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CONSUMPTION RISK AND  
INTERNATIONAL ASSET RETURNS:  
SOME EMPIRICAL EVIDENCE

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ABSTRACT

The paper examines if real stock returns in four countries are consistent with consumption-based models of international asset pricing. The paper finds that ex-ante real stock returns exhibit statistically significant fluctuations over time and that these fluctuations cannot be explained by consumption-based models when the conditional covariances between real stock returns and the rate of change of consumption are assumed to be constant over time. These conditional covariances are then modeled and the paper finds that they too exhibit statistically significant fluctuations over time. However, even when conditional covariances are allowed to change over time, the paper finds that the consumption-based models do not fully explain real stock returns.

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## Introduction

The bulk of empirical analysis of international asset returns has explored returns to speculation in the forward foreign exchange market or the expected rate of return differential between nominal deposits demoninated in different currencies. Considerably less work has examined stock returns internationally.<sup>1</sup> This paper departs from the practice of focusing exclusively on the behavior of returns on forward speculation or nominal assets and to examine the behavior of international stock market returns. In particular the paper asks if observed returns are consistent with the predictions of consumption-based models of international asset pricing.

The paper proceeds as follows. In section I the implications of the consumption-based international asset pricing model are described and tests of the restrictions on equilibrium expected real returns along the lines suggested by Hansen and Hodrick (1983) and Gibbons and Ferson (1985) are implemented. These tests require that the conditional covariances between real returns and the rate of change of consumption be proportional across all assets. In section II the estimation of these conditional covariances is discussed and a test of their proportionality across assets is implemented. In addition, a test of the consumption-based international asset pricing model that allows for changing conditional covariances is implemented.

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<sup>1</sup> In addition, much of the existing work analyzing equilibrium models of international asset returns such as Solnik (1974) and Stehle (1977) is a decade old. Problems with the tests in these papers are discussed in Adler and Dumas (1983).

## I. Testing the Consumption-Based International Asset Pricing Model

In this section we develop and implement a test of the consumption-based international asset pricing model proposed by Stulz (1981) using aggregate indexes from the stock markets of the major industrial countries. For convenience throughout we will examine equilibrium real returns from the point of view of a representative investor in the U.S., which is taken to be the home country. We begin by defining some notation. Let  $P_t^j$  be the dollar price of the  $j$ th asset at the end of period  $t$ . Next, let  $D_{t,k}^j$  be the dollar value of the  $j$ th asset's dividend or interest payments from the end of period  $t$  to the end of period  $t+k$ . The real rate of return earned on asset  $j$  from the end of period  $t$  to the end of period  $t+k$ ,  $r_{t,k}^j$ , is defined by,

$$(1) \quad 1 + r_{t,k}^j = [(P_{t+k}^j + D_{t,k}^j) / P_t^j] / (1 + \pi_{t+k}),$$

where  $\pi_{t,k}$  is the rate of inflation from the end of period  $t$  to the end of period  $t+k$  for a representative domestic investor. Stulz (1981), following Breeden (1979), derives a continuous time model of equilibrium returns. The assumption that trading takes place continuously allows him to move to the limit of continuous time and derive an equilibrium relationship between asset returns.<sup>2</sup> A discrete-time conditional consumption-based asset pricing model for a representative domestic consumer-investor can be obtained using the results in Hansen and Richard (1987), where a "generic" conditional asset pricing model is

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<sup>2</sup> Grossman and Shiller (1982) show how a consumption beta model can be derived from the discrete time first-order conditions of a representative consumer-investor's utility maximization problem by taking the limit as the trading interval goes to zero. They point out that distributional assumptions or assumptions about the functional form of the utility function are alternatives to the use of continuous time analysis.

examined.<sup>3</sup> While restrictions on equilibrium returns may be obtained in this way, the means by which Stulz's results on aggregation across countries can be obtained in a conditional discrete-time framework remains unknown. All that we require here, however, is that a consumption-based capital asset pricing model exist for a representative domestic consumer-investor. In Stulz's model, expected real returns will satisfy,

$$(2) \quad E_t(r_{t,k}^j - r_{t,k}^z) = \beta_{j,t} E_t[r_{t,k}^p - r_{t,k}^z]$$

where  $r_{t,k}^p$  is the real return on the benchmark portfolio,  $r_{t,k}^z$  is the real rate of return on a portfolio whose real return is conditionally uncorrelated with domestic real consumption growth, and  $\beta_{j,t}$  is the "consumption beta" of asset  $j$  from the point of view of a representative domestic investor.

$$\beta_{j,t} = \text{cov}_t(r_{t,k}^j, \Delta c_{t+k}/c_t) / \text{cov}_t(r_{t,k}^p, \Delta c_{t+k}/c_t)$$

$\Delta c_{t+k} = c_{t+k} - c_t$  is the change of consumption between  $t$  and  $t+k$ .

