NBER WORKING PAPER SERIES

AMBIGUITY AND THE TRADEOFF THEORY OF CAPITAL STRUCTURE

Yehuda Izhakian
David Yermack
Jaime F. Zender

Working Paper 22870
http://www.nber.org/papers/w22870

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2016, Revised March 2021

We are indebted to Adelina Barbalau, Tony Cookson, Rob Dam, Shaun Davies, Erica Soler-Hampejsek, Mervyn King, Chris Leach, Johann Reindl, and Ed Van Wesep for valuable discussions and seminar and conference participants at BI Oslo, Erasmus University, Manhattan College, the University of Mannheim, the University of Minnesota, the University of Oklahoma, the University of Western Australia, the 7th International Conference of the Financial Engineering and Banking Society, the Southern Finance Association Meeting, and the 44th European Finance Association Meeting for helpful comments. We also thank Gustavo Manso (the editor), two anonymous referees, and an anonymous associate editor for comments that have improved the paper. Izhakian acknowledges hosting by the Stern School of Business, New York University. Part of this paper was written while Yermack was a visiting Professor at Erasmus University Rotterdam. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Ambiguity and the Tradeoff Theory of Capital Structure
Yehuda Izhakian, David Yermack, and Jaime F. Zender
NBER Working Paper No. 22870
November 2016, Revised March 2021
JEL No. C65,D81,D83,G32

ABSTRACT

We examine the impact of ambiguity, or Knightian uncertainty, on the capital structure decision, using a static tradeoff theory model in which agents are both ambiguity and risk averse. The model confirms the well-known result that greater risk—the uncertainty over outcomes—leads firms to decrease leverage. Conversely, the model indicates that greater ambiguity—the uncertainty over the probabilities associated with the outcomes—leads firms to increase leverage. Using a theoretically based measure of ambiguity, our empirical analysis presents evidence consistent with these notions, showing that ambiguity has an important and distinct impact on capital structure.

Yehuda Izhakian
Zicklin College of Business
One Bernard Baruch Way
New York, NY 10010
yud@stern.nyu.edu

David Yermack
Stern School of Business
New York University
44 West Fourth Street, Suite 9-160
New York, NY 10012
and NBER
dyermack@stern.nyu.edu

Jaime F. Zender
Leeds School of Finance
University of Colorado at Boulder
419 UCB
Boulder, Colorado 80309-0419
jaime.zender@colorado.edu

1 Introduction

Every significant financial decision entails ambiguity, or Knightian uncertainty.[1] Ellsberg (1961) and others demonstrate that individuals act as if they are averse to this aspect of uncertainty. Nevertheless, the canonical results in finance rely upon expected utility theory, which can be interpreted as either assuming away ambiguity or assuming that individual preferences are neutral to ambiguity.[2] Ignoring ambiguity implies that our theoretical characterizations of optimal behavior are at best incomplete and at worst misleading.

We introduce ambiguity and aversion to ambiguity into the static tradeoff theory of corporate capital structure and present empirical tests of the model's predictions. While the market's valuation of the firm's debt and equity securities represents the focus of the capital structure question, most empirical studies of leverage (e.g., Titman and Wessels, 1988; Rajan and Zingales, 1995; Faulkender and Petersen, 2006; Lemmon et al., 2008; Frank and Goyal, 2009) do not consider ambiguity. In our tests, we adapt recent empirical models of the static tradeoff theory by including a measure of ambiguity derived from the underlying decision-theoretic framework. Our tests provide strong empirical evidence that ambiguity has an important and distinct impact on leverage.

The famous theorem of Modigliani and Miller (1958) asserts that with perfect capital markets, the capital structure choice does not influence firm value. In tradeoff theory models, frictions related to taxes, transaction costs, and imperfect information lead to optimal capital structure choices (e.g., Kraus and Litzenberger, 1973). We extend these theories to account for ambiguity and aversion to ambiguity. We show that in a perfect capital market, Modigliani and Miller's (1958) irrelevance proposition continues to hold in the presence of ambiguity. While the costs of debt and equity capital both increase with ambiguity, the weighted average cost of capital remains invariant

[1] Risk refers to a case in which an outcome to be realized is a-priori unknown, but the odds of all possible outcomes are perfectly known. Ambiguity, or Knightian uncertainty, refers to the case where not only is the outcome a-priori unknown, but also the odds of possible outcomes are either unknown or not uniquely assigned.

[2] The literature, however, extends expected utility theory to accommodate model (or parameter) uncertainty and explores its implications for asset pricing (e.g., Coles and Loewenstein, 1988; Coles et al., 1995; Avramov, 2002).

to changes in leverage for a given level of ambiguity.

Ambiguity's impact on the cost of equity highlights another prominent issue in capital structure research, the over-investment problem (Jensen and Meckling, 1976). For a given debt level, increasing the risk of the firm's cash flows increases the value of levered equity (e.g., Hartman, 1972; Abel, 1983), so that equity holders will accept negative net present value projects if they provide a sufficient increase in risk. Conversely, an increase in ambiguity reduces the value of levered equity. Since equity represents a call option on the assets of a levered firm, higher ambiguity reduces the value of this option, because ambiguity-averse investors reduce their perceived probabilities of events with high payoffs (e.g., Izhakian and Yermack, 2017; Augustin and Izhakian, 2020). For the same reason, the value of debt decreases, but to a smaller extent. Thus, consistent with Garlappi et al. (2017), investments that entail increased ambiguity may suffer from an under-investment problem.

Our tradeoff theory model highlights a sharp distinction in the impacts on capital structure choice of the two aspects of uncertainty, ambiguity and risk. Our model includes taxes and bankruptcy costs as frictions that lead a firm to an optimal leverage choice. At the optimum, the marginal expected cost of bankruptcy equals the marginal expected tax benefit of debt, and these two forces offset. In the model, an increase in the level of risk associated with the firm's cash flows results in less debt financing, while an increase in the level of ambiguity leads to greater use of debt. When risk increases, the marginal probability of bankruptcy rises, increasing the expected cost of debt and decreasing the expected tax benefit of debt. Obtaining the new optimum, therefore, entails reducing the use of debt. The economic intuition for why increased ambiguity results in higher optimal leverage is similar. Greater ambiguity causes ambiguity-averse investors to under-weight the perceived probabilities of future states, with high cash flow states being under-weighted more than low cash flow states. This decreases expected tax benefit of debt to a greater extent than the expected cost of debt is decreased, when the level of ambiguity increases. It also implies that the perceived marginal probability of the bankruptcy state decreases. The marginal expected bankruptcy cost is then lower than is the marginal expected tax benefit to debt financing. Only an increase in leverage can restore balance between these forces at the new optimum.

Because both risk and ambiguity impact the leverage decision, measuring ambiguity independent of aversion to ambiguity, risk, and aversion to risk is the main challenge to conducting empirical tests of the theoretical model. The empirical measure of ambiguity we employ is rooted in the decision theory framework of expected utility with uncertain probabilities (EUUP) (Izhakian, 2017, 2020). In that framework, aversion to ambiguity takes the form of aversion to mean-preserving spreads in probabilities. Therefore, the degree of ambiguity can be measured by the volatility of probabilities, just as the degree of risk can be measured by the volatility of outcomes. Our measure of ambiguity considers the variation in the probabilities of events without incorporating the magnitudes of the outcomes associated with those events (outcome-independence). In contrast, standard measures of risk consider the variation in the magnitudes of the outcomes without incorporating variation in the probabilities of the associated events.

We follow the methodology of recent studies and estimate firm-level ambiguity as the volatility of daily stock return probabilities, estimated from intraday returns data (e.g., Izhakian and Yermack, 2017). Studies by Brenner and Izhakian (2018) and Augustin and Izhakian (2020) conduct extensive tests, at the market level and the firm level, respectively, to allay concerns that this measure of ambiguity captures other well-known dimensions of uncertainty.

To test the hypothesis of a positive relation between ambiguity and leverage, we investigate whether a firm's leverage ratio is related to its lagged measure of ambiguity. We account for the unobservable firm-specific component of leverage, identified by Lemmon et al. (2008), by taking first differences of all the variables and examining the extent to which innovations in ambiguity are followed by changes in leverage in the subsequent year. In each regression specification, the estimated coefficient on the innovation in ambiguity is positive and significant, consistent with the model's prediction. Further, ambiguity exhibits economic significance of the same magnitude as risk, firm size, and asset tangibility, and greater than the economic significance of other standard control variables. Subgroup analysis, by firm size and by leverage, also provides findings consistent with the model's predictions, and our results are robust to including alternative uncertainty factors (e.g., the volatility of return means, the volatility of return volatilities, and disagreement among analysts) and market-microstructure factors (e.g., illiquidity and bid-ask spread) in the empirical model.

The literature on decision making under ambiguity has proposed different models, which are "seemingly different [...] rarely related to one another, and often expressed in drastically different formal languages" (Epstein and Schneider, 2010). Unlike our preference representation, most other representations of preferences toward ambiguity are outcome dependent, which may deliver different conclusions. For example, since preferences for ambiguity in the smooth model (Klibanoff et al., 2005) are outcome dependent, formed as preferences over expected utilities, they deliver conclusions different than ours (Lee, 2014). Preferences for ambiguity in the max-min model (Gilboa and Schmeidler, 1989) are also outcome dependent, which provides the different conclusions about optimal capital structure in Ban and Chen (2019). Inherently, outcome dependent preferences for ambiguity implies risk dependence. As a result, under outcome dependent preferences for ambiguity, the effect of ambiguity is confounded with the effect of risk, such that it mirrors the same effect as risk.

In a related recent paper, Malenko and Tsoy (2020) develop a model of security design when an outside investor faces ambiguity. Their model provides a signaling justification for choosing between external debt or external equity, and it delivers the intuition that for low ambiguity high leverage is optimal, while for high ambiguity low leverage is optimal (a negative relation between ambiguity and leverage). Our model implies the opposite, a positive relation between ambiguity and leverage. Our model examines different financial frictions than those in Malenko and Tsoy (2020). In the signalling model that Malenko and Tsoy (2020) propose, there are no taxes or bankruptcy costs, and asymmetric information is the primary friction. In our model, as in standard tradeoff theory, there is a tension between the tax benefits of debt and expected bankruptcy costs. Asymmetric information, however, plays no role in our model.

2 The model

We develop a static model of the impact of ambiguity and ambiguity aversion on corporate capital structure choice. The model has two important, distinguishing features. First, managers' and investors' preferences for ambiguity are outcome independent and, therefore, independent of risk and attitude toward risk. Outcome-independent preferences for ambiguity are necessary to differentiate completely the effects of ambiguity and risk. Second, agents are averse to both ambiguity and risk.

Capital structure decisions are determined by value (perceived expected utility), which is determined in part by the perceived expected returns. These perceived expected returns are affected by risk and ambiguity. To illustrate, suppose a firm considers investing \$400,000 where the payoff is determined by a flip of an unbalanced coin for which the manager does not know the odds. The payoff is given as \$1,000,000 for heads and \$0 for tails. Suppose now information arrives, raising the payoff for heads to \$2,000,000. In this case, risk and the expected payoff increase, such that the manager finds this investment more attractive. However, ambiguity has not changed since there is no new information regarding likelihoods. Suppose instead the new information indicates a greater assessed degree of ambiguity about the coin. An ambiguity-averse manager lowers her perceived probability of the high payoff and the investment opportunity becomes less attractive. In this case, the manager has no reason to change her assessment of the payoffs in the different events, as the new information concerns only the likelihoods.

This example highlights a critical property: ambiguity is outcome independent, up to a state-space partition. That is, if the outcomes associated with events change, while the induced partition of the state space into events (the set of events) remains unchanged, then the degree of ambiguity remains unchanged since the probabilities remain unchanged. Furthermore, outcome dependence implies risk dependence. To measure ambiguity independent of risk, the underlying preference for ambiguity must apply exclusively to the probabilities of events, independent of the outcomes associated with these events.

2.1 The decision-theoretic framework

To introduce ambiguity into the optimal capital structure model, we employ the preference representation in Choquet expected utility (CEU, Schmeidler, 1989), augmented with the theoretical framework contained in expected utility with uncertain probabilities (EUUP, Izhakian, 2017).[3] The EUUP model provides an axiomatic construct of the capacities (nonadditive probabilities) employed in the CEU model, based upon the ambiguity in the environment (beliefs) and the decision-makers' aversion to (attitude toward) ambiguity. Under the EUUP model, preferences for ambiguity apply exclusively to the uncertain probabilities of future events (are outcome independent), such that aversion to ambiguity is defined as an aversion to mean-preserving spreads in probabilities. Thereby, ambiguity can be examined independent of risk, as the volatility of the uncertain probabilities of future events. This separation allows theoretical examination of the distinct impacts of ambiguity and risk. It also provides a model-derived, risk-independent measure of ambiguity that can be employed to test the theory (Izhakian, 2020).[4]

Formally, the investor, who values a risky and ambiguous payoff $X : \mathcal{S} \to \mathbb{R}$, possesses a set $\mathcal{P}$ of prior probability distributions P over events (subsets of states in the set of states $\mathcal{S}$). The set $\mathcal{P}$ of prior distributions has a second-order prior probability distribution ξ (a distribution over

[3]Under some conditions, the max-min expected utility model (MEU, Gilboa and Schmeidler, 1989) can generate the same conclusions as CEU. However, MEU does not allow a separation of beliefs regarding ambiguity from attitudes toward ambiguity. Further, it is challenging to utilize the MEU or the smooth model of ambiguity aversion (Klibanoff et al., 2005) in comparative statics of the effect of ambiguity, since preferences for ambiguity in these models are outcome-dependent and, therefore, risk-dependent. As a result, ambiguity and risk in these models are confounded, such that the impact of a change in risk may mistakenly be attributed to a change in ambiguity.

[4]The measurement of ambiguity independent from risk poses a challenge for other frameworks that do not separate ambiguity from attitudes toward ambiguity (e.g., Gilboa and Schmeidler, 1989; Schmeidler, 1989) or in which preferences for ambiguity are outcome-dependent (e.g., Klibanoff et al., 2005; Chew and Sagi, 2008).

probability distributions).[5] Each probability distribution $\mathrm{P} \in \mathcal{P}$ is represented by a cumulative probability distribution $\mathrm{P}(\cdot)$ and a marginal probability function $\varphi(\cdot)$.[6] The investor evaluates the expected utility of a risky and ambiguous payoff by the CEU model (Schmeidler, 1989)

$$\mathrm{V}(X) \quad = \quad \int_{\mathcal{S}} \mathrm{U}(\cdot)\, d\mathrm{Q}, \tag{1}$$

where Q stands for the capacity.

In the EUUP model, a capacity is referred to as a perceived probability and is formed by the minimum unique certain probability value that the investor is willing to accept in exchange for the uncertain probability of a given favorable event, given her attitude toward uncertainty of probabilities. Accordingly, the perceived probability is assessed by

$$\mathrm{Q}(x) = \Upsilon^{-1}\left(\int_{\mathcal{P}} \Upsilon\left(1 - \mathrm{P}(x)\right) d\xi\right). \tag{2}$$

The marginal perceived probability can be usefully approximated as[7]

$$d\mathrm{Q} \quad = \quad \mathrm{E}\left[\varphi(x)\right]\left(1 + \frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}(x)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}(x)\right]\right)} \mathrm{Var}\left[\varphi(x)\right]\right), \tag{3}$$

where the function Υ, called the outlook function, captures the investor's attitude toward ambiguity;[8] the expected marginal and cumulative probabilities are computed using ξ, such that $\mathrm{E}\left[\varphi(\cdot)\right] = \int_{\mathcal{P}} \varphi(\cdot)\, d\xi$ and $\mathrm{E}\left[\mathrm{P}(\cdot)\right] = \int_{\mathcal{P}} \mathrm{P}(\cdot)\, d\xi$; and $\mathrm{Var}\left[\varphi(\cdot)\right] = \int_{\mathcal{P}} \left(\varphi(\cdot) - \mathrm{E}\left[\varphi(\cdot)\right]\right)^2 d\xi$ defines the variance of the marginal probability.

Using perceived probabilities, the investor assesses the expected utility of a risky and ambiguous

[5]Formally, there is a probability space $(\mathcal{S}, \mathcal{E}, \mathrm{P})$, where $\mathcal{S}$ is an infinite state space; $\mathcal{E}$ is a σ-algebra of subsets of the state space (a set of events); a λ-system $\mathcal{H} \subset \mathcal{E}$ contains the events with an unambiguous probability (i.e., events with a known, objective probability); and $\mathrm{P} \in \mathcal{P}$ is an additive probability measure. The set of all probability measures $\mathcal{P}$ is assumed to be endowed with an algebra $\Pi \subset 2^{\mathcal{P}}$ of subsets of $\mathcal{P}$ that satisfies the structure required by Kopylov (2010). Π is equipped with a unique countably-additive probability measure ξ that assigns each subset $A \in \Pi$ with a probability $\xi(A)$.

