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ARE INTEREST RATES REALLY LOW?

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Ivo Welch

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ABSTRACT

Contrary to common perception, many fixed-income investors have not suffered unusually low real interest rates in and after the Great Recession of 2008. This is because taxable investors must first pay taxes on nominal interest returns, before inflation further reduces their earned real interest rates. To obtain the same real after-tax yield, investors need more than one-to-one compensation for inflation. As a result, long-term Treasury bonds have been no less attractive for taxable investors in 2016 (with a 1.0% post-tax real yield) than they were in 2006 (0.5%), 1976 (–1.7%), 1966 (0.9%), and 1956 (0.8%), although they have been less attractive than they were in 1996 (2.4%) and 1986 (2.9%). Short-term Treasury bond yields have been on the low side but have also not been particularly unusual.

Daniel R. Feenberg
National Bureau of Economic Research
1050 Massachusetts Avenue
Cambridge, MA 02138
feenberg@nber.org

Clinton Tepper
Anderson School at UCLA
110 Westwood Plaza
LA CA 90095
clinton.tepper@gmail.com

Ivo Welch
Anderson School at UCLA (C519)
110 Westwood Place (951481)
Los Angeles, CA 90095-1482
and NBER
ivo.welch@anderson.ucla.edu

[Insert Figure 1 here: **Nominal Interest Rates on Short-Term and Long-Term Treasury Notes**]

Figure 1 plots the post-war history of nominal interest rates on short-term Treasury bills and long-run Treasury bonds. The low interest rates towards the end of the sample—seemingly the lowest since the Korean War—have raised widespread concern. High-ranking fed officials, like Eric Rosengren (Boston), John Williams (San Francisco), Chris Neely (St. Louis), Narayana Kocherlakota (Minneapolis), or Jerome Powell (the incoming Federal Reserve Chair), are on record discussing how low interest rates have caused investors to "reach for yield," and thus how low interest rates have at least been partly responsible for the high stock market. They are not alone. The perception that interest rates have been unusually low, perhaps because short-term yields have hit their nominal bound of zero, is also pervasive among journalists, foreign financial and non-financial policy makers, retail and professional investors—and academics. For example, the secular stagnation theory in Summers (2014) uses a decline in the real interest rate to make the case for unconventional monetary policy. Taylor (2014) posits that the Fed held interest rates too low for too long before the crisis. And so on.

Yet, there is an important aspect often overlooked. Most financial economists use canonical benchmark models, which are themselves based on "perfect market" assumptions—the equivalent of a friction-free environment in physics. Such a market ignores not only market power and information differences, but also all trading costs, liquidity, and taxation. Thus, many academic papers begin with the qualification that they assume away these "complications," and then they proceed to their analysis. In other words, many academic papers about interest rates and the economy reflexively ignore taxation. For many purposes, this does not greatly distort the insights of the model. However, in the case of assessing whether interest rates are low or high, it does. Taxation of nominal yields is a first-order concern.

For example, consider that the 20-year Treasury bond promised 5.0% in 2006 but "only" 2.2% in 2016. The prevailing inflation rates of 3.2% and 1.3%, respectively, eroded much of the difference, leaving real interest rates of 1.8% and 1.5%. Yet, even this difference is irrelevant for the average taxable retail investors in the U.S. The prevailing average marginal tax rate on interest in the economy was about 25% in both years. Assessed on 5.0% and 2.2%, investors had to pay taxes of 1.25% and 0.55% in 2006 and 2016, respectively, leaving them with 3.75% and 1.65% in nominal pre-tax terms. In real terms, investors in 2016 thus earned about 1%, which is not less but rather twice as much as the 0.5% that they earned in 2006. We will point out that similar calculations for the post World-War-2 sample show that short-term post-tax post-inflation interest have been on the low side since the financial crisis, but not unusually so by historical standards.

Of course, not all bondholders pay taxes on interest receipts. About 40% of government bonds are held by other U.S. government institutions themselves. Of the remaining 60%, about half is held by foreigners. Most foreigners are exempt from U.S. taxes on interest. Therefore, such foreigners may effectively earn higher rates of return on U.S. Treasuries than their U.S. retail investors counterparts. The remaining 30% of government debt could be held either in tax-exempt vehicles (such as in charitable endowments or in 401-K plans) or in taxable investment accounts.[1] Moreover, not all taxable investors are equally taxed. High-income investors generally pay higher taxes, as do investors in blue states. A single unique tax rate does not exist, much less a data source that makes it possible to extract the complete distribution of relevant tax rates. However, there are two reasonable methods to assess the financial-market relevant taxation on interest payments.

The first method relies on a large sample of anonymized and disclosure-proofed tax returns over-weighted with high income returns. While the samples are heavily redacted, the masking does not affect our ability to calculate average marginal tax rates by income type. [2] Simplifying, our model assesses an investor-averaged rate at which their last "marginal" dollar in interest receipts was taxed. The model is not only imperfect, but, as already noted, also applies only to the U.S.-taxed subset of investors, assuming that they did not have unusual but representative holdings in the Treasury bond market. Moreover, our main analysis considers only Federal taxation—the web appendix shows that state taxes

[1] In all cases, the U.S. effectively pays more in effective interest to foreign and untaxed investors than it pays to domestic taxable investors. From the perspective of the U.S. government, the real interest rate it pays is a weighted average of its non-tax-paying and tax-paying bond holders.

[2] A description of the methods can be found in Feenberg and Coutts (1993) and http://users.nber.org/~taxsim/allyup.

would add a further four to five percent on average. Including them would only strengthen our claim.

[Insert Figure 2 here: **Long-Term AAA Municipal and Corporate Yields**]

The second method relies on the interest rate differential between taxable Treasury bonds and tax-exempt highly-rated municipal bonds. If a Treasury bond offers 5% and an otherwise equivalent perfectly safe "Muni" bond offers 4%, it follows that an investor with a 20% tax rate on interest income is indifferent between holding either of the two. Unfortunately, municipal bonds are never exactly the same as Treasury bonds. The issuing municipal entity may default on repayment. Moreover, Munis have much lower liquidity (resellability before expiration), a problem that suddenly (and perhaps unexpectedly) became more acute in the financial crisis of 2009. Fortunately, it is possible to remove an estimate of the credit and liquidity spread components by comparing highly-rated muni bonds to highly-rated corporate bonds. Putting together Treasury, municipal AAA-rated, and corporate AAA-rated bonds allows extracting an "effective" implied tax rate on interest payments:

$$\text{Effective Tax Rate} = 1 - \frac{\text{Muni AAA Yield}}{\text{Treasury Yield } + \text{Corp AAA Yield Spread}} .$$

Of course, this implied effective tax rate itself depends on the supply and demand of tax-exempt and taxable investors and investment alternatives, both domestically and beyond. For example, if only the highest-taxed 1% of investors were holding Treasuries, the financial market series would indicate implied tax rates close to the maximum personal interest tax rate.[3] If only tax-exempts were holding Treasuries, the effective tax rate series would indicate values close to zero.

[3]The average economy-wide tax rate was significantly below the marginal economy-wide tax rate for interest income, suggesting that it was higher-earning and higher-taxed individuals who earned and paid taxes on interest receipts.

[Insert Figure 3 here: **Tax Rates**]

Figure 3 shows that the two tax-rate calculations are not perfectly congruent but generally economically quite similar when both are available.[4] The tax-rate of 20-25% on interest was also remarkably typical and stable over the sample. However, this tax rate was also very different from the top statutory rate on interest and/or ordinary income, which could reach as high as 90% in the 1950s! Even the 1986 Reagan Tax Reform Act (with lower tax rates, but fewer loopholes, exceptions, and exemptions) did not drastically reduce the effective tax rate.

The final series necessary to compute investor-relevant real interest rates is inflation. The relevant inflation rate should be the prevailing contemporary expected inflation rate over the life of the bond. Our proxy is the CPI-based geometric inflation rate in the year before, the same year, and the year after the interest rates are measured (reflecting at least some expectation of the future). Again, our key inference remains largely the same if we use just the contemporaneous interest rate or a multi-year ex-ante or ex-post inflation rate.

