NBER WORKING PAPER SERIES

FISCAL RULES, BAILOUTS, AND REPUTATION IN FEDERAL GOVERNMENTS

Alessandro Dovis
Rishabh Kirpalani

Working Paper 23942
http://www.nber.org/papers/w23942

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2017, Revised September 2018

We thank Fernando Alvarez, Marina Azzimonti, Marco Bassetto, Charlie Brendon, V.V. Chari, Russ Cooper, Pierre-Olivier Gourinchas, Marina Halac, Juan Carlos Hatchondo, Boyan Jovanovic, Patrick Kehoe, Ramon Marimon, Leonardo Martinez, Diego Perez, Debraj Ray, Thomas Sargent, and Pierre Yared for valuable comments. We also thank Ananya Kotia and Victor Duarte Lledó for sharing their dataset with us. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Fiscal Rules, Bailouts, and Reputation in Federal Governments
Alessandro Dovis and Rishabh Kirpalani
NBER Working Paper No. 23942
October 2017, Revised September 2018
JEL No. E40,E6,E61,F5,H6,H7

ABSTRACT

Expectations of transfers by central governments incentivize overborrowing by local governments. In this paper, we ask if fiscal rules can reduce overborrowing if central governments cannot commit. We study a model in which the central government's type is unknown and show that fiscal rules increase overborrowing if the central government's reputation is low. In contrast, fiscal rules are effective in lowering debt if the central government's reputation is high. Even when the central government's reputation is low, binding fiscal rules will arise in the equilibrium of a signaling game.

Alessandro Dovis
Department of Economics
University of Pennsylvania
445 McNeil Building
3718 Locust Walk
Philadelphia, PA 19104
and NBER
adovis@upenn.edu

Rishabh Kirpalani
Department of Economics
The Pennsylvania State University
606 Kern Building
University Park, PA 16802
rishabh.kirpalani@psu.edu

1 Introduction

There are numerous examples throughout history in which excessive spending and debt accumulation by subnational governments led to transfers or bailouts by central governments. Examples include provinces in Argentina, states in Brazil, länders in Germany, and most recently countries (Greece, Ireland, and Portugal) in the European Union.[1] One view of such events is that the inability of central governments to commit to not transfering resources to indebted regions leads to profligating fiscal policies ex-ante, which in turn justifies the transfers ex-post. This idea has been formally studied by Chari and Kehoe (2007), Chari and Kehoe (2008), and Cooper et al. (2008) in the economics literature and Rodden (2002) in political science. See also Sargent (2012).

A commonly held view is that *fiscal rules* can correct these incentives to overborrow. In practice, fiscal rules take the form of limits to debt-to-GDP or deficit-to-GDP ratios along with some penalty if these are violated. When thinking about the design of fiscal rules, a natural question that arises is why central governments can commit to enforcing these rules if they cannot commit to not bail out. In this paper, we ask if fiscal rules can be beneficial if central governments cannot commit to enforcing the fiscal rules and if these rules will arise in equilibrium.

We address these questions in a reputation model in the tradition of Kreps and Wilson (1982) and Milgrom and Roberts (1982). The type of the central government is uncertain: it can be either a commitment type or a no-commitment type. The reputation of a central government is the probability that local governments assign to it being a commitment type.

Our first main result is that if the reputation of the central government is low enough, then not only are fiscal rules ineffective but they actually lead to even more debt accumulation relative to the case with no rules. This is because the punishment associated with the fiscal rule enforcement makes it more attractive for the no-commitment type to reveal its type earlier relative to an environment without rules. This early resolution of uncertainty makes overborrowing more attractive for the local governments. In contrast, if the central government's reputation is sufficiently high, fiscal rules are effective in reducing borrowing by local governments. We show that these predictions are consistent with evidence from European countries. Our second main result is that despite promoting fiscal indiscipline when reputation is low, binding fiscal rules arise in an equilibrium of a signaling game because the commitment type wants to signal its type and it is optimal for the no-commitment type to initially mimic and then not enforce the rule once violated.

We show these results in a stylized three-period model populated by local governments and a benevolent central government. The local governments choose the provision

[1] See Rodden et al. (2003), Rodden (2006), and Bordo et al. (2013) for further documentation.

of a local public good and have access to local tax revenues. They can also borrow from the rest of the world at a given interest rate. The central government does not have tax revenues, but it can transfer resources from one local government to another. We consider an institutional setup in which there is a constitution that requires the central government to not impose such transfers (*no-bailout clause*) and the local governments to keep their debt below some level or face a penalty if they violate this rule (*fiscal rule*). The central government can either be a commitment type that enforces the fiscal constitution or a no-commitment type that can deviate from the constitution and choose a different policy. This type is initially unknown to the local governments, which learn about it through the actions of the central government.

We first consider the case in which the constitution contains only a no-bailout clause and no fiscal rules. When the central government's initial reputation level is low enough, there is a unique equilibrium in which the no-commitment type central government does not make transfers to the local governments in the intermediate period. Therefore, there is no revelation of the central government's type until the terminal period. The no-commitment central government prefers to delay revealing its type and maintain its reputation. The benefit is that a higher reputation of the central government reduces overborrowing by local governments and the cost is that without transfers public good provision might be unequal across local governments. For low levels of reputation, the benefits of maintaining reputation are first order, while the costs of not equalizing the provision of the local public good in the interim period via transfers are second order. When the local governments are homogenous, these costs are exactly zero on path. When the local governments are heterogeneous, the distribution of debt inherited in the interim period is non-degenerate; therefore, these costs are positive. However, if the probability of facing the commitment type is close to zero, the provision of the local public good across local governments is almost identical even without transfers in the interim period. This is because the more indebted governments borrow against the transfer they anticipate in the final period.

We next consider a constitution with both a no-bailout clause and a *binding* fiscal rule. Fiscal rules are binding if the debt limits are lower than the equilibrium debt levels without fiscal rules. If the central government's reputation and discount factor are low enough, there exists a unique equilibrium in which fiscal rules are violated in period 0 by the local governments and are not enforced ex-post by the no-commitment type central government. Therefore, in this equilibrium there is *early resolution of uncertainty* (i.e., the central government reveals its type in period 1). The intuition behind this result is that with fiscal rules, the costs of preserving reputation are higher. This is because the enforcement of the constitution now requires the no-commitment type central government to impose costly penalties on the local governments that violate the rule. In particular, unlike in the case without fiscal rules, the costs of enforcing the constitution are no longer second order.

We then compare the debt levels in the equilibrium outcomes with and without rules. Our main result is that if reputation is low enough, having fiscal rules in the constitution leads to even more debt accumulation relative to the case without rules. The key driver for this result is that the type of the central government is revealed in the interim period with rules (early resolution of uncertainty), and only in the terminal period without rules (late resolution of uncertainty). Knowing the type of the central government in period 1 allows the local governments to condition their new debt issuances on the government type. This, in turn, lowers the cost of servicing the debt inherited in period 1; hence, the local governments will issue more debt in period 0.

In contrast, if the central government's reputation is high enough, there exists a unique equilibrium in which the local governments obey the fiscal rule. This is because they anticipate facing a penalty for violating the rule with sufficiently high probability irrespective of the choice of the no-commitment type. Since fiscal rules are respected in equilibrium, debt levels are lower in the equilibrium with fiscal rules.

In summary, fiscal rules promote fiscal indiscipline if the central government's reputation is low, but they can be effective in imposing discipline if its reputation is high. We provide suggestive evidence that these predictions are consistent with the experience of a sample of European countries. We partition countries based on a proxy of the reputation of the central government from the World Bank's Worldwide Governance Indicators. We show that a tightening of subnational fiscal rules[2] is typically associated with increases in primary deficits for low-reputation countries, while the opposite is true for high-reputation countries.

Our analysis then raises the question of why we would ever see fiscal rules being instituted in practice when governments lack credibility. We study a signaling game in which rules are chosen at the beginning of time by the central government. We show that for intermediate values of the central government's discount factor, in the equilibrium of this game, the commitment type chooses to announce a fiscal rule, which is mimicked by the no-commitment type. However, in this equilibrium the rule is not enforced in period 1 by the no-commitment type, leading to early resolution of uncertainty and even more debt accumulation.

Fiscal rules in practice

Our analysis sheds light on historical and contemporary episodes when fiscal rules were instituted but were not enforced ex-post. A leading example is the Stability and Growth

[2] That is, rules that restrict fiscal policies of subnational governments.

Pact (SGP) in the Eurozone. The SGP calls for all EU member countries to keep budget deficits below 3% of GDP and public debt to below 60% of GDP. EU member countries are liable to financial penalties of up to 0.5% of GDP if they repeatedly fail to respect these limits. The SGP was instituted for the newly formed monetary union, under the pressure of Germany, with the intent of constraining fiscal policy in member countries to insulate the European Central Bank (ECB) from the pressure to inflate or monetize the debt of member countries. However, the enforcement of the SGP has been very lax. For example, in 2003 both Germany and France violated it and sanctions were not imposed. Through the lens of our theory, this corresponds to the case in which the central government reveals its type in the intermediate period. Consistent with our theory, after 2003, the power of the SGP in disciplining fiscal policy was arguably weakened. According to several commentators, this was a major factor in the current European debt crisis in which Greece, Ireland, and Portugal received bailout packages from the European Union and the ECB (the central government), as our theory predicts.

Arguably, after the bailouts to peripheral member countries, the reputation and credibility of the central European institutions were very low. EU member countries and European institutions agreed to impose tough fiscal rules by strengthening the SGP by introducing the so-called "Six-Pack" and "Fiscal Compact", consistent with the prediction of our signaling game. The provisions of the "Six-Pack" were soon violated by Spain and Portugal without any sanction being levied.[3] In 2016 the governor of the Bundesbank, Jens Weidmann, accused the Commission of not enforcing the fiscal rules: "My perception is that the European Commission has basically given up on enforcing the rules of the Stability and Growth Pact."[4]

Another leading example of federal governments with poor fiscal discipline among subnational governments is Brazil, the most decentralized state in the developing world. The fiscal behavior of the states and large municipal governments in Brazil was a major source of macroeconomic instability and resulted in subnational debt crises in 1989, 1993, and 1997. "The federal government took a variety of measures to control state borrowing in the 1990s, and at a first glance it would appear to have had access to an impressive array of hierarchical control mechanisms through the constitution, additional federal legislation, and the central bank. Most of these mechanisms have been undermined however, by loopholes or bad incentives that discourage adequate enforcement" (Rodden et al. (2003) page 222). In 1997, the federal government assumed the debts of 25 of the 27 states that were unable to service their debt—an amount equivalent to about 13% of GDP. By September 2001, 84% of state debt was held by the national treasury (see Rodden et al. (2003), page 234). After the bailouts in 1997, the Cardoso administration approved the

[3]See https://www.ft.com/content/f66a5c1d-b023-3d0f-ad02-767a9656d4f9

[4]See https://www.ft.com/content/95e7ee7e-ad8e-11e6-ba7d-76378e4fef24.

Fiscal Responsibility Law, which instituted "a rule-based system of decentralized federalism that leaves little room for discretionary policymaking at the subnational level. It has been motivated by the recognition that market control over subnational finances should be replaced, or strengthened, by fiscal rules as well as appropriate legal constraints and sanctions for noncompliance", Afonso and De Mello (2000). So, in a manner similar to Europe, the central government in Brazil imposed stringent fiscal rules when its reputation was arguably low.

Related literature

Our paper is related to several strands of literature. First, it is related to the literature that studies the free-rider problem in federal governments when the central government cannot commit (e.g., Chari and Kehoe (2007), Chari and Kehoe (2008), Cooper et al. (2008), Aguiar et al. (2015), Chari et al. (2016), and Rodden (2002)). The main result in this literature is that the inability of the central government (or monetary authority) to commit not to transfer ex-post leads to overborrowing ex-ante. In such settings, it is often argued that fiscal rules can improve outcomes by lowering the amount of debt issued (e.g., Beetsma and Uhlig (1999)). Our paper contributes to this literature by analyzing the effects of fiscal rules when the government cannot commit to enforcing them.

Fiscal rules have been studied in several environments as the solution to time inconsistency problems. In the context of delegation, see for instance Athey et al. (2005), Amador et al. (2006), Halac and Yared (2014), Halac and Yared (2017). In these papers, fiscal rules are typically thought of as a way to implement the solution to a mechanism design problem. In our paper, we take the set of policy instruments as given and study whether the presence of a fiscal rule allows for better outcomes (or not). Moreover, these papers assume full commitment to the rule, with the exception of Halac and Yared (2018), who study self-enforcing mechanisms. Under some conditions, the solution to the delegation problem can be implemented with rules that are violated in equilibrium with positive probability. Self-enforcing mechanisms are also the focus of Golosov and Iovino (2016) in the context of an insurance problem.

Hatchondo et al. (2015) and Alfaro and Kanczuk (2016) study fiscal rules in the context of sovereign default. Azzimonti et al. (2016) study the effects of introducing balanced budget rules in a political economy model. All these papers assume full commitment to these rules and do not analyze the enforcement problem, which is the main focus of our paper. Piguillem and Riboni (2018) study the role of fiscal rules as a default option in a legislative bargaining model.

The baseline model uses a reputational setup similar to Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982) with uncertainty about the type of the central government. Cole et al. (1995), Phelan (2006), and D'Erasmo (2008)) study environments in which a government with a hidden type interacts with a continuum of private agents. In contrast, in our paper the local governments are strategic and can incentivize the central government to reveal its type via its actions. In addition, we study how varying the costs of maintaining good reputation affects outcomes. In a companion paper, Dovis and Kirpalani (2018), we study an infinite horizon dynamic game where the local governments cannot commit to repaying their debt, but without fiscal rules, to study the joint dynamics of debt, central government's reputation, and interest rate spreads on local government debt.

Uncertainty about the type of the central government plays a key role in the provision of incentives to local governments. Nosal and Ordoñez (2016) also consider an environment in which uncertainty can mitigate the time inconsistency problem when a central government cannot commit not to bail out banks. The mechanism is very different: here uncertainty about the type of the central government curbs debt issuances by the local governments, while in their paper it is the uncertainty about the state of the economy that restrains the central government from not intervening ex-post.

Our paper is also related to the large empirical literature that studies the ability of fiscal rules to constrain fiscal policy. Heinemann et al. (2018) survey the literature and find mixed evidence for the efficacy of fiscal rules. Our paper can help rationalize this mixed evidence. Our findings suggest that a proxy for the central government's reputation is crucial to understanding the effects of fiscal rules. Using data on subnational fiscal rules, we provide suggestive evidence that fiscal rules can be effective if the central government's reputation is high, and detrimental if its reputation is low. Bergman et al. (2016) undertake a similar exercise but for national rules (rules imposed by the central government on itself). They find that that the effects of fiscal rules on primary balance depend on measures of government efficiency. In particular, they find that debt rules are effective only when government efficiency is larger than some threshold. Grembi et al. (2016) study the effect of a change in law that relaxed fiscal rules for certain Italian municipalities in 2001 and find that deficits increased. While they focus on the particular case of Italy, we look at the effects of fiscal rule changes for subnational governments across a variety of high- and low-reputation countries. While there are certainly cases in our sample in which fiscal rules seem to be effective for low-reputation countries, on average, the predictions are in line with our theory.

The rest of the paper is organized as follows. In Section 2 we present the model, and in Section 3 we analyze the equilibrium without fiscal rules. Section 4 demonstrates that if the central government's reputation is low enough then fiscal rules promote fiscal indiscipline relative to the benchmark without rules. Section 5 shows that rules are effective in reducing debt when the central government's reputation is high. Section 6 provides suggestive evidence supporting the theory. In Section 7, we show that rules can arise in the equilibrium of a signaling game. Finally, Section 8 concludes the paper.