A problem encountered when implementing tests of the consumption-beta model is that the benchmark portfolio is unobserved as is the portfolio whose return is uncorrelated with consumption. As a result, the two returns,  $r_{t,k}^z$  and  $r_{t,k}^p$  are unobserved. This problem can be circumvented by following the suggestion of Gibbons and Ferson (1985).<sup>4</sup> Since (2) must hold for all assets we can consider the equilibrium condition for

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<sup>3</sup> As Hansen and Richard (1987) point out, the consumption-based capital asset pricing model implies that the benchmark return in their analysis is the return on the aggregate consumption portfolio.

<sup>4</sup> Gibbons and Ferson (1985) propose the tests to be described as a means of testing the Sharpe-Lintner version of the CAPM. Extension to the case of consumption beta models involves no problems provided we are willing to make assumptions about the constancy of relative consumption betas over time. This assumption will be discussed below. The tests may be thought of as tests of any single-beta asset pricing model.

assets  $j$  and  $s$  and subtract the expected real return on asset  $s$  from the expected real return on asset  $j$ .

$$E_t(r_{t,k}^j - r_{t,k}^s) = (\beta_{j,t} - \beta_{s,t})E_t[r_{t,k}^p - r_{t,k}^z]$$

Next, we can divide this difference by the difference between expected return on an arbitrarily chosen reference asset,  $1$ , and the expected return on asset  $s$ . We then find that we have eliminated the unobserved returns.

$$(3) \quad E_t(r_{t,k}^j - r_{t,k}^s) / E_t(r_{t,k}^1 - r_{t,k}^s) = (\beta_{j,t} - \beta_{s,t}) / (\beta_{1,t} - \beta_{s,t}) \equiv \gamma_{j,t}$$

In order to simplify notation, we will suppress the terms involving  $r_{t,k}^s$  and will refer to the difference between  $r_{t,k}^j$  and  $r_{t,k}^s$  simply as  $r_{t,k}^j$ .

Next, we must model the expected returns as they too are unobservable. We assume the econometrician observes some data  $X_t$  that are included in the time- $t$  information set of agents, and consider the projection of expected real profits onto  $X_t$ .

$$(4) \quad E_t(r_{t,k}^j) = X_t \alpha_j + u_{t,k}^j,$$

where  $u_{t,k}^j$  is the projection equation error, which is orthogonal to  $X_t$  by construction. Since ex-ante returns are unobservable we work with observable realized profits, which can be decomposed into expected profits and a forecast error,  $\epsilon_{t,k}^j$ ,

$$(4') \quad r_{t,k}^j = X_t \alpha_j + u_{t,k}^j + \epsilon_{t,k}^j = X_t \alpha_j + \eta_{t,k}^j.$$

The key assumption that allows us to make inferences about the behavior of ex-ante returns based on the observed behavior of realized returns is the assumption that expectations are rational. That is we assume that forecast errors are unforecastable given information available at the time the forecast is made. Since  $X_t$  is assumed to be in the time  $t$  information set of agents,  $\epsilon_{t,k}^j$  is orthogonal to  $X_t$ . In addition,  $u_{t,k}^j$  is orthogonal to  $X_t$  by construction so that the composite error term is orthogonal to  $X_t$ . Estimating equation (4') by OLS thus produces consis-

tent estimates of the projection equation  $\alpha$ 's under the appropriate regularity conditions.<sup>5</sup> In order to test hypotheses concerning the  $\alpha$ 's, we must have a consistent estimate of their covariance matrix. In the empirical work carried out in this paper,  $k=3$ , and  $\epsilon_{t,3}^j$  is not realized until  $t+3$ . Thus,  $\epsilon_{t,3}^j$  will follow a second-order moving average process (MA(2)). In addition,  $u_{t,k}^j$  does not have to be serially uncorrelated so that the composite error will be serially correlated. Consistent estimates of the variance-covariance matrix of  $\alpha_j$  are calculated with a method outlined by Hansen (1982) and Cumby, Huizinga and Obstfeld (1983) that allow both for serial correlation and for conditional heteroscedasticity in the residuals. This is an attractive feature of this technique since an assumption of conditional homoscedasticity of asset returns is unlikely to be satisfied.<sup>6</sup>