[Insert Figure 4 here: **Post-Tax Real Yields on Short-Term 1-Year Treasuries**]
[Insert Figure 5 here: **Post-Tax Real Yields on Long-Term 20-year Treasuries**]

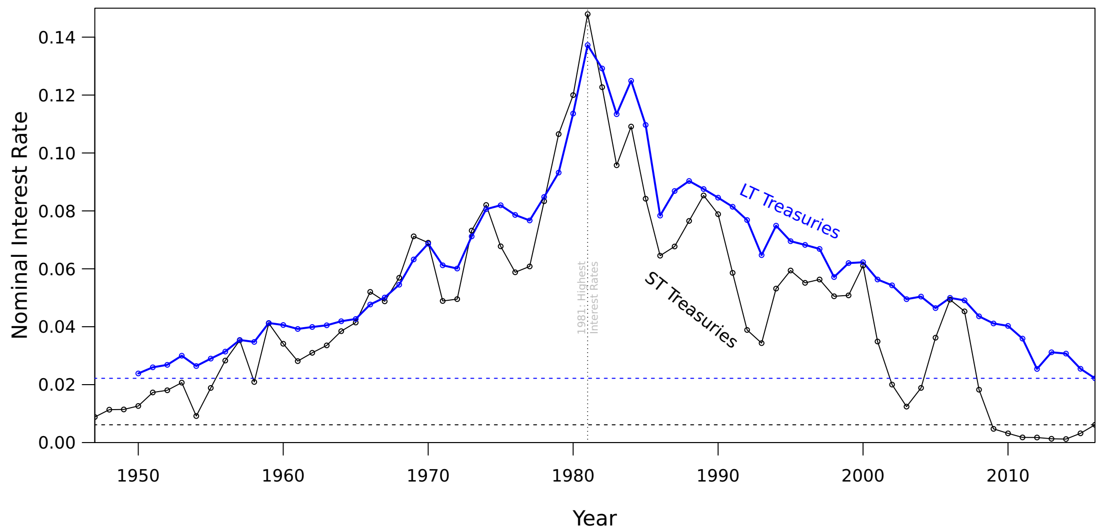
Figures 4 and 5 make our paper's key point. They plot the time-series of after-tax inflation-adjusted real interest rates on Treasury yields. The graphs show that real interest rates have not been unusually low after the crisis when put into the perspective of post-war history. The post-2008 interest rates have been well within the "ordinary" range. The 2016 yield on short-term Treasuries is only –0.10 standard deviations below its historical mean. The 2016 yield on long-term Treasuries is even +0.06 standard deviations above its historical mean.

We can conclude that the prevailing popular notion—that low interest rates have (and should have been) driving investors towards stocks and other risky investments—seems exaggerated. Of course, this is not to say that naïve investors after the financial crisis of

[4] Year-to-year changes in the tax rates correlate far less. This would be a concern for many other economic studies, but it is not of great concern in our own study.

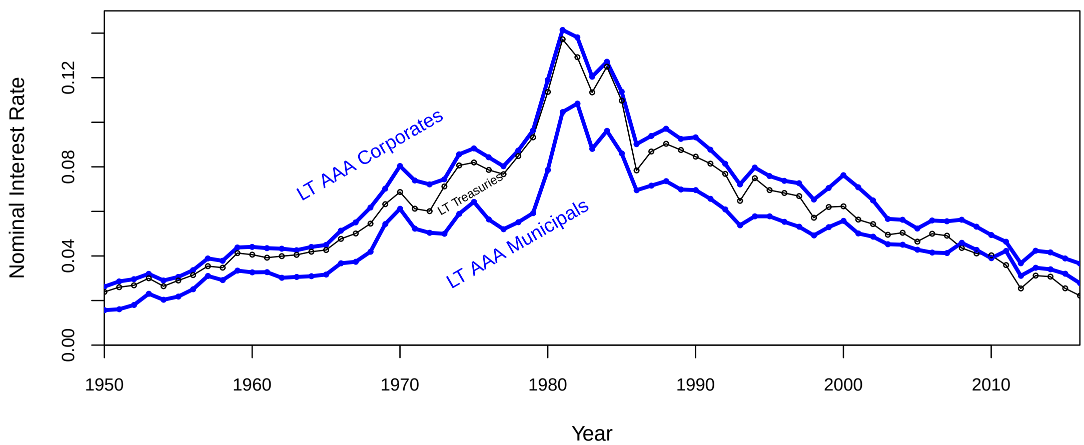
2008 may not have suffered from money illusion and fled the bond market to "reach for yield" in the stock market, after all. It is to say that sophisticated taxable investors should not have reached for yield any more than usual. For them, the "real" short- and long-term real interest rates in the wake of the Great Recession should have looked by-and-large mundane.

Figure 1: Nominal Interest Rates on Short-Term and Long-Term Treasury Notes



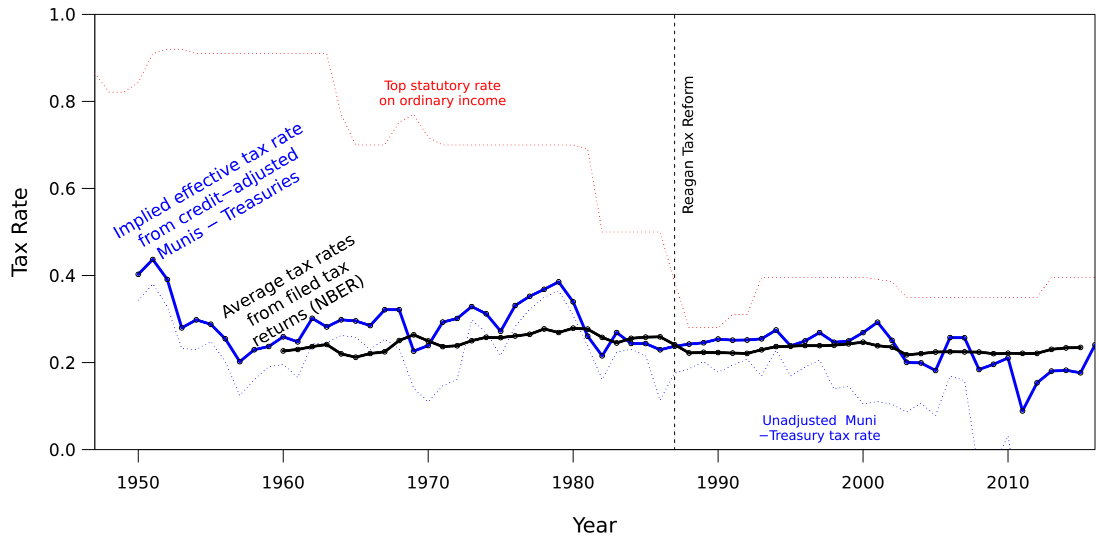
The graph shows that short-term Treasuries, represented by the 1-Year Treasury Note, have offered considerably lower yields than long-term Treasuries (20-Year) only in the second half of the sample. Interest rates peaked in 1981. All series originated from Global Financial Data (GFD), specifically series IGUSA1D and IGUSA20D, respectively, corresponding to 1-Year and 20-Year Treasuries. The data series are also listed in our web appendix.

Figure 2: Long-Term AAA Municipal and Corporate Yields



The graph shows that (1) a mixture of long-term (20+ year) AAA-rated corporate bonds offered increasingly higher promised (not expected!) yield spreads from 1965 to 1978 and after 1985; and (2) long-term 20-year AAA-rated municipal bonds have offered lower promised (not expected!) yield spreads until about 2008. All series originated from Global Financial Data (GFD). The corporate yield series is MOCAAAD, the "Moody's Corporate AAA Yield" index. The municipal bond yield series is MOWAAAW, the "Moody's 20Y AAA Muncipal Bond Yield." The series are also listed in our web appendix.

Figure 3: Tax Rates



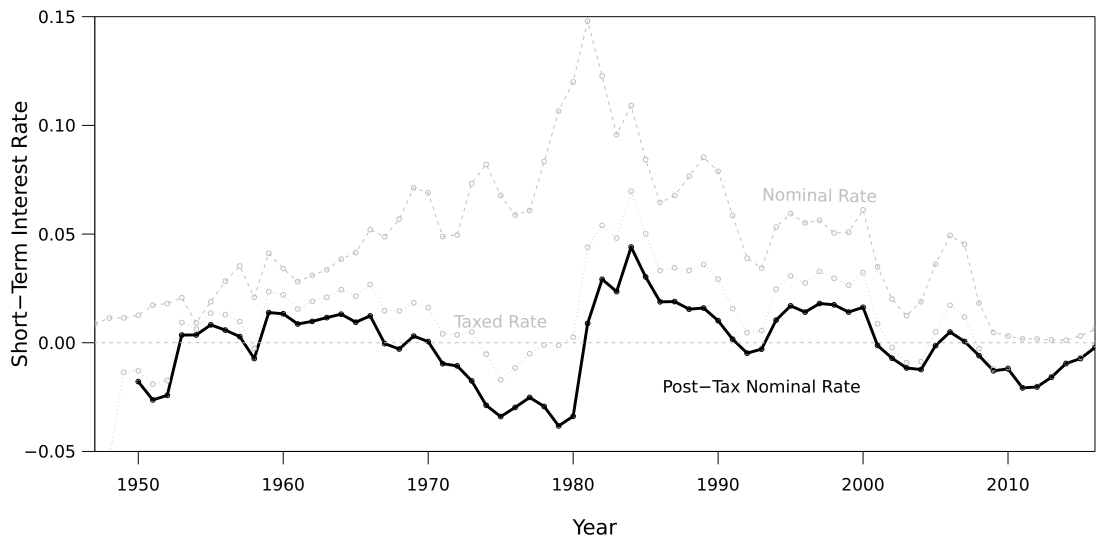
The black line shows the interest income weighted average marginal Federal (without any state) income tax rates on interest income, as calculated from the Taxsim model. This interest tax series is now available at `http://users.nber.org/~taxsim/marginal-tax-rates/af84.html`.

The blue lines are tax rates computed from the yield differential of non-taxable 20-year maturity-matched municipal bonds (series MOWAAAW) from GFD, and 20-year Treasury bonds (series code IGUSA20D), adjusted for the credit and liquidity spread (the difference between taxable Treasuries and 20+ year AAA corporate bonds (series MOCAAAD).

Despite originating from completely different methods, the NBER and financial tax series suggest similar marginal average level tax rates on interest—relatively stable and about 20% to 25%.

