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UNCERTAINTY AND LIQUIDITY

Alberto Giovannini

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ABSTRACT

This paper studies a model where money is valued for the liquidity services it provides in the future. These liquidity services cannot be provided by any other asset. Changes in expectations of the value of future liquidity services affect the desired proportions of money and other assets in agents' portfolios, and, as a result, they change nominal interest rates and real stock prices.

The paper concentrates on the effects of stochastic fluctuations in the distribution of exogenous shocks. I find that changes in dividend risk have effects opposite to those in standard dynamic portfolio models without money. Furthermore, shifts between money and other assets that are driven by precautionary liquidity demand make nominal interest rates capture information about the uncertainty in the economy more accurately than any other prices in the asset markets.

Alberto Giovannini
622 Uris Hall
Graduate School of Business
Columbia University
New York, NY 10027
212/280-3471

1. Introduction

This paper explores the role of precautionary demand for money in asset pricing models. I assume that money provides--together with the standard unit-of-account, store-of-value, and means-of-payments services--liquidity services that other assets do not possess. Portfolio shifts between money and other assets can take place in response to changes in expectations of future liquidity services of money. I call these portfolio shifts changes in "precautionary" demand for money. I argue in this paper that, when money demand has these special features, the effects of changes in risk in asset prices are quite different from those arising in "standard finance" or nonmonetary models.

The analysis of precautionary money demand with a model borrowed from finance theory is first performed by Tobin [1958, pp. 71-82]. He studies a setup where agents' portfolios of nominal assets are restricted to money and long-term bonds (consols). Since fluctuations in interest rates may make the return on consols negative with positive probability, the optimal share of money balances in agents' portfolios is shown to be greater than zero. Given that money is the only liquid asset available, money demand is determined by optimally trading-off its expected "liquidity services"[1] with its opportunity cost represented by foregone interest.[2]

In the model used in this paper, as in Tobin's [1958], agents' demand for money depends on its expected future liquidity services. I study the dynamic general equilibrium model with cash-in-advance constraints first introduced by Lucas [1982], Stockman [1980], and Svensson [1985]. I adopt Svensson's

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[1] The capital losses on consols can be interpreted as losses originating from the "lack of liquidity" of these assets.

[2] In the presence of one-period bonds, money demand would of course be zero in this model.

assumptions on the timing of transactions in goods and asset markets, which make money more "liquid" than stocks or bonds.

As it turns out, the role of precautionary demand for money is best illustrated by analyzing the effects of fluctuations in uncertainty about nominal and real disturbances. Increased volatility of US stock returns in the 1970s is documented and discussed by Pindyck [1984];[3] Mascaro and Meltzer [1983] provide evidence on the increased volatility of money growth in the United States after October 1979, and analyze its effects using an ISLM model. The analysis of fluctuations in uncertainty is of interest also because of the growing number of papers suggesting the crucial role played by fluctuations in conditional variances in asset pricing models.[4] This paper discusses the general-equilibrium effects of random fluctuations in the distributions of dividend payments and monetary disturbances. Abel [1987] and Barsky [1986] analyze the effects of changes in dividend risk in general-equilibrium models similar to the one used here. Unlike these authors, however, I concentrate on a monetary model. This allows me to uncover the special role of precautionary demand for liquidity, and to study the effects of changes in monetary uncertainty.

Section 2 describes the model and the way distributions of exogenous shocks fluctuate randomly over time. Section 3 characterizes the equilibrium with time-varying distributions. Section 4 analyzes the effects of changes in dividend risk, while section 5 discusses the effects of changes in monetary uncertainty. In section 6 I tackle some empirical questions related to the

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[3] See also Black [1976], Malkiel [1979], and Poterba and Summers [1985].

[4] See, for example, Bollerslev, Engle and Wooldridge [1985], Domowitz and Hakkio [1985], and Engle, Lilien and Robins [1987].

effects of persistence in changes in volatility and the information content of nominal interest rates. Section 7 contains a few concluding remarks.

2. The Model

I study a variation of the model of Svensson [1985]. The timing of transactions in the goods and asset markets is such that next-period's consumption can only be assured by accumulating monetary balances. This gives rise to a forward-looking precautionary motive in the demand for money. Money is an asset which provides liquidity services as dividends. After these liquidity services are specified, money is priced as any other asset.

