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ABSTRACT

How can mortgages be redesigned to reduce housing market volatility, consumption volatility, and default? How does mortgage design interact with monetary policy? We answer these questions using a quantitative equilibrium life cycle model with aggregate shocks, long-term mortgages, and an equilibrium housing market, focusing on designs that index payments to monetary policy. Designs that raise mortgage payments in booms and lower them in recessions do better than designs with fixed mortgage payments. The benefits are quantitatively substantial: In a simulated crisis under a monetary regime in which the central bank lowers real interest rates in a bust, house prices fall 2.24 percentage points less, 23 percent fewer households default, and consumption falls by 0.79 percentage points less with ARMs relative to FRMs. Among designs that reduce payments in a bust, we show that those that front-load the payment reductions and concentrate them in recessions outperform designs that spread payment reductions over the life of the mortgage. Front-loading alleviates household liquidity constraints in states where they are most binding, reducing default and stimulating housing demand by new homeowners. To isolate this channel, we compare an FRM with a built-in option to be converted to an ARM with an FRM with an option to be refinanced at the prevailing FRM rate. Under these two contracts, the present value of a lender's loan falls by roughly an equal amount, but the FRM that can be converted to an ARM, which front loads payment reductions, reduces the declines in prices and consumption six times as much, and reduces default three times as much.

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1 Introduction

The design of mortgages is crucial to both household welfare and the macroeconomy. Home equity is the largest component of wealth for most households, and mortgages tend to be their dominant source of credit, so the design of mortgages has an outsized effect on household balance sheets (Campbell, 2013). In the mid-2000s boom and subsequent bust, housing wealth extraction through the mortgage market boosted consumption in the boom and reduced consumption in the bust (e.g., Mian and Sufi, 2011; Mian, Rao, and Sufi, 2013). Mortgage debt also led to the wave of foreclosures that resulted in over six million households losing their homes, badly damaging household balance sheets and crippling the housing market (*e.g.*, Guren and McQuade, 2019; Mian, Sufi, and Trebbi, 2015). Finally, in the wake of the recession, there has been increased attention paid to the role that mortgages play in the transmission of monetary policy to the real economy through household balance sheets (*e.g.*, Auclert, 2019; Wong, 2019; Di Maggio et al., 2017; Beraja et al., 2019).

In this paper, we study how to best design mortgages in order to reduce household consumption volatility and default and to increase household welfare. There is considerable evidence that implementation frictions prevent financial intermediaries from modifying mortgages *ex post* in a crisis (*e.g.*, Agarwal et al., 2017a; Agarwal et al., 2017b). As a result, a better-designed *ex ante* contract can likely deliver significant welfare benefits (*e.g.*, Campbell and Cocco, 2015; Piskorski and Tchistyi, 2017; Greenwald, Landvoigt, and Van Nieuwerburgh, 2018; Piskorski and Seru, 2018). We are further motivated by the evidence that not just the level of household mortgage debt (*e.g.*, loan-to-value or payment-to-income ratio), but also the design of such debt and in particular the time path of payments, can impact household outcomes including consumption and default. For example, Fuster and Willen (2017) and Di Maggio et al. (2017) study cohorts of borrowers with hybrid adjustable rate mortgages contracted in the years before the crisis. Exploiting heterogeneity in the timing of monthly payment reductions as mortgages transitioned from initial fixed rates to adjustable rates during the crisis, these papers show that downward resets resulted in substantially lower defaults and increased consumption. Similarly, studies that exploit quasi-random variation in housing market interventions in the Great Recession such as the Home Affordable Refinance Program (HARP) (Agarwal et al., 2017b) and the Home Affordable Modification Program (HAMP) (Agarwal et al., 2017a; Ganong and Noel, 2019) have found that monthly payment reductions significantly reduced default and increased consumption.

Such empirical evidence suggests that given the cyclicality of interest rates, indexing mortgage payments to interest rates can improve household outcomes and welfare. We pursue this indexation question systematically using a quantitative equilibrium model featuring heterogeneous households, endogenous mortgage spreads, and endogenous house prices. In a crisis, default increases the supply of homes on the market, further pushing down prices, which in turn generates more default. Using this framework, we quantitatively assess a variety of questions related to mortgage design. How would consumption, default, home prices, and household welfare change if we were to alter the design of mortgages in the economy, particularly in a deep recession and housing bust like the one experienced during the Great Recession? In an economy that transits between booms, recessions, and crises, how well do different indexed mortgages perform? What is the most effective simple form of indexation?

Our main finding is that designs in which mortgage payments are higher in booms and lower in recessions do better than designs with fixed mortgage payments for risk and insurance reasons. Moreover, among such designs, the most effective ones front-load the payment reductions so that they are concentrated during recessions rather than spread out over the life of the mortgage. Front-loading payment relief smooths consumption and limits default for homeowners who are liquidity constrained and stimulates new housing demand by constrained current renters. The reduction in default and increase in demand helps short-circuit a price-default spiral. Consequently, the benefit of different designs depends largely on how effectively they deliver immediate payment reductions to highly constrained households.

Our model features overlapping generations of households subject to both idiosyncratic and aggregate shocks, making endogenous decisions over home purchases, borrowing, consumption, refinancing, and costly default. We consider different relationships between the interest rate and the exogenous aggregate state, reflecting alternative monetary policies. Competitive lenders set spreads for each mortgage to break even in equilibrium, so lenders charge higher interest rates when a mortgage design hurts their bottom line. We calibrate the lender stochastic discount factor (SDF) to options data so that lenders charge more for contracts that cost them more in crisis states. Equilibrium in the housing market implies that household decisions, mortgage spreads, and the interest rate rule influence the equilibrium home price process. Household expectations regarding equilibrium prices and mortgage rates feed back into household decisions, and we solve this fixed-point problem in a rich quantitative model using computational methods based on Krusell and Smith (1998).

A key aspect of our analysis is that mortgage design affects household default decisions and hence home prices, which in equilibrium feeds back to household indebtedness. The quantitative implications of our model depend on accurately representing the link between home prices and default. Consequently, after calibrating our model to match standard moments, monetary policy since the 1980s, and the empirical distributions of mortgage debt and assets, we evaluate its ability to quantitatively capture the effect of payment reductions on default by simulating the Fuster and Willen (2017) quasi-experiment in our model. The model does a good job matching their findings quantitatively. Simulating quasi-experiments in our calibration procedure is an innovation that ensures that our model accurately captures the effects of changes in LTVs and interest rates as we alter mortgage design.

The calibrated model provides a laboratory to assess the benefits and costs of different mortgage designs, shed light on the key economic channels through which they work, and provide magnitudes for how much different designs may stabilize prices, default, and consumption in a crisis.[1] Our primary application is the impulse response to a housing crisis, although we also consider the unconditional performance of different mortgages. We begin by comparing an economy with all fixed rate mortgages (FRMs) against one with all adjustable rate mortgages (ARMs). While ARMs

[1]As with any structural exercise, we have made modeling choices that trade off tractability and realism. It is likely that the model leaves out some aspects of reality, such as non-pecuniary default spillovers, household behavior that can generate momentum in home prices, investor behavior that can generate realistic movements in the term premium, frictional conversion between the owner-occupied and renter-occupied stock, or refinancing inertia. Thus, while our model fits many important moments of the data quite well, it misses on others, and as a result our analysis should be seen as providing a sense of the magnitudes involved in different mortgage designs.

and FRMs are not necessarily optimal contracts, they provide the simplest and starkest comparison for us to analyze the benefits of indexation in a world where the cyclicality of interest rates is aligned with the cyclicality of income and home prices. We find that in a counterfactual economy suffering a crisis similar to the 2007-2009 recession with all ARMs instead of all FRMs, house prices fall by 2.24 percentage points less, 23 percent fewer households default, consumption falls by 0.79 percentage points less. Young, liquidity constrained households benefit to an even greater extent, to the point that they are willing to give up 4 percentage points more of their remaining lifetime consumption to avoid a crisis under FRM than under ARM. Unconditional on a crisis, an economy with ARMs has 4.6% lower aggregate consumption volatility and 4.1% lower mean default rate than an economy with FRMs. The household welfare benefit of switching from FRM to ARM – not conditional on a crisis — is more modest at 0.12 percent of consumption.[2] This number is relatively small because most household consumption risk is idiosyncratic and we calibrate households to have a utility function with coefficient of relative risk aversion of only three.

ARMs alleviate the impact of a crisis for three reasons. First, ARMs deliver larger payment reductions to constrained homeowners due to front-loading. ARM rates fall significantly more than FRM rates during the crisis because FRM rates are determined by the long end of the yield curve, which falls by less due to expectations-hypothesis-type logic. With FRMs, the payment relief is spread out over the remaining life of the mortgage, but with ARMs it is concentrated in the crisis. We find this "yield curve channel" to be quantitatively the most important way in which ARMs stabilize a housing crisis. Second, ARMs automatically pass interest rate reductions through to households. By contrast, FRMs only pass-through rate reductions when households refinance, which is not possible for households that, due to the fall in house prices, have insufficient equity to satisfy the LTV constraint. Therefore, underwater homeowners who are most at risk of default and in need of liquidity relief are unable to receive any. Since ARMs provide greater hedging benefits against declining labor income during the crisis, there is less default by underwater homeowners which short-circuits the equilibrium price-default spiral and leads to a less-severe housing crisis. We find the improved passthrough of interest rate reductions to underwater households under ARM to be a quantitatively smaller channel than the yield curve channel. Third and finally, because ARM rates fall more than FRM rates, ARMs are more effective at stimulating housing demand by constrained renters in the crisis, which further limits price declines and the price-default spiral.

One issue with a pure ARM is that in an inflationary episode, real interest rates can spike up while real income falls, with potentially catastrophic consequences. We consequently consider a new mortgage design that partially protects against this scenario: A fixed rate mortgage with a one-time option to convert to an adjustable rate mortgage, as suggested by Eberly and Krishnamurthy (2014). Of course, borrowers pay for the prepayment option with a higher average loan rate, which is offset somewhat by banks anticipating fewer defaults and losses in a crisis. Despite this cost, this "EK convertible" mortgage delivers much better outcomes than a standard fixed rate mortgage in

[2]Throughout we use a welfare metric based on households' consumption. In the model, lenders break even subject to a calibrated SDF. Different mortgage designs leave lender utility unchanged, under this fixed SDF. The model leaves out the possibility that mortgage designs may alter the SDF, e.g, which would happen if the SDF was based on the equilibrium consumption stream of lenders, as in Campbell, Clara, and Cocco (2019). We do report the present value of losses to lenders over a crisis (using their SDF), and our mortgage pricing condition ensures they break even on average (again subject to their SDF).

a "typical" crisis, realizing the vast majority of the benefits of the all-ARM economy when rates fall in a downturn.At the same time, the EK mortgage significantly outperforms the standard ARM in an inflationary episode in which rates rise during a housing bust.

We also consider a "FRM with an underwater refinancing option" (FRMUR) in which households with a fixed-rate mortgage have an option to refinance in a crisis into another fixed-rate mortgage with equal principal regardless of their loan-to-value ratio. This is motivated by the fact that lower long rates were not passed through to underwater households, which also motivated the government's HARP program. While the FRMUR does help these households, it does so by relatively little because the long end of the yield curve does not fall by much, and so the payment relief provided by the FRMUR is limited. Indeed, the gains in price, consumption, and default stability are small relative to the EK convertible mortgage. This is the case despite the fact that the decline in the present value of the bank's mortgage portfolio when the crisis hits is similar under these two designs.

The comparison of the EK convertible mortgage with the FRM with an option to be refinanced underwater provides the sharpest example of our central finding that the best designs are those that deliver immediate payment relief to liquidity constrained households rather than spreading the relief over the entire term of the mortgage. Consistent with this, we show that an option ARM design, which allows households to negatively amortize the mortgage up to a cap when liquidity needs arise at a cost of higher payments in the future, delivers higher prices and consumption – but more default – than both EK and ARM. Unlike those designs, the option ARM allows borrowers to defer payments as a function of her idiosyncratic state. Our analysis quantifies the benefits of such insurance in the mortgage contract, although we do not model the adverse selection it can induce.

Our analysis also calls attention to an important externality. When deciding their personal debt position, households do not internalize the impact of their debt choice and liquid asset position on macro fragility. This has important consequences in our model. For instance, ARMs provide more relief relative to FRMs if they are introduced at the moment the crisis occurs rather than *ex ante*. This is the case because homeowners expect the central bank to provide insurance by reducing short rates in the ARM economy and take on more risk by levering up more and holding less liquid savings, undoing some of the insurance benefit. Similarly, the insurance benefits of an option ARM (OARM) design encourage households to take on more leverage risk *ex ante*, which creates a more fragile pre-crisis LTV distribution than would otherwise be the case and higher default. These results highlight that policy makers must account for the fact that households do not share their macro-prudential concerns and may take on too much debt when insurance is offered.

Finally, we find that monetary policy and mortgage design should not be studied in isolation. Indeed, monetary policy efficacy depends on mortgage design, and mortgage design efficacy depends on monetary policy. We highlight this interaction by considering the performance of various mortgage designs under alternate monetary policies. In particular, we allow monetary policy to directly impact term premia. The most prominent example of this is unconventional monetary policy, such as the Fed's quantitative easing (QE) purchases of mortgage-backed securities to lower long-term mortgage rates when the short rate constrained by the zero lower bound. Our analysis of term premia can also shed light on the case where term premia are highly sensitive to changes in the short rate, which is suggested by some empirical evidence (*e.g.,* Hanson and Stein, 2015) but absent in our baseline model. We show that mortgage designs tied to the long rate such as FRM and FRMUR are most effective in this case when monetary policy is able to impact the term premia. This result also implies that *ex-post* policies such as HARP need to be combined with policies targeting the term premium, such as QE, in order to be maximally effective. This highlights the importance of studying mortgage design and monetary policy jointly.

The remainder of the paper is structured as follows. Section 2 describes the relationship to the existing literature. Section 3 presents our model, and Section 4 describes our calibration procedure. Section 5 compares the performance of ARM-only and FRM-only economies to develop economic intuition. Section 6 compares more exotic mortgage designs that combine beneficial features of both FRMs and ARMs, and Section 7 considers the interaction of mortgage design with monetary policy. Section 8 concludes.

2 Related Literature

This paper is most closely related to papers that analyze the role of mortgages in the macroeconomy through the lens of a heterogeneous agents model. In several such papers, house prices are exogenous. Campbell and Cocco (2015) develop a life-cycle model in which households can borrow using long-term fixed- or adjustable-rate mortgages and face income, house price, inflation, and interest rate risk. They use their framework to study mortgage choice and the decision to default. In their model, households can choose to pay down their mortgage, refinance, move, or default, and mortgage premia are determined in equilibrium through a lender zero-profit condition. Our modeling of households shares many structural features with this paper, but while they take house prices as an exogenous process, we crucially allow for aggregate shocks and determine equilibrium house prices. This critical feature of our model allows us to study the interaction of mortgage design with endogenous price-default spirals. A prior paper, Campbell and Cocco (2003), uses a more rudimentary model without default and with exogenous prices to compare ARMs and FRMs and assess which households benefit most from each design. Similarly, Corbae and Quintin (2015) present a heterogeneous agents model in which mortgages are priced in equilibrium and households select from a set of mortgages with different payment-to-income requirements, but again take house prices as exogenous. They use their model to study the role of leverage in triggering the foreclosure crisis, placing particular emphasis on the differential wealth levels and default propensities of households that enter the housing market when lending standards are relaxed. Conversely, we focus on the impact of mortgage design and monetary policy on housing downturns, allowing for endogenous house price responses.

Other heterogeneous agent models of the housing market have endogenous house prices but lack aggregate shocks or rich mortgage designs. Kung (2015) develops a heterogeneous agents model of the housing market in which house prices are determined in equilibrium. His model, however, lacks aggregate shocks and household saving decisions. He focuses specifically on the equilibrium effects of the disappearance of non-agency mortgages during the crisis. By contrast, we include aggregate shocks and a rich set of household decisions that Kung assumes away. We also study a variety of mortgage designs and analyze how mortgage design interacts with monetary policy. Kaplan, Mitman, and Violante (2019) present a life-cycle model with default, refinancing, and moving in the presence of idiosyncratic and aggregate shocks in which house prices are determined in equilibrium. Their focus, however, is on explaining what types of shocks can explain the joint dynamics of house prices and consumption in the Great Recession. They simplify many features of the mortgage contract for tractability in order to focus on these issues, while our paper simplifies the shocks and consumption decision in order to provide a richer analysis of mortgage design.[3]

Our paper also builds on a largely theoretical literature studying optimal mortgage design. These papers identify important trade-offs inherent in optimal mortgage design in a partial equilibrium settings. Concurrent research by Piskorski and Tchistyi (2017) studies mortgage design in a setting with equilibrium house prices and asymmetric information in a two-period model. The intuition they develop about the insurance benefits of state contingent contracts is complementary to our own, which is is more focused on the timing of payments over the life of the loan. Concurrent research by Greenwald, Landvoigt, and Van Nieuwerburgh (2018) studies shared appreciation mortgages (SAMs) that index payments to aggregate house prices in a model with a fragile financial sector. They show that the losses incurred by banks in a deep recession quantitatively outweigh the benefits to household balance sheets under a SAM. Our papers are highly complementary. We highlight the benefits of front-loading payment relief in mortgage designs that shift risk from households to financial intermediaries to a much more limited degree than a SAM, while GLVN study whether such risk shifting would be beneficial. Indeed, we have experimented with SAMs in our framework and found that the losses that banks incur in a crisis are an order of magnitude larger than in the designs we consider. Finally, concurrent work Campbell, Clara, and Cocco (2018) uses a model similar to the one we study but with exogenous house prices and an endogenous lender SDF to study mortgage design. By contrast, our house prices are endogenous but our lender SDF is exogenously calibrated. We see our papers as complementary but with different emphases: We focus on the front-loading of payments and put greater emphasis on crisis episodes while CCC focus on the covariance of payment reductions with lender consumption.

Beyond these three closely-related papers, Piskorski and Tchistyi (2010; 2011) consider mortgage design from an optimal contracting perspective, finding that the optimal mortgage looks like an option ARM when interest rates are stochastic and a subprime loan when house prices are stochastic. Brueckner and Lee (2017) focus on optimal risk sharing in the mortgage market. Our paper is also related to a literature advocating certain macroprudential polices aimed at ameliorating the severity of housing crises. Mian and Sufi (2015) advocate for modifications through principal reduction, while Eberly and Krishnamurthy (2014) advocate for monthly payment reductions. Greenwald (2019) advocates for payment-to-income constraints as a macroprudential policy to reduce house price volatility.

To calibrate our model, we draw on a set of papers which document empirical facts regarding household leverage and default behavior. Foote et al. (2008) provide evidence for a "double trigger" theory of mortgage default, whereby most default is accounted for by a combination of

[3]For instance, Kaplan, Mitman, and Violante (2019) assume that all mortgages have a single interest rate and that lenders break even by charging differential up front fees. By contrast, we maintain each borrower's interest rate and contract choice as a state variable.

negative equity and an income shock as is the case in our model. Bhutta et al. (2010), Elul et al. (2010), and Gerardi et al. (2013) provide further support for illiquidity as the driving source of household default. Fuster and Willen (2017) and Di Maggio et al. (2017) show that downward rate resets lead to reductions in default and increases in household consumption, respectively. Agarwal et al. (2017b), Agarwal et al. (2017a), and Ganong and Noel (2019) study the HAMP and HARP programs and find similarly large effects of payment on default and consumption and limited effects of principal reduction for severely-underwater households, which they relate to the immediate benefits of payment relief versus the delayed benefits of principal reduction. This micro evidence motivates our focus on mortgage designs with state-contingent payments, and we use Fuster and Willen's evidence to evaluate the quantitative performance of our model.

Finally, our research studies how mortgage design interacts with monetary policy and thus relates to a literature examining the transmission of monetary policy through the housing market. Caplin, Freeman, and Tracy (1997) posit that in depressed housing markets where many borrowers owe more than their house is worth, monetary policy is less potent because individuals cannot refinance. Beraja, Fuster, Hurst, and Vavra (2019) provide empirical evidence for this hypothesis by analyzing the impact of monetary policy during the Great Recession and study the interaction of monetary policy with mortgage design. Relatedly, a set of papers have argued that adjustable-rate mortgages allow for stronger transmission of monetary policy since rate changes directly affect household balance sheets (Calza et al., 2013; Auclert, 2019; Cloyne et al., 2019). Garriga et al. (2016) provide a model with long-term debt that features a yield curve and is related to our findings about the differential effects of mortgage designs that are priced off the short end relative to the long end of the yield curve. Di Maggio et al. (2017) show empirically that the pass-through of monetary policy to consumption is stronger in regions with more adjustable rate mortgages. Finally, Wong (2019) highlights the role that refinancing by young households plays in the transmission of monetary policy to consumption.

3 Model

This section presents an equilibrium model of the housing market that we subsequently use as a laboratory to study different mortgage designs. Home prices and mortgage spreads are determined in equilibrium. Short-term interest rates, on the other hand, are exogenous to the model and depend on the aggregate state of the economy, reflecting an exogenous monetary policy rule. For ease of exposition, we present the model for the case of an all-FRM economy or an all-ARM economy, but consider other designs when presenting our quantitative results.

3.1 Setup

Time is discrete and indexed by t. The economy consists of a unit mass of overlapping generations of heterogeneous households of age $a = 1, 2, ..., T$ who make consumption, housing, borrowing, refinancing and default decisions over their lifetime. Household decisions depend both on aggregate state variables Σ_t and agent-specific state variables s_t^j, where j indexes agents. Unless otherwise stated, all variables are agent-specific, and to simplify notation, we suppress their dependency on

s_t^j.

The driving shock process in the economy is Θ_t, which is part of Σ_t. Θ_t follows a discrete Markov process over five states $\Theta_t \in$\{Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit\}. Θ_t is governed by a transition matrix Ξ^{Θ} described subsequently.

Each generation lives for T periods. At the beginning of a period, a new generation is born and shocks are realized. Agents then make decisions, and the housing market clears. Utility is realized and the final generation dies at the end of the period. Households enter period t with a state s_{t-1}^j and choose next period's state variables s_t^j in period t given the period t housing price p_t. Utility is based on period t choices, and the short rate r_t realized at time t is the interest rate between t and $t+1$.

Households receive flow utility from housing H_t and non-durable consumption C_t:[4]

$$U(C_t, H_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \alpha_a H_t.$$

In the last period of life, age T, a household with terminal wealth b receives utility:

$$\frac{C_t^{1-\gamma}}{1-\gamma} + \psi \frac{(b+\xi)^{1-\gamma}}{1-\gamma},$$

where $H_T = 0$ because the terminal generation must sell.[5] For simplicity, we assume that households use their wealth to finance housing and end-of-life care after their terminal period. Consequently, the wealth b is not distributed to incoming generations, who begin life with no assets.[6]

Households receive an exogenous income stream Y_t:

$$Y_t \equiv \exp\left(y_t^{agg}(\Theta_t) + y_t^{id}\right).$$

Log income is the sum of an aggregate component that is common across households and a household-specific idiosyncratic component. The aggregate component y_t^{agg} is a function of the aggregate state Θ_t. The idiosyncratic component y_t^{id} is a discrete Markov process over a set $\left\{Y_t^{id}\right\}$ with transition matrix $\Xi^{id}(\Theta_t)$.

Households retire at age $R < T$. After retirement, households no longer face idiosyncratic income risk and keep the same idiosyncratic income they had at age R, reduced by ρ log points to account for the decline in income in retirement. This can be thought of as a social security benefit that conditions on terminal income rather than average lifetime income for computational tractability, as in Guvenen and Smith (2014).

[4]The term α_a describes the utility from homeownership as a function of age. In our calibration, we will assume that α_a is decreasing in age so as to reflect the fact that at older ages the homeownership rate declines slightly.

[5]Including terminal wealth in the utility function is standard in OLG models of the housing market because otherwise households would consume their housing wealth before death. In the data, however, the elderly have substantial housing wealth which they do not consume. The functional form for the utility derived from terminal wealth is standard.

[6]We make this assumption for computational simplicity. In practice, few people receive significant bequests. Consequently, distributing bequests to younger generations would not significantly alter the distribution of wealth or our results.

There is a progressive tax system so that individuals' net-of-tax income is $Y_t - \tau(Y_t)$. The tax system is modeled as in Heathcote et al. (2017) so that:

$$\tau(Y_t) = Y_t - \tau_0 Y_t^{1-\tau_1}.$$

Houses in the model are of one size, and agents can either own a house ($H_t = 1$) or rent a house ($H_t = 0$).[7] There is a fixed supply of housing and no construction implying that the homeownership rate is constant.[8] Buying a house at time t costs p_t, and owners must pay a per-period maintenance cost of mp_t. With probability ζ, homeowners experience a life event that makes them lose their match with their house and list it for sale, while with probability $1-\zeta$, owners are able to remain in their house.

The rental housing stock is entirely separate from the owner-occupied housing stock. Rental housing can be produced and destroyed at a variable cost q, so in equilibrium renting costs q per period. Although this assumption is stark, it is meant to capture that while there is some limited conversion of owner-occupied homes to rental homes and vice-versa in practice, the rental and owner-occupied markets are quite segmented (Glaeser and Gyourko, 2009; Halket et al., 2015; Greenwald and Guren, 2019).[9] This assumption implies that movements in house prices are accompanied by movements in the price-to-rent ratio, as is the case in the data.

A household's end of period t mortgage balance is $M_t \geq 0$ and carries interest rate i_t. We make a timing assumption that the interest paid at date t is $i_t M_t$, so that households pay their interest between periods t and $t+1$ in advance at time t. With an annual calibration, this implies that the realization of interest rates immediately impacts payments for both an adjustable- and fixed-rate mortgage. An alternative timing convention would be to incur the payment of $i_t M_t$ at date $t+1$, which would imply that interest rate changes would affect FRMs a year after ARMs. In practice, mortgage payments are monthly and homeowners make decisions at an even higher frequency, so that our up-front payment timing is a better representation of reality within our model and puts ARMs and FRMs on equal footing. We assume that all debt in our model has this same timing

[7]We assume that houses are one size to maintain a computationally tractable state space in an environment with rich mortgage design. In practice, the average house size does grow over the life cycle with age (see *e.g.*, Li and Yao, 2007) and house size grows with income. Assuming one house size leads richer agents in our economy to have more liquid assets and lower LTVs than in the data. This is not problematic for our calibration as the marginal agents for purchasing and default are poorer.