2 Model

Environment The economy lasts for three periods indexed by $t = 0, 1, 2$.[5] Consider a small open economy consisting of N states or regions indexed by $i \in \{1, 2, ..., N\}$. We partition the local governments in two groups: the North, $i \in \mathcal{N} = \{1, ..., N_1\}$, and the South, $i \in \mathcal{S} = \{N_1 + 1, N_1 + 2, ..., N\}$. The representative citizen in region i has preferences over the local public good provision $\{G_{it}\}$ given by

$$U_i = \sum_{t=0}^{2} \beta^t u(G_{it}).$$

We make the following assumptions on the utility function throughout:

Assumption 1. *The period utility function* u *is strictly increasing, strictly concave,* $u \in C^1$, $\lim_{c \to 0} u'(c) = \infty$, *and* $u(0)$ *finite.*

The local public good provision is decided by a benevolent *local government* with local tax revenues $\{Y_{it}\}$. For all $n \in \mathcal{N}$ and $s \in \mathcal{S}$ we let[6]

$$Y_{n0} = Y_0 + \Delta \geqslant Y_0 - \Delta = Y_{s0}, \quad Y_{nt} = Y_{st} = Y \quad \text{for } t = 1, 2$$

with $\Delta \geqslant 0$ so the North is (weakly) richer at time 0 relative to the South. The local governments can borrow from the rest of the world at a rate $1 + r^*$. Let $q = 1/(1 + r^*)$ be the price of a bond that promises to pay one unit of the consumption good next period.

There is also a *central government*. The central government does not have tax revenues, but it can impose transfers from one region to another subject to a budget constraint

$$\sum_{i=1}^{N} T_{it} \leqslant 0, \tag{1}$$

where T_{it} is the transfer to region i in period t.

[5] In Section 4 we discuss how our results extend to any finite horizon model.
[6] Adding heterogeneity in tax revenues for $t > 0$ leaves the results unchanged.

Efficient allocation As a benchmark, we consider the efficient allocation for utilitarian Pareto weights in this environment. This allocation solves

$$\max_{\{G_{it}\}} \sum_{i=1}^{N} \frac{1}{N} \sum_{t=0}^{2} \beta^t u\left(G_{it}\right)$$

subject to the consolidated budget constraint

$$\sum_{t=0}^{2} \sum_{i=1}^{N} q^t \left[G_{it} - Y_{it}\right] \leqslant 0. \tag{2}$$

This allocation must satisfy

$$qu'\left(G_{it}\right) = \beta u'\left(G_{it+1}\right), \tag{3}$$

$u'\left(G_{it}\right) = u'\left(G_{jt}\right)$ for all i, j, t, and the consolidated budget constraint (2) with equality. Thus public good provision is equated across regions in every period, and it is efficiently smoothed over time.

Institutional setup and equilibrium Consider an institutional setup in which the central government is subject to a fiscal constitution. The fiscal constitution contains two clauses. The first clause states that the central government should not make any transfers, i.e., $T_{it} = 0$ for all i, t. We call such a provision the *no-bailout clause*. The second clause requires the local governments to keep their debt issued in period 0 below a cap $\bar{b}$. In case $b_{i1} > \bar{b}$, the central government must impose a penalty ψ on the region that violated the rule. We assume that the resources collected from penalties are thrown away.[7] We call this constitutional provision a *fiscal rule*. A fiscal rule is then fully described by $\left(\bar{b}, \psi\right)$. To simplify notation, we abstract from a cap on debt issued in period 1 and its associated penalty. All our propositions will extend to the case with a cap on debt issued in period 1.

The central government can be one of two types: a *commitment type*, which follows the prescriptions of the constitution, and a *no-commitment type*, which is not bound to follow the prescriptions of the constitution, as it chooses policies sequentially to maximize an

[7]This assumption ensures that the cost of imposing the fiscal rule is nonzero for the central government even if $\pi = 0$.

equally weighted average of the utility of citizens in both regions:[8]

$$W_t = \sum_{j \geqslant t}^{2} \frac{1}{N} \sum_{i=1}^{N} \beta^t u\left(G_{ij}\right)$$

for $t = 1, 2$. An alternative interpretation of these types is that the commitment type suffers a sufficiently large utility cost for violating the constitution, while the no-commitment type does not.

The type of the central government is drawn at the beginning of period 0 and is not known to the local governments. They have a common prior π that the central government is the commitment type. The timing is as follows: At $t = 0$, the local governments choose the local public good provision G_{i0} and debt b_{i1} subject to the budget constraints $G_{i0} \leqslant Y_{i0} + qb_{i1}$.

Period 1 can be divided into two sub-periods. In the first sub-period, the central government makes transfers $\{T_{i1}\}$ and decides whether to enforce the penalty if the fiscal rule is violated by a local government. The commitment type will always choose zero transfers and enforce the penalty. After observing the central government's actions, the local governments update their prior about the central government's type. In the second sub-period the local governments decide the provision of the local public good G_{i1} and new debt issuance b_{i2} subject to the budget constraints

$$G_{i1} + b_{i1} \leqslant Y + T_{i1} + qb_{i2} - \psi \mathbb{I}_{\{b_{i1} > \bar{b} \text{ and central government enforces fiscal rule}\}}.$$

At $t = 2$, the central government chooses transfers $\{T_{i2}\}$. As before, the commitment type will always choose zero transfers following the fiscal constitution. Next, the local governments choose G_{i2} subject to the budget constraint $G_{i2} + b_{i2} \leqslant Y + T_{i2}$.

We now comment on some of the assumptions in our model. First, we assume that the local governments can commit to repaying their debt. This can be motivated by the existence of high default costs, which makes repayment always optimal for the local government.

Second, under the fiscal constitution, the commitment type makes no transfers. Transfers may be valuable from an ex-ante utilitarian perspective in the presence of heterogeneity between regions.[9] Thus, in general, the ex-ante welfare associated with the com-

[8]The redistribution motive generates an incentive for the central government to bail out the local government with higher debt. We would obtain similar results if bailouts were motivated by spillovers, as in Tirole (2015).

[9]In general, an optimal transfer scheme that takes into account the benefits of redistribution would prescribe positive transfers that depend on (potentially stochastic) observables, such as tax revenues in our model, and not on inherited debt. Allowing for such transfers would not alter our results, as this case is equivalent to the case in which there is no heterogeneity.

mitment type need not be larger than that of the no-commitment type. This is because, on one hand, the commitment type minimizes the intertemporal distortion generated by the anticipation of future transfers, but on the other hand, the intratemporal distortion due to unequal consumption can be large. In this paper we will restrict attention to cases in which heterogeneity is small and so the value of redistribution is minimal. Thus, ex-ante welfare is higher if the local governments are facing the commitment type.

Third, in our model, in period 0, we allow for some heterogeneity in tax revenues but assume that the central government cannot make transfers. We can relax this assumption by allowing both the commitment and no-commitment type to make transfers in period 0. In this case our model will be equivalent to one in which $\Delta = 0$ and thus our results are unchanged.

We now define the states, payoffs, and beliefs at each node of the game tree.

Period 2 The state in the last period is the distribution of debt among the local governments, $b_2 = (b_{i2})_{i \in \{1,2,\ldots,N\}}$. If the central government is the no-commitment type, it will choose transfers $T_{i2}(b_2)$ such that the consumption of the local public good is equalized between regions[10]: $T_{i2}(b_2) = b_{i2} - \frac{\sum_{j=1}^{N} b_{j2}}{N}$ so that

$$G_{i2} = Y - \frac{\sum_{j=1}^{N} b_{j2}}{N},$$

We refer to this situation as *debt mutualization*. The value for the central government is

$$W_2(b_2) = \sum_{i=1}^{N} \frac{1}{N} u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right),$$

and the value for a local government is

$$V_{i2}(b_2) = u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right).$$

If instead the central government is the commitment type, transfers are zero and each region will consume $G_{i2} = Y - b_{i2}$. The value for the local government is then

$$V_{i2}^{c}(b_2) = u(Y - b_{i2}).$$

Period 1 The relevant state in the second sub-period of period 1 is the updated posterior about the central government's type and the distribution of total obligations owed

[10]Note that there is no benefit to preserving reputation, since the world ends after period 2.

by the local governments, $a_1 = \{a_{i1}\}_{i \in N}$. The total obligations for the local governments are debt owed to lenders minus transfers received from the central government plus the penalty if the local governments violated the fiscal rule (if enforced):

$$a_{i1} = b_{i1} - T_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b} \text{ and central government enforces fiscal rule}\}} \quad (4)$$

Facing this state, the local governments choose G_{i1}, b_{i2} to solve

$$V_{i1}(a_1, \pi) = \max_{G_{i1}, b_{i2}} u(G_{i1}) + \beta\pi V^c_{i2}(b_{i2}) + \beta(1-\pi) V_{i2}\left(b_{i2}, \left(b_{j2}(a_1, \pi)\right)_{j \neq i}\right) \quad (5)$$

subject to

$$G_{i1} + a_{i1} \leqslant Y + q b_{i2}$$

taking as given the strategy $b_{j2}(a_1, \pi)$ followed by the other local governments.

For later reference, the equilibrium outcome at this node will be given by $\{\mathbf{b}_{i2}(a_1, \pi)\}_{i=1}^{N}$, which solves for all i

$$q u'(Y - a_{i1} + q b_{i2}) = \beta\pi u'(Y - b_{i2}) + \beta(1-\pi) \frac{u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right)}{N}. \quad (6)$$

Unless the probability of facing the commitment type is one, the optimality condition (6) differs from the Euler equation (3) that characterizes the efficient allocation. In particular, if $\pi < 1$, there is overborrowing because each local government internalizes only $1/N$ of the marginal cost of repaying its debt if it anticipates a transfer when the central government is the no-commitment type.[11]

We now turn to the first sub-period. The state variables here are distribution of debt among the local governments, $b_1 = (b_{i1})_{i \in \{1,2,\ldots,N\}}$ and the prior on the type of the central government, π. We first describe the law of motion for beliefs of the central government's type. Let σ be the probability that the no-commitment type mimics the commitment type and follows the constitution in period 1. The law of motion for beliefs follows Bayes' rule and is given by

$$\pi'(\pi, \zeta; \sigma) = \begin{cases} \frac{\pi}{\pi + (1-\pi)\sigma} & \text{if } \zeta = 1 \\ 0 & \text{if } \zeta = 0 \end{cases}, \quad (7)$$

where $\zeta = 1$ denotes the event that the constitution is enforced and $\zeta = 0$ denotes the events in which either transfers are not positive or the penalty is not enforced. Note that

[11] Depending on the value of β and q, and the inherited debt, it may be optimal for the local government to save, $b_{i2} \leqslant 0$. When we refer to overborrowing, we also include situations in which the local governments save less than the efficient level. Clearly, we can guarantee that the debt levels are positive if $\beta \leqslant q$ and $Y_{i0} \leqslant Y$.

we can combine all events in which $T \neq 0$ or the fiscal rule is not enforced, because they signal that the central government is the no-commitment type for sure.

The problem of the no-commitment type is to choose transfers $\{T_{i1}\}$ and whether to enforce the penalty to maximize

$$\max \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - a_{i1} + q\mathbf{b}_{i2}\left(a_1, \pi'\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(a_1, \pi'\right)\right)\right]$$

subject to the definition of a_1, (4), the central government budget constraint, (1), the law of motion for beliefs (7), where $\zeta = 1$ if $\{T_{i1}\} = \mathbf{0}$ and the penalty is enforced, while $\zeta = 0$ otherwise. We let $W_1(\pi, b_1)$ denote the value of this program.

Next, we show that the problem of the no-commitment type can be transformed into one in which it makes a simple binary decision of whether to mimic the commitment type or not. In the latter case, its type is revealed, $\pi' = 0$, and a form of Ricardian equivalence holds in this environment, which implies that the payoffs are independent of the transfers chosen in period 1.[12]

Lemma 1. *If $\pi' = 0$, the continuation values and public good provisions for the local governments are independent of transfers in period 1: for any a_1, a_1' such that $\sum_i \frac{1}{N} a_{i1} = \sum_i \frac{1}{N} a_{i1}'$, we have that $V_{i1}(a_1, 0) = V_{i1}\left(a_1', 0\right)$.*

The proof of this lemma is provided in the Appendix. The main idea is that if local governments know they are facing the no-commitment central government type, then only the local government's consolidated budget constraint matters and thus transfering resources across regions in period 1 is irrelevant.

To see why, let us consider two extreme cases when the local governments are certain that they are facing the no-commitment type. In the first, the central government makes transfers to equalize the obligations of the local governments in period 1. In the second, it sets $T_{i1} = 0$ for all i. In the first case, it is easy to see that consumption of the local governments will be equalized in both periods 1 and 2. In the second case, absent transfers, the local governments with inherited debt above average will simply borrow more to keep current consumption at the level of other regions, expecting a transfer in the second period. On the other hand, the local governments with inherited debt below average, absent transfers, will reduce new debt issuances because they anticipate a negative transfer in period 2.

[12]Of course, the timing of transfers would matter if there were impediments to perfect capital markets, for example, borrowing constraints.

Formally, we can see this by setting $\pi = 0$ in equation (6):

$$qu'\left(Y - a_{i1} + qb_{i2}\right) = \beta \frac{u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right)}{N}. \tag{8}$$

Since the right side of the Euler equation is the same for all local governments that anticipate debt mutualization for sure, consumption in period 1 will be equalized even though transfers are zero and the distribution of a_{i1} is non-degenerate.

As a result of Lemma 1, we can then drop $\{T_{i1}\}$ as a choice variable and recast the problem as one in which the no-commitment type decides whether to enforce the constitution or not and thus reveal its type. The problem of the central government is

$$\begin{aligned} W_1\left(\pi, b_1\right) = \max_{\tilde{\sigma}\in[0,1]} \tilde{\sigma} \left(\sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - a_{i1} + q\mathbf{b}_{i2}\left(a_1, \pi'\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(a_1, \pi'\right)\right)\right] \right) \\ + \left(1 - \tilde{\sigma}\right) \left(\sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(b_1, 0\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(b_1, 0\right)\right)\right] \right) \end{aligned} \tag{9}$$

where $a_{i1} = b_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}$ and π' is given by (7). Let $\sigma\left(\pi, b_1\right)$ denote the solution to this problem.

Period 0 The state in period 0 is the prior on the type of the central government, π (the realization of Y_{i0} is incorporated by indexing the value functions by t and i). Each local government chooses the local public good provision and debt to solve

$$\begin{aligned} V_{i0}\left(\pi\right) = \max_{G_{i0}, b_{i1}} u\left(G_{i0}\right) + \beta\left[\pi + \left(1 - \pi\right)\sigma\left(\pi, b_1\right)\right] V_{i1}\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}, \pi'\right) \\ + \beta\left(1 - \pi\right)\left[1 - \sigma\left(\pi, b_1\right)\right] V_{i1}\left(b_1, 0\right) \end{aligned} \tag{10}$$

subject to the budget constraint, $G_{i0} \leqslant Y_{i0} + qb_{i1}$, the law of motion for beliefs, (7), taking as given the strategies $b_{-i1}\left(\pi\right)$ followed by other local governments, and $\sigma\left(\pi, b_1\right)$ followed by the central government.

