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ORDER BACKLOGS AND PRODUCTION SMOOTHING

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ABSTRACT

Empirical examination of some aggregate manufacturing data suggests that order backlogs may help explain two puzzling facts: (1)the variability of production appears to be greater than that of demand, and (2)inventories appear to be drawn down when demand is low, built up when demand is high.

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I. Introduction

The production smoothing model of inventories suggests that firms hold inventories mainly to smooth production in the face of random fluctuations in demand. It is well known, however, that some stylized facts appear to be inconsistent with both the spirit and the letter of the model. One such fact is that in virtually all manufacturing industries, the variability of production is greater than that of shipments (Blanchard (1983), Blinder (1986a), West (1986)). A second fact is that inventories tend to be accumulated when demand is high and decumulated when demand is low, precisely the opposite of the pattern predicted by the production smoothing model (Blinder (1986a), Summers (1981)).

All the studies just cited assume that physical inventories are the only buffer between demand and production. Backlogs of unfilled orders, however, might also serve as buffers. They might be built up when demand is high and drawn down when demand is low. If so, studies that ignore backlogs may be misleading.

Indeed, in the presence of backlogs, the anomalous stylized facts probably are not even directly relevant to at least some versions of the production smoothing model. As initially stated (Holt et al. (1961)) and recently generalized (Blinder (1982)), the model does not impose a nonnegativity condition on inventories. If demand is too high, orders are put on a backlog. Backlogged orders are implicitly considered negative inventories. If the model is taken literally, the implication is that empirical studies should follow Holt et al. (1961) and Belsley (1969) and use "net" inventories, i.e., physical inventories minus backlogs. If backlogs are substantial, the bias from using physical rather than net inventories may be large.

This paper considers the anomalous stylized facts for some industries where backlogs in fact are large. It assumes a model like that in Holt et al. (1961), Belsley (1969) or Blinder (1982). The model implies that the variance of production is less than the variance of new orders (rather than shipments). This is empirically true, for the data studied here. The model also implies that the net inventory stock should buffer production from demand. The stock should be decumulated when demand is high, accumulated when demand is low. This, too, holds empirically, in two senses. First, the covariance between new orders and investment in net inventories is negative. Second, a positive shock to new orders causes net inventories to be drawn down, with production rising only gradually. On the other hand, if one ignores backlogs, and examines physical inventories and shipments instead of net inventories and new orders, the usual stylized facts result. These facts are, however, irrelevant in the present production smoothing model.

Net inventories, then, appear to smooth production in the face of random fluctuations in demand. This suggests that production smoothing may indeed be a central determinant of production.

It should be emphasized, however, that this paper does not shed direct light on the determinants of physical inventories: the model used determines net inventories, with the individual levels of physical inventories and of backlogs indeterminate. This is, of course, a serious drawback in an inventory model. Moreover, common sense, as well as some formal time series evidence (Reagan and Sheehan (1985), West (1983b)), suggest that backlogs are not simply negative inventories. Further research is required to see whether backlogs and inventories play their prescribed roles when one allows them to affect costs in distinct ways. In addition, the evidence here is qualitative in the sense that while broad time series patterns are established, a precise model is never estimated, and standard errors are never calculated. I would therefore characterize the results in this paper as preliminary and suggestive.

Section II describes the model and tests performed. Section III presents empirical results. Section IV concludes. An appendix available on request contains some algebraic details and empirical results omitted to save space.

II. The Model and Tests

The empirical work requires data on backlogs. The Department of Commerce only collects such data for what are called "production to order" industries. The model used will therefore be one that is appropriate for such industries.

These are industries in which orders ordinarily arrive before production is completed. Storage costs for the finished product tend to be relatively large and the product line fairly heterogeneous (Abramovitz (1950), Zarnowitz (1973)). According to Belsley (1969), most two digit industries produce primarily to order, including virtually all durable goods industries. Backlogs tend to be substantial, relative either to shipments or to physical inventories. This is illustrated for aggregate durables in Figure 1, which plots backlogs, shipments, and two measures of inventories, finished goods and the sum of finished goods and works in progress. The backlog to shipment ratio, or the (backlog - physical inventories) to shipment ratio, suggests that customers typically wait anywhere from one to five months for shipment.

Let Q_t be production, I_t physical inventories, S_t shipments, B_t backlogs (unfilled orders) and N_t new orders. The variables are linked by the identities

$$(1) \qquad Q_t=S_t+\Delta I_t,$$

$$N_t=S_t+\Delta B_t,$$

$$\Longrightarrow \qquad Q_t=N_t+\Delta H_t, \; H_t\equiv I_t-B_t.$$

H_t is the net inventory stock, physical inventories minus unfilled orders.

The model I will use, which is developed in detail in the appendix, is a slightly modified version of the one in Belsley (1969). The representative firm minimizes the expected present discounted value of costs,

$$\text{(2)} \qquad \min E_0 \sum_{t=0}^{\infty} b^t C_t$$

E_0 is expectations conditional on the firm's period zero information, b is a discount rate, 0<b<1. Apart from inessential constant and linear terms, per period costs C_t are

$$\text{(3)} \qquad C_t = a_0(\Delta Q_t+u_{1t})^2 + a_1(Q_t+u_{2t})^2 + a_2(-H_t-a_3Q_t+u_{3t})^2.$$

The u_{it} are zero mean, white noise cost shocks. Apart, perhaps, from these shocks, the first two terms are standard. The cost of changing production, $a_0(\Delta Q_t+u_{1t})^2$, represents, for example, hiring and firing costs. The production cost, $a_1(Q_t+u_{2t})^2$, can be considered a Taylor series approximation to a concave cost function.

The final term in (3), $a_2(-H_t-a_3Q_t+u_{3t})^2$, is peculiar to a production to order firm. It balances two costs. The first is a cost of having a lengthy delivery period (bad customer relations, loss of reputation, etc.). Given the rate of production Q_t, this cost increases with $-H_t$ (=backlogs-physical inventories): the bigger the backlog or the smaller the stock of physical inventories, the lengthier the delivery period. The second is a cost of having to rush production (inefficient scheduling of batch production runs, etc.) Given Q_t, this cost decreases with $-H_t$: the bigger the backlog or the smaller the stock of physical inventories, the greater the flexibility in scheduling production. See Holt et al. (1961), Childs (1967) and Belsley (1969) for further discussion. It should be noted that all the tests in this paper are robust to the possibility that $a_3=0$, in which case the model is similar to that in Blinder (1982).

I will consider two empirical implications of the model. The first concerns production variability. The model implies that net inventories are used to buffer new orders. If variables are stationary around trend, this suggests

$$(4) \qquad 0 \le \text{var}(N)-\text{var}(Q),$$

where "var" is an unconditional variance. Inequality (4) follows under a variety of assumptions about market structure and demand, as long as any effects of net inventories on demand are captured by the $a_2(.)$ term in (2). In particular, (4) is implied even if prices adjust in response to demand fluctuations. See West (1986) and the appendix for a precise argument.[1]

If the variables are not stationary, var(N) and var(Q) do not exist. Related literature suggests that empirical tests that nonetheless assume that they exist may be seriously misleading (Fuller (1976), Marsh and Merton (1986)). By continuity, this also may be true in a given finite sample, if the variables are nearly nonstationary. The data used here in fact appear to be nonstationary or nearly so, even after growth is removed.