[8]We assume a fixed housing supply to keep the model tractable given the lags required to realistically model construction. Adding a construction response would dampen a boom but would not dramatically affect busts given the durability of housing (Glaeser and Gyourko, 2005).

[9]To maintain tractability, heterogeneous agent models of the type we consider here typically make the assumption of a perfectly segmented rental market or the polar opposite assumption of no segmentation and perfect arbitrage between own and rent (e.g., Kaplan, Mitman, and Violante, 2019). Greenwald and Guren (2019) analyze the importance of these assumptions. They show that perfect segmentation implies that changes in credit conditions – such as the tightening LTV constraint we use to start the crisis – will affect aggregate demand for housing and prices. Similarly, foreclosures will affect house prices. By contrast, with perfect arbitrage between renting and owning, credit conditions and foreclosure will change the homeownership rate but not house prices, which are pinned down by the present value of rents for landlords. Guren and Greenwald calibrate a model that can nest these extreme cases to match their micro evidence and find that in practice there is significant segmentation between owner-occupied and rental markets, which we see as supportive of our stark assumption. Nonetheless, adding a realistic amount of rental conversion would somewhat reduce the size of the equilibrium effects that foreclosures have on price and the price-foreclosure spiral at the heart of our model.

convention, and when we calibrate our model we convert interest rates to be consistent with the timing in our model.[10]

Mortgage interest is tax deductible, so that taxes are $\tau\left(\max\left\{Y_t - i_t M_t, 0\right\}\right)$. In order to economize on state variables, the mortgage amortizes over its remaining life as in Campbell and Cocco (2003, 2015). This rules out mortgage designs with variable term lengths, but still allows for the analysis of mortgage designs that rely on state-dependent payments. The appendix shows that the minimum payment on a mortgage for an agent who does not move or refinance at time t given our timing assumption is:

$$M_{t-1}\frac{\left(i_t(1+\frac{i_t}{1-i_t})^{T-a+1}\right)}{(1+\frac{i_t}{1-i_t})^{T-a+1}-1}. \tag{1}$$

For an FRM, the household keeps the same interest rate i_t determined at origination $i_t^{FRM}\left(\Theta_t\right)$, which is the same for all borrowers who originate in aggregate state Θ_t and determined competitively as described below. The interest rate on an adjustable-rate mortgage is $i_t = \frac{r_t}{1+r_t} + \chi_t^{ARM}\left(\Theta_t\right)$ where $\chi_t^{ARM}\left(\Theta_t\right)$ is a spread over the short rate that borrowers keep over the life of their loan, also determined at origination and dependent on origination state Θ_t.[11] This will also be the same for all borrowers in a given aggregate state Θ_t and determined competitively as described below. The short interest rate $r_t\left(\Theta_t\right)$ is a function of the exogenous and stochastic aggregate state Θ_t.

At origination, mortgages must satisfy a loan to value constraint:

$$M_t\left(a\right) \leq \phi_t p_t, \tag{2}$$

where ϕ_t parameterizes the maximum loan-to-value ratio. Mortgages are non-recourse but defaulting carries a utility penalty of d which is drawn each period i.i.d. from a uniform distribution over $[d_a, d_b]$.[12] Defaulting households lose their house today and cannot buy a new house in the period of default due to damaged credit. The default goes on their credit record, and they are unable to purchase until the default flag is stochastically removed.

Each period, homeowners can take one of four actions in the housing market: take no action with regards to their mortgage and make at least the minimum mortgage payment (N), refinance but stay in their current house (R), move to a new house and take out a new mortgage (M), or default (D). Note that if a household refinances or moves to a new house, they must take out an entirely new mortgage which is subject to the LTV constraint in equation (2). Moving has a cost of $k_m + c_m p_t$ for both buying and selling, while refinancing has a cost of $k_r + c_r M_t$.

Homeowners occasionally receive a moving shock that forces them to move with probability ζ.

[10] In particular, the interest rate i_t is a "pre-paid" interest rate (interest from t to $t+1$ paid at t), whereas the interest rate r_t and most interest rates observed in the data are "post-paid" interest rates (interest from t to $t+1$ paid at $t+1$). We convert post-paid interest rates r_t to pre-paid interest rates i_t using $i_t = \frac{r_t}{1+r_t}$ or back using $r_t = \frac{i_t}{1-i_t}$.

[11] When we refer to an ARM in this paper, we refer to a fully-adjustable-rate mortgage that adjusts every year. In many countries, hybrid ARMs that have several years of a fixed interest rate and float thereafter are known as "adjustable rate." Aside from replicating the Fuster-Willen quasi-experiment in evaluating our calibration, we do not consider hybrid ARMs to maintain a tractable state space.

[12] The assumption that d is drawn from a distribution rather than a single value helps smooth out the value functions in the numerical implementation, but is not crucial for our results. In practice, d_a and d_b are close and the model is essentially a single default cost model.

In this case, they cannot remain in their current house and either move or default, while agents who do not receive the moving shock are assumed to remain in their house and can either do nothing, refinance, or default. Households reaching the end of life must sell. Regardless of whether they receive a moving shock ζ, renters can either do nothing and pay their rent (N) or move into an owner-occupied house (M) each period.

3.2 Decisions and Value Functions

Consider a household at time t that enters the period with owned housing $H_{t-1} \in \{0, 1\}$, a mortgage with principal balance M_{t-1}, and $S_{t-1} \geq 0$ in liquid savings. The household may also have a default flag on its credit record $D_{t-1} = \{0, 1\}$. The state of the economy at time t, Θ_t, is realized. The household receives income Y_t. The agent-specific state households enter period t with is $s_{t-1}^j = \left\{S_{t-1}, H_{t-1}, M_{t-1}, i_{t-1}^{FRM}, Y_t, D_{t-1}, a_t\right\}$, a vector of the household's assets, liabilities, income, credit record default status, interest rate for an FRM (or spread χ_{t-1} for an ARM), and age. The vector of aggregate state variables Σ_t includes the state of the economy Θ_t, and $\Omega_t(s_{t-1}^j)$, the cumulative distribution of individual states s_{t-1}^j in the population. The home price p_t is a function of Σ_t.

The household faces two constraints. The first is a flow budget constraint:

$$
\begin{aligned}
Y_t - \tau\left(Y_t - i_t M_t\right) + S_{t-1} + \left(1 - i_t\right) M_t = C_t + \frac{S_t}{1 + r_t} + M_{t-1} + p_t\left(H_t - H_{t-1}\right) \\
+ q 1\left[H_t = 0\right] + m p_t 1\left[H_t = 1\right] + K\left(\mathit{Action}\right),
\end{aligned} \tag{3}
$$

where $K\left(\mathit{Action}\right)$ is the fixed or variable cost of the action the household takes. The left hand side of this expression is the sum of net-of-tax income, liquidated savings, and new borrowings. The right hand side is the sum of consumption, savings for the next period, payments on existing mortgage debt, net expenditures on owner-occupied housing, rental or maintenance costs, and the fixed and variable costs of the action that the household takes.

The second constraint addresses the evolution of a household's mortgage. Given a mortgage balance M_t, implicitly define ΔM_t as the change in the mortgage balance over and above the minimum payment:

$$
\Delta M_t = M_t - M_{t-1} + M_{t-1} \frac{\left(i_t (1 + \frac{i_t}{1 - i_t})^{T-a+1}\right)}{(1 + \frac{i_t}{1 - i_t})^{T-a+1} - 1} - M_t i_t \tag{4}
$$

If ΔM_t is positive, the mortgage balance has risen relative to the minimum payment and the homeowner has extracted equity, and if ΔM_t is negative the mortgage balance has prepaid. Thus, households that do not move, refinance, or default face a constraint of $\Delta M_t \leq 0$. If a household moves, it pays off its mortgage balance and chooses a new mortgage balance M_t, subject to the LTV constraint (2). Finally, a household may also choose to default, in which case it loses its house today and cannot buy, so $M_t = H_t = 0$. The household also receives a default on its credit record so $D_t = 1$ and cannot buy again until its credit record is cleared, which occurs each period with probability λ.

We write the household's problem recursively. Denote $V_a\left(s^j_{t-1};\Sigma_t\right)$ as the value function for a household, and $V^A_a\left(s^j_{t-1};\Sigma_t\right)$ as the values when following action $A=\{N,R,M,D\}$. Then,

$$V_a\left(s^j_{t-1};\Sigma_t\right)=\begin{cases}\zeta\max\left\{V^D_a\left(s^j_{t-1};\Sigma_t\right),V^M_a\left(s^j_{t-1};\Sigma_t\right)\right\}+ & \\ (1-\zeta)\max\left\{V^D_a\left(s^j_{t-1};\Sigma_t\right),V^R_a\left(s^j_{t-1};\Sigma_t\right),V^N_a\left(s^j_{t-1};\Sigma_t\right)\right\} & \text{if } H_{t-1}>0\\ \max\left\{V^M_a\left(s^j_{t-1};\Sigma_t\right),V^N_a\left(s^j_{t-1};\Sigma_t\right)\right\} & \text{if } H_{t-1}=0 \text{ and } D_{t-1}=0\\ V^N_a\left(s^j_{t-1};\Sigma_t\right) & \text{if } H_{t-1}=0 \text{ and } D_{t-1}=1.\end{cases}$$

On the top line, if the household receives the moving shock with probability ζ, it must decide whether to default on the existing mortgage and be forced to rent, or pay off the mortgage balance, in which case it can freely decide whether to rent or finance the purchase of a new home. On the second line, if the household does not receive the moving shock, it decides between defaulting, refinancing, or paying the minimum mortgage balance.[13] Finally, in the last two lines, a household that currently has no housing (currently a renter or just born) and does not have a default on their credit record can decide whether to purchase a house and take on a new mortgage or continue to rent. Renters with a default on their credit records $D_t=1$ cannot purchase.

We next define the value functions under each of the actions $A=\{N,R,M,D\}$. Households who pay their mortgage choose their mortgage payment, savings, and consumption and have a value function:

$$V^N_a\left(s^j_{t-1};\Sigma_t\right)=\max_{C_t,S_t,M_t}U\left(C_t,H_t\right)+\beta E_t\left[V_{a+1}\left(s^j_t;\Sigma_{t+1}\right)\right]\text{ s.t. (3)},$$
$$S_t\geq 0,$$
$$H_t=H_{t-1},$$
$$i_t=i_{t-1}\text{ for FRM, }\ \chi_t=\chi_{t-1}\text{ for ARM}$$
$$\Delta M_t<0.$$

Households who refinance make the same choices, but pay the fixed and variable costs of refinancing (which can be rolled into their new mortgage) and face the LTV constraint rather than the $\Delta M_t<0$

[13]Our model assumes that household refinance optimally. However, there is an empirical literature shows that there is significant inattention to refinancing (e.g., Keys, Pope and Pope (2016), Johnson, Meier, and Toubia (2019), and Andersen et al. (2018)). This literature implies that contracts that automatically index payments to the state of the economy, such as an ARM, are even more welfare enhancing relative to contracts that require a refinancing, such as FRM, than our model indicates.

constraint. They have a value function:

$$V_a^R\left(s_{t-1}^j;\Sigma_t\right)=\max_{C_t,S_t,M_t}\left\{U\left(C_t,H_t\right)+\beta E_t\left[V_{a+1}\left(s_t^j;\Sigma_{t+1}\right)\right]\right\} \text{ s.t. (3)},$$
$$S_t\geq 0,$$
$$M_t\leq \phi p_t \text{ if } H_{t+1}=1,$$
$$H_t=H_{t-1},$$
$$i_t=i_t^{FRM} \text{ for FRM}, \chi_t=\chi_t^{ARM} \text{ for ARM}.$$

Households who move choose their consumption, savings, and if buying, mortgage balance, as refinancers do, but also get to choose their housing H_{t+1}. They have a value function:

$$V_a^M\left(s_{t-1}^j;\Sigma_t\right)=\max_{C_t,S_t,M_t,H_t}\left\{U\left(C_t,H_t\right)+\beta E_t\left[V_{a+1}\left(s_t^j;\Sigma_{t+1}\right)\right]\right\} \text{ s.t. (3)},$$
$$S_t\geq 0,$$
$$M_t\leq \phi p_t \text{ if } H_{t+1}=1,$$
$$i_t=i_t^{FRM} \text{ for FRM}, \chi_t=\chi_t^{ARM} \text{ for ARM}.$$

Households who default lose their home but not their savings. The defaulting households choose consumption and savings and have a value function:

$$V_a^D\left(s_{t-1}^j;\Sigma_t\right)=\max_{C_t,S_t}\left\{-d+U\left(C_t,H_t\right)+\beta E_t\left[V_{a+1}\left(s_t^j;\Sigma_{t+1}\right)\right]\right\} \text{ s.t. (3)}$$
$$S_t\geq 0,$$
$$H_t=M_t=0$$
$$D_t=1.$$

In the final period, a household must liquidate its house regardless of whether it gets a moving shock, either through moving or defaulting:

$$V_T\left(s_t^j;\Sigma_t\right)=\max\left\{V_T^N\left(s_t^T;\Sigma_t\right),V_T^D\left(s_t^T;\Sigma_t\right)\right\}.$$

3.3 Mortgage Spread Determination

We assume that mortgages are supplied by competitive lenders who discount payoffs using a stochastic discount factor (SDF) $m_{t,t+1}$, which is a function of today's aggregate state Θ_t and tomorrow's state Θ_{t+1}. We do not endogenize the lender SDF but instead calibrate it using options market data in Section 4.[14] Importantly, our calibration sets high state prices for crisis states so that lenders charge for the insurance implicit in the different mortgage designs we consider.

Define the net present value of the expected payments made by an age a household with idiosyncratic state s_{t-1}^j and aggregate state Σ_t, which is the value of the mortgage to a lender, as

[14] See Campbell, Clara, and Cocco (2019) for a careful analysis of how an endogenous lender SDF affects mortgage design

$\Pi_a\left(s_{t-1}^j;\Sigma_t\right)$. This can be written recursively as:

$$\Pi_a\left(s_{t-1}^j;\Sigma_t\right)=\delta\left(s_{t-1}^j;\Sigma_t\right)\Upsilon p_t+\sigma\left(s_{t-1}^j;\Sigma_t\right)M_{t-1}+ \\ \left(1-\delta\left(s_{t-1}^j;\Sigma_t\right)-\sigma\left(s_{t-1}^j;\Sigma_t\right)\right)\left[\begin{array}{c}M_{t-1}-M_t\left(1-i_t\right)\\+m_{t,t+1}E_t\left[\Pi_{a+1}\left(s_t^j,\Sigma_{t+1}\right)\right]\end{array}\right], \tag{5}$$

where the household policy functions, $\sigma\left(s_{t-1}^j,\zeta;\Sigma_t\right)$ is an indicator for whether a household moves or refinances, $\delta\left(s_{t-1}^j,\zeta;\Sigma_t\right)$ is an indicator for whether a household defaults. In the event of default, the lender forecloses on the home, sells it in the open market, and recovers a fraction Υ of its current value. In the present period, the lender receives the recovered value in the event of a foreclosure, the mortgage principal plus interest in the event the loan is paid off, and the required payment on the mortgage plus any prepayments made by the borrower if the loan continues. The lender also gets the discounted expected continuation value of the loan at the new balance if the loan continues and discounts according to the SDF $m_{t,t+1}$.

We assume that the interest rate on an FRM originated at time t, i_t^{FRM}, and the spread over the short rate on an ARM originated at time t, χ_t^{ARM}, are determined competitively such that lenders break even on average in each aggregate state:

$$E_{\Theta_t=\Theta_i}\left[E_{\Omega_t^{orig}}\left[m_{t,t+1}\Pi_a\left(s_t^j;\Sigma_{t+1}\right)-\left(1-i_t\right)M_t\right]\right]=0\ \forall\ \Theta_i\in\Theta, \tag{6}$$

where Ω_t^{orig} is the distribution of newly originated mortgages at time t. The expectation integrates out over all periods in which Θ_t takes on a given value Θ_i and over all loans originated at time t.[15] This pools risk across borrowers but prices the mortgage to incorporate all information on the aggregate state of the economy, Θ_i. By allowing the pricing to depend on the aggregate state, we allow mortgage rates to depend on the interest rate in that state as well as the expected path of interest rates conditional on that state. We also allow pricing to depend on the endogenous prepayment risk and default risk of mortgages as originated in a given state. Thus, our assumptions imply that if a mortgage design shifts risks from borrowers to lenders in a given state, the spread rises until the lenders are compensated for this risk.

Our assumption of a single spread for each aggregate state implies that there is cross-subsidization in mortgage pricing. Because default is low on average in equilibrium, the amount of cross-subsidization is not substantial.[16] Moreover, in practice, there is cross-subsidization in GSE mortgage pricing: Hurst et al. (2016) document that GSE mortgage rates for otherwise identical loans do not vary spatially.

[15] M_t is the end of period mortgage balance. Given the timing, the household immediately makes a mortgage payment if i_tM_t, and so on net the bank gives the household $(1-i_t)M_t$.

[16] Banks would make large losses on low-income homeowners who cannot afford their mortgage payment. However, such households cannot make a down payment and cannot afford the fixed and variable costs of home purchase and thus do not purchase in equilibrium. Those homeowners that do obtain mortgages do vary in their default risk, but it is generally low.

3.4 Equilibrium

A competitive equilibrium consists of decision rules over actions $A = \{N, R, M, D\}$ and state variables C_t, S_t, M_t, H_t, a price function $p(\Sigma_t)$, an FRM rate $i^{FRM}(\Theta)$ or an ARM spread $\chi^{ARM}(\Theta)$ for each aggregate state Θ, and a law of motion for the aggregate state variable Σ_t. Decisions are optimal given the home price function and the law of motion for the state variable. At these decisions, the housing market clears at price p_t, the risk-neutral lenders break even on average according to (6), and the law of motion for Σ_t is verified.

Given the fixed supply of homes, market clearing equates supply from movers, defaulters, and investors who purchased last period with demand from renters, moving homeowners, and investors. Let $\eta\left(s^j_{t-1}, \zeta; \Sigma_t\right)$ be an indicator for whether a household moves and $\delta\left(s^j_{t-1}, \zeta; \Sigma_t\right)$ be an indicator for whether a household defaults. Movers and defaulters own $H_{t-1}\left(s^j_{t-1}; \Sigma_t\right)$ housing, while buyers purchase $H_t\left(s^j_t; \Sigma_t\right)$ housing. The housing market clearing condition satisfied by the pricing function $p\left(\Sigma_t\right)$ is then:

$$\int \delta\left(s^j_{t-1}, \zeta; \Sigma_t\right) H_{t-1}\left(s^j_{t-1}; \Sigma_t\right) d\Omega_t + \int \eta\left(s^j_{t-1}, \zeta; \Sigma\right) H_{t-1}\left(s^j_{t-1}; \Sigma_t\right) d\Omega_t \qquad (7)$$
$$= \int \eta\left(s^j_{t-1}, \zeta; \Sigma_t\right) H_t\left(s^j_{t-1}; \Sigma_t\right) d\Omega_t,$$

where the first line side is supply which includes defaulted homes and sales and the second line is demand.

3.5 Solution Method

Solving the model requires that households correctly forecast the law of motion for Σ_t which drives the evolution of home prices. Note that Σ_t is an infinite-dimensional object due to the distribution $\Omega_t(s^j_{t-1})$. To deal with this issue, we follow the implementation of the Krusell and Smith (1998) algorithm in Kaplan, Mitman, and Violante (2019). We focus directly on the law of motion for home prices and assume that households use a simple AR(1) forecast rule that conditions on the state of the business cycle today Θ_t and the realization of the state of the business cycle tomorrow Θ_{t+1} for the evolution of p_t:

$$\log p_{t+1} = f_{(\Theta_t, \Theta_{t+1})}\left(\log p_t\right) \qquad (8)$$

where $f_{(\Theta_t, \Theta_{t+1})}$ is a function for each realization of (Θ_t, Θ_{t+1}). We parameterize $f(\cdot)$ as a linear spline.[17] Expression (8) can be viewed either as a tool to compute equilibrium in heterogeneous-agent economies, following Krusell and Smith (1998) or as an assumption that households and investors are boundedly rational and formulate simple forecast rules for the aggregate state. To verify that the decision rule is accurate, we both compute the R^2 for each (Θ_t, Θ_{t+1}) realized in simulations and follow Den Haan (2010) by comparing the realized price with the 15, 30, 45, and

[17] We have found that a linear spline performs better than a linear function. The relationship is approximately linear in periods with no default and linear in periods with some default, although the line bends when default kicks in. A linear spline flexibly captures this relationship. For the numerical implementation, we use the discretized price grid points for our spline knot points.

100-year ahead forecasts given the realized process of aggregate shocks to verify that the forecast rule does a good job of computing expected prices many periods into the future and that small errors do not accumulate. The Appendix shows this is the case.

The model cannot be solved analytically, so a computational algorithm is used. First, the household problem is solved for a given forecast rule and mortgage spreads by discretization and backward induction. The model is simulated for 19,000 periods with the home price determined by (7). Given the distribution of mortgages originated in each state, the break-even spread for each aggregate state is determined according to (6), and the AR(1) forecast regression (8) is run in the simulated data for each (Θ_t, Θ_{t+1}). Finally, the forecast rule is updated based on the regression and the spread is updated based on the break-even spread, and the entire procedure is repeated until the forecast rules and spreads converge.

4 Calibration

Our calibration proceeds in three steps. First, we select the aggregate and idiosyncratic shocks to reflect modern business cycles in the United States. Second, we exogenously calibrate a number of parameters to standard values in the macro and housing literature. The final parameters are calibrated internally to match moments of the data. Our model does a good job of matching the life cycle and population distributions of assets and mortgage debt. Furthermore, as a validation exercise, we show that our model quantitatively matches quasi-experimental evidence on the effects of payment reductions on default.

Throughout, we calibrate to the data using a model in which all loans are fixed rate mortgages to reflect the predominant mortgage type in the United States. Table 1 summarizes the variables and their calibrated values. The calibration is annual.

4.1 Aggregate and Idiosyncratic Shocks

We consider an economy that occasionally experiences crises akin to what occurred in the Great Recession. To trigger such a downturn, we combine a deep and persistent recession – which lowers aggregate income and leads to more frequent negative idiosyncratic shocks – with a tightening of credit in the form of a stricter downpayment constraints. Several papers argue that tightening credit helped amplify the bust and model this as a tightening LTV constraint (*e.g.,* Favilukis et al., 2017; Justiniano et al., 2019). We consequently assume that credit always tightens in a crisis and then stochastically reverts to being loose in expansions (but not in recessions). Since there is insufficient data to evaluate how monetary policy differs in various credit regimes, we assume that income and monetary policy are identical in recessions with high and low credit and expansions with high and low credit. This implies that the transition matrix between the five aggregate states $\Theta_t \in$ {Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit} can be represented as a transition matrix between three states {Crisis, Recession, Expansion} along with a probability that credit switches form tight to loose in the tight credit expansion state.

We calibrate the Markov transition matrix between crisis, recession, and expansion based on

Table 1: Model Parameters in Baseline Parameterization

Param	Description	Value	Param	Description	Value
T	Years in Life	45	c_m	Variable Moving Cost as % of Price	3.0%
R	Retirement	35	k_m	Fixed Moving Cost	0.1
ρ	Log Income Decline in Retirement	0.35	c_r	Variable Refi Cost as % of Mortgage	1.0%
τ_0	Constant in Tax Function	0.8	k_r	Fixed Refi Cost	0.04
τ_1	Curvature Tax Function	0.18	d_a	Default Cost Dist Lower Bound	39.75
γ	CRRA	3.0	d_b	Default Cost Dist Upper Bound	49.75
ξ	Bequest Motive Shifter	0.570	q	Rent	0.20
ψ	Bequest Motive Multiplier	250	m	Maint Cost as % of Prices	2.5%
a	Utility From Homeownership	6.355	ζ	Prob of Moving Shock	1/9
β	Discount Factor	0.95	λ	Prob Default Flag Removed	1/3
Υ	Foreclosure Sale Recovery %	0.654		Homeownership Rate	65.0%
ϕ_{loose}	Max LTV, Loose Credit	95.0%	ϕ_{tight}	Max LTV, Tight Credit	80.0%
r	Short Rate	[0.26%, 1.32%, 3.26%] (crisis, recession, expansion)			
Y^{agg}	Aggregate Income	[0.0976, 0.1426, 0.1776]			
See appendix for transition matrix for Θ_t and Y_t^{id}.					

Note: This table shows parameters for the baseline calibration. Average income is normalized to one. There are five aggregate states, $\Theta_t \in$ {Crisis With Tight Credit, Recession With Tight Credit, Recession With Loose Credit, Expansion With Tight Credit, Expansion With Loose Credit}, but we assume that income and monetary policy are the same in a recession with loose or tight credit and in an expansion with loose or tight credit. The tuples of interest rates reflect the interest rate in a crisis, recession, and expansion, respectively.

the frequency and duration of NBER recessions and expansions. We use the NBER durations and frequencies to determine the probability of a switch between an expansion and crisis or recession, and we assume that crises happen every 75 years and that all other NBER recessions are regular recessions. We assume that every time the economy exits a crisis or recession it switches to an expansion and that crises affect idiosyncratic income in the same way as a regular recession but last longer and involve a larger aggregate income drop, with a length calibrated to match the average duration of the Great Depression and Great Recession. A regular recession reduces aggregate income by 3.5%, while a crisis reduces it by 8.0%, consistent with Guvenen et al.'s (2014) data on the decline in log average earnings per person in recessions since 1980. For the probability of reverting to loose credit from tight credit in an expansion, we choose 2.0%, so that when credit tightens it does so persistently but credit loosens quickly enough so that a large number of crises begin in the loose credit state. Our results are not sensitive to perturbing this target. The full transition matrices can be found in the appendix.

For the idiosyncratic income process, we match the countercyclical left skewness in idiosyncratic income shocks found by Guvenen et al. (2014). Left skewness is crucial to accurately capturing the dynamics of a housing crisis because the literature on mortgage default has found that large income shocks are crucial drivers of default. To incorporate left skewness, we calibrate log idiosyncratic income to follow a Gaussian AR(1) with an autocorrelation of 0.91 and standard deviation of 0.21 following Floden and Linde (2001) in an expansion but to have left skewness in the shock distribution in recessions and crises. We discretize the income process in an expansion by matching the mean and standard deviation of shocks using the method of Farmer and Toda (2017), which discretizes the distribution and optimally adjusts it to match the mean and variance of the distribution to be discretized. For the bust, we add the standardized skewness of the 2008-9 income change distribution from Guvenen et al. (2014) to moments to be matched, generating a shock distribution with left skewness. This gives a distribution with a negative mean income change in busts and leads to income being too volatile, so we shift the mean of the idiosyncratic shock distribution in busts to match the standard deviation of aggregate log income in the data. In doing so, we choose the income distribution of the newly born generations to match the lifecycle profile of income in Guvenen et al. (2019).[18] We normalize the income process so that average aggregate income is equal to one.