For later reference, we also define the value for the no-commitment type central government in period 0,

$$W_0\left(\pi\right) = \sum_{i=1}^{N} \frac{1}{N} u\left(G_{i0}\left(\pi\right)\right) + \beta W_1\left(\pi, b_1\right), \tag{11}$$

where $G_{i0}\left(\pi\right)$ and $b_1\left(\pi\right)$ are the decision rules in (10). The value for the commitment type is

$$W_0^c(\pi) = \sum_{i=1}^{N} \frac{1}{N} u(G_{i0}(\pi)) + \beta\sigma(\pi, b_1(\pi)) W_1^c\left(\pi', b_1(\pi) + \psi\mathbb{I}_{\{b_{i1}(\pi)>\bar{b}\}}\right) \quad (12)$$
$$+ \beta[1-\sigma(\pi, b_1(\pi))] W_1^c\left(1, b_1(\pi) + \psi\mathbb{I}_{\{b_{i1}(\pi)>\bar{b}\}}\right).$$

where

$$W_1^c(\pi, b_1) = \sum_{i=1}^{N} \frac{1}{N}\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, \pi')\right) + \beta V_{i2}^c\left(\mathbf{b}_{i2}(b_1, \pi')\right)\right].$$

Equilibrium definition We can now define a Perfect Bayesian Equilibrium for this institutional setup.

Definition. A Perfect Bayesian Equilibrium is a set of strategies and beliefs for the local governments, $b_{i1}(\pi)$, $\pi'(\pi, b_1, \zeta)$, $b_{i2}(b_1, \pi)$, a strategy for the no-commitment type central government, $\sigma(\pi, b_1)$, and associated value functions, such that i) given $b_{-i1}(\pi)$ and $\sigma(\pi, b_1)$, $b_{i1}(\pi)$ solves (10); ii) given $b_{-i2}(a_1, \pi)$, $b_{i2}(a_1, \pi)$ solves (5); iii) $\pi'(\pi, b_1, \zeta)$ satisfies (7); and iv) $\sigma(\pi, b_1)$ solves (9).

3 Equilibrium without fiscal rules

We start by characterizing the equilibrium when the fiscal constitution contains only a no-bailout clause and no fiscal rules. We show that if either the initial heterogeneity in tax revenues between regions or reputation is low enough, there exists a unique equilibrium outcome in which the central government's type is not revealed in period 1.

Proposition 1 (No revelation of central government type)**.** *Suppose the constitution has no fiscal rules. Then, for N large and either Δ or π sufficiently small, there exists a unique symmetric equilibrium in pure strategies in which the type of the central government is not revealed in period 1. Moreover, the debt issuances $\{b_1, b_2\}$ satisfy*

$$qu'(Y_{i0} + qb_{i1}) = \beta u'(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, \pi)) \quad (13)$$
$$+ \beta^2(1-\pi) u'\left(Y - \mathbf{b}_{j2}(b_1, \pi)\right) \frac{N-1}{N} \frac{\partial \mathbf{b}_{-i2}(b_1, \pi)}{\partial b_{i1}}$$

and $b_2 = \mathbf{b}_{i2}(b_1, \pi)$.

The proof of this and other propositions is provided in the Appendix. A key step in the proof of this result is to show that the no-commitment type does not want to implement any transfers along the equilibrium path in period 1. To understand this step, let us consider the *reputation benefits* and *inequality costs* associated with making no transfers in period 1. By making no transfers, the central government preserves its reputation. A higher reputation, in turn, promotes fiscal responsibility, because the local governments expect to repay their debt without a transfer from the central government with higher probability. Hence, the reputation benefits are associated with a reduction in the intertemporal distortions of the local government's Euler equations (6) relative to the efficient one (3). The inequality costs of making no transfers are associated with intratemporal distortions due to the inequality in the provision of the local public good in period 1. This inequality can be reduced by making transfers (or by the revelation that the central government is the commitment type, as shown in Lemma 1).

On one hand, if all regions are homogenous and have identical period 0 tax revenues, then along the equilibrium path each local government enters period 1 with the same amount of debt. Thus, the *inequality* costs are zero, but the reputational benefits are positive. Hence, it is optimal for the no-commitment type to make no transfer and maintain its reputation. However, off equilibrium, a local government could potentially increase the debt issued in period 0 to induce the central government to transfer resources to it in period 1. In the proof, we show that such a deviation is not profitable provided that N is sufficiently large.

On the other hand, if all regions are heterogeneous, they will enter period 1 with different levels of debt, which increases the *inequality* costs of making zero transfers. However, if Δ is sufficiently small, by continuity, it is profitable for the no-commitment type to not make any transfers in period 1, since the *inequality* costs are small relative to the reputation benefits.

Now suppose that Δ is fixed. We can still guarantee that the no-commitment will not make transfers when its reputation π is sufficiently low. To understand this notice that for π small enough, there is essentially no inequality in the local public good consumption even if the central government makes no transfers in period 1. This is related to Lemma 1. As illustrated by equation (8), since local governments expect debt mutualization with high probability in period 2, as $\pi \to 0$, Southern local governments borrow more to increase their consumption in period 1, while Northern governments borrow less expecting a negative transfer in period 2. This implies that the *inequality* costs of no transfers in period 1 are second order. However, the reputation benefits from inducing more fiscal discipline are first order, since the Euler equation is distorted relative to the efficient allocation. Hence, it is optimal for the central government to not make any transfers when its reputation is very low, for any $\Delta > 0$.

Thus, without fiscal rules, the central government does not reveal its type in period 1. In particular, the local governments' posterior that the central government is the commitment type is equal to its prior. So, when the local governments choose their debt issuance in period 1, they are still uncertain about the type of the central government and about the probability of receiving a transfer in the terminal period. Given these expectations, debt issuances along the equilibrium path are characterized by equation (13) and $b_2 = \mathbf{b}_{i2}(b_1, \pi)$. The first two terms of condition (13) resemble those in a standard intertemporal Euler equation, while the last term on the right hand side captures strategic effects in the debt issuance decision. Each local government understands that its choice of debt issuance in period 0 will affect the debt issuance decisions of the other $N-1$ local governments in period 1, which in turn affects the utility of the local government in period 2 in case of debt mutualization (which happens with probability $1-\pi$). Notice that this term vanishes as $N \to \infty$, since $(N-1)\,\partial \mathbf{b}_{-i2}\left(b_1, \pi\right)/\partial b_{i1} \to -1/q$, as shown in Lemma 3 in the Appendix.

4 Fiscal rules promote indiscipline when reputation is low

We now consider a constitution with fiscal rules and present the first main result of the paper: if the reputation of the central government is low enough, then binding fiscal rules are violated and lead to even more debt accumulation relative to the case with no rules. The key driver for this result is that with binding fiscal rules, the type of the central government is revealed in period 1 because the punishment associated with the fiscal rule enforcement makes it less attractive for the no-commitment type to enforce the constitution. This early resolution of uncertainty makes overborrowing more attractive for the local governments.

When the constitution has binding fiscal rules, i.e., $\bar{b} < b_{i1}^{\text{no-rules}}$ for all i, we have the following result:

Proposition 2 (Fiscal indiscipline with low reputation)**.** *Suppose the constitution has binding fiscal rules. Then, for N sufficiently large and β, Δ, and π sufficiently small there exists a unique symmetric equilibrium in pure strategies in which the fiscal rule is violated in period 0 and not enforced by the no-commitment type in period 1 so that the type of the central government is revealed in period 1. Moreover, the equilibrium debt issued in this equilibrium is larger than if the constitution did not contain fiscal rules.*

We first show that under these assumptions, for π close to zero there exists a unique equilibrium in which fiscal rules are violated. The key step to establish this result is to show that in period 1, if all local governments violate the fiscal rule, the no-commitment type central government prefers to not enforce the punishment ψ and reveal its type ($\pi' =$

0 thereafter) than to enforce the punishment and enjoy the reputation gain,[13]. The trade-off faced by the government is similar to the one described in the case without rules: by enforcing the rules, the central government enjoys reputation benefits, but it suffers inequality costs and the additional costs associated with imposing the penalty. This extra cost makes the total costs of enforcing the constitution not second order anymore. In the Appendix, we show that the government prefers not to enforce the constitution if β is below a threshold $\bar{\beta}$. Intuitively, a lower β implies a lower weight on the dynamic reputational benefits of enforcing the fiscal rule relative to the static costs of imposing the penalty and the inequality costs.

We next show that when the central government's reputation is low, binding fiscal rules promote *more* fiscal indiscipline than a constitution without fiscal rules. That is, the debt levels in this equilibrium are higher than in the equilibrium without fiscal rules. The debt issuances in period 0 in the equilibrium in Proposition 2 must satisfy the necessary condition

$$
\begin{aligned}
q u'\left(Y_{i0} + q b_{i1}\right) = {} & \beta\pi u'\left(Y - \left(b_1 + \psi\right) + q \mathbf{b}_{i2}\left(b_1 + \psi, 1\right)\right) \\
& + \beta\left(1 - \pi\right) u'\left(Y - b_1 + q\mathbf{b}_{i2}\left(b_1, 0\right)\right) \\
& + \beta^2\left(1 - \pi\right) u'\left(Y - \frac{\sum_{j=1}^{N} \mathbf{b}_{j2}\left(\left(b_1, b_{i1}\right), 0\right)}{N}\right) \sum_{j \neq i}^{N} \frac{1}{N} \frac{\partial \mathbf{b}_{j2}\left(\left(b_1, b_{i1}\right), 0\right)}{\partial b_{i1}},
\end{aligned}
\tag{14}
$$

where we have used the result that, when the local governments choose their new debt levels in period 1, they know with certainty the type of the central government they are facing. This shows up in equation (14), where the right side of the Euler equation is contingent on the type of the central government, in contrast to (13): with probability π, the local governments observe enforcement of the fiscal rule, learn that they are facing the commitment type, and the new debt issued is $\mathbf{b}_{i2}\left(b_1 + \psi, 1\right)$; with probability $1 - \pi$, they observe no enforcement of the fiscal rule, learn that they are facing the no-commitment type, and the new debt issued is $\mathbf{b}_{i2}\left(b_1, 0\right)$.

To compare the debt levels in period 0 with and without binding fiscal rules, it is useful to rewrite conditions (13) and (14) to make them more comparable. For the case without fiscal rules, we can combine (13) with (6) to obtain a condition that characterizes the debt

[13]The posterior jumps to one as the local governments expect only the commitment type to enforce the fiscal rule.

issuance in period 0:

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1,\pi))+\frac{\beta^2(1-\pi)}{qN}u'(Y-\mathbf{b}_{i2}(b_1,\pi))+\frac{\beta^2(1-\pi)}{N}u'(Y-\mathbf{b}_{i2}(b_1,\pi))\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,\pi)}{\partial b_{i1}}. \tag{15}$$

For the case with fiscal rules, we can combine (14) with (6) to obtain

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1+\psi,1))+\frac{\beta^2(1-\pi)}{qN}u'(Y-\mathbf{b}_{i2}(b_1,0))\;\frac{\beta^2(1-\pi)}{N}u'(Y-\mathbf{b}_{i2}(b_1,0))\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}. \tag{16}$$

These two optimality conditions are identical with the exception that with no fiscal rules (condition (15)), debt issued in period 1 is not conditional on the type of the central government, $b_{i2}=\mathbf{b}_{i2}(b_1,\pi)$. With binding fiscal rules (condition (16)), debt issued in period 1 is conditional on the type of the central government, and is either $\mathbf{b}_{i2}(b_1,1)$ if the central government is the commitment type (with probability π) or $\mathbf{b}_{i2}(b_1,0)$ if the central government is the no-commitment type. We next show that the early revelation of the central government's type in the equilibrium with fiscal rules induces the local governments to issue more debt.

Taking the limit as N goes to infinity for $\pi>0$ but small, since $\lim_{N\to\infty}u'(Y-\mathbf{b}_{i2}(b_1,\pi))<\infty$, as shown in Lemma 2 in the Appendix, condition (15) reduces to

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1,\pi)) \tag{17}$$

as the sum of the second and third terms on the right side converges to zero. Condition (16) instead reduces to

$$u'(Y+qb_1)\,q=\frac{\beta^2\pi}{q}u'(Y-\mathbf{b}_{i2}(b_1+\psi Y,1)), \tag{18}$$

because, as shown in Lemma 2 and 3 in the Appendix,

$$\lim_{N\to\infty}\frac{\beta u'(Y-\mathbf{b}_{i2}(b_1,0))}{N}\frac{1}{q}=-\lim_{N\to\infty}\frac{u'(Y-\mathbf{b}_{i2}(b_1,0))}{N}\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}.$$

We can then compare the right hand side of (17) and (18). We know that for π small enough, $\mathbf{b}_{i2}(b_1,\pi)>\mathbf{b}_{i2}(b_1+\psi Y,1)$, because as $\pi\to 0$, $\mathbf{b}_{i2}(b_1,\pi)\to Y$ but $\mathbf{b}_{i2}(b_1+\psi Y,1)$ is bounded away from Y (see Lemma 2 for details). This observation along with the concavity of u implies that

$$\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right)<\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right).$$

Therefore, from (17) and (18) we see that the expected marginal cost of issuing debt in period 0 is lower when there is early revelation of the central government's type. Hence, the local governments will issue more debt in period 0 because of the lower expected marginal cost.

Intuitively, if the central government reveals its type only in period 2, even if a local government is confident it will receive a transfer in period 2, it does not borrow a lot in period 0 because if the central government is the commitment type, consumption in period 2 will be very low. If instead the central government reveals its type in period 1, the local government will borrow more because in the unlikely event that the central government is the commitment type, the local government can spread the losses associated with not receiving a transfer over period 1 and period 2. The latter is preferable because of preferences for public consumption smoothing; so, the government has a higher incentive to borrow more in period 0 because it can better insure the risk of facing the commitment type.

Now consider debt issuances in period 1 if the central government is the no-commitment type. In this case, debt issued in period 1 is higher with rules than without for two reasons: first, the inherited debt is larger; second, the local governments face no uncertainty about the type of the central government and therefore internalize only $1/N$ of the cost of issuing debt, while without fiscal rules they internalize the full cost with probability π and $1/N$ of the cost with probability $1-\pi$.

Suppose next that local governments face the commitment type in period 1. Since they do not receive transfers in period 2, they issue less debt than if they faced the no-commitment type because they internalize the full cost of servicing the debt with probability 1. However, relative to the case without rules, we cannot sign the change in debt issued in period 1. This is because both reputation and inherited debt are higher, which has opposite effects on debt issuances.

Our characterization can extend to any finite horizon model. In particular, all else being equal, the introduction of fiscal rules increases the probability of early revelation of uncertainty since it increases the cost of maintaining reputation. To see this, consider a finite period environment in which $\Delta=0$ (no heterogeneity) and the constitution only contains a no-bailout clause. Clearly, there always exists an equilibrium with enforcement since on path all local governments will have the same public good provision and debt. Now consider the introduction of the fiscal rule. An almost identical argument to Proposition 2 implies that if β is low enough, there exists an equilibrium with early revelation of the central government's type. Notice that the $\bar{\beta}$ threshold required to get revelation in period 1 in a $T > 3$ period model will be lower than the corresponding threshold in a three-period model since the dynamic gains from enforcement are higher. However, for higher levels of the discount factor, the government will reveal its type before period T.

5 Fiscal rules promote discipline when reputation is high

We now show that when reputation levels are sufficiently high, fiscal rules are effective, since there exists a unique equilibrium in which these rules are followed. To do this we require the following assumption:

Assumption 2. $u\left(Y_{i0} + q\bar{b}\right) + \beta V_{i1}\left(\bar{b}, \pi\right) \geqslant \max_{b_i > \bar{b}} u\left(Y_{i0} + q b_i\right) + \beta V_{i1}\left(b_i + \psi, \bar{b}_{-i}, \pi\right)$

This assumption requires that the punishment must be large enough or the cap on debt must not be too restrictive so that for all π, if a local government believes that the fiscal rule will be enforced for sure the following period, it will prefer to respect the rule if all other local governments are doing so.

