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ACCOUNTING FOR BUSINESS CYCLES

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### **ABSTRACT**

We elaborate on the business cycle accounting method proposed by Chari, Kehoe, and McGrattan (2007), clear up some misconceptions about the method, and then apply it to compare the Great Recession across OECD countries as well as to the recessions of the 1980s in these countries. We have four main findings. First, with the notable exception of the United States, Spain, Ireland, and Iceland, the Great Recession was driven primarily by the efficiency wedge. Second, in the Great Recession, the labor wedge plays a dominant role only in the United States, and the investment wedge plays a dominant role in Spain, Ireland, and Iceland. Third, in the recessions of the 1980s, the labor wedge played a dominant role only in France, the United Kingdom, Belgium, and New Zealand. Finally, overall in the Great Recession the efficiency wedge played a more important role and the investment wedge played a less important role than they did in the recessions of the 1980s.

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In this paper we elaborate on the business cycle accounting method proposed by Chari, Kehoe, and McGrattan (2007), henceforth CKM, clear up some misconceptions about the method, and then apply it to compare the Great Recession across OECD countries as well as to the recessions of the 1980s in these countries. The goal of the method is to help guide researchers' choices about where to introduce frictions into their detailed quantitative models in order to allow the models to generate business cycle fluctuations similar to those in the data.

The method has two components: an equivalence result and an accounting procedure. The *equivalence result* is that a large class of models, including models with various types of frictions, is equivalent to a *prototype model* with various types of time-varying wedges that distort the equilibrium decisions of agents operating in otherwise competitive markets. At face value, these wedges look like time-varying productivity, labor income taxes, investment taxes, and government consumption. We labeled these wedges *efficiency wedges*, *labor wedges*, *investment wedges*, and *government consumption wedges*.

The *accounting procedure* also has two components. It begins by measuring the wedges, using data together with the equilibrium conditions of a prototype model. The measured wedge values are then fed back into the prototype model, one at a time and in combinations, in order to assess how much of the observed movements of output, labor, and investment can be attributed to each wedge, separately and in combinations.

Here we use this method to study the Great Recession in OECD countries. We also compare this recession with the recessions of the early 1980s. While the exact timing of the recessions of the early 1980s differs across countries in our OECD sample, most of the countries had a recession between 1980 and 1984. Throughout we refer to the recessions of the early 1980s as the *1982 recession*. We have four main findings. First, with the notable exception of the United States, Spain, Ireland, and Iceland, the Great Recession was driven primarily by the efficiency wedge. Second, in the Great Recession, the labor wedge plays a dominant role only in the United States, and the investment wedge plays a dominant role in Spain, Ireland, and Iceland. Third, in the recessions of the 1980s, the labor wedge played a dominant role only in France, the United Kingdom, Belgium, and New Zealand. Finally, overall in the Great Recession the efficiency wedge played a more important role and the

investment wedge played a less important role than they did in the recessions of the 1980s.

We now turn to elaborating on the equivalence results in CKM that link the four wedges to detailed models. We begin by showing that a detailed economy with fluctuations in investment-specific technological change similar to that in Greenwood, Hercowitz, and Krusell (1997) maps into a prototype economy with investment wedges. This result makes clear that investment wedges are by no means synonymous with financial frictions, a point stressed by CKM.

We then consider an economy that blends elements of Kiyotaki and Moore (1997) with that of Gertler and Kiyotaki (2009). The economy has a representative household and heterogeneous banks that face collateral constraints. We show that such an economy is equivalent to a prototype economy with investment wedges. This result makes clear that some ways of modeling financial frictions do indeed show up as investment wedges.

Finally, we turn to an economy studied by Buera and Moll (2015) consisting of workers and entrepreneurs. The entrepreneurs have access to heterogeneous production technologies that are subject to shocks to collateral constraints. We follow Buera and Moll (2015) in showing that this detailed economy is equivalent to a prototype model with a labor wedge, an investment wedge, and an efficiency wedge. This equivalence makes the same point as does the input-financing friction economy in CKM, namely, that other ways of modeling financial frictions can show up as efficiency wedges and labor wedges.

The point of the three examples just discussed is to help clarify how the pattern of wedges in the data can help researchers narrow down the class of models they are considering. If, for example, most of the fluctuations are driven by the efficiency and labor wedges in the data, then of the three models just considered, the third one is more promising than the first two.

We then turn to models with search frictions. We use these models to make an important point. Researchers should choose the baseline prototype economy that provides the most insights for the research program of interest. In particular, when the detailed economies of interest are sufficiently different from the one-sector growth model, it is often more instructive to adjust the prototype model so that the version of it without wedges corresponds to the planning problem for the class of models at hand. For example, when we

map the model with efficient search into the one-sector model, that model does have efficiency and labor wedges, but if we map it into a new prototype model with two capital-like variables, physical capital and the stock of employed workers, the new prototype model has no wedges.

We then consider a search model with an inefficient equilibrium. When we map this model into the new prototype model with two capital-like variables, then the prototype model has only labor wedges. But if we map it into the original prototype model, it has efficiency wedges and (complicated) labor wedges. These findings reinforce the point that it is often more instructive to adjust the prototype model so that the version of it without wedges corresponds to the planning problem for the class of models at hand.

Taken together, these equivalence results help clear up some common misconceptions. The first misconception is that efficiency wedges in a prototype model can only come from technology shocks in a detailed model. In our judgment, by far the least interesting interpretation of efficiency wedges is as narrowly interpreted shocks to the blueprints governing individual firm production functions. More interesting interpretations rest on frictions that deliver such high-frequency movements in this wedge. For example, the input-financing friction model in CKM shows how financial frictions in a detailed model can manifest themselves as efficiency wedges. Indeed, we think that exploring detailed models in which the sudden drops in efficiency wedges experienced in recessions come from frictions such as input-financing frictions is more promising than blaming these drops on abrupt negative shocks to blueprints for technologies. The second misconception is that labor wedges in a prototype model arise solely from frictions in labor markets in detailed economies. The Buera-Moll economy makes clear that this view is incorrect. The third misconception is that investment wedges arise solely due to financial frictions. Clearly, the detailed model with investment-specific technical change shows that this view is also incorrect.

We turn now to describing our procedure. This procedure is designed to answer questions of the following kind: How much would output fluctuate if the only wedge that fluctuated is the efficiency wedge and the probability distribution of the efficiency wedge is the same as in the prototype economy? If the wedges were independent at all leads and lags, the procedure can be implemented in a straightforward manner by letting only, say, the efficiency wedge fluctuate and setting all other wedges to constants. In the data, the wedges

are correlated with each other, so the straightforward implementation does not answer our question.

Our implementation views the wedges as being functions of underlying abstract events. In practice, we assume that the dimension of the underlying events is the same as the dimension of the wedges, namely four, and identify each event with one of the wedges. We then use the data to estimate the stochastic process for the underlying events. Given this estimated stochastic process, we can then answer our question by letting the wedge of interest vary with the underlying events in the same way as it did in the data but assuming that all other wedges are constant functions of the underlying events. The procedure ensures that the probability distribution over the wedge of interest is the same in the prototype economy with all wedges and in the experiment.

We then briefly discuss what at first seems to be an intuitive way to proceed: the wedges are identified with the underlying event not only in the estimation but also in the thought experiment. The problem with this procedure is that it does not make clear the conceptual distinction between underlying events and wedges. This distinction is apparent when the wedges are correlated. Indeed, in this case, this procedure makes it impossible to hold all but one wedge constant without changing the probability distribution over the wedge of interest. We note that not keeping clear the conceptual distinction between underlying events and wedges has been the source of some confusion in the literature (see, for example, Christiano and Davis (2006)).

Our business cycle accounting method is intended to shed light on promising classes of mechanisms through which primitive shocks lead to economic fluctuations. It is not intended to identify the primitive sources of shocks. Many economists think, for example, that shocks to the financial sector drove the Great Recession in developed economies, but these economists disagree about the details of the driving mechanism. Our analysis suggests that the transmission mechanism from shocks to the financial sector to broader economic activity must be different in the United States, Spain, Ireland, and Iceland than in the rest of the countries in the OECD. More precisely, our analysis shows that these shocks must manifest themselves as labor wedges in the United States, as investment wedges in Spain, Ireland, and Iceland, and as efficiency wedges in the rest of the OECD.

As CKM argue, the equivalence results provide the logical foundation for the way our accounting procedure uses the measured wedges. At a mechanical level, the wedges represent deviations in the prototype model's first-order conditions, in its relationship between inputs and outputs, and in a variable in the resource constraint. One interpretation of these deviations, of course, is that they are simply errors, so that their size indicates the goodness-of-fit of the model. Under that interpretation, however, feeding the measured wedges back into the model makes no sense. Our equivalence result leads to a more economically useful interpretation of the deviations by linking them directly to classes of models; that link provides the rationale for feeding the measured wedges back into the model.

Also in terms of method, the accounting procedure goes beyond simply plotting the wedges. Such plots, by themselves, are not useful in evaluating the quantitative importance of competing mechanisms of business cycles because they tell us little about the equilibrium responses to the wedges. Feeding the measured wedges back into the prototype model and measuring the model's resulting equilibrium responses is what allows us to discriminate between competing mechanisms.

*Related Literature.*

The paper most closely related to ours is Ohanian and Raffo (2012), who use a methodology similar to ours to study the Great Recession in 14 OECD countries and compare the peak-to-trough declines in output and hours across countries and recessions. In part, our findings are the same in spirit: we both find that in the Great Recession, the labor wedge plays a dominant role in the United States.

In part our findings are in contrast: they find that in Korea the labor wedge plays a large role in the Great Recession. We instead find that in Korea the efficiency wedge does. We note that both Ohanian and Raffo (2012) and Rodriguez-Lopez and Garcia (2016) find that the labor wedge rather than the investment wedge plays a dominant role in the Great Recession in Spain. Our findings differ from both studies in part because of differences in the treatment of the data, including, for example, how we treat consumer durables and how we deflate nominal variables to make them real. We also differ from Ohanian and Raffo (2012) in terms of methodology: we fit stochastic processes for the wedges, whereas they focus on perfect foresight models. For some related studies, see Mulligan (2009) and Ohanian (2010).

The business cycle accounting methodology has been used for many countries and time periods. For example, it has been used for Portugal by Cavalcanti (2007); for the economies of Brazil, Russia, India, and China by Chakraborty and Otsu (2013); for India by Chakraborty (2006); for the East Asian economies by Cho and Doblus-Madrid (2013); for the United Kingdom by Kersting (2008); for Japan by Kobayashi and Inaba (2006); for Asian economies by Otsu (2010); and for monetary economies by Sustek (2011) and Brinca (2013); and for a variety of countries by Brinca (2014).

## 1. Demonstrating the Equivalence Result

Here we show how various detailed models with underlying distortions are equivalent to a prototype growth model with one or more wedges.

### A. The Benchmark Prototype Economy

The *benchmark prototype economy* that we use later in our accounting procedure is a stochastic growth model. In each period  $t$ , the economy experiences one of finitely many events  $s_t$ , which index the shocks. We denote by  $s^t = (s_0, \dots, s_t)$  the history of events up through and including period  $t$  and often refer to  $s^t$  as the *state*. The probability, as of period 0, of any particular history  $s^t$  is  $\pi_t(s^t)$ . The initial realization  $s_0$  is given. The economy has four exogenous stochastic variables, all of which are functions of the underlying random variable  $s^t$ : the *efficiency wedge*  $A_t(s^t)$ , the *labor wedge*  $1 - \tau_{lt}(s^t)$ , the *investment wedge*  $1/[1 + \tau_{xt}(s^t)]$ , and the *government consumption wedge*  $g_t(s^t)$ .

In the model, consumers maximize expected utility over per capita consumption  $c_t$  and per capita labor  $l_t$ ,

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) U(c_t(s^t), l_t(s^t)) N_t,$$

subject to the budget constraint

$$c_t + [1 + \tau_{xt}(s^t)]x_t(s^t) = [1 - \tau_{lt}(s^t)]w_t(s^t)l_t(s^t) + r_t(s^t)k_t(s^{t-1}) + T_t(s^t)$$



and the capital accumulation law

$$(1) \quad (1 + \gamma_n)k_{t+1}(s^t) = (1 - \delta)k_t(s^{t-1}) + x_t(s^t),$$

where  $k_t(s^{t-1})$  denotes the per capita capital stock,  $x_t(s^t)$  per capita investment,  $w_t(s^t)$  the wage rate,  $r_t(s^t)$  the rental rate on capital,  $\beta$  the discount factor,  $\delta$  the depreciation rate of capital,  $N_t$  the population with growth rate equal to  $1 + \gamma_n$ , and  $T_t(s^t)$  per capita lump-sum transfers.

The production function is  $A(s^t)F(k_t(s^{t-1}), (1 + \gamma)^t l_t(s^t))$ , where  $1 + \gamma$  is the rate of labor-augmenting technical progress, which is assumed to be a constant. Firms maximize profits given by  $A_t(s^t)F(k_t(s^{t-1}), (1 + \gamma)^t l_t(s^t)) - r_t(s^t)k_t(s^{t-1}) - w_t(s^t)l_t(s^t)$ .

The equilibrium of this benchmark prototype economy is summarized by the resource constraint,

$$(2) \quad c_t(s^t) + x_t(s^t) + g_t(s^t) = y_t(s^t),$$

where  $y_t(s^t)$  denotes per capita output, together with

$$(3) \quad y_t(s^t) = A_t(s^t)F(k_t(s^{t-1}), (1 + \gamma)^t l_t(s^t)),$$

$$(4) \quad -\frac{U_{lt}(s^t)}{U_{ct}(s^t)} = [1 - \tau_{lt}(s^t)]A_t(s^t)(1 + \gamma)^t F_{lt}, \text{ and}$$

$$(5) \quad U_{ct}(s^t)[1 + \tau_{xt}(s^t)] \\ = \beta \sum_{s^{t+1}} \pi_t(s^{t+1}|s^t)U_{ct+1}(s^{t+1})\{A_{t+1}(s^{t+1})F_{kt+1}(s^{t+1}) + (1 - \delta)[1 + \tau_{xt+1}(s^{t+1})]\},$$

where, here and throughout, notations such as  $U_{ct}$ ,  $U_{lt}$ ,  $F_{lt}$ , and  $F_{kt}$  denote the derivatives of the utility function and the production function with respect to their arguments and  $\pi_t(s^{t+1}|s^t)$  denotes the conditional probability  $\pi_t(s^{t+1})/\pi_t(s^t)$ . We assume that  $g_t(s^t)$  fluctuates around a trend of  $(1 + \gamma)^t$ .

Notice that in this benchmark prototype economy, the efficiency wedge resembles a blueprint technology parameter, and the labor and investment wedges resemble tax rates on

labor income and investment. Other more elaborate models could be considered, such as models with other kinds of frictions that look like taxes on consumption or capital income. Consumption taxes induce a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor in the same way as do labor income taxes. Such taxes, if time-varying, also distort the intertemporal margins in (5). Capital income taxes induce a wedge between the intertemporal marginal rate of substitution and the marginal product of capital, which is only slightly different from the distortion induced by a tax on investment. We experimented with intertemporal distortions that resemble capital income taxes rather than investment taxes and found that our substantive conclusions are unaffected. (For details, see the Appendix.)

We emphasize that each of the wedges represents the overall distortion to the relevant equilibrium condition of the model. For example, distortions to labor supply affecting consumers and to labor demand affecting firms both distort the static first-order condition (4). Our labor wedge represents the sum of these distortions. Thus, our method identifies the overall wedge induced by both distortions and does not identify each separately. Likewise, liquidity constraints on consumers distort the consumer's intertemporal Euler equation, whereas investment financing frictions on firms distort the firm's intertemporal Euler equation. Our method combines the Euler equations for the consumer and the firm and therefore identifies only the overall wedge in the combined Euler equation given by (5). We focus on the overall wedges because what matters in determining business cycle fluctuations is the overall wedges, not each distortion separately.

For the equivalence results that follow, it is notationally convenient to work with the prototype model just described. For our quantitative results, we add investment adjustment costs by replacing the capital accumulation law (1) with

$$(6) \quad (1 + \gamma_n)k_{t+1}(s^t) = (1 - \delta)k_t(s^{t-1}) + x_t(s^t) - \phi \left( \frac{x_t(s^t)}{k_t(s^{t-1})} \right),$$

where  $\phi$  represents the per unit cost of adjusting the capital stock. We follow the macroeco-

conomic literature in assuming that the adjustment costs are parameterized by the function

$$\phi\left(\frac{x}{k}\right) = \frac{a}{2} \left(\frac{x}{k} - b\right)^2,$$

where  $b = \delta + \gamma + \gamma_n$  is the steady-state value of the investment-capital ratio.

## **B. The Mapping—From Frictions to Wedges**

Now we illustrate the mapping between detailed economies and prototype economies for several types of wedges. We show that investment-specific technical change in a detailed economy maps into investment wedges in our prototype economy. Likewise, bank collateral constraints also map into investment wedges in our prototype economy. We then consider an economy with heterogeneous productivity and collateral constraints and show that it maps into a prototype economy with efficiency, labor, and investment wedges. Finally, we consider a search model with efficient allocations and show that it maps into a prototype economy with a labor wedge and an efficiency wedge but no investment wedge. The four economies we use to illustrate this mapping are closed economies for which the associated government consumption wedge in the prototype economy is identically zero. Hence, we focus on the other three wedges and make no mention of the government consumption wedge.

We choose simple models in order to illustrate how the detailed models map into the prototypes. Since many models map into the same configuration of wedges, identifying one particular configuration does not uniquely identify a model; rather, it identifies a whole class of models that are consistent with that configuration. This point is seen clearly when comparing the prototype model associated with the economy with investment-specific technical change to that for the economy with bank collateral constraints. In this sense, our method does not uniquely determine the model most promising to analyze business cycle fluctuations. It does, however, guide researchers to focus on the key margins that need to be distorted in order to capture the nature of the fluctuations.

### ***An Equivalence Result for a Model with Investment-Specific Technical Change***

We begin with a two-sector model with investment-specific technical change and show how it maps into a prototype economy with only investment wedges.

*A Detailed Economy with Investment Specific Technical Change*

The detailed economy has consumption  $c_t(s^t)$  and investment  $x_t(s^t)$  produced according to

$$(7) \quad c_t(s^t) = A_t(s^t)F(k_{ct}(s^t), l_{ct}(s^t)) \text{ and } x_t(s^t) = A_{xt}(s^t)A_t(s^t)F(k_{xt}(s^t), l_{xt}(s^t)),$$

where  $k_{ct}(s^t)$  and  $l_{ct}(s^t)$  denote capital and labor used to produce consumption goods,  $k_{xt}(s^t)$  and  $l_{xt}(s^t)$  denote capital and labor used to produce investment goods,  $A_t(s^t)$  is neutral technical change,  $A_{xt}(s^t)$  denotes investment-specific technical change, and  $F$  satisfies constant returns to scale. The timing is that the (total) capital stock in use at period  $t$  is chosen at the end of period  $t - 1$  given the shock history  $s^{t-1}$ , whereas at the beginning of each period, after the current shock  $s_t$  is realized, labor and capital are allocated between sectors. This timing gives rise to a capital accumulation rule

$$(8) \quad k_{t+1}(s^t) = (1 - \delta)k_t(s^{t-1}) + x_t(s^t)'$$

constraints for sectoral capital allocation,

$$(9) \quad k_{ct}(s^t) + k_{xt}(s^t) \leq k_t(s^{t-1}),$$

and sectoral labor allocation,

$$(10) \quad l_{ct}(s^t) + l_{xt}(s^t) \leq l_t(s^t).$$

The planning problem is to choose allocations to solve

$$\max \sum_{s^t} \beta^t \mu(s^t) U(c_t(s^t), l_t(s^t))$$

subject to (7)–(10). Using that the production function  $F$  has constant returns to scale, the

first-order conditions imply that

$$\frac{k_{ct}(s^t)}{l_{ct}(s^t)} = \frac{k_{xt}(s^t)}{l_{xt}(s^t)} = \frac{k_t(s^{t-1})}{l_t(s^t)},$$

and hence

$$F_{kc}(k_{ct}(s^t), l_{ct}(s^t)) = F_{kx}(k_{xt}(s^t), l_{xt}(s^t)) \text{ and } F_{lc}(k_{ct}(s^t), l_{ct}(s^t)) = F_{lx}(k_{xt}(s^t), l_{xt}(s^t)),$$

and we can write these marginal products as  $F_k(k(s^{t-1}), l(s^t))$  and  $F_l(k(s^{t-1}), l(s^t))$ . The Euler equation is

$$\frac{U_{ct}(s^t)}{A_{xt}(s^t)} = \sum_{s_{t+1}} \beta \mu(s^{t+1}|s^t) \left[ U_{ct+1}(s^{t+1}) A_{t+1}(s^{t+1}) F_k(s^{t+1}) + (1 - \delta) \frac{U_{ct}(s^{t+1})}{A_{xt+1}(s^{t+1})} \right],$$

and the static first-order condition for labor is given by

$$-\frac{U_{lt}(s^t)}{U_{ct}(s^t)} = A_t(s^t) F_l(s^t).$$

If we express output in current consumption units, we can write

$$A_t(s^t) F(k_{ct}(s^t), l_{ct}(s^t)) + q_t(s^t) A_{xt}(s^t) A_t(s^t) F(k_{xt}(s^t), l_{xt}(s^t)) = A_t(s^t) F(k(s^{t-1}), l(s^t))$$

since the relative price of investment to consumption goods is  $q_t(s^t) = 1/A_{xt}(s^t)$ .

#### *The Associated Prototype Economy with Investment Wedges*

Now consider a prototype economy with just investment wedges. This prototype economy has a productivity shock  $A_t(s^t)$  equal to that in the consumption goods sector in the detailed economy, an investment wedge equal to the reciprocal of the level of investment-specific technical change, and no other wedges.

**PROPOSITION 1:** *The aggregate allocations in the detailed economy with investment-specific technical change coincide with those of the prototype economy if the efficiency wedge in the prototype economy equals the productivity shock in the consumption goods sector, the*

investment wedge is given by

$$1 - \tau_{xt}(s^t) = \frac{1}{A_{xt}(s^t)},$$

and the labor wedge is zero.

Note that if we measure output in the detailed economy at base period prices rather than at current prices, the map between the detailed economy and the prototype economy is more complicated.

### ***An Equivalence Result for an Economy with Bank Collateral Constraints***

Here we show the equivalence between an economy with bank collateral constraints and a prototype economy with only investment wedges.

#### *A Detailed Economy with Bank Collateral Constraints*

Consider an infinite horizon economy that blends elements of Kiyotaki and Moore (1997) with that of Gertler and Kiyotaki (2009) and is composed of a household that works and operates financial intermediaries, referred to as *banks*, together with firms and a government. Households elastically supply labor and save by holding deposits in banks and government bonds and receive dividends. Banks raise deposits from households and use these deposits plus retained earnings to invest in capital as well as to pay dividends to consumers. Firms rent capital and labor and produce output. The government finances an exogenous stream of government spending by taxing labor income and the capital stock and by selling government bonds.

Let the state of the economy be  $s_t \in S$  distributed according to  $\pi(s_t|s_{t-1})$ . Let  $s^t = (s_0, \dots, s_t)$ . The resource constraint is given by

$$(11) \quad C_t(s^t) + K_{t+1}(s^t) = A_t(s^t)F(K_t(s^{t-1}), L_t(s^t)),$$

where  $C_t$  is aggregate consumption,  $K_{t+1}$  is the capital stock,  $L_t$  is aggregate labor, and  $F$  is a constant returns to scale production function that includes the undepreciated capital stock. Throughout we use the convention that uppercase letters denote aggregates and lowercase letters denote the decisions of individual households or banks.

We follow Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012) in the formulation of households. The decision making in each household can be thought of as being made by different entities: a measure 1 of workers and a measure 1 of bankers. The workers supply labor and return their wages to the household while each banker manages a bank that transfers nonnegative dividends to the household. The household as a whole has preferences

$$(12) \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t | s_0) U(c_t(s^t), l_t(s^t)),$$

where  $c_t$  and  $l_t$  are an individual household's consumption and labor supply. Given initial asset holdings  $b_{H0}$  and  $d_0$ , the stand-in household in the economy maximizes this utility by choosing  $\{c_t, l_t, d_{t+1}\}$  subject to the budget constraint

$$c_t(s^t) + \sum_{s^{t+1}} q_{t+1}(s^{t+1}) d_{t+1}(s^{t+1}) \leq w_t(s^t) l_t(s^t) + d_t(s^t) + X_t(s^t) - \frac{1-\sigma}{\sigma} \bar{n}$$

and the restrictions that

$$(13) \quad d_{t+1}(s^{t+1}) \geq \bar{d},$$

where  $\bar{d}$  is a large negative number. Here  $d_{t+1}$  is the amount of deposits made by households in banks and  $q_{t+1}$  is the corresponding price. Also,  $w_t$  is the real wage,  $X_t$  are dividends paid by banks, and  $\bar{n}$  is the amount of initial equity given to each newly formed bank of which a measure  $(1-\sigma)/\sigma$  is formed each period.

The first-order conditions for the household's problem can be summarized by

$$(14) \quad -\frac{U_{L_t}(s^t)}{U_{C_t}(s^t)} = w_t(s^t)$$

$$(15) \quad q_{t+1}(s^{t+1}) = \frac{\beta \pi(s^{t+1} | s^t) U_{C_{t+1}}(s^{t+1})}{U_{C_t}(s^t)}.$$

A representative firm rents capital at rate  $R_t$  from banks and hires  $L_t$  units of labor

to maximize profits

$$(16) \quad \max_{K_t, L_t} AF(K_t, L_t) + (1 - \delta)K_t - R_t K_t - w_t L_t.$$

The first-order conditions to this problem imply

$$(17) \quad AF_K(s^t) + 1 - \delta = R_t(s^t) \text{ and } AF_L(s^t) = w_t(s^t).$$

Next consider the banks. At the beginning of each period, an idiosyncratic random variable is realized at each existing bank. With probability  $\sigma$ , the bank will continue in operation until the next period. With probability  $1 - \sigma$ , the bank ceases to exist and, by assumption, pays out all of its accumulated net worth as dividends to the household. Also at the beginning of each period, a measure  $(1 - \sigma)/\sigma$  of new banks is born, each of which is given an exogenously specified amount of initial equity  $\bar{n}$  from households. Since only a fraction  $\sigma$  of these newborn banks survive until the end of the period, the measure of surviving banks is always constant at 1. This device of having banks die is a simple way to ensure that they do not build up enough equity to make the financial constraints that we will next introduce irrelevant.

Turning to the budget constraint of an individual bank, note first that for any non-newborn bank the budget constraint at  $t$  is

$$(18) \quad x_t(s^t) + k_{t+1}(s^t) - \sum_{s^{t+1}} q_{t+1}(s^{t+1}) d_{t+1}(s^{t+1}) \leq R_t(s^t) k_t(s^{t-1}) - d_t(s^t),$$

where  $R_t$  is the rental rate for capital. We will let  $n_t(s^t) = R_t(s^t)k_t(s^{t-1}) - d_t(s^t)$  denote the right side of (18) and will refer to it as the *net worth* of the bank. For a bank that is newly born at  $t$ , the left side of the budget constraint is the same and the right side of (18) is replaced by initial net worth  $\bar{n}$ . Banks face a *collateral* constraint for each  $s^{t+1}$ ,

$$(19) \quad d_{t+1}(s^{t+1}) \leq \gamma R_{t+1}(s^{t+1}) k_{t+1}(s^t),$$



where  $0 < \gamma < 1$ , as well as non-negativity constraints on dividends and bond holdings,

$$(20) \quad x_t(s^t) \geq 0.$$

For notational simplicity only, consider the problem of a bank born in period 0. The bank chooses  $\{k_{t+1}(s^t), d_t(s^t), x_t(s^t)\}$  to solve

$$(21) \quad \max \sum_t \sum_{s^t} Q(s^t) \sigma^t [\sigma x_t(s^t) + (1 - \sigma)n_t(s^t)]$$

subject to (18)–(20) where  $n_t(s^t) = R_t(s^t) k_t(s^{t-1}) - d_t(s^t)$ ,  $n_0(s^0) = \bar{n}$ , and  $Q(s^t)$  is the price of a good at date  $t$  in units of a good at date 0 after history  $s^t$ . We assume that a bank that ceases to operate pays out its accumulated net worth as dividends. Since the bank is owned by the household, it values dividends at the marginal rates of substitution of consumers, so that

$$(22) \quad Q(s^t) = \beta^t \pi(s^t) U_C(s^t) / U_{C0}(s^0).$$

From the household's first-order condition, it follows that the discount factor used by the bank is consistent with the rate of return on deposits in that  $Q(s^t) = q_0(s^0) \cdots q_t(s^t)$ .

