G^A} = 0$$

$$-\frac{(\alpha_y^A + \alpha_y^B)}{\bar{Q}^B - G^B} + \frac{\beta^B}{G^B} = 0$$

the solutions to which are

$$G^A = \left(\frac{\beta^A}{\alpha_x^A + \alpha_x^B + \beta^A} \right) \bar{Q}^A; \quad G^B = \left(\frac{\beta^B}{\alpha_y^A + \alpha_y^B + \beta^B} \right) \bar{Q}^B \quad (25)$$

This is illustrated by the point B in Figure 1, which lies at the tangency points between the two indifference curves. Comparing (24) with (25) (or point A with point B), we see that cooperation leads to a reduction in the levels of government expenditure, consistent with the general result of Section 4.

An interesting issue concerns the magnitude of the difference between the two equilibria. One conclusion prevalent among the macroeconomic models dealing with this question, is that the non-cooperative and cooperative equilibria are typically closely clustered. The numerical gains from cooperation are small. In the present example we find that the respective percentage changes in the levels of G^A , G^B , (measured from the noncooperative equilibria as base) are

$$\frac{\Delta G^A}{G^A} = \frac{\alpha_x^B}{\alpha_x^A + \alpha_x^B + \beta^A}; \quad \frac{\Delta G^B}{G^B} = \frac{\alpha_y^A}{\alpha_y^A + \alpha_y^B + \beta^B}$$

Suppose, for example, one takes $\alpha_x^A = \alpha_y^B = .6$, $\alpha_y^A = \alpha_x^B = .4$, $\beta^A = \beta^B = .3$, suggesting that each economy has a preference for its own good. Then from (24), the ratios of government expenditure to output in the non-cooperative equilibrium in the two economies are $G^A/\bar{Q}^A = G^B/\bar{Q}^B = .333$. Moving to the cooperative equilibrium, reduces these shares to around .231 respectively (see (25)), which is a relative reduction in government expenditure of nearly 31%. While the ratios of consumptions to disposable income remain fixed (set by α_j^j), the absolute levels increase correspondingly by around 15%. Although the parameter values underlying these calculations are arbitrary, they are certainly plausible insofar as the implied shares of government expenditure in output, and consumption patterns are concerned.

These differences in the equilibria are substantially larger than those typically obtained from macroeconomic models. For example, the dynamic simulations of Oudiz and Sachs (1985) find that the optimal paths followed by output and inflation under noncooperative behavior change remarkably little when a cooperative equilibrium is considered. Turnovsky and d'Orey (1986) find the changes in the optimal level of money stock between the two equilibria to be typically small (around 2-4%) for simulations in which the elasticity of the demand for output with respect to the relative price is around unity, although it increases to around 15% as this elasticity falls to .1. They find utility gains to be almost negligible in the former case and to increase to around 10% (depending upon the disturbance), in the latter case. Similarly, Taylor (1985), whose numerical analysis is also based on a relative price elasticity of .1, finds utility gains to be around 10%. Taken together, these pieces of evidence suggest that the gains from cooperation as determined by these macro models are modest. The larger differences between the two equilibria suggested by the present analysis can be attributed to the strong international spillovers generated by relative price movements in the present framework. The utility gains, however, are arbitrary and are proportional to the choice of \bar{Q} .

6. THE EFFECTS OF A COALITION

Most of the formal literature deals with the gains from cooperation in a two country world. But in reality, cooperative arrangements are formed among groups of countries, with some countries being excluded from such coalitions. In this section we address an important issue, namely, how the gains from cooperation between two countries A and B say, impact on a third country C which produces a third good z say.⁸ For tractability, we consider the question within the logarithmic model of Section 5. In this analysis the number of economies and goods remains fixed (at three); what changes is the number of economies participating in the cooperative arrangement.

Augmented to three countries, the model contains the following relationships. The utility functions for Country i :

$$U^i = \alpha_x^i \ln C_x^i + \alpha_y^i \ln C_y^i + \alpha_z^i \ln C_z^i + \beta^i \ln G^i$$

$$\alpha_x^i + \alpha_y^i + \alpha_z^i = 1 \quad i = A, B, C$$
(26)

and corresponding budget constraints

$$C_x^A + C_y^A \sigma_y + C_z^A \sigma_x + G^A = \bar{Q}^A$$

$$C_x^B / \sigma_y + C_y^B + C_z^B \sigma_x / \sigma_y + G^B = \bar{Q}^B$$

$$C_x^C / \sigma_x + C_y^C \sigma_y / \sigma_x + C_z^C + G^C = \bar{Q}^C$$
(27)

where now

σ_y is the relative price of good y to good x ,

σ_x is the relative price of good z to good x .