4.2 Interest Rates and Lender SDF

Recall that the lender SDF, $m_{t,t+1}$, is a function of today's state, Θ_t, and tomorrow's state, Θ_{t+1}, so we must calibrate $m(\Theta_t, \Theta_{t+1})$. To do so, we match interest rate data, mortgage spread data, and options-pricing results from Backus, Chernov, and Martin (2011).

In calibrating the SDF, our goal is to ensure that lenders charge for mortgage contracts that provide insurance in crisis states. Because the economy only switches into a crisis from an expansion (with either tight or loose credit), we allow $m\,(\text{Expansion, Crisis})$ to differ from $m\,(\text{Expansion, Non-Crisis})$. Backus, Chernov, and Martin (2011) show that in order to match the prices of out-of-the-money S&P500 options, one needs a model with jumps, where the ratio of the risk-neutral jump intensity to the true jump intensity is 6.1 (see Table II of their paper). We will think of our crisis as a jump event and thus calibrate the risk price for the crisis state relative to non-crisis states as 6.1; that is, we set $m\,(\text{Expansion, Crisis}) = 6.1 \times m\,(\text{Expansion, Non-Crisis})$. In this way, the lenders charge a high price for insurance against crisis states consistent with the high price of out-of-the-money options.

To keep things simple, for $m\,(\text{Non-Expansion}, \Theta_{t+1})$ we assume that the SDF is constant regardless of tomorrow's state Θ_{t+1} and is moreover equal to $m\,(\text{Expansion, Non-Crisis})$. Finally, our lender SDF is required to satisfy:

$$E_t\left[m(\Theta_t, \Theta_{t+1})\right] = \frac{1}{1 + r_t + \kappa}.$$

Here r_t is the short-rate in state Θ_t. Lenders require that a sure payoff of one at date $t+1$ yields a return of $r_t + \kappa$, where κ captures the costs of making mortgage loans, including administrative and regulatory costs that may increase the lender cost of capital. We set κ so that in an all-FRM economy under our baseline monetary policy, the average spread between the FRM rate and a 10-year bond in the model is 1.65%, which is the average spread between the 30-year fixed mortgage rate and the 10-year bond in the data from 1983 to 2013 from FRED.[19] [20] We find κ to be 125

[18] Rather than including a deterministic income profile, we start households at lower incomes and let them stochastically gain income over time as the income distribution converges to its ergodic distribution. This does a good job of matching the age-income profile in the data as shown in the Appendix.

[19] The yield on the 10-year Treasury is computed internally within the model. The SDF used to price the 10-year Treasury reflects expected short rates and the higher risk price of the crisis state with $m\,(\text{Expansion, Crisis}) = 6.1 \times m\,(\text{Expansion, Non-Crisis})$. It does not reflect κ, which only impacts the lender SDF.

[20] FRED uses the Freddie Mac Primary Mortgage Market Survey to report 30-year fixed mortgage rates. This

basis points, and we impose this κ for all other mortgage designs.[21] In Appendix D.3, we consider a robustness check in which κ is time-varying and κ_t rises in a crisis to reflect a higher lender cost of capital.

We calibrate short rates during expansions and recessions to historical real rates from 1985-2007.[22] [23] We find that short rates equal 1.32% on average during recessions and 3.26% during expansions. For the crisis state, we assume that the real short rate is 3.0% less than during expansions, or 0.26%.[24]

To keep things simple, for $m\left(\text{Non-Expansion},\Theta_{t+1}\right)$ we assume that the SDF is constant regardless of tomorrow's state Θ_{t+1} and is moreover equal to $m\left(\text{Expansion, Non-Crisis}\right)$. Summarizing, we require that the SDF in expansions satisfies:

$$m\left(\text{Expansion, Crisis}\right) = 6.1 \times m\left(\text{Expansion, Non-Crisis}\right)$$
$$E_t\left[m_{t,t+1}\right] = \frac{1}{1+r_t+\kappa},$$

and in non-expansions satisfies only the second equation regardless of tomorrow's state. This SDF implies that lenders place considerably more weight on transitions to crisis states and thus charge a premium for insurance policies that pay off in crisis states.[25] [26]

We maintain these short rates, mortgage rates, and the SDF as we vary the mortgage contract and monetary policy to put different contracts on the same footing. In practice, mortgage design affects monetary policy, as we discuss in Section 7, and so with a different mortgage design the Central Bank may set different interest rates.

survey may under-represent subprime and non-conforming loans. To the extent that the overall mortgage spread should be a bit higher, this would be primarily reflected in a higher κ. Our key insights regarding mortgage design would not be materially impacted.

[21] The remaining 40 bps is due to prepayment risk and default risk. Prepayment risk accounts for 13.82 bps and default risk accounts for 26.18 bps.

[22] We use a real model to focus the model on the benefits of interest-rate indexation in a scenario like the Great Recession. Indeed, our central points are fairly orthogonal to the literature on "mortgage tilt" and inflation, with the exception of the possibility that adjustable-rate borrowers may see their payments rise if inflation is high in a crisis and the central bank raises interest rates. We consider such a scenario in Section 7.

[23] Our model abstracts from regional heterogeneity in the strength of housing cycles and recessions. Given such heterogeneity, monetary policy – and mortgage design – is a somewhat blunt instrument because it does not treat different regions differently. See Beraja et al. (2019) for evidence on heterogeneity in monetary policy across regions due to differences in the equity positions of households and Piskorski and Seru (2018) for a discussion of the potential gains from indexing mortgages to local economic conditions.

[24] In practice, interest rates adjust gradually to the aggregate state of the economy. We assume immediate adjustment to keep the number of aggregate states tractable. With gradual adjustment, ARMs would provide less insurance. This is another plus for the EK convertible mortgage , as agents may want to keep their mortgage as an FRM if ARM rates are not adjusting or are adjusting the wrong way.

[25] Since bonds hedge high marginal utility states of the world, our model implies a negative bond beta and a downward-sloping term structure of interest rates. Campbell et al. (2019) estimate a negative bond-stock beta for the post-2000 period. When we implement a Volcker-like monetary policy in Section 7.1, in which rates rise during the crisis instead of fall, our model instead features a positive bond beta and a positive term premium. This is consistent with the findings of Campbell et al. (2019), who document a positive bond-stock beta during the 1980s anti-inflationary Volcker regime.

[26] Our SDF also satisfies the Hansen-Jagannathan bound. The value of $\frac{\sigma(m)}{E[m]}$ is 0.55, whereas the Sharpe ratio on the equity market is around 0.4.

4.3 Other Calibration Targets

We set a number of parameters to standard values in the literature or to directly match moments in the data.

We assume households live for 45 years, roughly matching ages 25 to 70 in the data. Households retire after 35 years, at which point idiosyncratic income is frozen at its terminal level minus a 0.35 log point retirement decrease. The tax system is calibrated as in Heathcote et al. (2017), with $\tau_0 = 0.80$ and $\tau_1 = 0.18$. We use a discount factor of $\beta = 0.95$ and a CRRA of $\gamma = 3.0$.

Moving and refinancing involve fixed and variable costs. We set the fixed cost of moving equal to 10% of average annual income, or roughly $5,000. The proportional costs, paid by both buyers and sellers, equal 3% of the house value to reflect closing costs and realtor fees. Refinancing involves a fixed cost of 4% of average annual income, or roughly $2,000, as well as variable cost equal to 1% of the mortgage amount to roughly match average refinancing costs quoted by the Federal Reserve.[27]

Renters pay a rent of $q = 0.20$ to match a rent-to-income ratio of 20%. Homeowners must pay a maintenance cost equal to 2.5% of the house value every year. We calibrate the moving shock ζ so that homeowners move an average of every 9 years as in the American Housing Survey. The homeownership rate is set to match a long-run average homeownership rate of 65 percent in the United States.

Υ, the fraction of the price recovered by the bank after foreclosure, is set to 64.5 percent. This combines the 27 percentage point foreclosure discount in Campbell, Giglio and Pathak (2011) with the fixed costs of foreclosing upon, maintaining, and marketing a property, estimated to be 8.5% of the sale price according to Andersson and Mayock (2014).[28]

We assume that the maximum LTV at origination under loose credit is 95%, corresponding to the highest spike in the distribution of LTV at origination in the Great Recession, and under tight credit is 80%, which is the conforming loan limit LTV. This generates crises with a tightening of credit that feature a price decline similar to what we observed in the Great Recession.

We finally calibrate four parameters internally. We calibrate a, the utility benefit of owning a home, so that house prices are approximately five times the average pre-tax income in our economy.[29] We choose the bequest motive parameters ψ and ξ to match the ratio of total net worth at age 60 to age 45 in the SCF for the median and 10th percentile households. Intuitively, ψ, which controls the overall strength of the bequest motive, is pinned down by the median growth rate, while ξ, which controls the extent to which bequests are a luxury, is pinned down by the 10th percentile growth rate.[30] We finally calibrate $\bar{d}$, the average default cost, so that in simulations of the impulse response to a housing downturn akin to the Great Recession described below we

[27] Reassuringly, the preference and transaction cost parameters we use are similar to other papers in the literature, such as Chen, Michaux, and Roussanov (2019).

[28] Much of the literature calibrates to the "loss severity rate" defined as the fraction of the mortgage balance recovered by the lender. We calibrate to a fraction of the price because of a recent empirical literature that finds that in distressed markets, the loss recovery rate is much lower (e.g. Andersson and Mayock, 2014), which is consistent with a discount relative to price rather than a constant loss severity rate.

[29] In the SCF, the mean price to income ratio for homeowners is 4. Because homeowners in our model are richer than the average household, the ratio of the mean price to average income is 5.

[30] These two moments are similar to those used by Kaplan et al. (2019) to calibrate the parameters controlling the bequest motive and the extent to which it is a luxury in their paper.

Table 2: Consumption and House Prices In Model vs. Data

	SD of log (C)	SD of log (HPI)	Corr of log (C) and log (HPI)
Data	0.021	0.055	0.461
Model	0.031	0.070	0.511

Note: This table compares the model and data for the standard deviation of log consumption and log house prices. The data we use is annual from 1975 to 2018. For house prices we use the FHFA purchase-only index, FRED series USSTHPI. For consumption, we calculate real personal consumption expenditures net of housing expenditures by subtracting ,the "Services: Housing and Utilities" subcategory of real personal consumption expenditures, FRED series DHUTRC, from the Real PCE data, FRED series PCECCA. The data is HP filtered with a smoothing parameter of 100 so as to not over-smooth out the slow-moving house price series. The model uses 18,900 years of simulated data.

match that 8.00 percent of the housing stock was foreclosed upon from 2006 to 2013 (Guren and McQuade, 2019).[31] We match these moments within 0.15% of their target values as shown in the Appendix.

4.4 Performance of Model in Stochastic Simulations

The first step in evaluating the performance of our model is to consider how well endogenous aggregates in the model match time-series moments in the data. Table 2 compares aggregate consumption and house prices in our model relative to the data. The correlation of house prices and consumption is very close to the data, although each series is slightly too volatile. Overall, these series compare favorably to their counterparts in the data.

4.5 Lifecycle Patterns and Distributions Across the Population

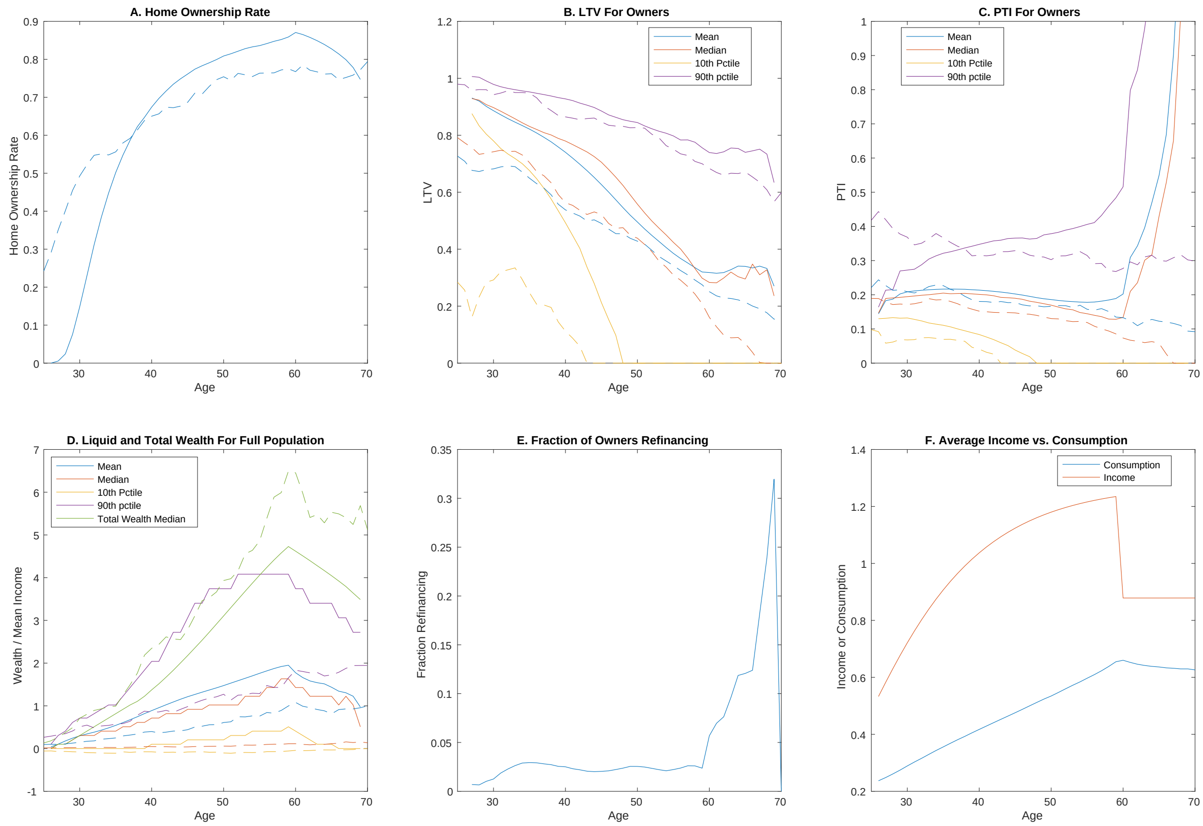
The model does a good job matching the lifecycle patterns and the overall distribution of debt and assets in the Survey of Consumer Finances for 2001, 2004, and 2007. Figure 1 shows the lifecycle patterns, while Figure 2 shows the distributions across the population. In both figures, the pooled SCF data for 2001 to 2007 is in dashed lines and the model analogues are in solid lines.

Panel A of Figure 1 shows the homeownership rate over the lifecycle. The model slightly underestimates the homeownership rate of the very young and over-estimates the homeownership rate of the middle aged.

Panels B and C of Figure 1 shows the mean, median, 10th percentile, and 90th percentile of the loan to value ratio (LTV) and payment to income ratio (PTI) by age, and panels A and B of Figure 2 shows the distribution of LTV and PTI across the population. The model somewhat over-predicts the LTV ratios of the young individuals in the bottom half of the LTV distribution. This has minimal impact on our quantitative results, however, since these homeowners are not at risk of default when the crisis hits. The model does reasonably well in capturing the LTV and PTI distributions across all ages, although it somewhat overstates the number of individuals with LTVs between 80 percent and 95 percent as is the case with any model with a single hard LTV constraint. We also find that PTIs that are too high for the old because mortgages amortize to the end of life. Because most of the equilibrium effects in our model come from the purchase, refinance,

[31] We choose d_a and d_b, the bounds of the uniform distribution from which d is chosen, to add a small bit of mass around $\bar{d}$. In the calibration, $\bar{d} = 44.75$, $d_a = 39.75$, and $d_b = 49.75$.

Figure 1: Lifecycle Patterns: SCF vs. Model



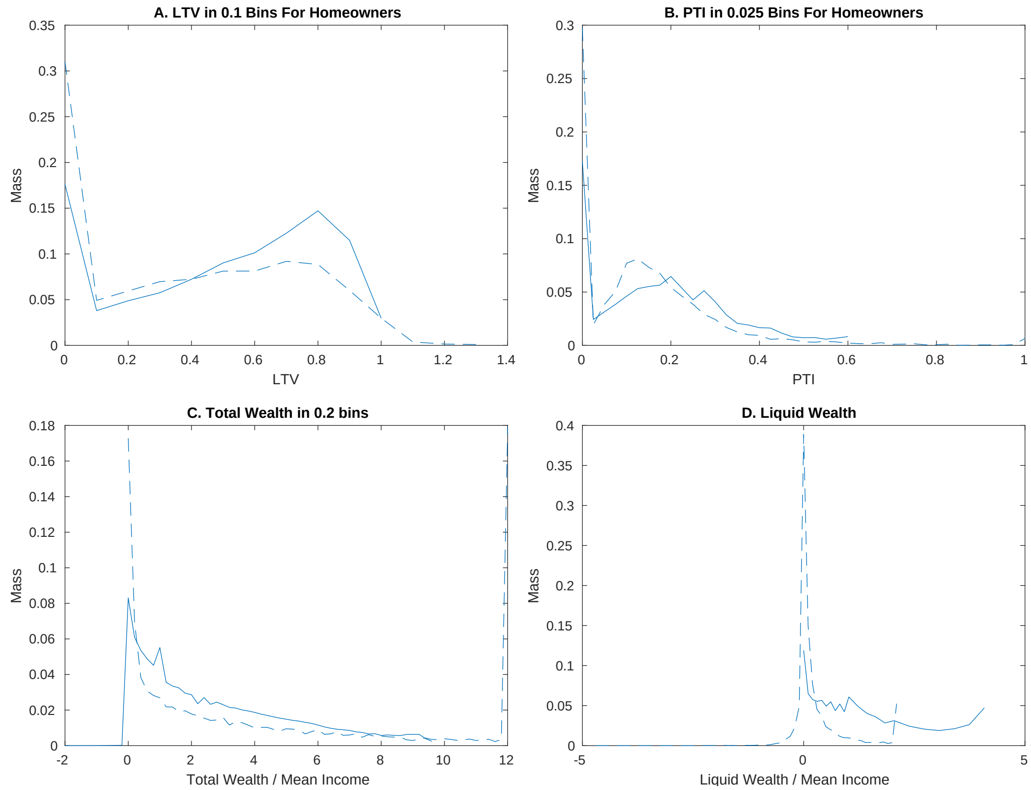
Note: This figure compares the baseline calibration of the model with all FRMs (solid lines) to SCF data from 2001 to 2007 (dashed lines) in panels A-D. Panels E and F are constructed based only on the model. Panel A shows the homeownership rate. Panel B shows the mean, median, 10th percentile, and 90th percentile of loan to value ratios for homeowners, and panel C shows the same statistics for the payment to income ratio. Panel D shows the mean, median, 10th percentile, and 90th percentile of liquid assets along with median total wealth. Panel E shows the refinance rate, and Panel F shows consumption and income in the model.

and default decisions of the young who have relatively high LTVs, the financial position of elderly homeowners has little impact on our results.

Panel D of Figure 1 shows percentiles of the liquid wealth and the median of total wealth by age, and panels C and D of Figure 2 show the distributions of total and liquid wealth in the population. The model does a reasonably good job matching median total wealth over the lifecycle and liquid wealth at young ages. Agents in the model accumulate more liquid assets in retirement than in the data. Again, this is not a significant issue, as the old do not play a crucial role in the housing market in our model. The data also has a thicker right tail of very wealthy individuals. Our model is designed to capture the impact of credit constraints and mortgages on housing markets, so capturing the extremely wealthy is not relevant for our exercise.

Finally, Panels E and F of Figure 1 show the fraction of owners refinancing and income and consumption over the life cycle, respectively. Most refinancing is of the cash-out variety because the FRM rate does not fluctuate dramatically due to expectations-hypothesis-type logic. Refinancing is relatively low until retirement, at which point it jumps so that agents can smooth their consumption. The excessive refinancing of the old is not crucial to our results because the old are not the marginal

Figure 2: Distributions Across Population: SCF vs. Model



Note: This figure compares the baseline calibration of the model with all FRMs (solid lines) to SCF data from 2001 to 2007 (dashed lines). Panel A shows loan to value ratios in 10 percentage point bins for homeowners, and panel B shows the payment to income ratio for homeowners in 0.025 bins. Panel C shows total wealth relative to mean income in bins of 0.2, while panel D shows liquid wealth relative to mean income in bins of 0.2. In all figures, the model and data are binned identically.

buyer or defaulter. Income follows a standard lifecycle profile, and consumption is smoother than income and increasing as individuals age, consistent with buffer stock models of consumption.

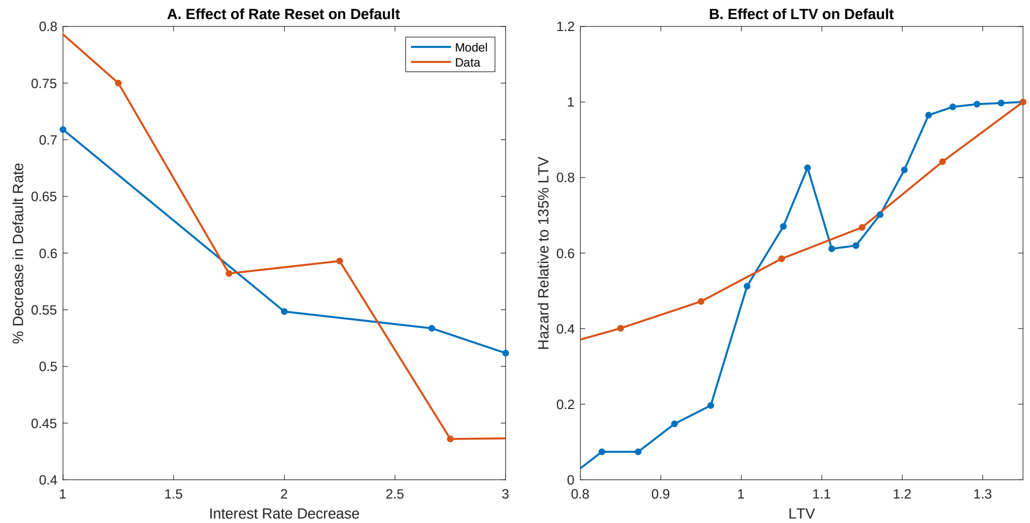
4.6 Calibration Evaluation Using Quasi-Experimental Evidence on Default

To evaluate the extent to which our model quantitatively captures the impact of payment reductions, we compare our model to quasi-experimental evidence from Fuster and Willen (2017). Fuster and Willen study a sample of homeowners who purchased ALT-A hybrid adjustable-rate mortgages during 2005-2008 period and quickly fell underwater as house prices declined. Under a hybrid ARM, the borrower pays a fixed rate for several years (typically five to ten) and then the ARM "resets" to a spread over the short rate once a year. These borrowers were unable to refinance because they were almost immediately underwater, so when their rates reset to reflect the low short rates after 2008, they received a large and expected reduction in their monthly payment.

Fuster and Willen provide two key facts for our purposes. First, they show that even for ALT-A borrowers – who have low documentation and high LTVs relative to the population – at 135 percent LTV the average default hazard prior to reset was only about 24 percent.[32] The fact that so many

[32]This figure is based on the default hazard in months 30 to 60 in Figure 1b. Fuster and Willen measure "default"

Figure 3: Fit to Fuster and Willen (2017) Natural Experiment



Note: The data from Fuster and Willen (107) come from column 1 of Table A.1 in their paper, which is also used in Figure 3 of their paper. The model estimates come form comparing a 2/1 ARM to a 1/1 ARM in our model.

households with significant negative equity do not default implies that there are high default costs. It is also consistent with a literature that finds evidence for a "double trigger" model of default whereby both negative equity and a shock are necessary to trigger default, as is the case for most default in our model.

Second, Fuster and Willen use an empirical design that compares households just before and after they get a rate reset and show that the hazard of default for a borrower receiving a 3.0 percent rate reduction falls by about 55 percentage points.

We evaluate the extent to which our model can match Fuster and Willen's estimates by simulating their rate reset quasi-experiment within our model. In particular, we compare the crisis default behavior of agents in our model with a 2/1 ARM that will reset next period with the behavior of an agent with a 1/1 ARM that has reset this period. This corresponds to the treatment and control used by Fuster and Willen. We assume that these borrowers are an infinitesimal part of the market, so we can consider them in partial equilibrium, and we compute their default rates at different LTVs with the 2/1 ARM and 1/1 ARM. To deal with the fact that the ALT-A sample used by Fuster and Willen is not representative of the population, we roughly match the assets, age, and income of the homeowners we consider to households with hybrid ARMs that have yet to reset in the 2007 Survey of Consumer Finances.[33] Finally, we assume that homeowners have a fixed rate corresponding to the FRM rate in the boom and reset to the ARM rate in the crisis

Figure 3 compares the calibrated model with the findings of Fuster and Willen (2015). Panel A shows the impact of rate reductions on default in the model and Fuster and Willen's estimates.

as becoming 60 days delinquent rather than an actual foreclosure, so the actual default rate might be slightly lower.

[33] In the SCF, we find that the ALT-A borrowers have low assets, are young, and have moderate-to-low income, as one would expect. Due to a limited number of observations, rather than using the ages and assets of households in the SCF, we assume a uniform distribution between the 25th and 75th percentiles of age and assets in the SCF data and assume that individuals in the Fuster-Willen experiment have moderate to low income.

Overall, the fit is quite good. The model slightly over-predicts the impact of small rate reductions on the default hazard and under-predicts the impact of large rate reductions. The right panel shows the baseline default rate under the 2/1 ARM at various LTVs relative to the default rate at 135 percent LTV. LTV reductions have modestly larger effects than in the data until one gets below 100 percent, at which point the default hazard falls off in the model but not in Fuster and Willen's data.[34] The model's fit to the Fuster and Willen quasi-experimental evidence suggests that the model will accurately capture the effect of payment and LTV changes on default.

