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FISCAL POLICIES AND THE STOCK MARKET: INTERNATIONAL DIMENSIONS

Assaf Razin

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ABSTRACT

The dynamic effects of fiscal policies on the real equilibrium have been the subject of a large body of recent research, emphasizing the intertemporal dimensions of tax and spending policies both in closed and open-economy contexts. The analysis in this paper extends the intertemporal analysis which was conducted under full certainty to uncertain enviroments. Specifically the paper uses a two-country stochastic general-equilibrium model of the world economy to address issues concerning the effects of government tax and spending policies on private sector consumption asset portfolios and stock market valuations. The key result of the paper is that the consequences of expected future policies and the charcateristics of their international transmission depend critically on the precise variability of these policies across states of nature. The effects of current policies on consumption savings and stock market prices are shown, however to conform closely to the predictions of the corresponding certainty intertemporal model.

Assaf Razin
Tel Aviv University
Ramat Aviv 69978
Tel Aviv, ISRAEL
972-03-420-733

This paper uses a two-country stochastic general-equilibrium model of the world economy to address issues concerning the effects of fiscal policies on the international allocation of consumption, saving and risk. Specifically, the paper deals with the effects of government spending and tax policies on private-sector consumption spending and asset portfolios. Consequently the analysis sheds light on the effects of these policies on the stock-market valuation of assets.

The dynamic effects of fiscal policies on the real equilibrium has been a subject of a large body of recent research. Examples of such research in a closed-economy framework are Barro (1981), Blanchard (1985) and Mankiw (1987), and in an open-economy framework are Buiter (1986) and Frenkel and Razin (1986, 1987a,b). This line of research, emphasizing intertemporal considerations, is confined to an environment of full certainty in which the private sector behavior is governed by perfect foresight. The open-economy research focused mainly on the international and intertemporal allocation of consumption and investment and their manifestations in trade imbalances. Consequently in this framework the terms of trade and the world rate of interest are the main channel through which fiscal policies in one country are transmitted to the rest of the world.

This paper extends this research to a stochastic environment in which the main channel of the international transmission of fiscal policies is the stock market, assumed to be freely accessible to both foreign and domestic residents.

Our interest in the effects of fiscal policies on the equilibrium prices of internationally traded equities of the home and foreign industries stems in part from the close connection existing between equity prices (the analogues of Tobin Qs) and levels of investment. Although investment in physical capital is not treated explicitly the analysis in this papaer provides bench-mark results on equity prices that can be used to assess the effects of fiscal policy on domestic and foreign investment levels. The key result of the paper is that the consequences of expected future government spending policies and the characterisitics of their international transmission depend critically on the precise variability of these policies across states of nature.

To analyze the effects of policies in such an environment it is necessary to extend the intertemporal model which is usefully employed in the deterministic framework, to allow for uncertainty and for the existence of risk-sharing securities that are competitively traded in the stock market. The stock market model used here is developed in Diamond (1967), Helpman and Razin (1978) and Lucas (1978). A recent application to international trade in assets is contained in Svensson (1987).

The paper is organized in the following manner. In Section I we briefly set out the stock-market model used in the remainder of the paper. Section II considers the effects of current and future government spending policies on asset prices and on levels of consumption. Section III considers the issue of government debt neutrality and Section IV considers the effect of government spending on the international allocation of risk and on the relative price of the domestic security in terms of the foreign security. Finally, Section V contains concluding remarks. Many of the derivations are contained in the appendix.

I. An International Stock Market Model

Consider a two-period model of a nonmonetary open economy producing and consuming one aggregate tradable good. The economy is assumed to be endowed with the sequence of endowments $\theta_0 Y_0$ and $\theta_1(\alpha)Y_1$ where the subscripts zero and one designate the corresponding periods indexed by the state of nature α . There are S possible states of nature, $\theta_0 Y_0$ and Y_1 are predetermined and $\theta_1(\alpha)$ corresponds to the state of technology in the home industries, $\alpha = 1,2,...S$. Similarly to the framework used in Helpman and Razin (1978) 1/ let q be the price of a unit of real equity in the representative home firm in terms of units of the aggregate consumption good. The home firms produce Y_1 units of these equities with a (gross) stock market value being equal to $q \cdot Y_1$. Let an asterisk denote foreign industries. Accordingly, foreign firms produce the sequence $\theta_0^* Y_0^*$ and $\theta_1^*(\alpha)Y_1^*$ in period zero and one, respectively, the price of a unit of foreign real equity is denoted by q* and the stock-market value of foreign industries is $q^* \cdot Y_1^*$.

In this two-period model the domestic private-sector's budget constraints are:

(1) $C_{H0} + qz_{H1} + q^* z_{H1}^* = qz_{H0} + q^* z_{H0}^* + \theta_0 z_{H0} + \theta_0^* z_{H0}^* - G_0$

(2) $C_{H1}(\alpha) = \theta_1(\alpha)z_{H1} + \theta_1^*(\alpha)z_{H1}^* - G_1(\alpha).$

1/ The present model is a two-period one-good version of the model which was developed in Helpman and Razin (1978, ch. 11). Labor income is contained in the initial endowment $\Theta_0 z_{H0}$. Bonds will be introduced later in Section IV.

In equations (1)-(2), C_{H0} and $C_{H1}(\alpha)$ denote first and (state-α) second-period consumption and z_{Ht} and z^*_{Ht} denote the shares of stocks in home and foreign firms in period t (t=0,1). The productivity parameters θ_t and θ^*_t indicate the amount of dividends, per share, paid by domestic and foreign firms respectively, and G_t denotes lump-sum taxes in period t (t=0,1). In equation (1) disposable income is equal to current dividends from the historically given portfolio (z_{H0}, z^*_{H0}), and the market value of this portfolio net of taxes, G_0. Income is spent on consumption C_{H0}, and on the new portfolio (z_{H1}, z^*_{H1}). Second-period disposable income, in equation (2), obtained from the dividends on the portfolio (net of taxes), includes no resale value of stocks since this period is the last period. For this reason there is also no acquisition of a new stocks in this period.

