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TAIL AND CENTER ROUNDING OF PROBABILISTIC EXPECTATIONS IN THE
HEALTH AND RETIREMENT STUDY

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ABSTRACT

A growing number of surveys elicit respondents' expectations for future events on a 0-100 scale of percent chance. These data reveal substantial heaping at multiples of 10 and 5 percent, suggesting that respondents round their reports. This paper studies the nature of rounding by analyzing response patterns across expectations questions and waves of the Health and Retirement Study. We discover a tendency by about half of the respondents to provide more refined responses in the tails of the scale than the center. Only about five percent provide more refined responses in the center than the tails. We find that rounding varies across question domains, which range from personal health to personal finances to macroeconomic events. We develop a two-stage framework to characterize person-specific rounding. The first stage uses observed responses to infer respondents' rounding practice in each question domain and scale segment. The second stage replaces each original point response with an interval, representing the range of possible values of the respondent's true latent belief implied by the degree of rounding inferred in the first stage. We study how the inferred rounding types in the first stage vary with respondent characteristics, including age and cognitive abilities.

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1. Introduction

Judgements about the likelihood of future events are an important input for predictions and decisions by citizens, policy makers, and researchers. From the early 1990s on, surveys designed by economists have increasingly measured respondents' subjective expectations for future events using a 0-100 scale of percent chance. This endeavor was prompted by earlier empirical evidence and theoretical arguments demonstrating the greater informativeness of elicited probabilities for binary events relative to "yes/no" intention measures (Juster, 1966; Manski, 1990). Manski (2004, 2017), Attanasio (2009), Hurd (2009), van der Klaauw (2012), Armantier *et al.* (2013), Delavande (2014), Schotter and Trevino (2014), and Giustinelli and Manski (2018) review the literature from various perspectives.

The Health and Retirement Study (HRS), whose data we analyze in this paper, has measured probabilistic expectations biannually since its start in 1992; Juster and Suzmann (1995) describe the initial design. Section P of the HRS core questionnaire has been devoted to expectations measurement, each wave including about 25 to 35 questions spanning different domains of personal and macroeconomic uncertainty. From 2002 on, expectations have been consistently elicited on a 0-100 percent-chance scale, with many questions repeated across waves.

Questions eliciting expectations on a 0-100 percent-chance scale in principle enable respondents to report beliefs to the nearest 1 percent, encouraging a common rounding convention with minimal data coarsening. But how do respondents use the scale in practice? The accumulated evidence reveals that respondents tend to round their responses. Responses that are not a multiple of 5 or 10 percent occur infrequently. When observed, they tend to occur near the endpoints of the scale to convey very small or large probabilities.

Some authors have devoted special attention to responses of 0, 50, and 100 percent. Fischhoff and Bruine de Bruin (1999) and Bruine de Bruin *et al.* (2002) hypothesize that some respondents use 50 percent to signal epistemic uncertainty. Lillard and Willis (2001) and Hudomiet and Willis (2013) conjecture that respondents form full subjective distributions for the probability of an event and then

report whichever of the values (0, 50, 100) is closest to the mode of their distribution. We analyze reports of (0, 50, 100) percent jointly with responses to the entire set of expectations questions asked. Systemic rounding of responses was observed as early as Dominitz and Manski (1997), who wrote (p. 272): “Most respondents do not round their responses to the values (0, 50, 100), but rather to the nearest multiple of five.”

Rounding of expectations poses a series of challenges for statistical inference. First, rounding generates greater data coarsening than intended by the measurement scale. Second, the extent of rounding is not directly observable and may vary across respondents and/or questions. Third, the reasons why respondents round are incompletely understood.

Observed response patterns carry information about respondents’ rounding practices, but they do not reveal why respondents round. Manski and Molinari (2010) hypothesize that respondents may round to simplify communication and/or to convey partial knowledge. If respondents round to simplify communication, rounding generates a form of measurement error. However, the structure of the errors produced by rounding is different from that occurring in the classical errors-in-variables model.

Manski and Molinari studied respondent-specific response patterns across all expectations questions asked in the 2006 HRS. They found strong evidence of rounding, with the extent differing across respondents. They proposed use of a person’s response pattern across questions to infer the person’s rounding practice, the result being interpretation of reported numerical values as interval data.

In this paper, we significantly expand study of respondent-specific rounding patterns by analyzing responses across all expectations questions asked in the core HRS questionnaire between 2002 and 2014. This enables us to learn important new features of rounding practices.

Section 2 presents the main findings of our data analysis, with Supplementary Appendices reporting further details. We initially study each wave of the HRS separately and find that the respondent-specific rounding patterns reported by Manski and Molinari (2010) are stable across waves. We then pool data

across waves. This yields rich respondent-specific data that enables us to probe more deeply than the earlier study.

We discover a tendency by about half of the respondents to provide more refined responses in the tails of the 0-100 scale than the center. In contrast, only about five percent of the respondents give more refined responses in the center than the tails. We find that respondents tend to report the values 25 and 75 more frequently than other values ending in 5. We also find that rounding practices vary somewhat across question domains, which range in the HRS from personal health to personal finances to macroeconomic events.

Based on our examination of rounding practices in Section 2, Section 3 develops a framework that interprets each numerical response given by a respondent as an interval. We propose a two-stage algorithm. The first stage classifies each respondent into one of a set of mutually exclusive and exhaustive rounding types and places an upper bound on the amount of rounding each respondent is inferred to apply when reporting their expectations. The second stage assigns an interval to each of the respondent's original point responses, which represents the range of values in which the respondent's underlying true belief is plausibly deemed to lie based on the respondent's inferred rounding type. These intervals can be interpreted as a measure of informativeness, or quality, of an increasingly used type of data (numerical probabilistic expectations) from an important data source (the HRS).

Our approach accommodates substantial heterogeneity in rounding practices. Within a specific question domain, a respondent's rounding type is a bivariate vector of the form (tail, center) rounding, partitioning the 0-100 scale into two symmetric tails (0-24 and 76-100) and a center (25-75). Thus, in addition to being person specific, the inferred degree of rounding may differ between the tails and the center of the scale and may vary across question domains. The assigned intervals vary across respondents and across values of the observed point responses.

We use our framework to study how rounding tendencies vary with observable characteristics of the respondents. We find that higher levels of educational attainment and of cognition are associated with a

tendency to give more refined responses (less rounding) across all scale segments and question domains. On the other hand, the association of rounding with age appears to be non-linear, with youngest (50-59) and oldest (80+) respondents displaying a higher tendency to round than respondents in the intermediate age groups (60-69 and 70-79).

These findings are substantively interesting, demonstrating systematic variation in how respondents answer expectations questions. They may also help to inform researchers working with other surveys that have less rich expectations data than the HRS. These researchers may know respondents' characteristics, but not have enough expectations data to apply our approach directly.

While this paper studies rounding as a subject of intrinsic interest, a reader may naturally ask how the interval data that our proposed approach generates from respondent point responses may be used in statistical analyses. In principle, empirical analysis with interval data is simply a matter of considering all points in the relevant interval to be feasible values of the quantity of interest. The practical feasibility of implementing this simple idea depends on the nature of the analysis. This matter has been addressed in the econometric literature studying conditional prediction with interval measurement of outcomes and/or covariates. See Manski and Tamer (2002) and Beresteanu and Molinari (2008). Manski and Molinari (2010) provide an illustrative application of best linear prediction with interval-measured expectations outcomes. Analogous applications to prediction with interval-measured covariates are feasible.

As far as we are aware, only two previous papers systematically study rounding of responses to probabilistic expectations questions. One is Manski and Molinari (2010), on whose work we build. The other is Kleijnans and van Soest (2014), who develop and estimate a panel-data structural econometric model to analyze response patterns to each of six expectations questions in the HRS. Their analysis aims to investigate the extent to which probability reports are determined by genuine underlying probabilistic beliefs, rounding, a tendency to give so-called "focal" responses of (0, 50, 100), and selective item non-response. Despite the very different approaches taken, their findings and ours reinforce each other in

some important respects. Specifically, they find that tendencies to round, give “focal” responses, and not respond tend to be persistent over time.

Beyond readers who have interest in expectations data, we anticipate that general statisticians concerned with survey research will find this paper useful. Our study of tendencies to round responses to expectations questions should heighten concern that respondents may round responses to numerical questions in other contexts. Consider, for example, questions asking respondents to state their income or the number of hours they worked in the past week. Respondents may round their responses, with the extent of rounding differing across persons. Examination of a person’s response pattern across different numerical questions, in the manner that we do here, may provide a credible way to infer that person’s rounding practice. One may then interpret reported numerical values as intervals.

Some surveys elicit interval data directly. For example, the HRS uses *unfolding bracket* questions to enable respondents who are not willing to provide exact information about their income and assets to indicate whether the quantities of interest lie above or below a sequence of specified thresholds. Similarly, the Occupation Employment Statistics (Bureau of Labor Statistics, 2018) collects wage information in interval form. These intervals, too, can be analyzed using econometric methods for interval data referenced earlier.

Our interpretation of rounded responses as interval data provides an interesting counterpoint to previous statistical research on data coarsening. Rounded numerical responses have a different structure from the data analyzed in the literature on data coarsening (e.g., Heitjan and Rubin (1991), Heitjan (1994), and Gill et al. (1997)). In that literature, it is assumed that the researcher observes a random set \mathcal{X} (an interval, a group, a partial categorization, etc.) to which an unobservable random variable of interest x belongs with probability one. An assumption of “coarsening at random” is imposed, which requires that the probability of observing $\mathcal{X} = A$ given $x = x_0$ is constant for all x_0 in A , where A denotes a subset of the support of x . In contrast, the HRS does not provide set-valued expectations data. The algorithm that we propose constructs sets \mathcal{X} based on respondents’ point responses and their tendencies

for rounding across the entire set of questions eliciting subjective beliefs. Our approach does not assume ignorability of the coarsening mechanism and it allows for a coarsening mechanism that differs among respondents.

2. Exploratory Analysis of Response Patterns across Questions and Waves in the HRS

Since 2002 the HRS has devoted Section P of its core questionnaire to measurement of expectations in the domains of personal health, personal finances, and general economic conditions. Across seven biannual waves spanning 2002 to 2014, expectations have been elicited on a 0-100 percent chance scale. Many questions have been repeated across multiple waves. Table 1 shows the questions, organized by domain and the waves in which they were asked.

Expectations questions in the HRS refer to future realizations of binary events. In some cases, respondents are asked to report the chance that the realization of a continuous variable will be above or below each of a sequence of thresholds. Answers to these questions can be used to measure a respondent's subjective distribution function for the variable in question. See Dominitz and Manski (1997) for an early example of elicitation and validation in this format.

The number of questions per wave ranges between a minimum of 22 in 2002 and a maximum of 38 in 2006. Most questions are in the personal finances domain (between 11 and 23 per wave, 31 overall), followed by the personal health domain (between 3 and 9 per wave, 10 overall), and the domain of general economic conditions (between 2 and 7 per wave, 12 overall). A subset of 12 questions across the three domains were asked in all waves.

The number of responses varies across questions and waves, ranging from about 5,000 to 30,000 responses per question in each wave. The variation across questions stems from the fact that the HRS makes extensive use of skip sequencing. Thus, whether a respondent is asked a specific question depends on the previous answers given by the respondent and on whether the event specified by the question is relevant to the respondent.

Table 1: Probabilistic Expectations Questions in the HRS (Section P, Waves 2002-2014)

#	Question	Wave						
		2002	2004	2006	2008	2010	2012	2014
PERSONAL HEALTH (3-9 Qs in each wave, 10 across waves)								
P19	Health limit work during next 10 years	Y	-	-	-	-	-	-
P28	Live to be 75 or more	Y	Y	Y	Y	Y	Y	Y
P29	Live to be age X or more	Y	Y	Y	Y	Y	Y	Y
P32	Move to nursing home ever (if age<65) / in the next 5 years (if age >= 65)	Y	Y	Y	Y	Y	Y	Y
P103	Live independently at 75	-	-	Y	Y	-	-	-
P104	Free of serious mental problems at 75	-	-	Y	Y	-	-	-
P106	Live independently at X	-	-	Y	Y	-	-	-
P107	Free of serious problems in thinking/reasoning at X	-	-	Y	Y	-	Y	Y
P108	Same health in 4 years	-	-	Y	Y	-	-	-
P109	Worse health in 4 years	-	-	Y	Y	-	-	-
PERSONAL FINANCES (11-23 Qs in each wave, 31 across waves)								
P4	Income keep up inflation for next 5 years	Y	Y	Y	-	-	-	-
P5	Leave inheritance >=\$10,000	Y	Y	Y	Y	Y	Y	Y
P6	Leave inheritance >=\$100,000	Y	Y	Y	Y	Y	Y	Y
P7	Leave any inheritance	Y	Y	Y	Y	Y	Y	Y
P8	Receive inheritance during next 10 years	Y	Y	Y	-	-	-	-
P14	Lose job next year	Y	Y	Y	-	Y	Y	Y
P15	Finding a job in few month in case of job-loss	Y	Y	Y	-	Y	Y	Y
P16	Working for pay in the future	Y	Y	Y	Y	Y	Y	Y
P17	Working full time after age 62	Y	Y	Y	Y	Y	Y	Y
P18	Working full time after age 65	Y	Y	Y	Y	Y	Y	Y
P20	Finding a job in few months if unemployed	Y	Y	Y	Y	Y	Y	Y
P30	Give \$5,000 to others over next 10 years	Y	Y	Y	-	-	-	-
P31	Receive \$5,000 from others over next 10 years	Y	Y	Y	-	-	-	-
P59	Leave inheritance >=\$500,000	Y	Y	Y	Y	Y	Y	Y
P70	Medical expenses use up savings in next 5 years	-	Y	Y	Y	-	-	-
P71	Give \$1,000 to others during next 10 years	-	Y	Y	-	-	-	-
P72	Give \$10,000 to others during next 10 years	-	Y	Y	-	-	-	-
P73	Give \$20,000 to others during next 10 years	-	Y	Y	-	-	-	-
P74	Receive \$2,500 from others over next 10 years	-	Y	Y	-	-	-	-
P75	Receive \$1,000 from others over next 10 years	-	Y	Y	-	-	-	-
P76	Receive \$10,000 from others over next 10 years	-	Y	Y	-	-	-	-
P111	Soc. Sec. will be worse over next 10 years - current own benefits	-	-	Y	Y	Y	Y	Y
P112	Soc. Sec. will be worse over next 10 years - future own benefits	-	-	Y	Y	Y	Y	Y
P166	Home worth more by next year	-	-	-	-	Y	Y	Y
P168	Home worth more/less by random "X" by next year	-	-	-	-	Y	Y	Y
P175	Out-of-pocket medical expense >\$1,500 during next year	-	-	-	-	Y	Y	Y
P176	Out-of-pocket medical expense >\$500 during next year	-	-	-	-	Y	Y	Y
P177	Out-of-pocket medical expense >\$3,000 during next year	-	-	-	-	Y	Y	Y
P178	Out-of-pocket medical expense >\$8,000 during next year	-	-	-	-	Y	Y	Y
P181	Any work after age 70	-	-	-	-	-	Y	Y
P182	Working full time after age 70	-	-	-	-	-	Y	Y
GENERAL ECONOMIC CONDITIONS (2-7 Qs in each wave, 12 across waves)								
P34	U.S. have economic depression during next 10 years	Y	Y	Y	Y	-	-	-
P47	Mutual funds increase in value by next year	Y	Y	Y	Y	Y	Y	Y
P110	Social Security in general will become worse in next 10 years	Y	-	Y	Y	Y	Y	-
P114	Mutual funds increase more than the cost of living over next 10 years	-	-	Y	-	-	-	-
P115	Mutual funds increase 8% more than the cost of living over next 10 years	-	-	Y	-	-	-	-
P116	Cost of living increases more than 5% over next 10 years	-	-	Y	Y	-	-	-
P150	Mutual funds increase by 20% (10%, or a random X%) by next year	Y	-	-	Y	Y	Y	Y
P180	Mutual funds decrease by 20% by next year	-	-	-	-	Y	Y	Y
P183	Medicare less generous in next 10 years	-	-	-	-	-	Y	Y
P190	Stock Market increase in value in 12 months of today	-	-	-	-	-	-	Y
P192	Stock Market increase by 20% (in 12 months)	-	-	-	-	-	-	Y
P193	Stock Market decrease by 20% (in 12 months)	-	-	-	-	-	-	Y
Total N of Questions		22	26	38	25	25	29	31

The total number of responses generated by a question across the seven waves varies because questions have been added and removed over time. It also varies due to changes in sample composition across waves. The HRS sample has periodically been augmented with new cohorts of respondents who joined the study in specific waves. Respondents exit the study due to attrition or death.