5 ARM vs. FRM: The Economics of State Contingent Mortgages

Having created and calibrated a laboratory to study mortgage design and its interaction with monetary policy, we now use our model to assess various mortgages. We focus on a crisis scenario that combines a housing bust and a deep recession as in the Great Recession. This allows us to analyze mortgage designs proposed to address the problems revealed by the Great Recession, which is the focus of the recent literature. Additionally, the equilibrium feedbacks that our model features are most interesting and potent in a downturn with a price-default spiral. However, we also consider the performance of various contracts unconditional on a crisis in Section 5.3. In this section we focus on conventional monetary policy that reduces real rates in a crisis, and we consider alternate monetary policies, including the case in which the central bank raises real rates in a bust, in Section 7.

To analyze a housing downturn, we simulate an impulse response where in the five years prior to the downturn the economy is in an expansion with loose credit and when the crisis hits the economy falls into a deep recession and the LTV constraint tightens. We assume that the crisis lasts at least three years, after which the economy stochastically exits according to the transition matrix so that the average crisis length is 5.66 years. We study the impulse responses of prices, default rates, and consumption to the resulting downturn, which we compute by averaging together 100 simulations with random shocks prior to the five-year expansion and subsequent to the first three years of the crisis. We also compare mortgage designs in stochastic simulations.

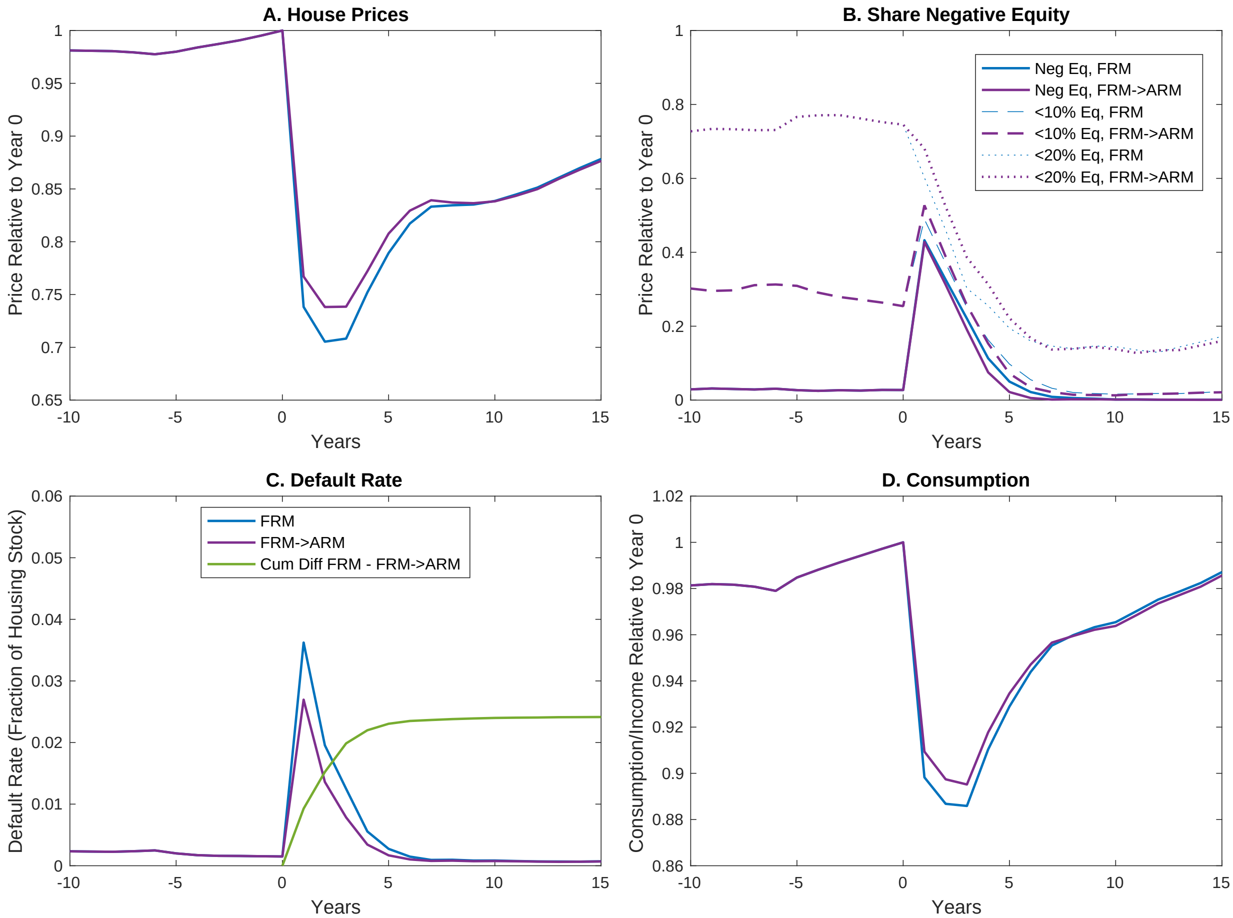
To analyze the effect of mortgage design in such a crisis, we first compare an economy with all FRM borrowers to an economy with all ARM borrowers. This provides us with most of the economic intuition regarding the benefits of adding state contingency to mortgages. In Section 6, we consider more complex mortgage designs.

5.1 Economic Intuition: FRM vs. ARM

Our baseline case is an economy in which the only available mortgage to home purchasers is a FRM. The results are illustrated by the blue lines in Figure 4.

[34] Fuster and Willen find a substantial default hazard below 100 percent LTV for three likely reasons. First, the combined LTV that Fuster and Willen use is likely measured with error both due to missing liens in the data and due to error in the automatic valuation model. Second, Fuster and Willen measure default as 60 day delinquency and not a final foreclosure. In areas with substantial foreclosure backlogs, borrowers who are above water can become delinquent before they sell. Finally, there are some frictions in terms of time to sell and the fixed costs of sale that may cause above-water households to default. In our model, there is minimal default for above water households

Figure 4: FRM vs. FRM→ARM: Housing Market Outcomes



Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and FRM→ARM. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

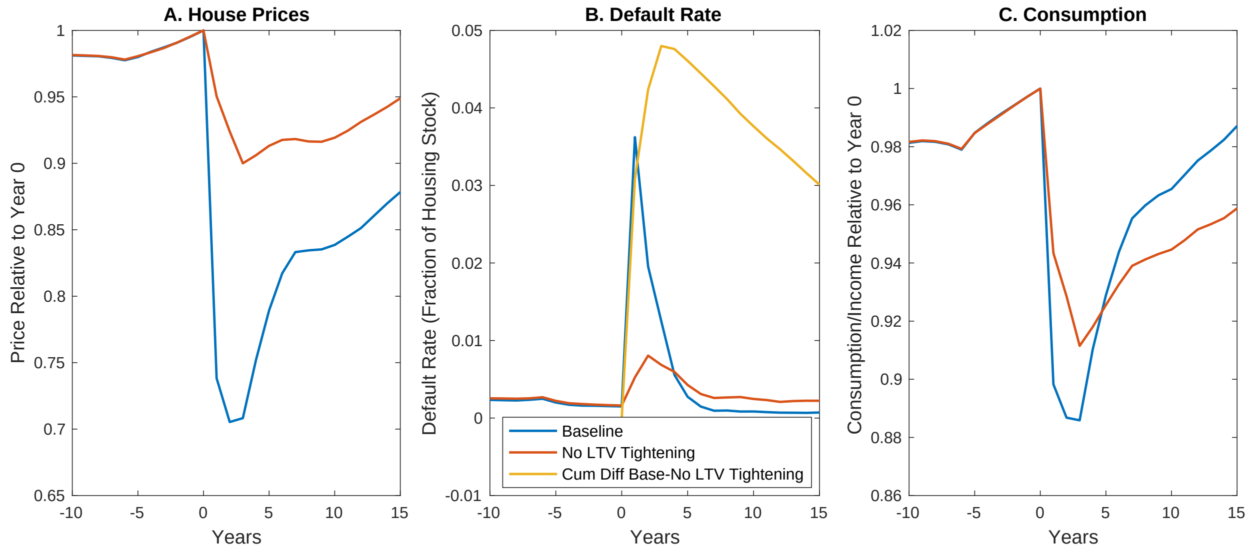
The model with all FRMs generates a housing crisis in the model of a similar magnitude to the the one experienced in the United States between 2006 and 2012 as shown in Figure 4. Prices fall by about a third, which closely matches the peak to trough decline in national repeat sales house price indices.[35] At the depths of the crisis, roughly 40 percent of homeowners are underwater and approximately 70 percent of homeowners have under 20 percent equity and cannot refinance given the tightened LTV constraint. The combination of negative equity and recession leads 8.00 percent of the housing stock to default (recall that matching the fraction of the housing stock that defaulted from 2006 to 2013 is a calibration target). Finally, consumption falls by 11.4 percent due to the sudden and persistent decline in income and the large number of constrained households. The decline is slightly higher than the decline in the data.[36]

because some households get a moving shock with very low equity and decide to default.

[35]Our model does not match the time series of house price indices because prices fall to their lowest level on the impact of the shock, while in the data they decline gradually. This is the case because our Walrasian model does not have any house price momentum. See Guren (2018) for a summary of the literature on momentum.

[36]With a 2.5 percent annual trend, real personal consumption expenditures fell 9.4 percent relative to trend peak

Figure 5: Baseline vs. Model With No LTV Tightening



Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM. In the baseline case, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years. In the case without LTV tightening, the maximum LTV stays at 95 percent but the same shocks hit the economy and the monetary policy is unchanged.

To elucidate the mechanisms behind the housing downturn, Figure 5 compares numerical results for the FRM economy in the baseline model where the LTV tightens to 80 percent in a crisis to a model in which the LTV constraint remains at 95 percent when a crisis hits. Without the LTV constraint tightening, house prices fall by 10.0 percent and gradually mean revert as opposed to falling 29.5 percent at the onset of the crisis, rising about 12.5 percent as the economy gets out of the crisis, and then slowly mean revert as credit stochastically loosens. Part of the reason that prices fall by less is there is no foreclosure crisis: After eight years only 3.87 percent of the housing stock has been foreclosed upon, as opposed to 8.00 percent in the baseline model. Consumption falls by 8.9 percent rather than 11.4 percent. These results show that most of the decline in house prices and defaults in our model can be attributed to tightening credit conditions.[37] The Appendix compares mortgage designs in this environment in which the LTV constraint does not tighten.

We examine the differential impacts of adjustable-rate mortgages through two experiments.

to trough in the recession. Our model may have a larger decline than in the data because we do not capture the upper tail of the income distribution well and because we do not model lenders.

[37]There is an active debate on whether the crisis is best explained by changes in credit conditions like the tightening LTV constraint of our analysis, or some other factor. Some papers such as Favilukis, Ludvigson, and Van Nieuwerburgh (2017) argue that a tightening LTV constraint explains the whole crisis. Others, such as Kaplan, Mitman, and Violante (2019) argue that credit conditions have essentially no effect on house prices. Our results provide a significant effect of credit conditions because rental markets are segmented from owner-occupied housing markets (Greenwald and Guren, 2019). Greenwald and Guren show that in practice housing markets are quite segmented, although not perfectly segmented. Accounting for imperfect segmentation would be computationally difficult and would change our results in two ways. First, we would need some other mechanism such as a shock to beliefs to generate the a decline in house prices of one third. Because the source of the decline in house prices is not central to the economics of our model, this change would not have a significant effect on our results. Second, with some conversion, the equilibrium effect of foreclosures on prices would be a bit smaller, making mortgage design slightly less powerful.

In the first, we assume that home purchasers have fixed-rate mortgages pre-crisis, but that when the crisis hits, all mortgages are unexpectedly converted to adjustable-rate mortgages with the spread based on the origination state of the mortgage. Because the central bank lowers the short rate in the crisis and this is fully passed through to households under ARM, mortgage payments fall dramatically. This experiment, which we call the FRM→ARM counterfactual, is a useful thought experiment to understand the mechanisms at work in our model because it holds fixed the distribution of individuals across idiosyncratic states when the crisis hits and isolates the *ex-post* effect of adjustable-rate mortgages on the severity of the crisis. In the second experiment, we consider an economy with ARMs both before and after the crisis hits, allowing for *ex-ante* behavior to affect the distribution of households across idiosyncratic states at the beginning of the crisis.

5.1.1 *Ex-Post* Effect of Switching From FRM to ARM

The FRM→ARM counterfactual is shown in purple lines in Figure 4. Relative to the baseline case, the housing crisis is less severe. House prices fall by 3.28 percentage points less and 29.8 percent fewer households default.

Under FRM, the rate reduction is not passed through to households to the same extent for two reasons. First, the long end of the yield curve moves by less, which we call the yield curve channel. Second, some homeowners do not have the means to satisfy the tighter LTV constraint and cannot refinance, which we call the underwater household channel.[38] Because of left-skewness of the income shock distribution in the crisis, a significant fraction of the underwater homeowners experience a drop in their income. Those with little savings default because they would have to cut their consumption substantially – and in many cases to zero – to make their mortgage payment, which causes them to be willing to bear the utility cost of default. Default increases the supply of homes on the market, further pushing down prices, which in turn leads to more default. This phenomenon is the canonical price-default spiral.

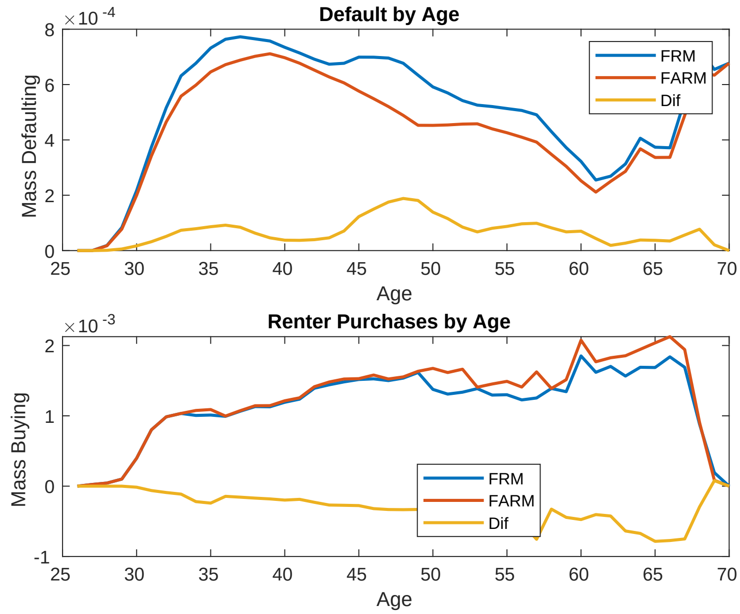
In the ARM economy, by contrast, the mortgage payment is pegged to the prevailing short rate in the market, so payments fall automatically. They also fall by more than under FRM because the short end of the yield curve adjusts by more than the long end. Many households that default with an FRM can avoid default, short-circuiting the default spiral and causing a less severe housing crisis.[39]

Additionally, because FRMs are priced off of long-term rates, which fall via by less due to expectations-hypothesis-type logic, buying is more affordable in the ARM economy for young renters during the crisis than in the FRM economy. This implies that the demand by renters is

[38] Refinancing is also costly in our model, which further works to limit monetary policy pass-through. Beyond the direct costs of refinancing, Johnson, Meier, and Toubia (2015), Keys, Pope, and Pope (2016), Andersen et al. (2018) and Berger et al. (2018) have documented apparent behavioral inertia in mortgage refinancing. While we do not formally model it, it is clear such behavioral inertia would further amplify the gains of the ARM mortgage design over the FRM design.

[39] Beraja et al. (2019) point out that while ARMs may lead to stronger effects of monetary policy on consumption in a crisis when lots of household are underwater, they may be weaker in normal times. This is because under FRM more households refinance when interest rates fall and liquidate home equity as long as they are paying the fixed costs of refinancing. While it is true that more households refinance and take out cash in a normal recession under FRM in our model, this effect on consumption is offset by the fact that ARM rates fall by more due to the yield curve effect. Thus, in a normal recession, we continue to find that ARMs help stabilize the housing market.

Figure 6: FRM vs. FRM→ARM: Default and Renter Demand By Age



Note: The figure shows the mass of households of each age defaulting and purchasing after renting last period in the first period of a simulated crisis under FRM and FRM→ARM along with the difference between the two lines. To ensure that the differences are entirely due to the policy functions, we compare FRM to FRM→ARM and evaluate the policy functions assuming the price is the realized FRM price.

higher when the economy switches to ARMs, which further ameliorates the impact of the housing crisis.

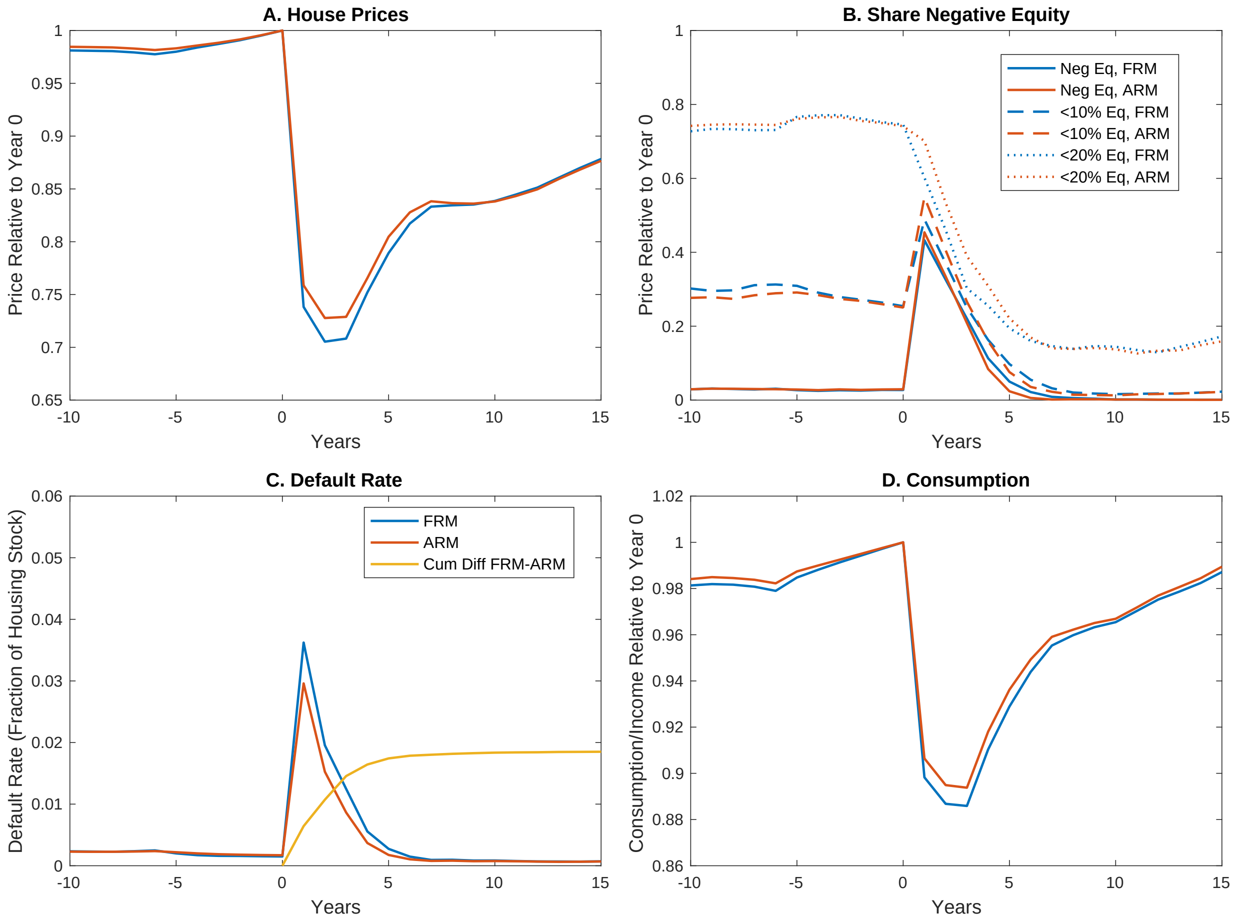
These effects are summarized in Figure 6, which plots the mass of homeowners defaulting and renters purchasing by age in the period in which the crisis begins. The results are plotted for both the baseline FRM economy and the FRM→ARM counterfactual, along with the difference. Because the pre-downturn distribution is the same, this figure only reflects differences in policy functions between the FRM and FRM→ARM economies. The gap in default is dominated by young and middle-aged households, and the gap in renter purchases is dominated by homebuyers around the age of first purchase. Similar comparisons by savings, income, and LTV reveal that the additional default under FRM comes from low income, low savings, and high LTV borrowers, while the additional demand comes from renters with the moderate savings and income reflective of first-time homebuyers.

5.1.2 *Ex-Ante* Effect of Switching From FRM to ARM

Of course, while useful for expositional purposes, the economy in which all mortgages suddenly switch to ARMs is unrealistic. We therefore now instead consider an economy in which all mortgages are adjustable-rate, both pre- and post-crisis. The results are presented in Figure 7, with the FRM economy shown in blue and the ARM economy shown in orange.

The benefits of adjustable-rate mortgages are reduced relative to the FRM→ARM counterfactual, with prices falling by 2.24 percentage points less and 23.0 percent fewer households defaulting than under FRM rather than 3.28 percentage points and 29.8 percent under FRM→ARM. These

Figure 7: FRM vs. ARM: Housing Market Outcomes

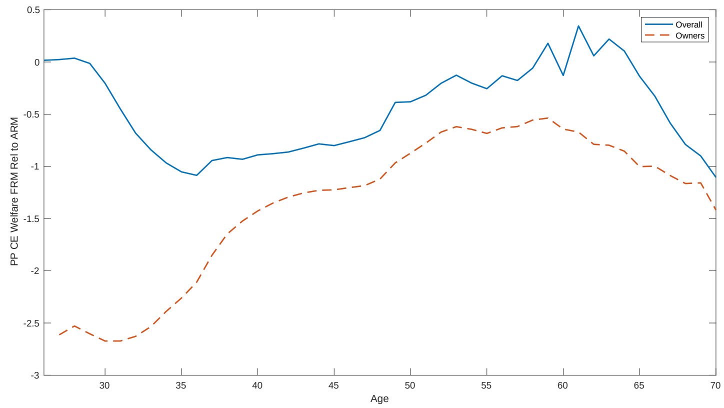


Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and ARM both *ex ante* and *ex post*. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

figures are summarized for ARM and the mortgage designs we consider subsequently in Table 3, which summarizes our main findings under the baseline monetary policy.

The crisis is worse in the ARM economy relative to the FRM→ARM economy because young homeowners understand the hedging properties of the ARM mortgage and take on more risk, primarily by holding less savings in order to increase their consumption in constrained states. Indeed, 2.3% more homeowners between the ages of 26 and 45 have less than a quarter of the average annual pre-tax income in savings under ARM than under FRM. This creates macro fragility, which households do not internalize. The degree to which the *ex-post* benefits of ARMs are undone by the *ex-ante* buildup of fragility – at least for house prices and default – is a numerical result. This highlights the need for realistic quantitative models of the type we analyze.[40]

Figure 8: FRM vs. ARM: Household Welfare Loss Conditional on Crisis by Age



Note: The figure shows the percentage point difference in consumption equivalent household welfare between FRM and ARM for each cohort alive when the crisis hits. Consumption equivalent household welfare is calculated as the equivalent variation amount each agent would be willing to reduce their consumption per year of their remaining life to avoid a crisis. We show the difference between FRM and ARM for generations alive when the crisis hits. The y axis is the difference between the FRM and ARM economies for this calculation, with a negative number indicating household are worse off under FRM. We repeat the calculations separately for owners, as indicated in the legend. For computational reasons, we cap the gain for any given individual at 100%. This restriction affects a tiny number of older people who have very little remaining lifetime consumption but derive significant utility from owning a house.

5.1.3 Household Welfare Benefits of Switching From FRM to ARM in a Crisis

To further evaluate the benefits of switching from FRM to ARM, we calculate the household consumption-equivalent welfare cost of the crisis under each mortgage. The conditional welfare cost is the amount an agent would be willing to reduce their consumption per year of their remaining life to avoid a crisis. This is calculated as equivalent variation and is aggregated by the pre-crisis distribution of individuals across states for agents of each age living when the crisis hits, as detailed in the Appendix. Because different cohorts have different remaining lives, we report results by cohort rather than an aggregate number. Importantly, the welfare metric we use is for households only and does not include lender consumption.[41] This is because in our model lenders break even subject to a calibrated stochastic discount factor, which means that we cannot calculate lender consumption or calculate lender welfare. Instead, we report in Table 3 the present value of losses to lenders over a crisis using the lender SDF, and our mortgage pricing condition ensures they break even on average, again subject to their SDF. Our welfare metric also does not include a social cost of foreclosure beyond the private utility cost of foreclosure. Estimates of the pecuniary social costs of foreclosure range from \$50,000 to \$80,000 and do not typically include the large non-pecuniary costs to foreclosed-upon households, which may or may not be part of the utility default cost (Diamond, Guren, and Tan, 2019).

Figure 8 shows the welfare difference separately by age for both renters and owners and for the overall population. A negative value indicates a larger welfare loss under FRMs than in the ARM counterfactual. The Figure shows that owners benefit from the switch to ARMs significantly: Young

[40] Allowing for multiple house sizes would likely further amplify the *ex-ante* buildup of fragility because households would upgrade to larger houses due to the insurance provided by ARM.

[41] See Campbell, Clara, and Cocco (2018) for a fuller discussion of issues related to lender consumption.

