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ABSTRACT

Watching TV and other forms of media consumption represent, after sleeping and working, the main activity that adults perform in developed countries. We present a dynamic theory of commercial broadcasting where the media trade utility-raising goods (programs, information, and services) with audiences in exchange for their exposure to advertisements (utility-decreasing bads), and where goods are otherwise free to the audience except for their opportunity cost of time. Goods and bads are dynamically arranged, and as such traded in an intertemporal bundle. No monetary transfers take place between media and audiences, and this barter exchange is not contractually sustained. We study this dynamic problem in a model that captures the central characteristics of how commercial media markets operate. The model is rich enough to account for a variety of disparate evidence in television, radio, print media and the web.

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1 Introduction

Using five decades of time-use surveys that document trends in the allocation of time in the United States, Aguiar and Hurst (2007) find that during the last half a century there was a "dramatic increase in leisure time." From 1965 to 2003, leisure increased by between 4.6 hours per week and 8.1 per week for the average non-retired adult "or, if one assumes a forty-hour work week, the increase in leisure is equivalent to 5.9 to 10.5 additional weeks of vacation per year." Not only in the US, but also in many other countries the time spent by adults on leisure activities has steadily increased during the last few decades, while at the same time the number of market hours worked has remained relatively stable.

A remarkable aspect of the data is that among all the potential leisure activities, there is one that systematically accounts for about *half* of all the leisure time: watching television. Several studies document that, during the past fifty years, in many developed countries watching TV represents a stable share of about 40 to 50 percent of all leisure time. Interestingly, Aguiar and Hurst (2007) find that in the US *more than 100 percent* of the increase in leisure from 1965 to 2003 can be accounted for by the increase in the time spent watching television. This huge increase in television viewing is mainly offset by declines in socializing (going to parties, bars, etc) and in reading (books, magazines, letters, etc). In absolute numbers, the average number of hours spent watching television per year for individuals aged 18 and over was estimated at 1,745 hours at the end of their period of study, 2003, and during the last decade is estimated to have remained roughly the same.[1] This amount is equivalent to approximately 35 hours per week. In other words, non-retired adults spend today just about as much time watching television as they do in the labor force.

Thus, television viewing is not only the main leisure activity in modern economies. After sleeping and working, it is also *the* main activity for adults in the US and other developed countries. This paper is concerned with this and similar activities, in particular with the distinct aspects that characterize how they are brought to the

[1] See US Census Bureau, *Statistical Abstract of the United States*, years 2004 to 2014, and Harold L. Vogel's *Entertainment Industry Economics*, 1st to 5th editions.

market. A focus of the analysis is commercial broadcasting which represents the main form of transmission of television programs for individual and general use.

Broadcasters sell audiences. This is the most important activity of the business, and virtually all other actions support this function. The people who buy these audiences are advertisers interested in capturing their attention in order to influence their behavior in certain ways. In the last few decades commercial mass media has greatly spread around the world and, accordingly, these audiences have become a very valuable commodity that is sold to advertisers for billions of dollars. Today media markets are a multibillion dollar industry that is mostly dependent on advertising revenues, often almost exclusively on them.[2]

The purpose of this paper is to formalize the unique characteristics that make commercial broadcasting fundamentally different from other markets. While there are important literatures on advertising, media, and leisure, the specific dynamic aspects we study have remained, to the best of our knowledge, mostly out of the scope of consumer theory analysis. Further, although we will focus on how commercial television bundles information and entertainment with advertising messages, the main characteristics that we study are also present, and similarly drive, traditional business models in radio, newspapers, magazines, and commercial websites, including search engines. As a result, much of our formal analysis for television will also have implications for these other media markets. The characteristics are the following:

First, we want to capture the fact that a large proportion of advertiser-supported programs and information is a "free" good to the audience, except for the opportunity cost of audience time and the potential increased costs of advertised products. Indeed, "free" television, radio, and internet do not charge individuals for advertisements or for the information, entertainment or services that they provide. The media usually pays for the cost of preparing and using the information, services, and ads.

Second, advertisements are typically provided *jointly* with information and programming in a "bundle." Thus, a natural question is that since television, radio, and

[2] Advertising revenues are estimated to represent about 92.5 percent of all revenues in the typical TV station (excluding pay-TV) and about 91.5 percent in the radio industry (see Vogel, 2011).

other media could technically provide advertisements without costly information and programming, why do they go to the expense of including them along with the ads? Why do they bundle costly information and services jointly with ads when they could be provided separately? Technological constraints and transaction costs often seem to make it too expensive to sell ads separately. Prior to pay television, ads on television or radio could simply not be sold directly since there was no way that the audience could be charged for what they watched or heard.[3] More importantly, if consumers were paid to take ads, it might not be profitable to allow them to take all they want at a fixed (negative) price per unit. The special monitoring problem in media markets in the consumption of ads means that consumers would surely want to "buy" a large number of them and ignore as many as possible. As a result of these difficulties, few advertisements are sold separately and directly to consumers in broadcasting markets. They may be given away, as in direct mail and in billboards, but they are typically sold jointly with television and radio programs in a bundle. The result is that, despite money's apparent advantage in trades, no direct monetary transfers between media and consumers take place. This is an important characteristic that we capture in our dynamic model of advertising-based broadcasting.

Third, we note that the bundle of "goods" and "bads" is simply a form of barter exchange. Since consumers do not have to be compensated for utility-raising information and services, the inference must be that most ads lower the utility of the marginal consumer, after netting out the value of the time spent performing the consumption. Clearly, the assumption that many ads lower utility is easier to reconcile with consumer behavior than is the assumption that they raise utility. For consumers do not seem to look forward to consuming advertisements, as indicated for instance by observed behaviors such as zapping, using the fast forward to speed through commercials in a taped program, or using pop-up blocking software to block intrusive advertising while surfing the web. As it were, in this barter exchange "free" information and services *generate* the audience for utility-reducing ads and *compensate* the individual in the audience for being exposed to the ads. This is the third characteristic

[3] Ads in print can definitely be sold separately but transaction costs are greatly reduced by selling them in newspapers and magazines along with the news and articles.

that we capture in the theoretical framework we develop.

Finally, the exchange of goods and bads is not sustained through any contractual agreement between broadcasters and audiences. As it were, in this barter exchange the utility gains from the consumption of goods must compensate individuals for the utility losses associated with the consumption of bads and the opportunity cost of time, and do so with no explicit agreement between the parties. The same monitoring problem that explains the lack of monetary exchange also explains the lack of contractual arrangement. In other words, in broadcast markets there is a tension between the interest of advertiser-supported media and the interest of individuals in receiving utility-raising goods that is not contracted away. This tension is a central component of our framework.

These are the distinct characteristics that make this market fundamentally different from other markets. The exclusive focus on these characteristics in a dynamic setting is what differentiates our analysis from previous work in the literature on advertising and media markets. Despite an extensive and important body of existing work, we are not aware of previous research that studies the dynamic bundling of goods and bads in a way that goods generate the audience for utility-reducing ads and compensate for the advertising, with no contractual agreements sustaining this exchange. In our framework, we use standard consumer theory and maintain the principle of consumer sovereignty, although consumers will not be able to buy all ads they want at a fixed price. As just argued, when consumers are paid to take ads, difficulties in monitoring the consumption of ads means that it might typically not be profitable to allow them to take all they want at a fixed (negative) price per unit of ads. We avoid this negative implication in our model since advertisers determine jointly and endogenously with consumers the dynamic bundle of advertising and utility-raising goods.

The result of the analysis is a dynamic framework that is rich enough to account for a variety of disparate evidence in the different media markets concerning the amount and allocation of information, programming, and advertising. As such the theory provides a unified way of thinking about the quality, quantity and positioning of ad and non-ad content in television and radio programs, as well as in print media and websites. It also provides new insights into the reasons why and how available content is organized and positioned differently across different programs, and generates a number of empirical predictions for possible extensions in future work.

2 Related Literature

Our analysis sets apart from recent literature on media markets in that it focuses on the dynamic media consumption decision. As such it abstracts from issues that have been already studied in important recent efforts (see Anderson, Waldfogel and Strömberg (2016)). Bagwell (2007) also presents a thorough review of the extensive literature on the economic analysis of advertising that virtually covers every topic and area of study. It includes the different views of advertising (positive and normative theories), the welfare economics of commercial broadcasting (Anderson and Coate, 2005), and several empirical regularities on the effects on price, quality, entry deterrence, market structure, and other aspects.[4] Caves (2005) presents an economic analysis of television broadcasting and the organizational changes that it has undergone in the past decades.

Barnett (1966) and Comanor and Wilson (1974) offer some of the first discussions of advertisements on radio and television as "bads," where a "bad" is something consumers are willing to pay for to have removed or must be compensated to accept.[5] As Becker and Murphy (1993) and others note, many early analyses of advertising in the literature often assumed without much justification that advertisements are given away at zero price, rather than sold to consumers, and that quantities are controlled by producers of the goods advertised. This treatment, which is intrinsically tied to what is called the "persuasive approach" that considers advertising as shifting tastes, cannot explain why consumers choose among different advertising choices that require

[4]Other aspects include the effect of advertising on sales and profits, the reaction of audiences to advertising, the influence of advertising on the media's programming content, and the number, and quality of programs offered under different market structures. See also Baker (1994), Nilssen and Sørgard (1998), Owen and Wildman (1992), Spence and Owen (1977), and other references therein.

[5]Depken and Wilson (2003) studies of the price of ads in magazines with and witout ads.

time, money, or other scarce resources, and thus among different bundles of goods and bads. A model with a zero price of advertising may apply to direct mail advertisements and billboards, although consumers can easily discard and ignore these ads. But it does not appear to explain ads in television, radio, and the internet. Consistent with this view, the demand for ad avoidance has seen the development of new technologies that allow consumers to siphon off content and strip out the advertising. In television, this is exemplified by the digital video recorder (TiVo is the most famous), which allows consumers to easily skip ads. Likewise, plug-ins for web browsers block advertising to provide clutter-free content. A recent literature studies the content provider's reactions to such technologies (see, e.g., Wilbur (2008a,b), Anderson and Gans (2011), Shah (2011), and other references therein). This is empirically important as the recent proliferation of ad-blocking software has stirred a debate about the implications for media companies that survive on ad revenue. How will markets react to ad blocking? Will ad-blocking over time decimate the "free" web?[6] Our model will bear on this literature and these questions.

Studying the dynamic bundle of goods and bads has implications not only for the quantity but also for the timing (in TV and radio) and positioning (in printed media and the internet) of ads. The timing and intensity of commercials has attracted recent attention in the literature, in particular the competition among platforms (e.g., see Peitz and Valletti (2008), Ambrus, Calvano and Reisinger (2014)) and the game theoretic aspects among multiple stations. Various studies on the radio industry, for example, focus on understanding under what conditions multiple competing stations may prefer coordination (choose the same times) or differentiation (different times). They have also been concerned with the empirical estimation of models with multiple equilibria.[7] For instance, Sweeting (2009, 2013) estimates a timing game with mul-

[6] In an American Economic Association research highlight, Hyde (2015) notes "When Apple's new mobile operating system was released on September 16, 2015, millions of iPhone users rushed to upgrade. But at least two groups weren't so thrilled about the new release: mobile advertisers and the publications who survive off them. That's because the new operating system included tools that made it easy to detect and filter out ads on mobile websites. Sure enough, ad-blocking apps rocketed to the top of the App Store in the hours after the new release."