PS: The red dotted line suggests that the top statutory tax-rate (IRS Historical Table 23 Series 5, available at `https://www.irs.gov/statistics/soi-tax-stats-historical-table-23`) should not be used as a proxy for the effective tax rate in the economy. It was neither greatly reflective of observed paid tax rates nor of the pricing of financial instruments. The blue dotted line suggests that an unadjusted Muni minus Treasury spread that is not credit- and liquidity- adjusted would yield highly misleading estimates, e.g., non-sensible negative estimates after the financial crisis. Our web appendix investigates this further using tracking regressions.

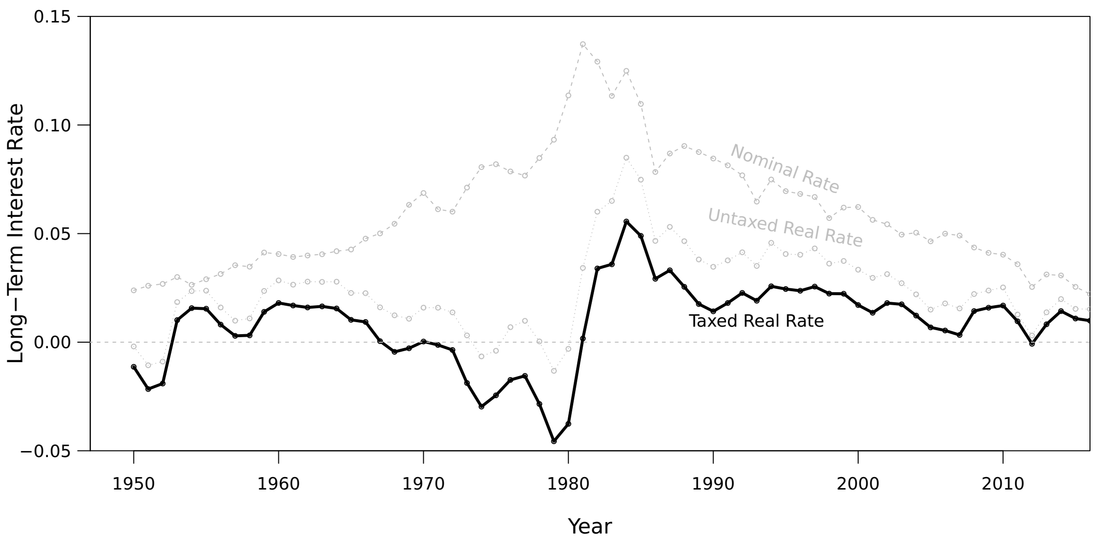
Figure 4: Post-Tax Real Yields on Short-Term 1-Year Treasuries



This graph plots short-term treasury yields after smoothed inflation and Federal taxes have been removed. Calculating the smoothed inflation rate involves averaging the previous, current, and subsequent year's CPI rate. The tax rate used in this graph is from the tax rate implied by the spread between the 20-year Treasury and municipal bonds, after adjusting for credit and liquidity, as measured by the spread between 20+ year AAA corporate bonds and 20-year Treasuries.

After-tax short-term (1-year) real Treasury yields are about zero as of 2016—but this is still higher than the rates from 1968-1980 and from 2001-2008. At 0%, the rate is more ordinary than extraordinary. The short-term real-after-tax rate in 2016 is **–0.10** standard deviations relative to its historical series since 1950, corresponding to the 43rd percentile.

Figure 5: Post-Tax Real Yields on Long-Term 20-year Treasuries



The figure is analogous to Figure 4, except that the focus is on longer-term Treasuries. After-tax long-term (20-year) real Treasury yields are well in line with common yield patterns—except for the 1982 to 1987 period which showed remarkably high yields. The long-term real-after-tax rate in 2016 is **+0.06** standard deviations relative to its historical series since 1950, corresponding to the 43rd percentile.

Appendix

A Not For Publication

In the interest of making our paper a "short note" quick read, it has omitted many finer details. The appendix here provides some of them.

A The Tax Code

A.1 The NBER Taxsim Model

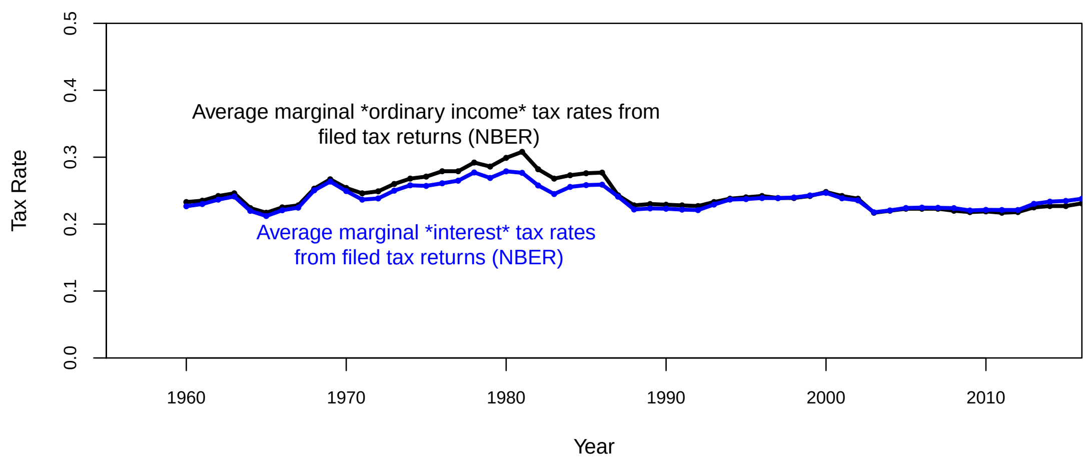
Each year since 1960 (except 1961, 1963, and 1965), the *Statistics of Income Division* of the IRS has released a public use file derived from the individual income tax filings for that year. Although the files are redacted to maintain confidentiality, they fairly represent the distribution of income and tax by component. We then use the NBER tax calculator to determine the tax liabilities from these files. This calculator takes into account the numerous details of the tax law, including the maximum tax on earned income, the minimum tax, special treatment of capital gains, the net investment income tax, income averaging, phase-ins and phase-outs of itemized deductions and income-based clawbacks of various credits deductions, and many other complexities, all of which can potentially influence the required tax.

The average marginal rate on *interest* income differs from the average marginal rate on *ordinary* income, not only because interest income is distributed differently, but also because some some features of the tax law treat interest income differently. (Even the maximum tax on earned income can effect the tax rate on interest income through the stacking rule.) Nevertheless, interest income was mostly treated the same as ordinary income. In 1971, a maximum tax rate on earned income resulted in a statutory difference between 8% to 20% for top income tax brackets. Specifically, the rule capped the tax on earned income at 60% in 1971 and 50% from 1972-1981 IRS (2016). Despite the seemingly significant statutory difference, the difference between the average marginal rates of interest and earned income reached a maximum of 3.1%.

Several other differences could lead to variations between taxes on earned and unearned income. The earned income tax credit can also lead to some differences in interest income treatment, primarily effecting lower tax brackets. Payroll (FICA) taxes exclusively effect earned income. A 3.8% surcharge on investment income was instituted in 2013. Finally, owners of Treasuries issued prior to 1941 typically received a credit of 3% of interest paid or, prior to 1955, would not pay the 3% "normal" tax on such income.

The average marginal rate[5] is computed over a finite difference in interest income of 1% of base interest income.

Figure: Average Marginal Total Ordinary *vs* Interest *Tax Rates*



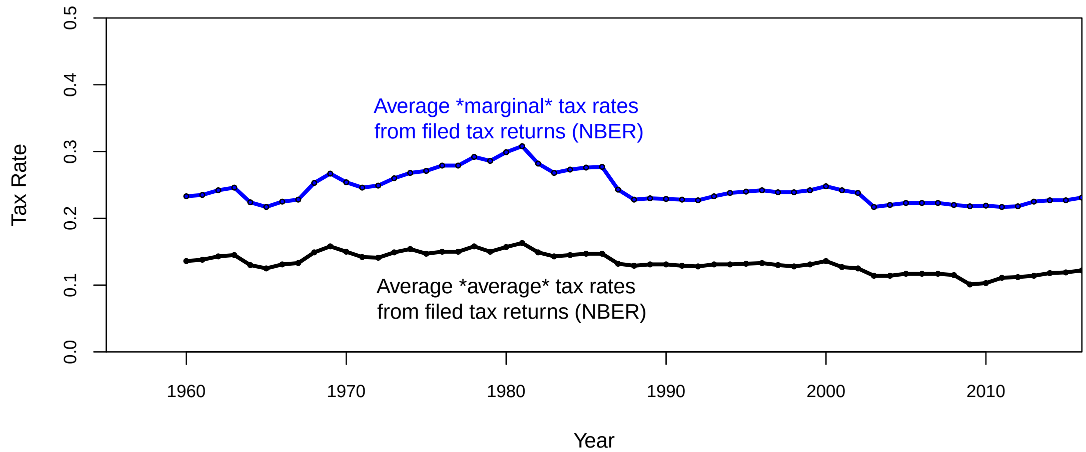
This graph plots the average marginal tax rate for all ordinary income as well as the average marginal interest-only tax rate over time. The differences between the graphs are generally small and transitory.