The first-order conditions to the bank's problem can be written as

$$Q(s^t) \sigma^{t+1} + \eta_{xt}(s^t) = \lambda_t(s^t)$$

$$\lambda_t(s^t) = \sum_{s^{t+1}} [Q(s^{t+1}) \sigma^{t+1} (1 - \sigma) R_{t+1}(s^{t+1}) + R_{t+1}(s^{t+1}) (\lambda_{t+1}(s^{t+1}) + \gamma \mu_{t+1}(s^{t+1}))]$$

$$(23) \quad -Q(s^{t+1}) \sigma^{t+1} (1 - \sigma) + \lambda_t(s^t) q_{t+1}(s^{t+1}) = \lambda_{t+1}(s^{t+1}) + \mu_{t+1}(s^{t+1}),$$

where  $\lambda_t(s^t)$ ,  $\mu_t(s^t)$ , and  $\eta_{xt}(s^t)$  are the multipliers on the bank budget constraint, the collateral constraint, and the nonnegative dividend constraint. We can manipulate these constraints to obtain

$$(24) \quad 1 = \sum_{s^{t+1}} \left[ R_{t+1}(s^{t+1}) q_{t+1}(s^{t+1}) \left( 1 - (1 - \gamma) \frac{\mu_{t+1}(s^{t+1})}{\lambda_t(s^t) q_{D,t+1}(s^{t+1})} \right) \right].$$

A competitive equilibrium is defined in the standard fashion.

*The Associated Prototype Economy with Investment Wedges*

Consider a version of the benchmark prototype economy that will have the same aggregate allocations as the banking economy just detailed. This prototype economy is identical to our benchmark prototype except that the new prototype economy has an investment wedge that resembles a tax on capital income rather than a tax on investment. Here the government consumption wedge is set equal to zero.

In the prototype economy, the consumer's budget constraint is

$$(25) \quad C_t(s^t) + K_{t+1}(s^t) = (1 - \tau_{Kt}(s^t))R_t(s^t)K_t(s^{t-1}) + (1 - \tau_{Lt}(s^t))w_t(s^t)L_t(s^t) + T_t(s^t).$$

The first-order condition for the investment wedge in this economy is given by

$$(26) \quad U_{Ct}(s^t) = \sum_{s^{t+1}} \beta \mu(s^{t+1}|s^t) U_{C_{t+1}}(s^{t+1}) [AF_{K_{t+1}}(s^{t+1}) + 1 - \delta](1 - \tau_{K_{t+1}}(s^{t+1})).$$

Comparing the first-order conditions in the detailed economy with bank collateral constraints to those of the associated prototype economy leads us to set

$$(27) \quad \tau_{Kt}(s^t) = (1 - \gamma) \frac{\mu_{t+1}(s^{t+1})}{\lambda_t(s^t) q_{Dt+1}(s^{t+1})}.$$

We then have the following proposition.

**PROPOSITION 2:** *The aggregate allocations in the detailed economy with bank collateral constraints coincide with those of the prototype economy if the efficiency wedge in the prototype economy  $A_t(s^t) = A$ , the labor wedge is zero, and the investment wedge is given by (27).*

Clearly, the efficiency wedge here is just the constant level of technology  $A$  in the detailed economy. To see why there is no labor wedge, note that combining (14) and (17) gives that

$$-\frac{U_L(s^t)}{U_C(s^t)} = AF_L(s^t).$$

To derive the expression for the investment wedge, substitute for  $R_{t+1}(s^{t+1})$  from the firm's

first-order condition (17) and for  $q_{t+1}(s^{t+1})$  from the consumer's first-order condition (15) to obtain

$$1 = \sum_{s^{t+1}} \left[ \beta \mu(s^{t+1}|s^t) \frac{U_{C_{t+1}}(s^{t+1})}{U_{C_t}(s^t)} [AF_{K_{t+1}}(s^{t+1}) + 1 - \delta] \left( 1 - (1 - \gamma) \frac{\mu_{t+1}(s^{t+1})}{\lambda_t(s^t) q_{D_{t+1}}(s^{t+1})} \right) \right]$$

and compare (26) to this equation.

### ***An Equivalence Result for an Economy with Heterogeneous Productivity and Collateral Constraints***

We use an example from Buera and Moll (2015) to illustrate how a model with fluctuations in financial frictions, modeled as shocks to a collateral constraint on entrepreneurs, is equivalent to a prototype model with a labor wedge, an investment wedge, and an efficiency wedge. We think of this example as making a point identical to that in Proposition 1 of Chari, Kehoe, and McGrattan (2007) but in a different context. That proposition showed how a detailed model with financial frictions modeled as input-financing frictions is equivalent to a prototype economy with a labor wedge, an investment wedge, and an efficiency wedge.

#### *A Detailed Economy with Heterogeneous Productivity and Collateral Constraints*

We consider an economy with only idiosyncratic shocks and exogenous incomplete markets against these shocks. A unit mass of identical workers, who can neither borrow nor lend, supply labor  $L_t$  at a wage  $w_t$ , to maximize

$$\sum_{t=0}^{\infty} \beta^t [\log(C_{Wt}) - V(L_t)]$$

subject to

$$(28) \quad C_{Wt} = w_t L_t.$$

The economy has a unit mass of entrepreneurs indexed by  $i \in [0, 1]$  and a unit mass of identical households. An entrepreneur of type  $i$  draws an idiosyncratic shock  $z_{it}$ , which is i.i.d. over time and across entrepreneurs and has density  $\psi(z)$ . This entrepreneur has a

technology to produce output of the form  $y_{it} = z_{it}^\alpha k_{it}^\alpha l_{it}^{1-\alpha}$  where  $k_{it}$  and  $l_{it}$  are the amounts of capital invested and labor hired by entrepreneur  $i$ .

The timing is that an entrepreneur's productivity in period  $t + 1$ , namely,  $z_{it+1}$ , is revealed at the end of period  $t$ , before the entrepreneur issues new debt  $d_{t+1}$ . Written in recursive form, an entrepreneur with utility function  $\sum \beta^t \log(c_t)$  solves

$$V_t(k, d, z_{-1}, z) = \max_{c, d', k'} \log c + \beta E [V_{t+1}(k', d', z, z')]$$

subject to a budget constraint

$$c + k' - d' = \Pi(z_{-1}, w, k) + (1 - \delta)k - (1 + r_t)d$$

and a collateral constraint

$$(29) \quad d' \leq \theta_t k' \text{ with } \theta_t \in [0, 1].$$

Note that (29) restricts the amount of leverage  $d'/k'$  to be less than some exogenous amount,  $\theta_t$ . We use the constant returns to scale production function and the multiplicative technology shock to write total profits  $\Pi(z_{-1}, w, k)$  as linear functions of the technology shock and the capital stock so that

$$\Pi(z_{-1}, w, k) = z\pi(w)k = \max_l (zk)^\alpha l^{1-\alpha} - wl,$$

where  $\pi(w) = \alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha}$ .

An equilibrium consists of sequences of prices  $\{r_t, w_t\}$  and quantities such that the allocations solve both the entrepreneur problem and the household problem, and markets clear in that

$$(30) \quad \int d_{it} di = 0 \text{ and } \int l_{it} di = L_t$$

$$C_{Et} + C_{Wt} + X_t = Y_t$$

$$K_{t+1} = X_t + (1 - \delta)K_t,$$

where  $X_t$  denotes aggregate investment. To characterize the equilibrium, we let  $m_{it}$  denote the entrepreneur's cash on hand given by

$$m_{it} \equiv z_{it}\pi_t k_{it} + (1 - \delta)k_{it} - (1 + r_t)d_{it},$$

and we let  $a_{it}$  denote the net worth of the entrepreneur,

$$a_{it} \equiv k_{it} - d_{it}.$$

We can use this notation to rewrite the dynamic programming problem of the entrepreneur as a two-stage budgeting problem: first choose how much net worth  $a'$  to carry over to the next period, and then in the second stage, conditional on  $a'$ , decide how to split this net worth between capital  $k'$  and bonds  $-d'$ . The two-stage problem is then to solve

$$v_t(m, z) = \max_{a'} [\log(m - a') + \beta E v_{t+1}(\tilde{m}_{t+1}(a', z), z')],$$

where

$$\tilde{m}_{t+1}(a', z) = \max_{k', d'} z\pi_{t+1}k' + (1 - \delta)k' - (1 + r_{t+1})d'$$

subject to

$$k' - d' = a',$$

and

$$(31) \quad k' \leq \lambda_t a' \text{ where } \lambda_t = \frac{1}{1 - \theta_t} \in [1, \infty).$$

This formulation immediately implies the following result.

**Lemma 1.** There is a productivity cutoff for being active  $\underline{z}_{t+1}$  defined by  $\underline{z}_{t+1}\pi(w_{t+1}) =$

$r_{t+1} + \delta$ . Given this cutoff, capital and debt holdings are given by

$$(32) \quad k_{it+1} = \left\{ \begin{array}{l} \lambda_t a_{it+1} \text{ for } z_{it+1} \geq \underline{z}_{t+1} \\ 0 \text{ otherwise} \end{array} \right\}, d_{it+1} = \left\{ \begin{array}{l} (\lambda_t - 1)a_{it+1} \text{ for } z_{it+1} \geq \underline{z}_{t+1} \\ -a_{it+1} \text{ otherwise} \end{array} \right\},$$

and entrepreneurs save a constant fraction of cash on hand  $a_{it+1} = \beta m_{it}$ .

Note that the optimal capital choice is always at one of two corners. Sufficiently unproductive entrepreneurs lend out all their net worth for use by other entrepreneurs and receive return  $r_{t+1} + \delta$ , whereas sufficiently productive entrepreneurs borrow the maximal amount allowed by the collateral constraint,  $\lambda_t a_{it+1}$ , and invest these funds in their own projects. The marginal entrepreneur has a productivity that makes the returns from investing in capital,  $\underline{z}_{t+1} \pi_{t+1}$ , just equal to the returns to lending out funds,  $r_{t+1} + \delta$ .

We can use this characterization of decision rules together with market clearing conditions to determine the cutoff  $\underline{z}_{t+1}$  as a function of the parameters of the economy. To do so, we aggregate over entrepreneurs to obtain

$$K_{t+1} = \beta [\alpha Y_t + (1 - \delta)K_t] \quad \text{and} \quad Y_t = A_t K_t^\alpha L_t^{1-\alpha},$$

where

$$(33) \quad A_t = \left( \frac{\int_{\underline{z}_t} z \psi(z) dz}{1 - \Psi(\underline{z}_t)} \right)^\alpha = (E[z|z \geq \underline{z}_t])^\alpha,$$

where  $\underline{z}_t$  is given by the solution to

$$(34) \quad \lambda_{t-1}(1 - \Psi(\underline{z}_t)) = 1.$$

To understand the determination of the cutoff productivity level, we use the results of Lemma 1 to obtain that

$$d_{it+1} = \left\{ \begin{array}{l} (\lambda_t - 1)\beta m_{it} \text{ for } z_{it+1} \geq \underline{z}_{t+1} \\ -\beta m_{it} \text{ otherwise} \end{array} \right\}.$$

Using the observation that  $m_{it}$  chosen before  $z_{it+1}$  is realized and is therefore independent of  $z_{it+1}$ , we can write the market clearing condition for debt given in (30) for period  $t + 1$  as

$$(\lambda_t - 1) \int_{z_{t+1}}^{\infty} \psi(z) dz = \int_0^{z_{t+1}} \psi(z) dz,$$

which, when rearranged, yields (34).

*The Associated Prototype Economy with Efficiency, Labor, and Investment Wedges*

Consider a version of the benchmark prototype economy that will have the same aggregate allocations as the banking economy just detailed. This prototype economy is identical to our benchmark prototype except that the new prototype economy has an investment wedge that resembles a tax on capital income rather than a tax on investment.

This economy can be mapped into our prototype economy with a period utility function of the form  $U(C_t, L_t) = \log C_t - V(L_t)$  as follows. The efficiency wedge is given by (33), the labor wedge is given by

$$(35) \quad \tau_{Lt} = -\frac{C_{Et}}{C_{Wt}},$$

and the investment wedge is defined recursively from

$$(36) \quad \frac{U_{ct}}{U_{ct+1}} \tau_{xt} = \beta(1 - \delta) \tau_{xt+1} + \frac{C_{Wt}}{C_t} \left( \frac{C_{Wt+1}}{C_{Wt}} - \frac{C_{Et+1}}{C_{Et}} \right)$$

with  $\tau_{x0} = 0$ . To derive (35), note that the labor wedge in the prototype economy is given by

$$(37) \quad C_t V'(L_t) = (1 - \tau_{Lt}) F_{Lt}.$$

Next, note that in the detailed economy, the first-order condition for the worker can be manipulated to yield  $L_t V'(L_t) = 1$ . Using this condition along with  $w_t = F_{Lt}$ ,  $C_t = C_{Wt} + C_{Et}$ , and  $C_{Wt} = w_t L_t$  in (37) yields (35). Note that (36) can be obtained by using the result that entrepreneurs save a constant fraction of their wealth.

PROPOSITION 3: *The aggregate allocations in the detailed economy with heterogeneous*

productivity and a collateral constraint coincide with those of the prototype economy if the efficiency wedge in the prototype economy is given by (33), the labor wedge is given by (35), and the investment wedge is given by (36).

### ***An Equivalence Result for an Economy with Efficient Search***

Consider the efficient outcomes from a standard search model. We will show that if we view the outcomes of this model through the lens of a prototype growth model, the prototype model has a labor wedge and an efficiency wedge but no investment wedge.

#### *A Detailed Economy with Efficient Search*

For simplicity, we focus on a version of the model without aggregate uncertainty. The technology is as follows. The population is normalized to 1. In each period, measure  $n_t$  of the population is employed and the rest are unemployed. Of the employed, measure  $v_t$  is used as recruiters and  $n_t - v_t$  are used in producing the single consumption-investment good. The matching technology depends on the measure of recruiters and the measure of unemployed,  $1 - n_t$ . The measure of new matches  $m_t$  created in any period is given by the constant returns to scale function  $G(v_t, 1 - n_t)$ . Existing matches dissolve at an exogenous rate  $\delta_n$  so that the law of motion for the measure of employed is given by

$$(38) \quad n_{t+1} \leq (1 - \delta_n)n_t + m_t$$

and the resource constraint for goods is

$$(39) \quad c_t + k_{t+1} \leq y_t + (1 - \delta)k_t,$$

where  $c_t$  is consumption,  $k_{t+1}$  is the capital stock,  $y_t = A_t F(k_t, n_t - v_t)$ , and  $\delta$  is the depreciation rate. We assume that  $F = k_t^\alpha (n_t - v_t)^{1-\alpha}$ . The utility of the stand-in household is given by

$$(40) \quad \sum \beta^t U(c_t, n_t).$$

The social planner's problem is to choose  $\{c_t, v_t, n_{t+1}, k_{t+1}\}$  to maximize utility subject to



(38) and (39). We summarize the key first-order conditions as

$$(41) \quad U_{ct} = \beta U_{ct+1} \left[ \frac{\alpha y_{t+1}}{k_{t+1}} + 1 - \delta \right],$$

$$(42) \quad U_{ct} \frac{F_{nt}}{G_{1t}} = \beta U_{ct+1} \left\{ \frac{F_{nt+1}}{G_{1t+1}} [(1 - \delta_n) - G_{2t+1}] + F_{nt+1} + \frac{U_{nt+1}}{U_{ct+1}} \right\},$$

and

$$y_t = A_t F(k_t, n_t - v_t).$$

*The Associated Prototype Economy with Efficiency and Labor Wedges*

Consider a prototype economy in which the production function is  $y_t = \hat{A}_t k_t^\alpha n_t^{1-\alpha}$  where

$$(43) \quad \hat{A}_t = A_t \left( \frac{n_t - v_t}{n_t} \right)^\alpha$$

and the resource constraint is the same as in (39). Lagging and manipulating (42) and using

$$A_t F_{nt} = (1 - \alpha) \frac{y_t}{n_t - v_t} = (1 - \alpha) \frac{y_t}{n_t} \frac{n_t}{n_t - v_t},$$

we obtain

$$(44) \quad \frac{U_{nt}}{U_{ct}(1 - \alpha)y_t/n_t} = \left( \frac{n_t}{n_t - v_t} \right) \left[ \frac{U_{ct-1} F_{nt-1}}{\beta U_{ct} A_t F_{nt}} \frac{1}{G_{1t-1}} - \frac{1}{G_{1t} A_t} [(1 - \delta_n) - G_{2t}] - \frac{1}{A_t} \right].$$

Since the labor wedge in the prototype economy is given by the right side of (44), we have the following result.

**PROPOSITION 4:** *The aggregate allocations in the efficient search economy coincide with those of the prototype economy if the efficiency wedge in the prototype economy is given by (43), the labor wedge  $1 - \tau_{lt}$  is given by the right side of (44), and the investment wedge is zero.*

### C. Adjusting the Prototype Economy

So far we have always established equivalence results between a given detailed economy and the prototype one-sector growth model. When using business cycle accounting logic, one can always do that. When the underlying economy is sufficiently different from the one-sector growth model, however, it is often more instructive to adjust the prototype model so that the version of it without wedges is the planning problem for the class of models at hand.

#### *An Equivalence Result for an Economy with Inefficient Search*

Here we illustrate what we mean by considering a version of the search model in which search is inefficient in that the equilibrium of the economy does not solve the planning problem just discussed. One alternative is to keep the prototype model as the one-sector growth model, in which case the wedges will simply be more elaborate versions of those just discussed. Here we illustrate an alternative: we now measure the wedges relative to a distorted version of the social planning problem just studied.

##### *A Detailed Economy with Inefficient Search*

Consider the decentralized equilibrium of a standard search model. The matching technology is as before: the measure of new matches  $m_t$  created in any period is given by the constant returns to scale function  $G(v_t, 1 - n_t)$ . Letting  $\theta_t = v_t/(1 - n_t)$  be the number of recruiters per unemployed worker, each firm that uses the recruiting technology attracts  $\lambda_f(\theta_t) = G(v_t, 1 - n_t)/v_t$  per recruiter to the firm. Thus, a measure of recruiters  $v_t$  attracts  $v_t\lambda_f(\theta_t)$  workers to the firm. The probability that an unemployed worker finds a job is  $\lambda_w(\theta) = G(v_t, 1 - n_t)/(1 - n_t)$ . Note for later that under constant returns to scale,  $\lambda_w(\theta) = \theta\lambda_f(\theta)$ .

Here, as is standard, we imagine that workers are part of a family that has idiosyncratic risk across its members. Since we abstract from aggregate shocks, the law of large numbers implies that the family solves a deterministic problem. As we did earlier, we assume that productivity deterministically varies over time. To keep notation simple, we only index the value function and the prices by time. The problem of a family written in recursive form is

$$V_t(a_t, n_t) = \max_{c, a'} U(c, n) + \beta V_{t+1}(a', n')$$

subject to the household budget constraint and the transition law for employed workers,

$$(45) \quad c + q_{t+1}a' = a + wn,$$

$$(46) \quad n' = (1 - \delta_n)n + \lambda_w(\theta)(1 - n).$$

In (45),  $a'$  is the quantity of goods saved at  $t$  and  $q_{t+1}$  is the price at  $t$  per unit of goods delivered at  $t + 1$ . In (46),  $\delta_n$  is the separation rate of employed workers and  $\lambda_w(\theta)(1 - n)$  is the measure of workers that transit from unemployment to employment.

The first-order condition for  $a'$  is

$$q_{t+1}U_{ct} = \beta V_{at+1},$$

and using the envelope condition for  $a$ , namely,  $V_{at} = U_{ct}$ , gives

$$(47) \quad q_{t+1}U_{ct} = \beta U_{ct+1}.$$

We can use the envelope condition to derive the marginal value to the household of an additional employed worker,

$$(48) \quad V_{nt} = U_{ct}w_t + U_{nt} + \beta [1 - \delta_n - \lambda_w(\theta_t)] V_{nt+1}(a', n'),$$

at the equilibrium wage  $w_t$  where  $n'$  is given from (46). The first term gives the marginal increase in utility from the increased consumption due to having an additional worker earning  $w_t$ . The second term gives the decrease in utility from increased work. The third term is the increase in the present value of utility from entering the next period with an additional worker.

In order to determine the wages in Nash bargaining, it is useful to define the value to the family of having an additional employed worker at an arbitrary current wage  $w$ . This worker will receive the equilibrium wage in all future periods if employed. This value is

$$\tilde{V}_{nt}(a, n, w) = U_c(w - w_t) + V_{nt}(a, n).$$

The problem of the firm with a current stock of employed workers  $n$  and a current stock of capital  $k$  can be written in recursive form as

$$J_t(n, k) = \max_{v, k'} \{z_t F(k, n - v) - [k' - (1 - \delta)k] - w_t n + q_{t+1} J_{t+1}((1 - \delta_n)n + v\lambda_f(\theta_t), k')\},$$

where the transition law from workers employed at this firm is

$$n' = (1 - \delta_n)n + v\lambda_f(\theta_t).$$

Here the flow profits at  $t$  are output,  $z_t F(k, n - v)$ , minus investment,  $[k' - (1 - \delta)k]$ , minus the wage bill,  $w_t n$ . The firm discounts the present value of future profits from  $t + 1$  on by  $q_{t+1}$ . The first-order condition for capital is  $q_{t+1} J_{kt+1}(n', k') = 1$ . Using the envelope condition for  $k$ ,  $J_{kt}(n, k) = z_t F_{kt} + (1 - \delta)$  in this first-order condition gives

$$(49) \quad 1 = q_{t+1} [z_{t+1} F_{kt+1} + (1 - \delta)].$$

The first-order condition for the mass of recruiters to deploy at  $t$  is

$$(50) \quad z_t F_{nt} = \lambda_f(\theta_t) q_{t+1} J_{nt+1}(n', k').$$

Using the envelope condition for  $n$ ,

$$(51) \quad J_{nt}(n, k) = z_t F_n - w_t + [1 - \delta_n] q_{t+1} J_{nt+1}(n', k'),$$

in the first-order condition for recruiters (50) gives

$$(52) \quad J_{nt}(n, k) = z_t F_n - w_t + [1 - \delta_n] \frac{z_t F_n}{\lambda_f(\theta_t)}.$$

The value of having an additional worker employed at an arbitrary wage  $w$  in the current period, who will receive the equilibrium wage in all future periods, is

$$\tilde{J}_{nt}(n, k, w) = w_t - w + J_{nt}(n_t, k_t).$$

Wages are determined according to Nash bargaining with the bargaining parameter  $\phi$  for the worker and  $1 - \phi$  for the firm. The bargained wage  $w$  maximizes the asymmetric Nash product by solving

$$\max_w \phi \log \left[ \tilde{V}_{nt}(a_t, n_t, w) \right] + (1 - \phi) \log \left[ \tilde{J}_{nt}(k_t, n_t, w) \right],$$

where the first term in brackets is the value to the family of having an additional worker employed rather than unemployed at an arbitrary wage  $w$ . The first-order condition is

$$\phi \frac{\tilde{V}_{nwt}}{\tilde{V}_{nt}} + (1 - \phi) \frac{\tilde{J}_{nwt}}{\tilde{J}_{nt}} = 0.$$

Using  $\tilde{V}_{nwt}(a_t, n_t, w) = U_{ct}$  and  $\tilde{J}_{nwt}(k_t, n_t, w) = -1$  and evaluating this first-order condition at equilibrium with  $w = w_t$  so that  $\tilde{V}_{nt} = V_{nt}$  and  $\tilde{J}_{nt} = J_{nt}$  gives

$$(53) \quad \phi \frac{U_{ct}}{V_{nt}} = (1 - \phi) \frac{1}{J_{nt}}.$$

Substituting for  $V_{nt}$  and  $V_{nt+1}$  from (53) into (48) and replacing  $J_{nt}$  with the right side of (52) gives

$$\phi \left[ \left( 1 + \frac{1 - \delta_n}{\lambda_f(\theta_t)} \right) z_t F_{nt} - w_t \right] = (1 - \phi) \left[ w_t + \frac{U_{nt}}{U_{ct}} \right] + \phi [1 - \delta_n - \lambda_w(\theta_t)] q_{t+1} J_{nt+1}.$$

Replacing  $J_{nt+1}$  using the first-order condition for recruiters (50), we can solve for the equilibrium wage,

$$(54) \quad w_t = \phi [1 + \theta_t] z_t F_{nt} + (1 - \phi) \left( -\frac{U_{nt}}{U_{ct}} \right).$$

Here, hiring an unemployed worker produces a marginal value to the firm that includes both the direct value of production and the savings on recruiters' time. The wage is a weighted average of this marginal value and the marginal rate of substitution between consumption and employment for the household. Substituting the wage equation into the recruiter's first-order

condition (50) gives

(55)

$$z_t F_{nt} U_{ct} = \beta U_{ct+1} \lambda_{ft} \left\{ z_{t+1} F_{nt+1} \left[ 1 + \frac{1 - \delta_n}{\lambda_{ft+1}} \right] - \phi [1 + \theta_{t+1}] z_{t+1} F_{nt+1} + (1 - \phi) \frac{U_{nt+1}}{U_{ct+1}} \right\}.$$

The corresponding first-order condition for recruiters for the planner (42) can be manipulated to be

$$(56) \quad z_t F_{nt} U_{ct} = \beta U_{ct+1} G_{1t} \left\{ z_{t+1} F_{nt+1} \left[ \frac{1 - \delta_n}{G_{1t+1}} - \frac{G_{2t+1}}{G_{1t+1}} \right] + z_{t+1} F_{nt+1} + \frac{U_{nt+1}}{U_{ct+1}} \right\}.$$

With a Cobb-Douglas matching function  $G(v, 1 - n) = Bv^{1-\eta}(1 - n)^\eta$ , we have that  $G_{1t} = (1 - \eta)\lambda_{ft}$  and  $G_{2t} = \eta\theta_t\lambda_{ft}$  so that (56) becomes

$$(57) \quad z_t F_{nt} U_{ct} = \beta U_{ct+1} \lambda_{ft} \left\{ z_{t+1} F_{nt+1} \left[ 1 + \frac{1 - \delta_n}{\lambda_{ft+1}} \right] - \eta [1 + \theta_{t+1}] z_{t+1} F_{nt+1} + (1 - \eta) \frac{U_{nt+1}}{U_{ct+1}} \right\}.$$

Clearly, these first-order conditions coincide if the Mortensen–Hosios condition is satisfied in that the worker’s bargaining weight  $\phi$  equals the elasticity of the matching function with respect to unemployment  $\eta$ . We can decentralize the solution to the planning problem as an equilibrium with a wage of

$$(58) \quad w_t^p = \eta [1 + \theta_t] z_t F_{nt} + (1 - \eta) \left( -\frac{U_{nt}}{U_{ct}} \right).$$

Notice that even in an efficient equilibrium, the wage typically equals neither the marginal product of labor  $z_t F_{nt}$  nor the marginal rate of substitution  $-U_{nt}/U_{ct}$ . Furthermore, the marginal rate of substitution is not equal to the marginal product of labor. These considerations suggest a different notion of a wedge relative to that used in the one-sector model. To that end, write the equilibrium wage as  $w_t = (1 - \tau_{lt})w_t^p$  where  $w_t^p$  is the planner’s wage. Hence,

$$(59) \quad 1 - \tau_{lt} = \frac{\phi [1 + \theta_t] z_t F_{nt} + (1 - \phi) \left( -\frac{U_{nt}}{U_{ct}} \right)}{\eta [1 + \theta_t] z_t F_{nt} + (1 - \eta) \left( -\frac{U_{nt}}{U_{ct}} \right)}.$$

Clearly, if the Mortensen–Hosios condition is satisfied, the wedge  $\tau_{lt} = 0$ .