[7] Epstein (1998), Zhou (2000) and Kadlec (2001), for instance, provide theoretical models where two television stations choose to have their commercial breaks at the same time in equilibrium.

tiple equilibria, and Sweeting (2006) empirically examines the timing of commercial breaks by music radio stations using panel data on more than one thousand stations.[8] Our framework contributes to this literature by formalizing the more fundamental situation where there is just a single station.

A more distant literature is concerned with how the bundling decisions of a monopolist providing two goods depend on the correlation between the consumer's valuation for the products, and their degrees of complementarity and substitutability.[9] We add to this literature the study not of two goods, but one good and one bad, in a dynamic bundle where the exchange is bartered (does not use monetary transfers) and it is not sustained through a contractual arrangement.

Finally, there is a literature on barter arrangements which treats barter as an inefficient form of exchange that is ultimately dominated by a commonly accepted form of fiat currency. We note that media markets represent perhaps the main form of barter exchange in modern economies, and that the special monitoring problem in broadcasting markets may make the barter exchange between media and audiences a more efficient form of trade than currency.

3 An Intertemporal Model of Bundling Goods and Bads

The model focuses on the microfoundations that determine the exchange between goods and bads between media and audiences in a dynamic sense. As such it abstracts from a large number of aspects and considerations already studied in the literature.

Consider one broadcasting firm (a television station) and one individual in the audience. The firm's problem is to offer a bundle of goods and bads that this individual will find at least as good as his opportunity cost of time. The individual derives utility from two goods at any point in time t, during a finite time interval $[0, T]$. The first consumption good $C(t)$ is the flow of information, entertainment and other

[8] Also, Sweeting (2010) is concerned with the effect of mergers between close competitors on product positioning.

[9] See Ibragimov and Walden (2010) for a review of this literature.

utility-raising goods that are provided by the firm. During the time the individual is watching a television program he may be exposed to some advertisements $A(t)$. The maximum amount of goods and advertisements per unit of time that can be provided is assumed to be 1, that is $C(t)+A(t)=1$, with $C(t)\geq 0$ and $A(t)\geq 0$ at all times.[10] When only advertisements are provided during time t the individual obtains $C(t)=0$ when exposed to the ads. In consequence, the bundle of goods and bads at time t can be simply denoted by $C(t)\in[0,1]$. Although other formulations can be readily considered, this one provides a convenient and tractable way of characterizing the bundle of goods and bads.

The second good from which the individual derives utility at time t is the stock of past consumption accumulated up to that point in time, $S\left(t\right)$. Consumption of information, services and entertainment builds into capital following the accumulation process:

$$\dot{S}\left(t\right)=C\left(t\right)-\delta S(t),$$

where $\delta\in(0,1)$ is an exogenous depreciation rate. Being exposed to advertisements, therefore, lowers individual's utility since $\dot{S}\left(t\right)<0$. The initial condition is given by $S(0)=S_0$. This initial stock of past consumption S_0 may be interpreted as "habitual capital," in the sense that audiences often seek out the same information sources over time as they develop a set of favorite TV programs, radio programs, and websites. Clearly, after the television and radio program has begun, $t\in(0,T]$, the stock $S(t)$ also includes the amount of previous consumption *within* the program. For example, when watching a movie, it includes as the "stock of movie" how much movie has been watched up to that time t.

Lastly, we assume that the individual derives utility not only from the flow of consumption of goods and from the stock of past consumption, but also from their interaction. The instantaneous utility function of the individual is then given by:

$$u\left[C\left(t\right),S\left(t\right)\right]=\alpha_C\cdot C\left(t\right)+\alpha_S\cdot S\left(t\right)+\alpha_{CS}\cdot C\left(t\right)\cdot S\left(t\right).$$

[10] Advertisements could also be readily assumed to directly reduce utility by a given amount per unit of time.

The parameters α_C and α_S are assumed positive. The interpretation of a positive α_C is straightforward, while a positive α_{CS} means that the marginal utility of consumption increases with the stock of past consumption, as in Becker and Murphy (1988) model of habits. Clearly, this interaction effect may well be quite different across different types of programs, audiences and, more generally, different media. In movies, dramas, and similar TV programs where there is, broadly speaking, a single theme or story running throughout the program, we may expect this interaction effect between the stock and flow of program to be important. In newspapers or in TV and radio news programs which contain different types of news (local, national, international, sports, weather, etc) that are largely independent of each other, we may expect α_{CS} to be much less relevant.

The preferences of the individual are denoted by:

$$U\left[C,S\right]=\int_0^T e^{-\rho t}u\left[C\left(t\right),S\left(t\right)\right]dt,$$

where the rate of time preference is ρ. The individual is assumed to have an outside opportunity worth $v(t)$ utils at time t. As a result, he may watch a TV program, listen to a radio program, read a newspaper, or consume the information and services provided by a specific website from $t=0$ to $t=T$ if and only if:

$$\int_\tau^T e^{-\rho t}u\left[C\left(t\right),S\left(t\right)\right]dt\geq\int_\tau^T e^{-\rho t}v\left(t\right)dt,\qquad\text{for all }\tau\in\left[0,T\right].$$

In other words, the individual knows the entire path and must be compensated for the value of his outside opportunity, in present value terms, at all times. If the bundle of goods and bads does not meet this condition at time τ, then the individual will leave the TV program, stop listening to the radio program or leave the website. Hence media companies must provide enough goods (quantity) and in such a way (timing, positioning) so as to meet the conditions that would keep the audience above his opportunity cost of time at all times in present value terms. Note that this formulation abstracts from modelling explicitly the individual's abilities or technologies at his disposal to try to avoid or ignore advertisements.[11] It simply

[11] Modelling these abilities (e.g., leaving to enjoy the outside option when ads are on and coming

requires the bundles $\{C(t)\}_{t=0}^{T}$ to be such that they provide the individual in the audience watching the ads with at least the utils that he could obtain in an alternative activity at all times.

The media firm has to decide the optimal combination of goods and bads that maximizes profits. We assume that income is only generated from advertising, and so it does not charge the individual for the content it consumes.[12] The firm is competitive in the price of the advertisements, and the revenue per unit of time per unit of advertisement at time t is $\eta(t)$.[13] Hence, his profit function can be denoted as:

$$\Pi[C] = \int_0^T e^{-\rho t} \eta(t) [1 - C(t)] \, dt.$$

The problem is how to choose the sequence of bundles of goods and bads over time $\{C(t)\}_{t=0}^{T}$ that maximize profits. The media's problem can be written as:

$$\max_{\{C(t)\}_{t=0}^{T}} \left\{ \int_0^T e^{-\rho t} \eta(t) [1 - C(t)] \, dt \right\},$$

subject to:

$$\int_\tau^T e^{-\rho t} u[C(t), S(t)] \, dt \geq \int_\tau^T e^{-\rho t} v(t) \, dt, \qquad \forall \tau \in [0, T],$$

with

$$\begin{aligned} \dot{S}(t) &= C(t) - \delta S(t) \\ S(0) &= S_0, \\ 0 &\leq C(t) \leq 1. \end{aligned}$$

back when they are gone) and modelling access to ad-avoidance technologies involves making specific assumptions about $v(t)$. Audiences do not need to be compensated for the ads they do not see, and so the outside option term $v(t)$ is interpreted as including the utility value of the individual's skills and technologies to avoid advertisements. See, e.g., Wilbur (2008a,b), and Anderson and Gans (2011).

[12] There is an important literature on two-sided markets. See, e.g., Armstrong (2006), Rochet and Tirole (2006), Anderson and Gabszewicz (2006), Rysman (2009), and other references therein.

[13] In television, advertising time is generally priced competitively and largely independently of its location during a program. Advertising rates basically depend on audience composition and ratings. Goettler (2012) reports how typically 70 to 80 percent of commercial time is sold in the *up-front* market during May for the upcoming in early September. The remainder is sold in the scatter market during the season, ocassionally even hours before the show airs. Contracts specify the prices to be paid for the commercial time and minimum guaranteed ratings, as measured in the US by Nielsen Media Research. Guaranteed ratings often correspond to particular demographic segments. When the guaranteed raitings are not attained, networks provide "make-goods" to compensate their short-changed advertisers.

For simplicity, we assume that $\eta(t) = 1$,[14] and that $v(t) = v$.[15] It is convenient to define the function:

$$w(\tau) = \int_{\tau}^{T} e^{\rho(\tau - t)} \left[\alpha_C \cdot C(t) + \alpha_S \cdot S(t) + \alpha_{CS} \cdot C(t) \cdot S(t) - v\right] dt, \qquad \forall \tau \in [0, T].$$

This function represents the discounted sum of utils net of opportunity costs that the individual will receive from τ until T. Thus, the condition that the individual may continue consuming the bundle of goods and bads at τ is that $w(\tau) \geq 0$ for all $\tau \in [0, T]$. This is the standard incentive compatibility constraint. The media's objective function attains its maximum at the point where:

$$\rho = \frac{w''(\tau)}{w'(\tau)},$$

and the participation constraint requires $\rho \cdot w(0) = v$.

The problem can thus be written as:

$$\max_{\{C(t)\}_{t=0}^{T}} \int_0^T e^{-\rho t}(-C(t))\, dt,$$

subject to:

$$\dot{S}(t) = C(t) - \delta S(t),$$

$$\dot{w}(t) = \rho w(t) + v - \alpha_C C(t) - \alpha_S S(t) - \alpha_{CS} C(t) S(t),$$

$$0 \leq C(t) \leq 1,$$

with $S(0) = S_0$, $w(\tau) \geq 0$ for all $\tau \in [0, T]$, and where $S(T)$ and $w(0)$ are free conditions. Obviously, $w(T) \equiv 0$ indicates that the individual receives no net gain or loss at T.

[14] Advertising pricing surveys in the television and radio industry show that commercial spots are sold according to expected thousand viewers/listeners of a program, with little regard for whether audiences are different at different points in the program. Similarly, for magazines and newspapers, publishers typically sell positions on a contractual basis based on multiple insertions and their circulation, not on the specific location where the ad will be placed.

[15] Goettler and Shachar (2001) report structural estimates of outside-alternative mean utilities for different demographic groups of television viewers.