Proposition 3 (Fiscal discipline with high reputation)**.** *Suppose the constitution has binding fiscal rules. Under Assumption 2, for N sufficiently large and Δ sufficiently small there exists a threshold $0 < \pi_2^* < 1$ such that for $\pi \geqslant \pi_2^*$, there exists a unique symmetric equilibrium in pure strategies in which the fiscal rule is enforced in period 1. Moreover, as $N \to \infty$ an equilibrium with enforcement exists for all π. Since fiscal rules are binding, the equilibrium level of debt issued is smaller than if the constitution did not contain fiscal rules.*

It follows directly from Assumption 2 that if $\pi = 1$, then the local governments will not deviate from the fiscal rule if all other governments are respecting the rule as well. It follows by continuity that when π is sufficiently close but less than 1, the local governments will also continue to respect the rule. Clearly, since fiscal rules are binding, the equilibrium level of debt issued will be lower than if there were no fiscal rules present.

To show the limiting result, notice that when $N = \infty$, since each local government is infinitesimal, there are no costs for the central government to enforce the penalty for a violation of the fiscal rule by an *individual* local government that has measure zero. Hence, if one local government expects that the other local governments will respect the fiscal rule, it is optimal for it to respect the rule as well; so, there always is an equilibrium in which fiscal rules can curb indebtedness and the local governments internalize the free-rider problem.

This result is fragile: for π low enough there is always an equilibrium where the rule is ignored by all the local governments and not enforced. In particular, if a government expects the other governments to violate the rule, it will find it optimal to violate the rule as well since it anticipates that the rule will not be enforced ex-post. This type of multiplicity is similar to the one in Farhi and Tirole (2012) and Chari and Kehoe (2015).

This result may help to rationalize why when two large countries such as Germany and France violated the SGP in 2003, no sanctions were imposed by the European institutions. More generally, Eyraud et al. (2017) provides suggestive evidence that compliance with the SGP rules has been lower among the largest countries. However, it may be possible for institutions such as the IMF to enforce penalties on a small country to preserve their reputation.

6 Empirical implications

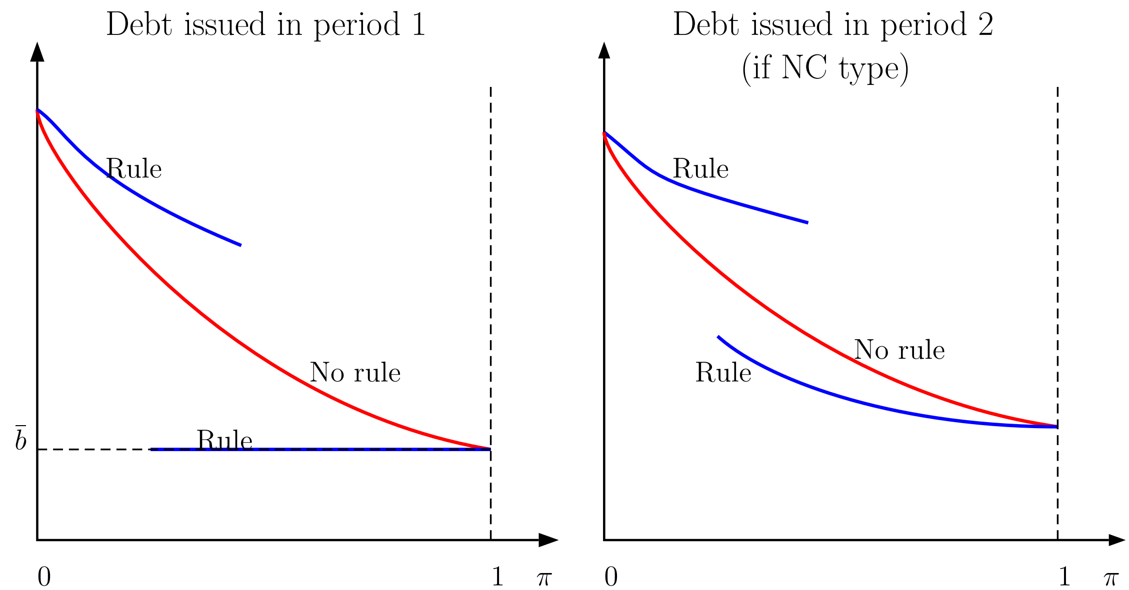
The main takeaway from the theory is that fiscal rules are likely to be effective in reducing debt when the central government's reputation is high, but detrimental when its reputation is low. We illustrate this in Figure 1. The two panels of Figure 1 display the debt issued by a representative local government along the equilibrium path without rules (blue line) and with rules (red line) as a function of the prior in period 0 that the central government is the commitment type.

When π is low enough, debt issuances in period 0 are higher with rules. The same is true in period 1 conditional on facing the no-commitment type. When π is above a threshold, there exists an equilibrium in which rules are followed, the central government does not reveal its type in period 1, and total indebtedness is lower than in the case without rules. Hence, fiscal rules may be effective in reducing debt only when the central government's reputation is sufficiently high. But when the central government's reputation is high, the gains from reducing indebtedness are smaller: debt is decreasing in π because the local governments expect that they will not receive a transfer with a high probability. Therefore, fiscal rules are detrimental exactly when the problem of overborrowing is most severe, while they are effective only when the gains from enforcement are relatively low.

Notice that our propositions prove the existence of unique equilibria when π is either close to zero or close to one. However, we know from Proposition 3 that if $N = \infty$, an equilibrium with enforcement exists for all π and thus for N large enough, an equilibrium (potentially multiple) is likely to exist for all π, as illustrated in Figure 1.

The dynamics in period 1 when the central government's reputation is low is consistent with the experience of several federal states in which fiscal rules are often violated by subnational governments. Our theory provides a rationale for why subnational governments kept on borrowing excessively after the central governments deviated from the fiscal constitution. Arguably, this is what happened in the European Monetary Union

Figure 1: **Equilibrium outcomes: Debt issued in periods 1 and 2**



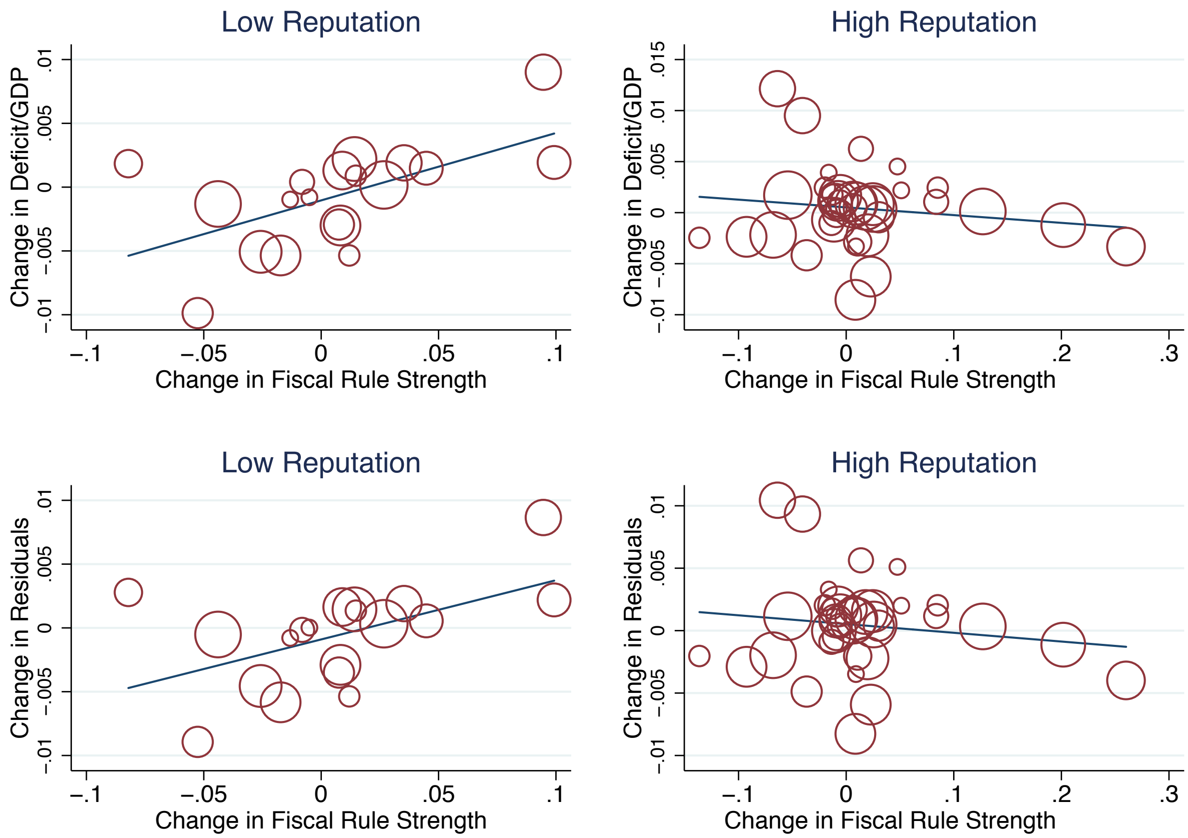
(EMU) after the violation of the Maastricht treaty in 2005 and the subsequent relaxation of the rules and penalties. This is also consistent with the experience in Brazil where "debt burden continued to grow in the 1990s. Despite the previous crises and bailouts - or perhaps because of them - the states continued to increase spending." (Rodden et al. (2003)).

We now provide evidence that accounting for the central government's reputation is crucial to understanding the effects of fiscal rules from an ex-ante perspective. In particular, tighter fiscal rules actually promote overborrowing when the central government's reputation is low. We consider a sample of European countries and study the changes in subnational primary deficits for European countries as a function of changes in fiscal rule strength for different values of government reputation. While it is challenging to directly measure reputation, we use data on government effectiveness from the World Bank's Worldwide Governance Indicators (WGI) as a proxy. The data on subnational primary deficits and fiscal rule strength are from Kotia and Lledó (2016). They construct an index of subnational fiscal rule strength using data from the European Commission. See the data appendix for more details.

We divide the countries into two groups based on the proxy for reputation. In particular, we designate low-reputation countries to be those that are at or below the 15th percentile, and high-reputation countries to be those that are at or above the 50th percentile.[14] To account for the fact that changes in fiscal rule strength can have lagged effects, we construct the average subnational fiscal deficits in a particular fiscal rule regime

[14]Our results are unchanged if we lower the low-reputation cutoff to below the 15th percentile or increase the high-reputation cutoff to above the 50th percentile.

Figure 2: **Changes in fiscal rule strength and primary deficits**



Note: The size of the circles corresponds to the average length of the regime across the two consecutive regimes considered.

(defined as periods in which the index for fiscal rule strength stays constant). Figure 2 displays changes in the mean subnational deficit/GDP against the change in the fiscal rule strength between two consecutive regimes, for the two groups of countries defined above. The top panels plot the average changes in deficits, while the bottom panels plot the average change in residuals after controlling for observables such as the cyclical component of GDP and unemployment, and also include country fixed effects.[15]

Figure 2 is consistent with our theoretical results: strengthening fiscal rules leads to lower deficits for local governments when the central government's reputation is high, but *higher* deficits when the central government's reputation is low. In the data appendix, we also do this exercise by considering only the contemporaneous effect of a change in

[15]In particular, in each period we run the following regression: $\mathtt{deficit}_{it} = \beta X_{it} + f_i + \varepsilon_{it}$, where $\mathtt{deficit}_{it}$ is the primary deficit, X_{it} is a vector of control variables, f_i is a country fixed effect, and $\varepsilon_{it}$is the residual from the regression. The figure plots the change in the average residual across two consecutive fiscal rule regimes.

fiscal rules and obtain similar results.

7 Equilibrium fiscal constitution

In this section, we ask why fiscal rules might be adopted when reputation is low even though their adoption might lead to higher debt than if there were no rules. We study the *equilibrium fiscal constitution*, that is, the fiscal constitution that arises as the outcome of a signaling game between the two types of government in period 0. We show that if the commitment type is sufficiently patient, it is optimal for it to announce fiscal rules that will promote early resolution of uncertainty in period 1, and the no-commitment type will choose to mimic the strategy of the commitment type in period 0 and also announce such rules (and violate them in period 1).

This result rationalizes why we often observe central governments with low reputation setting up tough fiscal rules. Examples include the case of the Eurozone after the European debt crisis and the bailouts in Greece, Portugal, Ireland, and Spain with the institution of the "Six-Pack", and the case of Brazil after the bailouts in 1997 and the Fiscal Responsibility Law approved by the Cardoso administration. In both cases, the reputation of the central government was low because of the recent bailouts to local governments.

More formally, we add an additional stage to the policy game described in Section 2. In the initial stage, given the prior π about the type of central government, the central government chooses to write a fiscal constitution. A fiscal constitution, denoted by α, has a no-bailout clause and a fiscal rule $\alpha = \left(\psi, \bar{b}\right)$ with $\psi \leqslant \bar{\psi}$. After observing the chosen fiscal constitution, the local governments update their prior about the type of the central government, and the subsequent equilibrium outcome is an equilibrium outcome of the policy game described in the previous sections.

In this section, we deviate from the reputation literature following Kreps et al. (1982) by allowing the commitment type to choose an action, the fiscal constitution in the initial stage. In this sense, the commitment type is no longer purely behavioral. We interpret the commitment type as a player that suffers a large utility cost from deviating from past promises. In the game below, the commitment type announces a constitution while internalizing that it will not violate it in the future due this utility cost. In contrast, the no-commitment type suffers no exogenous disutility from deviating from past promises. It does, however, suffer an endogenous cost due to the loss in reputation and trades this off with the static benefits of deviating from the constitution each period.

Definition (Equilibrium fiscal constitution.)**.** An equilibrium fiscal constitution is an equilibrium outcome of the signaling game between the two types of the central government. Given a prior π, an equilibrium of the signaling game is a strategy for the commitment type central government $\alpha^c = \left(\bar{b}^c, \psi^c\right)$, a strategy for the no-commitment type $\alpha^{nc} = \left(\bar{b}^{nc}, \psi^{nc}\right)$, and beliefs π_0' such that i) beliefs evolve according to

$$\pi_0'(\alpha, \pi) = \begin{cases} \pi & \text{if } \alpha = \alpha^{nc} = \alpha^c \\ 0 & \text{if } \alpha = \alpha^{nc} \neq \alpha^c \\ 1 & \text{if } \alpha = \alpha^c \neq \alpha^{nc} \\ 0 & \text{if } \alpha \notin \{\alpha^c, \alpha^{nc}\} \end{cases}, \tag{19}$$

ii) given α^{nc}, the strategy for the commitment type α^c is optimal, in that for all α

$$W_0^c\left(\pi_0'(\alpha^c, \pi); \alpha^c\right) \geqslant W_0^c\left(\pi_0'(\alpha, \pi); \alpha\right),$$

where W_0^c is defined in (12) ; iii) given α^c, the strategy α^{nc} for the no-commitment type is optimal, in that for all α

$$W_0\left(\pi_0'(\alpha^{nc}, \pi); \alpha^{nc}\right) \geqslant W_0\left(\pi_0'(\alpha, \pi); \alpha^{nc}\right),$$

where W_0 is defined in (11).

Note that in W_0^c and W_0 we highlight the dependence on α of this value function that was left implicit in the definitions (12) and (11). We will do this for all equilibrium objects from now onward.

