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INFLATION: THEORY AND EVIDENCE

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ABSTRACT

This survey attempts to cover an extremely broad topic by organizing around three sets of issues: ongoing (steady state) inflation; cyclical interaction of inflation with real variables; and positive analysis of monetary policy behavior. With regard to ongoing inflation, the paper demonstrates that the principal conclusions of theoretical analysis are not highly sensitive to details of model specification, provided that the latter posits rational agents free of money illusion. Whether one assumes finite-lived or infinite-lived agents, such models suggest that steady-state inflation rates will conform fairly closely to money stock growth rates, that superneutrality is not strictly implied but departures should be minor, and that socially optimal inflation rates correspond to the Chicago Rule. The first two of these conclusions are consistent with available evidence. With regard to the cyclical interaction of inflation with aggregate output and employment, there is much less professional agreement: four classes of aggregate-supply (or Phillips curve) theories are currently in use by researchers and at least two have been able thus far to withstand attempts at refutation. With regard to policy, a leading question is why the authorities have behaved, over the postwar era, in a manner that has resulted in a many-fold increase in the price level in most industrialized nations. A full answer will require a better theory of the political process than is now available, but an important insight regarding inflationary bias is suggested by models that focus on the effects of "discretionary" period-by-period decision making by a monetary authority that seeks to avoid unemployment as well as inflation.

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1. Introduction

Inflation is a topic that is so broad as to be almost co-extensive with monetary economics. Consequently, there are many ways in which the present chapter overlaps with others in this Handbook. In particular, various topics that are considered in detail in other chapters are treated more briefly here.[1] It needs to be emphasized, accordingly, that the main purpose of the present discussion is not to attempt authoritative treatments of those specialized topics, but instead to provide a moderately general overview of the subject of inflation. To a significant extent, the aim is offer a framework for coherent thought on that subject, both in terms of relevant theory and evidence regarding competing hypotheses.

During the dozen years that have passed since the preparation of the ambitious survey article by Laidler and Parkin (1975), the nature of research in monetary economics has changed considerably. Rational expectations has become the mainstream hypothesis concerning expectational behavior, the "Ricardian equivalence theorem" has become a familiar notion in policy discussions, issues involving overlapping-generations models and cash-in-advance constraints have become common fare, "bubble" and "sunspot" phenomena have been extensively investigated, and problems associated with dynamic inconsistency of policy have been pondered. It is natural, then, that the outline of a survey today would be different from that of the Laidler-Parkin contribution, just as its outline differed from those of earlier surveys (e.g., Bronfenbrenner and Holtzman (1963) and Johnson (1963).)

Nevertheless, one of the Laidler-Parkin organizational principles remains extremely useful. That principle is provided by the distinction between steady ongoing inflation, which will be anticipated by rational agents, and irregular cyclical outbursts of above- or below-normal inflation rates, which are likely to be unanticipated. While this distinction may be somewhat unclear in practice, it is an essential one in terms of theoretical analysis since the effects of anticipated and surprise inflation may be very different. Certain effects of the former, moreover, may be quantitatively important only if maintained over a long span of time. Accordingly, our discussion will make significant use of this distinction, with Sections 2 and 3 concerned with ongoing inflation and Section 4 pertaining to cyclical aspects of price level changes.

Of the various new directions in monetary economics alluded to above, there are two that are evidently of fundamental importance. The first of these is the increased tendency of theoretical researchers to conduct their analyses in general equilibrium models in which private agents are depicted as solving dynamic optimization problems. The second is the increased interest in understanding why macroeconomic policy makers--the monetary and fiscal authorities--behave as they do. The first of these two tendencies is clearly reflected in the present survey, with all of the analysis of Sections 2 and 3 being of a general equilibrium type. The second is also reflected to a substantial extent, though explicit recognition appears only toward the end of the paper. There, in Section 5, we consider a line of analysis that is designed to explain the fact that inflation rates in industrial nations have been, over the past 40 years, positive to an overwhelming extent.

There is one contentious issue that it will be useful to address at the outset, before beginning the main analytical discussion. That issue is the extent to which validity should be assigned to Milton Friedman's famous dictum that "Inflation is always and everywhere a monetary phenomenon" (1963, p. 17). That particular statement has been strongly disputed by leading economists, including one of the editors of this volume (Hahn, 1983). I would suggest, nevertheless, that there is in fact little professional disagreement with Friedman's position, when the latter is properly interpreted. In that regard it is essential to keep in mind that in the essay in question Friedman states that "inflation" will be taken to mean "a steady and sustained rise in prices" (1963, p. 1). Thus his proposition does not constitute a denial of the fact that a shock which reduces an economy's productive capacity--a drought, a capital-destroying earthquake, or an increase in the real price that must be paid for imported goods--will in the absence of any monetary response lead to an increase in the general price level.[2] Nor does the Friedman position imply that an economy's ongoing inflation rate is determined solely by the rate of monetary growth; certainly the normal rate of output growth (Friedman, 1963, p. 23) and the pace of technical change in the payments industry are relevant.[3] But neither of those factors can plausibly contribute more than a few percentage points per year, and their contributions tend to work in opposite directions. Thus their net effect can account for only (say) 0-2 percentage points per year on a sustained basis, a magnitude that is small in comparison to the contribution, during substantial inflations, of money growth.[4]

With respect to fiscal policy, matters cannot be summarized quite so briefly. But unless tax and spending patterns are such as to generate an unsustainable path--a possibility that will be described below in Section 2.3--different fiscal rules will imply different ongoing inflation rates (as distinct from price levels) only if they result in different money stock growth rates. Basically, Friedman's dictum relies only on the presumption that money demand behavior is reasonably stable in real terms and that the volume of real transactions does not respond on a sustained basis to changes in the rate of money creation.

The one notable way in which substantial long-lasting inflation could in principle result without excessive money growth is via the route of speculative "bubble" effects on the general price level. It is not clear that many economists actually believe in the empirical relevance of this type of phenomenon, but there has been considerable interest in it as a matter of theory. The idea will, accordingly, be briefly considered (in Section 3.2).

Another topic that can usefully be mentioned here is that of money stock exogeneity. In this regard, the emphasis given by Friedman and Schwartz (1963) to various historical episodes involving autonomous shifts in monetary policy has led some critics to conclude that Friedman and other "monetarist" economists hold the view that monetary policy actions are exogenous. But while it is understandable how such a conclusion could be drawn, the conclusion is nevertheless seriously mistaken. For monetary policy actions to be exogenous, they would have to be entirely unresponsive to current and past macroeconomic conditions. Whether there is <u>any</u> researcher who holds such a belief is doubtful, and certainly monetarist economists do not. Friedman (1960) and Brunner and Meltzer (1983) may believe that money stock exogeneity would be desirable, but--as their writings abundantly demonstrate--they do not believe that such unresponsiveness has in fact prevailed.

A limitation of the discussion in this chapter is that it pertains to a closed economy. From a practical point of view that limitation is serious, as policy toward inflation in actual economies is significantly intertwined with terms-of-trade and exchange-rate considerations. But space constraints dictate the elimination of many relevant topics, and it is at least possible that theoretical clarity on essentials is actually enhanced by the absence of open-economy complications.[5]

Organizationally, the paper proceeds as follows. Section 2 begins with the development of a simplified but dynamic general equilibrium framework for the analysis of steady-state inflation. Some welfare considerations are included. Next, Section 3 discusses an alternative framework, the possibility of price level bubbles, and some relevant empirical evidence. In Section 4 attention is then shifted to the case of cyclical fluctuations, the main emphasis being given to alternative theories of the link between real and nominal variables--i.e., to Phillips-curve relationships. Again empirical evidence is briefly reviewed. Section 5 is, as mentioned above, concerned with analysis of policy behavior and, in particular, the explanation of a possible inflationary bias in the policy process. Finally, a few concluding comments are provided in Section 6.

2. Basic Analysis of Ongoing Inflation

This section is devoted to a preliminary analysis of an economy with steady, ongoing inflation. But even though the analysis is concerned with alternative steady states, it begins with the specification of a general equilibrium model in which agents are depicted as solving dynamic optimization problems. Steady-state equilibria then emerge as special cases of more general dynamic equilibria. This procedure lessens the danger that agents' optimization problems are posed in a restrictive manner.

Two major simplifying devices will, on the other hand, be employed. First, all agents will be treated as alike in terms of preferences and production capabilities. Distributional matters are therefore neglected. Second, the model will be non-stochastic. Thus the effects of uncertainty on agents' choices and utility levels are also neglected. The presumption is that these two omissions--which permit substantial simplification of the analysis--will not seriously affect the principal conclusions concerning steady states. Some consideration of the effects of uncertainty will be provided in Subsection 2.5.

2.1 The Sidrauski Model

The framework to be utilized is a discrete-time, perfect-foresight version of the well-known model of Sidrauski (1967), modified in ways to be described as we proceed. To avoid inessential clutter, we shall utilize a version with no depreciation or population growth. Thus we consider an economy composed of a large number of similar households, each of which seeks at time t to maximize

$$(2.1)\quad u(c_t,m_t) + \beta\, u(c_{t+1},m_{t+1}) + \beta^2\, u(c_{t+2},m_{t+2}) + \cdots$$

Here c_t is consumption in period t while $m_t = M_t/P_t$, with M_t denoting the household's nominal money stock at the start of t and P_t the money price of the consumption good. The discount factor β equals $1/(1+\rho)$, where ρ is a positive time-preference parameter. Each household has access to a production function that is homogeneous of degree one in its two inputs, capital and labor. In the first version of the model labor is supplied inelastically, however, so the production function can be written as

$$(2.2) \quad y_t = f(k_t),$$

where y_t is output and k_t is capital held at the start of t. The function f is taken to be well-behaved (Sidrauski, 1967, p.535), so a unique positive value for k_{t+1} will be chosen in each period. Capital is output that is not consumed, so its price is the same as that of the consumption good. The real rate of return on capital held from t to t+1 is the marginal product $f'(k_{t+1})$.

An issue that arises immediately concerns the reason for the inclusion of m_t as an argument of the function u. The basic idea, described by many writers, is that holdings of the medium of exchange facilitate an agents' transactions. One way of expressing that idea is to assume that households derive utility only from consumption and leisure but the acquisition of consumption goods[6] requires "shopping" which reduces the time available for leisure or employment. In a monetary economy, however, the amount of shopping time required for a given amount of consumption depends negatively (up to some satiation level) upon the quantity of real money balances held by the household. If the shopping time is functionally related to c_t and m_t, substitution into the basic utility function yields an indirect utility function in which m_t appears.[7] This formalization does not justify every detail of Sidrauski's specification (2.1) as it brings in labor time as another argument and leads to somewhat different presumptions regarding the derivatives of u.[8] But it provides support for the general approach. For expositional reasons, our strategy will be to continue for the moment with Sidrauski's specification (2.1), in which u is assumed to be well-behaved, but to make modifications in what follows.

Regarding interactions with the government, it is assumed that in t each household receives transfers (net of taxes) in the real amount v_t. These are lump-sum in nature, i.e., the magnitude is regarded by each household as independent of its own assets and actions. A typical household's budget constraint for period t can then be written as

$$(2.3)\quad f(k_t) + v_t = c_t + k_{t+1} - k_t + (1+\pi_t)\, m_{t+1} - m_t$$

where $\pi_t = (P_{t+1}-P_t)/P_t$ is the inflation rate between t and t+1. At time t, the household maximizes (2.1) subject to a sequence of constraints like (2.3), one for each period.[9] Under the assumption that u and f are both well-behaved, the first-order Euler conditions for the maximum problem can be written as equalities holding for each period. In the perfect-foresight case, these conditions become

$$(2.4)\quad u_1(c_t,m_t) - \lambda_t = 0$$

$$(2.5)\quad \beta u_2(c_{t+1},m_{t+1}) - \lambda_t(1+\pi_t) + \beta\lambda_{t+1} = 0$$

$$(2.6)\quad -\lambda_t + \beta\lambda_{t+1}[f'(k_{t+1}) + 1] = 0.$$

In addition, there are two transversality conditions:[10]

$$(2.7)\quad \lim_{t\to\infty} m_{t+1}\, \beta^{t-1}\, \lambda_t(1+\pi_t) = 0$$

$$(2.8) \quad \lim_{t\to\infty} k_{t+1}\, \beta^{t-1}\, \lambda_t = 0.$$

In the setting that has been specified, conditions (2.3)-(2.6) are necessary for a maximum while (2.3)-(2.8) are jointly sufficient.[11] Thus if (2.7) and (2.8) are satisfied, the household's choices of c_t, m_{t+1}, k_{t+1}, and λ_t will be described (for given initial assets and paths of v_t and π_t) by the difference equations (2.3)-(2.6).