On one hand, I extend Svensson's work by allowing for time-varying returns distributions for the state variables. On the other hand, however, I impose some simplifying restrictions on tastes and distributions functions that allow me to concentrate exclusively on the effects of changing uncertainty.
Every period, the representative agent maximizes:

$$E_t \sum_{\tau=t}^{\infty} \frac{\beta^{\tau-t}}{1-\theta} c_\tau^{1-\theta} \tag{1}$$

where E_t stands for the expectations operator, conditional on information available at t. c is real consumption, θ is the elasticity of marginal utility, which implies an elasticity of intertemporal substitution $\sigma = 1/\theta$, and β is a utility discount factor ($0<\beta<1$). Maximization of (1) is subject to two constraints. The first is a liquidity constraint: every period the consumer enters the goods market with a predetermined amount of cash balances.

Consumption can only be purchased with cash:

$$c_t \leq \pi_t M_t \tag{2}$$

where π_t, the reciprocal of the nominal price level, is the purchasing power of money (the price of 1 dollar in terms of the consumption good); M_t is the predetermined amount of cash balances (which in equilibrium equals the exogenously given nominal money stock, $\bar{M}_t$).

After the goods market closes, the consumer enters the asset market. There he receives a money transfer equal to $(\omega_t-1)\bar{M}_t$, where ω_t is the stochastic gross rate of growth of the money stock, together with the gross return on his holdings of the real asset, which equals $(q_t+y_t)z_t$. q_t may be thought of as the price of the stock of a single representative firm producing GNP y_t, with a stochastic technology which uses no inputs. Thus y_t is also the dividend on the stock. z_t is the amount of the stock held by the consumer at the beginning of the period. These resources, together with the leftover money balances from the purchase of the consumption good, $\pi_t M_t - c_t$, are used to buy the real asset and the nominal money balances to be carried through next period, M_{t+1} and z_{t+1}. The budget constraint is thus:

$$\pi_t M_{t+1} + q_t z_{t+1} \leq (\pi_t M_t - c_t) + (q_t+y_t)z_t + \pi_t(\omega_t-1)\bar{M}_t \tag{3}$$

which implicitly defines wealth as follows:

$$w_t = \pi_t M_t + (q_t+y_t)z_t + \pi_t(\omega_t-1)\bar{M}_t$$

Every period, the clearing conditions for the goods market, the asset market, and the money market are:

$$M_{t+1} = \bar{M}_{t+1} = \omega_t\bar{M}_t \qquad z_t = 1 \qquad c_t = y_t \tag{4}$$

where $\bar{M}_t$ is the exogenously given money stock at the beginning of time t, before the money transfer. Let $s_t = [y_t, \omega_t]'$ denote the vector of supply innovations to the economy. I assume that every period the evolution of s is dictated by the following distribution function:

$$F(s_{t+1};\alpha_t) = \alpha_t F_b(s_{t+1}) + (1-\alpha_t)F_g(s_{t+1}) \tag{5}$$

with $\Pr(\alpha_{t+1} = 1) = \delta$, and $\Pr(\alpha_{t+1} = 0) = 1-\delta$, for every t, and α_t stochastically independent of s_t.

The distributions functions F_g and F_b obey the following restrictions:

$$\begin{array}{ll} \text{i.} & \omega_t \text{ and } y_t \text{ are stochastically independent;} \\ \text{ii.} & \int s_{t+1}dF_g(s_{t+1}) = \int s_{t+1}dF_b(s_{t+1}) = E(s); \\ \text{iii.} & dF_b(s_{t+1}) - dF_g(s_{t+1}) = MPS(s_{t+1}) \end{array} \tag{6}$$

Where $MPS(s_{t+1})$ is a mean preserving spread, as defined by Rotschild and Stiglitz [1970].

Assumption i. allows me to study changes in the distribution of y or ω in isolation. In section 4 I will restrict the distribution of ω to be constant over time. Section 5, conversely, will study time-varying risk in money surprises, and will restrict the y distribution to be invariant. These restrictions are imposed for expositional and analytical simplicity, but can be released with little consequences. Assumption iii. is the central feature of this paper. While the first moment of s is constant (from ii.), the probability density function of s is more "spread out" when α equals 1, than when α equals 0: this is consistent with stochastically changing variances of dividends or money supply.[5]

The i.i.d. "news" variable α_t is observed by the investor at the beginning of each period, together with the current realization of s, ω_t and y_t. The "news" variable tells him how much uncertainty there is in the economy. Although the distribution of exogenous shocks is stochastically changing over time, the investor--by observing α--knows for sure which distributions are next period's innovations drawn from. Finally, notice that since α is i.i.d. increases in uncertainty are purely temporary; and since F_g and F_b do not depend on s_t, variables in s_t are i.i.d. both conditional on α_t and unconditionally.