[6]To simplify notations, $\mathrm{P}(x)$ stands for $\mathrm{P}\left(\{s \in \mathcal{S} \,|\, X(s) \leq x\}\right)$ and $\varphi(x)$ for $\varphi\left(\{s \in \mathcal{S} \,|\, X(s) = x\}\right)$.

[7]Izhakian (2020) shows that the residual of this approximation is $R_2\big(\mathrm{P}(x)\big) = o\big(\mathrm{E}\left[|\mathrm{P}(x) - \mathrm{E}\left[\mathrm{P}(x)\right]|^3\right]\big)$ as $|\mathrm{P}(x) - \mathrm{E}\left[\mathrm{P}(x)\right]| \to 0$, which is negligible. Therefore, to simplify notation, we use the equal sign instead of the approximation sign. The use of the approximate perceived probabilities rather than the perceived probabilities has no impact on our results.

[8]The outlook function is assumed to satisfy $\left|\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\right| \leq \frac{1}{\mathrm{Var}[\varphi(x)]}$, which bounds the concavity and convexity of Υ to assure that the approximated marginal perceived probabilities are always positive and satisfy set monotonicity.

payoff by

$$\mathrm{V}(X) = \int \mathrm{U}(x) \underbrace{\mathrm{E}[\varphi(x)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}\mathrm{Var}[\varphi(x)]\right)}_{\text{Perceived Probability of Outcome } x} dx. \tag{4}$$

In this representation of expected utility, risk aversion is distinctly captured by a strictly-increasing, concave, and twice-differentiable continuous utility function $\mathrm{U}: \mathbb{R} \to \mathbb{R}$ applied to the uncertain outcomes. Ambiguity aversion is captured by a strictly-increasing, concave, and twice-differentiable continuous outlook function $\Upsilon: [0,1] \to \mathbb{R}$ applied to the uncertain probabilities.

The ambiguity-averse investor compounds the set of priors $\mathcal{P}$ using the second-order prior ξ over $\mathcal{P}$ in a nonlinear way, as reflected in her perceived probabilities (Equation (4)).[9] The (subadditive) perceived probabilities are a function of the extent of ambiguity, measured by $\mathrm{Var}[\varphi(\cdot)]$, and the investor's aversion to ambiguity, captured by $-\frac{\Upsilon''}{\Upsilon'} > 0$ (evaluated at the expected cumulative probability). A higher aversion to ambiguity or a greater extent of ambiguity result in lower (more underweighted relative to the expected probabilities) perceived probabilities.[10]

To simplify the analysis, we assume that relative risk aversion is non-increasing and that relative ambiguity aversion is non-increasing, i.e., $\left(\frac{\Upsilon''}{\Upsilon'}\right)' \geq 0$. The assumption of non-increasing risk aversion, as well as the assumption of moderate risk aversion, are widely used in the literature and are supported by behavioral evidence (e.g., Lengwiler, 2009, Box 4.7). The assumption of non-increasing ambiguity aversion, which is sufficient but not necessary for the results, is also supported by behavioral evidence (e.g., Baillon and Placido, 2019). Many classes of preferences for ambiguity satisfy this condition, including constant relative ambiguity aversion (CRAA) and constant absolute ambiguity aversion (CAAA).

Based on the functional form in Equation (4), the degree of ambiguity can be measured by

[9] Ambiguity aversion is reflected by the investor's preference for the expectation of an uncertain probability over an event drawn from the set of uncertain probabilities. This is analogous to the idea that risk aversion is exhibited when an investor prefers the expectation of the uncertain outcomes over an outcome drawn from the set of uncertain outcomes.

[10] When the investor is ambiguity neutral, Υ is linear, the perceived probabilities become the (additive) expected probabilities (the linear reduction of compound lotteries), and Equation (4) collapses to the standard expected utility. The same reduction of the model occurs in the absence of ambiguity, $\mathrm{Var}[\varphi(\cdot)] = 0$).

the expected probability-weighted average volatility of probabilities (across the relevant events). Formally, the measure of ambiguity is given by

$$\mho^2 [X] \quad = \quad \int \mathrm{E} [\varphi (x)] \, \mathrm{Var} [\varphi (x)] \, dx. \tag{5}$$

This statistic can be estimated using stock trading data. Risk independence represents another major advantage of $\mho^2$; in contrast to risk measures, it does not depend upon the magnitudes of the outcomes associated with events, only upon the partition they induce over the state space.

2.2 The asset pricing framework

We employ a standard framework, where the only variation in our structure is the specification of probabilities. There are two points in time, 0 and 1. The state at time 0 is known and the states at time 1 are ordered by their associated consumption levels from lowest to highest. Like much of the capital structure literature (e.g., Kraus and Litzenberger, 1973; Fischer et al., 1989; Leland, 1994; DeMarzo and Fishman, 2007), we assume a representative investor, endowed with initial wealth w_0 and time 1 wealth w_1, who trades in a perfectly competitive capital market. We assume the capital market is complete, that all participants are price takers, and that no arbitrage opportunities exist (i.e., the law of one price holds). Since the asset pricing framework is used only to extract the state prices, for simplicity we assume a single product in the economy, with an uncertain payoff x at time 1. The investor's objective function can then be written in the usual way

$$\max_{c_0, \theta} \quad \mathrm{V} (c_0) + \mathrm{V} (c_1) \tag{6}$$

subject to the budget constraints

$$c_0 = w_0 - \theta \int q(x) x dx \qquad \text{and} \qquad c_1 = w_1 + \theta x,$$

where $q(x)$ is the time 0 price of a pure state-contingent claim on state $s \in \mathcal{S}$ associated with outcome x; and θ is the investor's position of the single asset. Using the functional form of expected utility in Equation (4), the state prices can be extracted as follows.

Theorem 1. *Suppose a time-separable utility function. The state price of state x is then*[11]

$$q(x) \quad = \quad \pi\left(x\right)\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}, \tag{7}$$

where

$$\pi\left(x\right) \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right); \tag{8}$$

and $q(x)$ is unique and positive.

The state price $q(x)$ is the price of a claim to one unit of consumption contingent on the occurrence of state x (Arrow-Debreu security).[12] The assumption of a complete market implies the state prices are unique. Theorem 1 illustrates the distinct impacts that risk, ambiguity, and the attitudes toward these aspects of uncertainty have on market pricing. Ambiguity and aversion to ambiguity impact state prices through the consolidation of the uncertain probabilities to the perceived probability, $\pi\left(x\right)$, of each state x. Risk and aversion to risk impact the state price via the utility function U and the magnitude of outcomes x relative to consumption at time 0.[13,14]

The following corollary is an immediate consequence of Theorem 1.

Corollary 1. *The risk neutral probability of state x is*

$$\pi^{*}(x) \quad = \quad \frac{q(x)}{\displaystyle\int q(x)dx} \quad = \quad \frac{\pi\left(x\right)\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}}{\displaystyle\int\pi\left(x\right)\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}dx}, \tag{9}$$

and the risk-free rate of return is

$$r_{f} \quad = \quad \frac{1}{\displaystyle\int q(x)dx}-1 \quad = \quad \frac{1}{\displaystyle\int\pi\left(x\right)\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}dx}-1. \tag{10}$$

Ambiguity and aversion to ambiguity transform the standard representation of risk-neutral

[11] Henceforth, we identify a state of nature $s\in\mathcal{S}$ with its associated outcome x. We denote the partial derivative of the utility function with respect to x as $\partial_{x}\mathrm{U}$ and its partial derivative with respect to c_0 as $\partial_{0}\mathrm{U}$.

[12] Chapman and Polkovnichenko (2009) extract the state prices in a rank-dependent expected utility framework.

[13] Note that the separation of the impacts of risk and ambiguity, as illustrated by Theorem 1, is delivered by the separability inherent in the underlying preference representation. Such a separation is not feasible within other preference representations (e.g., Gilboa and Schmeidler, 1989; Klibanoff et al., 2005).

[14] In the absence of ambiguity (i.e., probabilities are perfectly known) state prices $q(x)$ reduce to the conventional representation $q(x)=\varphi\left(x\right)\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}$. Similarly, in the presence of ambiguity with ambiguity-neutral investors, consistent with the conventional representation, state prices are $q(x)=\mathrm{E}\left[\varphi\left(x\right)\right]\frac{\partial_{x}\mathrm{U}}{\partial_{0}\mathrm{U}}$.

probabilities via the term $\pi(x)$. Note that while they depend upon the subadditive perceived probabilities, the risk-neutral probabilities are additive.

2.3 The capital structure decision in a perfect capital market

Our capital structure model is based on the canonical tradeoff theory model of Kraus and Litzenberger (1973). Consider a one-period project that requires capital investment I at time 0. The payoff of this project, realized at time 1, is risky and ambiguous from the perspective of time 0. As discussed above, we assume that the owner of the project is a representative investor, whose set of priors, $\mathcal{P}$, is identical to the set of priors of each investor in the economy, and her second-order probability distribution, ξ over $\mathcal{P}$, is identical to that of each investor (homogeneous beliefs).

At time 0, investment in a project occurs if and only if

$$\frac{1}{1+r_f}\mathbb{E}^*[X] > I,$$

where $\mathbb{E}^*$ is the expectation taken using the risk-neutral probabilities, π^*, defined in Corollary 1.[15] Given the decision to invest in the project, its owner must decide at time 0 what mix of debt and equity financing the firm will use. The absence of managerial agency conflict implies that the objective of this decision is to maximize firm value:

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad S_0 &= \frac{1}{1+r_f}\mathbb{E}^*[\max(X-F\ ,\ 0)] \\ D_0 &= \frac{1}{1+r_f}\mathbb{E}^*[\min(F\ ,\ X)], \end{aligned}$$

where S_0 is the market value of the equity; D_0 is the market value of debt; and F is the debt's face value. Without loss of generality, the debt is assumed to be a one-period zero-coupon bond. The

[15] The double-struck capital font is used to designate expectation or variance of outcomes, taken using expected probabilities or risk-neutral probabilities, while the regular straight font is used to designate expectation or variance of probabilities, taken using the second-order probabilities, ξ.

optimization problem can be written more explicitly as

$$\begin{aligned}
\max_{F} \quad & S_0(F) + D_0(F) \\
\text{s.t.} \quad S_0 &= \frac{1}{1+r_f} \int_F^\infty \pi^*(x)(x-F)\,dx \\
D_0 &= \frac{1}{1+r_f} \left(\int_0^F \pi^*(x) x dx + \int_F^\infty \pi^*(x) F dx \right).
\end{aligned} \tag{11}$$

Any $x < F$ is considered a default state.

Consistent with the existing capital structure literature, at the optimum the face value of debt is assumed to be lower than the expected payoff of the project, $F < \mathbb{E}^*[x]$, such that $\frac{\partial q(F)}{\partial F} > 0$; otherwise, default is expected. This assumption implies that, at the optimum, the marginal default probability increases in F. It also coincides with the restrictions used to ensure a unique optimum by Scott (1976), Castanias (1983), and Kim (1978).

Theorem 2. *Suppose that there are no taxes, no bankruptcy costs, no asymmetric information and that all investors in the economy are averse to ambiguity. The market value of any firm is then independent of its capital structure.*

Theorem 2 shows that capital structure irrelevance proposition is maintained in the presence of ambiguity. Modigliani and Miller's proof is based on the idea that an investor can generate the return from holding an unlevered firm's equity by holding the equity of an equivalent levered firm and its debt in the proper proportions. The same argument holds in an ambiguous economy in which investors are ambiguity-averse.

2.4 The over-investment problem

In their seminal paper, Jensen and Meckling (1976) introduce the over-investment problem. Central to their idea is the notion that, in the investors' view, levered equity can be described as a call option on the firm, where the exercise price is the face value of debt (e.g., Merton, 1974). Maintaining the assumptions of an absence of taxes and bankruptcy costs, the current value of the equity is based on the future equity payoff. In our representation, the expected value of equity is assessed using the risk neutral probabilities, discounted at the risk-free rate (see Equation (11)). Thus, equity value is affected by ambiguity through π in π^*, and by risk through $\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}$.

As in Rothschild and Stiglitz (1970), we consider an asset as becoming riskier when new information arrives (a "comparative static shock") such that the distribution of the asset's payoffs after the information arrival can be written as a mean-preserving spread of its distribution before the information arrival. In this context, a mean-preserving spread of payoffs is taken using the expected probabilities and, therefore, may represent a mean-preserving spread of a single prior, multiple priors, or all the possible prior probability distributions in $\mathcal{P}$. The expected probability measure is an additive consolidated probability measure that compounds the first-order and second-order probabilities linearly. It can be viewed as the perceived probability measure facing an ambiguity-neutral Bayesian investor. The use of expected probabilities instead of known probabilities does not alter the classical intuition about the impact of increased risk on the value of an option-like payoff.

Proposition 1. *For a given face value of debt, the higher is the degree of risk, the higher is the value of the levered equity.*

Levered equity holders benefit from upside risk and are protected from downside risk. The value of levered equity, therefore, increases in the risk of the underlying cash flow. The over-investment problem follows from the recognition that levered equity holders benefit from an increase in risk even at the expense of some reduction in the expected value of the cash flow. Namely, levered equity holders benefit if the firm undertakes a negative net present value investment when it generates a sufficient increase in risk of the firm's cash flow. The next proposition shows that this incentive may be mitigated in a setting with ambiguity and ambiguity-averse investors. In Proposition 2, an increase in the degree of ambiguity occurs when new information arrives such that the possible probabilities associated with a given outcome after the information arrival can be written as a mean preserving spread of the possible probabilities prior to the arrival of the new information.

Proposition 2. *For a given face value of debt, the higher is the degree of ambiguity, the lower is* *the value of the levered equity.*

The intuition of Proposition 2 is equally straightforward. An increase in the level of ambiguity causes ambiguity-averse investors to heavily underweight the probabilities of "high" outcomes, reducing the value of the option-like payoff of levered equity. Similarly, as investors become more averse to ambiguity, the value of levered equity falls. Note that the same effect holds true for the value of debt, but to a smaller extent. The higher the ambiguity or aversion to ambiguity, the lower is the value of the debt and therefore the lower is the value of the firm.

A three-way tradeoff exists between value, risk, and ambiguity. Levered equity holders perceive a loss in value from an increase in ambiguity. Their incentive regarding an investment that increases both risk and ambiguity is therefore unclear. An oft-cited example of the over-investment problem is the incentive of a firm's managers to pursue a value-reducing acquisition if it increases the risk of the firm's cash flow. However, it is plausible that while a merger may increase the level of risk of a firm's cash flow, it may also increase the level of ambiguity. In such a case, the incentive of a manager, acting in the interests of levered equity holders, is unclear. At one extreme, a manager acting in the interest of levered equity holders would avoid a positive net present value acquisition that increases firm risk because it also increases the ambiguity of the firm, an under-investment problem.[16] A deeper examination of the effect of ambiguity on M&A decisions is the subject of ongoing research.[17]

A related study by Garlappi et al. (2017) models heterogeneous beliefs among a decision-making group that possesses a set of priors. In that case, the group can face ambiguity. Therefore, the greater is the disagreement (or, the more heterogenous are the beliefs) among the decision-

[16]The combination of ambiguity and individuals' aversion to ambiguity causes the change in perceived probabilities. If individuals are ambiguity-neutral, a change in ambiguity does not affect the expected probabilities and would have no impact on the perceived probabilities or the value of levered equity. Also, as noted in the introduction, the value of the debt and the value of the firm also fall as ambiguity is increased. The willingness to undertake a project with a negative stand-alone NPV if it reduces firm ambiguity sufficiently may not represent an agency problem. Nevertheless, this tradeoff does impact the standard over-investment problem.

[17]Herron and Izhakian (2019) find that firms facing high levels of ambiguity would be more acquisitive, select targets that face relatively low levels of ambiguity, and accomplish deals more quickly.

makers, the higher is the ambiguity of the "corporation." When the group evaluates a new project, Garlappi et al. (2017) show that even if individuals within the group independently believe that the investment provides value, collectively, the group may not invest.