If we combine (3) with the projection equations, (4) and (4'), that the econometrician uses to estimate conditional expectations based on the observable data  $X_t$  we obtain,

$$\begin{aligned} r_{t,k}^j &= r_{j,t} E_t(r_{t,k}^1) + \eta_{t,k}^1 \\ &= r_{j,t} (X_t \alpha_1 + u_{t,k}^1) + \eta_{t,k}^1 \end{aligned}$$

Thus  $\alpha_j = r_{j,t} \alpha_1$ . Gibbons and Ferson (1985) show that if we assume that  $r_{j,t}$ , the ratio of the consumption betas, is constant over time (or, equivalently, the conditional covariances between asset returns and the rate of change of real consumption are proportional across assets) a test of the asset pricing model (3) can be carried out by estimating a

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<sup>5</sup> In particular, we require  $X_t$  and  $\eta_j$  to be stationary and ergodic.

<sup>6</sup> Cumby and Obstfeld (1984), Hodrick and Srivastava (1984), and Giovannini and Jorion (1987) all find evidence of conditional heteroscedasticity in speculative returns in the foreign exchange market. Giovannini and Jorion also find similar conditional heteroscedasticity in U.S. stock market returns.

system of projection equations and testing the hypothesis that the coefficients in each equation are proportional to the coefficients in the first equation.<sup>7</sup> If there are  $N$  assets and  $k$  regressors in each of the projection equations, there will be  $Nk$  regressors in the system but only  $k + (N-1)$  parameters when the proportionality restrictions are imposed. There are thus  $Nk - (k+N-1)$  parameter restrictions that can be tested. If the model is correct and if the auxiliary assumptions concerning the constancy of the relative consumption betas and the rationality of expectations are correct, these parameter restrictions should be satisfied by the data.

Estimation of the restricted system of equations and testing of the parameter restrictions can be carried out using Hansen's (1982) generalized method of moments (GMM) procedure. Application of the GMM procedure begins by noting that each element of  $X_t$  is orthogonal to  $\eta_{t,k}^j$ , for all  $j$ . Thus there are  $Nk$  orthogonality conditions in the projection equations for the real returns. The GMM estimator is obtained by choosing the parameters of the system so as to make the sample versions of the orthogonality conditions as close to their population value of zero as possible by minimizing a quadratic form in the sample orthogonality conditions. In this case where there are  $Nk$  orthogonality conditions and  $k + N - 1$  parameters, there are  $k + N - 1$  first-order conditions for the minimization of the quadratic form and thus we can only set  $k + N - 1$  linear combinations of the sample orthogonality conditions to zero. Hansen (1982) notes that if the constraints of the model are true, the additional  $Nk - (k + N - 1)$  sample orthogonality conditions should be close to zero as well. However if the constraints are not satisfied, the

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<sup>7</sup> The assumption that the conditional covariances are proportional across assets is tested below.



additional sample orthogonality conditions should not be close to zero and thus the value of the criterion function, which is a quadratic form in these orthogonality conditions, should be large. Hansen (1982) proves that under the null hypothesis that the constraints are satisfied the criterion function is distributed as  $\chi^2$  with degrees of freedom equal to the number of parameter restrictions. Thus, estimation and testing may be carried out quite simply by using the appropriate GMM estimator.

Hansen and Hodrick (1983) test the restrictions implied by a single-beta CAPM model of the foreign exchange risk premium. The test they carry out is equivalent to the Gibbons-Ferson test. Perhaps this is not surprising since both are tests of single-beta asset pricing models. The fact that the two tests are identical is obscured somewhat by differences in interpretation in the motivation of the tests. Hansen and Hodrick assume that the betas are constant and treat the expected return on the benchmark portfolio as an unobserved latent variable assumed to be linearly related to some data  $X_t$ . Since the expected return on the benchmark portfolio enters the equilibrium condition for returns to speculation in each currency, they obtain a set of proportionality restrictions on a system of equations. Gibbons and Ferson, on the other hand substitute out the expected benchmark return by using an arbitrarily chosen reference asset and derive a set of proportionality restrictions that are identical to those obtained by Hansen and Hodrick.