A.2 Average Average Taxes versus Average Marginal Taxes

Most of the preceding discussions about statutory tax rates focused on the average marginal tax rate (on interest rates) of investors. The average total effective tax rate is an alternative taxation metric. Of course, each individual taxable investor should factor only the marginal tax rate into the decision making process. The usefulness of the average average tax rate would be only in assessing the tax burden in the economy. The graph below shows both the total average *marginal* statutory tax rate (not just for interest) and the total average *average* tax rate. Because of progressivity, the average marginal rate has always been about 10 percentage points higher than the average average rate. The difference was consistent over time.

[5]**Naming Convention:** The first *average* pertains across individuals. The second *average* or *marginal* pertains to one single investor's average or marginal tax over her own total income.

Figure: Marginal and Average Tax Rates



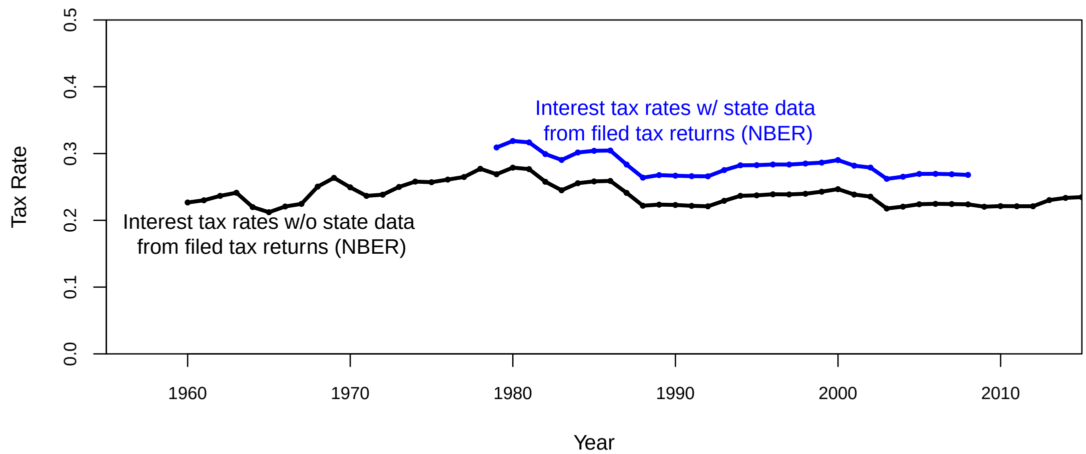
This graph plots the average marginal tax rate and the total average effective rate over time. At all times, the average effective rate is significantly smaller than the average marginal rate.

A.3 Inclusion of State Taxes

The tax data series inclusive of state taxes begins in 1978 and terminates in 2008. Because the point of this paper is about assessing the magnitude of current interest rates against interest rates from a historical perspective, the combined Federal plus state income tax data was simply too incomplete to be suitable for the results in the main text. In the appendix here, we can give some idea of the effect in the subsample in which both Federal and state tax data were available.

Our graph shows that state taxes increase the average marginal tax rate by about four to five percentage points. Although meaningful to investors, the impact on our main results is small: An after-tax marginal yield of about 1.5% would only be reduced by about 0.1%. Moreover, because the effect stays relatively even over time, the effect of state taxes on the relative ranking of current interest rates relative to historical interest rates is likely to be modest.

Figure: Total Taxes With and Without State Taxes



This graph plots the average marginal tax rate inclusive of and excluding state taxes. Excluding state taxes predictably leads to a lower effective tax rate. The effect is relatively uniform over time.

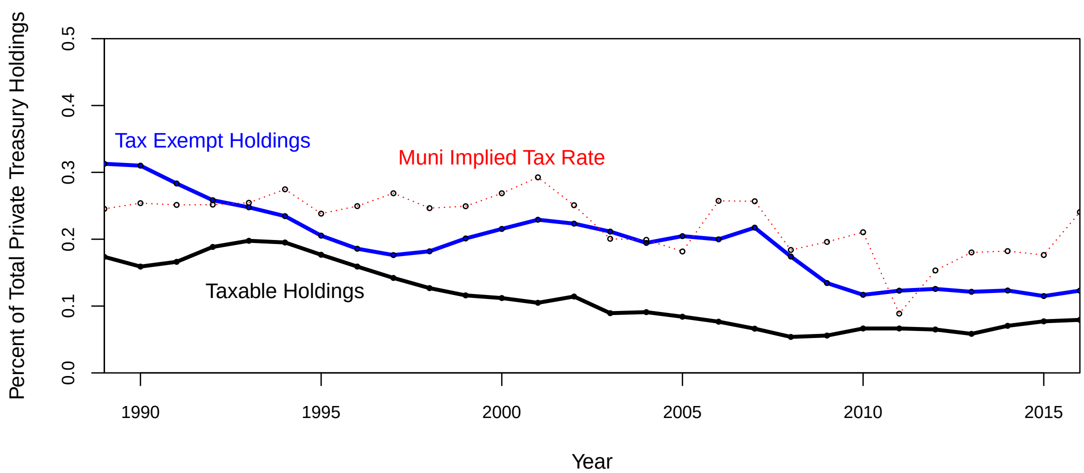
A.4 Tax Rates and Aggregate Substitution Between Taxable and Tax-Exempt Bonds

Remarkably, the evidence does not suggest large substitution effects by investors from taxable Treasuries towards non-taxable municipals in high-effective-tax environments.

In more detail, if municipal bond prices fail to adjust for changes in investor marginal tax rates, investors would likely substitute between municipal bonds and Treasuries until prices equilibrated. In other words, absent price adjustments, higher marginal tax rates should incentivize (some) taxable investors to substitute from taxable towards non-taxable municipal bonds. This should further increase the spread between the two. It should also induce the ratio of holdings of Treasury bonds by taxable investors over non-taxable investors to decrease.

However, we see little evidence of such substitution. In detail, as a crude measure, the plot below divides holders of privately held Treasuries into taxable, tax-exempt, and unclassified (not shown) investors as a total percentage of all Treasuries outstanding. The plot also includes the municipal implied tax rate. Unfortunately, this does not isolate the causal relationship between the municipal bond implied tax rate and Treasury flows, other than accounting for AAA corporate credit/liquidity spreads and inflation. Hence the absence of substitution flows when municipal bond implied tax rates changed provides little evidence that investors cared to substitute. Furthermore, Treasury ownership data is both coarse and of limited availability prior to 1990. Observation of the actual municipal and Treasury holdings of the counter-parties, and their respective marginal tax rates, would be a more direct means of observing the influence of marginal tax rates on security selection. We neither have the data nor is this our primary focus, so we leave this to others for future study.

Figure: Percent Holdings of Treasury Securities by Taxable and Tax-Exempt Investors



This graph plots the relative holdings of Treasury securities by tax-exempt investors and taxable investors. The series are derived from the "Estimated Ownership of Treasury Securities" tables available at `https://www.fiscal.treasury.gov/fsreports/rpt/treasBulletin/treasBulletin_home.htm`*. Taxable securities are defined as the sum of holdings of depository institutions and insurance companies. The sum of pension fund holdings and state and local government holdings together proxy tax-exempt holdings. Divide these values by total private Treasury holdings to calculate the values shown. The municipal bond implied tax rate is based on the spread of 20-year maturity municipal bonds and 20-year Treasuries after a credit and liquidity adjustment based on 20 to 30 year AAA corporate spreads over 20-year Treasuries.*

B The Credit- and Liquidity-Adjustment For Munis

B.1 Tracking Regressions

Our control approach to credit and liquidity spreads has been to presume that, for equal maturity bonds,

$$\text{T-Bond Yield} - \text{Muni Yield} \approx \text{Credit and Liquidity Spread} + \text{Tax Spread} + \text{etc}$$

We used the *20+ year AAA Moody's corporate spread over 20-year Treasuries* as our proxy for the credit and liquidity spread of *AAA Moody's 20-year municipal bonds over 20-year Treasuries*.

We can check whether the credit and liquidity characteristics between municipal bonds and corporate bonds were not one-to-one and/or greatly influenced by other time-varying effects, i.e., different from the implied tax, liquidity, and credit effects. We can use tracking regressions to test the efficacy of our corporate spread in representing the credit and liquidity components of municipal bonds, after adjusting for taxes. Similar behavior of AAA corporate spreads and AAA municipal bond spreads over time, with a coefficient of around 1, would support the hypothesis of an effective proxy of credit and liquidity effects by the corporate credit and liquidity spread (See our calculation of municipal implied tax rates in the main text). In brief, our results suggest good evidence in favor of an approximate 1-to-1 covariation of corporate spreads with municipal spreads. This mitigates concern about omitted (time-varying) distorting effects that would have been picked up by corporate bond spreads.

Our specification is a tracking regression of

$$[\text{20y T-Bond} - \text{20y Muni Yield}] \approx \gamma_0 + \gamma_1 \times [\text{20y T-Bond} - \text{20y Corporate AAA Yield}]$$
$$+\gamma_2 \times [\text{20y T-Bond} \cdot \tau] + \overline{\gamma} \cdot \text{ Covariates}$$

Covariates includes term structure effects and inflation, but could include other series (e.g. the S&P 500). Only the specification which is not differenced shows the CPI effect as significant.