Let $u(C_{Ht})$ be a concave von Neumann-Morgenstern utility function of the representative individual and let β denote his subjective discount factor. The expected utility is

(3) $U = u(C_{H0}) + \beta Eu[C_{H1}(\alpha)]$

where E is the expectation operator.

The consumption-portfolio choice problem of the individual is to maximize the expected utility (equation (3)) given (common) beliefs about the probability distribution of states of nature and subject to the budget constraints in equations (1) and (2). The resulting consumption and asset demands are functions of security prices, q and q^*, second-period taxes

G_1, and first period wealth, Wo:

$$(4) \quad C_{H0} = C_{H0}[q,q^*,G_1;\ W_0] ,$$

$$(5) \quad z_{H1} = z_{H1}[q,\ q^*,G_1;\ W_0] ,$$

$$(6) \quad z^*_{H1} = z^*_{H1}[q,q^*,G_1;\ W_0] ,$$

where the individual wealth defined by

$$(7) \quad W_0 = (q+\theta_0)z_{H0} + (q^*+\theta^*_0)z^*_{H0} - G_0 ,$$

is a function of q, q^* and G_0. The foreign private sector, indicated by the subscript F, solves essentially a similar problem. The model is general enough so that the utility functions u and u* can be different with respect to all parameters except that the probability assessments are identical across countries. The model is closed by the world market-clearing conditions for current-period consumption and equities of firms located in each of the countries. It is assumed in this section that the government runs a balanced budget so that tax revenue (G_0) is equal to government spendings. The world equilibrium is:

$$(8) \quad C_{H0}[q,q^*,G_1,W_0] + C_{F0}[q,q^*,W^*_0] = \theta_0 Y_0 + \theta^*_0 Y^*_0 - G_0$$

$$(9) \quad z_{H1}[q,q^*,G_1,W_0] + z^*_{F1}[q,q^*,W^*_0] = Y_1$$

$$(10) \quad z^*_{H1}[q,q^*,G_1,W_0] + z^*_{F1}[q,q^*,W^*_0] = Y^*_1$$

By Walras' law we can use only two equilibrium conditions. Accordingly, we choose to omit the current-consumption market-clearing condition, equation (8), from the analysis. The remaining two conditions in equations (9) and (10) are used to solve for the market-clearing prices q and q^*.

The nature of the world equilibrium, characterized by free financial capital movements evaluated at the steady state is best understood with the aid of Figure 1. The upward-sloping schedule ZZ describes different combinations of the security prices q and q^* which clear the domestic-asset market. Likewise, the upward sloping schedule Z^*Z^* describes different combinations of prices q and q^* which clear the foreign-security market. In the stationary state the portfolio composition does not change over time. Accordingly, $z_{H1} = z_{H0}$, $z^*_{H1} = z^*_{H0}$, $z_{F1} = z_{F0}$ and $z^*_{F1} = z^*_{F0}$. Around such an equilibrium the ZZ schedule is steeper than the Z^*Z^* schedule. To verify we differentiate equations (9)-(10) by using the definition of wealth in equation (7). Rearranging and making use of the Slutsky decomposition for asset demands the analysis yields

$$(11)\quad \frac{d\log q^*}{d\log q} = \frac{[z^s_{H1q} - (z_{H1}-z_{H0})z_{H1W}] + [z^s_{F1q} - (z_{F1}-z_{F0})z_{F1W}]}{[z^s_{H1q^*} - (z^*_{H1}-z^*_{H0})z_{H1W}] + [z^s_{F1q^*} - (z^*_{F1}-z^*_{F0})z_{F1W}]} \cdot \frac{q^*}{q}$$

along the ZZ schedule, and

$$(12)\quad \frac{d\log q^*}{d\log q} = \frac{[z^{*s}_{H1q} - (z_{H1}-z_{H0})z^*_{H1W}) + (z^{*s}_{F1q} - (z_{F1}-z_{F0})z^*_{F1W}]}{[z^{*s}_{H1q^*} - (z^*_{H1}-z^*_{H0})z^*_{H1W}] + [z^{*s}_{F1q^*} - (z^*_{F1}-z^*_{F0})z^*_{F1W}]} \cdot \frac{q^*}{q}$$

along the Z*Z* schedule.

A subscript denotes partial derivatives (e.g., $\partial z_{H1}/\partial q \equiv z_{H1q}$) and a superscript s indicates that the relevant price effect applies to the <u>compensated</u> asset demand. In Appendix I we derive the following conditions pertaining to compensated asset demands:

$$(13)\quad z^s_{j1q} < 0 \,, \quad z^s_{j1q^*} = z^{*s}_{j1q} > 0 \,, \quad z^{*s}_{j1q^*} < 0 \qquad j = H,F$$

$$(14)\quad q^* z^s_{j1q^*} < - q z^s_{j1q} \,, \quad q z^{*s}_{j1q} < - q^* z^{*s}_{j1q^*}, \qquad j = H,F \,.$$

This means that with an intertemporally separable expected utility function assets z and z^* are net substitutes and the (negative) own-price elasticity is larger (in absolute value) than the (positive) cross-price elasticity. Combining the sign conditions in equations (13) and (14) with the expressions in equations (11) and (12) it is evident that the ZZ and Z^*Z^* schedules are upward sloping and that the former is steeper than the latter around a steady state. In other words around the steady state individuals are neither net buyers nor are they net sellers in the security markets. Accordingly, security-price changes generate no real income effects on asset demands, and the characteristics of equlibrium are governed by asset substitutions.