[5] As the analysis below makes clear, the fact that E(s) is constant does not imply that equilibrium expected returns are constant. Thus both first and second moments of returns are randomly fluctuating over time.

3. Equilibrium with Time-Varying Distributions

To characterize equilibrium I make use of a value function, implicitly defined as follows:[6]

$$v(w_t, M_t, s_t, \alpha_t, \bar{M}_t) = \text{MAX}\ \{c_t^{1-\theta}/(1-\theta) + \beta\int v(w_{t+1}, M_{t+1}, s_{t+1}, \alpha_{t+1}, \bar{M}_{t+1})dF(s_{t+1};\alpha_t)\}:$$
$$\text{subject to (2) and (3).} \tag{7}$$

Maximization of (7) with respect to c_t, M_{t+1} and z_{t+1} subject to the constraints (2) and (3) leads to the standard relation between the partial derivatives of the value function and the multipliers associated with the liquidity and wealth constraint (μ and λ, respectively):

$$v_w(t) = \lambda_t \qquad\qquad v_M(t) = \mu_t \pi_t,$$

and to the following first-order conditions:

$$(c_t:) \qquad c_t^{-\theta} = \lambda_t + \mu_t \tag{8}$$

$$(M_{t+1}:) \qquad \lambda_t \pi_t = \beta E[(\mu_{t+1} + \lambda_{t+1})\pi_{t+1} \mid \alpha_t] \tag{9}$$

$$(z_{t+1}:) \qquad \lambda_t q_t = \beta E[(q_{t+1} + y_{t+1})\lambda_{t+1} \mid \alpha_t] \tag{10}$$

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6 The use of this value function presupposes the existence of a unique stochastic stationary rational expectations equilibrium. See the discussion in Svensson [1985] about the assumption that such an equilibrium exists.

and:

$$c_t \leq \pi_t M_t, \quad \mu_t \geq 0, \quad \mu_t(\pi_t M_t - c_t) = 0 \tag{11}$$

where $E[\cdot|\alpha_t]$ stands for the expectation operator, conditional on α_t. Equation (8) says that the increase in consumption by one unit at time t increases marginal utility, but at the expense of a tighter liquidity constraint measured by μ, and of less wealth carried over to the future (valued by λ). Equation (9) is perhaps the distinguishing feature of the model: since money balances are predetermined at the beginning of each period, and the money market opens after the goods market, the cost of accumulating one extra unit of money balances is just measured by the marginal utility of wealth; next period's benefit, however, is the result of both the liquidity and the store-of-value roles of money. Equation (10) is the standard dynamic asset pricing equation, similar to those obtained by Lucas [1978] and Breeden [1979], among others. Notice however an important difference between this monetary model and the real models as for example Lucas's [1978]: since the marginal utility of consumption and the marginal utility of wealth differ by the liquidity services of real balances μ, and given the special timing of transactions in the goods and asset markets, the consumption-oriented asset pricing equation does not hold in the presence of aprecautionary demand for money.

The solution of the model is obtained by substituting the equilibrium conditions (4) into (8)-(11). The solution also requires assumptions about the parameters in (6), which affect the probability that agents at any point in time are liquidity constrained. I follow Lucas [1982], and assume that these parameters are such that the liquidity constraint is always binding, thus μ is always strictly greater than zero. In the context of this work, the assumption considerably simplifies the analysis of the effects of changes in distributions.[7] Although velocity is now constant, this assumption does not imply that the precautionary demand effects that characterize this model disappear. As I show below, changes in precautionary demand for money affect the <u>ratio</u> of money to other assets, although the <u>level</u> of real money balances is pinned down by the transactions constraint. When the liquidity constraint is always binding, we can solve the quantity equation and make use of the goods market equilibrium condition to obtain the purchasing power of money:

$$\pi_t = y_t/\tilde{M}_t \tag{11'}$$

Equation (11') is then substituted into (9) to obtain an expression for the marginal utility of wealth:

$$\lambda_t = \beta\, E[y_{t+1}^{1-\theta} \mid \alpha_t]/(\omega_t y_t) \tag{12}$$

The marginal utility of wealth is proportional to the marginal utility value of tomorrow's income, and negatively proportional to a term resembling the inflation tax: the gross rate of growth of money balances times the stock of real balances at time t.

