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TESTS OF INTERNATIONAL CAPM
WITH TIME-VARYING COVARIANCES

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ABSTRACT

We perform maximum likelihood estimation of a model of international asset pricing based on CAPM. We test the restrictions imposed by CAPM against a more general asset pricing model. The "betas" in our CAPM vary over time from two sources -- the supplies of the assets (government obligations of France, Germany, Italy, Japan, the U.K. and the U.S.) change over time, and so do the conditional covariances of returns on these assets. We let the covariances change over time as a function of macroeconomic data. We also estimate the model when the covariances follow a multivariate ARCH process. When the covariance of forecast errors are time-varying, we can identify a modified CAPM model with measurement error -- which we also estimate. We find that the model in which the CAPM restrictions are imposed (which involve cross-equation constraints between coefficients and the variances of the residuals) perform much better when variances are not constant over time. Nonetheless, the CAPM model is rejected in favor of the less restricted model of asset pricing.

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1. Introduction

One branch of the "asset market approach" to exchange rates has focused on the demand by residents of an open economy for foreign currency denominated assets. In this view, people diversify their portfolios to hold a variety of domestic and foreign assets with the aim of getting the maximum return on their portfolios while taking into consideration the riskiness of the assets. In particular, foreign currency denominated assets are subject to exchange rate risk - or, perhaps more accurately, they might be subject to more purchasing power risk than domestic assets. In the general equilibrium of such a "portfolio balance" model, the supplies of outside assets affect macroeconomic variables, including the exchange rate. The portfolio balance approach to flexible exchange rates was pioneered by Black (1973), Kouri (1976), Branson (1977) and Girton and Henderson (1977).

Some general forms of the portfolio balance model have been treated empirically by, among others, Frankel(1982a, 1984), Branson, Halttunen and Masson (1977, 1979), Dooley and Isard (1979, 1983) and Lewis (1986). These models typically postulate that demand for domestic assets relative to foreign assets is a function of $i-i^{*}-\delta$, where i is the return on home assets, i^{*} is the return on assets from abroad, and δ is the expected rate of depreciation of the domestic currency. The quantity $i-i^{*}-\delta$ is often referred to as the risk premium in this literature. It represents the excess expected return the domestic asset must pay to compensate for its riskiness. (Of course, the risk premium as defined here could be negative, implying foreign assets pay a greater expected rate of return.) These papers proceed by making some assumption on how expectations of the future exchange rate are formed, and then estimate bond demand equations as functions of the risk premium, or else estimate a reduced form in which the exchange rate depends on asset supplies.

The portfolio balance approach can be seen as one possible explanation of the finding by many authors (including Geweke and Feige (1979), Frankel (1980), Hansen and Hodrick (1980) and Cumby and Obstfeld (1981)) that uncovered interest parity does not hold. That is, in the sample periods tested, the conditionally expected return differential between comparable assets across countries is non-zero ($i_{t+1}-i^*_{t+1}-\delta_{t+1}$ has a non-zero mean conditional on information available at time t). Although the presence of a risk premium is one explanation for this finding, others include the possibility of inefficient markets or a peso problem.

Numerous studies have tested whether the rejection of uncovered interest parity is attributable to a risk premium, without resorting directly to estimating bond demand equations. Various ingenious approaches have been taken by Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Domowitz and Hakkio (1985), Mark (1985), Cumby (1986), Giovannini and Jorion (1987a), and Kaminsky and Peruga (1986). These studies typically exploit the time series properties of asset returns (and sometimes other variables such as consumption) without relying on asset supply data.

One branch of this literature can be viewed as a refinement on the portfolio balance models, in that it derives asset demand equations (rather than asset pricing equations as in most of the literature cited in the previous paragraph) from an underlying utility maximization approach. Frankel (1982b) proposed that a popular and reasonable model of asset diversification -- the capital asset pricing model (CAPM) -- can be implemented for international asset data and estimated. Furthermore, the restrictions that CAPM places on more general bond demand equations can be tested. Papers that employ this type of international CAPM test include Frankel (1983), Frankel and Engel (1984), and Engel and Rodrigues (1986).

These papers demonstrate that CAPM implies an equation of the form:

(1) $$E_t z_{t+1} = c + \rho\Omega_t\lambda_t$$

where z_{t+1} is a vector of real rates of return between time t and t+1 relative to the real return on some numeraire asset (the jth real return is calculated as $[(1+i^j)(1+a^j)/(1+\pi)] - 1$, where i is the nominal rate of return of an asset in its own currency, a is the rate of appreciation of the currency relative to the currency of the numeraire asset and π is the rate of inflation of prices in the currency of the numeraire asset), c is a vector of constants, ρ is a constant that is a measure of relative risk aversion of the typical market participant, Ω is the conditional variance at time t of z_{t+1}, and λ is a vector whose jth element is the value of the total outstanding supply of the jth asset as a share of the value of all assets. The derivation of this equation assumes that all investors have the same consumption basket, and that the law of one price holds.

Frankel's key observation is that if $E_t z_{t+1}$ is replaced by the realized values of z_{t+1}, then under rational expectations, the variance of the vector of forecast errors $(z_{t+1}-E_t z_{t+1})$ should equal Ω. This suggests an empirical test of CAPM. Regress each relative ex-post rate of return on all of the asset shares in λ. If there are N assets (not counting the numeraire), this would yield an NxN matrix of regression coefficients that under the CAPM hypothesis should be proportional to the covariance matrix of the regression errors - with the constant of proportionality equal to ρ.

In the international finance context, this idea has been implemented by constructing aggregate asset data comprised of the outstanding obligations of governments from each of several countries. Dollar assets are chosen as numeraire, and average real returns for assets from each of the other countries relative to the real return on dollar assets are calculated. In the CAPM tests that have been performed using this technique there has been little support for the restrictions imposed by the CAPM theory. (See Frankel (1986) for a discussion of this literature.)

Typically the conditional variance, Ω, has been treated as a constant. However, several authors, including Hodrick and Srivastava (1984), Cumby and Obstfeld (1984), Hsieh (1984) and Diebold and Nerlove (1986) have noted that forecast errors in foreign exchange markets are notoriously heteroskedastic. Giovannini and Jorion (1987a) offer some "back of the envelope" calculations that suggest that the degree of variability over time in Ω is large enough to account for the empitical failure of the CAPM model. (However, see Giovannini and Jorion (1987b) and Frankel (1987)).

The international finance literature does not offer a very good guide to the determinants of the variance of the forecast error. It seems plausible that forecasts should have higher variance in times of economic turbulence. One approach we take in this paper is to let Ω vary over time as a function of macroeconomic data such as the U.S. money supply and oil prices. If, in fact shocks to the U.S. money supply and dollar oil prices increased the difficulty of forecasting foreign exchange rates, then the constant variance models of Frankel are misspecified. They base their measurement of the forecast variance at any time on the average of past squared forecast errors. However, if the money supply or oil prices have been behaving erratically in the recent past, it is likely that the exchange rate forecast variance will increase.

A method of modelling time-varying variances that does not rely on macroeconomic data has been suggested by Engle (1982). In essence, he postulates that the variance this period is likely to be large following a large error (positive or negative) in the previous period. In a univariate context, for example, we might see

$$\sigma_t^2 = \alpha + \beta\varepsilon_t^2,$$

where σ is the variance this period, and ε is the forecast error made at t for the time t-1 forecast. This modelling of the variance is labelled autoregressive conditional heteroskedasticity (ARCH) by Engle. ARCH models have been used in the foreign exchange literature by Domowitz and Hakkio (1985), Hsieh (1985) and Kaminsky and Peruga (1986).

In this paper we estimate and test a six-country international CAPM model, allowing for time-varying variances following both ARCH specifications and models relating the variance to macroeconomic data. We use aggregate asset data representing the nominal obligations of six governments - France, Germany, Italy, Japan, the U.K. and the U.S. - and the rates of return from Eurocurrency markets from April 1973 to December 1984. These CAPM tests can be viewed as a direct extension of Frankel's tests by allowing for heteroskedasticity. This work is also quite similar to that of Bollerslev, Engle and Wooldridge (1985), who estimate - but do not test the restrictions imposed by - one CAPM model for domestic U.S. assets, using nominal rates of return.

We also allow for a generalization of the Frankel-type CAPM model by introducing the possibility that the empirical CAPM equation (1) does not hold exactly. We imbed the model in the traditional measurement error framework and test the CAPM restriction under the assumption that the variance of the forecast error depends on observable data. We are able to identify the elements of the variance matrix of the measurement error because they are assumed to be time-invariant, as opposed to the variance of the forecast errors.

In section 2 we introduce the time-varying variance CAPM model and test CAPM under the assumption that the variance is a function of macroeconomic variables. In the next section, some multivariate ARCH models are presented and estimated. The CAPM restrictions are tested. Section 4 presents the measurement error model, and section 5 concludes. The appendices contain discussions of the Lagrange multiplier tests that appear throughout the paper, the sources of the data, and the estimation techniques.

2. CAPM with Time-Varying Variances

A general model of the real rates of return on N+1 assets might be that the rates of return are influenced by changes in the value of the outstanding supply of these assets. Thus, one might expect that for any given asset, its expected rate of return would rise if the supply of that asset increased. The expected return on this asset may also be influenced by an increase in the supplies of other assets.

Choosing one asset as numeraire, the expected rates of return on the N-vector for the other assets relative to the numeraire might be a function of the value of the supplies of these assets as a share of the total value of the N+1 assets. Thus, we could write:

$$E_t z_{t+1} = c + B_t \lambda_t, \tag{2}$$

where B is an NxN matrix of coefficients, and the other variables are defined as in section 1. In this general form of the equation, B could vary over time.

Equation (1) is clearly a restriction on equation (2) - it forces the matrix of coefficients in the N equations to be proportional to the covariance matrix of forecast errors of z_{t+1}. In the earlier papers that have tested international CAPM, B and Ω were treated as constants. Here, we allow both to vary over time. As in the previous literature, we can test the restriction imposed by CAPM that B is proportional to Ω.

In this paper, there are six aggregate assets. Each is essentially the outstanding debt at the end of period t of each of the six governments. The debt is calculated in such a way as to include only the value of debt in the hands of the public. For example, corrections are made for foreign exchange intervention by central banks that may remove some of the obligations of one government from public hands and replace it with another. The calculation of the data is described in detail in Frankel (1982b). The data set used here is an updated data set kindly provided to us by Alberto Giovannini. The asset share data are measured at the end of the month and run from April 1973 to December 1984. See Appendix 2 for a description of the data.

To produce an empirical model, it is assumed that expectations are formed rationally. Thus equations (1) and (2) can be transformed into regression equations by replacing the expected value of z_{t+1} with the ex post realized value of this variable, and appending an error term equal to the forecast error at the end of each equation. Thus, the ex post real rates of return are used to calculate z_{t+1}, with the dollar as the numeraire asset. The rate of return the asset of each country is calculated in dollar terms as $(1+i_{t+1})S_{t+1}/S_t$, where i is constructed for each country from spot and one-month forward exchange rates and the Eurodollar one-month rate assuming covered interest parity, and S is the end of period exchange rate in dollars per unit of each country's currency (e.g., the dollars per mark exchange rate).Each nominal rate is then deflated by the common deflator P_{t+1}/P_t, which is a dollar inflation index. P_t is a geometrically weighted average of price indices of the six countries (converted into dollar terms by multiplying by S_t). The ex post rates of return are measured from July 1973 to January 1985. These data are described in more detail in Appendix 2.

We assume that errors are distributed normally. The log likelihood for observation t is given by

$$(3) \qquad \ln L = -2.5\ln\pi - .5\ln|\Omega_t| - .5(z_t - c - B_t\lambda_t)'\,\Omega_t^{-1}(z_t - c - B_t\lambda_t).$$

When the CAPM constraint is imposed, $B_t = \rho\Omega_t$. We estimate the likelihood under alternative assumptions about the behavior of Ω over time.

Table 1 reports the maximum likelihood estimates of the model under the CAPM constraint and under the additional constraint that the variance matrix be constant over time. These estimates correspond to those reported by Frankel (1982b) and Frankel and Engel (1984).

Although the point estimate of ρ in this model is negative, its standard error is very large, so that essentially no economically reasonable value of ρ can be ruled out. In particular, $\rho = 0$ is not rejected, which would say that there is no evidence of a risk premium.

Table 1 also reports the Lagrange Multiplier (LM) statistic for a test of CAPM. (Our calculation of the Lagrange Multiplier statistics in this paper is described in detail in Appendix 1.) The alternative hypothesis is actually a somewhat restricted version of the general unrestricted version of the model in equation 2. In the test reported here, under the alternative hypothesis B is still restricted to be symmetric, although it is no longer proportional to the covariance matrix. This means that a one unit increase in the supply of asset j has the same effect on the relative return of asset i that a one unit increase in the supply of asset j has on the return of asset i. CAPM is rejected at the 1% level even against this restricted form of the alternative hypothesis. The test statistic reported is χ^2 with 14 degrees of freedom. (We are testing 15 proportionality restrictions while estimating the constant of proportionality, implying 14 unique restrictions.)