Consider the initial equilibrium point A in Figure 1. A rise in the domestic-security price, q, as indicated by the movement from point A to point B, generates an excess supply of domestic securities and since assets are pure substitutes the same price change generates also an excess demand for foreign securities. To restore equilibrium in the market for foreign securities, the foreign security price, q, rises to point B. Similarly to clear the domestic security market the price q^* is raised to point D. The reason that the latter rise in q^* is more pronounced than the former is that, as indicated by the demand condition in (14), the effect of this change on the demand for the domestic security is weaker than the corresponding effect on the demand for the foreign security. 1/

1/ These considerations need not hold in the presence of real-income effects of price changes which prevail outside of the steady state.

II. Government Spending and the World Stock Market

Government spending alters the amount of resources available to the private sector and thereby disturbs the world equilibrium in goods and assets. As a result asset prices must change to restore market clearing. In the presence of uncertainty a distinction between transitory and and permanent spending familiar from the analysis of government spending a non-stochastic environment (see Frenkel and Razin 1987a). To isolate the effects of government spending on the world stock market equilibrium we consider in this section only balanced-budget spending policies.

1. A Current Transitory Rise in Government Spending

Consider, first, the effect of a transitory rise in the home country's current government spending. The rise in spending which is associated with a corresponding tax hike creates an excess supply of assets. This excess supply is eliminated by a fall in the prices of both the domestic and the foreign securities. This analysis is illustrated in Figure 2. In Figure 2 the initial (stationary state) equilibrium obtains at point A at which the domestic-security price is q and the foreign-security price is q^*. A transitory balanced-budget rise in current government spending creates an excess supply in domestic securities and necessitates a corresponding rise in private sector demand. As implied by equation (3) the ZZ schedule shifts to the left to Z'Z'. The same rise in government spending creates also an excess supply in foreign securities and necessitates a corresponding increase in demand that is reflected by a downward shift of the Z^*Z^* schedule to $Z^{*\prime}Z^{*\prime}$. This result conforms with the one obtained in the nonstochastic analysis (see Frenkel and Razin [1987b] ch. 7) in which a transitory rise in government spending raises the equilibrium world rate of interest. Analogously, in the present case expected yields on both assets rise.

2. An Expected Rise in Future Government Spending

Consider now the effect of an expected transitory rise in the home country's future government spending. Suppose that the magnitude of this rise is known with certainty in the present period so that the rise is equal in all states of nature. The positive effect of the rise in G_1 (which lowers future disposable income in all states of nature) on the demand for assets is shown in Figure 3 in which the indifference curve UU describes the different combinations of savings and consumption that leave the level of expected utility unchanged. The budget line with a slope equal to 1 intersects the horizontal axis at point W, which is equal to the level of wealth, and point A describes the initial demand for consumption and savings. As shown in Appendix II a rise in G_1 switches the indifference curve from UU to U'U' and the asset demand point from A to B. As a result consumption falls while savings rise.

In other words, through the consumption-smoothing mechanism the fall in future disposable income enhances current-period savings. This result also conforms to the deterministic analysis in Frenkel and Razin (1987b). Even though the demand for assets as a group rises we cannot ascertain that the demand for each and every asset necessarily rises. If however the productivity parameters Θ and Θ^* are strongly positively correlated, it is likely that demand for every asset does increase. 1/ Another case in which the result holds is when the joint distribution of the domestic and foreign productivity shocks is symmetric so that (ex ante) the assets are substitutes.

The effect of the rise in future government spending on the world equilibrium under the assumption that assets are gross substitutes and demand for both of them increases following the future rise in government spending is illustrated in Figure 4 in which the initial equilibrium is indicated by point A. The rise in the asset demands consequent on the increase in government spending requires a rightward shift in the ZZ schedule and an upward shift in the Z*Z* schedule. As a result the equilibrium point shifts from A to C and the security prices rise from (q, q^*) to $(q', q^{*\prime})$. Similarly to the nonstochastic analysis in which an expected future increase in government spending lowers the world rate of interest, in the stochastic case average yields on both assets fall.

III. Government Spending Financed by a Stock Purchase

The introduction of state-dependent government spendings may alter the nature of equilibrium since the implied change in the time pattern and the state distribution of private-sector disposable incomes may affect significantly the characteristics of the demand for assets.

Consider, for concreteness, a policy in which the government makes a purchase of G units of the domestic security financed by current-period taxes. The dividends to be obtained from the government security holdings

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1/ See the extreme case of perfect correlation discussed in Appendix III.

are used as the government revenue which covers its (state-dependent) spending in the future period. This policy entails a current-period fall in the domestic private-sector wealth and a corresponding (state-dependent) fall in the world resources available for the domestic and foreign private-sector consumption in the future.

The policy generates an excess demand for the domestic security which is equal to $(1-z_{H1W})$ for each unit increase in government spending. At the same time, the policy also generates an excess supply for the foreign security which is equal to z^*_{H1W} per unit increase in government spending. In terms of the diagram in Figure 1, the ZZ schedule is displaced to the right while the Z*Z* schedule is downwardly displaced. A priori it is not possible to determine whether equilibrium stock prices rise or fall.

To demonstrate some specific effects of this policy on the equilibrium security prices consider the following example. Let the utility function be quadratic, $u(C_H) = \alpha C_H - \frac{1}{2} C_H^2$. As usual the quadratic utility is restricted to a range of consumption spendings in which $C_H < \alpha$. A similar utility function applies also to foreigners. The domestic and foreign measures of absolute risk aversion are $1/(\alpha - C_H)$ and $1/(\alpha^* - C_F)$ respectively. With this specification maximizing the domestic expected utility in equation (3) subject to the budget constraints in equations (1) and (2) yields

(15) $\beta[\alpha - (\sigma_{\theta\theta} + 1)z_{F1} - (\sigma_{\theta\theta^*} + 1)z^*_{H1} - G_1] = (\alpha - C_{H0})q$

(16) $\beta[\alpha - (\sigma_{\theta\theta^*} + 1)z_{H1} - (\sigma_{\theta^*\theta^*} + 1)z^*_{H1} - G_1] = (\alpha - C_{H0})q^*$.

where σ_{ij} denotes the covariance between i and j ($i=\theta,\theta^*$; $j=\theta,\theta^*$).