### *The Associated Prototype Economy with Efficiency and Labor Wedges*

Consider the following prototype model. In this model, the bargaining power of the worker is equal to the elasticity  $\eta$ , but workers have to pay a tax  $\tau_{lt}$  on their wages, investment is taxed at rate  $\tau_{xt}$ , and the productivity is given by  $\hat{A}_t$ . Next we compare the aggregate outcomes of the prototype model and the equilibrium search model. From (47) and (49), we immediately have that the Euler equation is undistorted so that the investment wedge  $\tau_{xt} = 0$ . Using the production function  $y_t = A_t F(k_t, n_t - v_t)$ , it is immediate that  $\hat{A}_t = A_t$ . Thus, we have the following proposition.

**PROPOSITION 5:** *The aggregate allocations in the equilibrium search economy coincide with those of the prototype economy if the efficiency wedge in the prototype economy is given by  $\hat{A}_t = A_t$ , the labor wedge  $1 - \tau_{lt}$  is given by (59), and the investment wedge is zero.*

Note that if search is efficient, the labor wedge is zero in the two-sector prototype economy.

## **2. The Accounting Procedure**

Having established our equivalence result, we now describe our accounting procedure at a conceptual level, discuss a Markovian implementation of it, and distinguish our procedure from others.

Our procedure is designed to answer questions of the following kind: How much would output fluctuate if the only wedge that fluctuated is the efficiency wedge and the probability distribution of the efficiency wedge is the same as in the prototype economy? Critically, our procedure ensures that agents’ expectations of how the efficiency wedge will evolve are the same as in the prototype economy. For each experiment, we compare the properties of the resulting equilibria to those of the prototype economy. These comparisons, together with our equivalence results, allow us to identify promising classes of detailed economies.

### **A. The Accounting Procedure at a Conceptual Level**

Recall that the state  $s^t$  is the history of the underlying abstract events  $s_t$ . Suppose for now that the stochastic process  $\pi_t(s^t)$  and the realizations of the state  $s^t$  in some particular episode are known. Recall that the prototype economy has one underlying (vector-valued)

random variable, the state  $s^t$ , which has a probability of  $\pi_t(s^t)$ . All of the other stochastic variables, including the four wedges—the efficiency wedge  $A_t(s^t)$ , the labor wedge  $1 - \tau_{lt}(s^t)$ , the investment wedge  $1/[1 + \tau_{xt}(s^t)]$ , and the government consumption wedge  $g_t(s^t)$ —are simply functions of this random variable. Hence, when the state  $s^t$  is known, so are the wedges.

To evaluate the effects of just the efficiency wedge, for example, we consider an economy, referred to as an *efficiency wedge alone economy*, with the same underlying state  $s^t$  and probability  $\pi_t(s^t)$  and the same function  $A_t(s^t)$  for the efficiency wedge as in the prototype economy, but in which the other three wedges are set to be constant functions of the state, in that  $\tau_{lt}(s^t) = \bar{\tau}_l$ ,  $\tau_{xt}(s^t) = \bar{\tau}_x$ , and  $g_t(s^t) = \bar{g}$ . Note that this construction ensures that the probability distribution of the efficiency wedge in this economy is identical to that in the prototype economy.

We compute the decision rules for the efficiency wedge alone economy, denoted  $y^e(s^t)$ ,  $l^e(s^t)$ , and  $x^e(s^t)$ . For a given initial value  $k_0$ , for any given sequence  $s^t$ , we refer to the resulting values of output, labor, and investment as the *efficiency wedge components* of output, labor, and investment.

In a similar manner, we define the *labor wedge alone economy*, the *investment wedge alone economy*, and the *government consumption wedge alone economy*, as well as economies with a combination of wedges, such as the *efficiency and labor wedge economy*.

## B. A Markovian Implementation

So far we have described our procedure assuming that we know the stochastic process  $\pi_t(s^t)$  and that we can observe the state  $s^t$ . In practice, of course, we need to either specify the stochastic process a priori or use data to estimate it, and we need to uncover the state  $s^t$  from the data. Here we describe a set of assumptions that makes these efforts easy. Then we describe in detail the three steps involved in implementing our procedure.

We assume that the state  $s^t$  follows a Markov process  $\pi(s_t|s_{t-1})$  and that the wedges in period  $t$  can be used to uniquely uncover the event  $s_t$ , in the sense that the mapping from the event  $s_t$  to the wedges  $(A_t, \tau_{lt}, \tau_{xt}, g_t)$  is one to one and onto. Given this assumption, without loss of generality, let the underlying event  $s_t = (s_{At}, s_{lt}, s_{xt}, s_{gt})$ , and let  $A_t(s^t) =$



$s_{At}, \tau_{lt}(s^t) = s_{lt}, \tau_{xt}(s^t) = s_{xt}$ , and  $g_t(s^t) = s_{gt}$ . Note that we have effectively assumed that agents use only past wedges to forecast future wedges and that the wedges in period  $t$  are sufficient statistics for the event in period  $t$ . This assumption is only to make our estimation easier, and it can be relaxed.

In practice, to estimate the stochastic process for the state, we first specify a vector autoregressive AR(1) process for the event  $s_t = (s_{At}, s_{lt}, s_{xt}, s_{gt})$  of the form

$$(60) \quad s_{t+1} = P_0 + P s_t + \varepsilon_{t+1},$$

where the shock  $\varepsilon_t$  is i.i.d. over time and is distributed normally with mean zero and covariance matrix  $V$ . To ensure that our estimate of  $V$  is positive semidefinite, we estimate the lower triangular matrix  $Q$ , where  $V = QQ'$ . The matrix  $Q$  has no structural interpretation. (Attempting to give  $Q$  such a structural interpretation is part of the source of some of the conceptual confusion about our approach. See Christiano and Davis (2006) for one such attempt.)

The first step in our procedure is to use data on  $y_t, l_t, x_t$ , and  $g_t$  from an actual economy to estimate the parameters of the Markov process  $\pi(s_t|s_{t-1})$ . We can do so using a variety of methods, including the maximum likelihood procedure described later.

The second step in our procedure is to uncover the event  $s_t$  by measuring the realized wedges. We measure the government consumption wedge directly from the data as the sum of government consumption and net exports. To obtain the values of the other three wedges, we use the data and the model's decision rules. With  $y_t^d, l_t^d, x_t^d, g_t^d$ , and  $k_0^d$  denoting the data and  $y(s_t, k_t)$ ,  $l(s_t, k_t)$ , and  $x(s_t, k_t)$  denoting the decision rules of the model, the realized wedge series  $s_t^d$  solves

$$(61) \quad y_t^d = y(s_t^d, k_t), \quad l_t^d = l(s_t^d, k_t), \quad \text{and} \quad x_t^d = x(s_t^d, k_t),$$

with  $k_{t+1} = (1 - \delta)k_t + x_t^d$ ,  $k_0 = k_0^d$ , and  $g_t = g_t^d$ . Note that we construct a series for the capital stock using the capital accumulation law (1), data on investment  $x_t$ , and an initial choice of capital stock  $k_0$ . In effect, we solve for the three unknown elements of the vector  $s_t$  using the three equations (3)–(5) and thereby uncover the state. We use the associated

values for the wedges in our experiments.

Note that the four wedges account for all of the movement in output, labor, investment, and government consumption, in that if we feed the four wedges into the three decision rules in (61) and use  $g_t(s_t^d) = s_{gt}$  along with the law of motion for capital, we simply recover the original data.

Note also that, in measuring the realized wedges, the estimated stochastic process plays a role only in measuring the investment wedge. To see that the stochastic process does not play a role in measuring the efficiency and labor wedges, note that these wedges can equivalently be directly calculated from (3) and (4) without computing the equilibrium of the model. In contrast, calculating the investment wedge requires computing the equilibrium of the model because the right side of (5) has expectations over future values of consumption, the capital stock, the wedges, and so on. The equilibrium of the model depends on these expectations and, therefore, on the stochastic process driving the wedges.

The third step in our procedure is to conduct experiments to isolate the marginal effects of the wedges. To do that, we allow a subset of the wedges to fluctuate as they do in the data while the others are set to constants. To evaluate the effects of the efficiency wedge, we compute the decision rules for the efficiency wedge alone economy, denoted  $y^e(s_t, k_t)$ ,  $l^e(s_t, k_t)$ , and  $x^e(s_t, k_t)$ , in which  $A_t(s^t) = s_{At}$ ,  $\tau_{lt}(s^t) = \bar{\tau}_l$ ,  $\tau_{xt}(s^t) = \bar{\tau}_x$ , and  $g_t(s^t) = \bar{g}$ . Starting from  $k_0^d$ , we then use  $s_t^d$ , the decision rules, and the capital accumulation law to compute the realized sequence of output, labor, and investment,  $y_t^e$ ,  $l_t^e$ , and  $x_t^e$ , which we call the *efficiency wedge components* of output, labor, and investment. We compare these components to output, labor, and investment in the data. Other components are computed and compared similarly.

Notice that in this experiment, we computed the decision rules for an economy in which only one wedge fluctuates and the others are set to be constants in all events. The fluctuations in the one wedge are driven by fluctuations in a four-dimensional state  $s_t$ .

By distinguishing the events to which the wedges are indexed from the wedges themselves, we can separate out the direct effect and the forecasting effect of fluctuations in wedges. As a wedge fluctuates, it directly affects either budget constraints or resource constraints. Whenever a wedge is not set to a constant, the fluctuations in the underlying state that lead to the fluctuations in the wedges also affect the forecasts of that wedge as well as

those of other wedges in the future. Our experiments are designed so that when we hold a particular wedge constant, we eliminate the direct effect of that wedge, but we retain the forecasting effect of the underlying state on the future evolution of the wedge. By doing so, we ensure that expectations of the fluctuating wedges are identical to those in the prototype economy.

### C. Distinguishing Our Procedure from Others

Since this way of separating the direct and forecasting effects of wedges is critical to our procedure, here we describe an alternative procedure that might, at first, seem like the intuitive way to proceed but does not answer the question that interests us.

Consider a simple example with just two wedges, an efficiency wedge and a labor wedge, denoted  $W_t = (A_t, \tau_{lt})'$ . Suppose that we used our prototype model to estimate the following vector process for them of the form  $W_{t+1} = PW_t + \varepsilon_{t+1}$  where  $E\varepsilon_t\varepsilon_t' = V$  :

$$(62) \quad \begin{bmatrix} A_{t+1} \\ \tau_{lt+1} \end{bmatrix} = \begin{bmatrix} P_{AA} & P_{Al} \\ P_{lA} & P_{ll} \end{bmatrix} \begin{bmatrix} A_t \\ \tau_{lt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{At+1} \\ \varepsilon_{lt+1} \end{bmatrix},$$

where we have suppressed the constant terms. Suppose also that we have decision rules of the form

$$(63) \quad y_t = y(W_t, k_t), \quad l_t = l(W_t, k_t), \quad \text{and} \quad x_t = x(W_t, k_t)$$

and that we have recovered the realized wedge series  $W_t^d$  along with the realized innovation series  $\varepsilon_{t+1}^d$ .

Now suppose we want to answer the question: How much would output fluctuate under the following three conditions? First, only the efficiency wedge fluctuates. Second, for the event, the realized sequence of the efficiency wedges coincides with that in the data. Third, the probability distribution of the efficiency wedge is the same as in the prototype economy.

A first attempt to answer this question is to simply feed a realized innovation series

$\hat{\varepsilon}_{t+1} = (\varepsilon_{At+1}^d, 0)$  for the event and to simulate the resulting shocks using

$$(64) \quad \begin{bmatrix} \hat{A}_{t+1} \\ \hat{\tau}_{lt+1} \end{bmatrix} = \begin{bmatrix} P_{AA} & P_{Al} \\ P_{lA} & P_{ll} \end{bmatrix} \begin{bmatrix} \hat{A}_t \\ \hat{\tau}_{lt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{At+1}^d \\ 0 \end{bmatrix}.$$

This attempt meets our first condition but does not meet our second condition if  $P$  or  $V$  has nonzero off-diagonal elements, as we show they do in the data. Indeed, with nonzero off-

diagonal elements, this procedure will not even produce a simulated  $\hat{A}_t$  series that agrees with  $A_t^d$ . Moreover, this attempt clearly does not meet our third condition.

For a second attempt, suppose we choose the sequence of innovations so that the first two conditions are met. That is, we choose the sequence  $\{\hat{\varepsilon}_{t+1}\}$  so that, in the event, the realized value of the efficiency wedge coincides with that in the data and the labor wedge is constant at, say, its mean value  $\bar{\tau}_l$ . Specifically, we choose  $\{\hat{\varepsilon}_{t+1}\}$  so that  $(\hat{A}_t, \hat{\tau}_{lt}) = (A_t^d, \bar{\tau}_l)$  in the event. The problem with this procedure is that agents' forecasts about future efficiency wedges are different under this procedure from what they are in the prototype economy. Hence, this procedure meets our first two conditions but not our third. To see why, note that the expected value of  $A_{t+1}$  in this procedure is given from

$$E_t \begin{bmatrix} A_{t+1} \\ \tau_{lt+1} \end{bmatrix} = \begin{bmatrix} P_{AA} & P_{Al} \\ P_{lA} & P_{ll} \end{bmatrix} \begin{bmatrix} A_t^d \\ \bar{\tau}_l \end{bmatrix}$$

so that

$$(65) \quad E_t A_{t+1} = P_{AA} A_t^d + P_{Al} \bar{\tau}_l \text{ and } E_t \tau_{lt+1} = P_{lA} A_t^d + P_{ll} \bar{\tau}_l.$$

The expectation of the underlying state  $s_{t+1}$  in the prototype economy, however, is calculated from

$$(66) \quad E_t \begin{bmatrix} s_{At+1} \\ s_{lt+1} \end{bmatrix} = \begin{bmatrix} P_{AA} & P_{Al} \\ P_{lA} & P_{ll} \end{bmatrix} \begin{bmatrix} s_{At}^d \\ s_{lt}^d \end{bmatrix}$$

to be

$$(67) \quad E_t s_{At+1} = P_{AA} s_{At}^d + P_{Al} s_{lt}^d \quad \text{and} \quad E_t s_{lt+1} = P_{lA} s_{At}^d + P_{ll} s_{lt}^d.$$

Since we have identified  $s_{At+1}$  with  $A_{t+1}$  and  $s_{lt+1}$  with  $\tau_{lt+1}$ , then (67) gives the expectations of the efficiency wedge and the labor wedge in the prototype economy during the event. Clearly, (65) and (67) do not agree when  $P_{Al}$  is not zero, so the procedure does not meet our third condition. Note that in some preliminary notes for Chari, Kehoe, and McGrattan (2007), while we were aware of the flaws in the second attempt, we followed a version of this second attempt as a quick approximation to get an initial set of answers. Christiano and Davis (2006) unfortunately did not realize that even in our NBER working paper version (Chari, Lehoe, McGrattan (2004)), we followed the correct procedure. We view their paper as a valuable exposition of why the second attempt is incorrect and of the flaws that arise when one follows it.

Next we show that our procedure meets our three conditions. In the efficiency wedge alone economy, the first two conditions are clearly met: only the efficiency wedge fluctuates, and in the event the realized efficiency wedge coincides with the measured efficiency wedge in the data. To see that the third, and more subtle, condition is met, note from (60) the probability distribution over  $s_{t+1}$ , and therefore  $A_{t+1}$  is the same in both the prototype economy and the efficiency wedge alone economy.

### 3. Applying the Accounting Procedure

Now we demonstrate how to apply our accounting procedure to the Great Recession and postwar data for the United States and a group of other OECD countries. (In the Appendix, we describe in detail our data sources, parameter choices, computational methods, and estimation procedures.)

#### A. Details of the Application

To apply our accounting procedure, we use functional forms and parameter values that are familiar from the business cycle literature. We assume that the production function has the form  $F(k, l) = k^\alpha l^{1-\alpha}$  and the utility function the form  $U(c, l) = \log c + \psi \log(1 - l)$ .

We choose the capital share  $\alpha$  to be one-third and the time allocation parameter  $\psi = 2.5$ . We choose the depreciation rate  $\delta$ , the discount factor  $\beta$ , and growth rates  $\gamma$  and  $\gamma_n$  so that, on an annualized basis, depreciation is 5%, the rate of time preference 2.5%, and the population growth rate and the growth of technology are country-specific and computed using OECD data. The adjustment cost parameter  $b = \delta + \gamma + \gamma_n$  is pinned down by the previous parameters and varies across countries. For the adjustment cost parameter  $a$ , we follow Bernanke, Gertler, and Gilchrist (1999) in choosing this parameter so that the elasticity,  $\eta$ , of the price of capital with respect to the investment-capital ratio is .25. In this setup, the price of capital  $q = 1/(1 - \phi')$ , so that, evaluated at the steady state,  $\eta = ab$ . Given  $\eta$  and  $b$ , we then set  $a$  accordingly.

Our prototype economy is a closed economy. When confronting the data, we let government consumption in the model correspond to the sum of government consumption and net exports in the data. The rationale for this choice is given in Chari, Kehoe, and McGrattan (2005), where we prove an equivalence result between an open economy model and a closed economy model in which government consumption is treated in this fashion. We then use a standard maximum likelihood procedure to estimate the parameters  $P_0$ ,  $P$ , and  $V$  of the vector AR(1) process for the wedges. In doing so, we use the log-linear decision rules of the prototype economy and data on output, labor, investment, and the sum of government consumption and net exports.

In confronting the theory with the data, we need to decide how to treat consumer durables and sales taxes. At a conceptual level, we think of current expenditures on consumer durables as augmenting the stock of consumer durables, which in turn provides a service flow of consumption to consumers. Based on this idea, we reallocate current expenditures of consumer durables from consumption to investment. We then add the imputed service flow from the stock of consumer durables to consumption and output. This imputed service flow is the rental rate on capital times the stock of durables. We assume that the stock of consumer durables depreciates at the same rate as the stock of physical capital. We also adjust the data to account for sales taxes. We assume that sales taxes are levied solely on consumption. This assumption leads us to subtract sales tax revenues from both consumption and measured output.

At a practical level, it turns out that while the U.S. NIPA accounts have quarterly data on consumer durable expenditures for the 1980:1-2014:4 sample we use, the OECD has more limited data. For some of the countries in our sample, data are only available annually or are missing. For countries for which we only have annual data, we fill in quarterly estimates using maximum likelihood estimates of a state space model. For countries for which we only have quarterly data for a subsample, we regress consumer durables on investment and output and use the coefficients to construct estimates of the missing data. Once we have the quarterly series on consumer durables, we construct estimates of the capital stock using the perpetual inventory method. The service flow of durables is assumed to be 4% of the stock of durables. (For details, see our Appendix.)

We express all variables in per capita form and deflate by the GDP deflator. We then estimate separate sets of parameters for the stochastic process for wedges (60) for each of the OECD countries after removing country-specific trends in output, investment, and government consumption. The other parameters are the same across countries. The stochastic process parameters for the Great Recession are estimated using quarterly data for 1980:1–2014:4. The stochastic process (60) with these values is used by agents in our economy to form their expectations about future wedges. In the Appendix, we give the details of the estimated values of the stochastic processes for each of the countries.

## **B. Findings**

Now we describe the results of applying our procedure to OECD countries for the Great Recession and the 1982 recession. Here we focus primarily on the fluctuations due to the efficiency, labor, and investment wedges.<sup>1</sup>

### ***The Great Recession***

Here we discuss our findings for the 24 OECD countries. The main finding is that in terms of accounting for the downturn, in the United States the labor wedge is by far the most important, in Spain, Ireland, and Iceland the investment wedge is the most important, and

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<sup>1</sup>We alert the reader that the quantitative results for Spain should be treated with caution. In some robustness analysis for Spain, we found that the nonlinear labor wedge computed directly from the consumer’s first-order condition (4) moved substantially more than the labor wedge computed using our log-linearization procedure.

in the rest of the countries, the efficiency wedge is the most important.

**Three Illustrative Recessions** Here we illustrate our findings for one country for which the efficiency wedge, labor wedge, and investment wedge, respectively, is the most important. In reporting our findings, we remove a country-specific trend from output, investment, and the government consumption wedge. Both output and labor are normalized to equal 100 in the base period 2008:1. Here we focus primarily on the fluctuations due to the efficiency, labor, and investment wedges. We discuss the government consumption wedge and its components in our Appendix.

**France: Primarily an Efficiency Wedge Recession** We begin with France. In Figure 1, panel A, we see that from 2008:1 to 2009:3, output fell about 7% while labor fell about 3% and investment fell about 18%. In Figure 1, panel B, we see that the efficiency wedge worsened by about 5%, the labor wedge worsened by about 1%, and the investment wedge worsened by about 5%. In Figure 1C we see that the efficiency wedge accounts for the bulk of the decline in output, namely, about 6% of the 7% decline. Figures 1D-E show that the labor and investment wedges play the most important roles in accounting for the declines in labor and investment.

Overall, these results imply that the Great Recession in France should be thought of as primarily an efficiency wedge recession with some role for the labor and investment wedges in accounting for the decline in hours and investment. This finding implies that models that emphasize fluctuations in the labor wedge in France are less promising than those that emphasize fluctuations in the efficiency and investment wedges.

**United States: Primarily a Labor Wedge Recession** Next consider the United States. In Figure 2, panel A, we see that output and labor both fell about 7% from 2008:1 to 2009:3 while investment fell about 23%. In Figure 2, panel B, we see that the efficiency wedge fell very modestly by only about 1%, while the labor wedge and the investment wedge both worsened dramatically, by about 8% and 9%, respectively. In Figure 2, panels C, D, and E, we see that the labor and investment wedges play the most important role in accounting for the downturn in output and labor, while the investment wedge accounts for the bulk of



the downturn in investment.

Overall, considering the period from 2008 until the end of 2011, these results imply that the Great Recession in the United States should be thought of as primarily a labor wedge recession, with an important secondary role for the investment wedge. This finding implies that the most promising models must yield significant fluctuations in the labor wedge, with some role for the investment wedge. Models that emphasize the efficiency wedge are less promising.<sup>2</sup>

**Ireland: Primarily an Investment Wedge Recession** Finally consider Ireland. In Figure 3, panel A, we see that from 2008:1 to 2009:3, output fell about 13%, labor about 11%, and investment almost 50%. Figure 3, panel B, shows that during this period, the efficiency wedge fell about 5%, the labor wedge worsened by about 10%, and the investment wedge worsened dramatically, that is, by about 20%.

In Figure 3, panels C, D, and E, we see that the investment wedge plays the largest role: it accounts for about half of the fall in output, about four-fifths of the fall in investment, and all of the fall in hours. Overall, these results imply that the Great Recession in Ireland should be thought of as primarily an investment wedge recession.

**Summary Statistics for our OECD Countries** So far we have described the Great Recession in three countries. Here we describe useful summary statistics over the period 2008:1 to 2011:3. One such statistic, referred to as the  $\phi$  statistic, is intended to capture how closely a particular component, say, the output component due to the efficiency wedge, tracks the underlying variable, say, output. For our decomposition of output, we let

$$\phi_i^Y = \frac{1/\sum_t (y_t - y_{it})^2}{\sum_j (1/\sum_t (y_t - y_{jt})^2)},$$

where  $y_{it}$  is the output component due to wedge  $i = (A, \tau_l, \tau_x, g)$ . We compute similar statistics for labor and investment. The  $\phi$  statistic has the desirable feature that it lies in

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<sup>2</sup>In the Appendix we show that if we estimated the stochastic process for the wedges from 1948 to 2015, the contribution of the labor wedge rises and that of the investment wedge falls. A similar change occurs if we decrease the investment adjustment cost parameter.

$[0, 1]$ , sums to one across the four wedges, and when a particular output component tracks output perfectly, in that if  $(y_t - y_{it}) = 0$  for all  $t$ , then  $\phi_i^Y = 1$ , that is, the  $\phi$  statistic for the wedge reaches its maximum value of 1. Note that this statistic is the inverse of the mean-square error for each wedge appropriately scaled so that the sum across wedges adds to one.

Now consider our main finding. In Figure 4, panel A, we display the  $\phi$  statistic for the efficiency wedge and labor wedge components of output. The downward-sloping lines represent combinations for which the sum of the labor wedge and efficiency wedge components is constant at 70% and 90%, respectively. This figure shows that the United States stands out from the other countries in that the labor wedge accounts for a much greater fraction of the movements in output than it does in any other country. Specifically, the labor wedge accounts for about 46% of the movements in output in the United States but no more than 22% in any other country. In all other countries except Iceland, Ireland and Spain, the efficiency wedge accounts for roughly 50% or more of the movements in output. In Table 1, we report the decompositions of output, labor, and investment for all countries. There we see that for Iceland, Ireland, New Zealand, and Spain, the investment wedge accounts for 51%, 48%, 42%, and 82% of the movements in output, respectively. In the other panels of Figure 4, we display the  $\phi$  statistics for the components of labor and investment.

Our main finding is also apparent if we use other ways to measure how important a given wedge is for the movements in output, labor, and investment. When we discussed the three illustrative recessions earlier, we compared simple peak-to-trough measures of output, labor, and investment to the corresponding measures for each of the components. In Tables 2A, 2B, and 2C, we report such measures for all of our countries. A quick perusal of these measures shows that they give the same message as the  $\phi$  statistics do. Consider France, for example. The  $\phi$  statistic indicates that the efficiency wedge accounts for the bulk of the movements in output, namely, about 92% of its decline. The peak-to-trough measure indicates that the efficiency wedge also accounts for the bulk of the peak-to-trough decline, namely about 5.9% of the 6.5% decline, or about 91% of the decline.

### *Comparing the Great Recession with Recessions of the Early 1980s*

The postwar era had essentially two periods during which most developed economies experienced recessions at roughly the same time: the early 1980s and the Great Recession of 2008. Here we compare the recessions of the early 1980s with the Great Recession. For the United States, we use the NBER business cycle dates; for the OECD countries, we use the business cycle dates as estimated by ECRI when available and otherwise use the CEPR Euro Area Business Cycle Dates. We use the stochastic process for wedges estimated over the 1980-2014 period for both episodes. (See the Appendix for details.)

In Figure 5, panel A, we compare the  $\phi$  statistics for the efficiency wedge component of output for the two recessions. This panel shows that for most of the countries, the efficiency wedge in the Great Recession played a more important role than it did during the recessions of the 1980s. In Figure 5, panel B, we compare the  $\phi$  statistics for the labor wedge component of output for the two recessions. This panel shows that in the Great Recession, the labor wedge accounts for over 40% of the fluctuations in output only in the United States, while in the 1982 recession it does so only in Belgium, the United Kingdom, and France. In Figure 5, panel C, we compare the  $\phi$  statistics for the investment wedge component of output for the two recessions. This panel shows that in most of the countries, the investment wedge played a larger role in the recessions of the 1980s than it did during the Great Recession.

In Table 3 we report the  $\phi$  statistics for the 1982 recessions. This table shows that the efficiency wedge played the most important role for ten countries, the labor wedge for three countries, and the investment wedge for seven countries. Together with Table 1, this table broadly reinforces our two main findings for the comparison. First, the labor wedge played an important role for output in the Great Recession only for the United States, and in the 1982 recession it played a dominant role only in Belgium, France, the United Kingdom, and New Zealand. Second, for most countries, in the Great Recession the efficiency wedge played a more important role and the investment wedge played a less important role than they did in the recessions of the 1980s.

In Table 4, panels A, B, and C, we report peak-to-trough results for the 1982 recession. Comparing Table 3 with the panels of Table 4, we see that the peak-to-trough results present the same overall picture as our  $\phi$  statistics do. If we compare the classification of the most

important wedge for each country using  $\phi$  statistics for output to that using the peak-to-trough decline for output, we see that they agree in all but three cases.