2.5 Taxes and bankruptcy costs

Consider now an economy with no asymmetric information or agency conflicts but with corporate taxes and bankruptcy costs as the highlighted frictions. In the event of default, we assume bankruptcy costs are proportional to the firm's output

$$B(x;F) = \begin{cases} \alpha x, & \text{if} \quad x < F \\ 0, & \text{if} \quad x \geq F \end{cases},$$

where $0 < \alpha < 1$. The tax benefit associated with debt is, for simplicity, based on the entire debt service in non-default states,

$$T(x;F) = \begin{cases} 0, & \text{if} \quad x < F \\ \tau F, & \text{if} \quad x \geq F \end{cases},$$

where τ is the corporate income tax rate. In such an economy, firm value is a function of the chosen capital structure. Specifically, for a given face value of debt, F, firm value is written

$$\begin{aligned} V(F) &= S_0(F) + D_0(F) \qquad (12) \\ &= \frac{1}{1+r_f}\left(\int_F^\infty \pi^*(x)(1-\tau)(x-F)\,dx + \int_0^F \pi^*(x)(1-\alpha)\,x dx + \int_F^\infty \pi^*(x)F dx\right). \end{aligned}$$

The capital structure choice problem can be written

$$\begin{aligned} \max_F \quad & S_0(F) + D_0(F) \\ \text{s.t.} \quad S_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)x - F + T(x;F)\ ,\ 0\right)\right] \\ D_0(F) &= \frac{1}{1+r_f}\mathbb{E}^*\left[\min\left(F\ ,\ x - B(x;F)\right)\right]. \end{aligned}$$

Notice that S_0 can also be written

$$S_0(F) = \frac{1}{1+r_f}\mathbb{E}^*\left[\max\left((1-\tau)(x-F)\ ,\ 0\right)\right].$$

The next theorem identifies the optimal leverage for the firm.[18]

Theorem 3. *The optimal leverage satisfies*

$$F \quad = \quad \frac{\tau}{\pi^*(F)\alpha} \int_F^\infty \pi^*(x)dx. \tag{13}$$

By Equation (13), a higher tax rate, τ, or lower bankruptcy cost, α, positively impact the use of debt. We focus on the implications of changes in ambiguity or risk on capital structure.

Proposition 3. *The higher is the degree of risk, the lower is the optimal leverage.*

Proposition 3 demonstrates that the well-known consequence of a comparative static change in risk remains in the current model. As above, an increase in risk is the application of a mean-preserving spread to the expected probability measure.

Proposition 4. *The higher is the degree of ambiguity, the higher is the optimal leverage.*

Proposition 4 describes the consequence of a comparative static change in ambiguity. As above, an increase in ambiguity is the application of a mean-preserving spread to the set of possible probabilities of a given outcome or set of outcomes.

Propositions 3 and 4 demonstrate that increased risk has a negative effect on the chosen level of leverage, while increased ambiguity has a positive effect. The intuition for Proposition 3 is standard: greater risk pushes probability mass from the center of the distribution to its tails, increasing the marginal expected bankruptcy cost and decreasing the marginal expected tax benefit. This effect is illustrated in Panel A of Figure 1. A reduction in the use of debt, to F', once again equates the marginal expected cost and benefit of debt financing.

[Figure 1]

It is tempting to suggest that this intuition can be applied to an increase in ambiguity as well; greater ambiguity might have the same effect as an increase in risk. As discussed above, this may be the case in models of ambiguity in which preferences for ambiguity are outcome-

[18] It is well-known that in a non-parametric representation as presented here, additional restrictions are required to ensure a unique optimum. As discussed above, we assume that at the optimal debt level $\frac{\partial q(F)}{\partial F} \geq 0$.

dependent, because in these models ambiguity is confounded with risk. An increase in ambiguity alone, however, represents solely increased uncertainty regarding *probabilities*. Thus, increased ambiguity is independent of the outcomes themselves, so the standard intuition does not apply.

More appropriate intuition derives from the notion that ambiguity-averse investors under-weight the probabilities of high output states more than they under-weight the probabilities of low output states. Therefore, increased ambiguity has an asymmetric effect on the perceived probability distribution, as illustrated in Panel B of Figure 1. The expected bankruptcy cost, $\mathbb{E}^*\left[B\left(x;F\right)\right] = \alpha\int_0^F \pi^*(x)xdx$, and the expected value of tax shields, $\mathbb{E}^*\left[T\left(x;F\right)\right] = \tau F\int_F^\infty \pi^*(x)dx$, both decline when ambiguity rises. Greater ambiguity decreases the perceived probabilities, with a stronger affect for larger x. Thus, the expected tax benefit decreases to a greater extent than the expected bankruptcy costs. However, the marginal expected tax benefit is higher than is the marginal expected bankruptcy costs, such that an increase in leverage is necessary to attain a new optimum. Figure OA.1 in the Online Appendix provides a visual representation of the effects of risk, aversion to risk, ambiguity, and aversion to ambiguity on optimal leverage.

Propositions 3 and 4 take ambiguity and risk as exogenous characteristics of the environment. It is possible that investors or management of a firm have incentives to reduce the ambiguity facing the firm through costly information acquisition. However, just as new information may act to reduce ambiguity it may also increase ambiguity (Epstein and Schneider, 2007). Therefore, the incentives for acquiring information toward reducing ambiguity are unclear. If there are incentives for behaviors aimed at reducing the ambiguity of high ambiguity firms, it may prove difficult to find a positive relation between leverage and ambiguity in the data.[19]

3 Empirical design

We now test the predictions in Propositions 3 and 4 empirically. For consistency with the majority of the empirical capital structure literature, we examine how annual leverage ratios are related to

[19]We thank an anonymous referee for noting this possibility.

ambiguity and risk. Similar results are obtained using quarterly data.

3.1 Sample selection

To construct our sample, we begin with all records from the Compustat fundamentals annual database. We eliminate financial firms and government entities. We drop observations missing any of the following descriptors: fiscal year-end (FYR), total assets (AT), long-term debt (DLTT) or stock price (PRCC). Observations with negative values for sales, total assets or debt, and observations with a negative leverage ratio or a leverage ratio greater than 100% (for both book and market leverage) are also excluded. In addition, we drop duplicate observations and those for which we cannot match identifiers to the CRSP or TAQ stock price databases. Observations for which the annual degree of ambiguity, risk, or cash flow volatility cannot be estimated as described below are also dropped. To limit the influence of outliers, variables taking only positive values are trimmed at 99%, and variables taking any value are trimmed at 1% and 99%. Applying these filters leaves $54,691$ observations for 5,092 unique companies between 1993 and 2018.

We estimate the annual degree of ambiguity and risk for each firm-year by computing their monthly values, as described below. We then use the average of these monthly values over the fiscal year as measures of the annual ambiguity and risk associated with each firm. As described below, firm levered and unlevered measures of ambiguity and risk are computed to ensure that the use of equity-based measures is not responsible for the reported findings.

3.2 Estimating ambiguity

The use of the CEU model augmented with the EUUP framework, Equation (4) naturally delivers a risk-independent measure of ambiguity (Izhakian, 2020). Within this framework, the degree of ambiguity can be measured by the volatility of the uncertain probabilities, just as the degree of risk can be measured by the volatility of the uncertain outcomes. Formally, the measure of ambiguity, denoted by $\mho^2$ and defined by Equation (5), represents an expected probability-weighted average of the variances of probabilities. We follow the recent literature (e.g., Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2020) and estimate the monthly degree of ambiguity for each firm using intraday stock return data from TAQ.[20]

To estimate ambiguity as implemented in Equation (14) below, the expectation of and the variation in return probabilities across the set of possible prior probability distributions must be measured. We assume that the intraday equity return distribution for each day in a given month represents a single distribution, P, in the set of possible distributions, $\mathcal{P}$, and the number of priors in the set is assumed to depend on the number of trading days in the month. The set of priors for each month, $\mathcal{P}$, therefore, consists of the 18-22 realized daily distributions in that month. For practical implementation, we discretize return distributions into n bins of equal size, and represent each distribution as a histogram, as illustrated in Figure OA.2 in the Online Appendix. Each bin $B_j = (r_{j-1}, r_j]$ represents an interval of return outcomes. The bin height is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of the returns in that bin. Equipped with these 18-22 daily return histograms, we compute the expected probability of each bin across the daily return distributions for each month, $\mathrm{E}\left[\mathrm{P}\left(B_j\right)\right]$, as well as the variance of these probabilities, $\mathrm{Var}\left[\mathrm{P}\left(B_j\right)\right]$, by assigning an equal likelihood (ξ) to each histogram.[21] Using these values, the monthly degree of ambiguity for firm i is computed as

$$\mho^2\left[r_i\right] = \frac{1}{\sqrt{w\left(1-w\right)}} \sum_{j=1}^{n} \mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right] \mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]. \tag{14}$$

To minimize the impact of the selected bin size on the value of ambiguity, we apply a variation of Sheppard's correction and scale the weighted-average volatilities of probabilities to the bin size by

[20] The measure of ambiguity, defined in Equation (5), is distinct from aversion to ambiguity. The former, which is a matter of beliefs (or information), is estimated from the data, while the latter is a matter of subjective attitudes. Baillon et al. (2018) suggest an elicitation of "matching probabilities," which are similar to the perceived probabilities in the EUUP model. Based upon matching probabilities, they suggest an ambiguity insensitivity index. By construction, this index supports only three possible events, and it is attitude dependent, making it less useful for this application. The measure employed here supports an unlimited number of events and is attitude independent.

[21] Equal likelihoods is consistent with: the principle of insufficient reason, which states that given n possibilities that are indistinguishable except for their names, each possibility should be assigned a probability equal to $\frac{1}{n}$ (Bernoulli, 1713; Laplace, 1814); the simplest non-informative prior in Bayesian probability (Bayes et al., 1763), which assigns equal probabilities to all possibilities; and the principle of maximum entropy (Jaynes, 1957), which states that the probability distribution which best describes the current state of knowledge is the one with the largest entropy.

$\frac{1}{\sqrt{w(1-w)}}$, where $w = r_{i,j} - r_{i,j-1}$.

In our implementation, we sample five-minute stock returns from 9:30 to 16:00, as this frequency has been shown to eliminate microstructure effects (Andersen et al., 2001; Bandi and Russell, 2006; Liu et al., 2015). Thus, we obtain daily histograms of up to 78 intraday returns, each of which represents a single $\mathrm{P} \in \mathcal{P}$. If we observe no trade in a specific time interval for a given stock, we compute returns based on the volume-weighted average of the nearest trading prices. We ignore returns between closing and next-day opening to eliminate the impact of overnight price changes and dividend distributions. We drop all days with less than 15 five-minute returns and months with less than 15 intraday return distributions. In addition, we drop extreme returns ($\pm 5\%$ log returns over five minutes), as many such returns occur due to improper orders subsequently canceled by the stock exchange. We normalize the intraday five-minute returns to daily returns.[22]

For the bin formation, we divide the range of daily returns into 162 intervals. We form a grid of 160 bins, from -40% to 40%, each of width 0.5%, in addition to the left and right tails, defined as $(-\infty, -40\%]$ and $[+40\%, +\infty)$, respectively. We compute the mean and the variance of probabilities for each interval by assigning equal likelihood to each histogram.[23] Some bins may not be populated with return realizations, which makes it difficult to compute their probability. Therefore, we assume a normal return distribution and use its moments to extrapolate the missing return probabilities. That is, $\mathrm{P}_i(B_j) = \Phi(r_j; \mu_i, \sigma_i) - \Phi(r_{j-1}; \mu_i, \sigma_i)$, where $\Phi(\cdot)$ denotes the cumulative normal probability distribution, characterized by its mean μ_i and the variance σ_i^2 of the returns. As in French et al. (1987), we compute the variance of returns by applying the adjustment for non-synchronous trading, as proposed by Scholes and Williams (1977).[24] This adjustment

[22] Our results are robust to the inclusion of extreme price changes, as well as for a cutoff of $\pm 1\%$ in terms of log returns over five minutes. Note that a $\pm 5\%$ five-minute return implies a $\pm 390\%$ daily return.

[23] Equal likelihoods are equivalent to daily ratios $\frac{\mu_i}{\sigma_i}$ that are Student-t distributed. When $\frac{\mu}{\sigma}$ is Student-t distributed, cumulative probabilities are uniformly distributed (e.g., Kendall and Stuart, 2010, page 21, Proposition 1.27).

[24] Scholes and Williams (1977) suggest adjusting the volatility of returns for non-synchronous trading as $\sigma_t^2 = \frac{1}{N_t}\sum_{\ell=1}^{N_t}\left(r_{t,\ell} - \mathrm{E}\left[r_{t,\ell}\right]\right)^2 + 2\frac{1}{N_t-1}\sum_{\ell=2}^{N_t}\left(r_{t,\ell} - \mathrm{E}\left[r_{t,\ell}\right]\right)\left(r_{t,\ell-1} - \mathrm{E}\left[r_{t,\ell-1}\right]\right)$. We also perform all estimations without the Scholes-Williams correction for non-synchronous trading. The findings are essentially the same.

further eliminates any microstructure effects caused by bid-ask bounce, although our use of five-minute returns already reduces microstructure effects.

The EUUP measure of ambiguity is outcome-independent (up to a state-space partition), which allows for a risk-independent examination of the impact of ambiguity on financial decisions. Specifically, the measure of ambiguity described above captures the variation in the frequencies (probabilities) of outcomes without incorporating the magnitudes of outcomes. In contrast, the measure of risk captures the variation in the magnitudes of outcomes without incorporating the variation in the frequencies of outcomes. Thus, the measure of ambiguity is risk-independent just as standard measures of risk are ambiguity-independent, implying that these two measures capture distinct aspects of uncertainty. Empirically, the monthly measure of ambiguity and the monthly measure of risk are weakly negatively correlated with a correlation coefficient of -0.06.

Other proxies for ambiguity in the literature include the volatility of mean return (Franzoni, 2017), the volatility of return volatility (Faria and Correia-da Silva, 2014), or the dispersion in (disagreement of) analyst forecasts (Anderson et al., 2009). As these measures are sensitive to changes in the set of outcomes (i.e., are outcome-dependent), they are therefore risk-dependent and less useful for this study.[25] Similarly, skewness, kurtosis, and other higher moments of the return distribution are outcome-dependent and so different from $\mho^2$. Jumps, time-varying mean and time-varying volatility are also outcome-dependent.

Brenner and Izhakian (2018) and Augustin and Izhakian (2020) show that $\mho^2$ does not simply reflect other well-known "uncertainty" factors including skewness, kurtosis, variance of mean, variance of variance, downside risk, mixed data sampling measure of forecasted volatility (MIDAS), jumps, or investors' sentiment, among many others. As in Augustin and Izhakian (2020), our robustness tests examine many of these uncertainty factors at the firm level. In particular, in Table 5, we examine the explanatory power of our measure of ambiguity relative to alternative uncertainty

[25] For example, Ben-Rephael and Izhakian (2020b) report a correlation between volatility of mean return and volatility of return (risk) of 0.81 and a correlation between volatility of return volatility and volatility of return (risk) equal to 0.72, implying that these proposed proxies for ambiguity are very closely related to risk.

factors and show that $\mho^2$ represents a distinct aspect of uncertainty in the present context.

Table OA.1 in the Online Appendix reports the autocorrelation of our annual measure of ambiguity, showing that the one-year autocorrelation is 67.2%. Table OA.2 in the Online Appendix reports the correlations of our ambiguity measure, computed using five-minute returns, with ambiguity computed using open-high-low-close prices and using daily returns over one, three and five year horizons. The correlations of the measures across frequencies and horizons are high, especially for annual ambiguity.

Leverage amplifies cross-sectional variation in stock return volatility, and leverage might also affect our measure of ambiguity. However, due to the outcome-independence of $\mho^2$, the nature of any such impact is unclear. Nevertheless, we estimate all regressions using both a measure of ambiguity based on firms' levered returns as well as one based on firms' unlevered returns. In these regressions, we use a measure of risk based on the firm's levered returns and one based on the firm's unlevered returns, respectively. We create these measures by combining the book value of total debt and the market value of equity to represent firm value for five-minute intervals.[26] The resulting firm values are used to estimate unlevered returns. Unlevered ambiguity and risk are computed from these unlevered returns.

3.3 Estimating risk

Following many earlier studies of leverage (e.g., Frank and Goyal, 2009), we use equity return volatility (or unlevered return volatility) as a proxy for default risk. Return volatility has also been used in the literature on pricing default risk for debt (e.g., Bharath and Shumway, 2008) and as an input for the "distance to default" measure derived from the Merton (1974) model. Other empirical studies of leverage have used cash flow volatility (e.g., the variance of operating income) rather than stock return volatility as the relevant proxy. Cash flow volatility has a more direct impact on default risk but does not take into account other relevant firm characteristics, including

[26]We consider three different frequencies of estimating the market value of the firm: updating the value every five-minutes, every day, and every month. Our results are robust across these alternatives.

the amount and timing of principal repayments, available cash, and other required debt service. We use return volatility as our primary measure of risk, however, in the Online Appendix, we present results using cash flow volatility as the measure of default risk.

For consistency we compute the variance of equity returns with the same five-minute returns used to estimate ambiguity. For each stock on each day, we compute the variance of intraday returns, applying the Scholes and Williams (1977) correction for non-synchronous trading and a correction for heteroscedasticity.[27] In a given year, we then compute the annual variance of stock returns using the average of daily variances, scaled to a monthly frequency. We also estimate risk as the average monthly variance of daily returns and find no meaningful change in results.