In this constant variance version there is not much encouraging news for the CAPM hypothesis. However, the CAPM model does not require that Ω be constant over time. Ω_t represents the variance conditional on information available at time t. It is a measure of dispersion of forecast errors for market participants. Frankel's formulation of CAPM is a significant advance on those empirical specifications that require the "beta" of each asset to be constant over time or else vary in a deterministic way. The "beta" for each asset will, in fact, vary as the supplies of the assets change over time. But there is another possible source of variation in the beta that Frankel does not allow for, and that is the change over time in Ω.

The CAPM hypothesis does not provide any particular clue to the source of change in the conditional variance of forecast errors over time. A more complete general equilibrium model of the economy would investigate the source of shocks to the economy and indicate what might cause fluctuations in the forecast variance. In the absence of such a model, we will test some plausible macroeconomic sources of variation.

Each element of the covariance matrix Ω could vary independently with the macroeconomic data (subject to the symmetry constraint on Ω). Thus, in our five equation model, each of the 15 elements of Ω might be a linear function of some variable or variables x. As a practical matter, this five equation system with the constraint imposed between coefficients and the variance matrix is very difficult to estimate. We find it desirable to parameterize the process parsimoniously, at least in our first pass at estimating the time-varying variance model. Thus, we let the variance depend on only one variable at a time, and initially we model the variance according to

$$\Omega_t = P'P + hh'x_t. \tag{4}$$

In this equation, P is an upper triangular matrix of parameters to be estimated and h is column vector of parameters. The macroeconomic variable is given by x_t. In all cases the values of the data are positive numbers, so the form of equation (4) guarantees that the estimated Ω matrix is positive semi-definite. In practice, imposing this positive-semi-definiteness constraint is useful in achieving convergence of the maximum likelihood estimates. Note that we are restricting the variance to depend only on one x_t (at a time) and that only twenty parameters are used to describe the relation between the variance and the macroeconomic data. The variance Ω_t refers to the variance of the forecast made at time t of t+1 variables, and that this variance is a function only of variables known at time t.

We have chosen macroeconomic variables that seem most likely to have influenced the variance of forecast errors of the exchange rate. We allow the variance to depend on stationary representations of the U.S. money supply and of the dollar price of oil. Both variables had very large economic effects during our sample period both in the U.S. and abroad. Furthermore, it is likely that the size and unpredictability of these variables over this span of time added to the variability of forecasts of many macroeconomic variables, including exchange rates. In tables 2 and 3 the variance is a function of the square of the change in the logs of money and oil prices, respectively.

The assumption of rational expectations might lead one to conclude that only unexpected changes in oil prices or the money supply would increase the difficulty of making forecasts. In this view, consumers are able to form expectations of the money supply and oil prices given the past behavior of these variables. A large, but expected, change in one of them will not add any uncertainty, or make forecasts of exchange rates more difficult. To allow for this possibility, we fit ARIMA models to the logs of U.S. M1 and dollar oil prices. We then squared the residuals, and allowed for the variance of the exchange rate forecast to increase if there were a large innovation in one of these variables in the previous month. Appendix 2 contains a description of the ARIMA models.

Table 2 reports the estimates of the model when the variance is allowed to depend on the square of the one month change in the log of the U.S. money supply. Three of the coefficients in the h vector are individually significantly different from zero, and the chi-square test indicates that they are jointly significant at the 5% level. Thus we can reject the constant Ω version of CAPM, in favor of a model in which the forecast variances are greater when there are large percentage changes in the money supply.

Given that this model is a significant improvement on the model whose estimates are reported in Table 1, we now want to test whether the restrictions imposed by CAPM are binding. In Table 5, the coefficients were constrained to be proportional to the variance matrix Ω. The unconstrained model in this case would let the coefficient matrix B (from equation (2)) vary over time as a function of the square of the change in the log of the money supply, and not be constrained to be related in any way to the Ω matrix. At the bottom of Table 5 is reported the LM statistic for the test of the CAPM restriction against the alternative that B is symmetric but not constrained to be proportional to the variance matrix of the residuals. The χ^2 statistic has 19 degrees of freedom and, as indicated in the table, shows that the CAPM restrictions are strongly rejected. Actually, visual inspection of Table 2 is enough to cast serious doubt on the CAPM hypothesis since the point estimate of ρ is negative (but not significantly different from zero).

A note of caution is in order here. Suppose the LM test had failed to reject the CAPM restrictions in Table 5. How should we interpret such a failure to reject? We estimate ρ quite imprecisely, and in fact cannot reject that ρ is zero. But this means that we cannot reject the hypothesis that our explanatory variables have no ability to explain our dependent variables, since all of the explanatory variables are multiplied by ρ. Thus, were we not to reject CAPM, probably the correct interpretation would be that we are unable to explain relative rates of return using asset shares -- but we seem to do about as badly whether or not the restrictions of CAPM are imposed.

Table 3 reports the results of the estimation in which the variance of the forecast errors is a linear function of the square of the change in the dollar price of oil. Here we do not quite reject the hypothesis that the macro data do not help explain changes in the variances and that Ω is constant at the 5% level. None of the elements of the h vector are individually significantly different from zero. The LM test at the bottom of the table indicates that the CAPM restrictions are strongly rejected.

The hypotheses that the variance in the forecast errors is related to the size of unanticipated changes in money and oil prices are tested in Tables 4 and 5. We in fact find that the squared ARIMA residuals for oil prices do a good job of explaining changes in the variance under the CAPM restrictions (in the sense that the likelihood is significantly improved over the constant Ω model), but the ARIMA residuals for money are not as successful. However, in both cases the point estimate of ρ is negative, and in both cases the LM test rejects the CAPM restrictions in favor of the more general model of equation 2. It is interesting that the unexpected squared changes in the oil prices do a better job of explaining the variance than the squared changes in the variable itself. However, since the restricted model is rejected, not too much significance can be attached to this outcome.

Equation (4) restricts the matrix of coefficients that multiplies the macroeconomic variable to have only five independent parameters. We also estimated the general form of equation (4) in which the matrix multiplying the macro variable has fifteen parameters. Specifically, we take the variance to be given by

$$\Omega_t = P'P + Q'Qx_t, \tag{5}$$

where Q is an upper triangular matrix. This formulation imposes the constraint that Ω_t be positive semi-definite. The five parameter version of this model given by equation (4) is a restriction on equation (5) which forces all but the top row of Q to equal zero -- providing ten zero restrictions.

In no case is the 15 parameter model a significant improvement on the 5 parameter model of equation (4). Table 6 reports the log likelihoods, the χ^2 statistics and the p-values for the null hypothesis that all but the top row of Q is zero. In no case is the null hypothesis rejected at the 5% level.

In this section we have found some evidence that the variance of the forecast error does change over time. In particular, we find that the square of the unanticipated monthly growth rate of dollar oil prices and of the monthly growth rate of U.S. M1 are significant explainers of the variance of the residuals. Thus, the constant Ω version of CAPM can be rejected. However, these models offer little consolation for the more general CAPM model, since the CAPM restrictions are in every case strongly rejected. The next section considers a more time series oriented model of the variance process - the ARCH model.

3. CAPM with ARCH

Often the economist does not know the true model of the variance. Without a full general equilibrium model of the economy, we do not know exactly which macroeconomic variables the variance should be related to. In such a case Engle's (1982) ARCH model is well-suited.

Engle's model does not require knowledge of the structure of the economy. Instead, it makes the reasonable postulation that, for example, if the absolute size of errors in t-1 are large, that the conditional variance at time t would be larger than average. This is the essence of the ARCH hypothesis.

In this section we apply the general idea of ARCH to our five equation CAPM system. We test two versions of ARCH. They take the general form:

$$\Omega_t = P'P + G\varepsilon_t\varepsilon_t'G, \tag{6}$$

where P is a constant upper triangular matrix, G is a constant symmetric matrix and ε_t represents the lagged forecast error (the error made in predicting the returns between t-1 and t). This formulation ensures that the estimated variance matrices are positive semi-definite. (This property is not necessarily satisfied in the ARCH formulation used by Bollerslev, Engle and Wooldridge.)

Equation (6) represents a particular form of a first-order ARCH. In our applications we will take G first to be diagonal (so that it has five independent non-zero elements) and then we will consider the general symmetric case for G (with fifteen independent elements). Even our "general" case is a quite parsimonious form of a first order ARCH. There are 15 elements in the time t conditional covariance matrix. In its most general formulation, each of those 15 elements could be related linearly to each of the 15 elements in the moment matrix for the lagged residual. Thus, for the general first-order ARCH, we could postulate a model with 225 parameters relating the current conditional variance to the lagged errors. Moreover, in generalized ARCH models (see Bollerslev (1986)), the variance matrix is essentially related to a distributed lag of error moment matrices. In a multiple equation model such as the one estimated here, the number of ARCH parameters increases quite quickly with the number of equations.

Two considerations motivate our model of equation (6). First, estimation of a five-equation ARCH system that imposes constraints between coefficients and elements of the variance matrix is very difficult. It is made much more manageable by choosing specifications with a low number of parameters. The less parsimoniously parameterized versions are not only more difficult to estimate, but given the limited data set from the floating rate period, they also leave too few degrees of freedom for meaningful estimation. The second advantage of our formulation is that it constrains the estimates of the variance to be positive definite (while the Bollerslev, Engle and Wooldridge set-up does not). This turns out to be of great practical importance in estimating a large multi-equation ARCH system.

In Table 7 we present the results of an ARCH estimation in which the matrix G is diagonal. Thus, there are five ARCH parameters to estimate. The constant variance model could be thought of as a constrained version of this time-varying variance formulation, in which the five ARCH parameters are forced to be zero. It is easy to perform a likelihood ratio test of this constraint. Doubling the difference between the log likelihood for the model reported in Table 7 and the one reported in Table 1 gives a χ^2 statistic with 5 degrees of freedom equal to 37.78. This is easily significant at the 1% level - so we can reject the constraint that the variance is constant over time.

The ARCH parameters range in size from about .23 to about .64, so the variance process appears stationary over time. (All the parameters are significantly different from one.) Four of the five parameters are significantly different from zero at the 1% level in a two-sided t test.

Allowing the variance to change over time greatly reduces the standard error on the estimated coefficient of relative risk aversion. It drops from 42.7 as reported in Table 1 to 15.0 in the ARCH model presented in Table 10. The point estimate of ρ is extremely high - around 13.6. In a one-sided t test it is not significantly different from zero at the 5% level.

This model with the CAPM constraint is tested against the alternative that the B matrix has the same form as Ω but is not constrained to be proportional to the covariance matrix. As the LM statistic in table 7 indicates, CAPM is still strongly rejected at the 1% level.

Figures 1a-1e are graphs of the estimated conditional variances from this model. Also drawn on the graph is an estimate of the unconditional variance. In most periods the conditional variances are much smaller than the unconditional one. Frankel (1986) argued that, given the size of his estimates of the unconditional variance, a change in the supply of outside assets is unlikely to generate much change in the risk premium, since the change in the risk premium is the product of the change in the asset supply and the variance of exchange rate forecasts (times ρ). Furthermore, he argues that the conditional variance will be smaller than the unconditional variance, so that even if the variance were allowed to vary over time the CAPM model still could not explain much of the exchange risk premium. Pagan (1986) has pointed out that, in fact, the conditional variance need not be smaller than the unconditional variance. The graphs in Figure 1 confirm that for some periods the conditional variances are much larger than the unconditional variance. However, on average they must be smaller, as Frankel (1987) indicates.

If the variance does vary over time, the variability of the variances themselves might explain the observed size of risk premia within the framework of CAPM. This point is brought out in the exchange between Giovannini and Jorion (1987a, 1987b) and Frankel (1987). The results from our ARCH estimation and CAPM tests, though, make this speculation moot. Our estimation does allow the variance to change, but CAPM is still strongly rejected.

Because the five parameter ARCH model of CAPM is such a large improvement on the CAPM model constrained to have a constant variance, we proceed to test a less sparsely parameterized ARCH model. Table 8 reports the results of the estimation in which the G matrix is allowed to be a general symmetric matrix. There are 15 independent elements in G.

The 5 parameter ARCH is a restricted version of the ARCH in this model in which all the off diagonal elements are forced to be zero. That imposes ten restrictions on this richer specification. The likelihood ratio test for these restrictions yields a χ^2 statistic with ten degrees of freedom of 43.34, which is easily significant at the 1% level. Thus, we seem to gain a lot by going to this more general specification.