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7 The assumption is <u>not</u> required to insure positive nominal interest rates, as Svensson [1985] shows. While the case where the liquidity constraint multiplier can also be zero is of interest because it is more general, the analysis of the model in that case would require to compute the probability that the liquidity constraint will be binding for all future dates. For this I would need other arbitrary assumptions about the distribution of s.

Finally, from (8) and (12), the value of the liquidity services of money is:

$$\mu_t = y_t^{-\theta} - \beta\, E[y_{t+1}^{1-\theta} \mid \alpha_t]/(\omega_t y_t) \tag{13}$$

4. Changes in Dividend Risk

In this section the distributions of nominal and real shocks are restricted as follows:

i. ω_t and y_t are stochastically independent;

ii. $\int s_{t+1} dF_g(s_{t+1}) = \int s_{t+1} dF_b(s_{t+1}) = E(s)$;

iii. $dF_b(s_{t+1}) - dF_g(s_{t+1}) = MPS(s_{t+1})$

iv. the distribution of ω_{t+1} is invariant across F_g and F_b.

By adding--for tractability--restriction iv., I explore in this section the effects of changes in dividend risk.

Increases in the volatility of future dividends affect the marginal utility of wealth λ. From equation (12) we obtain:

$$\frac{\beta\, E[y_{t+1}^{1-\theta} \mid \alpha_t = 1]}{\omega_t y_t} > \frac{\beta\, E[y_{t+1}^{1-\theta} \mid \alpha_t = 0]}{\omega_t y_t} \qquad \text{iff} \quad \theta > 1 \tag{14}$$

When the elasticity of intertemporal substitution $\sigma = 1/\theta$ is less than 1, the marginal utility of of wealth is a convex function of next period's dividends. An increase in the uncertainty of next period's dividends increases marginal utility of wealth. Thus, when present and future consumption are more "complementary" increases in dividend risk trigger a precautionary demand for savings. The intuition for this result can be obtained using the framework suggested by Selden [1979], who analyzes the relative role of risk aversion and intertemporal substitution in the problem of savings under uncertainty.[8] With positive risk aversion an increase in dividend (and consumption) risk decreases the certainty-equivalent level of future consumption. The response of current consumption depends on the degree of complementarity or substitutability of consumption in the two periods. When intertemporal substitution is low--$\theta>1$--a decrease in certainty-equivalent future consumption calls for a decrease in current consumption. Since current consumption is given from goods market equilibrium, the adjustment is through the marginal utility of wealth: λ increases. When intertemporal substitution is high--$\theta<1$-- a decrease in certainty-equivalent future consumption triggers an incipient increase in current consumption, that gives rise to a fall in the marginal utility of wealth λ.[9]

Notice in addition that since α_t is independent of y_t,

$$\mu_t(\alpha_t=1) < \mu_t(\alpha_t=0) \qquad \text{iff } 1/\theta < 1 \tag{15}$$

The liquidity value of money is lower at time t when $\alpha_t=1$ (and $\theta>1$). (15) is an

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8 In a utility function like (1) the elasticity of intertemporal substitution and the coefficient of relative risk aversion are represented by the same parameter. Therefore it is difficult to isolate the role of the two effects in the optimal response to increases in risk. Selden's analysis applies to the two-period consumption-saving problem. His main argument, however, applies also to the multi-period model used here.

9 See Barsky [1986] for a formal discussion of this issue in the context of a two-period model without money.

implication of (13) and (14). An increase in risk (with $\theta>1$), by increasing the demand for savings, increases the store-of-value services of money, and therefore decreases its liquidity value at time t.

I now turn to the response of the stock price and the nominal interest rate. A recursive application of equation (10) yields:

$$q_t = \frac{\beta}{\lambda_t} \sum_{j=0}^{\infty} \beta^j E[\lambda_{t+1+j} y_{t+1+j} \mid \alpha_t] \tag{16}$$

Since $\lambda_t y_t$ is a function of α_t and ω_t (from equation (12)), all terms of the summation in (16) are equal to the unconditional expectation, $E(\lambda y)$. Thus we have:

$$q_t = E(\lambda y)\beta/[(1-\beta)\lambda_t] \tag{17}$$

The stock price is inversely proportional to λ_t. Given that $\alpha_t=1$ is associated with higher values of λ_t, $\alpha_t=1$ implies a decrease of q_t, other things equal.