To complete the model we turn to the government. Abstracting temporarily from the possibility of borrowing, we write the government's budget constraint--in per-household terms--as follows:

$$M_{t+1} - M_t = P_t(g_t+v_t)$$

Here g_t is real government purchases of output during t. Dividing by P_t and using the definitions of π_t and m_t, this constraint can be expressed in real terms as

$$(2.9) \quad (1+\pi_t)\, m_{t+1} - m_t = g_t + v_t.$$

It will be noted that together the government and household budget constraints (2.9) and (2.3) imply the following overall resource constraint (or national income identity):

$$(2.10) \quad f(k_t) = c_t + k_{t+1} - k_t + g_t.$$

Because of this dependence, only two of these three constraints will be needed in the description of any equilibrium.

Consider now a situation in which time paths for M_t and g_t are chosen by the government.[12] Conditional upon those policy choices, competitive equilibrium paths for c_t, k_{t+1}, m_t, λ_t, π_t, P_t, and v_t are determined[13] by equations (2.3)-(2.6) and (2.9), plus the definitional identities

(2.11) $m_t = M_t/P_t$

(2.12) $\pi_t = (P_{t+1}-P_t)/P_t$.

2.2 Steady-State Analysis

Using the foregoing model, we now consider properties of steady states, i.e., dynamic equilibria in which every variable grows at some constant rate. Under present assumptions, with no technical progress or population growth, this condition requires that g_t, k_t, v_t, λ_t, c_t, and m_t must be constant over time, i.e., have growth rates of zero. To derive that implication, note that constant growth requires the ratio λ_{t+1}/λ_t to be constant, and that (by virtue of (2.6)) implies zero growth for k_t. Equation (2.10) then implies that $c_t + g_t$ must be constant, which can only be the case if both of those variables are constant.[14] Condition (2.4) then implies zero growth for m_t, while (2.9) does the same for v_t.

Given that these six variables must be constant over time, it is then possible to eliminate λ_t, M_t, and P_t and express the system as follows:

$$
\begin{aligned}
(2.13)\quad \beta\, u_2(c,m) &= u_1(c,m)\ [1+\pi-\beta] \\
1 &= \beta[f'(k) + 1] \\
f(k) &= c + g \\
\pi m &= g + v
\end{aligned}
$$

Here the first expression comes from (2.4) and (2.5); the second from (2.6); the third from (2.3) and (2.9); and the fourth from (2.9). With g and π given by policy,[15] these four relations determine the steady-state values of c,k,m, and v.

The implication that m_t is constant in a steady state means, of course, that the inflation rate will equal the (per-capita) money growth rate (denoted μ). That equality would not be implied, however, if there were technical change that progressively shifted the production function or the shopping-time function. If, for example, (2.2) were replaced with

$$(2.2') \quad y_t = (1+\alpha)^t f(k_t) \qquad \alpha > 0$$

then g_t, k_t, and c_t would be required to grow at some positive rate. The same would be true for m_t, moreover, implying that inflation would be smaller than money growth. The difference between π and μ would, nevertheless, be fixed by technological considerations that are independent of μ itself.

Of more interest, in the present context, is the question regarding "superneutrality": are the steady-state values of c, k, and y independent of π (and μ)? But in the present model it is a simple matter to determine by inspection of equations (2.13) that the answer is "yes". Specifically, the value of k is determined by the second of equations (2.13) alone. Then with g given by policy, the third equation determines c. Thus we see that k, c, and $y = f(k)$ can be solved for without reference to the value of π. Alternative settings for the latter will have no effect on steady-state values of the main real variables.

Of course there is one real variable that is affected by π, namely m. Since $u_1 > 0$, different settings for π will, via the first of equations (2.13), require different levels of real money holdings. Indeed, the comparative steady-state derivative $dm/d\pi$ is equal to $u_1(u_{22}-u_{12}u_2/u_1)/\beta$ so with $u_{22}-u_{12}u_2/u_1 < 0$[16] real balances will be smaller the higher is the rate of inflation (i.e., the cost of holding money). A terminological question that arises is whether this non-invariance of m to the inflation rate means that the Sidrauski model does not have the property of superneutrality. In my opinion, that interpretation of the term would render it almost useless, since the set of interesting monetary models in which m is invariant to π is probably empty. Thus the reasonable defining characteristic of superneutrality is the invariance[17] across steady states of all real variables excepting real money balances. This definition agrees with that of Patinkin (1987).

But while superneutrality thus defined is a property of the Sidrauski model as originally formulated, it does not survive even modest modifications. To demonstrate the non-robustness of the property, let us now drop the assumption--unreasonable in any event--that labor is supplied inelastically. Then the model's utility and production functions become

$$(2.1') \quad u(c_t, m_t, n_t) + \beta u(c_{t+1}, m_{t+1}, n_{t+1}) + \ldots$$

and

$$(2.3') \quad y_t = f(n_t, k_t)$$

where n_t is the quantity of labor expended in production by a typical household during period t. It is of course assumed that $u_3 < 0$ with $u_{33} < 0$, that $f_2 > 0$ with $f_{22} < 0$, and that the functions continue to yield interior solutions. In the presence of these modifications, the following equation must be appended to the Euler equations (2.3)-(2.6) above:

$$(2.14) \quad u_3(c_t, m_t, n_t) + \lambda_t f_1(n_t, k_t) = 0.$$

Reinterpretation of expressions involving u and f is also required, of course.

The resulting steady-state conditions that determine c,k,m,v, and n then become:

$$
\begin{aligned}
(2.15) \qquad \beta\, u_2(c,m,n) &= u_1(c,m,n)\ [1+\pi-\beta] \\
u_3(c,m,n) &= -u_1(c,m,n)\ f_1(n,k) \\
1 &= \beta[f_2(n,k) + 1] \\
f(n,k) &= c + g \\
\pi m &= g + v.
\end{aligned}
$$

The crucial difference is that the third of these relations no longer involves only one variable. Thus it alone cannot determine k. Indeed, there is no subset of equations (2.15) that can be solved for real variables without the involvement of π. Thus with this simple and appropriate modification, the Sidrauski model does not possess superneutrality--a point expressed clearly by Brock (1974).

Nevertheless, it warrants mention that if the ratio $u_3(c,m,n)/u_1(c,m,n)$ does not depend on m, then a subsystem of (2.15) can be solved for k, n, and c without involving m or π. Superneutrality will then prevail in this special case. Such a condition will obtain, moreover, if the function u(c,m,n) is of the Cobb-Douglas form in terms of c, m, and 1-n. As such a form seems often to provide a good approximation to the data, it is perhaps reasonable to conclude that the modified Sidrauski model does not imply strict superneutrality, but does suggest that departures from superneutrality may be quantitatively unimportant.

2.3 Extensions

At this point, it will be useful to add to the model a third asset. One obvious possibility would be private bonds, but with households all alike the equilibrium quantity held by each would have to be zero.[18] It is more interesting, therefore, to include government bonds. This inclusion, furthermore, has the benefit of modifying the government's budget constraint in a way that permits a clearer distinction to be drawn between monetary and fiscal policy actions.

To extend the model to include government bonds, let us specify that there are one-period securities that are sold in t at a money price of Q_t and redeemed in t+1 for one unit of money. Their nominal rate of return is then $R_t = (1-Q_t)/Q_t$ and their real rate r_t is defined by $1+r_t = (1+R_t)/(1+\pi_t)$. The number of such bonds purchased in t by a typical household is B_{t+1}. If we define $b_t = B_t/P_t$, the household budget constraint for t then becomes

$$(2.16) \qquad f(k_t) + v_t = c_t + k_{t+1} - k_t + (1+\pi_t)m_{t+1} - m_t + (1+r_t)^{-1} b_{t+1} - b_t$$

instead of (2.3). This change has no effect on the Euler conditions (2.4)-(2.6) but adds to that set the following:

$$(2.17) \qquad -\lambda_t(1+r_t)^{-1} + \beta\lambda_{t+1} = 0.$$

The latter condition is written as an equality to reflect an implicit assumption that households can choose positive or negative values for B_t-- i.e., can lend to or borrow from the government. In addition, the household's problem now features a new transversality condition, which is

$$(2.18) \qquad \lim_{t\to\infty} b_{t+1}\, \beta^{t-1} \lambda_t(1+r_t)^{-1} = 0.$$

This has the effect of placing limits on the household's willingness to accumulate bonds.

With the inclusion of government bonds, the government's budget constraint becomes

(2.19) $$(1+\pi_t)\, m_{t+1} - m_t + (1+r_t)^{-1}\, b_{t+1} - b_t = g_t + v_t,$$

while this condition and (2.16) imply (2.10) just as (2.3) and (2.9) did before. With the additional asset recognized, government policy can specify time paths for three of the four variables M_t, g_t, v_t, and B_t, with the fourth determined by (2.19).

Within this extended model, let us now consider a competitive equilibrium under the assumption that the government is specifying time paths for M_t, g_t, and v_t. Then, provided that the three transversality conditions are satisfied, equations (2.4), (2.5), (2.6), (2.11), (2.12), (2.16), (2.17), and (2.19) determine time paths for c_t, k_t, m_t, λ_t, π_t, P_t, r_t, and b_t. A compressed steady-state version of the system, analogous to (2.13), can moreover be written as

(2.20) $$\begin{aligned} \beta\, u_2(c,m) &= u_1(c,m)\ [1+\pi-\beta] \\ 1 &= \beta[f'(k) + 1] \\ r &= f'(k) \\ f(k) &= c+g \\ \pi m - br/(1+r) &= g+v. \end{aligned}$$

These five equations determine steady-state values of c,m,k,r, and b.

A striking property of the system just summarized is that the values of variables other than b are unaffected by the government's choice of v. That can be shown by noting that the first four of equations (2.20) can be solved for c,k,r, and m (given the policy-set values of g and π). Any change in v then merely implies a change in b as dictated by the fifth equation, the government budget restraint.[19] Even the time path for P_t is invariant to v, since the path of M_t is exogenous and m is determined in the previously-discussed block.[20]

This result--the invariance of other variables to bond-financed changes in tax receipts--is typically referred to as the "Ricardian Equivalence Theorem" since the offsetting nature of taxes and bonds was clearly recognized, as a matter of theory, by Ricardo (1817). Today's considerable interest in models with this Ricardian property stems primarily from the work of Barro (1974) (1984).[21]

That the Ricardian equivalence result does not hold when tax/transfer magnitudes are geared to income or factor payments may be shown as follows. Suppose that v_t is replaced in the household and government's budget constraints by $\tau\, f(k_t)$, with τ being a tax rate on production. Then the first-order condition (2.6) will have $(1-\tau)$ multiplying $f'(k_{t+1})$ so the second and third of equations (2.20) will have $(1-\tau)\, f'(k)$ instead of $f'(k)$. Thus the values of c,k,m, and r determined by the first four of equations (2.20) will depend on the value of τ. But that value will be linked to the value of b by the fifth of equations (2.20), so changes in b will lead to responses in c,k,m, and/or r.

Now consider again the case with lump-sum tax/transfer magnitudes. Since different paths of v and b imply different values for the government's budget position yet have no effect on other variables, the question arises whether it is possible to have an equilibrium with a permanent positive budget deficit. There is no problem if the deficit is financed by issuing money, but imagine an attempt to keep M_t constant and finance a positive deficit by the continuing sale of bonds. In this case, as it happens, the answer depends on the definition of "deficit" that is used. If the deficit is defined exclusive of interest payments, i.e., as the right-hand side of (2.19), then a positive value for $g_t + v_t$ and a constant M_t together imply that b_t must grow at a rate equal to the steady-state interest rate, which leads to the violation of the transversality condition (2.18). That fact is not itself conclusive, as transversality conditions are not in all cases necessary for optimality,[22] but it can be shown that in the case at hand the implied path for b_t is inconsistent with individual optimality (McCallum 1984a, p. 130). If, on the other hand, the "deficit" is defined more conventionally as inclusive of interest payments, as in

(2.21) $\quad d_t = g_t + v_t + b_t R_{t-1}/(1+R_{t-1}),$

then it is possible to have an equilibrium path with $d > 0$ in which all variables except v_t and b_t are constant. In this case b_t grows according to $b_{t+1} = b_t + d(1+r)$ so its growth is more than overcome by the geometric shrinkage of β^{t-1}, leading to the satisfaction of (2.18). Thus it is possible to have a zero-inflation equilibrium with $d > 0$ and an ever-growing stock of debt. But with d_t and g_t constant, (2.19) shows that taxes ($-v_t$) will also be ever-growing. Indeed, the result at hand, taken from McCallum (1984a), does not violate the condition that the budget must be balanced in present-value terms when revenue from money creation is regarded as a tax.

Each of the results in this subsection remains valid, it should be added, if the model is generalized to permit (i) depreciation, (ii) population growth within each household, and (iii) variable labor supply as in (2.1′) and (2.3′).

2.4 Welfare

We now wish to consider effects of alternative steady inflation rates on the utility of a typical household. To keep the results from being excessively special, let us use the version of the model that treats labor supply as a variable--i.e., the non-superneutrality version with utility and production functions (2.1´) and (2.3´). In this case, the steady-state values of c,k,m,n, and v are given (for policy-set values of g and π) by equations (2.15).