We now turn to the results from estimating the projection equations describing the behavior of real stock returns and the tests of the consumption beta model. We consider the returns on the aggregate market indexes in four countries, the United States, the United Kingdom, West Germany, and Japan taken from Morgan Stanley's Capital International Perspective. Together, these four represent 84% of the capitalized value

of the Capital International world index. A sample of January 1974 to December 1986 is chosen to coincide with the period of floating exchange rates. Three-month holding period returns measured in U.S. dollars are calculated for each of the aggregate portfolios. We take  $r_{t,k}^s$  to be the real return on a safe nominal dollar deposit and thus subtract the real three-month eurodollar deposit rate from each.<sup>8</sup>

In estimating the projection equations (4'), we need to consider what  $X_t$  variables to use in the regressions. In principle any variables in the information set are reasonable candidates. The  $X_t$  used in the projections are the dividend yield on each of the national market portfolios, the rate of change of consumption ( $c_{t+3}/c_t - 1$ ) lagged 3 months, terms of trade ( $TDT_t^j$ ) for each country, and inflation in each country lagged three months ( $\pi_{t-3,3}^j$ ). There are thus fourteen variables in  $X_t$ , including a constant term.<sup>9</sup> Consumption and the terms of trade are employed since various models suggest that these should affect savings and investment decisions and therefore affect equilibrium expected returns. Inflation is also included in  $X_t$  since several studies have pointed to a systematic relationship between stock returns and inflation.<sup>10</sup>

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<sup>8</sup> Exchange rates are end of month rates taken from International Financial Statistics. The three-month eurodollar rate is from Morgan Guaranty's World Financial Markets. U.S. inflation is calculated using the CPI-U, which is taken from the Survey of Current Business.

<sup>9</sup> The consumption data are real spending on nondurables and services per capita, taken from the Survey of Current Business. The terms of trade are calculated as the ratio of the unit value of exports to the unit value of imports, which are obtained from International Financial Statistics, as are the CPI data for all countries but the U.S.

<sup>10</sup> Solnik (1983) and Gultekin (1983) present international evidence on this relationship. Stulz (1986) presents an equilibrium model in which expected real stock returns are related to expected inflation.

Table 1 contains the results from the projections of real stock returns on  $X_t$ . The  $\chi^2$  statistics for testing the null hypothesis that all coefficients but the constant term are zero are presented along with their probability values and the  $R^2$  from each projection.<sup>11</sup> The results indicate that we can reject the null hypothesis that expected stock returns are constant at all reasonable significance levels.<sup>12</sup>

Table 2 contains the results of the tests of the consumption-based models when the relative consumption betas are assumed to be constant. Estimation of the full system of four equations each of which contains 14 regressors proved to be computationally infeasible. We therefore carry out the Gibbons-Ferson tests in reduced systems of two each.<sup>13</sup> The U.S. market index is chosen to be the reference asset in each case. In each of these reduced systems there are 28 orthogonality conditions and 15 parameters to be estimated. There are thus 13 parameter restrictions in each system. The values of the criterion functions, which are  $\chi^2$  random variables with 13 degrees freedom, are 24.3 for the first pair countries (U.S. and U.K), 16.2 for the second pair of countries (U.S. and West Germany), and 24.1 for the third pair of countries (U.S. and

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<sup>11</sup> The  $R^2$  is the squared correlation coefficient of the dependent variable and its fitted value.

<sup>12</sup> The autocorrelations of the residuals from the projections are generally consistent with a second-order moving average as would be expected if the projection error component of the composite error is small relative to the forecast error component, indicating that a reasonable information set has been chosen. All tests reported in the paper have also been carried out lagging  $X_t$  one period to accommodate possible reporting lags. In no instance does this affect the results of the hypothesis tests.

<sup>13</sup> Estimation requires the inversion of a matrix that is  $N_k \times N_k$ , which is in this case  $56 \times 56$ . Attempts to compute this inversion proved unsuccessful. If the restrictions are rejected by the data, the use of the three smaller systems does not present any problems in interpreting the test results since the full system test would simply provide stronger rejections.

Japan). The restrictions implied by the consumption beta model are then rejected at standard significance levels in two of the three cases.