Section 2.1 studies response patterns across questions in each wave, alternatively using all questions asked in the wave and the twelve questions asked in all waves. Focusing on the latter questions, we analyze the stability of response tendencies across pairs of waves. Supplementary Appendix 2.1 provides further detail, investigating patterns of response to specific questions.

Having established the temporal stability of rounding practices, Section 2.2 pools the HRS data across waves. This yields rich respondent-specific data that enables us to probe more deeply. We analyze response patterns separately by question domain. We pay particular attention to the location of responses inside the 0-100 scale and learn important features of respondents' response patterns in specific domains.

Throughout the paper, the notation M10 and M5 denotes responses that are multiples of 10 or 5 other than (0, 50, 100). Thus, $M10 \equiv \{10, 20, 30, 40, 60, 70, 80, 90\}$ and $M5 \equiv \{5, 15, 25, 35, 45, 55, 65, 75, 85, 95\}$. When responses are non-rounded values – neither (0, 50, 100) nor a multiple of 5 or 10 – we distinguish those in the outer tails of the scale, specifically 1-4 and 96-99, from those between 6 and 94. The abbreviation NR denotes nonresponse.

2.1. Temporal Stability of Response Tendencies

2.1.1. Response Tendencies in Each Wave

We first examine response tendencies in each HRS wave. Table 2 shows the fractions of respondents displaying each of seven mutually exclusive and exhaustive response patterns, progressing left to right from the most rounded to the least rounded. Column 3 gives the fraction of respondents who respond to no questions in the wave, coded in the HRS as “Don’t know” or “Refuse.” Column 4 gives the fraction

of respondents who, when they respond, only use the values 0 and 100 in the corresponding wave. Column 5 gives the fraction who only use the values (0, 50, 100). Columns 6 and 7 give the fractions of respondents who answer at least one question with a value in M10 and M5 respectively. Column 8 gives the fraction of respondents who respond to at least one question with a non-round value in the outer tails; that is, 1-4 or 96-99. Column 9, labelled “Some other,” gives the fraction who respond at least once with a non-round value in the range 6-94.

The set of expectations questions varies across waves. The top panel of Table 2 presents a version of the statistics where respondents are classified into one of the seven response patterns using only the twelve questions that were asked in all seven waves (i.e., P5, P6, P7, P16, P17, P18, P20, P28, P29, P32, P47, P59). The bottom panel uses the responses to all questions asked in a wave.

A very small fraction of respondents answer none of the questions posed to them. This fraction ranges between 0.009 and 0.027, depending on the set of questions used to classify respondents. Between 0.019 and 0.101 of respondents uses only the values (0, 100). Similar fractions of respondents use only the values (0, 50, 100). Most respondents give at least one answer in M10 or in M5. The fraction of M10 respondents ranges between 0.263 and 0.337 across waves when all questions asked in a wave are used for classification and between 0.392 and 0.458 when only the questions common to all waves are used. Similarly, the fraction of M5 respondents ranges between 0.427 and 0.513 when all questions are used for classification and between 0.295 and 0.353 when only the common set is used.

The fractions of respondents who give at least one response in the outer tails (1-4 or 96-99) or non-rounded values in 6-94 are sizeable but considerably smaller, especially the latter. The former fraction ranges between 0.101 and 0.144 when all questions are used for classification and between 0.054 and 0.092 when only the common set is used. The latter fraction ranges between 0.022 and 0.042 or between 0.011 and 0.020, depending on the set of questions used.

2.1.2. Transitions of Response Tendencies across Waves

The main message of Table 2 is that the response patterns found by Manski and Molinari (2010) in

Table 2: Response Tendencies in the 2002-2014 HRS

Wave	N	Response pattern						
		All NR	All 0 or 100	All 0, 50, or 100	Some M10	Some M5	Some 1-4 or 96-99	Some other
Based on the 12 questions asked in all waves								
2002	16032	0.022	0.101	0.101	0.392	0.320	0.054	0.011
2004	18250	0.015	0.062	0.084	0.418	0.353	0.056	0.013
2006	17191	0.027	0.072	0.077	0.409	0.336	0.065	0.014
2008	16060	0.021	0.068	0.063	0.417	0.340	0.072	0.018
2010	20400	0.010	0.053	0.050	0.426	0.350	0.092	0.020
2012	19360	0.015	0.051	0.058	0.445	0.328	0.083	0.020
2014	17647	0.012	0.065	0.062	0.458	0.295	0.090	0.018
Based on all questions asked in each wave								
2002	16032	0.014	0.023	0.039	0.324	0.459	0.119	0.022
2004	18250	0.010	0.019	0.032	0.337	0.467	0.108	0.026
2006	17191	0.025	0.019	0.023	0.263	0.513	0.117	0.039
2008	16060	0.021	0.025	0.019	0.290	0.511	0.101	0.033
2010	20400	0.009	0.029	0.022	0.316	0.442	0.144	0.038
2012	19360	0.014	0.027	0.021	0.317	0.443	0.139	0.038
2014	17647	0.012	0.026	0.022	0.329	0.427	0.142	0.042

NOTE: N = sample size, NR = nonresponse, M10 = multiple of 10 but not (0, 50, 100), M5 = multiple of 5 but not of 10. The following 12 questions were asked in all HRS waves between 2002 and 2014: P47: mutual fund increase in value; P28: live to be 75 or more; P29: live to be X or more; P5: live inheritance \geq \$10,000; P6: live inheritance \geq \$100,000; P59: live inheritance \geq \$500,000; P7: leave any inheritance; P16: work for pay in the future; P17: work full time after age 62; P18: work full time after age 65; P32: move to nursing home in 5 years; P20: finding a job in few months if unemployed.

the 2006 wave of the HRS hold throughout the seven waves between 2002 and 2014. However, these are aggregate patterns that may partly be susceptible to variation across wave in sample composition. To address this issue, we compute transition matrices of response tendencies across waves. Specifically, for each pair of waves indicated by column, Table 3 reports the fractions of respondents classified as belonging to any rounding category in the first wave who transitioned to the same rounding category in the second wave (1st row), who transitioned to a finer or coarser adjacent category (2nd row), and who transitioned to a more distant rounding category (3th row). The reported calculations use the twelve questions in common to the seven waves to classify respondents.

We find that between 0.406 and 0.436 of the respondents remain in the same rounding category across any pair of adjacent waves. Between 0.373 and 0.386 transition to an adjacent category. Thus, between 0.788 and 0.813 of the respondents transition to the same or an adjacent category. Even transitions between the first and last waves, with fourteen years separating them, display high

Table 3: Transitions of Response Tendencies across Waves

Transition waves:	2002 to 2004	2004 to 2006	2006 to 2008	2008 to 2010	2010 to 2012	2012 to 2014	2002 to 2014
Frequency (based on the 12 questions asked in all waves)							
% transitions to:							
same category	0.406	0.420	0.406	0.415	0.436	0.433	0.389
adjacent category	0.386	0.383	0.383	0.385	0.377	0.373	0.392
more distant category	0.209	0.197	0.212	0.201	0.187	0.194	0.218
N (100%)	14183	16126	15231	13732	18260	16923	8348
same or adjacent	0.792	0.803	0.788	0.800	0.813	0.806	0.782

NOTE: The percentages shown in the table are calculated from transition matrices of response tendencies defined in terms of the following categories: All NR, All (0, 100), All (0, 50, 100), Some M10, Some M5, Some 1-4 or 96-99, Some other. The following 12 questions were asked in all HRS waves between 2002 and 2014: P47: mutual fund increase in value; P28: live to be 75 or more; P29: live to be X or more; P5: live inheritance \geq \$10,000; P6: live inheritance \geq \$100,000; P59: live inheritance \geq \$500,000; P7: leave any inheritance; P16: work for pay in the future; P17: work full time after age 62; P18: work full time after age 65; P32: move to nursing home in 5 years; P20: finding a job in few months if unemployed.

persistence, with over 0.78 of the respondents transitioning to the same or an adjacent category.

The amount of temporal stability observed in Table 3 is remarkable. This is particularly true given the criteria used to classify respondents to this point. For example, consider a respondent whose most refined answer in 2002 is a multiple of 10 percent other than (0, 50, 100) and who is thus classified as “Some M10.” If, in 2004, the same respondent were observed to give a single answer that is a multiple of 5 percent but not of 10 percent, he would be now classified as “Some M5.”

2.2. Pooling Data across Waves to Probe More Deeply into Response Tendencies

With temporal stability established, we henceforth pool the HRS data across waves. This greatly increases the number of expectations responses observed per respondent, multiplying it sevenfold for respondents interviewed in all waves between 2002 and 2014. Across all questions and waves, the average number of responses per respondent is 106.8. By domain, this figure ranges from 19.1 for personal health to 66 for personal finances. With such rich respondent-specific data, we can probe more deeply into rounding practices than Manski and Molinari (2010) were able to with the 2006 wave alone.

To obtain further insight, we scrutinized the rounding behavior of 100 respondents drawn at random. We found two highly interesting patterns. First, a substantial fraction of respondents round more coarsely in the center of the 0-100 scale than in the tails. Second, respondents tend to use the percent-chance values 25 and 75 more than they do other values ending in 5. Supplementary Appendix 2.2 describes the analysis underlying these findings.

Our study of 100 randomly drawn respondents does not reveal how prevalent the discovered features are across the whole sample of HRS respondents. To answer this question, we now refine our earlier categorization of rounding patterns. We define the center (C) of the percent-chance scale to be values in the range 26-74 and the tails (T) to be values in the ranges 0-24 and 76-100. The values 25 and 75 form the boundary between the tail and center. We group responses into nine categories, defined by their presence in T or C and by the degree to which they are multiples of smaller numbers. The categories are: M1-T \equiv values in 1-24 or 76-99 that are not multiples of 5; M1-C \equiv values in 26-74 that are not multiples of 5; M5-T \equiv {5, 15, 85, 95}; M5-C \equiv {35, 45, 55, 65}; M10-T \equiv {10, 20, 80, 90}; M10-C \equiv {30, 40, 60, 70}; M25 \equiv {25, 75}; M100 \equiv {0, 100}; M50 \equiv {50}.

With this categorization, Table 4 shows the distribution of responses across respondents for each question asked in Section P between 2002 and 2014. The two main features detected by inspecting the random sample are decisively confirmed in the general HRS sample. Comparison of the frequencies of M25 responses (in column 5) with the frequencies of the remaining M5 responses (M5-C in column 9 and M5-T in column 8) reveals that the fraction of {25, 75} responses is always higher than the fraction of responses ending in 5 in the center of the scale; that is, responses in {35, 45, 55, 65}. For most questions across the three domains, the fraction of {25, 75} responses is higher than the fraction of responses ending in 5 in the tails of the scale; that is, responses in {5, 15, 85, 95}.

Even more striking is comparison of the frequencies of responses in the tails of the scale versus those in the center. The fractions of M10, M5, and M1 responses in the tails are higher than the corresponding fractions of M10, M5, and M1 responses in the center for nearly all questions in Table 4.

The only exceptions are questions P47 and P190, for which the fractions of M10-C responses are slightly higher than the fractions of M10-T responses.

3. Transforming Expectations Responses into Interval Data

The analysis of Section 2 reveals that HRS respondents differ systematically in their rounding practices, with a relatively small fraction habitually performing gross rounding and the majority sometimes giving more refined responses. We have established that the response tendencies are stable across waves. Furthermore, we have detected two patterns of responses that the earlier analysis by Manski and Molinari (2010) could not detect using only the 2006 data. One pattern is a relatively frequent use of 25 and 75 percent. The other is systematic use of more refined responses in the tails of the scale than in its center.