To evaluate the desirability of these equilibrium values, we now consider the "social planning" problem of choosing at t = 1, say, time paths of variables needed to maximize (2.1´) subject only to the economy's overall resource constraints. In per-household terms, these constraints are

$$(2.22) \qquad f(n_t,k_t) = c_t + k_{t+1} - k_t + g_t$$

for t = 1,2,.... The first-order conditions for this problem are (2.22) and those that follow, where $\phi_t \geq 0$ is the Lagrangian multiplier associated with (2.22):

$$(2.23) \qquad u_1(c_t,m_t,n_t) - \phi_t = 0$$

$$(2.24) \qquad u_2(c_t,m_t,n_t) = 0$$

$$(2.25) \qquad u_3(c_t,m_t,n_t) + \phi_t \, f_1(n_t,k_t) = 0$$

$$(2.26) \qquad -\phi_t + \phi_{t+1}\beta[f_2(n_{t+1},k_{t+1}) + 1] = 0$$

$$(2.27) \qquad -\phi_t \leq 0 \text{ with } -\phi_t g_t = 0.$$

Now the last of these implies, since $\phi_t > 0$ by (2.23), that g_t = 0--i.e., that government purchases of output must be zero. But this result obtains only because our setup has assigned no useful role to government purchases--by assumption these do not constitute capital or provide services valued by households. Consequently, since these assumptions are dubious at best, the $g_t = 0$ condition should not be taken literally as a conclusion regarding optimal fiscal policy. We set $g_t = 0$ in what follows only to assist in the investigation of the optimal inflation issue.

Proceeding then with $g = 0$, the steady-state version of the optimality conditions becomes

$$
\begin{aligned}
(2.28) \qquad & u_2(c,m,n) = 0 \\
& u_3(c,m,n) + u_1(c,m,n)\, f_1(n,k) = 0 \\
& 1 = \beta[f_2(n,k) + 1] \\
& [f(n,k) = c
\end{aligned}
$$

These determine optimal values of n,k,c, and m for $g = 0$ and the policy-set value of π. The question is, are these values the same as those provided by the competitive equilibrium? Inspection of (2.15) indicates readily that the answer will be "yes" if and only if the right-hand side of the first of equations (2.15) equals zero, thereby satisfying the first of equations (2.28). And since u_1 is strictly positive, this equality will obtain only if $1 + \pi - \beta = 1$. Thus social optimality requires an inflation rate of $\pi = \beta - 1$.

Interpretation of the latter is straightforward. In view of the steady-state condition $1 = \beta[f_2+1]$, the requisite inflation rate is given by $\pi = (P_{t+1}/P_t) - 1 = (1+f_2)^{-1} - 1$, or $P_t/P_{t+1} = 1 + f_2$, or

$$
(2.29) \qquad \frac{(P_{t+1}-P_t)}{P_{t+1}} = -f_2(n,k)
$$

But this is, of course, immediately recognizable as the famous "Chicago

Rule" developed most notably by Friedman (1969): deflate at a rate equal to the real rate of interest.[23] The logic of this requirement is simply that it is inefficient not to satiate agents with something--in this case, real money balances--that is socially costless to produce yet provides valuable services.

One matter that has received inadequate attention to this point is the behavior of the marginal-yield function $u_2(c,m,n)$. In particular, satiation with real money balances requires that $u_2 = 0$ for some adequately large value for m. The shopping-time parable of footnote 7 clearly supports the existence of such a value: it is possible to hold more money than would be useful in a period under any circumstances. It is unclear, however, whether it is better to think of u_2 as becoming negative or as remaining equal to zero for m in excess of the satiation level.

The optimality result that we have obtained relies, it should be said, on the assumption that transfers (taxes) are administered in a lump-sum fashion. If instead government revenues--it is certainly appropriate to presume that some will be needed to finance positive government spending--are raised by income or factor-payment taxes, conditions (2.15) would be altered and would fail to match conditions (2.28) even with $\pi = \beta - 1$. It has been argued by Phelps (1973) that in this type of situation, the optimal rate of inflation would be determined by the condition that all utilized revenue sources have the same marginal deadweight loss per unit of revenue. But this argument does not establish that a positive inflation rate would be optimal. First, it has been shown by Marty (1976) and Barro and Fischer (1976) that for the inflation tax this marginal "collection cost" is $-\eta/(1+\eta)$, where η is the elasticity of money demand with respect to the interest rate.[24] A value of -0.25 for η would then imply a marginal collection cost of 33%, which seems quite high. From this type of consideration, Barro and Fischer (1976, p. 146) conclude that "while the Phelps proposition that inflationary finance should be chosen as part of the optimal public finance package is incontestable in principle, it may be that, quantitatively, this argument would not lead to the choice of very much monetary expansion (and would likely lead to a negative rate of inflation)."[25]

Recently, a more definite and striking result has been developed. Specifically, Kimbrough (1986) has argued that with money helping to facilitate transactions, the inflation tax is analogous to a tax on an intermediate good and therefore does not belong in an optimal tax package. Assuming that money's transaction effects appear as described above in footnote 7, Kimborough shows this to be the case in a setting in which there is a consumption tax and a fixed capital stock.[26] A similar result was previously obtained by Lucas and Stokey (1983) using a variant of the cash-in-advance constraint. The Kimbrough and Lucas-Stokey arguments serve to restore the optimality of the Chicago Rule under the assumption that government revenue must be raised without lump-sum taxation.

The foregoing discussion has been concerned only with the "shoe-leather" cost of inflation, the failure to satiate agents with a service-yielding asset that is costless to produce. As many writers have emphasized, this cost is quite small in magnitude for inflation rates of (say) 20% per annum or below. Since rates well below that figure seem to give rise to considerable distress in actual economies, an important question is "why?" This topic has been extensively examined by Fischer in a series of papers that are summarized in Fischer (1984). Further summarization of the points is difficult, but in general the significant non-shoe-leather costs identified by Fischer are either due to the non-adaptation of institutional features designed for a noninflationary world (e.g., non-indexation of government debt and tax schedules) or to relative price variability that is not actually associated with inflation in any tight logical way. Thus anyone who believes that "inflation is associated with the decline of public morality, the rise and fall of nations, and more weighty matters than money triangles and the efficiency of the price system" (Fischer, 1984, pp. 45-6) will be somewhat disappointed by the outcome of Fischer's review. But, as he says, "with no long-run tradeoff between inflation and unemployment, there is nothing to be said for moderate rates of inflation except that they are costly to reduce" (1984, p. 46). Furthermore, the non-adaptations mentioned above may be of great practical importance. If, for example, the non-indexation of tax schedules is taken as given, then the cost of inflation might be regarded as including the resource misallocation--possibly quite substantial--induced by the interaction of inflation and inflation-sensitive taxes.

2.5 Stochastic Shocks

One significant limitation of the analysis of the previous four subsections is its neglect of uncertainty. It is my impression that propositions of the comparative steady-state type, with which this section is concerned, are not very sensitive to the presence or absence of uncertainty. A bit of interesting theoretical evidence relating to one of the topics--i.e., superneutrality--has recently been provided by Danthine, Donaldson, and Smith (1986). These authors investigate the effects of stochastic shocks to the production function in a model otherwise similar to that of subsection 2.1 above. They find that the existence of this type of technological uncertainty leads to the negation of the strict superneutrality result implied by equations (2.13) above.[27] The magnitude of this effect is, however, tiny. For a "representative" set of parameter values, stochastic simulations indicate that the mean value of the steady-state distribution of the capital stock changes only from 0.18485 to 0.18629 when the money growth rate is changed from zero to 500% per period.[28]

A consideration of the robustness of Ricardian equivalence results to the recognition of uncertainty has been undertaken by Chan (1983). His basic result is that debt-tax equivalence continues to hold in the presence of uncertainty if each household's <u>share</u> of an uncertain future tax burden is fixed and if there exist private securities that can be combined to act as a perfect substitute for the government's bonds. In cases in which these conditions do not obtain, a bond-financed tax change will typically have some effect on current aggregate demand, but the direction of the effect is--as conjectured by Barro (1974, p. 1115)--dependent upon the precise specification of the utility function and other aspects of the environment. Mention should also be made of the theoretical approach suggested by Blanchard (1985), which involves agents with uncertain lifetimes. If these agents do not have bequest motives, then some departure from Ricardian properties is implied.

In addition, it would be of considerable interest to know whether the "Chicago Rule" for optimal inflation would remain valid--as a prescription for the <u>average</u> inflation rate--in the presence of stochastic shocks to the system. Intuition suggests that the situation would be similar to that pertaining to superneutrality, i.e., that the conclusions based on deterministic models would be approximately valid. In terms of formal analysis, I have not found investigations in terms of the Sidrauski model itself but Lucas and Stokey (1983) have considered the matter using a model of the cash-in-advance type. Since they specify that the cash-in-advance constraint pertains to only a subset of the consumption goods--formally, to one of the model's two composite goods--with credit purchases possible for the other, their model is quite similar to a one-good Sidrauski setup; see their p. 80. In this setting, with randomly fluctuating government expenditures, Lucas and Stokey indicate that efficiency "requires ... a nominal interest rate identically zero, brought about by a deflation induced by continuous withdrawals of money from circulation" (1983, p. 82).[29] It must be said that the model in question includes no capital goods, but introduction of these would not seem to affect the necessity of monetary satiation, which would be induced by an inflation path that kept the private cost of holding money close to zero.

3. Issues Regarding Ongoing Inflation

Having presented a basic outline of one contemporary model for analyzing ongoing inflation, we now turn to important areas of disagreement and relevant evidence. First, in subsection 3.1 we consider the extent to which our previous conclusions are affected by adoption of an analytical framework in which agents have finite lifetimes, in contrast to the Sidrauski assumption of everlasting households. Second, in 3.2 the much-discussed possibility of "hyperinflationary" speculative bubbles is briefly reviewed. Then in 3.3 some consideration of existing empirical evidence--and difficulties in bringing evidence to bear on the outstanding issues--is provided.

3.1 Overlapping-Generations Models

The last decade has witnessed a significant volume of monetary analysis conducted in the context of models in which a new generation of individual agents, each with a finite life span of two (or more) periods, is born each period. These agents' perspectives on consumption versus saving naturally change as they age, so at any point in time the economy includes agents with different desires regarding the accumulation of wealth, even if agents are all born with the same lifetime utility function and production possibilities. This feature makes the overlapping-generations (OG) framework an attractive vehicle for the analysis of theoretical issues regarding saving and the accumulation of wealth.

Of the monetary analysis that has been conducted in OG models, a substantial fraction has incorporated the point of view according to which "it is not legitimate to take fiat money to be an argument of anyone's utility function or of any engineering production function" (Wallace, 1980, p. 49).

Adherents to that point of view have also avoided relationships such as the shopping-time function of footnote 7 above or the cash-in-advance constraint favored by Lucas (1980a) and others. The resulting models have generated predictions regarding inflation and other monetary phenomena that are very different from those of the Sidrauski-type framework.[30]

It has been argued, however, that most of the unusual features of these models stem from their neglect of the medium-of-exchange function of money. McCallum (1983a) shows, for example, that three of the most striking implications of the Wallace (1980) model vanish if it is modified, by the addition of shopping-time considerations, to reflect this function.[31] Incisive arguments of a similar nature have been put forth by Tobin (1980b) and Patinkin (1983).

The basic point of this line of argument is that it is the transaction-facilitating property of money that makes it a distinctive asset, so any model that totally neglects that property is apt to yield misleading conclusions regarding actual monetary phenomena. It is unfortunate, perhaps, that the representation of such phenomena cannot be incorporated in a more satisfying manner than by making money an argument of a production or utility function, but to "capture" this property in that inadequate way is better than to miss it entirely.[32]

Adopting this latter point of view, a number of writers have used OG models with cash-in-advance or money-in-the-utility-function features to analyze questions regarding inflation or other monetary phenomena.[33] The issue to be addressed here is whether these monetary OG models, with finite-lived agents, yield different conclusions than those obtained in

Section 2 above.

With respect to superneutrality, it is well-known to be the case that the OG model does not generally have that property; this is implied by the analyses of Stein (1971), Drazen (1981), and others. In fact, Drazen shows that under a fairly wide set of conditions the capital-labor ratio will be positively related to the inflation rate. It is possible to argue, however, that such effects are unlikely to be quantitatively important.[34] In any event, since the Sidrauski model implies exact superneutrality only with an unrealistic assumption, there is no major disagreement in this regard.

Next there is the Ricardian property of the Sidrauski model, i.e., the invariance of other variables to debt-financed alterations in the magnitude of lump-sum tax collections. In this case, the OG model does yield a different prediction.[35] In particular, as Diamond's (1965) pioneering analysis demonstrated in a non-monetary setting, the steady-state values of important macroeconomic variables will depend (for given time paths of the money stock and government purchases) upon the magnitude of a tax-transfer variable analogous to v of Section 2.