Similarly, the first-order conditions for the foreign-country optimization problem are:

(17) $\beta[\alpha^* - (\sigma_{\theta\theta} + 1)z_{F1} - (\sigma_{\theta\theta^*} + 1)z^*_{F1}] = (\alpha^* - C_{F0})q$

(18) $\beta[\alpha^* - (\sigma_{\theta\theta^*} + 1)z_{F1} - (\sigma_{\theta^*\theta^*} + 1)z^*_{F1}] = (\alpha^* - C_{F0})q^*$

Adding equation (15) to equation (17) and equation (16) to equation (18) and substituting the market-clearing conditions

(19) $C_{H0} + C_{F0} + G_0 = Y_0 + Y^*_0,\ z_{H1} + z_{F1} = Y_1 - G,\ z^*_{H1} + z^*_{F1} = Y^*_1$

yields the following expressions for the market-clearing prices.

$$(20)\ q = \beta \frac{\bar{\alpha} - (\sigma_{\theta\theta} + 1)Y_1 - (\sigma_{\theta\theta^*} + 1)Y^*_1 + G_1 + \sigma_{\theta\theta}G}{\bar{\alpha} - \bar{Y}_0 + G_0}$$

$$(21)\ q^* = \beta \frac{\bar{\alpha} - (\sigma_{\theta\theta^*} + 1)Y_1 - (\sigma_{\theta^*\theta^*} + 1)Y^*_1 + G_1 + \sigma_{\theta\theta^*}G}{\bar{\alpha} - \bar{Y}_0 + G_0}$$

where $\bar{\alpha} = \alpha + \alpha^*$ and $\bar{Y}_0 = Y_0 + Y^*_0$.

Evidently, as already stated in the preceding sections, a current-period transitory rise in government spending (a rise in G_0) lowers both the domestic and the foreign security prices while a future state-independent rise in government spending (a rise in G_1) raises these equlibrium prices. It is also evident from equations (20)-(21) that the expansionary fiscal policy charaterized by the government purchase of domestic securities (a rise in G) while raising the domestic-security price affect the foreign security price positively or negatively depending on the covariance between the domestic and the foreign productivity shocks. A positive correlation between the productivity shocks implies that the increase in domestic government spending raises foreign security price and conversely if the shocks are negatively correlated.

IV. The Neutrality of Government Finance

The preceding section demonstrates the significance of the means of government finance for the world equilibrium in some circumstances. The "claim"of the domestic private sector on domestic taxes can be viewed as a negative asset. Whether tax shift policies are relevant depends on whether the induced distribution of "returns" on this asset is already spanned by the equilibrium returns on the prevailing assets. A special case is the one in which taxes are state independent and a safe bond is traded in the world market. Consequently the neutrality of government debt emerges as in the certainty model. Accordingly, the budget constraints in equations (1) - (2) are modified as follows.

$$(22)\ C_{H0} + b_0 + qz_{H0} + q^*z_{H1} = (q + \theta_0)z_{H0} + (q^* + \theta_0^*)z_{H0}^* + R_{-1}b_{-1} - T_0$$

$$(23)\ C_{H1}(\alpha) = \theta_1(\alpha)z_{H1} + \theta_1^*(\alpha)z_{H1}^* + R_0b_0 - T_1$$

where b_t denotes a one-period fixed-return bond in period t (t=0,-1), R_0 denotes the interest factor (one plus the rate of interest) and T_t denotes lump-sum taxes in period t (t=0,1). Equations (22)-(23) are consolidated as usual into a single budget constraint (in terms of future value) to yield

$$(24)\ R_0C_{H0} + C_{H1}(\alpha) + R_0(qz_{H0} + q^*z_{H0}^*) = \overline{W} - (R_0T_0 + T_1)$$

where

$$\overline{W} = R_0(q + \theta_0)z_{H0} + R_0(q^* + \theta_0^*)z_{H0}^* + R_0R_{-1}b_{-1}\ .$$

It is clear that tax-shift policies which maintain the present value of taxes ($T_0 + T_1/R_0$) do not impact on the private-sector consumption possibility set. Accordingly government debt is neutral.

Obviously the neutrality of government finance need not require the existence of a safe bond. Consider the case in which only stocks are internationally traded. In this case the analogue of the consolidated budget constraint in equation (24) is

$$(24a)\quad \frac{1}{q} C_{HO} + C_{HI}(\alpha) = \Theta_1(\alpha)\ \left[z_{HO} + \frac{q^*}{q} z^*_{HO} + \frac{\Theta_0}{q}\ z_{HO} + \frac{\Theta^*_0}{q}\ z^*_{HO}\right] - \left[\frac{\Theta_1(\alpha)}{q}\ T_1 + T_0\right]$$

Now, if T_1 is state dependent any tax shift policy that maintains $\Theta_1(\alpha)T_1(\alpha) = -qT_0$ constant is neutral. Indeed, under such circumstances the return on the (negative) tax asset are proportional to the returns on the domestic security, state by state, and no new security is introduced by the government financial activities. Evidently, state independent future taxes are not neutral under these circumstances.

V. International Allocation of Risk and the Relative Security Price

To highlight the pure effect of government spending on the international allocation of risk and on the relative price of the domestic security we abstract in this section from intertemporal considerations. The minimal model for our purpose is the one with two states of nature and no initial consumption. This extreme case in which markets are complete yields sharp predictions for the effects of government spendings on stock prices.