Figure: Time Series Regressions in Levels: T-Bond Minus Muni Bond Spreads Explained By Statutory Taxes and Credit/Liquidity Spreads

Dependent Variable: 20-Year T-Bond Minus 20-Year AAA Muni Yield Spread, times 100					
20Y T-Bond Minus AAA Corp	113.2	106.6	124.9	109.4	*should be 100 (%)*
	(13.6)	(13.1)	(16.3)	(15.1)	
*20Y T-Bond*NBER Interest Tax*	102.7	83.1			*controls for real tax effects*
	(9.2)	(10.7)			
*20Y T-Bond*NBER CapGains Tax*			113.0	76.9	
			(11.5)	(12.5)	
10-Year Minus 20-Year T-Bond		18.2		29.3	
		(14.7)		(17.9)	
CPI Inflation		6.4		11.1	
		(2.2)		(3.2)	
Intercept	0.4	0.4	0.6	0.6	
	(0.2)	(0.2)	(0.2)	(0.1)	
Num Years	57	57	57	57	
R^2	0.91	0.93	0.80	0.88	

(Standard errors in parentheses)

Regressing the 20-year T-Bond – municipal bond spread against the corporate spread suggests a one-to-one relationship between corporate and municipal spreads. The independent variables, listed on the left hand side, include the focal 20-year T-Bond – 20+ year AAA corporate spread, as well as some additional covariates. The tax rate multiplied by the 20-year Treasury bond accounts for the tax-exempt status of muncipal bonds. The regression includes two different tax rates for robustness. Several other covariates were also included. If the relationship holds perfectly, the coefficient on the corporate AAA spread would be 100 percentage points. The regression results show that the spread is close to 100 in standard error terms.

Each regression decomposes the municipal treasury spread into a risk-less tax component and a corporate credit/liquidity component. Regressing the municipal bond spread against these components suggests how the corporate spread explains the residual difference between Treasuries after-tax and municipal bond rates. Some specifications include additional covariates to verify robustness. Finally, the relevance of the corporate spread is tested for resilience against several alternative statutory rates.

Figure: Time Series Regressions in Differences: T-Bond Minus Muni Bond Spreads Explained By Statutory Taxes and Credit/Liquidity Spreads

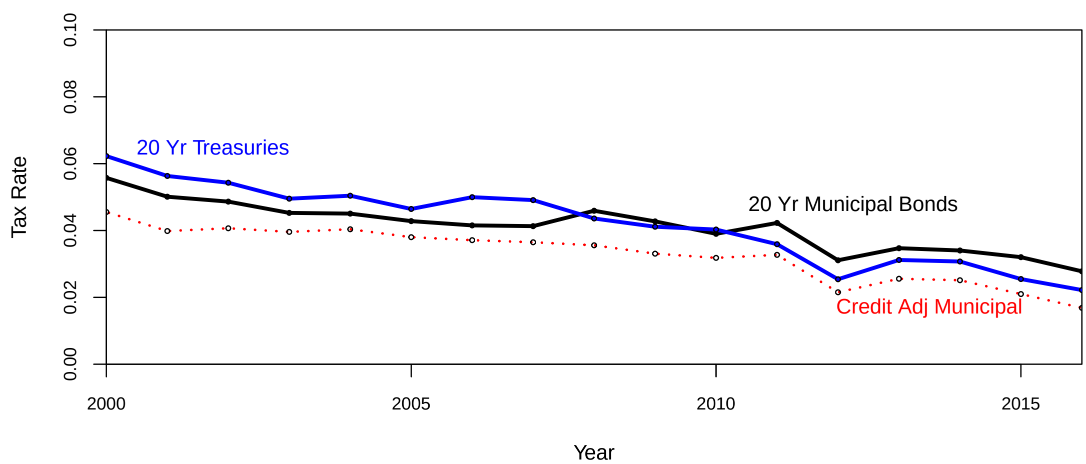
Dependent Variable: 20 Y T-Bond Minus 20Y Muni Yield Spread					
20Y T-Bond Minus AAA Corp	104.7	106.4	105.0	108.1	*should be 100 (%)*
	(15.8)	(17.9)	(15.9)	(17.2)	
*20Y T-Bond*NBER Interest Tax*	47.4	38.3			*controls for real tax effects*
	(24.9)	(34.6)			
*20Y T-Bond*NBER Gains Tax*			50.7	40.2	
			(23.4)	(25.6)	
10 Minus 20-Year T-Bond		3.5		10.6	
		(19.0)		(19.1)	
CPI Inflation		2.4		2.5	
		(4.3)		(3.1)	
Intercept	−0.0	−0.0	−0.0	−0.0	
	(0.0)	(0.0)	(0.0)	(0.0)	
Num Years	56	56	56	56	
R^2	0.64	0.64	0.63	0.64	

This is the same as the previous table except that the dependent and independent variables are differenced. Ideally, covariates other than the corporate spread and the tax effect should be near zero. In the level version presented in the previous table, the CPI is significant. Here, even the CPI term is always insignificant. The coefficient on the 20Y Treasury Note – 20+ corporate AAA yield spread *remains substantially unchanged.*

B.2 The Crossing of the Muni and Treasury Yields after the Crisis

During the financial crisis, the price of Treasuries rose above the price of AAA municipal bonds. A simple figure can show that this "anomaly" likely originated from changes in the market-wide prices for credit and liquidity, and not from a negative implied tax rate:

Figure: Treasury and Municipal Bond Yields



This graph shows that the raw municipal bond yield rose above the equivalent 20-year Treasury yield around the time of the financial crisis. The dotted line shows that this effect disappears after accounting for the credit and liquidity components of the municipal bond yield. The adjusted municipal bond yield is calculated by 20 Yr AAA Muni Yield $-(1-\tau)\cdot$ *(20+ Yr AAA Corp Rate* $-$ *20 Yr Treasury Yield), where* τ *is the 20-year municipal bond implied tax rate from the standard formula* $1-\frac{\text{20 Yr Muni AAA Yield}}{\text{20 Yr Treasury Yield + 20+ Yr Corp AAA Spread}}$. *Using the statutory tax rate instead of the municipal implied yield leads to a very similar result.*

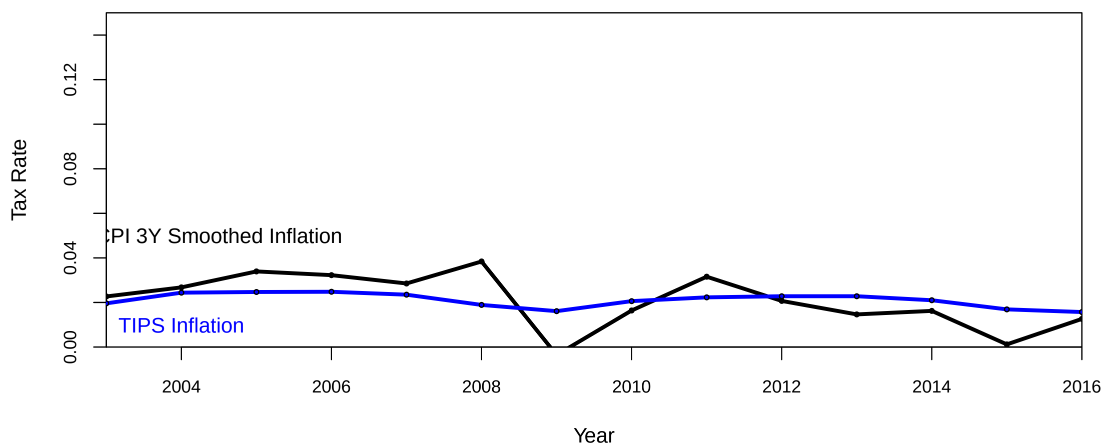
C Rates of Return and Inflation

C.1 Breakeven Inflation

Treasury Inflation Protected Securities (TIPS) seem superficially attractive as a measurement of inflation. Their yield can be inverted to imply a break-even level of inflation which would make an investor indifferent between TIPS and the corresponding nominal treasury security. Unfortunately, there are two problems with using TIPS in the main results. First, the inflation rate implied by TIPS can sometimes differ greatly from other empirical market metrics, as discussed in great detail in Fleckenstein, Longstaff, and Lustig (2014). Second, a continuous series for TIPS is only available beginning in 2003.

The following graph compares the 3-year CPI inflation used in our primary analysis with the TIPS-implied inflation rate from 2003 onward. Recall that our 3-year CPI inflation rate in the main text averaged CPI inflation across prior, current, and subsequent years. The graph suggest that the CPI smoothed rate is reasonably close to the TIPS implied rate in most years *for our purposes*.

Figure: Smoothed CPI Inflation vs Breakeven Inflation



This graph plots both the 10-year TIPS implied break-even inflation rate and the 3-year smoothed CPI inflation rate (averaged over t–1, t, and t+1). The breakeven rate is substantially similar to the CPI measure except in 2009. This suggests that the main results would be minimally affected by using breakeven as opposed to smoothed CPI inflation.