The third main conclusion of Section 2 was that, with lump-sum tax-transfer magnitudes, social optimality requires adherence to the Chicago Rule prescription: a rate of deflation equal to the marginal product of capital. For the OG model with money, McCallum (1987a) shows that this condition is again necessary for optimality in the following sense: if the economy does not have an overabundance of capital,[36] then unless the Chicago Rule condition obtains it would be possible to enhance one generation's utility without reducing the utility of any other generation. Since the analyses of Scheinkman (1980b), Tirole (1985), and McCallum (1987a) indicate that capital overaccumulation will not occur in a competitive equilibrium in any economy that possesses a positive quantity of a nonaugmentable and productive asset such as land, the conclusion regarding Pareto optimality is much the same as in Section 2.

It should be said that the foregoing conclusion pertains to a Pareto-type comparison recognizing different generations and an arbitrary initial stock of capital. If instead the analysis seeks to determine the inflation rate that will yield the highest steady-state utility--the same for all generations--then there will be a relevant tradeoff, with increased real money balances being associated with reduced levels of the capital stock. In this case, different specifications of utility and production functions will result in different optimal inflation rates but these will typically be greater than the Chicago Rule rate--see Fischer (1986). But Abel (1987) has demonstrated the following: if there is also a fiscal instrument available--i.e., lump-sum intergenerational transfers--then the optimal steady-state policy will involve Golden-Rule capital accumulation and monetary satiation. The latter is attained by equalization of the pecuniary rates of return on money and capital, just as called for by the Chicago Rule.

Before moving on, let us briefly return our attention to the class of OG models that does not give any transaction-facilitating role to the asset termed money. In such models it is possible, despite this omission, to devise assumptions that will permit the coexistence of valued money and other assets. That can be accomplished, for example, by assuming that money and other assets have different risk characteristics (Wallace, 1982) or by assuming that certain groups are prevented by law from holding particular assets (Sargent and Wallace, 1982). In such settings it will frequently be the case that open-market swaps of money for other assets, undertaken by the monetary policy authority, will have no effect on aggregate output or the price level--a result that Wallace (1981) attributes to a "Modigliani-Miller theorem." The important thing to recognize about these results is merely that money does not, in the relevant models, serve as a medium of exchange. If it did, then its rate of return would be lower (in the absence of satiation-inducing policy) than on other assets with similar risk characteristics, and open-market swaps for similar-risk assets would have the traditionally-posited type of effect on aggregate demand. One way of explaining this is to note that while a tax-financed increase in (e.g.) government bond holdings would have no effect on aggregate demand in a Ricardian model because of tax capitalization, a tax-financed increase in the money stock would have a positive effect (as there is no implied change in future taxes necessitated by the changed money stock). But an appropriate combination of the two operations is analytically equivalent to an open market operation, so it follows that an open market purchase of bonds has a positive net effect on nominal wealth. The same type of effect obtains, moreover, in a non-Ricardian model although its workings are weaker and the analysis less transparent. The upshot of these considerations is that nominal "Modigliani-Miller" results for open market operations do not obtain in models in which money has a transaction-facilitating role. Such results hold in the Wallace and Sargent-Wallace examples only because they pertain to assets swaps in which the two assets serve as media of exchange to precisely the same extent. These swaps are not open-market operations in the usual sense of the term.

A distantly related but quite distinct piece by Sargent and Wallace (1981), entitled "Some Unpleasant Monetarist Arithmetic," has been one of the most widely-discussed publications of recent years on the subject of inflation. The main reason for this attention is the apparent suggestion that an economy's monetary authority cannot, by its own base-money creation choices, prevent inflation if an irresponsible fiscal authority embarks upon a course of action that implies continuing deficits (defined net of interest). Formally, the paper's argument is only that paths of base money and fiscal variables are unavoidably related by the government budget constraint in a way that makes noninflationary base-money creation inconsistent with continuing real deficits. Whether the monetary authority has control over inflation thus depends on "which authority moves first, the monetary authority or the fiscal authority? In other words, who imposes discipline on whom?" (1981, p. 7). But the paper's analysis assumes that the fiscal authority "moves first," in the sense that a real deficit sequence is taken as given. In this way the paper seems to suggest that in fact a monetary authority, which can adjust the monetary base by open-market operations, may be technically dominated by a fiscal authority. But consideration indicates that this suggestion is misleading. To see this, suppose that the monetary authority seeks to avoid inflation (by creating base money slowly) while the fiscal authority attempts to follow a purchase/taxation plan that implies a continuing real deficit. In a case of this type, the monetary authority will be technically able to force the fiscal authority to submit to its discipline, in contrast with the Sargent-Wallace assumption. The reason is that the monetary authority has direct control over the monetary base while the fiscal authority does not have direct control over the deficit; it has direct control only over taxes and its bond offerings. In the case under consideration, then, the fiscal authority will be unable to carry out its plan because it will simply not have the requisite purchasing power. It can achieve its planned purchases, but only by increasing taxes and departing from its planned deficit path. Thus a truly determined monetary authority will always have its way. It is of course true that actual fiscal authorities often use political means to induce monetary authorities to cooperate in irresponsible undertakings, but the Sargent-Wallace (1981) analysis is not designed to investigate such political forces. The analysis provides, consequently, no reason for believing that a monetary authority cannot prevent inflation, if it wishes to do so.

In sum, the messages regarding inflation generated by OG models are not significantly different than those stemming from Sidrauski-type models, provided that the transaction-facilitating services of money are treated the same in each case.

3.2 Bubble Inflation

A great deal of professional attention has been given, during the last dozen years, to the possible existence of rational asset-price bubbles--i.e., to equilibria in which a component of the price process exists only because it is arbitrarily expected to exist, yet does so in a manner that does not violate expectational rationality.[37] In its simplest form, this sort of phenomenon can be represented as follows. Suppose that market clearing in period t requires that

$$(3.1) \quad \psi(P_t, P^e_{t+1}) = 0,$$

where ψ is an excess demand function, P_t is the current price in question, and P^e_{t+1} is the period-t expectation of P_{t+1}.[38] In a perfect-foresight context P^e_{t+1} will equal P_{t+1}, but (3.1) will remain a condition designed to explain P_t on the basis of the given expectational magnitude P_{t+1}. In particular, (3.1) is <u>not</u> an ordinary difference equation relating P_{t+1} to a given value of P_t. This has been stressed by Whiteman (1984).

One approach to solving the model (3.1) is to find a function that expresses P_t in terms of the system's relevant state variables. Since no other non-expectational variables appear in (3.1), a natural conjecture in this case is that P_t is constant over time.[39] The conjectured solution is $P_t = P$, with P to be determined. Since under this conjecture it will also be true that $P^e_{t+1} = P$, the relation $\psi(P,P) = 0$ can be solved for P.[40]

But suppose that instead the analyst conjectures that $P_t = \pi(P_{t-1})$, i.e., that P_t is functionally dependent on its most recent value. Then it must also be true that $P_{t+1} = \pi(P_t)$. Substitution into (3.1) in this case gives $\psi[P_t, \pi(P_t)] = 0$ which serves to determine the function π.[41] But adoption of this latter solution $P_t = \pi(P_{t-1})$ instead of $P_t = P$ is tantamont to assuming that agents base their expectations on "extraneous" state variables, for the implied dependence of P^e_{t+1} on P_t is not dictated by the model (3.1). In that sense, $P_t = \pi(P_{t-1})$ defines a family of bubble or bootstrap solutions, one for each conceivable initial value for P_{t-1}.[42] Unless that value happens by chance to equal P, the time path will then differ from $P_t = P$. And the discrepancy $P_t - P$ will in this case constitute the bubble or bootstrap component of the solution.

The sort of phenomena illustrated in the foregoing arises much more generally, in models in which the price at which some market clears is dependent upon current expectations of future values of that price. An issue that has been extensively investigated is whether the assumption of competitive equilibrium with optimizing, forward-looking agents is sufficient to rule out bubble equilibria of the type described. The answer seems to be that while some types of bubble phenomena are precluded--e.g., paths with exploding relative prices--others are not. In particular, it seems that in an economy with fiat money, optimizing behavior does not rule out bubbles in which real money balances fall continually, asymptotically approaching zero.[43] This implies the possibility of an ever-increasing price level in an economy with a constant money stock--a situation which may be termed an inflationary bubble. The existence of this logical possibility has been taken by Hahn (1983, pp. 11-13, 71) as grounds for objecting to the monetarist notion that, in his words, "a necessary and sufficient condition for inflation is an increasing stock of money."

That competitive theory fails to exclude inflationary bubbles does not mean, however, that they occur in actual economies. Accordingly, this becomes an appropriate point to begin our review of empirical evidence relating to the issues of Sections 2 and 3.

3.3 Empirical Evidence

Interesting attempts to determine whether bubble behavior prevailed during the famous German hyperinflation of 1920-23 have been conducted by Flood and Garber (1980) and Burmeister and Wall (1982). These test attempts are not entirely convincing, however, because of the restrictiveness of the utilized assumptions regarding behavior of the monetary authorities--in particular, the maintained assumption that the money supply is generated exogenously. In addition, they suffer from a technical problem, created by the existence of an exploding regressor, regarding the asymptotic distribution theory needed for formal tests. This problem is briefly mentioned by Flood and Garber (1980) in their footnote 18 (p. 754).[44]

One extremely simple test procedure, proposed by Diba and Grossman (1984), has been implemented for the German hyperinflation by Hamilton and Whiteman (1985). The basic idea is that the existence of an inflationary bubble implies that the bubble component of the price level process is explosive. In a model with a log-linear demand function, this implies that stationarity of the time series for log P_t will not be obtained by differencing that series the same number of times as is just adequate to induce stationarity of log M_t. Thus the graphs presented by Hamilton and Whiteman (1985, p. 369), which show that second differencing is just adequate to eliminate a growing mean for both log M_t and log P_t, constitute evidence against the hypothesis that the German hyperinflation represented an inflationary bubble. Unfortunately, the logic of this simple test seems to rely on the assumption of a log-linear money demand function. Researchers who consider that assumption dubious may then find the conclusion unpersuasive.

In any event, it cannot be claimed that any type of formal test for the presence of bubbles has been conducted for a wide variety of inflationary episodes. Thus there currently exists no compelling body of evidence adequate to firmly rule out bubble inflation. There is no formal evidence tending to support its existence, however. This writer would hazard a guess that continued study of the issue will not lend support to the notion that inflationary bubble phenomena are of empirical significance.

Let us now turn to issues raised in Section 2. In attempting to bring evidence to bear on propositions concerning steady-state properties of an economy, one is faced with the necessity of using data that do not conform neatly to comparative steady-state experiments. Instead, actual data sets reflect the experiences of economies undergoing fluctuations due to various types of shocks and administered by monetary authorities that rarely (if ever) make clear-cut policy regime changes of the type envisioned by comparative steady-state analysis. Now, in principle this fact need not deter the researcher, who can proceed by estimating a fully-specified dynamic model--one that tracks the economy's period-by-period fluctuations--and then determining its steady-state properties by analytical means. In practice, however, the approach is unappealing because of the necessity of specifying and estimating a model that satisfactorily reflects the economy's dynamic behavior. To do so, one must not only model portfolio balance and saving vs. consumption behavior, as in the examples of Section 2, but also the aggregate-supply or Phillips-curve relationship that we have yet to discuss. And, as Section 4 will indicate, there is little agreement concerning this critical component of any dynamic macroeconomic model.

Consequently, attempts have been made to reach conclusions regarding steady-state properties--in particular, superneutrality--by means of strategies that do not rely upon specification of the system's period-to-period dynamics. Most notably, Lucas (1980b) and Geweke (1986) have devised procedures for investigating low-frequency relationships among variables in analytical settings that are (except for the list of relevant variables) model-free. The general idea of the approach seems to be that "low frequency" corresponds to "long run," with the latter concept in turn presumably related to steady-state properties including superneutrality. Before discussing a weakness with the approach, let us note that both Lucas and Geweke report findings that are ostensibly supportive of the hypothesis of neutrality. In particular, Lucas (1980b) shows that inflation rates are related to money growth rates with a coefficient near to unity when a low-pass filter is used to remove high-frequency components of the two series.[45] Similarly, Geweke (1986) finds that output and ex post real interest rates are not significantly influenced by past money growth rates when only the lowest-frequency component of his measure of influence is considered.[46] His finding that real money balances are influenced by money growth is, as indicated in Section 2.2, consistent with an appropriate definition of superneutrality.

A difficulty with this type of test has been described in McCallum (1984b), which shows that the presumption that low-frequency measures will reflect comparative steady-state properties is not generally warranted. The problem is that the relevant steady-state relationships pertain to anticipated movements, while low-frequency statistics will usually[47] reflect a mixture of anticipated and unanticipated effects. The weak link in the test strategy is essentially the same as that discussed definitively by Sargent (1971): in any system in which responses are different to anticipated and unanticipated variations, sums of distributed-lag coefficients will not correspond to comparative steady-state effects. [Also, see Lucas (1972b).] McCallum's (1984b) demonstration merely emphasizes that this principle remains true if the data series are subjected to Fourier transforms prior to analysis.