Table 4: Responses by Question and across Waves in the 2002-2014 HRS

Question: percent chance that...	N total obs.	Percentage of responses in:									
		NR	M50	M100	M25	M10 T	M10 C	M5 T	M5 C	M1 T	M1 C
Personal Health											
P19: Health limit work next 10 years	5475	0.044	0.311	0.153	0.087	0.217	0.144	0.031	0.007	0.005	0.001
P28: Live to be age 75 or more	56497	0.038	0.219	0.204	0.082	0.270	0.120	0.042	0.010	0.013	0.001
P29: Live to be age X or more	118404	0.050	0.211	0.191	0.075	0.236	0.156	0.049	0.013	0.018	0.001
P32: Move to nursing home in 5 y	74696	0.059	0.120	0.426	0.039	0.206	0.062	0.060	0.003	0.023	0.001
P103: Live independently at 75	7590	0.031	0.190	0.136	0.115	0.292	0.152	0.056	0.016	0.012	0.001
P104: Free of serious mental... at 75	7590	0.034	0.210	0.099	0.130	0.259	0.183	0.052	0.020	0.011	0.002
P106: Live independently at X	15291	0.060	0.219	0.144	0.100	0.234	0.166	0.046	0.015	0.015	0.001
P107: Free of serious think/reason...	33518	0.062	0.227	0.135	0.088	0.229	0.179	0.049	0.014	0.016	0.001
P108: Same health in 4 years	16253	0.048	0.226	0.151	0.097	0.263	0.151	0.044	0.009	0.010	0.001
P109: Worse health in 4 years	16232	0.069	0.228	0.146	0.077	0.272	0.143	0.043	0.008	0.014	0.001
General Economic Conditions											
P34: U.S. have economic depression	50661	0.069	0.234	0.148	0.083	0.228	0.170	0.041	0.014	0.011	0.001
P47: Mutual funds up /next y	105714	0.157	0.247	0.093	0.076	0.185	0.193	0.025	0.014	0.008	0.001
P110: SS in general will be worse	71770	0.054	0.212	0.200	0.087	0.235	0.151	0.035	0.011	0.014	0.001
P114: Mutual fund up /more than living	16680	0.281	0.182	0.096	0.063	0.178	0.157	0.026	0.010	0.006	0.001
P115: Mutual fund up 8% /more than...	16652	0.307	0.162	0.076	0.061	0.187	0.150	0.033	0.010	0.012	0.001
P116: Cost living up /more than 5%	32431	0.077	0.151	0.210	0.089	0.252	0.152	0.045	0.010	0.013	0.001
P150: Mutual funds up by 20/10/ X%	42092	0.034	0.156	0.090	0.070	0.314	0.237	0.063	0.017	0.018	0.002
P180: Mutual funds down by 20%	31658	0.019	0.179	0.098	0.061	0.318	0.225	0.064	0.017	0.016	0.002
P183: Medicare less generous in 10 y	36524	0.039	0.219	0.216	0.075	0.246	0.150	0.032	0.008	0.014	0.001
P190: Stock market up by next year	8615	0.077	0.335	0.090	0.058	0.185	0.202	0.026	0.011	0.016	0.001
P192: Stock market up by 20%	5430	0.021	0.151	0.108	0.054	0.342	0.199	0.084	0.012	0.028	0.001
P193: Stock market down by 20%	5306	0.013	0.183	0.115	0.048	0.314	0.210	0.076	0.012	0.026	0.002

Table 4 (Continued): Responses by Question and across Waves in the 2002-2014 HRS

Question: percent chance that...	N total obs.	Percentage of responses in:									
		NR	M50	M100	M25 C	M10 T	M10 C	M5 T	M5 C	M1 T	M1 C
Personal Finances											
P4: Income keep up inflation in 5 y	51559	0.066	0.196	0.226	0.069	0.249	0.136	0.036	0.007	0.015	0.001
P5: Leave inheritance ≥ \$10K	116769	0.046	0.083	0.518	0.028	0.228	0.051	0.028	0.001	0.017	0.000
P6: Leave inheritance ≥ \$100K	95625	0.014	0.100	0.490	0.037	0.228	0.072	0.035	0.002	0.022	0.000
P7: Leave any inheritance	19716	0.020	0.053	0.763	0.013	0.098	0.021	0.020	0.001	0.012	0.000
P8: Receive inheritance in 10 y	51559	0.032	0.043	0.755	0.016	0.091	0.024	0.023	0.001	0.014	0.000
P14: Lose job next year	32743	0.017	0.129	0.405	0.028	0.261	0.060	0.067	0.003	0.031	0.000
P15: Find job in few months/loss	32727	0.015	0.158	0.276	0.056	0.287	0.128	0.053	0.004	0.022	0.000
P16: Work for pay in the future	66855	0.018	0.055	0.672	0.021	0.139	0.037	0.035	0.001	0.021	0.000
P17: Work full time after age 62	36603	0.011	0.144	0.333	0.055	0.268	0.120	0.043	0.006	0.020	0.001
P18: Work full time after age 65	37062	0.011	0.144	0.280	0.058	0.282	0.130	0.057	0.008	0.028	0.001
P20: Find job in few months/unemployed	8206	0.012	0.211	0.184	0.061	0.277	0.174	0.050	0.012	0.019	0.001
P30: Give \$5K to others in 10 y	50528	0.024	0.120	0.505	0.050	0.187	0.065	0.035	0.002	0.011	0.000
P31: Receive \$5K... in 10 y	50528	0.023	0.047	0.674	0.020	0.143	0.026	0.047	0.001	0.019	0.000
P59: Leave inheritance ≥ \$500K	73872	0.011	0.090	0.490	0.034	0.216	0.073	0.046	0.003	0.037	0.000
P70: Med expenses use up savings	50478	0.060	0.141	0.316	0.060	0.246	0.109	0.048	0.006	0.014	0.000
P71: Give \$1K to others in 10 y	21024	0.007	0.097	0.551	0.044	0.186	0.060	0.041	0.002	0.013	0.000
P72: Give \$10K to others in 10 y	12904	0.011	0.212	0.322	0.072	0.219	0.124	0.026	0.006	0.007	0.001
P73: Give \$20K to others in 10 y	11155	0.011	0.152	0.334	0.061	0.265	0.100	0.057	0.005	0.015	0.000
P74: Receive \$2.5K... in 10 y	30644	0.004	0.021	0.723	0.019	0.134	0.023	0.053	0.001	0.022	0.000
P75: Receive \$1K... in 10 y	30397	0.003	0.042	0.686	0.024	0.141	0.031	0.051	0.001	0.021	0.000
P76: Receive \$10K... in 10 y	3270	0.015	0.243	0.321	0.052	0.198	0.134	0.022	0.009	0.006	0.001
P111: SS worse/current own benefits	51023	0.036	0.246	0.197	0.080	0.246	0.138	0.037	0.007	0.012	0.001
P112: SS worse/future own benefits	26753	0.020	0.205	0.186	0.085	0.255	0.179	0.040	0.014	0.014	0.001
P166: Home worth more next year	28067	0.030	0.202	0.165	0.045	0.361	0.146	0.033	0.005	0.011	0.001
P168: Home worth more/less by X	26394	0.035	0.112	0.259	0.029	0.348	0.120	0.070	0.004	0.024	0.000
P175: OP med exp ≥ \$1.5K next year	56760	0.031	0.143	0.340	0.051	0.261	0.109	0.043	0.004	0.017	0.000
P176: OP med exp ≥ \$500 next year	10962	0.017	0.114	0.642	0.025	0.126	0.043	0.020	0.001	0.012	0.000
P177: OP med exp ≥ \$3K next year	44022	0.012	0.132	0.235	0.058	0.318	0.126	0.082	0.006	0.033	0.000
P178: OP med exp ≥ \$8K next year	36369	0.009	0.079	0.260	0.037	0.327	0.092	0.120	0.005	0.071	0.000
P181: Any work after age 70	17057	0.010	0.118	0.374	0.042	0.259	0.101	0.058	0.005	0.034	0.000
P182: Work full time after age 70	10384	0.003	0.100	0.264	0.038	0.323	0.108	0.097	0.007	0.060	0.000

NOTE: M50 ≡ {50}, M100 ≡ {0, 100}, M25 ≡ {25, 75}, M10-T ≡ {10, 20, 80, 90}, M10-C ≡ {30, 40, 60, 70}, M5-T ≡ {5, 15, 85,

95}, M5-C ≡ {35, 45, 55, 65}, M1-T ≡ non-round values in 1-24 or 76-99, M1-C ≡ non-round values in 26-74.

Generalizing the inferential approach proposed by Manski and Molinari (2010), this section develops a new algorithm that uses the response tendency of a respondent to characterize rounding of responses to particular questions. The algorithm classifies each respondent into one of a set of mutually exclusive and exhaustive rounding types and transforms each original point response into an interval where the true latent belief is deemed to lie. With this accomplished, statistical analyses of expectations may proceed using the intervals thus constructed in place of the observed point responses.

Our algorithm relies on considerably weaker and more credible assumptions than inference that uses expectations reports at face value. Nevertheless, we cannot be certain that the intervals we construct are

accurate. The algorithm is subject to two potential forms of misclassification. First, a given survey response may be less rounded than the interval assigned by the algorithm; that is, the actual rounding interval may be a subset of the algorithm's interval. Then our use of the data is correct, but it yields inference that is less sharp than it would be if the true degree of rounding were known. Second, the actual rounding interval may not be completely contained in the algorithm's interval. Then the actual belief may lie outside our interval, making our use of the data incorrect. Still, use of the algorithm substantially lowers the risk of the latter type of error relative to the standard approach that takes survey responses at face value.

The new algorithm embodies the data patterns documented in Section 2. Section 3.1 describes the determination of a respondent's rounding type. Section 3.2 presents the empirical distribution of the respondents' inferred rounding types and studies how rounding tendencies vary with observed characteristics of the respondents. Section 3.3 explains how a respondent's point response to a specific question and the respondent's inferred rounding type are used to construct the interval associated with the observed point response.

3.1. Determination of Respondent Rounding Types

Based on the evidence in Section 2, we allow a respondent's rounding type to vary across question domains and between the tails and center of the measurement scale. Thus, within a specific domain of questions, a respondent's rounding type is a bivariate vector of the form (tail, center) rounding, partitioning the 0-100 scale into two symmetric tails (0-24 and 76-100) and a center (25-75). We believe that our specific choice of tails and center reasonably reflects the empirical patterns of HRS responses, but judgments need not be uniform. The algorithm can be easily adapted to different definitions of tails and center or extended to accommodate finer partitions of the 0-100 scale (e.g., outer tails, inner tails, center).

The new algorithm refines the earlier one posed by Manski and Molinari (2010) in multiple ways. One refinement is to separate tail from center rounding. Another is to classify persons who only use the response values (0, 25, 50, 75, 100) as rounding to the nearest 25 percent rather than to the nearest 5 percent. A further difference between the two algorithms is that here we use a tighter criterion for assignment of a person to a more refined rounding type.

To explain the tighter criterion, consider categorization of a respondent as one who rounds to the nearest 10 percent (or to a more refined degree). Manski and Molinari assigned a respondent to this rounding type if all responses are multiples of 10 and at least one response is not a value in (0, 50, 100). We use here a tighter criterion that requires observation of at least two responses that are multiples of 10 other than (0, 50, 100), of which one must be in the domain under consideration and the other may be in a different domain and may also be a less rounded response; that is, a value that is not a multiple of 10.

Adding the new requirement reflects our desire for further credibility when assigning a person to a more refined rounding type. We want enhanced credibility because misclassification into an overly refined rounding category yields an inferential error, as the person's latent beliefs may not entirely lie within the overly refined interval. Misclassification of a person into a rounding category less refined than their actual one does not yield an inferential error, as the less refined interval includes the actual one as a subset.

The main criteria for classification of respondents are as follows:

- **Center rounding type** Define x_n in $\{1, 5, 10, 50\}$, with $n = 1, \dots, 4$. Respondent j is classified as rounding to the nearest x_n percent in the center within question domain l if one of the following two conditions holds: (i) they are observed to give at least two answers in the center that are multiples of x_n percent but not of $x_{n'}$ for any $n' < n$ within domain l ; or (ii) they are observed to give one answer in the center that is a multiple of x_n percent (but not of $x_{n'}$ for any $n' < n$) within domain l AND at least one answer in the center that is a multiple of $x_{n'}$ for any $n' \leq n$ within a second domain l' distinct from l .

- **Tail rounding type** Respondent j is classified as rounding to the nearest x_n percent in the tails within question domain l if one of the following two conditions holds: (i) they are observed to give at least two answers in the tails that are multiples of x_n percent but not of $x_{n'}$ for any $n' < n$ within domain l ; or (ii) they are observed to give one answer in the tails that is a multiple of x_n percent (but not of $x_{n'}$ for any $n' < n$) within domain l AND at least one answer in the tails OR center that is a multiple of $x_{n'}$ for any $n' \leq n$ within a second domain l' distinct from l .

To illustrate, consider a respondent who has answered four expectations questions in the domain of personal finances, either within the same wave or over multiple waves. Two of the observed responses belong to the tails, {5, 85}, and two to the center, {30, 60}. As the set of responses includes two multiples of 5 percent in the tails and two multiples of 10 percent in the center, our algorithm classifies this respondent as one rounding to the nearest 5 percent, *or to a finer degree*, in the tails ($\mathcal{M}5\text{-T}$) and to the nearest 10 percent, *or to a finer degree*, in the center ($\mathcal{M}10\text{-C}$).

In this example, our algorithm reaches conclusions that differ in important ways from those of the Manski and Molinari (2010) algorithm. Application of that algorithm would classify this respondent as rounding to the nearest 5 percent, *or to a finer degree*, everywhere within the personal finances domain. Equivalently, given that the Manski-Molinari algorithm does not allow for differential rounding between tails and center, it would classify the hypothesized respondent as ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$). Using our algorithm, the evidence generated by the respondent's response pattern across questions classifies the respondent as ($\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-C}$), with a less refined rounding type in the center. This difference reflects our new finding that respondents tend to round more coarsely in the center than in the tails, which could not have been reached using a single wave of the HRS data.

The Supplementary Appendix 3.1 provides additional and more complex examples. It also presents the complete algorithm in a formal and compact way.

3.2. Empirical Distribution of Rounding Types and Association with Observable Characteristics

We apply the algorithm to all HRS respondents who responded to at least one expectations question in any question domain and in any wave between 2002 and 2014. Table 5 reports the empirical distribution of rounding types for each domain of questions. Depending on the domain, between 40.40% and 61.03% of respondents are inferred to apply finer rounding in the tails than in the center. Between 28.49% and 38.73% of respondents apply the same degree of rounding in the tails and in the center. Between 2.90% and 6.71% of respondents apply coarser rounding in the tails than in the center.

The rounding type of a small minority of respondents could not be determined either in the tails or in the center or both. Most undetermined cases occur when, for a given respondent, we do not observe any answer in the relevant domain and scale segment. Among respondents for whom we observe at least one answer in the relevant domain and scale segment, all cases of undetermined tail rounding type disappear and only a few cases of undetermined center rounding type remain. The latter are respondents for whom we only observe one answer in the center in the relevant domain and no answers in the center in the remaining two domains.

We now investigate how rounding types vary with observable respondent characteristics. This exercise sheds some light on the heterogeneity of respondents' tendencies to round. In addition, it may inform researchers who analyze survey expectations in datasets other than the HRS. These researchers may know respondents' characteristics in their data sets, but not have enough expectations data to apply our proposed approach directly.