Strikingly, this result is opposite to those obtained in models without money, as the ones studied by Abel [1987] and Barsky [1986]. To illustrate the effects of increases in dividend risk in a model without money, I take an economy that is identical in all other respects to the one studied here, except that the liquidity constraint (2) is removed. In that case the marginal utility of wealth coincides with the marginal utility of consumption. The stock pricing equation becomes:

$$y_t^{-\theta} q_t = \beta\, E[(q_{t+1}+y_{t+1})y_{t+1}^{-\theta} \mid \alpha_t] \tag{18}$$

and can be written as follows:

$$y_t^{-\theta} q_t = \beta\, E[q_{t+1} y_{t+1}^{-\theta} \mid \alpha_t] + \beta\, E[y_{t+1}^{1-\theta} \mid \alpha_t] \qquad (19)$$

When $\theta > 1$ $\alpha_t = 1$ does increase the second term on the right-hand side of (19), but leaves the first term unaffected, as a recursive application of (19) can immediately show. Since the marginal utility of wealth is now given, the stock price <u>increases</u> in response to higher dividend risk, when intertemporal substitution is low ($\theta > 1$). The reason why stock prices increase is the same as the reason why the marginal utility of wealth increases in a monetary economy: in both cases individual's desire to postpone consumption to next period has gone up.

Why then do stock prices fall in the monetary economy? Unlike in the real model of (18) and (19), shares are less liquid than money, and cannot be used to provide for next period's consumption, since next period's asset market opens after the goods market is closed. Indeed, the only way to shift consumption from the current to the next period is to accumulate money balances.[10] Thus if intertemporal substitution is low the precautionary demand for money increases when next period's dividend risk increases. An increase in dividend risk brings about a portfolio shift out of stocks and into money. This effect is also illustrated by the change in expected future liquidity services of money and the nominal interest rate. A forward shift of equation (13) yields:

$$\mu_{t+1}\pi_{t+1} = \mu_{t+1} y_{t+1}/(\bar{M}_t \omega_t) = \{y_{t+1}^{1-\theta} - \beta\, E[y_{t+2}^{1-\theta} \mid \alpha_{t+1}]/\omega_{t+1}\}/(\bar{M}_t \omega_t) \qquad (20)$$

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[10] Of course the consumer can substitute present and future consumption beyond time t+1 by accumulating any asset besides money.

Taking expectations of (20) conditional on information at time t, establishes that the increase in expected liquidity services of money in response to higher dividend risk is identically equal to the increase in the marginal utility of wealth.

The increase in money demand generated by an increase in dividend risk--when intertemporal substitution is low--is also associated with higher nominal interest rates. Following the procedure outlined by Lucas [1984] we can price any asset in this economy, once its payoffs are clearly specified. In the case of a one-period nominal bond, we need to determine--as Svensson [1985]--the nominal price of an asset which is purchased in the asset market at time t, at the nominal price $(1+i_t)^{-1}$, and pays 1 unit of money when the asset market is open at time t+1. Applying the pricing equations presented above yields:

$$(1+i_t)\beta E[\lambda_{t+1}\pi_{t+1} | \alpha_t] = \lambda_t \pi_t \tag{21}$$

From equations (11') and (12) we have:

$$\lambda_t \pi_t = \beta E[y_{t+1}^{1-\theta} | \alpha_t]/(\bar{M}_t \omega_t) \tag{22}$$

Since $\lambda_{t+1}\pi_{t+1}$ depends on α_{t+1}, $E[\lambda_{t+1}\pi_{t+1} | \alpha_t]$ is independent of α_t. Thus from (17) we have:

$$i_t\ (\alpha_t=1) \ > \ i_t\ (\alpha_t=0) \qquad \text{iff } \theta > 1 \tag{23}$$

The intuition for this result can be obtained by solving (21) after substituting for $\lambda_t \pi_t$ with (9):

$$i_t = E[\mu_{t+1}\pi_{t+1} | \alpha_t]/E[\lambda_{t+1}\pi_{t+1} | \alpha_t] \quad (24)$$

Investors in nominal bonds are required to give up money balances at time t, to obtain principal and interest back when asset markets open at t+1. The opportunity cost of this contract is represented by the expected liquidity services of money at t+1. An increase in dividend risk increases the opportunity cost of holding a nominal bond (when intertemporal substitution is low) and therefore requires an increase in equilibrium nominal bond returns. Stock and bond prices fall since neither asset is liquid enough to be used to purchase consumption goods at t+1.