In order to characterize the optimal path we apply the standard Maximum Principle of Pontryagin et al. (1966). Let the functions $\psi(t)$ and $\varphi(t)$ represent the shadow prices of $S(t)$ and $w(t)$ respectively. The Hamiltonian is defined as:

$$\begin{aligned}\mathcal{H}\left[C(t),\psi(t),\varphi(t),S(t),w(t)\right] &= -e^{-\rho t}C(t)+\psi(t)\left[C(t)-\delta S(t)\right]+\\ &\quad+\varphi(t)\left[\rho w(t)+v-\alpha_C C(t)-\alpha_S S(t)-\alpha_{CS}C(t)S(t)\right],\end{aligned}$$

with

$$\mathcal{M}\left[\psi(t),\varphi(t),S(t),w(t)\right]\equiv\max_{0\leq C(t)\leq 1}\mathcal{H}\left[C(t),\psi(t),\varphi(t),S(t),w(t)\right].$$

The conditions for optimality are:

$$\begin{aligned}\mathcal{H}\left[C,\psi,\varphi,S,w\right] &= \mathcal{M}\left[\psi,\varphi,S,w\right],\\ \dot{\psi} &= \rho\psi-\frac{\partial\mathcal{M}}{\partial S},\\ \dot{\varphi} &= \rho\varphi-\frac{\partial\mathcal{M}}{\partial w},\\ \dot{\psi}(t) &\geq 0,\\ \dot{\varphi}(t) &\geq 0.\end{aligned}$$

The linearity of the Hamiltonian in $C(t)$ means that there are three possible phases:

1. In Phase I (Content Phase), the problem has a corner solution where only programming is provided:

$$\begin{aligned}\mathcal{H}_{C(t)}\left[.\right] &= -e^{-\rho t}+\psi(t)-\left[\alpha_C+\alpha_{CS}S(t)\right]\varphi(t)>0,\\ C(t) &= 1.\end{aligned}$$

2. In Phase II (Advertising Phase), it has a corner solution where only ads are provided:

$$\begin{aligned}\mathcal{H}_{C(t)}\left[.\right] &= -e^{-\rho t}+\psi(t)-\left[\alpha_C+\alpha_{CS}S(t)\right]\varphi(t)<0,\\ C(t) &= 0.\end{aligned}$$

3. In Phase III (Joint Content-Advertising Phase), the problem has an interior solution where $w(t) = 0$ and $0 < C(t) < 1$ at all times.

The Content and Advertising Phases I and II are relevant for understanding the amount, positioning and other aspects of advertising in TV and radio programs. Typically in these media either only programming or only advertisements can be provided at a given time: programming is interrupted with commercials and commercials are interrupted with programming.[16] The same occurs in radio programs. This means that these media firms must choose a sequence of bundles where $C(t) \in \{0, 1\}$ at all times. Phase III is relevant for print media and the web since in every page of a newspaper or a magazine, or in a computer screen, any proportion of goods and ads is feasible at any time.

We examine the optimal paths in each of these phases next: I and II for TV and radio, and III for print media and the internet, respectively.

4 Dynamics for Television and Radio

We first solve the model for the Content and Advertising phases I and II, and then study the conditions under which the switching between phases (from consumption to ads and viceversa) takes place. Finally, we provide some descriptive evidence.

4.1 Solving the Model

Content Phase I. In this phase the optimality conditions are:

$$\mathcal{M}\left[\psi(t), \varphi(t), S(t), w(t)\right] = -e^{-\rho t} + \psi(t)(1 - \delta S(t)) + \varphi(t)\left[\rho w(t) + v - \alpha_C - \alpha_S S(t) - \alpha_{CS} S(t)\right],$$

[16] This is not always true. In some TV programs, small commercial logos and ads may be shown during a program in parts of the screen. This does not happen in radio programs since it is not possible to listen simultaneously to programming and commercials.

with

$$\begin{aligned}
\dot{S}(t) &= 1-\delta S(t),\\
\dot{w}(t) &= \rho w(t)+v-\alpha_C-\left[\alpha_S+\alpha_{CS}\right]S(t),\\
\dot{\psi}(t) &= \rho\psi(t)+\delta\psi(t)+\varphi(t)\left[\alpha_S+\alpha_{CS}\right],\\
\dot{\varphi}(t) &= \rho\varphi(t)-\rho\varphi(t)=0.
\end{aligned}$$

Solving for $S(t)$ we have $\left[\dot{S}(t)+\delta S(t)\right]e^{\delta t}=e^{\delta t}$, which implies $S(t)e^{\delta t}=\frac{1}{\delta}e^{\delta t}+k_1$, that is,

$$S(t)=k_1e^{-\delta t}+\frac{1}{\delta}.$$

The solution for $w(t)$ can be found from the fact that:

$$\begin{aligned}
\left[\dot{w}(t)-\rho w(t)\right]e^{-\rho t} &= (v-\alpha_C)e^{-\rho t}-\frac{\alpha_S+\alpha_{CS}}{\delta}e^{-\rho t}-(\alpha_S+\alpha_{CS})k_1e^{-(\rho+\delta)t},\\
w(t)e^{-\rho t} &= (v-\alpha_C-\frac{\alpha_S+\alpha_{CS}}{\delta})(-\frac{1}{\rho})e^{-\rho t}+\frac{\alpha_S+\alpha_{CS}}{\rho+\delta}k_1e^{(\rho+\delta)t}+k_2.
\end{aligned}$$

This means that:

$$w(t)=\frac{\alpha_S+\alpha_{CS}+(\alpha_C-v)\delta}{\rho\delta}+\frac{\alpha_S+\alpha_{CS}}{\rho+\delta}k_1e^{-\delta t}+k_2\rho^{\rho t}.$$

Lastly, the solutions for the shadow prices of $S(t)$ and $w(t)$ in Phase I imply:

$$\begin{aligned}
\left[\dot{\psi}(t)-(\rho+\delta)\psi(t)\right]e^{-(\rho+\delta)t} &= (\alpha_S+\alpha_{CS})\varphi\left(t\right)e^{-(\rho+\delta)t},\\
\psi(t)e^{-(\rho+\delta)t} &= -\frac{\alpha_S+\alpha_{CS}}{\rho+\delta}\varphi(t)e^{(\rho+\delta)t}+k_3,
\end{aligned}$$

that is:

$$\psi(t)=k_3e^{(\rho+\delta)t}-\frac{\alpha_S+\alpha_{CS}}{\rho+\delta}k_4,$$

given that the condition $\dot{\varphi}\left(t\right)=0$ implies:

$$\varphi(t)=k_4.$$

Note that the locus $\dot{S}(t)=0$ is parallel to the $w(t)$–axe. The value of $S(t)$ at which $w(t)$ is zero is $\frac{1}{\delta}$. The locus $\dot{w}(t)=0$ crosses the $w(t)$–axe at the point $w(t)=\frac{\alpha_C-v}{\rho}$ and the $S(t)$–axe at the point $S(t)=\frac{v-\alpha_C}{\alpha_S+\alpha_{CS}}$. Figure 1 shows the phase diagram in this case.

[Figure 1 here]

The model generates a number of interesting features. The first is that, as we might expect, there are no stationary solutions. Consider, for instance, the case in which in the first instant of the program $w(0) = 0$. If the initial stock of past consumption S_0 is below $\frac{v-\alpha_C}{\alpha_S+\alpha_{CS}}$, then either the audience will receive the maximum consumption possible $C(t) = 1$ during the whole period (that is, $w(t) > 0$ with $\dot{w}(t) > 0$), or $w(t)$ will first increase for some time and then, after crossing the locus $\dot{w}(t) = 0$, will decrease up to at most the point $w(\tau) = 0$, since the condition $w(t) \geq 0$ must be met at all times. When $w(t) = 0$, $S(t)$ cannot be above $\frac{v-\alpha_C}{\alpha_S+\alpha_{CS}}$ if programming is to continue next period. In other words, given a stock $S(t)$, the parameters α_C, α_S, and α_{CS} cannot be too low, and v cannot be too high, if the audience member is to continue to receive programming.[17] This means, not suprisingly, that the preference parameters and $S(t)$ play an important role in the dynamics of programming and commercials. For example, take again the case that at the the beginning of a program $w(0) = 0$: the length of the first segment of programming (until it is interrupted with commercials) is shorter when the initial stock of past consumption S_0 is greater, when the opportunity cost v is lower, and when α_C, α_S, and α_{CS} are larger. These empirical implications accord well with intuition.

Lastly, the depreciation parameter δ determines the region where S increases or decreases with programming. Low values of δ make it less likely that S will decrease when consumers are enjoying the consumption good. Obviously, S can never decrease if $\delta = 0$. And if S_0 is above $1/\delta$, the stock of past consumption always decreases with time, implying $S(T) < S_0$. This situation, as the analysis of switching between content and advertising phases will show later, does not seem sustainable (unless the individual replenishes his "firm specific" stock when engaging in alternative occupations within or, more generally, outside times 0 to T).

Summing up, as expected, different combinations of the preference parameters and the initial stock contribute to determining the length of the programming segments, both at the beginning of a program and throughout the program, in an intuitive way.

[17] Obviously, the problem is meaningful only if $v > \alpha_C$. The qualitative discussion of case $w(0) > 0$ is similar.

ADVERTISING PHASE II. In this phase the conditions for optimality are:

$$\mathcal{M}\left[\psi(t),\varphi(t),S(t),w(t)\right] = -\psi(t)\delta S(t)+\varphi(t)\left[\rho w(t)+v-\alpha_S S(t)\right],$$

with

$$\begin{aligned}\dot{S}(t) &= -\delta S(t)\\ \dot{w}(t) &= \rho w(t)+v-\alpha_S S(t)\\ \dot{\psi}(t) &= \rho\psi(t)+\delta\psi(t)+\varphi(t)\alpha_S\\ \dot{\varphi}(t) &= \rho\varphi(t)-\rho\varphi(t)=0\end{aligned}$$

Solving for $S(t)$, we have $\left[\dot{S}(t)+\delta S(t)\right]e^{\delta t}=0$, which implies $S(t)e^{\delta t}=k_1$,that is:

$$S(t)=k_1 e^{-\delta t}.$$

For $w(t)$ we have:

$$\begin{aligned}\left[\dot{w}(t)-\rho w(t)\right]e^{-\rho t} &= ve^{-\rho t}-\alpha_S k_1 e^{-(\rho+\delta)t},\\ w(t)e^{-\rho t} &= -\frac{v}{\rho}e^{-\rho t}+\frac{\alpha_S\cdot k_1}{\rho+\delta}e^{(\rho+\delta)t}+k_2,\end{aligned}$$

which means that:

$$w(t)=-\frac{v}{\rho}+\frac{\alpha_S}{\rho+\delta}k_1 e{-}\delta t+k_2 e^{-\rho t}.$$

Lastly, since:

$$\begin{aligned}\left[\dot{\psi}(t)-(\rho+\delta)\psi(t)\right]e^{-(\rho+\delta)t} &= \alpha_S k_4 e^{-(\rho+\delta)t},\\ \psi(t)e^{-(\rho+\delta)t} &= -\frac{\alpha_S k_4}{\rho+\delta}e^{(\rho+\delta)t}+k_3,\end{aligned}$$

we have:

$$\psi(t)=k_3 e^{(\rho+\delta)t}-\frac{\alpha_S}{\rho+\delta}k_4,$$

given that $\dot{\varphi(t)}=0$ implies

$$\varphi(t)=k_4.$$

Note that the locus $\dot{S}(t)=0$ coincides with the $w(t)$-axe, and that the locus $\dot{w}(t)=0$ crosses the $w(t)$–axe at the point $w(t)=-\frac{v}{\rho}$ and the $S(t)$–axe at the point $S(t)=\frac{v}{\alpha_S}$. Figure 2 shows the trajectories in this case.