Cross-section evidence, pertaining to time-averaged experiences of different economies, is not subject to the above-mentioned difficulty. Thus it is arguable that such evidence provides a more reliable guide to comparative steady-state properties than single-economy studies. In this regard, the cross-country data sets compiled by Harberger (1978) and Barro (1984) exhibit a tendency to support the hypothesis that inflation rates vary approximately point-for-point with money growth rates. This type of examination has its own flaws, of course. For example, it relies on an implicit assumption that within a given economy the same money growth rate is achieved in each period included in the sample. Consequently, such evidence is certainly not adequate to sustain any conclusions regarding the rather delicate issue of superneutrality.

The other positive issue raised in Section 2 is the extent to which actual economies exhibit Ricardian properties. Empirical studies conducted prior to 1986 have recently been reviewed by Seater (1985), Bernheim (1987), and Leiderman and Blejer (1986). Seater concludes that "the [Ricardian] hypothesis is supported by virtually all the direct tests of it" (1985, p. 124) but recognizes that most of this "evidence consists of failures to reject the hypothesis and therefore may be of questionable power" (p. 125). In addition, he recognizes that contradictory indirect evidence has been provided by various studies of the consumption function which reject variants of the permanent-income hypothesis. (The latter topic, currently the subject of intensive investigation by a number of scholars, cannot adequately be reviewed here.) Bernheim (1987) expresses doubt "that it is possible to identify in a convincing way the relevant structural relationships through macro time series." But, partly on the basis of "indirect" and cross-country evidence, he finds the data to be strongly anti-Ricardian. By contrast, the conclusions of Leiderman and Blejer cannot be categorized so easily. They mention studies that seem to give both types of answers, and leave the reader with the impression that it is not yet settled whether the Ricardian proposition provides a reasonably good empirical approximation.

4. Inflation and Output Fluctuations

In this section we shift our attention from steady states to cyclical fluctuations. This shift leads to a variety of issues pertaining to Phillips-curve phenomena--i.e., to the relationship between inflation (or money growth) and employment (or output) levels measured relative to their normal or "natural-rate" values. Since it is by way of this relationship that monetary policy actions have their main effects on employment and output,[48] the precise nature of the relationship is of critical importance in the context of macroeconomic stabilization policy. Consequently, the area is one of the most extensively studied in all of economics. Nevertheless, there remains much disagreement concerning the Phillips (or aggregate supply) relationship. A cynic might guess that this lack of consensus stems from the desire on the part of researchers for intellectual product differentiation, but such a guess would in my opinion be unjustified. Instead, the main reason for a lack of consensus is the combined importance and difficulty of the subject. Specifically, it is inherently difficult to devise a theory to explain the nature of a relationship between real and nominal variables while respecting the axiom, fundamental to neoclassical economic theory, that rational agents are concerned only with real variables.

Relevant aspects of the story begin with A.W. Phillips's (1958) hypothesis that changes in money wage rates are induced primarily by recent values of the unemployment rate, the latter being a measure of the excess supply of labor. This hypothesis attracted much support because of its policy relevance, the interesting U.K. evidence reported by Phillips, and the fact that this sort of wage-change relation was just what was needed to convert the static Keynesian model--as interpreted by Modigliani (1944) and others, with its inexplicably given level of the money wage--into a usable dynamic framework. But, as is well known, Friedman (1966) (1968) and Phelps (1967) argued convincingly that the relationship should be expressed in terms of real, not nominal, wage changes. The modified relation would still be usable with the Keynesian model of aggregate demand, but would avoid an implausible implication of Phillips's original formulation, i.e., that unemployment could be kept permanently low (or output permanently high) by acceptance of a constant but "high" rate of inflation. The Friedman and Phelps versions involved expected changes in real wage rates and used the rather mechanical adaptive expectations formula to account for expectational behavior. Lucas then developed the case for rational expectations and explored its implications in papers (1972a) (1973) that set the stage for contemporary debates.

In the following sections we shall review the leading alternative Phillips curve hypotheses (as of 1986) as well as selected bits of evidence that are useful in discriminating among them. Because of the impossibility of covering the enormous literature in any detail in the space permitted, these reviews will be extremely brief and will mention only a small fraction of the worthwhile work that has been done in the area.

4.1 Alternative Theories

Currently there are four basic types of Phillips-curve or aggregate-supply theories that attract substantial support from knowledgeable economists. These may be categorized as follows.

(i) flexible-price, monetary misperception models

(ii) sticky-price expectational models

(iii) NAIRU models

(iv) Real business cycle models

The characteristics of the four classes will be described in turn.

Models of type (i), developed primarily by Lucas (1972a) (1973),[49] posit the existence of suppliers who base their production-rate decisions on the relative prices of their own products. The two cited papers rely upon different relative price variables. In particular, Lucas's (1972a) general-equilibrium model emphasizes the current own-product price in relation to the expected value of a future general price level, a comparison that reflects the expected rate of return from current savings.[50] By contrast, his (1973) model compares the current own-product price with the current general price level.[51] The two models are alike, however, in assuming that individual suppliers are ignorant of the current general price level and the current aggregate money stock. Their optimizing choices must accordingly be based on uncertain perceptions regarding these nominal aggregative magnitudes. Thus when a seller finds that his own product price--the "local" price, in one terminology--is unusually high, that may be because the aggregate money stock is unusually large or because relative demand conditions are unusually favorable to his product. The rational supply response, then, is a weighted average of the responses that would be appropriate to the two possibilities if known to prevail, with weights depending on the (known) extent to which local price variability is on average a consequence of the two possibilities.

In each of the Lucas models aggregate output responds to inflation only if it is unanticipated; high or even increasing rates of inflation will induce no output response if they are predictable on the basis of suppliers' knowledge of the economy's workings. For some plausible ways of completing the model, consequently, the Lucas supply theories both give rise to policy ineffectiveness propositions. Both result in models, that is, possessing the property that the stochastic behavior of output is entirely unaffected by the monetary authority's choice of parameters characterizing systematic aspects of policy behavior.[52]

This striking property induced, not surprisingly, a large volume of research designed to explore the robustness of the ineffectiveness proposition under the assumption of rational expectations.[53] One of the more notable contributions was Fischer's (1977) development of a model that is representative of our class (ii),[54] i.e., models with rational expectations but prices that are not free to adjust to market-clearing values within each period. In Fischer's specification, nominal wages for periods t and t+1 are set for half of the workforce at the start of each period t. The values pertaining to periods t and t+1 may be different and each of them is set, in light of existing price level expectations, so as to make the expected real wage for each period equal to its (expected) market-clearing value. Shocks occur, however, which typically result in price levels different from those anticipated, so real wages will usually be unequal to the market-clearing values. Employment and output are then determined so as to equate the marginal product of labor to the current real wage.[55]

With preset nominal wages, surprise inflation will result in a lower-than-expected real wage (for both groups of workers) and therefore, with Fischer's employment-determination assumption, a greater-than-normal level of aggregate output. But this model does not have the policy-ineffectiveness property: if demand shocks are serially correlated, then monetary policy rules can be designed to affect the variance of the output process--essentially because policy can in this case be made to respond to shocks that ocur after some currently-prevailing wages were set. The unconditional mean of the output process cannot be affected by the choice of policy-rule parameters, however, if effects of the type discussed in Section (2.2) are ruled out.[56] In this sense, then, the Fischer model satisfies the natural rate hypothesis as defined by Lucas (1972b): there is no monetary policy that will keep output permanently high in relation to its natural-rate path. The same will be true, moreover, for other models of class (ii) in which prices are preset at expected market-clearing levels.

A third category of aggregate supply models is one which builds on the concept of a "non-accelerating-inflation rate of unemployment" (NAIRU). Each such model posits a stable Phillips-type relation between unemployment (or output relative to its reference path) and the acceleration magnitude, i.e., the period-to-period change in the inflation rate. Distributed-lag specifications may be employed, as in Tobin's (1980a, p. 68) formulation, which explains each quarter's inflation rate by the previous quarter's unemployment rate and an average of the previous eight quarters' inflation rates. In NAIRU models it is not always clear whether past inflation values enter as proxies for inflationary expectations or to reflect "catch-up" or "inertia" effects that have no justification in terms of neoclassical theory. In any event, NAIRU models do not satisfy the natural-rate property: if there exists a stable negative relationship between unemployment and the change in inflation, then the unemployment rate can be permanently lowered by permanent acceptance of an increased value of the acceleration magnitude.[57]

The last category in our four-way classification scheme pertains to so-called "real business cycle" models. In these models the specification of the Phillips-curve relationship is a simple one; it is assumed that there is no such relation. There is, more precisely, no wage-price mechanism that would transmit monetary disturbances into output or unemployment effects.[58] Any observed correlations between output and (say) money growth are, according to this viewpoint, the consequence of "reverse causation," i.e., responses of the money stock to output fluctuations brought about by real shocks to technology or perhaps preferences. This line of research, initiated by Kydland and Prescott (1982), Long and Plosser (1983), and King and Plosser (1984), has been quite prominent in recent years (i.e., 1984-86). In part this popularity is no doubt due to the theoretical attractiveness of the basic notion that there is no Phillips relation; as mentioned before, it is difficult to account for such a relation on the basis of strict neoclassical reasoning.

There is one important formulation whose position in terms of the foregoing categorization is unclear, namely, the staggered-wage models of John Taylor (1979) (1980). In these models nominal wages (or prices) are set each period for a fraction of the sellers, as in the Fischer (1977) setup, and maintained for two or more periods. And again the preset values are selected on the basis of rational and forward-looking expectations. But the principle governing the level at which they are set is not to equate expected supply and demand quantities, but rather to keep in step with wages (or prices) pertaining to the other portion of the workforce, with an adjustment reflecting expected excess demand. In the two-group, two-period case this approach gives rise to a relation of the form

$$(4.1)\quad x_t = 0.5E_{t-1}[(x_{t+1}+x_{t-1})] + 0.5\gamma E_{t-1}[(y_t-\bar{y}_t) + (y_{t+1}-\bar{y}_{t+1})] \qquad \gamma > 0$$

where x_t is the log of the wage set (for half the workforce) at the start of t and $y_t - \bar{y}_t$ is a logarithmetic measure of output relative to normal. Now clearly this equation can be rearranged to yield

$$(4.2)\quad 0 = (E_{t-1}x_{t+1}-x_t) - (x_t-x_{t-1}) + \gamma E_{t-1}[(y_t-\bar{y}_t) + (y_{t+1}-\bar{y}_{t+1})].$$

And with rational expectations it must be true that realized values of x_t and $y_t - \bar{y}_t$ will differ only randomly from expected values. Consequently, the last equation can be seen to imply that different acceleration magnitudes will be permanently associated with different levels of excess demand. Thus Taylor's formulation, like NAIRU models, does not satisfy the natural rate hypothesis. The nature of the implied association is unlike that of the NAIRU models, however: higher values of $\Delta x_{t+1} - \Delta x_t$ give rise to <u>lower</u> values of $y_t - \bar{y}_t$. Thus Taylor's approach does not fall cleanly into either category (ii) or (iii).

4.2 Evaluation of Alternative Theories

A very large number of empirical studies have been conducted with the object of determining which type of Phillips-curve theory conforms most closely to the facts, but conclusions are not clear-cut. A major reason for this inconclusiveness is one that plagues attempts to test economic propositions of many types, namely, that any formal statistical test must rely upon maintained hypotheses that are about as dubious as the proposition under explicit scrutiny. In evaluating the evidence, consequently, it is necessary to utilize a subjective blend of statistical and theoretical findings.[59]

The empirical studies conducted by Barro (1977a) (1978) provided a substantial boost toward acceptance of the monetary misperception models of Lucas, since the results suggested that unanticipated changes in U.S. money growth rates have strong effects on employment and output with anticipated changes having insignificant effects. Barro's results relied, however, on some debatable assumptions concerning the basic specification of the monetary policy rule and the presence of lagged money surprise terms in the unemployment (or output) equation.[60] Both features can be defended, but the defense leaves room for skepticism. Later studies by Gordon (1982) and Mishkin (1982) yielded conclusions that conflict with Barro's by attributing significant explanatory power to anticipated money growth rates, a finding that is inconsistent with Lucas's models.

A line of argument that may be more convincing to some readers relies on the observation that information regarding aggregate money stock magnitudes (and price indices) is available to the public both promptly and cheaply. And since knowledge of current money stock magnitudes would eliminate the effect of monetary surprises on output in the Lucas models,[61] this observation tends to turn the misperceptions theory into one of the real business cycle class (iv).[62]

With regard to the latter, one reason for its recent popularity is the demonstration by Kydland and Prescott (1982) that simulations with a model, in which a stochastic technology shock provides the sole source of fluctuations, provides a fairly good match to actual U.S. data in several respects.[63] A second likely reason is Sims's (1980) demonstration that monthly money stock innovations explain very little of the variance of industrial production when a nominal interest rate is included in a small VAR system. It has been argued,[64] however, that this fact is easily reconciled with a belief in the potency of monetary policy surprises. Basically, the argument is that there is no reason to interpret money stock residuals in a VAR system with surprise actions of the monetary authority, especially when the latter typically focuses his attention on monthly interest rate movements.

