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ARRESTED DEVELOPMENT:
THEORY AND EVIDENCE OF SUPPLY-SIDE SPECULATION IN THE HOUSING MARKET

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ABSTRACT

This paper studies the role of disagreement in amplifying housing cycles. Speculation is easier in the land market than in the housing market due to frictions that make renting less efficient than owner-occupancy. As a result, undeveloped land both facilitates construction and intensifies the speculation that causes booms and busts in house prices. This observation reverses the standard intuition that cities where construction is easier experience smaller house price booms. It also explains why the largest house price booms in the United States between 2000 and 2006 occurred in areas with elastic housing supply.

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Asset prices go through periods of sustained price increases, followed by busts. To explain these episodes, economists have developed theories based on disagreement, speculation, and strategic trading. This literature focuses on the behavior of asset prices in stock markets, but it is natural to ask whether these ideas can explain housing markets as well. Like any other financial asset, housing is a traded, durable claim on uncertain cash flows. An enduring feature of housing markets is booms and busts in prices that coincide with widespread disagreement about fundamentals (Shiller, 2005), and there is a long history of investors using real estate to speculate about the economy (Kindleberger, 1978; Glaeser, 2013).

This paper incorporates disagreement into a neoclassical model of housing to examine whether the insight that disagreement raises stock prices generalizes to the housing market.[1] Housing differs in fundamental ways from the typical asset studied in finance. The typical financial asset is in fixed supply, and its dividends are worth the same to all buyers. Housing is a *good*—its value derives from the utility flows it delivers to end users. The dividends from housing have different values for different people. And because firms can respond to high prices with new construction, housing supply is not fixed.[2]

In our model, disagreement raises the price of housing only under certain conditions. In particular, we find a non-monotonic relationship between land supply and the effect of disagreement on house prices, with the price of housing being most sensitive to disagreement for cities at an intermediate level of development. This relationship contrasts with the intuition that disagreement's effect on asset prices should fall strictly with asset supply (Hong, Scheinkman and Xiong, 2006) and that cities where construction is easier experience smaller house price booms (Glaeser, Gyourko and Saiz, 2008). We also shed light on the mechanisms driving the house price boom by emphasizing speculation among developers on the supply-side of the market. Taken together, our findings can explain many puzzling aspects of the 2000-2006 US housing boom, including why the strongest house price growth occurred in cities with elastic housing supply.

We study a two-period model of a housing market with two classes of agents, potential residents and developers. Potential residents receive heterogeneous utility from consuming housing that accrues only when they own their houses. This non-transferable *ownership utility* captures the inefficiencies arising from the separation of ownership and control. Such

[1]Beginning with Miller (1977), a large literature has used models of disagreement to explain asset pricing patterns in the stock market. Hong and Stein (2007) survey this literature, which includes Harrison and Kreps (1978), Morris (1996), Diether, Malloy and Scherbina (2002), Scheinkman and Xiong (2003), Hong, Scheinkman and Xiong (2006), and Simsek (2013).

[2]Other papers have applied speculative finance models to housing. Piazzesi and Schneider (2009) and Burnside, Eichenbaum and Rebelo (2015) incorporate optimism and non-standard learning into search models of the housing market, Favara and Song (2014) incorporate a rental margin into a model of disagreement with short-selling constraints, and Giglio, Maggiori and Stroebel (2014) empirically evaluate whether rational bubbles exist in the housing market. Unlike those papers, our work focuses on housing supply.

moral hazard inefficiencies have long been recognized in corporate finance (Shleifer and Vishny, 1997), and Henderson and Ioannides (1983) use them to explain why some residents choose to own rather than rent. The equilibrium result of ownership utility is that home-ownership is dispersed among individual residents rather than concentrated among a few landlords who rent out the housing stock. Dispersed ownership is one of the most salient aspects of the US housing market, with over 60% of the housing stock owner-occupied.

Developers supply housing in a competitive market, buying land at market prices and converting it into housing for a constant resource cost. Construction is reversible, which leads land and house prices to comove in equilibrium. As in Saiz (2010), the amount of developable land is fixed due to geographic and regulatory constraints. Critically, developers are forward-looking: housing supply and end-of-period land holdings depend on their expectations of current and future land and house prices. Following the literature on disagreement in the stock market, we rule out short-selling in land and housing. The case for short-sale constraints is even stronger in real estate, where a lack of asset interchangeability makes it impossible to cover a short.

In this two-period setting, we study the effect of an information shock concerning the number of potential residents who will arrive in the final period. As in Miller (1977), agents may "agree to disagree" about the quantitative implications of the shock. Morris (1996) argues that such disagreement best fits unprecedented situations—like unanticipated secular shifts in housing demand—in which people have not yet been able to engage in rational learning. As Glaeser (2013) documents, housing booms have historically been accompanied by unanticipated events like the settlement of new cities or the discovery of new resources.

We first characterize how land and house prices aggregate disparate beliefs about the shock, partitioning cities in the initial period into three cases. In *unconstrained* cities, undeveloped land remains at the end of the initial period and commands a price of zero. Beliefs do not enter the pricing equation in unconstrained cities, and the price of housing simply equals the cost of construction. In *constrained* cities, no undeveloped land remains at the end of the initial period. The house price in constrained cities combines beliefs among all potential residents who choose to buy, including pessimists with high ownership utility whose participation mutes the impact of optimism on prices. Developer beliefs do not affect pricing in constrained cities because no developers hold land or housing at the end of the initial period. In *intermediate* cities, undeveloped land remains at the end of the initial period but commands a positive price, reflecting the possibility of future constraints. In this case, equilibrium accords most closely with the predictions in Miller (1977). Only the most optimistic developers hold land at the end of the initial period and, because they are risk-neutral and can borrow freely, they elastically demand land at their subjective valuation. Thus, only the most optimistic developer belief enters the equilibrium pricing equation.

Building on this result, we prove that an increase in disagreement raises the equilibrium price of housing most in intermediate cities. Two competing channels deliver this non-monotonic relationship between the effect of disagreement and land supply. Higher initial demand increases the importance of the information shock by raising the chance a city will be constrained in the final period. In opposition to this classical channel, higher initial demand can move a city from an equilibrium in which optimistic developers take large positions in the land market to one in which all space is dispersed among homeowners. As a result, higher initial demand decreases the effect of disagreement by reducing the influence of the most optimistic beliefs. These channels compete so that the price of housing reponds most to disagreement in intermediate cities in which land supply is nearly exhausted.

This non-monotonicity contrasts with the existing literature on the relationship between disagreement and supply in financial markets, in which disagreement's effect on prices strictly falls with supply.[3] We demonstrate the robustness of this result in several extensions that relax the model's assumptions in different ways. First, we consider the case where developers can issue equity and investors can short-sell that equity. Second, we consider an extension in which landlords can speculate in the housing market and rent out housing to pessimistic residents. In a final extension, we generalize the model to the case in which the supply elasticity declines continuously with the level of initial demand.[4] We also formally show how disagreement reduces welfare (in the sense of Brunnermeier, Simsek and Xiong, 2014) by reallocating space from high-flow-utility pessimists to low-flow-utility optimists and developers. Relative to the stock market, in which case disagreement may lead only to welfare-neutral transfers, this result demonstrates the potential welfare consequences of disagreement in the housing market.

In the last part of the paper, we use the model to understand the variation in price booms across US cities between 2000 and 2006, including those cities that appear as outliers in the classical supply elasticity framework. Understanding the factors driving these outlier cities is crucial for evaluating research using supply elasticity as an instrument for price growth.[5] The model predicts that an information-based house price boom is largest in cities

[3]Hong, Scheinkman and Xiong (2006) show that disagreement raises a stock's price more when the traded float is lower relative to the aggregate risk tolerance. Simsek (2013) predicts that the introduction of new non-redundant financial assets increases the "speculative variance" of portfolio values, but this result does not apply to our findings because the price of undeveloped land is perfectly correlated with the price of housing in our equilibrium.

[4]In this extension, a first-order approximation to the house price equation loads more heavily on the optimistic developer belief when the supply elasticity during the initial period is greater. The supply elasticity captures the degree to which developers are the marginal buyers of space, so a higher elasticity leads to a greater influence of optimistic developer beliefs on the price of housing.

[5]That the Saiz (2010) local housing supply elasticity predicts low house price growth for these cities during the boom has been offered as a challenge to studies using this instrument. Complementary to our analysis, Glaeser, Gyourko and Saiz (2008), Davidoff (2013), and Gao, Sockin and Xiong (2016) document the puzzling nature of these outlier cities. Many papers have used this identification strategy to instrument

where developers can build housing easily today but anticipate running out of land in the near future. As a result, the relationship between current supply elasticity and house price growth may be non-monotonic.

This theoretical condition—unconstrained in the present, possibly constrained in the future—likely characterized the set of puzzling high-price-growth, high-construction cities at the start of the US boom in 2000. Several of these cities face long-run limits to their growth but little regulation of current construction. For instance, Las Vegas is surrounded by land owned by the federal government, and Congress passed a law in 1998 prohibiting the sale of land outside a development ring depicted in Figure 1. During the boom, land investors acted as if they expected these governments to stop selling land and restrict future development.[6] Section 5 shows that land prices rose strongly in these cities, which would not have occurred had investors anticipated unlimited land in the long run.

We present evidence of land market speculation between 2000 and 2006 from US public homebuilders—behavior we term *supply-side speculation*. These firms tripled their land holdings during this time, while land prices rose significantly across the country. Statements by these firms in their financial reports confirm that perceived land supply constraints drove this behavior. At the same time, the homebuilding industry saw its stocks short-sold more frequently than 95% of the industries in the United States. Our model predicts these outcomes for developers in intermediate cities only when disagreement is present.

Our model also offers new predictions for the variation in house price booms within a city. When some potential residents prefer renting to owning, only the most optimistic landlords hold rental housing, just as only the most optimistic developers hold land in intermediate cities. We show that all else equal, the price of housing is larger in constrained cities in types of housing with greater underlying rental demand, such as condos and multifamily units. Similarly, neighborhoods where a greater share of housing is rented witness stronger price increases. This prediction matches the data: house prices increased more from 2000 to 2006 in neighborhoods where the share of rental housing in 2000 was higher.

for house price growth.

[6]Las Vegas provides a stark illustration of our model. The ample raw land available in the short run allowed Las Vegas to build more houses per capita than any other large city in the US during the boom. At the same time, speculation in the land market caused land prices to quadruple between 2000 and 2006, rising from $150,000 per acre to $650,000 per acre, and then lose those gains. This in turn led to a boom and bust in house prices. The high price of $150,000 for desert land before the boom and after the bust demonstrates the binding nature of the city's long-run development constraint. A *New York Times* article published in 2007 cites investors who believed the remaining land would be fully developed by 2017 (McKinley and Palmer, 2007). The dramatic rise in land prices during the boom resulted from optimistic developers taking large positions in the land market. In a striking example of supply-side speculation, a single land development fund, Focus Property Group, outbid all other firms in every large parcel land auction between 2001 and 2005 conducted by the federal government in Las Vegas, obtaining a 5% stake in the undeveloped land within the barrier. Focus Property Group declared bankruptcy in 2009.

1 A Housing Market with Disagreement

Our housing market model contains two periods, $t \in \{0, 1\}$. We study how news about a demand shock in period 1 affects the price of housing in period 0. In the model, agents can disagree about the shock's future size. Equilibrium outcomes depend on how the market aggregates these disparate beliefs. The central result is that the effect of disagreement on the price of housing in period 0 depends on a city's initial level of development.

Housing Supply and Developer Demand. The city we study has a fixed amount of space S. At the beginning of period 0, all of this space exists as undeveloped land that can be used for housing. Housing and land trade in spot markets each period but cannot be sold short.[7] The price of land and housing at t are p_t^l and p_t^h, respectively.

Developers are private firms endowed with the entire supply of land at the beginning of period 0. Developers can borrow or lend freely in global capital markets at an interest rate normalized to 0. In each period, a developer makes three decisions: (1) how much land to buy or sell, (2) how much housing to build, and (3) how much housing to sell. Building a unit of housing requires one unit of land and a resource cost k_t. To simplify equations and keep the user cost of housing constant over time, we set $k_0 = 2k$ and $k_1 = k$ for some $k > 0$.[8] At $t = 1$, the owners of each developer receive the proceeds from liquidation.

Each developer maximizes its subjective expectation of its liquidation value. Denote the holdings of housing, land, and bonds at the beginning of t by H_t, L_t, and B_t, respectively. Denote the control variables of home sales, land purchases, and home construction at each t by H_t^{sell}, L_t^{buy}, and H_t^{build}, respectively. At $t = 1$, the liquidation value π of a developer is the outcome of the constrained optimization problem:

$$\begin{aligned} \pi(p_1^h, p_1^l, H_1, L_1, B_1) = \max_{H_1^{sell}, L_1^{buy}, H_1^{build}} \quad & p_1^h H_1^{sell} - p_1^l L_1^{buy} - k H_1^{build} + B_1 \\ \text{subject to} \quad & H_1^{sell} \leq H_1 + H_1^{build} \\ & H_1^{build} \leq L_1 + L_1^{buy}. \end{aligned}$$

The actions $(H_1^{sell})^*$, $(L_1^{buy})^*$, and $(H_1^{build})^*$ chosen by the developer maximize this problem.

[7] Short-sale constraints in the housing market result from a lack of asset interchangeability. Although housing is homogeneous in the model, empirical housing markets involve large variation in characteristics across houses. This variation in characteristics makes it essentially impossible to cover a short.

[8] We abstract from the possibility of overbuilding by allowing developers to build negative amounts of housing by recouping k_t from turning a house into land.

At $t = 0$, the developer maximizes its subjective expectation of this liquidation value:

$$
\begin{aligned}
(H_0^{sell})^*, (L_0^{buy})^*, (H_0^{build})^* \in \underset{H_0^{sell}, L_0^{buy}, H_0^{build}}{\arg\max} \quad & \mathrm{E}\pi(p_1^h, p_1^l, H_1, L_1, B_1) \\
\text{subject to} \quad & H_0^{sell} \leq H_0^{build} \\
& H_0^{build} \leq L_0 + L_0^{buy} \\
& H_1 = H_0^{build} - H_0^{sell} \\
& L_1 = L_0 + L_0^{buy} - H_0^{build} \\
& B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2kH_0^{build}.
\end{aligned}
$$

At $t = 0$, developers may differ only in their land endowments L_0 and in their beliefs about p_1^h and p_1^l (as specified below). The sum of the land endowments across developers equals S. Developers take prices as given, which is consistent with evidence we discuss in Section 5 on perfect competition in the homebuilding industry.

Individual Housing Demand. *Potential residents* derive utility from consumption and from owning and occupying housing. There are two disjoint groups of potential residents: one arriving at $t = 0$ and one arriving at $t = 1$. Upon arrival, each potential resident decides whether to buy a house. Utility comes from consumption c at $t = 1$ and any housing services v received in the period of arrival. Utility is linear and separable in housing and consumption: $u = c + v$ if the potential resident owns a house in the period of her arrival, and $u = c$ otherwise.

A potential resident who buys at $t = 0$ decides whether to sell her house at $t = 1$. As with developers, potential residents at $t = 0$ may borrow in global capital markets at an interest rate of 0. Denote the control variables of whether or not to buy or sell a house by H_t^{buy} and H_t^{sell}, restricting these to equal 0 or 1. At $t = 1$, an arriving potential resident chooses

$$
\begin{aligned}
(H_1^{buy})^* \in \underset{H_1^{buy}}{\arg\max} \quad & H_1^{buy}(v - p_1^h) \\
\text{subject to} \quad & H_1^{buy} \in \{0, 1\},
\end{aligned}
$$

and the utility of potential residents who bought at $t = 0$ equals

$$
\begin{aligned}
u(p_1^h, B_1, v) = \max_{H_1^{sell}} \quad & H_1^{sell} p_1^h + B_1 + v \\
\text{subject to} \quad & H_1^{sell} \in \{0, 1\},
\end{aligned}
$$

where the choice $(H_1^{sell})^*$ maximizes this problem. At $t = 0$, arriving potential residents

maximize the subjective expectation of their utility:

$$\begin{aligned}(H_0^{buy})^* \in \arg\max_{H_0^{buy}} \quad & H_0^{buy}\,\mathrm{E}u(p_1^h, B_1, v) \\ \text{subject to} \quad & H_0^{buy} \in \{0,1\} \\ & B_1 \quad = -p_0^h H_0^{buy}.\end{aligned}$$

At $t = 0$, potential residents may differ only in their housing utility v and in their beliefs about p_1^h (as specified below). At $t = 1$, arriving potential residents may differ only in v.

Denote by $D(v)$ the complementary cumulative distribution function (1 minus the CDF) of v among arriving potential residents. $D(v)$ is a time-invariant function that encodes heterogeneity in housing flow utility. We make the following functional form assumption about $D(v)$:

Assumption 1. *There exists $\epsilon > 0$ such that*

$$D(v) = \begin{cases} 1 & \text{if } v < k \\ (k/v)^{\epsilon} & \text{if } v \geq k. \end{cases}$$

By Assumption 1, no potential residents have housing utility v less than k. This restriction implies all potential residents are willing to purchase housing at cost. As a result, no residents buy housing only because of expected capital gains. In the model, such pure speculators would instead be classified as developers.[9] Assumption 1 also invokes a constant elasticity of demand for housing, which allows us to derive simple analytic results.

Our utility specification makes two implicit assumptions about resident behavior. First, because utility is separable and linear in c, potential residents are risk-neutral. As a result, the purchase decisions of potential residents at $t = 0$ are not affected by the type of hedging motives studied by Piazzesi, Schneider and Tuzel (2007). Second, because potential residents receive utility from only one house, their housing utility displays diminishing marginal returns. This property leads homeownership to be dispersed among residents in equilibrium.[10]

Aggregate Demand and Beliefs. Aggregate resident demand for housing depends on the number of potential residents and the joint distribution of housing utility and beliefs.

[9] We explore the interaction between speculative and fundamental motives in related work (DeFusco, Nathanson and Zwick, 2016).

[10] This dispersion is partly due to the limitation that potential residents can buy at most one house. Section 4 considers an extension in which potential residents can buy unlimited amounts of housing and rent it to tenants.

The number of arriving potential residents at t equals $N_t S$, where $N_t > 0$. The growth in N_t between $t = 0$ and $t = 1$ is given by

$$\log(N_1/N_0) = \mu^{true} x,$$

where $x \geq 0$ is a shock and μ^{true} is some constant. At $t = 0$, all agents observe N_0 and x. They do *not* observe μ^{true}, the data needed to map the information shock x to the demand growth rate. Agents learn the value of μ^{true} at $t = 1$. The resolution of uncertainty at $t = 1$ is common knowledge at $t = 0$.

At $t = 0$, agents may disagree about the value of μ^{true}. Agent beliefs at $t = 0$ are indexed by $\theta \in \Theta \subset \mathbb{R}$. An agent of type θ believes with certainty that $\mu^{true} = \mu(\theta)$, where $\mu : \Theta \to \mathbb{R}$ is a weakly increasing function. When $\mu(\cdot)$ is not constant, beliefs vary across residents, and knowing the beliefs of other residents does not lead to any Bayesian updating. This "agree-to-disagree" assumption rules out any inference from prices at $t = 0$. Therefore, for instance, a developer who holds land in equilibrium can realize that it is the most optimistic developer, but this realization fails to change the developer's belief.

As argued by Morris (1996), this heterogeneous prior assumption is most appropriate when investors face an unusual, unexpected situation like the arrival of the shock we are studying. Examples in the housing market include the settlement of new cities (like Chicago in the 1830s) or the discovery of new resources (like the Texas oil boom of the 1970s). In the case of the US housing boom between 2000 and 2006, we follow Mian and Sufi (2009) in thinking of the shock as the arrival of new securitization technologies that expanded credit to homebuyers, although an equally valid interpretation would be demographic shifts leading to a secular increase in housing demand. The shock to housing demand between 2000 and 2006 is x, and μ^{true} represents the degree to which this shock persists after 2006. Even economists disagreed about μ^{true} during the boom (Gerardi, Foote and Willen, 2010).

Denote by f_d and f_r the distribution of θ across developers and residents, respectively. We allow these distributions to differ in order to study the equilibrium effects of developer and resident beliefs separately. We make two key assumptions about these distributions:

Assumption 2. *θ and v are independent for potential residents.*

Assumption 3. *$\theta_j^{max} \equiv \max \operatorname{supp} f_j$ exists for $j \in \{d, r\}$.*

Assumption 2 guarantees that the two sources of heterogeneity among potential residents at $t = 0$—their beliefs and their housing utility—are independent from one another.[11] Assumption 3 guarantees the existence of a most optimistic developer. Given that developers

[11] Although there exist theories of why consumer beliefs may be driven by preferences (Bénabou and Tirole, 2016), independence of beliefs and preferences seems like a reasonable starting point.

can access unlimited quantities of financing, an equilibrium would not exist without this regularity condition. We define $\mu_j^{max} \equiv \mu(\theta_j^{max})$ for $j \in \{d, r\}$.

To study the marginal effects of disagreement on equilibrium, we adopt the specification

$$\mu(\theta) = \overline{\mu} + z\theta,$$

where $\overline{\mu}, z \geq 0$.[12] To ensure that $z > 0$ generates disagreement, we assume that there exist agents with both positive and negative values of θ:

Assumption 4. *$\int_{\theta<0} f_j(\theta)d\theta > 0$ and $\int_{\theta>0} f_j(\theta)d\theta > 0$ for $j \in \{d, r\}$.*

Larger values of z generate more disagreement, and $z = 0$ delivers the special case in which all agents agree about μ^{true}.

Land and Housing Market Equilibrium. In an equilibrium, a city is *constrained* if all space is used for housing, *unconstrained* if some space remains as land and the price of land is 0, and *intermediate* if some space remains as land but the price of land is positive. This classification partitions all equilibrium outcomes and will prove useful for describing them.

The land market clears at t if the sum of $(L_t^{buy})^*$ across developers equals 0, and the housing market clears at t when the sum of $(H_t^{sell})^*$ across developers and potential residents equals the sum of $(H_t^{buy})^*$ across potential residents. The prices p_1^l and p_1^h constitute an equilibrium when the land and housing markets clear at $t = 1$. The following lemma characterizes this equilibrium:

Lemma 1. *Given N_1, a unique equilibrium at $t = 1$ exists and is given by*

$$p_1^h = \begin{cases} k & \text{if } N_1 < 1 \quad (\textit{unconstrained}) \\ kN_1^{1/\epsilon} & \text{if } N_1 \geq 1 \quad (\textit{constrained}) \end{cases}$$

and $p_1^l = p_1^h - k$.

Denote the prices in this equilibrium by $p_1^l(N_1)$ and $p_1^h(N_1)$.

These simple expressions for equilibrium prices depend on the model's assumptions in the following way. Because construction is reversible, $p_1^h = p_1^l + k$ and initial conditions such as endowments and the housing stock are irrelevant for prices. Thus only N_1, the number of arriving potential residents, matters for prices at $t = 1$. If $N_1 < 1$, then the available space S exceeds the number of potential residents who want to buy at $p_1^h = k$. As a result, land is free and $p_1^h = k$. If $N_1 > 1$, then more potential residents want to buy at $p_1^h = k$ than there

[12] The restriction $\overline{\mu} \geq 0$ implies that agents expect a non-negative growth rate without disagreement and simplifies our analysis by allowing us to focus on the case of a positive shock to the housing market.

is available space, so the price of housing rises. In this case, because the elasticity of $D(\cdot)$ for $v \geq k$ is assumed to be ϵ, the pass-through of more potential residents to the house price equals $1/\epsilon$, giving the formula in Lemma 1.

At $t = 0$, an agent of type θ believes with certainty that house and land prices at $t = 1$ will equal $p_1^h(e^{\mu(\theta)x}N_0)$ and $p_1^l(e^{\mu(\theta)x}N_0)$, respectively. Given these beliefs, p_0^l and p_0^h constitute an equilibrium when the land and housing markets clear at $t = 0$. The following lemma characterizes this equilibrium:

Lemma 2. *Given N_0, x, and z, a unique equilibrium at $t = 0$ exists, and in this equilibrium $p_0^l = p_0^h - 2k$.*

Denote prices in this equilibrium by $p_0^l(N_0, x, z)$ and $p_0^h(N_0, x, z)$. Sections 2 and 3 fully characterize these prices in the cases of agreement and disagreement, respectively.

As above, the result that $p_0^l = p_0^h - 2k$ follows from the reversibility of construction. If $p_0^l < p_0^h - 2k$, then developers would want to buy an infinite amount of land and build houses to sell; if $p_0^l > p_0^h - 2k$, developers would want to buy an infinite amount of housing to revert to land. Markets would not clear in either case, so neither inequality can hold in equilibrium.

2 Equilibrium with Agreement

The goal of this paper is to illustrate the effect of disagreement on prices in the housing market. In this section, we describe the equilibrium house price at $t = 0$ under agreement. This special case of the model, in which $z = 0$, provides the baseline to which we compare the equilibrium under disagreement, in which $z > 0$. Proposition 1 characterizes the equilibrium at $t = 0$ for land holdings, the price of housing, and the effect of the shock x on the price of housing, which we call the *house price boom*.

Proposition 1. *In equilibrium, when $z = 0$ developers hold land at $t = 0$ if and only if $N_0 < 1$. The house price at $t = 0$ equals*

$$p_0^h(N_0, x, 0) = \begin{cases} 2k & \text{if } N_0 \leq e^{-\overline{\mu}x} \quad (\textit{unconstrained}) \\ k + ke^{\overline{\mu}x/\epsilon}N_0^{1/\epsilon} & \text{if } e^{-\overline{\mu}x} < N_0 < 1 \quad (\textit{intermediate}) \\ k(1 + e^{\overline{\mu}x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \geq 1 \quad (\textit{constrained}) \end{cases}$$

and the house price boom

$$\frac{p_0^h(N_0,x,0)}{p_0^h(N_0,0,0)}-1=\begin{cases}0 & \text{if } N_0\leq e^{-\overline{\mu}x} \quad (\textit{unconstrained})\\ \frac{1}{2}(e^{\overline{\mu}x/\epsilon}N_0^{1/\epsilon}-1) & \text{if } e^{-\overline{\mu}x}<N_0<1 \quad (\textit{intermediate})\\ \frac{1}{2}(e^{\overline{\mu}x/\epsilon}-1) & \text{if } N_0\geq 1 \quad (\textit{constrained})\end{cases}$$

weakly increases in N_0.

The price at $t=0$ consists of two terms: one that reflects the housing utility for the marginal buyer today, and one that reflects the common expectation of this marginal utility tomorrow. Using Lemma 1, we may write $p_0^h(N_0,x,0)=p_1^h(N_0)+p_1^h(e^{\overline{\mu}x}N_0)$. When $N_0\leq e^{-\overline{\mu}x}$, agents expect that $N_1\leq 1$ and that $p_1^h=k$, leading to a price today of $2k$. In the intermediate case when $e^{-\overline{\mu}x}<N_0<1$, land is available today but agents agree it will not be tomorrow. When $N_0\geq 1$, housing is constrained both today and tomorrow.

Under agreement, the house price boom rises monotonically in the level N_0 of demand at $t=0$. When demand is low, the shock fails to raise prices because agents continue to expect the city will be unconstrained at $t=1$. Cities at an intermediate level of demand experience intermediate booms, with larger booms in places with more initial demand. Over the range $e^{-\overline{\mu}x}<N_0<1$, a larger N_0 indicates that the city is closer to being constrained so that a greater share of the shock x appears in prices at $t=1$. When demand is sufficiently high ($N_0\geq 1$), the shock passes through at a constant rate to the price of housing. Pass-through does not vary with N_0 over this range because of the constant elasticity specification of $D(v)$ in Assumption 1.