We henceforth suppress period subscripts. The maximization of the expected utility $\pi(0)u(C_H(0)) + \pi(1)u(C_H(1))$ (where $\pi(\alpha)$ is the probability of state α (α=0,1)) subject to equation (2) yields

$$(25)\ \frac{\pi(0)u'(0)\theta^*(0) + \pi(1)u'(1)\theta^*(1)}{\pi(0)u'(0)\theta(0) + \pi(1)u'(1)\theta(1)} = \tilde{q}^*$$

where $\tilde{q}^*$ is the relative price of the foreign security in terms of the domestic security. Denote the price of the state one contingent good in terms of the state zero contigent good by p. In an equilibrium the marginal rate of substitution between goods consumed in the two states is equal to the price. That is,

$$(26)\ \frac{\pi(1)u'(1)}{\pi(0)u'(0)} = p\ .$$

Equations (25) and (26) imply that

$$(27)\ \frac{\theta^*(0) + p\theta^*(1)}{\theta(0) + p\theta(1)} = \tilde{q}^*$$

Similar conditions apply to the foreign country since at equilibrium the relative price of the foreign security, and thereby also the relative price of goods consumed in state one, are equalized.

The equilibrium in the world economy is portrayed by the Edgeworth box in Figure 5. In this Figure the horizontal axis measures world output in state zero $(\theta(0)Y + \theta^*(0)Y^*)$ and the vertical axis measures the corresponding quantity in state one $(\theta(1)Y + \theta^*(1)Y^*)$. The international distribution of world claims to outputs is specified by point E. As usual, quantities pertaining to the home country are measured from point 0_H as an origin and quantities pertaining to the foreign country are measured by point 0_F as an origin. The equilibrium obtains at point A, where the slope of the domestic and the foreign indifference curves are equal to the equilibrium price of goods contingent on state one, p. Using equation (27) we can derive also the equilibrium relative price of the foreign security $\tilde{q}^*$.

Consider now the effect of a rise in the home country's government spending at the amount G(0) in state zero and G(1) in state one. As a result the size of the box (corresponding to the world output net of government spending) diminishes. Accordingly the length of the horizontal axis is reduced by G(0) and the length of the vertical axis is reduced by G(1). At the prevailing relative price p foreign demand remains unchanged as represented in Figure 5 by point B. This point is located on the foreign consumption expansion locus displaced to the new origin $0_F'$. By construction the parallel line segments $0_F'B$ and 0_FA are of equal length. Analogously, as long as the initial p remains unchanged the domestic consumption expansion locus also remains unchanged but in view of the fall of income the new reduced level of desired consumption is represented by B' instead of A. Observe that the fall in the domestic budget line measured by the line segment AB is equal to the shrinkage of the size of the world output box measured by the line segment, $0_F'0_F$. At the prevailing price there is an excess demand for state one goods and an excess supply for state zero goods indicated by the line segment BB'. Thus the relative price, p, must rise to restore equilibrium. From equation (27) it is evident that whether or not the foreign security price rises depends on whether the domestic productivity ratio, $\theta(1)/\theta(0)$, exceeds or falls short of the foreign productivity ratio $\theta*(1)/\theta*(0)$.

Although the diagram in Figure 5 is drawn for homothetic preferences with expansion schedules drawn as rays from the origin, the reader can verify that the transfer-problem criterion derived above is more general. Analogously to Frenkel and Razin (1987b) the basic criterion determining the effect of a rise in government spending on the relative price of goods consumed in state one involves a comparison between the government marginal propensity to spend in state one, indicated by the ratio MPG(1)/MPG(0) and the corrresponding ratio of the domestic private-sector propensities to consume indicated by MPC(1)/MPC(0), where MP denotes a marginal propensity. If the government propensity exceeds the private sector propensity to consume in state one the rise in government spending and the corresponding fall in the disposable income of the domestic private sector create an excess demand for state one goods and an excess supply of state zero goods. As a result p rises. If, on the other hand, the government propensity falls short of the corresponding private sector propensity the equilibrium price p must fall.

Given the induced change in p, to determine the effect of the rise in government spending on the relative price of the foreign security one should compare the domestic productivity-ratio $\theta(1)/\theta(0)$ to the corresponding foreign ratio $\theta*(1)/\theta*(0)$. If the domestic economy is relatively more productive in state one so that $\Theta(1)/\Theta(0)$ exceeds $\Theta^*(1)/\Theta^*(0)$ the domestic security price, q, rises relative to the foreign security price, q*, when state one contingent good price, p, rises and vice versa.

VI. Concluding Remarks

This paper extends the analysis of the effects of fiscal policies on the world equilibrium from the certain to stochastic environments. In such environments the key channel through which the effects of policies are transmitted internationally is the world stock market consisting of international trade in domestic and foreign equities.

As in the more standard intertemporal analysis of the issue the analysis in this paper distinguishes between temporary and permanent policies. But the unique feature of the present analysis is the precise specification of the variability of future government spending policies across states of nature, a key element in understanding the effects of expected future policies on the world economy.

The paper demonstrates that the effects of a current transitory rise in government spending on equilibrium consumption levels and on equilibrium mean returns on equities conform to the results obtained for the case of no uncertainty: domestic and foreign consumption fall while the mean equilibrium return on both domestic and foreign equity rises. In the presence of stochastic government spending policies the deterministic analysis also serves as a general guide for predicting the induced changes in current consumption and overall savings. But it provides no guide for understanding the effects of the policies on the composition of equilibrium portfolios and on the equilibrium prices of equities. The analysis shows that if the marginal propensity of the government to spend in the (worldwide) low productivity state exceeds the corresponding domestic private-sector propensity the equilibrium low productivity state-contingent price rises. Consequently, the domestic equity price rises relative to the foreign equity price if the domestic economy is relatively more productive in this state in comparison to the foreign economy and vice versa. Whether equity prices in terms of consumption rise or fall depends, among others, on the means of goverment finance and on the statistical correlation between asset returns.