A third boost for real business cycle models has come from recent recognition that arguments of Nelson and Plosser (1982) concerning alternative data detrending procedures--i.e., differencing versus linear trend removal--are conceptually interesting and quantitatively important. The Nelson-Plosser suggestion that it is possible to separate trend from cyclical components of observed time series, and that the cyclical component contributes comparatively little variability to GNP and employment series, is open to objection (McCallum, 1986). But it is possible to reject this specific suggestion and still view other considerations raised by Nelson and Plosser to be of considerable importance. Eichenbaum and Singleton (1986) have shown, for example, that analysis with differenced series indicates much weaker Granger causality from monetary to real variables than does analysis with series detrended by removal of linear trend terms. There is currently much activity in this area of research; some time will probably be needed before reliable conclusions can be drawn.

With regard to the NAIRU class of models, the main point would seem to be that their key implication--i.e., that output can be permanently raised (relative to normal) by monetary means--is implausible enough to warrant rejection on purely theoretical grounds. Empirically, furthermore, experiences with accelerating inflation seem not to have been accompanied by unusually high output levels.

The remaining class of theories is type (ii), sticky-price models that do not imply irrationality on the part of individual agents. In this case, the class is wide enough that it is difficult to conceive of evidence that would reflect badly on all its members. For example, to whatever extent the failure of real wages to move countercyclically tends to discredit the Fischer (1977) mechanism,[65] this failure is not at all inconsistent with specifications in which nominal wages are set a la Fischer but employment is demand-determined at preset <u>product</u> prices with current real wages irrelevant for employment decisions.[66] The main objection to the broad class of theories, consequently, has stemmed from the analytical difficulty of explaining why price stickiness relevant for quantity determination would pertain to nominal as opposed to real prices. In other words, if prices are to be preset (for whatever reason) why are they not preset in real terms by means of indexation (or linkage) arrangements? One possibility, suggested in McCallum (1986), is that for many specific product prices the benefits to individuals of the insurance provided by such arrangements would be extremely small. If this is so, then the transaction costs necessitated by such arrangements might be adequate to inhibit their use, even though these costs are themselves very small. And the aggregate consequences could be substantial, as the argument of Akerlof and Yellen (1985) illustrates.

On the basis of the foregoing considerations, empirical and theoretical, it would appear that there is at present no evidence or reason that clearly compels one to reject theories of either the sticky-price or real-business cycle type. Such evidence should in principle be obtainable, however, as the theories have implications that differ more markedly than is the case with sticky-price and monetary misperception models.

4.3 Other Sources of Nonneutrality

At this point we need briefly to consider cyclical output-inflation correlations that are brought about not via the Phillips-curve mechanisms discussed above, but by monetary "nonneutralities" that work by altering the natural-rate path of output. That such nonneutralities may exist should be apparent from the discussion in Sections 2 and 3, where it was concluded that precise steady-state superneutrality is not implied by OG models or by models of the Sidrauski type except under stringent restrictions. Clearly, the same behavior patterns that cause (per capita) values of capital, employment, and consumption to depend upon inflation rates, when comparisons are made across steady states, will give rise to related effects of anticipated inflation on a period-by-period basis. If, for example, the steady-state capital stock is increased by anticipated inflation, as in the model of Drazen (1981), then output may also be related to inflation at business-cycle frequencies.

There are two reasons, however, for devoting much less attention to these sources of nonneutrality than to those discussed in Section 4.1. The first of these concerns the fundamentally different nature of the implied cyclical variations. In particular, variations resulting from effects of anticipated inflation on the capital stock (or on the capital-labor ratio) are appropriately thought of as variations in the natural rate of output, rather than departures from the latter, for these variations would occur even with perfectly flexible prices and complete information on the part of individual agents.[67] They do not involve inefficiencies in the utilization of existing resources, as is the case with output variations of the Phillips-curve type.

Secondly, there are various reasons for believing that monetary effects on natural-rate values of output are not empirically of great importance. Of these reasons, three will be mentioned. First, the magnitude of the full effect of anticipated inflation on output is unlikely to be large. Even though precise superneutrality is not predicted by the models of Sections 2 and 3, it is apt to provide a good approximation to actuality. Second, such effects as do occur would tend to affect output slowly, for the existing capital stock is large in relation to plausible variations in annual investment flows. Finally, the direction of effect predicted by theoretical analysis is unclear. While higher inflation unambiguously increases output in Drazen's (1981) overlapping-generations setup, it does the opposite in other models such as those of Stockman (1981) and Kimbrough (1986).

In sum, it would appear that monetary effects on output at business cycle frequencies are probably due primarily to fluctuations in output in relation to its natural or normal values, rather than fluctuations in normal values themselves.

5. Positive Analysis of Monetary Policy

In this section we turn our attention to a topic of a different type, namely, why it is that substantial (positive) inflation has been a predominate feature of the postwar era.[68] Of course the models of Sections 2 and 3 suggest that sustained inflation will not occur in the absence of excessive money growth,[69] but acceptance of that view just alters the form of the question, which then becomes: why do current-day monetary authorities permit money growth rates that result on average in positive inflation? In response, it is tempting to point to the demise of commodity-money standards. But suppose that it was agreed that adherence to such a standard would prevent sustained inflation. This agreement would still leave unanswered the question of why the monetary authority, in an economy with a fiat money system, would not choose a zero or negative average rate of inflation.[70]

The most prominent attempt to address this issue appears in a line of work initiated by Barro (1983) and Barro and Gordon (1983a), who built upon insights developed by Calvo (1978) and especially Kydland and Prescott (1977). In this section we shall discuss the basic model used in this line of work, briefly consider extensions involving reputational considerations, and touch upon a few more general matters regarding the positive analysis of monetary policy.

5.1 Effects of Discretionary Policy Implementation

In the prototype model developed by Kydland and Prescott (1977) and spelled out by Barro and Gordon (1983a), the monetary authority's objectives are represented by a loss function in which the arguments are the squared deviations of unemployment and inflation from values determined by considerations of allocational efficiency.[71,72] It will simplify matters without distortion of the argument, however, if we simply take the loss function to be decreasing in the current money growth surprise (unanticipated money growth reduces unemployment) and increasing in the square of money growth itself.[73] There are also discounted values of similar terms included for all future periods, but for the moment these can be ignored. If, with such an objective function, the monetary authority were to adopt a <u>policy rule</u> by choosing among constant money growth rates, he would recognize that surprise values will average to zero whatever his choice so that the chosen money growth rate would be zero. Similarly, an <u>average</u> growth rate of zero would be implied by the optimal choice of a rule when a broader class of rules is considered.

But suppose that, instead, the authority implements his objectives in a so-called discretionary manner,[74] i.e., by selecting current money growth rates on a period-by-period basis. In each period, that is, the prevailing <u>expected</u> money growth rate is taken as a given piece of data--an initial condition. The current surprise is then apparently under the authority's control, so the loss-minimizing choice of the current money growth rate is that which just equates the marginal benefit of surprise money growth to the marginal cost of money growth per se. With the objective function as specified, this optimal value will be strictly positive. But rational private agents understand this policy process well enough that their expectations regarding money growth are correct on average. Thus the surprise magnitude is zero on average, over any large number of periods, even though the magnitude within each period is under the control of the monetary authority. Consequently, there is on average no benefit--no extra employment--materializing from surprises. On average, then, the discretionary regime features more money growth (i.e., inflation) but the same amount of surprise money growth (i.e., unemployment) as with an optimal rule.

As a matter of positive analysis, this model suggests that excessive money growth (i.e., positive inflation) is attributable to the fact that actual monetary authorities are not bound by rules, either self-imposed rules or ones stipulated externally. Instead, they conduct policy in a way that involves repeatedly taking account of the fact that for <u>given</u> expectations a lower money growth rate would result temporarily in more unemployment, while repeatedly ignoring the effect of these growth rates on the expectations that are subsequently given.

These points can be succinctly illustrated by means of a specific algebraic example, utilized by Barro and Gordon (1983b). In this example, the monetary authority seeks at time t to minimize the loss function

$$(5.1) \quad z_t + \beta z_{t+1} + \beta^2 z_{t+2} + \dots$$

where β is a discount factor ($0 < \beta < 1$). The per-period losses are given by

$$(5.2) \quad z_t = (a/2)\,\pi_t^2 - b(\pi_t - \pi_t^e), \qquad a,b > 0$$

where π_t represents inflation or money growth in t with π_t^e the previously-formed expectation of π_t. Rule-like optimization involves a once-and-for all choice of π_t values under the condition that $\pi_t - \pi_t^e$ will, for whatever choice, equal zero on average. In these circumstances, the optimal choice will be $\pi_t = 0$ for all t. But if π_t^e is taken as given in period t, and π_{t+j}^e

values are viewed as independent of π_t for $j = 1,2,\ldots$, then the value of π_t that will minimize (5.1) is the discretionary value $\pi_t = b/a > 0$. This will, under discretion, be chosen at each t.

There are alternative interpretations that can be given to the foregoing model. In particular, the beneficial aspect of actual money growth can be thought of as reflecting government revenues from money creation. And although the details of an appropriate specification would then be different, the conclusion regarding the inflationary tendency of a discretionary regime would again be obtained (Barro, 1983). An interpretive issue is whether the policymaker's objective function should be viewed as accurately reflecting preferences of the public. The affirmative position taken by Barro and Gordon (1983a, pp. 593-4) has been challenged by Cukierman (1986, p. 9). But whatever the outcome of that dispute, it is germane primarily to the normative uses of the model. From the perspective of understanding why inflation is observed, all that matters is whether the specification of the policymaker's objectives is sufficiently accurate, empirically.[75]

An interesting elaboration of the foregoing model has been provided by Cukierman and Meltzer (1986), who begin with a framework like that of Barro and Gordon (1983a) but extend it so as to accommodate imperfect control of, and noisy announcements about, money growth rates. In addition, Cukierman and Meltzer postulate stochastically-changing objectives of the monetary authority and assume that these fluctuations in objectives are not directly observable by the public. Two examples of the additional results that are obtained from this extended framework are, first, that the monetary authority will choose to have relatively looser control procedures if his rate of time preference is relatively high and, second, that looser control leads to a higher average rate of money growth.

There are a number of objections to the basic Barro-Gordon framework that could be raised. One, mentioned by Grossman and Van Huyck (1986), is that the excessive money-growth result is not obtained if the objective function is respecified to reflect a dislike for expected (rather than actual) inflation. This seems to be a criticism directed more at the model's normative merits, however, than at its merits as a positive theory of inflation, for actual policy makers seem to be concerned with costs of realized inflation, rather than the ones emphasized by economic theorists. From this positive perspective, however, another objection might be that actual policymakers have not based their period-by-period optimality calculations on models in which private agents' expectations are rational. While rationality may in fact have prevailed, so the argument goes, the Fed and other central banks have not recognized that to be the case and have modelled (perhaps implicitly) agents' expectations by means of fixed forecasting formulae.

To consider the force of this last objection, let us examine an example in which the monetary authority has the specific loss function (5.1) (5.2) but believes, in contrast to the previous example, that agents form expectations according to the fixed formula

(5.3) $\pi_t^e = 0.5(\pi_{t-1}+\pi_{t-2})$.

Now in this case the monetary authority believes that his choice of π_t will affect future expectations, i.e., π_{t+1}^e and π_{t+2}^e. His optimal choice at t of inflation or money growth is then as follows:

(5.4) $\tilde{\pi}_{t+j} = (b/a)\ [1-0.5(\beta+\beta^2)]\ j = 0,1,2,\ldots$

Thus instead of the value b/a chosen under the Barro-Gordon expectational assumption, the π_t value will depart from zero only to the extent that the authority's preferences exhibit impatience (i.e., $\beta < 1$). But it would seem extremely likely that the objectives of actual monetary authorities do have that property, so the point of view suggested by this objection continues to predict a positive inflation rate. That rate remains undesirable, moreover, in the sense that the experienced inflation induces no extra employment. It should be added that while this example does not feature the rules vs. discretion distinction, it does not contradict the normative force of the Kydland-Prescott demonstration as pertaining to an economy in which expectations are rational and the policy authority recognizes that rationality.

5.2 Reputational Considerations

The objection to the basic model that has been most prominent in the literature is neither of the ones mentioned above, but one that is based on its neglect of reputational effects. In particular, recent papers by Barro and Gordon (1983b), Barro (1986), Backus and Driffill (1985), Grossman and Van Huyck (1986), and others have proceeded in repeated-game formulations that explore the possibility that reputational forces can lead to outcomes closer to those obtained under rules than discretion in the basic Kydland-Prescott setup.

