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ABSTRACT

In this paper, we develop a new model for government cost-benefit analysis in the presence of risk. In our model, a benevolent government chooses the scale of a risky project in the presence of two key frictions. First, there are market failures, which cause the government to perceive project payoffs differently than private households do. This gives the government a "social risk management" motive: projects that ameliorate market failures when household marginal utility is high are appealing. The second friction is that government financing is costly because of tax distortions. This creates a "fiscal risk management" motive: incremental spending that occurs when total government spending is already high is particularly unattractive. A first key insight is that the government's need to manage fiscal risk frequently limits its capacity for managing social risk. A second key insight is that fiscal risk and social risk interact in complex ways. When considering many potential projects, government cost-benefit analysis thus acquires the flavor of a portfolio choice problem. We use the model to explore how the relative attractiveness of two technologies for promoting financial stability—bailouts and regulation—varies with the government's fiscal burden and characteristics of the economy.

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1 Introduction

In modern economies, a significant fraction of economy-wide risk is borne indirectly by taxpayers via the government. Governments have significant liabilities associated with retirement benefits, social insurance programs, and financial system backstops. These liabilities are large: the amount of credit risk explicitly recognized on the balance sheet of the U.S. federal government now exceeds $3 trillion, and implicit or off-balance sheet liabilities are even larger. For instance, off-balance sheet guarantees on mortgage-backed securities account for another $7 trillion. Moreover, the risk associated with the government's contingent liabilities is not idiosyncratic but varies systematically with economic conditions. For example, during the financial crisis, total off-balance sheet financial system backstops temporarily reached more than $6 trillion (Geithner [2014]). In addition, the U.S. debt-to-GDP ratio rose from 38% to 72% between 2007 and 2013 due to falling tax revenue and increasing expenditures on government programs that automatically expand in a recession.

Given the magnitude of these exposures, the set of risks the government chooses to bear and the way it manages those risks is of great importance. A vast literature in public economics studies the costs and benefits of various government programs such as unemployment insurance and social security (Baily [1978], Chetty [2006]). An equally vast literature studies optimal government financing policies—i.e., taxation and government debt management—that minimize costly distortions, holding fixed the set of programs the government wishes to undertake (Ramsey [1927], Diamond and Mirrless [1971], Mirrless [1971], Sandmo [1975]). In this paper, we bridge the gap between these two literatures, emphasizing the ways that government financing frictions impact the set of projects the government should undertake. The result is a flexible framework for conducting cost-benefit analysis in a stochastic environment where the government faces financing frictions.

In our model, the government chooses the scale of a program—designed to correct a specific market failure—whose social benefits and fiscal costs fluctuate over time and across states of the world. Our setup differs from the Ricardian framework, where the government is a veil for taxpayers, in two critical ways. First, we assume that government programs can generate social benefits that private actors are unable to generate on their own. While we model these social benefits in reduced form, we think of them as arising from the fact that the government often has unique technologies for addressing market failures. For instance, the government may be able to use price or quantity regulations to correct technological externalities (Weitzman [1974]) or pecuniary externalities in incomplete markets (Greenwald and Stiglitz [1986]), enforce contributions to address free-rider problems in the provision of public goods (Samuelson [1954]), or mandate participation to address market-unravelling issues (Rothschild and Stiglitz [1976]). Second, we assume that lump-sum taxation is not feasible and that the government can only raise tax revenue by levying proportional taxes that create deadweight efficiency losses (Ramsey [1927], Diamond and Mirrlees [1971], Mirrlees [1971], Saez [2001]). Specifically, we assume that the government raises revenue through an income tax that distorts the labor supply of households. As a result, each dollar of tax revenue the government raises costs society more than a dollar in pre-tax resources.

These two key frictions drive a rich set of interactions. In choosing the program's scale, the government has a "social risk management" motive: programs that generate large net benefits are attractive, particularly if those benefits accrue in recessions, when household marginal utility is high. An important subtlety that arises here is that the government cannot take marginal utility as given: by operating large programs, the government affects aggregate consumption and thus household utility. In addition, the fact that taxation is distortionary gives rise to a "fiscal risk management" motive: programs requiring large outlays are unattractive, particularly if those outlays tend to occur when spending on other government programs is elevated.

A first key insight to emerge from the model is that the government's need to manage fiscal risk frequently limits its capacity for managing social risk. We illustrate this idea using a numerical example of a single program: deposit insurance. In the example, deposit insurance is a social hedge, albeit one that is fiscally risky: we assume it generates large social benefits, raising output by preventing bank runs during recessions, but it also involves potentially large government outlays in the same bad states. The example highlights how the two risk management motives come into conflict. Most directly, because deposit insurance involves large government outlays and hence greater tax distortions in bad times, it creates additional fiscal risk. This reduces the government's desire to use deposit insurance to manage social risk, particularly when tax distortions are large or when the pre-existing fiscal burden is high.

Indeed, we show that the social and fiscal risk management motives typically pull in opposite directions as we vary the parameters of the economy or the program under consideration. For example, an increase in the volatility of exogenous private income makes deposit insurance more attractive from a social risk management standpoint: the value of preventing bank runs in bad times increases as marginal utility becomes more volatile. On the other hand, an increase in the volatility of private income also raises the volatility of tax rates, increasing fiscal risk. This tends to reduce the attractiveness of an expansive deposit insurance regime.

We then extend our framework to consider the case where the government must simulataneously choose the scales of multiple programs. A second key insight is that neither fiscal risk nor social risk can be judged in isolation. A program's fiscal risk depends on how its outlays covary with those of other programs. Similarly, a program's social risk depends, in part, on how its net benefits covary with those of other programs. These interpendencies imply that government cost-benefit analysis acquires the flavor of a classic portfolio choice problem (Markowitz [1952], Tobin [1958], Sharpe [1964], Linter [1965]).

We illustrate these portfolio intuitions by considering an example in which the government chooses between a "fiscally safe" and a "fiscally risky" program for promoting financial stability. Regulation that limits bank risk taking ex ante is a fiscally safe program because the associated expenditures vary little across states of the world. By contrast, bailouts in the form of ex post guarantees or capital injections are a fiscally risky program because the associated government outlays vary enormously across states and may surge in a deep recession. Since bailouts entail costly increases in taxes or cuts to other programs, the attractiveness of bailouts versus regulations depends on the government's other fiscal commitments.

When the government's fiscal burden is low, bailouts can be a relatively attractive way to promote financial stability. As the fiscal burden rises, it is optimal to substitute toward the less fiscally risky program, regulation. When the fiscal burden is high, it may be optimal to completely eschew bailouts and rely solely on regulation. For instance, if the government is also committed to a strong social safety net, which already requires large outlays in recessions, then ex post bailouts become less attractive relative to ex ante guarantees. These conclusions correspond to the classic portfolio choice intuition that an investor facing a higher level of "background risk" should choose a more conservative financial portfolio (Merton [1973], Campbell and Viceira [2002]). In addition, we show that when the distortionary costs of taxation rise, the optimal quantity of fiscally risky bailouts falls. This corresponds to the standard precept that the optimal portfolio allocation to risky assets falls as risk aversion rises.

Work in public finance typically considers individual government interventions in isolation. For instance, our paper is related to the public finance literature that studies the optimal provision of a single public good when the government must finance its expenditures using distortionary taxes. This literature, including Pigou (1947), Stiglitz and Dasgupta (1971), and Atkinson and Stern (1974), studies a static, deterministic setting. Our model generalizes this classic public finance problem to a multi-period, stochastic setting. This allows us to study how risk, both social and fiscal, impacts optimal program scale.

The literatures on optimal taxation and government debt management recognize that government expenditures are stochastic and, assuming that tax-smoothing is imperfect, that the tax burden will also be stochastic. However, this work typically treats the government as an exogenously given collection of programs. By contrast, our approach shows that the cyclicality of government expenditures has important implications for the set of programs that should be undertaken by the government.

Our model also reveals strong parallels between government cost-benefit analysis and modern theories of corporate investment. In our setting, the distortionary costs of taxation play a similar role to the one that costly external finance plays in a corporate setting. Specifically, the distortionary costs of taxation can lead the government to behave as though it is more risk-averse than the taxpayers it represents, just like financing frictions can lead firms to behave as though they are more risk-averse than shareholders. In a corporate finance setting, hedging and risk management activities can enhance firm value if external financing is costly (Froot, Scharfstein, and Stein [1993]), just as smoothing tax rates and debt management create value for taxpayers when taxation is distortionary (Barro [1979]). By the same logic, financing frictions have implications for the optimal scale and composition of government projects in our setting, much as they do for firm investment (Fazzari, Hubbard, and Petersen [1988], Kaplan and Zingales [1997], Bolton, Chen, and Wang [2011 and 2013]).

Our paper is also related to the literature on intermediary-based asset pricing (He and Krishnamurthy [2013], Brunnermeier and Sanikov [2014]). In those models, households indirectly hold financial assets via intermediaries. And, because there are frictions between the household sector and the financial sector, the stochastic discount factor that prices financial assets equals households' stochastic discount factors with an adjustment that captures these intermediation frictions. A similar result obtains in our model because tax distortions mean that the government perceives a financing wedge term between itself and the households it represents.

The plan for the paper is as follows. In Section 2 we develop the general model and characterize the optimal scale of a single potentially welfare-improving government program in the presence of distortionary taxation. We explore several special cases of the general model that help clarify the key intuitions. Section 3 extends the model to consider portfolios of multiple government programs. In particular, we explore the choice between a fiscally safe and a fiscally risky program for addressing a given market failure. Section 4 concludes.

2 Model

2.1 Setup

We consider a two-period model with dates $t = 0$ and 1. At time 0, a benevolent government chooses the scale of a government program, denoted q, and initial government borrowing D_0 to maximize the lifetime expected utility of a representative household.

By correcting a market failure, a government program of scale q generates a social payoff $W_t(q)$ at time t, where $W_t(0) = 0$, $W_t'(\cdot) > 0$, and $W_t''(\cdot) \leq 0$. $W_1(q)$ is stochastic, taking on different values at time 1 depending on the state of the world. For instance, deposit insurance may create broad financial stability benefits that raise private output in bad states at time 1, implying $W_1(q) > 0$ in bad states. In contrast, if there are no social benefits associated with the program, then $W_t(q) = 0$. If the government chooses quantity q of the program, this requires government outlays of $X_t(q)$ at time t, where $X_t(0) = 0$, $X_t'(\cdot) > 0$, and $X_t''(\cdot) \geq 0$. Like $W_1(q)$, $X_{\cdot}(q)$ can also be stochastic.

We assume that the government enters time 0 having previously accumulated debt $\overline{D}$. At time 0, the government issues default-free bonds in quantity D_0 that must be repaid at time 1. Letting R denote the gross riskless interest rate of interest between times 0 and 1, the government's budget constraints at time 0 and time 1 are

$$\begin{aligned} T_0 + D_0 &= G_0 + X_0(q) + \overline{D} \\ T_1 &= G_1 + X_1(q) + RD_0. \end{aligned} \tag{1}$$

T_t is tax revenue at time t; $X_t(q)$ is the endogenous level of expenditures associated with the specific program under consideration; G_t represents other exogenous government expenditures not associated with the specific program.