There are two bilateral balance of trade conditions:

$$C_y^A \sigma_y + C_z^A \sigma_x = C_x^B + C_x^C$$

$$C_x^B / \sigma_y + C_y^B \sigma_x / \sigma_y = C_y^A + C_y^C$$
(28)

together with three market clearing conditions:

$$C_j^A + C_j^B + C_j^C + G^i = \bar{Q}^i \quad i = A, B, C \quad j = x, y, z$$
(29)

although only two of these are independent.

Solving the system as before leads to

$$\sigma_y = \psi_y \left(\frac{\bar{Q}^A - G^A}{\bar{Q}^B - G^B} \right); \quad \sigma_x = \psi_x \left(\frac{\bar{Q}^A - G^A}{\bar{Q}^C - G^C} \right)$$

where

$$\psi_y = \frac{\alpha_y^C(1 - \alpha_x^A) + \alpha_x^C \alpha_y^A}{\alpha_x^B \alpha_y^C + \alpha_x^C(1 - \alpha_y^B)}; \quad \psi_x = \frac{\alpha_x^A(1 - \alpha_y^B) + \alpha_y^A \alpha_x^B}{\alpha_x^B \alpha_y^C + \alpha_x^C(1 - \alpha_y^B)}$$

and

$$C_x^A = \alpha_x^A(\bar{Q}^A - G^A); \quad C_y^A = \alpha_y^A(\bar{Q}^B - G^B) / \psi_y; \quad C_z^A = \alpha_z^A(\bar{Q}^C - G^C) / \psi_x$$

$$C_x^B = \alpha_x^B(\bar{Q}^A - G^A) \psi_y; \quad C_y^B = \alpha_y^B(\bar{Q}^B - G^B); \quad C_z^B = \alpha_z^B(\bar{Q}^C - G^C) \psi_y / \psi_x$$

$$C_x^C = \alpha_x^C(\bar{Q}^A - G^A) \psi_x; \quad C_y^C = \alpha_y^C(\bar{Q}^B - G^B) \psi_x / \psi_y; \quad C_z^C = \alpha_z^C(\bar{Q}^C - G^C)$$

The corresponding levels of utility are therefore of the form

$$U^i = K^i + \alpha_x^i \ln(\bar{Q}^A - G^A) + \alpha_y^i \ln(\bar{Q}^B - G^B) + \alpha_z^i \ln(\bar{Q}^C - G^C) + \beta^i \ln G^i \quad i = A, B, C \quad (30)$$

Suppose initially that all three countries act noncooperatively. The optimal levels of government expenditure for the three countries are of the form

$$G^A = \left(\frac{\beta^A}{\alpha_x^A + \beta^A} \right) \bar{Q}^A; \quad G^B = \left(\frac{\beta^B}{\alpha_y^B + \beta^B} \right) \bar{Q}^B; \quad G^C = \left(\frac{\beta^C}{\alpha_z^C + \beta^C} \right) \bar{Q}^C \quad (31)$$

which are analogous to (24), obtained previously.

Suppose now that Countries A and B form a coalition and cooperate to maximize their joint welfare $U^A + U^B$. The optimality conditions are

$$G^A = \frac{\beta^A}{\alpha_x^A + \alpha_x^B + \beta^A} \bar{Q}^A \quad (32a)$$

$$G^B = \frac{\beta^B}{\alpha_y^A + \alpha_y^B + \beta^B} \bar{Q}^B \quad (32b)$$

with both G^A and G^B being reduced from their respective noncooperative levels. Denoting these reductions by dG^A , dG^B , the welfare gains for Country A are

$$dU^A = -\frac{\alpha_y^A}{(\bar{Q}^B - G^B)} dG^B \quad (33)$$

where G^B is evaluated at the noncooperative level.

How does the formation of this coalition affect the welfare of the excluded Country C? The answer is given by the expression

$$dU^C = -\frac{\alpha_x^C}{(\bar{Q}^A - G^A)} dG^A - \frac{\alpha_y^C}{(\bar{Q}^B - G^B)} dG^B$$

which with dG^A , dG^B both being negative, implies an unambiguous welfare improvement. In fact, the welfare gains to Country C may quite plausibly exceed those accruing to either of the partners A and

B. This is so, for example, if all countries have identical tastes and the coefficients α_j^i are equal. Essentially the reason is that the coalition reduces both G^A and G^B and this has a favorable effect on the terms of trade for Country C with respect to both of its trading partners.⁹ Thus we find that at least for this logarithmic example, the formation of a coalition between two countries may yield greater benefit to an excluded country, than it does to either of the partners.