In the model of Barro and Gordon (1983b), it is assumed that the monetary authority announces the intention to create money or inflate at a specified rate, say $\pi^\star$, that is smaller than the discretionary value $\hat{\pi}$. Private agents expect this value to be chosen in each period so long as the authority's actual choices do not depart from $\pi_t = \pi^\star$. If, however, at some date t^0 there occurs a discrepancy, $\pi_{t^0} \neq \pi^\star$, then agents expect that the discretionary value $\hat{\pi}$ will prevail in $t^0 + 1$ and possibly for some additional periods, after which expectations revert to $\pi_t^e = \pi^\star$. Under these assumptions, the equilibrium outcomes tend to concentrate on the value $\pi^\star$, and that value is shown to lie between 0 and $\hat{\pi}$, i.e., between the values pertaining to the pure cases discussed previously. Thus reputational considerations are helpful, according to the analysis, but do not provide a complete substitute for the presence of a well-designed rule. Two problems with this particular model have, however, been pointed out by Barro (1986) and Rogoff (1986). First, it requires the monetary authority to have an infinite planning horizon: if the horizon is finite the purely discretionary outcome will prevail in each period. Second, the number of periods, for which the expected inflation rate is $\hat{\pi}$ (rather than $\pi^\star$) after the occurence of a $\pi_t \neq \pi^\star$ discrepancy, is arbitrary and the equilibrim value of $\pi^\star$ itself depends on that number. These problems are, as Barro and Rogoff recognize, quite serious in the context of the issues at hand.

Partly for that reason, Barro (1986) has considered an alternative approach that involves uncertainty on the part of agents about the "type" of policymaker that is in office as the monetary authority. Different types, in this context, correspond to different degrees of commitment to low inflation, with these differences apparently reflecting preferences--or some sort of political affiliation--since the same commitment technology would presumably be available to all potential policymakers. The model's attractiveness is considerably enhanced, in view of this interpretation, by

Rogoff's (1986) extension to a case with a continuum of policymaker types--an extension that has the virtue of eliminating the need to assume a randomized strategy on the part of the policymaker. Each new incumbent begins his term, according to the model, with zero money growth for a number (possibly zero) of periods. Afterwards, an uncommitted policymaker, who has been masquerading as a committed type in order to develop exploitable expectations of low inflation, switches to the discretionary value. During the initial interval, agents' expectations involve the subjective probability (which is revised as experience accumulates) of the incumbent policymaker being of the uncommitted type. Because the policymaker who is uncommitted succeeds in generating a positive monetary surprise at the end of his term, the model implies that, conditional upon the public's prior subjective probability regarding types, the expected value of the loss function is lower for the uncommitted type. Expected losses for either type are, however, smaller the greater is the subjective probability that the policymaker will be of the committed type. Institutions that more frequently place committed inflation-avoiders in the policymaking office therefore produce better outcomes on average (over a large number of terms), according to the assumed loss function, as well as lower average inflation rates.

While this alternative framework avoids the two particular problems noted above, it has weaknesses of its own. As Barro (1986, p. 20) notes, it "would seem preferable to generate predictions for inflation that depended less on individual traits of policymakers and more on basic institutional factors." To that might be added the related objection that the analysis is incomplete, so long as it includes no description of the process determining the "type" of policymaker that is selected each term. In a setting in which types matter, the type that attains office should be treated as an endogenous variable.

5.3 Private Objectives of the Monetary Authority

The models described above are open to the criticism that they unrealistically presume altruistic behavior on the part of the monetary authority. Actual policy decisions are made by purposeful individuals or groups of individuals whose actions are strongly influenced by matters affecting their own income, prestige, and working conditions--none of which are represented in the Barro-Gordon objective function. In this vein, insightful discussions of Federal Reserve behavior have been provided by Hetzel (1985), Lombra and Moran (1980), Kane (1982), and others.

It seems clear that the point of view represented by these authors has much merit; full understanding of policy behavior requires some attention to the actual motives of policymakers. But it also seems clear that a truly satisfactory analysis of this type will be extremely difficult. For policymakers' objectives are partly concerned with attainment and retention of policy positions, the filling of which is part of a nation's political process. Adequate treatment of this aspect of behavior then requires an adequate model of the political system--including voter behavior, if the nation in question is one in which the democratic process plays a significant role. And despite many worthwhile efforts,[76] the profession is currently a long way from having a widely-accepted model of that type.[77]

There is one way, nevertheless, in which reasoning about the private interest of policymakers seems highly relevant to the concerns of this chapter. In particular, Friedman (1985, pp. 60-61), has suggested that the prestige and subsequent income of the top monetary policymaker--e.g., the Chairman of the Board of Governors of the Federal Reserve System--depends strongly on the amount of attention accorded his actions by the news media. And this amount is certainty strongly increased by the perception that his office has discretionary power: a legislated rule for monetary policy would sharply reduce the media's attention to the Fed's Chairman. Now, the evidence to date indicates that it is wrong to think of this argument as applying literally to the subsequent pecuniary income of the Chairman; the personal histories of Eccles, Martin, and Burns do not conform. But in terms of prestige the suggestion is perhaps more convincing. And it is almost certainly better to interpret the argument in a manner that is not so highly personalized, but refers to many more individuals. Thus it would appear to be true that the utility--and in some cases subsequent income-- of Board Members and various professional employees of the Fed (including researchers) is enhanced by the public's perception that the Fed has important discretionary power. To the extent that this viewpoint is accepted, it seems unlikely that the Fed will willingly adopt behavioral rules that would eliminate the discretionary aspects--and, according to the analysis of Section 5.1, the inflationary bias--of U.S. monetary policy behavior.

6. Concluding Remarks

It may have been noted that the discussion in this chapter has devoted little explicit attention to the topic of hyperinflation, on which there is a substantial literature initiated by the famous study of Cagan (1956) and given a large boost by the innovative analysis of Sargent and Wallace (1973).[78] The reason for our lack of emphasis is that most of the relevant theoretical points are subsumed in our general discussion. Furthermore, much of the recent work on hyperinflation has been principally concerned with the development and application of econometric techniques appropriate under the hypothesis of rational expectations. Unless bubble phenomena were operative during the relevant episodes, which seems unlikely, the main substantive question that needs to be understood is why the monetary authorities permitted the outlandish money stock growth rates that occurred. Despite interesting historical investigations by Sargent (1982), Capie (1986), and others, this question remains unanswered.

Also slighted in our discussion has been the recent outburst of writings on commodity-money arrangements and, more generally, on monetary standards; a prominent example is provided by papers in the July 1983 issue of the Journal of Monetary Economics. Much of the work on monetary standards has been stimulating and continuing efforts may ultimately aid in the design of institutions that would avoid the inflationary tendency discussed above in Section 5.1. At present, however, it remains unclear how to achieve the adoption of such a standard or to prevent the violation of one that is nominally in force.

It is a difficult matter to summarize what is already a fairly compressed summarization. It may nevertheless be useful to conclude this chapter with an attempt briefly to identify the main themes that are implied by the discussion of Sections 1-5. The first of these is that, with regard to ongoing inflation, the principal conclusions of theoretical analyses are not very sensitive to details of model specification, so long as the latter posits rational agents devoid of money illusion. Whether one assumes finite-lived or infinite-lived agents, such models suggest (i) that steady-state inflation rates will conform fairly closely to money growth rates, (ii) that superneutrality is not strictly implied but departures should be minor, and (iii) that socially optimal inflation rates are probably zero or negative. Bubble inflation provides a possible exception to point (i) as a matter of theory, but there is little reason to believe that such a phenomenon is of significance empirically. With regard to irregular inflation, and the cyclical interaction of nominal and real variables, there is considerably less professional agreement. Four classes of aggregate-supply or Phillips-curve theories are currently in use by researchers and at least two of these have been able thus far to withstand attempts at refutation. Perhaps the most important issue regarding inflation is why policy authorities have behaved, over the last 40 years, in a manner that permitted a many-fold increase in the price level in most industrial nations. A full answer to that question will require a much better theory of the political process than is currently available. An important hypothesis regarding inflationary bias has been suggested, nevertheless, by models that focus on the effects of period-by-period (i.e., "discretionary") decision-making by a monetary authority that seeks, in a fiat-money regime, to avoid unemployment as well as inflation.

Footnotes

1. The relationship between monetary and fiscal policy, necessitated by the government budget constraint, is treated much more extensively in Brunner and Meltzer's Chapter 13, while bubbles and certain issues involving overlapping-generations models are explored in Brock's Chapter 7. Also Bewley's Chapter 9 provides a more detailed consideration of the optimal inflation rate while Chapters 19 and 21 by Blanchard and by King and Aschauer are concerned with aggregate supply or Phillips-curve issues.

2. An increase in monopoly power would, of course, do the same. But these are all examples of one-time effects on the price level, not the ongoing inflation rate. It is presumably agreed by both critics and supporters, incidentally, that Friedman's statement is a substantive proposition to the effect that inflation is brought about by money stock changes, not a tautological restatement of its definition as an ongoing decline in the value of money.

3. The point is that these are the only two determinants of inflation (beside the money stock growth rate) in a steady state. This is so because all sensible monetary models imply relationships--be they ad hoc money-demand functions in Keynesian macroeconometric models or Euler equations in optimizing general equilibrium models--that link money balances willingly held to a real transaction measure (like output) and an opportunity-cost interest rate, the last of which must be constant over time in a steady state. In the model of Section 2.1 below, for example, the Euler equations (2.4), (2.5) and (2.6) together imply the relation $u_2(c_{t+1},m_{t+1})/u_1(c_{t+1},m_{t+1}) = \pi_t + f'(k_{t+1}) + \pi_t f'(k_{t+1})$. (Symbols are defined below.) The

right-hand-side of the latter will be equal to the nominal interest rate R_t, so the steady-state relation is $u_2(c,m)/u_1(c,m) = R$. Technical change in the payments process would shift the u function, as explained in footnote 7.

4. This last statement might be disputed by pointing to the United States's experience of 1983-86, a period during which inflation rates averaged about 8% lower than money growth rates (measured by M1). A span of four years might arguably be considered as long enough to represent "sustained" in which case the evidence could appear to contradict Friedman's dictum. But it is important to recognize that the cost of holding money balances declined dramatically over this period, both because of interest rate reductions and regulatory changes that led to the introduction of checkable deposits (included in M1) on which interest is paid. Such a reduction would be expected to result in a sizeable increase in the quantity of real M1 balances held. If the money demand function were of the constant elasticity form and the elasticity with respect to the interest-cost variable were -0.20, for example, a reduction in the holding-cost measure from 0.12 to 0.02 would call for an increase in real balances of about 43%! ($-.2[\log .02 - \log .12] = \log 1.43$). The transitional effect is, in other words, large enough to keep inflation substantially below money growth rates for several years if the effects are spread over time.

5. Empirical results are, of course, impaired by this neglect.

6. The model as written explicitly recognizes the existence of only one good. It is intended to serve, however, as a simplified representation of an economy in which each household sells a single product and makes purchases (at constant relative prices) of a large number of distinct consumption goods.

That such an interpretation can be rigorously justified has been demonstrated by Lucas (1980a, p. 134), who remarks: "I imagine that this sort of elaboration is what we always have in mind when we work with aggregative models."

7. Suppose that the basic within-period utility function is $\tilde{u}(c_t,\ell_t)$ where ℓ_t is leisure in period t. Also suppose that shopping time in a period is $\psi(c_t,m_t)$ with $\psi_1 > 0$ and $\psi_2 < 0$. Then if the total time available per period is normalized at 1.0 and n_t is used to denote labor time, $\ell_t = 1 - n_t - \psi(c_t,m_t)$ and substitution yields $\tilde{u}(c_t,1-n_t-\psi(c_t,m_t)) \equiv u(c_t,m_t,n_t)$. Changes in payments technology will alter the function ψ and therefore u. It should be noted that the "cash-in-advance" constraint is simply a special case of the shopping-time model, one in which $\psi > 1$ when $m/c < 1$ and $\psi = 0$ for $m/c \geq 1$.

8. See McCallum (1983a, p. 30). A more complete discussion of related matters is provided by Feenstra (1986).

9. The general equilibrium nature of the model could be emphasized by permitting the household's supply of labor to differ from the amount used in production and likewise for capital, with discrepancies satisfied via competitive markets. But with all households alike, the equilibrium discrepancies would be zero. Consequently, the possibility is not recognized in (2.3) for the sake of notational simplicity.

10. The role of the transversality conditions is to rule out paths that satisfy (2.3)-(2.6) but are undesirable to the household on a longer-term basis. They prevent the household, for example, from indefinitely accumulating assets at a rate so high that "future" consumption benefits are never

obtained.

11. See Brock (1975). It should be mentioned that the contributions of Brock (1974) (1975) to the perfect-foresight analysis of the present type of model are so extensive that the class might justifiably be termed Sidrauski-Brock models.

12. Other possibilities are that the government chooses paths for M_t and v_t or for g_t and v_t. Choice of a path for M_t is equivalent, it should be noted, to the choice of a path for the money-stock growth rate μ_t, defined as $\mu_t = (M_{t+1} - M_t)/M_t$.