3 Equilibrium with Disagreement

This section describes the equilibrium at $t=0$ under disagreement about future demand growth, which holds when $x,z>0$ as assumed throughout this section. We study the effect of disagreement on land holdings and the price of housing, the aggregation of beliefs into the price of housing, and the variation in house price booms across cities depending on their initial level of development. The key result is that the house price boom for cities at intermediate levels of development can exceed the boom for cities with high initial demand. Disagreement alters the equilibrium allocation of land holdings among agents and can reduce welfare, as optimistic developers crowd out potential residents.

3.1 Dispersed Homeownership, Land Speculation, and Belief Aggregation

Proposition 2 formally describes the equilibrium allocation of land and housing at $t=0$ as well as the equilibrium house price under disagreement.

Proposition 2. *In equilibrium, housing is held at $t = 0$ by potential residents of each type $\theta \in \operatorname{supp} f_r$. For land holdings, there exists $N_0^*(x, z) \in \mathbb{R}_{>1} \cup \{\infty\}$ such that:*

- *(Unconstrained) If $N_0 \leq e^{-\mu_d^{max} x}$, then some developers hold land at $t = 0$, and these developers may be of any type $\theta \in \operatorname{supp} f_d$.*
- *(Intermediate) If $e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z)$, then some developers hold land at $t = 0$, and all of these developers have type $\theta = \theta_d^{max}$. Furthermore, there exists $L^* \in (0, S)$ such that if the sum of L_0 across developers for whom $\theta = \theta_d^{max}$ is less than L^*, then the sum of $(L_0^{buy})^*$ across these developers exceeds the sum of $(H_0^{build})^*$ across them.*
- *(Constrained) If $N_0 \geq N_0^*(x, z)$, then no developers hold land at $t = 0$.*

The equilibrium house price at $t = 0$ equals

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \leq e^{-\mu_d^{max} x} \\ k + ke^{\mu_d^{max} x/\epsilon} N_0^{1/\epsilon} & \text{if } e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z) \\ k(1 + e^{\mu_r^{agg}(N_0, x, z) x/\epsilon}) N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z), \end{cases}$$

where $\mu_r^{agg}(N_0, x, z)$ is defined for $N_0 \geq N_0^*(x, z)$ as the unique solution to

$$1 = N_0 \int_\Theta D\left(k(1 + e^{\mu_r^{agg} x/\epsilon}) N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta) x} N_0)\right) f_r(\theta) d\theta.$$

$N_0^*(x, z) = \infty$ if and only if $\int_{\theta \geq \theta_d^{max}} f_r(\theta) d\theta = 0$ and $\int_{\theta < \theta_d^{max}} (e^{\mu_d^{max} x/\epsilon} - e^{\mu(\theta) x/\epsilon})^{-\epsilon} f_r(\theta) d\theta \leq 1$.

In equilibrium, homeownership at $t = 0$ is dispersed among potential residents of all beliefs. A potential resident buys a house at $t = 0$ if $v > p_0^h - \mathrm{E} p_1^h$. The number of homebuyers of type θ equals $N_0 S D(p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta) x} N_0))$, which is positive for all $\theta \in \Theta$. Positivity depends on Assumption 1, which guarantees that $D(v) > 0$ for any argument. There exist potential residents with arbitrarily high flow utility, so no matter how expensive housing appears to them, some potential residents of each type choose to buy.

This point relates to the work of Cheng, Raina and Xiong (2014), who find that securitized finance managers did not sell off their personal housing assets during the boom. They interpret this result as evidence that these managers had the same beliefs as the rest of the market about future house prices. An alternative interpretation is that the managers did doubt market valuations, but continued to own housing because they derived sufficiently high utility from housing to compensate for low expected capital gains.

Developers choose not to hold housing at the end of $t = 0$ because it is cheaper to hold land and build a house at $t = 1$ for k instead of paying $2k$ at $t = 0$. In the land market, a

developer wants to purchase an infinite amount of land if $p_0^l < \mathrm{E}p_1^l$. This situation cannot hold in equilibrium, so for all $\theta \in \operatorname{supp} f_d$,

$$p_0^l(N_0, x, z) \geq p_1^l(e^{\mu(\theta)x} N_0). \tag{1}$$

For θ such that (1) holds with equality, a developer of type θ is indifferent to holding land at $t = 0$. For θ such that (1) is an inequality, a developer of type θ chooses not to hold land at the end of $t = 0$, either by selling it or by building houses using the land and selling the houses. As a result, only developers for whom $p_1^l(e^{\mu(\theta)x} N_0) = p_1^l(e^{\mu_d^{max} x} N_0)$ may hold land at $t = 0$. This is a simple statement of the result that prices in asset markets with disagreement and limited short-selling tend to reflect the beliefs of the most optimistic agents (Miller, 1977).

Under two conditions, undeveloped land remains at the end of $t = 0$ and is held only by developers for whom $\theta = \theta_d^{max}$. First, developers must disagree about their expectations of p_1^l so that $p_1^l(e^{\mu(\theta)x} N_0) < p_1^l(e^{\mu_d^{max} x} N_0)$ when $\theta < \theta_d^{max}$. Because $p_1^l(\cdot)$ strictly increases only on $[1, \infty)$, this monotonicity condition holds if and only if $N_0 > e^{-\mu_d^{max} x}$. The second condition is that some undeveloped land remains at the end of $t = 0$. This condition is met if S exceeds potential resident demand at the price that attracts optimistic developers to hold land:

$$S > N_0 S \int_\Theta D(k + p_1^h(e^{\mu_d^{max} x} N_0) - p_1^h(e^{\mu(\theta)x} N_0)) f_r(\theta) d\theta.$$

The proof of Proposition 2 shows there exists a cutoff $N_0^*(x, z) > 1$ such that the above inequality holds if and only if $N_0 < N_0^*(x, z)$. When $N_0^*(x, z) < \infty$, a sufficiently large number of potential residents will always outbid the most optimistic developers for space.

Proposition 2 shows how the housing and land markets differ in the concentration of ownership among optimists. While potential residents of all beliefs own housing, land is owned only by the most optimistic developers in cities at which the initial level of demand takes on intermediate values. The idea that real estate speculation transpires largely in land markets departs from the literature, which has focused mostly on investors in houses.[13]

Developers can carry land over between $t = 0$ and $t = 1$ and thus care about future prices. This feature raises the possibility they buy land in advance of their immediate construction needs according to their beliefs about future demand. We refer to this behavior as *supply-side speculation*. In the model, if undeveloped land remains and the most optimistic developers own all of it, then they must have bought more than they used for homebuilding (unless they were initially endowed with enough land). Proposition 2 formally states this prediction, which we explore in Section 5 by examining the balance sheets of US public homebuilders

[13] See, for example, Barlevy and Fisher (2011), Haughwout et al. (2011), Bayer et al. (2015), and Chinco and Mayer (2015).

during the boom and bust of the early 2000s.

Proposition 2 shows how the allocation of land and housing among developers and potential residents affects the price of housing at $t = 0$. There are two important differences between cities with different initial demand levels in terms of how prices aggregate beliefs. First, prices in cities with intermediate demand reflect only the beliefs of developers. Recent research has measured owner-occupant beliefs about the future evolution of house prices.[14] In intermediate cities, developer rather than owner-occupant beliefs determine prices. Data on the expectations of homebuilders would supplement the research on owner-occupant beliefs to explain price movements in such areas. In cities with high levels of initial demand, developers are crowded out of the market, so only potential resident beliefs matter for prices.

The second difference between intermediate-demand and high-demand cities is how they aggregate the beliefs of the relevant class of agents. In intermediate cities, prices reflect the most optimistic belief μ_d^{max}. Other than the maximal value of its support, all other information encoded in the distribution f_d of θ across developers is irrelevant for prices. In contrast, the entire distribution f_r of potential resident beliefs matters for house prices when $N_0 \geq N_0^*(x, z)$. This stark contrast depends on the absence of other constraints on developer size such as risk aversion or capital constraints. The more general point is that when land is held by potential residents, prices need not reflect the most optimistic belief. This is because residents derive utility from housing that may not be correlated with expected capital gains. Moreover, because these utility benefits exhibit diminishing returns, homeownership will tend to be more dispersed than land ownership.

For $N_0 \geq N_0^*(x, z)$, the formula for $p_0^h(N_0, x, z)$ is identical to the formula without disagreement given by Proposition 1 except with $\overline{\mu}$ replaced by $\mu_r^{agg}(N_0, x, z)$. Here, $\mu_r^{agg}(N_0, x, z)$ aggregates the disparate beliefs of potential residents in the $N_0 \geq N_0^*(x, z)$ regime. This aggregate belief is always less than the most optimistic potential resident belief μ_r^{max}. Under the following assumption, $\mu_r^{agg}(N_0, x, z)$ is also less than the most optimistic developer belief μ_d^{max}:

Assumption 5. $e^{\mu_r^{max} x/\epsilon} - e^{\mu_d^{max} x/\epsilon} < 1$ *and* $\int_\Theta \left(1 + e^{\mu_d^{max} x/\epsilon} - e^{\mu(\theta) x/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta < 1$.

Assumption 5 holds when $f_r = f_d$, so that the distribution of beliefs is the same for each class of agents, but it may hold even if some potential residents are more optimistic than the most optimistic developer. The assumption fails if there is a sufficiently large group of residents with very optimistic beliefs. We invoke Assumption 5 for the purpose of analyzing the effect of disagreement on the house price boom at $t = 0$.

[14] See Landvoigt (2014), Case, Shiller and Thompson (2012), Burnside, Eichenbaum and Rebelo (2015), Soo (2013), Suher (2014), and Cheng, Raina and Xiong (2014).

3.2 The Effect of Disagreement on House Prices

We turn now to the model's main result, which concerns the effect of disagreement on the price of housing at $t = 0$. Proposition 3 formally states the results.

Proposition 3. *The effect of disagreement on the house price at $t = 0$ is given by*

$$\frac{p_0^h(N_0, x, z)}{p_0^h(N_0, x, 0)} - 1 = \begin{cases} 0 & \text{if } N_0 \leq e^{-\mu_d^{max} x} \\ \frac{1}{2}(e^{\mu_d^{max} x/\epsilon} N_0^{1/\epsilon} - 1) & \text{if } e^{-\mu_d^{max} x} < N_0 \leq e^{-\overline{\mu} x} \\ \frac{(e^{\mu_d^{max} x/\epsilon} - e^{\overline{\mu} x/\epsilon}) N_0^{1/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}} & \text{if } e^{-\overline{\mu} x} < N_0 \leq 1 \\ \frac{(e^{\mu_d^{max} x/\epsilon} - e^{\overline{\mu} x/\epsilon}) N_0^{1/\epsilon} - (N_0^{1/\epsilon} - 1)}{(1 + e^{\overline{\mu} x/\epsilon}) N_0^{1/\epsilon}} & \text{if } 1 < N_0 < N_0^*(x, z) \\ \frac{e^{\mu_r^{agg}(N_0, x, z) x/\epsilon} - e^{\overline{\mu} x/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon}} & \text{if } N_0 \geq N_0^*(x, z). \end{cases}$$

The increase is positive for $N_0 \in (e^{-\mu_d^{max} x}, 1]$ and is strictly maximized at $N_0 = 1$. If $\operatorname{supp} f_r \subset [-\overline{\mu}/z, \theta_d^{max}]$ *and $\int_\Theta \theta f_r(\theta) d\theta = 0$, then the increase is also positive for all $N_0 > 1$, and the marginal effect of small disagreement on the price of housing,*

$$\frac{\partial p_0^h(N_0, x, 0)/\partial z}{p_0^h(N_0, x, 0)} = \begin{cases} 0 & \text{if } N_0 < e^{-\overline{\mu} x} \\ \frac{\theta_d^{max} x}{\epsilon} \frac{e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}}{1 + e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon}} & \text{if } e^{-\overline{\mu} x} \leq N_0 \leq 1 \\ 0 & \text{if } N_0 > 1, \end{cases}$$

is positive only for $e^{-\overline{\mu} x} \leq N_0 \leq 1$.

The first part of Proposition 3 calculates the relative effect of disagreement on $p_0^h(N_0, x, z)$ by comparing the price formulas in Propositions 1 and 2. We now describe the effect of disagreement on the price of housing in each regime using the expressions in Proposition 3.

When $N_0 \leq e^{-\mu_d^{max} x}$, all developers agree that $p_1^h = k$ because the city will remain unconstrained at $t = 1$. Disagreement between developers on how to interpret the shock x is irrelevant for today's price. Consistent with basic intuition, a fully unconstrained city reacts similarly to an expected shock under agreement and disagreement.

When $e^{-\mu_d^{max} x} < N_0 \leq e^{-\overline{\mu} x}$, the most optimistic developers expect the city to be constrained so that $p_1^h > k$, but a developer with the average belief does not. As a result, disagreement raises the price of housing. This increase is larger when N_0 is greater because the price the most optimistic developer expects at $t = 1$ rises with the level of demand.

The analysis of the $e^{-\overline{\mu} x} < N_0 \leq 1$ case is similar, except that now the average developer believes the city will be constrained in the future. Within this range, $p_1^h > k$ under both

the average and most optimistic developer belief. The effect of disagreement reflects the extent to which the optimistic developer belief of p_1^h exceeds the average belief, with this difference appearing in the numerator. As N_0 increases, $p_0^h(N_0, x, 0)$ places more weight on beliefs about $t = 1$ relative to the user cost at $t = 0$, so the effect of disagreement on this price increases as well.

The effect of disagreement is most subtle when $1 < N_0 < N_0^*(x, z)$. In this range, disagreement changes the equilibrium from one in which potential residents own all space to one in which the most optimistic developers hold some land. This change alters the house price in two opposing ways, corresponding to the two terms in the numerator in Proposition 3. The first term is positive, as beliefs about p_1^h rise from $ke^{\overline{\mu}x}N_0^{1/\epsilon}$ to $ke^{\mu_d^{max}x}N_0^{1/\epsilon}$. The second term is negative, as the flow valuation of the marginal buyer at $t = 0$ falls from $kN_0^{1/\epsilon}$ to k. These terms reflect a change in land ownership at the margin: under agreement, the marginal buyer is a potential resident whose housing utility equals $kN_0^{1/\epsilon}$; under disagreement, the marginal buyer is a developer whose flow value of housing is k. The net price effect strictly decreases with N_0 because utility crowd-out increases relative to $N_0^{1/\epsilon}$ as N_0 gets larger.

When $N_0^*(x, z) < \infty$, the effect of disagreement eventually reaches the level given in the final regime of Proposition 3. The effect is positive when $\mu_r^{agg}(N_0, x, z) > \overline{\mu}$ for all $N_0 \geq N_0^*(x, z)$, the conditions for which are provided in the statement of the proposition.[15] When $N_0^*(x, z) = \infty$, for large N_0 the effect of disagreement on the house price asymptotes to $(e^{\mu_d^{max}x/\epsilon} - e^{\overline{\mu}x/\epsilon} - 1)/(1 + e^{\overline{\mu}x/\epsilon})$, which the proof of Proposition 3 shows is positive.

The key result is that the effect of disagreement in the $N_0 \geq N_0^*(x, z)$ regime is less than that at $N_0 = 1$. This comparison depends on Assumption 5, which guarantees that $\mu_r^{agg}(N_0, x, z) < \mu_d^{max}$, and on Assumption 2, which leads to dispersion of homeownership among potential residents of all beliefs.

The last part of Proposition 3 isolates the effect of disagreement by studying a small increase in z from $z = 0$. The restrictions on $\operatorname{supp} f_r$ imply that the aggregate belief $\mu_r^{agg}(N_0, x, z)$ always exceeds the average belief $\overline{\mu}$. However, because homeownership is dispersed among residents with different beliefs, the appendix shows that when disagreement is small, the gap between aggregate and average beliefs exists only to the second order, such that $\mu_r^{agg}(N_0, x, z) - \overline{\mu} = o(z)$. In this case, a small increase in disagreement acts as a mean-preserving spread that does not alter the aggregate resident belief. As a result, the marginal effect of small disagreement is only positive in the intermediate region of N_0.

In summary, disagreement raises the price of housing everywhere except cities where the

[15] The conditions imply that the mean of $\mu(\theta)$ among potential residents is $\overline{\mu}$ and that $0 \leq \mu(\theta) \leq \mu_d^{max}$ for all potential residents, which leads the demand curve for housing to be globally convex with respect to θ. Global convexity implies that disagreement (holding the price constant) stimulates the demand of optimists more than it attenuates the demand of pessimists. Thus, Jensen's inequality implies that the price that clears the market under disagreement exceeds the market-clearing price with agreement, meaning that $\mu_r^{agg}(N_0, x, z) > \overline{\mu}$.

level of demand is very low and possibly cities where the level of demand is very high but many extreme optimists and pessimists exist. For specifications of the joint distribution of resident and developer beliefs that satisfy Assumption 5 (such as identical distributions in the two subpopulations), disagreement raises the price most in cities at an intermediate level of development. This non-monotonic effect of disagreement on the price of housing is the model's main result.

3.3 The Variation in Price Booms across Cities

Proposition 4 characterizes how the house price boom under disagreement varies across cities.

Proposition 4. *The house price boom under disagreement,*

$$\frac{p_0^h(N_0,x,z)}{p_0^h(N_0,0,z)} - 1 = \begin{cases} 0 & \text{if } N_0 \leq e^{-\mu_d^{max}x} \\ \frac{1}{2}(e^{\mu_d^{max}x/\epsilon}N_0^{1/\epsilon} - 1) & \text{if } e^{-\mu_d^{max}x} < N_0 \leq 1 \\ \frac{1}{2}(e^{\mu_d^{max}x/\epsilon} - 2 + N_0^{-1/\epsilon}) & \text{if } 1 < N_0 < N_0^*(x,z) \\ \frac{1}{2}(e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - 1) & \text{if } N_0 \geq N_0^*(x,z), \end{cases}$$

is strictly maximized at $N_0 = 1$.

As in Proposition 1, we define the price boom as the effect of the shock x on $p_0^h(N_0,0,z)$. This boom can be decomposed into the product of $p_0^h(N_0,x,0)/p_0^h(N_0,0,0)$ (the marginal effect of x when $z=0$ given by Proposition 1) and $p_0^h(N_0,x,z)/p_0^h(N_0,x,0)$ (the marginal effect of z given by Proposition 3).[16] The former monotonically increases in the level of initial demand N_0, whereas the latter strictly peaks at $N_0 = 1$. Proposition 4 shows that the combined effect also strictly peaks at $N_0 = 1$, meaning that with disagreement the result that demand shocks raise prices the most in constrained cities no longer holds.

The intuition behind Proposition 4 is similar to that of Proposition 3. Cities with low initial demand experience no price boom because all developers agree that they will remain unconstrained at $t=0$. For intermediate cities where $e^{-\mu_d^{max}x} < N_0 \leq 1$, prises rise according to the beliefs of the most optimistic developer. For intermediate cities with $1 < N_0 < N_0^*(x,z)$, prices rise less when $N_0 > 1$ because of the utility crowd-out of homeowners by developers. Finally, prices rise for $N_0 \geq N_0^*(x,z)$ according to the aggregate beliefs of all potential residents. Under Assumption 5, this aggregate belief falls short of the most optimistic developer belief, so the price boom is largest when $N_0 = 1$.

To illustrate the variation in the price boom across cities, Figure 2 plots the expression from Proposition 4 across different values of N_0, both for a positive value of z and for $z=0$. We set $f_r = f_d$ so the conditions of Assumption 5 hold. As can be seen in the figure,

[16] Here we are using the fact that $p_0^h(N_0,0,z) = p_0^h(N_0,0,0)$ because z becomes irrelevant when $x=0$.

disagreement amplifies the boom everywhere except in cities with small initial demand where disagreement has no effect. The amplification is largest in cities with intermediate values of initial demand, leading the boom to be largest in the case of disagreement at $N_0 = 1$. The boom in the case of agreement rises monotonically with respect to the level of initial demand.

3.4 Disagreement and Welfare

Disagreement can reallocate space from pessimists to optimists. This reallocation destroys welfare if some of the pessimists are potential residents with high flow utility and some of the optimists are developers or potential residents with lower flow utility. To formally analyze the effect of disagreement on welfare, we adopt the "belief-neutral Pareto efficiency" criterion proposed by Brunnermeier, Simsek and Xiong (2014) as a welfare measure for models with heterogeneous beliefs. An allocation is belief-neutral Pareto efficient if it is Pareto efficient under all linear combinations of agent beliefs. Proposition 5 shows that disagreement reduces welfare in intermediate and constrained cities.

Proposition 5. *The equilibrium allocation is belief-neutral Pareto efficient if* $z = 0$ *or* $N_0 \leq e^{-\mu_d^{max} x}$ *and is belief-neutral Pareto inefficient otherwise.*

The reallocations that improve welfare when $z > 0$ are as follows. When $e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z)$, there exists a potential resident who chooses not to buy despite having flow utility $v > k$. The resource cost of building a house at $t = 0$ instead of $t = 1$ is k, so there exists a cash transfer from this potential resident to a developer that makes them both better off if the developer builds a house and gives it to the potential resident. For $N_0 \geq N_0^*(x, z)$, there exist potential residents with flow utilities v^1 and v^2 such that $v^1 < v^2$ and the potential resident with $v = v^1$ buys whereas the one with $v = v^2$ does not. With a suitable cash transfer, changing which potential resident owns the house improves the welfare of both. Under the $z = 0$ equilibrium allocation, these situations never occur.

4 Extensions and Additional Predictions

Equity Financing. The developers in the baseline model raise any needed funds at $t = 0$ using debt. Appendix B presents an extension in which developers may raise funds only through equity offerings. The analysis formalizes results that we explore empirically in Section 5, in which we examine the market value and short-selling of the equity of public developers during the 2000-2006 US housing boom.

In this extension, *equity investors* constitute a third class of agents. Across equity investors, the distribution of beliefs f_i about μ^{true} satisfies Assumption 3, which guarantees

the existence of a most optimistic equity investor, and Assumption 4, which ensures disagreement when $z > 0$. Some of the developers endowed with land at $t = 0$ may raise funds by selling claims on their $t = 1$ liquidation values to these investors.[17] Each equity investor may borrow freely at a rate of 0 to finance positive purchases of these claims. In contrast, to sell these claims short, equity investors must pay a positive proportional fee. Furthermore, each equity investor may sell short a limited number of claims. In equilibrium, a price exists for the claim on each developer able to access the equity market such that the value of the claims sold by the developer equals the net quantity demanded by equity investors.

The following proposition characterizes the price of housing, the allocation of land holdings, the price of developer equity, and the total short position by equity investors at $t = 0$.[18] We define $p_0^h(N_0, x, z, f_r, f_d)$ to be the equilibrium value of p_0^h in Proposition 2 given the potential resident belief distribution f_r and the developer belief distribution f_d, and we denote θ_i^{max} and $\mu_i^{max} = \mu(\theta_i^{max})$ to be the type and belief of the most optimistic equity investor.

Proposition 6. *If $xz = 0$, then the aggregate value of short claims equals zero, and there exists an equilibrium in which no equity issuance nor land purchases occur. If $xz > 0$:*

- *if $\sum_{\theta > \theta_i^{max}} L_0 = 0$ then the equilibrium house price equals $p_0^h(N_0, x, z, f_r, f_i)$;*
- *if $\sum_{\substack{\theta < \theta_i^{max} \\ \text{developers w/o} \\ \text{access to equity}}} L_0/S > e^{-\mu_i^{max} x}$ then there exists N_0 for which the following all hold:*
  - *(a) some developers issue equity with positive value,*
  - *(b) $\sum (L_0^{buy})^* > \sum (H_0^{build})^*$ across developers issuing equity,*
  - *(c) the total short position in this equity is positive for some values of the short fee,*
  - *(d) the equity price for each such developer exceeds the price under $x = 0$, and*
  - *(e) the equity price for each such developer falls from $t = 0$ to 1 iff $\mu_i^{max} > \mu^{true}$.*

If none of the developers endowed with land are more optimistic than the most optimistic equity investor (for example, if the developer and investor belief distributions coincide), the pricing formula in Proposition 2 carries over to the model with equity financing with one difference: now the most optimistic equity investor belief replaces the most optimistic developer belief. The non-monotonicity of the house price boom and disagreement price effect

[17] The developers who cannot access the equity market represent small firms and nonprofit landowning entities like governments and Native American tribes that are not able to issue equity.

[18] In this extension, land that remains undeveloped at the end of $t = 0$ pays a small positive dividend at the beginning of $t = 1$. Proposition 6 reports the limiting equilibrium price as this dividend goes to 0. The sole purpose of this dividend is to ensure the existence of equilibrium when $p_0^h(N_0, x, z) = 2k$.

also carry over as long as $\theta \leq \theta_i^{max}$ for non-landowning developers.[19] The most optimistic investor prices all developer equity because long positions are unlimited while short positions are costly. When $\sum_{\theta > \theta_i^{max}} L_0 = 0$, all developers are willing to sell equity backed by their landholdings to investors or to sell their landholdings at the optimistic investor valuation. As a result, the equilibrium land price coincides with the optimistic investor valuation, leading to an equilibrium house price of $p_0^h(N_0, x, z, f_r, f_i)$.[20]

Proposition 6 also characterizes quantities in the equity and land markets without and with disagreement. Without disagreement, short-selling never occurs because all investors agree on the equity valuations and the short fee is positive. Equity issuance and land purchases by developers are not guaranteed to occur because developers are indifferent between selling land, selling equity, and holding land until $t = 1$.

With disagreement, equity issuance, land purchases by equity-issuing developers, and short-selling of that equity all occur in equilibrium as long as the short fee is small enough and pessimistic developers without equity market access hold enough land. With enough pessimistic developers, other developers must raise funds from optimistic investors to buy out the pessimists and satisfy the expected demand of potential residents. Equity-issuing developers buy land in excess of their immediate construction needs, resembling the optimistic developers characterized by Proposition 2.

Housing Rental Market. In the baseline model, potential residents derive housing utility only from owning and may own only one house. To explore the importance of these restrictions, Appendix C presents an extension in which rental contracts are available and potential residents may operate as landlords.

In this extension, potential residents may buy any positive amount of housing. They choose how much housing to lease as landlords and how much to keep as owner-occupied housing. Potential residents may also choose to rent housing as tenants. A fraction χ of potential residents receive housing utility v if and only if they occupy at least one unit of housing as a tenant, and the remaining potential residents receive v if and only if they occupy at least one unit of housing as an owner-occupant. Rental prices at $t = 0$ and $t = 1$ clear the market, so that the quantity of housing chosen to be leased by landlords equals the quantity chosen to be rented by tenants.

We define $N_0^*(x, z, f_r)$ to be the value of N_0^* in Proposition 2 given the potential resident belief distribution f_r, and we define the distribution f_r^χ by $f_r^\chi(\theta) = \chi \mathbf{1}_{\theta_r^{max}} + (1 - \chi) f_r(\theta)$.

[19] In this case $\theta_d^{max} \leq \theta_i^{max}$, so Assumption 5 (on which Propositions 3 and 4 rely) holds with μ_i^{max} in place of μ_d^{max}.

[20] Appendix B analyzes in detail the case in which $\sum_{\theta > \theta_i^{max}} L_0 > 0$. In this case, some landowning developers are more optimistic than the most optimistic investor. There may exist values of N_0 at which these optimistic developers hold land in equilibrium, with the $t = 0$ house price independent of the beliefs and endowments of all other developers.