A key feature of our setup is that we assume taxation is distortionary. Specifically, we assume that households choose their labor supply ℓ_t in period t to maximize period t utility, which we refer to as "consumption"[1]:

$$C_t(\ell_t) = Y_t\left(\overbrace{\ell_t(1-\tau_t)}^{\text{After-tax income}} - \overbrace{\frac{(\ell_t - 1 + \eta)^2 - \eta^2}{2\eta}}^{\text{Disutility from labor}}\right) + W_t(q) + (\text{Net trade in govt bonds})_t. \quad (2)$$

Here Y_t is the exogenous level of productivity at time t, which we refer to as the "tax base," τ_t is the proportional income tax levied by the government at time t, and $\eta \geq 0$ governs the elasticity of labor with respect to τ_t.

By Eq. (2), the optimal labor supply when households face an income tax rate of τ_t is $\ell_t^* = 1 - \eta\tau_t$, which compares to the first-best labor supply of $\ell_t^{**} = 1$ under lump-sum taxation. Thus, an income tax at rate τ_t generates total tax revenues of

$$T_t = \tau_t \ell_t^* Y_t = \tau_t(1 - \eta\tau_t) Y_t \leq \tau_t Y_t. \quad (3)$$

(Note that Eq. (3) implicitly links the level of tax revenue T_t to the tax rate τ_t.[2]) However, because income taxation disincentivizes labor, it generates a deadweight efficiency loss of

$$C_t(\ell_t^{**}) - C_t(\ell_t^*) - T_t = Y_t \frac{\eta}{2}\tau_t^2, \quad (4)$$

as in Harberger (1962). Naturally, the deadweight loss is greater when tax rates are higher or when the elasticity of labor supply with respect to tax rates is larger—i.e., when η is larger. When $\eta = 0$, income taxation generates no deadweight losses and is equivalent to lump-sump taxation.

When $\eta > 0$, each dollar of tax revenue costs society more than a dollar in pre-tax resources. Specifically, the cost of public funds (Browning [1976])—i.e., the total cost of raising T_t dollars of tax revenue in terms of household consumption—is equal to the amount of tax revenue raised *plus* the deadweight loss: $T_t + Y_t(\eta/2)\tau_t^2$. This implies that the marginal

[1] Formally, Eq. (2) means that period t utility, C_t, takes a quasi-linear form with private consumption serving as the numeraire. Thus, $Y_t((\ell_t - 1 + \eta)^2 - \eta^2)/(2\eta)$ captures households' disutility from supplying ℓ_t units of labor, and $W_t(q)$ is the additional utility households derive when the government chooses program scale q. This quasilinear specification for period utility is similar to the one used in Atkinson (1990) and Diamond (1998). The substance of the assumption in this case is that program benefits $W_t(q)$ do not impact labor supply choices.

[2] Because income taxation disincentivizes labor, the government faces a "Laffer curve." Specifically, for a given Y_t, Eq. (3) says that tax revenue is an inverse U-shaped function of the tax rate, which is maximized when $\tau_t = 1/(2\eta)$. Since the government will always choose to be on the upward sloping part of the Laffer curve where $\tau_t < 1/(2\eta)$, the tax rate at time t is $\tau_t = (1 - \sqrt{1 - 4\eta T_t/Y_t})/(2\eta)$.

cost of public funds is[3]

$$\frac{\partial}{\partial T_t}\left(T_t + Y_t\frac{\eta}{2}\tau_t^2\right) = 1 + \frac{\eta\tau_t}{1-2\eta\tau_t} \geq 1. \tag{5}$$

In what follows, we use the notation

$$h'(\tau_t) \equiv \frac{\eta\tau_t}{1-2\eta\tau_t} > 0 \tag{6}$$

to denote the extent to which the marginal cost of public funds exceeds one. In other words, $h'(\tau_t)$ is the marginal deadweight cost of raising additional tax revenue—i.e. the cost of raising an additional dollar of revenue via distortionary taxation.[4] Naturally, we have $h''(\tau_t) > 0$: the marginal cost of public funds is increasing in the income tax rate because higher tax rates imply greater labor supply distortions.

The lifetime utility of the representative household is

$$U = u(C_0) + \beta E[u(C_1)], \tag{7}$$

where $0 < \beta \leq 1$, $u'(\cdot) > 0$, and $u''(\cdot) \leq 0$. Households choose their bond holdings (B_0) taking taxes, program scale, and the interest rate as exogenously given. Thus, the Euler condition for household bond holdings is

$$u'(C_0) = R\beta E\left[u'(C_1)\right]. \tag{8}$$

Imposing market clearing for government bonds ($B_0 = D_0$) and using the government's budget constraint to substitute out debt and taxes, we find that household consumption at time t is

$$C_t = Y_t(1 - \frac{\eta}{2}\tau_t^2) + W_t(q) - X_t(q) - G_t. \tag{9}$$

The government's problem is to choose program scale q and initial borrowing D_0, which together determine the path of taxes, to maximize the lifetime utility of the representative household. The government takes the path of $\{Y_t, W_t, X_t, G_t\}$ and $\overline{D}$ as given. Formally, the

[3] The marginal cost of public funds exceeds one if the elasticity of labor supply with respect to taxes is negative. In a more general model, the sign of this elasticity is ambiguous. It is the sum of a negative substitution effect and a positive wealth effect: higher taxes make households poorer and thus motivate them to work more. Given our quasi-linear specification of per-period utility, there are no wealth effects, and only the substitution effect is present.

[4] The term $(1 - 2\eta\tau_t)$ appears in the denominator of (5) and (6) because the government faces a Laffer curve. As a result, a given change in required tax revenue has a larger effect on the required tax rate when the rate is closer to the revenue-maximizing tax rate of $1/(2\eta)$.

planner solves

$$\max_{D_0,q}\left\{u\left(Y_0(1-\frac{\eta}{2}\tau_0^2)+W_0(q)-X_0(q)-G_0\right)+\beta E\left[u\left(Y_1(1-\frac{\eta}{2}\tau_1^2)+W_1(q)-X_1(q)-G_1\right)\right]\right\}, \tag{10}$$

subject to the non-negativity constraint that $q \geq 0$.[5] In choosing D_0 and q, the government recognizes that tax rates τ_0 and τ_1 depend on its choices of D_0 and q and are given by

$$\tau_0 = \frac{1-\sqrt{1-4\eta\frac{\overline{D}-D_0+G_0+X_0(q)}{Y_0}}}{2\eta} \text{ and } \tau_1 = \frac{1-\sqrt{1-4\eta\frac{RD_0+G_1+X_1(q)}{Y_1}}}{2\eta}. \tag{11}$$

Somewhat more subtly, the government also recognizes that it cannot take the riskless interest rate as given because its decisions affect aggregate consumption. The riskfree interest rate R depends on its choices and is implicitly defined by

$$u'\left(Y_0(1-\frac{\eta}{2}\tau_0^2)+W_0(q)-X_0(q)-G_0\right)=R\beta E\left[u'\left(Y_1(1-\frac{\eta}{2}\tau_1^2)+W_1(q)-X_1(q)-G_1\right)\right]. \tag{12}$$

Ignoring the non-negativity constraint, the government solves an unconstrained problem where it recognizes that tax rates and the riskfree rate are implicitly defined by Eqs. (11) and (12).

Examples To make the setup concrete, we briefly discuss two government programs as examples. Internet Appendix A formally maps these two examples into our framework. Our first example builds on Stein (2012) and considers the value of government interventions to prevent economically destabilizing bank runs on financial intermediaries. In Stein's (2012) model, intermediaries hold long-term risky assets, which they finance by issuing short-term debt. While households value this short-term debt because it is safe and liquid, short-term financing exposes intermediaries to bank runs that can force them to liquidate financial assets in an economic downturn. These asset fire sales have real economic costs because, instead of investing in new real projects, other savers must use their scarce capital to purchase liquidated financial assets. As a result, fire sales raise the equilibrium hurdle rate on new real projects and lead to an additional drop in total private output that exacerbates incipient economic downturns. Because financial intermediaries do not internalize this pecuniary externality, they are overly reliant on short-term financing from a social point of view, and

[5]The constraint that $q \geq 0$ reflects the fact that the government typically cannot choose a negative program scale. Thus, our framework can be used to study both the extensive and intensive margins of government policy.

there is scope for welfare-improving government policies.

Consider a program in which the government guarantees the short-term debt of financial intermediaries in order to prevent these socially costly bank runs. The government outlays associated with this guarantee program (X) are the realized fiscal costs of the guarantees net of any insurance premia paid by financial intermediaries. Thus, outlays would be positive in states of the world where financial intermediaries suffer large losses on their risky assets and the government has to make short-term creditors whole. The social payoff (W) from this program is the gain in net private income that stems from the fact that stopping bank runs allows savers to use their capital for productive new projects, net of any output that is lost due to moral hazard distortions.

Our second example is unemployment insurance. Consider the Rothschild-Stiglitz (1976) model in which two types of agents have different probabilities of becoming unemployed. When agents know their types and insurers do not, a competitive equilibrium in which different types pool together cannot exist: insurers will always find it profitable to offer an insurance contract that appeals only to low-risk types. In the separating equilibrium that may or may not exist, agents with a low probability of future unemployment must be less than fully insured in order to prevent high-risk agents from mimicking them and purchasing cheap insurance. This candidate separating equilibrium will not exist—indeed no competitive equilibrium exists and the insurance market will completely shutdown—when low-risk types find it costly to separate from high-risk types (or find it attractive to pool with high-risk types).

A government-run mandatory insurance program can lead to a Pareto improvement by enforcing pooling. The social payoff W from a mandatory insurance program is the welfare gain associated with the move from the separating equilibrium (or, when no equilibrium exists, from an outcome with no insurance) to the pooling outcome, again, net of any moral hazard distortions.[6] As in the previous example, the net government outlay X is the difference between government insurance payouts and insurance premia collected (i.e., payroll taxes).

Discussion of model setup Several features of the model setup deserve discussion. First, we adopt a representative agent perspective, assuming that the social benefits of any program accrue to a representative household. However, if market failures create scope for government

[6] See Baily (1978) and Chetty (2006) for classic examinations of the trade-off between the insurance benefits and moral hazard costs of social insurance programs.

policies to generate Pareto improvements, a representative agent may fail to exist.[7] Thus, in cases where no representative agent exists, our framework should be viewed as a short-hand for a setting in which the government maximizes a more complicated social welfare function.

Second, the source of fiscal frictions in our model is the incentive distortions stemming from proportional taxation (Ramsey [1927], Diamond and Mirrlees [1971], Mirrlees [1971], Saez [2001]). Taken literally, this means that, with lump-sum taxation, there would be no fiscal frictions in our model. More generally, while we refer to distortions $h'(\tau)$ as "tax distortions," they are best seen as a short-hand for a host of frictional costs that may arise when the government faces a significant fiscal burden. For instance, there may be costs associated with the risk of sovereign default (Borensztein and Panizza [2009]) or costs associated with high rates of nominal price inflation that are often triggered by large government debt burdens, as in Leeper's (1991) fiscal theory of the price level.

Third, the setup largely abstracts from the fact that government programs may distort the behavior of private agents in undesirable ways. For instance, government insurance programs may create moral hazard problems (Baily [1978], Allen et al [2015]). Conceptually, these distortionary costs should be folded into the Ws, as we noted above in our two examples.