There is one troubling aspect to this particular form of the model.The characteristic roots of the estimated G matrix are -.646, .198, .305, .421 and

1.144. This last root indicates the possibility of non-stationarity in the variance process.

If indeed the ARCH process is not stationary, Engle and Bollerslev (1986) suggest that the limiting distributions of the coefficients and test statistics as reported here may not be correct. Not much is known about the asymptotic properties of ARCH models with unit roots. We will proceed here by reporting our test statistics, but they should be interpreted with caution.

Figures 2a-2e show the estimates of the conditional variances for the five relative real rates of return in this model. As in the five-parameter ARCH model, there are a few periods in which the conditional variances are very large relative to the remainder of the time. We do not include an estimate of the unconditional variance here, because the non-stationarity of the variance in our estimates indicates that the unconditional variance does not exist.

This ARCH specification once again reduces the standard error of the estimate of ρ - from 15.0 in the 5 parameter ARCH to 9.4 here. The estimate of ρ is negative and significantly different from zero. This would be very troubling were it not for the fact that the restrictions of CAPM relative to a formulation of asset demands with a symmetric B are very strongly rejected. The rejection is much stronger than in any of our earlier specifications of the CAPM model. So, the model reported in Table 8 is not capturing the true behavior of investors in international financial markets.

It is not clear why CAPM fares so badly in this case. Perhaps the non-stationarity in the variance is distorting the test statistics, but more likely the CAPM restrictions are just very strong relative to a model that allows returns to depend in a relatively unrestricted way on asset shares and lagged covariances.

It is possible that we have not considered a general enough ARCH specification. Perhaps the variance is related to the moment matrix of forecast errors at lags greater than 1. We tested for this possibility following a suggestion of Bollersev (1986). We performed some time-series identification on the squared deviations from means of the relative rates of return. We found in all cases either no serial correlation or support for the MA1 specification. These results are consistent with our choice of ARCH models. In particular, there was no evidence of an AR1 component in any of the series, which would support Bollerslev's GARCH specification.

In sections 2 and 3 we have estimated a variety of CAPM models with time-varying variances. There seems to be abundant evidence that the explanatory power of the model is increased by letting the conditional covariances of the asset returns to change over time. Yet, these more sophisticated models still provide no support for the CAPM restrictions between coefficients and variances when tested against the more general unconstrained asset pricing models. We perform six LM tests of the capital asset pricing model in sections 2 and 3, and they all reject the restrictions. In the next section we take up the possibility of some specific cases of misspecification in equation (1).

4. Measurement Error

The model presented in equation (1) and estimated in sections 2 and 3 is a model in which it is assumed that the CAPM equation holds exactly and all variables are measured without error, so that the residual in the estimating equation can be identified with the forecast error. There is a large class of empirical macroeconomic models that make the same assumptions in order to exploit "orthogonality conditions". These assumptions are particularly convenient because they rule out any simultaneous equations problem. The right hand side variables in equation (1) are uncorrelated with the error term because under rational expectations all currently known variables are orthogonal to forecast errors.

Suppose that equation (1) did not exactly describe investors' behavior, but instead it was only correct on average. Preference shocks, for example, would be represented by a disturbance appended to the true model of equation (1). Then when we replace the expectation of z_{t+1} with its realization, the disturbance term in the estimating equation will be the sum of the forecast error and the preference shocks. In this case, the explanatory variables -- the asset shares -- would be correlated with the error, since we would expect that as risk preferences change the values of the outstanding stocks of the assets would also change. Thus, the estimation techniques undertaken here would be rendered invalid. It should be understood, then, as Frankel and Engel discuss, that the CAPM tests discussed previously in this paper are tests that the CAPM equation (1) holds exactly.

There is another problem that the addition of demand errors cause that is special to this problem and would not necessarily appear in some other test of "orthogonality conditions". It is also present if there is measurement error in either the dependent or explanatory variables. When some source of error other than the forecast error is present in the residual, it is no longer the case that under CAPM the coefficients on the asset shares should be proportional to the variance of the residual. They should only be proportional to the variance of the forecast error.

Although we offer no general solution to the problems of misspecification, we can generalize the tests in one direction when forecast errors have a time-varying variance. We can identify and estimate equation (1) even if there is measurement error in the rates of return. This identification is possible when the variance of the forecast errors is a function of observable data, and the variance of the measurement error is constant.

Our economic model is still given by equation (1), but the model to estimate no longer simply replaces $E_t z_{t+1}$ with $z_{t+1} - \varepsilon_{t+1}$. We now have:

$$(7) \qquad z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1} + u_{t+1},$$

where u is the measurement error of z. The variance of ε is given by Ω while the variance of u is equal to Σ. We assume that u is uncorrelated with ε and λ, and that Σ is constant.

The likelihood for the model in this case is given by:

$$(8) \qquad \ln L = -2.5\ln\pi - .5\ln|\Phi_t| - .5(z_t - c - B_t\lambda_t)'\,\Phi_t^{-1}(z_t - c - B_t\lambda_t),$$

where $\Phi_t = Var_t(\varepsilon_{t+1} + u_{t+1}) = \Omega_t + \Sigma$. When the CAPM constraint is imposed, the matrix of coefficients is proportional to the variance of the forecast errors: $B_t = \rho\Omega_t$.

It is useful to consider for a moment why Σ is identified in this model. Intuitively, identification comes because the part of the variance that also appears in the asset pricing equations (Ω_t) is time-varying, while the measurement error variance (Σ_t) is not. Suppose the forecast error variance were constant as in the Frankel empirical models. Then it would be impossible to distinguish between the forecast error variance and the measurement error variance. Suppose we had some estimate of this hypothetically constant Ω as well as estimates of ρ, Σ, and Φ that maximized the likelihood. We could obtain the same value of the likelihood with, for example, a $\Omega^{*} = 2\Omega$, a value of $\rho^{*} = .5\rho$, a value of $\Sigma^{*} = \Phi - \Omega^{*}$, and the same estimate of Φ. However, when Ω_t is time-varying, we can no longer perturb its value without changing the value of the likelihood, since the forecast error variance is the only part of the total error variance that changes with time.

We only consider models of the forecast error variance in which the variance depends on macroeconomic data, as in equation (4). While we are able to separate out the variance of the forecast errors from the variance of the measurement error, we cannot distinguish between the forecast error and the measurement error themselves. The ARCH model, however, requires that the forecast errors be identifiable, if we hypothesize that the variance of the forecast error depends on the lagged moment matrix of realized forecast errors. So, we must abandon ARCH as our model of the time-varying variance of forecast errors, and instead rely on equation (4).

The models of section 2 are restricted forms of equation (7). They force the matrix Σ to be zero. As in the previous sections, we choose a form for the variance that ensures it is positive semi-definite. Equation (4) defines our model for Ω_t. For the variance of the measurement error we impose

(9) $$\Sigma = Q'Q$$

where Q is an upper triangular matrix. The models of section 2 force Q to equal zero -- a total of 15 restrictions.

We calculate maximum likelihood estimates for the model using the two macroeconomic variables that seemed to do the best job in explaining the variances in sections 2 and 4 of this paper -- the square of the change in the log of money and the squared residual from the oil price ARIMA. Results of this estimation are reported in tables 9 and 10.

These measurement error models do not significantly improve the explanatory power of the models estimated in section 2 where the asset pricing equation was assumed to hold identically. Table 9, which contains the estimates for the model in which money helps explain forecast error variances, reports a log likelihood of 1652.55. The restricted form of this model, reported in table 2, has a log likelihood of 1647.63. While the log likelihood of the restricted model is of necessity lower than of the unrestricted, the chi-square test of the 15 restrictions yields a statistic of 9.84 which is far less than the 5% critical level of 25.0.

The same conclusions can be drawn from the model in which the square of the innovation of the oil price is a determinant of the conditional variance of the forecast errors, as indicated in Table 10. The log of the likelihood for this model is 1651.08, compared to the log likelihood for its corresponding restricted model reported in Table 5 of 1645.89. The chi-square statistic with 15 degrees of freedom is 9.38, which again is insignificant.

In general, the measurement error model does very poorly. The point estimates of ρ are quite unappealing. There seems to be a high degree of correlation between the elements of P and Q, as evidenced by the high standard errors on the individual components of the two matrices (while in the restricted models of Tables 2 and 5, the elements of P are estimated with a great deal of precision). While the evidence of sections 2 and 3 has provided little support for the CAPM model, it is clear that the measurement error model is not the solution to the misspecification.

6. Conclusion

The CAPM model is in many respects a very attractive model for pricing of international assets. It can be derived from a (restrictive) utility maximizing framework, and the asset demand equations that it produces are a special case of those studied in the portfolio balance literature. Unfortunately, the restrictions that CAPM places on those more general asset demand equations -- restrictions that imply that the returns on assets are functions not only of the supplies of assets, but also of the variances and covariances of the asset returns -- have universally been rejected in previous literature. Although this paper allows for substantial generalization of the CAPM model, its restrictions are still not accepted.

One might think of several reasons why previous papers have rejected CAPM -- but some of the most important problems with past models are taken care of here. One of the most obvious problems, as pointed out by Giovannini and Jorion (1987a) among others, is that in practice those who have implemented the Frankel test have assumed the forecast error variances and covariances to be constant over time. Given the abundant evidence of heteroskedasticity in these errors, this seemed like a natural point of misspecification in the empirical CAPM models. However, we test several versions of CAPM with conditionally heteroskedastic errors but still find little support for the model.

It is sometimes argued that a deficiency in the Frankel approach is that the only way it can provide a measure of the variance of expectational errors is by extrapolating from past expectational errors. It is argued that if there is a policy change or some other economic shock that increases uncertainty, that there is no way to pick this up reliably from the data. This argument carries much less weight when we allow the variance of the forecast error to be a function of economic variables whose changes are apt to be important generators of uncertainty -- such as money growth or oil prices. Our models of sections 2 and 3 allow for just such a possibility, yet again CAPM gets no support.

Another potential problem with the Frankel set-up is that it requires the CAPM equation to hold exactly. We have relaxed this constraint in section 4 by allowing for an error that is uncorrelated with the forecast error and the right-hand-side variables. However, this model is no improvement empirically.

This paper extends the frontiers of estimation of international CAPM to consider some of the most important possibilities that have been suggested for the empirical failure of that model, yet the model still does not get a shred of support.

Appendix 1

Lagrange Multiplier Test

In several places in this paper, we have presented maximum likelihood estimates of a model and wish to compare the model to a less restricted model. In every case the estimates reported could have been obtained by estimating the less restricted model subject to some restrictions. (These are zero restrictions exclusively in this paper.) We obtained our estimates by maximizing the restricted likelihood:

(A) $$\hat{\theta}_R \text{ solves } \max_{\theta} \mathcal{L}_R(\theta)$$

where $\hat{\theta}_R$ is the vector of restricted maximum likelihood estimates and $\mathcal{L}_R(\theta)$ is the log likelihood function for the restricted model. A mathematically equivalent way to obtain the reported estimates would be by obtaining:

(B) $$\hat{\theta}_R \text{ solves } \max_{\theta} \mathcal{L}(\theta) \text{ subject to } H_o\text{: } \mathbf{h}(\theta)=0$$

where $\mathcal{L}$ represents the unrestricted log likelihood function and $\mathbf{h}(\theta)$ is the vector valued function of constraints. For convenience, the solutions to problems (A) and (B) are represented by the same vector although the solution to (B) has higher dimension than the solution to (A).

One asymptotic test of the null hypothesis H_o: $\mathbf{h}(\theta)=0$ is the Lagrange multiplier test based on the statistic:

$$-\frac{\partial \mathcal{L}}{\partial \theta'} \left[\frac{\partial^2 \mathcal{L}}{\partial \theta\, \partial \theta'}\right]^{-1} \frac{\partial \mathcal{L}}{\partial \theta}$$

evaluated at the restricted maximum likelihood estimate, $\hat{\theta}_R$. See Amemiya (1985) page 142 for details. We used a variant of the asymptotically equivalent test statistic:

$$T^{-1} \frac{\partial \mathcal{L}}{\partial \theta'} \left[T^{-1}\, \mathbf{I}\right]^{-1} \frac{\partial \mathcal{L}}{\partial \theta} \quad \text{where } \mathbf{I} = -\,\mathcal{E}\left[\frac{\partial^2 \mathcal{L}}{\partial \theta\, \partial \theta'}\right]$$

discussed by Silvey (1975), pages 118-119. We replaced $T^{-1}I$ with the following asymptotically equivalent form suggested by Berndt, Hall, Hall, and Hausman (1974):

$$T^{-1} \sum_{t=1}^{T} \frac{\partial \ell_t}{\partial \theta} \frac{\partial \ell_t}{\partial \theta'}$$

where ℓ_t represents the log likelihood for observation t.