Using breakeven rates in our calculations would likely not change our conclusion that real after-tax rates are not extraordinarily low. In fact, breakeven inflation rates in 2016 were slightly lower than they have been at any other point in the 14-year sample. Naively plugging in the 2016 breakeven inflation rate instead of smoothed CPI rate to calculate the 2016 real after-tax interest yield gives an indication as to the impact on the results. Using the higher breakeven rate leads to a value for the real after-tax 20-year bond of approximately 0, corresponding to the 27th percentile of after-tax rates dating back to

1950. Although lower than the 43rd percentile calculated in the main results, the value is hardly without precedent.

D Data

D.1 Summary Statistics

This section provides additional detail on sources and derivations behind the key data series. The bulk of the statutory tax rates originated from the NBER TAXSIM database at [http://users.nber.org/~taxsim/]. The average marginal tax rate is available at [http://users.nber.org/~taxsim/allyup/], where the procedure for calculating the average marginal rate from the raw tax filings is extensively documented. Feenberg and Coutts (1993) contains additional information on the TAXSIM model. The *topord* series originates from the IRS's website [https://www.irs.gov/statistics/soi-tax-stats-historical-table-23]. Treasury holding data came from the Department of Treasury (DOT) Treasury Bulletin Publication, specifically from the 2017, 2012, 2007, 2002, 2000, and 1996 publications.

Global Financial Data (GFD) provided most of our financial series. The nominal one-year Treasury yields came from series IGUSA1D. The 20-year Treasury yields came from series IGUSA20D. The principle AAA municipal bond yields are found under GFD ticker MOMAAAW. This series is also a Moody's index. Finally, CPI data and information on breakeven inflation rates originated from FRED.

Many of the remaining series are derived. To calculate the real Treasury yields, the CPI index for each year was averaged over the past and subsequent years, thus forming a three-year moving average. The equation $\frac{1+\text{ Treasury Yield}}{1+\text{ CPI rate}}-1$ provides the real Treasury yield from the nominal rate, while the after-tax real Treasury rate similarly stems from $\frac{1+\text{ Treasury Yield}\cdot(1-\tau)}{1+\text{ CPI rate}}-1$.

The implied municipal bond tax rate may be calculated from the below equation. The corporate spread consists of the GFD series MOCAAAD net of the 20-year T-Bond nominal rate. The *0credit* series re-calculates the municipal bond implied tax rate with the corporate credit and liquidity spread set to zero:

$$\text{Effective Tax Rate} = 1 - \frac{\text{Muni AAA Yield}}{\text{Treasury Yield } + \text{ Corp AAA Spread}}$$

The next table links each figure presented in the main section and appendix with its respective data series. The subsequent table summarizes the source or derivation for data series used in the main section and appendix. The last table in this section contains basic summary information for each data series. In order of the column headings, the summary table lists the number of data points from 1950 onward, the mean, standard deviation, percentile measures, percentile of the 2016 data point (if available), and number of data points greater than the 2016 value $+2\sigma$.

Figure or Table	*Data Series Used*
Nominal Interest Rates	*tnote20, tnote1*
LT AAA municipal and Corp. Rates	*corplt, tnote20, muni20*
Tax Rates	*munitax, munitax0credit, interesttax, topord*
Post-tax Real Rates (ST)	*tnote1, tnote1r, tnote1at*
Post-tax Real Rates (LT)	*tnote20, tnote20r, tnote20at*
Average Marg. Ordinary and Interest Taxes	*avgord, interesttax*
Average Average and Average Marginal Taxes	*avgord, avgordtotal*
Taxes Net and Gross of State Taxes	*interesttax, interesttaxstate*
Treasury Holdings by Investor Tax Status	*hightaxholdings, taxexemptholdings, munitax*
Tracking Regressions	*tnote20, tnote10, muni20, corplt, interesttax, gainstax, inflation*
Treasury and municipal Rates	*muni20, tnote20, muni20minusspread*
Smoothed CPI vs Breakeven Inflation	*inflation3y, tipsinflation*

Series	Description	Vendor	Series Code	Formula
cpi	Consumer price index for all urban consumers (avg)	FRED	CPIAUCNS	
inflation	Consumer price index for all urban consumers (avg YoY)	FRED	CPIAUCNS	
tipsinflation	10-year breakeven inflation rate (avg)	FRED	T10YIE	
inflation3y	Average inflation over three years			$\left(\frac{\text{cpi}_{t+1}}{\text{cpi}_{t-2}}\right)^{1/3}$
tnote1	United States 1-year Treasury (avg)	GFD	IGUSA20D	
tnote20	United States 20-year Treasury (avg)	GFD	IGUSA20D	
muni20	Moody's 20-year AAA municipal bond yield (avg)	GFD	MOMAAAW	
corplt	Moody's 20+ year corporate AAA yield (avg)	GFD	MOCAAAD	
topord	Top marginal tax rate	IRS	Historical Table 23 (6)	
avgord	Federal average marginal tax rate (deflated)	NBER TAXSIM		
avgordtotal	Federal average average tax rate (deflated)	NBER TAXSIM		
interesttax	Federal average marginal tax rate on interest (deflated)	NBER TAXSIM		
interesttaxstate	Federal + state average marginal tax rate on interest (deflated)	NBER TAXSIM		
munitax0credit	Implied muncipal bonds tax, no credit or liquidity adj			$1-\frac{\text{muni20}}{\text{tnote20}}$
munitax	Implied muncipal bonds tax with credit and liquidity adj			$1-\frac{\text{muni20}}{\text{tnote20}+(\text{corplt}-\text{tnote20})}$
tnote1r	1-year Treasury yield after inflation			$\frac{1+\text{tnote1}}{1+\text{inflation3y}}-1$
tnote1at	1-year Treasury yield after-taxes and inflation			$\frac{1+\text{tnote1}(1-\text{munitax})}{1+\text{inflation3y}}-1$
tnote20r	20-year Treasury yield after inflation			$\frac{1+\text{tnote20}}{1+\text{inflation3y}}-1$
tnote20at	1-year Treasury yield after-taxes and inflation			$\frac{1+\text{tnote20}(1-\text{munitax})}{1+\text{inflation3y}}-1$
tbankamt	Depository Treasury holdings (avg quarterly)	DOT	Table OFS-2 (4)	
tinsuranceamt	Insurance Treasury holdings (avg quarterly)	DOT	Table OFS-2 (8)	
tpensionlocalamt	Public pension Treasury holdings (avg quarterly)	DOT	Table OFS-2 (7)	
tpensionprivateamt	Private pension Treasury holdings (avg quarterly)	DOT	Table OFS-2 (6)	
tlocalgovamt	Local government Treasury holdings (avg quarterly)	DOT	Table OFS-2 (10)	
ttotalprivateamt	Total non-Federal Treasury holdings (avg quarterly)	DOT	Table OFS-2 (3)	
hightaxholdings	Taxable Treasury holdings			$\frac{\text{tbankamt}+\text{tinsuranceamt}}{\text{ttotalprivateamt}}$
taxexemptholdings	Tax-exempt Treasury holdings			$\frac{\text{tpensionlocalamt}+\text{tpensionprivateamt}+\text{tlocalgovamt}}{\text{ttotalprivateamt}}$
residualholdings	Unclassified Treasury holdings			$1-\text{hightaxholdings}-\text{taxexemptholdings}$
muni20minusspread	The credit and liquidity spread after-taxes			$\text{muni20}-(\text{corplt}-\text{tnote20})(1-\text{munitax})$

Figure: Data Series Summary Statistics

	Summary Statistics			Percentiles						
	N	*mean*	σ	*0 %*	*25 %*	*50 %*	*75 %*	*100 %*	$\%_{2016}$	$> x_{2016} + 2\sigma$
cpi	67	108	74	24	33	100	169	240	1.000	*none*
inflation	67	0.035	0.028	−0.004	0.016	0.029	0.043	0.135	0.164	7
inflation3y	67	0.035	0.025	0.003	0.020	0.028	0.043	0.117	0.045	10
tnote20	67	0.060	0.028	0.022	0.040	0.055	0.078	0.137	0.015	17
tnote1	67	0.048	0.033	0.001	0.020	0.049	0.066	0.148	0.119	13
tnote10	67	0.057	0.029	0.018	0.037	0.051	0.075	0.139	0.030	17
muni20	67	0.049	0.020	0.016	0.033	0.049	0.058	0.108	0.119	11
corplt	67	0.066	0.028	0.026	0.044	0.065	0.083	0.141	0.119	11
topord	67	0.580	0.224	0.280	0.388	0.500	0.735	0.920	0.448	13
avgord	57	0.245	0.023	0.216	0.228	0.239	0.260	0.307	0.404	6
avgordtotal	57	0.134	0.014	0.102	0.125	0.131	0.146	0.163	0.246	5
munitax0credit	67	0.161	0.139	−0.257	0.119	0.190	0.235	0.381	0.030	59
munitax	67	0.263	0.060	0.089	0.237	0.255	0.294	0.437	0.313	5
gainstax	57	0.194	0.038	0.140	0.165	0.181	0.228	0.257	0.667	*none*
tnote1r	67	0.012	0.020	−0.021	−0.003	0.012	0.026	0.070	0.328	5
tnote1at	67	−0.001	0.017	−0.038	−0.012	0.001	0.013	0.044	0.433	1
tnote20r	67	0.024	0.019	−0.013	0.013	0.023	0.035	0.085	0.313	4
tnote20at	67	0.009	0.019	−0.046	0.000	0.014	0.018	0.056	0.433	2
interesttax	57	0.239	0.017	0.212	0.224	0.237	0.250	0.279	0.544	3
interesttaxstate	30	0.286	0.016	0.266	0.272	0.287	0.294	0.321		
tbankamt	28	269	118	115	182	257	326	582	1.000	*none*
tinsuranceamt	28	200	64	107	151	196	240	334	1.000	*none*
tpensionlocalamt	28	171	26	128	151	166	187	217	0.714	*none*
tpensionprivateamt	28	208	128	117	136	147	185	543	1.000	*none*
tlocalgovamt	28	453	138	241	335	430	588	697	1.000	*none*
ttotalprivateamt	28	5013	3014	1947	2991	3401	6999	11548	1.000	*none*
tipsinflation	14	0.021	0.003	0.016	0.019	0.022	0.023	0.025	0.071	7
hightaxholdings	28	0.112	0.048	0.054	0.069	0.098	0.159	0.198	0.393	4
taxexemptholdings	28	0.088	0.026	0.043	0.064	0.100	0.108	0.127	0.250	2
residualholdings	28	0.800	0.070	0.699	0.732	0.795	0.866	0.898	0.679	*none*
muni20minusspread	67	0.044	0.020	0.014	0.029	0.040	0.054	0.101	0.060	15