Fourth, while program scale impacts household utility, it does not impact the tax base in our model. This is an appropriate assumption in the case where the government raises revenue to produce a classic public good such as public infrastructure or national security.[8] However, in other cases, the $W_t(q)$ may add to the tax base. This might be the case for a financial stability program that helps prevent further collapses in private output due to bank runs. In this case, Eq. (9) should be replaced with $C_t = (Y_t + W_t(q))(1 - \frac{\eta}{2}\tau_t^2) - X_t(q) - G_t$ and Eq. (3) for tax revenue should be replaced with $T_t = (Y_t + W_t(q))(1 - \eta\tau_t)\tau_t$. In this case, the need to manage fiscal risk can reinforce the government's desire to manage social risk, partially alleviating the usual tension between fiscal and social risk management. Specifically, programs that help to keep the tax base high when Y_t falls also help to keep tax rates and the associated deadweight losses low.

A final feature of our set up is that the government makes a one-shot choice about program scale at time 0 and can credibly commit to this scale at time 1. Thus, the model is best seen as applying to non-discretionary budgeting where, for reasons of efficiency, fairness,

[7] In the case of incomplete markets problems like pollution externalities, a representative agent will typically exist. However, in other cases where market failures are generated by information problems among heterogeneous agents such as the insurance example above, one may not exist (Huang and Litzenberger [1988], Duffie [2001]).

[8] In that case, $W_t(q)$ can be interpreted as household utility from consuming the public good, and C_t is interpretable as a quasi-linear period utility function with private consumption serving as the numeraire.

or political economy, program scale is stable over time.

2.2 Optimal government policy

2.2.1 Model solution

We now characterize optimal government policy in the model. We first explain how government policies impact the riskless interest rate R. We then turn to the optimality conditions for government borrowing D_0 before finally solving for the optimal program scale q.

When households are risk-neutral, $R = \beta^{-1}$ irrespective of the government's choices of q and D_0. If households are risk averse ($u''(\cdot) < 0$), however, $\partial R/\partial q$ will be non-zero: the scale of the program impacts the interest rate. Specifically, we have

$$\frac{\partial R}{\partial q} = -\frac{R\beta E\left[u''(C_1)\left(W_1'(q) - X_1'(q)\left(1 + h'(\tau_1)\right)\right)\right] - u''(C_0)\left(W_0'(q) - X_0'(q)\left(1 + h'(\tau_0)\right)\right)}{\beta E\left[u'(C_1) - RD_0 u''(C_1)\, h'(\tau_1)\right]}. \tag{13}$$

The sign of $\partial R/\partial q$ is ambiguous and depends on the nature of the program under consideration. For example, if a program is expected to raise C_1 relative to C_0,[9] households will want to borrow more at time 0 to smooth consumption, causing the interest rate to rise. On the other hand, if a program is expected to lower C_1 relative to C_0, doing more of the program will lower the interest rate.

If households are risk-averse and taxation is distortionary, $\partial R/\partial D_0$ will be non-zero. Specifically, we have

$$\frac{\partial R}{\partial D_0} = \frac{R^2\beta E\left[u''(C_1)\, h'(\tau_1)\right] + u''(C_0)\, h'(\tau_0)}{\beta E\left[u'(C_1) - RD_0 u''(C_1)\, h'(\tau_1)\right]} \leq 0. \tag{14}$$

The intuition is straightforward. When taxes are distortionary ($h'(\cdot) > 0$), borrowing more today lowers current taxes and tax distortions, thereby raising current consumption. It also raises future taxes and tax distortions, lowering future consumption in expectation. When $u''(\cdot) < 0$, this means that current marginal utility ($u'(C_0)$) falls and future marginal utility ($u'(C_1)$) rises in expectation. As a result, households want to save more at time 0, so R must fall.

With these two comparative statics in hand, we now turn to optimal borrowing at time

[9] This would be the case if, for instance, we had $W_0 = X_0 = 0$, $E\left[W_1'(q) - X_1'(q)\left(1 + h'(\tau_1)\right)\right] > 0$, and $Cov\left[u''(C_1), \left(W_1'(q) - X_1'(q)\left(1 + h'(\tau_1)\right)\right)\right] = 0$.

0. The first order condition for D_0 can be written as

$$u'(C_0)\,h'(\tau_0) = \left(R + D_0\frac{\partial R}{\partial D_0}\right)\beta E\left[u'(C_1)\,h'(\tau_1)\right]. \tag{15}$$

To understand this condition, suppose the government issues more debt D_0 at time 0 and reduces taxes T_0 by the same small amount. This deviation reduces tax distortions by $h'(\tau_0)$ at time 0, which raises utility at time 0 by $u'(C_0)\,h'(\tau_0)$ at the margin. Since this deviation raises taxes by $(R + D_0\frac{\partial R}{\partial D_0})$ at time 1, it raises future tax distortions by $(R + D_0\frac{\partial R}{\partial D_0})h'(\tau_1)$ at time 1, which lowers discounted expected utility by $(R + D_0\frac{\partial R}{\partial D_0})\beta E[u'(C_1)\,h'(\tau_1)]$. Eq. (15) says that, at an optimum, such a deviation must have zero effect on expected lifetime utility.

We now turn to the optimal scale of the government program, q. The first-order condition for q is given by

$$0 = u'(C_0)\left(\frac{\partial C_0}{\partial q} + \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial q}\right) + \beta E\left[u'(C_1)\left(\frac{\partial C_1}{\partial q} + \frac{\partial C_1}{\partial \tau_0}\frac{\partial \tau_1}{\partial q}\right)\right]. \tag{16}$$

We can write the effect of changing q on household consumption at times 0 and 1 as:

$$\begin{aligned}
\frac{\partial C_0}{\partial q} + \frac{\partial C_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial q} &= W_0'(q) - X_0'(q) - h'(\tau_0)\,X_0'(q) \\
\frac{\partial C_1}{\partial q} + \frac{\partial C_1}{\partial \tau_0}\frac{\partial \tau_1}{\partial q} &= W_1'(q) - X_1'(q) - h'(\tau_1)\,X_1'(q) - h'(\tau_1)\,D_0\frac{\partial R}{\partial q}.
\end{aligned}$$

Increasing program scale directly alters time t consumption by $W_t'(q) - X_t'(q)$ and increases the deadweight loss from distortionary taxation by $h'(\tau_t)\,X_t'(q)$. Thus, the optimal amount of government activity satisfies

$$\begin{aligned}
0 &= u'(C_0)\left(W_0'(q) - X_0'(q) - h'(\tau_0)\,X_0'(q)\right) \\
&\quad + \beta E\left[u'(C_1)\left(W_1'(q) - X_1'(q) - h'(\tau_1)\,X_1'(q) - h'(\tau_1)\,D_0\frac{\partial R}{\partial q}\right)\right].
\end{aligned} \tag{17}$$

Proposition 1 *An optimum is a pair (D_0^*, q^*) such that D_0^* and q^* satisfy Eqs. (15) and (17), and where τ_0, τ_1, and R are implicitly defined by Eqs. (11) and (12).*

2.2.2 A decomposition

To interpret (17), let

$$M_1 = \beta \frac{u'(C_1)}{u'(C_0)} = \beta \frac{u'\left(Y_1(1-\frac{\eta}{2}\tau_1^2) + W_1(q) - X_1(q) - G_1\right)}{u'\left(Y_0(1-\frac{\eta}{2}\tau_0^2) + W_0(q) - X_0(q) - G_0\right)}. \tag{18}$$

denote the representative household's stochastic discount factor. Note that the optimal scale of the government program satisfies

$$\begin{aligned} 0 \;=\; & \overbrace{\left(W_0'(q) - X_0'(q)\right) + R^{-1}E\left[W_1'(q) - X_1'(q)\right]}^{\text{Expected net marginal benefit}} + \overbrace{Cov\left[M_1, W_1'(q) - X_1'(q)\right]}^{\text{Net marginal benefit risk premium}} \qquad (19) \\ & \underbrace{-\left(h'(\tau_0)X_0'(q) + R^{-1}E\left[h'(\tau_1)\left(X_1'(q) + D_0\frac{\partial R}{\partial q}\right)\right]\right)}_{\text{Expected tax cost}} - \underbrace{Cov\left[M_1, h'(\tau_1)\left(X_1'(q) + D_0\frac{\partial R}{\partial q}\right)\right]}_{\text{Tax risk premium}}. \end{aligned}$$

We now discuss the four terms in Eq. (19) that impact the optimal scale of a government program.

The first term in Eq. (19) is the *expected net marginal benefit* from the government program, discounted at the risk-free rate. This term reflects the way that the *costs* of credit and guarantee programs are accounted for in the Federal budget. Specifically, under the Federal Credit Reform Act (FCRA) of 1990,[10] the cost of a guarantee program equals the expected net present value of government outlays discounted at the risk-free rate. Obviously, the total net benefit, equal to the increase in private income (W_t) minus the cash outlays associated with the program (X_t), should be taken into account from a cost-benefit perspective.

The second term in (19) is the *risk premium* associated with these *net marginal benefits* and reflects the government's "social risk management" motive. It is commonly argued—see e.g., Lucas (2012) and the citations within—that the government should charge the same risk premium as the private sector because it is acting on behalf of risk-averse tax payers. Specifically, the government should charge a risk premium $Cov\left[M_1, X_1'(q)\right]$ for bearing the risk of cash outlays $X_1(q)$, just as private investors would. However, our model clarifies that the risk premium should be assessed on the net marginal gain or loss from the program, $W_1'(q) - X_1'(q)$. For instance, to the extent that the social gains from correcting some market failure accrue primarily in bad times, the $Cov\left[M_1, W_1'(q)\right]$ term will be positive, reflecting the government's "social risk management" motive.

One subtlety regarding this second term is that the stochastic discount factor M_1 itself

[10] See http://www.fas.org/sgp/crs/misc/R42632.pdf

(and therefore risk premia) depends on the scale of the government program q and government borrowing D_0 (see Eq. (18)). Government projects have the potential to alter aggregate consumption and therefore cannot be treated as if they are "marginal" in the traditional sense, a point first noted by Dasgupta, Sen, and Marglin (1972) and Little and Mirrlees (1974) and recently emphasized by Martin and Pindyck (2015).[11] Put differently, the existence of government programs itselfs alters the demand for government programs. For instance, if government policies reduce (increase) the volatility of aggregate consumption and, hence, the volatility of marginal utility, risk premia will be smaller (larger) than they would in the corresponding economy where $q = 0$.

The third term in (19) captures *marginal distortionary tax costs* generated by the program and reflects the government's "fiscal risk management" motive. When $h'(\tau_t) > 0$, tax distortions lead the government to act as if it is more risk-averse than the taxpayers it represents. Specifically, this term makes programs less desirable if they tend to raise taxes on average (i.e., $E[X_1'(q) + D_0 \partial R/\partial q]$ is large) or tend to raise taxes in states of the world where tax rates are already elevated (i.e., $Cov[h'(\tau_1), X_1'(q)]$ is large). By contrast, if $h'(\cdot) = 0$, the model collapses to the Ricardian case in which the government is a veil for taxpayers.

A critical tension that emerges from our framework is the conflict between the social risk management motive captured by the second term in (19) and the fiscal risk management motive captured by the third term. Programs like deposit insurance and automatic stabilizers that have significant social risk management benefits tend to involve government expenditures and, hence, higher tax distortions in bad times, creating greater fiscal risk.