The Lagrange multiplier test statistic used in this paper:

$$\frac{\partial \mathfrak{L}}{\partial \theta'} \left[\sum_{t=1}^{T} \frac{\partial \ell_t}{\partial \theta} \frac{\partial \ell_t}{\partial \theta'} \right]^{-1} \frac{\partial \mathfrak{L}}{\partial \theta}$$

has an asymptotic χ^2 distribution with degrees of freedom equal to the number of unique restrictions imposed by H_o. See Silvey (1975), pp. 118-119 or Judge et al (1985), pp. 182-184 for detailed derivations of these distribution results.

Appendix 2

Data Appendix

Our analysis relies on rate of return and aggregate asset data for the six countries in the study: France, Germany, Italy, Japan, United Kingdom, and United States. The assets studied are publicly held outstanding government debt denominated in the six currencies: francs, yen, lire, marks, pounds, and dollars. The calculation method, designed to measure debt at the end of each month, is described in detail in Frankel (1982). We used an updated data set provided by Alberto Giovannini. Our other major source of data was the Data Resources Inc. DRIFACS data base. (We are indebted to Ken Froot and Susan Collins for porviding us access to this data.) We also obtained data from the Citibank Citibase tape and the International Monetary Fund's IFS tape.

Asset Shares

A complete description of how the values of the assets is calculated is included in Frankel (1982). Briefly, for each country the asset data starts with the value of outstanding debt reported by each government. To this figure is added the cumulative value of foreign exchange purchases by the central bank of that country (which has the effect of exchanging foreign denominated assets for domestic denominated assets in the hands of the public.) Subtracted from this total is the value of assets held in that currency by central banks (a figure which is obtainable through numbers available in the IMF Annual Report). Because no correction is made for debt held by the central bank of its own government, these figures for the values of the outstanding assets are the values of debt and monetary base held by the public for each country.

The analysis in the paper utilizes asset shares rather than the level of assets. The asset shares were computed from the asset levels, measured in dollars. The asset share data covered the period from June 1973 to December 1984.

Rates of Return

The nominal rate of return for dollar assets was taken as the average of the bid and ask Eurodollar rates (DRIFACS series USD01B and USD01A) on one-month securities, measured on the last day of the month that was not a holiday.

The nominal rate of return for the other five currencies was calculated assuming covered interest parity:

$$1 + i^{*}_{t+1} = (S_t/F_t)(1 + i_{t+1})$$

where i^{*} is the interest rate for those five currencies, i is the one-month Eurodollar rate, S is the dollars per unit of foreign currency exchange rate and F is the dollars per unit of foreign currency one-month forward exchange rate, measured on the last day of the month. The interest rate calculated on the last day of the month gives the return (known with certainty) for assets held for the forthcoming month (hence the t+1 subscript). The exchange rates are averages of bid and ask rates (DRIFACS series FRC00B, FRC00A, WGC00B, WGC00A, ITC00B, ITC00A, JAC00B, JAC00A, UKC00B, and UKC00A) as were the forward rates (DRIFACS series FRC01B, FRC01A, WGC01B, WGC01A, ITC01B, ITC01A, JAC01B, JAC01A, UKC01B, and UKC01A).

The nominal rates of return are converted to ex-post real rates by a common price index., P_t:

$$\frac{P_t}{P_{t+1}} \quad \frac{S_{ct+1}}{S_{ct}} \left[1 + \frac{i_{ct+1}}{100} \right]^{1/12}$$

This formula gives, for time t, the ex-post real return from holding the government assets of country c from the end of period t to the end of period t+1.

The common price index, P_t, is computed as a geometric mean of the consumer price indices, IFS item 64, for the six countries at time t after they are converted to dollars by multiplying by the end of period exchange rate.

The weights used in the geometric mean are the 1984 shares of total GNP in dollars. The individual country GNP are measured by IFS item 99a (except for France where GDP, IFS item 99b, was used because current GNP was not available.) These are converted into dollars by dividing by the end of period exchange rates, IFS item ae. In 1984 the shares were:

France	.0682847
Germany	.0851713
Italy	.0479105
Japan	.181802
United Kingdom	.0568336
United States	.559998

Economic Variables

In Section 2 we allowed the variance of the forecast errors to be a function of an oil price index and the United States money supply. These data were obtained from the Citibase data tape for the period January 1973 to December 1984. The money supply measure used was Citibase variable FM1, United States M1 measured in current dollars as a seasonally adjusted monthly average of daily figures. The oil price index used was the Citibase variable PW561, the producer price index for crude petroleum products (1967=100), not seasonally adjusted.

To measure the surprise component of the money supply and oil prices, we estimated ARIMA models for the natural logarithm of each series over the period January 1973 to December 1984. The autocorrelations of each series indicated that both the money supply and the oil price index were nonstationary in levels and stationary in first differences. Identification indicated that the money supply followed a moving average process with one and four lags. The estimated model for the differences in natural logs, DM_t, is shown below along with the Box-Pierce statistic calculated from the residuals (standard errors of the coefficients are shown in parentheses):

$$DM_t = \underset{(0.0003996)}{0.0055398} + \underset{(0.0843936)}{0.2759634}\, a_t - \underset{(0.0862326)}{0.2574734}\, a_{t-4}$$

$R^2 = 0.115483$ Q-Statistic (24 lags) = 22.24084
S.E. of $a_t = 0.004795$ S.D. of dependent var = 0.005063
144 Observations (January 1973 to December 1984)

The first difference of the logarithm of the oil price series, DP_t, was identified as an AR(1). The estimated model is presented below along with the Box-Pierce Q statistic calculated from the residuals (standard errors of the coefficients are shown in parentheses):

$$DP_t = \underset{(0.0043962)}{0.0119926} + \underset{(0.0766518)}{0.4095051}\, DP_{t-1} + a_t$$

$R^2 = 0.167357$ Q-Statistic (24 lags) = 21.89200
S.E. of $a_t = 0.031151$ S.D. of dependent var = 0.034019
144 Observations (January 1973 to December 1984)

Both ARIMA models appear to fit the series well.

Appendix 3
Estimation Techniques

This appendix contains a brief discussion of the techniques we used in order to obtain the maximum likelihood estimates reported in the tables. We used various hill-climbing methods available in the Gauss programming language. In general we found the Berndt, Hall, Hall and Hausman (1974) routine led to some problems. For the models in which the variance depends only on economic variables, we primarily used the method of Broyden, Fletcher, Goldfarb and Shanno. This algorithm seemed to work quickly and reliably. We actually used BHHH for the first and last iterations. The BHHH method provides a consistent estimator of the information matrix, while BFGS and the other methods available do not. Furthermore, according to the Gauss manual, BHHH seems to provide more reliable numerical estimates of the Hessian matrix.

For the ARCH models we found the Davidon, Fletcher and Powell algorithm to be more trustworthy, though very slow. Typically we would proceed with the DFP method until the likelihood was changing by small amounts. Then we would switch to the Broyden, Fletcher, Goldfarb and Shanno algorithm, which seemed to work more quickly. (Both the DFP and BFGS algorithms are described in Dennis and Schnabel (1983).) We used BHHH for the first and last iterations.

In all cases we used numerical estimates of the derivatives. The analytic derivatives of the likelihood function in the ARCH models with respect to the parameters are expressions involving all the lagged variables for any observation, so they are quite complicated. Hence, we have to rely on numeric derivatives.

We also found it useful to give the maximization routine "good" starting values. To estimate the models in which the variance depends only on economic variables we picked as starting values the 21 parameters from the constant variance model and then more or less arbitrarily picked starting values for the five remaining parameters. In the versions in which 15 parameters relate the economic data to the variance, we took as starting values the 26 parameters from the version in which 5 parameters relate the variance to economic variables, and then picked arbitrary starting values for the other 10 variables. Analogously, to estimate the 5 parameter ARCH, we took as starting values the estimates of the 21 parameters from the constant variance CAPM model, and took .4 to be the starting values for the ARCH parameters. In the 15 parameter ARCH specification, our starting values were the estimates from five parameter ARCH,with off-diagonal elements of G equal to .01. We proceeded in the same manner for the models of sections 4 and 5.

At each iteration, an initial value for the first lagged error in the ARCH models must be specified. This was always taken to be zero.

All estimation was performed on a Compaq 386 computer equipped with an Intel 80387 coprocessor. Each model estimated required from about 40 iterations to as many as 400 iterations to converge. Depending on the complexity of the problem and how good the initial guesses were, the model would take anywhere from 1 hour to 20 hours to converge. (We actually switched to the 80387 midway through our work. On the 80287 coprocessor these times are approximately doubled.)

References

Amemiya, Takeshi, 1985. Advanced Econometrics, Cambridge: Harvard University Press.

Berndt, Ernst; Bronwyn Hall; Robert Hall; and Jerry Hausman, 1974. "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement 4: 653-665.

Black, Stanley, 1973. "International Money Markets and Flexible Exchange Rates," Studies in International Finance, no. 25, Princeton University.

Bollerslev, Tim, 1986. "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics 31, 307-327.

Bollerslev, Tim; Robert Engle; and Jeffrey Wooldridge, 1985. "A Capital Asset Pricing Model with Time Varying Covariances," University of California, San Diego.

Branson, William, 1977. "Asset Markets and Relative Prices in Exchange Rate Determination," Sozialwissenschaftliche Annalen 1 (January): 69-89.

Branson, William; Hannu Halttunen; and Paul Masson, 1977. "Exchange Rates in the Short Run: The Dollar-Deutschemark Rate," European Economic Review 10 (October): 303-24.

Branson, William; Hannu Halttunen; and Paul Masson, 1979. "Exchange Rates in the Short Run: Some Further Results," European Economic Review 12 (October): 395-402.

Cumby, Robert, 1986. "Consumption Risk and International Asset Returns : Some Empirical Evidence," New York University.

Cumby, Robert and Maurice Obstfeld, 1981. "A Note on Exchange-Rate Expectations and Nominal Interest Differentials: A Test of the Fisher Hypothesis," Journal of Finance 36 (June): 697-704.

Cumby, Robert and Maurice Obstfeld, 1984. "International Interest Rate and Price Level Linkages under Flexible Exchange Rates: A Review of Recent Evidence," in J. Bilson and R. Marston, eds., Exchange Rate Theory and Practice, University of Chicago.

Dennis and Schnabel, 1983. Numerical Methods for Unconstrained Optimization and Nonlinear Equations, New York: Prentice-Hall.

Diebold, F. and M. Nerlove, 1986. "The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model," Federal Reserve Board.

Domowitz, Ian and Craig Hakkio, 1985. "Conditional Variance and the Risk Premium in the Foreign Exchange Market," Journal of International Economics 19 (August): 47-66.

Dooley, Michael and Peter Isard, 1979. "A Portfolio Balance Rational Expectations Model of the Dollar-Mark Rate," Journal of International Economics 12 (November): 257-76.

Dooley, Michael and Peter Isard, 1983. "The Portfolio Balance Model of Exchange Rates and Some Structural Estimates of the Risk Premium," IMF Staff Papers (December): 683-702.

Engel, Charles and Anthony Rodrigues, 1986. "A Test of International CAPM," National Bureau of Economics Working Paper #2054.

Engle, Robert, 1982. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," Econometrica 50: 987-1008.

Engle, Robert and Tim Bollerslev, 1986. "Modelling the Persistence of Conditional Variances," University of California, San Diego, forthcoming in Econometric Reviews.

Frankel, Jeffrey, 1980. "Tests of Rational Expectations in the Forward Exchange Market," Southern Economic Journal 46 (April): 1083-1101.

Frankel, Jeffrey, 1982a. "A Test of Perfect Substitutability in the Foreign Exchange Market," Southern Economic Journal 48 : 406-416.

Frankel, Jeffrey, 1982b. "In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization," Journal of International Money and Finance 1 (December): 255-74.

Frankel, Jeffrey, 1983. "Estimation of Portfolio Balance Functions that are Mean-Variance Optimizing: The Mark and the Dollar," European Economic Review 23: 315-27.

Frankel, Jeffrey, 1984. "Tests of Monetary and Portfolio Balance Models of Exchange Rate Determination," in J. Bilson and R. Marston, eds., Exchange Rate Theory and Practice, University of Chicago.

Frankel, Jeffrey, 1986. "The Implications of Mean-Variance Optimization for Four Questions in International Macroeconomics," Journal of International Money and Finance 5 (March): S53-S75.

Frankel, Jeffrey, 1987. "Recent Work on Time-Variation in the Conditional Variance and in the Exchange Risk Premium," University of California, Berkeley.