3.4 Leverage and firm specific characteristics

The annual leverage ratio of each firm is the main dependent variable in our empirical tests. We consider both book value and market value versions of this ratio. Book leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total book value of assets. Market leverage is computed as "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt."

Our regressions include annual firm characteristics widely used in the empirical capital structure literature. Firm size is measured by the log of one plus the firm's sales.[28] Profitability is measured by operating income before depreciation divided by book assets. Asset tangibility is measured by property, plant and equipment divided by book assets. The market-to-book ratio is measured by the market value of equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is measured by R&D expenses relative to sales, normalized by $10,000$. In addition, to explore the firms' leverage

[27] See, for example, French et al. (1987).

[28] Alternatively, firm size can be measured by the log of book assets. We conduct all regression tests using this alternate measure of firm size, and the findings are virtually identical.

ratios relative to their industries, we include the industry median annual leverage ratio, with firms classified by four-digit SIC codes. Expected marginal tax rates are obtained from John Graham's website. When the expected marginal tax rate is missing, we use the industry median annual marginal tax rate if available or the market-wide median annual marginal tax rate if necessary.

3.5 Summary statistics

Table 1 presents summary statistics for the key variables used in the paper, which indicate our sample is broadly consistent with those used in recent studies of leverage.

[Table 1]

In untabulated results, we find that the monthly degree of our measure of ambiguity and the measure of risk computed on a monthly basis are weakly negatively correlated; a correlation coefficient of -0.06 in our sample. This suggests that the two variables capture distinct aspects of financial uncertainty, which coincides with comprehensive tests in the recent literature (e.g., Izhakian and Yermack, 2017; Brenner and Izhakian, 2018; Augustin and Izhakian, 2020).

Table OA.3 in the Online Appendix report cross-correlations for the variables used in the regression tests. Ambiguity or unlevered ambiguity show only small correlations with the other explanatory variables. It has a small negative correlation with risk, profitability, tangibility, the market to book ratio, and the marginal tax rate. It has a small positive correlation with cash flow volatility, R&D expenditures, firm size, and firm age. Because the standard control variables are included in our tests implies that ambiguity is not simply a proxy for previously known explanators.[29]

Table OA.4 in the Online Appendix presents a quintile analysis, sorting firms on ambiguity and reporting the average level of firm characteristics for each quintile. This analysis reflects very

[29]The positive correlation between ambiguity and firm size (age) may be related to the stage of the firm. Large and old firms may exhaust their low-ambiguity organic growth opportunities and what remains are relatively high-ambiguity growth opportunities, which reflects on the firm ambiguity. These firms are likely to reduce investments and increase payout (Herron and Izhakian, 2018) or look to acquire low-ambiguity targets (Herron and Izhakian, 2019).

similar information to that contained in the correlation matrix. It adds that ambiguity has a small negative relation to idiosyncratic risk and the kurtosis of returns and a small positive correlation with the skewness of returns.

Similar to risk, ambiguity has many drivers, both economy-wide and firm-specific. At the economy level, for example, the political environment, elections, FMOC announcements, and tariffs all affect ambiguity. Uncertainty about new regulations increases ambiguity in related firms in an industry; for example, smoking restrictions impact all companies in the tobacco industry. At the firm-level, companies with low organic growth opportunities likely have more ambiguity. Firm with intensive R&D activity, especially small bio-medical firms and highly innovative firms, has greater ambiguity. Different events also affect ambiguity (Doan et al., 2018; Ben-Rephael and Izhakian, 2020a). Some events increase ambiguity (e.g., M&A, credit rating watches, debt increases, and Form 8-K filings), while other events reduce ambiguity (e.g., earnings announcements, analyst recommendations, and credit rating changes).

3.6 Regression tests

The main focus of our empirical tests is Proposition 4. Our main empirical model considers a version of the time series, cross-sectional regression that is standard in the empirical capital structure literature, augmented to include the measure of ambiguity as an additional explanatory variable:

$$L_{i,t} = \alpha + \beta \cdot \mho^2_{i,t-1} + \gamma \cdot Z_{i,t-1} + \varepsilon_{i,t}, \tag{15}$$

where L is the leverage ratio and Z is a vector of common explanatory variables. To address the possible omitted variables problem identified by Lemmon et al. (2008), we utilize first differences of the dependent and independent variables. To this end, we estimate

$$\Delta L_{i,t} = \alpha + \beta \cdot \Delta\mho^2_{i,t-1} + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}, \tag{16}$$

where $\Delta L_{i,t}$ is the change in variable L between time $t-1$ and t, $\Delta\mho^2_{i,t-1}$ and $\Delta Z_{i,t-1}$ represent the changes in $\mho^2$ and the vector of control variables Z between time $t-2$ and time $t-1$. This approach also addresses concerns regarding reverse causality.

Lemmon et al. (2008) employ firm fixed effects to control for possible omitted variables bias.[30] Using first differences in the model in Equation (16) is conceptually analogous to adding firm fixed effects to Equation (15). If the unobserved variation is truly time-invariant, then using firm fixed effects offers a more robust solution than taking first differences. However, recent evidence suggests that the unobserved variation in leverage highlighted by Lemmon et al. (2008) may not be entirely time-invariant. DeAngelo and Roll (2015) show that interaction terms between firm fixed effects and decade fixed effects have coefficient estimates that are statistically significant. Their finding suggests that the use of first differences offers a superior solution (e.g., Grieser and Hadlock, 2019; Wooldridge, 2010) to the omitted variables problem in this context.[31]

The regression specification in Equation (16) is estimated using both book and market leverage ratios as the dependent variables. Each model is also tested using both levered and unlevered ambiguity (and risk) at the firm level as the explanatory variable of interest. All reported standard errors are clustered by firm and are robust to within-firm heteroskedasticity. For purpose of comparison, in the Online Appendix, we present our main analysis using firm and year fixed effects rather than first differences. The findings regarding ambiguity and risk are qualitatively the same.

4 Empirical findings

4.1 Main findings

To examine Proposition 4, we estimate the model in Equation (16). Estimates for our main empirical specification appear in Table 2, with book leverage as the dependent variable in Panel A and market leverage in Panel B. Consistent with our theoretical predictions, in every specification the

[30]See also the extension of this analysis proposed by Coles and Li (2020).

[31]Furthermore, Wooldridge (2010) notes that, in an instrumental variables context, the use of first-difference two-stage least squares has a weaker exogeneity restriction than does a fixed-effects two-stage least squares estimator. In the Online Appendix, we present an instrumental variables, first-difference, two-stage analysis in order to provide external validity for the ambiguity measure employed in this paper. The instrumental variables approach presented in the Online Appendix shows that variation in the ambiguity measure explained by the instruments is also significantly positively related to the firm's leverage ratio, consistent with the prediction in Proposition 4.

ambiguity (or unlevered ambiguity) variable has a positive and statistically significant coefficient estimate. In addition, all coefficient estimates for the measure of risk are negative and significant.

Coefficient estimates for control variables in Table 2 are generally in line with prior studies. Leverage ratios tend to be higher in larger firms and for firms with more tangible assets, while firms with higher market-to-book ratios tend to use less leverage. Consistent with earlier studies, more profitable firms use less leverage; however, the estimated coefficient on profitability is an order of magnitude smaller than those reported in recent studies (e.g. Lemmon et al., 2008). Further examination shows that the cause for the difference in the magnitude is the use of first difference transformation and not the inclusion of ambiguity as an explanatory variable.

Interestingly, asset tangibility and firm size are the only reliably significant control variables in these regression estimations. In Panel A, in which book leverage is the dependent variable, only the coefficient estimates for asset tangibility, market-to-book ratio, and firm size have statistically significance. In Panel B, in which market leverage is the dependent variable, the coefficient estimate for R&D expenditures becomes statistically significant, while the coefficient estimate for the market-to-book ratio becomes insignificant. It is also somewhat surprising that the estimated relation between R&D expenditures and the leverage ratios (market and book) is essentially zero.[32] Prior studies have found a negative relation (e.g., Mackie-Mason, 1990; Berger et al., 1997), in line with the Myers (1977) prediction that growth opportunities will be financed primarily by equity. This, again, is the result of the first differences specification.

[Table 2]

The reported findings are robust across a variety of restrictions on the sample or alternative measures of the explanatory variables. When we winsorize rather than trim outliers in the data, require a greater number of consecutive data points to emphasize the time-series component to the estimates, or use quarterly rather than annual data, the estimated coefficient on ambiguity remains positive and significant. When using the volatility of cash flow (see Table OA.5 in the

[32] Coiculescu et al. (2019) find that ambiguity negatively affects R&D expenditures.

Online Appendix) or a measure of the "distance to default" derived from the Merton (1974) model to measure risk (e.g., Bharath and Shumway, 2008), or measuring firm size using the natural log of book assets rather than sales, the estimated coefficient on ambiguity remains positive and significant. However, the relation between risk and the leverage ratio varies depending upon the specification. The somewhat erratic significance of risk as an explanatory variable for leverage is consistent with Frank and Goyal (2009), who find that risk, measured as the variance of equity returns, is not one of the "core factors" determining leverage.

Table OA.6 in the Online Appendix replicates the main analysis using a firm and year fixed effects specification (Lemmon et al., 2008). The findings confirm the conclusion that both book leverage and market leverage are negatively affected by risk and positively affected by ambiguity.

To examine further the relation between ambiguity and leverage, Table 3 estimates the model in Equation (16) using sub-samples partitioned by the sample medians of firm size and leverage ratio. Panel A of Table 3 reports estimates for book leverage, and Panel B reports estimates for market leverage. The findings for book leverage partially support our theoretical predictions and the estimates on the full sample. In Columns (1) and (2) of Panel A, ambiguity is significantly positively related to the leverage ratio, and risk is significantly negative, for subgroups of firms with both low and high leverage. In Columns (3) and (4), however, unlevered ambiguity is significantly positively related to leverage only for the low leverage firms. Conversely, Columns (3) and (4) show that unlevered risk is significantly negatively related to leverage only for the high leverage firms. Columns (5) and (6) show that ambiguity is significantly positively related to leverage only among large firms, while indicating that risk is significantly negatively related to leverage only for small firms. Similarly, Columns (7) and (8) show that unlevered ambiguity is significantly positively related to leverage only for large firms. Unlevered risk is significantly negatively related to leverage in both subgroups.

Panel A of Table 3 also shows that the standard control variables do not exhibit significant relations with leverage in all subgroups. Asset tangibility has significantly positive estimates across all subgroups, but it is the only control variable with a consistently significant estimate. Firm size has significant estimates in only half the reported tests. The market-to-book ratio is significantly negatively related to leverage in most of the subgroups, while R&D expenditures have insignificant estimates in all subgroups.

Panel B of Table 3 indicates that for market leverage, the results are much more consistent. Ambiguity (and unlevered ambiguity) have significantly positive relation to leverage across all subgroups, and risk (and unlevered risk) is significantly negatively related to leverage. Estimates for the standard control variables in Panel B of Table 3 are also more consistent with those reported in Table 2. Specifically, asset tangibility and firm size have significantly positive relations with leverage across all subgroups.

[Table 3]

Some firms may have highly concentrated ownership (mainly small firms) while others may have dispersed ownership. This raises a question about the implications of ambiguity diversification. Theoretically, there is a tradeoff between risk diversification and ambiguity diversification, such that the maximum risk diversification in not optimal (Izhakian, 2019). Therefore, optimal portfolios have both risk and ambiguity. The sub-sample analysis by size in Table 3 demonstrates that our findings are consistent across firm with different sizes. Thereby, it shows that our finding are consistent across different levels of ambiguity (and risk) diversification.[33]

4.2 Economic significance

In Table 4, we investigate the economic significance of ambiguity in explaining capital structure. The table is constructed by multiplying the standard deviation of each independent variable, shown in the first column, by its corresponding coefficient estimate from Table 2, shown in the second column. The product of these two quantities is displayed in the third column under the heading

[33] We thank an anonymous referee for this insight.

"Significance."[34] In the fourth and fifth columns, we report the significance of ambiguity and unlevered ambiguity relative to the significance of the other variables, to aid in interpretation.

Table 4 reveals three interesting findings. First, ambiguity or unlevered ambiguity have levels of economic significance similar to risk or unlevered risk. This indicates that the two aspects of uncertainty have roughly equal importance in determining the capital structure choices made by firms. Second, ambiguity displays a greater level of economic significance than does unlevered ambiguity. Together with the consistency in the sign and significance of ambiguity and unlevered ambiguity reported in Table 2, this finding suggests that ambiguity of equity returns does not provide a distorted view of the relation between leverage and ambiguity. Third, depending on whether the dependent variable is market or book leverage, ambiguity and unlevered ambiguity have comparable or superior levels of economic significance to the other control variables. For both market and book leverage, ambiguity and unlevered ambiguity display greater levels of economic significance than the industry median leverage ratio, profitability, R&D expenditures, and the expected corporate tax rate. Ambiguity and unlevered ambiguity also have greater economic significance in the estimates using market leverage than asset tangibility, the market-to-book ratio, and firm size. For book leverage, the economic significance of ambiguity is roughly equivalent to the significance of the market-to-book ratio and firm size, with only asset tangibility displaying a greater level of significance than the ambiguity measure.

[Table 4]

5 Robustness

We next examine the robustness of our analysis to alternative measures of ambiguity.

[34] Recall that we use first differences of the variables in our regression tests. Therefore, the standard deviation reported in the table, "Std," is the standard deviations of the innovations or changes in the explanatory variables rather than the standard deviations of the variables themselves.

5.1 Alternative uncertainty factors

Table 5 reports robustness tests using alternative uncertainty factors from the capital structure literature. We test each of the following four factors as a substitute for our ambiguity measure as well as a factor alongside our measure. The variance of the mean (Var Mean) is the variance of daily mean returns (computed from 5-minute returns) over a month and averaged over the year. The variance of the variance (Var Var) is the variance of the daily variance of returns (computed from 5-minute returns) over a month and averaged over the year. Bid-ask spread is the annual average of the effective bid-ask spread for the firm's equity. Disagreement or dispersion of analysts' forecasts is the variance among analyst forecasts regarding the future stock price.

As above, we run our regression tests using the first differences of book and market leverage ratios, with one year lagged differences in the independent variables. All regression tests include an intercept and first differences of the standard control variables that are included in the model presented in Table 2. When the alternative uncertainty factors are substituted for our measure of ambiguity in the regression tests explaining book leverage (Columns (2) - (5) of Panel A), only the bid-ask spread and the dispersion of analyst forecasts are significantly related to leverage, both with negative coefficient estimates. When the alternative uncertainty factors are substituted for our ambiguity measure in explaining market leverage (Columns (2) - (5) of Panel B), only the bid-ask spread has a significant coefficient estimate, again negative.

The final four columns in Panels A and B of Table 5 include both our measure of ambiguity and each of the uncertainty factors in regression tests as well as the measure of risk and the control variables. Our measure of ambiguity remains positive and statistically significant in all the regressions for both book and market leverage. The other uncertainty factors all have either insignificant or significantly negative coefficient estimates. The variance of the mean is insignificantly related to both book and market leverage ratios when included alongside ambiguity. Since the variance of the mean has been considered a proxy for time-varying probability distributions, this finding rules out the possibility that the significance of our measure of ambiguity is driven solely by a time-varying mean. Similarly, the variance of the variance continues to be insignificantly related to the leverage ratio when it appears alongside ambiguity in the regression tests. The bid-ask spread has a significantly negative relation to both book and market leverage ratios when included alongside ambiguity. Finally, the disagreement among analysts is significantly negatively related to the book leverage ratio and insignificantly related to the market leverage ratio when included alongside ambiguity.

These findings indicate that our measure of ambiguity and the various uncertainty factors capture distinct aspects of uncertainty. We also conclude that the other uncertainty factors, while plausibly reflecting aspects of ambiguity, are either too closely related to risk or do not capture salient features of ambiguity in the context of the leverage decision.[35] The alternative factors we examine are outcome-dependent and are therefore risk-dependent, which may explain their negative relations to leverage. Our measure is outcome-independent and therefore risk independent, so it captures the relation between leverage and ambiguity independent of risk.