The final term in (19), $Cov[M_1, h'(\tau_1)(X_1'(q) + D_0 \partial R/\partial q)]$, is the risk premium stemming from the cyclicality of taxes and reflects the interaction between the "social risk management" and "fiscal risk management" motives. Specifically, if a program leads to increased taxes in bad economic times, the distortions reduce private consumption precisely when it is most valuable, leading the government to do less of the program than it otherwise might.

2.2.3 Relation to public finance

Our model is related to several strands of the public finance literature.

[11] There is also a burgeoning literature on how government projects affect the prices of private assets. Pastor and Veronesi (2012, 2013) and Kelly, Pastor, and Veronesi (2016) note that the effect of government projects on aggregate consumption itself generates risk and study the effect of this risk on corporate equities and debt. Bond and Goldstein (2015) argue that if the government relies on market prices for information, it alters private incentives for information production, which can ultimately make prices less informative.

The Samuelson criterion One strand of the public finance literature has focused on project selection in non-stochastic enviornments. To highlight the relationship between our model and this literature, we rewrite Eq. (17) using the definition of household's stochastic discount factor in Eq. (18) to obtain

$$\begin{aligned} W_0'(q) + E\left[M_1 W_1'(q)\right] &= X_0'(q) + E\left[M_1 X_1'(q)\right] \\ &\quad + \left\{ h'(\tau_0) X_0'(q) + E\left[M_1 h'(\tau_1) \left(X_1'(q) + D_0 \frac{\partial R}{\partial q} \right) \right] \right\}. \end{aligned}$$

Here $W_t'(q)$ is the marginal rate of substitution between direct private consumption and the benefits of the government program. And $X_t'(q)$ is the marginal rate of transformation of private output into public output. Specifically, Samuelson (1954) argued that optimal program scale should equate the marginal rate of substitution $(W_t'(q))$ with the marginal rate of transformation $(X_t'(q))$. Inspired by Pigou (1947), later work, including Diamond and Mirrless (1971), Stiglitz and Dasgupta (1971), and Atkinson and Stern (1974), argues that Samuelson's criterion no longer holds when the government must finance its expenditures using distortionary taxes. Marginal tax distortions $(h'(\tau_t))$ must be taken into account.

Our framework shows how these intuitions translate into a dynamic, stochastic setting. The previous equation shows that optimal scale equates the stochastically discounted marginal rate of substitution (the left-hand-side) with the stochastically discounted marginal rate of transformation (the right-hand-side), plus an additional term (in curly braces) that accounts for the incremental tax distortions generated by the government program. The presence of this additional term means that marginal program costs should be discounted at a *lower rate* than marginal program benefits. And, because $h'(\tau_1)$ is unknown as of time 0, this additional term means that *fiscal risk considerations* now impact optimal program scale.

The Arrow-Lind result A second strand of the public finance literature studies project selection with stochastic payoffs that are only subject to idiosyncratic risk. Specifically, Arrow and Lind (1970) argued that if the marginal net benefits of a program $(W_1'(q) - X_1'(q))$ are only subject to idiosyncratic risk, then those net benefits should be discounted at the riskless rate.

Using the fact that $E[M_1] = R^{-1}$, we can rewrite Eq. (17) as

$$\begin{aligned} 0 &= \left(W_0'(q) - X_0'(q) - h'(\tau_0) X_0'(q)\right) \\ &+R^{-1}E\left[W_1'(q) - X_1'(q) - h'(\tau_1) X_1'(q) - h'(\tau_1) D_0 \frac{\partial R}{\partial q}\right] \\ &+Cov\left[M_1, W_1'(q) - X_1'(q) - h'(\tau_1) X_1'(q) - h'(\tau_1) D_0 \frac{\partial R}{\partial q}\right]. \end{aligned}$$

This equation shows that a modified version of the Arrow-Lind (1970) theorem holds in our model when a program's marginal net benefits *and* marginal tax distortions are only subject to idiosyncratic risk—i.e., when $Cov\left[M_1, W_1'(q) - X_1'(q) - h'(\tau_1) X_1'(q) - h'(\tau_1) D_0 \partial R/\partial q\right] = 0$. In this case, the marginal net benefit generated by the program, *including* the relevant marginal tax distortions, should be discounted at the riskless rate. By contrast, if a program's marginal net benefits or tax distortions are subject to systematic risk, then it is not appropriate to use a riskless discount rate.

Optimal debt management Finally, our model is related to the literature on optimal debt management. The first order condition for taxes, Eq. (15), captures the idea that the government should smooth taxes over time when there are convex distortionary costs of taxation (Barro [1979], Aiyagari, Marcet, Sargent, and Seppala [2002]). We can rewrite the equation as:

$$h'(\tau_0) = \frac{R + D_0 \frac{\partial R}{\partial D_0}}{R} \frac{E\left[M_1 h'(\tau_1)\right]}{E\left[M_1\right]}.$$

When households are risk neutral, this equation reduces to $h'(\tau_0) = E\left[h'(\tau_1)\right]$: the government fully smooths marginal tax distortions between times 0 and 1. When households are risk-averse, the government smoothes risk-adjusted marginal tax distortions, recognizing that its borrowing affects the riskfree rate.

In addition, our model embeds the core intuitions present in the state-contingent debt management problem studied in Bohn (1990), Aiyagari, Marcet, Sargent, and Seppala (2002), and Bhandari, Evans, Golosov, and Sargent (2016). In this problem, the government chooses riskless debt and its issuance q of a risky security with time 0 price $P = -X_0$ and state-contingent time 1 payoffs X_1 to minimize the cost of distortionary taxes.

This problem can be studied in our framework by setting $W_t = X_t$, so that the net benefits of the project are zero. In this case, the choice of q only impacts household consumption insofar as it impacts tax revenues. As shown in the Internet Appendix, the optimality condition for q then provides intuitions similar to those in the state-contingent debt management literature. Specifically, it says that the government wants to hedge background fiscal risk by issuing risky securities that have low returns (i.e., low X_1/P) in states where other government spending (G_1) is unexpectedly high, so that the tax rate τ_1 is unexpectedly high.

2.3 Approximate solutions

To clarify the key economic intuitions present in the model, we compute approximate solutions to the above problem by replacing $h'(\tau_t)$ and $u'(C_t)$ in the two optimality conditions with first-order Taylor series approximations. We also assume that the government program has constant returns to scale so $W_t(q) = qW_t$ and $X_t(q) = qX_t$. These approximations yield a system of quadratic equations that we can solve in closed form. The details are given in the Appendix. First, we can approximate the marginal deadweight cost of taxation as

$$h'(\tau_t) \approx \bar{h}' + \bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{T_t}{\overline{T}} - \frac{Y_t}{\overline{Y}}\right),$$

where $\bar{h}' = \eta\bar{\tau}/(1-2\eta\bar{\tau})$, $\bar{\eta} = \eta/(1-2\eta\bar{\tau})^3 > \eta$, and $\bar{\tau}$ denotes the tax rate when required tax revenue is $\overline{T}$ and income is $\overline{Y}$. Thus, when $\eta = 0$, we have $\bar{h}' = \bar{\eta} = 0$.

Next, we approximate consumption using

$$C_t \approx \tilde{C}_t \equiv Y_t - \left(\bar{Y}\frac{\eta}{2}\bar{\tau}^2 + \bar{h}'\left(T_t - \overline{T}\right) - \hat{\eta}_Y\left(Y_t - \overline{Y}\right)\right) + W_t q - X_t q - G_t,$$

where $\hat{\eta}_Y$ is a parameter of the linearization that is zero when $\eta = 0$. We then approximate marginal utility by taking a Taylor series approximation about the consumption level $\overline{C}$ that satisfies $u'\left(\overline{C}\right) = 1$. This yields

$$u'(C_t) \approx 1 - \gamma\left(C_t - \overline{C}\right) \approx 1 - \gamma(\tilde{C}_t - \overline{C}),$$

where $\gamma = -u''\left(\overline{C}\right)$.

Finally, we approximate the riskless interest rate as

$$R \approx \hat{R} \equiv \beta^{-1}\frac{1-\gamma(Y_0 - \frac{\eta}{2}\overline{Y}\bar{\tau}^2 - G_0 - \overline{C})}{1-\gamma(E[Y_1 - \frac{\eta}{2}\overline{Y}\bar{\tau}^2 - G_1] - \overline{C})},$$

time 1 tax revenues as $T_1 \approx G_1 + X_1 q + \hat{R}D_0$, and $\partial R/\partial D_0$ and $\partial R/\partial q$ using the constants δ_{D_0} and δ_q, which are defined in the Appendix.

Combining the above, we replace the optimality condition for D_0 in Eq. (15) with the approximate condition

$$
\begin{aligned}
&\left(1-\gamma(\tilde{C}_0-\overline{C})\right)\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_t}{\overline{\overline{T}}}-\frac{Y_t}{\overline{\overline{Y}}}\right)\right) \\
=\;& (1+\beta D_0\delta_{D_0})\,E\left[\left(1-\gamma(\tilde{C}_1-\overline{C})\right)\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_t}{\overline{\overline{T}}}-\frac{Y_t}{\overline{\overline{Y}}}\right)\right)\right].
\end{aligned}
\tag{20}
$$

Similarly, we replace the optimality condition for q in Eq. (17) with the approximate condition

$$
\begin{aligned}
0 \;=\;& \left(1-\gamma(\tilde{C}_0-\overline{C})\right)\left(W_0-X_0-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_0}{\overline{\overline{T}}}-\frac{Y_0}{\overline{\overline{Y}}}\right)\right)X_0\right) \\
&+\beta E\left[\left(1-\gamma(\tilde{C}_1-\overline{C})\right)\left(W_1-X_1-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_1}{\overline{\overline{T}}}-\frac{Y_1}{\overline{\overline{Y}}}\right)\right)(X_1+D_0\delta_q)\right)\right].
\end{aligned}
\tag{21}
$$

This is a system of two quadratic equations in D_0 and q. When $\gamma > 0$ and $\eta > 0$, this system can be reduced to a single quartic equation in q that can be solved in closed form. And, with either (i) risk-neutral households ($\gamma = 0$) and tax distortions ($\eta > 0$) or (ii) risk-averse households ($\gamma > 0$) and no tax distortions ($\eta = 0$), we obtain a system of two linear equations in D_0 and q.

2.3.1 Approximate solution with risk-neutral households ($\gamma = 0$ and $\eta > 0$)

To build intuition, we first consider the special case where households are risk-neutral ($\gamma = 0$), but government taxation is distortionary ($\eta > 0$). This case, where the government seeks to generate large net benefits for households while limiting fiscal risk, allows us to identify the key determinants of fiscal risk in the model.