### *Summary Statistics for the Entire Period*

In Tables 5A, 5B, and 5C, we present some summary statistics for the entire period 1980:1–2014:3 about the importance of the various wedges in accounting for the movements in output, labor, and investment. In Table 5A, for example, we report the standard deviation of the output component due to each wedge relative to the standard deviation of output during entire period, along with the correlation of each such output component with output. In Tables 5B and 5C, we report similar statistics for labor and its components and for investment and its components.

Using these statistics to infer the importance of various wedges is more subtle than using the  $\phi$  statistics. The  $\phi$  statistic captures in one statistic how much the component due to a wedge moves, as well as how closely this component tracks the underlying variable. Instead, to evaluate the importance of a wedge using the statistics in this table, we need to jointly consider the relative standard deviations and the correlations.

Consider France, for example. Viewing the relative standard deviations alone suggests that the labor and investment wedges play roughly the same role in accounting for the movement in output. Indeed, the relative standard deviations of the labor and investment components of output are 93% and 92%, respectively. But the correlations of these variables with output suggests that the investment wedge plays a much more important role. Indeed, the labor component of output comoves negatively with output, whereas the investment component of output comoves positively with output.

With this perspective in mind, the averages across countries show that the efficiency wedge plays the most important role in accounting for output. The standard deviation of the efficiency component of output is 92% of output, and its correlation with output is 0.77. Even though the labor component of output is 89% as variable as output itself, it is essentially uncorrelated with output. In this sense, the labor wedge does not account for much of the movements in output.

### *The Importance of the Classification of Consumer Durables*

Macroeconomists have long argued that theory implies it is appropriate to treat the expenditures on consumer durables as a form of investment that yields a flow of consumption services. This treatment requires adjustments to the national income accounts classification of consumption and investment to make them consistent with the theory.

Here we show that while this adjustment is quantitatively important for some countries, for most countries it does not change the overall findings. In Figure 6, panel A, we contrast the  $\phi$  statistic for the efficiency wedge component of output when this consistent adjustment is made and when it is not. Clearly, the countries with statistics most affected by this adjustment are Iceland and Spain. In Iceland, for example, the contribution of the efficiency wedge falls from 26% when durables are correctly accounted for to 12% when they are not. In Spain, the contribution of the efficiency wedge increases from 11% when durables are correctly accounted for to 29% when they are not.

In Figure 6, panels B and C, we contrast the analogous  $\phi$  statistics for the labor wedge component of output and for the investment wedge component of output. In panel C we see that in Iceland and Spain, the contribution of the investment wedge to output is 51% and 82% when durables are correctly accounted for and 65% and 35% when they are not.

### *Comparing our Procedure with a Perfect Foresight Procedure*

Some authors implement a perfect foresight version of our procedure in which agents have perfect foresight about the future evolution of the wedges. The equilibrium conditions for the deterministic version of our prototype model are

$$(68) \quad c_t + x_t + g_t = y_t,$$

$$(69) \quad y_t = A_t F(k_t, (1 + \gamma)^t l_t),$$

$$(70) \quad -\frac{U_{lt}}{U_{ct}} = [1 - \tau_{lt}] A_t (1 + \gamma)^t F_{lt}, \text{ and}$$

$$(71) \quad U_{ct}[1 + \tau_{xt}] = \beta U_{ct+1} \{A_{t+1} F_{kt+1} + (1 - \delta)[1 + \tau_{xt+1}]\}.$$

Clearly, the efficiency wedge, the labor wedge, and the government consumption wedge can be recovered from the static relationships in (68), (69), and (70). Recovering the investment wedge, however, requires solving the difference equation implied by the Euler equation (71). To do so we need to impose either an initial condition or a terminal condition. In practice, we imposed an initial condition that the investment wedge begins at zero.

In Figure 7, panels A, B, and C, we plot the  $\phi$  statistics for the perfect foresight procedure against the same statistics for our procedure. These panels show that for a significant number of the countries, the  $\phi$  statistics are very different. In particular, the perfect foresight procedure greatly exaggerates the importance of the labor wedge for the United States and Spain. Under perfect foresight, the labor wedge accounts for 92% and 72% of the movements in output for the United States and Spain, while under the standard business cycle accounting procedure, the labor wedge accounts for only 46% and 5%, respectively.

We highlight two important sources for these differences. One is that in the perfect foresight procedure, private agents anticipate the evolution of future wedges perfectly and thus react in the current period to actual future worsening or improvement of the wedges. In this sense, the perfect foresight procedure brings with it all the undesirable properties of the simple “news” models by which an anticipated worsening of, say, the labor wedge leads to a current boom as households choose to increase labor supply before times worsen. The other is that, as we noted earlier, the perfect foresight procedure uses the nonlinear version of the first-order conditions (68)-(71) to compute the wedges, whereas our procedure uses log-linearized versions of these conditions.

## 4. Conclusion

We have elaborated on the business cycle accounting method proposed by CKM, cleared up some misconceptions about the method, and applied it to compare the Great Recession across OECD countries as well as to the recessions of the 1980s in these countries.

We documented four findings. First, with the notable exception of the United States, Spain, Ireland, and Iceland, the Great Recession was driven primarily by the efficiency wedge. Second, in the Great Recession, the labor wedge plays a dominant role only in the United States, and the investment wedge plays a dominant role in Spain, Ireland and Iceland. Third,

in the recessions of the 1980s, the labor wedge played a dominant role only in France, the United Kingdom, Belgium, and New Zealand. Finally, overall in the Great Recession the efficiency wedge played a much more important role and the investment wedge played a less important role than they did in the recessions of the 1980s.

## 5. Appendix

### A. Data and Sources

The data used for the business cycle accounting exercises throughout the paper come mainly from the OECD (variable codes in parentheses). The time span is from 1980 to the end of 2014 and, unless mentioned otherwise, at the quarterly frequency. For some countries (such as Germany, Ireland, Israel, and Mexico), data for most series were only available starting later than 1980Q, and thus the business cycle accounting exercises were performed for shorter samples. We obtained data from Economic Outlook 98 for the following variables and have indicated the mnemonic for each variable in parentheses: Gross domestic product, value, market prices (GDP), GDP deflator, market prices (PGDP), Gross capital formation, current prices (ITISK), Government final consumption expenditures, value, expenditure approach (CG), Exports of goods and services, value, national accounts basis (XGS), Imports of goods and services, value, national accounts basis (MGS), Hours worked per employee, total economy (HRS), Total employment (ET). For durable goods, we obtained data from the System of Quarterly National Accounts. These data are a subcategory of CQRSA: private final consumption expenditure by durability, national currency, current prices. For taxes on goods and services, we used tax on goods and services as a share of GDP, annual (TAXGOODSERV, PCGDP). For Population and Labor Force, we used Population 15-64, persons, annual.

All data are deflated by the GDP deflator. Data on durables are available for different time spans and frequency. When data were available at a quarterly frequency, the series of durables were computed by regressing durables on a constant, Gross Capital Formation (ITISK) and Gross Domestic Product (GDP) in logs, for the available time span, and then using the coefficient estimates to compute the series for durables from the beginning of the sample. When data on durables were only available at an annual frequency, quarterly observations were estimated using maximum likelihood estimates of a state space model and, as before, series on gross capital formation and gross domestic product. Once we get durables at

the quarterly frequency, we extend the series to the beginning of the sample by the method described earlier. Population data are available at an annual frequency and are interpolated to quarterly frequency using cubic splines. All other transformations are standard and constructed as follows: per capita output ( $y$ ) is given by real GDP – sales taxes + services from consumer durables (with return = 4%) + depreciation of durables (at an annualized rate of 25%), deflated by the GDP deflator and divided by population 16-64; per capita hours ( $h$ ): hours worked\*total employment, divided by population 16-64; per capita investment ( $x$ ): gross capital formation + personal consumption expenditures on durables net of sales taxes, all deflated by the GDP deflator and divided by population 16-64; per capita government consumption ( $g$ ): government final consumption expenditures + Exports of goods and services – Imports of goods and services, all deflated by the GDP deflator and divided by population 16-64.

## B. Parameterization and Calibration

The period utility function is

$$\log c + \psi \log(1 - l).$$

The parameters held fixed across countries are as follows: the annualized discount factor  $\beta = 0.975$ , the annualized depreciation rate  $\delta = 0.05$ ,  $\psi = 2.5$ , and the capital share  $\theta = 0.33$ .

Other parameters are specific to each country and shown in the following table, where  $\gamma_n$  is the average growth rate of population,  $\gamma$  the growth rate of labor-augmenting technology, and  $a$  the adjustment costs coefficient. To compute  $\gamma$ , we set it so that detrended log output is mean zero over the sample period.

The other parameters that are country specific are the stochastic processes for the wedges, which are obtained from the maximum likelihood estimation procedure. These estimates are available online in a longer appendix, which includes all tables and figures in the paper and auxiliary reports for each country with additional tables and figures. (The website is <http://pedrobrinca.pt/2016-accounting-for-business-cycles>.) Replication files are also available at this website.



Table 1: Parameters Specific to Each Country

Country	$\gamma_n$	$\gamma$	$a$
Australia	0.014	0.021	11.646
Austria	0.005	0.022	12.901
Belgium	0.003	0.020	13.427
Canada	0.011	0.017	12.694
Denmark	0.003	0.020	13.639
Finland	0.002	0.030	12.058
France	0.005	0.018	13.672
Germany	-0.001	0.019	14.451
Iceland	0.012	0.024	11.516
Ireland	0.014	0.046	9.066
Israel	0.020	0.021	10.872
Italy	0.002	0.015	14.715
Japan	-0.001	0.021	14.121
Korea	0.013	0.051	8.822
Luxembourg	0.013	0.036	10.060
Mexico	0.018	0.007	13.141
New Zealand	0.012	0.016	12.696
Netherlands	0.005	0.024	12.597
Norway	0.008	0.023	12.151
Spain	0.007	0.022	12.394
Sweden	0.004	0.022	13.060
Switzerland	0.009	0.014	13.666
United Kingdom	0.003	0.025	12.824
USA	0.010	0.019	12.551