[Table 5]

5.2 Alternative ambiguity estimates

To construct our ambiguity measure, we assume that intraday returns are normally distributed and therefore fully characterized by the first and the second moments of the return distribution. For robustness, we explore alternative parametric assumptions of the intraday return distribution, allowing for skewness, kurtosis, or both. First, we compute ambiguity, assuming that intraday returns follow a Laplace distribution. Second, to eliminate jump effects, we also compute both measures of ambiguity (normal and leptokurtic) by truncating five-minute intraday returns larger than 1% instead of 5%. Third, we compute ambiguity non-parametrically by constructing the

[35] Using daily measures, Ben-Rephael and Izhakian (2020b) report a correlation between the volatility of the mean and risk equal to 0.81, and a correlation between the volatility of volatility and risk equal to 0.72, implying that these uncertainty factors are very closely related to risk.

statistical histograms of the intraday return distributions without imposing a particular distributional form. Fourth, we compute ambiguity using thirty-second and ten-minute return frequencies, as alternatives to the five-minute return frequency used in the results reported here. Fifth, we compute ambiguity using 82, 322 and 1002 bin histograms, instead of the 162 bin histograms used to generate our main results.[36] We run the main regression specifications using these variations, and the untabulated findings are essentially the same. The only meaningful difference is that the economic significance weakens for the non-parametric distribution, as many empty histogram bins are not extrapolated as compared to the use of the parametric distributions. These alternative estimates of ambiguity indicate that the significance of our results is not tied to a specific parametric assumption regarding the intraday return distribution.

6 Conclusion

Uncertainty plays an important role in the capital structure decision, as in many other financial decisions. Until recently, most studies examined uncertainty only by considering risk, the uncertainty of outcomes. However, ambiguity, or Knightian uncertainty, represents a separate and distinct aspect of uncertainty, the uncertainty of outcome probabilities.

We introduce a model that identifies a positive relation between ambiguity and leverage and a negative relation between risk and leverage. Our empirical analysis provides evidence in support of these relations using pooled time-series cross-sectional data covering more than 54,000 firm-year observations from more than 5,000 individual firms. Consistent with the model, we find positive associations between ambiguity and leverage. The empirical findings suggest that the ambiguity facing the firm has an economically meaningful impact on the capital structure decision. The economic significance of our estimates for ambiguity is equivalent to those for risk and is similar to or larger than the economic significance of commonly employed explanatory variables. Robustness

[36]For each bin grid, we examine intraday returns sampled at thirty-second, five-minute, and ten-minute intervals. For consistency, alongside each alternative ambiguity measure, the measure of risk was also computed using the same frequency of returns.

tests demonstrate that other uncertainty factors are not, due to their risk dependence, meaningful explanatory variables for the leverage decision.

Our findings are consistent with other recent papers analyzing financial decisions affected by ambiguity, including the pricing of credit default swaps and the timing of the exercise of executive stock options. Together, these studies suggest that the role of ambiguity in financial decisions is richer and more nuanced than previously understood.

References

Abel, A. B., 1983. Optimal investment under uncertainty. The American Economic Review 73, 228.

Andersen, T. G., Bollerslev, T., Diebold, F. X., Ebens, H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics 61, 43–76.

Anderson, E. W., Ghysels, E., Juergens, J. L., 2009. The impact of risk and uncertainty on expected returns. Journal of Financial Economics 94, 233–263.

Angrist, J. D., Pischke, J.-S., 2008. Mostly harmless econometrics: An empiricist's companion. Princeton university press.

Atanasov, V. A., Black, B. S., 2016. Shock-based causal inference in corporate finance and accounting research. Critical Finance Review 5, 207–304.

Augustin, P., Izhakian, Y., 2020. Ambiguity, volatility, and credit risk. The Review of Financial Studies 33, 1618–1672.

Avramov, D., 2002. Stock return predictability and model uncertainty. Journal of Financial Economics 64, 423–458.

Baillon, A., Huang, Z., Selim, A., Wakker, P. P., 2018. Measuring ambiguity attitudes for all (natural) events. Econometrica 86, 1839–1858.

Baillon, A., Placido, L., 2019. Testing constant absolute and relative ambiguity aversion. Journal of Economic Theory 181, 309–332.

Baker, S. R., Bloom, N., Davis, S. J., 2016. Measuring economic policy uncertainty. The Quarterly Journal of Economics 131, 1593–1636.

Ban, M., Chen, C. C., 2019. Ambiguity and capital structure adjustments. International Review of Economics and Finance 64, 242 – 270.

Bandi, F. M., Russell, J. R., 2006. Separating microstructure noise from volatility. Journal of Financial Economics 79, 655–692.

Bayes, T., Price, R., Canton, J., 1763. An essay towards solving a problem in the doctrine of chances. C. Davis, Printer to the Royal Society of London London, U. K.

Ben-Rephael, A., Izhakian, Y., 2020a. Do I realy want to hear the news? public information arrival and investor beliefs. SSRN eLibrary 3631760.

Ben-Rephael, A., Izhakian, Y., 2020b. Should I stay or should I go? trading behavior under ambiguity. SSRN eLibrary 3628757.

Berger, P. G., Ofek, E., Yermack, D. L., 1997. Managerial entrenchment and capital structure decisions. The Journal of Finance 52, 1411–1438.

Bernoulli, J., 1713. Ars Conjectandi (The Art of Conjecturing).

Bharath, S. T., Shumway, T., 2008. Forecasting default with the merton distance to default model. The Review of Financial Studies 21, 1339–1369.

Brenner, M., Izhakian, Y., 2018. Asset prices and ambiguity: Empirical evidence. Journal of Financial Economics 130, 503–531.

Castanias, R., 1983. Bankruptcy risk and optimal capital structure. The Journal of Finance 38, 1617–1635.

Chapman, D. A., Polkovnichenko, V., 2009. First-order risk aversion, heterogeneity, and asset market outcomes. The Journal of Finance 64, 1863–1887.

Chew, S. H., Sagi, J. S., 2008. Small worlds: modeling attitudes toward sources of uncertainty. Journal of Economic Theory 139, 1–24.

Coiculescu, G., Izhakian, Y., Ravid, S. A., 2019. Innovation under ambiguity and risk. SSRN eLibrary 3428896.

Coles, J., Li, Z. F., 2020. An empirical assessment of empirical corporate finance. SSRN library 1787143.

Coles, J., Loewenstein, U., 1988. Equilibrium pricing and portfolio composition in the presence of uncertain parameters. Journal of Financial Economics 22, 279–303.

Coles, J., Loewenstein, U., Suay, J., 1995. On equilibrium pricing under parameter uncertainty. Journal of Financial and Quantitative Analysis 30, 347–364.

DeAngelo, H., Roll, R., 2015. How stable are corporate capital structures? The Journal of Finance 70, 373–418.

DeMarzo, P. M., Fishman, M. J., 2007. Agency and optimal investment dynamics. The Review of Financial Studies 20, 151–188.

Doan, B., Foster, F. D., Yang, L., 2018. Do investors differentiate between uncertainty and risk: Evidence from trading around US macroeconomic news .

Ellsberg, D., 1961. Risk, ambiguity, and the savage axioms. Quarterly Journal of Economics 75, 643–669.

Epstein, L. G., Schneider, M., 2007. Learning under ambiguity. Review of Economic Studies 74, 1275–1303.

Epstein, L. G., Schneider, M., 2010. Ambiguity and asset markets. Annual Review of Financial Economics 2, 315–346.

Faria, G., Correia-da Silva, J., 2014. A closed-form solution for options with ambiguity about stochastic volatility. Review of Derivatives Research 17, 125–159.

Faulkender, M., Petersen, M. A., 2006. Does the source of capital affect capital structure? The Review of Financial Studies 19, 45–79.

Fischer, E. O., Heinkel, R., Zechner, J., 1989. Dynamic capital structure choice: Theory and tests. The Journal of Finance 44, 19–40.

Frank, M. Z., Goyal, V. K., 2009. Capital structure decisions: which factors are reliably important? Financial management 38, 1–37.

Franzoni, L. A., 2017. Liability law under scientific uncertainty. American Law and Economics Review 19, 327–360.

French, K. R., Schwert, G. W., Stambaugh, R. F., 1987. Expected stock returns and volatility. Journal of Financial Economics 19, 3–29.

Garlappi, L., Giammarino, R., Lazrak, A., 2017. Ambiguity and the corporation: Group disagreement and underinvestment. Journal of Financial Economics 125, 417–433.

Gilboa, I., Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. Journal of Mathematical Economics 18, 141–153.

Grieser, W. D., Hadlock, C. J., 2019. Panel-data estimation in finance: Testable assumptions and parameter (in)consistency. Journal of Financial and Quantitative Analysis 54, 1–29.

Hartman, R., 1972. The effects of price and cost uncertainty on investment. Journal of economic theory 5, 258–266.

Herron, R., Izhakian, Y., 2018. Ambiguity, risk, and dividend payout policy. SSRN eLibrary 2980600.

Herron, R., Izhakian, Y., 2019. Mergers and ambiguity: The role of ambiguity. SSRN eLibrary 3100611.

Ingersoll, J. E., 1987. Theory of financial decision making, vol. 3. Rowman & Littlefield.

Izhakian, Y., 2017. Expected utility with uncertain probabilities theory. Journal of Mathematical Economics 69, 91–103.

Izhakian, Y., 2019. Knight meets sharpe: Capital asset pricing under ambiguity. SSRN eLibrary 2007815.

Izhakian, Y., 2020. A theoretical foundation of ambiguity measurement. Journal of Economic Theory Forthcoming.

Izhakian, Y., Yermack, D., 2017. Risk, ambiguity, and the exercise of employee stock options. Journal of Financial Economics 124, 65–85.

Jaynes, E. T., 1957. Information theory and statistical mechanics. Physical review 106, 620.

Jensen, M. C., Meckling, W. H., 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of Financial Economics 3, 305–360.

Kendall, M., Stuart, A., 2010. The Advanced Theory of Statistics. vol. 1: Distribution Theory. London: Griffin, 2010, 6th ed. 1.

Kim, E. H., 1978. A mean-variance theory of optimal capital structure and corporate debt capacity. The Journal of Finance 33, 45–63.

Klibanoff, P., Marinacci, M., Mukerji, S., 2005. A smooth model of decision making under ambiguity. Econometrica 73, 1849–1892.

Kopylov, I., 2010. Simple axioms for countably additive subjective probability. Journal of Mathematical Economics 46, 867–876.

Kraus, A., Litzenberger, R. H., 1973. A state-preference model of optimal financial leverage. The Journal of Finance 28, 911–922.

Laplace, P. S., 1814. Théorie analytique des probabilitiés.

Lee, S., 2014. Knightian uncertainty and capital structure: Theory and evidence. SSRN eLibrary 2374679.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. The Journal of Finance 49, 1213–1252.

Lemmon, M. L., Roberts, M. R., Zender, J. F., 2008. Back to the beginning: persistence and the cross-section of corporate capital structure. The Journal of Finance 63, 1575–1608.

Lengwiler, Y., 2009. Microfoundations of financial economics: an introduction to general equilibrium asset pricing. Princeton University Press.

LeRoy, S. F., Werner, J., 2014. Principles of financial economics. Cambridge University Press.

Liu, L. Y., Patton, A. J., Sheppard, K., 2015. Does anything beat 5-minute rv? a comparison of realized measures across multiple asset classes. Journal of Econometrics 187, 293–311.

Mackie-Mason, J. K., 1990. Do taxes affect corporate financing decisions? The Journal of Finance 45, 1471–1493.

Malenko, A., Tsoy, A., 2020. Asymmetric information and security design under knightian uncertainty. SSRN library 3100285.

Merton, R. C., 1974. On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance 29, 449–470.

Modigliani, F., Miller, M. H., 1958. The cost of capital, corporation finance and the theory of investment. The American economic review 48, 261–297.

Myers, S. C., 1977. Determinants of corporate borrowing. Journal of Financial Economics 5, 147–175.

Rajan, R. G., Zingales, L., 1995. What do we know about capital structure? some evidence from international data. The Journal of Finance 50, 1421–1460.

Rothschild, M., Stiglitz, J. E., 1970. Increasing risk: I. A definition. Journal of Economic Theory 2, 225–243.

Schmeidler, D., 1989. Subjective probability and expected utility without additivity. Econometrica 57, 571–587.

Scholes, M., Williams, J., 1977. Estimating betas from nonsynchronous data. Journal of Financial Economics 5, 309–327.

Scott, J. H., 1976. A theory of optimal capital structure. The Bell Journal of Economics 7, 33–54.

Titman, S., Wessels, R., 1988. The determinants of capital structure choice. The Journal of Finance 43, 1–19.

Wooldridge, J. M., 2010. Econometric Analysis of Cross Section and Panel Data. The MIT Press, second ed.

Appendix

Proof of Theorem 1. Substituting the budget constraints into the objective function in Equation (6) and solving the maximization problem using point-wise optimization (i.e., differentiation with respect to θ conditional on a given event associated with x), as is standard in the literature (e.g., Lengwiler, 2009; LeRoy and Werner, 2014), provides

$$q(x)\partial_0 \mathrm{U} \quad = \quad \mathrm{E}\left[\varphi\left(x\right)\right]\left(1+\frac{\Upsilon''\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1-\mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}\mathrm{Var}\left[\varphi\left(x\right)\right]\right)\partial_x \mathrm{U}.$$

Since markets are complete and the law of one price holds, the payoff pricing functional assigns a unique price to each state contingent claim. Organizing terms completes the proof. $\square$

Proof of Theorem 2. Obtained from the same arguments in Proposition 1 of Modigliani and Miller (1958). $\square$

Proof of Theorem 3. The first-order condition, obtained by differentiating the firm value in Equation (12) with respect to its leverage, is

$$\frac{\partial V(\cdot)}{\partial F} \quad = \quad \frac{1}{1+r_f}\left(\ -\int_F^\infty \pi^*(x)\left(1-\tau\right)dx + \pi^*(F)F\left(1-\alpha\right) + \int_F^\infty \pi^*(x)dx - \pi^*(F)F\ \right) \quad = \quad 0.$$

Rearranging terms provides

$$F \quad = \quad \frac{\tau}{\pi^*(F)\alpha}\int_F^\infty \pi^*(x)dx.$$

The second-order condition for this problem is written

$$\frac{\partial^2 V(\cdot)}{\partial F^2} = -\tau\pi^*(F) - \alpha\pi^*(F) - \alpha F\frac{\partial \pi^*(F)}{\partial F}.$$

A sufficient condition for $\frac{\partial^2 V(\cdot)}{\partial F^2}$ to be negative is that $\frac{\partial \pi^*(F)}{\partial F} \geq 0$, which is delivered by $\frac{\partial q(F)}{\partial F} \geq 0$. $\square$

Proof of Proposition 1. By the "representation theorem" (Ingersoll, 1987), without the loss of generality, state prices can be considered directly instead of risk-neutral probabilities.[37] Suppose an event associated with a payoff $x \geq F$. A mean-preserving spread of size Δ_D and Δ_U is applied to x, such that the payoffs $x - \Delta_D$ and $x + \Delta_U$ are both in the set of possible outcomes, X. This

[37] Note that state prices are determined by aggregate consumption in the economy. Since the payoff of any project is small relative to aggregate consumption (the firm acts as a price taker), the altered payoffs are in the set of possible outcomes. Thus, introducing a mean-preserving spread to the outcome of a particular project does not affect the state prices.

implies that the conditional probabilities of these payoffs are, respectively, $\frac{\Delta_U}{\Delta_D+\Delta_U}$ and $\frac{\Delta_D}{\Delta_D+\Delta_U}$.

Consider first the case where $\Delta_D \leq x - F$. Risk neutral preferences imply that the value of the equity is not affected. Risk aversion with convex marginal utility (non-increasing relative risk aversion) implies that the value of the equity increases, due to the convex transformation of the state prices. That is, by the Jensen inequality,

$$\pi(x)\frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}}(x-F) \leq \pi(x)\frac{\Delta_U}{\Delta_D+\Delta_U}\frac{\partial_{x-\Delta_D}\mathrm{U}}{\partial_0 \mathrm{U}}(x-F-\Delta_D) + \pi(x)\frac{\Delta_D}{\Delta_D+\Delta_U}\frac{\partial_{x+\Delta_U}\mathrm{U}}{\partial_0 \mathrm{U}}(x-F+\Delta_U). \quad (17)$$

Consider the case of $\Delta_D > x - F$. Equation (17) holds true for every $\Delta_D \geq 0$. Thus, since $x - F - \Delta_D < 0$, it also holds true for $\max(x-F-\Delta_D, 0) = 0$ (i.e., limited liability in default states). Therefore, the value of the equity increases in risk for $\Delta_D > x - F$.