Frankel, Jeffrey and Charles Engel, 1984. "Do Asset-Demand Functions Optimize over the Mean and Variance of Real Returns? A Six-Currency Test," Journal of International Economics 17 (December): 309-323.

Geweke, John and Edgar Feige, 1979. "Some Joint Tests of the Efficiency of Markets for Forward Foreign Exchange," Review of Economics and Statistics 61: 334-41.

Giovannini, Alberto and Philippe Jorion, 1987a. "Interest Rates and Risk Premia in the Stock Market and in the Foreign Exchange Market," Journal of International Money and Finance 6 (March): 107-124.

Giovannini, Alberto and Philippe Jorion, 1987b. "Foreign-Exchange Risk Premia Volatility Once Again," Columbia University.

Girton, Lance and Dale Henderson, 1977. "Central Bank Operations in Foreign and Domestic Assets under Fixed and Flexible Exchange Rates," in P. Clark, D. Logue and R. Sweeney, eds., The Effects of Exchange Rate Adjustment, Washington: U.S. Treasury.

Hamilton, James, 1985. "Uncovering Financial Market Expectations of Inflation," Journal of Political Economy 93: 1224-1241.

Hansen, Lars and Robert Hodrick, 1980. "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy 88 (October): 829-853.

Hansen, Lars and Robert Hodrick, 1983. "Risk Averse Speculation in the Forward Exchange Market: An Econometric Analysis of Linear Models," in J. Frenkel, ed., Exchange Rates and International Macroeconomics, University of Chicago.

Hodrick, Robert and Sanjay Srivastava, 1984. "Risk and Return in Forward Foreign Exchange," Journal of International Money and Finance 3 (April): 5-29.

Hsieh, David, 1984. "Tests of Rational Expectations and No Risk Premium in Forward Exchange Markets," Journal of International Economics 17 (August): 173-184.

Hsieh, David, 1985. "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," University of Chicago.

Judge, George; W.E. Griffiths; R. Carter Hill; Helmut Lütkepohl; and Tsoung-Chao Lee, 1985. The Theory and Practice of Econometrics, Second Edition, New York: John Wiley and Sons.

Kaminsky, Graciela and Rodrigo Peruga. "Risk Premium and the Foreign Exchange Market," University of California, San Diego.

Kouri, Pentti, 1976. "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach," Scandinavian Journal of Economics, 78 (May): 280-308.

Lewis, Karen, 1986. "Testing the Portfolio Balance Model: A Multi-Lateral Approach ," New York University, forthcoming in Journal of International Economics.

Mark, Nelson, 1985. "On Time Varying Risk Premia in the Foreign Exchange Market," Journal of Monetary Economics 16 (July): 3-18.

Pagan, Adrian, 1986. "A Note on the Magnitude of Risk Premia," University of Rochester.

Silvey, S.D., 1975. Statistical Inference, New York: Chapman and Hall.

Table 1

CAPM Estimation, constant Ω

$$z_{t+1} = c + \rho(P'P)_t\lambda_t + \varepsilon_{t+1}$$

$$Var_t(\varepsilon_{t+1}) = P'P$$

*** Log Likelihood : 1639.789088 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
.0039629	.0028871	.0044543	.0052062	.0020641
(.0168931)	(.0172629)	(.0140137)	(.0173972)	(.0133013)

The estimate of the coefficient ρ:

-19.287650
(42.676856)

The estimate of the upper triangular matrix P:

.0322951	.0288285	.0223217	.0179637	.0161121
(.0019034)	(.0018396)	(.0024820)	(.0024482)	(.0025755)
0	.0165643	.0018479	.0034280	.0046026
	(.0010655)	(.0017589)	(.0030412)	(.0023601)
0	0	.0181945	.0009552	.0030412
		(.0006800)	(.0040832)	(.0025898)
0	0	0	.0270647	.0051627
			(.0015644)	(.0018011)
0	0	0	0	.0237012
				(.0014565)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 33.221925

marginal significance level = 0.002676 (14 d.f.)

Table 2

CAPM Estimation, Variance a Function of Square of Change in Log of U.S. M1

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$var_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$

x_t = square of change in log U.S. M1

*** Log Likelihood : 1647.625183 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.005956	0.004988	0.006123	0.006953	0.003338
(0.013681)	(0.013695)	(0.011231)	(0.014585)	(0.011425)

The estimate of the coefficient ρ:

-23.934543
(36.290540)

The estimate of the upper triangular matrix P:

0.030219	0.025191	0.020613	0.018436	0.019729
(0.002054)	(0.002419)	(0.002908)	(0.003298)	(0.003370)
0	0.015772	0.001692	0.005086	0.009071
	(0.001358)	(0.002119)	(0.003297)	(0.002637)
0	0	0.018193	0.001027	0.003226
		(0.000716)	(0.004228)	(0.002686)
0	0	0	0.026389	0.001813
			(0.001625)	(0.002341)
0	0	0	0	0.018658
				(0.002559)

The estimate of the vector h' :

1.590317	2.077879	1.201510	0.287016	-0.917194
(0.713113)	(0.645719)	(0.622900)	(0.607644)	(0.559770)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 56.225719

marginal significance level = 0.000015 (19 d.f.)

Table 3

CAPM Estimation, Variance a Function of Square of Change in Log of Oil Prices

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$

$$x_t = \text{square of change in log of oil prices}$$

*** Log Likelihood : 1644.831445 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.001267	0.000111	0.002179	0.002574	-0.000065
(0.015653)	(0.016243)	(0.012810)	(0.016369)	(0.011975)

The estimate of the coefficient ρ:

-12.116358
(39.781602)

The estimate of the upper triangular matrix P:

0.031925	0.029232	0.021589	0.018180	0.014923
(0.001909)	(0.001905)	(0.002446)	(0.002405)	(0.002522)
0	0.015851	0.002886	0.002749	0.006823
	(0.001026)	(0.002252)	(0.002975)	(0.002348)
0	0	0.017783	0.001374	0.001311
		(0.000614)	(0.004243)	(0.002801)
0	0	0	0.026995	0.005779
			(0.001667)	(0.002130)
0	0	0	0	0.022210
				(0.001541)

The estimate of the vector h' :

0.166874	-0.015146	0.222064	-0.001335	0.308111
(0.226000)	(0.294538)	(0.206733)	(0.264205)	(0.184717)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 41.783589

marginal significance level = 0.001895 (19 d.f.)

Table 4

CAPM Estimation, Variance a Function of Squared Residuals from U.S. M1 ARIMA

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$

x_t = squared residuals from U.S. M1 ARIMA

*** Log Likelihood : 1642.882930 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.001309	0.000170	0.002201	0.002633	-0.000140
(0.009329)	(0.009666)	(0.007950)	(0.010075)	(0.008103)

The estimate of the coefficient ρ:

-11.979929
(25.283582)

The estimate of the upper triangular matrix P:

0.029774	0.026267	0.019919	0.014512	0.017220
(0.002634)	(0.003280)	(0.002598)	(0.003762)	(0.002958)
0	0.016545	0.001767	0.003173	0.004906
	(0.001151)	(0.001874)	(0.003399)	(0.002458)
0	0	0.018123	0.005011	0.003619
		(0.000818)	(0.004116)	(0.002703)
0	0	0	0.026596	0.006513
			(0.001811)	(0.002158)
0	0	0	0	0.022410
				(0.001939)

The estimate of the vector h' :

2.809936	2.674950	2.294778	2.656054	0.137049
(1.502476)	(1.237809)	(1.128216)	(1.282874)	(1.276563)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 43.015021

marginal significance level = 0.001290 (19 d.f.)

Table 5

CAPM Estimation, Variance a Function of Squared Residuals from Oil Price ARIMA

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$

x_t = squared residuals from oil price ARIMA

*** Log Likelihood : 1645.886909 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.007924	0.006649	0.007915	0.008725	0.005302
(0.014236)	(0.014920)	(0.011401)	(0.015060)	(0.010897)

The estimate of the coefficient ρ:

-28.590147
(36.423752)

The estimate of the upper triangular matrix P:

0.031231	0.029080	0.020707	0.018182	0.014499
(0.001860)	(0.001971)	(0.002482)	(0.002438)	(0.002571)
0	0.015867	0.002965	0.002615	0.006072
	(0.001043)	(0.002033)	(0.003054)	(0.002477)
0	0	0.017710	0.001530	0.001757
		(0.000666)	(0.004738)	(0.002904)
0	0	0	0.026918	0.005706
			(0.001623)	(0.001911)
0	0	0	0	0.026918
				(0.001616)

The estimate of the vector h' :

0.288975	0.1019411	0.315394	0.060141	0.285922
(0.250094)	(0.238875)	(0.218500)	(0.368671)	(0.177539)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 53.858668

marginal significance level = 0.000035 (19 d.f.)

Table 6

CAPM Estimation, General Parameterization of Variance as a Function of Macroeconomic Data

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\text{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + Q'Qx_t$$

Square of Change in Log of U.S. M1

Log of likelihood = 1649.718806
Chi-square statistic (10 d.f.) = 4.187246
Marginal significance level =0.93850486

Square of Change in Log of Price of Oil

Log of likelihood = 1645.836216
Chi-square statistic (10 d.f.) = 2.009542
Marginal significance level =0.99626650

Square of Residual from U.S. M1 ARIMA

Log of likelihood = 1643.892504
Chi-square statistic (10 d.f.) = 2.019542
Marginal significance level =0.99618817

Square of Residual from Oil Price AR!MA

Log of likelihood = 1646.620739
Chi-square statistic (10 d.f.) = 1.467660
Marginal significance level =0.99903254

Table 7

CAPM Estimation, 5 Parameter ARCH

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t'\varepsilon_t'G$$

*** Log Likelihood : 1658.682319 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
-0.006027	-0.007428	-0.004122	-0.004009	-0.006882
(0.006596)	(0.006869)	(0.005514)	(0.006892)	(0.005812)

The estimate of the coefficient ρ:

13.270001
(15.036330)

The estimate of the upper triangular matrix P:

0.030631	0.027416	0.021628	0.015429	0.015644
(0.002278)	(0.002153)	(0.002339)	(0.003147)	(0.002867)
0	0.015277	0.002311	0.002357	0.004313
	(0.001201)	(0.001541)	(0.003246)	(0.002660)
0	0	0.015184	-0.000316	0.002262
		(0.000578)	(0.003692)	(0.003157)
0	0	0	0.022775	0.005693
			(0.001720)	(0.002368)
0	0	0	0	0.023374
				(0.001620)

The estimates of the diagonal elements of G:

0.425252	0.384488	0.478617	0.642844	0.227064
(0.103266)	(0.106483)	(0.082283)	(0.103571)	(0.116554)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 45.846083

marginal significance level = 0.000521 (19 d.f.)

Table 8

CAPM Estimation, 15 Parameter ARCH

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t'\varepsilon_t'G$$

*** Log Likelihood : 1680.349931 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.005302	0.004158	0.002732	0.005903	0.001906
(0.005211)	(0.004958)	(0.004468)	(0.004900)	(0.003912)

The estimate of the coefficient ρ:

-18.815029
(9.366542)

The estimate of the upper triangular matrix P:

0.030750	0.028908	0.023892	0.018148	0.016024
(0.002431)	(0.002272)	(0.002072)	(0.003315)	(0.003051)
0	0.015238	-0.001725	0.003238	0.004367
	(0.001211)	(0.001288)	(0.003848)	(0.002422)
0	0	0.007587	0.004764	0.007590
		(0.001389)	(0.005438)	(0.004048)
0	0	0	0.025977	0.003559
			(0.001905)	(0.002234)
0	0	0	0	0.020771
				(0.001922)

The estimate of the symmetric matrix G:

0.551903	0.018319	-0.334667	0.092724	-0.106092
(0.133893)	(0.081518)	(0.083272)	(0.073811)	(0.086655)
0.018319	0.276057	-0.180710	0.053321	0.033104
	(0.103955)	(0.084837)	(0.075722)	(0.080986)
-0.334667	-0.180710	0.689377	-0.322378	0.349194
		(0.122514)	(0.083084)	(0.095240)
0.092724	0.053321	-0.322378	0.251767	0.321065
			(0.154097)	(0.130203)
-0.106092	0.033104	0.349194	0.321065	-0.347991
				(0.155485)

(standard errors in parentheses)

Test of CAPM Restrictions

lagrange multiplier test statistic = 88.6136637
marginal significance level = 0.000000 (29 d.f.)