Specifically, when $\gamma = 0$ and $\eta > 0$, the approximate first order condition for D_0 in (20) collapses to

$$
\frac{T_0}{\overline{\overline{T}}}-\frac{Y_1}{\overline{\overline{Y}}}=E\left[\frac{T_1}{\overline{\overline{T}}}-\frac{Y_1}{\overline{\overline{Y}}}\right],
\tag{22}
$$

which says that government borrowing is chosen to smooth expected tax rates over time. As shown in the Appendix, the approximate first order condition for q in (21) can be solved to obtain:

$$q^* = \frac{1}{\bar{\eta}/\overline{Y}} \frac{(W_0 - X_0) + \beta E[W_1 - X_1]}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]}$$
$$- \frac{\bar{h}'/\left(\bar{\eta}/\overline{Y}\right) + (1+\beta)^{-1}\left(\overline{D} + (G_0 + \beta E[G_1]) - \frac{\overline{T}}{\overline{Y}}(Y_0 + \beta E[Y_1])\right)}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]}(X_0 + \beta E[X_1])$$
$$- \frac{\beta Cov[G_1, X_1] - \frac{\overline{T}}{\overline{Y}}\beta Cov[Y_1, X_1]}{(1+\beta)^{-1}(X_0 + \beta E[X_1])^2 + \beta Var[X_1]}. \tag{23}$$

The first term in Eq. (23) is proportional to *expected marginal program net benefits*, i.e., $(W_0 - X_0) + \beta E[W_1 - X_1]$. The denominator of this term is the direct deadweight costs from increasing program expenditures at the margin times the effective degree of "fiscal risk aversion." The effective degree of fiscal risk aversion is $\bar{\eta}/\overline{Y}$, which is increasing in the labor supply elasticity with respect to the tax rate (η) and decreasing in the tax base ($\overline{Y}$).

The second term in (23) reflects the marginal *expected deadweight tax costs* of the program that arise in the presence of other government expenditures. Specifically, the second term is equal to expected discounted marginal outlays (i.e., $X_0 + \beta E[X_1]$) times the expected marginal deadweight cost of taxation when $q = 0$. The third term reflects *fiscal risk* and captures the additional deadweight costs that arise if time 1 spending on the program covaries positively with time 1 tax rates. Naturally, the covariance of program spending with tax rates is higher when $Cov[G_1, X_1]$ is larger or when $Cov[Y_1, X_1]$ is smaller.

The next proposition provides a set of intuitive comparative statics, showing how the optimal scale of the project q^* depends on the exogenous parameters.

Proposition 2 *Consider the case with risk-neutral households ($\gamma = 0$) and distortionary taxation ($\eta > 0$). Assume that $G_0 + \beta E[G_1] > 0$, $X_0 + \beta E[X_1] > 0$, and $q^* > 0$. Then we have the following comparative statics for optimal program scale:*

- $\partial q^*/\partial W_0 > 0$ *and* $\partial q^*/\partial E[W_1] > 0$;
- $\partial q^*/\partial X_0 < 0$ *and* $\partial q^*/\partial E[X_1] < 0$;
- $\partial q^*/\partial G_0 \propto -(X_0 + \beta E[X_1]) < 0$ *and* $\partial q^*/\partial E[G_1] \propto -(X_0 + \beta E[X_1]) < 0$;
- $\partial q^*/\partial \overline{D} \propto -(X_0 + \beta E[X_1]) < 0$;
- $\partial q^*/\partial \eta \propto -((W_0 - X_0) + \beta E[W_1 - X_1])$;
- $\partial q^*/\partial Corr[X_1, G_1] < 0$;

- $\partial q^*/\partial Corr\left[X_1, Y_1\right] > 0$;
- $\partial q^*/\partial Var\left[X_1\right] < 0$.

Proof. Differentiation of Eq. (23). ■

These comparative statics identify the drivers of fiscal risk in the mode. For instance, the optimal scale of a program with positive outlays ($X_0 + \beta E\left[X_1\right] > 0$) declines with other government spending (G_0 or $E\left[G_1\right]$). Intuitively, increasing other government spending raises the fiscal burden and tax distortions. By decreasing the scale of a positive-outlay program, the government can reduce the need for distortionary taxation, partially offsetting the effect of increased spending.

Similar logic applies to the effect of the severity of marginal tax distortions, η. At the optimum, $\partial q^*/\partial \eta$ is proportional to $-((W_0 - X_0) + \beta E\left[W_1 - X_1\right])$, so an increase in η, which controls the deadweight loss from taxation, leads the government to cut back on attractive projects with large discounted net benefits. In addition, all else equal, the government should choose a smaller scale for programs whose outlays are more variable. Finally, the government should choose a smaller scale for programs whose outlays covary positively with other spending (G_1) or negatively with the tax base (Y_1).

Rather than considering the optimal program scale q^*, we can instead fix program scale and consider the *initial fee* that that the government should charge for the program. Specifically, we fix the scale of the program at some level $\overline{q}$ and then allow the initial fee the government charges, $P = -X_0$, to adjust so that the optimal scale, q^*, equals $\overline{q}$. For instance, in the case of deposit insurance, the fee P corresponds to the deposit insurance premia that the government charges financial institutions at time 0. In the case of unemployment insurance, P corresponds to the level of payroll taxes at time 0. Using (23), the optimal program fee satisfies

$$P^* = -\frac{W_0 + \beta E\left[W_1 - X_1 - \left(\bar{h}' + \bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{T_1}{\overline{T}} - \frac{Y_1}{\overline{Y}}\right)\right) X_1\right]}{1 + \bar{h}' + \bar{\eta}\frac{\overline{T}}{\overline{Y}}\left(\frac{T_0}{\overline{T}} - \frac{Y_0}{\overline{Y}}\right)}. \tag{24}$$

The following proposition characterizes the behavior of the initial fees the government should charge for specific programs.

Proposition 3 *Consider the case with risk-neutral households ($\gamma = 0$) and distortionary taxation ($\eta > 0$). Assume that $G_0 + \beta E\left[G_1\right] > 0$, $-P^* + \beta E\left[X_1\right] > 0$, and $\overline{q} > 0$. Then we have the following comparative statics for optimal program fees, P^*:*

- $\partial P^*/\partial W_0 < 0$ and $\partial P^*/\partial \left(E\left[W_1\right]\right) < 0$;
- $\partial P^*/\partial E\left[X_1\right] > 0$;
- $\partial P^*/\partial G_0 \propto \left(-P^* + \beta E\left[X_1\right]\right) > 0$ and $\partial P^*/\partial E\left[G_1\right] \propto \left(-P^* + \beta E\left[X_1\right]\right) > 0$;
- $\partial P^*/\partial \overline{D} \propto \left(-P^* + \beta E\left[X_1\right]\right) > 0$;
- $\partial P^*/\partial \eta \propto W_0 + P^* + \beta E\left[W_1 - X_1\right]$;
- $\partial P^*/\partial Corr\left[X_1, G_1\right] > 0$;
- $\partial P^*/\partial Corr\left[X_1, Y_1\right] < 0$;
- When $Corr\left[X_1, G_1\right] > 0$ and $Corr\left[X_1, Y_1\right] < 0$, we have $\partial P_0^*/\partial Var\left[X_1\right] > 0$;
- $\partial P^*/\partial \overline{q} \propto \left(-P^* + \beta E\left[X_1\right]\right)^2/(1+\beta) + \beta Var\left[X_1\right] > 0$.

Naturally, the optimal fee is smaller for programs that generate greater social benefits and is larger for programs with greater expected future outlays. When the program involves a net outlay in the sense that $\left(-P^* + \beta E\left[X_1\right]\right) > 0$, the optimal fee is larger when other government spending (G_0 or $E\left[G_1\right]$) is larger. All else equal, fees should be higher for programs whose time 1 outlays covary more strongly with tax rates and, under plausible conditions, whose time 1 outlays are more volatilte. Finally, all else equal, the optimal fee is increasing in the size of the program.

2.3.2 Approximate solution with no tax distortions ($\eta = 0$ and $\gamma > 0$)

We next consider the special case where households are risk-averse ($\gamma > 0$) and there are no tax distortions ($\eta = 0$). This case, where the government seeks to generate large net benefits for households and to manage social risk, allows us to identify the key drivers of social risk in the model.

Because $\eta = 0$, Ricardian equivalence holds: the level of debt is irrelevant and is not pinned down at the optimum. In this case, the Appendix shows that the approximate first order condition for q in (21) can be solved to obtain:

$$q^* = \frac{1-\gamma\left(E\left[Y_1-G_1\right]-\overline{C}\right)}{\gamma}\frac{\left(W_0-X_0\right)+\beta E\left[W_1-X_1\right]}{\left(W_0-X_0\right)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)} \quad (25)$$
$$-\frac{\left(\left(Y_0-G_0\right)-E\left[Y_1-G_1\right]\right)\left(W_0-X_0\right)}{\left(W_0-X_0\right)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)}$$
$$-\frac{\beta Cov\left[Y_1-G_1,W_1-X_1\right]}{\left(W_0-X_0\right)^2+\beta\left(E\left[W_1-X_1\right]\right)^2+\beta\left(Var\left[W_1-X_1\right]\right)}.$$

The numerator of the first term in Eq. (25) is proportional to time 1 expected marginal utility when $q=0$ (i.e., $1-\gamma\left(E\left[Y_1-G_1\right]-\overline{C}\right)$) times expected program net benefits for risk-neutral households (i.e., $\left(W_0-X_0\right)+\beta E\left[W_1-X_1\right]$). The denominator captures the "wealth effect" that arises because the chosen level of q affects the marginal utility of risk-averse households. Projects that generate large expected benefits lower marginal utility, making further projects with positive expected benfits less appealing to households. The second term in (25) arises only when $Y_0-G_0 \neq E\left[Y_1-G_1\right]$ and reflects any benefits of smoothing consumption between time 0 and time 1.

The final term in Eq. (25) captures the pure social risk management motive of smoothing consumption across states at time 1. This risk management term is proportional to $-Cov\left[Y_1-G_1,W_1-X_1\right]$. In other words, the government likes programs that are a hedge against exogenous shocks, both those emanating from the private economy (Y_1) and those emanating from other government expenditures (G_1).

In this Ricardian case, comparative statics with respect to the Ws and Xs are generally ambiguous due to competing substitution and wealth effects. These wealth effects arise because the government program impacts aggregate consumption and hence aggregate marginal utility. For instance, the impact of program net benefits at time 0 on optimal scale is

$$\partial q^*/\partial\left(W_0-X_0\right) \propto \left(1-\gamma C_0\right)-\gamma q^*\left(W_0-X_0\right),$$

which is ambiguous. Holding marginal utility fixed, an increase in $\left(W_0-X_0\right)$ leads to a substitution effect that makes the government want to do more of the program. But there is a competing wealth effect: the increase in $\left(W_0-X_0\right)$ reduces marginal utility and lowers the government's willingness to pay, pushing it to do less of the program.

In contrast, comparative statics with respect to the Ys and Gs will be unambiguous because they only involve wealth effects, which alter the government's willingness to pay for a particular program. For instance, we have $\partial q^*/\partial Y_0 \propto -\left(W_0-X_0\right)<0$.

Finally, we have $\partial q^*/\partial\gamma \propto -\left((W_0 - X_0) + \beta E\left[W_1 - X_1\right]\right)$, so an increase in risk aversion leads the government to do less of programs that have positive expected net benefits when discounted at the risk-free rate.

2.3.3 Solution with both risk aversion and tax distortions

Finding the approximate solution in the general case where $\gamma > 0$ and $\eta > 0$ requires solving a system of two quadratic equations in q and D_0, namely Eqs. (21) and (20). These two equations can be combined to yield a quartic equation in q alone that can be solved in closed form. We do not pursue this approach here because the general formula is unwieldly and adds little additional insight.