13. Provided that the transversality conditions are satisfied.

14. This follows from the useful fact that if (say) $z_t = x_t + y_t$, then z_t, x_t, and y_t can all grow at constant rates only if x_t and y_t each grow at the same rate. That is so because $\Delta z_t/z_t \equiv (x_t/z_t)\ \Delta x_t/x_t + (y_t/z_t)\ \Delta y_t/y_t$. Thus the faster-growing of x_t and y_t has an increasing influence on the growth rate of z_t.

15. The sense in which π is given by government choice will be explained in the next sentence.

16. This condition is implied by Sidrauski's (1967, p. 535) assumption that neither c nor m is inferior.

17. Invariance, that is, to alternative inflation rates.

18. That would not prevent determination of the rate of interest on such bonds, however. The additional Euler equation would give rise to a steady-state requirement that the nominal rate of interest be equal to $f'(k) + \pi + \pi f'(k)$.

19. We are still assuming that the system's transversality conditions are satisfied.

20. A similar result holds, it should be added, outside the steady state. The set of equations (2.4) (2.5) (2.6) (2.10) (2.17) and the identities for m_t and π_t make no reference to either v_t or b_t. Those seven equations therefore determine values of c_t, k_t, m_t, λ_t, π_t, P_t, and r_t without reference to v_t when the government sets paths for g_t, M_t, and either v_t or b_t.

21. In particular, Barro (1974) shows that the result may hold with finite-lived individuals if they care about the utility of their offspring and leave non-zero bequests.

22. In the case at hand, however, the production and utility functions possess enough concavity that Weitzman's (1973) conditions are satisfied and the transversality conditions are in fact necessary.

23. While an early and incisive discussion of the Chicago Rule was provided by Marty (1961), the earliest statement that I have found is that of Friedman (1960, p. 70).

24. Both Marty's analysis and that of Auernhumer (1974), on which Barro and Fischer draw, rely upon areas under money demand functions for their cost estimates. The agreement between our general-equilibrium result and that of Friedman (1969) leads me to guess that the money-demand approach is not misleading.

25. In general, the Barro-Fischer (1976) paper provides an excellent brief summary of matters discussed in the present section. That paper asks, however, "why would the private sector hold any real capital at all when the opportunity cost of holding money is driven all the way down to zero?" (1976, p. 144). The answer is that any reduction in the stock of capital would raise its yield above that on money and lead asset holders to move back into capital.

26. The significance of the latter condition warrants investigation.

27. This result can be obtained by noting that the relevant version of (2.6) becomes $E\lambda = \beta E[\lambda f'(k) + \lambda]$. Since λ_{t+1} and k_{t+1} are not independent, it is not possible to cancel out $E\lambda$. Consequently, the system does not decompose in the way that (2.13) does.

28. More precisely, the comparison is for $\mu = 0$ and $\mu = 5$; see Danthine, Donaldson, and Smith (1986, p. 23). The case under discussion features a Cobb-Douglas production function and non-separable preferences with constant relative risk aversion.

29. A similar result has been obtained by Krugman, Persson, and Svensson (1985).

30. See, for example, Wallace (1980) or Bryant and Wallace (1979).

31. Wallace's three implications are that money will not be demanded if its growth rate exceeds that of output; that steady-states with valued money will be Pareto-optimal if and only if money growth is non-positive; and that open-market operations have no effect on the price level.

32. This position is developed most extensively by McCallum (1983a).

33. Examples include Stein (1971), Drazen (1981), Helpman and Sadka (1979), Weiss (1980), McCallum (1983a), Lucas (1980a), and Woodford (1987).

34. If the ratio of the marginal utility of consumption (MUC) when old to the MUC when young is independent of money holdings, then departures from superneutrality will occur only to the extent that money is a significant form of wealth. But in the U.S., the value of outside money is only about 1% of the value of tangible real assets.

35. Here the term "OG model" is being used in a sense that does not include setups, like those of Barro (1974), that feature operative intergenerational altruism.

36. That is, does not have so much capital that the net marginal product is less than the rate of aggregate output growth. The possibility of such a situation has been stressed by Diamond (1965), Phelps (1966), Cass and Yaari (1967), and many others.

37. I know of no general-purpose survey of the topic. Specific aspects have been usefully discussed by Taylor (1986), Brock (1987), and Woodford (1984).

38. An example is provided by the Cagan (1956) model of the price level with rational expectations and a constant money stock.

39. More generally, the conjecture would be that P_t depends only on the variables that explicitly appear in the model at hand.

40. Here I am assuming that this solution for P exists and is unique. The non-uniqueness that results from bubbles is of a different type, one that will be described momentarily.

41. There may be more than one solution for π, only one of which will normally give the same stationary value for P_t as the P defined above.

42. This multiplicity arises because the model (3.1) itself does not refer to any lagged price; it suggests that historical initial conditions are irrelevant.

43. Important contributions in this line of analysis have been made by Gale (1973), Brock (1975), Calvo (1979), and Obstfeld and Rogoff (1985).

44. Recently, a test strategy that avoids this problem has been implemented in a preliminary manner by Casella (1986). The strategy was previously developed in a different context by West (1985).

45. Lucas's sample consists of M1 and CPI rates of change for the U.S. over the period 1955-1975. One observation, for the second quarter, was used for each year.

46. Geweke's concept, termed "feedback," is essentially a measure of the extent to which Granger causality occurs, one designed so as to permit a decomposition by frequency. Geweke's (1986) data is also for the United States, but his tests involve quarterly postwar time series as well as annual observations for 1870-1978, with tests pertaining to subperiods designed to reflect different policy regimes.

47. Reference here is to results based on model-free procedures of the type under discussion.

48. This claim will be discussed in Section 4.3.

49. Significant elaborations were provided by Barro (1976) and Cukierman (1979).

50. In the (1972a) model, non-interest bearing money is the only store of value.

51. The idea behind this particular comparison, together with the informational discrepancy mentioned below, has been described by Sargent (1979, p. 381) as reflecting the notion that "the labor supplier works in one market but shops in many other markets" so that he cares about the price of his own services in relation to the current price of "an economy-wide bundle of goods." There seems, however, to be a logical flaw with this interpretation: how can the purchaser of these various goods fail to discover their current prices? Also, Lucas's (1973) model posits market clearing only in the aggregate, not in each market separately. In general, the logic of the (1973) model is different from, and less satisfactory than, that of the (1972a) model.

52. This property was brought to the profession's attention most notably by Sargent and Wallace (1975). The proposition presumes that systematic components of policy can be related to past, but not current, values of variables.

53. Reviews of that literature are provided by Barro (1981), McCallum (1980), and Taylor (1985).

54. Others include Phelps and Taylor (1977), Gray (1976), and--subject to caveats mentioned below--Taylor (1979) (1980).

55. Barro (1977b) has emphasized that this assumption regarding employment determination is as critical for the model's properties as the wage-setting feature. He has pointed out that other employment determination rules could be combined with the staggered wage process and that some would yield higher levels of utility, ex post, for workers and employers.

56. It is entirely appropriate to rule out such effects in the context of the issue at hand. The point is not that such effects are non-existent, but rather that they should be thought of as affecting the "normal" or "natural rate" level of output, not the deviation of output from that reference value. The issue at hand is the effectiveness of stabilization policy, which is concerned with these deviations.

57. It might be objected that in actual practice accelerations are never permanent. But that does not constitute a denial that the NAIRU models possess a distinctive feature. And <u>if</u> that feature is judged plausible (implausible), then it constitutes a mark in favor of (against) the class of models even if the hypothetical experiment used in defining the feature never occurs.

58. Thus we are again abstracting from effects of the type discussed in Section 2.2.

59. For a thoughtful alternative evaluation, see King and Dotsey (1987).

60. Lagged surprise terms need to be rationalized by some argument that has them entering to reflect adjustment-cost effects, which might be more directly expressed in terms of a lagged value of the employment or output variable. Sargent (1976) showed that if lagged money surprises are permitted, identification of unanticipated money changes must rely upon exclusion restrictions that presume considerable knowledge of monetary policy behavior.

61. It has been suggested that this difficulty might not prevail if the "true" monetary aggregate were unobservable and thus measured with error. It was shown by King (1981), however, that if observations are available on a proxy variable that differs only randomly from the true aggregate, then output (or employment) should be unrelated to movements in measured monetary aggregates. This suggestion seems unsatisfactory, therefore, for one who accepts the facts to be as indicated by the Barro (1977a) (1978), Gordon (1982), or Mishkin (1982) studies.

62. It should be kept in mind that this objection to Lucas's theory would not be applicable to prewar periods, when aggregate data was much more difficult to obtain. The greater availability of such data may, in accordance with the Lucas models, be one reason for the reduced severity of business cycle fluctuations in the postwar period.

63. In particular, real GNP autocorrelations, correlations of GNP with other variables, and variances of other variables are reasonably well matched provided that the variance of GNP is itself consistent with actual data. The latter condition is obtained in the Kydland-Prescott (1982) study by choice of the variance of the (unobserved) technology shock. There are various respects in which the Kydland-Prescott model does not provide a good match with actual data. On this subject, see Summers (1987) and McCallum (1987b).

64. See, e.g., McCallum (1983b). The point, that a neglect of monetary policy operating procedures may seriously distort econometric results, is applicable in a variety of issues.

65. This failure can not entirely discredit the Fischer model, as technology shocks will lead to procyclical real wages even under its assumptions. If these predominate over demand shocks, then the gross correlations could be as observed.

66. This sort of scheme has been described by McCallum (1982). Reasons why current wages may be unimportant in employment determination have been explored at length by Hall (1980), whose analysis complements that of Barro (1977b).

67. Typically, "natural rate" values would be defined relative to such reference conditions. No general definition is here attempted, however, for the appropriate definition will differ from model to model.

68. For the United States, for example, the price level in mid 1986 (as measured by the CPI) stands at 5.6 times its 1946 value. By way of comparison, it is interesting to note that the 1940 price level, as measured by the WPI, was only 1.3 times its value as of 1776. (This calculation splices the official WPI to the Warren and Pearson values in 1890.)

69. What rate is excessive depends, as mentioned in Section 1, on output growth and the rate of technical progress in transaction technology. But these factors are both small and tend to oppose each other so a non-inflationary money growth rate will be within 1 or 2 (annual) percentage points of zero.

70. That this choice lies within the monetary authority's power seems indisputable, for large economies or ones with floating exchange rates, since the time span under discussion is a matter of decades.

71. The unemployment term is of the form $(Un_t - k\bar{U}n_t)^2$, with $\bar{U}n_t$ the natural-rate value of Un_t and $k < 1$. The latter condition reflects the assumption tht the monetary authority's "target" value of unemployment is below the natural-rate value. Barro and Gordon (1983a) interpret this as reflecting some externality that makes the socially optimal value of Un_t less than $\bar{U}n_t$, and are consequently able to claim that there is no discrepancy between the policymaker's objectives and private agents' preferences. The analysis would remain the same, of course, if the $k < 1$ condition was interpreted as merely reflecting a desire by the policymaker for an excessively low rate of unemployment.

72. The analysis relies upon the plausible assumption that deviations of inflation from the efficient rate are increasingly costly at the margin. Use of the squared deviation is designed to reflect that condition in a tractable manner.

73. In this setup there is no need to distinguish between money growth and inflation rates. Accordingly, we shall here use the terms interchangably. The allocationally-efficient rate of growth is taken to be zero only for convenience; in principle it would be whatever rate leads to the optimal steady inflation rate.

74. This terminology is due to Kydland and Prescott (1977). It does not agree with that used in earlier versions of the "rules vs. discretion" debate, which were (in today's terms) actually concerned with non-activist vs. activist policy. That a <u>rule</u> can be activist--i.e., be responsive to recent conditions--should need no explanation here.

75. With regard to actual Federal Reserve objectives, interesting support for the view that the principal aims are avoidance of inflation and unemployment is provided by Piece (1974).

76. A recent example is provided by Alesina (1987).

77. Cukierman and Meltzer (1986) interpret the stochastic objectives of the policymaker in their model as reflecting desires to remain in office, with the chances of doing so believed by the policymaker to depend on inflation and employment. Further, the relative importance of these two determinants of popularity "shifts in unpredictable ways as individuals within the decision-making body of government change their positions, alliances, and views" (1986, p. 1103). Treatment of political influences as stochastic and exogenous illustrates the absence of a well-developed theory of such influences.

78. As is well-known, Cagan's study attempted to lend support to the hypothesis that money demand functions do not shift about erratically by showing that such functions remained in place through the exceptionally stressful periods of seven 20th century European hyperinflations (Austria 1921-22; Germany 1922-23; Greece 1943-44; Hungary 1923-24; Hungary 1945-46; Poland 1923-24; and Russia 1921-24). Cagan's work was remarkable for its time, but his principal conclusion was somewhat undermined by econometric procedures that would today be judged as flawed; the basic regression specification included an endogenous variable as a regressor and took no account of severly autocorrelated residuals. Also, as Benjamin Friedman (1978) has noted, Cagan's dynamic stability analysis incorrectly applied a stability condition appropriate for a continuous-time

formulation to an empirical model estimated with discrete-time data.

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