Finally, suppose that Country C joins the coalition, so that all three act cooperatively. Maximizing the aggregate utility $U^A + U^B + U^C$, we find

$$G^i = \left(\frac{\beta^i}{\alpha_j^A + \alpha_j^B + \alpha_j^C + \beta^i} \right) \bar{Q}^i \quad i = A, B, C \quad j = x, y, z$$

This involves a further reduction in government expenditure and the mutual gains from cooperation increase. For Country A, for example,

$$dU^A = -\frac{\alpha_y^A}{(\bar{Q}^B - G^B)} dG^B - \frac{\alpha_z^A}{(\bar{Q}^C - G^C)} dG^C$$

which now gains from the reduction in public expenditure in the new member Country C, as well as in that of G^B . Country C gains from joining because of the additional reductions in G^A , G^B this induces. Thus, at least in this example, we find that the gains from cooperation increase with the number of countries cooperating.

7. CONCLUSIONS: REINTERPRETATION AS A TWO-PERIOD ANALYSIS

This paper has analyzed the gains from fiscal cooperation, within the context of the standard two commodity real trade model. Using this framework, we have shown how the adjustment in the terms of trade is the critical factor in determining the effects of moving from a noncooperative equilibrium. In general, a noncooperative equilibrium leads to an overexpansion of government expenditure on the export good and an underexpansion on the import good, relative to an equilibrium in which both governments act cooperatively. Aggregate world welfare will be increased by a reduction in government expenditure on the former and an increase in government expenditure on the latter. The

specific example of a logarithmic economy has been considered. We have demonstrated how, at least within such an economy, the formation of a coalition among two countries may confer welfare gains on an excluded third country, which exceed those accruing to either partner.

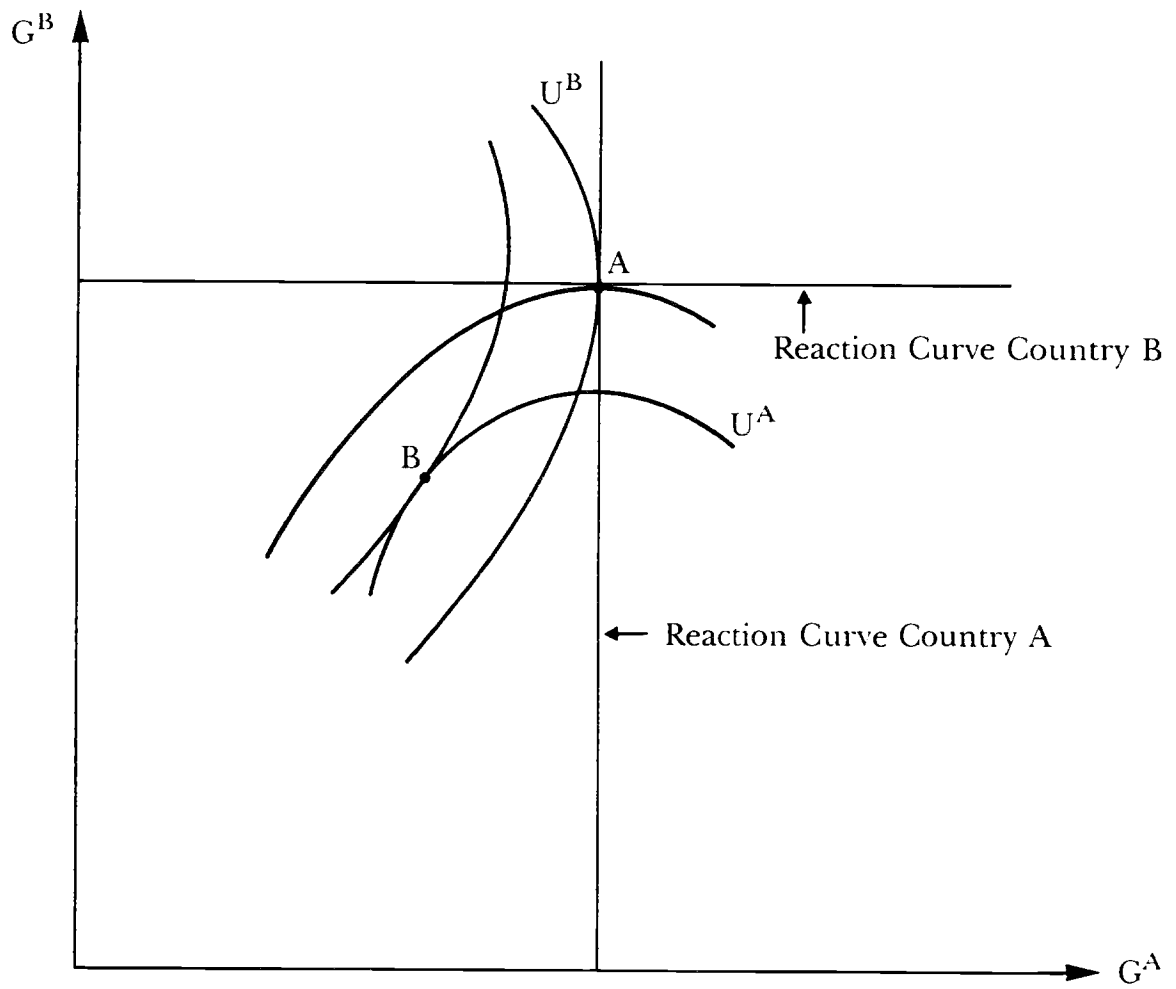
It is straightforward to reinterpret the framework in terms of a two-period single commodity model, such as that used by Hamada (1986). The subscripts x, y may now be reinterpreted as two periods (the present and future say), so that the utility functions (1) and the production possibilities curves (3) apply intertemporally. Furthermore, combining equations (5) and (6) yields