Instead, we explore the general case where $\gamma > 0$ and $\eta > 0$ using numerical examples. As these examples highlight, a key takeaway is that the government's social risk management and fiscal risk management motives often pull in opposite directions. Which motive dominates depends on the parameters of the economy and the project.

Table 1 lists the baseline parameters underlying our numerical example. At time 0, exogenous private income is $Y_0 = 1$ and exogenous government spending is $G_0 = 0$. At time 1, a high state occurs with probability $p = 50\%$. In the high state, private income is $Y_{1H} = 1.2$, and government spending is $G_{1H} = -0.1$. With probability $1 - p = 50\%$, a low state occurs. We interpret the low state as a severe recession that leads to a large rise in government spending on automatic stabilizer programs. In the low state, exogenous income is $Y_{1L} = 0.8$, and government spending is $G_{1L} = 0.1$. Thus, the expected growth rates of private income and other government spending are both zero.

Three parameters control household preferences: β, γ, and $\overline{C}$. We set $\beta = 1$, so the risk-free rate would be zero in the absence of risk aversion. We set $\gamma = 0.25$ and $\overline{C} = 1$, so that marginal utility equals one when $C_t = 1$ and declines to 0 when $C_t = 5$. Turning to the fiscal parameters, we set $\overline{D} = 0.6$—a debt-to-GDP ratio of 60%—to capture the case where a government faces a high accumulated fiscal burden. We set $\eta = 0.05$. Thus, in the benchmark case where $T_0 = T_1 = 0.3$, the marginal cost of social funds is roughly 1.015 because marginal tax distortions are $1.5\% = 5\% \times 30\%$.

The program we consider is meant to represent a financial stability intervention such as deposit insurance. The program requires government outlays of $X_0 = 0.024$ at time 0, and creates no additional private income at time 0, so $W_0 = 0$. The project requires no outlays and generates no additional income in the high state at time 1, so $X_{1H} = W_{1H} = 0$. In the low state at time 1, the project requires large outlays of $X_{1L} = 0.05$, but generates signifiant additional private income of $W_{1L} = 0.1$. Overall, the net benefits for risk-neutral households are quite small: $(W_0 - X_0) + \beta E[W_1 - X_1] = 0.0014$.

Table 2 reports the exact optimal government policies (D_0, q, R) in this example obtained from solving (1), (3), (12), (15), and (17), assuming a quadratic utility function of the form $u(C_t) = C_t - (\gamma/2)\left(C_t - \overline{C}\right)^2$. Table 3 reports the approximate optimal government policies (D_0, q) in this example obtained from solving our system of quadratics given in (20) and (21). The approximate solutions given in Table 3 are close to the exact solutions in Table 2. Thus, our discussion focuses on Table 2.

Panel A of Table 2 reports the optimal scale of the financial stability program q, Panel B reports optimal time 0 debt, and Panel C reports the risk-free rate $R - 1$. The ten rows in each panel show how optimal policy varies with parameters of the economy and the program under consideration. The five columns show how optimal policy varies with household risk aversion (γ) and fiscal costs (η). Specifically, column (1) reports our baseline results with $\gamma = 0.25$ and $\eta = 0.05$—i.e., with both risk aversion and tax distortions. Column (2) shows the risk-neutral solution with only tax distortions: $\gamma = 0$ and $\eta = 0.05$. Column (3) shows the Ricardian solution with $\gamma = 0.25$ and $\eta = 0$. Because Ricardian equivalence holds in this case, debt is indeterminate in Panel B. Column (4) shows the effect of increasing risk-aversion relative to the baseline, setting $\gamma = 0.35$ and $\eta = 0.05$. Finally, collumn (5) shows the effect of increasing tax distortions relative to the baseline, setting $\gamma = 0.25$ and $\eta = 0.075$.

We start with the results in row (1) and column (1). The optimal scale of the project in Panel A is $q = 3.73$. Although the net benefits for risk-neutral households are small, the financial stability program is an attractive "social hedge" that delivers additional consumption in the low state when marginal utility is high. To smooth taxes over time, Panel B shows that the government borrows $D_0 = 0.25$ at time 0. Panel C shows that the risk-free rate is 4.6%, reflecting the fact that marginal utility is expected to decline slightly between time 0 and time 1.

Moving across the first row, column (2) shows that optimal program scale declines when households are risk-neutral: risk-neutral households naturally have a lower willingness to pay for this social hedge. Similarly, optimal program scale rises when there are no tax distortions in column (3). Finally, q rises in column (4) when risk aversion rises to $\gamma = 0.50$ and q falls in column (5) when tax distortions increase to $\eta = 0.075$. Taken together, the results in row (1) illustrate a key lesson from our model: *the need to manage fiscal risk can significantly reduce the government's ability to manage social risk.* Specifically, when taxes are distortionary $(\eta > 0)$, the government should only choose a large amount of a program if it has large net benefits in expectation ($(W_0 - X_0) + \beta E[W_1 - X_1]$ is large) or if it is a strong social hedge ($Cov[Y_1 - G_1, W_1 - X_1]$ is large). Put differently, the distortionary costs of taxation argue in favor of fiscal conservatism, raising the hurdle that needs to be cleared before government initiates a candidate program designed to correct a market failure or to manage social risk.

The remaining rows in Table 2 show how optimal policy varies with parameters of the background economy and the program. In row (2), we increase the goverment's accumulated deficit by 20% from $\overline{D} = 0.6$ to $\overline{D} = 0.72$. When taxation is distortionary—i.e., in all columns other than (3), this leads to a decline in optimal program scale. Intuitively, raising the government's accumulated deficit increases tax distortions. By decreasing the scale of a program with positive outlays, the government can reduce the need for distortionary taxation, partially offsetting the effect of the rise in $\overline{D}$. Put differently, our model is consistent with a "fiscal austerity" logic under which a high accumulated deficit reduces the attractiveness of most government programs at the margin.

In row (3), we raise expected private income at time 1 by 0.05 relative to the baseline in row (1)—i.e., we set $Y_{1H} = 1.25$ and $Y_{1L} = 0.85$. The rise in $E[Y_1]$ has two competing effects. When households are risk averse, the increase in $E[Y_1]$ lowers marginal utility at time 1, reducing the willingness to pay for the financial stability program. However, the increase in $E[Y_1]$ also lowers expected tax rates, leading to a decline in the marginal deadweight costs from taxation. This force pushes the government to do more of the project. This can be see in column (2), which shows that tax distortions have an offsetting effect, so q^* actually rises when households are risk neutral ($\gamma = 0$) and taxes are distortionary ($\eta > 0$).

In row (4), we make the economy riskier by raising the volatility of time 1 income by 0.05 relative to the baseline, setting $Y_{1H} = 1.25$ and $Y_{1L} = 0.75$. Again, the rise in $Var[Y_1]$ has two competing effects. Higher volatility makes the financial stability program more valuable as a social hedge, increasing the optimal program scale. However, the increase in $Var[Y_1]$ raises the volatility of tax rates at time 1. Since $Cov[X_1, Y_1] < 0$, the resulting increase in fiscal risk pushes the government to do less of the program. This can be seen in column (2), which shows that tax distortions have an offsetting effect, so q^* falls when households are risk neutral ($\gamma = 0$) and taxes are distortionary ($\eta > 0$).

In row (5), we increase expected time 1 government spending by 0.05, setting $G_{1H} = -0.05$ and $G_{1L} = 0.15$. The rise in $E[G_1]$ has two competing effects. When households are risk averse, this change increases expected marginal utility at time 1, pushing the government to do more of the program. Put differently, increased government spending can create additional demand for other government projects. However, when taxes are distortionary, raising $E[G_1]$ also increases expected taxes, pushing the government to reduce the scale of the program. Therefore, the effect on an increase in $E[G_1]$ is generally ambiguous. However, in our example, the former force outweighs the latter.

In row (6), we increase the volatility of time 1 government spending holding fixed the mean, setting $G_{1H} = -0.15$ and $G_{1L} = 0.15$. As above, the rise in $Var[G_1]$ works through two competing channels. When households are risk averse, more volatile government spending makes marginal utility more volatile, pushing the government to do more of the program on social risk management grounds. When taxation is distortionary, more volatile government spending increases fiscal risk, pushing the government to reduce the scale of the program. Again, in our example, the first channel outweighs the second.

In row (7), we increase the expected benefits of the program, settuing $W_{1L} = 0.0025$ and $W_{1H} = 0.1025$. Although changes in $E[W_1]$ have competing substitution and wealth effects, the substitution effect dominates in our example, so optimal program scale rises with $E[W_1]$. In row (8), we increase the variance of the project payoffs, holding fixed the mean, setting $W_{1L} = -0.005$ and $W_{1H} = 0.105$. Holding fixed marginal utility, the resulting substitution effect makes the project more desirable as a social hedge when households are risk averse. Again, there is a competing wealth effect because this change lowers expected marginal utility for a given level of q. As shown in Table 2, the overall impact on program scale is ambiguous.

In row (9), we increase the expected time 1 program outlays to $X_{1L} = 0.001$ and $X_{1H} = 0.051$. In our example, the resulting substitution effect dominates, so this change always reduces the optimal program scale. Finally, in row (10), we increase the variance of program outlays, holding fixed the mean, at time 1, so that $X_{1L} = 0.0025$ and $X_{1H} = 0.0525$. The optimal scale of the project falls. Distortionary costs are convex of function of tax revenue, so increasing the variance of taxes raises expected tax distortions, leading the government to reduce the scale of the program. By contrast, as column (3) shows, when there are no tax distortions increasing the variance of program outlays has only a neglible effect on program scale.

Overall, Table 2 illustrates how the government's need to manage fiscal risk limits its capacity to manage social risk. The table also highlights the fact that the government's social risk and fiscal risk management motives often pull in opposite directions as the characteristics of the economy or the government program change.

3 Portfolios of government programs

We now extend our framework to characterize the *optimal portfolio* of government programs. The same basic tradeoffs between social and fiscal risk management that we emphasized above continue to apply. However, these tradeoffs now acquire a portfolio management flavor. Specifically, distortionary taxation and household risk aversion create interdependencies amongst otherwise unrelated government programs. For instance, when taxes are distortionary, the fiscal risk of a program depends on how its required outlays covary with those of the government's overall portfolio of programs. Similarly, when households are risk averse, the social risk of a program depends (in part) on how its net benefits covary with the net benefits of the government's portfolio of programs. As a result, government programs cannot be evaluated in isolation. Instead, proper cost-benefit analysis needs to explicitly take these fiscal risk and social risk interdependencies into account.