## II. Modeling Conditional Covariances

The behavior of the conditional covariance between asset returns and the rate of change of consumption plays a central role in consumption-based models asset pricing. In this section we discuss modeling this conditional covariance with two goals in mind. First, it may be possible that the restrictions implied by the consumption-beta model are rejected due to time varying relative consumption betas. If the conditional covariances can be modeled, the constancy of the relative consumption betas can be tested. Second, if we find that the relative consumption betas change over time, we want to determine if the movement they exhibit can account for ex-ante real stock returns.

The estimation of the conditional covariance may be carried out by extending the results of Amemiya (1977) and Hasbrouck (1985) who consider a regression model in which the variance of the disturbance term is linearly related to a set of observable data and derive the large sample distribution of estimators of the parameters of the linear variance function.<sup>14</sup> The extension to the case at hand is derived in Cumby (1986).

We are interested in estimating the conditional covariance between the rate of change of consumption and real aggregate stock returns.

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<sup>14</sup> Hasbrouck (1985) extends the results in Amemiya (1977) in several important directions. Most importantly, he allows the regressors to be stochastic, does not require that the regression disturbance be normal, and allows the addition of a stochastic disturbance to the linear variance function.

$$\begin{aligned} \sigma_{j,t} &= \text{cov}_t(r_{t,k}^j - r_{t,k}^s, \Delta c_{t+k}/c_t) \\ &= E_t[(r_{t,k}^j - r_{t,k}^s - E_t(r_{t,k}^j - r_{t,k}^s))(\Delta c_{t+k}/c_t - E_t(\Delta c_{t+k}/c_t))] \end{aligned}$$

where  $\Delta c_{t+k} = c_{t+k} - c_t$ . The econometrician, who is assumed to observe a set of variable,  $X_t$ , can use as an estimate of the conditional covariance the projection of  $\sigma_{j,t}$  onto  $X_t$ ,

$$\sigma_{j,t} = X_t \theta_j + \omega_t^j$$

It will prove convenient to rewrite the projection as,

$$\eta_{t,k}^j \eta_{t,k}^c = X_t \theta_j + \omega_t^j + (\eta_{t,k}^j \eta_{t,k}^c - \sigma_{j,t}) = X_t \theta_j + \omega_t^j$$

Where  $\eta_{t,k}^j$  and  $\eta_{t,k}^c$  are the disturbances from projections of  $r_{t,k}^j$  and  $\Delta c_{t+k}/c_t$  onto  $X_t$ , respectively. Since  $\eta_{t,k}^j$  and  $\eta_{t,k}^c$  unobservable, we need to work with the residuals,  $\hat{\eta}_{t,k}^j = \eta_{t,k}^j - X_t(\hat{\alpha}_j - \alpha_j)$  and  $\hat{\eta}_{t,k}^c = \eta_{t,k}^c - X_t(\hat{\alpha}_c - \alpha_c)$ . The projection can then be rewritten in terms of observables,

$$(5) \quad \hat{\eta}_{t,k}^j \hat{\eta}_{t,k}^c = X_t \theta_j + \omega_t^j - \eta_{t,k}^c X_t(\hat{\alpha}_j - \alpha_j) - \eta_{t,k}^j X_t(\hat{\alpha}_c - \alpha_c) + X_t(\hat{\alpha}_c - \alpha_c) X_t(\hat{\alpha}_j - \alpha_j)$$

In Cumby (1986) it is shown that the OLS estimate of  $\theta_j$  is consistent and asymptotically normal with a covariance matrix that can be consistently estimated using the techniques described in Hansen (1982) and Cumby, Huizinga, and Obstfeld (1983).<sup>15</sup>

Once a consistent estimate of  $\theta_j$  and its asymptotic covariance matrix are obtained, we can test hypotheses about the conditional covariance of

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<sup>15</sup> It should be pointed out that since we are examining the conditional covariances of the  $\eta$  and not the conditional covariances of the  $\epsilon$ , the covariances we estimate are the sum of the covariances of the projection errors and the covariances real returns and real consumption. Therefore any inference about the movement of the conditional covariance of real returns and real consumption over time based on the evidence presented here is conditional on assumptions we make concerning the movements of covariance of the projection errors over time. If the data do a good job of describing the movements of real returns so that the projection errors are small, we may reasonably assume that the covariance of projection errors is small. The estimates will then be dominated by movements in the conditional covariances of real returns and real consumption.

real consumption and real stock returns. The first of these hypotheses is the constancy of this conditional covariance. This hypothesis is the hypothesis that all elements of  $\theta_j$  are zero except for the constant term. Next, if the hypothesis of a constant conditional covariance is rejected, we need to determine if the comovements of the conditional covariance and real stock returns are consistent with the consumption-based international asset pricing model. We proceed in two steps. First, we can test the assumption of constant relative consumption betas required for the Gibbons-Ferson test by using the projection equations (5). If relative consumption betas are constant over time, the conditional covariances must all move together over time. The hypothesis that the conditional covariances move together can be tested by determining if the coefficients in the projection equations (5) are proportional across assets. This test can be carried out in the manner described above.