Even if the data have unit roots, ΔH_t is stationary. Since $Q_t=N_t+\Delta H_t$, N_t and Q_t are cointegrated (Engle and Granger (1987)), and a slightly more cumbersome restatement of (4) is valid. We have $Q_t=N_t+\Delta H_t$, so $N_t^2-Q_t^2=-2N_t\Delta H_t-\Delta H_t^2$. Let "cov" denote an unconditional covariance. Under fairly general statistical conditions, $\text{cov}(N_t,\Delta H_t)$ exists, even if N_t has a unit root (e.g., if $(\Delta N_t,\Delta H_t)$ follows a finite parameter ARMA process; see Fuller (1976) and West (1987)). Whether or not there are unit roots, then, one can test

$$(5) \qquad 0 \le -2\text{cov}(N_t,\Delta H_t)-\text{var}(\Delta H_t).$$

If there are unit roots, one must not estimate $\mathrm{cov}(N_t,\Delta H_t)$ as a sample moment in the usual way. This would just reduce (5) to (4). Section IIA explains how to get an estimate that (a)is consistent if N_t has a unit root, and (b)is asymptotically the same as (4) if the data are stationary.

The second of the model's empirical implications that I will consider concerns whether net inventories buffer production. One test of this is whether the covariance between new orders and investment in net inventories is negative (Blinder (1986a)). If so, inventories tend to be decumulated when demand is high, accumulated when demand is low. Note, however, that $\mathrm{cov}(N_t,\Delta H_t)<0$ is necessary (but not sufficient) for (4) and (5). Since, as we shall see, (4) and (5) hold in these data, no separate empirical work will be needed to test this proposition.

A second test of whether net inventories buffer production concerns the response of production and net inventories to a shock to new orders (Blinder (1986a)). This is conveniently analyzed under the (over) simplifying assumptions that the firm uses just lagged new orders to forecast future new orders, and that the univariate new order process follows an AR(q):

$$(6) \qquad N_t = \phi_1 N_{t-1} + \dots + \phi_q N_{t-q} + v_t .$$

In (6), unit roots are allowed (e.g., if q=1, $N_t=N_{t-1}+v_t$ is allowed). Deterministic terms are suppressed in (6) and below, for notational simplicity.

By algebra such as in Blanchard (1983) or Eichenbaum (1984), (2) and (6) imply that the decision rule for H_t is

$$(7) \qquad H_t = \rho_1 H_{t-1} + \rho_2 H_{t-2} + \delta_0 N_t + \dots + \delta_{q-1} N_{t-q+1} + u_t .$$

The disturbance u_t is a linear combination of the cost shocks u_{it}, i=1 to 3. The ρ_i depend on b and the a_i in a complicated way, the δ_i depend on b, the a_i and the ϕ_i in a complicated way. The exact formulas are not of interest, except perhaps to note that ρ_2 is zero of the cost of changing production a_0 is zero. Parameter estimates are consistent even if the variables have unit roots (Sims, Stock and Watson (1986)).

Under the identifying assumption that the demand shock v_t and the cost shock u_t are uncorrelated, one can estimate not only (6) but (7) as well by least squares. One can then trace out an impulse response function, for how production and net inventories respond to a demand shock v_t: $\partial H_t/\partial v_t=\delta_0$, $\partial Q_t/\partial v_t=1+\delta_0$, $\partial H_{t+1}/\partial v_t=\rho_1\delta_0+\delta_0\phi_1+\delta_1$, etc. The model suggests that H_t will be drawn down in response to a positive demand shock ($\delta_0<0$), with production rising gradually to meet the increased demand.

III. Empirical Results

A. Data

The data were monthly and seasonally adjusted, 1967-1984. (Data that are not seasonally adjusted might be preferable (Miron and Zeldes (1986)) but are not available for backlogs.) Nominal backlog data were conveniently available from CITIBASE for aggregate durables and six two digit manufacturing industries: stone, clay and glass (SIC 32), primary metals (SIC 33), fabricated metals (SIC 34), non-electrical machinery (SIC 35), electrical machinery (SIC 36), transportation equipment (SIC 37), and instruments (SIC 38). BEA constant (1972) dollar inventory data on finished goods and works in progress inventories and shipments were kindly supplied by Jeff Miron. Inventory data were converted from cost to market as in West (1983a) and Blinder and Holtz-Eakin (1983).

Constant dollar backlog data were not available. The discussion in Foss et al. (1980, pp156-57), as well as a reading of Bureau of the Census's Form M-3 (Appendix I in Foss et al. (1980)) suggests that it is reasonable to assume that firms value the entire backlog at current delivery prices. Real backlogs were therefore obtained by deflating the BEA figure for the nominal stock of backlogs by the ratio of (nominal shipments/real shipments). New orders were calculated from the identity $N_t=S_t+\Delta U_t$. Two net inventory series were used: finished goods - backlog, and finished goods + works in progress - backlog. Production was calculated as $Q_t=N_t+\Delta H_t$. As a check on the deflation procedure, real backlogs were also obtained for aggregate durables by deflating by the producer price index. The resulting second moments of the data were very similar to those reported in Table 1 below.

Before any estimation, a common geometric trend was removed from all variables. (This is consistent with the model, as shown in the appendix.) The estimated common growth rates for finished goods inventories, backlogs and shipments, in percent per month, for aggregate durables and SIC codes 32 to 38 were: .18, -.01, -.00, -.03, .29, .38, .04, .40. The estimated rates for finished goods + works in progress, backlogs and shipments were: .17, -.01, .01, -.04, .28, .40, .07, .42. Before any of the computations reported below were done, all variables were scaled to remove this growth. For example, all durables data were divided by $(1.0018)^t$ when net inventories = finished goods inventories - backlogs, by $(1.0017)^t$ when net inventories = finished goods inventories + works in progress - backlogs. Variances and covariances of the resulting data were calculated around a constant mean. Constant terms were used in estimation of (6) and (7). To make sure that inference was not sensitive to the exact estimate of growth rates, the second moments reported in Table 1 below were recalculated for aggregate durables, with growth rates half again as big or half as small (i.e., for growth rates of $.17 \pm (.17/2)$ and $.18 \pm (.18/2)$). Results were similar.

The Durbin-Watson of each of the regressions to estimate a common trend was very low, typically under .10. This suggests possible nonstationarity of the geometrically detrended variables. To guard against possible resulting biases, the cov(N,ΔH) term that appears in equation (5) was calculated as follows. Let T be the sample size. Ignore constant terms for notational simplicity. If N_t has a unit root, $T^{-1}\Sigma N_t \Delta H_t$ has a nondegenerate limiting distribution, and thus is not a consistent estimate of $\mathrm{cov}(N_t,\Delta H_t)$ (Fuller (1976), West (1987)). We have $N_t = \Delta N_t + \Delta N_{t-1} + \Delta N_{t-2} + \ldots$. This suggests calculating $\mathrm{cov}(N_t,\Delta H_t)$ as $\mathrm{cov}(\Delta N_t,\Delta H_t) + \mathrm{cov}(\Delta N_{t-1},\Delta H_t) + \ldots$. Let $\hat{c}_j$ be an estimate of $\mathrm{cov}(\Delta N_{t-j},\Delta H_t)$, $\hat{c}_j=T^{-1}\Sigma_{t=j+1}^{T}\Delta N_{t-j}\Delta H_t$. Consider estimating cov $(N_t,\Delta H_t)$ as $\Sigma_{j=0}^{m}\hat{c}_j$, and letting $m \to \infty$ as $T \to \infty$. The literature on estimation of spectral densities (Hannan (1970,p280)) indicates that if $(m/T^{1/2}) \to 0$ as $m, T \to \infty$, $\Sigma_{j=0}^{m}\hat{c}_j$ consistently estimates cov(N,ΔH). I set m=20 in the results reported below. (If N_t is stationary, one could of course set m=T, and just calculate $T^{-1}\Sigma N_t \Delta H_t$.)