3.1 Optimal portfolios

Different government programs are indexed by $j = 1, ..., J$, and we let q_j denote the chosen scale of program j. For simplicity, we focus on the case where each program has constant returns to scale. Thus, for $t = 0$ and 1, the government outlays for program j are $q_j X_{tj}$ and the additional private income generated by program j is $q_j W_{tj}$. Adopting the vector notation that $[\mathbf{q}]_j = q_j$, $[\mathbf{x}_t]_j = X_{tj}$, and $[\mathbf{w}_t]_j = W_{tj}$, this means that the first order condition for $\mathbf{q}$ is now

$$\begin{aligned} 0 \; = \; & u'(C_0)\left(\mathbf{w}_0 - \mathbf{x}_0 - h'(\tau_0)\,\mathbf{x}_0\right) \\ & + \beta E\left[u'(C_1)\left(\mathbf{w}_1 - \mathbf{x}_1 - h'(\tau_1)\,\mathbf{x}_1 - h'(\tau_1)\,D_0 \frac{\partial R}{\partial \mathbf{q}}\right)\right], \end{aligned} \tag{26}$$

where $\partial R/\partial \mathbf{q}$ is the vector analog of Eq. (13). A solution is a tuple $(D_0^*, \mathbf{q}^*)$ that satisfies Eqs. (15) and (26), and where τ_0, τ_1, and R are implicitly defined by the vector analogs of Eqs. (11) and (12).[12]

[12] As in the single program case, one can also include a set of non-negativity constraints to study the extension margin of government program choice.

3.2 Approximate solutions

As above, we can approximate Eq. (26) using:

$$
\begin{aligned}
0 &= \left(1-\gamma(\tilde{C}_0-\overline{C})\right)\left(\mathbf{w}_0-\mathbf{x}_0-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_1}{\overline{\overline{T}}}-\frac{Y_1}{\overline{\overline{Y}}}\right)\right)\mathbf{x}_0\right) \qquad (27)\\
&+\beta E\left[\left(1-\gamma(\tilde{C}_1-\overline{C})\right)\left(\mathbf{w}_1-\mathbf{x}_1-\left(\bar{h}'+\bar{\eta}\frac{\overline{T}}{\overline{\overline{Y}}}\left(\frac{T_1}{\overline{\overline{T}}}-\frac{Y_1}{\overline{\overline{Y}}}\right)\right)(\mathbf{x}_1+D_0\boldsymbol{\delta}_q)\right)\right],
\end{aligned}
$$

where $\tilde{C}_t \equiv Y_t - \left(\eta\overline{Y}\bar{\tau}^2/2+\hat{\eta}_T\left(T_t-\overline{T}\right)-\hat{\eta}_Y\left(Y_t-\overline{Y}\right)\right)+(\mathbf{w}_t-\mathbf{x}_t)'\,\mathbf{q}-G_t$. To see the intuition behind Eq. (27), we consider two special cases.

In the risk-neutral case where $\gamma=0$ and $\eta>0$, the approximate first order condition for program j can be written as

$$
\begin{aligned}
0 &= \overbrace{(W_{0j}-X_{0j})+\beta E\left[W_{1j}-X_{1j}\right]}^{\text{Expected net benefit of program } j}\\
&\underbrace{-\left[\bar{h}'+\frac{\bar{\eta}}{\overline{\overline{Y}}}\left(\frac{\overline{D}+(G_0+\beta E\left[G_1\right])+\sum_j q_k(X_{0k}+\beta E\left[X_{1k}\right])}{1+\beta}-\frac{T}{\overline{\overline{Y}}}\frac{Y_0+\beta E[Y_1]}{1+\beta}\right)\right](X_{0j}+\beta E\left[X_{1j}\right])}_{\text{Expected fiscal cost of program } j}\\
&\underbrace{-\frac{\bar{\eta}}{\overline{\overline{Y}}}\beta Cov\left[G_1+\sum_k q_k X_{1k}-\frac{T}{\overline{\overline{Y}}}Y_1, X_{1j}\right]}_{\text{Fiscal risk of program } j}.
\end{aligned}
$$

Thus, when $\eta>0$, the desire to manage fiscal risk means that the government dislikes programs that require large outlays in states where the portfolio of government programs also requires large outlays. In this case where $\gamma=0$ and $\eta>0$, the vector of approximately optimal program scales, $\mathbf{q}^*$, is given by the vector analog of Eq. (23).

In the Ricardian case where $\gamma>0$ and $\eta=0$, the first order condition for program j is

$$
\begin{aligned}
0 &= \overbrace{(W_{0j}-X_{0j})+R^{-1}E\left[W_{1j}-X_{1j}\right]}^{\text{Expected net benefit}}\\
&\underbrace{-\frac{\gamma}{1-\gamma\left(C_0-\overline{C}\right)}\beta Cov\left[Y_1-G_1+\sum_k q_k\left(W_{1k}-X_{1k}\right), W_{1j}-X_{1j}\right]}_{\text{Net benefit risk premium}}.
\end{aligned}
$$

Thus, when $\gamma>0$, the desire to manage social risk means that the government likes individual programs that deliver large net benefits in states where the portfolio of government programs delivers small net benefits. The complex interdependence that arises in the Ricardian limit of our model with risk-averse households is akin to the interdependence recently emphasized by Martin and Pindyck (2015). In this case, where $\gamma > 0$ and $\eta = 0$, the vector of approximately optimal program scales, $\mathbf{q}^*$, is given by the vector analog of Eq. (23).

3.3 Ex-ante regulation versus ex-post bailouts

We now use the multi-program extension to explore the optimal mix of programs the government should use to promote financial stability. Specifically, we consider the choice between *ex-ante* regulations and *ex-post* bailout programs, sometimes referred to as the "lean versus clean" tradeoff.[13] Both financial regulations and bailouts may be beneficial from a financial stability standpoint, helping to reduce the likelihood or severity of financial crises. *Ex-ante r*egulation can rein in risk-taking by financial intermediaries, reducing the probability of financial crises. However, regulation may inefficiently reduce *ex ante* economic growth to the extent that it chokes off useful financial innovations or leads intermediaries to unduly restrict the supply of credit. Alternatively, the government can use bailouts in the form of debt guarantees or capital injections to clean up financial crises *ex-post*. *Ex-post* bailouts leave *ex-ante* growth unfettered, but require a larger use of government fiscal capacity in the event of a crisis.

Our framework suggests that the optimal mix between these two interventions varies with the extent of tax distortions and the government's preexisting fiscal commitments.[14] We illustrate these ideas formally in the risk-neutral case where $\gamma = 0$ and $\eta > 0$. Let program $j = 1$ denote ex-ante regulations and program $j = 2$ denote ex-post bailouts. Without loss of generality, we focus on benefits and outlays at time 1, assuming that $W_{0j} = 0$ and $X_{0j} = 0$ for $j = 1, 2$, $G_0 = E[G_1] = 0$, and $Y_0 = E[Y_1] = \overline{Y}$. To ease notation, we drop the time subscripts so that, for example, X_1 denotes outlays associated with regulatory program 1 and X_2 denotes outlays associated with bailout program 2.

We assume that regulation is a fiscally riskless program in the sense that outlays are constant across states: $Var[X_1] = 0$. Regulatory outlays can be thought of as the costs of paying regulators and conducting bank examinations. Obviously, bailouts are a fiscally risky program so $Var[X_2] > 0$. For example, bailout outlays can be thought of as the realized fiscal costs of a program that guarantees short-term debt issued by financial intermediaries, net of any insurance premia paid by intermediaries. Furthermore, we assume that spending on bailouts tends to be high in states where tax rates are high, so $Cov\left[G - \frac{\overline{T}}{\overline{Y}}Y, X_2\right] > 0$.

[13] http://www.federalreserve.gov/newsevents/speech/stein20131018a.htm

[14] de Faria e Castro, Martinez, and Philippon (2014) also study the relationship between fiscal capacity and financial stability interventions. However, their focus is on government policies that disclose information about asset quality in the financial sector.

At an interior optimum, the optimal program mix satisfies:

$$\begin{bmatrix} q_1^* \\ q_2^* \end{bmatrix} = \frac{\left(1+\beta^{-1}\right)}{\bar{\eta}/\overline{Y}} \left(\frac{E\left[W_1 - X_1\right]}{E\left[X_1\right]} - \bar{h}' \right) \begin{bmatrix} \left(E\left[X_1\right]\right)^{-1} \\ 0 \end{bmatrix} \tag{28}$$

$$+ \frac{1}{\bar{\eta}/\overline{Y}} \frac{E\left[X_2\right]}{Var\left[X_2\right]} \left(\frac{E\left[W_2 - X_2\right]}{E\left[X_2\right]} - \frac{E\left[W_1 - X_1\right]}{E\left[X_1\right]} \right) \begin{bmatrix} -\left(E\left[X_1\right]\right)^{-1} E\left[X_2\right] \\ 1 \end{bmatrix}$$

$$-\beta^{-1}\overline{D} \begin{bmatrix} \left(E\left[X_1\right]\right)^{-1} \\ 0 \end{bmatrix} - \frac{Cov\left[G - \frac{\overline{T}}{\overline{Y}} Y, X_2\right]}{Var\left[X_2\right]} \begin{bmatrix} -\left(E\left[X_1\right]\right)^{-1} E\left[X_2\right] \\ 1 \end{bmatrix}.$$

This formula illustrates the forces that determine q_1^* and q_2^*.[15] The first term in Eq. (28) says that an increase in the expected returns to ex-ante regulation, $E\left[W_1 - X_1\right]/E\left[X_1\right]$, raises q_1^*. The second term in Eq. (28) says that an increase in the differential expected returns to ex-post bailouts versus ex-ante regulation, $E\left[W_2 - X_2\right]/E\left[X_2\right] - E\left[W_1 - X_1\right]/E\left[X_1\right]$, leads the government to substitute from ex-ante regulation towards ex-post bailouts. (Indeed, since bailouts are fiscally risky, the government should only choose $q_2^* > 0$ if the expected returns to ex-post bailouts are sufficiently greater than the returns to ex-ante regulation.) Given the desire to smooth tax rates, any substitution from regulation to bailouts is stronger when $Var\left[X_2\right]$ is small. The last two terms in Eq. (28) capture the way that background fiscal risk impacts the choices of q_1 and q_2. Specifically, an increase in accumulated deficits $\overline{D}$ reduces the desirability of regulation. Finally, when bailout spending covaries with other fiscal risks—i.e., when $Cov[G - \frac{\overline{T}}{\overline{Y}} Y, X_2]$ is large, the desire to manage fiscal risk argues against bailouts and in favor of regulation. The following proposition describes the behavior of the optimal mix at an interior optimum.