Table 9

Measurement Error, Variance of Forecast Error a Function of Square of Change in Log of U.S. M1

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1} + u_{t+1}$$

$$\mathrm{var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$

$$\mathrm{var}_t(u_{t+1}) = Q'Q$$

x = square of change in log of U.S. M1

*** Log Likelihood : 1652.550603 ***

The estimate of the vector c' :

FRA	GER	ITA	JAP	UK
0.035629	0.035035	0.040648	0.033120	-0.005391
(0.041356)	(0.034263)	(0.041738)	(0.033050)	(0.027549)

The estimate of the coefficient ρ:
-825.095579
(991.230505)

The estimate of the upper triangular matrix P:

0.016300	0.005168	0.009236	0.005001	-0.002644
(0.014090)	(0.012205)	(0.016772)	(0.005378)	(0.011199)
0	0.011229	0.004178	0.006745	0.000330
	(0.012680)	(0.018539)	(0.007675)	(0.012587)
0	0	0.014596	-0.003179	-0.004776
		(0.013975)	(0.009402)	(0.012804)
0	0	0	0.006733	0.005633
			(0.011472)	(0.014762)
0	0	0	0	0.000000
				(533.9825)

The estimate of the vector h' :

0.872287	1.333254	0.487075	-0.291049	-1.271120
(0.584745)	(0.521342)	(0.598958)	(0.366732)	(0.365828)

The estimate of the upper triangular matrix Q:

0.026724	0.028347	0.019949	0.018372	0.023666
(0.009158)	(0.009604)	(0.010363)	(0.006283)	(0.013799)
0	0.004564	-0.008531	-0.011976	-0.004068
	(0.073867)	(0.203270)	(0.261557)	(0.159312)
0	0	0.005144	-0.014921	0.010971
		(0.362406)	(1.950776)	(0.399700)
0	0	0	-0.014441	-0.004597
			(2.229423)	(2.374180)
0	0	0	0	-0.000005
				(3045.827)

(standard errors in parentheses)

Table 10

Measurement Error, Variance of Forecast Error a Function of Square Residuals from Oil Price ARIMA

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1} + u_{t+1}$$
$$var_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$
$$var_t(u_{t+1}) = Q'Q$$

x = squared residuals from oil price ARIMA

*** Log Likelihood : 1651.080910 ***

The estimate of the vector c':

FRA	GER	ITA	JAP	UK
0.040010	0.043604	0.044028	0.032435	-0.011706
(0.043150)	(0.035958)	(0.039476)	(0.031975)	(0.027287)

The estimate of the coefficient ρ:
-3615.193209
(7848.353986)

The estimate of the upper triangular matrix P:

0.012727	0.000795	0.003390	0.001432	-0.002299
(0.014722)	(0.003466)	(0.006885)	(0.001774)	(0.004887)
0	0.008435	0.003630	0.002548	-0.002014
	(0.009878)	(0.005947)	(0.003732)	(0.004835)
0	0	0.007502	-0.001283	-0.002294
		(0.009197)	(0.002988)	(0.005944)
0	0	0	0.004311	0.003621
			(0.005121)	(0.005422)
0	0	0	0	-0.000000
				(556.1796)

The estimate of the vector h':

0.077217	-0.116686	0.076154	-0.005022	-0.002702
(0.083384)	(0.146096)	(0.113008)	(0.044544)	(0.088752)

The estimate of the upper triangular matrix Q:

0.029282	0.031929	0.022481	0.018738	0.018558
(0.006678)	(0.006995)	(0.003786)	(0.004458)	(0.007404)
0	0.004618	-0.012402	-0.014263	0.001691
	(0.063832)	(0.223960)	(0.255834)	(0.087216)
0	0	0.008931	-0.020890	0.001934
		(0.323567)	(1.478327)	(0.106485)
0	0	0	-0.004544	-0.021624
			(7.615409)	(35.99457)
0	0	0	0	-0.001627
				(478.4125)

(standard errors in parentheses)

Figure IA: Variances-France

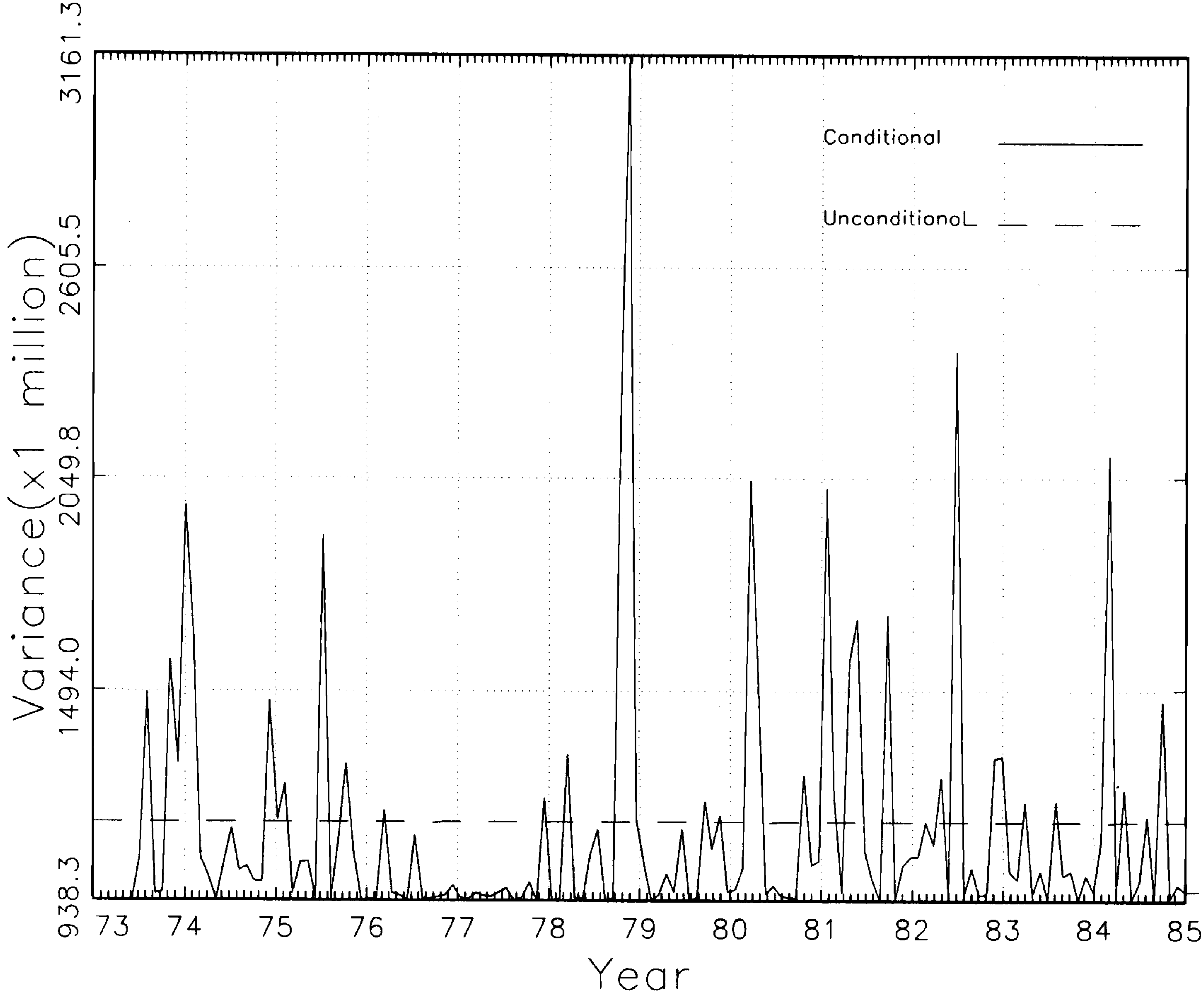


Figure IB: Variances-Germany

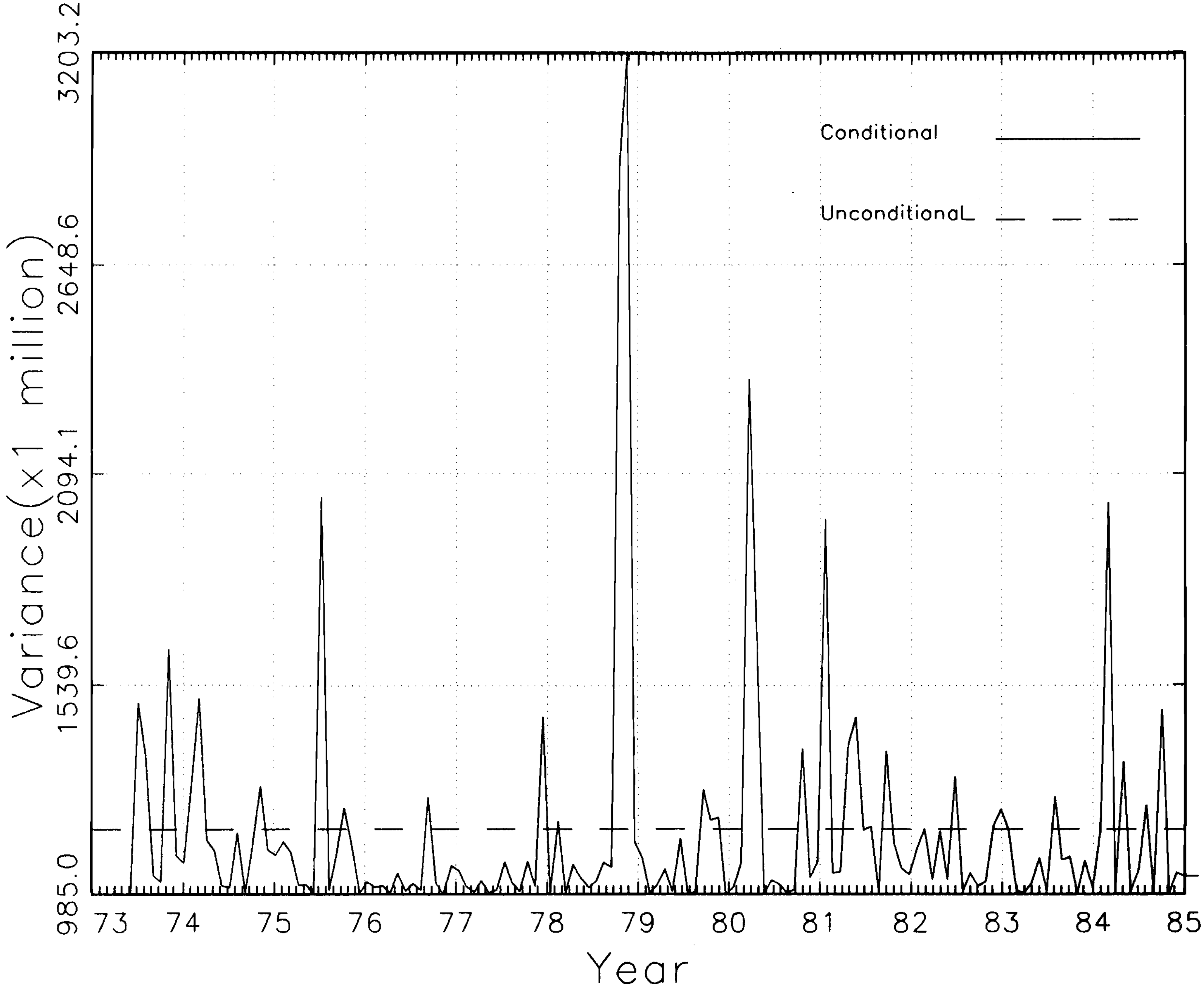
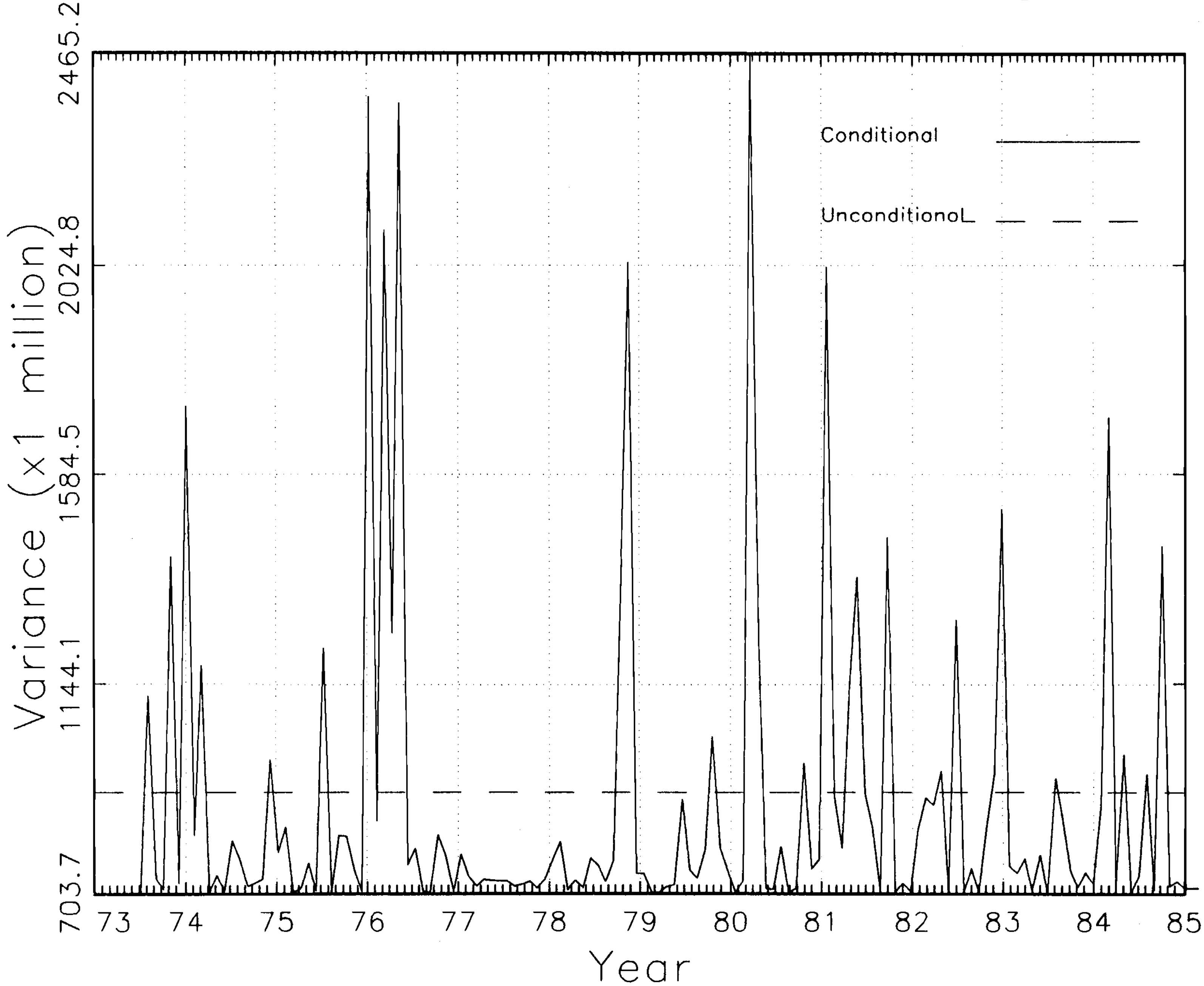