Since an increase in risk is considered with respect to the expected probabilities $\frac{\Delta_U}{\Delta_D+\Delta_U}$ and $\frac{\Delta_D}{\Delta_D+\Delta_U}$, the arguments above hold true for a mean-preserving spread in outcomes to a single prior, multiple priors, or all priors in the set of priors $\mathcal{P}$. □

Proof of Proposition 2. By the "representation theorem" (Ingersoll, 1987), without loss of generality, state prices can be considered directly instead of risk-neutral probabilities. Consider a mean preserving spread in the probabilities of a given event associated with outcome x; i.e., $\mathrm{Var}[\varphi(x)]$ increases. As a function of the expected probabilities, which are invariant to a mean preserving spread in probabilities, the coefficient of absolute ambiguity aversion, $\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}$, remains unchanged. By Equation (8), the perceived probability $\pi(x)$ and the associated state price decreases in $\mathrm{Var}[\varphi(x)]$. This holds true for any x. Ambiguity, which is the weighted sum of the variances of the probabilities across the different events, increases when the variance of probabilities of at least one event increases. Thus, an increase in ambiguity weakly decreases the state prices for all states and, by Equation (11), weakly decreases S_0. □

Proof of Proposition 3. Substitute the explicit expression of $\pi^*(\cdot)$ into the optimal leverage in

Equation (13) to obtain

$$F = \frac{\tau}{\alpha q(F)} \int_F^{\infty} q(x)dx. \tag{18}$$

The first-order condition can be defined by the implicit function

$$G\left(F, \mho^2, \mathcal{R}\right) = \tau \int_F^{\infty} q(x)dx - \alpha q(F)F, \tag{19}$$

where $\mathcal{R}$ stands for the level of risk. By the second-order condition, at a maximum

$$\frac{\partial G}{\partial F} < 0.$$

Suppose that for any event associated with an outcome x the outcome is instead $x \times t$, where $t > 0$. An increase in t increases the risk and also the expected payoff. Differentiating the implicit function in Equation (19) with respect to t provides

$$\frac{\partial G}{\partial t} = \tau \int_F^{\infty} \mathrm{E}\left[\varphi\left(x\right)\right] \left(1 + \frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)} \mathrm{Var}\left[\varphi\left(x\right)\right]\right) \frac{\partial_{xx}\mathrm{U}}{\partial_0 \mathrm{U}} x dx - \alpha F \frac{\partial}{\partial t} q(F).$$

Neither the perceived probability nor ambiguity changes as a consequence of the change in outcomes associated with events, as they are outcome-independent (up to a state space partition). By risk aversion, $\partial_{xx}\mathrm{U} < 0$. Thus, since $\frac{\partial q(F)}{\partial F} \geq 0$, $\frac{\partial G}{\partial t} \leq 0$. By the implicit function theorem,

$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt = 0.$$

Therefore,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \leq 0.$$

This implies that the increase in risk reduces leverage despite the accompanying increase in the expected payoff. $\square$

Proof of Proposition 4. The mean-preserving spreads in probabilities can be written as $\mathrm{Var}\left[\varphi\left(x\right)\right] \times t$ where $t > 0$ (i.e., an increase in t increases ambiguity). Differentiating the implicit function in Equation (19) with respect to t, provides

$$\frac{\partial G}{\partial t} = \tau \int_F^{\infty} \mathrm{E}\left[\varphi\left(x\right)\right] \frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(x\right)\right]\right)} \mathrm{Var}\left[\varphi\left(x\right)\right] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx - \alpha \mathrm{E}\left[\varphi\left(F\right)\right] \frac{\Upsilon''\left(1 - \mathrm{E}\left[\mathrm{P}\left(F\right)\right]\right)}{\Upsilon'\left(1 - \mathrm{E}\left[\mathrm{P}\left(F\right)\right]\right)} \mathrm{Var}\left[\varphi\left(F\right)\right] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}} F.$$

Subtracting the first-order condition, which is equal to zero as defined by Equation (19), provides

$$\frac{\partial G}{\partial t} = -\tau \int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx + \alpha \mathrm{E}\left[\varphi\left(F\right)\right] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}} F.$$

Thus, $\frac{\partial G}{\partial t} \geq 0$ when

$$F \geq \frac{\tau}{\alpha} \frac{1}{\mathrm{E}\left[\varphi\left(F\right)\right] \frac{\partial_F \mathrm{U}}{\partial_0 \mathrm{U}}} \int_F^\infty \mathrm{E}\left[\varphi\left(x\right)\right] \frac{\partial_x \mathrm{U}}{\partial_0 \mathrm{U}} dx. \tag{20}$$

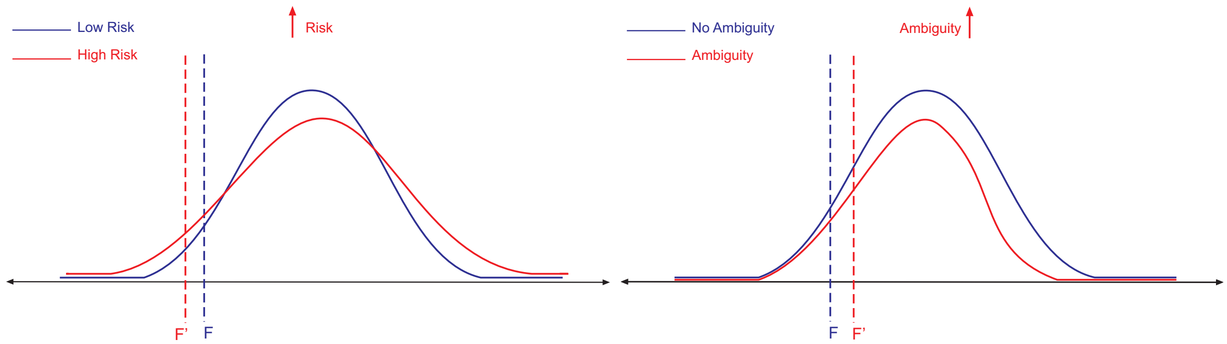
The right hand side of this inequality is the optimal F assuming there is no ambiguity. Since $\left(\frac{\Upsilon''}{\Upsilon'}\right)' \geq 0$, then $0 \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(F)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(F)])} \geq \frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(x)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(x)])}$ for any $x \geq F$. Therefore, the inequality $\frac{\partial G}{\partial t} \geq 0$ holds true when ambiguity is present. By the implicit function theorem,

$$\frac{\partial G}{\partial F} dF + \frac{\partial G}{\partial t} dt = 0.$$

Therefore, since $\frac{\partial G}{\partial F} < 0$,

$$\frac{dF}{dt} = -\frac{\frac{\partial G}{\partial t}}{\frac{\partial G}{\partial F}} \geq 0.$$

This holds true for a mean-preserving spread in probabilities of a single state or multiple states. $\square$



Panel A: Increasing Risk

Panel B: Increasing Ambiguity

Figure 1: **Optimal leverage**

This figure depicts the effect of an increase in risk (left panel) and an increase in ambiguity (right panel) on optimal leverage. Each panel represents a stylized perceived probability distribution in which the x-axis illustrates the range of the firm's possible future values. The point F is the optimal choice for the firm's leverage, at which the expected marginal benefit of interest tax shields just equals the expected marginal cost of bankruptcy. In the left panel, the blue series shows a lower-risk perceived probability distribution, which is assumed to shift to the red series, representing a higher risk. The optimal choice of leverage then moves to the left on the x-axis. In the right panel, an increase in ambiguity is assumed to occur, resulting in lower perceived probabilities of expected outcomes, as shown by the change from the blue to the red probability distribution. Note that in the presence of ambiguity and aversion to ambiguity, probabilities are sub-additive. In this case, the optimal choice of leverage moves to the right.

Table 1: **Descriptive statistics**

The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website.

Statistic	N	Mean	St. Dev.	Median	Min	Max
Book Leverage	54,691	0.201	0.202	0.157	0.000	0.968
Market Leverage	54,691	0.176	0.206	0.102	0.000	0.959
Ambiguity	54,691	0.020	0.021	0.013	0.001	0.180
Ambiguity Unlevered.	54,691	0.029	0.035	0.016	0.001	0.288
Risk	54,691	0.108	0.148	0.039	0.001	1.042
Risk Unlevered.	54,691	0.091	0.133	0.029	0.001	0.894
Cash Flow Volatility	47,297	0.042	0.694	0.0002	0.000	44.967
Median Book Leverage	54,691	0.171	0.152	0.139	0.000	0.950
Median Market Leverage.	54,691	0.152	0.162	0.100	0.000	0.953
Profitability	54,691	0.048	0.270	0.111	−3.242	0.787
Tangibility	54,691	0.251	0.229	0.174	0.000	0.958
Market to Book	54,691	1.998	2.038	1.368	0.002	28.783
Research & Development	54,691	16.031	49.013	0.700	0.000	589
Sales	54,691	1.025	0.203	1.080	0.000	1.234
Tax Rate	54,691	0.147	0.152	0.044	0.000	0.395
Firm Age	54,691	16.823	12.527	13.000	2.000	61.000

Table 2: **First difference regression estimates of capital structure**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.109^{***}		0.129^{***}	0.144^{***}				
	(0.033)		(0.032)	(0.032)				
Ambiguity Unlevered					0.056^{***}		0.058^{***}	0.068^{***}
					(0.021)		(0.021)	(0.021)
Risk		-0.028^{***}	-0.031^{***}	-0.038^{***}				
		(0.008)	(0.008)	(0.008)				
Risk Unlevered						-0.019^{**}	-0.020^{**}	-0.027^{***}
						(0.009)	(0.009)	(0.009)
Median Book Leverage				-0.005				-0.006
				(0.007)				(0.007)
Profitability				-0.003				-0.003
				(0.005)				(0.005)
Tangibility				0.080^{***}				0.078^{***}
				(0.012)				(0.012)
Market to Book				-0.001^{***}				-0.001^{***}
				(0.000)				(0.000)
Research & Development				0.000				0.000
				(0.000)				(0.000)
Sales				0.031^{***}				0.032^{***}
				(0.011)				(0.011)
Tax Rate				-0.000				-0.000
				(0.004)				(0.004)
Constant	0.006^{***}	0.006^{***}	0.006^{***}	0.005^{***}	0.006^{***}	0.006^{***}	0.006^{***}	0.006^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	38,813	38,813	38,813	38,813	38,813	38,813	38,813	38,813
R^2	0.000	0.001	0.001	0.004	0.000	0.000	0.000	0.004

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.388***		0.484***	0.488***				
	(0.040)		(0.040)	(0.041)				
Ambiguity Unlevered					0.370***		0.385***	0.385***
					(0.028)		(0.029)	(0.029)
Risk		−0.138***	−0.149***	−0.154***				
		(0.009)	(0.009)	(0.009)				
Risk Unlevered						−0.121***	−0.125***	−0.128***
						(0.009)	(0.009)	(0.009)
Median Market Leverage				−0.009				−0.012
				(0.007)				(0.007)
Profitability				−0.006				−0.004
				(0.004)				(0.004)
Tangibility				0.087***				0.084***
				(0.013)				(0.013)
Market to Book				−0.000				−0.000
				(0.000)				(0.000)
Research & Development				0.000**				0.000**
				(0.000)				(0.000)
Sales				0.037***				0.039***
				(0.008)				(0.008)
Tax Rate				0.003				0.004
				(0.005)				(0.005)
Constant	0.007***	0.007***	0.006***	0.006***	0.007***	0.007***	0.006***	0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	38,813	38,813	38,813	38,813	38,813	38,813	38,813	38,813
R^2	0.003	0.009	0.013	0.016	0.005	0.006	0.010	0.013

Table 3: **First difference regression estimates of capital structure on subgroups by median**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables using subgroups of above and below median size and above and below median leverage. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		Leverage Subgroups				Size Subgroups		
	Low	High	Low	High	Small	Large	Small	Large
Ambiguity	0.105**	0.130***			0.116	0.168***		
	(0.043)	(0.045)			(0.083)	(0.034)		
Ambiguity Unlevered			0.064*	−0.017			0.097	0.065***
			(0.036)	(0.026)			(0.069)	(0.022)
Risk	−0.024***	−0.101***			−0.036***	−0.026		
	(0.007)	(0.018)			(0.009)	(0.023)		
Risk Unlevered			−0.008	−0.121***			−0.023**	−0.053*
			(0.007)	(0.026)			(0.009)	(0.032)
Median Book Leverage	−0.014	−0.027***	−0.015	−0.030***	−0.016	0.005	−0.018	0.004
	(0.010)	(0.009)	(0.010)	(0.009)	(0.012)	(0.008)	(0.012)	(0.008)
Profitability	0.001	−0.014	0.002	−0.014	−0.003	−0.010	−0.002	−0.010
	(0.004)	(0.015)	(0.004)	(0.015)	(0.006)	(0.013)	(0.006)	(0.013)
Tangibility	0.061***	0.097***	0.059***	0.095***	0.070***	0.099***	0.068***	0.099***
	(0.012)	(0.019)	(0.012)	(0.019)	(0.017)	(0.015)	(0.017)	(0.015)
Market to Book	−0.001***	−0.003**	−0.001***	−0.003*	−0.001***	−0.001	−0.001***	−0.001
	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
Research & Development	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.011	0.103**	0.012	0.103**	0.030***	0.079	0.031***	0.074
	(0.008)	(0.040)	(0.008)	(0.041)	(0.011)	(0.075)	(0.011)	(0.075)
Tax Rate	−0.001	0.002	−0.000	0.002	0.004	−0.005	0.004	−0.005
	(0.004)	(0.006)	(0.004)	(0.006)	(0.006)	(0.005)	(0.006)	(0.005)
Constant	−0.012***	0.023***	−0.012***	0.023***	0.007***	0.004***	0.007***	0.004***
	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)	(0.001)	(0.000)
Observations	19,404	19,405	19,404	19,405	19,404	19,405	19,404	19,405
R^2	0.005	0.008	0.004	0.008	0.004	0.005	0.004	0.004

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Leverage Subgroups				Size Subgroups			
	Low	High	Low	High	Small	Large	Small	Large
Ambiguity	0.173***	0.670***			0.574***	0.358***		
	(0.034)	(0.064)			(0.091)	(0.045)		
Ambiguity Unlevered			0.099***	0.405***			0.752***	0.251***
			(0.026)	(0.038)			(0.079)	(0.030)
Risk	−0.070***	−0.335***			−0.133***	−0.375***		
	(0.007)	(0.021)			(0.010)	(0.038)		
Risk Unlevered			−0.054***	−0.353***			−0.115***	−0.428***
			(0.006)	(0.026)			(0.010)	(0.050)
Median Market Leverage	−0.023***	−0.015	−0.025***	−0.023**	−0.011	−0.004	−0.010	−0.009
	(0.008)	(0.010)	(0.008)	(0.010)	(0.013)	(0.009)	(0.013)	(0.009)
Profitability	−0.001	−0.016	−0.000	−0.016	−0.006	−0.021	−0.004	−0.018
	(0.002)	(0.013)	(0.002)	(0.013)	(0.004)	(0.017)	(0.004)	(0.017)
Tangibility	0.022**	0.138***	0.018*	0.136***	0.088***	0.097***	0.083***	0.095***
	(0.010)	(0.021)	(0.010)	(0.021)	(0.015)	(0.023)	(0.015)	(0.023)
Market to Book	0.000***	−0.004***	0.000***	−0.004***	−0.000	0.001**	−0.000	0.001**
	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
Research & Development	0.000*	0.000	0.000*	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.019***	0.164***	0.020***	0.163***	0.028***	0.785***	0.029***	0.787***
	(0.006)	(0.045)	(0.006)	(0.045)	(0.008)	(0.114)	(0.008)	(0.114)
Tax Rate	0.007*	0.002	0.008*	0.004	0.019***	−0.015**	0.020***	−0.014**
	(0.004)	(0.008)	(0.004)	(0.008)	(0.007)	(0.007)	(0.007)	(0.007)
Constant	−0.012***	0.024***	−0.012***	0.024***	0.007***	0.002***	0.007***	0.002***
	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	19,404	19,405	19,404	19,405	19,404	19,405	19,404	19,405
R^2	0.016	0.033	0.010	0.027	0.017	0.024	0.016	0.021

Table 4: **Economic significance of the estimates of capital structure**

The levels of economic significance for the measures of ambiguity relative to the economic significance of the control variables. The first column reports the standard deviation of the first differences of each variable, the second column reports the coefficient estimates from Table 2, and the third column reports their product. The final two columns report the levels of the economic significance of the measures of ambiguity relative to the economic significance of the control variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Expected marginal tax rates are obtained from John Graham's website.