Proposition 4 *Suppose that $\gamma = 0$ and $\eta > 0$ and that regulation (program 1) and bailouts (program 2) have the characteristics assumed above. At an interior optimum where both $q_1^* > 0$ and $q_2^* > 0$, we have the following comparative statics:*

[15] When $E\left[W_2 - X_2\right]/E\left[X_2\right] - E\left[W_1 - X_1\right]/E\left[X_1\right] < \eta Cov\left[G, X_2\right]/E\left[X_2\right]$ the constraint that $q_2^* = 0$ binds and we have $q_1^* = (1+R)\left(\eta E\left[X_1\right]\right)^{-1} E\left[W_1 - X_1\right]/E\left[X_1\right] - R\overline{D}/E\left[X_1\right]$. By contrast, when

$$E\left[W_1 - X_1\right]/E\left[X_1\right] < \frac{E\left[W_2 - X_2\right]/E\left[X_2\right] + \eta R\overline{D}/E\left[X_2\right] - \eta Cov\left[X_2, G\right]/Var\left[X_2\right]}{E\left[X_2\right]/Var\left[X_2\right] + (1+R)/E\left[X_2\right]},$$

the constraint that $q_1^* = 0$ binds and we have

$$q_2^* = \frac{\eta^{-1} E\left[W_2 - X_2\right] - R\overline{D}E\left[X_2\right]/(1+R) - Cov\left[X_2, G\right]}{\left(E\left[X_2\right]\right)^2/(1+R) + Var\left[X_2\right]}.$$

- $\partial q_1^*/\partial \overline{D} = -R/E\left[X_1\right] < 0$ *and* $\partial q_2/\partial \overline{D} = 0$;

- $\partial q_1^*/\partial Corr[G - \frac{\overline{T}}{\overline{Y}}Y, X_2] > 0$ *and* $\partial q_2^*/\partial Corr[G - \frac{\overline{T}}{\overline{Y}}Y, X_2] < 0$; *and*

- $\partial q_1^*/\partial \eta \propto -\left(1+\beta^{-1}\right)\left\{E\left[W_1 - X_1\right]/E\left[X_1\right]\right\}$
 $+\left(\left(E\left[X_2\right]\right)^2/Var\left[X_2\right]\right)\left\{E\left[W_2 - X_2\right]/E\left[X_2\right] - E\left[W_1 - X_1\right]/E\left[X_1\right]\right\}$ *and*
 $\partial q_2^*/\partial \eta \propto -\left\{E\left[W_2 - X_2\right]/E\left[X_2\right] - E\left[W_1 - X_1\right]/E\left[X_1\right]\right\} < 0$.

The comparative statics have strong analogies to portfolio choice logic. As the level of accumulated deficits $\overline{D}$ rises, all adjustment takes place by reducing the amount of the fiscallly riskless program—i.e., ex-ante regulation. This is analogous to the portfolio choice logic that dictates that with constant absolute risk aversion, the total dollar amount invested in risky assets does not vary with total wealth. Only the amount invested in riskless assets varies.

Since taxation is distortionary, raising the correlation between bailout spending and tax rates, $Corr[G - \frac{\overline{T}}{\overline{Y}}Y, X_2]$, or the variance of tax rates, $Var[G - \frac{\overline{T}}{\overline{Y}}Y]$, makes bailouts less attractive and regulation more attractive. These results can also be interpreted as motivating "financial repression" at high levels of government debt. Reinhart and Sbrancia (2011) argue that at high levels of government debt, financial regulation is used to force financial intermediaries to hold government debt, providing a captive buyer. Our model makes the point that financial repression may be optimal in high debt situations, not just because the government needs a buyer of debt but because the government cannot afford the costs of the alternative financial stability policy—namely, ex post bailouts.

Finally, we consider the effect on changing the extent of tax distortions reflected in η. An increase in tax distortions always leads to a reduction in the amount of fiscally risky bailout programs, $\partial q_2^*/\partial \eta < 0$. To the extent that ex-post bailouts are highly attractive relative to ex-ante regulation (i.e., $E\left[W_2 - X_2\right]/E\left[X_2\right] - E\left[W_1 - X_1\right]/E\left[X_1\right]$ is large), the increase in distortions also leads the government to substitute to fiscally riskless regulation. However, to the extent that ex-ante regulation is highly attractive (i.e., $E\left[W_1 - X_1\right]/E\left[X_1\right]$ is large), there is an offsetting effect: in this case regulatory expenditures are already quite large, so the increase in distortions leads the government to cut back on regulation as well.

We have discussed comparative statics in terms of quantities here. As pointed out above, if we fix the quantities of regulation and bailouts, comparative statics in terms of their prices will simply have the opposite signs. Thus, the price the government charges intermediaries for financial guarantees should rise with the scale of existing fiscal commitments and tax distortions.

4 Conclusion

We present a model in which the distortionary taxation makes financing costly for the government. We explore the consequences of this assumption for the set of programs that the government should choose to undertake. As in the corporate finance literature on costly external finance, we show that distortionary taxation impacts the optimal scale and pricing of government programs. In particular, the government has both social and fiscal risk management motives. The social risk management motive arises from the fact that some government programs deliver large benefits in bad states when household marginal utility is high. The fiscal risk management motive arises from the government's desire to avoid raising distortionary taxes further in states where taxes are already high. Neither fiscal risk nor social risk can be judged in isolation. For example, a program's fiscal risk depends on how its outlays comove with those of other programs.

We highlight the interaction between the social and fiscal risk management motives. These motives frequently come into conflict because programs with significant social risk management benefits often entail large government expenditures and, hence, higher tax distortions in bad times, adding to total fiscal risk.

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Table 1: Model parameters for numerical example. This table presents the baseline model parameters that we use in our numerical example.

Parameter	Description	Value
Background economy		
Y_0	Private income at time 0	1
G_0	Other government spending at time 0	0
P	Probability of the "high" state at time 1	50%
Y_{1H}	Private income in "high" state at time 1	1.2
G_{1H}	Other government spending in "high" state at time 1	-0.1
Y_{1L}	Private income in "low" state at time 1	0.8
G_{1L}	Other government spending in "low" state at time 1	0.1
Household preferences		
β	Discount factor due to household time preference	1
γ	Household risk aversion	0.25
$\overline{C}$	Baseline level of consumption	1
Fiscal parameters		
η	Parameter governing tax distortions	0.05
$\overline{D}$	Initial accumulated debt	0.6
Government program under consideration		
W_0	Additional private income at time 0	0
X_0	Additional government spending at time 0	0.0236
W_{1H}	Additional private income in "high" state at time 1	0
X_{1H}	Additional government spending in "high" state at time 1	0
W_{1L}	Additional private income in "low" state at time 1	0.1
X_{1L}	Additional government spending in "low" state at time 1	0.05

Table 2: Optimal fiscal policies in numerical example. This table illustrates optimal fiscal policies in our numerical example. The ten rows show how optimal policy varies with parameters of the background economy and the program under consideration. The five columns show how optimal policy varies with household risk aversion (γ) and fiscal costs (η).

		(1) Baseline γ=0.25, η=0.05	(2) Risk-neutral γ=0, η=0.05	(3) Ricardian γ=0.25, η=0	(4) Higher γ γ=0.50, η=0.05	(5) Higher η γ=0.25, η=0.075
		Panel A: Optimal program scale (q)				
(1)	Baseline	3.73	2.97	7.25	3.82	2.33
(2)	Higher $\overline{D}$	3.32	1.41	7.25	3.55	1.76
(3)	Higher $E[Y_1]$	3.29	3.41	6.56	3.27	1.98
(4)	Higher $Var[Y_1]$	4.14	2.52	7.94	4.34	2.64
(5)	Higher $E[G_1]$	4.12	2.32	7.94	4.34	2.59
(6)	Higher $Var[G_1]$	4.10	2.24	7.94	4.33	2.56
(7)	Higher $E[W_1]$	5.35	10.52	9.20	4.68	3.78
(8)	Higher $Var[W_1]$	3.93	2.97	7.01	4.03	2.64
(9)	Higher $E[X_1]$	3.56	2.15	7.05	3.72	2.17
(10)	Higher $Var[X_1]$	3.48	2.60	7.32	3.59	2.02
		Panel B: Optimal government debt (D_0)				
(1)	Baseline	0.25	0.27	N/A	0.24	0.26
(2)	Higher $\overline{D}$	0.31	0.33	N/A	0.30	0.32
(3)	Higher $E[Y_1]$	0.26	0.28	N/A	0.25	0.27
(4)	Higher $Var[Y_1]$	0.24	0.26	N/A	0.22	0.24
(5)	Higher $E[G_1]$	0.22	0.24	N/A	0.21	0.23
(6)	Higher $Var[G_1]$	0.24	0.26	N/A	0.22	0.25
(7)	Higher $E[W_1]$	0.24	0.23	N/A	0.23	0.25
(8)	Higher $Var[W_1]$	0.25	0.27	N/A	0.24	0.26
(9)	Higher $E[X_1]$	0.25	0.27	N/A	0.24	0.26
(10)	Higher $Var[X_1]$	0.25	0.27	N/A	0.24	0.26
		Panel C: Riskless interest rate ($R-1$)				
(1)	Baseline	4.6%	0.0%	9.2%	9.7%	2.9%
(2)	Higher $\overline{D}$	4.1%	0.0%	9.2%	9.0%	2.2%
(3)	Higher $E[Y_1]$	5.4%	0.0%	9.7%	11.1%	3.7%
(4)	Higher $Var[Y_1]$	5.1%	0.0%	10.2%	11.1%	3.3%
(5)	Higher $E[G_1]$	3.8%	0.0%	8.7%	8.3%	1.9%
(6)	Higher $Var[G_1]$	5.1%	0.0%	10.2%	11.1%	3.1%
(7)	Higher $E[W_1]$	6.8%	0.0%	12.1%	12.3%	4.8%
(8)	Higher $Var[W_1]$	4.9%	0.0%	8.9%	10.3%	3.3%
(9)	Higher $E[X_1]$	4.4%	0.0%	8.9%	9.4%	2.7%
(10)	Higher $Var[X_1]$	4.3%	0.0%	9.3%	9.1%	2.5%

Table 3: Approximate optimal fiscal policies in numerical example. This table illustrates optimal fiscal policies in our numerical example. The ten rows show how optimal policy varies with parameters of the background economy and the program under consideration. The five columns show how optimal policy varies with household risk aversion (γ) and fiscal costs (η).

		(1) Baseline γ=0.25, η=0.05	(2) Risk-neutral γ=0, η=0.05	(3) Ricardian γ=0.25, η=0	(4) Higher γ γ=0.50, η=0.05	(5) Higher η γ=0.25, η=0.075
		Panel A: Optimal program scale (q)				
(1)	Baseline	4.00	4.21	7.25	3.97	2.65
(2)	Higher $\overline{D}$	3.64	2.59	7.25	3.74	2.15
(3)	Higher $E[Y_1]$	3.48	4.41	6.56	3.38	2.20
(4)	Higher $Var[Y_1]$	4.51	4.00	7.94	4.56	3.09
(5)	Higher $E[G_1]$	4.44	3.54	7.94	4.52	2.98
(6)	Higher $Var[G_1]$	4.43	3.52	7.94	4.51	2.97
(7)	Higher $E[W_1]$	5.70	14.30	9.20	4.86	4.22
(8)	Higher $Var[W_1]$	4.18	4.21	7.01	4.17	2.95
(9)	Higher $E[X_1]$	3.82	3.18	7.05	3.88	2.49
(10)	Higher $Var[X_1]$	3.76	3.72	7.32	3.76	2.34
		Panel B: Optimal government debt (D_0)				
(1)	Baseline	0.30	0.30	N/A	0.30	0.30
(2)	Higher $\overline{D}$	0.36	0.36	N/A	0.37	0.36
(3)	Higher $E[Y_1]$	0.30	0.30	N/A	0.30	0.30
(4)	Higher $Var[Y_1]$	0.30	0.30	N/A	0.30	0.30
(5)	Higher $E[G_1]$	0.28	0.27	N/A	0.29	0.28
(6)	Higher $Var[G_1]$	0.30	0.30	N/A	0.30	0.29
(7)	Higher $E[W_1]$	0.31	0.29	N/A	0.31	0.30
(8)	Higher $Var[W_1]$	0.30	0.30	N/A	0.31	0.30
(9)	Higher $E[X_1]$	0.30	0.30	N/A	0.30	0.30
(10)	Higher $Var[X_1]$	0.30	0.30	N/A	0.30	0.30