In equations (6) and (7) the length of the autoregression was set to four. It should be noted that the assumption that firms use only lagged new orders to forecast future new orders is consistent with a comment in Blinder (1986a) suggesting that inventories tend not to Granger cause sales.

B. Empirical Results

Table 1 contains point estimates of the right hand sides of (4) and (5) when net inventories = finished goods inventories - backlogs, Table 2 when net inventories = finished goods + works in progress - backlogs. Units are billions of 1972 dollars, squared. As may be seen, the production variance is less than the new order variance, in all specifications except instruments (columns (4) and (6)).[2] As in Blinder (1986a), however, the production variance is almost always greater than the shipment variance (columns (5) and (7)).

Since column (4) is less than one and column (6) is positive, it follows that $cov(N_t, \Delta H_t)<0$. Net inventories therefore on average are accumulated during expansions, decumulated during contractions. This is illustrated in Figure 2, which plots detrended aggregate durables data, for net inventories = finished goods + works in progress - backlog. The tendency for H to be built up when N is low, to be drawn down when N is high, is quite apparent. The plots of B and works in progress + finished goods inventories indicate that the theoretically predicted pattern of fluctuations for H essentially reflects procyclical accumulation of backlogs but not countercyclical accumulation of physical inventories. It is worth noting that while the model does not formally determine a level of inventories separate from that of backlogs, the actual inventory behavior probably is consistent with production smoothing behavior in production to order industries. Abramowitz (1950) and Belsley (1969) suggest that finished goods inventories, at least, are built up in part because of unavoidable delays in transit. One might therefore expect inventories to be built up when shipments are high.

Additional evidence on the role of net inventories in buffering production may be found in the impulse response functions in Table 3. The functions are calculated from estimates of equations (6) and (7). (These estimates are available on request. Regression estimates and impulse response functions were also calculated for net inventories = finished good - backlogs, but are not reported because they were quite similar to those in Table 3.) Since the period is a month, the entry for period 12 indicates the response one year after the shock, for 24 two years after, and so on.

The estimates indicate that from 40 to 80 percent of the initial impact of a demand shock is absorbed by net inventories, with production adjusting gradually.

Figure 3 contains a plot for the aggregate durables entry in Table 3. Production is built up gradually to meet the increased demand. If the data are stationary, all variables return to their steady state levels, with production meeting the increased demand ($\Sigma_{j=0}^{\infty}(\partial Q_{t+j}/\partial v_t) = \Sigma_{j=0}^{\infty}(\partial N_{t+j}/\partial v_t)$; v_t is the demand shock.) Note, however, that the return is painfully slow, indicating the borderline nonstationary behavior of inventories and new orders. In fact, the roots of $(1-\rho_1 L-\rho_2 L^2)$, with ρ_1 and ρ_2 defined in equation (6), were outside the unit circle for two data sets (fabricated metals and transportation).

Figure 4 contains the comparable plot for a shock to shipments, when physical inventories alone are assumed to buffer production. Little buffering is evident.

IV. Conclusions

A production smoothing model is qualitatively consistent with some aggregate data when it is assumed that net inventories (physical inventories minus backlogs), rather than physical inventories, buffer production. The variance of production is less than that of new orders, so production is smoother than demand. The covariance of new orders and investment in net inventories is negative, so that net inventories are accumulated during contractions, decumulated during expansions. A positive shock to new orders is buffered by net inventories, so that production rises only gradually to meet increases in demand.

These results are in no sense definitive. The model that I used assumed rather implausibly that backlogs are negative inventories. No standard errors were calculated in any of the tests. The data were purely for production to order industries.

One therefore cannot jump to the conclusion that production smoothing is the major determinant of both backlogs and inventories. Nonetheless, in conjunction with the conclusions of other papers, the present results seem highly suggestive. Theoretical work using more carefully formulated models than mine indicates that the presence of backlogs may indeed explain apparently anomalous production behavior (Kahn (1986), Maccini (1973)). Empirical work at least since Lovell's (1961) seminal research has found an important role for backlogs; recent contributions include Blinder (1986b) and Maccini and Rossana (1984). Large and volatile backlogs are perhaps more pervasive than many researchers, including myself (West (1986)) have assumed: of the six two digit manufacturing industries classified by Belsley (1969) as production to stock, two (apparel [SIC 23] and chemicals [SIC 28]) in fact are or have become largely production to order (Foss et al. (1980, pp158)).

The fundamental question is whether firms systematically use backlogs as a buffer between production and demand. If so, it is premature to conclude from, say, a comparison of production and shipment variances that firms do not smooth production in the face of fluctuations in demand. Whether or not backlogs can save the production smoothing model is therefore an important task for future research.

Footnotes

1. Technically, this requires $a_3=0$ and no cost shocks. If, say, the penalty for having a large backlog is prohibitive, demand shocks may be passed directly to production. In addition, if costs vary stochastically, the firm will tend to produce a relatively large amount when costs are low, thereby inducing extra variability in production. The spirit of the model, however, is that the primary role of net inventories is to buffer production from demand. It therefore seems reasonable to expect (4) to hold, even if $a_3 \neq 0$ and there are cost shocks.

2. The only reason the entries for var(N) and var(S) are different in the two tables is the slightly different estimates of growth rates.

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Appendix

This appendix contains algebraic details and empirical results omitted from the paper to save space. It has three parts:

I. Algebraic details on the model

II. Second moments for aggregate durables, under different growth rates and different deflator (Table A1)

III. Estimates of equations (6) and (7) (Table A2)

I. Algebraic details on the model

This considers (A)derivation of (4) and (5); (B)accounting for growth; and (C)derivation of (7).

(A)Let p_t be the real price of output. The firm maximizes

$$\text{(A1)} \qquad \max E_0 \sum_{t=0}^{\infty} b^t (p_t N_t - C_t)$$

For simplicity it is assumed that the firms gets revenue when an order is placed, rather than when it is shipped.

Let C_t be as in (3), with $a_3=u_{0t}=u_{1t}=u_{2t}=0$. Assume that all variables have zero mean. (See West (1986) for why this is an innocuous assumption.) Let V_0^* be the value of (A1), under the optimal policy. Consider an alternative policy in which $Q_t^A=N_t$ and $H_t^A=0$ for all t. Costs C_t^A thus are $a_0(\Delta N_t)^2 + a_1(N_t)^2$. Under the assumption that all effects of net backlogs on demand are adequately captured by the cost function, it will still be feasible to obtain revenue p_tN_t under this alternative policy. Let V_t^A be the expected present discounted value of cash flows under this alternative,

$$V_0^A = E_0 \sum_{t=0}^{\infty} b^t (p_t N_t - C_t^A).$$

Since the policy actually followed is assumed optimal, we have

$$\text{(A2)} \quad V_0^A \le V_0^* \implies 0 \le E_0 \sum b^t (C_t^A - C_t)$$
$$= E_0 \sum b^t a_0(\Delta N_t^2 - \Delta Q_t^2) + E_0 \sum b^t a_1(N_t^2 - Q_t^2) - E_0 \sum b^t a_2 H_t^2$$

The third term is nonnegative by construction, since it is the expectation of a sum of nonnegative random variables. A necessary condition for (A2), then, is that

$$(A3)\qquad 0 \le E_0\ \Sigma b^t a_0(\Delta N_t^2-\Delta Q_t^2) + E_0\ \Sigma b^t a_1(N_t^2-Q_t^2)$$
$$\Longrightarrow\qquad 0 \le E\Sigma b^t a_0(\Delta N_t^2-\Delta Q_t^2) + E\Sigma b^t a_1(N_t^2-Q_t^2)$$
$$= (1-b)^{-1}\{\ a_0[var(\Delta N)-var(\Delta Q)] + a_1E(N_t^2-Q_t^2)\ \}$$

The first implication follows from the law of iterated expectations, since, under the assumption that at most one difference is required to induce stationarity, each term in each infinite sum has a finite unconditional expectation.