[Figure 2 here]

There are again a few features of interest. If the initial stock of consumption is below $\frac{v}{\alpha_s}$ with $w(t) \geq 0$, then providing only advertisements increases the media's "debt" w over time. This is more likely the case, given initial values of S and w, the lower the time preference parameter ρ, the greater the value v of the outside opportunity, and the lower α_S. For initial stocks of consumption above $\frac{v}{\alpha_s}$, w will decrease either permanently or at most until reaching $\dot{w}(\tau) = 0$, after which the media's debt increases again. Obviously, S can never increase when only advertising is provided. Again, these dynamics appear to accord well with intuition.

4.2 Switching Between Content and Ads

To understand the switching between programming and advertising that takes place during a TV or radio program, we need to study when the "jump conditions" that determine when switching takes place from one phase to another are satisfied. These conditions require that both the Hamiltonian and the shadow prices be equal in the two phases at the points of switching. The Hamiltonian in the Content phase is:

$$\mathcal{H}_I\left[.\right] = -e^{-\rho t} + \psi(t)\left[1 - \delta S(t)\right] + \varphi(t)\left[\rho w(t) + v - \alpha_C - \alpha_S S(t) - \alpha_{CS} S(t)\right],$$

and in the Advertising phase:

$$\mathcal{H}_{II}\left[.\right] = -\delta\psi(t)S(t) + \varphi(t)\left[\rho w(t) + v - \alpha_S S(t)\right].$$

The condition that $\mathcal{H}_I\left[.\right] - \mathcal{H}_{II}\left[.\right] = 0$ implies:

$$\mathcal{H}_{C(t)}\left[.\right] = -e^{-\rho t} + \psi(t) - \left[\alpha_C + \alpha_{CS} S(t)\right]\varphi(t) = 0.$$

Note that:

$$\frac{d}{dt}\mathcal{H}_{C(t)} = \rho e^{-\rho t} + \dot{\psi}(t) - \left[\alpha_C + \alpha_{CS} S(t)\right]\dot{\varphi}(t) - \alpha_{CS}\varphi(t)\dot{S}(t)$$

which in the two cases of interest is:

$$\begin{aligned}
\left.\frac{d}{dt}\mathcal{H}_{C(t)}\right|_{C(t)=1} &= \rho e^{-\rho t} + (\rho + \delta)\,\psi(t) + (\alpha_S + \alpha_{CS})\,\varphi(t) - \alpha_{CS}\varphi(t)\left[1 - \delta S(t)\right], \\
\left.\frac{d}{dt}\mathcal{H}_{C(t)}\right|_{C(t)=0} &= \rho e^{-\rho t} + (\rho + \delta)\,\psi(t) + \alpha_S\varphi(t) + \alpha_{CS}\varphi(t)\delta S(t).
\end{aligned}$$

Note that:

$$\frac{d}{dt}\mathcal{H}_{C(t)}\Big|_{C(t)=1} - \frac{d}{dt}\mathcal{H}_{C(t)}\Big|_{C(t)=0} = \alpha_{CS}\varphi(t) - \alpha_{CS}\varphi(t) = 0.$$

This means that:

$$\frac{d}{dt}\mathcal{H}_{C(t)} = \rho e^{-\rho t} + (\rho+\delta)\,\psi(t) + (\alpha_S + \alpha_{CS}\delta S(t))\,\varphi(t),$$

where $\varphi(t) = \varphi$ is constant over time. If we study the movements of $\psi(t)$ and $S(t)$ in phases I and II around their steady states, we find that switching between $C(t) = 1$ and $C(t) = 0$ is feasible when $S(t)$ is in the interval $\left(0, \frac{1}{\delta}\right)$ and $\psi(t)$ is in the interval $\left(-\frac{\alpha_S+\alpha_{CS}}{\rho+\delta}\varphi, -\frac{\alpha_S}{\rho+\delta}\varphi\right)$, as indicated in Figures 3-4.

[Figures 3-4 here]

The exponential behavior of $S(t)$ and $\psi(t)$ in each of these phases means that they move relatively slow when close to the equilibrium, and relatively fast as we move away from the equilibrium. Importantly, it is definitely possible to find a constant φ that makes $\mathcal{H}_{C(t)} = 0$. For instance, consider the values $S(t) = \frac{1}{2\delta}$ and $\psi(t) = -\frac{1}{\rho+\delta}\left(\alpha_S + \frac{\alpha_{CS}}{2}\right)\varphi$ in the admissible intervals. It is not difficult to show that there is a constant φ that solves:

$$\mathcal{H}_{C(t)} = -e^{-\rho t} - \frac{1}{\rho+\delta}\left(\alpha_S + \frac{\alpha_{CS}}{2}\right)\varphi - \left[\alpha_C + \alpha_{CS}\frac{1}{2\delta}\right]\varphi = 0.$$

Chattering is thus perpetual in these circumstances. In order to avoid chattering, costs of switching between phases may be introduced. In the context of radio and television, these costs may simply be interpreted as the minimum amount of time necessary to deliver and absorb "meaningful" programming and commercials. What is important is that it is certainly feasible for the system to move from:

I.1. Phase I: $C(t) = 1$, $\mathcal{H}_{C(t)} > 0$ with $\frac{d}{dt}\mathcal{H}_{C(t)} > 0$, to

I.2. Phase I: $C(t) = 1$, $\mathcal{H}_{C(t)} > 0$ with $\frac{d}{dt}\mathcal{H}_{C(t)} < 0$, to

II.1. Phase II: $C(t) = 0$, $\mathcal{H}_{C(t)} < 0$ with $\frac{d}{dt}\mathcal{H}_{C(t)} < 0$, to

II.2. Phase II: $C(t) = 0$, $\mathcal{H}_{C(t)} < 0$ with $\frac{d}{dt}\mathcal{H}_{C(t)} > 0$, and then back to subphase I.1, I.2, II.1, and so on.

With respect to the duration of the dynamics, different patterns are obviously possible depending upon the specific set of parameters. Needless to say, there may be instances empirically in which Phases I or II may alternate with Phase III, which we study in the next section. Before that we present some descriptive evidence.

4.3 Descriptive Evidence

We next report some data showing how in fact a large variety of patterns is observed in media markets. This serves to stress the idea that a dynamic model of bundling ads, which we have argued must include the ingredients we are studying, should account for a variety of disparate evidence.

We present data from TV programs that are different in many respects (content, audience, duration), as these differences may be helpful to provide intuition into the potential determinants of the different bundles in the context of the model. The unit of time in the programs in Figures 5-7 is a minute. There are two types of minutes: "programming minutes," defined as those where only utility-raising program was shown to the audience, and "commercial minutes," which are those that had some advertising time. The figures plot for each minute of the program the number of programs that had a "commercial minute."[18]

[Figures 5 to 7 here]

Among 30-minute programs, we see very different distributions of commercial time both in terms of quantity and timing. Sitcoms and news are similar in the percentage of ads and number of cuts, but strikingly different in the distribution of their commercial time. Their distributions of ads are also different from those in soap operas and children programs,[19] which are also rather different from each other.

[18] The data come from 1,017 programs shown on 62 different channels during the period September 1 - December 31, 2012 in Providence, RI. These channels are those included in the basic cable service provided by Cox Communications New England in Providence, RI, for zip code 02906.

[19] Children programs are regulated by the Federal Communications Commission (FCC). In 1990, Congress enacted the Children's Television Act of 1990 (CTA), which currently regulates advertising. Among other requirements, the CTA imposes that television broadcast licensees and cable

Within news programs, there are some interesting differences. For instance, weekend and non-evening news appear to take several more minutes before interrupting the program with the first segment of commercial time, and weekend news typically have one more segment of commercials. In the one specific station we look at (CNN), the first commercial in sports news comes about two minutes earlier than in general news, and its last commercial comes about three minutes later. Cartoons for children are also substantially different from *The Simpsons*, often considered the classic cartoon par excellence for adults, whose distribution of commercial time is similar to that of a sitcom.

Among 1-hour programs the differences in ad distributions are also quite substantial. Dramas and series are similar to sitcoms, and very different from the rest. Soap operas are somewhat similar to talk shows in the second half of the program, but quite different in the first half. The figures also show interesting differences between weekend movies and weekday movies, and between "older movies" (which we define as those first shown in theaters at least 15 years ago) and "new movies" (those first shown in theaters no more than 5 years ago).

This variety of programs and ad distributions may be intuitively linked to the parameters of the model although, needless to say, the evidence that we observe is the result of not only the ingredients we have studied but also different strategic and market structure considerations outside our setting. While it is beyond this discussion to provide a careful empirical analysis, here are some conjectures. For soap operas, which are stories that last for months and some even for years, we may expect a low degree of complementarity between the stock and the flow of program within a given episode α_{CS} and a large degree of complementarity across episodes, as captured by the stock of initial capital S_0. In news programs, which are, broadly speaking, a collection of shorter and independent news (national, international, local, weather, sports) with

operators must limit the amount of commercial matter that may be aired during children's programs to not more than 10.5 minutes per hour on weekends and not more than 12 minutes per hour on weekdays. In 2000, the FCC stated its expectation that the industry would honor the voluntary advertising guidelines under which broadcasters were to air no more than 9.5 minutes of advertising per hour on weekends. Further details are available at the FCC's Media Bureau web site at www.fcc.gov/mb/policy/cetv.html.

no single story running through the program, we may expect α_C to be an important determinant of the characteristics of the bundle, whereas α_S and especially α_{CS} much less so. The relevance of S_0 will probably be quite different in movies (stories that begin and end in the same program) than in sitcoms and dramas (different stories involving the same characters shown over time, typically on a weekly basis, building initial stock of capital). Similarly, we can think of the differences between "older movies" vs "new movies", and between weekday and weekend movies.[20] For children cartoons, the fact that children appear to have a lower attention span (δ) than adults, and perhaps a greater complementarity parameter α_{CS} too, should help explain the difference with respect to cartoons appealing to adults, as *The Simpsons*, whose distribution of commercials is, unsurprisingly, similar to that in sitcoms.