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FIGURE 1A  
OUTPUT, LABOR, AND INVESTMENT FOR FRANCE, 2008:1-2014:4

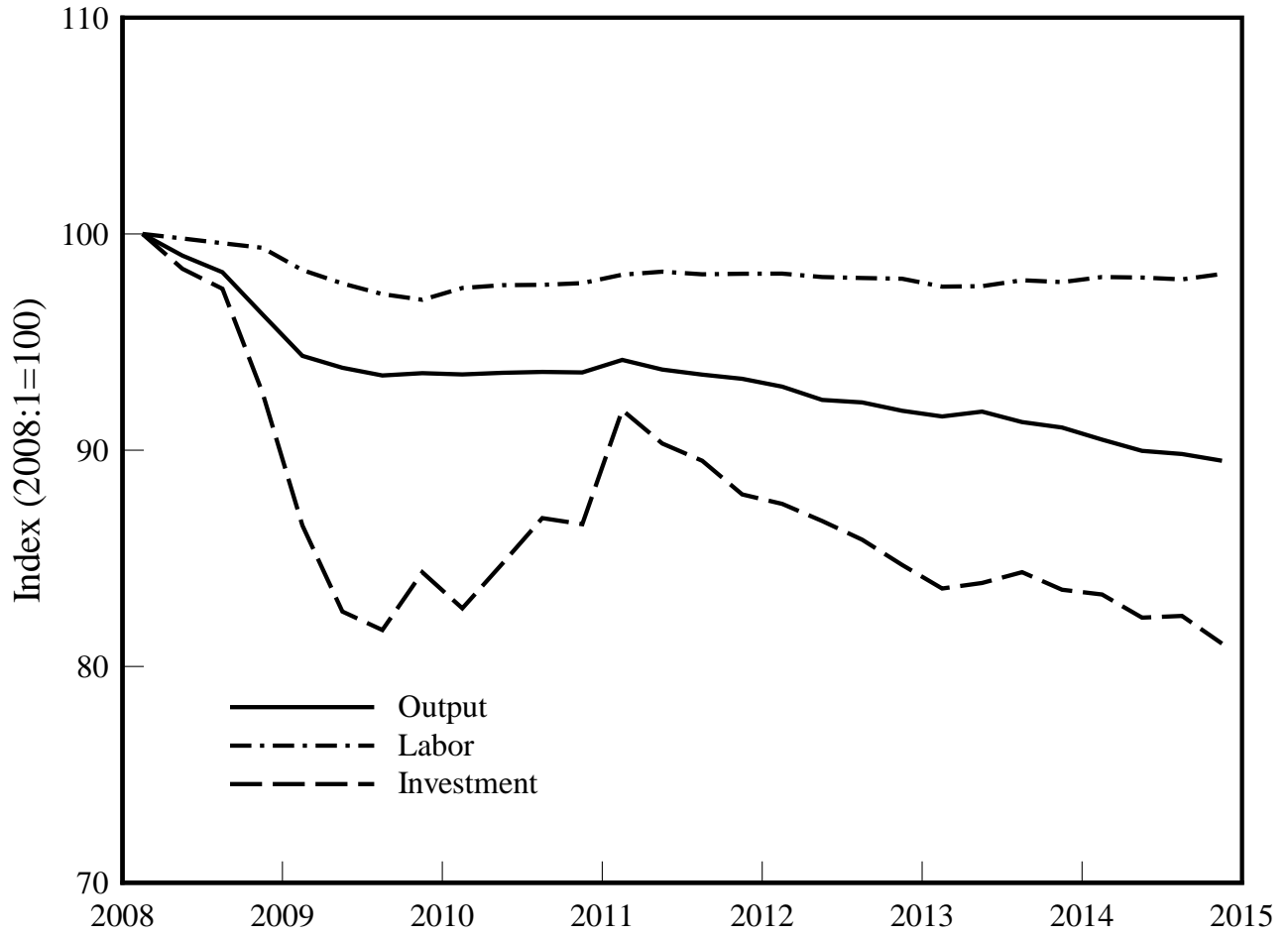


FIGURE 1B  
OUTPUT AND THREE WEDGES FOR FRANCE, 2008:1-2014:4

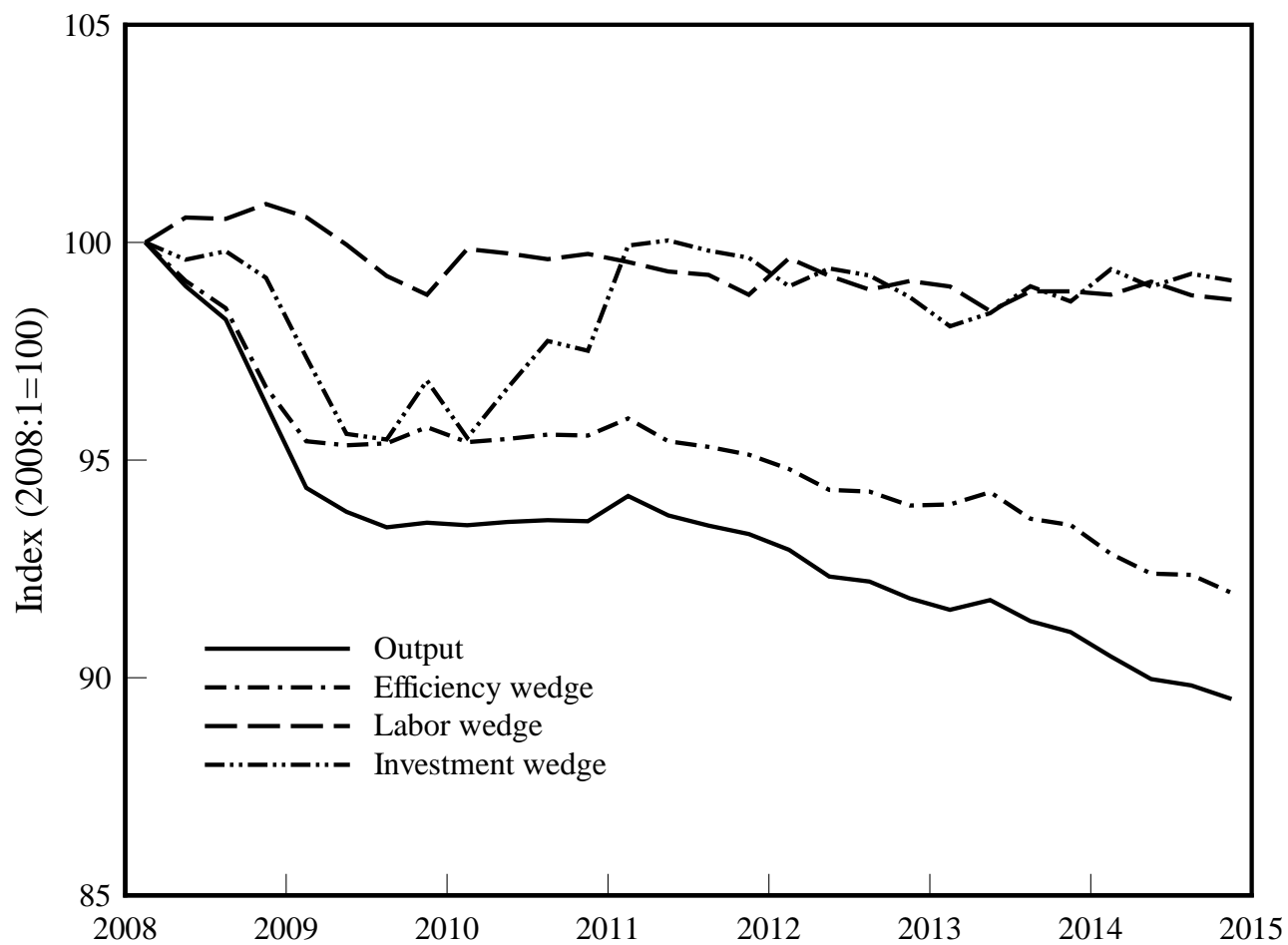


FIGURE 1C  
OUTPUT AND OUTPUT COMPONENTS FOR FRANCE, 2008:1-2014:4

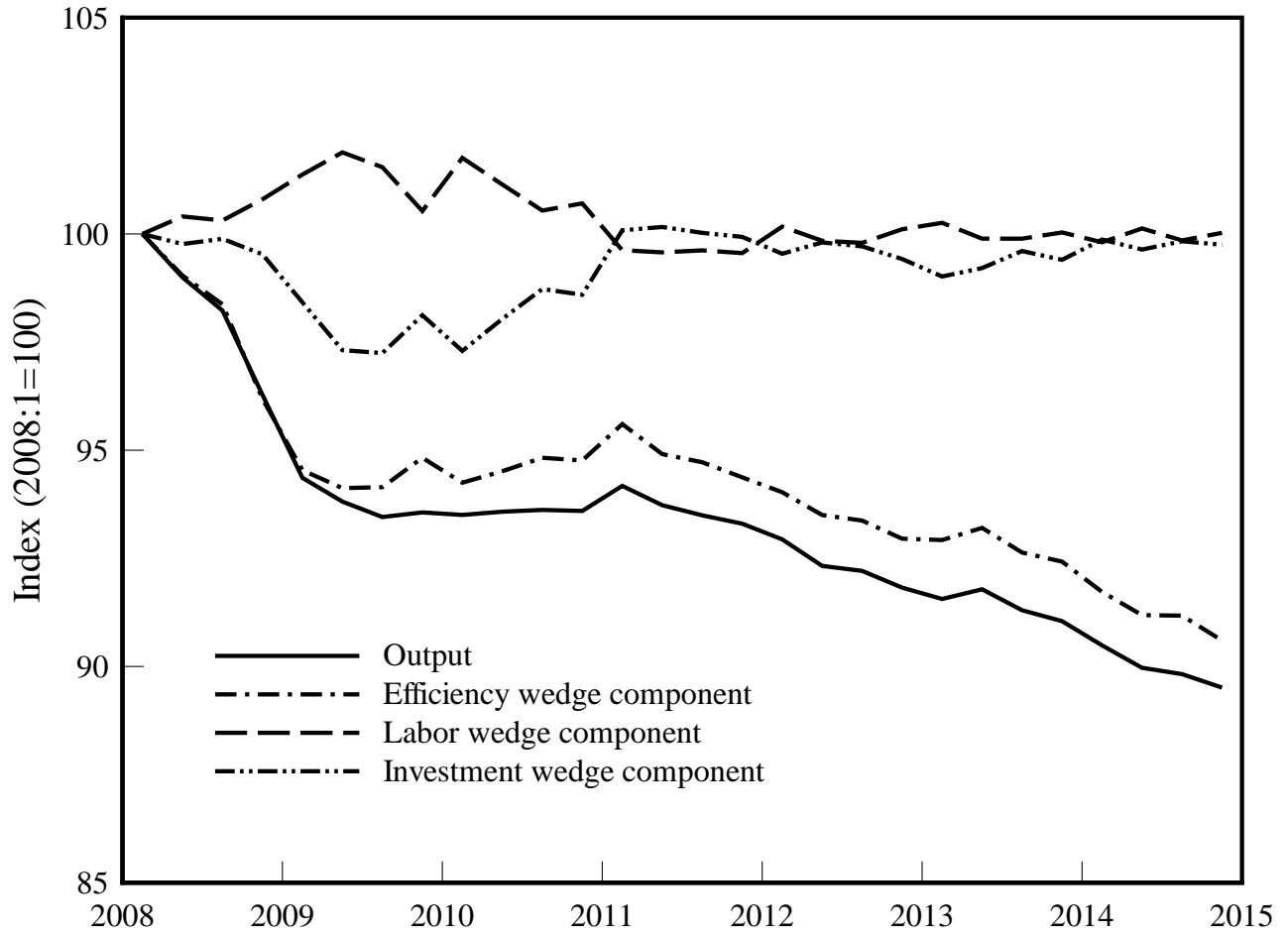


FIGURE 1D  
LABOR AND LABOR COMPONENTS FOR FRANCE, 2008:1-2014:4

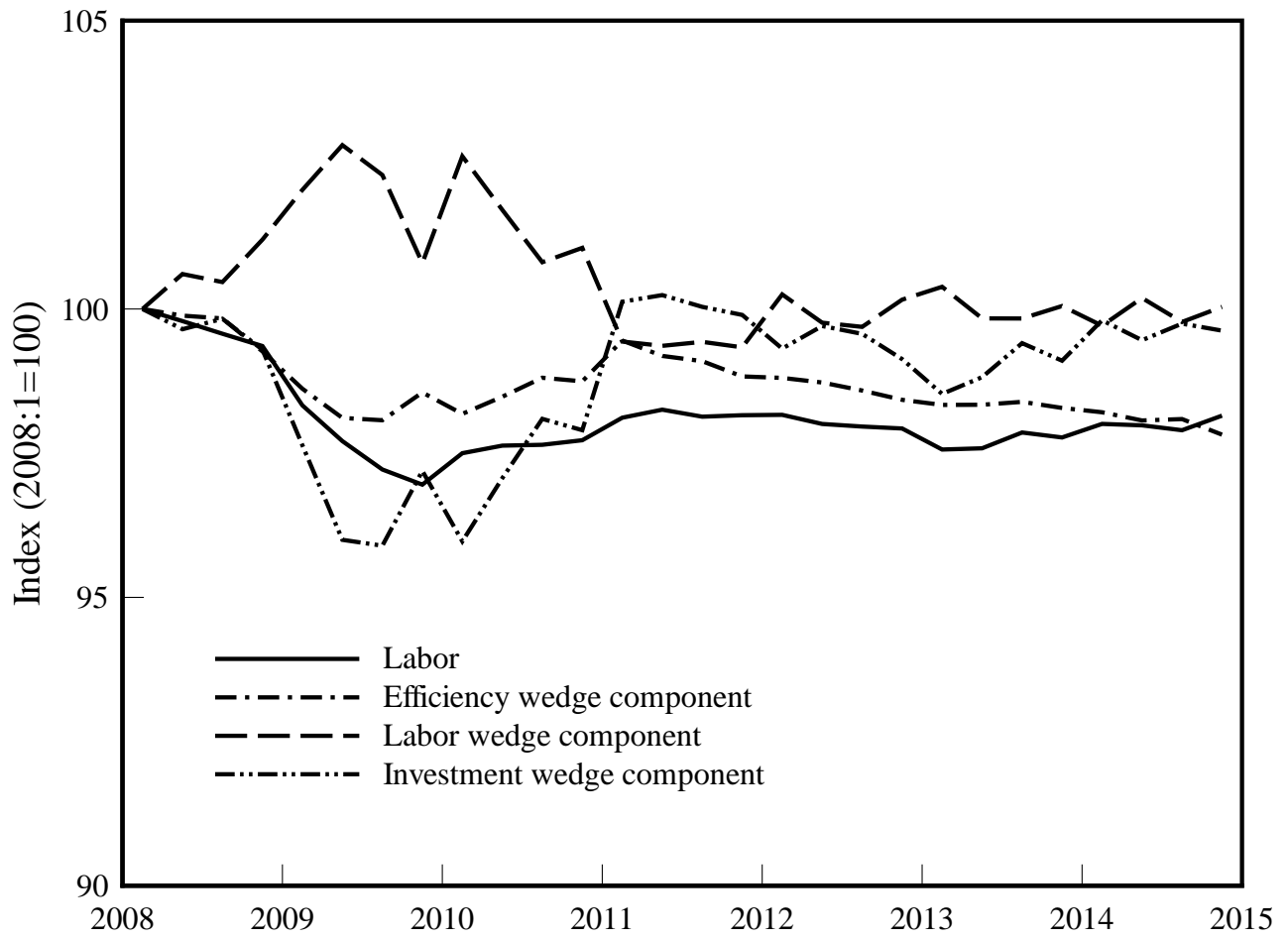


FIGURE 1E  
 INVESTMENT AND INVESTMENT COMPONENTS FOR FRANCE, 2008:1-2014:4

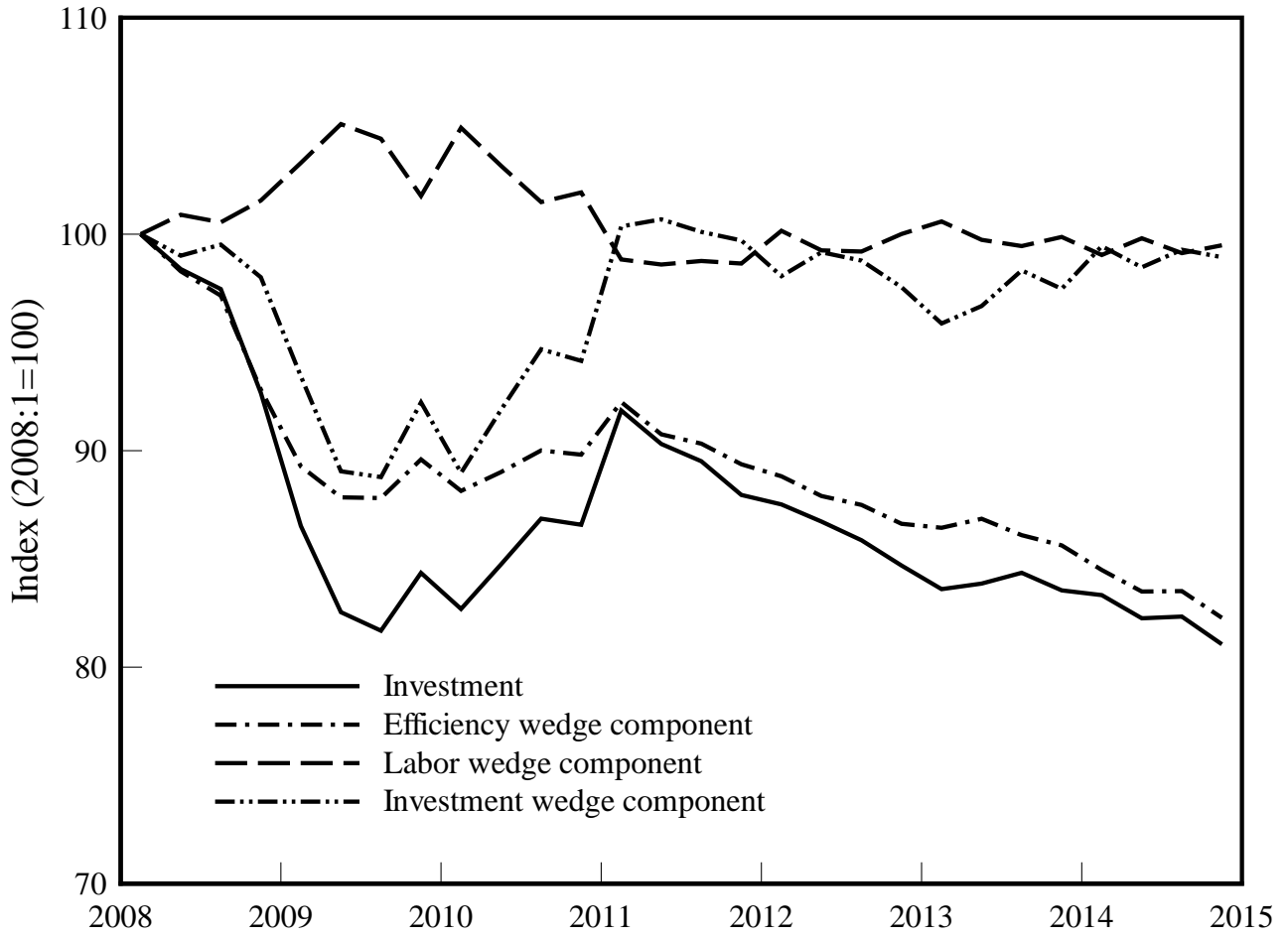




FIGURE 2A  
OUTPUT, LABOR, AND INVESTMENT FOR THE UNITED STATES, 2008:1-2014:4

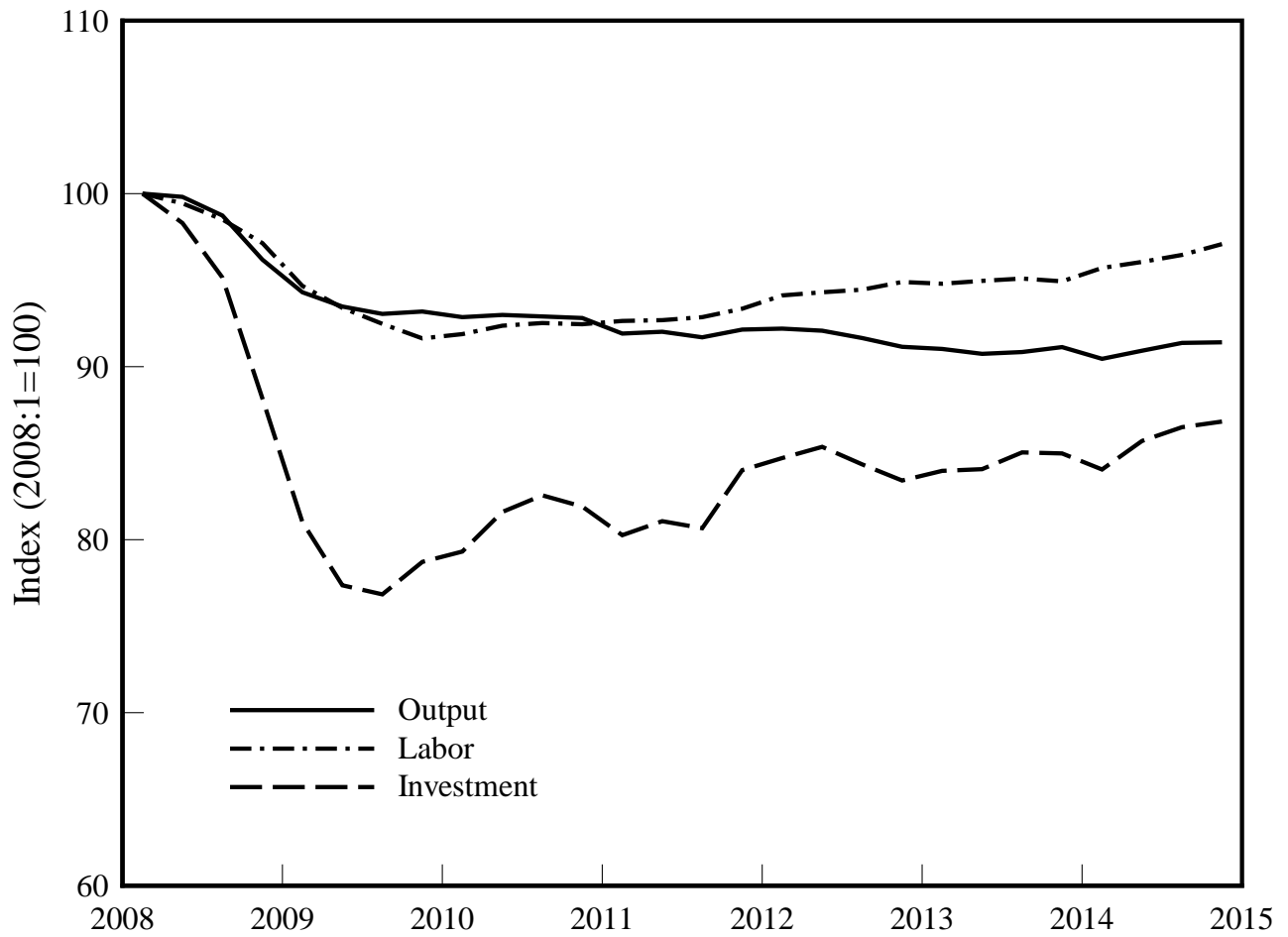


FIGURE 2B  
OUTPUT AND THREE WEDGES FOR THE UNITED STATES, 2008:1-2014:4

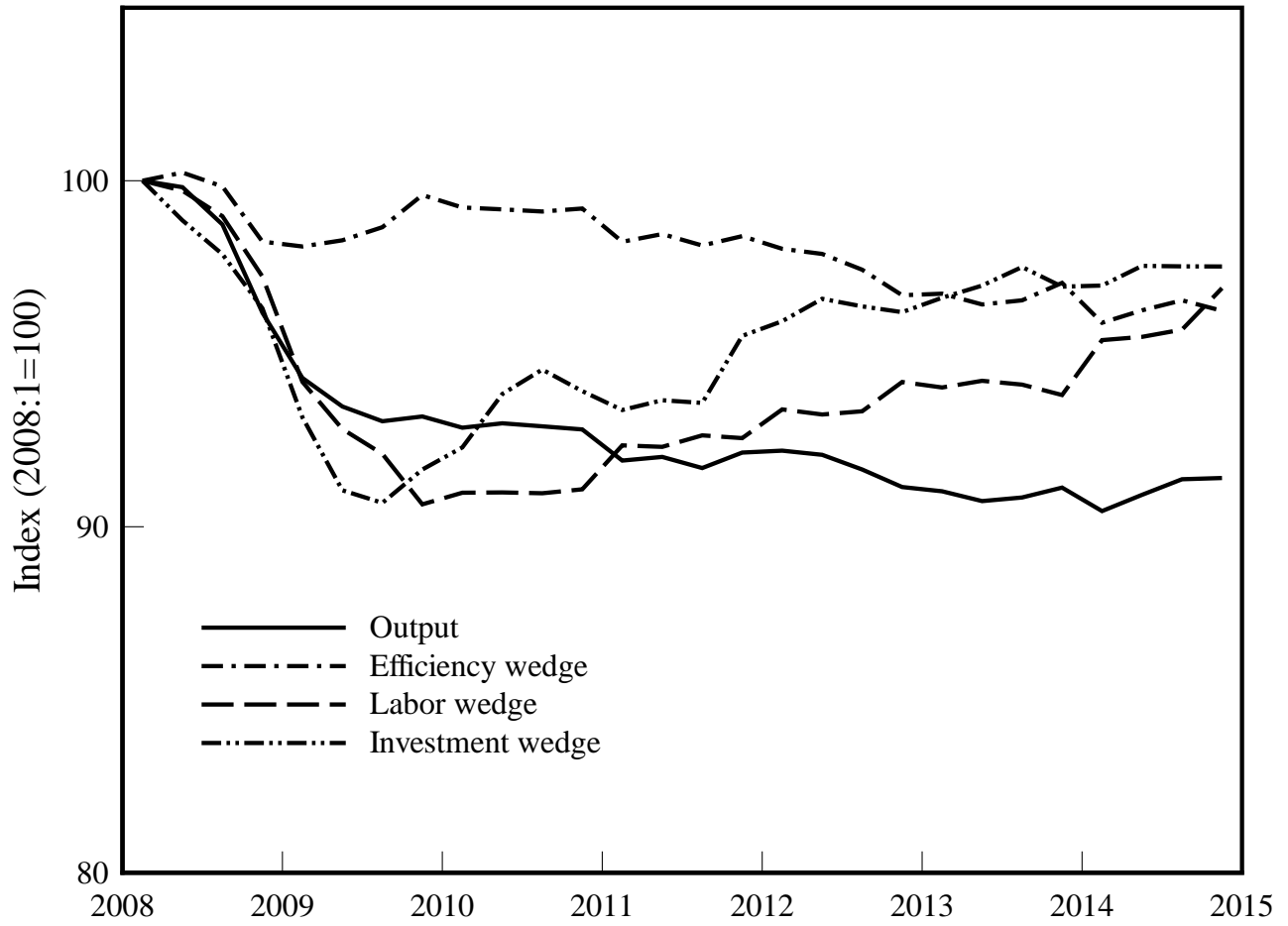


FIGURE 2C

OUTPUT AND OUTPUT COMPONENTS FOR THE UNITED STATES, 2008:1-2014:4

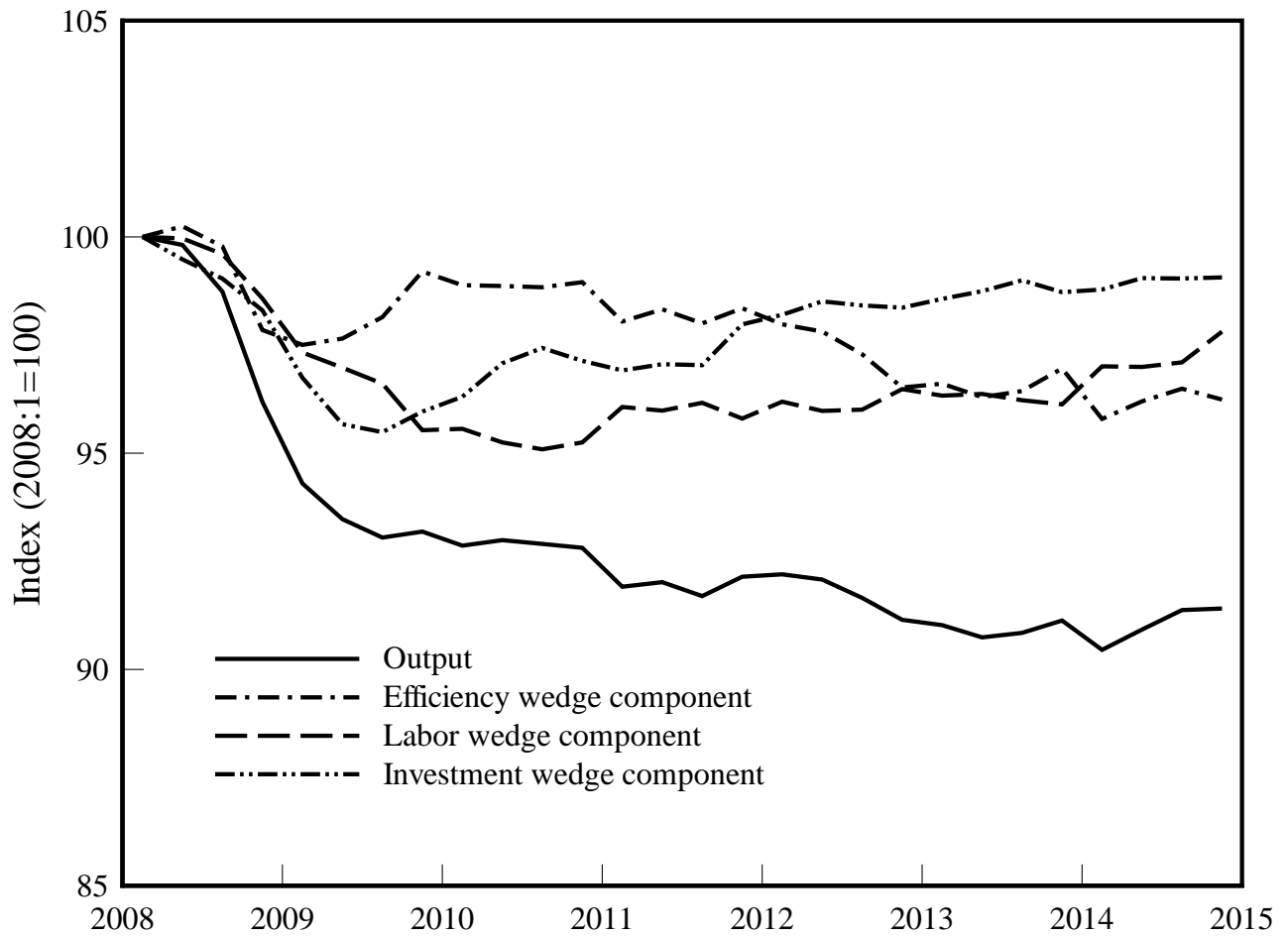


FIGURE 2D  
LABOR AND LABOR COMPONENTS FOR THE UNITED STATES, 2008:1-2014:4

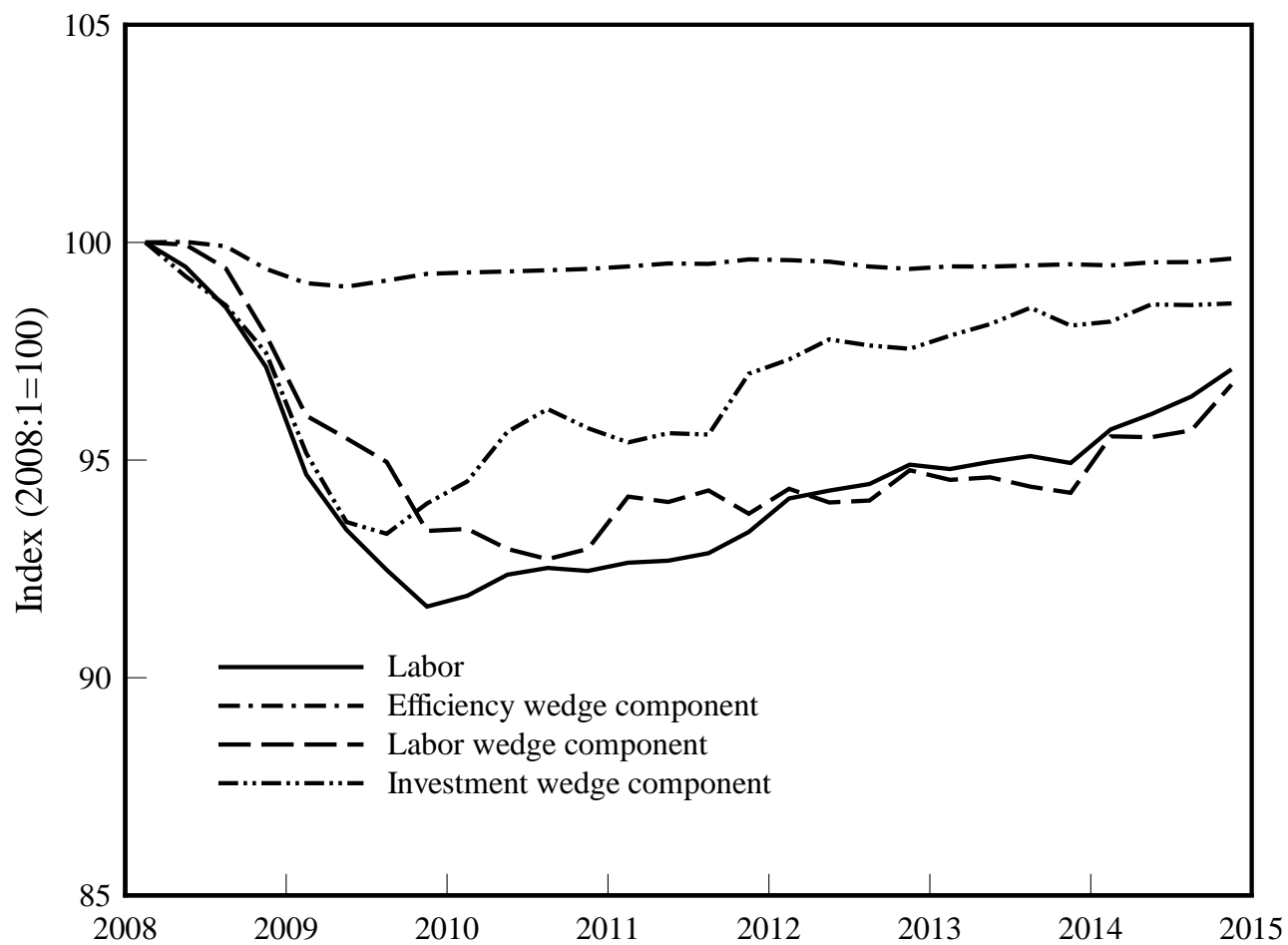


FIGURE 2E

INVESTMENT AND INVESTMENT COMPONENTS FOR THE UNITED STATES, 2008:1-2014:4

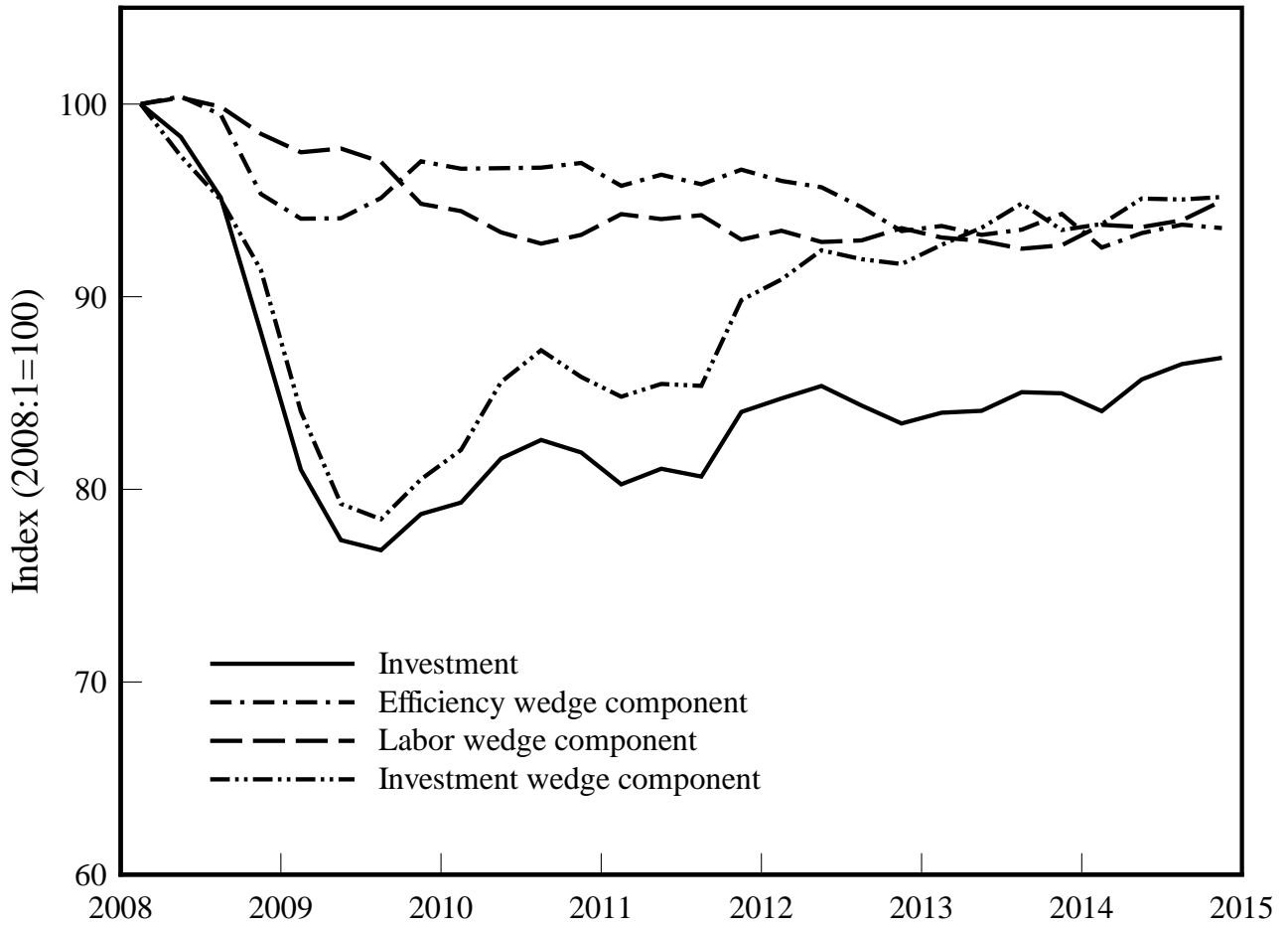


FIGURE 3A  
OUTPUT, LABOR, AND INVESTMENT FOR IRELAND, 2008:1-2014:4

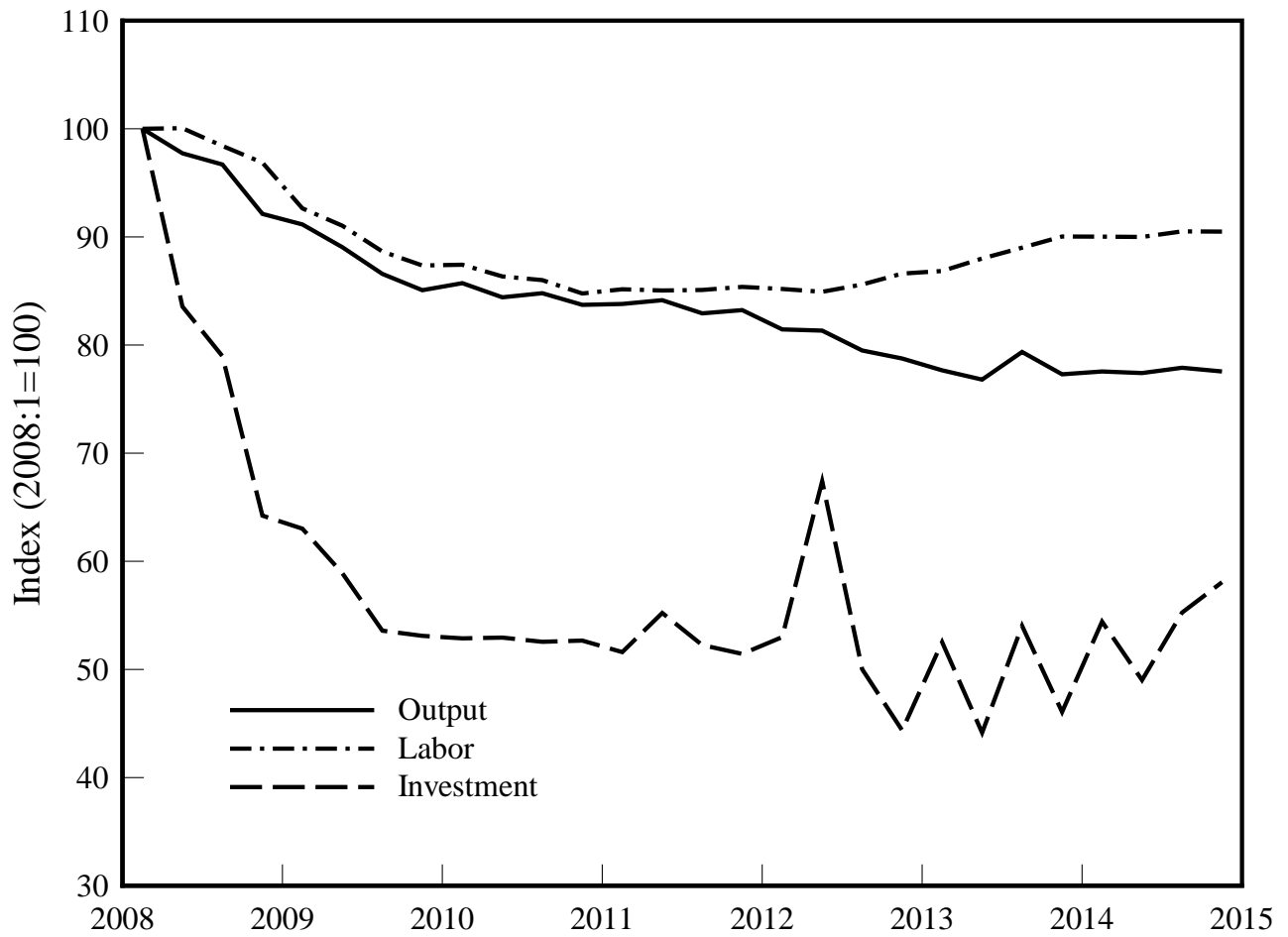


FIGURE 3B  
OUTPUT AND THREE WEDGES FOR IRELAND, 2008:1-2014:4

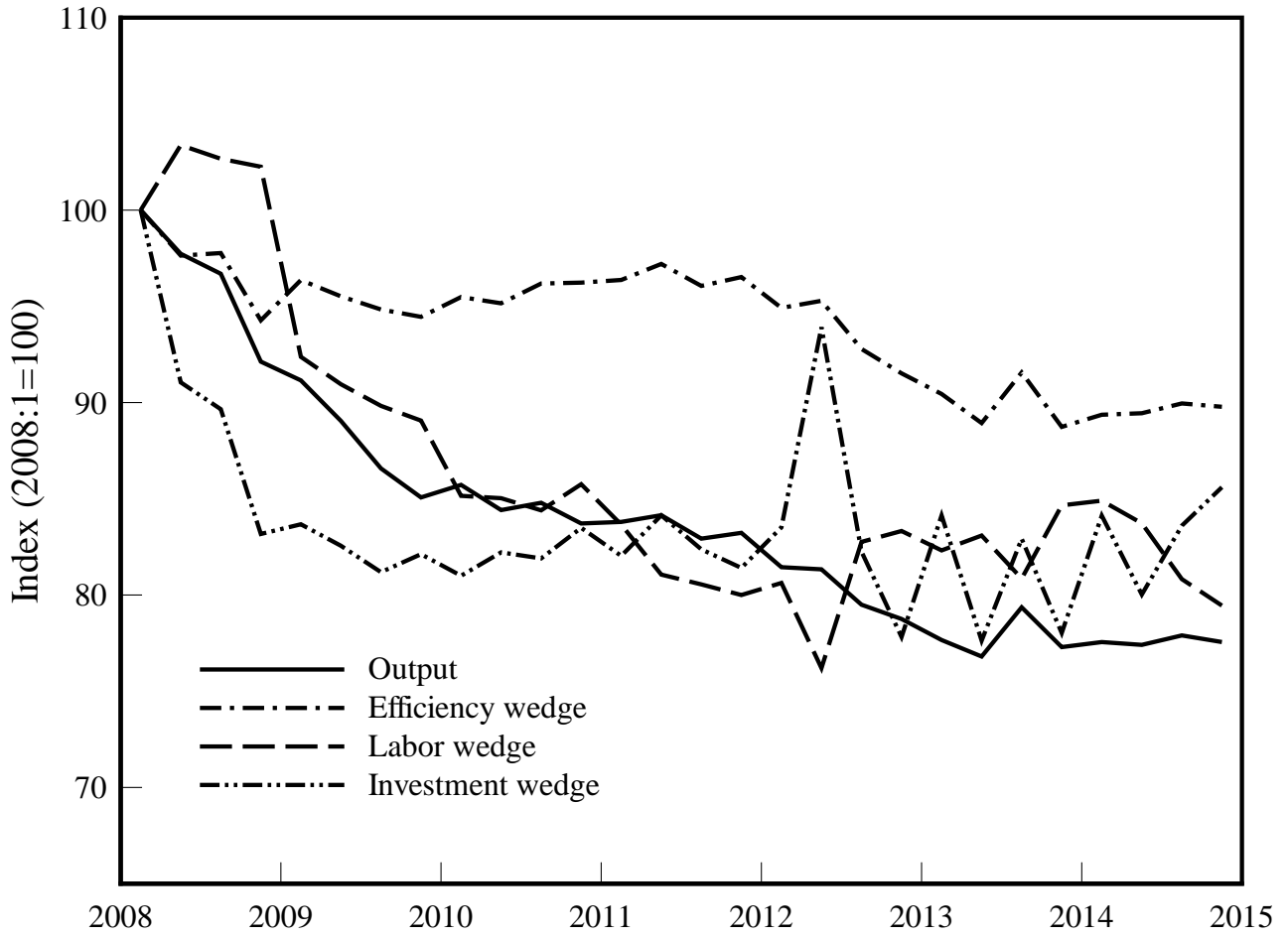


FIGURE 3C  
OUTPUT AND OUTPUT COMPONENTS FOR IRELAND, 2008:1-2014:4

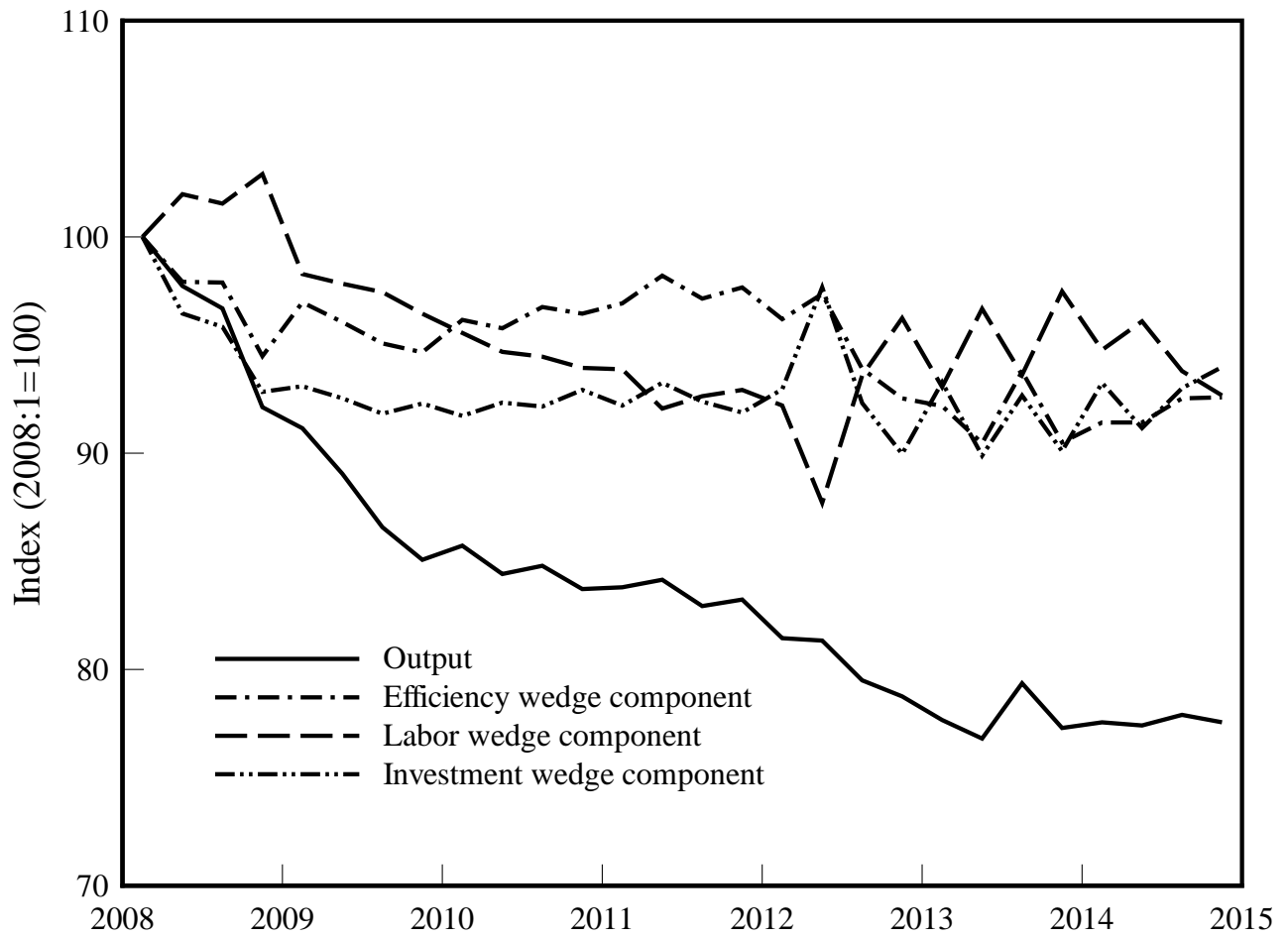




FIGURE 3D  
 LABOR AND LABOR COMPONENTS FOR IRELAND, 2008:1-2014:4

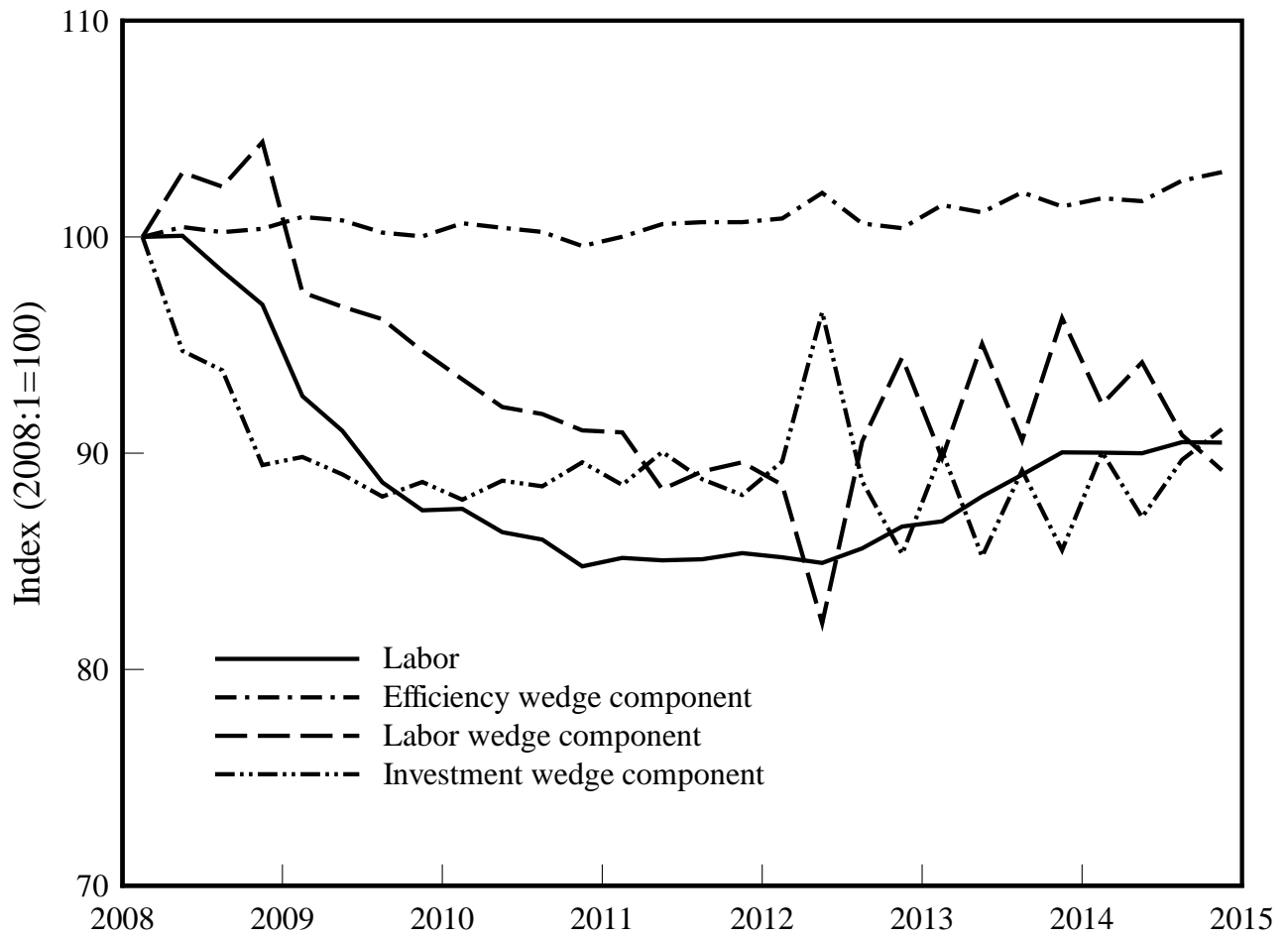


FIGURE 3E

INVESTMENT AND INVESTMENT COMPONENTS FOR IRELAND, 2008:1-2014:4

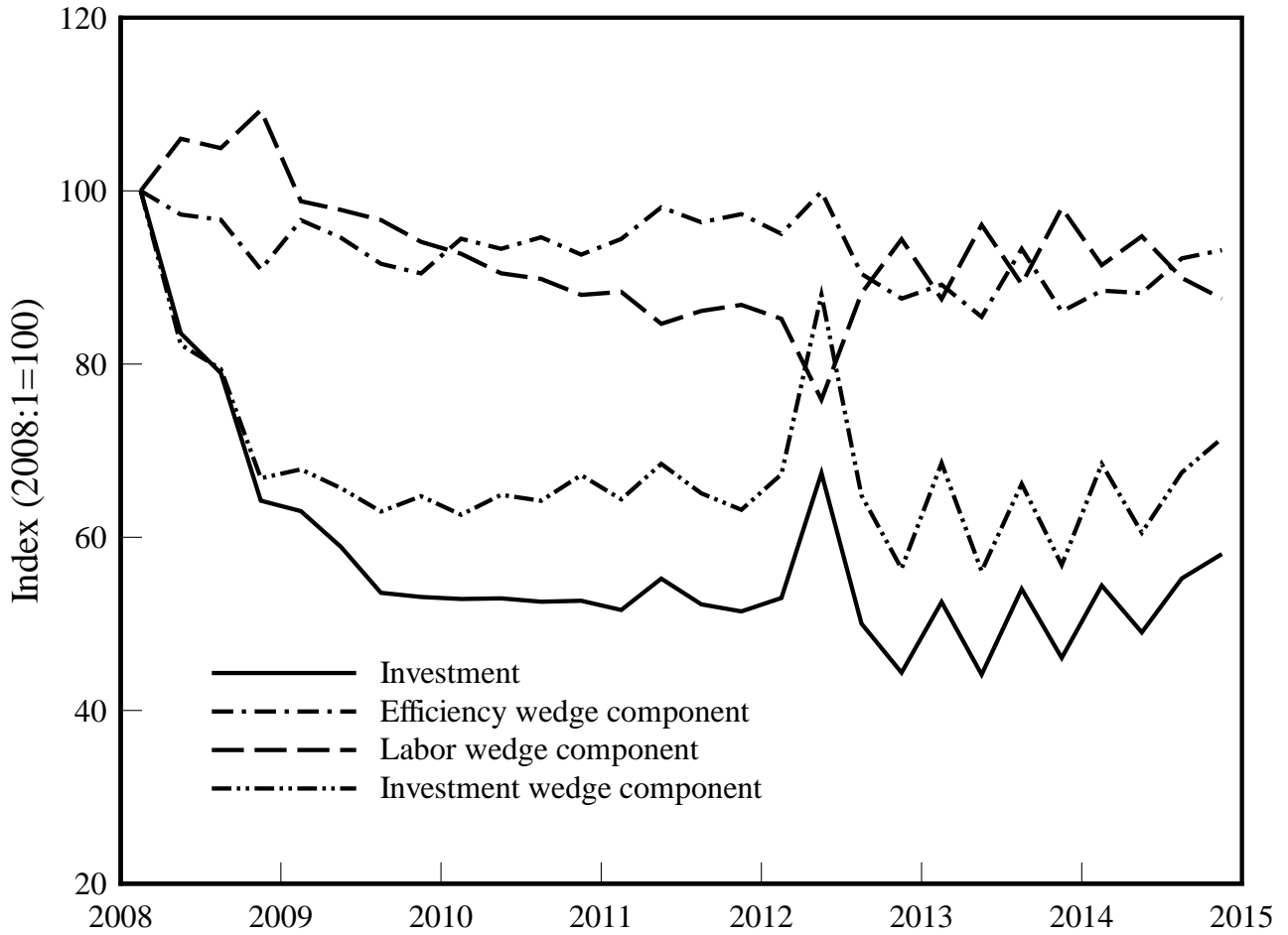


FIGURE 4A  
 DECOMPOSITION OF OUTPUT, 2008:1-2011:3

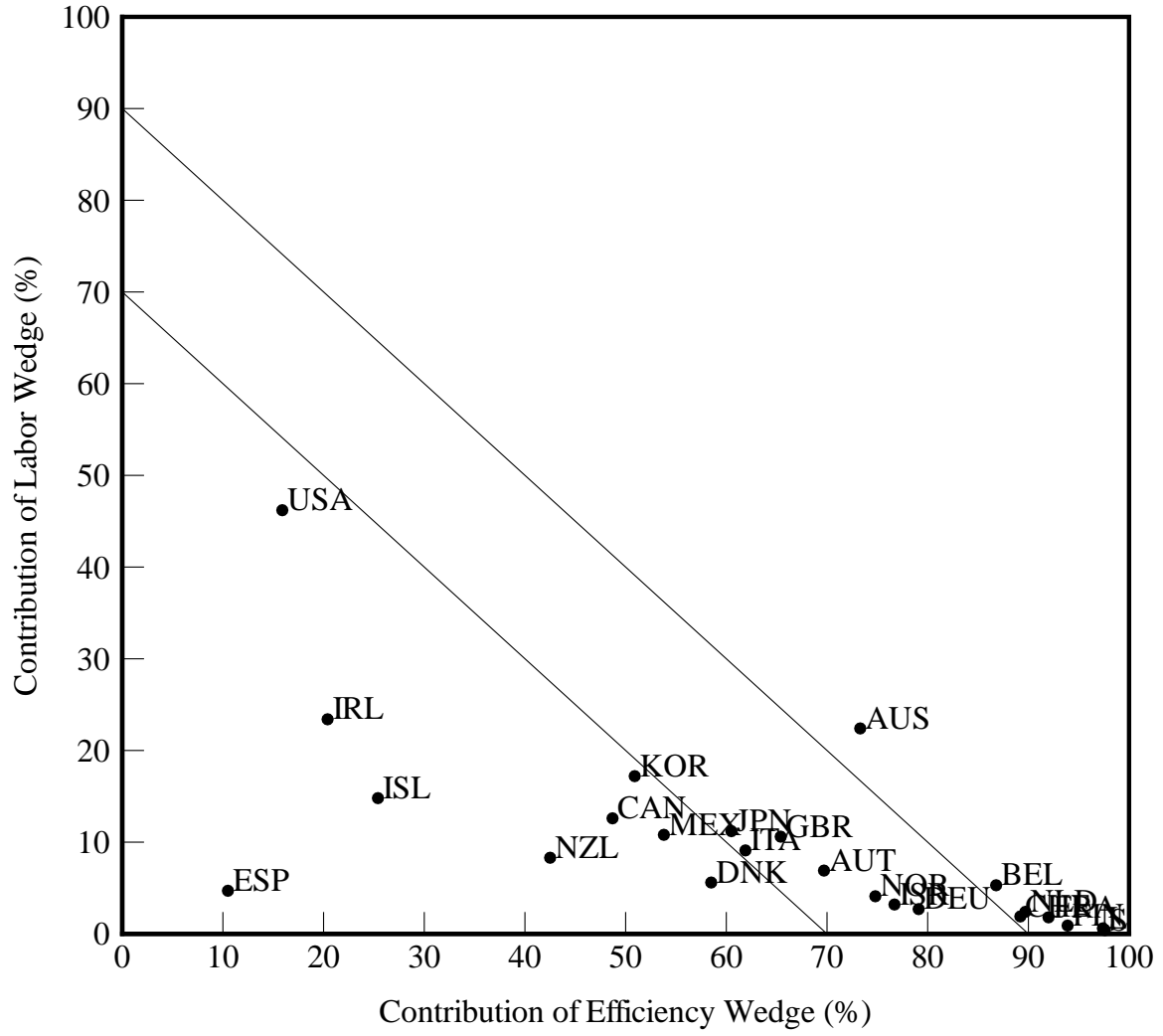


FIGURE 4B  
 DECOMPOSITION OF LABOR, 2008:1-2011:3

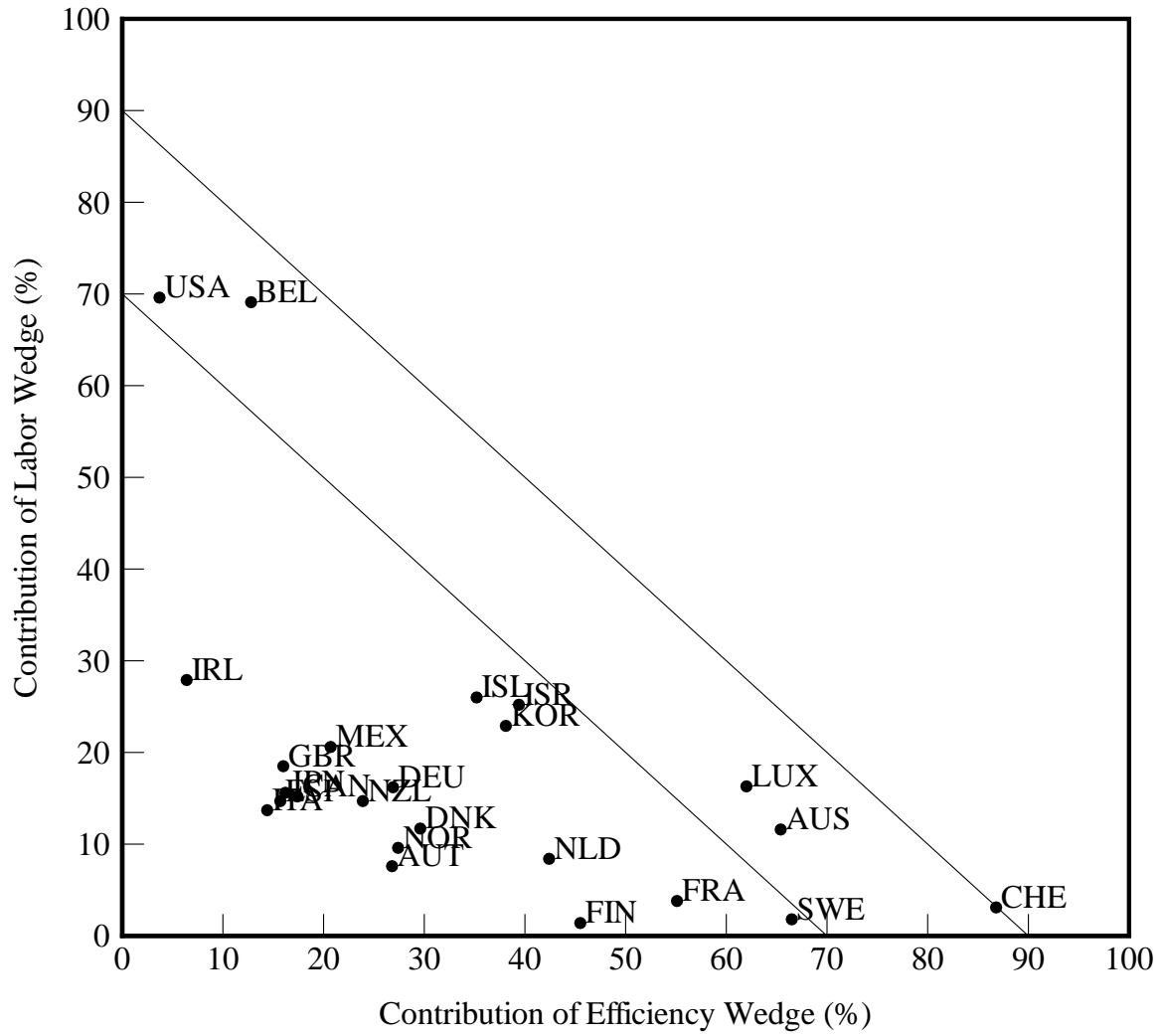


FIGURE 4C  
 DECOMPOSITION OF INVESTMENT, 2008:1-2011:3

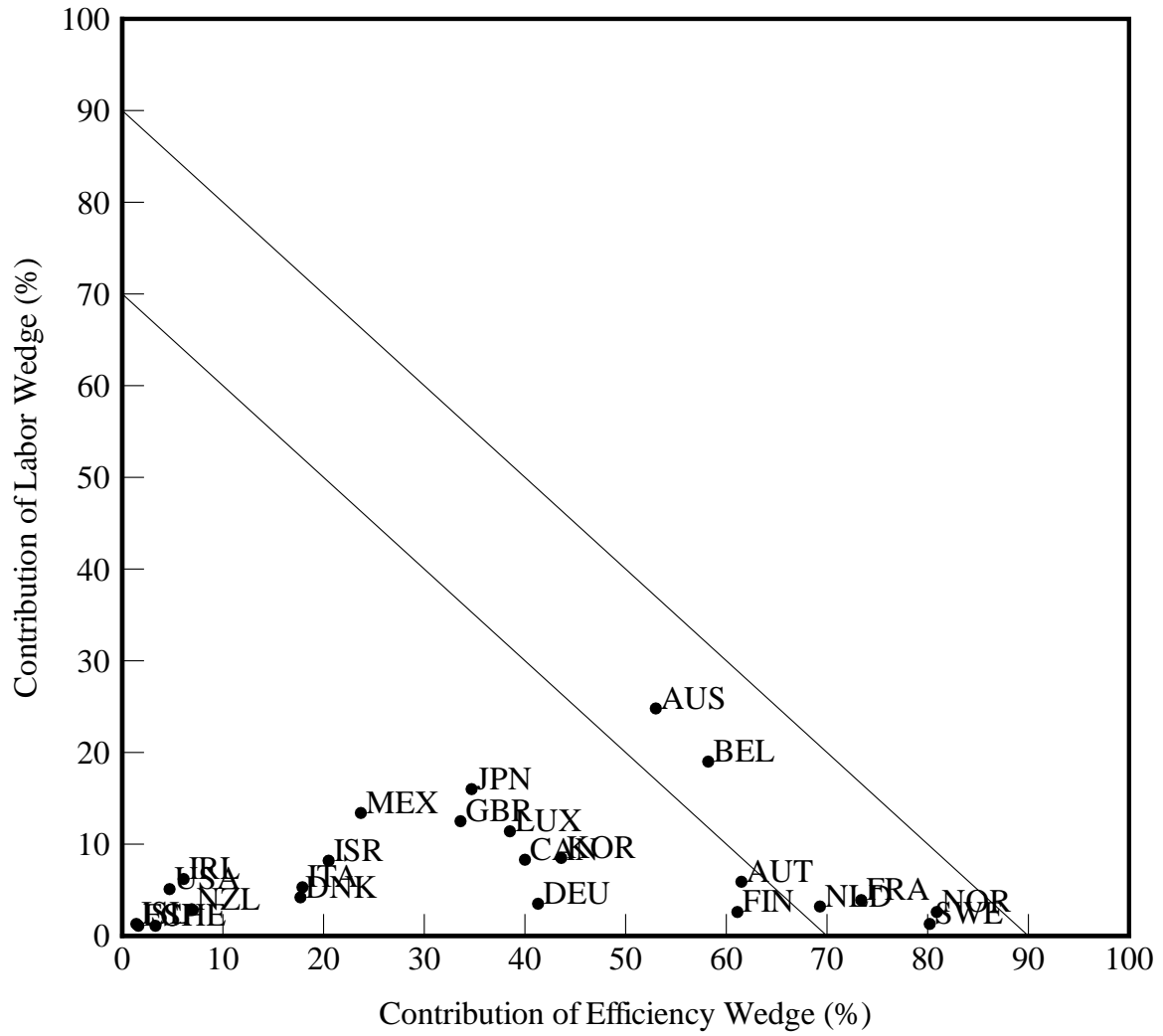


FIGURE 5A  
 EFFICIENCY COMPONENT OF OUTPUT FOR TWO RECESSIONS

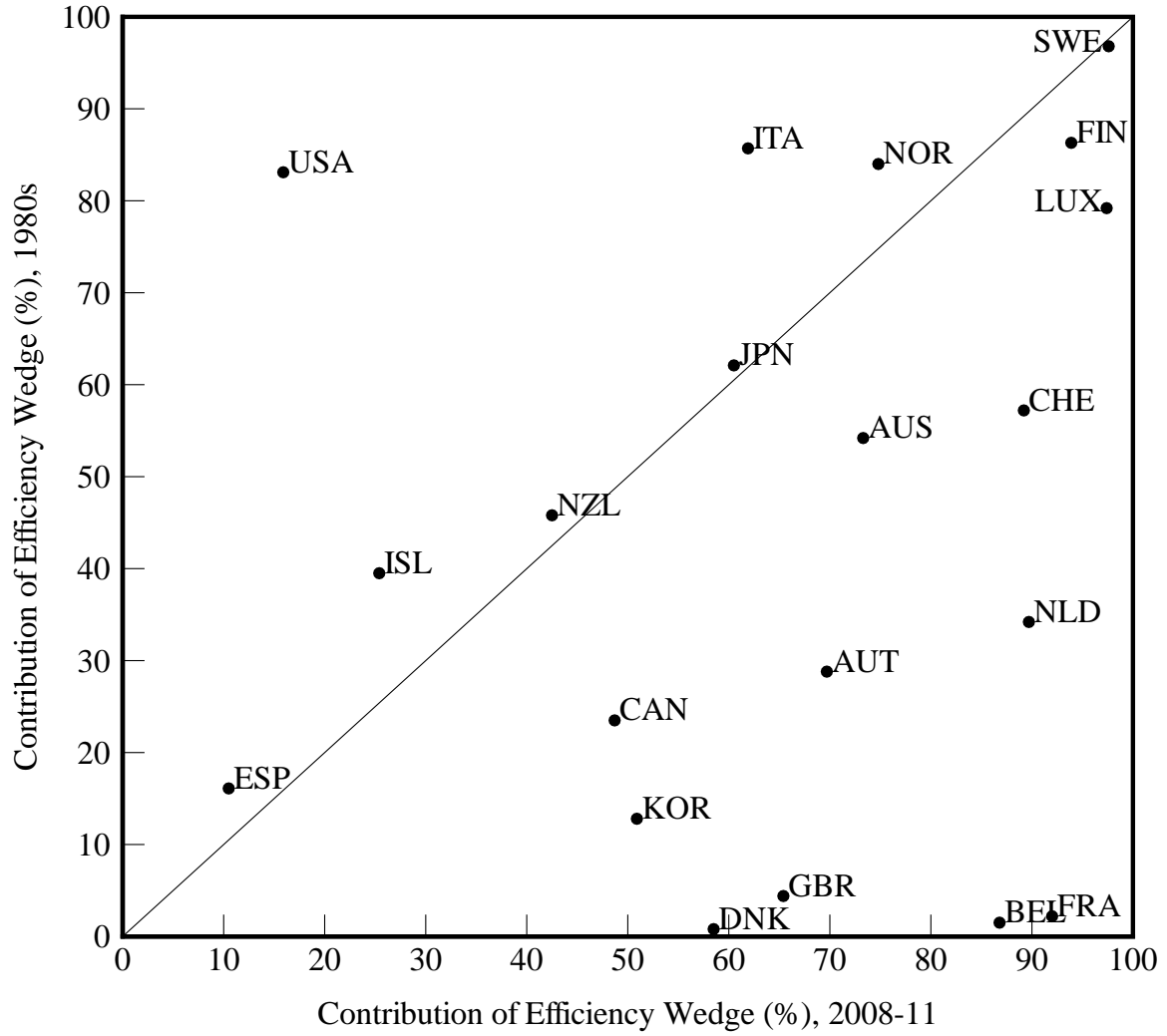


FIGURE 5B  
LABOR COMPONENT OF OUTPUT FOR TWO RECESSIONS

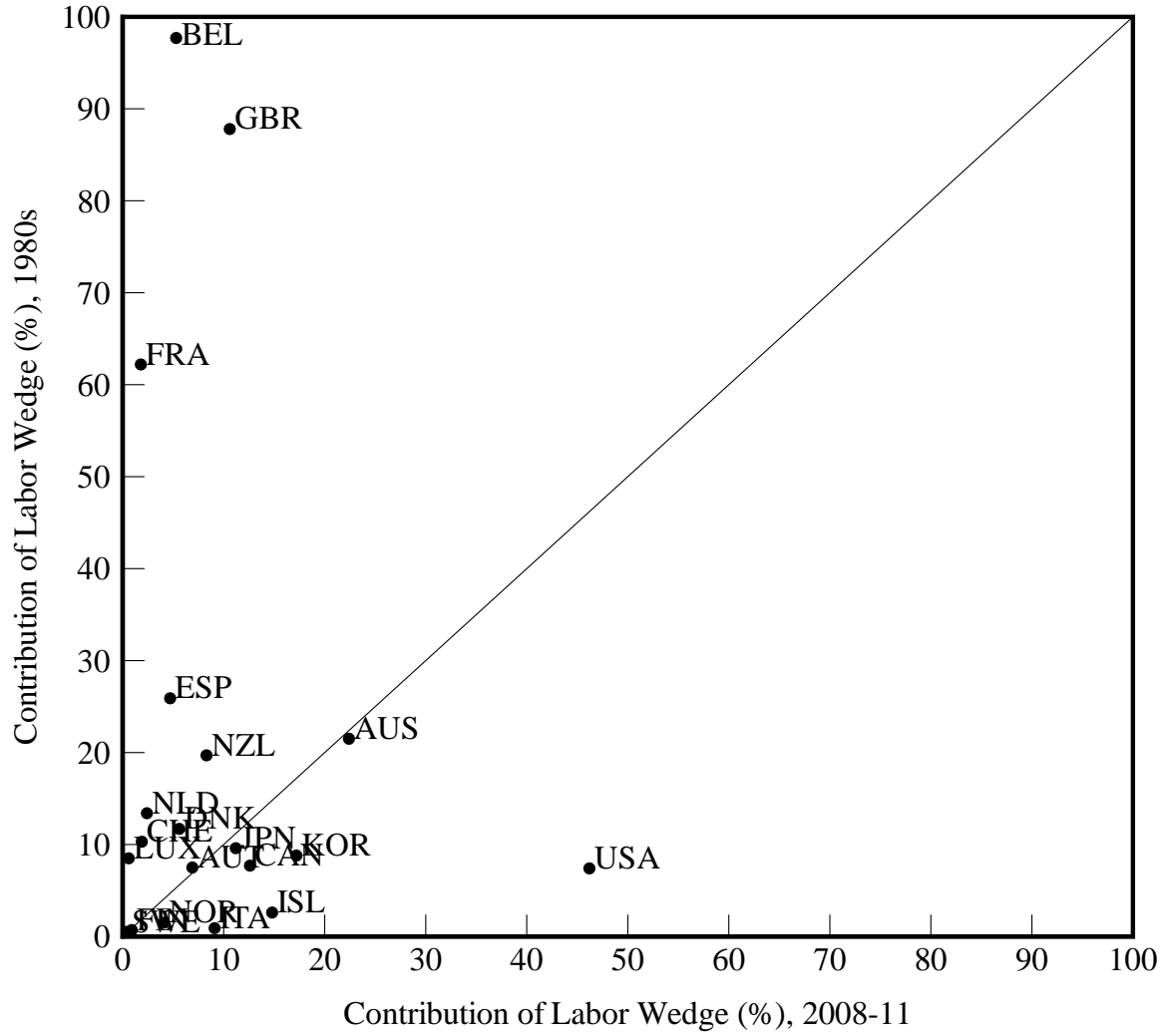


FIGURE 5C

INVESTMENT COMPONENT OF OUTPUT FOR TWO RECESSIONS

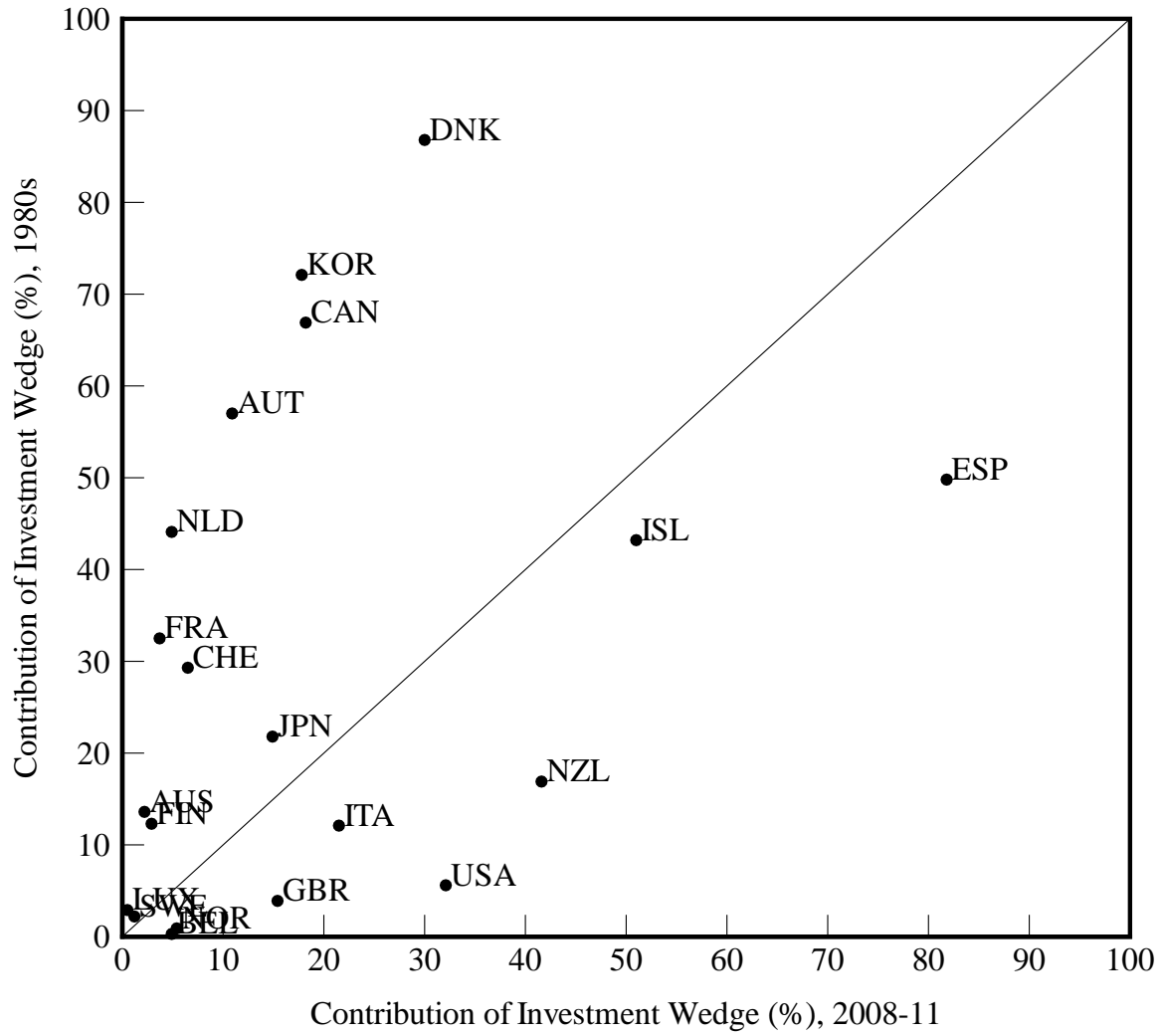




FIGURE 6A

EFFICIENCY COMPONENT OF OUTPUT FOR TWO INVESTMENT MEASURES

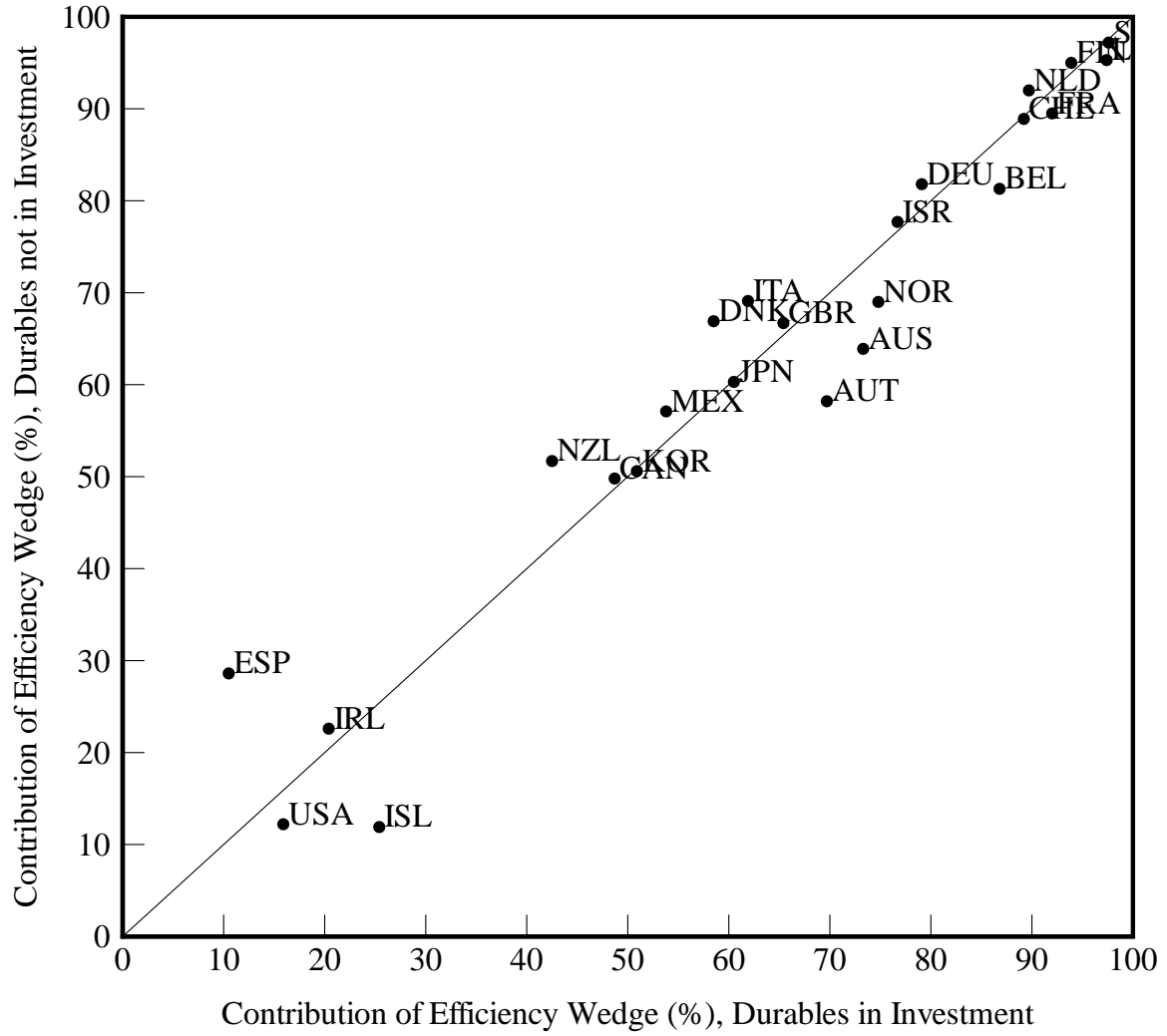


FIGURE 6B

LABOR COMPONENT OF OUTPUT FOR TWO INVESTMENT MEASURES

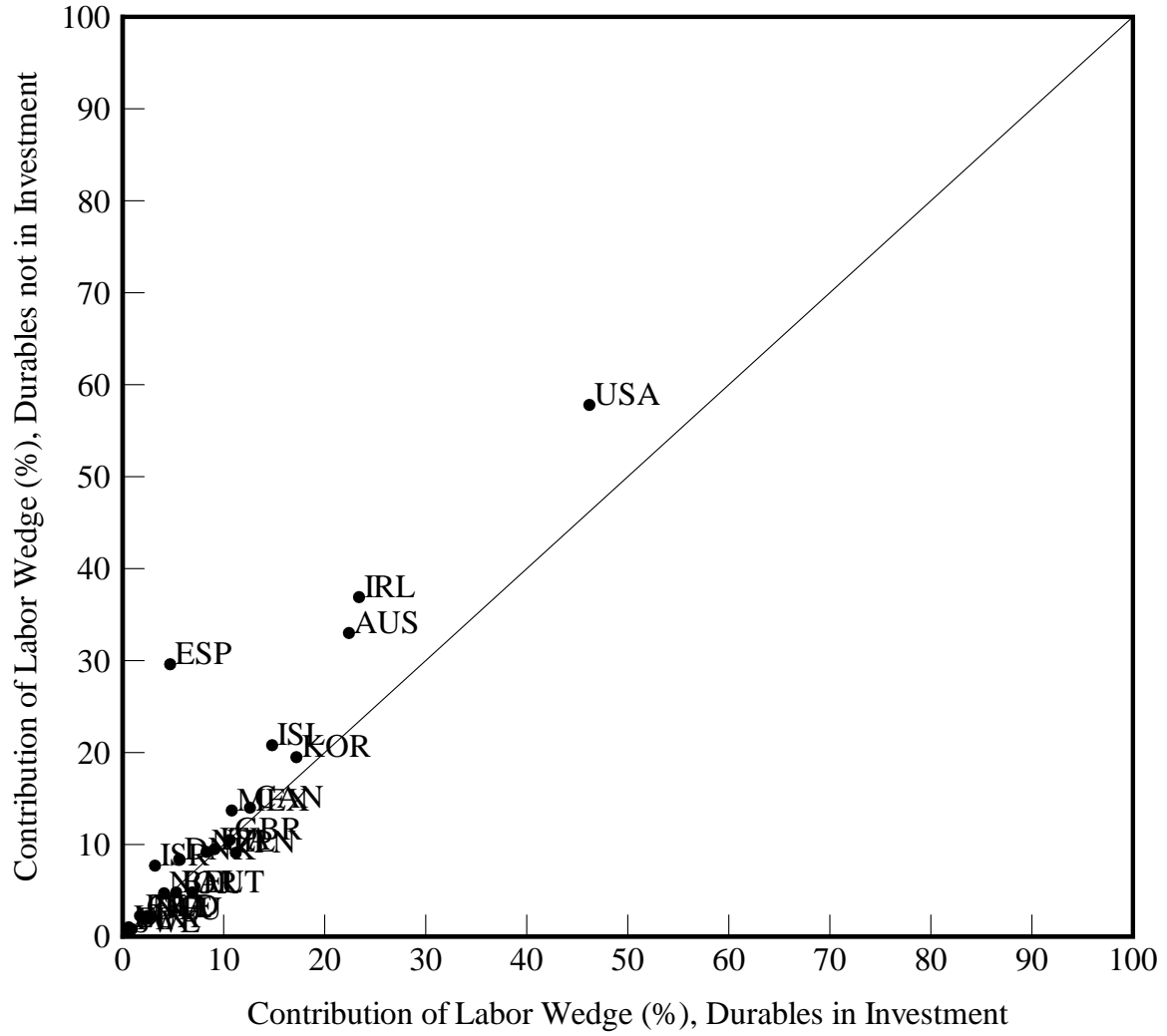


FIGURE 6C

INVESTMENT COMPONENT OF OUTPUT FOR TWO INVESTMENT MEASURES

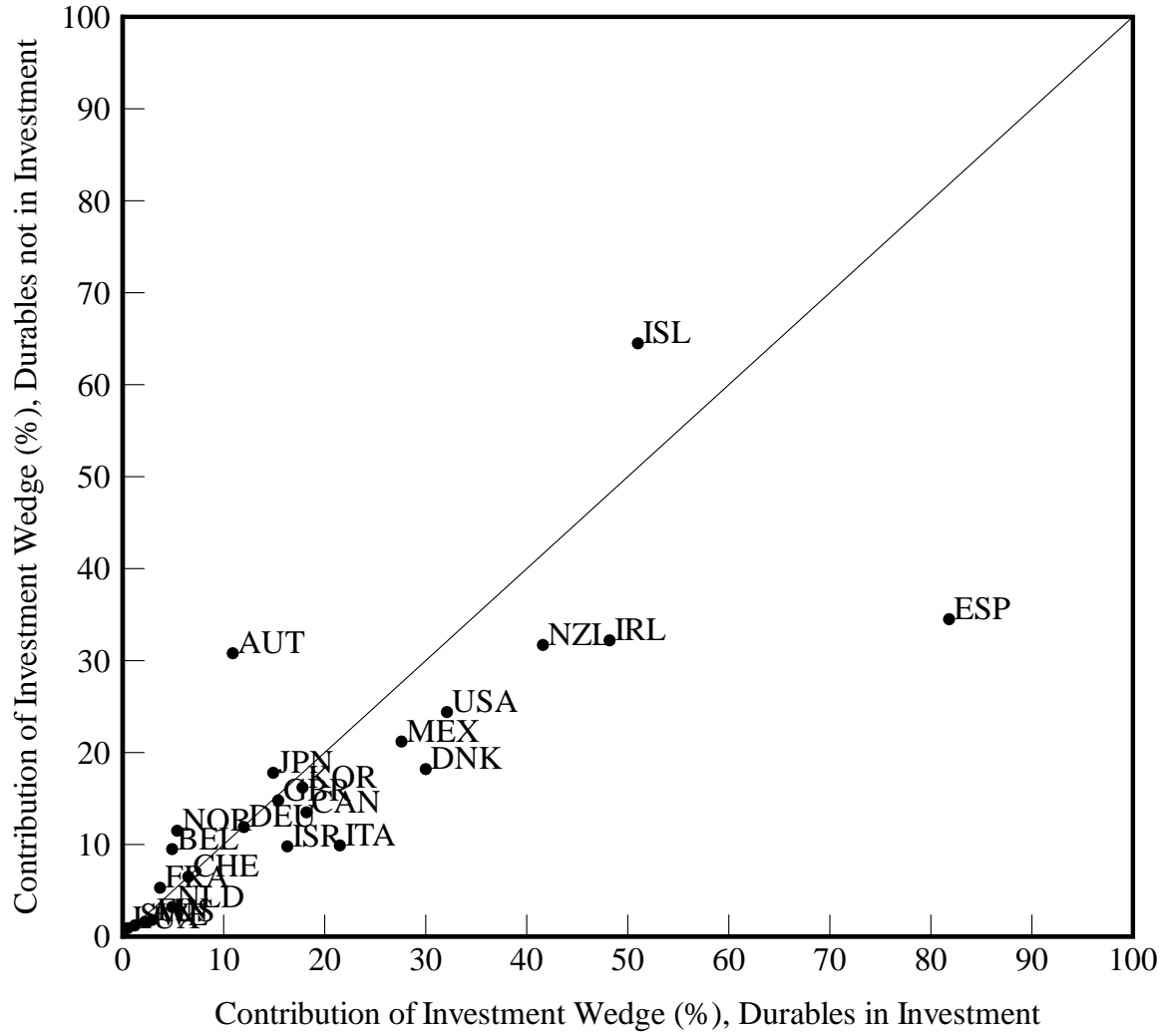


FIGURE 7A

EFFICIENCY COMPONENT OF OUTPUT FOR TWO EXPECTATIONAL ASSUMPTIONS

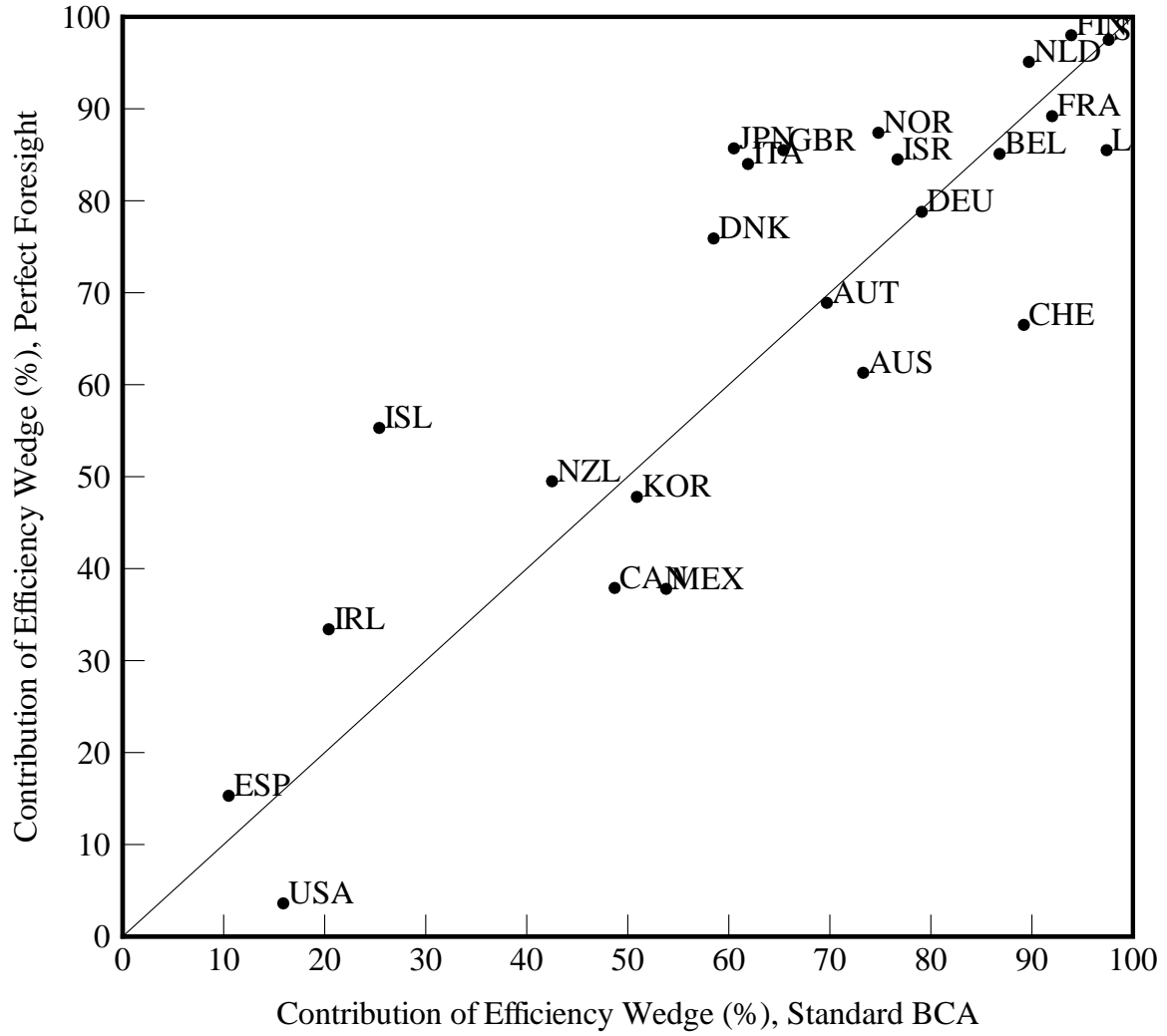


FIGURE 7B

LABOR COMPONENT OF OUTPUT FOR TWO EXPECTATIONAL ASSUMPTIONS

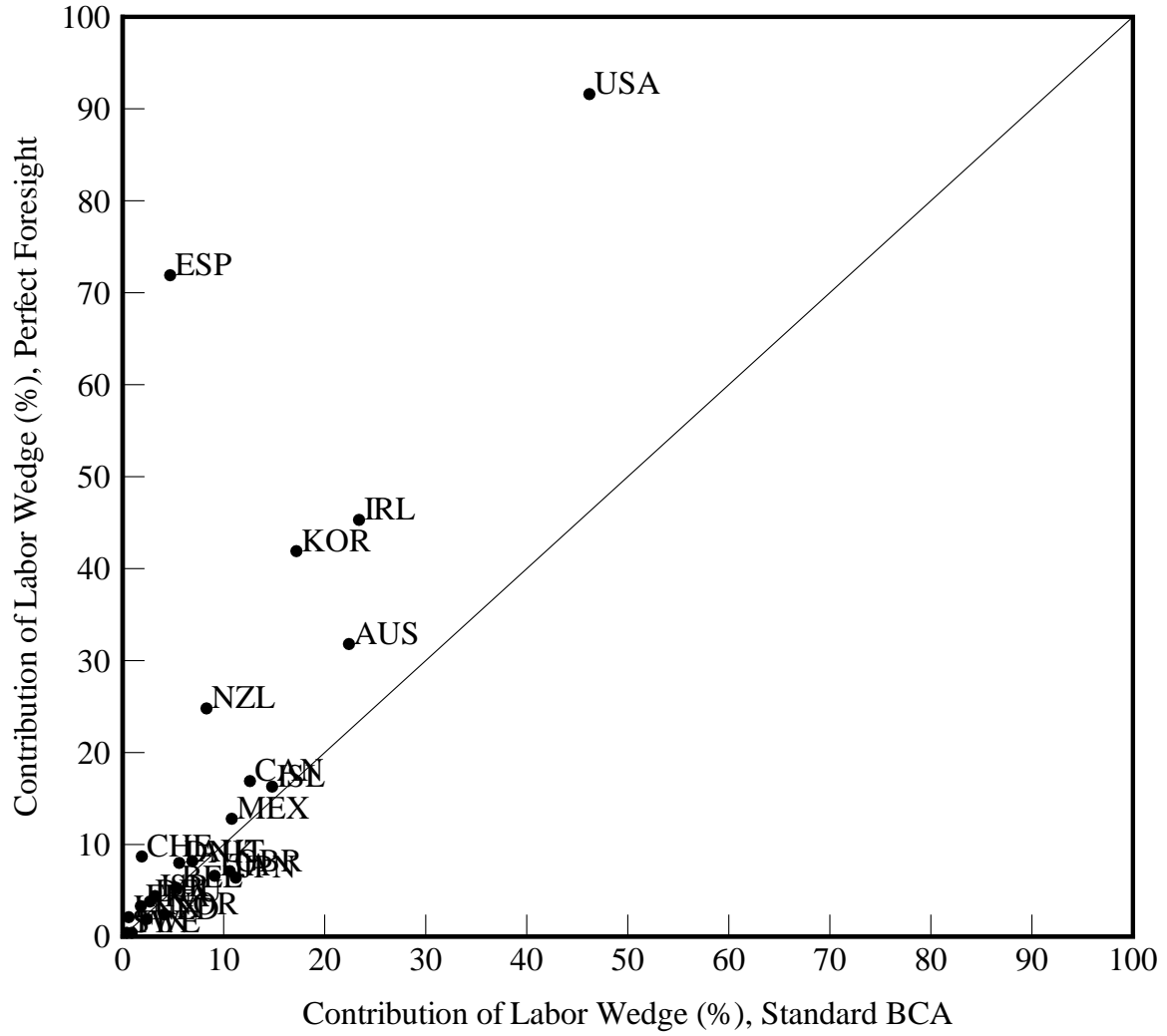


FIGURE 7C

INVESTMENT COMPONENT OF OUTPUT FOR TWO EXPECTATIONAL ASSUMPTIONS

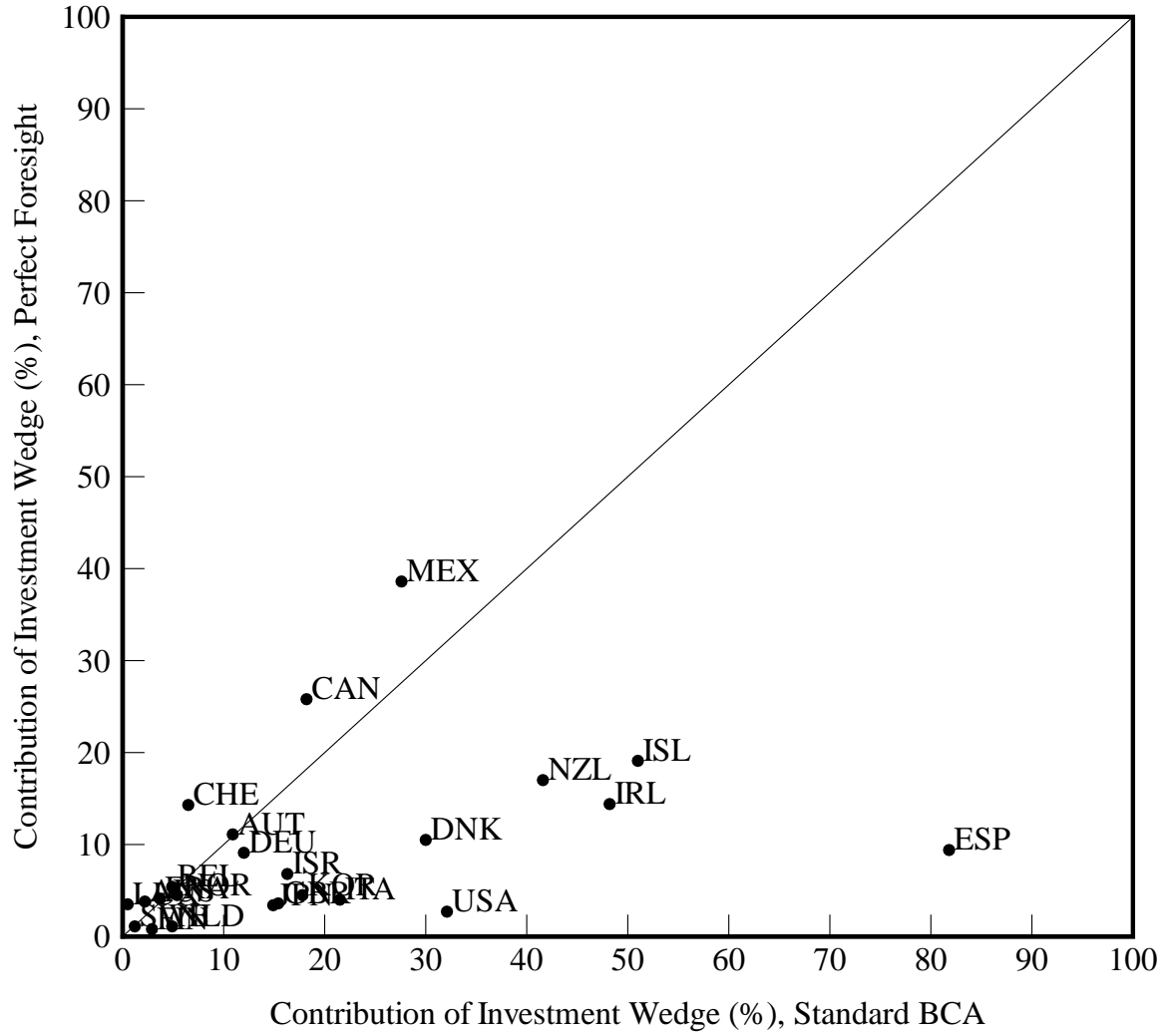


TABLE 1

 $\phi$ -STATISTICS FOR OUTPUT, LABOR, AND INVESTMENT COMPONENTS, GREAT RECESSION

Countries:	Output components			Labor components			Investment components		
	$\phi_A^Y$	$\phi_{\tau_l}^Y$	$\phi_{\tau_x}^Y$	$\phi_A^L$	$\phi_{\tau_l}^L$	$\phi_{\tau_x}^L$	$\phi_A^X$	$\phi_{\tau_l}^X$	$\phi_{\tau_x}^X$
Australia	0.73	0.22	0.02	0.65	0.12	0.13	0.53	0.25	0.04
Austria	0.70	0.07	0.11	0.27	0.08	0.19	0.61	0.06	0.21
Belgium	0.87	0.05	0.05	0.13	0.69	0.14	0.58	0.19	0.15
Canada	0.49	0.13	0.18	0.17	0.15	0.32	0.40	0.08	0.47
Denmark	0.58	0.06	0.30	0.30	0.12	0.47	0.18	0.04	0.72
Finland	0.94	0.01	0.03	0.46	0.01	0.07	0.61	0.03	0.30
France	0.92	0.02	0.04	0.55	0.04	0.30	0.73	0.04	0.17
Germany	0.79	0.03	0.12	0.27	0.16	0.33	0.41	0.04	0.50
Iceland	0.25	0.15	0.51	0.35	0.26	0.27	0.01	0.01	0.95
Ireland	0.20	0.23	0.48	0.06	0.28	0.62	0.06	0.06	0.82
Israel	0.77	0.03	0.16	0.39	0.25	0.08	0.20	0.08	0.60
Italy	0.62	0.09	0.22	0.14	0.14	0.64	0.18	0.05	0.74
Japan	0.60	0.11	0.15	0.16	0.16	0.45	0.35	0.16	0.32
Korea	0.51	0.17	0.18	0.38	0.23	0.16	0.44	0.09	0.34
Luxembourg	0.97	0.01	0.01	0.62	0.16	0.15	0.39	0.11	0.07
Mexico	0.54	0.11	0.28	0.21	0.21	0.49	0.24	0.13	0.51
Netherlands	0.90	0.02	0.05	0.42	0.08	0.25	0.69	0.03	0.24