The following proposition describes the $t = 0$ equilibrium in the rental extension:

Proposition 7. *If $xz = 0$, χ equals the share of the housing stock that is rented. If $xz > 0$:*

- *the equilibrium house price equals $p_0^h(N_0, x, z, f_r^\chi, f_d)$;*
- *the house price boom is strictly maximized at $N_0 = 1$ for some $\chi > 0$;*
- *the house price boom depends on χ only if $N_0 \geq N_0^*(x, z, f_r^\chi)$, in which case it increases.*

As shown by Proposition 7, the equilibrium house price assigns additional weight to the optimistic resident belief in proportion to the underlying share of potential residents who prefer renting. This skewing occurs because only the most optimistic potential residents become landlords in equilibrium.

Proposition 7 further shows that the non-monotonicity of the house price boom characterized by Proposition 4 carries over to this setting for certain values of χ. As shown in Appendix C, this restriction on χ operates as follows. If $\mu_r^{max} < \mu_d^{max}$, then the house price boom remains maximized at $N_0 = 1$ for all χ because the developers pricing housing at that point are more optimistic than the residents pricing housing in markets where developers do not participate. In the more interesting case when $\mu_r^{max} \geq \mu_d^{max}$, the price boom is maximized at $N_0 = 1$ only for $\chi < \chi^*(x, z)$, where $\chi^*(x, z) > 0$. The $\chi^*(x, z)$ constraint need not be very restrictive—if $\mu_r^{max} = \mu_d^{max}$, then $\chi^*(x, z) = 1$, so the boom remains non-monotonic as long as some positive measure of potential residents prefer owner-occupancy to renting.

The last part of Proposition 7 delivers the empirical prediction that among identical cities where no land remains, cities in which a higher share of housing is rented without disagreement experience larger house price booms with disagreement. We explore this prediction in Section 5.

Continuous Housing Supply Elasticity. A key statistic used to analyze house price booms is the elasticity of housing supply (Glaeser, Gyourko and Saiz, 2008; Mian and Sufi, 2009). In the baseline model, this elasticity is zero when all land is used for housing and infinite when some undeveloped land remains. Disagreement amplifies the house price boom most when $e^{-\mu_d^{max} x} < N_0 < N_0^*(x, z)$, the parameter region in which supply is perfectly elastic at $t = 0$ but is expected by the optimistic developers to be perfectly inelastic at $t = 1$. This characterization suggests that disagreement amplifies price booms most when supply is elastic today but expected to be inelastic tomorrow. To see how robust this conclusion is, Appendix D extends the rental market model to the case in which the supply elasticity declines continuously with the level of initial demand.

In this extension, developers may rent out undeveloped land on spot markets each period to firms that use the city's land as an input. The land demand from these firms at $t = 0$ and

$t = 1$ is given by a continuously differentiable, decreasing, positive function of the spot rental rate. The limiting values of this function for small and large rents are sufficiently extreme that a unique equilibrium exists, and this function becomes weakly more inelastic for larger rents.

Given a house price p_t^h, a unique partial equilibrium exists in which developers optimize and the land markets clear. We define the elasticity of supply to be the partial derivative of the log of the housing stock chosen by developers with respect to p_t^h, all normalized by the house rent r_t^h.[21] As shown in Appendix D, there exists a continuous, decreasing function $\epsilon^s : \mathbb{R}_+ \to \mathbb{R}_+$ such that the supply elasticity when $z = 0$ equals $\epsilon^s(N_t)$.

It is difficult to provide an exact solution for the house price boom (a counterpart for Proposition 4) in this setting. Instead, the following proposition solves for the marginal impact of the shock x on the equilibrium house price $p_0^h(N_0, x, z, \chi)$ when x and z are small:

Proposition 8. *As* $x, z \to 0$

$$\frac{\partial p_0^h(N_0, x, z, \chi)/\partial x}{p_0^h(N_0, x, z, \chi)} = \frac{\epsilon^s(N_0)\mu_d^{max} + \chi\epsilon\mu_r^{max} + (1-\chi)\epsilon\overline{\mu}}{\epsilon^s(N_0) + \epsilon} \frac{(\epsilon^s(e^{\overline{\mu}x}N_0) + \epsilon)^{-1}}{1 + e^{-\int_0^x (\epsilon^s(e^{\overline{\mu}x'}N_0)+\epsilon)^{-1}\overline{\mu}dx'}}$$

holds to the first order in x *and* z.

Proposition 8 confirms the intuition from the main model on the distinct roles played by the supply elasticities at $t = 0$ and $t = 1$. The first fraction on the right aggregates beliefs across market participants. When the current elasticity $\epsilon^s(N_0)$ is higher, more weight is placed on the optimistic developer belief μ_d^{max} because these developers constitute a larger share of the marginal buyers in the market. A larger $\epsilon^s(N_0)$ implies a greater influence of disagreement on the house price today. Similarly, a higher χ increases the share of marginal buyers who are landlords and raises the weight on the optimistic resident belief μ_r^{max}.

In contrast, a greater value of $\epsilon^s(e^{\overline{\mu}x}N_0)$—the supply elasticity at $t = 1$ under the mean belief—lowers the house price at $t = 0$. A larger supply elasticity at $t = 1$ implies smaller pass-through of the shock x to the price at $t = 1$. This pass-through is given by $(\epsilon^s(e^{\overline{\mu}x}N_0) + \epsilon)^{-1}$. Similarly, a greater value of $\epsilon^s(e^{\overline{\mu}x}N_0)$ lowers the value of the integral, which captures the expected price tomorrow relative to today. When this ratio is lower, the pass-through of the shock x to the price at $t = 0$ is lower.

In summary, the price boom is largest when $\epsilon^s(N_0)$ is large but $\epsilon^s(e^{\overline{\mu}x}N_0)$ is small. This combination occurs when the shock x pushes the city from having a high supply elasticity

[21] Normalizing by r_t^h instead of p_t^h allows us to use the same function to analyze the supply elasticity at $t = 0$ and $t = 1$. Given the finite horizon of the model, the price at $t = 0$ is roughly double the price at $t = 1$, so supply would be about twice as elastic at $t = 0$ if we normalized by p_t^h. A unique equilibrium rent exists for $\chi > 0$; when $\chi = 0$ we choose the rent that obtains as the limit as $\chi \to 0$, which is a valid equilibrium rent for $\chi = 0$.

in the present to possibly having a low supply elasticity in the future. The possibility of such a transition amplifies the role of disagreement relative to cases in which transition to low supply elasticity is unlikely or has already occurred. Thus the intuition from the baseline model that disagreement affects the size of the house price boom most for cities at an intermediate level of development translates to this more general setting.

5 Stylized Facts of the US Housing Boom and Bust

This section uses data from the US housing boom between 2000 and 2006 to provide evidence consistent with the model's predictions about the house price boom at $t = 0$. We first document the importance of the relationship between the price of raw land and the price of housing across cities, which supports the model's focus on how disagreement interacts with potential land constraints. We then describe the speculative behavior in land markets among public homebuilders, who resemble the optimistic developers in the model extension in which developers issue equity. Last, we show how the model can be used to understand the booms in the cross-section of US cities—including those cities that appear as outliers in the classical supply elasticity framework—as well as across neighborhoods within cities.

5.1 The Central Importance of Land Prices

A key assumption of the model is that housing supply is limited in the long run by development constraints. These constraints lead land prices to rise during a housing boom, as developers anticipate the exhaustion of land. As a result, house prices and land prices rise in unison as shown by the result in Lemma 2 that $p_0^h = p_0^l + 2k$ in equilibrium.

Tracing house price increases to land prices distinguishes our model from "time-to-build." Traditionally, housing supply has been modeled as inelastic in the short run and elastic in the long run (DiPasquale and Wheaton, 1994; Mayer and Somerville, 2000). This paradigm described the US housing market very well for a time. Topel and Rosen (1988) show that essentially all variation in house prices between 1963 and 1983 in the US came from changes to the construction cost of structures. Temporary shortages of inputs needed to build a house, such as drywall and skilled labor, could explain this pattern, with the fluctuations in these input prices causing house price cycles.

Between 1983 and 2000, a secular shift occurred in housing supply in the US. Land prices became a much larger share of house prices (Davis and Heathcote, 2007), especially in certain cities (Davis and Palumbo, 2008). A large literature, surveyed by Gyourko (2009), has attributed this change to the rise of government regulations restricting housing supply. These rules bound city growth by limiting the number of building permits that are issued to developers. When demand to live in the city rises, land prices increase because the city

cannot expand.

Developers in a city without supply restrictions today might expect them to arrive in the future. Anticipating these regulatory changes, developers bid up land prices immediately after a demand shock, even in the absence of current building restrictions. In such a city, supply is elastic in the short run, but inelastic in the long run—it's "arrested development." In the baseline model, cities at an intermediate development level exhibit an extreme case of arrested development, with an infinite supply elasticity in period 0 and a zero supply elasticity in period 1 if all land is developed. The equilibrium presented in Proposition 8 considers a more general case in which the supply elasticity declines continuously with the level of initial demand.

Under arrested development, a nationwide housing demand shock can increase land prices *everywhere*, not just in cities where regulations currently restrict supply. Land prices rise even in areas with rapid construction. In contrast, time-to-build predicts construction cost increases and not land price increases. If temporary input shortages are driving house prices, then land prices, which are fully forward looking, should remain flat.

To assess the relative importance of land prices, we gather data on land prices and construction costs at the city level between 2000 and 2006. We use land prices measured directly from parcel transactions during this time. This approach contrasts with that used by Davis and Heathcote (2007) and Davis and Palumbo (2008), who measure land prices as the residual when construction costs are subtracted from house prices. A direct measure of land prices addresses concerns that such residuals capture something other than land prices between 2000 and 2006. The land price data we use are the indices constructed by Nichols, Oliner and Mulhall (2013). Using land transaction data, they regress prices on parcel characteristics and then derive city-level indices from the coefficients on city-specific time dummies.

We measure construction costs using the R.S. Means construction cost survey. This survey asks homebuilders in each city to report the marginal cost of building a square foot of housing, including all labor and materials costs. Survey responses reflect real differences across cities. In 2000, the lowest cost is \$54 per square foot and the highest is \$95; the mean is \$67 per square foot and the standard deviation is \$9. This survey has been used to study the time series and spatial variation in residential construction costs (Glaeser and Gyourko, 2005; Gyourko and Saiz, 2006; Gyourko, 2009).

As shown by Lemmas 1 and 2, the assumptions of our model imply that house prices must equal land prices plus construction costs: $p_t^h = p_t^l + k_t$. Log-differencing this equation between 2000 and 2006 yields

$$\Delta \log p^h = \alpha \Delta \log p^l + (1 - \alpha) \Delta \log k,$$

where Δ denotes the difference between 2000 and 2006 and α is land's share of house prices in 2000. The factor that matters more should vary more closely with house prices across cities. Because α and $1-\alpha$ are less than 1, the critical factor should also rise more than house prices do.

Figure 3 plots for each city the real growth in construction costs and land prices between 2000 and 2006 against the corresponding growth in house prices. Construction costs did rise during this period, but they rose substantially less than land prices, and construction cost increases display very little variation across cities. The time-to-build hypothesis, then, does explain some of the level of house price increases in the US during the boom, but none of the cross-sectional variation. Land prices display the opposite pattern, rising substantially and exhibiting a high correlation with house prices. Each city's land price increase also exceeds its house price increase. This evidence underscores the central importance of land prices for understanding the cross-section of the house price boom, and broadly supports the relative contribution of arrested development over time-to-build.

5.2 Supply-Side Speculation by Homebuilders

Proposition 2 predicts that as long as they are not endowed with too much land, optimistic developers amass land beyond their immediate construction needs at $t = 0$ in intermediate cities. Proposition 6 extends this result to the case in which developers finance themselves with equity and offers additional predictions about the developer equity market. We examine these predictions among a class of developers for whom rich data are publicly available: public homebuilders. We focus on the eight largest firms and hand-collect landholding data from their annual financial statements between 2001 and 2010.[22]

The eight equity-financed large firms we study nearly tripled their landholdings between 2001 and 2005, as shown in Figure 4(a). Consistent with Propositions 2 and 6(a) and 6(b), these land acquisitions far exceed land needed for new construction. Annual home sales increased by 120,000 between 2001 and 2005, while landholdings increased by 1,100,000 lots. One lot can produce one house, so landholdings rose more than *nine times* relative to home sales. In 2005, Pulte changed the description of its business in its 10-K to say, "We consider land acquisition one of our core competencies." This language appeared until 2008, when it was replaced by, "Homebuilding operations represent our core business."

Having amassed large land portfolios, these firms subsequently suffered significant capital losses. Figure 4(b) documents the dramatic rise and fall in the total market equity of these homebuilders between 2001 and 2010. Homebuilder stocks rose 430% and then fell 74% over this period. By Proposition 6(d), the rise is consistent with a positive shock $x > 0$ at $t = 0$.

[22]Our analysis complements Haughwout et al. (2012), an empirical study of the homebuilding industry that presents similar facts from different data.

If we interpret the period between 2006 and 2010 as that between $t = 0$ and $t = 1$, then by Proposition 6(e) the losses are consistent with a realization of μ^{true} below the optimistic investor belief μ_i^{max}.

The majority of the losses borne by homebuilders arose from losses on the land portfolios they accumulated from 2001 to 2005. In 2006, these firms began reporting write-downs to their land portfolios. At $29 billion, the value of the land losses between 2006 and 2010 accounts for 73% of the market equity losses over this time period. The homebuilders bore the entirety of their land portfolio losses. The absence of a hedge against downside risk supports the theory that homebuilder land acquisitions represented optimistic beliefs.

It is hard to argue that this rise and fall of equity prices reflects any monopoly rents homebuilders earned by building houses during this period. During the boom, homebuilding was extremely competitive. Haughwout et al. (2012) document that the largest ten homebuilders had less than a 30% market share throughout the boom, with firms outside the largest sixty constituting over half of market share. Although some consolidation occurred between 2000 and 2006, these numbers portray an extremely competitive market. If anything, consolidation may reflect purchases by optimistic firms of pessimists who chose to abstain from land speculation.

Consistent with Prediction 6(c), these homebuilders witnessed heightened short-selling of their equity during the boom. Figure 4(c) plots the distribution of the average monthly short interest ratio, defined as the ratio of shares currently sold short to total shares outstanding, across all industries between 2000 and 2006. The short interest of homebuilder stocks lies in the 95th percentile, meaning that investors short-sold this industry more than nearly all others during the boom. As a point of comparison, the short interest in homebuilders was triple that in investment banks, another industry exposed to housing at this time. The short interest in homebuilders provides direct evidence of disagreement over the value of their land portfolios.[23]

Several recent papers argue that optimism about house prices was widespread between 2000 and 2006. For instance, Foote, Gerardi and Willen (2012) document twelve facts about the mortgage market during this time inconsistent with incentive problems between borrowers and lenders, but consistent with beliefs of borrowers and lenders that house prices would continue to rise. Case, Shiller and Thompson (2012) directly survey homeowners during the boom and find that they expected continued appreciation in house prices over the next decade, as opposed to the bust that eventually occurred. Cheng, Raina and Xiong

[23] An earlier draft of this paper provided the time series of short interest in homebuilder stock from 2001 to 2010. Short interest rose from 2001 to 2006, but rose even further from 2006 to 2009. Homebuilder short interest was highest as the bust was beginning. This peak may indicate that disagreement reached its peak after the boom, complicating the idea that disagreement was high during the boom. Alternatively, the late peak could indicate that shorting is more attractive for pessimists when they anticipate a bust in the near future.

(2014) find that securitized finance managers did not sell off their personal housing assets during the boom, indicating that these managers had similarly optimistic beliefs relative to the rest of the market. The disagreement our model relies on is in fact consistent with such widespread optimism. Homeowners and investors can be optimistic on average, with dispersion in beliefs around this optimistic mean. Furthermore, only the *most* optimistic investors price land in the model. Thus, a few extraordinarily optimistic investors have a large price impact, even when nearly all people agree about the future of house prices.

5.3 The Cross-Section of Cities

House price increases differed markedly across cities during the 2000-2006 US housing boom. Propositions 1 and 4 derive house price increases as a function of city development levels and disagreement. We test these predictions by interpreting them as comparative statics and then examining them against the empirical variation in house price increases across cities. Not only are these predictions borne out, but they explain some of the most puzzling aspects of this cross-city variation.

We document these puzzling cross-sectional facts using city-level house price and construction data. House price data come from the Federal Housing Finance Agency's metropolitan statistical area quarterly house price indices. We measure the housing stock in each city at an annual frequency by interpolating the US Census's decadal housing stock estimates with its annual housing permit figures. Throughout, we focus on the 115 metropolitan areas for which the population in 2000 exceeds 500,000. The boom consists of the period between 2000 and 2006, matching the convention in the literature to use 2006 as the end point (Mian, Rao and Sufi, 2013).[24]

Figure 5(a) plots construction and house price increases across cities. The house price increases vary enormously across cities, ranging from 0% to 125% over this brief six-year period. The largest price increases occurred in two groups of cities. The first group, which we call the Anomalous Cities, consists of Arizona, Nevada, Florida, and inland California. The other large price increases happened in the Inelastic Cities, which comprise Boston, Providence, New York, Philadelphia, and the west coast of the United States.

The history of construction and house prices in the Anomalous Cities before 2000 constitutes the first puzzle. As shown in Figures 5(b) and 5(c), from 1980 to 2000 these cities provided clear examples of very elastic housing markets in which prices stay low through rapid construction activity. Construction far outpaced the US average while house prices

[24]Davidoff (2013) also documents these facts, and we use his approach of comparing construction and house prices before 2000 to during the boom. Gao, Sockin and Xiong (2015) show that price growth during the boom display a non-monotonic relationship with respect to Saiz (2010)'s measures of long-run supply elasticity. They develop a model in which intermediate levels of supply elasticity impede information aggregation in housing markets, leading the intermediate cities to experience the greatest house price volatility.

remained constant. In a model like Glaeser, Gyourko and Saiz (2008), in which each city is characterized by a constant housing supply elasticity, the subsequent surge in house prices in these cities is impossible. Perfectly elastic supply should meet whatever housing demand shock arrived in 2000 with higher construction, holding down house prices.

Our model explains this pattern by distinguishing short-run and long-run housing supply elasticities. Proposition 1 shows that prices rise in intermediate cities in which vacant land remains because supply is expected to become constrained soon. This phenomenon depends on the way we have modeled housing supply and holds even without disagreement. Figure 1 demonstrates long-run barriers in Las Vegas. More broadly, the land price increases across the country shown in Figure 3 indicate the presence of these constraints in other cities, or at least developers' anticipation of them.

The second puzzle is that the price increases in the Anomalous Cities were as large as those in the Inelastic Cities. The Inelastic Cities consist of markets where house prices rise because regulation and geography prohibit construction from absorbing higher demand. We document this relationship in Figures 5(b) and 5(c), which show that construction in these cities was lower than the US average before 2000 while house price growth greatly exceeded the US average. As shown in Figure 5(a), house prices increased as much in the Anomalous Cities as they did in the Inelastic Cities. This pattern poses a puzzle for models without disagreement. Proposition 1 shows that, without disagreement, constrained cities experience larger house price booms than intermediate and unconstrained cities. Even models such as Hong, Scheinkman and Xiong (2006), in which investors disagree but the marginal buyer type does not vary with asset float, cannot easily explain the non-monotonicity in house price increases.

Proposition 4 explains the pattern. With disagreement $z > 0$, the same demand shock $x > 0$ raises the $t = 0$ house price most in intermediate cities, not in constrained cities. According to our model, land availability in the Anomalous Cities facilitated speculation and thus amplified the increase in house prices. This amplification effect was smaller in the Inelastic Cities, which featured less undeveloped land. Evidence of disagreement during the boom comes from the stylized facts about public homebuilders in Section 5.2, as Proposition 6 shows that these facts are guaranteed to hold only with disagreement.

The final puzzle is that some elastic cities built housing quickly during the boom but, unlike the Anomalous Cities, experienced stable house prices. These cities appear in the bottom-right corner of Figure 5(a), and are located mostly in the southeastern United States (e.g., Texas and North Carolina).[25] Their construction during the boom quantitatively matches that in the Anomalous Cities, but the price changes are significantly smaller. Why

[25]The cities with annual housing stock growth above 2% and cumulative price increases below 25% are Atlanta, Austin, Charlotte, Colorado Springs, Columbus, Dallas, Denver, Des Moines, Fort Collins, Fort Worth, Houston, Indianapolis, Lexington, Nashville, Ogden, Raleigh, and San Antonio.

was rapid construction able to hold down house prices in some cities and not others?

Propositions 1 and 4 explain that what distinguishes these cities are their long-run supply elasticities. A city can have perfectly elastic short-run supply, yet its long-run supply is indeterminate. Among the cities with elastic short-run supply, the intermediate cities face constraints soon while the unconstrained cities do not. The model's explanation of Figure 5(a) is that the Anomalous Cities are the ones approaching the long-run constraints, whereas the cities in the bottom right did not face development barriers in the foreseeable future.

Some evidence consistent with this argument comes from the financial statements of Pulte, one of the homebuilders studied in Section 5.2. In a February 2004 presentation to investors, Pulte listed several of the Anomalous Cities as "supply constrained markets you may not have expected": West Palm Beach, Orlando, Tampa, Ft. Myers, Sarasota, and Las Vegas (Chicago was also listed). In contrast, Pulte stated that Texas was "the only area of the country without supply constraints in some form," and listed many of the non-anomalous elastic cities (Atlanta, Charlotte, and Denver) as "not supply constrained overall," although "supply issues in preferred submarkets" were noted.[26] The slides are presented in Appendix E.[27]

An alternate explanation for these facts is that the Anomalous Cities simply experienced much larger demand growth between 2000 and 2006 than the rest of the country. Abnormally large demand growth would increase prices and construction, leading the Anomalous Cities to occupy the top-right part of Figure 5(a).[28] While our discussion above considers the case of a common demand shock, Figure 5 makes it clear that demand growth did differ across cities. The cluster of cities in the bottom left of the graph likely saw low price growth and construction because demand was flat during this time. The experiences of these cities raise the possibility that the Anomalous Cities saw abnormally large demand growth just as these cities saw abnormally small growth.[29]

We examine whether the Anomalous Cities experienced abnormally large demand shocks,

[26]Other cities with supply constraints only in submarkets were Phoenix, Jacksonville, Detroit, and Minneapolis.

[27]The Pulte slides provide narrative support for some of the other assumptions and predictions of the model. Pulte stated that "[Anti-growth efforts] are not new for heavily populated areas (Northeast, California) but now are widespread across the country." This statement indicates that at least one major developer—the largest public homebuilder at the time—recognized the rise of supply restrictions throughout the country, consistent with our assumption of finite long-run land supply.

[28]Another explanation is that the value of the option, described by Titman (1983) and Grenadier (1996), to develop land with different types of housing may have been largest in the anomalous cities, but many of these areas consist of homogeneous sprawl (Glaeser and Kahn, 2004), lessening this concern.

[29]Section 3.3 was silent on the model's predictions for construction, which we present in Appendix F. The model is ill-suited to explain construction between 2000 and 2006 because it considers only a shock to news about future demand and not a shock to current demand. As shown in Appendix F, the shock x does not alter the equilibrium housing stock, except in intermediate cities with disagreement where $x > 0$ lowers the stock relative to the case without disagreement.

and whether these large shocks are sufficient to account for the extreme price movements in these cities. Mian and Sufi (2009) argue that the shock was the expansion of credit to low-income borrowers. It is possible that this shock affected the Anomalous Cities more than the rest of the nation, for instance because they contained greater shares of low income individuals, and that this greater exposure to the shock led to abnormally large price increases.

To address this possibility, we calculate the house price booms that would be predicted from each city's supply elasticity and relevant demographics in 2000. We construct the predicted price increases in the following manner. Suppose that between 2000 and 2006, each city experienced a permanent increase in log housing demand equal to x_j. From Proposition 8, the resulting increase in house prices when $z = 0$ equals

$$\Delta \log p_j^h = \frac{\overline{\mu} x_j}{2(\epsilon_j^s + \epsilon)} \tag{2}$$

to the first order in x, where ϵ_j^s is city j's housing supply elasticity. Because we are exploring a counterfactual without disagreement, this specification assumes that $\overline{\mu}$ does not vary across cities.

Mian and Sufi (2009) show that the following demographic variables predict the presence of subprime borrowers at the ZIP-code level: household income (negatively), poverty rate, fraction with less than high school education, and fraction nonwhite. We measure these variables at the metropolitan area level in the 2000 US Census, and use them to predict the unobserved shock x_j. We denote this vector of demographics, plus a constant and log population, by $\mathbf{d}_j$. Under the null hypothesis that these demographics alone predict the shock, we may write $\overline{\mu} x_j / 2 = \beta \mathbf{d}_j + \eta_j$, where β is the same across cities, and $\mathbf{d}_j \perp \eta_j$. Substituting this expression into equation (2) yields the estimating equation

$$(\epsilon_j^s + \epsilon) \Delta \log p_j^h = \beta \mathbf{d}_j + \eta_j. \tag{3}$$

Estimating β using this equation allows us to calculate the house price boom predicted by the supply elasticity ϵ_j^s and the demographics $\mathbf{d}_j$. In equation (3), the left represents the house price increase adjusted by the elasticity of supply, while $\beta \mathbf{d}_j$ is the housing demand shock predicted by the city's exposure to subprime. We use Saiz (2010)'s supply elasticity estimates for ϵ_j^s, and a value of 0.6 for the housing demand elasticity ϵ. This value lies in the range of estimates calculated by Hanushek and Quigley (1980). Using these data, we produce an estimate $\widehat{\beta}$ using ordinary least squares on equation (3). The resulting house

price boom predicted from demographics and supply elasticity equals

$$\mathbf{E}\left(\Delta \log p_j^h \mid \mathbf{d}_j, \epsilon_j^s\right) = \frac{\widehat{\beta}\mathbf{d}_j}{\epsilon_j^s + \epsilon}.$$

Figure 6 plots the actual house price growth against the predicted price growth for each city in Figure 5(a). The Anomalous Cities remain clear outliers. Abnormal demand growth from low-income borrowers does not explain the extreme experiences of these cities. In theory, these predicted price increases could have lined up well with the actual increases in the Anomalous Cities. This alignment would have held if the subprime demographics predicted the shocks, these cities were very exposed to subprime, and their housing supply were inelastic enough. This story fails to explain the anomalous house price booms, which experienced higher price growth despite elastic supply and even conditioning on observable drivers of demand. Furthermore, the growth in subprime credit was widespread, with high-housing supply elasticity cities experiencing large expansions in subprime credit without house price growth (Mian and Sufi, 2009, Table VII).

5.4 Variation in House Price Booms Within Cities

Proposition 7 of the model predicts larger price increases in market segments within a city that attract more renters than owners. A sufficient statistic for this effect is χ, the share of the housing stock that is rented. Proposition 7 holds only among segments with the same N_0, x and z. This "all else equal" assumption is unlikely to hold empirically, so our discussion focuses on the conceptual predictions about within-city variation made by the model.

We first consider variation in χ across neighborhoods. Neighborhoods provide an example of market segments because they differ in the amenities they offer. For instance, some areas offer proximity to restaurants and nightlife, while others provide access to good public schools. These amenities appeal to different groups of residents. Variation in amenities hence leads χ to vary across space. Neighborhoods whose amenities appeal relatively more to renters than to owner-occupants are characterized by a higher value of χ.