5. Changes in the Volatility of Money Growth

In this section the distributions of nominal and real shocks are restricted as follows:

i. ω_t and y_t are stochastically independent;

ii. $\int s_{t+1} dF_g(s_{t+1}) = \int s_{t+1} dF_b(s_{t+1}) = E(s)$;

iii. $dF_b(s_{t+1}) - dF_g(s_{t+1}) = MPS(s_{t+1})$

iv. the distribution of y_{t+1} is invariant across F_g and F_b.

This section analyzes the effects of changes in the distribution of monetary innovations: restriction iv. now allows me to study the effects of increases in money-growth risk in isolation from dividend risk.

From equation (12) we know that, as long as nominal and real shocks are stochastically independent, the distribution of future monetary shocks does not affect the relative valuation of current and next-period consumption, and therefore λ_t is unaffected by an increase in the volatility of ω_{t+1}.

Consider now the effects of increases in uncertainty of monetary shocks on future wealth and liquidity. Shifting equation (9) forward one period and taking expectations conditional on α_t we have:[11]

$$E(\lambda_{t+1}\pi_{t+1} \mid \alpha_t) = \frac{\beta}{M_t\omega_t} E\,[y_{t+2}^{1-\theta} \mid \alpha_t]\, E\,[\frac{1}{\omega_{t+1}} \mid \alpha_t] \tag{25}$$

From Jensen's inequality, an increase of uncertainty in monetary innovations increases (at time t) the expected purchasing power of monetary balances at time t+2, and therefore increases the expected marginal utility of nominal wealth at time t+1, $\lambda_{t+1}\pi_{t+1}$. Since π_{t+1} is given by M_t, ω_t, and y_{t+1}, all of which are independent of α_t and, of course, ω_{t+1}, it follows that the expected marginal utility of real wealth, λ_{t+1}, also increases. Notice that this result,[12] unlike in the case of changes in dividend risk, is independent of preferences.

The expected liquidity value of money at t+1 decreases, as can be verified below:

$$E[(\mu_{t+1}+\lambda_{t+1})\pi_{t+1} \mid \alpha_t] = E[\, y_{t+1}^{1-\theta} \mid \alpha_t\,]/(M_t\,\omega_t) \tag{26}$$

Since the right-hand side of (26)--under the assumptions of this section--is

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[11] Here I exploit the assumption that ω and y are stochastically independent.

[12] This result is well known from the literature on bond prices with price-level risk, see Fischer [1975], Gertler and Grinols [1982], and Stulz [1986].

independent of α_t, the increase in the expectation of $\lambda_{t+1}\pi_{t+1}$ has to be matched by a decrease in the expectation of $\mu_{t+1}\pi_{t+1}$. On the other hand, somewhat surprisingly, the expected liquidity value of money at time t+2 remains constant. Note, by shifting equation (25) forward one period, that the expected wealth value of money at t+2 increases as much as the left-hand-side of equation (25): this happens simply because the expected purchasing power of money at t+2 has increased.[13] The constancy of the expected liquidity value of money at t+2 is proved using the following equation:

$$E[\lambda_{t+1}\pi_{t+1} \mid \alpha_t] = \beta E[\lambda_{t+2}\pi_{t+2} \mid \alpha_t] + \beta E[\mu_{t+2}\pi_{t+2} \mid \alpha_t] \qquad (27)$$

Since both the left-hand side and the first term on the right-hand side of (27) increases with $\alpha_t = 1$ by the same amount, the expected liquidity value of money at t+2 is unaffected by α_t: the increase in the expected purchasing power of money at t+2 decreases the multiplier for the liquidity constraint by a proportional amount.

Fluctuations in the riskiness of monetary innovations have also interesting effects on the term structure of interest rates. From the analysis in the previous section, we know that nominal interest rates at different maturities are determined by $\lambda_t\pi_t$ and the expectations of $\lambda_{t+i}\pi_{t+i}$ for bonds of maturity i periods. The results above imply that both the 1-period and the 2-period nominal bond rates decrease when uncertainty about next period's money growth rate increases. The 1-period bond price increases because the wealth-value of its payoff is expected to increase, whereas the 2-period bond price increases

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[13] It is easy to prove that $E(\lambda_{t+2}y_{t+2})$ is unaffected by a change in α_t.

because the expected purchasing power of 1 unit of money at t+2 has increased. This last result is qualitatively identical to that obtained by Stulz [1986] using a continuous-time model. The implication is that the expected future 1-period interest rate is unaffected: this can be proved by applying equation (24).