Table 3: Moments From Downturn Simulations For Various Mortgage Designs

Design	FRM	ARM	EK	FRMUR	Option ARM
Pct Point Reduction in Max ΔP Rel to FRM		2.24	2.90	0.45	6.10
Pct Point Reduction in Max ΔC Rel to FRM		0.79	0.63	0.05	1.81
Share Defaulting over 8 Years	8.00%	6.16%	6.20%	7.37%	6.52%
Decline in PV of Mortgages When Entering Crisis	2.55%	3.32%	2.52%	2.62%	3.62%

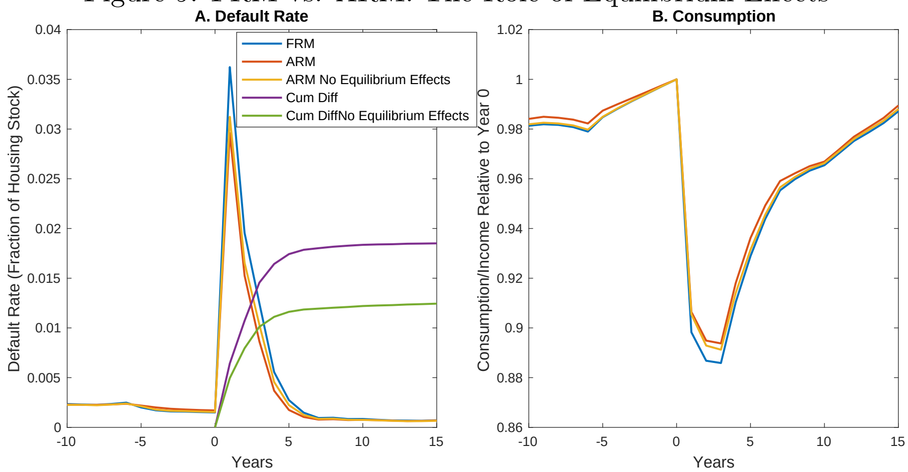
Note: The Table shows the indicated statistics for each mortgage design averaging across 100 simulations of a crisis that lasts 5.66 years after an expansion with loose credit as described in the main text. The top row shows the percentage point change in the peak-to-trough change in price relative to FRM, with a positive number indicating a less severe price decline. The second row shows the percentage point change in the peak-to-trough change in consumption relative to FRM, with a positive number indicating a less severe decline in consumption. The third row shows the share of the housing stock defaulting after 8 years. The fourth row shows the percentage point decline in the present value of the mortgages in the economy when the crisis occurs, which represents the deterioration of lender balance sheets in the crisis.

owners would be willing to give up nearly 3 percent more of their remaining lifetime consumption to avoid a crisis under FRM relative to ARM, while older owners would be willing to give up just over one percent. The number is particularly high for younger owners because they tend to have lower savings, lower income, and higher LTVs. These households cannot refinance when the crisis hits due to the LTV constraint, are stuck at a high interest rate, and do not have much liquid savings to help cushion an income shock. Additionally, they have lower consumption and expect their income to rise later in life, so they are borrowing constrained and their baseline marginal utility of consumption is high. Consequently, they are willing to pay a substantial amount of annual consumption to switch to an ARM, as the lower interest rate translates to a much smaller minimum payment and boosts consumption substantially. By contrast, renters, and particularly older renters with high savings, lose under ARM because they face higher house prices. On net, most generations see a welfare benefit under ARM relative to FRM of between 0 and 1 percent.

5.2 ARM vs. FRM: The Importance of Equilibrium Effects

An important feature of our analysis that differentiates it from the preceding literature is that house prices and mortgage spreads are determined in equilibrium. Figure 9 shows the impact of these equilibrium effects on consumption and default in a simulated downturn in our model. To calculate the impulse response in the ARM economy with no equilibrium effects, we take the price path and distribution of agents across states from the FRM model as given and calculate default and consumption under ARM using ARM policy functions computed with the FRM forecast rule. Figure 9 reveals that the equilibrium feedbacks account for about 34.3% of the difference in default and 32.1% of the difference in consumption between the ARM and FRM counterfactuals. This result also highlights an externality that is present in our model: Agents do not take into account the impact of their mortgage choices on equilibrium prices when making their decisions, yet their choices have a large impact on equilibrium prices.

Figure 9: FRM vs. ARM: The Role of Equilibrium Effects



Note: The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and ARM both *ex ante* and *ex post*. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years. For the "no equilibrium effects" counterfactual, we take the price path and distribution of agents across states from the FRM model as given and report default and consumption using policy functions computed with ARMs with the FRM forecast rule.

5.3 ARM vs. FRM Not Conditional on a Crisis

We now ask whether ARMs deliver benefits unconditionally rather than focusing on results conditional on a crisis. The first column of Table 4 shows the standard deviation of price, default, aggregate consumption, and idiosyncratic consumption for stochastic simulations of an all-ARM relative to an all-FRM economy. It also shows unconditional household welfare, which is calculated in terms of equivalent variation as the percent increase in per-period consumption that would be required in the FRM economy to make an agent indifferent between being born in a random period in the FRM economy and a random period in the ARM economy. This welfare computation corresponds to the change in household utility; lenders break even on average in our economy discounting using their calibrated SDF, so unconditional lender welfare is the same across mortgage designs.

The ARM economy features 1.4% lower house price volatility, 4.1% less default, 21.5% lower default volatility, and 4.6% lower aggregate consumption volatility. While these are significant improvements, the household consumption-equivalent welfare benefits are more modest: An agent would be indifferent between the ARM and FRM economies if their consumption rose by 0.12 percent per year in the FRM economy. This is the case because while aggregate consumption volatility, default, and default volatility are reduced, the households only have a CRRA of 3, which limits the welfare consequences of consumption volatility. Furthermore, idiosyncratic shocks dominate consumption volatility at the household level.[42] Since ARMs only provide insurance against aggregate shocks, the consumption insurance benefits in normal times are modest. This does, however, open the door for mortgage designs that provide insurance against idiosyncratic shocks, a point we

[42] A variance decomposition makes this point clearly: in the FRM economy, 77.9% of consumption volatility across individuals is across individuals within periods and within generations, 21.8% is across generations within periods, and only 0.3% is due to aggregate consumption across periods.

Table 4: Moments From Stochastic Simulations For Various Mortgage Designs Relative to FRM

Design	ARM	EK	FRMUR	Option ARM
Std Dev of Price Rel to FRM	98.6%	96.7%	98.3%	94.3%
St Dev of Default Rate Rel to FRM	78.5%	81.2%	93.1%	73.1%
Mean Default Rate Rel to FRM	95.9%	98.2%	99.1%	106.4%
St Dev of Agg Consumption Rel to FRM	95.4%	96.1%	99.4%	91.8%
Increase in Household Welfare Rel to FRM	0.12%	0.07%	0.02%	0.42%

Notes: Each cell shows the mortgage design indicated by each column as a percentage of FRM for the statistic indicated in each row. Price, default, and aggregate consumption are calculated from aggregate 19,000 year simulations. Idiosyncratic consumption is calculated by simulating 25,000 individuals each periods for 6,300 years. Household welfare is calculated as the equivalent variation in terms of the percent increase in annual consumption an agent would require in the FRM economy to be indifferent between being born in a random period in the indicated economy instead of the FRM economy.

return to in the next section.

5.4 Interest Rate Movement Orthogonal To The State of the Economy

In our main analysis, we have assumed that interest rates vary with economic fundamentals. There are many reasons why this may not be the case; most importantly, inflation may move real rates and create an output-inflation tradeoff for the central bank that precludes dropping rates in a crisis. Importantly, this may affect the relative benefits of FRM and ARM because FRM would insure against interest rate risk, while ARM would expose households to interest rate risk. To evaluate this quantitatively, in this section we introduce movements in interest rates orthogonal to the state of the economy.

To do so while remaining as close to the original calibration as possible, we generalize the aggregate state Θ_t to have 10 values instead of 5. As before, there is a crisis state with tight credit, a recession state with either tight or loose credit, and an expansion state with either tight or loose credit. However, all five states may have high or low short interest rates, with an independent and identically distributed 50 percent probability of being in the high or low interest rate state. We assume that the interest rates in the high interest rate state are $\Delta/2$ above our baseline calibration rates, while the interest rates in the low interest rate state are $\Delta/2$ below our baseline calibration rates. Δ is thus the rate difference between interest rates in the high and low states. Rather than taking a stand on Δ, we vary Δ and report results for ARM and FRM. To create a comparison to our baseline results that is as clear as possible, we maintain our original calibration throughout, so the models with orthogonal rate risk do not match our internal calibration targets.

Table 5.4 shows moments of price, consumption, and default as well as the increase in household equivalent welfare for FRM relative to ARM for $\Delta = 0\%$ (baseline), $\Delta = 0.5\%$, $\Delta = 1.0\%$, and $\Delta = 1.5\%$. The Table shows that adding variation in interest rates orthogonal to the state of the economy reduces the unconditional household welfare benefits of the ARM. This is because the FRM provides insurance against variation in interest rates, while the ARM exposes the household to interest rate risk. As the Table shows, in equilibrium households behave to smooth the consumption risk, and so this additional risk mostly shows up in higher default and higher volatility of default

Table 5: Moments From Stochastic Simulations With Different Amounts of Interest Rate Movement Orthogonal To The State of the Economy

Δ	0% (Baseline)	0.5%	1.0%	1.5%
Std Dev of Price Rel to FRM	98.6%	97.3%	97.9%	97.9%
St Dev of Default Rate Rel to FRM	78.6%	76.8%	77.3%	78.8%
Mean Default Rate Rel to FRM	95.9%	95.9%	96.5%	97.9%
St Dev of Agg Consumption Rel to FRM	95.4%	94.6%	95.1%	95.1%
Mean of Agg Consumption Rel to FRM	99.9%	99.8%	99.8%	99.8%
Increase in ARM Unconditional Household Welfare Rel to FRM	0.12%	0.05%	0.04%	0.00%

Notes: Each cell shows ARM as a percentage of FRM for the statistic indicated by each row under the amount of orthogonal interest rate movement indicated in each column. In particular, Δ is the gap between interest rates in the high interest rate state and the low interest rate state, with the midpoint corresponding to our baseline calibration. There is an iid 50 percent chance of being in the high or low state. Price, default, and aggregate consumption are calculated from aggregate 19,000 year simulations. Household welfare is calculated as the equivalent variation in terms of the percent increase in annual consumption an agent would require in the FRM economy to be indifferent between being born in a random period in the indicated economy instead of the FRM economy.

as Δ rises. The additional interest rate risk causes the unconditional household welfare gain of switching from FRM to ARM to decline as Δ rises to the point that the unconditional welfare benefits of ARM are completely eliminated with $\Delta = 1.5\%$ basis points. Given that the real short rate drops 3.0% in a crisis, these results show that a large amount of orthogonal rate variation is necessary to eliminate the unconditional benefits of ARMs.

6 Evaluating New Mortgage Designs

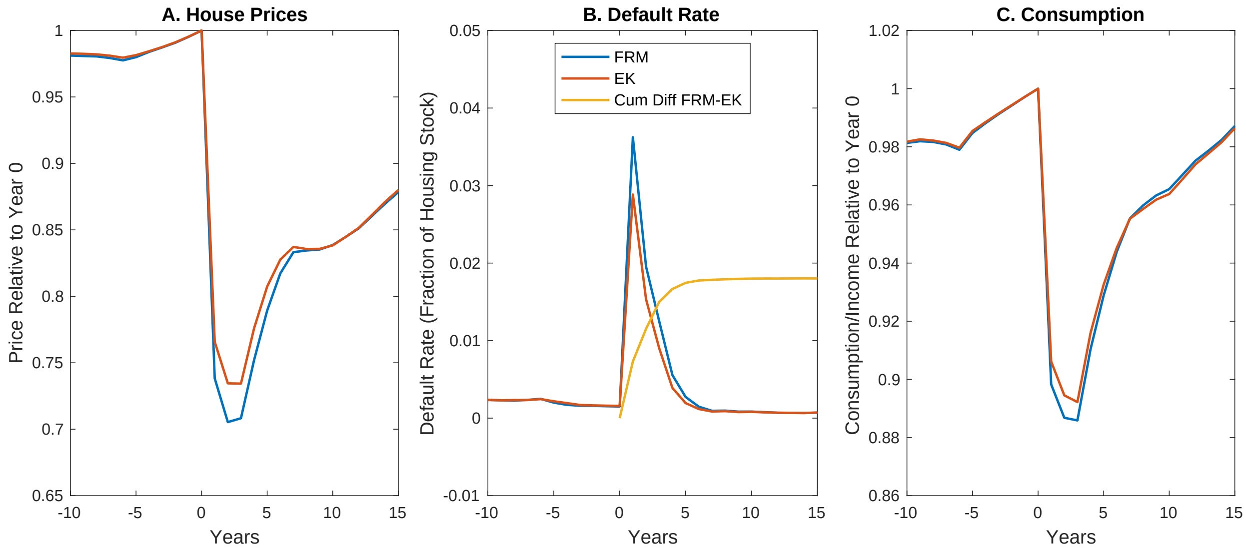
We have so far focused on comparing ARMs and FRMs to clearly elucidate the insurance benefits of indexation. However, ARMs have some drawbacks. In particular, if the central bank raises interest rates to fight an inflationary recession, our assumption that in recessions short rates fall may be violated. In these cases, ARMs are *worse* from an insurance perspective, the reverse of our main results because the covariance of interest rates and income shocks switches sign. We show this quantitatively in Section 7. Furthermore, by revealed preference, most Americans prefer fixed rate mortgages, which on average account for roughly 80 percent of mortgage originations.

Given these downsides, in this section we evaluate several mortgage designs that allow for state contingency in a crisis while preserving the benefits of FRMs in normal times. We also evaluate a contract that provides more idiosyncratic insurance than a standard ARM. Our main finding is that mortgage designs that front-load payment relief in the crisis outperform mortgages that spread payment relief over the life of the mortgage, even though the impact on lender balance sheets during the crisis is similar.

6.1 Eberly-Krishnamurthy Convertible Mortgage

Eberly and Krishnamurthy (2014) propose a fixed-rate mortgage that can at any time be costlessly converted to an adjustable-rate mortgage, but not back. This is similar to an economy in which

Figure 10: Eberly-Krishnamurthy Convertible Mortgage: Housing Market Outcomes



The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and the Eberly-Krishnamurthy convertible mortgage both *ex ante* and *ex post*. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

one can choose between ARM and FRM with two important distinctions. First, homeowners who do not satisfy the LTV constraint can still switch to an ARM. Since in our simulated crisis 70 percent of homeowners have under 20 percent equity at the beginning of the crisis, this is likely to be significant. Second, homeowners who are so highly constrained that they cannot afford to pay the fixed costs of refinancing can convert to an ARM costlessly. We introduce an EK mortgage into our model by adding an additional agent-specific state that indicates whether an agent has a mortgage functioning as an ARM or as an FRM and allowing agents to choose to convert their loan from an FRM to an ARM but not back.[43] New mortgages always start as an FRM, so one can always obtain an FRM by refinancing. Otherwise, the model is unchanged.

This mortgage has two added benefits. First, introducing it in practice is likely to be less disruptive, as it can act exactly like a FRM and does not take away the FRM option. Second, in the event that the covariance of interest rates and monetary policy changes, it performs more like an FRM, as we show in Section 7.

The convertible mortgage does, however, come at a cost, as borrowers will convert to an ARM when it is beneficial for them to do so and then refinance into a new loan that starts as an FRM when this is no longer the case. This implies that lenders will take losses in crisis states in which the SDF is high, which will drive up spreads. However, in our calibration this cost is small: The EK convertible mortgage on average has a 6 basis point higher spread over the riskless 10-year bond rate than an FRM.[44] The increase in the spread is quantitatively small because there is an

[43]There are five equations to price the EK mortgage but ten rates: An FRM rate and an ARM spread for each state. One must thus make a further assumption about the relationship between the FRM rate and ARM spread. We assume that when converted to ARM, the EK mortgage has the same spread over the short rate as the EK mortgage acting as an FRM has over the 10-year risk-free bond rate in the model.

[44]The average spread is are calculated as an average of the spread for originating in each state weighted by the

offsetting effect of reduced losses due to default in crisis states.

Figure 10 shows a simulated downturn under the convertible mortgage. The crisis is roughly 80 percent of the way to the all-ARM economy and realizes many of the consumption-smoothing and macroprudential benefits of ARMs. This is the case because on average the share of mortgages that are functioning as ARMs rises from under 10 percent pre crisis to near 100 percent during the crisis as households who need the payments relief the most convert. The convertible mortgage does not look fully like an ARM because of the higher spread. Table 4 compares the Eberly-Krishnamurthy mortgage with an FRM in stochastic simulations and also finds it is most of the way to the ARM economy. The unconditional household welfare benefits of EK relative to FRM are 0.07% of annual consumption, 60% of the welfare gain of ARM.

6.2 Fixed Rate Mortgage With an Underwater Refinancing Option

The second mortgage that we consider is a fixed-rate mortgage that has a built-in underwater refinance option (FRMUR), which can be exercised even if the origination LTV constraint is no longer satisfied.[45] We introduce this in our model by adding an additional agent-specific state that indicates whether an agent has exercised the built-in refinance option. We assume the structure is such that the option can only be exercised in the crisis state.

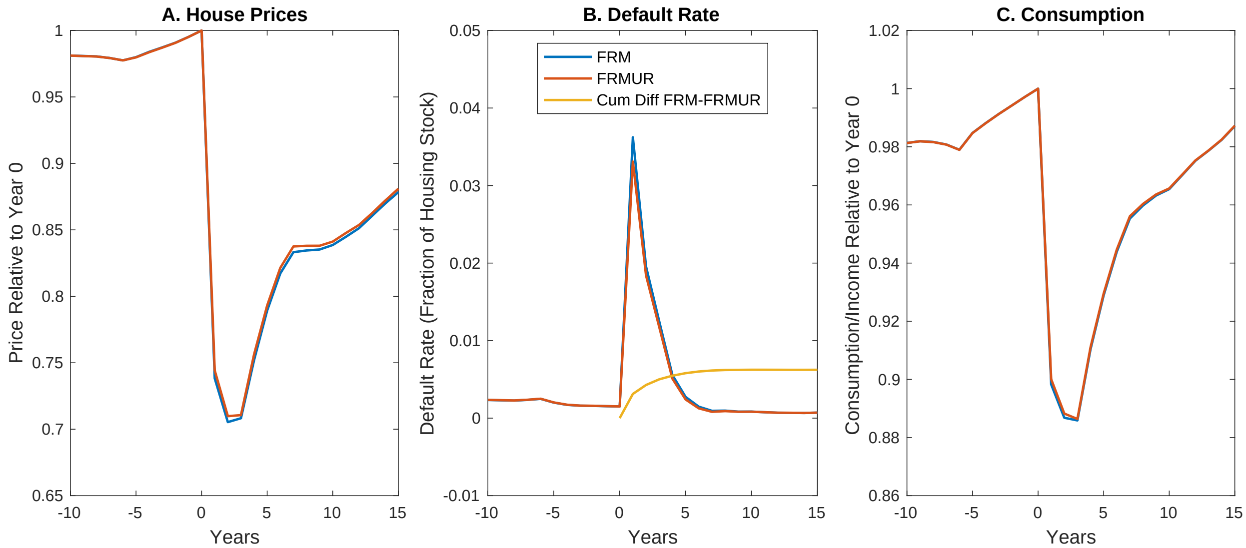
FRMUR acts like an FRM except it directly addresses the issue of not being able to refinance when one is underwater. Because payments fall in bad states, it helps smooth consumption and reduce default losses for lenders. Of course, the option itself is costly for lenders. On net, in our model, the FRM with an underwater refinancing option has a 0.7 basis point higher spread than an FRM. Again, there are two offsetting effects: lenders provide insurance to underwater households who take advantage of the underwater refinancing option during the crisis which pushes spreads up, but there is less default in equilibrium which pushes down spreads.

Figure 11 shows the crisis under FRMs with an underwater refinancing option. The FRMUR is priced off the long end of the yield curve, which falls less than the short end in a crisis. Because the insurance it provides is minimal, consumption is roughly the same as FRMs in the crisis. The initial decline in prices is 0.45 percentage points smaller, only 7.9% fewer households default, and consumption only rises by 0.05 percentage points. All of these figures are far smaller than under the EK mortgage. In stochastic simulations, the FRMUR option behaves much like an FRM with slightly lower default volatility and has only a 0.02% household welfare gain relative to FRM, as shown in Table 4. Overall, allowing for underwater refinancing provides some limited macroprudential benefits in a crisis but only limited consumption-smoothing benefits because the

share of loans originating in each state.

[45] We model this mortgage as an FRM with a built-in option to convert to a new FRM. Upon exercise of the option, the homeowner continues to pay the same spread over the 10-year bond rate as at origination but instead over the current 10-year risk-free bond rate, which the household locks in for the remainder of the mortgage. Since the 10-year bond rate falls during the crisis, the borrower has a lower minimum payment. These choices put the FRMUR mortgage on an equal footing with the EK mortgage in the sense that the original lender prices the built-in option to refinance into the pre-crisis mortgage rate. If we were to require that a new lender take on the additional risk through refinancing, the FRMUR would look worse because rates would rise in the crisis as borrowers with a high risk of default would refinance. With higher rates in the crisis, the payment relief from being able to refinance would be reduced. See the Appendix for a quantitative comparison.

Figure 11: FRM With an Underwater Refinancing Option: Housing Market Outcomes



The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM and the FRM with underwater refinancing both *ex ante* and *ex post*. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

FRM is priced off the long end of the yield curve. The fact that FRMUR affects prices, default, and consumption so little reveals that most of the benefits of ARMs are due to the yield curve channel rather than the underwater homeowner channel, which, while active, is quantitatively weak in our baseline model.

The comparison of the Eberly-Krishnamurthy convertible mortgage and the FRM with an underwater refinancing option provides the clearest contrast for our central finding: It is best to "front load" payment relief so that it is concentrated in a crisis. Both the EK and FRMUR provide insurance to underwater borrowers, however the EK front loads the relief by delivering maximal relief in a recession while the FRMUR gives households a new lower fixed rate that they keep for longer, thereby spreading the payment relief over the length of the mortgage. From the perspective of a lender entering the crisis state, these two mortgages are quite similar. Indeed, the present value of outstanding mortgages falls by 2.62 percent upon entering the crisis under FRMUR and by 2.52 percent under EK. However, the convertible mortgage helps liquidity constrained households in the crisis by three times as much and stems the tide of default and the price-default spiral. It also makes buying much more affordable for new homeowners, helping put a floor under house prices. The convertible mortgage does hit household balance sheets more when rates rise, but by then the macroeconomy and housing market have stabilized, and house prices have recovered so that most households can refinance back into a new mortgage that initially functions as an FRM if they so choose. Consequently, designs that front-load the benefits of rate reductions so that they are concentrated in recessions do best.

One aspect of refinancing that this analysis misses is the ability of underwater homeowners to extend their mortgage, as all mortgages amortize over the remaining lifetime of the borrower. For, example, refinancing a loan that amortizes over 10 remaining years into a 30 year loan would substantially lower monthly payments, even if the loans are at the same interest rate and have the same principal (Lucas et al., 2011). Since the key issue during the crisis is liquidity, our analysis shows that such a reduction in payments could deliver benefits. However, there are a few important caveats. First, we have shown that the benefits are largest when payment reductions are front-loaded, and extending an FRM's term reduces payments over the remaining life of the mortgage rather than front-loading them. Moreover, the benefits of obtaining a new 30-year mortgage depend on how many years the borrower has left on her current mortgage. The households most at risk of default and in need of liquidity in the model are largely young households who have recently purchased a home and for whom the term extension would have a small impact, as their existing mortgage is already a relatively long-term mortgage. Indeed, in Agarwal et al.'s (2017b) analysis of the HARP program, the average refinanced loan under HARP received a term extension of only 4.7 years. Since little principal is paid down in the first several years of the loan, this implies that over three quarters of the payment relief from HARP came from rate reductions and less than one quarter from principal reductions.

Finally, the FRMUR is similar to *ex post* policies that refinance borrowers who are underwater, such as the Home Affordable Refinance Program pursued in the Great Recession. However, the HARP program occurred at least in part while the Federal Reserve pursued quantitative easing, and Agarwal et al. (2017b) report that the average HARP borrower experienced a 1.4 percent reduction in their mortgage rate, while in our baseline calibration the reduction is 0.62 percent. We show how FRMUR interacts with quantitative easing in Section 7.2.

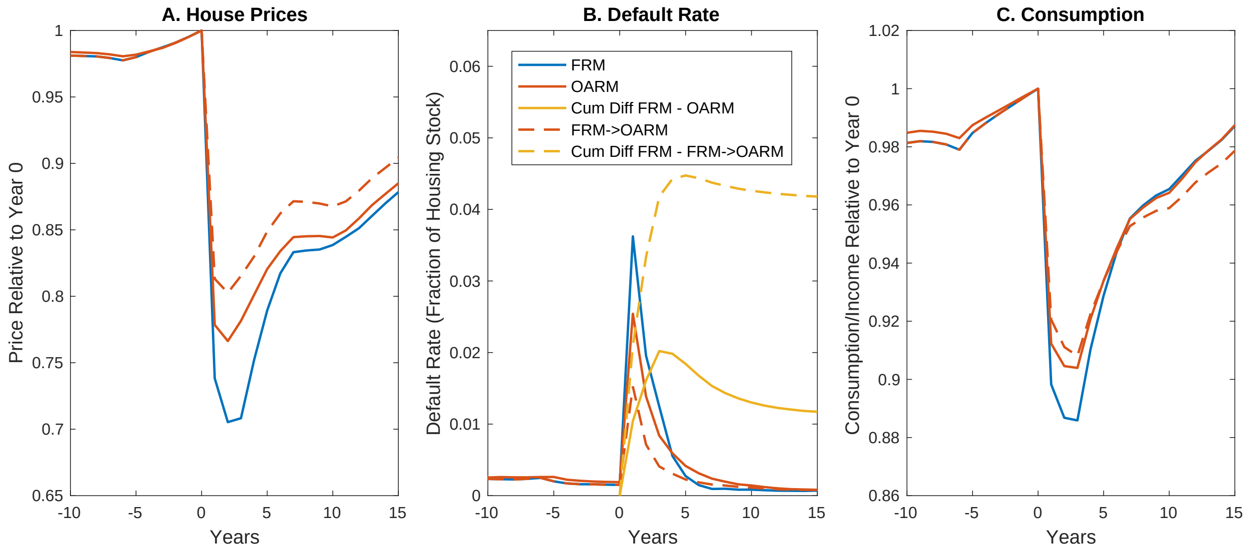
6.3 Option ARM

We now consider a mortgage design that takes maximal advantage of front-loading benefits, an option ARM. This is an ARM mortgage that became popular in the early 2000s boom allowing the borrower to make one of three payments: a fully amortizing payment, an interest only payment, and a potentially negatively amortizing payment equal to the minimum payment based on the interest rate at origination. The negative amortization is allowed up to a ceiling, and after several years, the option ARM converts to a fully amortizing ARM. The key feature of the option ARM is the ability to delay payments, which is potentially beneficial to a homeowner in a crisis. Piskorski and Tchistyi's (2010) theoretical analysis of mortgage contracts with exogenous house prices highlights the benefits of an option ARM.