Key Characteristics of Asset Demands

In this appendix we derive the key characteristics of the asset-demand functions which underlie the slopes of the schedules ZZ and Z*Z* in Figure 1. To simplify the notation we suppress period and country subscripts. Using equation (2) the utility function in equation (3) can be expressed as a function of c_0, z, z^* and G as follows:

$$\text{(A-1)} \quad U(C_0,z,z^*,G) = u(C_0) + \beta E\{u[\theta(\alpha)z + \theta^*(\alpha)z^* - G]\}$$
$$= u(C_0) + v(z,z^*,G)$$

consider the inverse of the function $u(C_0)$, that is $C_0 = u^{-1}(u) \equiv \phi(u)$, the expenditure function $\mu(q,q^*,G,v)$ associated with $v(z,z^*,G)$ and the expenditure function associated with $U(C_0,z,z^*,G)$. They are related to each other as follows:

$$\text{(A-2)} \quad m[1,q,q^*,G,U] = \min_{u,v} \{1\cdot\phi(u) + \mu(q,q^*,G,v) \quad \text{s.t. } u+v=U\}$$
$$= \min_{u,v} [1\cdot \phi(U-v) + \mu(q,q^*,G,v)]$$

The first-order condition of the optimization problem in (A-2) is

$$\text{(A-3)} \quad -1\cdot \phi'(U-v) + \mu_v(q,q^*,G,v) = 0$$

As usual the compensated-asset demands are the partial derivatives of the expenditure function (A-2) with respect to prices (1, q, q*). Thus differentiating equations (A-2), using equation (A-3), yields

$$\text{(A-4)} \quad \text{(a)} \quad C_0 = m_1 = \phi(U-v)$$
$$\text{(b)} \quad z = m_q = \mu_q(q,q^*,G,v)$$
$$\text{(c)} \quad z^* = m_{q*} = \mu_{q*}(q,q^*,G,v)$$

Suppressing the arguments of the functions for convenience and differentiating (A-4) yields

(A-5) (a) $m_{ql} = \mu_{qu} v_l$

(b) $m_{qq} = \mu_{qq} + \mu_{qv} v_q$

(c) $m_{qq*} = \mu_{qq*} + \mu_{qv} v_{q*}$

To obtain explicit expressions for v_l, v_q, and v_{q*} we differentiate (A-3). This yields

(A-6) $$v_l = \frac{\phi'}{\phi'' + \mu_{vv}} > 0$$

(A-7) $$v_q = -\frac{\mu_{qv}}{\phi'' + \mu_{vv}} < 0$$

(A-8) $$v_{q*} = -\frac{\mu_{q*v}}{\phi'' + \mu_{vv}} < 0$$

The signs of the expressions in (A-6) - (A-8) indicated above follow from the assumption of normality (implying that μ_{qv} and μ_{q*v} are positive) and from the convexity of the expenditure functions ϕ and μ which reflect the assumption of risk aversion (implying that ϕ'' and μ_{vv} are positive.)

Substituting (A-6) - (A-8) into (A-4a) - (A-4c) yields

(A-9) $$m_{ql} = \mu_{qv} \frac{\phi'}{\phi'' + \mu_{vv}} > 0$$

(A-10) $$m_{qq} = \mu_{qq} - \frac{\mu_{qv}^2}{\phi'' + \mu_{vv}} < 0$$

(A-11) $$m_{qq*} = \mu_{qq*} - \frac{\mu_{qv}\mu_{q*v}}{\phi'' + \mu_{vv}} > 0$$

The negative sign in (A-10) follows from the fact that $\mu_{qq} < 0$ (because the demand for the domestic demand falls when the domestic security price rises while holding the welfare level v constant). The positive sign in (A-11) is implied from the convexity of the expenditure function so that

$$\mu_{vv}\mu_{qq*} > \mu_{qv}\mu_{q*v}.$$

Equation (A-10) implies that the domestic security demand is negatively related to q and (A-11) implies that the assets z and z* are pure substitutes.

Multiplying equation (A-10) by q and adding the resulting expression to equation (A-11), multiplied by q*, yields

$$\text{(A-12)} \qquad q^*m_{qq^*} + qm_{qq} = -\frac{\mu_v\mu_{qv}}{\phi''+\mu_{vv}} < 0$$

where use has been made of the familiar properties of expenditure function: μ_q is homogenous of degree zero in the prices q and q* and μ_v is homogenous of degree one in these prices. The negative sign in (A-12) implies that

$$\text{(A-13)} \qquad q^*m_{q^*q} < -\, qm_{qq}$$

Expressed in terms of the price elasticity of the compensated demand this yields

$$\text{(A-14)} \qquad \frac{\partial \log z^{*s}}{\partial \log q} < -\frac{\partial \log z^{s}}{\partial \log q}$$

This means that a rise in the domestic security price, q, exerts a more pronounced effect on the demand for this security than on the demand for the other security. Similarly, it can be verified that a rise in q* exerts a weaker effect on z^s than on z^{*s}.

The Effect of a Future Transitory Rise in Government Spending on Consumption and Savings.

In this appendix we derive explicit expressions for the effects of a future state-independent rise in government spending on consumption and savings which underlie the diagrams in Figure 3.

Consider first explicitly the expenditure function $\mu(q,q^*,G,v)$ introduced in equation (A-1):

$$\text{(A-15)} \qquad \mu(q,q^*,G,v)= \min_{z,z^*} qz+q^*z^* + \lambda[v-\beta EU(\Theta(\alpha)z+\Theta^*(\alpha)z^*-G)]$$

where λ is a Lagrangian multiplier. The first-order conditions are given by

(A-16) (a) $q-\lambda\beta EU'\Theta=0$

(b) $q^*-\lambda\beta EU'\Theta^*=0$

(c) $v-\beta EU=0$

Using the envelope theorem the derivatives of μ in equation (A-15) are

(A-17) (a) $\mu_v = \lambda$

(b) $\mu_G = \lambda\beta EU'$

(c) $\mu_q = z$

(d) $\mu_{q^*} = z^*$

Differentiating (A-16) with respect to v and G yields

(A-18) (a) $\frac{\partial z}{\partial v} \equiv \mu_{qv} = -\frac{\beta^2}{\Delta}(EU''\Theta\Theta^* EU'\Theta^* - EU''\Theta^{*2} EU'\Theta)$