The second step is to determine if observed returns are consistent with the consumption-based international asset pricing model when we allow for variation over time in the conditional covariances. In order to see how we can do this, rewrite (3), using the definition of the consumption betas to obtain,

$$(3') E_t(r_{t,k}^j)/E_t(r_{t,k}^1) = (\sigma_{j,t}/\sigma_{1,t}).$$

Next, substitute realized real returns less a forecast error for the expected returns and impound the forecast errors into the error term.

Finally, add and subtract  $(\hat{\sigma}_{j,t}/\hat{\sigma}_{1,t})$  and rewrite to obtain.

$$(6) (\hat{\sigma}_{j,t}/\hat{\sigma}_{1,t}) = a + br_{t,k}^j/r_{t,k}^1 - bv_{t,k}^j - [(\sigma_{j,t}/\sigma_{1,t}) - (\hat{\sigma}_{j,t}/\hat{\sigma}_{1,t})]$$

where  $v_{t,k}^j$  is the difference between  $E_t(r_{t,k}^j)/E_t(r_{t,k}^1)$  and  $r_{t,k}^j/r_{t,k}^1$ . If stock returns are consistent with the consumption based model of international asset pricing, we expect to find  $a=0$  and  $b=1$ . Two problems

arise when estimating the parameters of (13). First, the realized real return on the right hand side of (13) will be correlated with the forecast error,  $v_{t,k}^1$ . We therefore need to use an instrumental variables procedure to obtain consistent estimates of  $a$  and  $b$ . Second, since the left-hand-side variable uses the fitted values from the projections rather than the true conditional covariances and expected return, an additional component arises in the error term. This additional component may introduce both heteroscedasticity and serial correlation to the error in (6) that we need to take into account in estimation. Instrumental variables estimation of (6) taking account of possible heteroscedastic and serially correlated errors can be carried out using the two-step two-stage least squares technique of Cumby, Huizinga, and Obstfeld (1983). The choice of instruments is straightforward since the variables  $X_t$  used in the real return projection equations will be uncorrelated with the error term. Since the serial correlation in the error term is of unknown order, the spectral estimator of the covariance matrix of the parameters of (6) is used.

In estimating and testing hypotheses about the conditional covariances, the choice of the data to include in  $X_t$  must again be made. It seems natural to use the same information to estimate the behavior of conditional first moments and conditional second moments so the same set of  $X_t$  as is used above will again be employed. Prior to proceeding with estimation, a problem with consumption data should be confronted. Published data measure consumption over an interval rather than at a point in time. Using the results in Breeden, Gibbons, and Litzenberger (1986), it can be shown that if monthly data sampled quarterly are used and if the covariance between real returns and real consumption growth is constant, the estimate of the covariance obtained using interval con-

sumption data will understate the true "spot" covariance by twenty percent. In order to correct for this bias the dependent variables in the projections (12) could be multiplied by 1.2 prior to estimation. Doing so would, of course, leave the test statistics unchanged. Since such a correction would also leave relative consumption betas unchanged, this correction is pursued here.

The results of estimating the conditional covariances can be found in Table 3, where the  $\chi^2$  statistics for testing the the constancy of conditional covariances are reported along with their probability values and the  $R^2$  from each projection. In all cases but the West German market index, the hypothesis of constant conditional covariances can be rejected at standard significance levels. In the West German case rejection is at the eleven percent level.