5 Dynamics for Print Media and Internet

In these industries any proportion of goods and bads is feasible at any point in time: Goods and bads may share the same computer screen, and may also share the same page in a newspaper or in a magazine.[21] The analysis of phase III is much simpler. We have an interior solution where $w(t) = 0$ and $0 < C(t) < 1$ for all t. In this case we have:

$$\begin{aligned} 0 &< C(t) = \frac{v - \alpha_S S(t)}{\alpha_C + \alpha_{CS} S(t)} < 1, \\ \dot{S}(t) &= \frac{v - \alpha_S S(t)}{\alpha_C + \alpha_{CS} S(t)} - \delta S(t). \end{aligned}$$

[20] For example, older movies, which are more likely to have *already* been watched when they were first shown in theatres, in videos or previously on TV, are never interrupted with commercials in the last eight minutes. Their distribution of ads may be driven by different parameters, particularly α_{CS} and δ, e.g., lower values would help to explain why they are never interrupted at the end, and why new movies have one more commercial segments.

[21] The temporal dimension of the model when applied to the web is not much different from that in TV and radio, and so it is straightforward to interpret it (though, of course, the web is more interactive and the consumer may move back and forth, up and down). With respect to print media, the interpretation is not trivial as consumers have even more freedom to move in different directions; they can even choose the initial period (the place when their consumption starts, e.g., the international news in a newspaper), and the final period. Hence the temporal dimension applies to each "program" (section or set of sections).

These conditions imply the following two restrictions on the parameters of the model:

$$S(t) < \frac{v}{\alpha_S},$$

$$S(t) > \frac{v - \alpha_C}{\alpha_S + \alpha_{CS}}.$$

which, not surprisingly, are the points at which $\dot{w}(t) = w(t) = 0$ in the Content and Advertising phases I and II. These restrictions imply $\alpha_C \cdot \alpha_S + \alpha_{CS} \cdot v > 0$. Note that the loci $\dot{S}(t) = 0$ and $\dot{w}(t) = \rho w(t) = 0$ cross at the point where:

$$\delta S(t) = \frac{v - \alpha_S S(t)}{\alpha_C + \alpha_{CS} S(t)},$$

that is:

$$S(t) = \frac{-\left(\delta \cdot \alpha_C + \alpha_S\right) \pm \left[\left(\delta \cdot \alpha_C + \alpha_S\right)^2 + 4\delta \cdot \alpha_{CS} \cdot v\right]^{1/2}}{2\delta \cdot \alpha_{CS}}.$$

Figure 8 shows the dynamics in this case.

[Figure 8 here]

A number of features accord well with intuition. The media should offer more programming, information or services the higher the depreciation rate (to keep the stock high enough at every moment). Also, the stronger the individual's taste for current and past consumption, including the extent to which they are complements, the lower the steady state levels of programming. Similarly, the higher the outside option, the lower the quantity of ads.

Finally, we note the relevance of the model in the context of a novel idea in online advertising, which accounts for 20% of the ad industry's total spending and over 90% of the revenue for internet giants such as Google and Facebook. This idea concerns the recent developments around the concepts of "engaged *time*" and "*time* metrics," the biggest buzzwords of late. Traditionally, ads are being bought and sold based on CPI (cost per impression) and clicks. But the technology is ripe, and so it is the idea that if an ad is not seen then it should not be paid for. This represents a big breakthrough in the current online media world (italics ours):

> "The Financial Times recently announded that it will sell display ads based off the *time* its audience spends with content. The British newspaper hopes it will solve the viewability problems that plague the online advertising industry .[...] This shift is part of the broader vision for the future of advertising. Using *time* as an advertising currency fits that mission perfectly." (January 2015)

> "The Guardian overhauls its site in an anticipation of publishers selling ads based on *time* in ongoing efforts to sell advertisers on reader attention not clicks [...] The biggest question around view time is how do you insert ads *dynamically* down in the page." (December 2014)

Consistent with this recent idea, in the model we have developed "time" is precisely the advertising currency, and time determines the performance of an ad in terms of users' attention.

6 Discussion, Applications and Extensions

The framework we have developed captures the tension between the interests of advertiser-supported media and audiences interested in receiving utility-raising goods in an intertemporal barter exchange that is not contractually sustained. This tension is intuitive for audiences do not seem to look forward to consuming ads. In broadcast television, zapping and using the fast forward on a DVD to speed through commercials in a taped program are common behaviors. Modern technologies help these efforts to avoid ads. In television the most famous is perhaps TiVo, which offers a digital video recorder that automatically finds and records one's favorite programs allowing viewers to automatically skip the commercials when viewing the recorded program.[22] A similar mechanism is often used in certain radio programs, not by audiences but by the radio stations themselves, to push more commercials into their audiences. Radio stations are able to speed up programming by applying time compression mechanisms to make room for more advertising.[23]

[22]Subscribers may also sit down to watch, for instance, a 60-minute "live" program 20 minutes past the start time so they can skip past the commercials and catch up to "real" time by the program's end.

[23]For instance, the early sound processor called "Cash" snips out the silent pockets between words, shortening the pauses, and generally speeding up the pace of the speech in recorded talk programs.

Relative to TV and radio, the internet is more interactive on both the demand and supply sides of goods and bads. Users can easily move away from passively receiving information and entertainment, and actively seek the consumption of their goods of interest. They can also block ads. Thus we would expect the tension in the internet to be more apparent.[24] In fact, with the advent of new technologies, it is. On the one hand, the current situation pushing ads to viewers, seems to have exceeded "reasonable" levels with consumers complaining about garish ads and intrusive trackers, which make many sites bloated and increasingly slow to load:

> "The ads that are being shown now on the Web have just gone overboard and people don't want to see them anymore in contrast to ads in other mediums like TV and on radio. We've created this problem together, we created a bad experience and users are now voting everybody off the island." *Adweek*, October 2015.

On the other hand, with the advent of ad-blocking technologies into the web, in particular to mobile devices, current discussion about ad-blockers are all the rage:

> "Will ad-blocking over time decimate the "free" web? What role and form will advertising play on the web in the near future?" (Cowen, 2015)

The *Reuters Institute Digital News Report* (2015) reports that almost *half* of all US internet users block ads, and that the ad revenue globally being blocked for 2016 may exceed $40 billion. To thwart some of the growth of ad blockers publishers appear to have been banking on the growth of mobile devices, where the bulk of

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Radio stations may add as many as four seconds per minute, that is four minutes of commercial time per hour. The way it works is similar to TiVo. Instead of broadcasting a program live, the radio station first inserts four additional minutes of commercials. After these extra commercials, the station starts playing back the program after removing the dead air spacing words closer together. Within an hour, the clipped version catches up to the live feed. This type of device is used primarily by local radio stations that apply it to syndicated programs (see *The New York Times* (2000)).

[24] This tension has experienced substantial changes over time. In the 1990's websites used to give individuals free information and services with virtually no ads and let users freely go their preferred way with little or no attempts to manipulate their attention. Beginning in the early 2000's companies began to "push" increasing amounts of advertising at them and actively engage in manipulation. The movement away from "pull technologies" towards "push technologies," also referred to as "screen bias," was well documented in a number of prominent legal cases where websites engaged excessively in the manipulation of users' attention. Even initially neutral search engines now direct individuals toward specific web sites that pay for prominent billing.

today's web audiences are and where, until recently, ad-blocking plugins either didn't work or were cumbersome to install. But this is no longer the case. Popular software such as AdBlock Plus is becoming available on mobiles, and Apple's new operating system iOS9, released last fall, allows consumers to put content blockers on its web browser. Perhaps not surprisingly, a day after iOS9's launch, ad blockers topped the App Store worldwide. It is unclear how media companies and consumers with ad blockers will behave,[25] but the model we have developed helps us think about the different issues and gives us a way to study the trade-offs. As modeled in our dynamic framework, both people and media want complete control to choose how they interact with each other to determine digital advertising. *Adweek* (2015, italics added):

> "The entire industry is predicated on a *value exchange* where everyone has to win–consumers, brands and publishers. But the *pact* has been corrupted with too much and ubiquitous, and frankly, terrible advertising.
>
> Our stance in general is that there are ads out there that can provide value online–it is getting to those ads that should be the goal. ...The point is, we've got to find a middle ground that allows consumers to get the content they want as fast as they want in all the ways that they want and still be able to create great advertising connected to it."

This "middle ground" or "pact" (equilibrium) is precisely what we have attempted to formalize. As such, the framework we have presented provides a way to approach and investigate many different matters that future research should examine. For instance, it is straightforward to modify the preference specification to reflect the "even heftier" ads by assuming that ads reduce utility by an amount $a \geq 0$ per unit of time. The utility flow of consumption becomes $\alpha_C + a$ and this impacts the "experience," as well as the stock of initial and past consumption. Parametrizing how ads may reduce utility is also useful to evaluate changes in regulatory standards with interesting regulatory implications.[26] More challenging undertakings may include the study of changes in targeting technology to improve the quality of the match between advertisers and the intended recipient of ads, including more general selection

[25] See, e.g., Gans (2015) and Hyde (2105).

[26] See Meyer (2015) on the new set of standards currently being developed for the digital advertising industry, in particular to guide "reasonable" ad development on the mobile web.

issues, which may also be modeled through the preference specification (Athey and Gans (2010), Athey, Calvano, Gans (2013)).[27] The price of ads $\eta(t)$, which we assumed given, is definitely determined under strategic and intertemporal competition (Athey, Calvano and Gans, 2014) and the extent of coordination (Sweeting, 2006, 2009, 2013). Insight into the impact of changes in the technology to measure ads consumption (from CPI and clicks to the concept of "engaged time" discussed earlier) can be studied by examining changes in η. And access to and new developments in ad-avoiding technologies, including time-varying outside options, may be modeled through $v(t)$. Similarly, time-varying consumption intensity (or quality), perhaps including "hooks" as in surprise and suspense effects (Ely, Frankel and Kamenica, 2015) will operate through the process $\{C(t)\}$.

7 Concluding Remarks

We have developed a theory that formalizes the dynamic tension between the interests of advertiser-supported media and consumers interested in receiving utility-raising goods in a setting where the barter exchange between goods and bads is not contracted away. We have argued that understanding how media markets operate, and how media audiences form, requires a dynamic framework that includes these central characteristics, and that such a framework is currently lacking in the literature.

The paper's contribution is essentially methodological. We believe our theory is useful because it captures indispensable attributes that should be present in dynamic models. As such we definitely hope the theory will prove fruitful in future theoretical and empirical work, in particular when integrated into models of strategic and intertemporal competition. Our results may also help study various policy questions which typically hinge on our understanding of the relevant trade-offs. We leave these and other open extensions to future research.

[27] Non-linear utility effects of ads (ads that build their own stock of past ads) could also be studied.