Panel A: Book Leverage

	Std	Coefficient	Significance	Ambiguity Relative To	Unlevered Ambiguity Relative To
Ambiguity	0.0130	0.1412	0.0018	1	0.6833
Ambiguity Unlevered	0.0183	0.0687	0.0013	1.4635	1
Risk	0.0698	-0.0389	-0.0027	0.6778	0.4631
Risk Unlevered	0.0617	-0.0275	-0.0017	1.0838	0.7406
Median Book Leverage	0.0675	-0.0069	-0.0005	3.9353	2.6890
Profitability	0.1470	-0.0034	-0.0005	3.6407	2.4877
Tangibility	0.0500	0.0803	0.0040	0.4586	0.3134
Market to Book	1.6978	-0.0012	-0.0020	0.9379	0.6408
Research & Development	62.6167	0.0000	0.0006	3.1542	2.1552
Sales	0.0576	0.0321	0.0018	0.9960	0.6806
Tax Rate	0.1109	-0.0006	-0.0001	26.7791	18.2979

Panel B: Market Leverage

	Std	Coefficient	Significance	Ambiguity Relative To	Unlevered Ambiguity Relative To
Ambiguity	0.0130	0.4879	0.0064	1	1.1146
Ambiguity Unlevered	0.0183	0.3869	0.0071	0.8972	1
Risk	0.0698	-0.1549	-0.0108	0.5872	0.6545
Risk Unlevered	0.0617	-0.1304	-0.0080	0.7904	0.8810
Median Market Leverage	0.0830	-0.0072	-0.0006	10.5815	11.7945
Profitability	0.1470	-0.0053	-0.0008	8.1474	9.0814
Tangibility	0.0500	0.0899	0.0045	1.4154	1.5777
Market to Book	1.6978	-0.0003	-0.0005	12.9858	14.4744
Research & Development	62.6167	0.0000	0.0006	11.4156	12.7242
Sales	0.0576	0.0385	0.0022	2.8692	3.1981
Tax Rate	0.1109	0.0031	0.0003	18.3298	20.4311

Table 5: **Robustness tests**

Linear model regression estimates of the changes in annual leverage ratio explained by one-year lagged changes in the variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Each regression includes an intercept and the following control variables (whose coefficient estimates are unreported). Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. The variance of the mean (Var mean) is the variance of daily mean returns (computed from 5-minute return) over the month. The variance of the variance (Var var) is the variance of daily variance returns (computed from 5-minute return) over the month. Bid-ask spread is the effective bid-ask spread. Analysts disagreement is the variance among analyst forecasts of the stock price. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ambiguity	0.137***					0.136***	0.137***	0.135***	0.162***
	(0.033)					(0.033)	(0.033)	(0.033)	(0.032)
Risk	−0.039***	−0.039***	−0.036***	−0.013	−0.044***	−0.042***	−0.039***	−0.017	−0.046***
	(0.009)	(0.010)	(0.009)	(0.010)	(0.010)	(0.010)	(0.009)	(0.010)	(0.010)
Var Mean		0.016				0.011			
		(0.022)				(0.022)			
Var Var			0.000				0.000		
			(0.004)				(0.004)		
Bid-ask Spread				−0.335***				−0.332***	
				(0.088)				(0.088)	
Analysts Disagreement					−0.006**				−0.006**
					(0.003)				(0.003)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	39,317	39,317	39,317	39,317	33,331	39,317	39,317	39,317	33,331
R^2	0.004	0.004	0.004	0.004	0.005	0.004	0.004	0.005	0.005

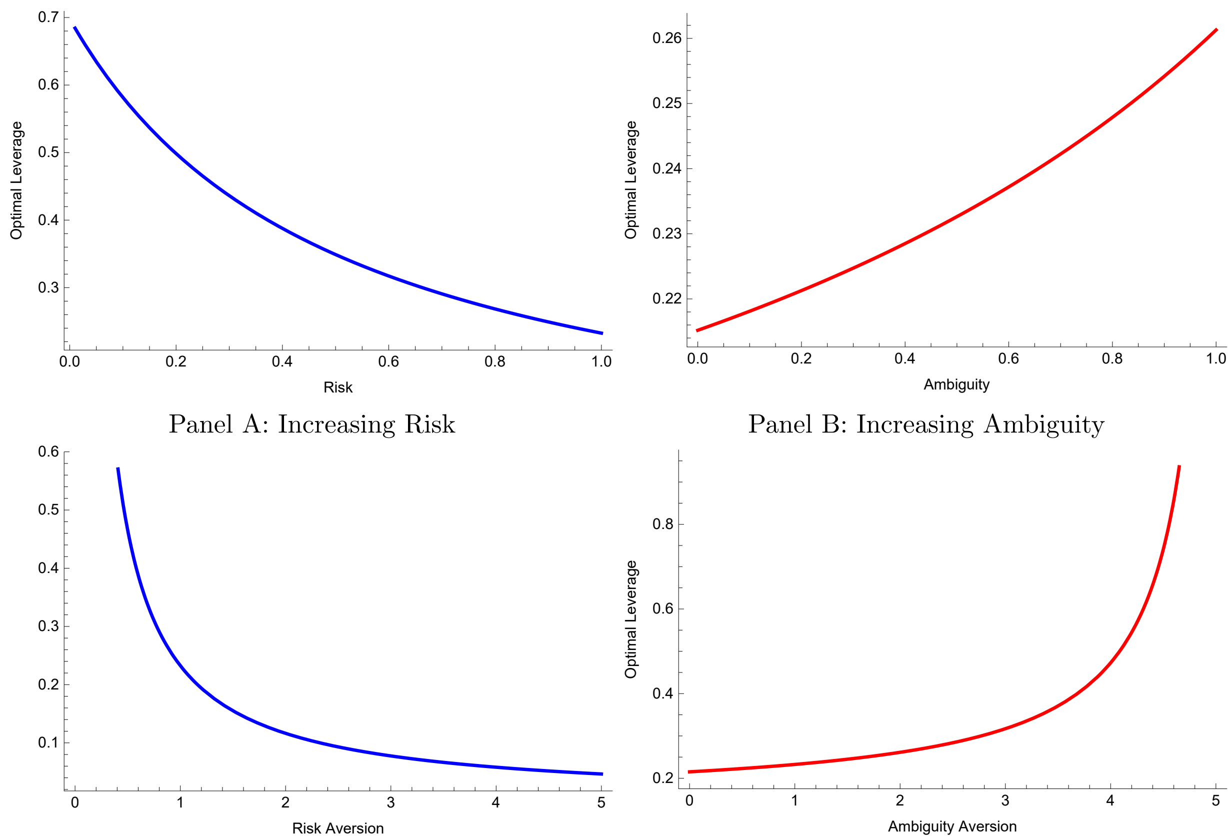
Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ambiguity	0.489^{***}					0.490^{***}	0.489^{***}	0.484^{***}	0.494^{***}
	(0.043)					(0.044)	(0.043)	(0.043)	(0.042)
Risk	-0.157^{***}	-0.145^{***}	-0.144^{***}	-0.087^{***}	-0.180^{***}	-0.154^{***}	-0.157^{***}	-0.101^{***}	-0.187^{***}
	(0.011)	(0.013)	(0.011)	(0.017)	(0.013)	(0.013)	(0.011)	(0.017)	(0.013)
Var Mean		0.006				−0.012			
		(0.024)				(0.026)			
Var Var			0.002				0.001		
			(0.007)				(0.008)		
Bid-ask Spread				-0.831^{***}				-0.821^{***}	
				(0.174)				(0.176)	
Analysts Disagreement					−0.001				−0.001
					(0.002)				(0.002)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	39,317	39,317	39,317	39,317	33,331	39,317	39,317	39,317	33,331
R^2	0.016	0.012	0.012	0.015	0.013	0.016	0.016	0.019	0.017

Online Appendix

OA.1 Simulations

Figure OA.1 below provides a general visual representation of the comparative statics conducted in Section 2.5. It depicts the effects of risk, aversion to risk, ambiguity, and aversion to ambiguity on optimal leverage. These visual comparative statics illustrate that the quantitative effect of each variable is meaningful. Figure OA.2 below provides a general visual representation of the methodology employed to compute ambiguity from the data. To simplify the illustration, it assumes six-bin histograms.



Panel A: Increasing Risk

Panel B: Increasing Ambiguity

Panel C: Increasing Risk Aversion

Panel D: Increasing Ambiguity Aversion

Figure OA.1: **Optimal leverage**

This figure simulates the effect of an increase in risk (Panel A), an increase in ambiguity (Panel B), an increase in risk aversion (Panel C), and an increase in ambiguity aversion (Panel D) on optimal leverage. It simulates the optimal leverage as delivered by Equation (13) in Theorem 3. To this end, it assumes investors with constant absolute risk aversion (CARA) and constant absolute ambiguity aversion (CAAA). It also assume a set of priors contracted around the standard normal probability distribution to generate an increase in ambiguity. Risk is generated in the same way as in Proposition 3. Panel A assumes, a CARA coefficient of 1, a CAAA coefficient of 1, and an ambiguity factor of 0.5. Panel B assumes, a CARA coefficient of 1, a CAAA coefficient of 1, and a risk factor of 1. Panel C assumes, a CAAA coefficient of 1, a risk factor of 1, and an ambiguity factor of 0.5. Panel D assumes, a CARA coefficient of 1, a risk factor of 1, and an ambiguity factor of 0.5.

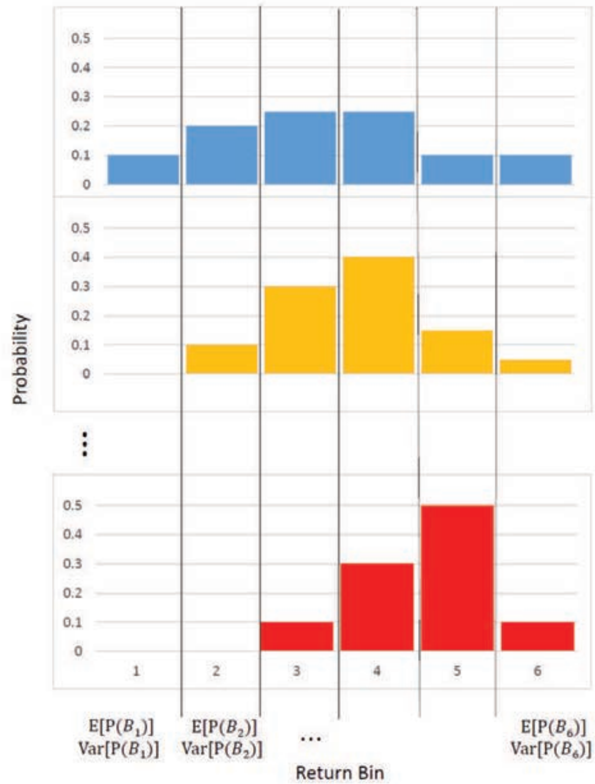


Figure OA.2: **Ambiguity measurement**

This figure illustrates the computation of the ambiguity measure, which is derived for each firm-month based on intraday stock-returns sampled at five-minute frequencies from 9:30 to 16:00. Thus, we obtain up to 22 daily histograms of up to 78 intraday returns in each month. We discretize the daily return distributions into n bins of equal size $B_j = (r_j, r_{j-1}]$ across histograms. The height of the histogram for a particular bin is computed as the fraction of daily intraday returns observed in that bin, and thus represents the probability of that particular bin outcome. We compute the expected probability of a particular bin across the daily return distributions, $\mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right]$, as well as the variance of these probabilities, $\mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]$. Ambiguity is then computed as $\mho^2\left[r_i\right] \equiv 1/\sqrt{w\left(1-w\right)}\sum_{j=1}^{n}\mathrm{E}\left[\mathrm{P}_i\left(B_j\right)\right]\mathrm{Var}\left[\mathrm{P}_i\left(B_j\right)\right]$, where we scale the weighted-average volatilities of probabilities to the bins' size $w = r_{i,j} - r_{i,j-1}$.

OA.2 Additional analyses

This section provides additional summary statistics. Table OA.1 below reports the autocorrelation of the annual ambiguity measure used in the paper. It shows that the one-year autocorrelation is very high, 67.2%, but decays over a five-year horizon. Table OA.2 below reports the cross correlations of our ambiguity measure computed in five different ways over different horizons. It computes ambiguity using five-minute returns, using daily open-high-low-close prices, and using daily returns. Using these data, ambiguity is computed over one, three and five years. This table demonstrates that the correlations across different frequencies and horizons are high, especially for annual ambiguity. Table OA.3 below reports the standard matrix of cross correlations between the variables used in the empirical analysis. Table OA.4 below reports a quintile analysis of the variables used in the empirical analysis.

Table OA.1: **Ambiguity Autocorrelation**

The annual ambiguity autocorrelation. Ambiguity, based on TAQ intraday stock price records, is calculated according to procedures described in the text.

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
0.672	0.614	0.560	0.578	0.504	0.600	0.488	0.570	0.526	0.555

Table OA.2: **Ambiguity Correlation**

The cross correlation of ambiguity measured with different return frequencies over different time horizons.
(1) is the ambiguity computed from five-minute return for each month and averaged over a year.
(2) is the ambiguity computed from open, high, low and close prices for each month and averaged over a year.
(3) is the ambiguity computed from daily returns over a year.
(4) is the ambiguity computed from daily returns over three years.
(5) is the ambiguity computed from daily returns over five years.

	(1)	(2)	(3)	(4)	(5)
(1)	1				
(2)	0.672	1			
(3)	0.557	0.563	1		
(4)	0.442	0.458	0.873	1	
(5)	0.418	0.450	0.823	0.911	1

Table OA.3: **Correlation matrix**

The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(1) Book Leverage	1													
(2) Market Leverage	0.817	1												
(3) Ambiguity	-0.003	-0.005	1											
(4) Ambiguity Unlev.	0.033	0.017	0.878	1										
(5) Risk	0.125	0.186	-0.022	-0.123	1									
(6) Risk Unlev.	0.038	0.110	-0.022	-0.118	0.971	1								
(7) Cash Flow Vol.	0.032	0.040	0.059	0.076	-0.166	-0.173	1							
(8) Profitability	-0.175	-0.257	-0.047	-0.053	-0.049	-0.044	-0.040	1						
(9) Tangibility	0.133	0.137	-0.062	-0.072	0.227	0.219	-0.059	-0.068	1					
(10) Market to Book	-0.130	-0.442	-0.008	-0.004	-0.087	-0.073	-0.056	0.293	-0.019	1				
(11) RD	0.041	0.052	0.166	0.200	-0.275	-0.272	0.213	-0.167	-0.053	-0.093	1			
(12) Sales	0.042	0.048	0.170	0.211	-0.457	-0.463	0.224	0.178	-0.167	-0.086	0.353	1		
(13) Tax Rate	-0.045	-0.077	-0.055	-0.059	0.024	0.027	-0.065	0.255	-0.011	0.076	-0.144	0.011	1	
(14) Firm Age	0.009	0.073	0.184	0.233	-0.458	-0.455	0.236	-0.194	-0.236	-0.193	0.397	0.489	-0.107	1

Table OA.4: **Ambiguity Quintile**

This table sorts on firm annual ambiguity into 5 quintile. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Ivol1 is the idiosyncratic risk based on the single factor model. Ivol3 is the idiosyncratic risk based on the Fama-French three-factor model. Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales.

Ambiguity Quintile	Risk	IdioV1	IdioV3	skew	kurt	Market to Book	Sales	Rsearch and Devlopment	Profitability	Tangibility
1	0.191	0.205	0.177	-0.011	8.621	2.591	0.912	12.189	-0.064	0.211
2	0.146	0.146	0.126	-0.005	8.576	2.172	0.967	12.622	0.002	0.242
3	0.109	0.118	0.103	0.004	8.3	1.923	1.023	16.344	0.044	0.261
4	0.09	0.086	0.075	0.016	7.865	2.136	1.068	26.898	0.104	0.276
5	0.084	0.075	0.066	0.021	7.41	2.829	1.113	100.869	0.193	0.263

OA.3 Cash flow volatility

Table OA.5 reports findings for the relation between the first differences of leverage and first differences of the explanatory variables using the variance of cash flow as the measure of risk. These findings are provided as a robustness check regarding the measure of risk. The coefficient estimates on the measures of ambiguity remain uniformly positive and highly significant in each of the specifications. Cash flow volatility, however, is insignificant in most of the regressions. It may seem unsurprising that this measure of risk is not a significant explanatory variable for leverage in the first differences estimation because, by construction, there is limited time-series variation in the change in cash flow volatility. Very similar results are found using different time periods over which cash flow volatility is measured or when the variation in cash flow is measured using quarterly data.