We obtain ZIP-level data on χ from the US Census, which reports the share of occupied housing that is rented, as opposed to owner-occupied, in each ZIP code in 2000. χ varies considerably within cities. Its national mean is 0.29 and standard deviation is 0.17, while the R^2 of regressing χ on city fixed-effects is only 0.12. We calculate the real increase in house prices from 2000 to 2006 using Zillow.com's ZIP-level house price indices. We regress this price increase on χ and city fixed-effects, and find a positive and highly significant coefficient of 0.10 (0.026), where the standard error is clustered at the city level. Thus, consistent with Proposition 7, house prices increased more between 2000 and 2006 in neighborhoods where χ was higher in 2000.

This positive relationship between χ and price increases may not be causal. Housing demand shocks in the boom were larger in neighborhoods with a higher value of χ. The housing boom resulted from an expansion of credit to low-income households (Mian and Sufi, 2009; Landvoigt, Piazzesi and Schneider, 2015). As a result, the strong covariance of ZIP-level income with χ will tend to bias our estimates.[30] Furthermore, a city-wide demand shock might raise house prices most strongly in cheap areas due to gentrification dynamics (Guerrieri, Hartley and Hurst, 2013), and χ covaries negatively with the level of house prices within a city.

The appeal of χ is that it predicts price increases in any housing boom in which there is disagreement about future fundamentals. In general, χ predicts price increases because it is positively correlated with speculation, not because it is correlated with demand shocks. Empirical work can test Proposition 7 by examining housing booms in which the shocks are independent from χ.

The second approach to measuring χ is to exploit variation across different types of housing structures. According to the US Census, 87% of occupied detached single-family houses in 2000 were owner-occupied rather than rented. In contrast, only 14% of occupied multifamily housing was owner-occupied. According to Proposition 7, the large difference in χ between these two types of housing causes a larger price boom in multifamily housing, all else equal.

This result squares with accounts of heightened investment activity in multifamily housing during the boom.[31] For instance, a consortium of investors—including the Church of England and California's pension fund CalPERS—purchased Stuyvesant Town & Peter Cooper Village, Manhattan's largest apartment complex, for a record price of $5.4 billion in 2006. Their investment went into foreclosure in 2010 as the price of this complex sharply fell (Segel et al., 2011). Multifamily housing attracts speculators because it is easier to rent out than single-family housing. During periods of uncertainty, optimistic speculators bid up multifamily house prices and cause large price booms in this submarket.

6 Conclusion

In this paper, we argue that disagreement explains an important part of housing cycles. Disagreement amplifies house price booms by biasing prices toward optimistic valuations. Our emphasis on how disagreement interacts with long run development constraints allows us to explain aspects of the boom that are at odds with existing theories of house prices.

[30] The IRS reports the median adjusted gross income at the ZIP level. We take out city-level means, and the resulting correlation with χ is -0.40.

[31] Bayer et al. (2015) develop a method to identify speculators in the data. A relevant extension of their work would be to look at the types of housing speculators invest in.

Many of the largest price increases occurred in cities that were able to build new houses quickly. This fact poses a problem for theories that stress inelastic housing supply as the sole source of house price booms. But it sits well with our theory, which instead emphasizes speculation. Undeveloped land facilitates speculation due to rental frictions in the housing market. In our model, large price booms occur in elastic cities facing a development barrier in the near future.

Introducing key aspects of the housing market—heterogeneous ownership utility and the nature of asset supply—extends and clarifies past work in finance that has focused on disagreement in financial asset markets. In our model, disagreement raises the price of housing only under certain conditions and the relationship between disagreement and asset supply can be non-monotonic. The particular setting of housing markets also presents an important case in which disagreement reduces welfare, as pessimists with high flow utility may be replaced by optimists with low flow utility.

We document the central importance of land price increases for explaining the US house price boom between 2000 and 2006. These land price increases resulted from speculation directly in the land market. Consistent with this theory, homebuilders significantly increased their land investments during the boom and then suffered large capital losses during the bust. Many investors disagreed with this optimistic behavior and short-sold homebuilder equity as the homebuilders were buying land.

In one of the model's extensions, price booms are larger in submarkets within a city where a greater share of housing is rented. We present some evidence for this prediction, but further empirical work is needed to test it more carefully. We also look forward to work exploring these findings to understand cycles outside the US, in historical episodes, and in other markets with similar features to housing.

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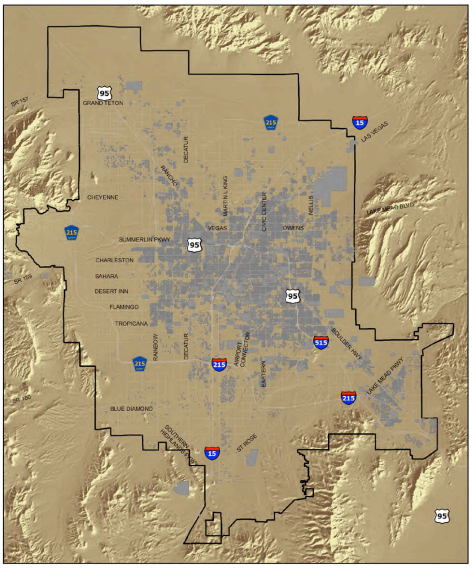
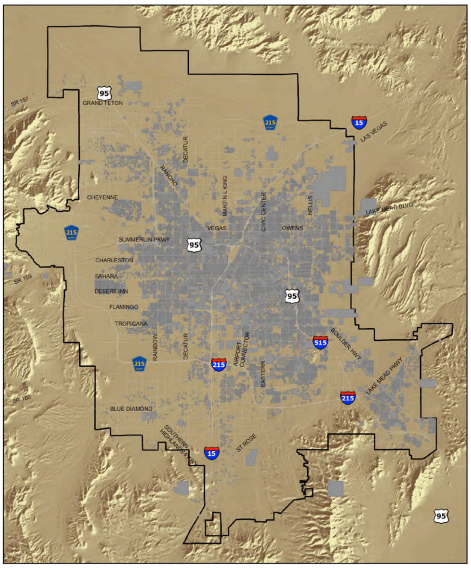
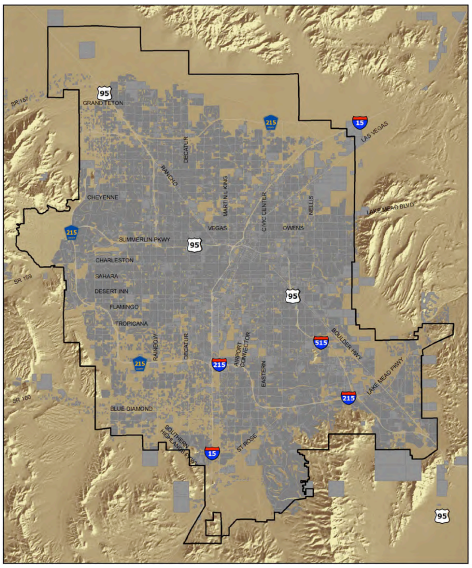
FIGURE 1
Long-Run Development Constraints in Las Vegas



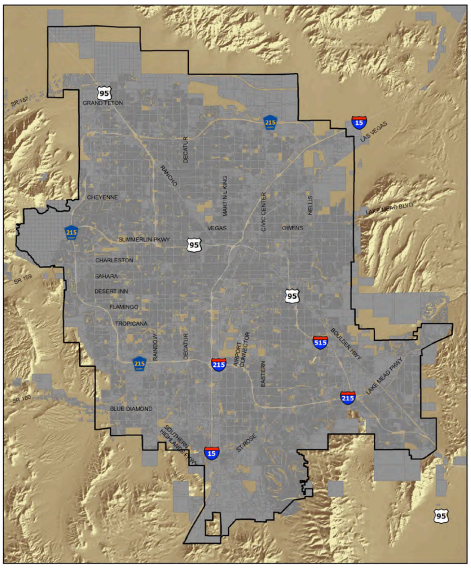
1980



1990



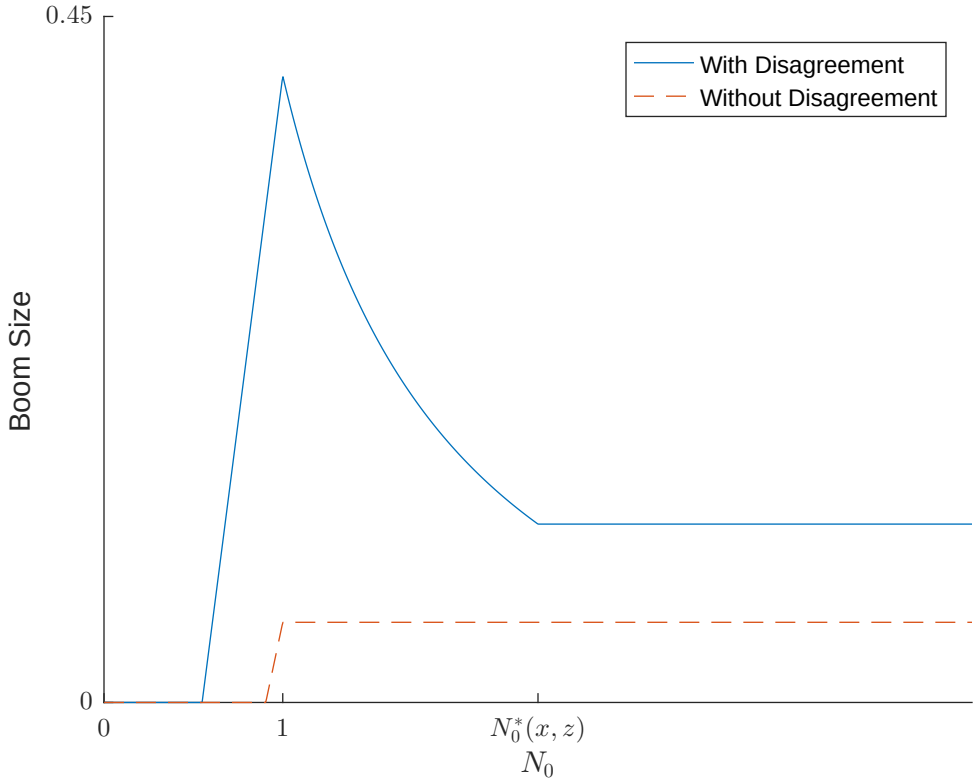
2008



2030

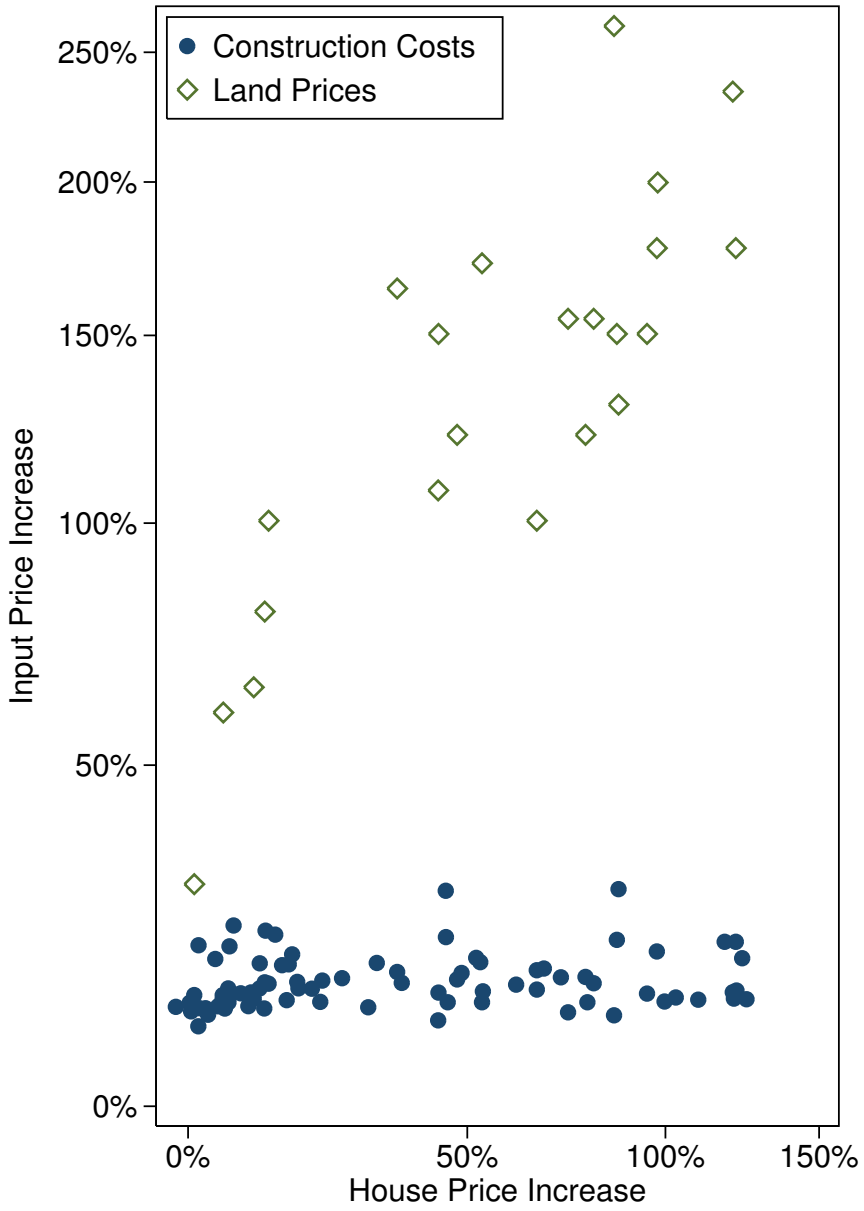
Notes: This figure comes from page 51 of the Regional Transportation Commission of Southern Nevada's *Regional Transportation Plan 2009-2035* (RTCSNV, 2012). The first three pictures display the Las Vegas metropolitan area in 1980, 1990, and 2008. The final picture represents the Regional Transportation Commission's forecast for 2030. The boundary is the development barrier stipulated by the Southern Nevada Public Land Management Act. The shaded gray region denotes developed land.

FIGURE 2
House Price Boom for Different Initial Demands



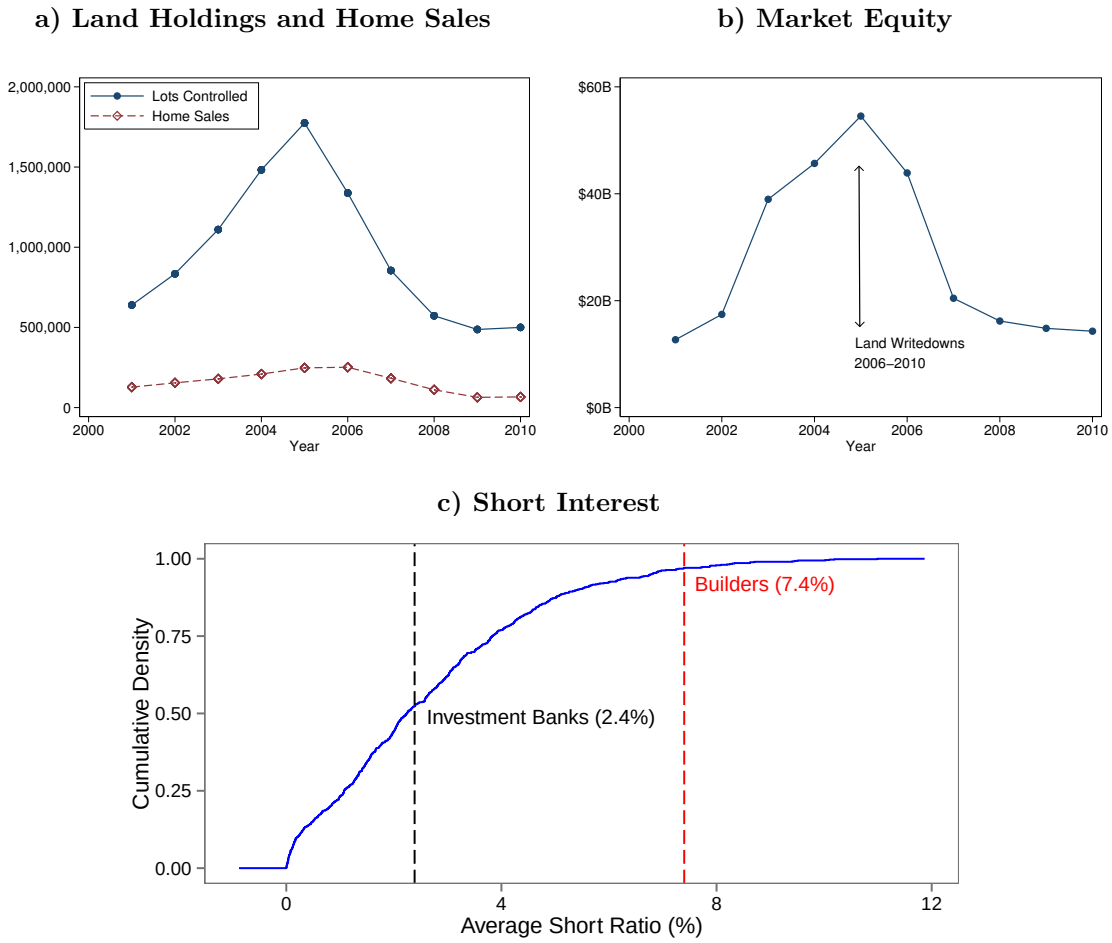
Notes: The boom size equals $p_0^h(N_0,x,z)/p_0^h(N_0,0,z)-1$ with disagreement and $p_0^h(N_0,x,0)/p_0^h(N_0,0,0)-1$ without disagreement. N_0 equals the number of potential residents at $t=0$ relative to the amount of space in the city. The parameter values used to generate this figure are $x=0.5$, $z=1$, $\epsilon=1$, $\overline{\mu}=0.2$, and $f_r=f_d$ with 90 percent of agents having $\theta=-1/9$ and 10 percent having $\theta=1$. These parameters are defined in Section 1.

FIGURE 3
Input Price and House Price Increases Across Cities, 2000-2006



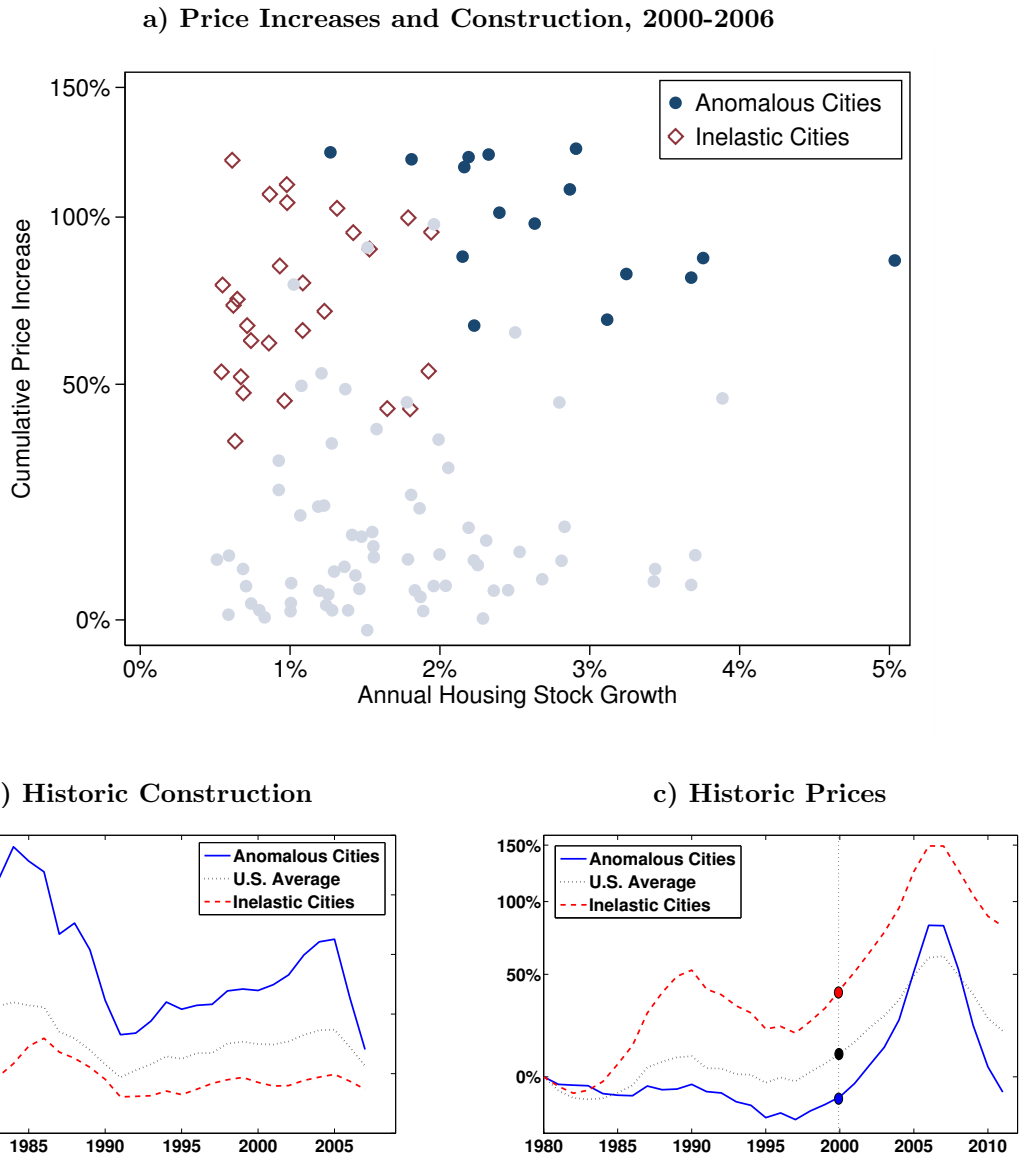
Notes: We measure construction costs for each city using the R.S. Means survey figures for the marginal cost of a square foot of an average quality home, deflated by the CPI-U. Gyourko and Saiz (2006) contains further information on the survey. Land price changes come from the hedonic indices calculated in Nichols, Oliner and Mulhall (2013) using land parcel transactions, and house prices come from the second quarter FHFA housing price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. For land prices, we have data for Atlanta, Baltimore, Boston, Chicago, Dallas, Denver, Detroit, Houston, Las Vegas, Los Angeles, Miami, New York, Orlando, Philadelphia, Phoenix, Portland, Sacramento, San Diego, San Francisco, Seattle, Tampa, and Washington D.C.

FIGURE 4
Supply-Side Speculation Among U.S. Public Homebuilders, 2001-2010



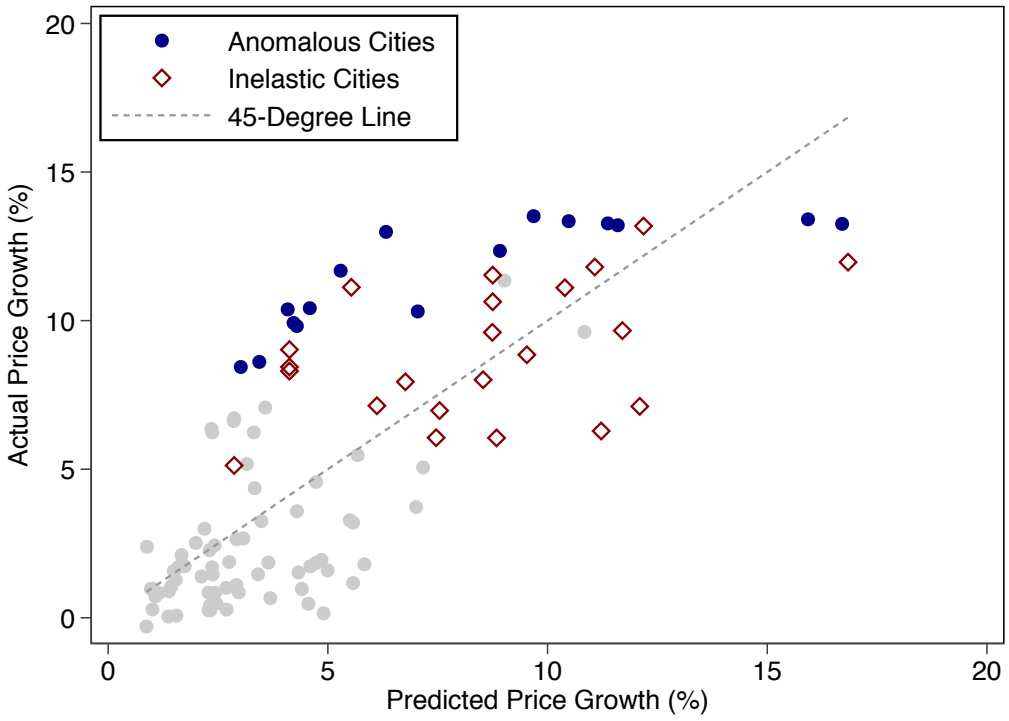
Notes: **(a), (b)** Data come from the 10-K filings of Centex, Pulte, Lennar, D.R. Horton, K.B. Homes, Toll Brothers, Hovnanian, and Southern Pacific, the eight largest public U.S. homebuilders in 2001. "Lots Controlled" equals the sum of lots directly owned and those controlled by option contracts. The cumulative writedowns to land holdings between 2006 and 2010 among these homebuilders totals $29 billion. **(c)** Short interest is computed as the ratio of shares currently sold short to total shares outstanding. Monthly data series for shares short come from COMPUSTAT and for shares outstanding come from CRSP. We compute mean short interest between 2000 and 2006 for each six-digit NAICS industry and plot the cumulative distribution of these means. Builder stocks are classified as those with NAICS code 236117 and investment bank stocks are those with NAICS code 523110.

FIGURE 5
The U.S. Housing Boom and Bust Across Cities



Notes: Anomalous Cities include those in Arizona, Nevada, Florida, and inland California. Inelastic Cities are Boston, Providence, New York, Philadelphia, and all cities on the west coast of the United States. We measure the housing stock in each city at an annual frequency by interpolating the U.S. Census's decadal housing stock estimates with its annual housing permit figures. House price data come from the second quarter FHFA house price index deflated by the CPI-U. The figure includes all metropolitan areas with populations over 500,000 in 2000 for which we have data. **(a)** The cumulative price increase is the ratio of the house price in 2006 to the house price in 2000. The annual housing stock growth is the log difference in the housing stock in 2006 and 2000 divided by six. **(b), (c)** Each series is an average over cities in a group weighted by the city's housing stock in 2000. Construction is annual permitting as a fraction of the housing stock. Prices represent the cumulative returns from 1980 on the housing in each group.

FIGURE 6
Anomalous Cities and Differential Demand Shocks, 2000-2006



Notes: This plot compares actual price growth during the boom to predicted price growth as a function of city level demographics, where predicted price growth proxies for differential demand shocks. Actual price growth is the log change in the second quarter FHFA house price index deflated by the CPI-U. We compute predicted price growth from a cross sectional regression of actual price growth on a set of city level demographics: log population, log of median household income, percent white, percent white and not hispanic, percent with less than 9th grade education, percent with less than 12th grade education, percent unemployed, and percent of families under the poverty line. Demographics come from the 2000 Census.

A Proofs

Lemma 1

If $p_1^h > p_1^l + k$, then each developer wants to buy an infinite amount of land, build houses with the land, and then sell the houses. As a result, the land market cannot clear. If $p_1^h < p_1^l + k$, then the reverse holds, meaning that developers want to sell an infinite amount of land. The land market cannot clear in this case either. In equilibrium, the only possibility is that $p_1^h = p_1^l + k$. At $t = 1$, demand from arriving potential residents equals $N_1 SD(p_1^h)$. Supply from outgoing residents equals 0 if $p_1^h < 0$ and Q_r if $p_1^h > 0$, where Q_r is the number of potential residents who bought at $t = 0$. Developers are indifferent to how much housing they sell because $p_1^h = p_1^l + k$, but the most they can sell emerges from summing across the two developer constraints to obtain $\sum H_1^{sell} \leq \sum H_1 + L_1 = S - Q_r$. The sum of H_1^{sell} across developers and potential residents cannot exceed S.