We can characterize the equilibrium of this game by considering the problem for the commitment type given the prior π. To do so, it is useful to define the value for the no-commitment type of enforcement if the inherited debt is b_1 and the posterior after enforcing equals π':

$$\begin{aligned} \omega^e\left(b_1, \pi'; \alpha\right) = \sum_{i=1}^{N} \frac{1}{N} \Big[& u\left(Y - \left(b_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + q\mathbf{b}_{i2}\left(\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}\right), \pi'\right)\right) \\ & + \beta W_2\left(\mathbf{b}_2\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}\right)\right)\Big] \end{aligned} \tag{20}$$

and the value of non-enforcement:

$$\omega^{ne}(b_1) = \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) + \beta W_2\left(\mathbf{b}_2(b_1, 0)\right)\right] \tag{21}$$

Clearly, if there is no enforcement, then the posterior jumps to zero.

The problem for the commitment type in period 0 is

$$W_0^c = \max\left\{W_0^{c,sep}, W_0^{c,pool}\right\},$$

where $W_0^{c,sep}$ is the value for the commitment type if it chooses a fiscal rule that ensures separation in period 1, and $W_0^{c,pool}$ is the value for the commitment type if the fiscal constitution it chooses is such that the no-commitment type enforces the rule in period 1. The value for $W_0^{c,sep}$ is given by

$$\begin{aligned} W_0^{c,sep} &= \max_{\alpha} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi,\alpha)\right) + \\ &+ \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}} - b_{i1}^{er}(\pi,\alpha) + q \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}}, 1\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}}, 1\right)\right) \end{array} \right] \end{aligned}$$

subject to

$$\omega^{ne}\left(b_{i1}^{er}(\pi,\alpha)\right) \geqslant \omega^{e}\left(b_{i1}^{er}(\pi,\alpha), 1; \alpha\right),$$

where $b_{i1}^{er}(\pi,\alpha)$ is the debt issued in period 0 when the local governments expect to learn the central government's type in period 1 defined in (14) given $\alpha = \left(\bar{b}, \psi\right)$. The last constraint requires that the punishment induces the no-commitment to prefer not to enforce the penalty and lose its reputation rather than enforce and have its reputation jump to 1.

The value for $W_0^{c,pool}$ is given by

$$\begin{aligned} W_0^{c,pool} &= \max_{\alpha} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi,\alpha)\right) + \\ &+ \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1}^{lr}(\pi,\alpha) + q \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{lr} > \bar{b}}, \pi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\psi) + \psi \mathbb{I}_{b_{i1}^{lr} > \bar{b}}, \pi\right)\right) \end{array} \right] \end{aligned}$$

subject to

$$\omega^{e}\left(b_{i1}^{lr}(\pi,\alpha), \pi; \alpha\right) \geqslant \omega^{ne}\left(b_{i1}^{lr}(\pi,\alpha), 0\right),$$

where $b_{i1}^{lr}(\pi,\alpha)$ is the debt issued in period 0 when the local governments do not expect to learn the central government's type in period 1 defined in (13) given $\alpha = \left(\bar{b}, \psi\right)$. The last constraint requires that the no-commitment type prefers to mimic the commitment type in period 1.

In setting up these problems we assumed that it was optimal for the no-commitment type to mimic the strategy of the commitment type in period 0. In the next proposition we prove that this is the case.

We assume that the commitment type can only choose between two levels of penalties,

$\psi \in \{0, \bar{\psi}\}$. Furthermore, assume that the discount factor for the central government is less than $\bar{\beta}$ in Proposition 2 so that in period 1 is not optimal for the no-commitment type to enforce the penalty if the fiscal constitution has $\psi = \bar{\psi}$. Moreover, we assume that the initial reputation is sufficiently close to zero and N is sufficiently large. Under these assumptions, the next proposition shows that if the commitment type central government is sufficiently patient, then there exists a unique equilibrium fiscal constitution that has fiscal rules. Moreover, the no-commitment type central government prefers to mimic the strategy of the commitment type in period 0 and chooses a constitution with fiscal rules despite knowing that it will not enforce the constitution in period 1. If, instead, the commitment type central government is not patient enough, the equilibrium constitution has no fiscal rules:

Proposition 4. *If* N *is sufficiently large, and* Δ *and* π *are sufficiently small, then there exist two cutoffs* $\underline{\beta} \leqslant \bar{\beta}$ *such that:*

1. *For* $\beta \in [\underline{\beta}, \bar{\beta}]$*, there exists a unique fiscal constitution with fiscal rules that are violated by the local governments, and there is early resolution of uncertainty in period 1. If* $\Delta > 0$*, then* $\bar{\beta} > \underline{\beta}$*.*

2. *For* $\beta < \underline{\beta}$*, there exists a unique fiscal constitution with no fiscal rules and* $\psi = 0$*.*

When the central government's reputation is sufficiently close to zero, for intermediate values of the discount factor β, fiscal rules arise in equilibrium even if they are going to be violated by the local governments. The commitment type chooses to do so to reveal its type in period 1. From its perspective, this has benefits, because in period 1 the reputation of the central government will jump from almost zero to one, promoting fiscal discipline going forward. In particular, the local government's decision will satisfy the Euler equation and so is efficient from period 1 onward.[16] But this also has costs. As we have shown in Proposition 2, instituting fiscal rules promotes overborrowing and fiscal indiscipline in period 0. Moreover, the commitment type will suffer the costs of punishment when the local governments violate the rule. When β is above the cutoff $\underline{\beta}$ defined in the Appendix, the benefits outweigh the costs. Conditional on the commitment type announcing a fiscal rule, for π close to zero, the no-commitment type always prefers to mimic the strategy of the commitment type in period 0. Intuitively, the reputation cost of not mimicking the strategy of the commitment type is of first order, while the benefit of equalizing consumption is of second order when π is close to zero, using a logic similar to the one in Lemma 1.

[16] Of course, the commitment type central government would like to redistribute resources from the North to the South, but in our setup it has no instruments to do so.

Finally, for the no-commitment type to not enforce the rule in period 1, we need to impose an upper bound on the discount factor. In Proposition 2, we define such an upper bound $\bar{\beta}$. In the Appendix we show that $\underline{\beta} < \bar{\beta}$ when countries are heterogeneous in period 0, i.e., $\Delta > 0$. This is because there is an additional benefit to the no-commitment type of not enforcing the constitution, namely, that it can equalize consumption across regions. This implies that it requires a larger discount factor in order to prefer enforcement. As $\Delta \to 0$, this additional benefit shrinks to zero and so $\overline{\beta} \to \underline{\beta}$.

If, instead, β is below $\underline{\beta}$, the commitment type prefers not to institute the rule, and clearly the no-commitment type chooses to do the same.

8 Conclusion

Fiscal rules are often thought to be useful in federal states when the central government cannot commit to no-bailout clauses. In this paper, we ask if this is indeed the case when the central government also cannot commit to imposing these rules. We show that in a reputation model in which the local governments are uncertain whether the central government can commit or not, outcomes with rules attain higher debt levels than outcomes without rules when the central government's reputation is low. Our results shed light on the multitude of examples throughout history when fiscal rules were instituted but not enforced. Our analysis of the equilibrium constitution suggests that stringent fiscal rules can arise when the central government's reputation is low even though they are not optimal under the veil of ignorance in that they increase local governments' debt when such governments are already overborrowing.

In this paper, we assumed that the central government is benevolent and maximizes the utility of the local governments. Another possibility is to study institutional settings where local governments' representatives vote to impose sanctions on the local governments that violate the rule. This is left for future research.

Finally, in our analysis we take as given the policy instruments available to the central government, such as the form of the fiscal rules. It would be interesting to study the optimal design of these rules from an ex-ante perspective taking into account reputation-building incentives.

References

AFONSO, J. R. AND L. DE MELLO (2000): "Brazil: an evolving federation," *Managing fiscal decentralization*, 265–285. 6

AGUIAR, M., M. AMADOR, E. FARHI, AND G. GOPINATH (2015): "Coordination and Crisis in Monetary Unions," *The Quarterly Journal of Economics*, qjv022. 6

ALFARO, L. AND F. KANCZUK (2016): "Fiscal Rules and Sovereign Default," . 6

AMADOR, M., I. WERNING, AND G.-M. ANGELETOS (2006): "Commitment vs. flexibility," *Econometrica*, 74, 365–396. 6

ATHEY, S., A. ATKESON, AND P. J. KEHOE (2005): "The optimal degree of discretion in monetary policy," *Econometrica*, 73, 1431–1475. 6

AZZIMONTI, M., M. BATTAGLINI, AND S. COATE (2016): "The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy," *Journal of Public Economics*, 136, 45–61. 6

BEETSMA, R. AND H. UHLIG (1999): "An analysis of the Stability and Growth Pact," *The Economic Journal*, 109, 546–571. 6

BERGMAN, U. M., M. M. HUTCHISON, AND S. E. H. JENSEN (2016): "Promoting sustainable public finances in the European Union: The role of fiscal rules and government efficiency," *European Journal of Political Economy*, 44, 1–19. 7

BORDO, M. D., L. JONUNG, AND A. MARKIEWICZ (2013): "A fiscal union for the euro: Some lessons from history," *CESifo Economic Studies*, ift001. 2

CHARI, V., A. DOVIS, AND P. J. KEHOE (2016): "A Journey Down the Slippery Slope to the European Crisis: A Theorist's Guide," . 6

CHARI, V. AND P. J. KEHOE (2015): "Bailouts, time inconsistency, and optimal regulation: A macroeconomic view," *American Economic Review*. 22

CHARI, V. V. AND P. J. KEHOE (2007): "On the need for fiscal constraints in a monetary union," *Journal of Monetary Economics*, 54, 2399–2408. 2, 6

——— (2008): "Time Inconsistency and Free-Riding in a Monetary Union," *Journal of Money, Credit and Banking*, 40, 1329–1356. 2, 6

COLE, H. L., J. DOW, AND W. B. ENGLISH (1995): "Default, settlement, and signaling: Lending resumption in a reputational model of sovereign debt," *International Economic Review*, 365–385. 7

COOPER, R., H. KEMPF, AND D. PELED (2008): "Is it is or is it ain't my obligation? Regional debt in a fiscal federation," *International Economic Review*, 49, 1469–1504. 2, 6

DOVIS, A. AND R. KIRPALANI (2018): "Reputation, Bailouts, and Interest Rate Spread Dynamics," . 7

D'ERASMO, P. (2008): "Government reputation and debt repayment in emerging economies," *Manuscript*. 7

EYRAUD, L., V. GASPAR, AND T. POGHOSYAN (2017): "Fiscal Politics in the Euro Area," . 22

FARHI, E. AND J. TIROLE (2012): "Collective moral hazard, maturity mismatch, and systemic bailouts," *The American Economic Review*, 102, 60–93. 22

GOLOSOV, M. AND L. IOVINO (2016): "Social Insurance, Information Revelation, and Lack of Commitment," . 6

GREMBI, V., T. NANNICINI, AND U. TROIANO (2016): "Do fiscal rules matter?" *American Economic Journal: Applied Economics*, 8, 1–30. 7

HALAC, M. AND P. YARED (2014): "Fiscal rules and discretion under persistent shocks," *Econometrica*, 82, 1557–1614. 6

——— (2017): "Fiscal Rules and Discretion in a World Economy," . 6

——— (2018): "Fiscal Rules and Discretion under Self-Enforcement," . 6

HATCHONDO, J. C., L. MARTINEZ, AND F. ROCH (2015): "Fiscal rules and the sovereign default premium," *Available at SSRN 2625128*. 6

HEINEMANN, F., M.-D. MOESSINGER, AND M. YETER (2018): "Do fiscal rules constrain fiscal policy? A meta-regression-analysis," *European Journal of Political Economy*, 51, 69–92. 7

KOTIA, A. AND V. D. LLEDÓ (2016): *Do Subnational Fiscal Rules Foster Fiscal Discipline? New Empirical Evidence from Europe*, International Monetary Fund. 23, 48

KREPS, D. M., P. MILGROM, J. ROBERTS, AND R. WILSON (1982): "Rational cooperation in the finitely repeated prisoners' dilemma," *Journal of Economic Theory*, 27, 245–252. 6, 25

KREPS, D. M. AND R. WILSON (1982): "Reputation and imperfect information," *Journal of economic theory*, 27, 253–279. 2, 6

MILGROM, P. AND J. ROBERTS (1982): "Predation, reputation, and entry deterrence," *Journal of economic theory*, 27, 280–312. 2, 6

Nosal, J. B. and G. Ordoñez (2016): "Uncertainty as commitment," *Journal of Monetary Economics*, 80, 124–140. 7

Phelan, C. (2006): "Public trust and government betrayal," *Journal of Economic Theory*, 130, 27–43. 7

Piguillem, F. and A. Riboni (2018): "Fiscal Rules as Bargaining Chips," Tech. rep., Einaudi Institute for Economics and Finance (EIEF). 6

Rodden, J. (2002): "The dilemma of fiscal federalism: grants and fiscal performance around the world," *American Journal of Political Science*, 670–687. 2, 6

——— (2006): *Hamilton's paradox: the promise and peril of fiscal federalism*, Cambridge University Press. 2

Rodden, J., G. S. Eskeland, and J. I. Litvack (2003): *Fiscal decentralization and the challenge of hard budget constraints*, MIT press. 2, 5, 23

Sargent, T. J. (2012): "Nobel Lecture: United States Then, Europe Now," *Journal of Political Economy*, 120, 1–40. 2

Tirole, J. (2015): "Country solidarity in sovereign crises," *The American Economic Review*, 105, 2333–2363. 10

A Appendix: Omitted Proofs

A.1 Proof of Lemma 1

Let $\pi' = 0$ and consider two vectors a_1 and a_1' that differ only transfers. We know that debt issuances $\{b_{i2}\}$ must satisfy

$$qu'\left(Y - a_{i1} + qb_{i2}\left(a_1, 0\right)\right) = \frac{\beta}{N} u'\left(Y - \frac{\sum_{j=1}^{N} b_{i2}\left(a_1, 0\right)}{N}\right) \quad \text{for all } i$$

We can then see that if $\{b_{i2}\left(a_1, 0\right)\}$ solves the system given a_1 then

$$b_{i2}\left(a_1', 0\right) = b_{i2}\left(a_1, 0\right) - \frac{1}{q}\left(T_{i1} + T_{i1}'\right) \quad \text{for all } i$$

solves the system given a_1' and leaves public good provisions in period 1 and 2 unchanged. Hence the value is unaffected by transfers in period 1 when $\pi = 0$. A straightforward extension of thes arguments implies that this result holds more generally for any two sequences a_1 and a_1' such that $\sum \frac{1}{N} a_{i1} = \sum \frac{1}{N} a_{i1}'$. Q.E.D.

A.2 Preliminary results for proof of Proposition 1-4

For the following proofs it is useful to define the value of enforcing if the posterior equals π'

$$\omega^{e}\left(b_1, \pi'\right) = \sum_{i=1}^{N} \frac{1}{N} \Big[u\left(Y - b_{i1} - \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}} + q\mathbf{b}_{i2}\left(b_1, \pi'\right)\right) + \beta W_2\left(\mathbf{b}_2\left(b_1 + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}, \pi'\right)\right)\Big] \tag{22}$$

and the the value of non-enforcement

$$\omega^{ne}\left(b_1\right) = \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(b_1, 0\right)\right) + \beta W_2\left(\mathbf{b}_2\left(b_1, 0\right)\right)\right] \tag{23}$$

To prove Proposition 2 we use the following two lemmas:

Lemma 2. *As* $N \to \infty$*, the continuation equilibrium in period 1 given inherited debt* b_1 *and posterior* π *is such that:*

1. *If* $\pi > 0$, $\lim_{N \to \infty} \mathbf{b}_{i2}\left(b_1, \pi\right) \to b_{i2} < Y$;

2. *If* $\pi = 0$, $\lim_{N\to\infty} \sum_i \frac{\mathbf{b}_{i2}(b_1,0)}{N} \to Y$ *and*

$$\lim_{N\to\infty} \frac{1}{N} u'\left(Y - \sum_i \frac{\mathbf{b}_{i2}(b_1,0)}{N}\right) = \frac{q}{\beta} u'\left((1+q)Y - \sum_i \frac{b_{i1}}{N}\right) > 0.$$

Moreover, $\lim_{N\to\infty} V_{i1}(b_1,0) = u(Y(1+q) - b_1) + \beta u(0)$.