In versions of the model in which $a_0=0$ (e.g., Blinder (1986a)), equations (4) and (5) follow from (A3). If $a_0\neq 0$ (e.g., Belsley (1969)), (4) and (5) follow if $0<var(\Delta N)-var(\Delta Q)$. Although not reported in the paper, the sample variances of ΔN and ΔQ obeyed this inequality for all eight data sets.

(B)As stated in the text, the data actually used in the regressions were scaled by a growth rate of $(1+g)^t$. It is assumed that $b(1+g)<1$. In explaining how this fits into the model, it is convenient to call h_t, q_t and n_t the original data in levels and H_t, Q_t and N_t the scaled data (e.g., $H_t=h_t/(1+g)^t$). The full cost function, including deterministic terms, is

$$(A4)\qquad C_t = k_t + c_{0t}(\Delta q_t-m_{0t}) + c_{1t}(q_t-m_{1t}) + c_{2t}(-h_t-a_3q_t-m_{2t})$$
$$+ a_0(\Delta q_t-m_{0t})^2 + a_1(q_t-m_{1t})^2 + a_2(-h_t-a_3q_t-m_{2t})^2$$

k_t is a purely deterministic term that grows no faster than $(1+g)^t$. The m_{it} shift the minimum cost points for each of the three types of costs. Each m_{it} has both deterministic and stochastic components, $m_{it} = (1+g)^t m_i - (1+g)^t u_{it}$. The u_{it} are the white noise cost shocks in equation (2). The c_{it} grow deterministically, $c_{it}=(1+g)^t c_i$.

Using the (A4) definition of costs, differentiate (A1) with respect to h_t, divide by 2 and rearrange to get

$$
\begin{aligned}
\text{(A5)}\quad 0 = E_t\{\ & b^2 a_0 h_{t+2} - [a_0(2b+2b^2)+ba_1+ba_2a_3(1+a_3)]h_{t+1} \\
& + [a_0(1+4b+b^2)+a_1(1+b)+a_2(1+a_3)^2+ba_2a_3^2]h_t \\
& - [a_0(2+2b)+a_1+a_2a_3(1+a_3)]h_{t-1} + a_0h_{t-2} \\
& + b^2a_0n_{t+2} - [a_0(b^2+2b)+ba_1+ba_2a_3^2]n_{t+1} \\
& + [a_0(1+2b)+a_1+a_2a_3(1+a_3)]n_t - a_0n_{t-1} \\
& + (1+g)^t c + (1+g)^t m - (1+g)^t v_t \ \},
\end{aligned}
$$

$$
\begin{aligned}
c &\equiv c_0(1-2b+b^2) + c_1(1-b) + c_2[-(1+a_3)+ba_3], \\
m &\equiv a_0m_0[1-2b(1+g)+b^2(1+g)^2] + a_1m_1[1-b(1+g)] \\
&\quad + a_2m_2[-(1+a_3)+ba_3(1+g)], \\
v_t &\equiv a_0u_{0t} + a_1u_{1t} - a_2(1+a_3)u_{2t}.
\end{aligned}
$$

Dividing by $(1+g)^t$ and rearranging terms gives

$$
\begin{aligned}
\text{(A6)}\quad E_t\{\ & [(1+g)b]^2a_0H_{t+2} - (1+g)[a_0(2b+2b^2)+ba_1+ba_2a_3(1+a_3)]H_{t+1} \\
& + [a_0(1+4b+b^2)+a_1(1+b)+a_2(1+a_3)^2+ba_2a_3^2]H_t \\
& - (1+g)^{-1}[a_0(2+2b)+a_1+a_2a_3(1+a_3)]H_{t-1} + (1+g)^{-2}a_0H_{t-2} = Z_{t+2}\},
\end{aligned}
$$

$$
\begin{aligned}
Z_{t+2} \equiv & - [(1+g)b]^2a_0N_{t+2} + (1+g)[a_0(b^2+2b)+ba_1+ba_2a_3^2]N_{t+1} \\
& - [a_0(1+2b)+a_1+a_2a_3(1+a_3)]N_t + (1+g)^{-1}a_0N_{t-1}
\end{aligned}
$$

$- c - m + v_t$

It follows from algebra in part (C) below that if N_t is stationary in levels or some difference, then so is H_t. Thus, if N_t grows at rate 1+g, so does H_t.

(C)Call λ_1 and λ_2 the two smallest (in modulus) roots to the fourth degree (A5) lag polynomial in H_t. The comparable polynomial in (A6) has roots $\lambda_1/(1+g)$, $\lambda_2/(1+g)$, $1/[b(1+g)\lambda_1]$ and $1/[b(1+g)\lambda_2]$. So if $|\lambda_1|<1$ and $|\lambda_2|<1$, the (A6) lag polynomial has exactly two stable and two unstable roots. Let $\rho_1=-(\lambda_1+\lambda_2)/(1+g)$, $\rho_2=(\lambda_1\lambda_2)/(1+g)^2$. Solving the stable roots backwards, the unstable roots forwards, we obtain

$$\text{(A7)} \qquad H_t = \rho_1 H_{t-1} + \rho_2 H_{t-2} + dE_t\{ b(1+g)\lambda_1 \Sigma[b(1+g)\lambda_1]^j Z_{t+j+2} - b(1+g)\lambda_2 \Sigma[b(1+g)\lambda_2]^j Z_{t+j} \},$$

$$d \equiv \lambda_1\lambda_2/[(\lambda_1-\lambda_2)b(1+g)a_0].$$

It follows from the argument in Blanchard (1983) that if the firm uses only past new orders to forecast future new orders, and that equation (6) represents the univariate new orders process, then equation (7) is the closed form solution to (A7).

Table A1

Second Moments for Aggregate Durables

Specification	(1) var(Q)	(2) var(N)	(3) var(S)	(4) var(Q)/var(N)	(5) var(Q)/var(S)	(6) -2cov(N,ΔH) - var(ΔH)	(7) -2cov(S,ΔI) - var(ΔI)
(1)	5.170	9.084	5.008	.57	1.03	6.733	-.172
(2)	6.161	9.235	5.124	.67	1.20	5.433	-1.502
(3)	6.157	9.498	5.989	.65	1.03	5.765	-.105
(4)	9.046	14.324	8.922	.63	1.01	8.264	-.225
(5)	6.743	9.322	5.778	.72	1.17	4.752	-1.147
(6)	10.198	14.534	9.106	.70	1.12	6.720	-1.764

Lines (1) and (2) are as in the first lines of Tables 1 and 2, except that the PPI is used to deflate. Lines (3) and (4) are the same as the first line of Table 1, except that the growth rates used in scaling the variables are .27 percent (line (3)) and .09 percent (line (4)) per month. Lines (5) and (6) are the same as the first line in Table 2, except that the growth rates used in scaling the variables are .26 percent (line (5)) and .09 per cent (line (6)).