D.2 Data

Table: Key Series Comprising Graphs in the Paper, Quoted in Percent

	(Figure 3)				(Figures 1, 4, and 5)					
	Tax Rates				1-Year T-Note			20-Year T-Bond		
year	munitax	0credit	interesttax	topord	Nominal	Real	Txd-Real	Nominal	Real	Txd-Real
1950	40.3	34.3		84.4	1.3	−1.3	−1.8	2.4	−0.2	−1.1
1951	43.7	38.1		91.0	1.7	−1.9	−2.6	2.6	−1.1	−2.2
1952	39.1	32.9		92.0	1.8	−1.7	−2.4	2.7	−0.9	−1.9
1953	28.0	23.2		92.0	2.1	0.9	0.4	3.0	1.8	1.0
1954	29.8	23.0		91.0	0.9	0.6	0.4	2.6	2.4	1.6
1955	28.8	24.9		91.0	1.9	1.4	0.8	2.9	2.4	1.5
1956	25.5	20.2		91.0	2.8	1.3	0.6	3.1	1.6	0.8
1957	20.2	12.4		91.0	3.5	1.0	0.3	3.5	1.0	0.3
1958	22.9	16.1		91.0	2.1	−0.3	−0.7	3.5	1.1	0.3
1959	23.7	19.0		91.0	4.1	2.3	1.4	4.1	2.4	1.4
1960	25.9	19.6	22.7	91.0	3.4	2.2	1.3	4.1	2.8	1.8
1961	24.8	16.6	23.0	91.0	2.8	1.6	0.9	3.9	2.6	1.7
1962	30.2	24.3	23.7	91.0	3.1	1.9	1.0	4.0	2.8	1.6
1963	28.2	24.5	24.1	91.0	3.4	2.1	1.2	4.0	2.8	1.6
1964	29.8	26.2	22.0	77.0	3.8	2.4	1.3	4.2	2.8	1.6
1965	29.6	25.8	21.2	70.0	4.1	2.1	0.9	4.3	2.3	1.0
1966	28.5	23.0	22.1	70.0	5.2	2.7	1.2	4.8	2.3	0.9
1967	32.1	25.4	22.5	70.0	4.9	1.5	−0.0	5.0	1.6	0.0
1968	32.1	23.2	25.1	75.2	5.7	1.5	−0.3	5.5	1.2	−0.4
1969	22.6	14.1	26.4	77.0	7.1	1.8	0.3	6.3	1.1	−0.3
1970	23.9	10.9	24.9	71.8	6.9	1.6	0.1	6.9	1.6	0.0
1971	29.3	14.6	23.7	70.0	4.9	0.4	−1.0	6.1	1.6	−0.1
1972	30.1	16.1	23.8	70.0	5.0	0.4	−1.1	6.0	1.4	−0.4
1973	32.9	29.8	25.0	70.0	7.3	0.5	−1.8	7.1	0.3	−1.9
1974	31.2	26.9	25.8	70.0	8.2	−0.5	−2.9	8.1	−0.7	−3.0
1975	27.2	21.6	25.7	70.0	6.8	−1.7	−3.4	8.2	−0.4	−2.4
1976	33.1	28.3	26.1	70.0	5.9	−1.2	−3.0	7.9	0.7	−1.7
1977	35.2	32.2	26.5	70.0	6.1	−0.5	−2.5	7.7	1.0	−1.6
1978	36.8	35.0	27.7	70.0	8.3	−0.1	−2.9	8.5	0.0	−2.8
1979	38.5	36.5	26.9	70.0	10.7	−0.1	−3.8	9.3	−1.3	−4.6
1980	33.9	30.8	27.9	70.0	12.0	0.3	−3.4	11.4	−0.3	−3.8
1981	26.1	23.8	27.7	69.1	14.8	4.4	0.9	13.7	3.4	0.2
1982	21.5	16.1	25.8	50.0	12.3	5.4	2.9	12.9	6.0	3.4
1983	26.9	22.4	24.5	50.0	9.6	4.8	2.4	11.3	6.5	3.6
1984	24.4	23.0	25.6	50.0	10.9	7.0	4.4	12.5	8.5	5.6
1985	24.4	21.6	25.8	50.0	8.4	5.0	3.0	11.0	7.5	4.9
1986	22.9	11.3	25.9	50.0	6.5	3.3	1.9	7.8	4.7	2.9
1987	23.7	17.6	24.1	38.5	6.8	3.5	1.9	8.7	5.3	3.3
1988	24.2	18.6	22.2	28.0	7.7	3.3	1.5	9.0	4.7	2.5
1989	24.5	20.2	22.4	28.0	8.5	3.6	1.6	8.8	3.8	1.8
1990	25.4	17.7	22.3	28.0	7.9	2.9	1.0	8.5	3.5	1.4
1991	25.1	19.4	22.2	31.0	5.9	1.6	0.2	8.1	3.8	1.8

(Table continued on following page.)

	(Figure 3)				(Figures 1, 4, and 5)					
	Tax Rates				1-Year T-Note			20-Year T-Bond		
year	munitax	0credit	interesttax	topord	Nominal	Real	Txd-Real	Nominal	Real	Txd-Real
1992	25.2	20.8	22.1	31.0	3.9	0.5	−0.5	7.7	4.1	2.3
1993	25.5	16.9	22.9	39.6	3.4	0.6	−0.3	6.5	3.5	1.9
1994	27.5	22.8	23.7	39.6	5.3	2.5	1.0	7.5	4.6	2.6
1995	23.8	16.9	23.7	39.6	5.9	3.1	1.7	7.0	4.1	2.4
1996	25.0	19.0	23.9	39.6	5.5	2.8	1.4	6.8	4.0	2.4
1997	26.9	20.6	23.9	39.6	5.6	3.3	1.8	6.7	4.3	2.6
1998	24.7	13.9	24.0	39.6	5.1	3.0	1.7	5.7	3.6	2.2
1999	24.9	14.6	24.3	39.6	5.1	2.7	1.4	6.2	3.7	2.2
2000	26.9	10.5	24.7	39.6	6.1	3.2	1.6	6.2	3.3	1.7
2001	29.3	11.0	23.9	39.1	3.5	0.9	−0.1	5.6	3.0	1.4
2002	25.1	10.4	23.6	38.6	2.0	−0.2	−0.7	5.4	3.1	1.8
2003	20.1	8.6	21.8	35.0	1.2	−0.9	−1.2	5.0	2.7	1.7
2004	19.9	10.6	22.1	35.0	1.9	−0.9	−1.2	5.0	2.2	1.2
2005	18.2	7.8	22.4	35.0	3.6	0.5	−0.1	4.6	1.5	0.7
2006	25.7	16.9	22.5	35.0	4.9	1.7	0.5	5.0	1.8	0.5
2007	25.7	15.9	22.4	35.0	4.5	1.2	0.1	4.9	1.6	0.3
2008	18.4	−5.3	22.4	35.0	1.8	−0.3	−0.6	4.4	2.2	1.4
2009	19.6	−3.9	22.0	35.0	0.5	−1.2	−1.3	4.1	2.4	1.6
2010	21.0	3.2	22.1	35.0	0.3	−1.1	−1.2	4.0	2.5	1.7
2011	8.9	−17.7	22.1	35.0	0.2	−2.1	−2.1	3.6	1.3	1.0
2012	15.3	−22.3	22.1	35.0	0.2	−2.0	−2.0	2.5	0.3	−0.1
2013	18.0	−11.3	23.0	39.6	0.1	−1.6	−1.6	3.1	1.4	0.8
2014	18.2	−10.7	23.4	39.6	0.1	−0.9	−1.0	3.1	2.0	1.4
2015	17.6	−25.7	23.5	39.6	0.3	−0.7	−0.7	2.5	1.5	1.1
2016	24.1	−25.3	23.8	39.6	0.6	−0.1	−0.2	2.2	1.5	1.0

This tables presents the key data series used in our main analysis. The series "munitax" corresponds to the tax rate implied by the spread of 20-year AAA municipal bonds over 20-year Treasuries after adjusting for credit and liquidity using AAA 20+ year corporate bonds. "0Credit" is the same series absent the credit and liquidity adjustment. "Interesttax" contains the average tax on interest income. "Topord" is the top statutory tax rate. Real Treasury yields refer to the nominal yield less the average of the previous, current, and subsequent years' CPI inflation rates. "Txd-real" refers to the real yield after the application of the tax listed in the first column.