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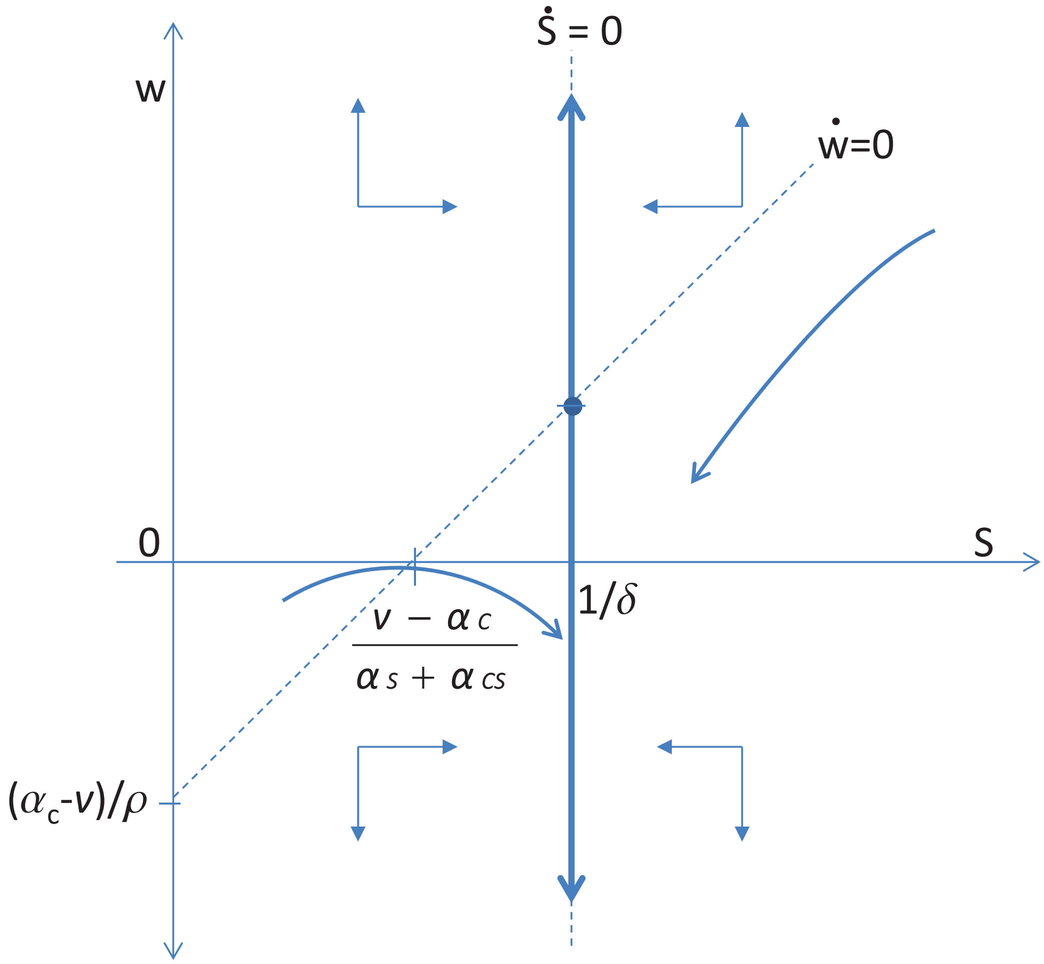


Figure 1. Dynamics in TV and Radio when Programming is Provided.

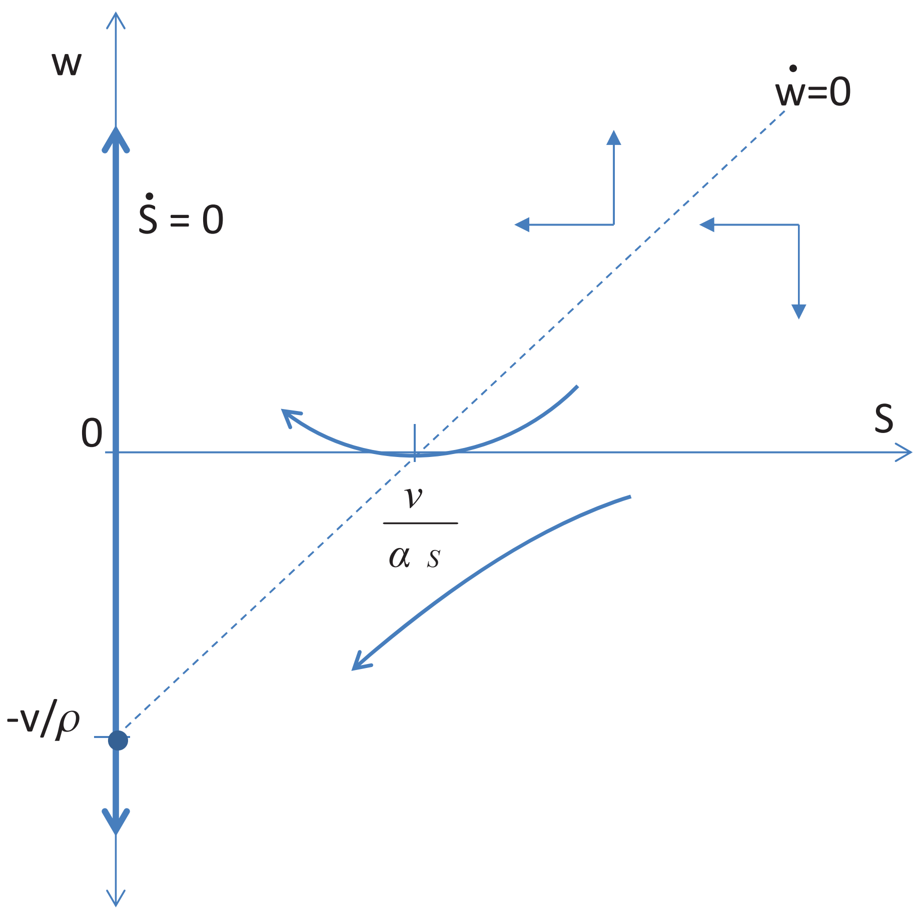
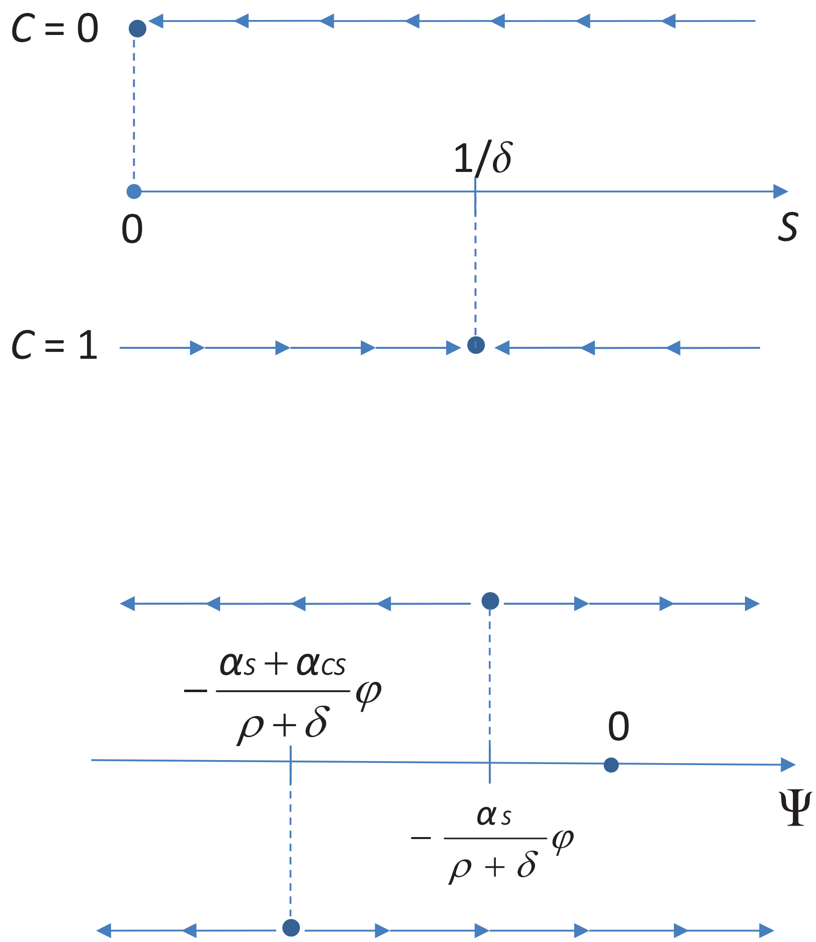


Figure 2. Dynamics in TV and Radio when Advertising is Provided.



Figures 3-4. Switching Dynamics Between Programming and No Programming in TV and Radio