Figure IC: Variances-Italy



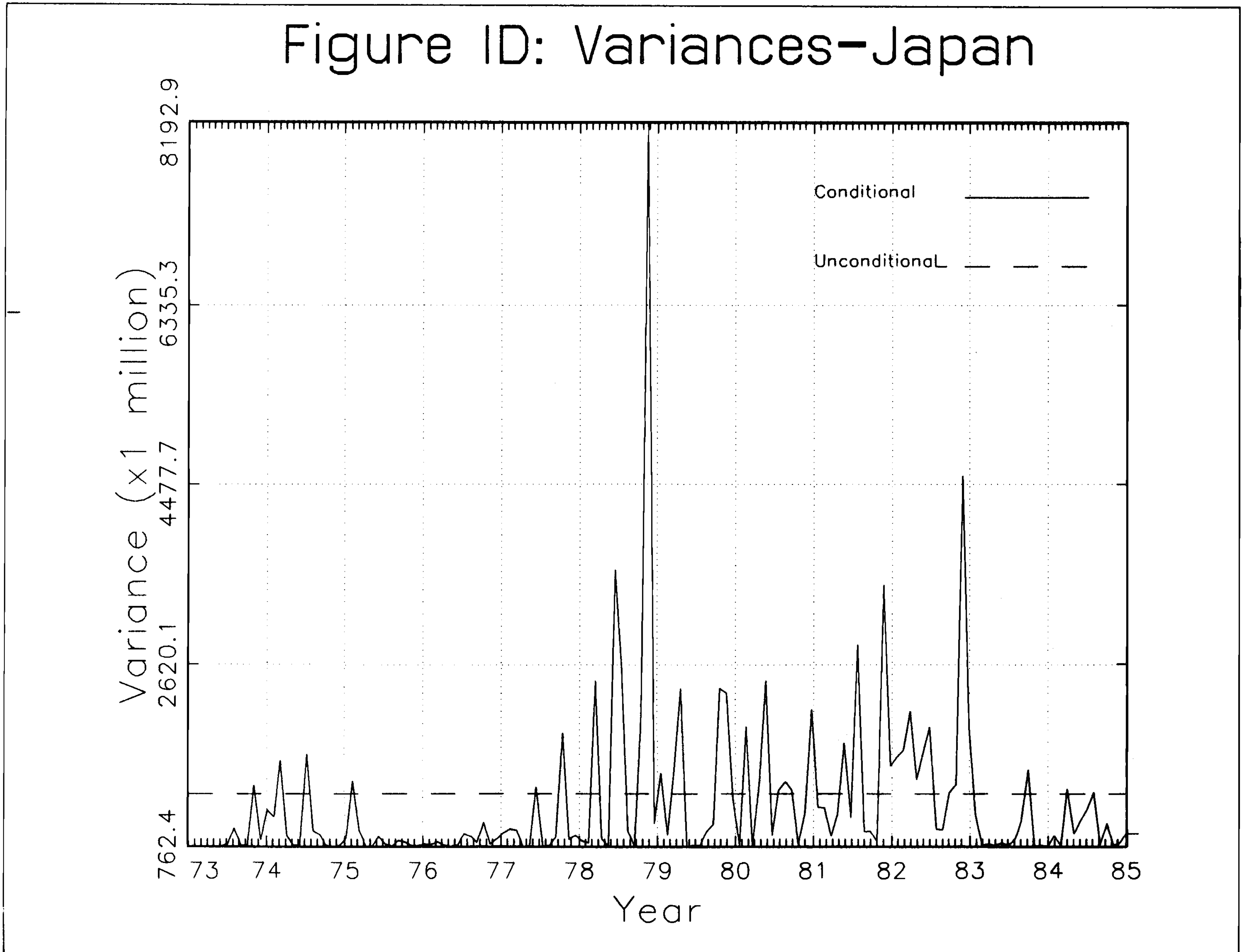


Figure ID: Variances-Japan
Variance (x1 million)
8192.9
6335.3
4477.7
2620.1
762.4
Conditional
UnconditionaL
73
74
75
76
77
78
79
80
81
82
83
84
85
Year

Figure IE: Variances - U.K.