E Literature

E.1 Other Academic Papers Relating to Taxes and Municipal Bonds

- Elton, Gruber, Agrawal, et al. (2001) discusses the components of the corporate bond less Treasury yield spread. They use a transition matrix approach to estimate a default premium, Fama-French factors to estimate a risk premium, and a range of state tax rates to determine the tax effects. The authors quantify a significant risk premium, in addition to notable tax and default effects.

- Severn and Stewart (1992) analyze the tax effects on the Treasury corporate spread. They find that in most states circa 1992, investors benefit from holding state-tax deductible treasuries. However, the implied tax rate determined from the spread varies widely over their sample range.

- Elton and Green (1998) study tax and liquidity effects in the pricing of treasuries using inter-dealer data. They identify a small on-the-run premium due to their value in the repo market. Their analysis also uncovers limited evidence of a small tax effect on prices.

- Green and Odegaard (1997) test a null hypothesis of no tax effect in the relative pricing of treasuries by estimating a structural model. They find evidence of a tax effect prior to 1986 but no tax effect in subsequent years. Their tests exploited the difference in the tax treatments of discount and premium bonds, with the difference mostly eliminated from 1986 tax legislation.

- Elton, Gruber, and Blake (2005) compare the price effects of distributions made by tax-exempt closed end funds versus those of taxable closed end funds. They find ceteris paribus that the price drops by a greater amount for tax-exempt funds relative to taxable funds, with the price of tax-exempt funds falling by more than the dividend. The authors also determine that tax estimates from the implied dividend rate vary with the capital gains rate.

- Chalmers (1998) takes an innovative approach in isolating the default risk from municipal bond yields. Comparing the yields of pre-refunded default free municipal bonds with ordinary municipal bonds, he finds that default risk does not explain the difference in after-tax yields between Treasuries and municipal bonds. Municipal bond yields are generally higher than would be predicted by default risk and tax effects.

- Starks, Yong, and Zheng (2006) examine the behavior of investors in municipal bond closed-end funds near year end. Their regressions associate January effect abnormal returns with tax-loss harvesting. Municipal bond CEFs are chosen to isolate tax-sensitive investors. The results support the tax sensitivity of municipal bond fund investors.
- Elton and Gruber (1970) create a parsimonious model for the effect of taxes on price variation at the time of a dividend distribution. They find that the drop increases with the capital gains rate and decreases with the ordinary tax rate. The authors interpret the result as a clientele effect, where investors pick firms with dividend policies that correspond with the respective investor's tax situation.
- Harris and Piwowar (2006) quantifies liquidity effects in municipal bond markets. They uncover evidence that municipal bond transactions are expensive, particularly for taxable investors. The identified costs decline with credit quality.
- Ang, Bhansali, and Xing (2010) back out implicit tax rates on municipal bonds by studying transactions of discount securities. They calculate that discount municipal bonds trade at a higher yield after accounting for default risk and liquidity effects. The results imply tax rates higher than 70% for inter-dealer transactions.

E.2 Some Academic Papers Emphasizing Unusually Low Interest Rates

- Maggio and Kacperczyk (2017) discuss the effect of very low interest rates on the product offerings of financial institutions. Their analysis principally concerns money market funds and the degree to which such funds "reach for yield." Generally funds affiliated with large institutions are more likely to exit the market, while funds manged by independent investment firms demonstrate increased tendency to invest in riskier assets.
- Fischer (2016) examines the persistence of the zero lower bond and the implications of the persistence on policy. He discusses the effects of negative interest rates and other central bank monetary tools. The paper also includes his views on stability regulation.
- Gust et al. (2017) quantify the impact of the zero lower bound by estimating a DSGE model. They incorporate five types of shocks into the model, including TFP , fiscal,

monetary and two types of financial shocks. Their model shows that the lower bound led to an extra 2% in output contraction given a total estimated contraction of 6%.

- Negro et al. (2017) likewise calibrate a DSGE model, focusing their analysis on financial frictions and the effects of government policy. They demonstrate that government intervention mitigated reduced a potential –5.8% drop in output to –4.4%. The authors further discuss the amplifying effects of the zero lower bound on several components of the crisis, including deflation expectations and a decline in demand.

- Gourinchas and Rey (2016) take a global perspective, analyzing the implications of low real and natural rates across advanced economies. They identify two periods of low consumption wealth ratios, including the 1920s and the 2000s. Using predictive regressions, they uncover evidence of the ratio as a leading indicator of low real rates, and further estimate that real interest rates will remain low until 2021.

- Filipović, Larsson, and Trolle (2017) present a term structure model engineered to account for the current environment of low interest rates and the issues created by the zero lower bound. The results are achieved via a Linear-Rational Square Root Model. Their approach contributes effective simulation of persistent low interest rates.

- Korinek and Simsek (2016) focuses on the effectiveness of macroprudential policies. They propose a model where tighter borrowing constraints drive the economy into a liquidity trap and further force households with accumulated borrowing to de-lever. The inefficiencies brought by the liquidity trap imply more aggressive policy measures to insure borrows and a higher inflation target.

- Dell'Ariccia, Laeven, and Suarez (2017) analyze the effect of low short term rates by measuring the riskiness of new loans from 1997-2011. They find evidence that reduced short term interest rates leads to more aggressive risk taking. Moreover, the empirical results indicate that the negative effect increases with bank capital.

- Summers (2014) famous secular stagnation theory uses the decline in the real interest rate to make the case for unconventional monetary policy. He begins by arguing that economic growth over the past several decades failed to meet expectations. He then connects the underwhelming post-crisis recovery with a reduction in the real interest rates. The analysis drives a recommendation to boost demand via private and public investment.

- Eggertsson, Mehrotra, and Robbins (2017) quantifies Summers's hypothesis of a low negative real interest rate with an overlapping generations model. They find a natural rate between –1.5% and –2.2% and further simulate a permanently negative neutral rate using standard macro parameters. Major contributors to the decline in the natural rate include reductions in productivity growth and the fertility rate.

- Holston, Laubach, and Williams (2017) apply a Kalman filter to economic data in order to estimate, among other measures, the natural rate of interest. They identify a decline in the neutral interest rate in the US and three other developed economies. Their approach estimates that the neutral rate fell in the US between 1.5% and 2% between 2007 and 2016. Their work implies a greater frequency of periods where monetary policy is constrained by the lower bound.

- Mehrotra (2017) studies how low growth and real interest rates influence the cost of debt servicing. He finds that the real rate of interest is less than GDP growth, suggesting a negative debt service cost. The results are tempered by his calculation that debt servicing costs could turn positive with approximately a 30% probability.

- Taylor (2014) uses the Taylor rule as a benchmark to posit that the Fed held rates too low for too long before the crisis. Moreover, the low short-term rates fueled origination of adjustable rate mortgages with low teaser rates. Taylor further claims that increased regulation, quantitative easing, and zero-rate forward guidance impaired the post-crisis recovery.

- Cochrane (2013) examines the results of New Keynesian DSGE models, and finds that the policy recommendations following a period of negative interest rates can change depending on the selected equilibrium. His critique centers on the premise that the choice of equilibria and hence the predictions of the models are suspect.

- Eggertsson and Krugman (2012) discusses the consequences of rapid deleveraging event in an economy where agents have substantial debt. Their results suggest that in a period of low interest rates, government spending should inordinately increase output. The authors advocate a higher inflation target as a policy measure for overcoming the liquidity trap.

E.3 Various Officials and Others Emphasizing Unusually Low Interest Rates

- Smialek and Mayeda (2017) "Boston Fed President Eric Rosengren told Bloomberg Television on Friday in Boston that he frets low rates spur a reach for yield, leaving investors more exposed to a shock."
- McGeever (2017) notes that Hyun Song Shin, head of research at the Bank for International Settlements, are pushing investors farther out on the term structure as they chase for yield.
- In Miller (2017) John Williams of the San Francisco Fed discusses how persistently low rates could incentivize investors to take greater risk in their search for yield.
- Weissmann (2016) reports on how then candidate Donald Trump stated that Yellen is using low interest rates to keep the stock market high.
- In Federal-News-Service (2014), Representative Dennis Ross (R-FL) discusses how the Fed is forcing people to buy stocks by keeping rates low. Yellen also discusses how low the interest rates acts as an incentive for individuals to invest in higher yielding securities.
- Appelbaum (2017) summarizes Yellen's discussions on how the Fed's yield lowering mechanisms aided the economy and increased growth.
- Neely (2014) from the St. Louis Fed writes on how QE reduced yields and increased the price of equities.
- In Crutsinger (2015), Yellen cites low rates of return on bonds as a cause of high equity prices.
- Hilsenrath (2016) writes in the Wall Street Journal that Fed officials believe low rates may cause investors to under price risk and thus create a financial bubble.
- Belz (2014) reports how Narayana Kocherlakota of the Minneapolis Fed views low interest rates as one of many contributors to high asset prices.
- Powell (2017) Powell's speech to the 77th Annual Meeting of the American Finance Association includes the view that low rates have supported asset prices, albeit not to the point of creating a bubble. He also discusses how long-term nominal and real rates have declined for the past 30 years.

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