Proof. We know from the text, equation (6), that along a symmetric equilibrium outcome, it must be that

$$qu'(Y - b_{i1} + q\mathbf{b}_{i2}(b_1,\pi)) = \beta\pi u'(Y - \mathbf{b}_{i2}(b_1,\pi)) + \beta(1-\pi)\frac{1}{N}u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b_1,\pi)}{N}\right)$$

whenever $\sum_i \mathbf{b}_{i2}(b_1,\pi)/N < Y$.

Consider part 1 and let $\pi > 0$. Clearly, for each finite N, $\mathbf{b}_{i2} < Y$ due to the Inada condition and so the Euler equation above holds. Suppose by way of contradiction that $\mathbf{b}_{i2}(b_1,\pi) \to Y$ as $N \to \infty$. Then the right side goes to ∞ while the left side goes to $qu'(Y - b_1 + qY)$ which is finite. This is a contradiction.

Consider part 2 and let $\pi = 0$. For all finite N, because of the Inada condition, it must be that $\sum_i \mathbf{b}_{i2}/N < Y$ and so the following Euler equation must hold:

$$qu'(Y - b_{i1} + q\mathbf{b}_{i2}(b_1,0)) = \beta\frac{1}{N}u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b_1,0)}{N}\right) \tag{24}$$

Suppose by way of contradiction that $\frac{\sum \mathbf{b}_{i2}(b_1,0)}{N} \to B_2 < Y$. Then the left side converges to a positive number, $qu'(Y(1+q) - b_1)$, while the right side converges to zero. This is a contradiction. In particular, since the right side is identical for all i,

$$Y - b_{i1} + q\mathbf{b}_{i2}(b_1,0) \to (1+q)Y - \frac{\sum_i b_{i1}}{N}$$

Therefore, it must be that

$$\lim_{N\to\infty} \frac{1}{N}u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b_1,0)}{N}\right) = \frac{q}{\beta}u'(Y - b_{i1} + qY).$$

It follows that, if the posterior equals zero, the value of a continuation equilibrium is

$$u(Y(1+q) - b_1) + \beta u(0).$$

□

Lemma 3. *Suppose* $\pi = 0$. *Then for all* i,

$$\lim_{N\to\infty} \sum_{j\neq i} \frac{\partial \mathbf{b}_{j2}(b_1, 0)}{\partial b_{i1}} = -\frac{1}{q}$$

Proof. Step 1:$\lim_{N\to\infty} G_{i1}(\pi = 0) = 0$.

We know from Lemma 1 that the equilibrium allocations are identical whether or not there are transfers by the central government in period 1. In the case in which there are transfers $T_{i1} = b_{i1} - \sum_i \frac{1}{N} b_{i1}$, the first order conditions for b_{i1} and b_{i2} respectively are

$$u'(G_{i0})\, q = \beta \left[\frac{1}{N} u'(G_{i1}) + \frac{\beta}{N} u'(G_{i2}) \sum_{j\neq i} \frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}} \right] \tag{25}$$

$$u'(G_{i1})\, q = \frac{\beta}{N} u'(G_{i2}) \tag{26}$$

where the superscript tr denotes outcomes with transfers. Therefore

$$\sum_{j\neq i} \frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}} = \frac{u'(G_{i0})\frac{qN}{\beta} - u'(G_{i1})}{\beta u'(G_{i2})} = \frac{u'(G_{i0})\frac{qN}{\beta} - u'(G_{i1})}{N u'(G_{i1})\, q} = \frac{\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta} - \frac{1}{N}}{q} \tag{27}$$

We know from Lemma 2 that $\lim_{N\to\infty} G_{i2}(0) = 0$. Now suppose by way of contradiction that $\lim_{N\to\infty} G_{i1}(0) > 0$. Then from (27) we see that

$$\lim_{N\to\infty} \sum_{j\neq i} \frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}} = \frac{u'(G_{i0})}{\beta u'(G_{i1})} > 0$$

Next, we can combine (25) and (26) to obtain

$$u'(G_{i0})\, q = \beta \frac{u'(G_{i1})}{N} \left[1 + q \sum_{j\neq i} \frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}} \right] \tag{28}$$

If $G_{i1} > 0$ then the term $\frac{u'(G_{i1})}{N}$ converges to zero as $N \to \infty$, while the argument above establishes that the limit of $q \sum_{j\neq i} \frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}}$ is finite. Therefore, as $N \to \infty$, the right side of (28) converges to zero while the left side is finite. This is a contradiction. Since the equilibrium outcome with transfers in period 1 and the one without are equivalent when $\pi = 0$ then $\lim_{N\to\infty} G_{i1}(\pi = 0) = 0$.

Step 2: $\lim_{N\to\infty} \sum_{j\neq i} \frac{\partial \mathbf{b}_{j2}(b_1, 0)}{\partial b_{i1}} = -\frac{1}{q}$.

Now consider the case in which there are no transfers in period 1. In this case the first order conditions imply that

$$\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}\left(b_1,0,N\right)}{\partial b_{i1}}=\frac{u'\left(G_{i0}\right)\frac{qN}{\beta}-u'\left(G_{i1}\right)N}{\beta u'\left(G_{i2}\right)}=N\left(\frac{u'\left(G_{i0}\right)\frac{q}{\beta}-u'\left(G_{i1}\right)}{Nu'\left(G_{i1}\right)q}\right)=\frac{\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}-1}{q}$$

Since we just established that $\lim_{N\to\infty}G_{i1}=0$ taking limits on both sides of the above equation yields the result since $\lim_{N\to\infty}\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}=0$. □

Lemma 4. *If* $b_1=\{b_{i1}\}$ *is degenerate in that* $b_{i1}=b_{j1}$ *for all* i,j *then* $\lim_{N\to\infty}\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial\pi}<\infty$.

Proof. By applying the implicit function theorem to (6) we obtain

$$\frac{\partial \mathbf{b}_{i2}\left(b_1,0\right)}{\partial\pi}=\frac{\beta\frac{N-1}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{\left[q^2u''\left(Y-b_1+q\mathbf{b}_{i2}\left(b_1,0\right)\right)+\frac{\beta}{N}u''\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)\right]}$$

so

$$\frac{1}{N}\frac{\partial \mathbf{b}_{i2}\left(b_1,0\right)}{\partial\pi}=\left(1-\frac{1}{N}\right)\frac{\beta\frac{1}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{\left[q^2u''\left(Y-b_1+q\mathbf{b}_{i2}\left(b_1,0\right)\right)+\frac{\beta}{N}u''\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)\right]}$$

As $N\to\infty$, the above converges to

$$\frac{\beta\frac{1}{N}\sum_{j\neq i}u'\left(0\right)}{\left[q^2u''\left(Y-b_1+qY\right)+\beta\frac{u''(0)}{N}\right]}$$

We know that $\beta\frac{1}{N}\sum_{j\neq i}u'\left(G_{i2}\right)$ converges to a finite number. If $\beta\frac{u''(G_{i2})}{N}$ converges to a finite constant or zero then the above converges to a finite number. If it converges to ∞ then the above converges to zero. In both cases, as $N\to\infty$, $\frac{1}{N}\frac{\partial \mathbf{b}_{i2}(b_1,0)}{\partial\pi}$ converges to a finite number. □

Lemma 5. *i) For all* π, $\omega^e\left(\cdot,\pi\right)$ *is continuous and differentiable.*
ii) For all b, *for* π *small enough,* $\omega^e\left(b,\cdot\right)$ *is increasing in* π.

Proof. For convenience, rewrite (22):

$$\omega^e\left(b,\pi\right)=\sum_i\frac{1}{N}\left[u\left(Y-b_i+q\mathbf{b}_{i2}\left(b,\pi\right)\right)+\beta u\left(Y-\frac{\sum_i\mathbf{b}_{i2}\left(b,\pi\right)}{N}\right)\right]$$

Part i). The fact that ω^e_1 is continuous and differentiable in b follows from continuity and differentiability of u and $\mathbf{b_2}$.

Part ii). Consider the derivative with respect to π:

$$\frac{\partial \omega^e(b,\pi)}{\partial \pi} = \sum_i \frac{1}{N}\left[qu'(G_{i1})\frac{\partial \mathbf{b}_{i2}}{\partial \pi} - \beta \frac{u'(G_{i2})}{N}\frac{\partial \sum_i \mathbf{b}_{i2}(b,\pi)}{\partial \pi}\right]$$

While we cannot sign this term in general, at $\pi = 0$, since $qu'(G_{i1}) = \frac{\beta}{N}u'(G_{i2})$, we have

$$\frac{\partial \omega^e(b,\pi)}{\partial \pi} = -\beta \sum_i \frac{u'(G_{i2})}{N^2} \sum_{j \neq i} \frac{\partial \mathbf{b}_{-i2}}{\partial \pi} = -\beta \frac{u'(G_{i2})}{N} \frac{(N-1)}{N} \frac{\partial \mathbf{B}_2}{\partial \pi}$$

where $\mathbf{B}_2 \equiv \sum_i \mathbf{b}_{i2}$ and so if $\frac{\partial \mathbf{B}_2}{\partial \pi} < 0$, then $\frac{\partial \omega^e(b,\pi)}{\partial \pi} > 0$.

We now turn to show how $\mathbf{B}_2(b_1,\pi)$ varies with π. Recall the first order condition in period 1, (6), rewritten here for convenience:

$$qu'(Y - b_{i1} + q\mathbf{b}_{i2}) = \beta\pi u'(Y - \mathbf{b}_{i2}) + \beta(1-\pi)\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{i2}}{N}\right)}{N} \tag{29}$$

First define

$$\Delta MU_i \equiv \beta\left[u'(Y - b_{i2}) - \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}}{N}\right)}{N}\right]$$

Clearly, if local governments are homogeneous, $b_{i2} = \sum_j \frac{b_{j2}}{N}$ and so $\Delta MU_i > 0$ for all i. Consider next the case with heterogeneous local governments, $\Delta > 0$. Let s index a local government in the South, and n index a local government in the North. Since $\mathbf{b}_{s2} \geqslant \frac{\mathbf{b}_{s2}+\mathbf{b}_{n2}}{2} \geqslant \mathbf{b}_{n2}$, $\Delta MU_s > 0$. The sign of ΔMU_n is in general ambiguous but it is positive for π close to zero. To see this, note that from the foc (29), as $\pi \downarrow 0$,we have

$$qu'(Y - b_{n1} + q\mathbf{b}_{n2}) - \beta\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}}{N}\right)}{N} \to 0$$

and so

$$\lim_{\pi \downarrow 0}\left[\beta u'(Y - \mathbf{b}_{n2}) - \beta\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}}{N}\right)}{N}\right] = \lim_{\pi \downarrow 0}\left[\beta u'(Y - \mathbf{b}_{n2}) - qu'(Y - b_{n1} + q\mathbf{b}_{n2})\right] > 0$$

Also notice that

$$\Delta MU_s - \Delta MU_n = \beta\left[u'(Y - b_{s2}) - u'(Y - b_{n2})\right] \geqslant 0$$

Define

$$A_i \equiv \left[-\beta\pi u''\left(G_{i2}^c\right) - \frac{\beta\left(1-\pi\right)}{2N}u''\left(G_{i2}\right) - qu''\left(G_{i1}\right)\right] > 0$$
$$a_i \equiv \frac{2N}{\beta\left(1-\pi\right)}A_i > 0$$

where $G_{i2}^c = Y - \mathbf{b}_{i2}$. Using the implicit function theorem we have

$$A_i d\mathbf{b}_{i2} = \frac{\beta\left(1-\pi\right)}{2N}u''\left(G_{i2}\right) d\mathbf{b}_{-i2} - \Delta MU_i d\pi$$

and so

$$\frac{\partial \mathbf{b}_{i2}}{\partial \pi} = \frac{1}{1 - \frac{u''(G_{i2})}{a_i}\frac{u''(G_{-i2})}{a_{-i}}}\frac{-\Delta MU_i}{A_i} + \frac{u''\left(G_{i2}\right)}{a_i}\frac{-\Delta MU_{-i}}{A_i}.$$

Next, we have

$$\begin{aligned}\frac{\partial \mathbf{B}_2}{\partial \pi} = &\frac{1}{1 - \frac{u''(G_{s2})}{a_s}\frac{u''(G_{n2})}{a_n}}\frac{-\Delta MU_s}{A_s} + \frac{u''\left(G_{s2}\right)}{a_s}\frac{-\Delta MU_n}{A_s} \\ &+ \frac{1}{1 - \frac{u''(G_{s2})}{a_s}\frac{u''(G_{n2})}{a_n}}\frac{-\Delta MU_n}{A_n} + \frac{u''\left(G_{n2}\right)}{a_n}\frac{-\Delta MU_s}{A_n}\end{aligned}$$

At $\pi = 0$,

$$A_i = \left[-\frac{\beta}{4}u''\left(G_{i2}\right) - qu''\left(G_{i1}\right)\right] = A > 0$$
$$a_i = \frac{4}{\beta}A_i = a > 0$$

Therefore evaluating $\frac{\partial \mathbf{B}_2}{\partial \pi}$ at $\pi = 0$, we obtain

$$\frac{d\mathbf{B}_2}{d\pi} = \left[-\frac{1}{1 - \frac{u''(G_{s2})}{a}\frac{u''(G_{s2})}{a}} - \frac{u''\left(G_{n2}\right)}{a}\right]\frac{1}{A}\left[\Delta MU_s + \Delta MU_n\right] \tag{30}$$

We know that

$$\frac{1}{1 - \frac{u''(G_{s2})}{a}\frac{u''(G_{n2})}{a}} > 1$$

and

$$\frac{u''\left(G_{n2}\right)}{a} = \frac{u''\left(G_{n2}\right)}{\left[-u''\left(G_{n2}\right) - q\frac{4}{\beta}u''\left(G_{n1}\right)\right]} > -1$$

Therefore

$$-\frac{1}{1-\frac{u''(G_{s2})}{a}\frac{u''(G_{n2})}{a}}-\frac{u''(G_{n2})}{a}<-1+1=0$$

Next, notice that

$$\Delta MU_s+\Delta MU_n=\beta\left[u'(Y-\mathbf{b}_{s2})+u'(Y-\mathbf{b}_{n2})-u'\left(Y-\frac{\mathbf{b}_{s2}+\mathbf{b}_{n2}}{2}\right)\right]$$

Clearly, if $\Delta=0$ then $\Delta MU_s+\Delta MU_n=\beta u'(Y-\mathbf{b}_{s2})>0$. Thus, by continuity, $\Delta MU_s+\Delta MU_n>0$ if Δ is small enough.[17] Therefore, for π close to zero, $\frac{\partial \mathbf{B}_2}{\partial \pi}\leqslant 0$ because all three terms in (30) are positive. □

A.3 Proof of Proposition 1

Assume first that the local governments expect that the central government will not make any transfers in period 1 and will mutualize debt in period 2 with probability $1-\pi$. The optimality condition of problem (10) and the envelope condition from problem (5) imply that debt issuance in period 0 satisfies (13) and the debt issuance in period 1 is $b_2^{\text{no-rules}}=\mathbf{b}_{i2}\left(b_1^{\text{no-rules}},\pi\right)$. We are going to denote the proposed equilibrium outcome with a superscript "no-rules."