Note that relative to the contracts we have considered thus far, the option ARM allows the payment reduction to be a function of the borrower's idiosyncratic state and not just the aggregate state. In other words, a borrower who receives a negative income shock in an expansion state can choose to defer payments. The ARM or the EK contract only allow for reduced payments in recession or crisis states when interest rates are low. Thus our analysis of the option ARM also captures the benefits of mortgage contracts whose payments can be indexed to idiosyncratic states.

To introduce the option ARM in our model, we assume that the mortgage behaves like a normal ARM for households in the last 25 years of their life. However, households in the first 20 years of life are able to choose a mortgage balance next period equal to the maximum of their current balance and the maximum balance allowed under the LTV constraint ϕ in the period in which they

Figure 12: Option ARM: Housing Market Outcomes



The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under FRM, the option ARM mortgage, and an FRM→OARM counterfactual. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

took out their mortgage given today's price. This allows for some negative amortization up to a ceiling defined by ϕ, and makes it so that households are not forced to pay down principal when the LTV constraint tightens at the beginning of the crisis.

Figure 12 compares the crisis under FRM and the option ARM. Under the option ARM, as prices fall by 6.10 percentage points rather than 2.24 percentage points under ARM. The decline in aggregate consumption is also 1.81 percentage points smaller than FRM rather than 0.79 percentage points under ARM. However, there is more default than under ARM.

The option ARM is successful because in addition to indexing payments to the short rate as with a standard ARM, it delivers additional front-loaded relief by allowing homeowners to negatively amortize up to a cap during the crisis and defer mortgage payments until after the crisis. There is an offsetting effect because homeowners understand the additional insurance benefits the option ARM design provides and they take on more leverage when purchasing a home. Some also take advantage of the negative amortization option before the crisis to deal with an idiosyncratic shock. This implies that households hold fewer liquid assets and have higher LTV ratios in non-crisis periods relative to an economy with a standard ARM, which creates macro fragility that undoes some of the benefits of the option ARM when the crisis hits. To illustrate this, Figure 12 shows in dashed lines an FRM→OARM counterfactual that preserves the pre-crisis distribution of individuals across states from the FRM model by switching the economy from FRM to OARM by surprise when the crisis hits, as with the FRM→ARM counterfactual in Section 5.1. One can see that absent the endogenous increase in leverage and decrease in liquid asset holdings at the beginning of the crisis under OARM, the price and consumption declines would have been ameliorated even more, and the default rate would have been much lower under the option ARM. Indeed, the fact that the option ARM preforms worse than the ARM for default is due entirely to the rightward shift in the LTV distribution and reduction in liquid asset holdings.

It is also worthwhile to note that the option ARM provides substantially higher unconditional household welfare benefits than the other mortgage designs, as reported in Table 4. This once again is intuitive. The other mortgage designs only provide state-contingent insurance against aggregate shocks, since changes in interest rates are tied to aggregate movements in the economy. However, households are exposed to substantial idiosyncratic risks. Unlike the other mortgage designs, the option ARM provides for insurance against such shocks because young households can negatively amortize in response to a purely idiosyncratic income shock.

These latter observations also hint at the Achilles' heel of the option ARM (or any similar contract with insurance against idiosyncratic income shocks). In a model with heterogeneity in types in which some households have low default utility costs and others have high default costs, the option ARM will attract types with low default costs who will take the mortgage, continually delay payments, and eventually default. Such behavior will raise the cost of the option ARM and even unravel the market so that there is little insurance benefit from the contract. On the other hand, with the EK mortgage or a regular ARM, the ability to delay payments is contingent on the aggregate state, which is less subject to gaming. Our purpose in studying the option-ARM is not to identify it as the optimal design but simply to quantify the benefit of the additional insurance it provides absent adverse selection concerns.

7 The Interaction of Mortgage Design and Monetary Policy

We now turn to examining how monetary policy interacts with mortgage design in a crisis. We begin by comparing the polar opposite case from our main analysis in which the real interest rate rises in a recession. We then turn to more aggressive monetary policy in a crisis both through more aggressive short rate reductions and quantitative easing. Our model allows us to quantitatively compare how different combinations of monetary policy and mortgage designs affect the economy. This analysis highlights the value of studying mortgage design and monetary policy jointly.

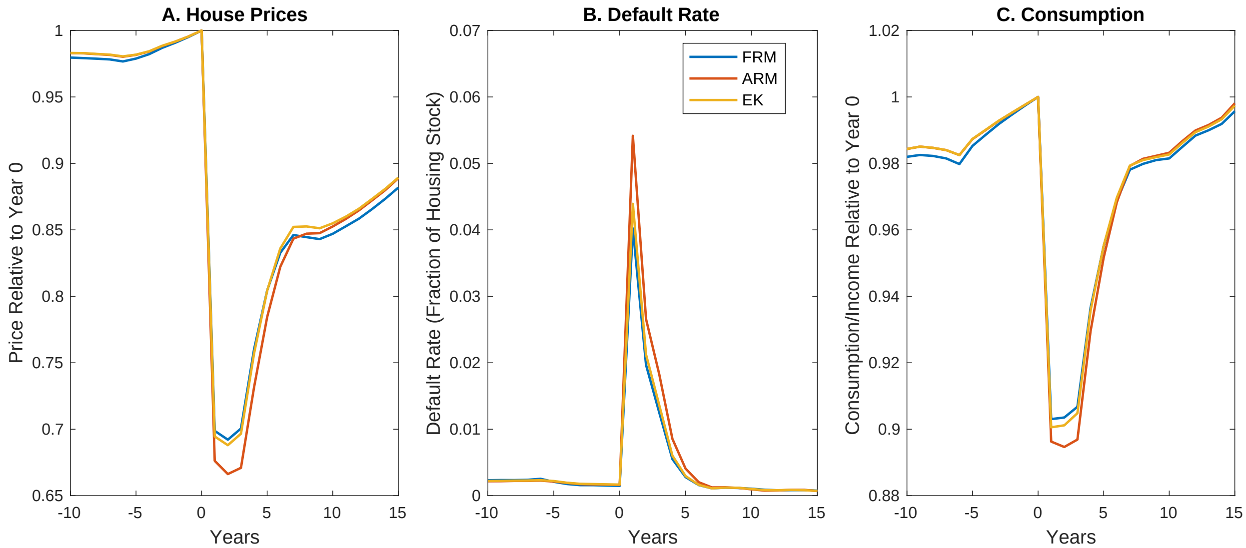
7.1 Fighting Inflation: Rising Real Rates in a Crisis

In this subsection we consider a housing-led recession with substantial inflation that causes the central bank to raise the real interest rate to contain inflationary pressures. One can think of this as a combination of the housing bust in the Great Recession and the inflation experienced in the 1981-2 recession. To consider this case, we keep the same calibration but increase the real short rate in a crisis to 5.6 percent, which is the average real rate in the 1981-2 recession instead of 0.26 percent.[46] [47] We call this the "Volcker monetary policy."

[46]Combining the 2006-8 credit crunch with the 1981-2 recession results in rather extreme event, especially given that the Federal Reserve may not have raised rates as much in the 1981-2 recession in an all-ARM economy. We nonetheless evaluate this extreme event in order to stress-test our mortgage designs.

[47]Many ARMs have a ceiling on the amount that the interest rate can rise over the course of the loan. In this counterfactual, we assume that there are no such ceilings. If there were, the increase in the ARM interest rate would be smaller, but knowing this banks would increase the *ex ante* mortgage spread.

Figure 13: Short Rate Rises in Crisis as in 1981-2 Recession

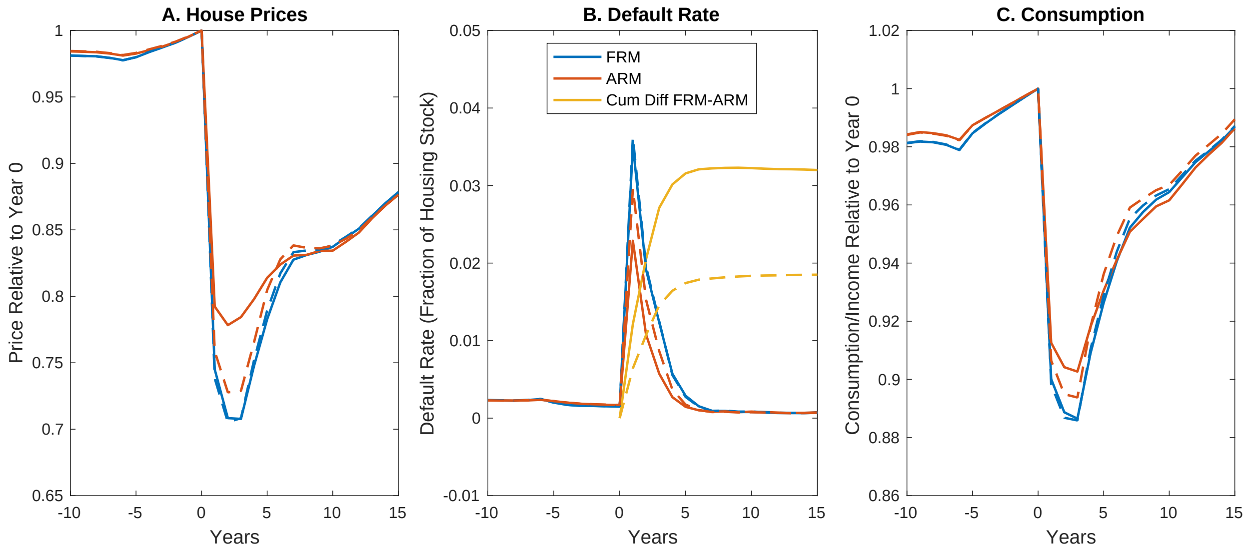


Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95 percent to 80 percent and there is a deep downturn lasting an average of 5.66 years under an FRM, ARM, EK, and FRMUR for the case where the central bank raises the short rate to 5.6 percent, the average real rate in the 1981-2 recession, in crisis states. Monetary policy in other states is unchanged.

Figure 13 shows ARM, FRM, and EK, in a simulated crisis under this alternate monetary policy. FRMUR is not shown because it is indistinguishable from FRM when rates rise in a crisis. Because the covariance of the short rate and income has the opposite sign from under the baseline monetary policy, an ARM makes the household's income stream net of mortgage payments more volatile and leads to higher price volatility and default and lower consumption. Quantitatively, prices fall by 2.59 percentage points more under ARM, and 11.62 percent of the housing stock defaults over eight years under ARM relative to 8.46 percent under FRM.

The EK convertible mortgage performs closer to an FRM under the Volcker monetary policy. Prices fall by roughly the same amount peak to trough as FRM although they fall slightly more in the first period, and the cumulative default rate over eight years is 9.17 percent. Most households that take out a new mortgage or refinance do not convert their EK mortgage from fixed-rate to adjustable-rate, and the ARM share at the beginning of the crisis is approximately 10 percent. However, a few households convert their EK mortgage to adjustable-rate and do not refinance into a new EK mortgage that begins as a fixed-rate prior to the crisis. Some of these households experience an adverse income shock in the crisis, are underwater, and end up defaulting, which is why default is slightly higher under EK. That being said, the crisis is nowhere near as severe as under ARM because the vast majority of households have an EK mortgage that is functioning as a FRM, which is why EK provides a nice balance of insuring against catastrophe in an inflationary environment while providing insurance when real rates fall in a crisis. Indeed, the EK achieves most of the gains of ARM under the baseline monetary policy but only a fraction of the downside under an inflationary monetary policy.

Figure 14: Monetary Policy: Short Rate Falls By More in Crisis



Note: The figure shows in solid lines the outcomes in a simulated downturn in which the maximum LTV falls from 95 percent to 80 percent and there is a deep downturn lasting an average of 5.66 years under an FRM and ARM or the case where the central bank further reduces the short rate by 100 basis points relative to the baseline monetary policy. The baseline scenario without the additional 100 basis point reduction is shown in dashed lines for comparison.

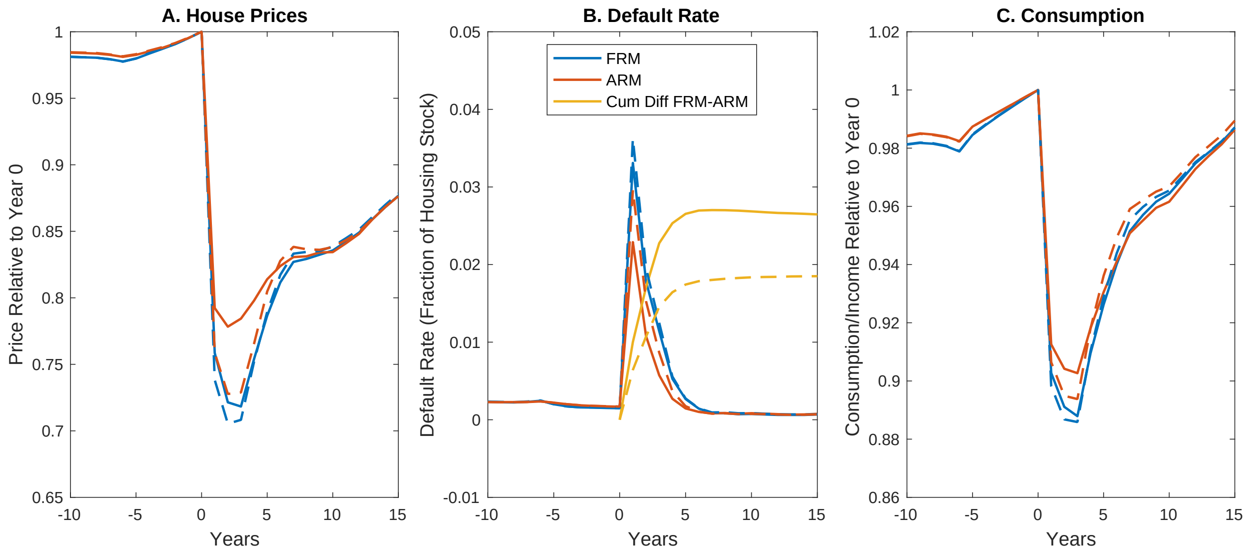
7.2 Additional Monetary Experiments

We now consider two modifications to the central bank's baseline monetary policy. In the first modification, we consider expansionary monetary policy in crisis states whereby the central bank lowers the short rate and consequently ARM mortgage rates by an additional 100 basis points. Long interest rates fall less since they adjust according to expectations-hypothesis-type logic. We think of this as corresponding to more aggressive traditional monetary policy, with long rates still governed by the lender SDF.

In the second modification, we assume that in addition to reducing the short rate an additional 100 basis points, in crisis states the central bank is able to further reduce long rates. This experiment can be viewed through two lenses. There is evidence that long rates react more to monetary policy shocks than might be expected under the expectations hypothesis. See, for example, Hanson and Stein (2015). Thus with this experiment, we provide some sense of how more responsive long rates alter our baseline conclusions. Alternatively, the Fed pursued quantitative easing in the crisis and its aftermath due to hitting the zero lower bound on short rates, and its policy was directly targeted at the long end of the yield curve and mortgage term premia. Thus our exercise can shed light on the interaction between unconventional monetary policy and mortgage design.

We implement the movement in long-rates by assuming that the central bank reduces the bank cost of capital for the FRM and FRMUR in the crisis state by enough so that interest rate differential between the expansion with loose credit and crisis states for a risk-free 10-year bond is 1.4 percent, which is the average reduction in interest rates for HARP borrowers reported by Agarwal et al. (2017b). This requires a 208 basis point capital subsidy in the crisis state. An alternative approach would be to model the SDF as depending on unconventional monetary policy,

Figure 15: Monetary Policy: Further Reduction in Long Rates (QE)



Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95 percent to 80 percent and there is a five year deep downturn under an FRM and ARM, for the case where the central bank both further reduces the short rate relative to the baseline monetary policy by 100 basis points and additionally pursues a policy that subsidizes the long rate so that a 10-year risk-free bond has a 1.4% interest rate differential between the expansion and crisis states. The baseline scenario without is shown in dashed lines for comparison.

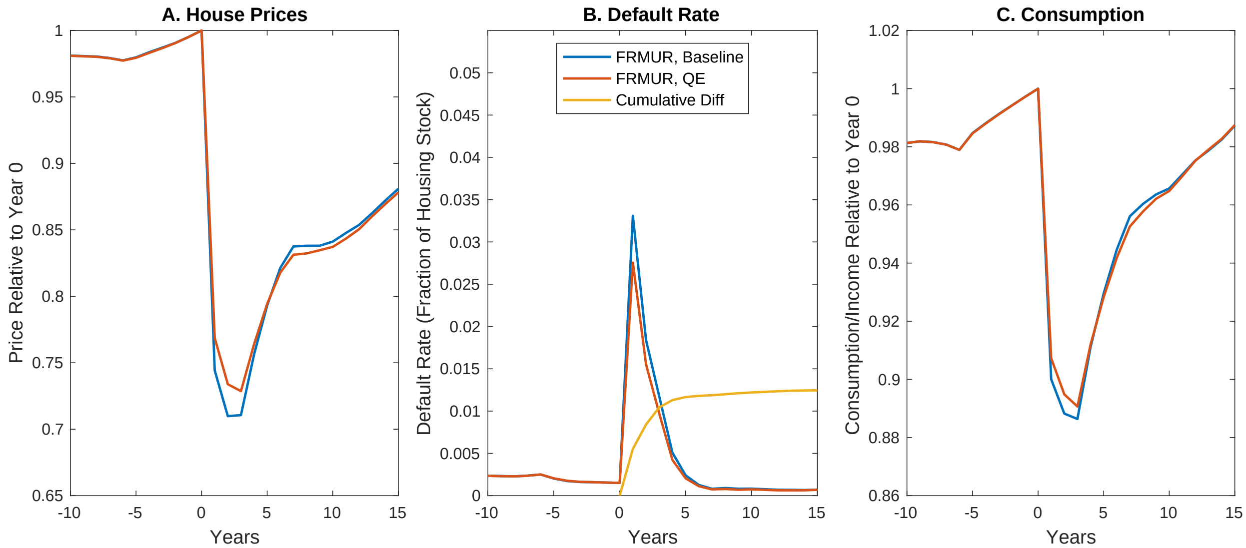
but our investigations suggest that such an approach would substantially increase the complexity of the model.

The results of the first modification, which reduces only short rates, are shown in Figure 14, and key moments are summarized in the first two columns of Table 6. Traditional monetary policy has very little impact on the severity of the crisis under FRM. While the return to saving in liquid assets changes, the FRM interest rate does not change appreciably because FRMs are priced off the long end of the yield curve. Since few households have liquid assets and most saving in the economy is for retirement, housing demand by young households and the behavior of high-LTV households that are primarily young is unchanged, and so the housing market equilibrium is not substantially affected.

On the other hand, more aggressive monetary policy that affects short rates is useful when homeowners have ARMs. Price declines and default are substantially lower and consumption is slightly higher due to the more aggressive monetary policy. This is, of course, not surprising. Lower short-rates leads to lower mortgage payments which leads to less default and a smaller price-default spiral, and lower short rates also stimulate new homeowner demand more.

When the risk-free 10-year long rate falls 1.4 percent in the crisis relative to an expansion, due either to a more responsive term premium or unconventional monetary policy such as QE, the FRM economy performs better than under the baseline monetary policy, as shown in Figure 15 and summarized in the last three columns of Table 6. Prices fall by 1.29 percentage points less than with FRM under the baseline monetary policy, and 7.35 percent of the housing stock defaults over eight years instead of 8.00 percent. These figures are an improvement but still fall short of the ARM under the baseline monetary policy. As with the ARM, new homeowners can now lock in cheap

Figure 16: Monetary Policy: FRM With Underwater Refi Option With and Without QE



Note: The figure shows the outcomes in a simulated downturn in which the maximum LTV falls from 95 percent to 80 percent and there is a five year deep downturn under an FRM with an underwater refinancing option for the baseline monetary policy and for the case where the central bank both further reduces the short rate relative to the baseline monetary policy by 100 basis points and additionally pursues a policy that subsidizes the long rate so that a 10-year risk-free bond has a 1.40% interest rate differential between the expansion and crisis states.

financing, which stimulates demand for housing and boosts house prices. However, there is still limited passthrough of the lower rates to existing homeowners due to refinancing constraints, which keeps the default rate high. Thus while a more responsive term premium or forms of unconventional monetary policy which push down long rates mitigates the differences between the FRM and ARM economies, they do not resolve them.

This suggests that *ex post* policies such as HARP or the FRMUR mortgage design, which resolve the underwater refinancing frictions, may be quite effective when combined with monetary policies that push down long rates, such as QE.[48] This can be seen in two ways. First, Figure 16 compares the FRM with underwater refinancing under the baseline policy and the more aggressive short rate policy coupled with quantitative easing to generate a 1.4 percent drop in the risk-free 10-year long rate – and thus a 1.4 percent decline in the FRM rate for homeowners who exercise their refinancing option that is tied to this rate – in the crisis. Prices fall by 1.88 percentage points less than the baseline policy, consumption falls 0.43 percentage points less, and the default rate is 6.17 percent over eight years rather than 7.37 percent. Second, one can compare how much of an improvement FRMUR is over FRM under the baseline monetary policy (Table 3) and the monetary policy with a more-responsive term premium (Table 6). With a more responsive term premium, there is now a more significant difference between the FRM and FRMUR designs. Whereas under the baseline policy FRMUR caused prices to fall 0.45 percentage points less, consumption to fall 0.05 percentage points less, and default to decline by 0.63 percentage points more than under FRM, with a more-responsive term premium, prices fall 1.04 percentage points less, consumption falls 0.28 percentage

[48]Our results here are related to those in Beraja et al. (2019), who in a more stylized model with a fixed mortgage design show that HARP is more effective with monetary policies like QE.

Table 6: Downturn Moments Under Short Rate and More Response Term Premium Policies

Design	FRM	ARM	FRM	ARM	FRMUR
Monetary Policy	Short Rate		Short Rate + Term Premium/QE		
Pct Point Change in Max ΔP Rel to FRM		7.05		6.00	1.04
Pct Point Change in Max ΔC Rel to FRM		1.62		1.48	0.28
Share Defaulting over 8 Years	7.87%	4.63%	7.35%	4.63%	6.17%

Note: Table shows the indicated statistics for each mortgage design averaging across 100 simulations of a crisis that lasts 5.66 years after an expansion with loose credit as described in the main text. under two monetary policies. The first policy reduces the short rate relative to the baseline monetary policy by 100 basis points and lets the long rate adjust according to expectations-hypothesis-type logic, but discounting according to the lender SDF. The second policy reduces the short rate relative to the baseline monetary policy by 100 basis points and additionally pursues a policy that subsidizes the long rate so that a 10-year risk-free bond has a 1.40% interest rate differential between the expansion and crisis states. The top row shows the percentage point change in the peak to trough change in price relative to FRM, with a positive number indicating a less severe price decline. The second row shows the percentage point change in the peak to trough change in consumption relative to FRM, with a positive number indicating a less severe decline in consumption. The third row shows the share of the housing stock defaulting after 8 years.

points less, and default declines 1.18 percentage points more than under FRM. The FRMUR design, coupled with a more responsive term premium, provides significant liquidity relief to constrained households, and furthermore, allows them to take advantage of it. Furthermore, this exercise shows resolving the underwater refinancing frictions becomes more important as the amount of liquidity relief available increases.

8 Conclusion

We assess how mortgages can be redesigned or modified in a crisis to reduce housing market volatility, consumption volatility, and default and how mortgage design interacts with monetary policy. To do so, we construct a quantitative equilibrium life cycle model with aggregate shocks in which households have realistic long-term mortgages that are priced by risk-neutral and competitive lenders and household decisions aggregate up to determine house prices. We calibrate the model to match aggregate moments as well as quasi-experimental evidence on the effect of payment size and LTV on default so that our model is tailored to qualitatively assess the benefits of adding simple state contingency to mortgage contracts.

We use the model to assess the performance of various mortgage contracts in a realistic, recession-driven housing crisis. In our model, indexing payments to monetary policy significantly reduces household consumption volatility and default. If the central bank reduces interest rates in response to the crisis, mortgage payments fall both by more and regardless of whether a household refinances, which helps to smooth consumption, limits default by relaxing budget constraints in bad states, and stimulates housing demand by new homeowners. These hedging benefits are quite large for constrained, high LTV households who bear the brunt of the housing bust. Quantitatively, under ARM relative to FRM house prices fall by 2.24 percentage points less, 23 percent fewer households default, consumption falls by 0.79 percentage points less. These benefits depend on the extent to which the insurance provided by ARMs is anticipated by households, as households take on more debt and hold fewer liquid assets when they expect their payments to fall in a crisis, leading to more macro fragility.

Our main conclusion is that mortgages that front-load payment relief provide much better outcomes for households and can substantially improve household welfare. Reducing monthly payments in a crisis alleviates liquidity constraints when they are most binding, limits default, and stimulates housing demand by renters, which stems house price declines and ameliorates pecuniary externalities that work through the price of housing. The clearest example of the benefits of front-loading payment reductions comes from our comparison of a FRM with an option to convert to an an ARM to an FRM with an option to refinance underwater. Although both mortgages have similar effects on the balance sheets of lenders when a crisis hits, the EK convertible mortgage front-loads payment reductions during the crisis, which makes it far more successful at stabilizing prices, consumption, and default. The FRMUR does worse because it is priced off the long end of the yield curve and thus smooths payment relief over the life of the loan, providing households payment relief in states where they need it less.

While the comparison of these two loans gives the starkest contrast, there are a number of different ways in which one might front-load payment relief. We consider quantitatively the option ARM, which allows households to insure idiosyncratic shocks. One might also consider an ARM with a cap or an FRM or ARM that can have its payments cut in a recession through term extension. We leave the analysis of the best way to front-load payment relief for future research.

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A Numerical Implementation

We solve the model numerically by discretizing the state space. Because the aggregate state Θ_t has five values, we use five values for the interest rate. We use seven grid points for income. We use 22 grid points for savings, with a denser grid at low savings. We use 41 grid points for the mortgage balance, with two balances for renters (with and without the default flag), and 39 values for the mortgage including zero. The grid is denser for higher balances that are close to the equilibrium price given that amortization schedules feature smaller principal payments earlier in the mortgage. Finally, we use eight equally-spaced grid points for the house price and interpolate on the price grid when the price falls between grid points. We have experimented with finer grids and have found the results are not sensitive to the grids we choose.