In principle, nonparametric regression of rounding type on respondent characteristics would give the most complete information on observable heterogeneity in tendencies to round. However, we distinguish twenty rounding types (4 tail categories times 5 center categories) and observe multiple respondent characteristics. This makes nonparametric regression too cumbersome to be appealing. A pragmatic

Table 5: Distribution of Rounding Types by Domain

Rounding Type	Percent Personal Health	Percent Personal Finances	Percent General Economic Conditions
(M1-T, M1-C)	0.17	0.33	0.26
(M1-T, M5-C)	1.07	3.03	1.22
(M1-T, M10-C)	6.08	15.84	5.73
(M1-T, M25)	1.33	1.72	0.80
(M1-T, M50)	1.27	1.31	0.86
(M1-T, None/Undet.)	1.02	0.50	0.42
(M5-T, M1-C)	0.07	0.08	0.11
(M5-T, M5-C)	2.60	2.97	3.65
(M5-T, M10-C)	16.05	23.47	16.98
(M5-T, M25)	3.20	2.95	2.29
(M5-T, M50)	2.53	1.75	1.35
(M5-T, None/Undet.)	1.39	0.53	0.55
(M10-T, M1-C)	0.13	0	0.16
(M10-T, M5-C)	1.84	0.73	2.47
(M10-T, M10-C)	25.92	22.75	32.50
(M10-T, M25)	5.91	5.09	5.24
(M10-T, M50)	7.98	5.88	5.93
(M10-T, None/Undet.)	4.35	2.36	2.70
(M100, M1-C)	0	0	0.01
(M100, M5-C)	0.16	0.03	0.14
(M100, M10-C)	2.89	1.04	1.96
(M100, M25)	1.62	1.01	1.08
(M100, M50)	3.90	2.45	2.32
(M100, None/Undet.)	4.74	3.42	2.47
(None/Undet., M1-C)	0.01	0	0.01
(None/Undet., M5-C)	0.20	0.01	0.24
(None/Undet., M10-C)	1.27	0.01	2.50
(None/Undet., M25)	0.47	0.00	0.92
(None/Undet., M50)	0.92	0	2.06
(None/Undet., None/Undet.)	0.91	0.75	3.06
Total	100	100	100
Sample size	28044	28252	28172
Tails finer than center	45.42	61.03	40.40
Tails same as center	32.60	28.49	38.73
<i>Tails coarser than center</i>	<i>6.71</i>	<i>2.90</i>	<i>5.94</i>
No/Undet. T and/or C	15.27	7.58	14.93

approach is to summarize the data using parametric bivariate ordered probit regression, which embodies the basic ordinal property that our rounding categories display across different degrees of rounding.

Table 6 presents estimated coefficients of three bivariate ordered probit regressions, one per question domain. The outcome variables are the respondent's bivariate vectors of tail and center rounding categories in each domain. As predictors, we use binary variables for respondent's gender (male, with

female omitted), educational attainment (high school, some college, bachelor, and graduate, with less than high school omitted), and race (black and other, with white omitted).

We also include information on individual's age and cognitive functioning. While these variables are time-varying for each respondent, our analysis in Section 2.1.2 and 2.2 supports treating respondent rounding behavior as fixed over time. We therefore account for age variation across respondents by incorporating in our bivariate ordered probit regressions an indicator of whether each respondent's cross-wave average age lies in the categories 60-69, 70-79, and 80+ years, with 50-59 the omitted category. We account for variation in cognitive functioning across respondents by including each respondent's cross-wave average cognitive score. See Fisher et al. (2012) and Crimmins et al. (2011) for a description and an empirical assessment of the HRS cognitive measures.

The cognitive score has a range of 0-35. In our data, the respondent-specific cross-wave average cognitive score has a mean of 23 and a standard deviation of 4.11 across respondents. The respondent-specific cross-wave standard deviation in cognitive score has a mean of 2.9 across respondents. The fact that the standard deviation of the cross-wave average score is larger than the average cross-wave standard deviation in the score lessens our concerns for using a time-fixed measure of cognitive functioning in our bivariate ordered probit regressions. Nonetheless, the time variation in cognitive score and its association with rounding warrant study in future research.

The model permits the error terms of the latent variables underlying the inferred tail and center rounding categories to be correlated with each other. The correlation parameter, ρ , is estimated along with the other coefficients. The rounding categories are ordered from least coarse to most coarse. Thus, positive associations indicate a tendency to round more coarsely.

Formally, within each domain, the bivariate ordered probit model specifies two seemingly unrelated latent equations for the respondent's tendency to round, one for the tail and one for the center, where $y^{*,C} = X\beta^C + \varepsilon^C$ and $y^{*,T} = X\beta^T + \varepsilon^T$. The pair $(\varepsilon^C, \varepsilon^T)$ is assumed to have the standardized bivariate normal distribution with means zero, both variances equal to one, and correlation ρ .

Table 6: Bivariate Ordered Probit Model Predicting Rounding Type

	Personal Health		Personal Finances		Gen. Econ. Conditions	
	Tail Type	Center Type	Tail Type	Center Type	Tail Type	Center Type
Male	0.0047 (0.0149)	-0.0497 (0.0155)	-0.0032 (0.0142)	-0.0154 (0.0153)	-0.0070 (0.0151)	-0.0693 (0.0157)
Age 60-69	-0.1961 (0.0180)	-0.1436 (0.0194)	-0.0116 (0.0174)	0.0145 (0.0189)	-0.1090 (0.0185)	-0.1049 (0.0195)
Age 70-79	-0.1639 (0.0199)	0.0481 (0.0206)	0.1466 (0.0189)	0.1987 (0.0204)	-0.0941 (0.0199)	0.0232 (0.0208)
Age 80+	0.1092 (0.0266)	0.4465 (0.0261)	0.4934 (0.0246)	0.5658 (0.0258)	0.1718 (0.0266)	0.3209 (0.0266)
High school	-0.0842 (0.0224)	-0.0864 (0.0221)	-0.1277 (0.0208)	-0.1579 (0.0219)	-0.0614 (0.0226)	-0.1115 (0.0227)
Some college	-0.0642 (0.0362)	-0.0758 (0.0379)	-0.1688 (0.0342)	-0.1948 (0.0372)	-0.0588 (0.0364)	-0.1487 (0.0389)
Bachelor	-0.2027 (0.0288)	-0.2432 (0.0301)	-0.2677 (0.0277)	-0.3073 (0.0296)	-0.1726 (0.0292)	-0.2692 (0.0305)
Graduate	-0.2818 (0.0319)	-0.3658 (0.0337)	-0.3367 (0.0307)	-0.3549 (0.0332)	-0.2438 (0.0320)	-0.3454 (0.0341)
Black	0.0188 (0.0220)	0.1148 (0.0226)	-0.1507 (0.0203)	-0.0798 (0.0220)	-0.0562 (0.0219)	-0.0456 (0.0228)
Other race	0.1136 (0.0303)	0.1374 (0.0322)	0.0604 (0.0289)	0.0173 (0.0310)	0.0887 (0.0314)	0.0477 (0.0322)
Avg. Cog.	-0.0261 (0.0022)	-0.0339 (0.0023)	-0.0368 (0.0020)	-0.0373 (0.0022)	-0.0202 (0.0022)	-0.0370 (0.0023)
Rho	0.2595 (0.0081)		0.3848 (0.0087)		0.2897 (0.0093)	
N	22,447		24,541		22,593	

NOTES: (i) Respondents with undetermined tail or center rounding type are excluded from this analysis. (ii) Predictors are dummies for gender, age (averaged across waves), education, and race, plus average cognition score across waves. (iii) Omitted dummies are ‘Female,’ ‘Age in 50-59,’ ‘No degree,’ and ‘White.’ (iv) ‘Rho’ is the parameter capturing the correlation between the error terms of the tail and center latent equations. (v) Standard errors are in parentheses.

For each $k = C, T$, the model assumes that respondent rounds to multiples of 1 if $y^{*,k} < \delta_1^k$, to multiples of 5 if $\delta_1^k \leq y^{*,k} < \delta_2^k$, and so on, for a total of four categories in the tails (M1-T, M5-T, M10-

T, M100) and five categories in the center (M1-C, M5-C, M10-C, M25, M50). It is then possible to obtain the likelihood function for the rounding types. For $l = 1, \dots, 5$ and $m = 1, \dots, 4$, the probability that a person has rounding type (l, m) is

$$\begin{aligned} \Pr(y_i^C = l, y_i^T = m) = & \Phi\left((\delta_l^C - X\beta^C), (\delta_m^T - X\beta^T), \rho\right) - \Phi\left((\delta_{l-1}^C - X\beta^C), (\delta_m^T - X\beta^T), \rho\right) \\ & - \Phi\left((\delta_l^C - X\beta^C), (\delta_{m-1}^T - X\beta^T), \rho\right) + \Phi\left((\delta_{l-1}^C - X\beta^C), (\delta_{m-1}^T - X\beta^T), \rho\right), \end{aligned}$$

where Φ is the standardized bivariate normal distribution function with mean zero, both variances equal to one, and correlation ρ .

We estimate the parameters by maximum likelihood using the Stata package described in Sajaia (2008). The bivariate probit model and other multivariate discrete outcome models are discussed, for example, in Amemiya (1981). Estimated coefficients with standard errors are reported in Table 6. Table 7 reports predicted probabilities of selected tail and center rounding types for persons with specified covariate values.

We find that higher levels of educational attainment and of person-specific average (cross-wave) cognitive score are associated with a tendency to give more refined responses across all scale segments and question domains. The patterns for the other predictors are more varied.

For example, respondents in the oldest age category (80+) tend to give more rounded responses than respondents belonging to the youngest one (50-59) across all scale segments and questions domains. On the other hand, respondents in the two intermediate age groups (i.e., 60-69 and 70-79) belong to rounding categories that may be more refined, coarser, or statistically indistinguishable from those characterizing younger respondents, depending on the specific domain or scale segment.

A potential interpretation of the observed age patterns is that individuals belonging to the intermediate age groups may have more direct experience and hence better knowledge of the topics covered by the questions than younger respondents, generating more refined responses among the middle groups. On the other hand, individuals of older age might already, on average, have lower

cognitive functioning, leading to coarser responses. This pattern, however, continues to hold after conditioning on the respondent's average (across waves) cognitive score. Parameter estimates for a specification without cognitive score are shown in the Supplementary Appendix.

Male respondents tend to round more coarsely than female respondents in the personal health and personal finances domains, but only in the tails. On the other hand, male respondents tend to round less coarsely than women respondents in the center in the domain of general economic conditions. While respondents belonging to the residual race category (including Hispanic, Asian, and Pacific Islander) tend to round more coarsely than white respondents, the differential rounding tendencies of black respondents relative to white respondents vary across question domains and scale segments.

The large, positive, and statistically significant estimates of the correlation parameter ρ reveal that rounding tendencies are positively correlated across scale segments. Hence, respondents who give coarser responses in the tails are more likely to do so in the center.

3.3. Using Survey Responses and Rounding Types to Form Expectations Intervals

As we have shown, rounding varies substantially across respondents and the tendency to round is systematically associated with respondents' observable characteristics. Yet, it is natural to wonder the extent to which failing to account for rounding might lead to inaccurate conclusions when analyzing data. A simple numerical illustration pertaining to the analysis of the effect of longevity expectations on hours worked shows that ignoring rounding may yield highly inaccurate conclusions.

Suppose that two respondents both round their response to the longevity expectation question to the closest multiple of 25. Suppose that one respondent views their probability to live past age 75 to be forty percent while the other respondent views it to be sixty percent, with the latter working significantly more hours as a consequence. With rounding, both respondents report their probability to live past age 75 as fifty percent. The notable difference in hours-worked outcomes with apparently the same

expectations may be misinterpreted as caused by unobserved heterogeneity in labor-leisure preferences, when the actual cause is different longevity expectations.

Table 7. Predicted Probabilities of Rounding Types for Selected Covariate Profiles

Panel A. Personal Health –(Female, White, Bachelor Degree) Respondents								
Average Cognition Across Waves								
		Mean -1 SD	Mean	Mean +1 SD				
		Prob. of Type ($\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-C}$)			Prob. of Type ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)			
Average Age Across Waves	50-59	0.1846	0.2036	0.2198	50-59	0.3118	0.3123	0.3064
	60-69	0.2136	0.2289	0.2402	60-69	0.2971	0.2897	0.2767
	70-79	0.2008	0.2194	0.2347	70-79	0.2784	0.2768	0.2696
	80+	0.1433	0.1658	0.1878	80+	0.2494	0.2623	0.2701
			Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}25\text{-C}$)			Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}50\text{-C}$)		
	50-59	0.0199	0.0157	0.0121	50-59	0.0312	0.0221	0.0153
	60-69	0.0135	0.0103	0.0077	60-69	0.0192	0.0133	0.0090
	70-79	0.0151	0.0119	0.0091	70-79	0.0256	0.0180	0.0124
80+	0.0247	0.0207	0.0170	80+	0.0583	0.0433	0.0316	
Panel B. Personal Finances –(Female, White, Bachelor Degree) Respondents								
Average Cognition Across Waves								
		Mean -1 SD	Mean	Mean +1 SD				
		Prob. of Type ($\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-C}$)			Prob. of Type ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)			
Average Age Across Waves	50-59	0.2634	0.2724	0.2731	50-59	0.2483	0.2248	0.1976
	60-69	0.2632	0.2722	0.2728	60-69	0.2440	0.2209	0.1942
	70-79	0.2453	0.2621	0.2715	70-79	0.2583	0.2415	0.2191
	80+	0.1887	0.2162	0.2402	80+	0.2665	0.2665	0.2586
			Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}25\text{-C}$)			Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}50\text{-C}$)		
	50-59	0.0072	0.0049	0.0032	50-59	0.0107	0.0065	0.0038
	60-69	0.0071	0.0048	0.0031	60-69	0.0107	0.0065	0.0038
	70-79	0.0102	0.0071	0.0048	70-79	0.0175	0.0110	0.0067
80+	0.0196	0.0149	0.0109	80+	0.0443	0.0298	0.0194	

Panel C. General Economic Conditions –(Female, White, Bachelor Degree) Respondents

		Average Cognition Across Waves							
		Mean -1 SD	Mean	Mean +1 SD		Mean -1 SD	Mean	Mean +1 SD	
		Prob. of Type ($\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-C}$)			Prob. of Type ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)				
Average Age Across Waves	50-59	0.2031	0.2170	0.2273	50-59	0.3733	0.3724	0.3647	
	60-69	0.2201	0.2315	0.2387	60-69	0.3625	0.3562	0.3435	
	70-79	0.2157	0.2298	0.2401	70-79	0.3509	0.3495	0.3415	
	80+	0.1671	0.1858	0.2027	80+	0.3524	0.3658	0.3725	
		Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}25\text{-C}$)			Prob. of Type ($\mathcal{M}100\text{-T}$, $\mathcal{M}50\text{-C}$)				
	50-59	0.0111	0.0088	0.0068	50-59	0.0165	0.0116	0.0080	
	60-69	0.0086	0.0067	0.0051	60-69	0.0119	0.0082	0.0056	
	70-79	0.0094	0.0074	0.0051	70-79	0.0145	0.0101	0.0070	
	80+	0.0166	0.0138	0.0112	80+	0.0317	0.0233	0.0167	

NOTES: (i) ($\mathcal{M} 5\text{-T}$, $\mathcal{M}10\text{-C}$) denotes rounding to the nearest 5 percent or a finer degree in the tails and rounding to the nearest 10 percent or a finer degree in the center. ($\mathcal{M} 10\text{-T}$, $\mathcal{M}10\text{-C}$) denotes rounding to the nearest 10 percent or a finer degree in both the tails and the center. ($\mathcal{M} 100\text{-T}$, $\mathcal{M}25\text{-C}$) denotes rounding to any degree in the tails and to the nearest 25 percent or a finer degree in the center. ($\mathcal{M} 100\text{-T}$, $\mathcal{M}50\text{-C}$) denotes rounding to any degree in both the tails and the center. (ii) Predicted probabilities are evaluated at the mean value of average cognition across waves (denoted Mean), at the mean minus one standard deviation value of average cognition across waves (denoted Mean – 1 SD), and as the mean plus one standard deviation Value of average cognition across waves. Predicted probabilities are evaluated at average age across waves falling in each of the categories 50-59, 60-69, 70-79, and 80+.