New Zealand	0.42	0.08	0.42	0.24	0.15	0.51	0.07	0.03	0.86
Norway	0.75	0.04	0.05	0.27	0.10	0.23	0.81	0.03	0.05
Spain	0.11	0.05	0.82	0.16	0.15	0.62	0.02	0.01	0.96
Sweden	0.98	0.00	0.01	0.67	0.02	0.17	0.80	0.01	0.17
Switzerland	0.89	0.02	0.07	0.87	0.03	0.03	0.03	0.01	0.94
United Kingdom	0.65	0.11	0.15	0.16	0.19	0.55	0.34	0.13	0.42
United States	0.16	0.46	0.32	0.04	0.70	0.25	0.05	0.05	0.88
Average	0.64	0.09	0.20	0.33	0.19	0.31	0.36	0.07	0.48

TABLE 2A  
PEAK TO TROUGH DECLINES IN OUTPUT AND COMPONENTS, GREAT RECESSION

Countries:	Trough	Changes in Output and its Components			
		$\Delta Y$	$\Delta Y_A$	$\Delta Y_{\tau_l}$	$\Delta Y_{\tau_x}$
Australia	2011:1	-5.6	-5.6	-2.0	0.7
Austria	2010:1	-9.2	-6.2	3.3	-4.7
Belgium	2010:1	-7.4	-5.8	-2.3	-0.1
Canada	2009:3	-6.5	-3.2	0.0	-2.1
Denmark	2009:4	-9.9	-6.9	1.4	-5.3
Finland	2010:1	-14.1	-12.5	4.2	-3.3
France	2009:3	-6.5	-5.9	1.5	-2.8
Germany	2009:2	-8.6	-7.2	2.3	-3.5
Iceland	2011:1	-14.3	-4.6	2.2	-15.5
Ireland	2009:4	-14.9	-5.3	-3.6	-7.7
Israel	2009:2	-4.8	-3.3	-1.6	-0.8
Italy	2010:1	-10.5	-6.7	-0.7	-3.5
Japan	2009:1	-10.0	-8.3	-0.4	0.4
Korea	2009:2	-7.4	-6.1	4.5	-5.6
Luxembourg	2009:4	-15.6	-16.5	1.2	5.9
Mexico	2009:2	-5.4	-4.7	0.5	-2.0
Netherlands	2010:3	-8.5	-7.4	1.2	-2.3
New Zealand	2010:4	-7.6	-5.3	-0.2	-2.2
Norway	2011:2	-11.9	-8.8	1.1	0.5
Spain	2013:4	-19.7	-9.2	-0.6	-10.8
Sweden	2009:4	-10.5	-9.5	2.9	-2.7
Switzerland	2009:2	-5.7	-5.7	3.3	-4.8
United Kingdom	2012:2	-14.8	-10.3	0.1	-2.9
United States	2009:3	-7.0	-1.9	-3.4	-4.5
Average		-9.9	-7.0	0.6	-3.3

*Note:* The date of the peak is 2008:1 for all countries.



TABLE 2B  
PEAK TO TROUGH DECLINES IN LABOR AND COMPONENTS, GREAT RECESSION

Countries:	Trough	Changes in Labor and its Components			
		$\Delta L$	$\Delta L_A$	$\Delta L_{\tau_l}$	$\Delta L_{\tau_x}$
Australia	2011:1	-0.5	-1.0	-3.1	1.1
Austria	2010:1	-4.9	-1.3	5.0	-7.0
Belgium	2010:1	-3.2	-0.8	-3.4	-0.1
Canada	2009:3	-5.7	-0.7	0.0	-3.1
Denmark	2009:4	-5.3	-1.1	2.2	-7.9
Finland	2010:1	-2.9	-1.3	6.3	-4.9
France	2009:3	-2.8	-1.9	2.3	-4.1
Germany	2009:2	-3.6	-2.0	3.5	-5.2
Iceland	2011:1	-9.1	1.0	3.4	-22.4
Ireland	2009:4	-12.6	0.0	-5.3	-11.3
Israel	2009:2	-1.6	0.1	-2.3	-1.3
Italy	2010:1	-5.2	-0.5	-1.0	-5.1
Japan	2009:1	-3.4	-1.2	-0.6	0.6
Korea	2009:2	-2.9	-1.7	6.8	-8.3
Luxembourg	2009:4	3.7	0.0	1.7	9.0
Mexico	2009:2	-2.5	-1.1	0.7	-3.0
Netherlands	2010:3	-1.1	-0.5	1.9	-3.4
New Zealand	2010:4	-3.3	-1.2	-0.3	-3.3
Norway	2011:2	-3.3	1.0	1.7	0.8
Spain	2013:4	-14.8	-3.7	-0.8	-15.7
Sweden	2009:4	-3.2	-2.0	4.3	-4.1
Switzerland	2009:2	-1.2	-1.3	5.1	-7.1
United Kingdom	2012:2	-3.8	-1.0	0.1	-4.2
United States	2009:3	-7.5	-0.9	-5.0	-6.7
Average		-4.2	-1.0	1.0	-4.9

*Note:* The date of the peak is 2008:1 for all countries.

TABLE 2C

## PEAK TO TROUGH DECLINES IN INVESTMENT AND COMPONENTS, GREAT RECESSION

Countries:	Trough	Changes in Investment and its Components			
		$\Delta X$	$\Delta X_A$	$\Delta X_{\tau_l}$	$\Delta X_{\tau_x}$
Australia	2011:1	-13.0	-9.8	-3.5	3.1
Austria	2010:1	-19.6	-10.2	8.4	-16.2
Belgium	2010:1	-21.8	-11.9	-10.1	-0.3
Canada	2009:3	-13.9	-7.0	-0.1	-9.7
Denmark	2009:4	-33.1	-14.7	5.0	-23.8
Finland	2010:1	-23.9	-19.7	8.2	-12.6
France	2009:3	-18.3	-12.2	4.4	-11.2
Germany	2009:2	-19.9	-14.2	4.6	-14.5
Iceland	2011:1	-56.6	-4.6	6.5	-55.0
Ireland	2009:4	-46.9	-9.5	-5.9	-35.3
Israel	2009:2	-14.9	-5.6	-4.8	-4.1
Italy	2010:1	-18.4	-9.6	-2.2	-12.7
Japan	2009:1	-15.4	-13.1	-2.2	1.7
Korea	2009:2	-23.2	-9.8	9.0	-20.5
Luxembourg	2009:4	-13.2	-28.2	-2.7	30.2
Mexico	2009:2	-18.3	-8.7	-0.4	-9.7
Netherlands	2010:3	-16.6	-13.4	4.2	-10.0
New Zealand	2010:4	-16.7	-9.8	1.5	-9.9
Norway	2011:2	-16.6	-14.4	4.5	2.0
Spain	2013:4	-47.9	-17.8	2.2	-38.9
Sweden	2009:4	-21.8	-21.4	8.4	-12.7
Switzerland	2009:2	-18.7	-10.4	8.6	-21.8
United Kingdom	2012:2	-28.0	-17.5	-1.5	-12.7
United States	2009:3	-23.2	-4.9	-3.0	-21.6
Average		-23.3	-12.4	1.6	-13.2

*Note:* The date of the peak is 2008:1 for all countries.

TABLE 3

 $\phi$ -STATISTICS FOR OUTPUT, LABOR, AND INVESTMENT COMPONENTS, 1982 RECESSION

Countries:	Output components			Labor components			Investment components		
	$\phi_A^Y$	$\phi_{\tau_l}^Y$	$\phi_{\tau_x}^Y$	$\phi_A^L$	$\phi_{\tau_l}^L$	$\phi_{\tau_x}^L$	$\phi_A^X$	$\phi_{\tau_l}^X$	$\phi_{\tau_x}^X$
Australia	0.54	0.22	0.14	0.22	0.39	0.23	0.43	0.18	0.24