Table A2

Estimates of Equations (6) and (7)

AGGREGATE DURABLES

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99597896 RBAR**2 .99586127
SSR 57702705. SEE 530.54369
DURBIN-WATSON 2.15804691
Q(42)= 50.4496 SIGNIFICANCE LEVEL .174082

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	2926.664	542.2134	5.397623
2	H	26	1	1.494685	.5888146E-01	25.38464
3	H	26	2	-.5053061	.5744315E-01	-8.796629
4	N	27	0	-.6382120	.3195819E-01	-19.97022
5	N	27	1	.4695861	.5204124E-01	9.023346
6	N	27	2	.5072586E-01	.3824902E-01	1.326200
7	N	27	3	.8775555E-02	.3628758E-01	.2418336

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .84938348 RBAR**2 .84647302
SSR .28882997E+09 SEE 1181.2340
DURBIN-WATSON 1.99959603
Q(42)= 50.0075 SIGNIFICANCE LEVEL .185298

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	2163.817	867.3513	2.494741
2	N	27	1	.6543254	.6801315E-01	9.620572
3	N	27	2	.2730040	.8212626E-01	3.324199
4	N	27	3	.1941789	.8545563E-01	2.272278
5	N	27	4	-.1919794	.7082341E-01	-2.710678

SIC 32

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99702008 RBAR**2 .99693286
SSR 372690.67 SEE 42.638049
DURBIN-WATSON 2.07243596
Q(42)= 47.8775 SIGNIFICANCE LEVEL .246478

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	18.16011	25.51898	.7116314
2	H	26	1	1.273020	.6586822E-01	19.32678
3	H	26	2	-.2726517	.6617350E-01	-4.120255
4	N	27	0	-.4229396	.3846323E-01	-10.99595
5	N	27	1	.2513702	.5327903E-01	4.717996
6	N	27	2	.9339714E-01	.4593459E-01	2.033264
7	N	27	3	.7285187E-01	.4042233E-01	1.802268

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .90009804 RBAR**2 .89816757
SSR 1247176.8 SEE 77.620929
DURBIN-WATSON 1.99274176
Q(42)= 59.2499 SIGNIFICANCE LEVEL .406397E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	83.88150	43.86879	1.912100
2	N	27	1	.6228767	.6930236E-01	8.987814
3	N	27	2	.2304147	.8169732E-01	2.820346
4	N	27	3	.3109415E-01	.8174648E-01	.3803730
5	N	27	4	.7554613E-01	.6931645E-01	1.089873

SIC 33

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99708798 RBAR**2 .99700275
SSR 4188275.7 SEE 142.93570
DURBIN-WATSON 1.96751237
Q(42)= 30.0658 SIGNIFICANCE LEVEL .915647

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	389.5945	127.0239	3.067097
2	H	26	1	1.615117	.4558927E-01	35.42758
3	H	26	2	-.6321131	.4318655E-01	-14.63681
4	N	27	0	-.7479607	.3301801E-01	-22.65311
5	N	27	1	.6048886	.5584720E-01	10.83114
6	N	27	2	.7399748E-02	.4478908E-01	.1652132
7	N	27	3	.5191541E-01	.3827991E-01	1.356205

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .83239651 RBAR**2 .82915779
SSR 18817841. SEE 301.50861
DURBIN-WATSON 1.98682059
Q(42)= 38.4423 SIGNIFICANCE LEVEL .627928

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	389.6951	146.1376	2.666631
2	N	27	1	.8844811	.6997483E-01	12.63999
3	N	27	2	.4174468E-01	.9333905E-01	.4472370
4	N	27	3	.3522219E-03	.9360508E-01	.3762850E-02
5	N	27	4	-.9371970E-02	.7049919E-01	-.1329373

SIC 34

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99713524 RBAR**2 .99705140
SSR 5985290.7 SEE 170.86995
DURBIN-WATSON 2.15419265
Q(42)= 80.7023 SIGNIFICANCE LEVEL .306484E-03

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	850.4326	138.1033	6.157946
2	H	26	1	1.292611	.6735152E-01	19.19201
3	H	26	2	-.2798400	.6856604E-01	-4.081322
4	N	27	0	-.6198806	.4908722E-01	-12.62815
5	N	27	1	.4213045	.6884974E-01	6.119188
6	N	27	2	.6018794E-01	.5574024E-01	1.079793
7	N	27	3	-.2585401E-01	.4938159E-01	-.5235557

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .78432160 RBAR**2 .78015390
SSR 12807049. SEE 248.73641
DURBIN-WATSON 1.98017670
Q(42)= 58.0853 SIGNIFICANCE LEVEL .503526E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	298.4133	159.6477	1.869199
2	N	27	1	.5082484	.6945684E-01	7.317471
3	N	27	2	.3246834	.7861740E-01	4.129917
4	N	27	3	.7831119E-01	.7828451E-01	1.000341
5	N	27	4	.2511398E-01	.7088995E-01	.3542671

SIC 35

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99736883 RBAR**2 .99729182
SSR 4869035.8 SEE 154.11487
DURBIN-WATSON 2.43301991
Q(42)= 80.3215 SIGNIFICANCE LEVEL .338651E-03

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	400.9500	117.5866	3.409826
2	H	26	1	1.552202	.5292142E-01	29.33033
3	H	26	2	-.5584924	.5199758E-01	-10.74074
4	N	27	0	-.8110167	.2884205E-01	-28.11924
5	N	27	1	.5822032	.4756996E-01	12.23888
6	N	27	2	.1105920	.3076776E-01	3.594412
7	N	27	3	.2834249E-01	.3410362E-01	.8310699

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .65350319 RBAR**2 .64680760
SSR 28852582. SEE 373.34228
DURBIN-WATSON 1.99459187
Q(42)= 78.3388 SIGNIFICANCE LEVEL .565677E-03

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	491.6597	230.3257	2.134628
2	N	27	1	.1479388	.6976116E-01	2.120647
3	N	27	2	.3613292	.6634616E-01	5.446120
4	N	27	3	.4032880	.6850286E-01	5.887171
5	N	27	4	-.1213699E-01	.7279230E-01	-.1667346

SIC 36

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99617943 RBAR**2 .99606760
SSR 2487445.1 SEE 110.15389
DURBIN-WATSON 2.20857829
Q(42)= 59.0752 SIGNIFICANCE LEVEL .419842E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	242.1957	88.41002	2.739460
2	H	26	1	1.409585	.6329802E-01	22.26901
3	H	26	2	-.4097124	.6373124E-01	-6.428753
4	N	27	0	-.7465420	.4002698E-01	-18.65097
5	N	27	1	.4398262	.6106390E-01	7.202720
6	N	27	2	.1140802	.4285989E-01	2.661701
7	N	27	3	.1209181	.4388313E-01	2.755457

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .71028777 RBAR**2 .70468947
SSR 7909549.4 SEE 195.47476
DURBIN-WATSON 1.98392087
Q(42)= 46.2011 SIGNIFICANCE LEVEL .302871

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	328.7746	144.8035	2.270488
2	N	27	1	.3953073	.6968295E-01	5.672942
3	N	27	2	.2638675	.7263991E-01	3.632541
4	N	27	3	.2418712	.7373056E-01	3.280473
5	N	27	4	.5811976E-02	.7037213E-01	.8258918E-01