We solve the household's problem by backward induction, starting with an initial guess for the mortgage spreads and the forecast rules. We then simulate 19,000 years of data and throw out the first 100 years. In each period of the simulation, a new generation is born and shocks hit the economy. We then calculate the supply and demand schedules, calculate the equilibrium price, and finally update the distribution of individuals across states according to the policy functions, interpolating between price grid points.

We then follow the Krusell and Smith-style algorithm described in the main text. We run forecast rule regressions:

$$\log p_{t+1} = f_{(\Theta_t,\Theta_{t+1})}\left(\log p_t\right)$$

where $f_{(\Theta_t,\Theta_{t+1})}$ is a function for each realization of (Θ_t, Θ_{t+1}). Because Θ_t has five values, there are potentially 25 forecast rules, but 12 relate to (Θ_t, Θ_{t+1}) transitions that do not occur in practice (*e.g.*, one only transitions to the tight credit state after a crisis) so there are only 13 forecast rules that must converge. With orthogonal interest rate movement, there are 52 (the same 13 for high-high, high-low, low-high, and low-low rates). We parameterize $f(\cdot)$ as a linear spline with knot points located at the price grid points. After running the regressions on the simulated data, we update the forecast rules to a convex combination of the original forecast rules and the regressions.[49] We also solve for the equilibrium spreads given κ and update the spreads to a convex combination of the original forecast spreads and the new zero profit spreads. We iterate until the forecast rules and spreads converge. Three criteria must be simultaneously met for convergence. First, the spread must be within one basis point of the actual break-even spread. Second, the forecast rule R-squareds must converge so that the difference is less than 0.0025 between iterations. Third, for all forecast rules, the distance between the regression and the forecast rule must be less than 0.005 at all prices for which we observe data in our 19,000-period simulation. In practice, the third criterion is the most stringent, and our spreads and forecast rules are highly accurate as described below.

For the FRM in the baseline calibration, we must solve for the κ that implies that the average spread between the spread in the model and a 10-year bond is 1.65 percent as in the data. For this, we follow a similar algorithm to the one described above, but in addition to adjusting the spread

[49] One could in principle simply use the regression as the new forecast rule, but we have found that this leads to oscillating behavior. We consequently use a convex combination which, while slower, provides better convergence properties.

Table 7: Den Haan Tests: R^2 Of Predicted vs. Actual Price N Years Ahead

R^2 Over Horizon:	15 Years	30 Years	45 Years	100 Years
Baseline, FRM	95.22%	95.06%	95.08%	95.10%
Baseline, ARM	93.80%	93.78%	93.88%	93.88%
Baseline, EK	93.47%	93.32%	93.42%	93.43%
Baseline, FRMUR	94.86%	94.71%	94.74%	94.77%
Baseline, OARM	92.48%	92.40%	92.52%	92.47%
Volcker, FRM	95.65%	95.58%	95.65%	95.66%
Volcker, ARM	95.20%	95.32%	95.48%	95.55%
Volcker, EK	94.98%	95.08%	95.22%	95.27%
Volcker, FRMUR	95.67%	95.61%	95.57%	95.68%
Volcker, OARM	93.19%	93.24%	93.43%	93.48%
Short Rate Only, FRM	95.12%	94.96%	94.99%	95.00%
Short Rate Only, ARM	92.19%	92.13%	92.32%	92.33%
QE, FRM	94.71%	94.52%	94.55%	94.55%
QE, ARM	92.19%	92.13%	92.32%	92.33%
QE, FRMUR	94.19%	94.03%	94.07%	94.07%

Note: The table shows the R^2 of a regression of the forecast price on the actual price 15, 30, 45, and 100 years ahead. To create the table, for each period of the simulation we calculate the expected price based on the realized price N periods ago, the forecast rule, and the realized sequence of aggregate states in the intervening N years. We then regress the actual price on the predicted price and report the R^2.

and forecast rules, we adjust κ to hit the 1.65 percent average spread between the 10 year mortgage and a synthetic 10-year bond rate calculated according to the lender SDF. κ converges along with the spreads.

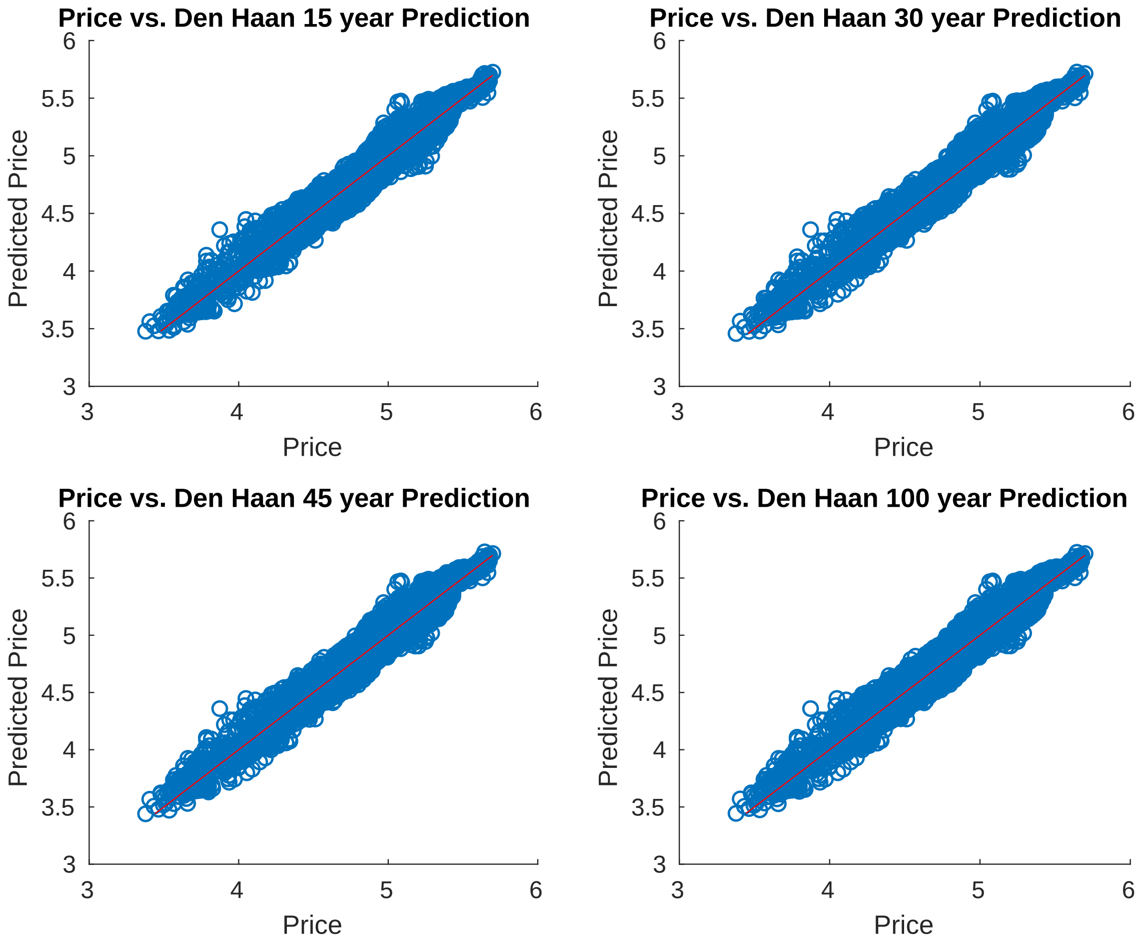
To assess the accuracy of the forecast rules, we follow Den Haan (2010) and calculate the predicted price using the realized shocks 15, 30, 45, and 100 years into the future and compare the realized price to the predicted price. Table 7 shows the results for each of the models we consider in the paper. The R^2 ranges from 92% to 96%, indicating the forecast rules are accurate and that errors do not accumulate over longer horizons. Figure 7 shows the scatter plot of the predicted versus the actual price at each horizon for FRM.

In the text, we report results from three exercises. In the first, we report unconditional aggregate statistics from simulations of 19,000 years of data.

In the second exercise, we report the volatility of consumption at the individual level. To do so, we track 25,000 individuals per generation for 6,300 years and report the standard deviation of consumption. We conduct a standard variance decomposition that divides individual consumption into the variance of aggregate consumption across periods, the variance of generational average consumption within periods, and the variance of individual consumption across generations.

In the third exercise, we calculate the impulse response of our economy to a crisis. We run 100 simulations in which we seed the economy to be in a loose credit expansion for the last 5 periods. We then seed the economy to be in a crisis for at least 3 periods, after which is stochastically exits. We average over the 100 simulations to obtain the impulse response. In doing so, we also calculate the impact of the initial switch into the crisis state on the present value of the bank's portfolio.

Figure 17: Den Haan Tests: Scatter Of Predicted vs. Actual Price N Years Ahead For FRM



Note: The figure shows a scatter plot of the forecast price 15, 30, 45, and 100 years ahead versus the actual price. To create the figure, for each period of the simulation we calculate the expected price based on the realized price N periods ago, the forecast rule, and the realized sequence of aggregate states in the intervening N years.

B Household Welfare

As discussed in the main text, our approach to welfare is to calculate it for the households ignoring the lenders. This is because we abstract from explicitly modeling the lenders and instead calibrate a lender SDF using options data, and so we cannot calculate lender welfare. We do, however, report the present value loss to lenders (using their SDF) in a crisis, and our pricing implies that lenders break even according to their lender SDF on average. Our welfare approach also neglects any social costs of foreclosure, as we detail in the main text.

The value function in our model for an age a individual at time t in idiosyncratic state s_j^t and aggregate state Σ_t is:

$$V_t^a\left(s_t^j, \Sigma_t\right) = E_t\left\{\sum_{t=0}^{T-a} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \alpha_a H_t - 1\left[Default_t\right] E\left\{d\right\}\right] + \beta^{T-a}\psi\frac{(b+\xi)^{1-\gamma}}{1-\gamma}\right\},$$

where c_t is consumption at time t, b is the bequest at time T, H_t is housing at time t, and $Default_t$ is an indicator for default at time t. Define the value function under two different economies 0 and 1 as $V_t^{a,0}$ and $V_t^{a,1}$.

We want to calculate the consumption-equivalent household welfare change, which we denote by $\Delta^a\left(s_t^j, \Sigma_t\right)$. We calculate the household welfare change as equivalent variation, that is the amount consumption would have to increase by in economy 0 to make the agent indifferent between economy 0 and economy 1. $\Delta^a\left(s_t^j, \Sigma_t\right)$ is thus defined implicitly as:

$$
\begin{aligned}
V_t^{a,1}\left(s_t^j \Sigma_t\right) &= E_t \left\{ \sum_{t=0}^{T-a} \beta^t \left[\frac{\left(c_t^0\left(1+\Delta^a\left(s_t^j, \Sigma_t\right)\right)\right)^{1-\sigma}}{1-\sigma} + \alpha H_t^0 - 1\left[Default_t^0\right] E\left\{d\right\} \right] + \beta^{T-a}\psi \frac{\left(b^0+\xi\right)^{1-\gamma}}{1-\gamma} \right\} \\
&= V_t^{a,0}\left(s_t^j, \Sigma_t\right) + \left[\left(1+\Delta^a\left(s_t^j, \Sigma_t\right)\right)^{1-\sigma} - 1\right] E_t \left\{ \sum_{t=0}^{T-a} \beta^t \left[\frac{\left(c_t^0\right)^{1-\sigma}}{1-\sigma}\right] \right\}.
\end{aligned}
$$

Consequently,

$$
\Delta^a\left(s_t^j, \Sigma_t\right) = \left(1 + \frac{V_t^{a,1}\left(s_t^j, \Sigma_t\right) - V_t^{a,0}\left(s_t^j, \Sigma_t\right)}{E_t\left\{\sum_{t=0}^{T-a} \beta^t \left[\frac{\left(c_t^0\right)^{1-\sigma}}{1-\sigma}\right]\right\}}\right)^{\frac{1}{1-\sigma}} - 1.
$$

We report two distinct household welfare calculations. The first is unconditional welfare. This is defined as the consumption-equivalent equivalent variation of being born in economy 0 relative to economy 1. Because we are using equivalent variation, we use the ergodic distribution for the initial generation over both micro and macro states in the 0 economy, which we denote by ω^0, to weight the value functions in the 0 economy and the 1 economy. The aggregate unconditional welfare difference between economies 0 and 1 is then:

$$
\Delta = \int \Delta^a\left(s_t^j, \Sigma_t\right) d\omega^0.
$$

The second household welfare calculation is conditional welfare, which corresponds to the aggregate welfare loss experienced when the economy switches from a loose-credit expansion to the crisis state. We follow Krueger et al. (2016) in calculating the welfare losses from a switch in the aggregate state as "the permanent percentage increase in consumption that a...household would require so that its welfare in the transition is the same as the welfare when the transition does not happen." Given this, we can define the consumption-equivalent equivalent variation welfare change for a household with idiosyncratic state s_t^j of going from aggregate state Σ_t^0 to aggregate state Σ_t^1 as:

$$
\Delta^a_{\Sigma_t^0 \Sigma_t^1}\left(s_t^j\right) = \left(1 + \frac{V_t^{a}\left(s_t^j, \Sigma_t^1\right) - V_t^{a}\left(s_t^j, \Sigma_t^0\right)}{E_t\left\{\sum_{t=0}^{T-a} \beta^t \left[\frac{\left(c_t^0\right)^{1-\sigma}}{1-\sigma}\right]\right\}}\right)^{\frac{1}{1-\sigma}} - 1.
$$

We report results aggregating up conditional welfare across age groups. We do this because different cohorts have different remaining lives, so one cannot compare Δ easily across age groups.

The concept we use to aggregate is also from Krueger et al.: "Suppose households are randomly placed into the pre-recession cross-sectional distribution over individual characteristics. Under the veil of ignorance of not knowing where in the distribution one would be placed, we ask by what percentage would lifetime consumption of everyone need to be increased to be compensated from the loss of falling into a recession." Define Ω_t^a as the distribution of households of age a across states at the beginning of period t, when the shock hits. Consumption equivalent welfare for age a is then:

$$\Delta^a_{\Sigma_t^0 \Sigma_t^1} = \int \Delta^a_{\Sigma_t^0, \Sigma_t^1} d\Omega_t^a$$

Rather than reporting $\Delta^a_{\Sigma_t^0 \Sigma_t^1}$ directly, we report the difference between $\Delta^a_{\Sigma_t^0 \Sigma_t^1}$ under FRM and ARM.

C Calibration Details

C.1 Income Process

This appendix details the calibration of the income process.

Our first task is to create a discretized income process with left skewness in busts. We assume that the income process of Floden and Linde (2001) holds in an expansion and add left skewness in a recession to match the left skewness observed in 2008-9 by Guvenen et al. (2014). We then discretize the resulting distribution using the method of Farmer and Today (2017) modified slightly to discretize two idiosyncratic shock distributions on the same grid. Because there are multiple ways to add this amount of left skewness, we need an additional moment to match. In particular, we match the standard deviation of aggregate log income of 0.0215. Finally, we adjust the distribution of initial income for incoming distributions to best match the lifecycle profile of income in Guvenen et al. (2019) and adjust the aggregate income level so that we obtain an average income of one.

We now describe how we implement this procedure in detail. Our first step is to create a mixture of normals to target in the Farmer and Toda (2017) procedure. We assume that the distribution of idiosyncratic shocks is the same in expansions regardless of whether credit is loose or tight. Similarly, the distribution of idiosyncratic shock is the same in recessions and crises, again regardless of whether credit is loose or tight. In expansions, we use the income process of Floden and Linde (2001), which is an AR(1) process with a standard deviation of 0.21 log points and an annual persistence of 0.91. In a crisis or recession, we add the standardized skewness of the 2008-9 income change distribution from Guvenen et al. (2014) to the mixture of normals. To do this, we fit a mixture of normals to the histogram underlying Figure 12 in Guvenen et al. (2014). We then calculate the standardized skewness of this distribution. We then adjust the Floden and Linde (2001) income process by adding mass to the left tail to match the same standardized skewness we found in the Guvenen et al. data.

With these mixtures of normals in hand, we then adjust three parameters to target three moments: the standard deviation of aggregate log income, the fit between the model and data age-income profiles, and an aggregate income of one. We adjust three parameters in a minimum distance routine. First, we shift the mean of the bust idiosyncratic shock distribution, which allows us to target the standard deviation of log income in the data. Second, we adjust fraction of the population that starts at each initial income which allows us to target the age-income profile. Third, we adjust the level of the aggregate income vector y^{agg}, which allows us to target an average income of one. For each vector of parameters, we apply a modified version of the Farmer and Toda (2017) algorithm using code from Toda's website. The modification is to calculate the idiosyncratic income grid using only the boom distribution and then discretizing both the boom and bust distributions on the same grid. Given the left skewness, we use a grid with grid-points spaced from 2.5 standard deviations below the mean to 1.5 standard deviations above the mean. After discretizing, we simulate income, generate simulated data and calculate the average aggregate income, the standard deviation of log income, and the mean squared error between the age-income profile in the data and the model, and then search over the parameter space to find the optimal. We match the standard deviation of log income and aggregate income nearly exactly, and the fit to the age-income profile is close as shown in Figure 18.

We obtain from this calibration procedure four objects objects: a transition matrix that is a function of the aggregate state $\Xi^{id}(\Theta_t)$, a vector $y^{agg}(\Theta_t)$ which gives log aggregate income as a function of the aggregate state, a log idiosyncratic income vector y^{id} for non-retirement (and implicitly a log idiosyncratic income vector for retirement that is equal to $y^{id}-0.35$ by our assumption of a 0.35 log point income decline in retirement), and an entrant distribution over the idiosyncratic states. Recall that we discretize to seven idiosyncratic incomes and have five aggregate states. For these vectors and matrices, we let state 1 be the crisis, state 2 be a recession with loose credit, state 3 be an expansion with loose credit, state 4 be a recession with tight credit, and state 5 be an expansion with tight credit.

The aggregate income vector is

$$y^{agg} = \begin{bmatrix} 0.0976 \\ 0.1426 \\ 0.1776 \\ 0.1426 \\ 0.1776 \end{bmatrix}$$

We calibrate so that aggregate income falls by 8.0 percent in a crisis and 3.5 percent in a recession, so this is just a normalization of the resulting vector ensuring that aggregate income is on average equal to one.

The idiosyncratic income vector is:

$$y^{id} = \begin{bmatrix} -1.2663 \\ -0.9286 \\ -0.5909 \\ -0.2533 \\ 0.0844 \\ 0.4221 \\ 0.7598 \end{bmatrix}$$

The distribution of entrants has 92.69 percent weight on the second to bottom grid point and 7.31

percent weight on the third to bottom grid point.

By assumption, the idiosyncratic income distribution is the same in the crisis state, the recession state with loose credit, and the recession state with tight credit. It is equal to:

$$\Xi^{id}\,(\text{Crisis or Recession}) = \begin{bmatrix} 0.8183 & 0.1593 & 0.0103 & 0.0003 & 0.0001 & 0.0001 & 0.0116 \\ 0.1820 & 0.5046 & 0.2847 & 0.0279 & 0.0007 & 0.0001 & 0 \\ 0.0563 & 0.0783 & 0.6038 & 0.2611 & 0.0005 & 0 & 0 \\ 0.0123 & 0.0209 & 0.1493 & 0.6123 & 0.2011 & 0.0041 & 0 \\ 0.0021 & 0.0091 & 0.0227 & 0.1939 & 0.6093 & 0.1595 & 0.0034 \\ 0.0001 & 0.0019 & 0.0099 & 0.0268 & 0.2391 & 0.5904 & 0.1318 \\ 0.0033 & 0.0049 & 0.0067 & 0.0083 & 0.0140 & 0.1648 & 0.7980 \end{bmatrix}$$

In this matrix, the probability listed is the probability of a transition from the row i to the column j, so that rows add to a probability of one. By assumption, the idiosyncratic income distribution is the same in the expansion state regardless of the state of credit and is equal to:

$$\Xi^{id}\,(\text{Expandsion}) = \begin{bmatrix} 0.7059 & 0.2663 & 0.0235 & 0.0005 & 0 & 0 & 0.0038 \\ 0.0911 & 0.5794 & 0.3204 & 0.0091 & 0 & 0 & 0 \\ 0.0021 & 0.1138 & 0.6156 & 0.26116 & 0.0069 & 0 & 0 \\ 0 & 0.0030 & 0.1478 & 0.6327 & 0.2116 & 0.0049 & 0 \\ 0 & 0 & 0.0042 & 0.1887 & 0.6361 & 0.1674 & 0.0036 \\ 0 & 0 & 0 & 0.0032 & 0.2462 & 0.6104 & 0.1402 \\ 0.0070 & 0 & 0 & 0.0001 & 0.0073 & 0.1456 & 0.8400 \end{bmatrix}.$$

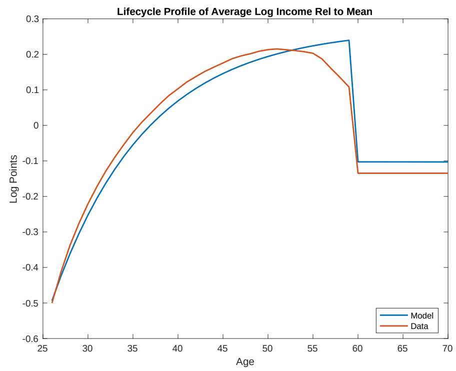
Our calibration procedure does a good job of replicating key facts in the data that we target. Figure 18 shows the age-income distribution in the model relative to the data. We impose on the data that income falls 0.35 log points at retirement, so the retirement numbers are equal to average income at 60 minus 0.35. The model does a good job of capturing the age-income profile at younger ages. Figure 19 shows the distribution of idiosyncratic income shocks in the boom relative to the bust. Our income shocks capture well the left skewness found by Guvenen et al. (2014).

C.2 Calibration Targets and Procedure

Most of the external calibration targets are described in the main text. There are a few details that we relegate to this appendix. First, although the transition matrix is described in the main text, we do not detail it there. Letting state 1 be the crisis, state 2 be a recession with loose credit, state 3 be an expansion with loose credit, state 4 be a recession with tight credit, and state 5 be an expansion with tight credit, the transition matrix is:

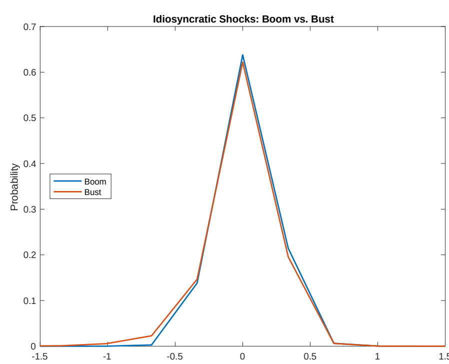
$$\Xi^{\Theta} = \begin{bmatrix} 0.6364 & 0 & 0 & 0 & 0.3636 \\ 0 & 0.1011 & 0.8989 & 0 & 0 \\ 0.0133 & 0.1832 & 0.8035 & 0 & 0 \\ 0 & 0 & 0 & 0.1011 & 0.8989 \\ 0.0133 & 0 & 0.0200 & 0.1832 & 0.7835 \end{bmatrix}$$

Figure 18: Age-Income Profile in Model and Data



Note: This figure shows the age-income distribution. The x axis is ages from 26 to 70 in our model. The y axis shows average log income by age, with zero being the average log income in the model. The data come from Guvenen et al. (2016). The retirement data is equal to the average income at 60 minus 0.35 to be consistent with retirement in our model. The model is simulated from our model.

Figure 19: Distribution of Idiosyncratic Income Shocks in Boom vs. Bust



Note: This figure shows the distribution of idiosyncratic income shocks in the boom and the bust calculated from model simulations. The results compare favorably to the evidence in Guvenen et al. (2014).

Table 8: Moments Matched in Calibration Procedure

Moment	Data	Model
Mean Price	5.00	4.994
Ratio of Net Worth at 60 45 at 10th Percentile Net Worth	3.576	3.472
Ratio of Net Worth at 60 45 at Median Net Worth	2.096	2.094
Cumulative Default Over 8 Year Crisis	8.00%	8.00%

Note: The four moments used in the calibration of a, ψ, ξ, and $\bar{d}$. The model column indicates the moments in the model and the data column indicates their values in the data that we use as targets.

In this matrix, the probability listed is the probability of a transition from the row i to the column j, so that rows add to one. The economy remains in the crisis state with a probability 63.64% of the time and exits with a probability of 36.36%. The expansion and recession states look identical for loose and tight credit except that in a tight-credit expansion the probability of remaining is 2% lower. There is a 2% probability the economy transitions to a loose credit expansion. The economy enters a crisis from an expansion with 1.33% probability. The transition matrices between expansion and recession are based on NBER dates as described in the main text.

The second detail relegated to the main text is how we reduce a_a, the age-dependent valuation of a house, in old age. We assume that this is constant at its calibrated value until retirement, at which point it falls by 1/15 of its initial value each year until death, which is 10 years after retirement. Our results are not sensitive to this specification.

We match four internal calibration targets. To match these four moments, we alter the parameters and solve the FRM economy under the baseline monetary policy. The average price is fairly insensitive to ψ, ξ, and $\bar{d}$ and can be set using a alone, and we target a house price of five times the average pre-tax income in the economy. We then alter ψ, ξ, and $\bar{d}$ to target three moments. The first two are the ratio of total net worth at age 60 to age 45 in the SCF for the median and 10th percentile households by net worth. To smooth out noise in the SCF for any particular age, we calculate total net worth at age 60 as the mean total net worth from ages 58 to 62 and total ent worth at age 45 as the mean total net worth from age 43 to 47. We find a 10th percentile ratio of 3.576 in the SCF and a ratio at the median of 2.096. The final moment is the cumulative default rate over a eight years in our main impulse response, which we choose to target eight percent. The four moments are matched quite well, as indicated in Table 8. All four moments are within 0.15% of their target values.