We now consider three possible equilibria. In the first, $p_1^l < 0$. This inequality cannot hold in equilibrium because developer land demand would be infinite for each developer, and the land market would not clear. The next possibility is that $p_1^l = 0$. In this case, demand from arriving potential residents equals $N_1 S$. If $N_1 > 1$, then this equilibrium fails because maximal aggregate home sales equal S. If $N_1 \leq 1$, then we construct an equilibrium as follows. We cannot have $N_1 S > L_1 + H_1 + Q_r$ for all developers (summing across them delivers a contradiction when $N_1 \leq 1$), so consider a developer for whom $N_1 S \leq L_1 + H_1 + Q_r$. This developer sets $L_1^{buy} = 0$, $H_1^{build} = N_1 S - Q_r - H_1$, and $H_1^{sell} = N_1 S - Q_r$. All other developers set $L_1^{buy} = 0$, $H_1^{build} = -H_1$, and $H_1^{sell} = 0$. All developer constraints and optimality conditions are satisfied under these choices, and both the housing and land markets clear. Finally, we consider the possibility that $p_1^l > 0$. Because $p_1^h > 0$, the first constraint for the developers binds, so we can rewrite the developer objective function as $p_1^h H_1 + p_1^l (H_1^{build} - L_1^{buy})$. Because $p_1^l > 0$, the developer maximizes this objective by satisfying the second constraint and setting $H_1^{build} - L_1^{buy} = L_1$. Because both constraints are satisfied with equality, summing across them yields $\sum H_1^{sell} = \sum H_1 + \sum L_1 + \sum L_1^{buy}$. If the land market clears, the last sum equals 0. The housing market clears when $N_1 SD(p_1^h) = \sum H_1 + \sum L_1 + Q_r = S$. If $N_1 < 1$, then no solution for p_1^h exists. If $N_1 = 1$, then the only solutions have $p_1^h \leq k$, contradicting the assumption that $p_1^l > 0$. This equilibrium is possible if and only if $N_1 > 1$, in which case the unique solution is $p_1^h = k N_1^{1/\epsilon}$. The optimality conditions and constraints for all developers are satisfied if they set $H_1^{build} = L_1$, $H_1^{sell} = H_1 + L_1$, and $L_1^{buy} = 0$. The land and housing markets clear under these choices. In summary, a unique equilibrium exists for each value of N_1. When $N_1 \leq 1$, only $p_1^l = 0$ and $p_1^h = k$ are possible, whereas when $N_1 > 1$, only $p_1^l = k N_1^{1/\epsilon} - k$ and $p_1^h = N_1^{1/\epsilon}$ are possible.

Lemma 2

The utility at $t = 1$ of a resident who bought at $t = 0$ equals $p_1^h - p_0^h + v$ if $p_1^h \geq 0$. Housing demand from potential residents at $t = 0$ equals $\int_\Theta N_0 SD(p_0^h - p_1^h(e^{\mu(\theta)x} N_0)) f_r(\theta) d\theta$. For the same argument given in the proof of Lemma 1 that $p_1^h = p_1^l + k$, $p_0^h = p_0^l + 2k$ in equilibrium: developers would want to buy or sell infinite land otherwise. In all of the equilibria characterized in the proof of Lemma 1, $\pi = p_1^h H_1 + p_1^l L_1 + B_1$. By making substitutions using the constraints of the $t = 0$ developer problem, we see that the objective at $t = 0$ is to choose $H_1, L_1 \geq 0$ to maximize $(p_1^h(e^{\mu(\theta)x} N_0) - p_0^h) H_1 + (p_1^h(e^{\mu(\theta)x} N_0) - p_0^h + k) L_1 + p_0^l L_0$. In all equilibria, all developers choose finite values of H_1 and L_1, so the first order conditions imply $p_1^h(e^{\mu(\theta)x} N_0) - p_0^h + k \leq 0$ for all

$\theta \in \text{supp}\, f_d$. Because $p_1^h(\cdot)$ increases, either $p_0^h = p_1^h(e^{\mu_d^{max} x} N_0) + k$ in which case developers with $\theta = \theta_d^{max}$ may choose any $L_1 \geq 0$, or $p_0^h > p_1^h(e^{\mu_d^{max} x} N_0) + k$ in which case $L_1 = 0$ for all developers.

We now consider these two possible equilibria. The first may hold only if potential resident housing demand does not exceed S (developers cannot build more than this quantity of housing, and the housing market must clear). This condition reduces to

$$1 \geq N_0 \int_\Theta D(p_1^h(e^{\mu_d^{max} x} N_0) + k - p_1^h(e^{\mu(\theta) x} N_0)) f_r(\theta) d\theta. \tag{A1}$$

If (A1) holds, then we construct an equilibrium as follows. Denote $Q_r = \int_\Theta N_0 S D(p_1^h(e^{\mu_d^{max} x} N_0) + k - p_1^h(e^{\mu(\theta) x} N_0)) f_r(\theta) d\theta$. For one developer for whom $\theta = \theta_d^{max}$, we set $H_0^{build} = Q_r$, $H_0^{sell} = Q_r$, and $L_0^{buy} = S - L_0$. For all other developers, we set $H_0^{build} = 0$, $H_0^{sell} = 0$, and $L_0^{buy} = -L_0$. All developer constraints and optimality conditions hold, and the housing and land markets clear.

If (A1) fails, then we must have $N_0 > 1$ because $D \leq 1$. As a result, we may define $p_0^h(N_0, x, z)$ to be the unique solution to

$$1 = N_0 \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta) x} N_0)) f_r(\theta) d\theta. \tag{A2}$$

To see that a solution to (A2) exists, consider that when $p_0^h = 2k$ the right side of the equation equals $N_0 > 1$. Because the right side goes to 0 (pointwise) as $p_0^h \to \infty$, by the intermediate value theorem we may find p_0^h satisfying (A2). This solution is unique because the integrand strictly decreases if $p_0^h - p_1^h(e^{\mu(\theta) x} N_0) \geq k$; this equation must hold at $p_0^h = p_0^h(N_0, x, z)$ for at least some $\theta \in \text{supp}\, f_r$ for otherwise the right side of (A2) exceeds 1. Because the right side of (A2) weakly decreases in p_0^h, if (A1) fails then $p_0^h(N_0, x, z) > p_1^h(e^{\mu_d^{max} x} N_0) + k$. As a result, developer constraints and optimality conditions are satisfied when for all developers $L_0^{buy} = 0$, $H_0^{build} = L_0$, and $H_0^{sell} = L_0$; housing and land markets clear as well.

In summary, if (A1) holds then the unique equilibrium price is $p_0^h(N_0, x, z) = p_1^h(e^{\mu_d^{max} x} N_0) + k$; if (A1) fails, then the unique equilibrium price $p_0^h(N_0, x, z)$ is the unique solution to (A2).

Proposition 1

From the proof of Lemma 2, developers hold at the end of $t = 0$ if and only if (A1) holds without equality. When $z = 0$, $\mu(\theta) = \mu_d^{max}$ for all $\theta \in \Theta$ so (A1) reduces to $1 \geq N_0$. As a result, developers hold land at the end of $t = 0$ if and only if $N_0 < 1$, as claimed. In this case, $p_0^h(N_0, x, 0) = p_1^h(e^{\overline{\mu} x} N_0) + k$. If $N_0 < e^{-\overline{\mu} x}$, then (from Lemma 1) $p_0^h(N_0, x, 0) = 2k$; if $e^{-\overline{\mu} x} \leq N_0 < 1$, then $p_0^h(N_0, x, 0) = k(e^{\overline{\mu} x/\epsilon} N_0^{1/\epsilon} - 1)$. If $N_0 \geq 1$, then $p_0^h(N_0, x, 0) = k N_0^{1/\epsilon} + p_1(e^{\overline{\mu} x} N_0)$ is the unique solution to (A2). Because $\overline{\mu} x \geq 0$, in this case $p_0^h(N_0, x, 0) = k(1 + e^{\overline{\mu} x/\epsilon}) N_0^{1/\epsilon}$, as claimed. The final equation in the proposition follows from the solution for $p_0^h(N_0, x, 0)$; note that in the intermediate case, if $x = 0$, then $p_0^h = 2k$ as in the unconstrained case.

Proposition 2

The number of potential residents of type θ who purchase housing equals $N_0 S D(p_0^h(N_0, x, z) - p_1(e^{\mu(\theta) x} N_0)) > 0$, so Assumptions 1 and 2 guarantee that residents of each type $\theta \in \text{supp}\, f_r$ hold housing. To prove the other parts of the proposition about quantities, we show that there exists a unique $N_0^*(x, z) \in \mathbb{R}_{>1} \cup \{\infty\}$ such that (A1) holds strictly if and only if $N_0 < N_0^*(x, z)$ and with

equality if and only if $N_0 = N_0^*(x,z)$. We may rewrite (A1) as

$$1 \geq \begin{cases} N_0 & \text{if } N_0 \leq e^{-\mu_d^{max}x} \\ \displaystyle\int_{\Theta_1(N_0,x,z)} N_0 f_r(\theta)d\theta + \int_{\Theta_2(N_0,x,z)} e^{-\mu_d^{max}x} f_r(\theta)d\theta & \\ \quad + \displaystyle\int_{\Theta_3(N_0,x,z)} \left(N_0^{-1/\epsilon} + e^{\mu_d^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon} f_r(\theta)d\theta & \text{if } N_0 > e^{-\mu_d^{max}x}, \end{cases}$$

where $\Theta_1(N_0,x,z) = \{\theta \mid \theta \geq \theta_d^{max}\}$, $\Theta_2(N_0,x,z) = \{\theta \mid \theta < \theta_d^{max}\} \cap \{\theta \mid e^{\mu(\theta)x}N_0 \leq 1\}$, and $\Theta_3(N_0,x,z) = \{\theta \mid \theta < \theta_d^{max}\} \cap \{\theta \mid e^{\mu(\theta)x}N_0 > 1\}$. For notational ease, we name the right side of this inequality $\phi(N_0)$. We have $\lim_{N_0 \to 0} \phi(N_0) = 0$, and ϕ strictly increases for $0 < N_0 \leq e^{-\mu_d^{max}x}$. The integrands coincide for θ in the boundary of Θ_2 and Θ_3, so $\phi'(N_0)$ for $N_0 \geq e^{-\mu_d^{max}x}$ equals the sum of the derivatives under the integral signs (the changing limits of integration cancel out). Therefore ϕ strictly increases in N_0 for all $N_0 > 0$ except those for which $\Theta_2(N_0,x,z) = \operatorname{supp} f_r$. For any such N_0, $1 > \phi(N_0)$ because $\mu_d^{max} > \overline{\mu} \geq 0$ given Assumption 4 and given that $z > 0$. The increasing nature of ϕ means that there may exist at most one solution to $1 = \phi(N_0)$, and that (A1) is satisfied strictly for any N_0 less than this solution and is not satisfied for any N_0 greater than this solution. We deem the solution $N_0^*(x,z)$. Note that $\phi(1) < 1$ unless $\int_{\Theta_1(1,x,z)} f_r(\theta)d\theta = 1$, which is impossible by Assumption 4. Therefore $\phi(1) < 1$ and $N_0^*(x,z) > 1$. For later proofs, we note here that $\lim_{z \to 0} N_0^*(x,z) = 1$, which is evident because for any $N_0 > 1$, $\lim_{z \to 0} \phi(N_0) > 1$, while $\phi(1) < 1$ for all $z > 0$.

The existence of $N_0^*(x,z)$ implies that some developers hold land if and only if $N_0 < N_0^*(x,z)$. If $N_0 < N_0^*(x,z)$, then the proof of Lemma 2 shows that $p_0^h = p_1^h(e^{\mu_d^{max}x}N_0) + k$ and that a developer of type θ may hold land if and only if $p_1^h(e^{\mu(\theta)x}N_0) + k \leq p_0^h(N_0,x,z)$. If $N_0 \leq e^{-\mu_d^{max}x}$, then $p_1^h(e^{\mu(\theta)x}N_0) = k$ for all $\theta \in \operatorname{supp} f_d$, so any developer may hold land at the end of $t = 0$, as claimed. If $e^{-\mu_d^{max}x} < N_0 < N_0^*(x,z)$, then the only developers for whom $p_1^h(e^{\mu(\theta)x}N_0) \leq p_1^h(e^{\mu_d^{max}x}N_0)$ are those for whom $\theta = \theta_d^{max}$ due to Lemma 1. As a result, only these developers hold land when N_0 satisfies these constraints, as claimed.

We now prove the result on excess land holdings by developers in the intermediate case. Define Q_r to be the quantity of housing held by potential residents in equilibrium. From Lemma 2, Q_r does not depend on the developer land endowments L_0, and by the first part of the proposition, $0 < Q_r < S$ for $e^{-\mu_d^{max}x} < N_0 < N_0^*(x,z)$. Summing across the constraint on L_1 for developers at $t = 0$ with $\theta = \theta_d^{max}$ yields $S - Q_r = \sum L_1 = \sum(L_0 + (L_0^{buy})^* - (H_0^{build})^*)$. As a result, $\sum((L_0^{buy})^* - (H_0^{build})^*) = S - Q_r - \sum L_0$, which exceeds zero as long as $\sum L_0 < S - Q_r \equiv L^*$.

We now derive the pricing equations. If $N_0 < N_0^*(x,z)$, then (A1) holds and $p_0^h(N_0,x,z) = p_1^h(e^{\mu_d^{max}x}N_0) + k$. By applying Lemma 1, we arrive at the first two pricing equations in Proposition 2. The equation for $N \geq N_0^*(x,z)$ follows immediately from (A2). To prove the existence and uniqueness of $\mu_r^{agg}(N_0,x,z)$, consider that for $N_0 \geq N_0^*(x,z)$, $p_0^h(N_0,x,z) \geq p_1^h(e^{\mu_d^{max}x}N_0) + k$ as shown in the proof of Lemma 2. Because $\mu_d^{max} > \overline{\mu} \geq 0$, $p_1^h(e^{\mu_d^{max}x}N_0) = ke^{\mu_d^{max}x/\epsilon}N_0^{1/\epsilon}$, so $p_0^h(N_0,x,z) > kN_0^{1/\epsilon}$. It follows that $p_0^h(N_0,x,z) = k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon}$ has a unique solution for $\mu_r^{agg}(N_0,x,z)$, as the right side strictly increases from $kN_0^{1/\epsilon}$ to ∞ as $\mu_r^{agg}(N_0,x,z)$ goes to ∞ (which holds due to the assumption that $x > 0$).

Finally, if $\int_{\theta \geq \theta_d^{max}} f_r(\theta)d\theta > 0$, then $\lim_{N_0 \to \infty} \phi(N_0) = \infty$, leading to $N_0^*(x,z) < \infty$. If $\int_{\theta \geq \theta_d^{max}} f_r(\theta)d\theta = 0$, then $\lim_{N_0 \to \infty} \phi(N_0) = \int_{\theta < \theta_d^{max}} (e^{\mu_d^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon}) f_r(\theta)d\theta$, so $N_0^*(x,z) < \infty$ in this case if and only if this integral exceeds 1.

Proposition 3

Substituting the formulas for $p_0^h(N_0, x, z)$ from Proposition 2 and for $p_0^h(N_0, x, 0)$ from Proposition 1 yield the equations in the first part of Proposition 3. For clarity, we prove the remainder of the claims in Proposition 3 in three parts.

Part 1: Disagreement effect is maximized at $N_0 = 1$

The effect of disagreement on the house price at $t = 0$ weakly increases in N_0 up to $N_0 = 1$ and decreases for $1 \leq N_0 \leq N_0^*(x, z)$. The maximum for $0 < N_0 \leq N_0^*(x, z)$ therefore equals the value at $N_0 = 1$, which is $(e^{\mu_d^{max} x/\epsilon} - e^{\overline{\mu} x/\epsilon})/(1 + e^{\overline{\mu} x/\epsilon})$. This value exceeds the disagreement effect for all $N_0 \geq N_0^*(x, z)$ if and only if $\mu_d^{max} > \mu_r^{agg}(N_0, x, z)$ for all $N_0 \geq N_0^*(x, z)$. We prove this in two steps. We first show that $p_0^h(N_0, x, z)$, and hence $\mu_r^{agg}(N_0, x, z)$, weakly increases in N_0 for $N_0 \geq N_0^*(x, z)$. Second, to show that $\mu_r^{agg}(N_0, x, z) < \mu_d^{max}$ for all $N_0 \geq N_0^*(x, z)$, we show that $\lim_{N_0 \to \infty} \mu_r^{agg}(N_0, x, z)$ exists and is less than μ_d^{max}.

We may rewrite (A2) as

$$\begin{aligned} 1 = &\int_{\Theta_1(N_0,x,z)} N_0 D(p_0^h - k) f_r(\theta) d\theta \\ &+ \int_{\Theta_2(N_0,x,z)} N_0 \left(\frac{k}{p_0^h - k e^{\mu(\theta) x/\epsilon} N_0^{1/\epsilon}} \right)^{\epsilon} f_r(\theta) d\theta + \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta) d\theta, \end{aligned} \tag{A3}$$

where $\Theta_1(N_0, x, z) = \{\theta \mid e^{\mu(\theta) x} N_0 < 1\}$, $\Theta_2(N_0, x, z) = \{\theta \mid 1 \leq e^{\mu(\theta) x} N_0 \leq (p_0^h/k - 1)^{\epsilon}\}$, and $\Theta_3(N_0, x, z) = \{\theta \mid e^{\mu(\theta) x} N_0 > (p_0^h/k - 1)^{\epsilon}\}$. For a given p_0^h, the right side of (A3) weakly increases in N_0: each integrand weakly increases in N_0 (for each θ), and the integrands coincide at the boundaries of the limits of integration, meaning that the marginal effect from changing the limits of integration equals 0. Because the right side of (A2) weakly decreases in p_0^h (as shown in the proof of Lemma 2), it follows that $p_0^h(N_0, x, z)$ weakly increases in N_0.

This monotonicity means that for all $N_0 \geq N_0^*(x, z)$, $\mu_r^{agg}(N_0, x, z) \leq \lim_{N_0' \to \infty} \mu_r^{agg}(N_0', x, z)$. Substituting $p_0^h(N_0, x, z) = k(1 + e^{\mu_r^{agg}(N_0,x,z) x/\epsilon}) N_0^{1/\epsilon}$ into (A3) yields

$$\begin{aligned} 1 = &\int_{\Theta_1(N_0,x,z)} \left(1 + e^{\mu_r^{agg}(N_0,x,z) x/\epsilon} - N_0^{-1/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta \\ &+ \int_{\Theta_2(N_0,x,z)} \left(1 + e^{\mu_r^{agg}(N_0,x,z) x/\epsilon} - e^{\mu(\theta) x/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta + \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta) d\theta. \end{aligned} \tag{A4}$$

Because $\mu_r^{agg}(N_0, x, z)$ increases in N_0, $\lim_{N_0 \to \infty} \mu_r^{agg}(N_0, x, z)$ either exists and is finite or it equals ∞. In the latter case, because $\mu(\theta) \leq \mu_r^{max}$ for all $\theta \in \operatorname{supp} f_r$, each integral goes to 0 as $N_0 \to \infty$, leading to a contradiction. (That the last integral $\to 0$ follows because $\theta \in \Theta_3(N_0, x, z)$ if and only if $e^{\mu(\theta) x} N_0 \geq ((1 + e^{\mu_r^{agg}(N_0,x,z) x/\epsilon}) N_0^{1/\epsilon} - 1)^{\epsilon}$, which implies that $\mu(\theta) \geq \mu_r^{agg}(N_0, x, z)$ because $N_0 \geq N_0^*(x, z) > 1$. For all N_0 such that $\mu_r^{agg}(N_0, x, z) > \mu_r^{max}$, $\int_{\Theta_3(N_0,x,z)} f_r(\theta) d\theta = 0$.) Thus, $\lim_{N_0 \to \infty} \mu_r^{agg}(N_0, x, z) < \infty$. In this case, $\lim_{N_0 \to \infty} \Theta_3(N_0, x, z) = \left\{\theta \mid e^{\mu(\theta) x/\epsilon} \geq 1 + e^{\lim_{N_0 \to \infty} \mu_r^{agg}(N_0,x,z) x/\epsilon}\right\}$, whose measure under f_r must equal 0 for otherwise $\lim_{N_0 \to \infty} \int_{\Theta_3(N_0,x,z)} N_0 f_r(\theta) = \infty$, a contradiction due to (A4). Because $\lim_{N_0 \to \infty} \Theta_1(N_0, x, z) = \emptyset$, taking the limit of (A4) as $N_0 \to \infty$ yields $1 = \int_{\Theta} \left(1 + e^{\lim_{N_0 \to \infty} \mu_r^{agg}(N_0,x,z) x/\epsilon} - e^{\mu(\theta) x/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta$. By Assumption 5, this equation implies that $\lim_{N_0 \to \infty} \mu_r^{agg}(N_0, x, z) < \mu_d^{max}$.

Part 2: Positivity of the disagreement effect

Because $\mu_d^{max} > \overline{\mu}$, the effect of disagreement on prices is positive for $e^{-\mu_d^{max}x} < N_0 \leq 1$. Because the effect decreases for $1 \leq N_0 \leq N_0^*(x,z)$, it is positive for $N_0 > 1$ if it is positive for $N_0 \geq N_0^*(x,z)$ in the case that $N_0^*(x,z) < \infty$, or if its asymptote as $N_0 \to \infty$ is positive in the case that $N_0^*(x,z) = \infty$.

In the second case, by Proposition 2 and Jensen's inequality, $1 > (e^{\mu_d^{max}x/\epsilon} - e^{\overline{\mu}x/\epsilon})^{-\epsilon}$, which implies that $e^{\mu_d^{max}x/\epsilon} > 1 + e^{\overline{\mu}x/\epsilon}$. When $N_0^*(x,z) = \infty$, the effect of disagreement on the house price asymptotes to $(e^{\mu_d^{max}x/\epsilon} - e^{\overline{\mu}x/\epsilon} - 1)/(1 + e^{\overline{\mu}x/\epsilon}) > 0$.

In the first case, we prove that positivity obtains when $\text{supp}\, f_r \subset [-\overline{\mu}/z, \theta_d^{max}]$ and $\int_\Theta f_r(\theta)d\theta = 0$. To demonstrate positivity, we show that $\mu_r^{agg}(N_0,x,z) > \overline{\mu}$ for $N_0 \geq N_0^*(x,z)$. Adopting the notation from above, for $N_0 \geq N_0^*(x,z)$, $\text{supp}\, f_r \cap \Theta_1(N_0,x,z) = \emptyset$ because $\mu(\theta) \geq 0$. Because $\mu(\theta) \leq \mu_d^{max}$ for all $\theta \in \text{supp}\, f_r$, Assumption 5 implies that $\mu_r^{agg}(N_0,x,z) < \mu_d^{max}$. It follows that $\text{supp}\, f_r \cap \Theta_3(N_0,x,z)$ is nonempty if and only if $e^{\mu_d^{max}x/\epsilon}N_0^{1/\epsilon} > e^{\mu_r^{max}x/\epsilon}N_0^{1/\epsilon} > (1+e^{\mu_d^{max}x/\epsilon})N_0^{1/\epsilon} - 1$, but this inequality chain always fails because $N_0 > 1$. As a result, $\text{supp}\, f_r \cap \Theta_3(N_0,x,z) = \emptyset$. It follows that $\text{supp}\, f_r \subset \Theta_2(N_0,x,z)$ and that $\mu_r^{agg}(N_0,x,z)$ satisfies

$$1 = \int_{\Theta_2(N_0,x,z)} \left(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon} f_r(\theta)d\theta. \tag{A5}$$

The argument of this integral is convex in θ for $\theta \in \Theta_2(N_0,x,z)$, so Jensen's inequality implies that $1 > (1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - e^{\overline{\mu}x/\epsilon})^{-\epsilon}$, from which it follows that $\mu_r^{agg}(N_0,x,z) > \overline{\mu}$.

Part 3: Marginal effect of small disagreement

For any $N_0 < e^{-\overline{\mu}x}$, we may find $z > 0$ small enough so that $N_0 < e^{-\mu_d^{max}x}$ because $\lim_{z\to 0} \mu_d^{max} = \overline{\mu}$. By the first part of Proposition 3, for such small z, $\partial p_0^h(N_0,x,z)/\partial z = 0$, proving the first part of formula.

For $e^{\overline{\mu}x} \leq N_0 \leq 1$, we divide the formula in the first part of Proposition 3 by z and take the limit as $z \to 0$ to obtain the expression in the second part of the formula.

Last, for each $N_0 > 1$, because $\lim_{z\to 0} N_0^*(x,z) = 1$ as shown in the proof of Proposition 2, for small enough $z > 0$ we have $N_0 \geq N_0^*(x,z)$. To show that $\partial p_0^h(N_0,x,0)/\partial z = 0$, we demonstrate that $\mu_r^{agg}(N_0,x,z) = \overline{\mu} + o(z)$ as $z \to 0$. We continue to assume that $\text{supp}\, f_r \subset [-\overline{\mu}/z, \theta_d^{max}]$, so (A5) continues to hold. Taking the derivative of (A5) with respect to z and simplifying yield

$$\frac{\partial \mu_r^{agg}(N_0,x,z)}{\partial z} = \frac{\int_{\Theta_2(N_0,x,z)} \theta e^{\mu(\theta)x/\epsilon} \left(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon-1} f_r(\theta)d\theta}{\int_{\Theta_2(N_0,x,z)} e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} \left(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon} - e^{\mu(\theta)x/\epsilon}\right)^{-\epsilon-1} f_r(\theta)d\theta}.$$

As $z \to 0$, the denominator goes to $e^{\overline{\mu}x/\epsilon}$, whereas the numerator goes to $\int_\Theta e^{\overline{\mu}x/\epsilon}\theta f_r(\theta)d\theta$, which equals 0 because $\int_\Theta \theta f_r(\theta)d\theta = 0$ by assumption. Therefore $\mu_r^{agg}(N_0,x,z) = \overline{\mu} + o(z)$ as $z \to 0$.

Proposition 4

As noted in the text, $p_0^h(N_0,0,z) = p_0^h(N_0,0,0)$ because when $x = 0$, $\mu(\theta) = \overline{\mu}$ for all $\theta \in \Theta$, so z becomes irrelevant for the equilibrium. As a result, we may take the formula for $p_0^h(N_0,0,0)$ given by Proposition 1 in the special case when $x = 0$ and use it for $p_0^h(N_0,0,z)$. Combining this formula with that for $p_0^h(N_0,x,z)$ given by Proposition 3 yields the formulas in Proposition 4. The boom strictly increases for $e^{-\mu_d^{max}x} \leq N_0 \leq 1$ and strictly decreases for $1 \leq N_0 \leq N_0^*(x,z)$. Therefore, it is strictly maximized at $N_0 = 1$ as long as its value at $N_0 = 1$, which equals $(e^{\mu_d^{max}x/\epsilon} - 1)/2$, exceeds the boom for all $N_0 \geq N_0^*(x,z)$, which occurs as long as $\mu_d^{max} > \mu_r^{agg}(N_0,x,z)$ for all $N_0 \geq N_0^*(x,z)$. This inequality obtains by Assumption 5, as shown in the proof of Proposition 3.