Figure 5
Distribution of Commercials in 30-Minute TV Programs

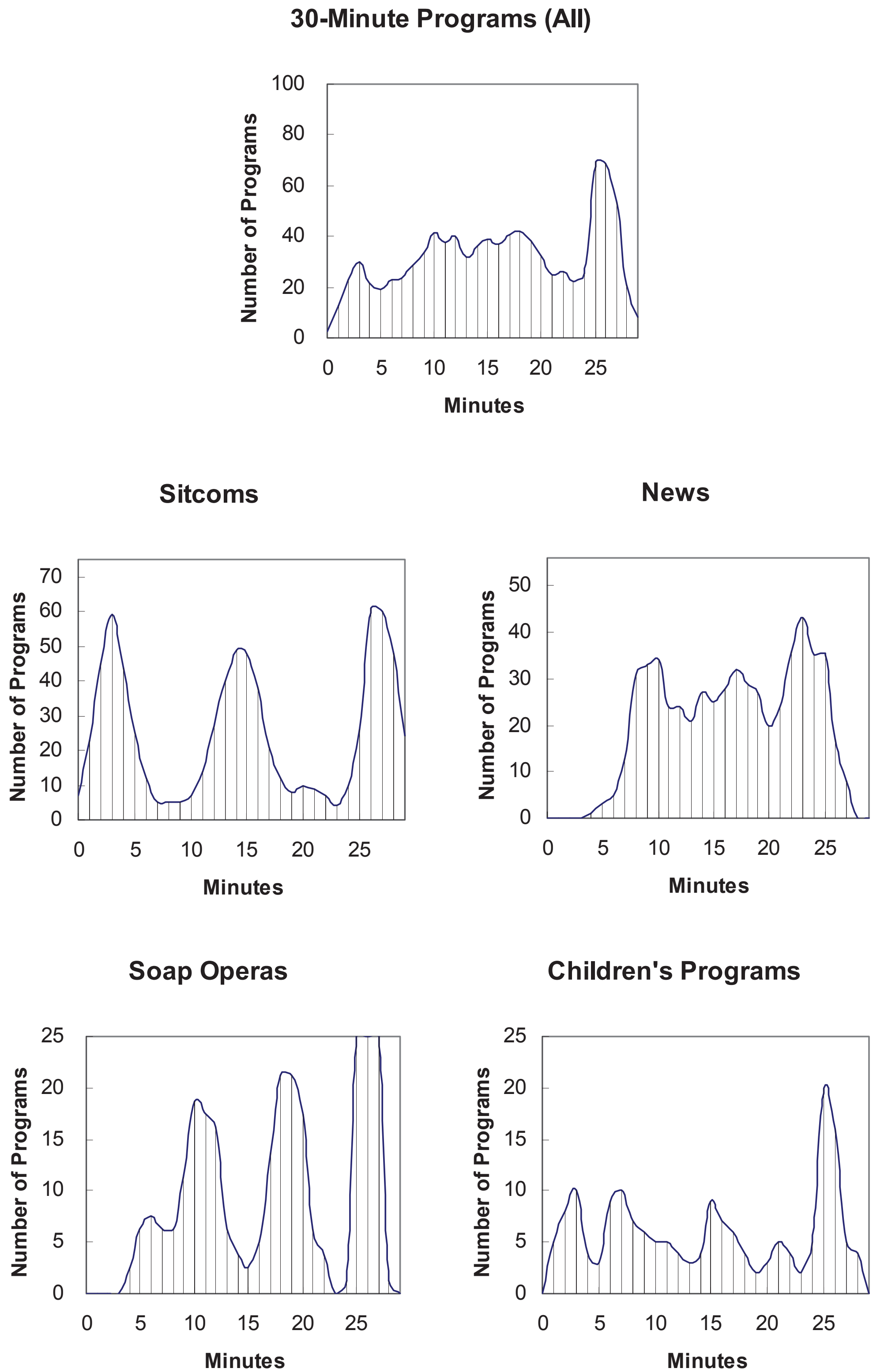
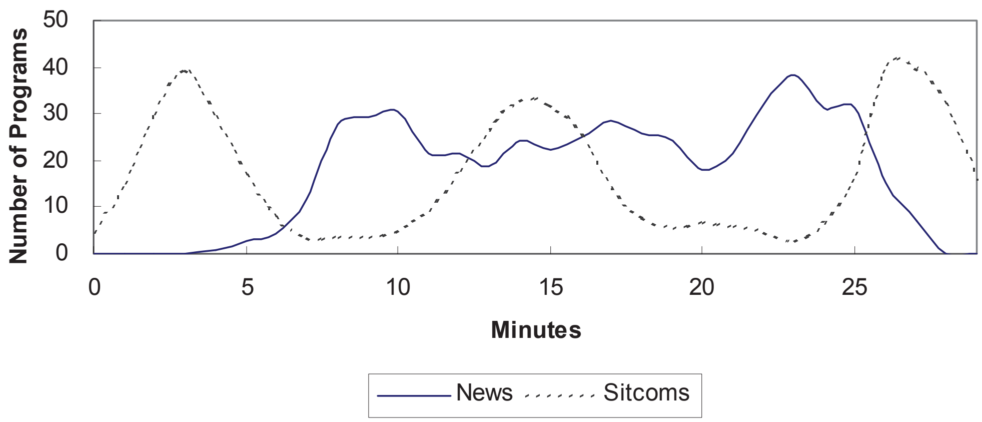


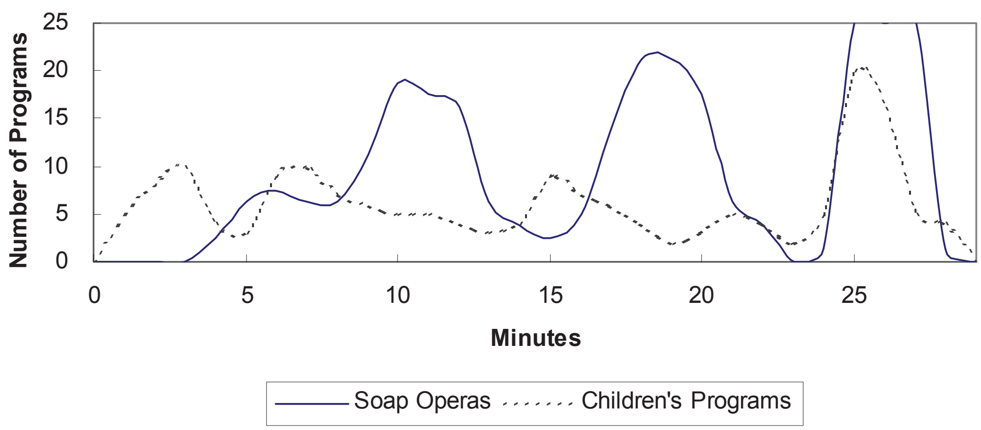
Figure 5a

Comparing the Distribution of Commercials in 30-Minute Programs

News vs. Sitcoms



Soap Operas vs. Children's Programs



Cartoons vs. Simpsons

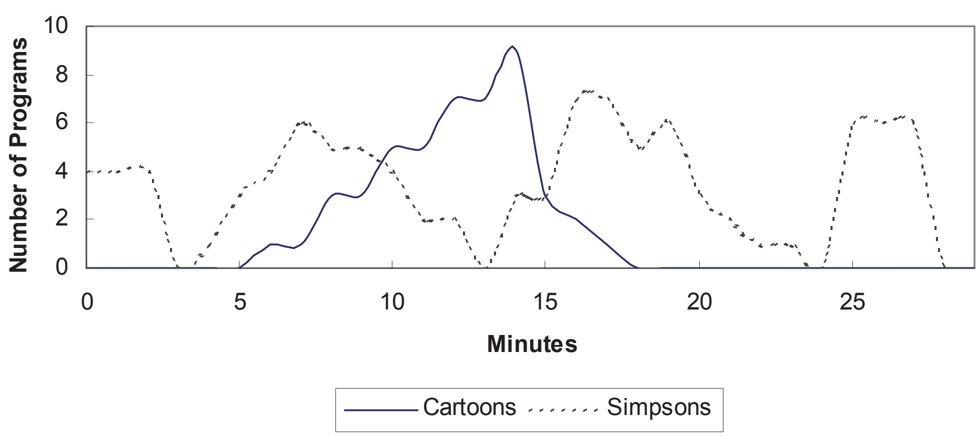
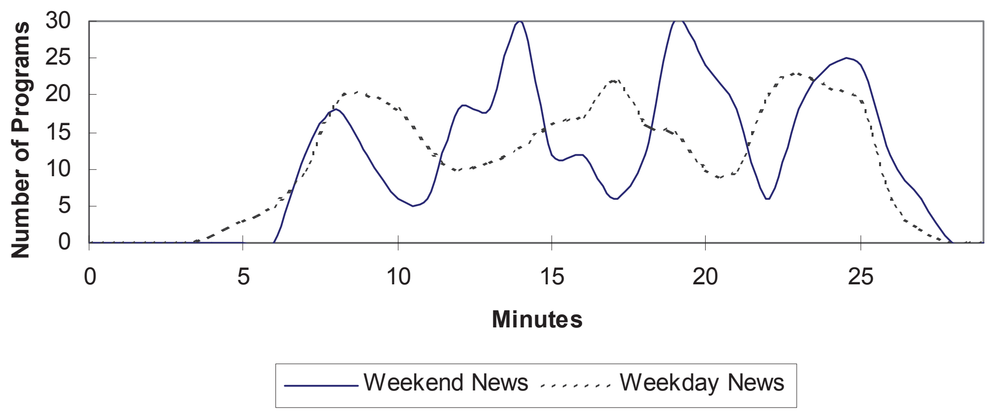


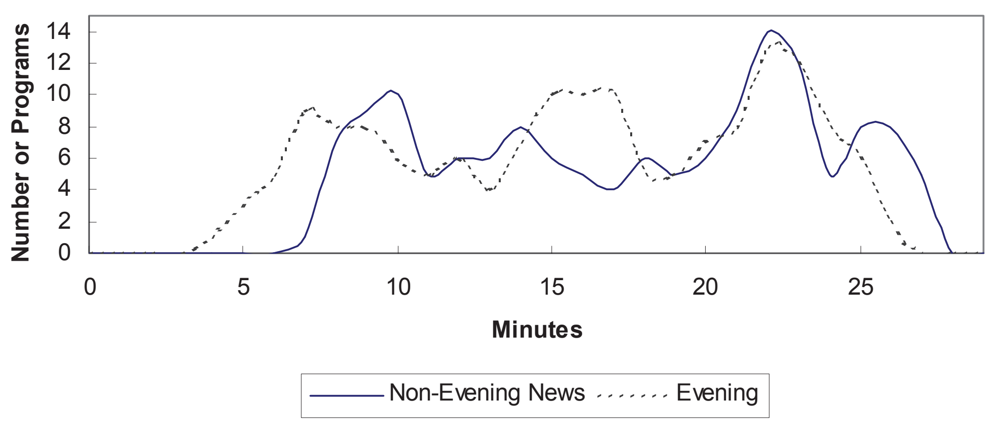
Figure 5b

Comparing the Distribution of Commercials in 30-Minute News Programs

News (Weekday vs. Weekend)



News (Non-Evening vs. Evening)



News vs. Sports News (CNN)