SIC 37

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .99645620 RBAR**2 .99635248
SSR 27019308. SEE 363.04476
DURBIN-WATSON 2.00011997
Q(42)= 49.9536 SIGNIFICANCE LEVEL .186699

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	1294.413	237.0771	5.459882
2	H	26	1	1.441687	.6605471E-01	21.82564
3	H	26	2	-.4391590	.6620127E-01	-6.633695
4	N	27	0	-.7982100	.2735219E-01	-29.18268
5	N	27	1	.5295063	.5752871E-01	9.204209
6	N	27	2	.7459051E-01	.2926520E-01	2.548778
7	N	27	3	.3475817E-01	.2842091E-01	1.222979

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .52107518 RBAR**2 .51182060
SSR .20287503E+09 SEE 989.98617
DURBIN-WATSON 2.00436204
Q(42)= 54.7768 SIGNIFICANCE LEVEL .893759E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	1310.556	440.2129	2.977096
2	N	27	1	.3818978	.6970998E-01	5.478381
3	N	27	2	.1742641	.7291481E-01	2.389969
4	N	27	3	.2243827	.7344729E-01	3.055017
5	N	27	4	.5057087E-01	.7032173E-01	.7191357

SIC 38

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 205
R**2 .96203321 RBAR**2 .96092198
SSR 451454.92 SEE 46.927807
DURBIN-WATSON 2.01439173
Q(42)= 31.4343 SIGNIFICANCE LEVEL .883422

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	43.19661	54.68571	.7899068
2	H	26	1	.9647422	.6952590E-01	13.87601
3	H	26	2	.2419852E-01	.7009648E-01	.3452173
4	N	27	0	-.5624113	.7201133E-01	-7.810040
5	N	27	1	.1549046	.7995057E-01	1.937505
6	N	27	2	.2177688	.7171438E-01	3.036613
7	N	27	3	.1603241	.7323630E-01	2.189134

FROM 1967: 5 UNTIL 1984: 12
OBSERVATIONS 212 DEGREES OF FREEDOM 207
R**2 .70981433 RBAR**2 .70420687
SSR 430802.71 SEE 45.619870
DURBIN-WATSON 1.99710127
Q(42)= 39.5400 SIGNIFICANCE LEVEL .579515

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*******	***	***	************	************	************
1	CONSTANT	0	0	88.12154	42.76069	2.060807
2	N	27	1	.2539888	.6927458E-01	3.666407
3	N	27	2	.3422988	.6833741E-01	5.008952
4	N	27	3	.3102004	.6820658E-01	4.547953
5	N	27	4	.8778049E-02	.6871102E-01	.1277531

Table 1

Second Moments, H = Finished goods - Backlogs

Industry	(1) var(Q)	(2) var(N)	(3) var(S)	(4) var(Q)/var(N)	(5) var(Q)/var(S)	(6) -2cov(N,ΔH) - var(ΔH)	(7) -2cov(S,ΔI) - var(ΔI)
Aggregate	4.709	8.856	4.540	.53	1.04	6.875	-.156
Stone, Clay Glass	.060	.062	.059	.96	1.02	.006	-.002
Primary Metals	.359	.525	.366	.68	.98	.303	.017
Fabricated Metals	.167	.272	.160	.62	1.04	.196	-.012
Non-electrical Machinery	.177	.375	.161	.47	1.10	.372	-.021
Electrical Machinery	.081	.143	.076	.57	1.06	.061	-.008
Transportation Equipment	.880	2.161	.866	.41	1.02	1.305	-.011
Instruments	.006	.008	.006	.78	1.08	.000	-.001

Table 2

Second Moments, H = Finished goods + WIP - Backlogs

Industry	(1) var(Q)	(2) var(N)	(3) var(S)	(4) var(Q)/var(N)	(5) var(Q)/var(S)	(6) -2cov(N,ΔH) - var(ΔH)	(7) -2cov(S,ΔI) - var(ΔI)
Aggregate	5.604	8.940	4.585	.63	1.22	5.631	-1.417
Stone, Clay Glass	.060	.062	.058	.97	1.03	.006	-.002
Primary Metals	.375	.525	.370	.72	1.01	.280	.008
Fabricated Metals	.193	.279	.168	.69	1.15	.164	-.044
Non-electrical Machinery	.232	.388	.172	.60	1.35	.290	-.108
Electrical Machinery	.094	.128	.070	.74	1.35	.049	-.034
Transportation Equipment	.904	1.983	.791	.46	1.14	1.115	-.146
Instruments	.008	.007	.005	1.11	1.57	-.000	-.004

In Tables 1 and 2, columns (6) and (7) essentially calculate var(N)-var(Q) and var(S)-var(Q) in a fashion that is robust to the presence of unit roots. See the text.

Table 3

Response to Unit Demand Shock, H = Finished goods + WIP - Backlogs

Durables

Period	N	Q	H
0	1.00	.36	-.64
1	.65	.39	-.90
12	.42	.40	-1.92
24	.19	.20	-1.91
60	.02	.04	-1.07
120	.00	.01	-.29

Stone, Clay and Glass

Period	N	Q	H
0	1.00	.58	-.42
1	.65	.50	-.55
12	.47	.47	-.49
24	.34	.35	-.41
60	.13	.13	-.28
120	.03	.03	-.22

Primary Metals

Period	N	Q	H
0	1.00	.25	-.75
1	.89	.37	-1.26
12	.36	.42	-1.74
24	.13	.18	-1.03
60	.01	.02	-.18
120	.00	.00	-.01

Fabricated Metals

Period	N	Q	H
0	1.00	.38	-.62
1	.51	.43	-.70
12	.38	.27	-1.98
24	.23	.13	-3.24
60	.06	-.09	-7.36
120	.00	-.37	-21.49

Machinery

Period	N	Q	H
0	1.00	.19	-.81
1	.15	.16	-.80
12	.25	.24	-1.27
24	.15	.15	-1.31
60	.03	.04	-.98
120	.00	.01	-.43

Electrical Machinery

Period	N	Q	H
0	1.00	.25	-.75
1	.40	.23	-.91
12	.28	.28	-1.16
24	.15	.15	-1.23
60	.02	.02	-1.28
120	.00	.00	-1.28

Transportation

Period	N	Q	H
0	1.00	.20	-.80
1	.38	.25	-.92
12	.16	.13	-1.49
24	.05	.04	-1.74
60	.00	-.00	-2.14
120	.00	-.00	-2.80

Instruments

Period	N	Q	H
0	1.00	.44	-.56
1	.25	.29	-.53
12	.29	.30	-.40
24	.17	.18	-.30
60	.04	.04	-.16
120	.00	.00	-.07