Proposition 5

As shown in the proof of Lemma 2, in equilibrium the profit (utility for firm owners) of a developer equals $p_0^l L_0 + (p_1^h - p_0^h + k)L_1$ and the utility of potential residents equals $(v + p_1^h - p_0^h)H_0^{buy}$. The set of possible changes to the allocation are summarized by a cash transfer τ (which may vary across agents) with $\sum \tau = 0$ and changes ΔL_1 for each developer and ΔH_0^{buy} for each potential resident such that $\sum \Delta L_1 = \sum \Delta H_0^{buy}$. For a given realization of p_1^h, this change is a Pareto improvement only if $(p_1^h - p_0^h + k)\Delta L_1 + \tau \geq 0$ for all developers and $(v + p_1^h - p_0^h)\Delta H_0^{buy} + \tau \geq 0$ for all potential residents, with at least one strict inequality. Summing these inequalities across agents gives $\sum k\Delta L_1 + \sum v\Delta H_0^{buy} > 0$.

We now show that the $z = 0$ equilibrium (described in the proof of Lemma 2) is Pareto efficient for any p_1^h. If $N_0 < 1$, a potential resident buys if and only if $v \geq p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta)x}N_0) = k$. The only feasible ΔH_0^{buy} are -1 for someone with $H_0^{buy} = 1$ and 1 for someone with $H_0^{buy} = 0$. Because $\sum \Delta L_1 = \sum \Delta H_0^{buy}$, either one of these changes does not increase the welfare criterion given above. When $N_0 \geq 1$, $L_1 = 0$ for all developers and a potential resident buys only if $v \geq p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta)x}N_0) = kN_0^{1/\epsilon} \geq k$. The only feasible ΔL_1 are positive, and the only feasible changes to ΔH_0^{buy} are 1 for $v \leq kN_0^{1/\epsilon}$ and -1 for $v \geq kN_0^{1/\epsilon}$. The change to the welfare criterion above can never be positive resulting from these changes. As a result, the allocation under the $z = 0$ equilibrium is Pareto efficient for any p_1^h, meaning that it is belief-neutral Pareto efficient.

When $N_0 \leq e^{-\mu_d^{max}x}$, potential residents buy when $v \geq p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta)x}N_0)$. This difference is no greater than k because $p_0^h(N_0, x, z) = 2k$ and $p_1^h(e^{\mu(\theta)x}N_0) \geq k$. Because all potential residents have $v \geq k$ by Assumption 1, buyers all have $v \geq k$, and all potential residents with $v > k$ buy. For the same argument given in the $z = 0$ equilibrium above, there does not exist a reallocation that improves welfare for each p_1^h, meaning that the equilibrium under the equilibrium with $z > 0$ is belief-neutral Pareto efficient when $N_0 \leq e^{-\mu_d^{max}x}$.

When $e^{-\mu_d^{max}x} < N_0 < N_0^*(x, z)$, $L_1 > 0$ for at least one developer. If $z > 0$, then by Assumption 4 there exists a positive measure of potential residents for whom $\theta < \theta_d^{max}$. These potential residents buy only if $v \geq p_0^h(N_0, x, z) - p_1^h(e^{\mu(\theta)x}N_0) > p_0^h(N_0, x, z) - p_1^h(e^{\mu_d^{max}x}N_0) = k$, where the latter equality uses an equilibrium condition from the proof of Lemma 2. It follows that there exists a positive measure of potential residents with $v > k$ who do not buy. For a given p_1^h, we improve the allocation by setting $\Delta L_1 = -1$ and $\tau = \tau^*$ for a developer holding land and setting $\Delta H_0^{buy} = 1$ and $\tau = -\tau^*$ for a potential resident with $v > k$ who does not buy a house, where $k + p_1^h - p_0^h \leq \tau^* \leq v + p_1^h - p_0^h$.

When $N_0 \geq N_0^*(x, z)$, potential residents buy only if $v \geq k(1+e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} - p_1^h(e^{\mu(\theta)x}N_0)$. Due to Assumption 4, $z > 0$ implies that $\mu(\theta)$ varies across potential residents. Because $N_0 > 1$ and $\overline{\mu} \geq 0$, $p_1^h(e^{\mu(\theta)x}N_0)$ varies across potential residents. It follows that the purchase cutoff varies across potential residents, meaning that we can find a potential resident with $H_0^{buy} = 1$ and $v = v^1$ and a potential resident with $H_0^{buy} = 0$ and $v = v^2$ with $v^1 < v^2$. Setting $\Delta H_0^{buy} = -1$ and $\tau = \tau^*$ for the first potential resident and $\Delta H_0^{buy} = 1$ and $\tau = -\tau^*$ for the second potential resident strictly increases the welfare objective if $v^1 + p_1^h - p_0^h \leq \tau^* \leq v^2 + p_1^h - p_0^h$.

Internet Appendix

B Equity Extension

Developers who can access the equity market choose a share $\alpha^{sell} \in [0,1]$ of the claim to their total $t=1$ liquidation value to sell at $t=0$. The price of this claim equals p_0^π, which may vary across developers. Each of these developers may also pay itself a dividend δ at $t=0$ using its available cash flow. Finally, land that remains undeveloped at the end of $t=0$ pays a dividend $k_l > 0$ at $t=1$; we focus on the limiting equilibria as $k_l \to 0$.[32] The optimal behavior for such a developer is to choose δ^*, $(\alpha^{sell})^*$, $(H_0^{sell})^*$, $(L_0^{buy})^*$, and $(H_0^{build})^*$ from

$$\begin{aligned}
\underset{\delta,\alpha^{sell},H_0^{sell},L_0^{buy},H_0^{build}}{\arg\max} \quad & \delta + (1-\alpha^{sell})\mathrm{E}\pi(p_1^h, p_1^l, H_1, L_1, B_1) \\
\text{subject to} \quad & \alpha^{sell} \in [0,1] \\
& H_0^{sell} \leq H_0^{build} \\
& H_0^{build} \leq L_0 + L_0^{buy} \\
& H_1 = H_0^{build} - H_0^{sell} \\
& L_1 = L_0 + L_0^{buy} - H_0^{build} \\
& B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2kH_0^{build} + \alpha^{sell} p_0^\pi - \delta \\
& 0 \leq B_1 \\
& 0 \leq \delta.
\end{aligned}$$

Developers who cannot access the equity market face the same problem with the additional constraint $\alpha^{sell} = 0$. For all developers, the $t=1$ problem remains the same as before.

A unit measure of equity investors chooses a share α^{buy} of the claim to each developer's $t=1$ liquidation value to buy at $t=0$. The chosen α^{buy} may differ for each investor-developer pair. Each investor faces a proportional cost $k_s \in (0,1)$ for each dollar invested in a negative position, and the most negative position that can be taken is $-\overline{\alpha}$, where $\overline{\alpha} > 0$. For a given developer, an equity investor chooses $(\alpha^{buy})^*$ from

$$\begin{aligned}
\underset{\alpha^{buy}}{\arg\max} \quad & \alpha^{buy}\mathrm{E}\pi(p_1^h, p_1^l, H_1, L_1, B_1) - \max(\alpha^{buy}, (1-k_s)\alpha^{buy})p_0^\pi \\
\text{subject to} \quad & -\overline{\alpha} \leq \alpha^{buy} \\
& H_1 = (H_0^{build})^* - (H_0^{sell})^* \\
& L_1 = L_0 + (L_0^{buy})^* - (H_0^{build})^* \\
& B_1 = p_0^h (H_0^{sell})^* - p_0^l (L_0^{buy})^* - 2k(H_0^{build})^* + (\alpha^{sell})^* p_0^\pi - \delta^*,
\end{aligned}$$

where E denotes the equity investor's expectation and δ^*, $(\alpha^{sell})^*$, $(H_0^{sell})^*$, $(L_0^{buy})^*$, and $(H_0^{build})^*$ denote the actions chosen by the developer.

The potential resident problems remain the same. Prices p_0^h, p_0^l, and p_0^π constitute an equilibrium when, in addition to the clearing of land and housing markets described in Section 1, the

[32] This dividend leads to a positive land price at $t=0$ that guarantees the existence of equilibrium when $\mathrm{E}p_1^l = 0$ for all equity investors but $\mathrm{E}p_1^l > 0$ for some developers. The proof of Proposition 6 further discusses this issue.

following holds: for each developer, $(\alpha^{sell})^*$ equals the sum across equity investors of $(\alpha^{buy})^*$.

We now characterize equilibrium. The first lemma simplifies the objective of each developer:

Lemma B1. *In equilibrium, each developer chooses* α^{sell} *and* $L_1 \geq 0$ *such that* $p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi \geq 0$ *to maximize* $p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi + (1 - \alpha^{sell})\mathrm{E}(p_1^l + k_l)L_1$.

Proof. In all of the $t = 1$ equilibria characterized in the proof of Lemma 1, $\pi = p_1^h H_1 + (p_1^l + k_l)L_1 + B_1$ (p_1^l is the ex-dividend price). At $t = 0$, the developer maximizes $\delta + (1 - \alpha^{sell})\mathrm{E}(p_1^h H_1 + (p_1^l + k_l)L_1 + B_1)$. From substituting the H_1 and L_1 constraints into the B_1 constraint, we have $B_1 = -p_0^h H_1 + p_0^l(L_0 - L_1) + (p_0^h - p_0^l - 2k)H_0^{build} + \alpha^{sell} p_0^\pi - \delta$. In equilibrium $p_0^h = p_0^l + 2k$, for otherwise each developer would want to build a positively or negatively infinite amount of housing. Therefore $B_1 = -p_0^h H_1 + p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi - \delta$. The developer maximizes $\delta + (1 - \alpha^{sell})\mathrm{E}((p_1^h - p_0^h)H_1 + (p_1^l + k_l - p_0^l)L_1 + p_0^l L_0 + \alpha^{sell} p_0^\pi - \delta)$ by choosing $H_1, L_1 \geq 0$, $\alpha^{sell} \in [0, 1]$, and δ such that $B_1 \geq 0$. Because $p_1^h - p_0^h = p_1^l - p_0^l - k < p_1^l + k_l - p_0^l$, in equilibrium all developers set $H_1 = 0$ (if $H_1 > 0$ is optimal, then the developer wants an infinite L_1). The objective weakly increases in δ for $\alpha^{sell} \in [0, 1]$, so it is maximized at $\delta = -p_0^h H_1 + p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi$, the largest possible value given the $B_1 \geq 0$ constraint. The $\delta \geq 0$ constraint produces $p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi \geq 0$. The objective simplifies to $p_0^l(L_0 - L_1) + \alpha^{sell} p_0^\pi + (1 - \alpha^{sell})\mathrm{E}(p_1^l + k_l)L_1$, as claimed. □

The developer objective consists of three terms: profits from current land sales, revenues from equity offerings, and profits expected at $t = 1$ from end-of-period land holdings. The next lemma delivers the equilibrium price of equity:

Lemma B2. *In equilibrium,* $p_0^\pi = (p_1^l(e^{\mu_i^{max} x} N_0) + k_l)L_1$ *for any developer for whom* $(\alpha^{sell})^* > 0$.

Proof. As shown in the proof of Lemma B1, each developer sets $H_1 = 0$ and sets $B_1 = 0$ when $\alpha^{sell} > 0$. The liquidation value of the developer becomes $\pi = (p_1^l + k_l)L_1$. If $p_0^\pi < (p_1^l(e^{\mu_i^{max} x} N_0) + k_l)L_1$, then the equity investors for whom $\theta = \theta_i^{max}$ want to set α^{buy} arbitrarily large. The equity market cannot clear in this case because the maximal aggregate short position across equity investors is bounded at $-\overline{\alpha}$. Therefore $p_0^\pi \geq (p_1^l(e^{\mu_i^{max} x} N_0) + k_l)L_1$. If this inequality is strict, then $(\alpha^{buy})^* \leq 0$ for all equity investors, preventing clearing in the equity market. The only equilibrium outcome is the one given in the lemma. □

The price of any traded claim equals the most optimistic equity investor valuation of the land held by that developer at the end of $t = 0$. In this sense, traded developers act like land hedge funds by raising equity against speculative land investments. To make this point clear, the following lemma relates the equilibrium prices of developer equity and the land they hold:

Lemma B3. *In equilibrium,* $p_0^\pi = p_0^l L_1$ *for any developer for whom* $(\alpha^{sell})^* > 0$.

Proof. We prove this claim by delineating all possible choices by developers in equilibrium. By substituting Lemma B2 into Lemma B1, we rewrite the developer problem as choosing

$$
\begin{aligned}
L_1^*, (\alpha^{sell})^* \in \underset{L_1, \alpha^{sell}}{\arg\max} \quad & p_0^l L_0 + \left(\alpha^{sell} p_1^l(e^{\mu_i^{max} x} N_0) + (1 - \alpha^{sell}) p_1^l(e^{\mu(\theta) x} N_0) + k_l - p_0^l\right) L_1 \\
\text{subject to} \quad & p_0^l L_1 \leq p_0^l L_0 + \alpha^{sell}(p_1^l(e^{\mu_i^{max} x} N_0) + k_l)L_1 \\
& 0 \leq L_1 \\
& \alpha^{sell} \in [0, 1] \text{ (with access to equity market)} \\
& \alpha^{sell} = 0 \quad \text{(without access to equity market).}
\end{aligned}
$$

A developer that cannot access the equity market sets $(\alpha^{sell})^* = 0$ and chooses

$$\begin{aligned} L_1^* &= L_0 && \text{if } p_0^l < p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &\in [0, L_0] && \text{if } p_0^l = p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &= 0 && \text{if } p_0^l > p_1^l(e^{\mu(\theta)x}N_0) + k_l \end{aligned}$$

if $p_0^l > 0$. If $p_0^l \leq 0$, then L_1^* does not exist because the developer always increases its objective function without violating the constraints by increasing L_1 beyond L_0. Similarly, if $p_0^l < p_1^l(e^{\mu_i^{max}x}N_0) + k_l$ then L_1^* does not exist for developers with access to the equity market. With $\alpha^{sell} = 1$, increasing L_1 always increases the objective function while obeying the constraints. If $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$, then the optimal choices for developers with access to the equity market are

$$\begin{aligned} L_1^* &= \frac{L_0}{1 - (\alpha^{sell})^*} && \text{and } (\alpha^{sell})^* \in [0, 1) && \text{if } p_1^l(e^{\mu_i^{max}x}N_0) < p_1^l(e^{\mu(\theta)x}N_0) \\ L_1^* &\geq 0 && \text{and } (\alpha^{sell})^* = 1 && \\ &\text{or} && && \text{if } p_1^l(e^{\mu_i^{max}x}N_0) = p_1^l(e^{\mu(\theta)x}N_0) \\ L_1^* &\in \left[0, \frac{L_0}{1 - (\alpha^{sell})^*}\right] && \text{and } (\alpha^{sell})^* \in [0, 1) && \\ L_1^* &= 0 && \text{and } (\alpha^{sell})^* \in [0, 1) && \\ &\text{or} && && \text{if } p_1^l(e^{\mu_i^{max}x}N_0) > p_1^l(e^{\mu(\theta)x}N_0). \\ L_1^* &\geq 0 && \text{and } (\alpha^{sell})^* = 1 && \end{aligned}$$

The first case follows because if $\alpha^{sell} < 1$, the objective strictly increases in L_1 and so is maximized at $L_1^* = L_0/(1 - \alpha^{sell})$ with a value of $(p_1^l(e^{\mu(\theta)x}N_0) + k_l)L_0$. This value exceeds $p_0^l L_0$, the objective function value obtained when $\alpha^{sell} = 1$. In the second case of the optimal developer choices, the objective is independent of L_1 and α^{sell}, so the developer may choose any feasible combination. In the third case, the objective decreases in L_1 if $\alpha^{sell} < 1$, leading to $L_1^* = 0$; if $\alpha^{sell} = 1$, then the objective is independent of L_1, permitting the developer to choose any feasible value for L_1^*. Finally, if $p_0^l > p_1^l(e^{\mu_i^{max}x}N_0) + k_l$ then

$$\begin{aligned} L_1^* &= L_0 && \text{and } (\alpha^{sell})^* = 0 && \text{if } p_0^l < p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &\in [0, L_0] && \text{and } (\alpha^{sell})^* = 0 && \\ &\text{or} && && \text{if } p_0^l = p_1^l(e^{\mu(\theta)x}N_0) + k_l \\ L_1^* &= 0 && \text{and } (\alpha^{sell})^* \in [0, 1] && \\ L_1^* &= 0 && \text{and } (\alpha^{sell})^* \in [0, 1] && \text{if } p_0^l > p_1^l(e^{\mu(\theta)x}N_0) + k_l \end{aligned}$$

are the optimal choices for developers with access to the equity market. In the first case, the value of the objective function at the given choices equals $(p_1^l(e^{\mu(\theta)x}N_0) + k_l)L_0$. For $\alpha^{sell} \geq (p_1^l(e^{\mu(\theta)x}N_0) + k_l - p_0^l)/(p_1^l(e^{\mu(\theta)x}N_0) - p_1^l(e^{\mu_i^{max}x}N_0))$, the coefficient in the objective function on L_1 is non-positive, meaning that it is maximized at $L_1^* = 0$ with a value of $p_0^l L_0$, which is less than the maximized value when $L_1^* = L_0$ and $(\alpha^{sell})^* = 0$. For $\alpha^{sell} \in (0, (p_1^l(e^{\mu(\theta)x}N_0) +$

$k_l - p_0^l)/(p_1^l(e^{\mu(\theta)x}N_0) - p_1^l(e^{\mu_i^{max}x}N_0)))$, the coefficient on L_1 in the objective function is positive, meaning that it is maximized at $L_1 = p_0^l L_0/(p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l))$ with a value of $p_0^l L_0(1 - \alpha^{sell})(p_1^l(e^{\mu(\theta)x}N_0) + k_l)/(p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l))$. This value is less than the maximized value when $(\alpha^{sell})^* = 0$ because for such α^{sell}, $(1 - \alpha^{sell})p_0^l < p_0^l - \alpha^{sell}(p_1^l(e^{\mu_i^{max}x}N_0) + k_l)$. We have proved that the given choices are optimal in the first case. The proof that the choices are optimal in the second case is similar. The maximized objective equals $p_0^l L_0$. If $\alpha^{sell} > 0$, then the coefficient on L_1 in the objective is negative, leading to $L_1^* = 0$. If $\alpha^{sell} = 0$, then the coefficient on L_1 in the objective is 0, leading to any feasible choice of L_1^*. Finally, in the third case, the coefficient on L_1 in the objective is negative for all α^{sell}, leading to $L_1^* = 0$, in which case $(\alpha^{sell})^*$ does not affect the objective.

In all of the equilibrium choices we have just listed, $(\alpha^{sell})^* > 0$ only if $L_1^* = 0$ or if $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$. In either case, $p_0^\pi = p_0^l L_1$ by Lemma B2. □

We now use Lemmas B1 and B2 to formulate and prove a lemma that characterizes the equilibrium house price at $t = 0$ as $k_l \to 0$. The lemma relies on the following definitions: $\mu_i^{max} = \mu(\theta_i^{max})$ is the belief of the most optimistic equity investor, $\theta_d^{sup} = \sup\{\theta \in \text{supp}\, f_d \mid L_0 > 0\}$ is the least upper-bound of the beliefs of developers endowed with land, and $N_0^*(x, z, f_r, \theta_d^{max})$ is the value of $N_0^*(x, z)$ in Proposition 2 given f_r and θ_d^{max}.

Lemma B4. *Suppose that $x, z > 0$. If $\sum_{\theta > \theta_i^{max}} L_0 = 0$, then the limit of the equilibrium house price at $t = 0$ as $k_l \to 0$ equals*

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \le e^{-\mu_i^{max}x} \\ k + ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^*(x, z, f_r, \theta_i^{max}) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \ge N_0^*(x, z, f_r, \theta_i^{max}). \end{cases}$$

If $\sum_{\theta > \theta_i^{max}} L_0 > 0$, then the limit of the equilibrium house price at $t = 0$ as $k_l \to 0$ equals

$$p_0^h(N_0, x, z) = \begin{cases} 2k & \text{if } N_0 \le \min(e^{-\mu_i^{max}x}, N_0^{**}(x, z)) \\ k + ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^{**}(x, z) \\ k + ke^{\mu_d^{agg}(N_0,x,z)x/\epsilon}N_0^{1/\epsilon} & \text{if } N_0^{**}(x, z) < N_0 < N_0^*(x, z, f_r, \theta_d^{sup}) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \ge N_0^*(x, z, f_r, \theta_d^{sup}). \end{cases}$$

Here $\mu_d^{agg}(N_0, x, z)$ increases in N_0 and depends on the beliefs and endowments of only those developers for whom $\theta > \theta_i^{max}$ and $L_0 > 0$, and $N_0^*(x, z, f_r, \theta_d^{sup}) \ge N_0^{**}(x, z) \in \mathbb{R}_{\ge 0} \cup \{\infty\}$ with equality if and only if $N_0^{**}(x, z) = \infty$, which occurs if and only if $\int_{\theta \ge \theta_i^{max}} f_r(\theta)d\theta = 0$ and $\int_{\theta < \theta_i^{max}} (e^{\mu_i^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta)d\theta \le \sum_{\theta \le \theta_i^{max}} L_0/S$.

Proof of Lemma B4. The proof of Lemma B3 fully characterized developer choices of end-of-period landholdings at $t = 0$ given p_0^l. The land price constitutes an equilibrium when the space demanded by potential residents given p_0^l plus the sum of L_1 across developers equals S (the proof of Lemma B1 shows that $H_1 = 0$ for all developers). If $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$, then the total L_1 across developers can take on any value at least $\sum_{\theta | p_1^l(e^{\mu_i^{max}x}N_0) < p_1^l(e^{\mu(\theta)x}N_0)} L_0$. Equilibrium holds in this case if and only if

$$\sum_{\theta | p_1^h(e^{\mu_i^{max}x}N_0) \ge p_1^h(e^{\mu(\theta)x}N_0)} L_0/S \ge \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \quad \text{(B1)}$$

By the same argument about the right side of (A1) in the proof of Proposition 2, the right side of (B1) weakly and continuously increases in N_0 and $\to 0$ as $N_0 \to 0$. The left side of (B1) equals

$$\sum_{\theta|p_1^h(e^{\mu_i^{max}x}N_0)\geq p_1^h(e^{\mu(\theta)x}N_0)} L_0/S = \begin{cases} 1 & \text{if } N_0 \leq e^{-\mu_d^{max}x} \\ 1-\sum_{\theta|e^{\mu(\theta)x}N_0>1} L_0/S & \text{if } e^{-\mu_d^{max}x} \leq N_0 \leq e^{-\mu_i^{max}x} \\ 1-\sum_{\theta>\theta_i^{max}} L_0/S & \text{if } N_0 \geq e^{-\mu_i^{max}x}, \end{cases}$$

which weakly decreases in N_0 and is left-continuous. As a result, there is $N_0^{**}(x,z,k_l) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ such that (B1) holds if and only if $N_0 \leq N_0^{**}(x,z,k_l)$. Because the right side of (B1) decreases in k_l, $N_0^{**}(x,z,k_l)$ increases in k_l, meaning that $N_0^{**}(x,z) \equiv \lim_{k_l \to 0} N_0^{**}(x,z,k_l)$ exists.

We pause here to prove two needed facts about $N_0^{**}(x,z,k_l)$. As a point of notation, define $N_0^*(x,z,f_r,\theta_d^{max},k_l)$ to be the value of $N_0^*(x,z)$ obtained from (A1) with $k+k_l$ in place of k inside the integral. First: if $\sum_{\theta>\theta_i^{max}} L_0 = 0$, then the left side of (B1) reduces to 1. It follows from comparison with (A1) that $N_0^{**}(x,z,k_l) = N_0^*(x,z,f_r,\theta_i^{max},k_l)$ and $N_0^{**}(x,z) = N_0^*(x,z,f_r,\theta_i^{max})$ in this case. Second: by the same argument used in Proposition 2 to analyze (A1), the limit of the right side of (B1) as $N_0 \to \infty$ equals ∞ if $\int_{\theta \geq \theta_i^{max}} f_r(\theta)d\theta \geq 0$ and equals $\int_{\theta<\theta_i^{max}} (e^{\mu_i^{max}x/\epsilon} - e^{\mu(\theta)x/\epsilon})^{-\epsilon} f_r(\theta)d\theta$ otherwise. It follows that $N_0^{**}(x,z) = \infty$ if and only if the conditions given in Lemma B4 hold.

In the second possible equilibrium, $p_0^l > p_1^l(e^{\mu_i^{max}x}N_0) + k_l$. In this case, the total L_1 across developers may take on any value between $\sum_{\theta|p_0^l<p_1^l(e^{\mu(\theta)x}N_0)+k_l} L_0$ and $\sum_{\theta|p_0^l\leq p_1^l(e^{\mu(\theta)x}N_0)+k_l} L_0$. Equilibrium holds if potential residents demand for space at $p_0^h = p_0^l + 2k$ equals the remaining land not held by developers; that is, if p_0^h satisfies

$$\sum_{\theta|p_0^h>p_1^h(e^{\mu(\theta)x}N_0)+k+k_l} L_0/S \leq \int_\Theta N_0 D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta \leq \sum_{\theta|p_0^h\geq p_1^h(e^{\mu(\theta)x}N_0)+k+k_l} L_0/S. \tag{B2}$$

Such a p_0^h exists if and only if (B1) fails. Indeed, suppose (B1) holds. The left side of (B2) weakly increases in p_0^h, while the middle strictly decreases for $p_0^h \geq p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$ because $\theta < 0 < \theta_i^{max}$ for a positive measure of potential residents (Assumption 4). If (B1) holds, then the left side of (B2) is at least the middle when $p_0^h = p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$, meaning that for larger p_0^h, the left strictly exceeds the middle in violation of (B2). Now suppose that (B1) fails. Then the middle of (B2) exceeds the right side at $p_0^h = p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$. Because the middle strictly and continuously decreases to 0 with $p_0^h \geq p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$, there exists a unique solution to (B2), which we deem $p_0^h(N_0,x,z,k_l)$. Existence and uniqueness follow from the fact that the greatest lower bound of the p_0^h for which the left inequality fails equals the lowest upper bound of the p_0^h for which the right inequality fails.

We further partition this possible equilibrium into two cases. Set $\mu_d^{sup} = \mu(\theta_d^{sup})$. In the first case,

$$1 \geq \int_\Theta N_0 D(p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta. \tag{B3}$$

At $p_0^h = p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$, the right side of (B2) equals 1. As a result, if (B3) fails, then $p_0^h(N_0,x,z,k_l)$ satisfies (A2). If (B3) holds, then if $p_0^h > p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$, the left and right of (B2) equal 1 while the middle is less than 1. As a result, $p_0^h(N_0,x,z,k_l) \leq p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$. By the same argument given in the proof of Proposition 2 concerning (A1), (B3) holds if and only if $N_0 \leq N_0^*(x,z,f_r,\theta_d^{sup},k_l)$.

In summary, a unique equilibrium house price at $t=0$ exists. If $N_0 \leq N_0^{**}(x,z,k_l)$, then we have $p_0^h(N_0,x,z,k_l) = p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l$. If $N_0^{**}(x,z,k_l) < N_0 < N_0^*(x,z,f_r,\theta_d^{sup},k_l)$,

then $p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l < p_0^h(N_0, x, z, k_l) \leq p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$. If $N_0 > N_0^{**}(x, z, k_l)$ and $N_0 \geq N_0^*(x, z, f_r, \theta_d^{sup}, k_l)$, then $p_0^h(N_0, x, z, k_l)$ satisfies (A2).