(b) $\frac{\partial z^*}{\partial v} \equiv \mu_{q^*v} = -\frac{\beta^2}{\Delta}(EU''\Theta\Theta^* EU'\Theta - EU''\Theta^2 EU'\Theta^*)$

(c) $\frac{\partial \lambda}{\partial v} \equiv \mu_{vv} = -\frac{\beta^2}{\Delta}(EU''\Theta^2 EU''\Theta^{*2} - (EU''\Theta\Theta^*)^2)$

$$(d)\ \frac{\partial\lambda}{\partial G} \equiv \mu_{vG} = -\beta^3[\lambda EU''\Theta\Theta^*EU'\Theta^* - EU''\Theta^{*2}EU'\Theta)$$

$$+ \lambda EU''\Theta^*(EU''\Theta\Theta^*EU'\Theta - EU''\Theta^2EU'\Theta^*)+EU'(EU''\Theta^2EU''\Theta^{*2}-(EU''\Theta\Theta^*)^2)]$$

where $\Delta = \beta^3[EU''\Theta^2(EU'\Theta^*)^2 - 2EU''\Theta\Theta^*EU'\Theta EU'\Theta^* + EU''\Theta^{*2}(EU'\Theta)^2] > 0.$

Equations (A-18) (a)-(d) imply that

$$\text{(A-19)} \qquad \mu_{vG} = \beta\{\lambda[(EU''\Theta)\mu_{qv} + (EU''\Theta^*)\mu_{q^*v}] + (EU')\mu_{vv}\}$$

Next we evaluate the effect of the rise in G on the utility levels U, v and u. For this purpose consider the demand system from appendix I:

$$\text{(A-20)} \qquad 1\cdot\phi'(U-v) - \mu_v(q,q^*,G,v) = 0$$

$$\text{(A-21)} \qquad 1\cdot\phi(U-v) + \mu\,(q,q^*,G,v) = W_0.$$

where W_0 denotes current wealth. This system can be solved for the endogenous variables U and v given the levels of the exogenous variables q,q^* and G.

Differentiating equations (A-20) - (A-21) yields:

$$\text{(A-22)} \qquad \frac{dU}{dG} = -\frac{\mu_G}{\phi'}$$

$$\text{(A-23)} \qquad \frac{dv}{dG} = -\frac{\phi''\mu_G + \phi'\mu_{vG}}{\phi'(\mu_{vv} + \phi'')}$$

Since u = U-v substracting (A-23) from (A-22) using (A-20) yields

$$\text{(A-24)} \qquad \frac{du}{dG} = \frac{\mu_{vG}\mu_v - \mu_G\mu_{vv}}{\phi'(\mu_{vv} + \phi'')}$$

Finally, substituting equations (A-19) into equation (A- 24) yields

$$\text{(A-25)} \qquad \frac{du}{dG} = \beta\lambda^2\,\frac{(EU''\Theta)\mu_{qv} + (EU''\Theta^*)\mu_{q^*v}}{\phi'(\mu_{vv} + \phi'')}$$

We conclude that if the assets z and z* are normal "goods" (so that $\mu_{qv}>0$ and $\mu_{q*v}>0$) then a transitory future rise in government spending, G, lowers first period utility level $U(C_o)$ and thereby also lowers first-period consumption C_o.

Perfect Correlation Between the Returns of the Domestic and the Foreign Securities

If there exists a perfect (positive) correlation between the returns of the two securities, there is essentially just one tradable security whose price is denoted by q. From the budget constraint and (A-25) we have

$$\text{(A-26)} \quad \frac{\partial z}{\partial G} = -\frac{1}{q}\frac{dCo}{dG} > 0.$$

Thus a future rise in government spending raises the demand for the security.

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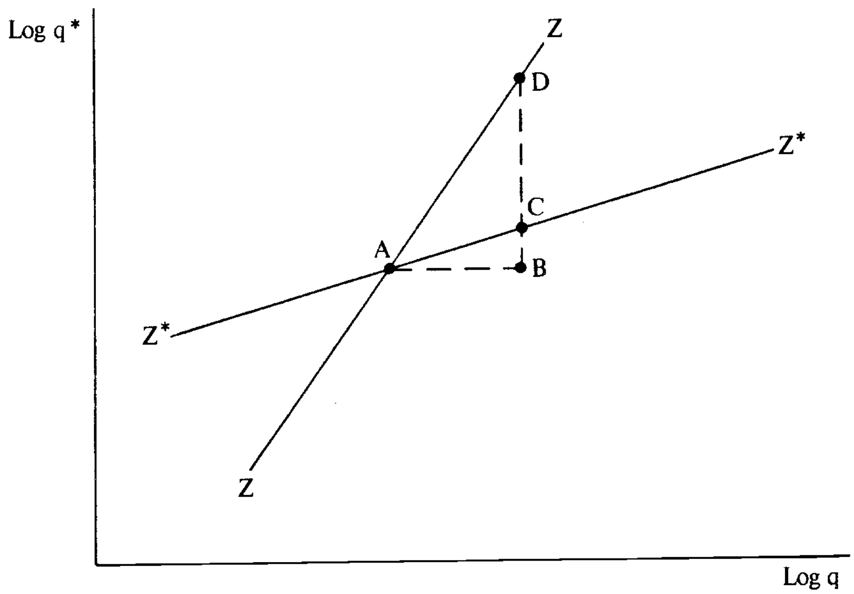
Figure 1

Stationary-State Equilibrium Equity Prices



Note: The stationary-state portfolio is characterized by $z_{H1} = z_{H0}$, $z^*_{H1} = z^*_{H0}$, $z_{F1} = z_{F0}$, and $z^*_{F1} = z^*_{F0}$.