Given that the covariances change over time, do they do so in a way that explains the behavior of ex-ante real returns? Table 3 also contains the results of the tests of proportionality of the conditional covariances for the three pairs of assets examined above. The tests are carried out using the GMM procedure used in carrying out the Gibbons-Ferson tests. Again there are 28 orthogonality conditions in the each system and 15 parameters to be estimated in each so that each system has 13 restrictions to be tested. The table reports the  $\chi^2(13)$  statistics for the hypothesis that the conditional covariances are proportional along with the corresponding probability values. We can see that the proportionality constraints are rejected at the six percent level for the U.S., Japan combination, and at the 12 percent level for the U.S., U.K. combination. There is no evidence that the constraints do not hold for the U.S., West Germany combination. Thus a violation of the assumption of proportional conditional covariances may account for the rejection



tion of the restrictions implied by the single-beta model with constant relative betas for the U.S. and Japan, but is unlikely to account for the rejection for the U.S. and the U.K. Since the single-beta model with constant relative betas is not rejected for the U.S. and Germany, failing to reject the proportionality of conditional covariances is reassuring.

Table 4 contains the results of the tests of the consumption-based model of international asset pricing with conditional covariances that are allowed to vary over time. Recall that under the null hypothesis we expect to find a slope of one and an intercept of zero. In all three cases we find that while the slope coefficients are positive, they are less than one. The largest slope coefficient (approximately .5) is found in the U.S. - Japan combination, the combination that exhibited the strongest evidence against the hypothesis of constant relative consumption betas. The hypothesis that the slope coefficient is one is rejected at reasonable significance levels in all cases. Although movements in relative consumption betas and relative returns are positively correlated as the consumption-based model predicts, rejection of the null hypothesis that the slope coefficients are one implies that even when conditional covariances are allowed to change over time, the consumption-based international asset pricing model falls short of fully explaining observed real stock returns.

Since evidence against the model is found only when U.K. and Japanese returns are examined, barriers to international investment may be behind the results. Capital controls were in place in the U.K. until October 1979 and restrictions on capital movements in Japan were not eased until after 1980. An attempt to carry out the tests using data only from the latter part of the sample was made. However, the sample turned out to be

too short to obtain reliable estimates.

### III. Concluding remarks

The paper examines if real stock returns in four countries are consistent with consumption-based models of international asset pricing. Tests such as those suggested by Gibbons and Ferson (1985) are considered first. These tests require that we assume that the conditional covariance of real stock returns and the rate of change of real consumption move together over time for all assets. The restrictions implied by the consumption-based model assuming the proportionality of the conditional covariances are rejected by the data when combinations of the U.S. and the U.K. and the U.S. and Japan are examined. No evidence against the restrictions is found when the U.S. and Germany are considered. Next, estimates of these conditional covariances are presented and the evidence shows that, like the ex-ante real stock returns, they exhibit statistically significant fluctuations over time. The assumption of proportional conditional covariances necessary for implementation of the Gibbons-Ferson test is also tested and is rejected in the two cases in which the model is rejected by the Gibbons-Ferson test. This raises the possibility that the rejections may be due to the auxiliary assumption of constant relative consumption betas rather than a failure of the model itself. A final set of tests is carried out in which conditional covariances are allowed to change over time and the results indicate that observed real stock returns cannot be explained fully by consumption-based models of international asset pricing.

References

- Adler, Michael and Bernard Dumas, 1983, International portfolio choice and corporation finance: A synthesis, Journal of Finance, 38, 925-977.
- Amemiya, Takeshi, 1977, A note on a heteroscedastic model, Journal of Econometrics, 6, 365-370.
- Breeden, Douglas T., 1979, An intertemporal -asset pricing model with stochastic consumption and investment opportunities, Journal of Financial Economics, 7, 265-196.
- Cumby, Robert E., 1986, Is it risk? Explaining deviations from uncovered interest parity, working paper, New York University Graduate School of Business Administration.
- Cumby, Robert E., John Huizinga, and Maurice Obstfeld, 1983, Two-step two-stage least squares estimation in models with rational expectations, Journal of Econometrics, 11, 333-355.
- Cumby Robert E. and Maurice Obstfeld, 1984, International interest rate and price level linkages under floating exchange rates: A review of recent evidence, in J. F. O. Bilson and R. Marston (eds), Exchange Rate Theory and Practice, Chicago: University of Chicago Press for the National Bureau of Economic Research.
- Gibbons, Michael R. and Wayne Ferson, 1985, Testing asset pricing models with changing expectations of an unobservable market portfolio, Journal of Financial Economics, 14, 217-236.
- Giovannini, Alberto and Philippe Jorion, 1987, Interest rates and risk premia in the stock market and in the foreign exchange market, Journal of International Money and Finance, 6, 107-123.
- Grossman, Sanford J. and Robert J. Shiller, 1982, Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information, Journal of Financial Economics, 10, 195-210.
- Gultekin, N. Bulent, 1983, Stock market returns and inflation: Evidence from other countries, Journal of Finance, 38, 49-65.
- Hansen, Lars P., 1982, Large sample properties of generalized method of moments estimators, Econometrica, 50, 1029-1054.
- Hansen, Lars P. and Robert J. Hodrick, 1983, Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models, in J.A. Frenkel (ed), Exchange Rates and International Macroeconomics, Chicago: University of Chicago Press for the National Bureau of Economic Research.
- Hansen, Lars P. and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, Econometrica, 55, 587-613.