If $\sum_{\theta > \theta_i^{max}} L_0 = 0$, then $\mu_d^{sup} \leq \mu_i^{max}$. In this case, $N_0^{**}(x, z, k_l) = N_0^*(x, z, f_r, \theta_i^{max}, k_l) \geq N_0^*(x, z, f_r, \theta_d^{sup}, k_l)$, where the equality was proved earlier and the inequality follows because N_0^* increases in its fourth argument. As a result, the equilibrium house price in this case equals

$$p_0^h(N_0, x, z, k_l) = \begin{cases} 2k + k_l & \text{if } N_0 \leq e^{-\mu_i^{max}x} \\ k + ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} + k_l & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^*(x, z, f_r, \theta_i^{max}, k_l) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_i^{max}, k_l). \end{cases}$$

Taking the limit as $k_l \to 0$ yields the formula in Lemma B4.

If $\sum_{\theta > \theta_i^{max}} L_0 > 0$, then $\mu_d^{sup} > \mu_i^{max}$. From comparing (B1) to (B3), we see that $N_0^{**}(x, z, k_l) \leq N_0^*(x, z, f_r, \theta_d^{sup}, k_l)$ and $N_0^{**}(x, z) \leq N_0^*(x, z, f_r, \theta_d^{sup})$, with equality in each if and only if the respective left side equals ∞. If $N_0^{**}(x, z, k_l) < N_0 \leq N_0^*(x, z, f_r, \theta_d^{sup}, k_l)$, then $p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l < p_0^h(N_0, x, z, k_l) \leq p_1^h(e^{\mu_d^{sup}x}N_0) + k + k_l$. Over this range, the only developers on which (B2) depends are those with positive land holdings and beliefs in $\{\theta \mid p_1^h(e^{\mu_i^{max}x}N_0) < p_1^h(e^{\mu(\theta)x}N_0)\} = \{\theta \mid \theta > \theta_i^{max}\}$. It follows that $p_0^h(N_0, x, z, k_l)$ in this range depends on only these developers. Because $p_1^h(e^{\mu_d^{sup}x}N_0) = ke^{\mu_d^{sup}x/\epsilon}N_0^{1/\epsilon}$ in this range, there exists a unique $\mu_d^{agg}(N_0, x, z, k_l) \in [-\log(N_0)/x, \mu_d^{sup}]$ such that on this range of N_0, $p_0^h(N_0, x, z, k_l) = k + ke^{\mu_d^{agg}(N_0,x,z,k_l)x/\epsilon}N_0^{1/\epsilon} + k_l$. Because $p_0^h(N_0, x, z, k_l)$ increases in k_1 and is bounded on this range, $\lim_{k_l \to 0} \mu_d^{agg}(N_0, x, z, k_l)$ exists; we deem it $\mu_d^{agg}(N_0, x, z)$. The middle of (B2) increases in N_0, as shown in the the proof of Proposition 2, so $\mu_d^{agg}(N_0, x, z)$ increases in N_0. Putting everything together, we have that

$$p_0^h(N_0, x, z, k_l) = \begin{cases} 2k + k_l & \text{if } N_0 \leq \min(e^{-\mu_i^{max}x}, N_0^{**}(x, z, k_l)) \\ k + ke^{\mu_i^{max}x/\epsilon}N_0^{1/\epsilon} + k_l & \text{if } e^{-\mu_i^{max}x} < N_0 < N_0^{**}(x, z, k_l) \\ k + ke^{\mu_d^{agg}(N_0,x,z,k_l)x/\epsilon}N_0^{1/\epsilon} + k_l & \text{if } N_0^{**}(x, z, k_l) < N_0 < N_0^*(x, z, f_r, \theta_d^{sup}, k_l) \\ k(1 + e^{\mu_r^{agg}(N_0,x,z)x/\epsilon})N_0^{1/\epsilon} & \text{if } N_0 \geq N_0^*(x, z, f_r, \theta_d^{sup}, k_l). \end{cases}$$

when $\sum_{\theta > \theta_i^{max}} L_0 > 0$. Taking the limit as $k_l \to 0$ yields the formula in Lemma B4.

The only point at which $k_l > 0$ was used is for the existence of equilibrium when $p_0^h(N_0, x, z, k_l) = 2k + k_l$. In this case, $p_0^l(N_0, x, z, k_l) = k_l$, but we showed earlier that $p_0^l = 0$ never can be an equilibrium. This equilibrium exists only as a limit as $k_l \to 0$. □

The case when $\sum_{\theta > \theta_i^{max}} L_0 > 0$ and $N_0^{**}(x, z) < N_0 < N_0^*(x, z, f_r, \theta_d^{sup})$ deserves some explanation, as the equilibrium house price in this region looks quite different than any of the prices in Proposition 2. This case occurs when demand from potential residents is at least equal to the space held by developers for whom $\theta \leq \theta_i^{max}$, but is not as large as the entire space S. In such an equilibrium, developers for whom $\theta > \theta_i^{max}$ become the marginal owners of space and hold some land in equilibrium. The equilibrium house price aggregates the beliefs of such landowning developers through μ_d^{agg}. This case always occurs unless demand from potential residents when the optimistic equity investors price space is never large enough to cut into the landholdings of these very optimistic developers; this condition is precisely the one at the end of Lemma B4.

Finally, we build on the proof of Lemma B4 to prove Proposition 6.

Proof of Proposition 6. The claim that the equilibrium house price equals $p_0^h(N_0, x, z, f_r, f_i)$ when $\sum_{\theta > \theta_i^{max}} L_0 = 0$ follows immediately from comparing the pricing formula in Lemma B4 to that in Proposition 2.

To prove the remaining claims, we first solve for the optimal equity purchases for investors. By Lemma B2, the objective function for an equity investor with respect to a given developer is to maximize $\alpha^{buy}(p_1^l(e^{\mu(\theta)x}N_0)+k_l)L_1 - \max(\alpha^{buy},(1-k_s)\alpha^{buy})(p_1^l(e^{\mu_i^{max}x}N_0)+k_l)L_1$ subject to $\alpha^{buy} \geq -\overline{\alpha}$. If $L_1 > 0$, then the optimal choice for the equity investor is

$$\begin{aligned}
&(\alpha^{buy})^* \geq 0 && \text{if } p_1^l(e^{\mu(\theta)x}N_0) = p_1^l(e^{\mu_i^{max}x}N_0)\\
&(\alpha^{buy})^* = 0 && \text{if } p_1^l(e^{\mu(\theta)x}N_0) \in \left((1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_sk_l, p_1^l(e^{\mu_i^{max}x}N_0)\right)\\
&(\alpha^{buy})^* \in [-\overline{\alpha},0] && \text{if } p_1^l(e^{\mu(\theta)x}N_0) = (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_sk_l\\
&(\alpha^{buy})^* = -\overline{\alpha} && \text{if } p_1^l(e^{\mu(\theta)x}N_0) < (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_sk_l.
\end{aligned}$$

When $xz = 0$, $p_1^l(e^{\mu(\theta)x}N_0) = p_1^l(e^{\mu_i^{max}x}N_0) > (1-k_s)p_1^l(e^{\mu_i^{max}x}N_0) - k_sk_l$ because $k_s > 0$, so $(\alpha^{buy})^* \geq 0$ for all equity investors and developers for whom $L_1 > 0$. The claim that the aggregate value of short claims equals zero when $xz = 0$ is proved. For the second claim about the $xz = 0$ case, first consider the possibility that $p_0^l > p_1^l(e^{\overline{\mu}x}N_0) + k_l$. Then the proof of Lemma B4 shows that $L_1^* = 0$ for all developers and that $(\alpha^{sell})^* = 0$ is possible for all developers, meaning that an equilibrium exists in which no equity issuance occurs and in which $(H_0^{build})^* = L_0$ and $(L_0^{buy})^* = 0$ for all developers. Now consider the other possibility, that $p_0^l = p_1^l(e^{\overline{\mu}x}N_0) + k_l$. Then by the proof of Lemma B4, each developer may choose $(\alpha^{sell})^* = 0$ and $L_1^* \leq L_0$. As a result, no equity is issued, and the sum of L_1^* across developers can take on any value between 0 and S, meaning that we may find an equilibrium in which $(L_0^{buy})^* = 0$ for all developers and $(H_0^{build})^*$ is chosen to clear the housing market.

We turn now to the remaining claims about the $xz > 0$ case. We define $N_0^{***}(x,z,k_l)$ to be the least upper bound of N_0 such that

$$\sum_{\substack{\theta<\theta_i^{max}\\ \text{developers w/o}\\ \text{access to equity}}} L_0/S > \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \tag{B4}$$

As discussed in the proof of Lemma B4, the right side of (B4) continuously increases in N_0 and limits to 0 as $N_0 \to 0$, so $N_0^{***}(x,z,k_l) \in \mathbb{R}_{\geq 0} \cup \{\infty\}$ exists. Because the right side of (B4) is continuous in k_l, we may define $N_0^{***}(x,z) = \lim_{k_l \to 0} N_0^{***}(x,z,k_l) = N_0^{***}(x,z,0)$. Furthermore, substituting $N_0 = e^{-\mu_i^{max}x}$ into the right side of (B4) when $k_l = 0$ yields $e^{-\mu_i^{max}x}$, so because the left side exceeds $e^{-\mu_i^{max}x}$, we must have $N_0^{***}(x,z,k_l) \geq N_0^{***}(x,z) > e^{-\mu_i^{max}x}$ (N_0^{***} decreases in k_l). The left side of (B4) is less than or equal to the left side of (B1) as shown in the analysis after (B1), so $N_0^{***}(x,z,k_l) \leq N_0^{**}(x,z,k_l)$ and $N_0^{***}(x,z) \leq N_0^{**}(x,z)$.

We prove the remaining claims about the $xz > 0$ case for N_0 such that $e^{-\mu_i^{max}x} < N_0 < N_0^{***}(x,z)$. Such N_0 satisfy $e^{-\mu_i^{max}x} < N_0 < N_0^{**}(x,z,k_l)$ given the inequalities above. By the proof of Lemma B4, $p_0^l = p_1^l(e^{\mu_i^{max}x}N_0) + k_l$ in equilibrium for such N_0. Assume for a contradiction that $(\alpha^{sell})^* L_1^* = 0$ for all developers. The largest possible sum of L_1^* across all developers equals $\sum_{\theta \geq \theta_i^{max}} L_0$. An equilibrium is possible only if the housing demand from potential residents is at least equal to the remaining land. This condition is

$$\sum_{\theta<\theta_i^{max}} L_0/S \leq \int_\Theta N_0 D(p_1^h(e^{\mu_i^{max}x}N_0) + k + k_l - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta,$$

which fails for $N_0 < N_0^{***}(x,z)$ due to (B4), providing the necessary contradiction and proving that

the aggregate value of issued equity is positive.

From one of the developer constraints, $(L_0^{buy})^* - (H_0^{build})^* = L_1^* - L_0$, so the sum of the former across equity-issuing developers equals the sum of the latter across them. Assume for a contradiction that the latter sum is ≤ 0. For all developers not issuing equity, $L_1^* \leq L_0$, with $L_1^* = 0$ for developers without access to the equity market for whom $\theta < \theta_i^{max}$. As a result, the total demand for space may equal S only if the precise opposite of (B4) holds. Because $N_0 < N_0^{***}(x, z)$, we have a contradiction that proves that developers who issue equity in the aggregate buy land beyond construction needs.

We now prove the statement about shorting of equity-issuing developers. Pick any $\theta' < \theta_i^{max}$ such that $\int_{\theta \leq \theta'} f_i(\theta) d\theta > 0$, where f_i is the distribution of θ across equity investors (Assumption 4 guarantees the existence of θ'). We will show that we can find k_s small enough so that $(\alpha^{buy})^* = -\overline{\alpha}$ for all $\theta \leq \theta'$. If $e^{\mu(\theta)x} N_0 \leq 1$, then $(\alpha^{buy})^* = -\overline{\alpha}$ if and only if $k < (1 - k_s) k e^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} - k_s k_l$. As $k_s \to 0$, the right side limits to something greater than k, so we can find $k_s > 0$ small enough so that $(\alpha^{buy})^* = -\overline{\alpha}$ for all θ with $e^{\mu(\theta)x} N_0 \leq 1$. Now consider $\theta \leq \theta'$ with $e^{\mu(\theta)x} N_0 > 1$. An equity investor with such θ sets $(\alpha^{buy})^* = -\overline{\alpha}$ if and only if $k e^{\mu(\theta)x/\epsilon} N_0^{1/\epsilon} < (1 - k_s) k e^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} - k_s k_l$. This equation holds if $k e^{\mu(\theta')x/\epsilon} N_0^{1/\epsilon} < (1 - k_s) k e^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} - k_s k_l$. Because $\theta' < \theta_i^{max}$, the right side limits to something greater than the left as $k_s \to 0$, so we may choose k_s small enough so that the inequality holds. We may pick k_s small enough so that $(\alpha^{buy})^* = -\overline{\alpha}$ for all $\theta \leq \theta'$, as desired.

From Lemma B2, the price of the claim on a developer for whom $(\alpha^{sell})^* > 0$ and $L_1^* > 0$ equals $p_0^{\pi} = (k e^{\mu_i^{max} x/\epsilon} N_0^{1/\epsilon} + k_l) L_1^*$. This expression increases strictly in x, as claimed. The price at $t = 1$ equals $p_1^{\pi} = (p_1^l (e^{\mu^{true} x} N_0) + k_l) L_1^*$, which is strictly less than p_0^{π} if and only if $\mu^{true} < \mu_i^{max}$.

□

C Rental Extension

A share $\chi \in [0, 1)$ of residents are of type $a = 1$ and get flow utility only from renting; the remainder are of type $a = 0$ and get flow utility only from owning.[33] The type a is distributed independently from v and θ. All residents can act as landlords, but developers cannot (the developer problem remains the same as before). We denote R_t^{buy} the quantity of housing rented as a tenant and R_t^{sell} the quantity rented as a landlord. The rental price of housing equals p_t^r. At $t = 1$, an arriving potential resident chooses $(H_1^{buy})^*, (R_1^{buy})^*, (R_1^{sell})^*$ from

$$\begin{aligned} \underset{H_1^{buy}, R_1^{buy}, R_1^{sell}}{\arg\max} \quad & \left(a\iota(R_1^{buy}) + (1 - a)\iota(H_1^{buy} - R_1^{sell}) \right) v - p_1^h H_1^{buy} - p_1^r (R_1^{buy} - R_1^{sell}) \\ \text{subject to} \quad & 0 \leq H_1^{buy} \\ & 0 \leq R_1^{buy} \\ & 0 \leq R_1^{sell} \\ & R_1^{sell} \leq H_1^{buy}, \end{aligned}$$

[33] We rule out $\chi = 1$ because f_r^{χ} does not satisfy Assumption 4 when $\chi = 1$, meaning that the expressions $p_0^h(N_0, x, z, f_r^{\chi}, f_d)$ and $N_0^*(x, z, f_r^{\chi})$ that appear in Proposition 7 are not well-defined. The existence of equilibrium does not depend on $\chi \neq 1$, so by continuity the $\chi = 1$ equilibrium equals the limiting equilibrium as $\chi \to 1$.

where $\iota(R) = 1$ if $R \geq 1$ and 0 otherwise. The utility $u(p_1^h, B_1, v, a, H_0^{buy}, R_0^{buy}, R_0^{sell})$ at $t = 1$ of a potential resident of type a and v who arrived at $t = 0$ and chose H_0^{buy}, R_0^{buy}, and R_0^{sell} equals

$$\begin{aligned} \underset{H_1^{sell}}{\arg\max} \quad & \left(a\iota(R_0^{buy}) + (1-a)\iota(H_0^{buy} - R_0^{sell})\right) v + H_1^{sell} p_1^h + B_1 \\ \text{subject to} \quad & 0 \leq H_1^{sell} \\ & H_1^{sell} \leq H_0^{buy}. \end{aligned}$$

At $t = 0$, arriving potential residents maximize the subjective expectation of their utility by choosing $(H_0^{buy})^*$, $(R_0^{buy})^*$, and $(R_0^{sell})^*$ from

$$\begin{aligned} \underset{H_0^{buy}, R_0^{buy}, R_0^{sell}}{\arg\max} \quad & \mathrm{E}u(p_1^h, B_1, v, a, H_0^{buy}, R_0^{buy}, R_0^{sell}) \\ \text{subject to} \quad & 0 \leq H_0^{buy} \\ & 0 \leq R_0^{buy} \\ & 0 \leq R_0^{sell} \\ & R_0^{sell} \leq H_0^{buy} \\ & B_0 = -p_0^h H_0^{buy} - p_0^r (R_0^{buy} - R_0^{sell}). \end{aligned}$$

Equilibrium is the same as before with the addition of the condition that the sum of $(R_t^{buy})^*$ across all residents equals the sum of $(R_t^{sell})^*$ across them at each t. The following lemma characterizes this equilibrium at $t = 1$:

Lemma C1. *A unique equilibrium at $t = 1$ exists and coincides with that given by Lemma 1.*

Proof. A potential resident arriving at $t = 1$ of type $a = 1$ gets utility $v - p_1^r$ from setting $R_1^{buy} = 1$ and utility 0 from setting $R_1^{buy} = 0$ (all other choices are dominated). The sum of $(R_1^{buy})^*$ therefore equals $\chi N_1 SD(p_1^r)$.

Increasing H_1^{buy} and R_1^{sell} the same amount increases utility of $p_1^r > p_1^h$ and decreases utility if $p_1^r < p_1^h$. The former cannot hold in equilibrium, as it leads to unlimited housing demand, which cannot be matched by the limited supply. The latter cannot hold in equilibrium if $\chi > 0$, as it leads to zero rental supply, which cannot be matched by rental demand, which is positive if $\chi > 0$. If $\chi = 0$, then $p_1^r < p_1^h$ can hold in equilibrium if $(R_0^{sell})^* = 0$ for all potential residents. Therefore $p_1^h = p_1^r$ or $\chi = 0$ and $p_1^r < p_1^h$.

If $\chi > 0$, then a potential resident arriving at $t = 1$ of type $a = 1$ sets $(H_1^{buy})^* = (R_1^{sell})^*$. Arriving potential residents of type $a = 0$ set $(R_1^{buy})^* = 0$ because $p_1^r = p_1^h \geq k > 0$ in the case that $\chi > 0$, or because clearing of the rental market in the case that $\chi = 0$ and $p_1^r < p_1^h$ requires it (the equilibrium possibilities are then $p_1^r \in [0, p_1^h)$). Setting $R_1^{sell} = 0$ in the case that $\chi = 0$ and $p_1^r < p_1^h$, arriving potential residents of type $a = 0$ get utility $v - p_1^h$ if $H_1^{buy} - R_1^{sell} = 1$ and utility 0 if $H_1^{buy} - R_1^{sell} = 0$ (all other choices of $H_1^{buy} - R_1^{sell}$ are dominated). The total of $(H_1^{buy})^* - (R_1^{sell})^*$ across these potential residents equals $(1 - \chi) N_1 SD(p_1^h)$.

The total of $(H_1^{buy})^* - (R_1^{sell})^* + (R_1^{buy})^*$ across all residents equals $N_1 SD(p_1^h)$. Because the rental market clears, the total of $(H_1^{buy})^*$ equals $N_1 SD(p_1^h)$, which coincides with housing demand in the model of Section 1. As shown in the proof of Lemma 1, the sum of $(H_1^{sell})^*$ across departing residents is irrelevant for equilibrium prices at $t = 1$, so we are done. □

We now prove Proposition 7.

Proof of Proposition 7. For clarity, we divide the proof into three parts.

Part 1: Equilibrium house price at $t = 0$

Consider potential residents for whom $a = 1$. If $R_0^{buy} \in [0, 1)$, then utility is $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$. If $p_0^r < 0$, then R_0^{buy} cannot be chosen to maximize utility, so $p_0^r \geq 0$ in equilibrium. As a result, utility weakly increases in R_0^{sell}, so it is maximized when $R_0^{sell} = H_0^{buy}$ and $R_0^{buy} = 0$ at $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. If $R_0^{buy} \geq 1$, then utility equals $v + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$, which is maximized when $R_0^{buy} = 1$ and $R_0^{sell} = H_0^{buy}$ at $v - p_0^r + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. Thus, unless $p_0^r = 0$ (which we consider below), the sum of $(R_0^{buy})^*$ across potential residents of type $a = 1$ equals $\chi N_0 SD(p_0^r)$, and the sum of $(R_0^{sell})^*$ across them equals the sum of $(H_0^{buy})^*$ across them.

Consider the problem for potential residents with $a = 0$. If $H_0^{buy} - R_0^{sell} \in [0, 1)$, then utility equals $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$. Utility is maximized when $R_0^{sell} = H_0^{buy}$ at $H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. If $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h > 0$ for any $\theta \in \operatorname{supp} f_r$, then utility cannot be maximized. As a result, $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h \leq 0$ for all $\theta \in \operatorname{supp} f_r$, and utility is maximized at 0. If $H_0^{buy} - R_0^{sell} \geq 1$, then utility equals $v + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h) + (R_0^{sell} - R_0^{buy})p_0^r$, which weakly rises in R_0^{sell}. Utility is maximized when $R_0^{sell} = H_0^{buy} - 1$ at $v - p_0^r + H_0^{buy}(p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h)$. Because $p_1^h(e^{\mu(\theta)x}N_0) + p_0^r - p_0^h \leq 0$ for all $\theta \in \operatorname{supp} f_r$, utility is maximized when $R_0^{sell} = 0$ and $H_0^{buy} = 1$ at $v - p_0^h + p_1^h(e^{\mu(\theta)x}N_0)$. Thus, unless $p_0^r = 0$, the sum of $(R_0^{buy})^*$ across these potential residents equals 0, and the sum of $(H_0^{buy})^*$ across them exceeds the sum of $(R_0^{sell})^*$ across then by $(1 - \chi)N_0 S \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$.

Combining the two cases, we see that if $p_0^r > 0$, the sum of $(H_0^{buy})^*$ across potential residents equals $\chi N_0 SD(p_0^r) + (1 - \chi)N_0 S \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$ due to the clearing of the rental market.

If $p_1^h(e^{\mu_r^{max}x}N_0) + p_0^r - p_0^h < 0$, then $(R_0^{sell})^* = 0$ for all potential residents. The rental market can clear only if $\chi N_0 SD(p_0^r) = 0$, which can hold only if $\chi = 0$. In this case, rental supply and demand equals 0 for all potential residents (or $p_0^r = 0$), in which case the rental market becomes irrelevant and the equilibrium reduces to that analyzed in Section 3. Therefore, for the rest of the proof we assume that $\chi > 0$. In this case, $(R_0^{sell})^* > 0$ for some potential residents, so $p_1^h(e^{\mu_r^{max}x}N_0) + p_0^r - p_0^h = 0$.

As shown in the proof of Lemma 2, either $p_0^h = k + p_1^h(e^{\mu_d^{max}x}N_0)$ in which case developers for whom $p_1^h(e^{\mu(\theta)x}N_0) = p_1^h(e^{\mu_d^{max}x}N_0)$ may choose any $L_1 \geq 0$, or $p_0^h > k + p_1^h(e^{\mu_d^{max}x}N_0)$ in which case $L_1 = 0$ for all developers. The former may hold in equilibrium if and only if the resulting housing demand falls short of S:

$$\begin{aligned} 1 \geq \chi N_0 D(p_1^h(e^{\mu_d^{max}x}N_0) - p_1^h(e^{\mu_r^{max}x}N_0)) \\ + (1 - \chi)N_0 \int_\Theta D(p_1^h(e^{\mu_d^{max}x}N_0) - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \end{aligned} \tag{C1}$$

This inequality is the same as (A1) but with f_r replaced by f_r^χ, so (C1) holds if and only if $N_0 \leq N_0^*(x, z, f_r^\chi)$. For such N_0, $p_0^h = k + p_1^h(e^{\mu_d^{max}x}N_0) = p_0^h(N_0, x, z, f_r^\chi, f_d)$. If $N_0 > N_0^*(x, z, f_r^\chi)$, then p_0^h must equate total housing demand with S, meaning that it is the unique value satisfying

$$1 = \chi N_0 D(p_0^h - p_1^h(e^{\mu_r^{max}x}N_0)) + (1 - \chi)N_0 \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta. \tag{C2}$$

This equation coincides with (A2) but with f_r^χ in place of f_r, so the equilibrium house price at $t = 0$ equals $p_0^h(N_0, x, z, f_r^\chi, f_d)$ for $N_0 \geq N_0^*(x, z, f_r^\chi)$.

Part 2: Non-monotonicity of house price boom

According to Proposition 4, the boom is strictly maximized at $N_0 = 1$ if Assumption 5 holds when applied to f_r^χ in place of f_r. The first condition in the assumption, $e^{\mu_d^{max} x/\epsilon} > e^{\mu_r^{max} x/\epsilon} - 1$, continues to hold because the maxima of f_r and f_r^χ coincide. The second condition applied to f_r^χ is

$$1 > \chi \left(1 + e^{\mu_d^{max} x/\epsilon} - e^{\mu_r^{max}/\epsilon}\right)^{-\epsilon} + (1-\chi) \int_\Theta \left(1 + e^{\mu_d^{max} x/\epsilon} - e^{\mu(\theta) x/\epsilon}\right)^{-\epsilon} f_r(\theta) d\theta. \tag{C3}$$

This inequality holds for $\chi = 0$ by assumption. The right side of (C3) increases continuously in χ because $\mu_r^{max} > \mu(\theta)$ for all $\theta < \theta_r^{max}$, so (C3) holds for all χ if and only if it holds for $\chi = 1$. When $\chi = 1$, (C3) reduces to $\mu_r^{max} < \mu_d^{max}$. If $\mu_r^{max} \geq \mu_d^{max}$, then by the intermediate value theorem there exists $\chi^*(x, z) \in (0, 1]$ such that (C3) holds if $\chi < \chi^*(x, z)$. When $\mu_r^{max} = \mu_d^{max}$, (C3) holds as an equality, so $\chi^*(x, z) = 1$.

Part 3: House price boom and rental share

As shown earlier in this proof,

$$\frac{\sum (R_0^{buy})^*}{\sum (H_0^{buy})^*} = \frac{\chi D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max} x} N_0))}{\int_\Theta D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta) x} N_0)) f_r^\chi(\theta) d\theta}.$$

If $xz = 0$, then $\mu(\theta) = \mu_r^{max}$ for all θ, so this fraction equals χ.

We now fix a value of N_0 and consider the case when $xz > 0$. The right side of (C1) weakly increases in χ, so $N_0^*(x, z, f_r^\chi)$ weakly and continuously decreases in χ. As a result, if $N_0 < N_0^*(x, z, \chi)$, then a marginal increase in χ has no bearing on $p_0^h(N_0, x, z, f_r^\chi, f_d)$, as this equilibrium price is independent of χ for $N_0 < N_0^*(x, z, f_r^\chi)$. If $N_0 \geq N_0^*(x, z, f_r^\chi)$, then $p_0^h(N_0, x, z, f_r^\chi, f_d)$ solves (C2). Because $N_0 \geq N_0^*(x, z, f_r^\chi) > 1$, the integral in (C2) evaluated at $p_0^h = p_0^h(N_0, x, z, f_r^\chi, f_d)$ must be less than 1. It follows that $D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max} x} N_0)) > \int_\Theta D(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta) x} N_0)) f_r(\theta) d\theta$, so an increase in χ increases the right side of (C2) holding $p_0^h = p_0^h(N_0, x, z, f_r^\chi, f_d)$ constant. Because the right side of (C2) weakly decreases in p_0^h, it follows that $p_0^h(N_0, x, z, f_r^\chi, f_d)$ strictly increases in χ, as desired.