Finally, changes in volatility of money growth affect also stock prices. The discussion above has shown that increases in uncertainty on next period's money growth ω_{t+1} leaves λ_t and $\lambda_{t+i}y_{t+i}$ (i=2,3,...) unaffected, but increases the expectation of $\lambda_{t+1}y_{t+1}$. An application of the stock pricing equation (16) shows that stock prices increase as a result. The response in stock prices, as well as that of bond prices, is explained by the desire of consumers to accumulate those assets whose expected return has increased in response to an increase in money-growth uncertainty.[14]

In summary, while an increase in dividend risk (with low intertemporal substitution) brings about an incipient portfolio readjustment out of stocks and bonds and into money, an increase in money-growth uncertainty unambiguously triggers an increase in relative demand for stocks and bonds.

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[14] Notice from equation (26) that the expected return of money balances at time t for t+1 is unaffected by an increase in money growth risk.

6. Two Empirical Questions

The analysis of the effects of changes in uncertainty of nominal and real shocks reveals a number of interesting and potentially important effects which are linked to the "forward-looking" or "precautionary" demand for money. The most striking feature of the results is that the typical predictions of models without money on the effects of changes in dividend risk are exactly reversed in the presence of a precautionary demand for money.

In this section I explore the empirical implications of the model by concentrating on two questions. The first regards the i.i.d. assumption on changes in distributions functions. The increase in precautionary demand for money when dividend risk increases is intuitively justified by the observation that the increase in risk is <u>immediate</u> (occurring before next period's asset markets open) and <u>transitory</u> (in that it does not persist after next periods assets are traded). Thus it is important to explore the role of persistence in changes in the distributions of exogenous shocks.

The second question is about the information content of nominal interest rates. I discuss whether the model can rationalize some empirical regularities suggesting that nominal interest rates have significant incremental predictive power for both real variables and financial variables.

6.1 Temporary vs. Permanent Changes in Uncertainty

Rather than parametrizing explicitly the persistence of changes in volatility I explore the effects of permanent changes in volatility by studying two economies, which are identical in all respects, but have different distributions of exogenous shocks. As above, I discuss the effects of higher risk in dividend and money-supply shocks separately.

Equation (12) implies that in an economy where dividend risk is high, λ_t is higher for every t, if the elasticity of intertemporal substitution is low, i.e. $\theta>1$. The interpretation of this result is identical to that of temporary changes in risk studied above in section 4. What happens to stock prices? Since all expected future λs are higher with higher dividend risk,[15] equation (16) shows that stock prices are unaffected by permanently higher dividend risk. As in section 4., a higher λ_t arising from higher dividend risk--other things equal--provokes a fall in stock prices at t, since shares are less liquid than money. In economies with permanently higher dividend risk this liquidity loss is however fully compensated by future gains, since the wealth-value of all expected future dividend payments is proportionately higher.

The conclusion is that a fall in stock prices in response to an increase in dividend risk discussed in section 4 is indeed due to the temporary nature of the risk increase. The analysis of economies affected by higher dividend risk suggests that the longer the persistence of dividend volatility changes, the smaller should be the effect on stock prices (This effect is positive or negative depending on intertemporal substitution). Notice however that changing the persistence of volatility fluctuations does not appear to reconcile the results I obtain in a monetary economy with those obtained in nonmonetary models by Abel [1987] and Barsky [1986]. Thus the importance of precautionary money demand effects in finance models is further stressed.

An application of equations (13) and (24) shows that economies with higher dividend risk should not display higher nominal interest rates. The intuition is similar to that provided for stock prices. The expected liquidity value of

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[15] This is also shown by recursively applying equation (12): the change in λ_t and $E(\lambda_{t+j}y_{t+j})$ for j=1,2,.. is the same.

money is unchanged because the expected utility of consumption is affected as much as the marginal utility of wealth at all periods. Thus from equation (24) we know that nominal interest rates should be the same. When dividend risk increases permanently the incipient increase in precautionary money demand is exactly offset by the increase in the expected wealth-value of interest payments.