Hasbrouck, Joel, 1985, Ex ante uncertainty and ex post variance in linear return models: An econometric analysis, working paper, New York University, Graduate School of Business Administration.

Hodrick, Robert J. and Sanjay Srivastava, 1984, An investigation of risk and return in forward foreign exchange, Journal of International Money and Finance, 3, 5-29.

Solnik, Bruno, 1974, The international pricing of risk: An empirical investigation of the world capital market structure, Journal of Finance, 29, 48-54.

Solnik, Bruno, 1983, The relationship between stock prices and inflationary expectations: The international evidence, Journal of Finance, 38, 35-48.

Stehle, Richard, 1977, An empirical test fo the alternative hypotheses of national and international pricing of risky assets, Journal of Finance, 32, 493-502.

Stulz, Rene M., 1981, A model of international asset pricing, Journal of Financial Economics, 9, 383-406.

Stulz, Rene M., 1986, Asset pricing and expected inflation, Journal of Finance, 41, 209-223.

Table 1: Ex Ante Real Stock Returns\*

Chi-Square Tests with Dividend Yield, Consumption Growth, Terms of Trade, and Inflation

$$r_{i,t+k}^e = a_i + \sum b_{ji} (D_{i,t}/P_{i,t} + d_{j,t}/c_{t-3} + \sum e_{ji} TOT_{i,t} + \sum f_{ji} P_{i,t}/P_{i,t-3})$$

Country	$\chi^2(13)$	$R^2$
U.S.	140.76 (0.00)	.471
U.K.	71.13 (0.00)	.446
Germany	45.26 (.19E-04)	.339
Japan	119.80 (0.00)	.506

Table 2: Gibbons-Ferson Tests of Single Beta Model of Stock Returns\*

Countries	$\chi^2(13)$
U.S., U.K.	24.32 (0.03)
U.S., Germany	16.17 (0.24)
U.S., Japan	24.08 (0.03)

\* The sample period is 1974:1 to 1986:12.  $R^2$  is defined as the squared correlation coefficient of the dependent variable and its fitted value. Marginal significance levels are in parentheses below the  $\chi^2$  statistics.

Table 3: Conditional Covariances

A. Chi-Square Tests with Dividend Yield, Consumption Growth, Terms of Trade, and Inflation\*

$$\eta_{t,k}^j \eta_{t,k}^c = a_j + \sum b_{ji} (D_{it}/P_{it} + d_{jct}/c_{t-3} + \sum e_{ji} TOT_{it} + \sum f_{ji} p_{it}/p_{it3})$$

Country	$\chi^2(13)$	$R^2$
U.S.	76.89 (0.00)	.119
U.K.	48.04 (.64E-05)	.269
Germany	19.44 (.11)	.057
Japan	75.21 (0.00)	.148

B. Tests of the Proportionality of Conditional Covariances

Countries	$\chi^2(13)$
U.S., U.K.	19.24 (0.12)
U.S., Germany	11.69 (0.55)
U.S., Japan	21.76 (0.06)

\* The sample period is 1974:1 to 1986:12.  $R^2$  is defined as the squared correlation coefficient of the dependent variable and its fitted value. Marginal significance levels are in parentheses below the  $\chi^2$  statistics.

Table 4: Tests of Consumption Based Model with  
Time-Varying Conditional Covariances

$$\hat{\sigma}_{j,t} / \hat{\sigma}_{US,t} = a_j + b_j r_{t,k}^j / r_{t,k}^{US}$$

Country	$a_j$	$b_j$
U.K.	0.144 (0.009)	0.105 (0.023)
Germany	-0.082 (0.035)	0.263 (0.179)
Japan	-0.064 (0.020)	0.498 (0.167)

\* The sample period is 1974:1 to 1986:12. Asymptotic standard errors are in parentheses.