We now study the incentives for the central government to implement positive transfers in period 1 on-path. First, fix some $\pi>0$. Clearly, for $\Delta=0$, the central government strictly prefers to not transfer due the reputational benefits because the inherited debt distribution is degenerate. By continuity, for Δ small but positive, it will also strictly prefer to implement zero transfers and enforce the constitution.

Next, fix some $\Delta>0$. We now show that even though the central government faces a non-degenerate distribution of debt $\left\{b_{i1}^{\text{no-rules}}\right\}$ in period 1, it does not have incentives to implement positive transfers if π is small enough. Define the difference between the value of enforcement if $\pi'=\pi$ and not for a central government that inherits debts $b_1^{\text{no-rules}}(\pi)=\left\{b_{i1}^{\text{no-rules}}\right\}$ as

$$\mathcal{W}(\pi)\equiv\omega^e\left(b_1^{\text{no-rules}}(\pi),\pi\right)-\omega^{ne}\left(b_1^{\text{no-rules}}(\pi)\right)$$

where since there are no fiscal rules we set $\psi=0$ in the definition of ω^e in (22). Note that for an equilibrium with enforcement to exist, it must be that $\mathcal{W}(\pi)\geqslant 0$. Since the utility and policy functions are continuous in π, $\mathcal{W}$ is continuous in π. Moreover $\mathcal{W}(0)=0$.

[17]One can prove the same result for arbitrary Δ if $u'''>0$.

Differentiating $\mathcal{W}$ we obtain:

$$\mathcal{W}'(\pi) = \sum_i \left(\left[\frac{\partial \omega^e \left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial b_{i1}} - \frac{\partial \omega^{ne} \left(b_1^{\text{no-rules}}(\pi)\right)}{\partial b_{i1}} \right] \frac{\partial b_{i1}^{\text{no-rules}}(\pi)}{\partial \pi} \right) + \frac{\partial \omega^e \left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial \pi}$$

Evaluating the expression above at $\pi = 0$, using that $\omega^{ne}(\cdot) = \omega^e(\cdot, \pi = 0)$ when $\psi = 0$ and so $\partial \omega^e \left(b_1^{\text{no-rules}}(0), 0\right) / \partial b_{1i} = \partial \omega^{ne} \left(b_1^{\text{no-rules}}(0)\right) / \partial b_{1i}$, we obtain

$$\mathcal{W}'(0) = \frac{\partial \omega^e \left(b_1^{\text{no-rules}}(0), 0\right)}{\partial \pi} > 0$$

as wanted. That ω^e is increasing in π for π close to zero is established in Lemma 5 part ii).

We are left to show that an individual government has no incentives to increase its debt and force the central government to make a transfer. Suppose local government i chooses $b_{i1} > b_1^{\text{no-rules}}$ to induce the central government to make a transfer to region i in period 1 with some positive probability. The value for the best deviation for such local government is:

$$V_i^{dev} = \max_{b_{i1}} u\left(Y_{i0} + q b_{i1}\right) + \beta \left[\pi + (1-\pi)\,\sigma\left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right] V_{i1}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi'\right) + \beta(1-\pi)\left[1 - \sigma\left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right] V_{i1}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, 0\right)$$

subject to

$$\omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) \leqslant \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right) \tag{31}$$

Let V_i be the value along the conjectured equilibrium and $\Delta V_i = V_i - V_i^{dev}$. At $\pi = 0$ Note that by construction, $b_1^{\text{no-rules}}$ solves (10) or

$$V_i = \max_{b_{i1}} u\left(Y + q b_{i1}\right) + \beta\pi\left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right)\right] + \beta(1-\pi)\left[u\left(Y - b_{i1} + q \mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)}{N}\right)\right]$$

Note that for $\pi = 0$, $\Delta V_i = 0$. Now suppose that $\pi > 0$. Notice that as N gets large, b_{i1} needs to increase in order to induce the central government to make a transfer. In particular, for any finite b_{i1}, as $N \to \infty$ then, eventually, $\omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) > \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right)$.

This is because

$$
\begin{aligned}
&\lim_{N\to\infty}\left(\omega^{e}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) - \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right) \\
&= u\left(Y - b_1^{\text{no-rules}} + q\mathbf{b}_{-i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), \pi\right)\right) \\
&- \left[u\left(Y - b_1^{\text{no-rules}} + q\mathbf{b}_{i2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_1^{\text{no-rules}}, b_{i1}\right), 0\right)\right)\right] \\
&> 0
\end{aligned}
$$

As a result, a necessary condition for $\omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right) \geqslant \omega^{e}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right)$ as $N \to \infty$ is that $b_{i1} \to \infty$ which violates feasibility. For each π there exists $N(\pi)$ such that for $N > N(\pi)$, the deviation is infeasible. And so for $N > \max_{\pi} N(\pi)$, the constructed outcome is an equilibrium outcome.

We are left to show that such an equilibrium is unique (among symmetric pure strategy equilibria). First, fix some $\Delta > 0$. Suppose there exists an interval $(0, \pi_1)$ such that for all $\pi \in (0, \pi_1)$, there exists an equilibrium in which the no-commitment type implements positive transfers with strictly positive probability. Then, it must be that $\mathcal{W}(\pi) \leqslant 0$. However, this contradicts our earlier argument that $\mathcal{W}(\pi) > 0$ for π sufficiently close to zero. As a result, an equilibrium in which $\sigma > 0$ cannot exist for π sufficiently small.

Next, fix some $\pi > 0$. We know that for $\Delta = 0$, in any symmetric equilibrium, $\mathcal{W}(\pi) > 0$. Therefore, by continuity this inequality will continue to hold for Δ sufficiently small by positive. As a result, an equilibrium in which $\sigma > 0$ cannot exist for Δ sufficiently small. Q.E.D.

A.4 Proof of Proposition 2

Consider first the problem a local government i that expects that i) other local governments are going to violate the fiscal rule, ii) the no-commitment type central government is not going to enforce the fiscal rule punishment in period 1. Consequently, local government i expects to learn the type of the central government in period 1. The problem for the local government at time 0 is then:

$$
\Omega(\pi) = \max_{b_{i1}} u\left(Y_{i0} + qb_{i1}\right) + \beta\pi V_{i1}\left(\left(b_1^{\text{rules}} + \psi, b_{i1} + \psi\right), 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)
$$

where $b_1^{\text{rules}} > \bar{b}$ is the debt chosen by the other local governments. The optimality condition is:

$$
qu'\left(Y_{i0} - qb_{i1}\right) = \beta\pi \frac{\partial V_{i1}\left(\left(b_1^{\text{rules}} + \psi, b_{i1} + \psi\right), 1\right)}{\partial b_{i1}} - \beta(1-\pi)\frac{\partial V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)}{\partial b_{i1}}
$$

and using the envelope conditions for $V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 1\right)$ and $V_{i1}\left(\left(b_1^{\text{rules}}, b_{i1}\right), 0\right)$ we obtain

$$\begin{aligned} qu'\left(Y_{i0}+qb_{i1}\right) = {} & \beta\pi u'\left(Y-\left(b_{i1}+\psi\right)+q\mathbf{b}_{i2}\left(b_1+\psi,1\right)\right) \\ & +\beta\left(1-\pi\right)u'\left(Y-b_{i1}+q\mathbf{b}_{i2}\left(\left(b_1^{\text{rules}}, b_{i1}\right),0\right)\right) \\ & +\beta^2\left(1-\pi\right)u'\left(Y-\frac{\sum_{j=1}^{N}\mathbf{b}_{j2}\left(\left(b_1^{\text{rules}}, b_{i1}\right),0\right)}{N}\right)\sum_{j=1,j\neq i}^{N}\frac{1}{N}\frac{\partial \mathbf{b}_{j2}\left(\left(b_1^{\text{rules}}, b_{i1}\right),0\right)}{\partial b_{i1}}, \end{aligned} \tag{32}$$

which is equation (14) in the text. Note that for Δ small enough, $b_{i1}^{\text{rules}} > \bar{b}$ for all i.

We now show that for N large enough and π small enough no individual local government has an incentive to deviate from b_{i1}^{rules} and choose $b_{i1} = \bar{b}$ to attain value

$$\begin{aligned} \bar{\Omega}\left(\pi\right) = {} & u\left(Y_{i0}+q\bar{b}\right)+\beta\left[\pi+\left(1-\pi\right)\sigma\left(\pi,\bar{b},b_{-i1}^{\text{rules}}\right)\right]V_{i1}\left(\bar{b},b_{-i1}^{\text{rules}},\pi'\right) \\ & +\beta\left(1-\pi\right)\left[1-\sigma\left(\pi,\bar{b},b_{-i1}^{\text{rules}}\right)\right]V_{i1}\left(\bar{b},b_{-i1}^{\text{rules}},0\right) \end{aligned}$$

First notice that as $N\to\infty$,

$$\omega^{e}\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right),1\right)-\omega^{ne}\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right)\right)\to\omega^{e}\left(b_{-i1}^{\text{rules}}\left(\pi\right),1\right)-\omega^{ne}\left(b_{-i1}^{\text{rules}}\left(\pi\right)\right)<0$$

since we assume that the central government does not enforce. Therefore there exists $\tilde{N}_1$ such that for $N \geqslant \tilde{N}_1$, $\sigma\left(\pi,\bar{b},b_{-i1}^{\text{rules}}\right)=0$. Next, we have that

$$\begin{aligned} \Omega\left(\pi\right)-\bar{\Omega}\left(\pi\right) = {} & \left[u\left(Y_{i0}+qb_{i1}^{\text{rules}}\left(\pi\right)\right)-u\left(Y_{i0}+q\bar{b}\right)\right] \\ & +\beta\pi\left[V_{i1}\left(\left(b_{i1}^{\text{rules}}\left(\pi\right)+\psi,b_{-i1}^{\text{rules}}\left(\pi\right)+\psi\right),1\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right)+\psi\right),1\right)\right] \\ & +\beta\left(1-\pi\right)\left[V_{i1}\left(\left(b_{i1}^{\text{rules}}\left(\pi\right),b_{-i1}^{\text{rules}}\left(\pi\right)\right),0\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right)\right),0\right)\right] \end{aligned}$$

Clearly, since $b_{i1}^{\text{rules}}\left(\pi\right)>\bar{b}$ we know that

$$\begin{aligned} \left[u\left(Y_{i0}+qb_{i1}^{\text{rules}}\left(\pi\right)\right)-u\left(Y_{i0}+q\bar{b}\right)\right]&>0, \\ \left[V_{i1}\left(\left(b_{i1}^{\text{rules}}\left(\pi\right)+\psi,b_{-i1}^{\text{rules}}\left(\pi\right)+\psi\right),1\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right)+\psi\right),1\right)\right]&<0. \end{aligned}$$

Notice that as $N\to\infty$, $\left[V_{i1}\left(\left(b_{i1}^{\text{rules}},b_{-i1}^{\text{rules}}\right),0\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{\text{rules}}\right),0\right)\right]\to 0$. Let $\tilde{N}_2^{*}$ be the threshold, such that for $N\geqslant\tilde{N}_2^{*}$,

$$\left[u\left(Y_{i0}+qb_1\right)-u\left(Y_{i0}+q\bar{b}\right)\right]+\beta\left[V_{i1}\left(\left(b_{i1}^{\text{rules}}\left(\pi\right),b_{-i1}^{\text{rules}}\left(\pi\right)\right),0\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{\text{rules}}\left(\pi\right)\right),0\right)\right]>0$$

Therefore, for $N\geqslant\tilde{N}_2$, there exists a $\tilde{\pi}_1$ such that for $\pi\leqslant\tilde{\pi}_1$, $\Omega\left(\pi\right)-\bar{\Omega}\left(\pi\right)>0$, and thus a local government has no incentives to satisfy the rule in the conjectured equilibrium.

The next step in establishing that the conjectured equilibrium exists is to show that the no-commitment type central government when faced with debt $b_1 = b_1^{\text{rules}}$ for all i prefers to not enforce the punishment ψ and reveal its type ($\pi' = 0$ thereafter) than enforce the punishment and have the posterior jump to one (as the local governments expect only the commitment type to enforce the fiscal rule). That is, it must be that

$$\omega^e \left(b_1^{\text{rules}} (\pi) + \psi, 1 \right) \leqslant \omega^{ne} \left(b_1^{\text{rules}} (\pi) \right)$$

which is true if π and β is sufficiently small. In particular, this is true for $\beta \leqslant \bar{\beta} (\pi, N)$ where $\bar{\beta} (\pi, N) \equiv$

$$\frac{\sum_{i=1}^{N} \frac{1}{N} \left[u \left(Y - b_{i1}^{\text{rules}} (\pi) + q\mathbf{b}_{i2} \left(b_1^{\text{rules}} (\pi), 0 \right) \right) - u \left(Y - \left(b_{i1}^{\text{rules}} (\pi) + \psi \right) + q\mathbf{b}_{i2} \left(b_1^{\text{rules}} (\pi) + \psi, 1 \right) \right) \right]}{u \left(Y - \frac{\sum \mathbf{b}_{i2} \left(b_1^{\text{rules}} (\pi) + \psi, 1 \right)}{N} \right) - u \left(Y - \frac{\sum \mathbf{b}_{i2} \left(b_{i1}^{\text{rules}} (\pi), 0 \right)}{N} \right)}.$$

The right side of the expression above implicitly defines the maximal discount factor under which it is optimal not to enforce . Therefore, if $\beta < \bar{\beta} (\pi, N)$, $\omega^e \left(b_1^{\text{rules}} (\pi) + \psi, 1 \right) \leqslant \omega^{ne} \left(b_1^{\text{rules}} (\pi) \right)$.

Next, we ask if an equilibrium with enforcement can exist for for π small. For this to be an equilibrium, it must be that if all other regions are following the rule, no single region has an incentive to deviate and violate it. The value of such a deviation is given by

$$\begin{aligned} V_i^{dev} (\pi) = \max_{b_{i1} > \bar{b}} \; & u (Y_{i0} + q b_{i1}) + \beta \left[\pi + (1 - \pi) \sigma \left(\pi, b_{i1}, \bar{b}_{-i} \right) \right] V_{i1} \left(b_{i1} + \psi, \bar{b}_{-i}, \pi' \right) \\ & + \beta (1 - \pi) \left[1 - \sigma \left(\pi, b_{i1}, \bar{b}_{-i} \right) \right] V_{i1} \left(b_{i1}, \bar{b}_{-i}, 0 \right) \end{aligned}$$

First, notice that

$$\lim_{\pi \to 0} \left(\omega^e \left(b_{i1}^{\text{rules}} + \psi, \bar{b}, \pi \right) - \omega^{ne} \left(b_{i1}^{\text{rules}} + \psi, \bar{b} \right) \right) < 0$$

so that $\lim_{\pi \to 0} \sigma \left(\pi, b_{i1}, \bar{b}_{-i} \right) = \sigma_0 < 1$. But then

$$\lim_{\pi \to 0} V_i^{dev} (\pi) = u (Y_{i0} + q b_{i1}) + \beta \sigma_0 V_{i1} \left(b_{i1} + \psi, \bar{b}_{-i}, 0 \right) + \beta [1 - \sigma_0] V_{i1} \left(b_{i1}, \bar{b}_{-i}, 0 \right)$$

where we used that $V_{i1} \left(b_{i1} + \psi, \bar{b}_{-i}, \pi' \right) = V_{i1} \left(b_{i1} + \psi, \bar{b}_{-i}, 0 \right)$ since

$$\lim_{\pi \to 0} \pi' = \lim_{\pi \to 0} \frac{\pi}{\pi + (1 - \pi) \sigma} = 0.$$

Next, recall from Lemma 1, that the value $V_{i1} \left(b_{i1} + \psi, \bar{b}_{-i}, 0 \right)$ depends on the average level of debt $\frac{1}{N}\left(b_{i1}+\psi\right)+\frac{(N-1)}{N}\bar{b}$. Therefore, as $N\to\infty$, $V_{i1}\left(b_{i1}+\psi,\bar{b}_{-i},0\right)\to V_{i1}\left(\bar{b},0\right)$ which implies that value of punishment for the deviating local government shrinks to zero. Therefore, this deviation is strictly profitable. And so there exists some $\tilde{N}_4$ such that for $N\geqslant\tilde{N}_4$ there exists $\tilde{\pi}_2$ such that for $\pi\geqslant\tilde{\pi}_2$, this deviation is strictly profitable. Now choose $N\geqslant\max\left\{\tilde{N}_1,\tilde{N}_2\tilde{N}_3,\tilde{N}_4\right\}$ and $\pi\leqslant\min\{\tilde{\pi}_1,\tilde{\pi}_2\}$. This proves the result. Q.E.D.Proof of Proposition 3

We first show that for π close to 1, there exists an equilibrium with enforcement. At $\pi=1$, the value for a local government of respecting the fiscal rule is $u\left(Y_{i0}+q\bar{b}\right)+\beta V_{i1}\left(\bar{b},\pi\right)$ while the value of violating is $\max_{b_i>\bar{b}} u\left(Y_{i0}+qb_i\right)+\beta V_{i1}\left(b_i+\psi,\bar{b}_{-i},\pi\right)$. That the latter is larger than the former follows directly from Assumption 2. By continuity, there exists some $\tilde{\pi}_1<1$ such that for $\pi\geqslant\tilde{\pi}_1$, the inequality continues to hold.