Austria	0.29	0.07	0.57	0.19	0.13	0.59	0.04	0.02	0.91
Belgium	0.01	0.98	0.00	0.09	0.82	0.03	0.04	0.91	0.01
Canada	0.23	0.08	0.67	0.11	0.07	0.82	0.13	0.04	0.80
Denmark	0.01	0.12	0.87	0.02	0.28	0.68	0.01	0.02	0.96
Finland	0.86	0.01	0.12	0.87	0.06	0.02	0.04	0.01	0.94
France	0.02	0.62	0.33	0.07	0.63	0.25	0.04	0.10	0.82
Iceland	0.40	0.03	0.43	0.41	0.11	0.07	0.13	0.03	0.77
Italy	0.86	0.01	0.12	0.97	0.01	0.01	0.10	0.02	0.85
Japan	0.62	0.10	0.22	0.15	0.15	0.62	0.29	0.13	0.47
Korea	0.13	0.09	0.72	0.09	0.12	0.72	0.02	0.02	0.94
Luxembourg	0.79	0.09	0.03	0.05	0.72	0.01	0.10	0.73	0.02
Netherlands	0.34	0.13	0.44	0.13	0.28	0.50	0.03	0.01	0.94
New Zealand	0.46	0.20	0.17	0.22	0.47	0.09	0.17	0.06	0.61
Norway	0.84	0.01	0.01	0.66	0.23	0.07	0.11	0.34	0.10
Spain	0.16	0.26	0.50	0.12	0.28	0.54	0.04	0.04	0.90
Sweden	0.97	0.01	0.02	0.85	0.04	0.06	0.52	0.05	0.34
Switzerland	0.57	0.10	0.29	0.22	0.59	0.16	0.04	0.03	0.92
United Kingdom	0.04	0.88	0.04	0.06	0.85	0.05	0.17	0.49	0.15
United States	0.83	0.07	0.06	0.21	0.54	0.19	0.64	0.14	0.15
Average	0.42	0.22	0.29	0.28	0.33	0.30	0.17	0.16	0.59

TABLE 4A  
PEAK TO TROUGH DECLINES IN OUTPUT AND COMPONENTS, 1982 RECESSION

Countries:	Peak	Trough	Changes in Output and its Components			
			$\Delta Y$	$\Delta Y_A$	$\Delta Y_{\tau_i}$	$\Delta Y_{\tau_x}$
Australia	1981:3	1983:2	-10.4	-5.9	-1.4	-3.6
Austria	1980:1	1983:1	-7.2	-2.0	0.5	-6.4
Belgium	1980:1	1983:2	-8.6	-3.6	-7.9	2.4
Canada	1981:2	1982:4	-8.7	-5.1	-1.0	-6.5
Denmark	1980:1	1981:2	-5.4	0.4	-2.7	-4.8
Finland	1980:3	1984:2	-8.3	-7.0	0.9	-5.4
France	1982:1	1984:4	-4.4	1.5	-3.5	-2.5
Iceland	1980:1	1983:4	-10.5	-13.2	7.3	-5.5
Italy	1980:2	1983:2	-9.2	-8.3	6.1	-9.2
Japan	1991:2	1995:1	-5.8	-3.7	-0.9	-1.9
Korea	1997:3	1998:3	-11.5	-3.4	-2.2	-7.1
Luxembourg	1980:1	1983:1	-13.2	-9.7	-3.7	3.4
Netherlands	1980:1	1982:3	-11.2	-5.2	-3.0	-3.9
New Zealand	1981:3	1983:1	-5.1	-3.3	-0.9	-1.9
Norway	1980:1	1982:3	-7.7	-6.7	0.3	3.6
Spain	1980:1	1984:2	-13.9	0.3	-5.9	-10.8
Sweden	1980:1	1983:1	-6.3	-6.2	1.4	-2.3
Switzerland	1981:3	1982:4	-6.6	-6.2	-0.3	-2.6
United Kingdom	1980:1	1982:2	-8.7	-1.1	-8.9	1.7
United States	1980:1	1982:4	-9.1	-6.8	-1.3	-1.6
Average			-8.1	-4.2	-1.7	-3.2

TABLE 4B  
PEAK TO TROUGH DECLINES IN LABOR AND COMPONENTS, 1982 RECESSION

Countries:	Peak	Trough	Changes in Labor and its Components			
			$\Delta L$	$\Delta L_A$	$\Delta L_{\tau_l}$	$\Delta L_{\tau_x}$
Australia	1981:3	1983:2	-7.7	-1.1	-2.1	-5.3
Austria	1980:1	1983:1	-6.3	-0.1	0.7	-9.4
Belgium	1980:1	1983:2	-8.9	-1.4	-11.6	3.7
Canada	1981:2	1982:4	-8.4	-3.6	-1.6	-9.7
Denmark	1980:1	1981:2	-8.0	0.2	-4.0	-7.1
Finland	1980:3	1984:2	-1.2	0.3	1.3	-8.0
France	1982:1	1984:4	-7.3	-0.6	-5.2	-3.7
Iceland	1980:1	1983:4	3.9	-1.0	11.2	-8.2
Italy	1980:2	1983:2	-2.7	-2.9	9.4	-13.4
Japan	1991:2	1995:1	-4.8	-0.8	-1.3	-2.9
Korea	1997:3	1998:3	-11.8	-0.1	-3.2	-10.4
Luxembourg	1980:1	1983:1	-5.1	-0.3	-5.5	5.1
Netherlands	1980:1	1982:3	-7.0	1.0	-4.5	-5.8
New Zealand	1981:3	1983:1	-3.9	-0.6	-1.4	-2.8
Norway	1980:1	1982:3	-0.5	1.3	0.4	5.5
Spain	1980:1	1984:2	-15.8	1.3	-8.7	-15.7
Sweden	1980:1	1983:1	-0.4	-0.6	2.0	-3.4
Switzerland	1981:3	1982:4	-1.5	-1.1	-0.5	-3.9
United Kingdom	1980:1	1982:2	-9.5	-0.1	-13.0	2.5
United States	1980:1	1982:4	-4.2	-1.0	-2.0	-2.4
Average			-5.7	-0.6	-2.4	-4.6

TABLE 4C  
PEAK TO TROUGH DECLINES IN INVESTMENT AND COMPONENTS, 1982 RECESSION

Countries:	Peak	Trough	Changes in Investment and its Components			
			$\Delta X$	$\Delta X_A$	$\Delta X_{\tau_l}$	$\Delta X_{\tau_x}$
Australia	1981:3	1983:2	-25.1	-10.3	-2.1	-14.3
Austria	1980:1	1983:1	-21.0	-2.9	2.8	-21.5
Belgium	1980:1	1983:2	-29.9	-9.2	-25.4	12.8
Canada	1981:2	1982:4	-35.4	-15.5	0.3	-27.7
Denmark	1980:1	1981:2	-25.6	1.2	-4.4	-21.7
Finland	1980:3	1984:2	-23.3	-9.8	4.6	-20.0
France	1982:1	1984:4	-14.2	1.4	-5.3	-10.3
Iceland	1980:1	1983:4	-25.7	-20.7	14.9	-23.6
Italy	1980:2	1983:2	-32.2	-15.1	11.5	-30.9
Japan	1991:2	1995:1	-17.1	-6.4	-2.6	-8.2
Korea	1997:3	1998:3	-31.1	-3.9	-1.3	-25.2
Luxembourg	1980:1	1983:1	-9.5	-17.6	-7.9	16.6
Netherlands	1980:1	1982:3	-23.2	-7.3	-2.9	-16.8
New Zealand	1981:3	1983:1	-14.4	-6.0	-0.4	-8.4
Norway	1980:1	1982:3	-1.2	-10.4	1.2	15.4
Spain	1980:1	1984:2	-39.5	3.0	-8.0	-38.9
Sweden	1980:1	1983:1	-20.8	-13.2	4.7	-10.8
Switzerland	1981:3	1982:4	-13.2	-10.6	1.0	-12.5