Figure 2

The Effect of a Transitory Balanced-Budget Rise in Current Government Spending

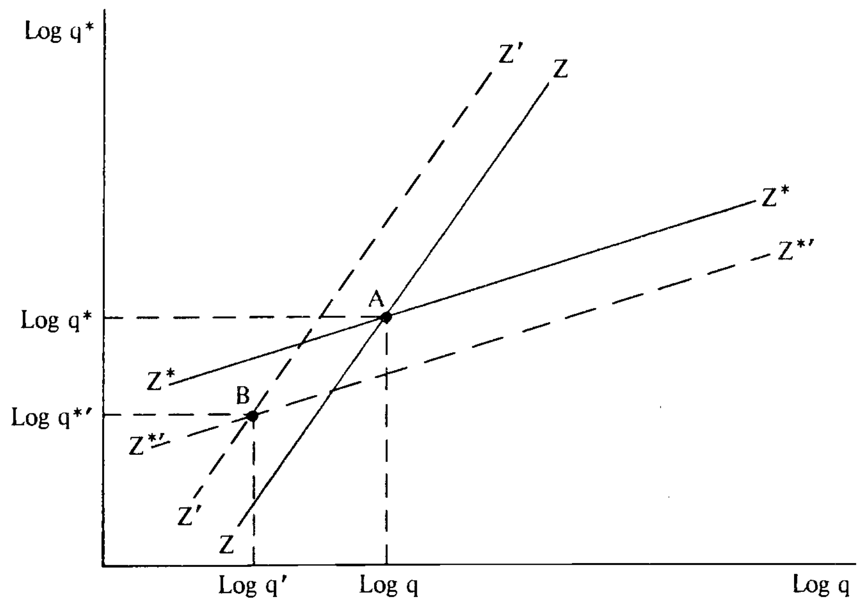


Figure 3

The Effect of a Rise in Future Government Spending on Consumption and Asset Demands ($G_1' > G_1$)

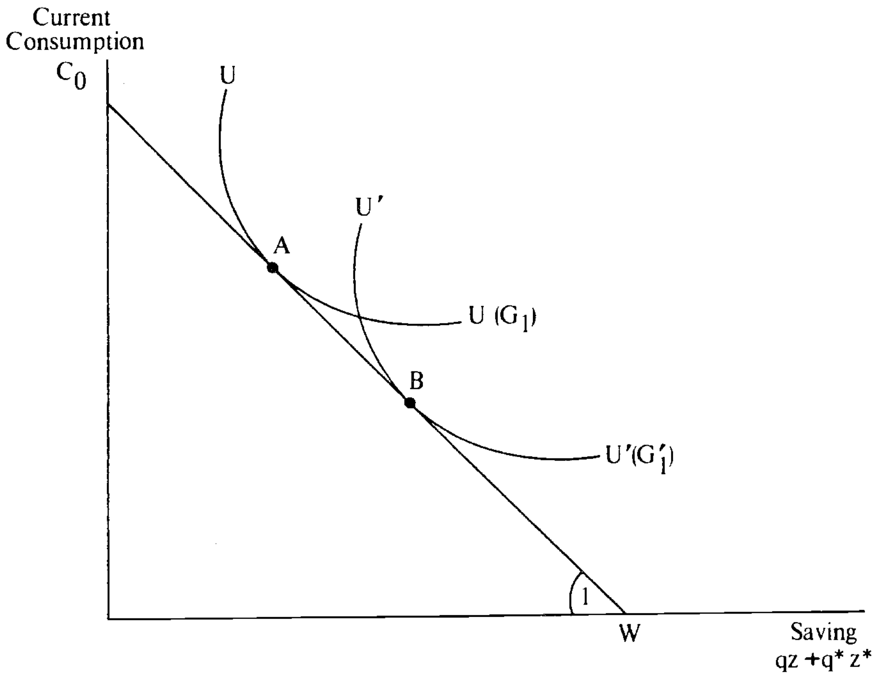


Figure 4

The Effect of a Transitory Balanced-Budget Rise in Future Government Spending on Equity Prices

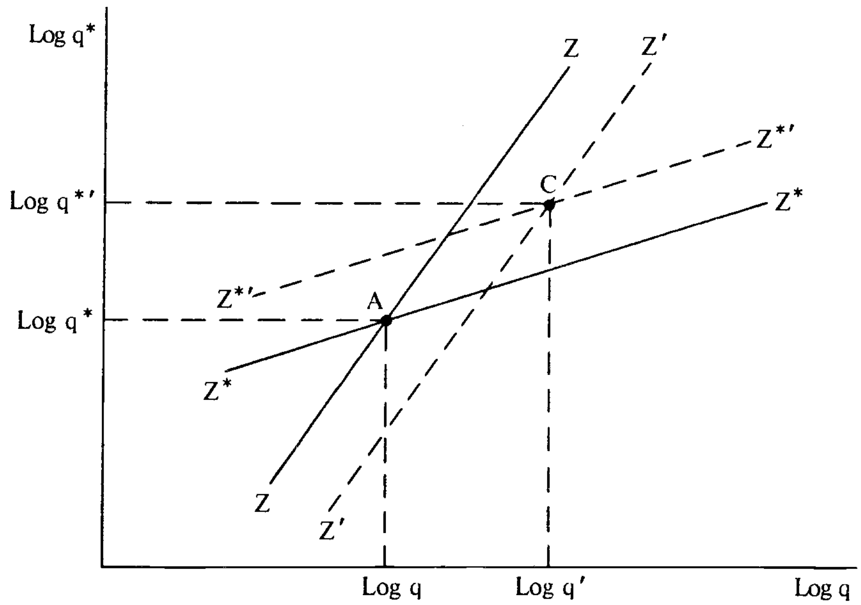


Figure 5

The Effect of a State-Dependent Rise in Government Spending on International Risk Sharing