Table OA.5: **First difference regression estimates of capital structure**

Linear model regression estimates of the annual leverage ratio explained by one-year lagged variables using first differences. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity, based on TAQ intraday stock price records, is calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Cash flow volatility is the standard deviation of the firm's operating income before depreciation measured over the proceeding three fiscal years. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.117***		0.118***	0.126***				
	(0.035)		(0.035)	(0.035)				
Ambiguity Unlevered					0.055**		0.055**	0.063***
					(0.022)		(0.022)	(0.022)
Cash Flow Volatlity		0.001	0.001	0.001*		0.001	0.001	0.001*
		(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Median Book Leverage				−0.008				−0.009
				(0.007)				(0.007)
Profitability				−0.000				−0.000
				(0.006)				(0.006)
Tangibility				0.076***				0.076***
				(0.013)				(0.013)
Market to Book				−0.001				−0.001
				(0.000)				(0.000)
Research & Development				0.000				0.000
				(0.000)				(0.000)
Sales				0.034***				0.034***
				(0.013)				(0.013)
Tax Rate				−0.000				−0.000
				(0.004)				(0.004)
Constant	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***	0.006***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	34,025	34,025	34,025	34,025	34,025	34,025	34,025	34,025
R^2	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.003

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ambiguity	0.402***		0.402***	0.389***				
	(0.042)		(0.042)	(0.042)				
Ambiguity Unlevered					0.368***		0.368***	0.361***
					(0.030)		(0.030)	(0.030)
Cash Flow Volatility		−0.000	0.000	0.000		−0.000	0.000	0.000
		(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	(0.001)
Median Market Leverage				−0.019**				−0.016**
				(0.008)				(0.008)
Profitability				0.004				0.004
				(0.004)				(0.004)
Tangibility				0.066***				0.068***
				(0.014)				(0.014)
Market to Book				0.001***				0.001***
				(0.000)				(0.000)
Research & Development				0.000**				0.000**
				(0.000)				(0.000)
Sales				0.047***				0.046***
				(0.009)				(0.009)
Tax Rate				0.004				0.004
				(0.005)				(0.005)
Constant	0.007***	0.008***	0.007***	0.007***	0.007***	0.008***	0.007***	0.007***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Observations	34,025	34,025	34,025	34,025	34,025	34,025	34,025	34,025
R^2	0.003	0.000	0.003	0.005	0.005	0.000	0.005	0.007

OA.4 Level regression tests

For robustness, Table OA.6 below replicates our main empirical specifications using a firm and year fixed effects approach rather than first differences. The same qualitative results for ambiguity and risk are obtained.

Table OA.6: **Level regression estimates of capital structure**

Linear model regression estimates of the level of annual leverage ratio explained by one-year lagged variables. The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)	(5)	(6)
Ambiguity	0.236***	0.258***	0.167***			
	(0.022)	(0.030)	(0.034)			
Ambiguity Unlevered				0.215***	0.221***	0.161***
				(0.014)	(0.021)	(0.023)
Risk	−0.029***	−0.035***	−0.045***			
	(0.004)	(0.007)	(0.007)			
Risk Unlevered				−0.021***	−0.025***	−0.038***
				(0.005)	(0.007)	(0.008)
Past Book Leverage	0.858***	0.642***	0.641***	0.849***	0.634***	0.633***
	(0.004)	(0.007)	(0.007)	(0.004)	(0.007)	(0.007)
Median Book Leverage	0.052***	0.040***	0.024***	0.050***	0.039***	0.023***
	(0.005)	(0.007)	(0.007)	(0.005)	(0.007)	(0.007)
Profitability	−0.002	−0.004	−0.006	−0.002	−0.004	−0.006
	(0.004)	(0.005)	(0.005)	(0.004)	(0.005)	(0.005)
Tangibility	0.025***	0.036***	0.030***	0.027***	0.038***	0.031***
	(0.002)	(0.009)	(0.009)	(0.002)	(0.009)	(0.009)
Market to Book	−0.001*	−0.001	−0.001***	−0.001*	−0.001	−0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Rsearch and Devlopment	0.000	0.000***	0.000***	0.000	0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.002	0.022**	0.025***	0.003	0.025**	0.027***
	(0.004)	(0.010)	(0.010)	(0.004)	(0.010)	(0.010)
Tax Rate	−0.013***	0.000	−0.002	−0.013***	0.001	−0.002
	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.004)
Firm Age	−0.000***	0.001***		−0.000***	0.001***	
	(0.000)	(0.000)		(0.000)	(0.000)	
Constant	0.023***			0.023***		
	(0.005)			(0.005)		
Firm Fixed Effects	No	Yes	Yes	No	Yes	Yes
Year Fixed Effects	No	No	Yes	No	No	Yes
Observations	47,917	47,917	47,917	47,917	47,917	47,917
R^2	0.784	0.823	0.825	0.785	0.823	0.825

Panel B: Market Leverage

	(1)	(2)	(3)	(4)	(5)	(6)
Ambiguity	0.148***	0.352***	0.209***			
	(0.024)	(0.035)	(0.038)			
Ambiguity Unlevered				0.176***	0.316***	0.223***
				(0.015)	(0.024)	(0.025)
Risk	−0.041***	−0.101***	−0.103***			
	(0.005)	(0.007)	(0.007)			
Risk Unlevered				−0.041***	−0.096***	−0.100***
				(0.005)	(0.007)	(0.008)
Past Market Leverage	0.860***	0.620***	0.624***	0.853***	0.610***	0.615***
	(0.005)	(0.008)	(0.007)	(0.005)	(0.008)	(0.007)
Median Market Leverage	0.024***	0.011	0.011	0.024***	0.013*	0.013*
	(0.005)	(0.007)	(0.007)	(0.005)	(0.008)	(0.007)
Profitability	−0.000	−0.014***	−0.014***	−0.000	−0.015***	−0.014***
	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)
Tangibility	0.037***	0.045***	0.041***	0.037***	0.046***	0.042***
	(0.003)	(0.009)	(0.009)	(0.003)	(0.009)	(0.009)
Market to Book	−0.001***	−0.001***	−0.002***	−0.002***	−0.001***	−0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Rsearch and Devlopment	−0.000	0.000***	0.000***	−0.000	0.000***	0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.007**	0.037***	0.033***	0.004	0.038***	0.034***
	(0.003)	(0.008)	(0.008)	(0.004)	(0.008)	(0.008)
Tax Rate	−0.002	−0.004	−0.005	−0.002	−0.003	−0.004
	(0.003)	(0.005)	(0.005)	(0.003)	(0.005)	(0.005)
Firm Age	−0.000***	−0.001***		−0.001***	−0.001***	
	(0.000)	(0.000)		(0.000)	(0.000)	
Constant	0.027***			0.029***		
	(0.004)			(0.004)		
Firm Fixed Effects	No	Yes	Yes	No	Yes	Yes
Year Fixed Effects	No	No	Yes	No	No	Yes
Observations	47,917	47,917	47,917	47,917	47,917	47,917
R^2	0.747	0.796	0.809	0.748	0.797	0.809

OA.5 Instrumental variable regression findings

To provide further evidence regarding the relation between ambiguity and leverage, Table OA.7 presents the findings of a first-difference two-stage least squares estimate of the model specified by Equations (OA.1) and (OA.2) below. The instrumental variables estimation provides external validation for the ambiguity measures we employ. The consideration of only the variation in our ambiguity measures that is explained by the instruments provides further evidence regarding the relation between the ambiguity facing the firm and the firm's capital structure choice. The empirical model we estimate is

$$\Delta \mho^2_{i,t} \quad = \quad \alpha + \beta_1 \cdot DIVC_t + \beta_2 \cdot DEF_{i,t} + \gamma \Delta Z_{i,t} + \epsilon_{i,t}, \tag{OA.1}$$

and

$$\Delta L_{i,t} \quad = \quad \alpha + \beta \cdot \widehat{\Delta \mho}^2_{i,t-1} + \gamma \cdot \Delta Z_{i,t-1} + \varepsilon_{i,t}. \tag{OA.2}$$

In the first stage, we instrument for the first difference in the ambiguity measures using lagged values of two proxies: the number of lawsuits in which the firm is named as a defendant in each year as a proxy for firm level ambiguity (DEF), and an indicator variable equal to one if the U.S. House of Representatives and U.S. Senate are controlled by different political parties than that of the President, and zero otherwise, as a proxy for economy-wide ambiguity (Divided Congress, $DEFC$).[38,39] In the second stage, the differences in the fitted values of ambiguity and unlevered ambiguity are used as explanatory variables for leverage.

These instruments are chosen because they are plausibly exogenous to the firm's leverage decisions. The instrument Divided Congress is the result of popular elections. It is not likely that the

[38] The Defendant variable is constructed from the data provided in AuditAnalytics database. The Divided Congress indicator is constructed from the data provided at
https://en.wikipedia.org/wiki/Divided_government_in_the_United_States.

[39] The use of variables in levels as instruments for the first-difference of ambiguity is a benefit of using a first-differences model to control for the unobserved heterogeneity. Doing so implies that the exogeneity required for the instrument is only contemporaneous exogeneity rather than strict exogeneity as would be required in a fixed-effects two-stage least squares estimation (Wooldridge, 2010). Using the differences of the instrumental variables (rather than their levels) to instrument for ambiguity the same qualitative results are obtained.

ambiguity of an individual firm would cause a divided congress. The instrument Defendant is the result of the unintended consequences of past operating decisions. Because lawsuits are the result of outcomes, it is not likely that ambiguity causes lawsuits. Both of these instruments are, however, expected to influence the level of ambiguity firms face. The variable Divided Congress reflects the level of policy uncertainty—uncertainty regarding the "rules of the game" for economic activity for a specific period (Baker et al., 2016). The idea is that a unified government is more likely to be able to enact significant policy changes, decreasing the level of policy uncertainty. The Defendant variable is a reflection of the level of uncertainty regarding the firm's cash flow and potentially the nature of restrictions on its operations in the near future.

As with all instrumental variables analysis in a corporate context, exclusion is difficult to establish. The primary difficulty in identifying instruments for ambiguity is that these instruments may also affect the firm's risk. Both of the chosen instruments have significant correlations with ambiguity or unlevered ambiguity and their correlations with risk are smaller by at least an order of magnitude. An additional concern is the possibility that ambiguity affects lawsuits. Lawsuits are filed and litigated when there is uncertainty regarding the probability the suit will be successful. While the probability a lawsuit will be successful is different than the probability distribution of future returns, there is no reason that variation in the uncertainty regarding these different probabilities cannot be related. It is, therefore, difficult to establish that the "exclusion restriction" holds. A further concern with the use of these instruments in an empirical examination of leverage is the unobserved heterogeneity inherent in these models (Lemmon et al., 2008). Identifying whether our chosen instruments also influence leverage via one of the omitted variables (Angrist and Pischke, 2008; Atanasov and Black, 2016)) is, of course, infeasible. We, therefore, consider our instrumental variables analysis as providing external validity to the measure of ambiguity employed in this study, rather than establishing a causal relationship between ambiguity and leverage.

In each panel of Table OA.7, the first and the third columns present the findings from the first-stage regression estimating the first difference of ambiguity or unlevered ambiguity, respectively. The second and fourth columns present the findings from the second-stage regression tests. As before, Panel A examines book leverage, and Panel B examines market leverage.

As in our other empirical models, the estimated coefficients are consistent with the predictions of the theoretical model. In the first-stage regressions, the coefficient estimates on the instruments for ambiguity and unlevered ambiguity are consistently positive and highly significant for both book and market leverage ratio. The coefficient estimates indicate that our measure of ambiguity is indeed higher when firms are named as defendants in a greater number of lawsuits and when there is a greater level of policy uncertainty. The level of significance of these variables and the reported F-statistics for the first-stage regressions suggest that the model does not suffer from a weak instruments problem.

In each second-stage regression test, the coefficient estimates for the fitted values of the first difference in ambiguity or unlevered ambiguity indicate a positive and highly significant relation with the first-difference in both book and market leverage ratios. The F-statistics are highly significant in each second-stage regression. These estimates again demonstrate that our measures of ambiguity are positively related to firms' subsequent leverage choices. The second-stage regressions also confirm that risk (equity return volatility or unlevered return volatility) continues to have a significantly negative relation to the leverage ratio.

Table OA.7: **First difference 2-stage least squares regression estimates of capital structure**

The full sample includes 54,691 annual observations associated with 5,092 individual firms between 1993 and 2018, using records from the Compustat database. Ambiguity and risk, based on TAQ intraday stock price records, are calculated according to procedures described in the text. Unlevered ambiguity is computed using the book value of total debt and the market value of equity to represent firm value for every five-minute interval. Book leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total book assets. Market leverage ratio is "debt in current liabilities" plus "long-term debt" divided by the total market value of assets, where the latter is the market value of the equity at the end of the fiscal year plus "debt in current liabilities" plus "long-term debt." Sales is the log of one plus the firm's reported sales. Profitability is operating income before depreciation divided by book assets. Tangibility is property, plant and equipment divided by book assets. The market-to-book ratio is market equity plus total debt plus preferred stock liquidation value minus deferred taxes and investment tax credits divided by book assets. Research and development is R&D expenses relative to sales. Median leverage is the median of the annual leverage ratio in the industry. Expected marginal tax rates are obtained from John Graham's website. The instrumental variable Defendant is the number of lawsuits in which the firm is named as a defendant in each year, taken as a proxy for the firm-level ambiguity. The instrumental variable Divided Congress is an indicator variable set equal to one if the US house of representatives and the US senate are controlled by different political parties, and zero otherwise, taken as a proxy for economy wide ambiguity. Standard errors are clustered by firm and robust to heteroscedasticity. Standard errors are in parentheses, *p<0.1; **p<0.05; ***p<0.01

Panel A: Book Leverage

	(1)	(2)	(3)	(4)
Ambiguity (Fit)		0.820^{***}		
		(0.199)		
Ambiguity Unlevered (Fit)				0.658^{***}
				(0.151)
Divided Congress	0.005^{***}		0.006^{***}	
	(0.000)		(0.000)	
Defendant	0.000^{***}		0.000^{***}	
	(0.000)		(0.000)	
Risk	0.027^{***}	-0.055^{***}		
	(0.002)	(0.010)		
Risk Unlevered			0.016^{***}	-0.027^{***}
			(0.002)	(0.010)
Median Book Leverage	-0.007^{***}	-0.015^{**}	-0.005^{***}	-0.018^{**}
	(0.001)	(0.008)	(0.002)	(0.008)
Profitability	-0.000	-0.001	-0.001	0.000
	(0.000)	(0.006)	(0.001)	(0.006)
Tangibility	-0.007^{***}	0.081^{***}	-0.011^{***}	0.080^{***}
	(0.001)	(0.014)	(0.002)	(0.014)
Market to Book	0.000^{***}	-0.001^{***}	0.000^{***}	-0.001^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
Research & Development	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.003^{**}	0.022^{*}	0.004^{***}	0.023^{*}
	(0.001)	(0.013)	(0.001)	(0.013)
Tax Rate	0.001	-0.001	0.001	-0.001
	(0.001)	(0.004)	(0.001)	(0.004)
Observations	38,813	38,813	38,813	38,813
R^2	0.092	0.066	0.082	0.059
F-Stat	586.35	9.82	503.12	9.14
Kleibergen-Paap Wald F-Stat	587.16	587.158	503.71	503.714
Stock-Yogo weak ID test:				
10% maximal IV size		19.93		19.93
15% maximal IV size		11.59		11.59
20% maximal IV size		8.75		8.75
25% maximal IV size		7.25		7.25
Hansen J-Stat		2.041		3.705
Hausman Stat ($Prob > \chi^2$)		11.52 (0.174)		13.81 (0.087)

Panel B: Market Leverage

	(1)	(2)	(3)	(4)
Ambiguity (Fit)		4.028^{***}		
		(0.270)		
Ambiguity Unlevered (Fit)				3.152^{***}
				(0.209)
Divided Congress	0.005^{***}		0.006^{***}	
	(0.000)		(0.000)	
Defendant	0.000^{***}		0.000^{***}	
	(0.000)		(0.000)	
Risk	0.028^{***}	-0.255^{***}		
	(0.002)	(0.013)		
Risk Unlevered			0.018^{***}	-0.173^{***}
			(0.002)	(0.011)
Median Market Leverage	-0.020^{***}	0.047^{***}	-0.029^{***}	0.054^{***}
	(0.001)	(0.010)	(0.001)	(0.011)
Profitability	-0.000	-0.002	-0.001^{**}	0.001
	(0.000)	(0.004)	(0.001)	(0.004)
Tangibility	-0.006^{***}	0.111^{***}	-0.010^{***}	0.111^{***}
	(0.001)	(0.015)	(0.002)	(0.015)
Market to Book	0.000^{***}	-0.001^{***}	0.000^{***}	-0.001^{***}
	(0.000)	(0.000)	(0.000)	(0.000)
Research & Development	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Sales	0.003^{***}	0.017^{*}	0.004^{***}	0.019^{**}
	(0.001)	(0.010)	(0.001)	(0.010)
Tax Rate	0.001	0.001	0.001	0.004
	(0.001)	(0.006)	(0.001)	(0.006)
Observations	38,813	38,813	38,813	38,813
R^2	0.106	-0.111	0.097	-0.158
F-Stat	586.35	48.89	503.12	46.79
Kleibergen-Paap Wald F-Stat	587.16	587.158	503.71	503.714
Stock-Yogo weak ID test:				
10% maximal IV size		19.93		19.93
15% maximal IV size		11.59		11.59
20% maximal IV size		8.75		8.75
25% maximal IV size		7.25		7.25
Hansen J-Stat		41.077		51.591
Hausman Stat ($Prob > \chi^2$)		196.52 (0.000)		189.10 (0.000)