United Kingdom	1980:1	1982:2	-10.9	-2.0	-17.8	8.1
United States	1980:1	1982:4	-20.2	-12.2	-3.2	-8.1
Average			-20.4	-7.4	-2.7	-11.8

TABLE 5A  
 PROPERTIES OF THE OUTPUT COMPONENTS, ENTIRE SAMPLE

Countries:	Standard Deviations			Correlations		
	$\sigma_{Y_A}/\sigma_Y$	$\sigma_{Y_{\tau_l}}/\sigma_Y$	$\sigma_{Y_{\tau_x}}/\sigma_Y$	$\rho_{Y_A,Y}$	$\rho_{Y_{\tau_l},Y}$	$\rho_{Y_{\tau_x},Y}$
Australia	0.92	0.94	0.85	0.67	-0.10	0.71
Austria	1.06	0.98	1.05	0.82	-0.32	0.37
Belgium	0.77	1.00	0.44	0.72	0.68	-0.34
Canada	0.67	0.42	0.63	0.89	-0.03	0.79
Denmark	1.18	0.95	0.89	0.58	-0.15	0.72
Finland	0.74	0.72	0.89	0.80	-0.33	0.71
France	1.11	0.93	0.92	0.88	-0.45	0.64
Germany	0.74	0.34	0.61	0.87	0.02	0.69
Iceland	0.97	1.19	1.44	0.75	-0.15	0.27
Ireland	0.84	0.92	0.92	0.62	-0.02	0.53
Israel	0.83	0.58	0.59	0.92	0.08	0.40
Italy	0.99	1.03	1.39	0.85	-0.32	0.51
Japan	0.97	0.48	0.46	0.85	0.01	0.35
Korea	1.04	0.99	0.90	0.69	-0.12	0.58
Luxembourg	1.14	1.01	1.14	0.95	-0.18	-0.20
Mexico	0.97	0.69	0.68	0.91	0.15	0.21
Netherlands	0.99	0.87	1.06	0.72	-0.27	0.50
New Zealand	1.06	0.83	0.88	0.66	-0.14	0.58
Norway	1.08	2.15	1.35	0.71	-0.21	0.24
Spain	0.72	1.15	1.29	0.34	0.35	0.35
Sweden	0.93	0.53	0.40	0.93	-0.28	0.84
Switzerland	1.13	1.15	1.32	0.90	-0.25	0.35
United Kingdom	0.73	0.85	0.55	0.61	0.50	0.43
United States	0.60	0.58	0.61	0.76	0.64	0.74
Average	0.92	0.89	0.89	0.77	-0.04	0.46

*Notes:* The entire sample is 1980:1–2014:4. Series are first logged and detrended with the filter of Hodrick and Prescott (1997).

TABLE 5B  
 PROPERTIES OF THE LABOR COMPONENTS, ENTIRE SAMPLE

Countries:	Standard Deviations			Correlations		
	$\sigma_{L_A}/\sigma_L$	$\sigma_{L_{\tau_l}}/\sigma_L$	$\sigma_{L_{\tau_x}}/\sigma_L$	$\rho_{L_A,L}$	$\rho_{L_{\tau_l},L}$	$\rho_{L_{\tau_x},L}$
Australia	0.27	1.20	1.08	0.39	0.42	0.50
Austria	0.28	1.77	1.90	-0.14	0.36	0.20
Belgium	0.26	1.40	0.61	0.36	0.95	-0.50
Canada	0.39	0.66	0.99	0.75	0.36	0.82
Denmark	0.23	1.10	1.03	-0.44	0.73	0.53
Finland	0.16	1.25	1.56	0.19	0.05	0.61
France	0.63	1.90	1.87	0.25	0.20	0.38
Germany	0.27	0.63	1.13	0.40	0.31	0.78
Iceland	0.22	2.05	2.47	-0.33	0.29	0.37
Ireland	0.21	1.23	1.24	0.30	0.53	0.39
Israel	0.09	1.69	1.74	-0.88	0.38	0.33
Italy	0.55	2.15	2.90	0.07	0.15	0.29
Japan	0.49	1.06	1.02	-0.05	0.46	0.51
Korea	0.45	1.48	1.35	-0.28	0.49	0.34
Luxembourg	0.46	3.22	3.63	-0.18	0.39	0.08
Mexico	0.38	1.64	1.62	0.17	0.39	0.29
Netherlands	0.39	1.45	1.76	-0.35	0.39	0.41
New Zealand	0.28	1.16	1.23	-0.43	0.47	0.55
Norway	0.58	3.49	2.20	-0.13	0.31	0.25
Spain	0.31	1.19	1.33	0.10	0.49	0.42
Sweden	0.75	0.93	0.69	0.83	0.16	0.70
Switzerland	0.38	2.62	3.00	-0.03	0.30	0.13
United Kingdom	0.12	1.16	0.75	-0.27	0.81	0.29
United States	0.14	0.84	0.89	0.64	0.83	0.75
Average	0.35	1.55	1.58	0.04	0.43	0.39

*Notes:* The entire sample is 1980:1–2014:4. Series are first logged and detrended with the filter of Hodrick and Prescott (1997).



TABLE 5C  
 PROPERTIES OF THE INVESTMENT COMPONENTS, ENTIRE SAMPLE

Countries:	Standard Deviations			Correlations		
	$\sigma_{X_A}/\sigma_X$	$\sigma_{X_{\tau_l}}/\sigma_X$	$\sigma_{X_{\tau_x}}/\sigma_X$	$\rho_{X_A,X}$	$\rho_{X_{\tau_l},X}$	$\rho_{X_{\tau_x},X}$
Australia	0.38	0.38	0.77	0.78	-0.31	0.87
Austria	0.62	0.71	1.35	0.53	-0.71	0.89
Belgium	0.39	0.76	0.47	0.84	0.91	-0.69
Canada	0.43	0.16	0.75	0.89	-0.28	0.97
Denmark	0.54	0.42	0.86	0.44	-0.32	0.97
Finland	0.34	0.39	0.95	0.73	-0.66	0.98
France	0.63	0.58	0.97	0.90	-0.72	0.91
Germany	0.53	0.22	0.93	0.58	-0.12	0.96
Iceland	0.29	0.40	1.12	-0.17	-0.36	0.93
Ireland	0.34	0.40	0.92	0.49	-0.36	0.95
Israel	0.39	0.33	0.79	0.69	-0.03	0.83
Italy	0.47	0.48	1.37	0.54	-0.73	0.90
Japan	0.70	0.37	0.74	0.65	-0.01	0.71
Korea	0.56	0.50	1.01	0.57	-0.59	0.93
Luxembourg	0.58	0.58	1.33	0.23	-0.92	0.87
Mexico	0.50	0.36	0.92	0.67	-0.12	0.72
Netherlands	0.60	0.54	1.30	0.20	-0.70	0.96
New Zealand	0.51	0.40	0.96	0.36	-0.47	0.94
Norway	0.48	0.88	1.05	-0.06	0.20	0.44
Spain	0.38	0.49	1.24	0.13	-0.36	0.90
Sweden	0.74	0.32	0.51	0.94	-0.36	0.97
Switzerland	0.35	0.41	1.10	0.27	-0.81	0.99
United Kingdom	0.39	0.50	0.73	0.42	0.23	0.84
United States	0.35	0.29	0.92	0.79	0.15	0.94
Average	0.48	0.45	0.96	0.52	-0.31	0.82

*Notes:* The entire sample is 1980:1–2014:4. Series are first logged and detrended with the filter of Hodrick and Prescott (1997).