FIGURE 1

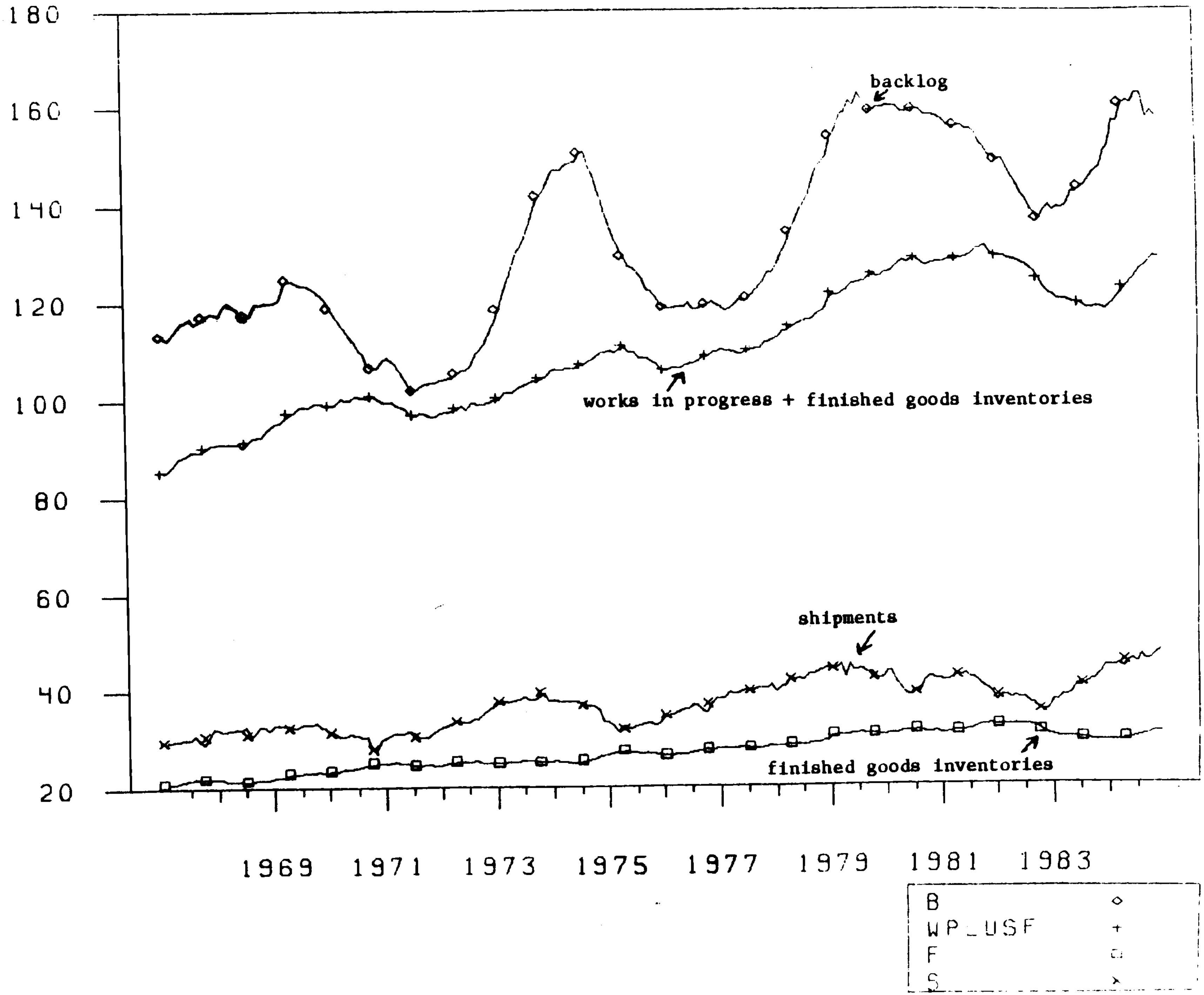
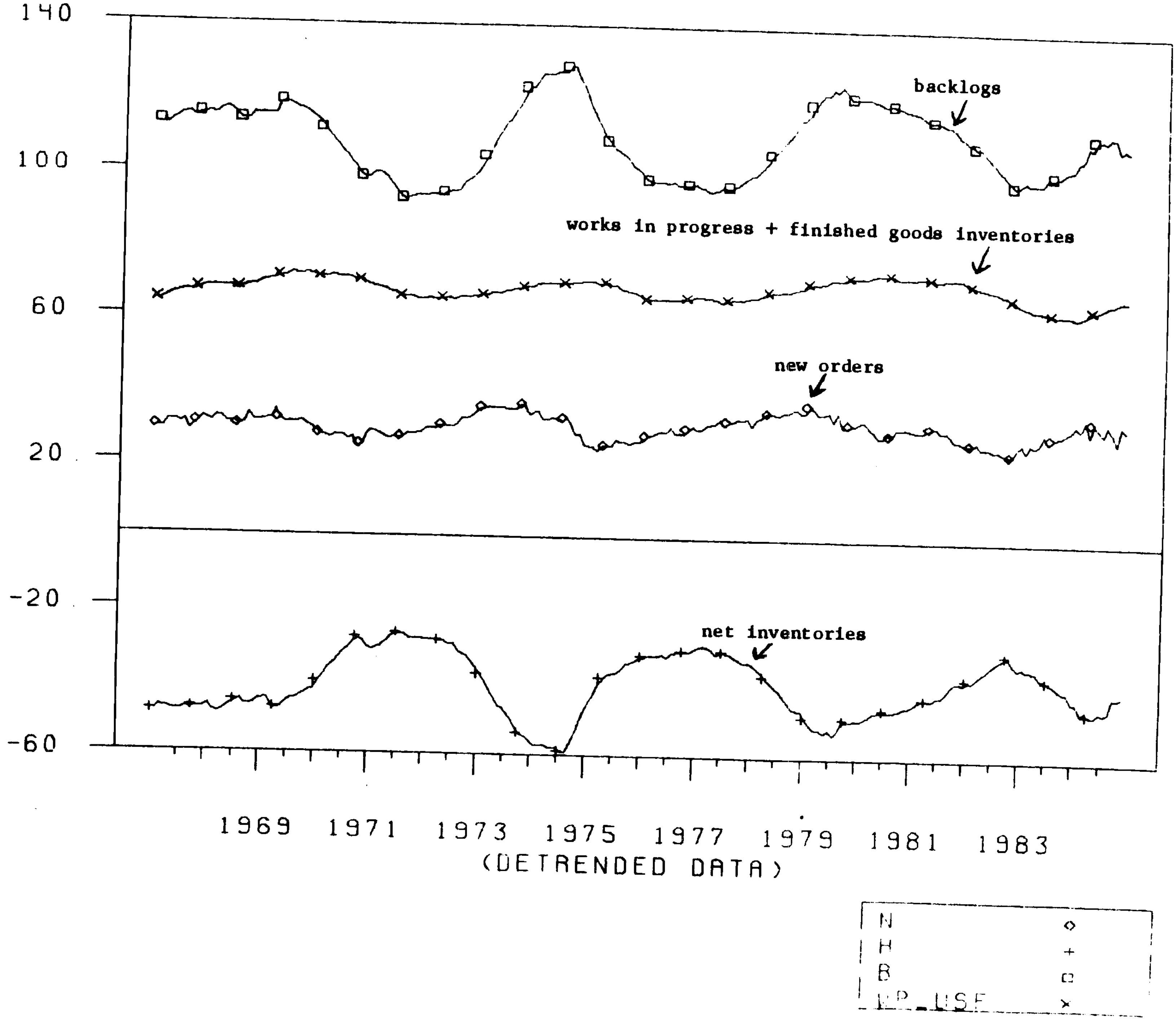
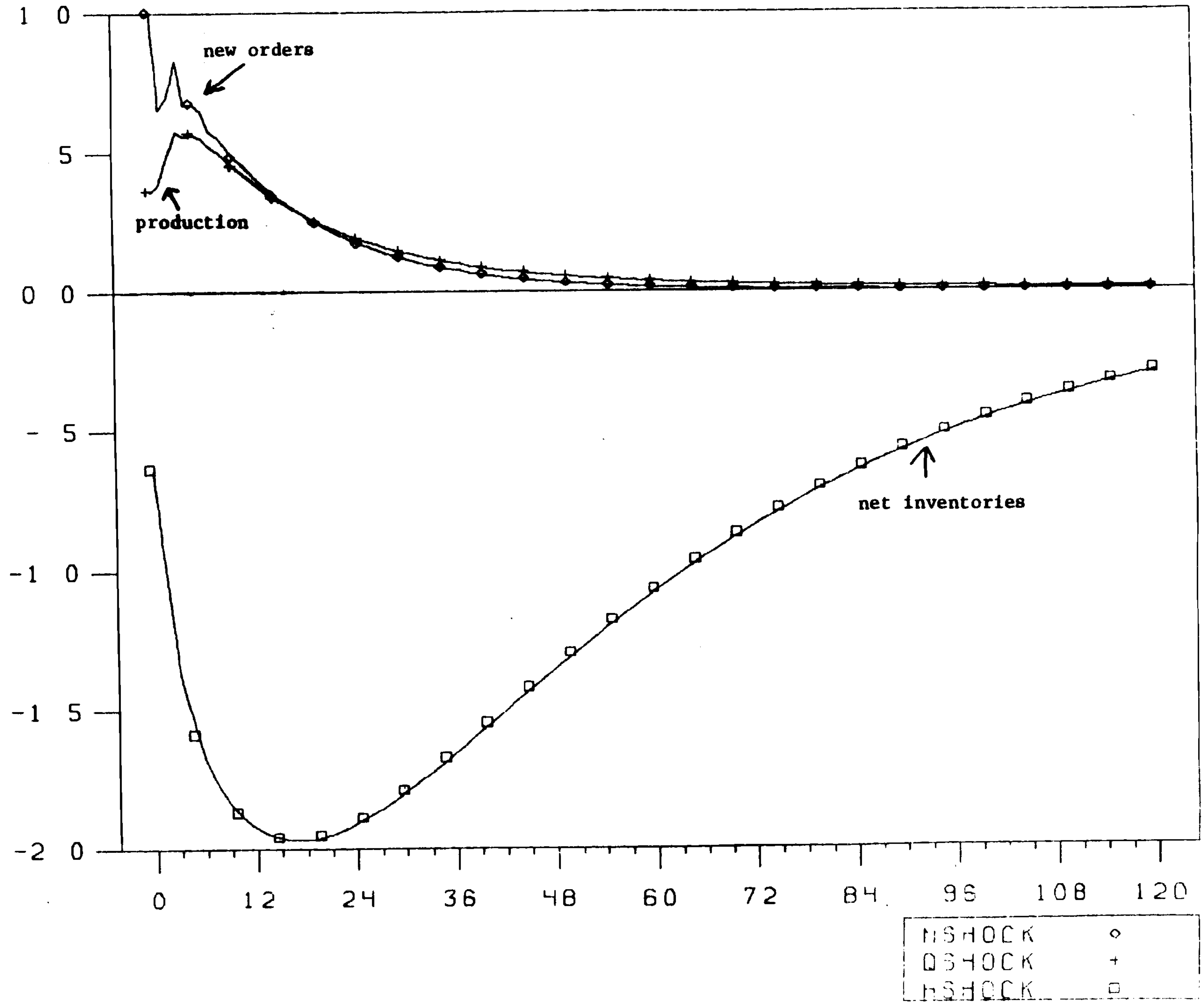