Next, we want show that there is an interval around $\pi=1$ for which the enforcement equilibrium is unique. For an equilibrium with non-enforcement $\left(b_1=b_1^{rules}\right)$ to exist, it must be that it is optimal for a local government to violate the fiscal rule rather than obeying the rule when all other local governments are violating the rule. That is, $\Omega\left(\pi\right)\geqslant\bar{\Omega}\left(\pi\right)$ where these objects were defined in the proof of Proposition 2. Note that

$$
\begin{aligned}
\bar{\Omega}\left(1\right) &= u\left(Y_{i0}+q\bar{b}\right)+\beta V_{i1}\left(\bar{b},1\right)\\
&> \max_{b_i>\bar{b}} u\left(Y_{i0}+qb_i\right)+\beta V_{i1}\left(b_i+\psi,\bar{b}_{-i},1\right)\\
&= \max_{b_i>\bar{b}} u\left(Y_{i0}+qb_i\right)+\beta V_{i1}\left(b_i+\psi,b_{-1}^{rules}+\psi,1\right)\\
&= \Omega\left(1\right)
\end{aligned}
$$

where the first line is the definition of $\bar{\Omega}\left(1\right)$, the second line follows from Assumption 2, the third line follows since the debt holdings of other regions is irrelevant if the central government is the commitment type for sure ($\pi=1$), and the last line is the definition of $\Omega\left(1\right)$. Hence, by continuity, if π is sufficiently close to 1, $\bar{\Omega}\left(\pi\right)>\Omega\left(\pi\right)$, and the local government i will prefer to deviate from b_{i1}^{rules} and not violate the fiscal rule. Therefore there exists some $\tilde{\pi}_2$ such that $\pi\geqslant\tilde{\pi}_2$, an equilibrium with non-enforcement cannot exist. Choose $\pi\geqslant\max\{\tilde{\pi}_1,\tilde{\pi}_2\}$ .Then there exists a unique equilibrium with enforcement. Q.E.D.

A.5 Proof of Proposition 4

Recall that $b_{i1}^{er}\left(\pi,\alpha\right)$ denotes the debt issued in period 0 when the local governments expect to learn the central government type in period 1 defined in (14) given $\alpha=\left(\bar{b},\psi\right)$; $b_{i1}^{lr}\left(\pi,\alpha\right)$ denotes the debt issued in period 0 when the local governments do not expect to learn the central government type in period 1 defined in (13) given $\alpha=\left(\bar{b},\psi\right)$.

Given the punishment ψ and $\beta \leqslant \bar{\beta}$ where $\bar{\beta}$ is defined in the proof of Proposition 2, we know that for π small enough the only two possible equilibria are i) the debt limit is never binding and ii) there is separation in period 1 and early resolution of uncertainty. Thus we can write $W_0^{c,sep}$ as

$$W_0^{c,sep} = \max_{\bar{b}} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}>\bar{b}}\right) + q\mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) \\ +\beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) \end{array} \right]$$

subject to $\bar{b} < b_{s1}^{\text{no-rules}}(\pi)$, where $b_{s1}^{\text{no-rules}}(\pi)$ is the equilibrium debt level in an economy with no rules given π for a local government in the South. That is, the commitment type is choosing a debt limit that is binding so that by Proposition 2 there is separation in period 1. (Note that the value is $W_0^{c,sep}$ is constant for all $\bar{b} < b_{s1}^{\text{no-rules}}(\pi)$.) Also, we can write $W_0^{c,pool}$ as the the value for the commitment type if it imposes no fiscal rules:

$$W_0^{c,pool} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - b_{i1}^{lr}(\pi,\alpha) + q\mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \\ +\beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \end{array} \right].$$

The commitment type will then impose a binding rule if and only if $W_0^{c,sep} \geqslant W_0^{c,pool}$. Let $\Gamma(\pi,\alpha) = W_0^{c,sep} - W_0^{c,pool}$. As $\pi \to 0$, $\Gamma(\pi,\alpha) \to$

$$\beta \sum_i \frac{1}{N} \left[u\left(Y - \psi \mathbb{I}_{b_{i1}>\bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right)\right] \\ -\beta \sum_i \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) + \beta u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]$$

since $b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha) = b_{i1}$. (From now on we use $b_{i1} = b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha)$.) Rearranging the expression above we obtain

$$\frac{\beta^2}{N} \sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right] \\ -\frac{\beta}{N} \sum_i \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi \mathbb{I}_{b_{i1}>\bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right)\right].$$

Note that both terms in square brackets are positive, thus we can define the cutoff β such that the expression above equals zero:

$$\underline{\beta}(\pi, N) \equiv \frac{\sum_i \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi\mathbb{I}_{b_{i1}>\bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right)\right]}{\sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]}$$

Then for $\beta < \underline{\beta}(\pi, N)$, $\Gamma(0, \psi) < 0$. Thus, for π small, $W_0^{c,sep} < W_0^{c,pool}$ and the unique constitution will feature no fiscal rules. Conversely, for $\beta > \underline{\beta}(\pi, N)$, $\Gamma(0, \psi) > 0$. Thus, for π small, $W_0^{c,sep} > W_0^{c,pool}$ and the unique constitution will feature fiscal rules.

To show that this is an equilibrium, we need to show that the no-commitment type does indeed not want to enforce the constitution in period 1 (and induce separation) for $\beta > \underline{\beta}$. We know from the proof of Proposition 2 that if $\beta < \bar{\beta}(\pi, N)$, where $\bar{\beta}(\pi, N) \equiv$

$$\frac{\sum_{i=1}^{N} \frac{1}{N}\left[u\left(Y - b_{i1}^{rules}(\pi) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi), 0\right)\right) - u\left(Y - \left(b_{i1}^{rules}(\pi) + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi) + \psi, 1\right)\right)\right]}{u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_1^{rules}(\pi) + \psi, 1\right)}{N}\right) - u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_{i1}^{rules}(\pi), 0\right)}{N}\right)},$$

then for π close to zero, the no-commitment will strictly prefer to not enforce the rule at $t = 1$. Thus we have our desired result for $\beta \in \left[\underline{\beta}(\pi, N), \bar{\beta}(\pi, N)\right]$. To show that this a well defined interval, we need to show that $\bar{\beta}(0, N) > \underline{\beta}(0, N)$. This is true if

$$0 > N\left[u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right)\right] - \left[Nu\left(Y - \frac{\sum \mathbf{b}_{i2}(b_1, 0)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]$$

For this to be true we need $\mathbf{b}_{s2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right) - \mathbf{b}_{n2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right) < \mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0)$. From the first order conditions for $\mathbf{b}_{i2}(b_1, 0)$ we have

$$u'\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) q = \frac{\beta}{N} u'\left(Y - \frac{\sum \mathbf{b}_{i2}(b_1, 0)}{N}\right)$$

This implies that

$$\mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0) = \frac{b_{s1} - b_{n1}}{q} \tag{33}$$

Next from the first order conditions for $\mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)$ we have

$$u'\left(Y - \psi\mathbb{I}_{b_{i1}>\bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right) q = \beta u'\left(Y - q\mathbf{b}_{i2}\left(b_1 + \psi\mathbb{I}_{b_{i1}>\bar{b}}, 1\right)\right)$$

Then, if the rule is not binding for the North:

$$\begin{aligned}&u'\left(Y-\psi-b_{s1}+qb_{s2}\right)-u'\left(Y-b_{n1}+qb_{n2}\right)\\&=\beta u'\left(Y-qb_{s2}\right)-\beta u'\left(Y-qb_{n2}\right)>0\end{aligned}$$

and so

$$\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1,1\right)<\frac{\psi+b_{s1}-b_{n1}}{q} \tag{34}$$

If instead the rule is binding for the North as well we have

$$\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1,1\right)<\frac{b_{s1}-b_{n1}}{q} \tag{35}$$

So from (33) and (34)-(35) it follows that for ψ small enough, $\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)<\mathbf{b}_{s2}\left(b_1,0\right)-\mathbf{b}_{n2}\left(b_1,0\right)$ and so $\bar{\beta}\left(0,N\right)>\underline{\beta}\left(0,N\right)$. Therefore, for β in this range and π small enough, we have an equilibrium in which $\psi>0$ and the rules not being enforced in period 1 by the no-commitment type.

Finally, we need to show that the no-commitment type will mimic the commitment type in period 0 and announce the same rule anticipating it will not enforce it in period 1. The value of choosing the same constitution as the commitment type in period 0 is given by

$$\begin{aligned}W_0^m\left(\pi,\alpha\right)&=\sum_i u\left(Y_{i0}+qb_{i1}^{er}\left(\pi,\alpha\right)\right)+\beta W_1^{er}\left(b_1^{er}\left(\pi,\alpha\right)\right)\\&=\sum_i \left[u\left(Y_{i0}+qb_{i1}^{er}\left(\pi,\alpha\right)\right)+\beta u\left(Y-\psi-b_{i1}^{er}\left(\pi\right)+q\mathbf{b}_{i2}\left(b_{i1}^{er}\left(\pi,\alpha\right),0\right)\right)\right.\\&\quad\left.+\beta^2u\left(Y-\frac{\sum_j\mathbf{b}_{j2}\left(b_{i1}^{er}\left(\pi,\alpha\right),0\right)}{N}\right)\right]\end{aligned}$$

while the value of not choosing a different constitution is $W_0^m\left(0,\alpha\right)$ because the local governments learn that they are facing the no-commitment type. We will establish that $\frac{\partial}{\partial\pi}W_0^m\left(\pi,\alpha\right)>0$, at $\pi=0$ which in turn implies that if π is close to 0, the no-commitment type will always find it optimal to mimic. Differentiating $W_0^m\left(\pi,\alpha\right)$ with respect to π and evaluating at $\pi=0$ yields

$$\begin{aligned}\frac{\partial}{\partial\pi}W_0^m\left(0,\alpha\right)=\sum_i&\left[u'\left(G_{i0}\right)q\frac{\partial b_{i1}^{er}\left(0\right)}{\partial\pi}-\beta u\left(G_{i1}\right)\frac{\partial b_{i1}^{er}\left(0\right)}{\partial\pi}+\right.\\&\left.+u'\left(G_{i1}\right)q\frac{\partial\mathbf{b}_{i2}}{\partial b_{j1}}\frac{\partial b_{j1}^{er}\left(0\right)}{\partial\pi}-\frac{\beta^2}{N}u'\left(G_{i2}\right)\frac{\partial\mathbf{B}_2}{\partial b_{j1}}\frac{\partial b_{j1}^{er}\left(0\right)}{\partial\pi}\right]\end{aligned}$$

Recall the first order conditions for the local government in periods 1 and 2

$$u'(G_{i0})\,q = \beta u'(G_{i1}) + \frac{\beta^2}{N}u'(G_{i2})\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}}{\partial b_{i1}}$$
$$u'(G_{i1})\,q = \frac{\beta}{N}u'(G_{i2})$$

Substituting these into the previous equation yields

$$\begin{aligned}\frac{\partial}{\partial \pi}W_0^m(0,\alpha) =& \sum_i u'(G_{i1})\,q\frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}}\frac{\partial b_{-i1}^{er}(0)}{\partial \pi}\\ =& u(G_{i1})\,q\frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}}\frac{\partial B_1^{er}(0)}{\partial \pi} > 0\end{aligned}$$

since at $\pi = 0$, $\frac{\partial}{\partial b_{N1}}\mathbf{b}_{S2}(b_1,0) = \frac{\partial}{\partial b_{S1}}\mathbf{b}_{N2}(b_1,0) < 0$ and $\partial B_1^{er}(0)/\partial\pi < 0$. Q.E.D.

B Appendix: Data underlying Figure 2

We use two datasets:

1. Dataset used in Kotia and Lledó (2016). They construct an index for the strength of subnational fiscal rules using a database from the European Commission (EC), measuring the strength of all the fiscal rules present in each EU country. The EC dataset includes all types of numerical fiscal rules—budget balance rules, debt rules, expenditure rules, and revenue rules—covering different levels of government—central, regional, and local—in force since 1990 across EU countries. They then weight the scores for the components applicable at the subnational level: regional and local. See Appendix B in Kotia and Lledó (2016) for details about the construction of the index.

   The dataset also contains information on

   (a) subnational primary balances—based on authors' own consolidation of total revenue and expenditures across local and (when applicable) state or regional governments using non-consolidated fiscal data from Eurostat;

   (b) output gap from the World Economic Outlook;

   (c) population above 65 years of age from the World Development Indicators;

   (d) unemployment from the World Economic Outlook;

(e) legislative election dummy taking the value of 1 if a national legislative election was held in that year, and zero otherwise, from the Database for Political Institutions (DPI).

2. World Bank's Worldwide Governance Indicators (WGI) data. This dataset consists of data on the quality of governance provided by a large number of enterprise, citizen, and expert survey respondents in industrial and developing countries. The WGI consists of aggregate indicators of six broad dimensions of governance: (i) Voice and Accountability, (ii) Political Stability and Absence of Violence/Terrorism, (iii) Government Effectiveness, (iv) Regulatory Quality, (v) Rule of Law, and (vi) Control of Corruption. The governance indicator ranges from around -2.5 to 2.5, with higher values implying better outcomes. The data on government efficiency are biannual from 1996 until 2002 and then annual. We use linear interpolation to add observations in 1997, 1999, and 2001. Our preferred measure of reputation, π, is Government Effectiveness.

In figure 3 we plot the raw data and look at the changes in deficits for contemporaneous changes in fiscal rules.

In the bottom panels of Figure 2 and 3, we report the change in residuals after controlling for an estimated fiscal reaction function. In particular, we run the following regression

$$\mathtt{deficit}_{it} = \beta X_{it} + f_i + \varepsilon_{it},$$

where $\mathtt{deficit}_{it}$ is the primary deficit; X_{it} is a vector of control variables including output gap, population above 65 years of age, unemployment, legislative election dummy, and inflation; f_i is a country fixed effect; and ε_{it} is the residual from the regression. The figures plot the change in the average residual across two consecutive fiscal rule regimes.

Figure 3: Scatter plot of changes in primary deficits to changes in fiscal rule strength