D Additional Numerical Results (Online Only)

D.1 Downturn Moments for Alternate Monetary Policies

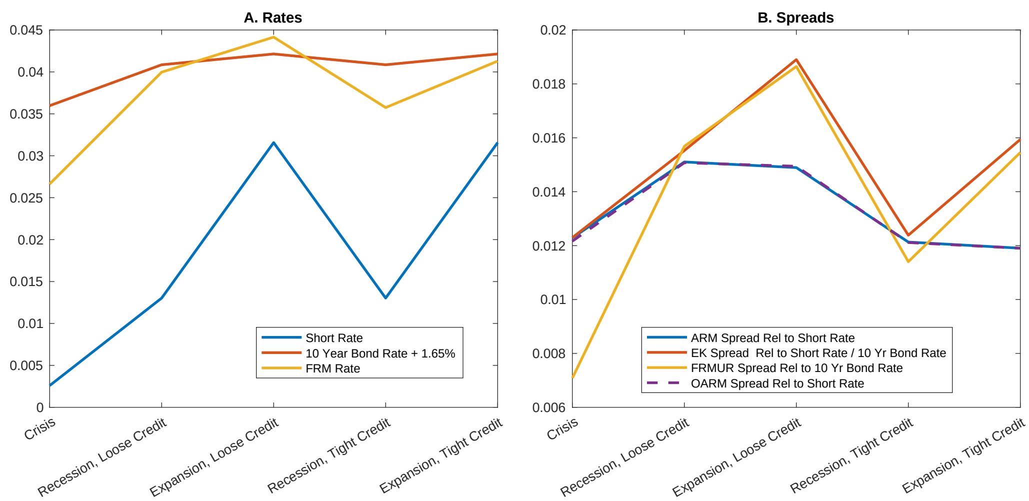
Table 3 in the main text summarizes the performance of the various mortgage designs we consider under the baseline monetary policy, and Table 6 reports the same statistics this for the short rate reduction and the short rate reduction with quantitative easing monetary policies. Table 9 reports the same statistics this for the Volcker monetary policy.

Table 9: Downturn Moments For Various Mortgage Designs Under Volcker Monetary Policy

Design	FRM	ARM	EK	FRMUR	Option ARM
Pct Point Reduction in Max ΔP Rel to FRM		-2.59	-0.42	0.01	-2.83
Pct Point Reduction in Max ΔC Rel to FRM		-0.84	-0.25	0.00	-0.59
Share Defaulting over 8 Years	8.46%	11.62%	9.17%	8.46%	13.27%

Note: Table shows the indicated statistics for each mortgage design averaging across 100 simulations of a crisis that lasts 5.66 years after an expansion with loose credit as described in the main text under the Volcker monetary policy. The top row shows the percentage point change in the peak to trough change in price relative to FRM, with a positive number indicating a less severe price decline. The second row shows the percentage point change in the peak to trough change in consumption relative to FRM, with a positive number indicating a less severe decline in consumption. The third row shows the share of the housing stock defaulting after 8 years.

Figure 20: Yield Curves and Mortgage Spreads: Baseline Monetary Policy



Note: The figure shows the short rate, 10-year bond rate, and FRM mortgage rates in the left panel and the ARM, FRMUR, EK, and OARM spreads in the right panel. All interest rates are for pre-paid interest timing.

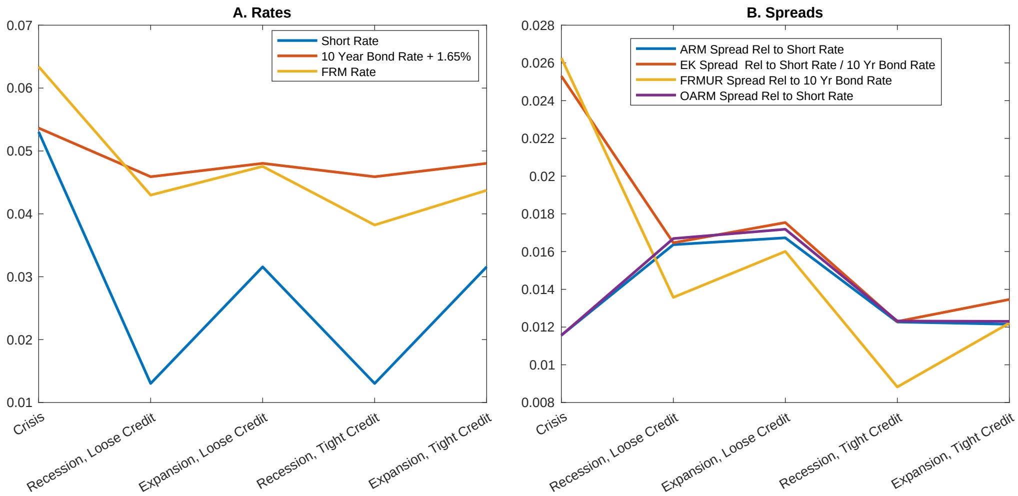
D.2 Mortgage Spreads and Yield Curves

This section documents the equilibrium yield curves and mortgage spreads in our model.

We consider five main mortgage designs. The FRM has an interest rate that depends on the aggregate state in which the mortgage is originated. The ARM has a spread over the short rate that depends on the aggregate state in which the mortgage is originated. The EK mortgage has a spread over the short rate if it is converted to an ARM or the 10-year bond rate if it is an FRM based on the aggregate state in which the mortgage is originated. The FRM with an underwater refinancing option can be written as a spread over the initial 10-year bond rate that is preserved when the underwater refinancing option is exercised in the crisis state. Finally, the option ARM has a spread just like the ARM but allows negative amortization up to an LTV defined by the LTV constraint at origination.

The left panel of Figure 20 shows the short rate, the 10-year bond rate plus 1.65 percent, and

Figure 21: Yield Curves and Mortgage Spreads: Volcker Monetary Policy



Note: The figure shows the short rate, 10-year bond rate, and FRM mortgage rates in the left panel and the ARM, FRMUR, EK, and OARM spreads in the right panel. All interest rates are for pre-paid interest timing.

the FRM rate for the baseline calibration. Recall that the calibration is set such that the average FRM rate is equal to the average 10 year bond rate plus 1.65 percent. There is a positive spread relative to the 10-year bond in expansions and a negative spread in recessions and crises. This is because mortgages originated in a recession or crisis tend to be “safer” in terms default probabilities since prices are expected to recover, which lowers spreads.

The right panel of Figure 20 shows the spreads for the ARM, EK, FRMUR, and OARM. Like the FRM, the spreads are higher in expansions. This is especially true for the EK and FRMUR, since in expansions the spread prices in the option to convert to an ARM or refinance when underwater.

Figure 21 shows the same figures for the Volcker monetary policy. One can see that the short rate now rises in crises and the FRM rate also rises, although not by as much. Because of high default in the crisis, spreads are higher in the crisis for EK and FRMUR.

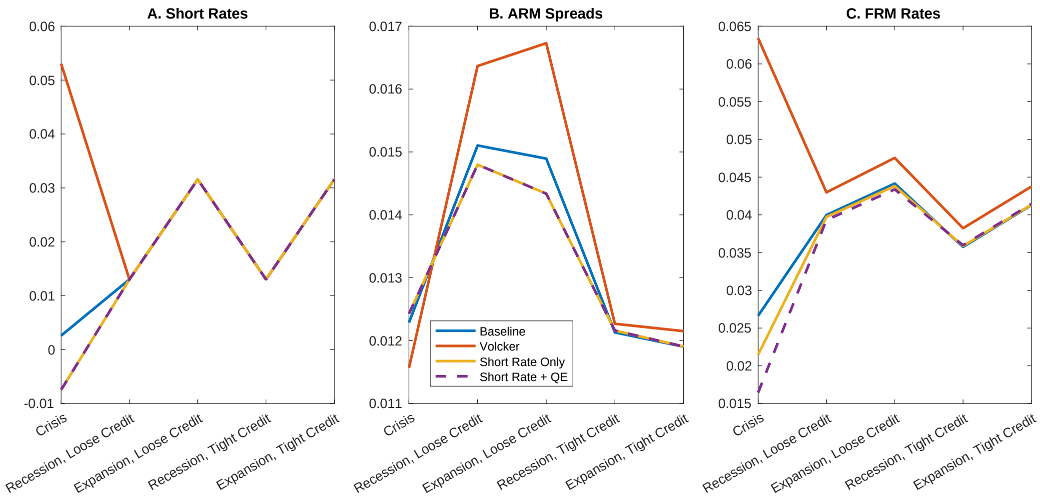
Finally, Figure 22 shows the short rate, ARM spread, and FRM rate for the four different monetary policies we consider: baseline, Volcker, the short rate reduction only, and the short rate reduction with QE. For the short rate and ARM spread, the short rate only and short rate plus QE lines overlap. There is a 1.00 percent reduction in the short rate in this state. In the FRM rates, however, the long rate is subsidized enough so that the difference between the expansion and crisis risk-free 10-year bond rate is 1.40 percent.

D.3 Lender Cost of Capital Rises in Crisis

This section considers an extension in which the lender cost of capital κ rises in a crisis. The model easily generalizes to the case in which κ_t is state-dependent. We keep our baseline calibration (that is we do not attempt to re-match the internally-calibrated moments) and κ at 125 basis points for non-crisis states.

Anderson, Duffie, and Song (2019) show that in a model of debt overhang, shareholders of a bank will require a return on a riskless loan of approximately the credit spread of the bank. In the

Figure 22: Yield Curves and Mortgage Spreads: Comparing Monetary Policies



Note: The figure shows the short rate in the first panel, the ARM spreads in the second panel, and the FRM rate in the third panel for the baseline, Volcker, short rate only, and short rate with quantitative easing policies.

crisis, credit spreads on banks rise about 1.5%. Hence we consider a case in which κ rises by 1.5% in the crisis state. This is a somewhat extreme case as our crisis lasts 5.7 years while the spike in credit spreads on banks in the crisis was far more short-lived, but we use this value in the spirit of an aggressive robustness check.

The results are shown in Tables 10 and 11. A few things are worth noting. First, the price and default benefits of alternative mortgage designs relative to FRM shrinks from the baseline. This is the case because the cost of originating a new mortgage in the crisis rises, which reduces demand by new homebuyers under the alternative mortgage designs. However, the reason why this happens differentially for ARM and FRM is somewhat subtle. Recall that with the FRM households keep the interest rate over the life of the loan, while with the ARM, households keep the spread over the life of the loan, so the increase in κ affects FRM and ARM equally absent a behavioral response by the borrowers. This occurs due to prepayment behavior. The fixed FRM rate is still low relative to the other states so there is not an incentive to refinance out of mortgages originated in the crisis. However, now the spread at which one originates an ARM rises for loans originated in the crisis, creating an differential incentive to refinance out of a loan originated in the crisis under ARM. This prepayment risk pushes up spread under ARM relative to FRM, which is what reduces demand by new homeowners. This effect is what eliminates some of the benefits of ARM and EK for price and default

In terms of unconditional household welfare, ARM, EK, and FRMUR all look roughly similar to FRM. Intuitively, the prepayment risk drives up the cost of these mortgages, which eliminates some of the benefits.

Overall, the results for this aggressive robustness check shows that while our results are quan-

Table 10: Moments From Downturn Simulations For Various Mortgage Designs With High Crisis Lender Cost of Capital

Design	FRM	ARM	EK	FRMUR	Option ARM
Pct Point Reduction in Max ΔP Rel to FRM		1.33	1.06	0.45	1.56
Pct Point Reduction in Max ΔC Rel to FRM		0.85	0.60	0.19	1.20
Share Defaulting over 8 Years When Enter Crisis	8.50%	6.81%	7.06%	7.81%	7.89%

Note: Table shows the indicated statistics for each mortgage design averaging across 100 simulations of a crisis that lasts 5.66 years after an expansion with loose credit as described in the main text. The top row shows the percentage point change in the peak-to-trough change in price relative to FRM, with a positive number indicating a less severe price decline. The second row shows the percentage point change in the peak-to-trough change in consumption relative to FRM, with a positive number indicating a less severe decline in consumption. The third row shows the share of the housing stock defaulting after 8 years.

Table 11: Moments From Stochastic Simulations For Various Mortgage Designs Relative to FRM With High Crisis Lender Cost of Capital

Design	ARM	EK	FRMUR	Option ARM
Std Dev of Price Rel to FRM	98.0%	98.6%	99.9%	95.7%
St Dev of Default Rate Rel to FRM	80.4%	85.1%	93.0%	80.8%
St Dev of Agg Consumption Rel to FRM	96.4%	98.3%	101.0%	91.5%
Increase in Household Welfare Rel to FRM	0.01%	0.01%	0.04%	0.33%

Notes: All series are percentages relative to the same statistic for the indicated FRM model. Price, default, and aggregate consumption are calculated from aggregate 6,400 year simulations. Idiosyncratic consumption is calculated by simulating 25,000 individuals each periods for 6,300 years. Household welfare is calculated as the equivalent variation in terms of the percent increase in annual consumption an agent would require in the FRM economy to be indifferent between being born in a random period in the indicated economy instead of the FRM economy.

titatively smaller if lenders have a significantly higher cost of capital in crisis states, they are qualitatively similar.

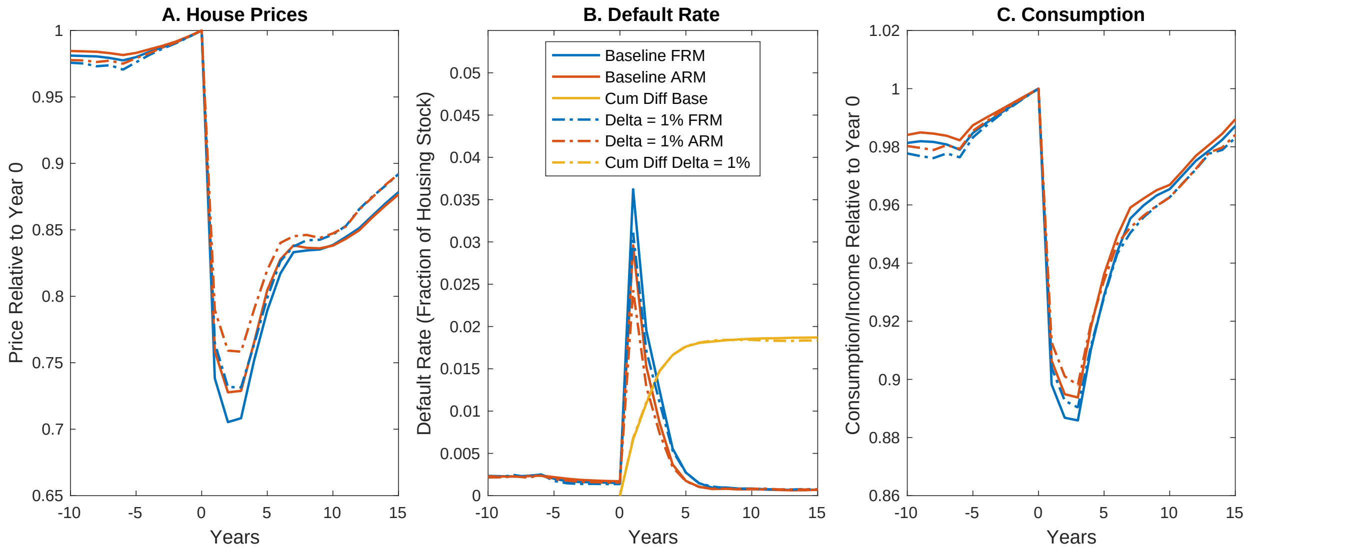
D.4 Movements In Interest Rates Orthogonal To the State of the Economy

Subsection 5.4 introduced interest rate variation orthogonal to the state of the economy. As a reminder, to do so we generalize the aggregate state Θ_t to have 10 values instead of 5. As before, there is a crisis state with tight credit, a recession state with either tight or loose credit, and an expansion state with either tight or loose credit. However, all five states may have high or low short interest rates, with an independent and identically distributed 50 percent probability of being in the high or low interest rate state. We assume that the interest rates in the high interest rate state are $\Delta/2$ above our baseline calibration rates, while the interest rates in the low interest rate state are $\Delta/2$ below our baseline calibration rates. Δ is thus the difference between interest rates in the high and low states.

This section shows downturn simulations for these models. Because things look similar for the $\Delta = 0.5\%$, $\Delta = 1.0\%$, and $\Delta = 1.5\%$ figures (albeit shifted by different amounts) we show only the $\Delta = 1.0\%$ basis points case below.

Figure 23 shows the results. One can see that with variation in interest rates orthogonal to the state of the economy, the downturn is slightly smaller for both ARM and FRM in terms of

Figure 23: Movements In Interest Rates Orthogonal to State of the Economy



Note: The figure shows in solid lines the outcomes in a simulated downturn in which the maximum LTV falls from 95 percent to 80 percent and there is a deep downturn lasting an average of 5.66 years under an FRM and ARM. The dashed lines show the case with orthogonal rate movement with a 1.0% difference between the high and low interest rate states, with the midpoint corresponding to our baseline calibration. There is an iid 50 percent chance of being in the high or low state.

prices, default rates, and consumption. In fact, the FRM economy with $\Delta = 1.0\%$ orthogonal rate movement is close to the ARM economy with no orthogonal rate movement. However, the differences between FRM and ARM are quite similar with and without orthogonal rate movement. This shows that our main results are robust to allowing for additional variation in interest rates that does not depend on the state of the economy.

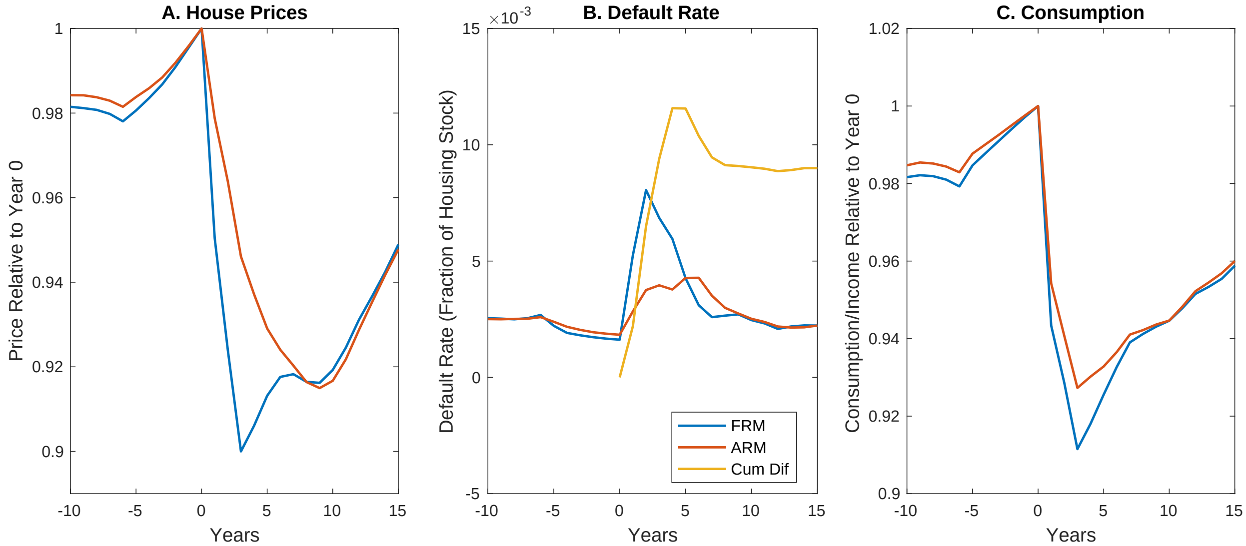
D.5 Crisis Without Tightening LTV Constraint

This appendix compares ARM and FRM in the version of the model in which the LTV constraint does not tighten to complement Figure 5, which compares the baseline FRM model to an FRM model without a tightening LTV constraint. One can see that ARMs still deliver higher house prices, less default, and higher consumption. However, the magnitudes on default and house prices are much smaller without a significant housing bust.

D.6 FRM With Underwater Refinancing Without Built-In Option

In the main text, we model the FRM with an underwater refinancing option (FRMUR) as an FRM with a built-in option to lower the FRM rate in a crisis. We do so in order to put the FRMUR on equal footing with the EK mortgage in the sense that the original lender prices the option to refinance into the pre-crisis mortgage rate. In this appendix, we show quantitative results for an alternate specification: an FRM that can be refinanced underwater without an increase in principal (which we call a "refinancing FRMUR" rather than an "option FRMUR" in the main text). Under this specification, the additional risk of refinancing the FRM underwater is priced by the new bank that originates the loan in the crisis rather than by the original lender. We introduce this into our

Figure 24: ARM vs. FRM With No LTV Tightening



Note: The figure shows the impulse response to a simulated downturn preceded by an expansion. In this version of the model the maximum LTV does not drop from 95 percent to 80 percent and instead stays at 80 percent, but the economy does fall into a deep downturn for an average of 5.66 years. Monetary policy is unchanged from the baseline. The blue line shows FRM, while the orange line shows ARM.

model by altering the refinancing constraint so that the LTV constraint only has to be satisfied if the principal balance is being increased.

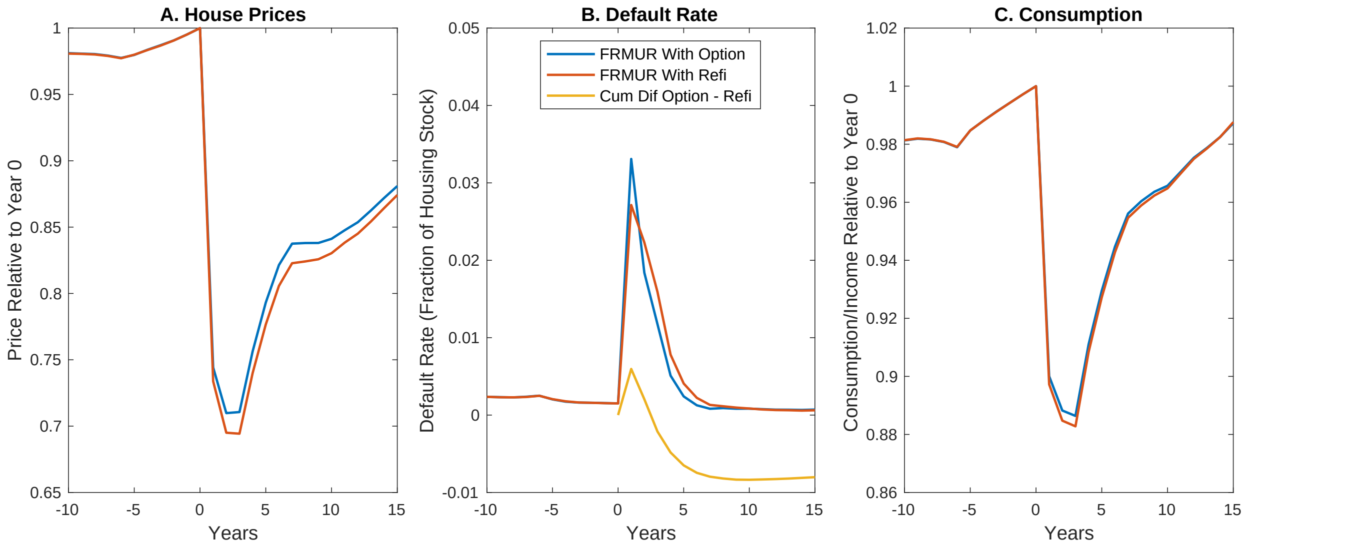
Figure 25 quantitatively compares the option FRMUR with the refinancing FRMUR. One can see that the results are qualitatively and quantitatively similar, although the refinancing FRMUR performs slightly more poorly than the option FRMUR. Indeed, the FRM that requires a refinancing has a 1.55 percentage point larger decline in prices, has less initial default but more subsequent default so that over eight years an additional 0.82 percent of homeowners default, and consumption falls by 0.36 percentage points more. To understand the results, recall that with the option FRMUR, the option to refinance is priced into the loan *ex ante*, which drives up the spread in expansion state. By contrast, with the refinancing FRMUR, borrowers are allowed to refinance underwater and their default risk is priced by the new lender that originates the refinanced loan in the crisis. These borrowers are riskier because they have a high LTV, and this drives up the spread in the crisis state. Relative to the option FRMUR, the gap between interest rates in the expansion and crisis state shrinks and the amount of relief provided by the FRMUR is lower, leading to a slightly larger price declines and more default.

E Mortgage Amortization Schedule (Online Only)

In this appendix, we derive the amortization formula that defines the required minimum payment for a mortgage in our setting with pre-paid interest.

Assume a mortgage is taken out with principal M_0 at time 0 and amortizes over T periods at an interest rate i. Define P_t as the principal payment at time t and I_t as the interest payment at

Figure 25: Option FRMUR vs. Refinancing FRMUR



The figure shows the impulse response to a simulated downturn preceded by an expansion with loose credit under an "option FRMUR" as in the main text where the initial lender gives the borrower and option to refinance in the crisis and prices it into the up-front rate and a "refinancing FRMUR" where the subsequent lender charges the underwater refinancing borrower a higher rate for their default risk. In the downturn, the maximum LTV falls from 95 percent to 80 percent and the economy falls into a deep downturn for an average of 5.66 years.

time t. Finally, let the constant annual payment be x.

Let us begin with the "standard" or "post-paid" timing whereby the interest between time t and $t-1$ is paid at time t. The principal at time t is then:

$$P_t = (x - iM_0)(1+i)^{t-1}.$$

It also must be that the sum of the mortgage payments is equal to the initial principal plus interest payments:

$$Tx = M_0 + \sum_{t=1}^{T} I_t = M_0 + \sum_{t=1}^{T} (x - P_t).$$

These two equations can be combined to obtain:

$$M_0 = (x - iM_0) \sum_{t=0}^{T-1} (1+t)^t,$$

which when solved for x yields the standard amortization formula:

$$x = M_0 \frac{i(1+i)^T}{(1+i)^T - 1}.$$

We now wish to define a similar formula for the case of prepaid interest whereby the interest between time t and $t+1$ is paid at t. To maintain symmetry with the standard formula, we assume that the household takes out a mortgage of size M_0 and in period one makes an interest payment of $iM_0 + P_0$. As above, $x = P_t + I_t$ but now $I_T = 0$ and $x = P_T$. Consequently:

$$P_t = \frac{(x - iM_0)\left(1 + \frac{i}{1-i}\right)^t}{1 - i}$$

and

$$Tx = M_0 + \sum_{t=0}^{T-1} I_t = M_0 \sum_{t=0}^{T-1} (x - P_t).$$

Combining these two equations gives:

$$M_0 = \frac{(x - iM_0)}{1 - i} \sum_{t=0}^{T-1} \left(1 + \frac{i}{1 - i}\right)^t$$

and solving for x yields:

$$x = M_0 \frac{i\left(1 + \frac{i}{1-i}\right)^T}{\left(1 + \frac{i}{1-i}\right)^T - 1}.$$

This is the same amortization formula as is standard replacing i with $i/(1-i)$ in the recursive part. This makes sense because pre-paying an interest at rate i at time t is the same as paying a rate of $i/(1-i)$ at $t+1$.