180
160
140
120
100
80
60
40
20
1969
1971
1973
1975
1977
1979
1981
1983
backlog
works in progress + finished goods inventories
shipments
finished goods inventories
B
WP_USF
F
S

FIGURE 2



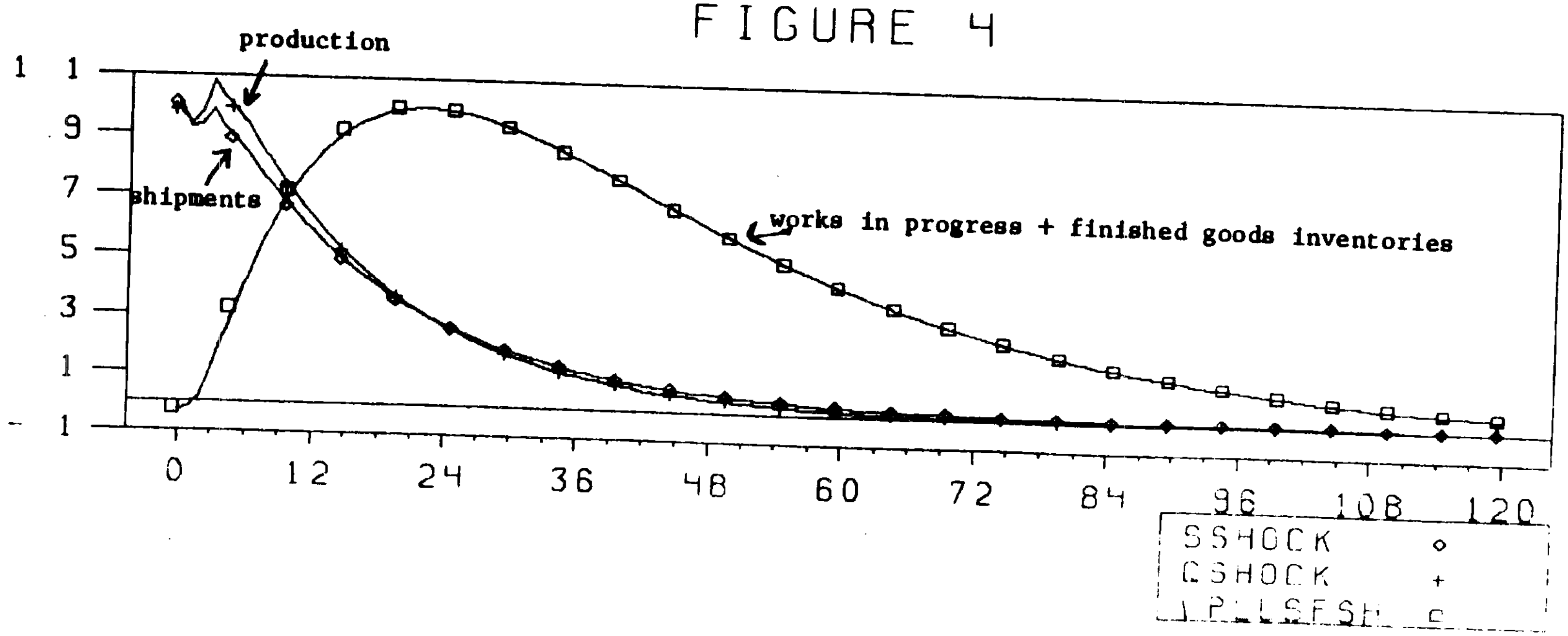
140
100
60
20
-20
-60
backlogs
works in progress + finished goods inventories
new orders
net inventories
1969 1971 1973 1975 1977 1979 1981 1983
(DETRENDED DATA)
N
H
B

FIGURE 3



1 0
5
0 0
- 5
-1 0
-1 5
-2 0
new orders
production
net inventories
0
12
24
36
48
60
72
84
96
108
120
NSHOCK ◇
QSHOCK +
HSHOCK □

FIGURE 4


production
shipments
works in progress + finished goods inventories
SSHOCK
CSHOCK
0
12
24
36
48
60
72
84
96
108
120