$$C_x^i + \sigma C_y^i = (Q_x^i - G_x^i) + \sigma(Q_y^i - G_y^i)$$

which now can be interpreted as being a two-period intertemporal budget constraint in which σ denotes the intertemporal price (the interest rate), rather than the intratemporal terms of trade.¹⁰ Formally, everything else applies as before. The assumptions $E_y^A > 0$, $E_x^B > 0$ can now be interpreted that Country A has an excess demand for future goods and is therefore a lender, while Country B has an excess demand for current goods and is a borrower.

With this interpretation, the effects of fiscal cooperation depend upon the adjustment of the interest rate. In what we have called the normal case, an increase in government expenditure on the current good raises the current interest rate, while an increase in government expenditure in the future period lowers it. In general, we find that a noncooperative equilibrium leads to an overexpansion of government expenditure on present goods and an underexpansion on future goods, by the lender, relative to the cooperative equilibrium. The reverse holds true with respect to the borrower.

Figure 1
Noncooperative and Cooperative Equilibria:
Logarithmic Case



* I wish to thank Michael Devereux and the referee for helpful comments.

1. See, e.g., Canzoneri and Gray (1985), Oudiz and Sachs (1985), Miller and Salmon (1985), Currie and Levine (1985), Taylor (1985), Turnovsky and d'Orey (1986).
2. This change of emphasis is apparent from the recent NBER Conference on the international aspects of fiscal policy; see Frenkel (1987).
3. Our choice of import and export goods implies $E_y^A > 0$, $E_x^A < 0$, $E_x^B > 0$, $E_y^B < 0$.
4. Recall that the formula for the optimal tariff is independent of the utility function, or marginal utility. The reason for this is that, unlike government expenditure, the tariff rate does not enter the utility function directly. If, to complete the analogy, one assumes that G_x^A does not generate independent utility, then (13) reduces to $1 + E_y^A \partial \sigma / \partial G_x^A = 0$. This can be rewritten as

$$\frac{G_x^A}{E_x^A} = \frac{\partial \sigma}{\partial G_x^A} \frac{G_x^A}{\sigma}$$

That is, the ratio of government expenditure to the excess supply for the export good equals the elasticity of the relative price with respect to government expenditure on the export good.

5. Two conditions assumed include: (i) That an increase in the relative price of good x in either country reduces the excess demand for that good, while increasing it for the other good; (ii) that the Marshall-Lerner condition holds. Condition (i) in turn requires the usual assumption that the substitution effect of a price change dominates the income effect.