As far as nominal disturbances are concerned, equations (25) and (27) can be used to show that the transitory or permanent nature of volatility shocks should only affect the term structure of interest rates. While transitory volatility shocks in money supply innovations leave long-term interest rates unaffected in an economy where nominal and real disturbances are conditionally i.i.d., economies with higher volatility of monetary innovations display higher stock prices and higher prices for short-term and long-term discount bonds.

6.2 The Information Content of Nominal Interest Rates

The number of empirical papers documenting the non-constancy of returns distributions for several different assets has been growing in the recent past. A partial list of references documenting the time variation of first and second moments of returns for a large spectrum of assets includes Hansen and Singleton [1982], Bollerslev, Engle and Wooldridge [1985], Cumby [1986], Hansen and Hodrick [1980], Christie [1982], and Campbell [1985]. The most interesting finding in this literature is that nominal interest rates help predict changes in volatility. Christie [1982] shows that the nominal interest rate has additional explanatory power over the debt/equity ratio in 354 out of 379 projection equations of the volatility of individual stock returns: an increase in nominal interest rates is associated with a predictable increase in volatility of stock returns. Giovannini and Jorion [1987] show that nominal interest rates are positively correlated with the volatility of returns and negatively correlated with nominal risk premia, both in the US stock market and in the foreign exchange market. Additional evidence on the correlation of nominal interest rates--or interest rate differentials--with volatility of returns is provided by Cumby and Obstfeld [1984], and Hodrick and Srivastava [1984]. Finally, Litterman and Weiss [1985] document an apparently puzzling phenomenon, whereby nominal interest rates have incremental predictive power for future macroeconomic variables, in such a way that cannot be easily justified with the currently available families of macroeconomic models.

The model in this paper has important implications for the ability of nominal interest rates to predict movements of conditional distributions of exogenous shocks, and of the endogenous conditional second moments of asset returns. Like Fama [1982] and Litterman and Weiss [1985], I consider a world where the relevant information exceeds current and past observations of all real and monetary variables: current information includes the knowledge of the degree of uncertainty about future exogenous shocks. Changes in uncertainty, however, are not directly observable.

The most important result is that, with i.i.d. shocks, nominal interest rates are a sufficient statistic for the uncertainty in the economy, represented by the uncertainty of nominal and real shocks. Indeed, nominal interest rates are the best (and perfect) predictors of transitory changes in uncertainty. The reason is that with constant i.i.d. distributions nominal interest rates are constant in this model, as appears from equation (24). While this result would-

-of course--not hold in its current form if the i.i.d. assumption was relaxed, its importance is that it suggests an economic interpretation of the evidence from atheoretical projection equations. If intertemporal substitution is less than 1, more uncertainty about future dividends induces agents to stay more "liquid," by shifting out of stocks and bonds, and into money. As a result, nominal interest rates increase, and stock prices fall. Thus the model can reproduce and explain the findings by Christie [1982] and Giovannini and Jorion [1987] that nominal interest rates are positively correlated with conditional variances of returns.[16]

7. Concluding Remarks

This paper has studied some of the effects associated with the presence of precautionary demand for money in a general-equilibrium asset pricing model. Changes in dividend risk produce stock-price and interest-rate movements that differ markedly from those that occur in the absence of precautionary money demand. With low interemporal substitution, more dividend risk makes investors want to stay more "liquid" and thus prompts a portfolio shift out of stocks and into money. In the model, this effect makes interest rates predict next-period dividend uncertainty more accurately than any other asset price. I also find that changes in monetary-policy uncertainty affect both stock prices and nominal interest rates.

The effects I illustrate in the paper appear worth exploring further, given the nature of the empirical regularities that standard asset pricing models

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[16] A positive relationship between nominal interest rates and returns volatility is additional evidence supporting the hypothesis that intertemporal substitution is empirically very low.

cannot fully explain. The most important of these empirical facts is of course the predictive content of nominal interest rates. One shortcoming, however, is that some of the assumptions used in the model are arguably little plausible empirically. Perhaps liquidity services are performed in reality by short-term assets rather than by money. My objective in this paper was not to criticize current monetary models, or to propose new ones: rather I wanted to identify shifts in agents' portfolios associated with phenomena that are not commonly discussed in finance models. If liquidity services were offered by assets other than cash, like Treasury bills or overnight deposits, the qualitative portfolio shifts I study in this paper would most likely still be present in some form, although their quantitative impact on short- and long-term interest rates might be somewhat different.

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