Next, consider a scenario where the first respondent views their probability to live past age 75 to be thirty-seven percent while the other respondent views it to be thirty-eight percent, with the latter working slightly more hours. With rounding, the first respondent reports a probability to live past age 75 of twenty-five percent, and the second respondent reports fifty percent. The slight difference in outcomes with an apparent large difference in expectations may be misinterpreted as evidence of minimal effect of expectations on labor supply.

These examples, while stylized, illustrate that ignoring rounding might lead to “boundary mistakes;” that is, to significantly underestimating or overestimating an effect of interest. We therefore propose an algorithm that uses the information contained in each respondent’s reporting behavior across the survey, as analyzed in the preceding sections, to transform observed percent-chance point reports into intervals.

Here we present the construction of interval data within the context of the illustration introduced in Section 3.1. The Supplementary Appendix 3.3 discusses more complex cases, presents the complete

algorithm formally, and reports the distributions of interval width for the responses given to specific questions.

In the example introduced in Section 3.1, the respondent is observed to answer with {5, 30, 60, 85} to four expectations questions concerning personal finances and is classified to be of rounding type ($\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-C}$) in that domain. Because the respondent is classified to round to the nearest 5 percent in the tails, the algorithm assigns to each of the respondent's point responses in the tails an interval of width 5 centered around the point response. Specifically, the algorithm assigns the interval [2.5, 7.5] to response 5 (i.e., 5 ∓ 2.5) and the interval [82.5, 87.5] to response 85 (i.e., 85 ∓ 2.5). Similarly, as the respondent is classified to round to the nearest 10 percent in the center, the algorithm assigns interval [25, 35] to the 30 percent response (i.e., 30 ∓ 5) and the interval [55, 65] to the 60 percent response (i.e., 60 ∓ 5).

In general, construction of intervals around point responses near the thresholds which separate the center from the tails---that is, near 25 and 75 percent---requires specific "boundary conditions." Such conditions are not binding in this example. We explain them in the Appendix.

By construction, each interval contains the point response because the former is centered around the latter. Moreover, the interval is assumed to cover the unobserved true latent belief with certainty. However, no assumption is made about the location of the true latent belief inside the interval.

Our algorithm relies on considerably weaker and hence more credible assumptions than inference using expectations reports at face value. At the opposite extreme, one could be ultraconservative, maintaining that each point response is consistent with any amount of rounding. One would then replace all reported expectations with a [0, 100] interval. Obviously, doing this empties the data of any information content.

Our choice of assumptions used to identify respondents' rounding types and bound their unobserved true beliefs strikes a balance between those two extremes and is informed by the respondents' response patterns across HRS questions and waves, which we have documented in this paper. A researcher

entertaining a different set of assumptions about how survey respondents round their expectations reports could easily apply our framework by simply replacing our assumptions with theirs. In general, stronger and/or more numerous assumptions will yield (weakly) narrower intervals.

4. Conclusion

We have studied the nature of rounding in numerical reports of probabilistic expectations, a type of survey measure that has become widely used in empirical economic analysis of individual and household decisions under uncertainty. Our analysis of the responses to all expectations questions asked in the HRS core questionnaire between 2002 and 2014 confirms earlier findings based on analysis of the 2006 waves of data and establishes important new findings. We propose an inferential approach that interprets expectations reports as interval data and that explicitly incorporates the documented patterns of responses across waves, question domains, and location within the measurement scale.

The main tenet of the analysis is that observed response patterns across questions and waves carry information about individual respondents' rounding practices. Observed response patterns, however, do not reveal whether individual respondents round their reports to simplify communication or to convey partial knowledge. Consistent with the first interpretation, we have assumed that respondents have well-formed latent point beliefs. If instead the relevant latent objects were sets or ranges of beliefs, the algorithm would still work as intended as long as the algorithm's interval completely includes the latent interval.

If respondents round to convey partial knowledge about the likelihood of future events of the kind HRS expectations questions refer to, it would be better to allow them to express their ambiguity directly. This could be achieved by allowing respondents to give either a single percent-chance value or a range as they see fit. Then range measures of subjective expectations may be analyzed using existing econometric tools for interval data. See Manski and Molinari (2010) and Giustinelli and Pavoni (2017) for exploratory data collection and analysis of this type.

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SUPPLEMENTARY MATERIAL

Supplementary Appendix to Section 2

SA2 Exploratory Analysis of Response Patterns Across Questions and Waves in the HRS

Since 2002 the HRS has devoted an entire section of its core questionnaire to measurement of respondents' expectations in the domains of personal health, personal finances, and general economic conditions. Figure 1 in the main text shows the list of expectations questions asked in Section P of the HRS core questionnaire between 2002 and 2014 organized by domain.

As documented in Table S1, the number of responses varies across expectations questions. This occurs for several reasons. First, questions have been added and removed over time.

Second, the HRS makes extensive use of skip sequencing. In particular, whether a specific question is asked or not to a certain respondent may depend on the previous answers given by the respondent and on whether the event specified by the question is relevant to the respondent. For example, respondents who are older than 62 are not asked their subjective probability of working full-time past 62. Similarly, respondents who are older than 75 are not asked their subjective probability of living past 75, and so on. Moreover, respondents who respond 'Don't know' (DK) or 'Refuse' (RF) to three consecutive expectations questions are skipped to the next section.

Third, sample composition may change over time. In particular, the HRS sample has been augmented with new cohorts of respondents who joined the study in specific waves. On the other hand, respondents may exit the study due to attrition or death.

Table S1: Number of Waves, Observations, and Respondents by Question

Question: percent chance that...	N waves asked	N total obs. (across waves)	N Rs asked (across waves)
Personal Health			
P19: Health limit work next 10 years	1	5,475	5,475
P28: Live to be age 75 or more	7	56,497	17,868
P29: Live to be age X or more	7	118,404	27,638
P32: Move to nursing home in 5 y	7	74,696	26,095
P103: Live independently at 75	2	7,590	5,693
P104: Free of serious mental... at 75	2	7,590	5,693
P106: Live independently at X	2	15,291	13,228
P107: Free of serious think/reason...	4	33,518	15,599
P108: Same health in 4 years	2	16,253	12,509
P109: Worse health in 4 years	2	16,232	12,512
General Economic Conditions			
P34: U.S. have economic depression	4	50,661	19,598
P47: Mutual funds up /next y	7	105,714	27,279
P110: SS in general will be worse	5	71,770	24,868
P114: Mutual fund up /more than living	1	16,680	16,680
P115: Mutual fund up 8% /more than...	1	16,652	16,652
P116: Cost living up /more than 5%	2	32,431	17,781
P150: Mutual funds up by 20/10/ X%	5	42,092	20,051
P180: Mutual funds down by 20%	3	31,658	17,826
P183: Medicare less generous in 10 y	2	36,524	19,938
P190: Stock market up by next year	1	8,615	8,615
P192: Stock market up by 20%	1	5,430	5,430
P193: Stock market down by 20%	1	5,306	5,306

NOTE: N of total observations includes all answers by any respondent in any wave to the corresponding question, including don't know/refuse. The set of questions each respondent is asked and observed to answer may vary across waves as a function of aspects of survey design such as the decision of designers to introduce new questions or to eliminate existing ones, the respondent's time-varying characteristics used for skip logic, etc. Additionally, new cohorts of respondents have been added over time, while a portion of respondents from the initial cohorts have left the study due to death or other reasons.

Table S1 (Continued): Number of Waves, Observations, and Respondents by Question

Question: percent chance that...	N waves asked	N total obs. (across waves)	N Rs asked (across waves)
Personal Finances			
P4: Income keep up inflation in 5 y	3	51,559	20,852
P5: Leave inheritance \geq \$10K	7	116,769	28,252
P6: Leave inheritance \geq \$100K	7	95,625	25,360
P7: Leave any inheritance	7	19,716	9,426
P8: Receive inheritance in 10 y	3	51,559	20,852
P14: Lose job next year	6	32,743	12,220
P15: Find job in few months/loss	6	32,727	12,220
P16: Work for pay in the future	7	66,855	20,902
P17: Work full time after age 62	7	36,603	13,325
P18: Work full time after age 65	7	37,062	13,158
P20: Find job in few months/unemployed	7	8,206	5,182
P30: Give \$5K to others in 10 y	3	50,528	20,633
P31: Receive \$5K... in 10 y	3	50,528	20,633
P59: Leave inheritance \geq \$500K	7	73,872	21,339
P70: Med expenses use up savings	3	50,478	19,583
P71: Give \$1K to others in 10 y	2	21,024	13,717
P72: Give \$10K to others in 10 y	2	12,904	8,981
P73: Give \$20K to others in 10 y	2	11,155	7,838
P74: Receive \$2.5K... in 10 y	2	30,644	18,014
P75: Receive \$1K... in 10 y	2	30,397	17,924
P76: Receive \$10K... in 10 y	2	3,270	2,786
P111: SS worse/current own benefits	5	51,023	16,477
P112: SS worse/future own benefits	5	26,753	10,599
P166: Home worth more next year	3	28,067	11,422
P168: Home worth more/less by X	3	26,394	11,168
P175: OP med exp \geq \$1.5K next year	3	56,760	21,771
P176: OP med exp \geq \$500 next year	3	10,962	7,482
P177: OP med exp \geq \$3K next year	3	44,022	19,526
P178: OP med exp \geq \$8K next year	3	36,369	17,453
P181: Any work after age 70	2	17,057	9,915
P182: Work full time after age 70	2	10,384	6,856

NOTE: N of total observations includes all answers by any respondent in any wave to the corresponding question, including don't know/refuse. The set of questions each respondent is asked and observed to answer may vary across waves as a function of aspects of survey design such as the decision of designers to introduce new questions or to eliminate existing ones, the respondent's time-varying characteristics used for skip logic, etc. Additionally, new cohorts of respondents have been added over time, while a portion of respondents from the initial cohorts have left the study due to death or other reasons.

SA2.1 Temporal Stability of Response Tendencies

We start by investigating the empirical distributions of responses to each of the questions listed in Table S1 above separately for each wave between 2002 and 2014. To reduce length, in Table S2 we present the response patterns for a subset of 9 questions in different domains. We focus on questions that were asked in at least 4 waves.

For each of the 9 questions selected and for each of the waves in which those questions were posed, the columns of Table S2 show the fractions of respondents who do not respond (NR), who respond 0, 50, or 100, who respond with any other multiple of 10 percent (i.e., in $M10 = \{10, 20, 30, 40, 60, 70, 80, 90\}$), who respond with any multiple of 5 percent that is not a multiple of 10 percent (i.e., in $M5 = \{5, 15, 25, 35, 45, 55, 65, 75, 85, 95\}$), and who respond in two ranges of multiples of 1 percent that are not multiples of 5 or 10 percent (i.e., in 1-4 and in 96-99). In the column “Other” we report the residual fraction of respondents who respond with a multiple of 1 percent that does not lie in the 1-4 or 96-99 range.

By and large, HRS expectations questions feature low rates of item nonresponse in the personal health and personal finances domains (below 0.05) and higher rates of item nonresponse in the general economic conditions domain (typically between 0.05 and 0.10), with peaks of 0.25-0.30 rates of nonresponse to specific questions eliciting respondents’ expectations of future performance of the stock market (e.g., see question P47 in Table S2).

The rates of 0, 50, and 100 vary across questions. For example, the fraction of 50 percent responses tends to be higher in the general economic conditions domain, where they range between 0.20 and 0.30, than in the remaining domains. Among the 9 questions shown in Table S2, the fractions of 0 and 100 are highest for specific questions belonging to the personal finances and personal health domains. For example, the fraction of 0 ranges between 0.35 and 0.50 for P14 (probability of losing own job during the next year) and for P32 (probability of moving to a nursing home in 5 years); whereas the fraction of

100 percent is highest for P5 (probability of leaving an inheritance of at least \$10K), ranging between 0.324 and 0.447 across waves.

The high rates of 0, 50, and 100 in response to specific questions do not suggest any particular degree of rounding. For example, responses of 50 percent are consistent with any degree of rounding. Respondents who answered P47 (probability that the mutual fund will increase in value in the next year) might genuinely believe that it is equally likely that the stock market will increase or decrease in value in a 1-year time; they might mean that the chances that the stock market will go up are between 40 and 60 percent; or they might have epistemic uncertainty, using 50 percent to indicate a complete lack of knowledge.

Consistently high fractions of responses across questions and waves are multiples of 10 percent and, to a lesser extent, of 5 percent. For the 9 questions shown in Table S2, the fractions of M10 and M5 responses range respectively between 0.20 and 0.45 and between 0.05 and 0.15 across questions and waves. On the other hand, the fractions of cases where the response takes the value 1-4 or 96-99 are substantially smaller and range respectively between 0.002 and 0.035 and between 0.000 and 0.010 across questions and waves. Responses in the “Other” category occur even more infrequently and usually constitute 0.006 or less of cases.

The main takeaway from Table S2 is that the basic patterns found by Manski and Molinari (2010) using the 2006 data are confirmed for the remaining waves as well. Hence, these patterns are stable across waves.