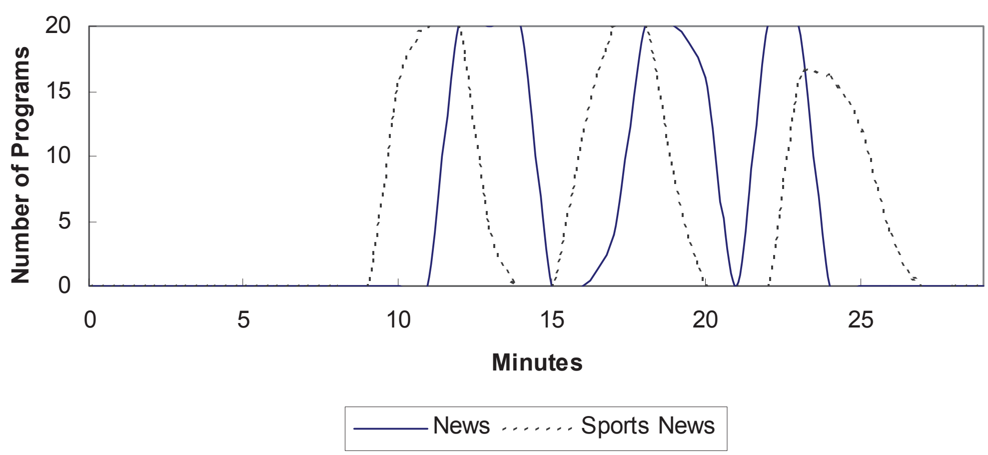


Figure 6

Distribution of Commercials in 1-Hour TV Programs

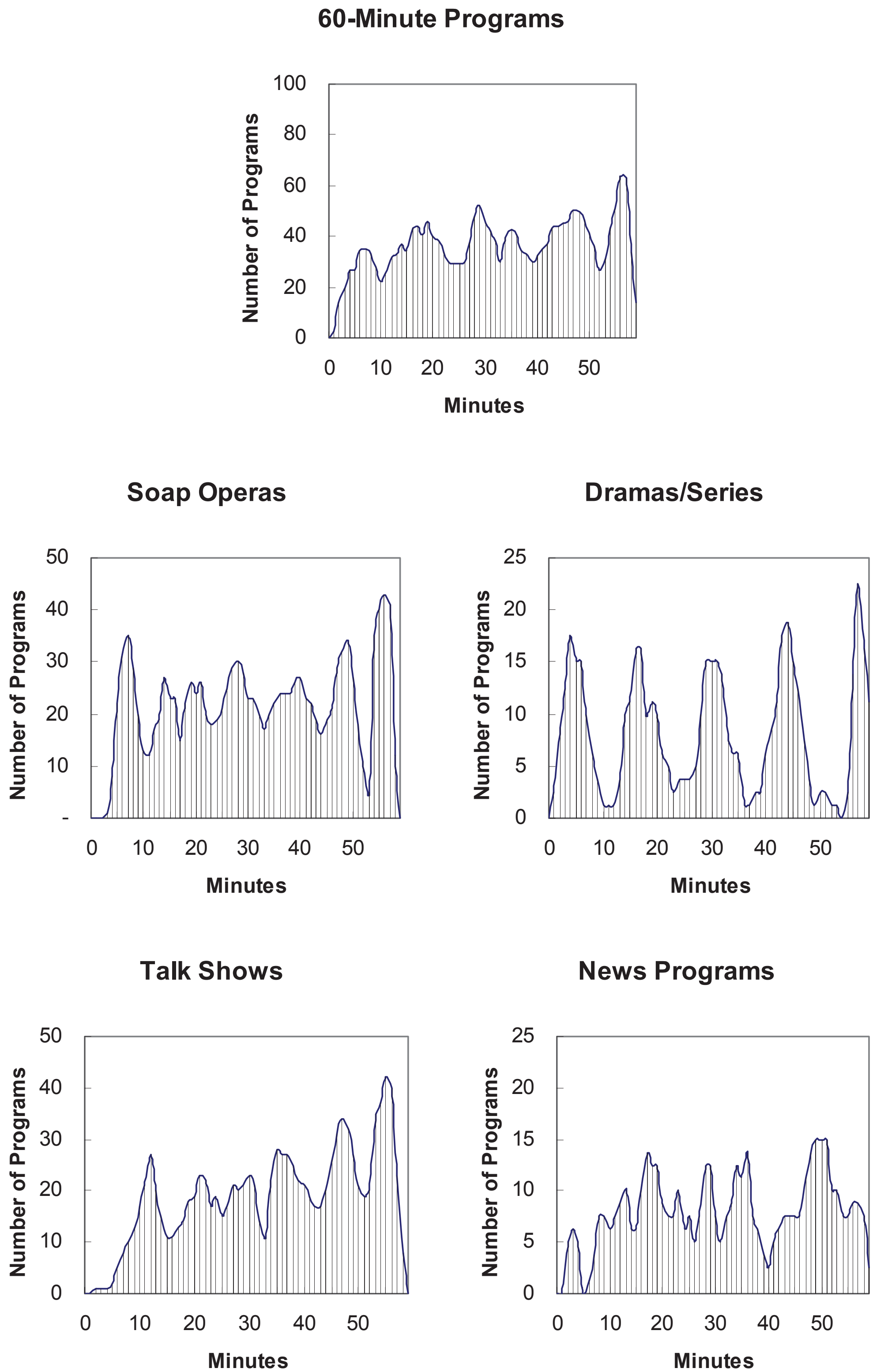
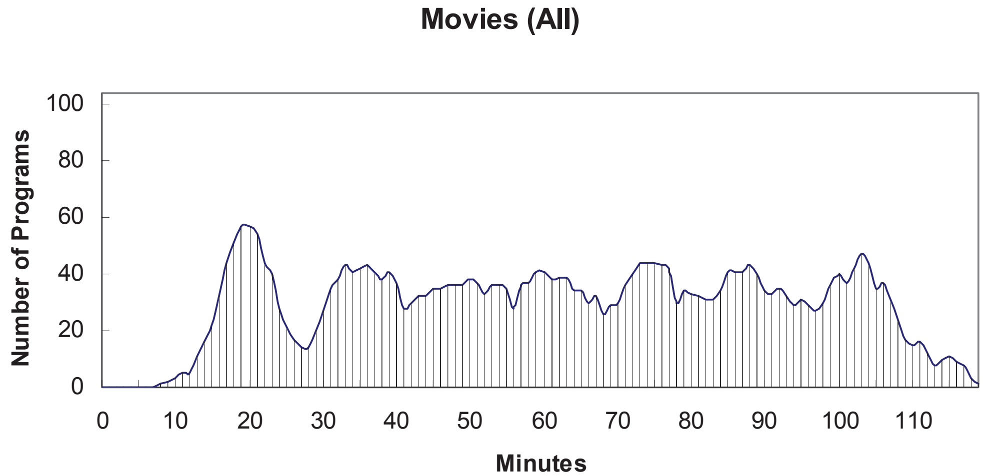
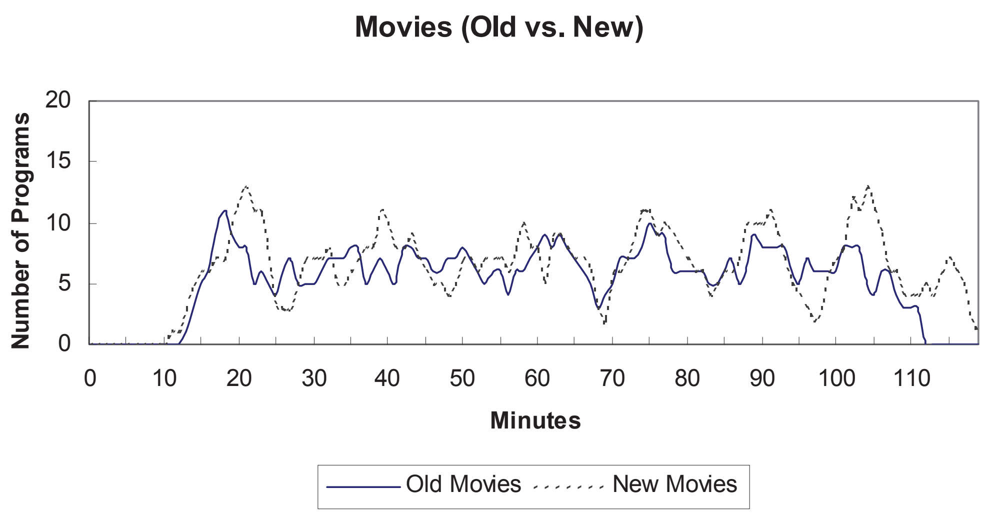
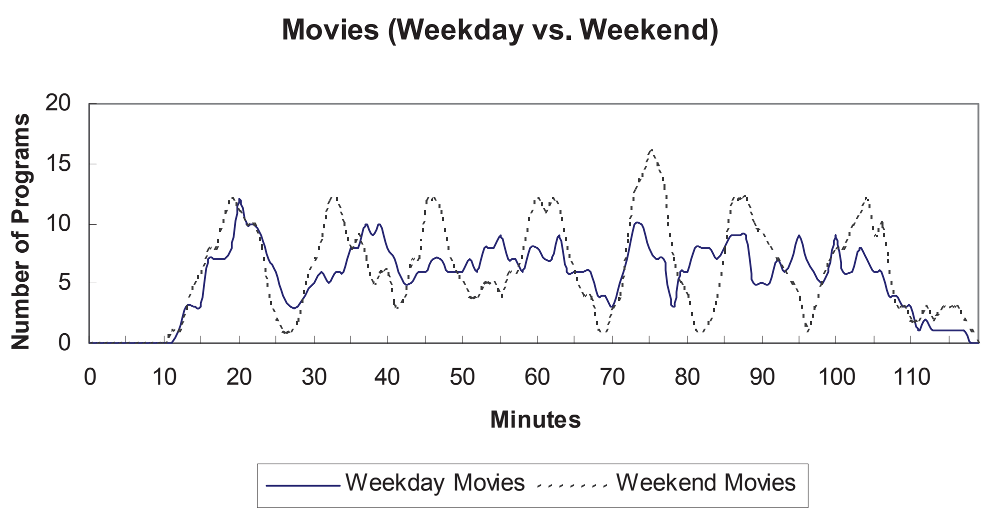


Figure 7

Distribution of Commercials in TV Movies







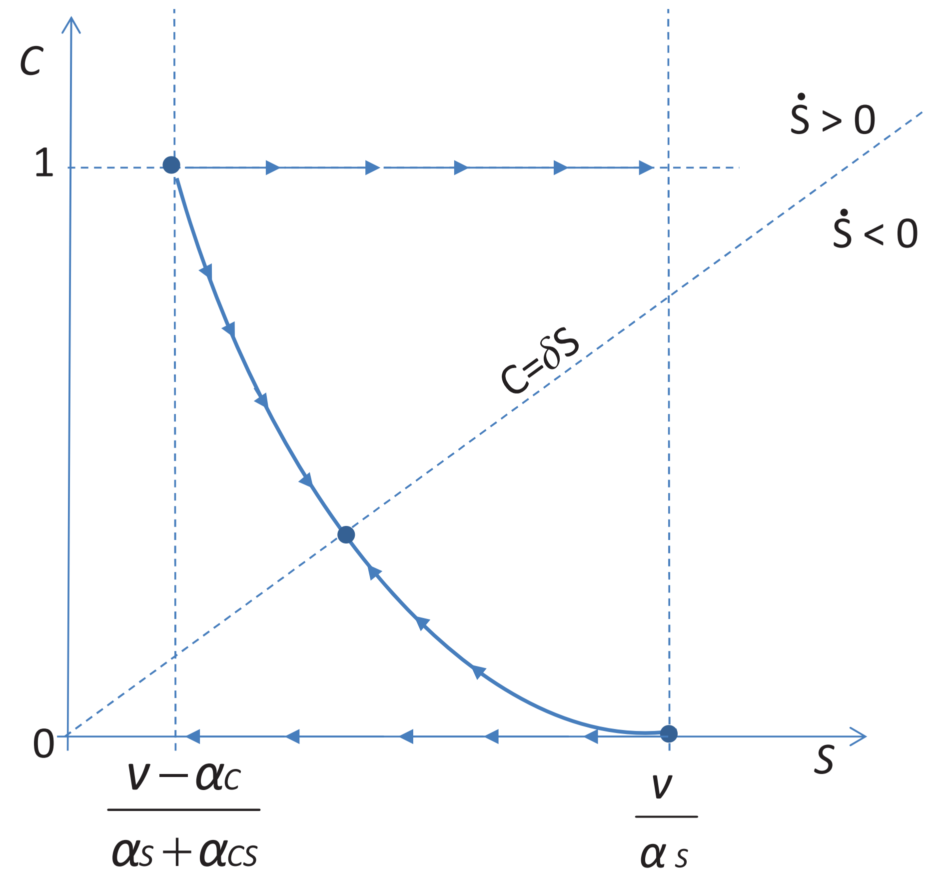


Figure 8. Advertising Dynamics in Internet and Print Media