□

D Supply Elasticity Extension

Developers may rent out undeveloped land on spot markets each period to firms, such as banana stands, that use the city's land as an input. We denote the land rent r_t^l. Spot land demand of firms equals $SD^l(r_t^l)$, where D^l satisfies the following:

Assumption D1. *$D^l : \mathbb{R}_+ \to \mathbb{R}_+$ is continuously differentiable and decreases, $-r(D^l)'(r)/D^l(r)$ weakly decreases, and $\lim_{r \to 0} D^l(r) \geq 1 > \lim_{r \to \infty} D^l(r)$.*

The positivity of D^l guarantees that some vacant land exists in equilibrium, a property that makes analyzing the equilibrium easier. The condition on $r(D^l)'/D^l$ means that land demand becomes weakly less elastic as its spot price rises so that it is weakly costlier to use each marginal unit of land. The first limit implies that land demand is at least equal to available space when land is free

and leads to a positive spot price in equilibrium. The second limit implies that land demand falls below available space at a high enough price and leads to the existence of equilibrium.

Each developer chooses the quantity L_t^{rent} of land to rent on the spot market. At $t = 1$, the liquidation value π of a developer is the outcome of the constrained optimization problem

$$\begin{aligned}\pi(p_1^h, p_1^l, r_1^l, H_1, L_1, B_1) = \max_{H_1^{sell}, L_1^{buy}, H_1^{build}, L_1^{rent}} \quad & p_1^h H_1^{sell} - p_1^l L_1^{buy} - kH_1^{build} + r_1^l L_1^{rent} + B_1 \\ \text{subject to} \quad & H_1^{sell} \leq H_1 + H_1^{build} \\ & H_1^{build} \leq L_1 + L_1^{buy} - L_1^{rent}.\end{aligned}$$

The actions $(H_1^{sell})^*$, $(L_1^{buy})^*$, $(H_1^{build})^*$, and $(L_1^{rent})^*$ chosen by the developer maximize this problem. At $t = 0$ each developer chooses $(H_0^{sell})^*, (L_0^{buy})^*, (H_0^{build})^*$, and $(L_0^{rent})^*$ from

$$\begin{aligned}\arg\max_{H_0^{sell}, L_0^{buy}, H_0^{build}, L_0^{rent}} \quad & \mathrm{E}\pi(p_1^h, p_1^l, r_1^l, H_1, L_1, B_1) \\ \text{subject to} \quad & H_0^{sell} \leq H_0^{build} \\ & H_0^{build} \leq L_0 + L_0^{buy} - L_0^{rent} \\ & H_1 = H_0^{build} - H_0^{sell} \\ & L_1 = L_0 + L_0^{buy} - H_0^{build} \\ & B_1 = p_0^h H_0^{sell} - p_0^l L_0^{buy} - 2kH_0^{build} + r_0^l L_0^{rent}.\end{aligned}$$

The potential resident problems are the same as in Appendix C. Equilibrium is the same as before with the addition of the condition that the sum of $(L_t^{rent})^*$ across developers equals $SD^l(r_t^l)$ at each t. The following lemma characterizes this equilibrium at $t = 1$:

Lemma D1. *Given N_1, a unique equilibrium at $t = 1$ exists, and in it $p_1^h - k = p_1^l = r_1^l > 0$.*

Proof. If $r_1^l \neq p_1^l$, then developers cannot maximize π because holding $L_1^{buy} - L_1^{rent}$ constant and increasing L_1^{buy} and L_1^{rent} always increases π if $r_1^l > p_1^l$ and decreases π if $r_1^l < p_1^l$. So $p_1^l = r_1^l$ in any equilibrium. For the same reasons given in the proof of Lemma 1, $p_1^h = p_1^l + k$ in any equilibrium.

From the proof of Lemma C1, housing demand from arriving potential residents at $t = 1$ equals $SN_1D(p_1^h)$. Land demand from firms equals $SD^l(r_1^l)$. If $r_1^l \leq 0$, then demand for space is either not defined or exceeds S. Therefore, in any equilibrium $r_1^l > 0$. It follows that $(L_1^{rent})^* + (H_1^{build})^* = L_1 + (L_1^{buy})^*$ for all developers. Similarly, $(H_1^{sell})^* = H_1 + (H_1^{build})^*$ for all developers. Therefore the sum of $(H_1^{sell})^* + (L_1^{rent})^*$ across developers equals the sum of $H_1 + L_1$ across them. All space other than $H_1 + L_1$ is owned by departing potential residents at the beginning of $t = 1$. The clearing of the land spot market and the housing market therefore imply that in equilibrium, $1 = D^l(r_1^l) + N_1D(p_1^h)$. Substituting $r_1^l = p_1^h - k$ yields

$$1 = D^l(p_1^h - k) + N_1D(p_1^h). \tag{D1}$$

Because $r_1^l > 0$, $p_1^h > k$, so both D^l and D strictly decrease for possible p_1^h. The right side exceeds 1 as $p_1^h \to k$. As $p_1^h \to \infty$, the right side limits to something less than 1 by Assumption D1. It follows that a unique value of p_1^h satisfies this equation. □

We denote equilibrium prices $p_1^h(N_1)$, $p_1^l(N_1)$, and $r_1^l(N_1)$. The first two should not be confused with the functions defined after Lemma 1 that use the same notation.

The next lemma establishes the existence of a unique equilibrium at $t = 0$.

Lemma D2. *Given N_0, x, z, and χ, a unique equilibrium at $t = 0$ exists.*

Proof. For the same reasons given in the proof of Lemma D1, $r_0^l > 0$ in any equilibrium. In the equilibrium at $t = 1$, $\pi = p_1^h H_1 + p_1^l L_1 + B_1$. By making substitutions using the constraints of the $t = 0$ developer problem, we see that the objective at $t = 0$ is to choose $H_1, L_1 \geq 0$ to maximize $(p_1^h(e^{\mu(\theta)x}N_0) - p_0^h)H_1 + (p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k + r_0^l)L_1 + p_0^l L_0$ and that $(L_0^{rent})^* = L_1$ for each developer. Because $k+r_0^l > 0$, it follows that $H_1 = 0$ for all developers. If $p_1^h(e^{\mu(\theta)x}N_0)-p_0^h+k+r_0^l < 0$ for all developers, then $L_1 = 0$ for all of them, but then $(L_0^{rent})^* = 0$, leading to a failure of market-clearing in the land spot market because $D^l(r_0^l) > 0$. If $p_1^h(e^{\mu(\theta)x}N_0) - p_0^h + k + r_0^l > 0$ for any developer, then the objective function cannot be maximized. It follows that $p_0^h = p_1^h(e^{\mu_d^{max}x}N_0) + k + r_0^l$. Market-clearing in all markets implies that spot land demand plus total housing demand from arriving potential residents equals S. Using the equations for housing demand from the proof of Proposition 7, we form the equilibrium condition

$$\begin{aligned} 1 = {} & D^l(p_0^h - p_1^h(e^{\mu_d^{max}x}N_0) - k) + \chi N_0 D(p_0^h - p_1^h(e^{\mu_r^{max}x}N_0)) \\ & + (1-\chi)N_0 \int_\Theta D(p_0^h - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta) d\theta. \end{aligned} \tag{D2}$$

The right side strictly decreases in p_0^h wherever it is defined. It is defined for $p_0^h > k+p_1^h(e^{\mu_d^{max}x}N_0)$. As p_0^h approaches this value, D^l is at least 1, whereas the remainder of the right side is positive. It follows that the entire right side exceeds 1 in the limit. As $p_0^h \to \infty$, the terms involving D go to 0, and the term involving D^l limits to something less than 1 according to Assumption D1. It follows that a unique solution exists to this equation. □

We denote this unique equilibrium price $p_0^h(N_0, x, z, \chi)$, which should not be confused with the equilibrium price given by Proposition 7.

We turn now to defining the elasticity of housing supply. The proof of Lemma C1 showed that $r_1^h = p_1^h(N_1)$ is the unique equilibrium rent when $\chi > 0$ and is an equilibrium rent when $\chi = 0$. We define $r_1^h(N_1) = p_1^h(N_1)$. Because the housing stock at $t = 1$ equals $S - SD^l(p_1^h - k)$, the elasticity of housing supply at $t = 1$ is

$$\epsilon_1^s(N_1) = -\frac{r_1^h(N_1)(D^l)'(p_1^h(N_1) - k)}{1 - D^l(p_1^h(N_1) - k)}.$$

Similarly, the proof of Proposition 7 showed that $r_0^h = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max}x}N_0)$ is the unique equilibrium rent when $\chi > 0$ and is an equilibrium rent when $\chi = 0$. We define $r_0^h(N_0, x, z, \chi) = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max}x}N_0)$. Because the housing stock equals $S - SD^l(p_0^h - p_1^h(e^{\mu_d^{max}x}N_0) - k)$ at $t = 0$, the elasticity of housing supply at $t = 0$ equals

$$\epsilon_0^s(N_0, x, z, \chi) = -\frac{r_0^h(N_0, x, z, \chi)(D^l)'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max}x}N_0) - k)}{1 - D^l(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max}x}N_0) - k)}.$$

The next lemma characterizes these elasticities.

Lemma D3. *There exists a continuous, decreasing function $\epsilon^s : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\epsilon_0^s(N_0, x, 0, \chi) = \epsilon^s(N_0)$ and $\epsilon_1^s(N_1) = \epsilon^s(N_1)$.*

Proof of Lemma D3. We define the function $\epsilon^s(\cdot)$ by

$$\epsilon^s(N) \equiv -\frac{p_1^h(N)(D^l)'(p_1^h(N) - k)}{1 - D^l(p_1^h(N) - k)}. \tag{D3}$$

Given (D1), the denominator equals $ND(p_1^h(N)) > 0$. As shown by Lemma D1, $p_1^h(N) > k$, so the numerator is negative and well-defined. It follows that $\epsilon^s(N) > 0$ for all $N > 0$. Because $p_1^h(N) > k$, the implicit function theorem applied to (D1) implies that $p_1^h(\cdot)$ is continuous; Assumption D1 them implies that $\epsilon^s(\cdot)$ is continuous. To show that ϵ^s decreases, we rewrite (D3) as

$$\epsilon^s(N) = \frac{-(p_1^h(N)-k)(D^l)'(p_1^h(N)-k)}{D^l(p_1^h(N)-k)} \frac{p_1^h(N)}{p_1^h(N)-k} \frac{D^l(p_1^h(N)-k)}{1-D^l(p_1^h(N)-k)}. \tag{D4}$$

It is clear from (D1) that $p_1^h(N)$ strictly increases in N because D^l and D both strictly decrease over the domains relevant in that equation. It follows that each fraction on the right of (D4) strictly decreases in N, with the result about first fraction following from Assumption D1. Because $r_1^h(N_1) = p_1^h(N_1)$, $\epsilon_1^s = \epsilon^s(N_1)$. When $z = 0$, it is clear from (D1) that $p_0^h = p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)$ solves (D2). Therefore $r_0^h(N_0, x, 0, \chi) = p_1^h(N_0)$ and $p_0^h(N_0, x, 0, \chi) - p_1^h(e^{\mu_x^{max}}N_0) = p_1^h(N_0)$ when $z = 0$. it follows that $\epsilon_0^s(N_0, x, 0, \chi) = \epsilon^s(N_0)$. □

Next, we prove Proposition 8.

Proof of Proposition 8. Differentiating (D1) and simplifying yields

$$\frac{\partial p_1^h(e^{\mu(\theta)x}N_0)}{\partial x} = \frac{\mu(\theta)p_1^h(e^{\mu(\theta)x}N_0)}{\epsilon^s(e^{\mu(\theta)x}N_0)+\epsilon}.$$

Using this equation, we differentiate (D2) with respect to x and simplify to obtain

$$\begin{aligned}\frac{\partial \log p_0^h(N_0, x, z, \chi)}{\partial x} = &\frac{c_1 p_1^h(e^{\mu_d^{max}x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu_d^{max}}{\epsilon^s(e^{\mu_d^{max}x}N_0)+\epsilon} + \frac{c_2 p_1^h(e^{\mu_r^{max}x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu_r^{max}}{\epsilon^s(e^{\mu_r^{max}x}N_0)+\epsilon} \\ &+ \int_\Theta \frac{c_3 p_1^h(e^{\mu(\theta)x}N_0)}{p_0^h(N_0, x, z, \chi)} \frac{\mu(\theta)}{\epsilon^s(e^{\mu(\theta)x}N_0)+\epsilon} f_r(\theta)d\theta,\end{aligned}$$

where $c_i = \gamma_i/(\gamma_1+\gamma_2+\gamma_3)$ for $i \in \{1,2,3\}$ and the γ_i are defined as follows:

$$\begin{aligned}\gamma_1 &= (D^l)'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_d^{max}x}N_0)) \\ \gamma_2 &= \chi N_0 D'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu_r^{max}x}N_0)) \\ \gamma_3 &= (1-\chi)N_0 \int_\Theta D'(p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta)x}N_0)) f_r(\theta)d\theta.\end{aligned}$$

To prove that the equation in the proposition holds to the first order, we show that it holds exactly when $x = 0$ or $z = 0$. When $x = 0$, $r_0^h(N_0, x, z, \chi) = p_0^h(N_0, x, z, \chi) - p_1^h(e^{\mu(\theta)x}N_0)$ for all θ, so $\gamma_1 = \epsilon^s(N_0)/(\epsilon^s(N_0)+\epsilon)$, $\gamma_2 = \chi\epsilon/(\epsilon^s(N_0)+\epsilon)$, and $\gamma_3 = (1-\chi)\epsilon/(\epsilon^s(N_0)+\epsilon)$. We also have $p_0^h(N_0, 0, z, \chi) = p_1^h(N_0)/2$. It follows that

$$\frac{\partial \log p_0^h(N_0, 0, z, \chi)}{\partial x} = \frac{1}{2} \frac{\epsilon^s(N_0)\mu_d^{max} + \chi\epsilon\mu_r^{max} + (1-\chi)\epsilon\overline{\mu}}{\epsilon^s(N_0)+\epsilon} \frac{1}{\epsilon^s(N_0)+\epsilon},$$

which coincides with the formula in the text. When $z = 0$, $\mu(\theta) = \overline{\mu}$ for all θ and $p_0^h(N_0, x, z, \chi) = p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)$ as shown in the previous proof. It follows that

$$\frac{\partial \log p_0^h(N_0, x, 0, \chi)}{\partial x} = \frac{p_1^h(e^{\overline{\mu}x}N_0)}{p_1^h(N_0) + p_1^h(e^{\overline{\mu}x}N_0)} \frac{\overline{\mu}}{\epsilon^s(e^{\overline{\mu}x}N_0)+\epsilon}.$$

This expression coincides with the formula in the text because

$$\frac{p_1^h(e^{\overline{\mu}x}N_0)}{p_1^h(N_0)} = \exp\left(\int_0^x \frac{\partial \log p_1^h(e^{\overline{\mu}x'}N_0)}{\partial x'}dx'\right) = \exp\left(\int_0^x \frac{\overline{\mu}dx'}{\epsilon^s(e^{\overline{\mu}x'}N_0)+\epsilon}\right).$$

□

Figure D1 plots the $t = 0$ pass-through $1/(\epsilon^s(N_0)+\epsilon)$ and $t = 1$ pass-through $1/(\epsilon^s(e^{\overline{\mu}x}N_0)+\epsilon)$ as well as the pass-though of x to $\log p_0^h(N_0, x, z, \chi)$ with and without disagreement. Disagreement amplifies the price impact of x most when the short-run elasticity is high and the long-run elasticity is low.

E Pulte Investor Presentation

Figure E1 presents slides from a 2004 presentation to investors by Pulte, one of the large public homebuilders studied in Section 5.2. These slides provide some evidence that builders viewed supply constraints as binding in the long run across many cities during the housing boom, and also that our partition of cities in Figure 5 matches that considered by builders contemporaneously with the boom.

F Construction Analysis

To analyze the effect of the shock x on construction, we define $Q_r(N_0, x, z)$ to be the quantity of housing held by potential residents at $t = 0$ in equilibrium. The following lemma characterizes the response of Q_r to x:

Lemma F1. *$Q_r(N_0, x, z) < Q_r(N_0, 0, z)$ if $e^{-\mu_d^{max}x} < N_0 < N_0^*(x, z)$ and $z = 0$. $Q_r(N_0, x, z) = Q_r(N_0, 0, z)$ otherwise.*

Proof. By Proposition 2, $Q_r(N_0, x, z) = 1$ when $N_0 \geq N_0^*(x, z)$. By (A1) in the proof of Lemma 2, $Q_r(N_0, x, z) = SN_0 \int_\Theta D(p_1^h(e^{\mu_d^{max}x}N_0) + k - p_1^h(e^{\mu(\theta)x}N_0))f_r(\theta)d\theta$ when $N_0 < N_0^*(x, z)$.

When $z = 0$, $p_1^h(e^{\mu_d^{max}x}N_0) = p_1^h(e^{\mu(\theta)x}N_0)$ for all $\theta \in \Theta$, so $Q_r(N_0, x, 0) = SN_0$ for $N_0 < N_0^*(x, 0)$. Because $N_0^*(x, 0) = 1$ as shown by Proposition 1, $Q_r(N_0, x, 0) = Q_r(N_0, 0, 0)$.

When $z > 0$ and $N_0 \leq e^{-\mu_d^{max}x}$, $p_1^h(e^{\mu(\theta)x}N_0) \geq p_1^h(e^{\mu_d^{max}x}N_0)$ for all $\theta \in \Theta$, so by Assumption 1 $Q_r(N_0, x, z) = SN_0$. Thus $Q_r(N_0, x, z) = Q_r(N_0, 0, z)$ in this case as well.

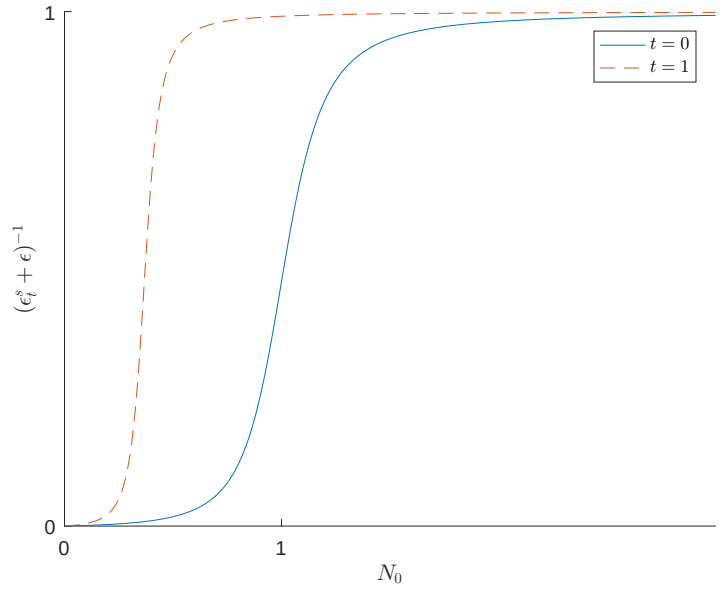
When $z > 0$ and $N_0 \geq N_0^*(x, z)$, $Q_r(N_0, x, z) = 1$ and $Q_r(N_0, x, 0) = 1$ because $N_0 \geq N_0^*(x, z) > N_0^*(x, 0) = 1$. Again $Q_r(N_0, x, z) = Q_r(N_0, 0, z)$ in this case.

We divide the final case in which $e^{-\mu_d^{max}x} < N_0 < N_0^*(x, z)$ and $z > 0$ into two subcases. If $1 \leq N_0 < N_0^*(x, z)$, then $1 = N_0^*(0, z) \leq N_0 < N_0^*(x, z)$. It follows that $Q_r(N_0, x, z) < 1 = Q_r(N_0, 0, z)$, as claimed. If $e^{-\mu_d^{max}x} < N_0 < 1$, then $Q_r(N_0, 0, z) - Q_r(N_0, x, z) = SN_0 \int_{\theta<\theta_d^{max}}(1 - D(p_1^h(e^{\mu_d^{max}x}N_0) + k - p_1^h(e^{\mu(\theta)x}N_0)))f_r(\theta)d\theta$. For all $\theta < \theta_d^{max}$, the integrand is positive because $e^{\mu_d^{max}x}N_0 > 1$. By Assumption 4, $\int_{\theta<\theta_d^{max}} f_r(\theta)d\theta > 0$, so $Q_r(N_0, 0, z) > Q_r(N_0, x, z)$, as claimed. □

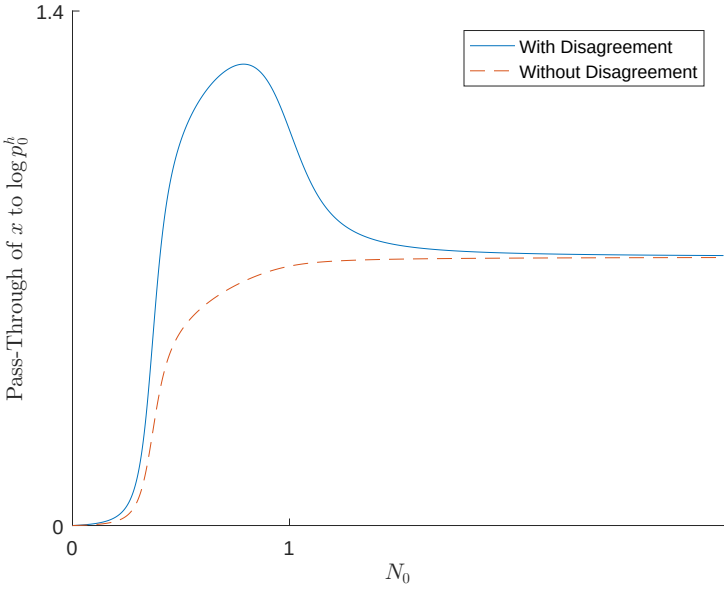
Lemma F1 shows that the shock x only affects the equilibrium quantity of housing in intermediate cities with disagreement, in which case the shock lowers the housing stock. Because the shock does not change the current demand N_0, it does not alter housing supply in most cases. It only does so when optimistic developers set prices so high that the number of potential residents choosing to buy falls. This scenario occurs in intermediate cities with disagreement.

FIGURE D1
Comparative Statics with Respect to Initial Demand

a) Supply Elasticity

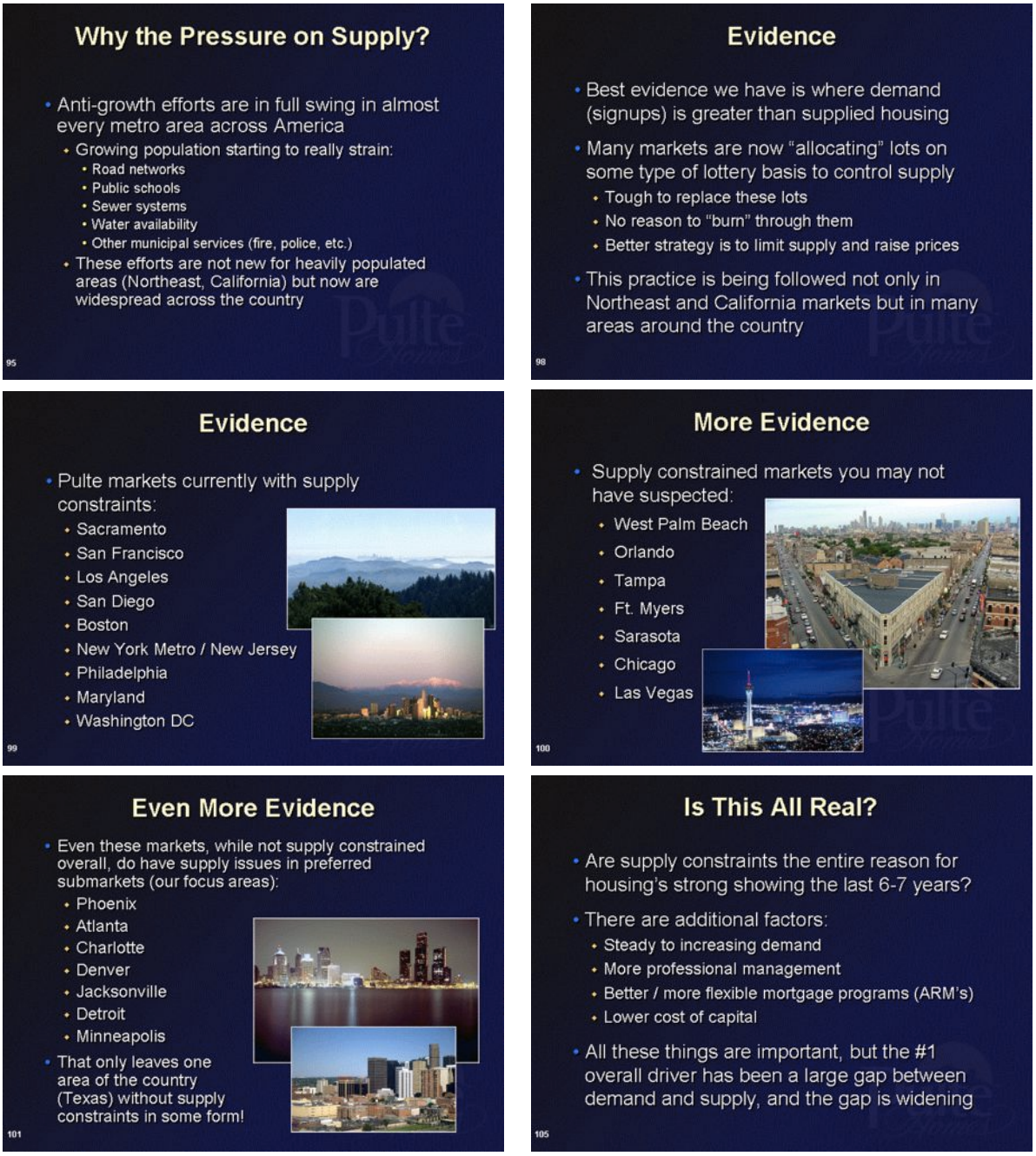


b) Price Increase



Notes: N_0 equals the number of potential residents at $t = 0$ relative to the city size. Supply elasticities are given by $\epsilon_0^s = \epsilon^s(N_0)$ and $\epsilon_1^s = \epsilon^s(e^{\overline{\mu}x}N_0)$, where $\epsilon^s(\cdot)$ is defined by Lemma D3. The pass-through of x to $\log p_0^h$ equals the expression for $\partial \log p_0^h(N_0, x, z, \chi)/\partial x$ given by Proposition 8. The parameters used to generate this figure are $k = 1$, $x = 1$, $z = 1$, $\epsilon = 1$, $\chi = 0$, $\overline{\mu} = 1$, $f_r = f_d = (0.9)\mathbf{1}_{-1/9} + (0.1)\mathbf{1}_1$, and $D^l(r) = 0.01k/r$, with $z = 0$ used in the "without disagreement" graph.

FIGURE E1
Land Supply Slides from Pulte's 2004 Investor Conference



Notes: Slides excerpted from a presentation by Pulte Homes, Inc., on February 26, 2004, to investors and disclosed under SEC Regulation FD requirements. Last accessed on March 15, 2015, at `http://services.corporate-ir.net/SEC.Enhanced/SecCapsule.aspx?c=77968&fid=2633894`.