Table S2: Responses by Question and Wave in the 2002-2014 HRS

Question: percent chance that...	Wave	N	Fraction of responses equal to or in:								
			NR	0	1-4	50	96-99	100	M10	M5	Other
P5: leave inheritance ≥ \$10,000 (personal finances)	2002	16,119	0.050	0.154	0.004	0.074	0.007	0.443	0.205	0.060	0.002
	2004	18,249	0.037	0.162	0.004	0.083	0.008	0.404	0.241	0.059	0.002
	2006	17,191	0.053	0.159	0.004	0.067	0.008	0.447	0.209	0.052	0.001
	2008	16,060	0.050	0.153	0.004	0.067	0.010	0.431	0.236	0.046	0.002
	2010	20,397	0.037	0.172	0.007	0.080	0.009	0.344	0.296	0.053	0.003
	2012	19,359	0.039	0.170	0.007	0.085	0.009	0.329	0.306	0.053	0.003
	2014	17,647	0.037	0.167	0.006	0.086	0.008	0.324	0.319	0.050	0.003
P14: lose job during next year (personal finances)	2002	4,220	0.022	0.479	0.021	0.122	0.002	0.018	0.244	0.091	0.002
	2004	5,629	0.013	0.450	0.021	0.128	0.000	0.019	0.277	0.091	0.001
	2006	4,797	0.020	0.461	0.026	0.107	0.001	0.018	0.274	0.090	0.003
	2010	6,785	0.018	0.323	0.028	0.141	0.001	0.022	0.356	0.106	0.004
	2012	6,093	0.017	0.322	0.033	0.140	0.001	0.022	0.363	0.099	0.002
	2014	5,219	0.015	0.323	0.035	0.126	0.001	0.018	0.376	0.103	0.003
	P15: find equally good job (personal finances)	2002	4,220	0.022	0.183	0.009	0.165	0.006	0.142	0.353	0.120
2004		5,629	0.013	0.176	0.012	0.158	0.003	0.138	0.387	0.112	0.002
2006		4,797	0.017	0.173	0.014	0.152	0.004	0.143	0.383	0.112	0.003
2010		6,769	0.013	0.188	0.022	0.148	0.004	0.069	0.435	0.118	0.004
2012		6,093	0.014	0.166	0.018	0.164	0.003	0.076	0.447	0.108	0.003
2014		5,219	0.014	0.141	0.016	0.166	0.002	0.083	0.463	0.112	0.003
P17: work full time after age 62 (personal finances)		2002	3,219	0.012	0.194	0.005	0.139	0.005	0.220	0.312	0.111
	2004	4,528	0.007	0.161	0.008	0.156	0.004	0.163	0.387	0.112	0.003
	2006	5,238	0.011	0.299	0.011	0.133	0.004	0.142	0.305	0.093	0.002
	2008	3,870	0.026	0.160	0.012	0.134	0.006	0.202	0.357	0.099	0.004
	2010	7,828	0.008	0.152	0.014	0.151	0.006	0.143	0.415	0.108	0.004
	2012	6,647	0.010	0.148	0.016	0.147	0.005	0.136	0.434	0.098	0.005
	2014	5,294	0.006	0.147	0.015	0.142	0.005	0.137	0.443	0.099	0.005

NOTE: N = sample size, NR = nonresponse, M10 = multiple of 10 but not (0, 50, 100), M5 = multiple of 5 but not of 10.

Table S2 (Continued): Responses by Question and Wave in the 2002-2014 HRS

Question: percent chance that...	Wave	N	Fraction of responses equal to or in:								
			NR	0	1-4	50	96-99	100	M10	M5	Other
P28: live to be 75 or more (personal health)	2002	7200	0.048	0.038	0.002	0.223	0.005	0.178	0.359	0.144	0.003
	2004	9037	0.035	0.049	0.003	0.230	0.004	0.165	0.372	0.139	0.002
	2006	6713	0.040	0.053	0.004	0.222	0.005	0.152	0.375	0.144	0.004
	2008	5567	0.038	0.041	0.004	0.207	0.005	0.156	0.394	0.148	0.006
	2010	10498	0.041	0.059	0.005	0.206	0.006	0.143	0.402	0.133	0.006
	2012	9482	0.035	0.064	0.006	0.221	0.006	0.135	0.406	0.124	0.004
	2014	8084	0.029	0.064	0.006	0.226	0.006	0.136	0.414	0.115	0.004
P32: move to nursing home in 5 years (personal health)	2002	9177	0.082	0.491	0.014	0.111	0.001	0.006	0.207	0.088	0.002
	2004	12629	0.063	0.444	0.012	0.144	0.001	0.008	0.232	0.095	0.002
	2006	10044	0.075	0.463	0.021	0.101	0.000	0.007	0.231	0.100	0.002
	2008	10106	0.061	0.433	0.020	0.089	0.000	0.007	0.281	0.106	0.002
	2010	15512	0.045	0.393	0.025	0.130	0.001	0.016	0.284	0.103	0.003
	2012	9870	0.046	0.402	0.023	0.120	0.000	0.012	0.289	0.105	0.003
	2014	9367	0.037	0.400	0.028	0.113	0.000	0.013	0.304	0.102	0.003
P34: U.S. have economic depression (general economic conditions)	2002	184	0.103	0.054	0.016	0.299	0.000	0.082	0.359	0.071	0.016
	2004	17996	0.069	0.084	0.005	0.264	0.002	0.056	0.384	0.134	0.003
	2006	16754	0.078	0.066	0.006	0.238	0.002	0.060	0.404	0.142	0.004
	2008	15727	0.060	0.044	0.005	0.194	0.006	0.137	0.409	0.141	0.004
P110: Social Security will be less generous (general economic conditions)	2006	16754	0.065	0.048	0.003	0.231	0.005	0.120	0.387	0.139	0.002
	2008	15727	0.064	0.049	0.002	0.223	0.006	0.111	0.395	0.147	0.003
	2010	20208	0.046	0.048	0.005	0.191	0.010	0.187	0.379	0.130	0.005
	2012	19081	0.043	0.051	0.004	0.210	0.008	0.175	0.387	0.118	0.004
P47: mutual fund increase in value (general economic conditions)	2002	7260	0.206	0.079	0.004	0.239	0.000	0.040	0.306	0.122	0.003
	2004	17996	0.148	0.058	0.004	0.264	0.001	0.041	0.359	0.121	0.004
	2006	16754	0.240	0.042	0.003	0.231	0.001	0.036	0.339	0.106	0.003
	2008	15727	0.197	0.057	0.004	0.216	0.001	0.028	0.374	0.119	0.004
	2010	20208	0.111	0.062	0.006	0.238	0.001	0.037	0.420	0.122	0.005
	2012	19081	0.119	0.058	0.005	0.271	0.000	0.033	0.401	0.108	0.005
	2014	8828	0.097	0.052	0.007	0.273	0.000	0.041	0.414	0.109	0.006

SA2.2 Pooling Data across Waves to Probe More Deeply into Response Tendencies

The exploratory analysis presented in Section 2.1 of the main text describes the relative prevalence of rounding patterns aggregated across the HRS respondents. To obtain further insight, we examine in depth the rounding behavior of particular respondents across questions and waves. This kind of exploration is possible in the HRS, as each respondent has been asked and answered many expectations questions. Table S3 displays the average numbers of expectations questions asked and answered by HRS respondents, in total and by wave and question domain. This exploration yields important new findings, which we describe next.

We proceed by drawing a random subset of 100 HRS respondents and by generating histograms of the responses each respondent thus selected gave in each of the three question domains. Figure S1 illustrates using the respondent selected by the 9th random draw.

Inspection of the histograms across the 100 randomly drawn respondents suggests that many of them may be applying weakly coarser rounding in the middle of the 0-100 percent chance scale than in its tails. To better visualize this pattern we report a grouped version of the histograms. For example, Figure S2 presents the grouped versions of the histograms shown in Figure S1 for respondent #9. Specifically, in Figure S2 responses are grouped according to the following partition of the 0-100 scale, where 25 and 75 are used as the thresholds separating the center from the two symmetric tails: $M1\text{-Tail} = \text{values in } 1\text{-}24 \cup 76\text{-}99 \text{ that are not divisible by } 5$; $M1\text{-Center} = \text{values in } [26, 74] \text{ that are not divisible by } 5$; $M5\text{-Tail} = \{5, 15, 85, 95\}$; $M5\text{-Center} = \{35, 45, 55, 65\}$; $M10\text{-Tail} = \{10, 20, 80, 90\}$; $M10\text{-Center} = \{30, 40, 60, 70\}$; $M25 = \{25, 75\}$; $M100 = \{0, 100\}$; $M50 = \{50\}$.

There are two notable features in the distributions of responses given by respondent #9 in Figure S2. First, the high frequencies of 25 and 75 percent responses (grouped in $M25$) relative to other multiples of 5 (grouped in $M5\text{-T}$ and $M5\text{-C}$) suggest that 25 and 75 may have special status among multiples of 5.

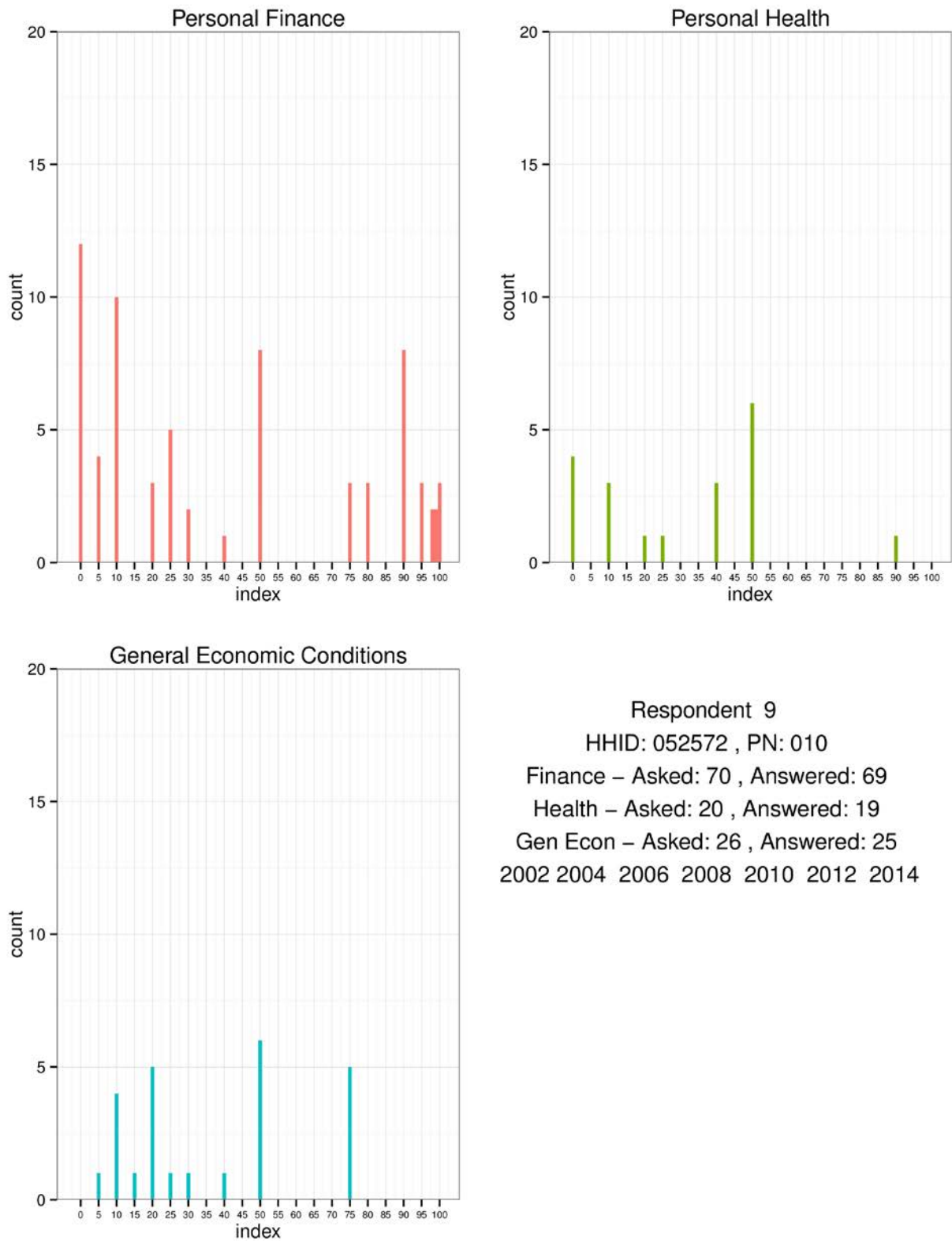
These percentages correspond respectively to “1 in 4” and “3 in 4” chances. Thus, they might be viewed by respondents as more rounded than other multiples of 5.

The second important feature emerging from the histograms shown in Figure S2 is that the relative frequencies of refined responses in the tail segments of the scale are generally higher than the frequencies of such responses in the corresponding center segment. For instance, the heights of the bars corresponding to M10-T responses are systematically higher than those corresponding to M10-C responses in all three question domains. The same pattern applies to the remaining response categories. This suggests that the more frequent use of multiples of 1 percent near the endpoints of the scale than toward the middle of the scale documented by earlier analyses of rounding might be the expression of a more general tendency of respondents to round more coarsely around the middle of the 0-100 scale than in its tails.

Table S3: Numbers of Questions Asked and Answered by Wave and Question Domain

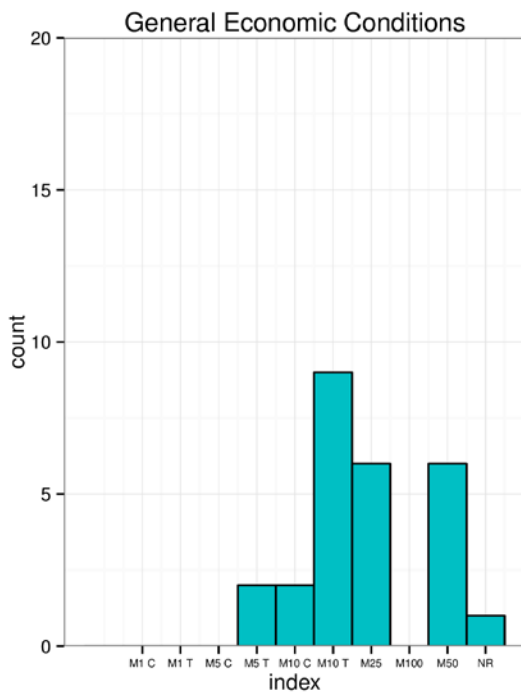
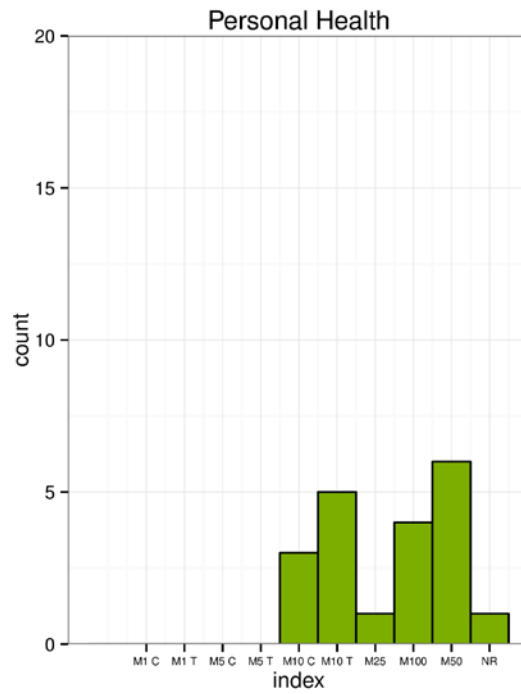
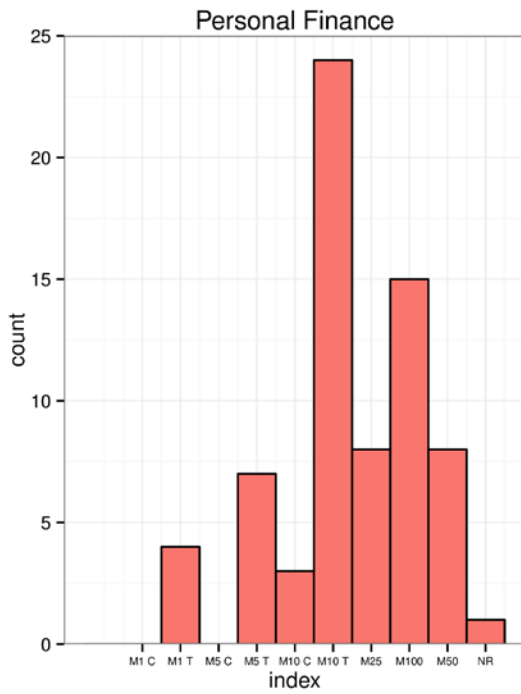
Wave	2002	2004	2006	2008	2010	2012	2014	All Waves	
Question Domain									
	Number of Questions								
personal finances	14	21	23	11	18	20	20	127	
personal health	4	3	9	9	3	4	4	36	
gen. economic cond.	3	2	6	5	4	5	7	32	
total	21	28	38	25	25	29	31	197	
	Average Number of Questions Asked								
personal finances	8	12.4	13.2	5.6	9	9.7	9.7	67.6	
personal health	2.3	2.1	3.5	5.1	2.2	2.4	2.5	20.1	
gen. economic cond.	1	2	5.8	4.6	3.3	4.2	3.3	24.2	
total	11.3	16.5	22.5	15.3	14.5	16.3	15.5	111.9	
	Average Number of Questions Answered								
personal finances	7.8	12.1	12.8	5.4	8.9	9.5	9.5	66	
personal health	2.2	2	3.3	4.8	2.1	2.3	2.4	19.1	
gen. economic cond.	0.8	1.8	4.8	4.2	3	4	3.1	21.7	
total	10.8	15.9	20.9	14.4	14	15.8	15	106.8	