In an Appendix to this paper, we show how invoking (i) and (ii) implies

$$\text{sgn} \left[\frac{\partial \sigma}{\sigma G_x^A} \right] = \text{sgn} \left[\frac{\partial E_y^A}{\partial G_x^A} \right] = \text{sgn} \left[-\frac{d}{dC_x^A} \left[\frac{\partial U^A / \partial C_y^A}{\partial U^A / \partial C_x^A} \right] + \frac{d}{dG_x^A} \left[\frac{\partial U^A / \partial C_y^A}{\partial U^A / \partial C_x^A} \right] \right]$$

The first term in the parentheses can be shown to be proportional to the effect of a decrease in income on the excess demand E_y^A and is presumably negative. This reflects the fact that an increase in G_x^A raises taxes, thereby lowering disposable income, private consumption of good y and excess demand for that good. The second term describes how the change in government

expenditure on good x affects the marginal rate of substitution between the two private goods. Under mild restrictions, including weak separability of the utility function in private and public goods, when this latter term is zero, the first term dominates, implying inequality (9).

6. One can also show that in the "normal" case, the reduction in G_x^A as one moves from the non-cooperative to the cooperative equilibrium will raise real national output in both countries and the volume of real trade, both measured in terms of good x .
7. That is, the Nash noncooperative equilibrium is also a consistent conjectural variations equilibrium.
8. My interest in analyzing this issue grew out of remarks made at a recent conference on the international effects of monetary policy, held recently at the University of Illinois at Urbana-Champaign.
9. The terms of trade are,

$$\frac{1}{\sigma_x} = \frac{1}{\psi_x} \left(\frac{\bar{Q}^C - G^C}{Q^A - G^A} \right); \quad \frac{\sigma_y}{\sigma_x} = \frac{\psi_y}{\psi_x} \left(\frac{\bar{Q}^C - G^C}{Q^B - G^B} \right)$$

It is clear that these both improve through reduction in dG^A , dG_B .

10. More precisely, we redefine $\sigma = 1/(1 + r)$, where r is the interest rate.

REFERENCES

- Bhagwati, J. N. and T. N. Srinivasan, 1983, *Lectures on International Trade*, MIT Press, Cambridge, Mass.
- Canzoneri, M. and J. A. Gray, 1985, "Monetary Policy Games and the Consequences of Noncooperative Behavior," *International Economic Review*, 26, 547-564.
- Chari, V. V. and P. J. Kehoe, 1987, "International Coordination of Fiscal Policy in Limiting Economies," Working Paper 317, Federal Reserve Bank of Minneapolis.
- Currie, D. and P. Levine, 1985, "Macroeconomic Policy Design in an Interdependent World," in W. H. Buiter and R. C. Marston (eds.), *International Economic Policy Coordination*, Cambridge University Press, Cambridge, U.K.
- Devereux, M., 1986, "The International Coordination of Fiscal Policy and the Terms of Trade: An Example," unpublished manuscript, University of Toronto.
- Frenkel, J. A. (ed.), 1987, *International Aspects of Fiscal Policy*, NBER, University of Chicago Press, Chicago.
- Frenkel, J. A. and A. Razin, 1985, "Government Spending, Debt, and International Economic Interdependence," *Economic Journal*, 95, 619-636.
- Frenkel, J. A. and A. Razin, 1986, "Fiscal Policies in the World Economy," *Journal of Political Economy*, 94, 564-594.
- Hamada, K., 1976, "A Strategic Analysis of Monetary Interdependence," *Journal of Political Economy*, 84, 677-700.
- Hamada, K., 1986, "Strategic Aspects of International Fiscal Interdependence," *Economic Studies Quarterly*, 37, 165-180.
- Johnson, H. G., 1954, "Optimum Tariffs and Retaliation," *Review of Economic Studies*, 21, 142-153.
- Kehoe, P. J., 1987, "Coordination of Fiscal Policies in a World Economy," *Journal of Monetary Economics*, 19, 349-376.
- Oudiz, G. and J. Sachs, 1985, "International Policy Coordination in Dynamic Macroeconomic Models," in W. H. Buiter and R. C. Marston (eds.), *International Economic Policy Coordination*, Cambridge University Press, Cambridge, U.K..
- Miller, M. H. and M. Salmon, 1985, "Policy Coordination and Dynamic Games," in W. H. Buiter and R. C. Marston (eds.), *International Economic Policy Coordination*, Cambridge University Press, Cambridge, U.K.
- Takayama, A., 1974, *International Trade*, Holt, Rinehart and Winston, New York.
- Taylor, J. B., 1985, "International Coordination in the Design of Macroeconomic Policy Rules," *European Economic Review*, 28, 53-82.
- Turnovsky, S. J. and V. d'Orey, 1986, "Monetary Policies in Interdependent Economies with Stochastic Disturbances: A Strategic Approach," *Economic Journal*, 96, 696-721.