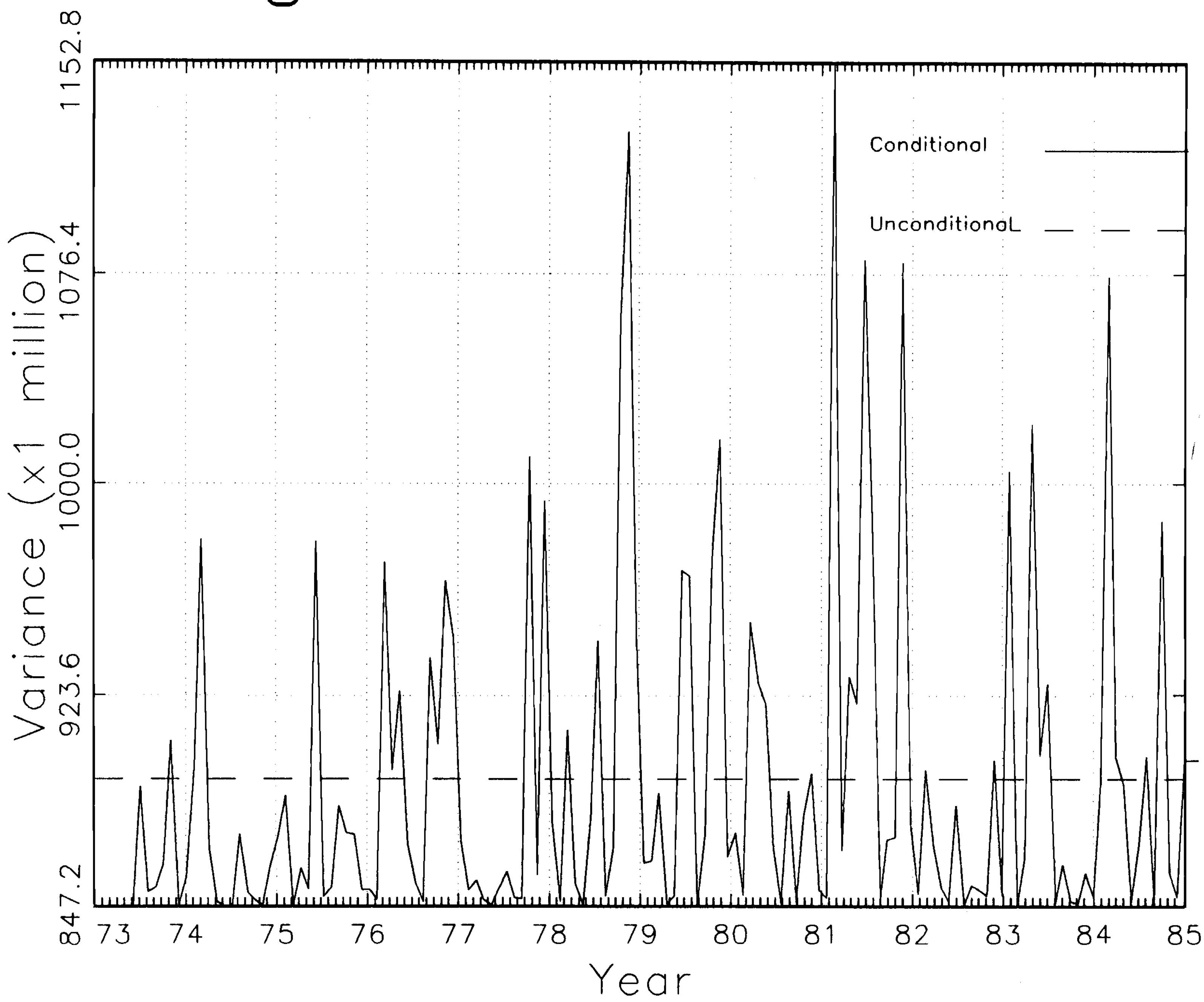


Figure IIA: Variances-France

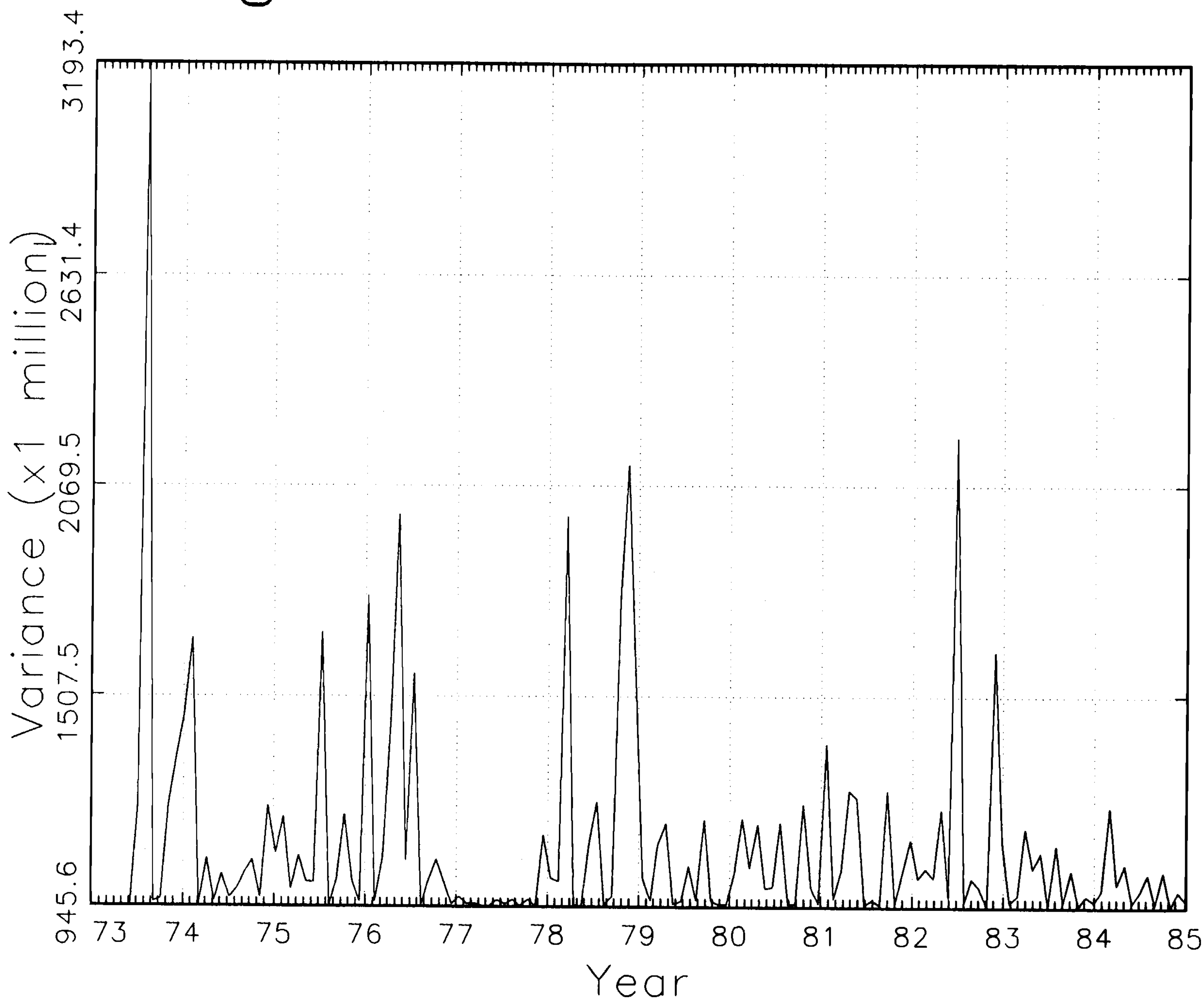


Figure IIB:Variances-Germany

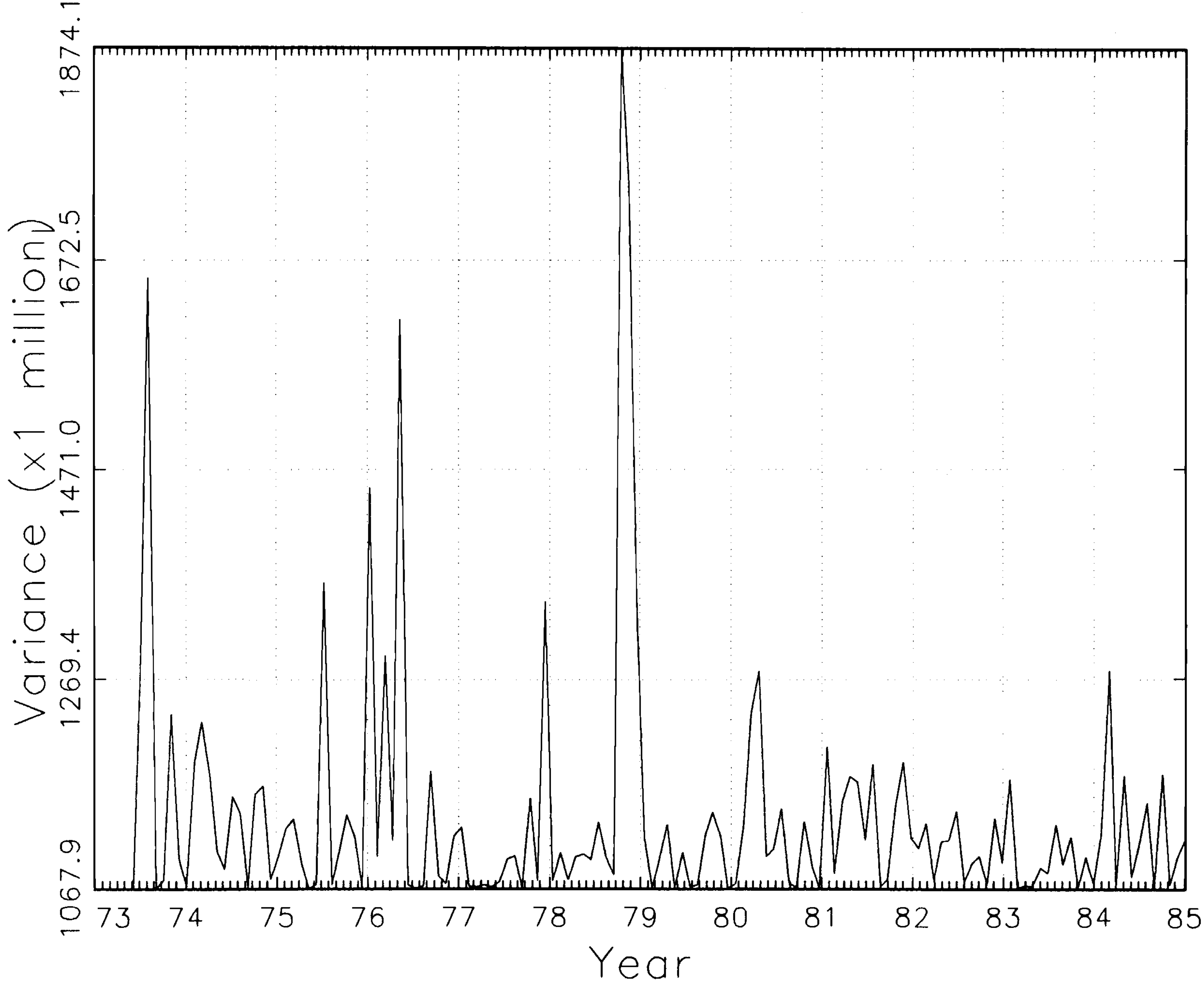


Figure IIC: Variances-Italy

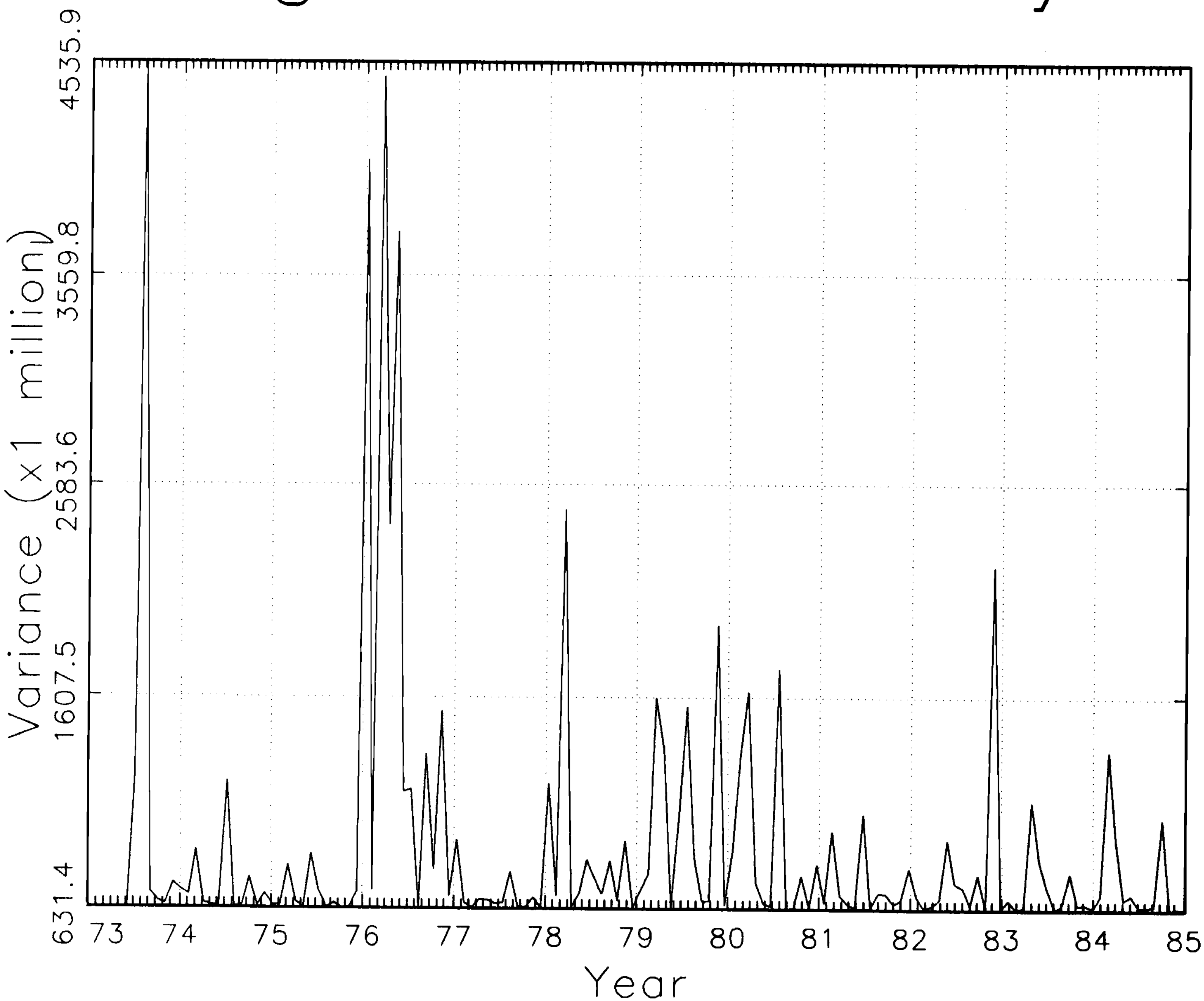


Figure IID: Variances-Japan

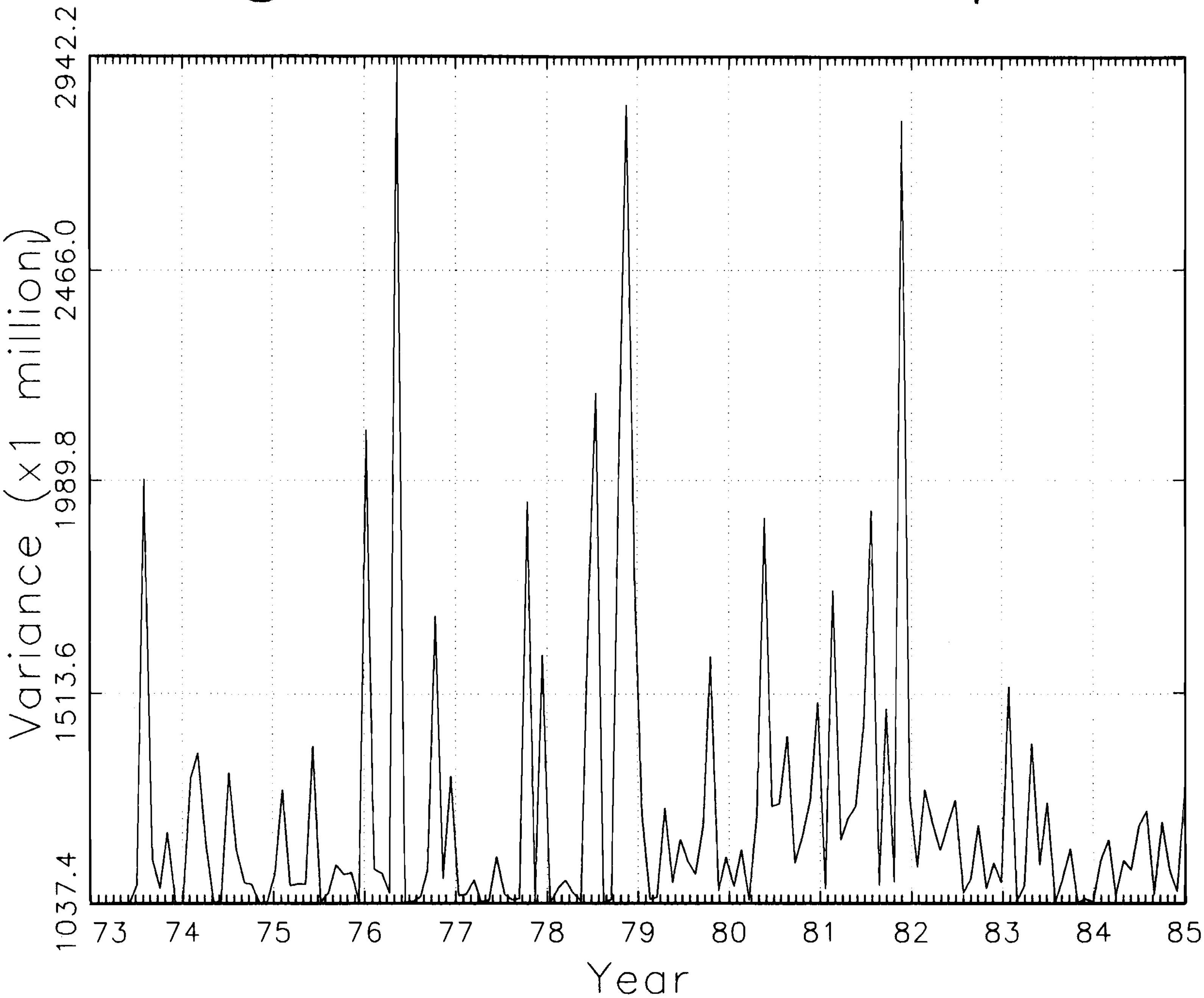


Figure IIE: Variances - U.K.