Figure S1: Distribution of Responses across Waves (2002-2014) of an Individual Respondent by Domain



Respondent 9
 HHID: 052572 , PN: 010
 Finance – Asked: 70 , Answered: 69
 Health – Asked: 20 , Answered: 19
 Gen Econ – Asked: 26 , Answered: 25
 2002 2004 2006 2008 2010 2012 2014

Figure S2: Distribution of Responses across Waves (2002-2014) of an Individual Respondent by Domain: Grouped Version



Respondent 9
 HHID: 052572 , PN: 010
 Finance – Asked: 70 , Answered: 69
 Health – Asked: 20 , Answered: 19
 Gen Econ – Asked: 26 , Answered: 25
 2002 2004 2006 2008 2010 2012 2014

Supplementary Appendix to Section 3

SA3.1 Determination of Respondent Rounding Types

Table S4 presents in a formal and compact way the complete algorithm used to determine a respondent's rounding type in the center of the 0-100 scale (panel A) and in its tails (panel B) within a given question domain. Specifically, Table S4A maps all logically possible response tendencies that may be observed in the center of the 0-100 scale into corresponding center rounding types. Table S4B maps all logically possible response tendencies that may be observed in the tails of the 0-100 scale into corresponding tail rounding types. For each question domain, each respondent is assigned a bivariate (tails, center) rounding type belonging to the cross product of the tail and center rounding types listed in the two panels of Table S4. Both panels make use of the partition of the 0-100 scale described in Table S5.

In Section 3.1, we present an example where a respondent is observed to answer four expectations questions in the domain of personal finances. The respondent's answers are {5, 30, 60, 85}. As the set includes 2 multiples of 5 percent in the tails and 2 multiples of 10 percent in the center, the respondent is classified as rounding to the nearest 5 percent *or finer degree* in the tails ($\mathcal{M}5\text{-T}$) and to the nearest 10 percent *or finer degree* in the center ($\mathcal{M}10\text{-C}$).

We now discuss additional cases to further illustrate the logic of our proposed algorithm. Let us first consider an alternative scenario where the respondent is asked an additional question in the domain of personal finances and answers it with a value in the center that is either a multiple of 10 percent or 50 percent. Under this scenario, our conclusion about the respondent's rounding type in the center for the finances domain does not change. If, on the other hand, the respondent were to answer the additional question with a multiple of 5 percent in the center, our conclusion might change as it would depend on the respondent's response pattern in the two domains other than personal finances. For example, if in a second domain (say personal health), the respondent gave at least one center response that is a multiple

of 5 percent or finer (i.e., a multiple of 1 percent), then the respondent would be classified as rounding to the nearest 5 percent (rather than 10 percent) in the center within the personal finances domain.

Moving now to the tails, let us imagine that the respondent is asked an additional question in the class of personal finances and answers it with a value in the tails that is a multiple of 5 percent, a multiple of 10 percent, or a focal response of 0 or 100. In this case, our conclusion about the respondent's rounding type in the tails for the finances domain does not change. If, on the other hand, the respondent were to answer the additional question with a multiple of 1 percent in the tails, our conclusion might change depending on the respondent's response pattern in the other two domains. Specifically, if in a second domain (say general economic conditions), the respondent gave at least one response — either in the tails or in the center — that is a multiple of 1 percent, then the respondent would be classified as rounding to the nearest 1 percent in the tails within the personal finances domain.

Building on the example introduced in Section 3.1, in Section 3.3 we explain how to assign probability intervals to the respondents' point responses. Here we discuss additional cases to further illustrate the logic of our algorithm, particularly the application of the boundary conditions in construction of the intervals.

Let us first consider a case where the respondent is asked an additional question (relative to the example discussed in Section 3.1) and were observed to answer with a multiple of 1 percent in the tails (say 2 percent). The respondent is still classified as $\mathcal{M}5$ -T in the tails, as long as they did not use any multiple of 1 percent to answer questions in the remaining domains. Under this scenario, construction of the interval around 2 percent requires a "boundary condition," whereby the lower bound of the assigned interval cannot be smaller than 0 percent. Hence, if the respondent were observed to respond with 2 percent to one question in the finances domain, while still being classified as $\mathcal{M}5$ -T, 2 percent would be assigned the interval $[0, 4.5]$ or $[\max(0, 2 - 2.5), 2 + 2.5]$. In the right tail of the scale, a response of 98

percent would be handled symmetrically and would be assigned a range of $[95.5, 100]$ or $[98 - 2.5, \min(100, 98 + 2.5)]$.

Let us now consider an alternative scenario where the respondent is asked two additional questions in the personal finances domain and is observed to answer both of them with a multiple of 1 percent in the tails (say 2 percent and 98 percent). We now classify the respondent as $\mathcal{M}1\text{-T}$. Under this scenario, all of the respondent's tail answers in the personal finances domain are taken at face value. Hence, 2 percent is assigned the range $[2, 2]$, 5 percent is assigned the range $[5, 5]$, and so on. Finally, regardless of the respondent's rounding type, any NR is assigned an interval of $[0, 100]$.

Let us now entertain a final situation where the respondent's highest response in the left tail is 24 percent. In this case, the boundary condition to the left of 30 might bind, depending on the respondent's rounding type in the tails. Specifically, if the respondent is still $\mathcal{M}5\text{-T}$ — as it would happen if 24 percent were the only multiple of 1 percent (but not of 5 percent) used by the respondent in any domain — then the boundary condition to the left of 30 percent would bind, since $24 + 2.5 > 30 - 5$. In this case, the probability interval assigned to the response of 30 percent in the center would be $[26.5, 35]$ instead of $[25, 35]$. On the other hand, if the respondent were classified to be $\mathcal{M}1\text{-T}$ — as it would happen if they gave a second response, in addition to 24 percent, that is a multiple of 1 percent (but not of 5 percent) in any domain — then the boundary condition to the left of 30 percent would not bind, since $24 < 30 - 5$.

Table S4A: Portion of the Algorithm Determining the Rounding Type of Respondent j in the Center for Questions of Domain l

START: IF	AND \exists domain $l' \neq l$ s.t.	$\#(Y_l \cap$ M1-C) ≥ 1	$\#(Y_l \cap$ M1-C) $= 0$	$\#(Y_l \cap$ M5-C) ≥ 1	$\#(Y_l \cap$ M5-C) $= 0$	$\#(Y_l \cap$ M10-C) ≥ 1	$\#(Y_l \cap$ M10-C) $= 0$	$\#(Y_l \cap$ M25) ≥ 1	$\#(Y_l \cap$ M25) $= 0$	$\#(Y_l \cap$ M50) ≥ 1	$\#(Y_l \cap$ M50) $= 0$	All NR
$\#(Y_l \cap \text{M1-C}) \geq 2$		j is $\mathcal{M}1\text{-C}$										
$\#(Y_l \cap \text{M1-C}) = 1$	$\mathcal{M}1\text{-C}$	IF j is still UNCLASSIFIED, GO to the NEXT row										
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C}\}) \geq 2$		j is $\mathcal{M}5\text{-C}$										
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C}\}) = 1$	$\mathcal{M}5\text{-C}$		$\mathcal{M}5\text{-C}$		IF j is still UNCLASSIFIED, GO to the NEXT row							
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C}\}) \geq 2$		j is $\mathcal{M}10\text{-C}$										
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C}\}) = 1$	$\mathcal{M}10\text{-C}$		$\mathcal{M}10\text{-C}$		$\mathcal{M}10\text{-C}$		IF j is still UNCLASSIFIED, GO to the NEXT row					
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C} \cup \text{M25}\}) \geq 2$		j is $\mathcal{M}25$										
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C} \cup \text{M25}\}) = 1$	$\mathcal{M}25$		$\mathcal{M}25$		$\mathcal{M}25$		$\mathcal{M}25$		IF j is still UNCLASSIFIED, GO to the NEXT row			
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C} \cup \text{M25} \cup \text{M50}\}) \geq 2$		j is $\mathcal{M}50$										
$\#(Y_l \cap \{\text{M1-C} \cup \text{M5-C} \cup \text{M10-C} \cup \text{M25} \cup \text{M50}\}) = 1$	$\mathcal{M}50$		$\mathcal{M}50$		$\mathcal{M}50$		$\mathcal{M}50$		$\mathcal{M}50$		j type is Undetermined , END	
All NR		j type is Undetermined , END										

NOTE: Y_l is the set of responses given by a hypothetical respondent j in domain l . M1-C, M5-C, M10-C, M25, and M50 are sets partitioning the center of the 0-100 scale, defined in Table 6. $\mathcal{M}1\text{-C}$, $\mathcal{M}5\text{-C}$, $\mathcal{M}10\text{-C}$, $\mathcal{M}25$, $\mathcal{M}50$, and ‘Undetermined’ denote rounding types in the center. $\mathcal{M}1\text{-C}$ denotes a respondent who rounds to the nearest 1 percent in the center, $\mathcal{M}5\text{-C}$ denotes a respondent who rounds to the nearest 5 percent or finer in the center, and so on. **Undetermined** denotes respondents who could not be classified to belong to any of the preceding center types.

Table S4B: Portion of the Algorithm Determining the Rounding Type of Respondent j in the Tails for Questions of Domain l

START: IF	AND \exists domain $l \neq l$ s.t.	$\#(Y_l \cap$ $\{M1-T$ $\cup M1-$ $C\}) \geq 1$	$\#(Y_l \cap$ $\{M1-T$ $\cup M1-$ $C\}) = 0$	$\#(Y_l \cap$ $\{M5-T$ $\cup M5-$ $C\}) \geq 1$	$\#(Y_l \cap$ $\{M5-T$ $\cup M5-$ $C\}) = 0$	$\#(Y_l \cap$ $\{M10-T$ $\cup M10-$ $C\}) \geq 1$	$\#(Y_l \cap$ $\{M10-T$ $\cup M10-$ $C\}) = 0$	$\#(Y_l \cap$ $M25)$ ≥ 1	$\#(Y_l \cap$ $M25)$ $= 0$	$\#(Y_l \cap$ $\{M100$ $\cup M50\})$ ≥ 1	$\#(Y_l \cap$ $\{M100$ \cup $M50\}) = 0$	All NR
$\#(Y_l \cap M1-T) \geq 2$		j is $M1-T$										
$\#(Y_l \cap M1-T) = 1$	$M1-T$	IF j is still UNCLASSIFIED, GO to NEXT row										
$\#(Y_l \cap \{M1-T \cup$ $M5-T\}) \geq 2$		j is $M5-T$										
$\#(Y_l \cap \{M1-T \cup$ $M5-T\}) = 1$	$M5-T$		$M5-T$	IF j is still UNCLASSIFIED, GO to NEXT row								
$\#(Y_l \cap \{M1-T \cup$ $M5-T \cup M10-T\}) \geq 2$		j is $M10-T$										
$\#(Y_l \cap \{M1-T \cup$ $M5-T \cup M10-T\}) = 1$	$M10-T$		$M10-T$		$M10-T$	IF j is still UNCLASSIFIED, GO to NEXT row						
$\#(Y_l \cap \{M1-T \cup$ $M5-T \cup M10-T \cup$ $M25 \cup M100\}) \geq 2$		j is $M100$										
$\#(Y_l \cap \{M1-T \cup$ $M5-T \cup M10-T \cup$ $M25 \cup M100\}) = 1$	$M100$		$M100$		$M100$		$M100$		$M100$	j type is Undetermined , END		
All NR		j type is Undetermined , END										

NOTE: Y_l is the set of responses given by a hypothetical respondent j in domain l . M1-T, M5-T, M10-T, and M100 are sets partitioning the tails of the 0-100 scale, defined in Table 6. $M1-T$, $M5-T$, $M10-T$, $M100$, and ‘Undetermined’ denote rounding types in the tails. $M1-T$ denotes a respondent who rounds to the nearest 1 percent in the tails, $M5-T$ denotes a respondent who rounds to the nearest 5 percent or finer in the tails, and so on. **Undetermined** denotes respondents who could not be classified to belong to any of the preceding types.

Table S5: Partition of the 0-100 Percent Chance Scale in Two Symmetric Tails and a Center

	LT (Left Tail)	RT (Right Tail)	T (Tail)	C (Center)	Union
(M100,M50)	{ 0 }	{ 100 }	M100-LT \cup M100-RT	{ 50 }	M100 \cup M50
M25	\emptyset	\emptyset	\emptyset	{ 25, 75 }	M25
M10	{ 10, 20 }	{ 80, 90 }	M10-LT \cup M10-RT	{ 30, 40, 60, 70 }	M10-T \cup M10-C
M5	{ 5, 15 }	{ 85, 95 }	M5-LT \cup M5-RT	{ 35, 45, 55, 65 }	M5-T \cup M5-C
M1	1-4 \cup 6-9 \cup 11-14 \cup 16-19 \cup 21-24	76-79 \cup 81-84 \cup 86-89 \cup 91-94 \cup 96-99	M1-LT \cup M1-RT	26-29 \cup 31-34 \cup 36-39 \cup 41-44 \cup 46-49 \cup 51-54 \cup 56-59 \cup 61-64 \cup 66-69 \cup 71-74	M1-T \cup M1-C
Union	M100-LT \cup M10-LT \cup M5-LT \cup M1-LT	M100-RT \cup M10-RT \cup M5-RT \cup M1-RT	M100 \cup M10-T \cup M5-T \cup M1-T	M50 \cup M25 \cup M10-C \cup M5-C \cup M1-C	0-100 (entire scale)

SA3.3 Variation of Rounding Types with Respondent Characteristics

Before describing how probability intervals are formed based on respondents' point responses and their inferred rounding types, we investigate whether the latter vary systematically by respondents' characteristics. To this end, in Section 3.3 we estimate three bivariate ordered probit models, one per question domain, where the outcome variables are the respondent's bivariate vectors of tail and center rounding categories in the corresponding domains and the predictors are respondent's gender, age, educational attainment, race, and cognitive score.

Here we provide additional estimates from a specification that excludes cognitive scores. These estimates are shown in Table S6. We do so as we believe that this part of our analysis may yield useful information about likely characteristics of respondents that are associated with coarser or more refined rounding behavior to researchers who analyze survey expectations but do not have access to: (a) a sufficiently large number of expectations questions per respondent to directly apply our method; (b) a sufficiently rich or specialized set of relevant covariates as in the HRS.

The main patterns are analogous to those observed in the specification including cognitive scores. In particular, higher levels of educational attainment are still unambiguously and statistically significantly associated with a tendency to give more refined responses (less rounding) across all scale segments and question domains. Similarly, the dummies continue to display a non-linear effect. Respondents belonging to the oldest age category (80+) have a statistically significant tendency to give more rounded responses than respondents belonging to the youngest one (50-59) across all scale segments and questions domains. On the other hand, respondents in the two intermediate age groups (i.e., 60-69 and 70-79) belong to rounding categories that may be more refined, coarser, or statistically indistinguishable from those characterizing younger respondents, depending on the specific domain or scale segment. Gender and race continue to features a somewhat mixed pattern. As before, rounding tendencies are

positively correlated across scale segments. Hence, respondents who give coarser responses in the tails are more likely to do so in the center and *vice versa*.

Table S6: Bivariate Ordered Probit of (Tail, Center) Rounding Categories on Respondent's Characteristics, by Question Domain

	Personal Health		Personal Finances		Gen. Econ. Conditions	
	Tail Type	Center Type	Tail Type	Center Type	Tail Type	Center Type
Male	0.0306 (0.0146)	-0.0203 (0.0152)	0.0321 (0.0139)	0.0166 (0.0149)	0.0137 (0.0147)	-0.0346 (0.0154)
Aged 60-69	-0.1860 (0.0177)	-0.1343 (0.0191)	-0.0062 (0.0171)	0.0217 (0.0186)	-0.1064 (0.0182)	-0.0962 (0.0192)
Aged 70-79	-0.1409 (0.0196)	0.0784 (0.0203)	0.1732 (0.0187)	0.2271 (0.0201)	-0.7937 (0.0196)	0.0562 (0.0205)
Aged 80+	0.1768 (0.0257)	0.5320 (0.0252)	0.5862 (0.0237)	0.6615 (0.0248)	0.2228 (0.0258)	0.4162 (0.0257)
High school	-0.1749 (0.0210)	-0.1996 (0.0206)	-0.2507 (0.0194)	-0.2776 (0.0203)	-0.1250 (0.0211)	-0.2324 (0.0210)
Some college	-0.1607 (0.0346)	-0.2081 (0.0359)	-0.2969 (0.0326)	-0.3290 (0.0351)	-0.1289 (0.0347)	-0.2820 (0.0367)
Bachelor	-0.3400 (0.0264)	-0.4218 (0.0276)	-0.4566 (0.0253)	-0.4950 (0.0271)	-0.2714 (0.0268)	-0.4588 (0.0277)
Graduate	-0.4362 (0.0290)	-0.5580 (0.0311)	-0.5459 (0.0281)	-0.5586 (0.0306)	-0.3513 (0.0294)	-0.5527 (0.0313)
Black	0.0846 (0.0211)	0.1947 (0.0216)	-0.0548 (0.0193)	0.0212 (0.0209)	-0.0036 (0.0209)	0.0477 (0.0217)
Other race	0.1586 (0.0296)	0.2031 (0.0315)	0.1264 (0.0280)	0.0897 (0.0302)	0.1220 (0.0306)	0.1128 (0.0312)
Rho	0.2698 (0.0086)		0.3799 (0.0073)		0.2985 (0.0092)	
N	22,821		25,016		22,983	

NOTES: (i) Respondents whose tail or center rounding category is undetermined are excluded from this analysis. (ii) Omitted dummies are 'Female,' 'Aged 50-59,' 'No degree,' and 'White.' 'Rho' is the parameter capturing the correlation between the error terms of the tail and center latent equations. (iii) Standard errors are reported in parentheses.

SA3.4 Using Survey Responses and Rounding Types to Form Expectations Intervals

Table S7 (making use of the partition of the 0-100 scale described in Table S5) presents in a formal and compact way the complete portion of the algorithm used to assign intervals to observed point responses in the scale tails (panel A) and in the its center (panel B) within a given domain. Specifically, Table S7A maps all logically possible rounding types and responses that may be observed in the tails of the 0-100 scale into corresponding tail intervals. Similarly, Table S7B maps all logically possible rounding types and responses that may be observed in the center of the 0-100 scale into corresponding center intervals.

Table S7A: Portion of the Algorithm Assigning Probability Intervals, $[\mathbf{v}_{jktL}^T, \mathbf{v}_{jktU}^T]$, to Point Responses in the Tails by Respondent j to Questions in Domain l , \mathbf{v}_{jkt}^T , by Rounding Type

Center Type \ Tails Type	$\mathcal{M}1\text{-C}$	$\mathcal{M}5\text{-C}$	$\mathcal{M}10\text{-C}$	$\mathcal{M}25$	$\mathcal{M}50$	No or Undetermined center type
$\mathcal{M}1\text{-T}$	\mathbf{v}_{jkt}^T	\mathbf{v}_{jkt}^T	\mathbf{v}_{jkt}^T	\mathbf{v}_{jkt}^T	\mathbf{v}_{jkt}^T	\mathbf{v}_{jkt}^T
$\mathcal{M}5\text{-T}$	SAME AS ($\mathcal{M}1\text{-T}$, $\mathcal{M}1\text{-C}$)	$[\max(0, \mathbf{v}_{jkt}^T - 2.5), \min(\mathbf{v}_{jkt}^T + 2.5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 2.5), \min(\mathbf{v}_{jkt}^T + 2.5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 2.5), \min(\mathbf{v}_{jkt}^T + 2.5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 2.5), \min(\mathbf{v}_{jkt}^T + 2.5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 2.5), \min(\mathbf{v}_{jkt}^T + 2.5, 100)]$
$\mathcal{M}10\text{-T}$	SAME AS ($\mathcal{M}1\text{-T}$, $\mathcal{M}1\text{-C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	$[\max(0, \mathbf{v}_{jkt}^T - 5), \min(\mathbf{v}_{jkt}^T + 5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 5), \min(\mathbf{v}_{jkt}^T + 5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 5), \min(\mathbf{v}_{jkt}^T + 5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 5), \min(\mathbf{v}_{jkt}^T + 5, 100)]$
$\mathcal{M}100$	SAME AS ($\mathcal{M}1\text{-T}$, $\mathcal{M}1\text{-C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	SAME AS ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)	$[\max(0, \mathbf{v}_{jkt}^T - 12.5), \min(\mathbf{v}_{jkt}^T + 12.5, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 25), \min(\mathbf{v}_{jkt}^T + 25, 100)]$	$[\max(0, \mathbf{v}_{jkt}^T - 50), \min(\mathbf{v}_{jkt}^T + 50, 100)]$
No or Undet. tail type	SAME AS ($\mathcal{M}1\text{-T}$, $\mathcal{M}1\text{-C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	SAME AS ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)	SAME AS ($\mathcal{M}100$, $\mathcal{M}25$)	SAME AS ($\mathcal{M}100$, $\mathcal{M}50$)	$[0, 100]$
All NR responses regardless of type	$[0, 100]$	$[0, 100]$	$[0, 100]$	$[0, 100]$	$[0, 100]$	$[0, 100]$

NOTE: $\mathcal{M}1\text{-T}$, $\mathcal{M}5\text{-T}$, $\mathcal{M}10\text{-T}$, $\mathcal{M}100$, and ‘**Undetermined**’ denote rounding types in the tails. \mathbf{v}_{jkt}^T denotes a hypothetical response respondent j gave in the tails of the 0-100 scale when answering a question in domain l . $[\mathbf{v}_{jktL}^T, \mathbf{v}_{jktU}^T]$ denotes the probability interval assigned to the point response by the algorithm. The boundary conditions ensure that the lower and upper bounds of the probability interval lie in the tails of the 0-100 scale.

Table S7B: Portion of the Algorithm Assigning Probability Intervals, $[\nu_{jktL}^C, \nu_{jktU}^C]$, to Point Responses in the Center by Respondent j to Questions in Domain l , ν_{jkt}^C , by Rounding Type

Center Type \ Tails Type	$\mathcal{M}1\text{-C}$	$\mathcal{M}5\text{-C}$	$\mathcal{M}10\text{-C}$	$\mathcal{M}25$	$\mathcal{M}50$	No or Undet. center type or any NR
$\mathcal{M}1\text{-T}$	ν_{jkt}^C	$[\max(\max \Upsilon_j^{LT}, \nu_{jkt}^C - 2.5), \min(\nu_{jkt}^C + 2.5, \min \Upsilon_j^{RT})]$	$[\max(\max \Upsilon_j^{LT}, \nu_{jkt}^C - 5), \min(\nu_{jkt}^C + 5, \min \Upsilon_j^{RT})]$	$[\max(\max \Upsilon_j^{LT}, \nu_{jkt}^C - 12.5), \min(\nu_{jkt}^C + 12.5, \min \Upsilon_j^{RT})]$	$[\max(\max \Upsilon_j^{LT}, \nu_{jkt}^C - 25), \min(\nu_{jkt}^C + 25, \min \Upsilon_j^{RT})]$	$[0, 100]$
$\mathcal{M}5\text{-T}$	AS ($\mathcal{M}1\text{T}$, $\mathcal{M}1\text{C}$)	$[\max(\max \Upsilon_j^{LT} + 2.5, \nu_{jkt}^C - 2.5), \min(\nu_{jkt}^C + 2.5, \min \Upsilon_j^{RT} - 2.5)]$	$[\max(\max \Upsilon_j^{LT} + 2.5, \nu_{jkt}^C - 5), \min(\nu_{jkt}^C + 5, \min \Upsilon_j^{RT} - 2.5)]$	$[\max(\max \Upsilon_j^{LT} + 2.5, \nu_{jkt}^C - 12.5), \min(\nu_{jkt}^C + 12.5, \min \Upsilon_j^{RT} - 2.5)]$	$[\max(\max \Upsilon_j^{LT} + 2.5, \nu_{jkt}^C - 25), \min(\nu_{jkt}^C + 25, \min \Upsilon_j^{RT} - 2.5)]$	$[0, 100]$
$\mathcal{M}10\text{-T}$	AS ($\mathcal{M}1\text{T}$, $\mathcal{M}1\text{C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	$[\max(\max \Upsilon_j^{LT} + 5, \nu_{jkt}^C - 5), \min(\nu_{jkt}^C + 5, \min \Upsilon_j^{RT} - 5)]$	$[\max(\max \Upsilon_j^{LT} + 5, \nu_{jkt}^C - 12.5), \min(\nu_{jkt}^C + 12.5, \min \Upsilon_j^{RT} - 5)]$	$[\max(\max \Upsilon_j^{LT} + 5, \nu_{jkt}^C - 25), \min(\nu_{jkt}^C + 25, \min \Upsilon_j^{RT} - 5)]$	$[0, 100]$
$\mathcal{M}100$	AS ($\mathcal{M}1\text{T}$, $\mathcal{M}1\text{C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	SAME AS ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)	$[\nu_{jkt}^C - 12.5, \nu_{jkt}^C + 12.5]$	$[\max(25, \nu_{jkt}^C - 25), \min(\nu_{jkt}^C + 25, 75)]$	$[0, 100]$
No or Undet. tail type	AS ($\mathcal{M}1\text{T}$, $\mathcal{M}1\text{C}$)	SAME AS ($\mathcal{M}5\text{-T}$, $\mathcal{M}5\text{-C}$)	SAME AS ($\mathcal{M}10\text{-T}$, $\mathcal{M}10\text{-C}$)	SAME AS ($\mathcal{M}100$, $\mathcal{M}25$)	SAME AS ($\mathcal{M}100$, $\mathcal{M}50$)	$[0, 100]$

NOTE: $\mathcal{M}1\text{-C}$, $\mathcal{M}5\text{-C}$, $\mathcal{M}10\text{-C}$, $\mathcal{M}50$, and ‘Undetermined’ denote rounding types in the tails. ν_{jkt}^C denotes a hypothetical response respondent j gave in the center of the 0-100 scale when answering a question in domain l . $[\nu_{jktL}^C, \nu_{jktU}^C]$ denotes the probability interval assigned to the point response by the algorithm. The boundary conditions ensure that the lower and upper bounds of the probability interval lie in the center of the 0-100 scale. Υ_j^{LT} denotes the set of responses respondent j gave in the left tail (i.e., in 0-24) when answering questions in domain l . Υ_j^{RT} denotes the set of respondent j ’s responses in the right tail (i.e., in 76-100).

We apply the algorithm described in Table S7 to all responses by HRS respondents who responded to at least one expectations question in any question domain and in any wave between 2002 and 2014. For the purpose of constructing the intervals, respondents who were classified as rounding more coarsely in the tails than in the center are now treated as respondents who were classified as rounding to the same degree in the tails and in the center.

Table S8 reports the distributions of interval width for the responses given in wave 2014 to the following three questions: the percent chance that the respondent will live to be 75 or older (P28), the percent chance that the respondent will work full time past age 62 (P17), and the percent chance that a mutual fund will increase in value within the next year (P47).

The distribution of interval width for the probability of working past 62 displayed in the middle column of the table displays higher frequencies at lower width values than the distributions shown in the remaining columns, consistent with the pattern shown in Table 5 of the main text.

Table S8: Distribution of Range Size for Specific Expectations Questions in the 2014 HRS

Range Size	Percent Live to be 75 or older (P28 in Personal Health)	Percent Work full time past age 62 (P17 in Personal Finances)	Percent Mutual funds increase in value (P47 in General Economic Conditions)
0	7.17	20.95	6.04
2.5	3.71	9.05	2.02
3.5	0.09	0.09	0
4.5	0.04	0.08	0.02
5	27.72	31.72	23.82
6	0.01	0.02	0
7.5	0.99	1.38	1.55
9	0.02	0.02	0
10	42.96	32.58	48.11
12.5	1.53	0.34	0.77
15	0.38	0.19	0.36
17.5	0.06	0.13	0.11
20	0.05	0.02	0.02
22.5	0.06	0.11	0.09
25	4.40	1.57	3.77
27.5	0.02	0	0
30	0.02	0.02	0.01
32.5	0	0.02	0
35	0.01	0	0
40	0	0	0.02
42.5	0.01	0	0
50	7.71	1.1	3.56
60	0.01	0	0
100	2.99	0.62	9.72
Total	100	100	100
Sample size	8,084	5,294	8,828